The claimed non-signalling proof of a thought experiment

written report by

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Quantum correlation and generalized probabilistic theories

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Contents

1 Introduction

In the scope of the lecture "Quantum correlations and generalized probabilistic theories", given by Markus Müller in the summer term 14, the fact of non-signalling, sending information not faster than light, has been presented and proven. The present paper focuses on the work of R. Srikanth, who presented a suggested experiment to achieve superluminal communication in his article with the title "Noncausal Superluminal Nonlocal Signalling" [1]. However, the axioms and theorems from the lecture will be used to prove that the theoretical argumentation from R. Srikanth to achieve an information transfer faster than light, is not valid. If not mentioned otherwise author and paper refer to R. Srikanth and his paper.

2 Non-signalling

In the lecture [2] , signalling conditions are introduced for two observers i.e. Alice and Bob in different laboratories. In each laboratory, one system can be analyzed, which is prepared in a specific input state (a for Alice and b for Bob system). The measurement outcome for Alice is x and the one for Bob is y. Giving these two distinct observers, the non-signalling theorem states that any kind of information cannot be exchanged faster than light. In a more rigorous way, Alice cannot send a signal to Bob in the sense that she cannot influence the measurement outcome from Bob. Thus, Bobs measurement outcome probability $P(y|b)$ does only depend on the preparation b of his own measurement device and is independent of those of Alice labeled a and a'.

$$P(y|a,b) = \sum_x P(x,y|a,b) = \sum_x P(x,y|a',b) = P(x|a',b) \qquad \forall x,a,a',b \tag{2.1}$$

In the same way the probability for Alice $P(x|a)$ does not depend on the preparation b or b'.

$$P(x|a,b) = \sum_y P(x,y|a,b) = \sum_y P(x,y|a,b') = P(x|a,b') \qquad \forall x,a,b,b' \tag{2.2}$$

Hence no information can be transferred from Bob to Alice because their measurement probabilities remain unchanged if in the other setup different preparations are realized. In general, the entanglement between two particles forbids communication between the two parts and sending of information gets impossible if just the entangled states are included.

3 Basic concepts

We begin with a summary of the gedankenexperiment describing a signalling process which transmits information faster than light. The theoretical setup make use of the entangling behaviour of two parts. If one knows the possible states of both parts, i.e. when they are prepared in such a way, the measurement outcome from the first part will immediately specify the outcome from the other part. The

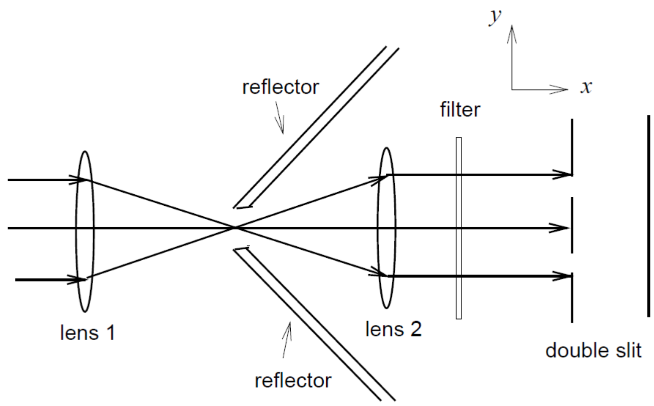

Figure 3.1: Bobs double slit measurement device: Just radiation parallel to the x-axis will be focused by the first lens such that the photons pass the reflector. The second lens undo the focusing and if the rays have the proper wavelength, they also pass the spectral filter and fall onto the double slit. The result of the measurement will be an intensity modulation on the screen. (from: [1])

author presents a slight modification of the experiment of Einstein, Podolsky and Rosen. Here, an entangled photon pair is used which is observed in two different labs (A and B). At these positions the two observers Alice and Bob have a measurement apparatus to determine the state of their photon. Alice has an equipment to measure position or momentum of the photon, whereas Bob sends his received photon through a double slit interferometer.

3.1 Effect on Bob´s interference pattern

In this part the key question of the paper is presented and discussed: Will a measurement on Alice's part affect the measurement outcome of Bobs device?
In general, Bob measures the properties of his photon with a double slit with which he tries to identify effects of Alice's measurement on his side. The suggested measurement device for Bob to see this effect is arranged such that only radiation of a direction perpendicular to the double slit plane can enter into it. Therefore, lenses and a spectral filter are placed in front of the double slit and additionally a reflector shields the device from other radiation. With such a setup (sketched in figure 3.1) it is ensured that just monochromatic light of a perpendicular incidence direction passes onto the double slit. From now on one considers that Alice makes the measurement at time t_1 and the measurement from Bob follows at a later time $t_2 > t_1$. For Alice measurement choice there are two possibilities; either she measures the transverse position y and the longitudinal momentum component p_x or she gets information about the two momentum components p_x and p_y. If Alice makes a momentum measurement, the photon wave function in A will collapse into a momentum eigenstate. Immediately, the second photon will also be in a momentum eigenstate. In general, one of the photons out of the pair is measured in A, before the other photon reaches the measurement apparatus in B. The arriving photons may have collapsed into different momentum states. In an experimental setup, lenses, filter and reflectors ensure that only horizontal

momentum eigenstates can pass through the double slits. Due to the uncertainty relation, the total wave function is imprecise in the spacial coordinates. Thus, it remains unknown which slit the photon will passes before falling onto the screen. According to a general double slit experiment with light, only probabilistic predictions can be made and on a screen one can measure an interference pattern with intensity: $I(y) \propto (|\Psi_1| + |\Psi_2|)^2$.

In the other case, Alice sets via her measurement the photon state into a transverse position and longitudinal momentum eigenfunction and thus the photon in B is also well defined in the position along the double slit and has negative momentum compared to the one in A. If the momentum and the position of the photon is such that it hits the first lens perpendicularly and corresponds to the allowed wavelength, it is passed through the reflectors and likewise through the filter. Consequently, the photons carries a well-defined position at the double slit. Thus, one can say deterministically which slit the photon passes and the photon track is well known. The resulting signal does not contain interference terms and the intensity on the screen is the sum of the two probabilities coming from the two different slits: $I(y) \propto |\Psi_1|^2 + |\Psi_2|^2$

The author uses these intensity relations and the mentioned measurement tools to claim an overcome of the non-signalling theorem. It is easy to see that the missing of interference terms is leading to a distinction of the two types. Carrying on the argumentation, the entangled particles are represented by momentum eigenstates or by position eigenstates before any observer interacts with them. After Alice measurement, the system is just in one of the mentioned states.

3.2 Mathematical approach

The reduced density operator for Bob's particle is now compared for both possible measurements from Alice in the case that he has made different measurements before the particle gets into the interferometer. The following paragraph refers to the question: How will the reduced matrix representation change under Bob's operation?

For simplicity, the position and the momentum of the particles A and B have just two values as the following: $|y_i\rangle\,(i = 1, 2)$ for the position eigenstates and $|p_j\rangle\,(j = q, r)$ for the transverse momentum eigenstates. The entangled state wave function is represented in ket-products, in which the first ket denotes the state of A and the second ket belongs to B:

$$\Psi_{1,2} = \frac{1}{\sqrt{2}} \left(|y_1\rangle\,|y_2\rangle - |y_2\rangle\,|y_1\rangle \right) = \frac{1}{\sqrt{2}} \left(|p_r\rangle\,|p_q\rangle - |p_q\rangle\,|p_r\rangle \right) \tag{3.3}$$

The part on the right hand side is derived from the transformation between the position and momentum bases. Therefore, the author uses the following relations between both bases to transfer an arbitrary position state into the momentum representation.

$$|y_1\rangle = \frac{1}{\sqrt{2}} \left(|p_q\rangle + |p_r\rangle \right), \qquad |y_2\rangle = \frac{1}{\sqrt{2}} \left(|p_q\rangle - |p_r\rangle \right) \tag{3.4}$$

According to the non-signalling theorem, the following two density matrices should be the same: The first one is the reduced density operator D_p which describes the subsystem of Bob after Alice has done a momentum detection and afterwards is collapsed into a momentum eigenstate.

$$D_p = \frac{1}{2} \left(|p_q\rangle \langle p_q| + |p_r\rangle \langle p_r| \right) \tag{3.5}$$

This result is compared with the density operator of Bob achieved after a position measurement in the lab of Alice:

$$D_y = \frac{1}{2} \left(|y_1\rangle \langle y_1| + |y_2\rangle \langle y_2| \right) = D_p \tag{3.6}$$

In the last step of eq. 3.6 the position states are transformed into momentum states according to eq. 3.4. The cross terms of D_y cancel because the coefficients in the y_2 transformation have opposite signs whereas the coefficients in y_1 has equal signs. Finally, the author comes to the result that with the given transformation between momentum and position basis both density operators are the same.
To describe the resulting signal on a screen, a calculation of the new wave function which evolves after the two slits must be done. A momentum eigenstate is not confined in position and thus a wave hits both slits when it reaches the double slit. From both slits (1,2) a new wave function evolves with the same momentum properties as before (q,r) and leads to superposition. The following transformation expresses the new state of two monochromatic wave fronts (one of each slit):

$$|p_j\rangle \rightarrow |p_j'\rangle \equiv \frac{1}{\sqrt{2}} \left(|p_{1j}\rangle + |p_{2j}\rangle \right) \qquad j = q, r \tag{3.7}$$

In a position eigenstate just at one of the slits appears a propagating wave and consequently the position state transforms into a sum of waves with momentum q and r, which evolve just from one slit i:

$$|y_i\rangle \rightarrow |y_i'\rangle \equiv \frac{1}{\sqrt{2}} \left(|p_{iq}\rangle + |p_{ir}\rangle \right) \tag{3.8}$$

Therefore the new momentum and position density operators D_p' and D_y' are:

$$D_p' = \frac{1}{2} \left(|p_q'\rangle \langle p_q'| + |p_r'\rangle \langle p_r'| \right), \qquad D_y' = \frac{1}{2} \left(|y_1'\rangle \langle y_1'| + |y_2'\rangle \langle y_2'| \right) \tag{3.9}$$

The operator D_y was transformed in his original state (see therefore left hand side of eq. 3.6) into D_y' by applying eq. 3.8. Thus, if the density operator D_y' is transformed into a momentum basis both density operators can be distinguished. The difference between both operators given in the paper is:

$$D_p' - D_y' = \frac{1}{4} \left[\left(|p_{1q}\rangle - |p_{2r}\rangle \right) \left(\langle p_{2q}| - \langle p_{1r}| \right) \right] \tag{3.10}$$

The author argues that Bob can evaluate this difference in the density operators to get information about the measurement that Alice had done.

4 Why the non-signalling theorem still holds

The main idea of the paper is to circumvent the non-signalling proofs by using the filters and double slit. It claims that these would allow Bob to distinguish between a state that collapsed into a momentum eigenstate and a state that collapsed into a position eigenstate. In the following argumentation, one can easily see, that this can not be true. The general non-signalling argument, which is also accepted by the author of the paper, states that (without the use of filters and a double slit) the density matrix ρ_B of the subsystem of Bob B does not change under a measurement on the subsystem of Alice A. Therefore, at the time t_1 at which the photons have not reached the filters or the double slit ρ_B is always the same no matter what Alice does. Since the filters and the double slit can be considered as a local quantum operation which acts on $\rho_B(t_1)$ the state after time t_2 at which all photons have passed the filters and the double slit is given by:

$$\rho_B(t_2) = \Psi(\rho_B(t_1)) \tag{4.11}$$

Here, Ψ is a completely positive, trace non-increasing map. Because non of the terms on the right hand site of eq. 4.11 depend on the measurement choice of Alice, also $\rho_B(t_2)$ will always be the same, no matter what Alice does. Therefore, including the filters and the double slit does not change anything at Bob´s situation.

After mentioning that there has to be a mistake in the argumentation of the author we are going to look at this in more detail to find the mistake. Actually, there are two different argumentation in the paper why superluminal signalling should work and therefore also two mistakes, because both are wrong. The first argumentation is based on the interference pattern after the double slit, while the second one is based on a mathematical description of superluminal signalling for a simplified system. At first we are going to look at the latter case.

4.1 The simplified system

The density matrices calculated in the paper differ depending on the measurement choice of Alice. The reason for that lies in the assumption how the double slit affects the momentum and position eigenstates, which is given in eqs. 3.7 and 3.8. By comparing eq. 3.8 with eq. 3.4 one can see that the $|y_2\rangle$ state would change under the double slit, what obviously makes no sense. Also the change of the momentum eigenstate given in the paper is not right. To find out how the momentum state does really change under the double slit one has to transform it first into the position eigenstates, which will not be affected by the double slit, and then back into the momentum eigenstates:

$$|p_q\rangle = \frac{1}{\sqrt{2}}\left(|y_1\rangle + |y_2\rangle\right) \rightarrow \frac{1}{2}\left(|p_{1q}\rangle + |p_{1r}\rangle + |p_{2q}\rangle - |p_{2r}\rangle\right) \tag{4.12}$$

$$|p_r\rangle = \frac{1}{\sqrt{2}}\left(|y_1\rangle - |y_2\rangle\right) \rightarrow \frac{1}{2}\left(|p_{1q}\rangle + |p_{1r}\rangle - |p_{2q}\rangle + |p_{2r}\rangle\right) \tag{4.13}$$

If one calculates D_p with Eqs. 4.12 and 4.13 one sees that it is the same as D_y and that therefore there is no superluminal signalling.

4.2 The interference pattern after the double slit

In the paper it is claimed, that depending on in which eigenstate the photons in subsystem B are, the interference pattern after the double slit would differ. The argumentation of the author is that a photon which is in a position eigenstate passes just one slit and thus, does not interfere with itself. On the other hand a photon in a momentum eigenstate is in a superposition of going to the one or the other slit and thus will self interfere. While it is true, that just a photon in the momentum eigenstate is able to self interfere, it is not true that Bob would see a double slit interference pattern. The reason for that is that the interference pattern (especially the position of the peak of the pattern) is different for different momentum eigenstates. If one adds up the interference pattern of all momentum eigenstates, they just cancel out each other, so that no interference pattern will appear. In the case of the simplified system with two momentum and position eigenstates one sees quite easily that this is true: That a momentum eigenstate is equal to a superposition of position eigenstates follows out of eq. 3.4. Therefore the momentum eigenstate will self interfere. A maximally mixed momentum state on the other hand is equal to a maximally mixed position state (see eq. 3.6) and therefore no interference will be observed. The only possible way without a disagreement between these two facts is that the sum over all interference patterns of the momentum eigenstates cancel each other out.

5 Summary

All in all the claimed superluminal signalling was based on wrong assumptions. We have seen, that it is not possibly to overcome the non-signalling theorem, by just modifying Bob´s measurement apparatus, because there will be always a time point after Alice, but before Bob, has done the measurements. At this time point the non-signalling proof will be valid and therefore no matter what Bob does after this time, the non-signalling theorem will always hold.

References

[1] R. Srikanth, "Noncausal superluminal nonlocal signaling," 1999.

[2] M. P. Müller, "Quantum correlations and generalized probabilistic theories: an introduction." Lecure notes, May 2014.