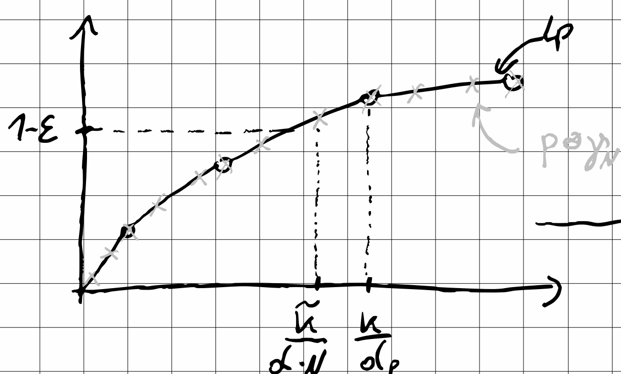
4.5 Smoothed non-uniformity monotones

i) Show that I_0^ε is not a non-uniformity monotone.

$$I_0^\varepsilon(p) = -\log\frac{k}{d_p} \quad \text{for } k \geq 1 \text{ smallest s.t. } \sum_{i=1}^{k} p_i \geq 1-\varepsilon$$

but $I_0^\varepsilon(p\otimes\gamma_N) \geq I_0^\varepsilon(p)$ because



$$I_0^\varepsilon(p) = -\log\left(\frac{k}{d_p}\right)$$

$$I_0^\varepsilon(p\otimes\gamma_N) = -\log\left(\frac{\tilde{k}}{d_p\cdot N}\right)$$

$$\longrightarrow \frac{\tilde{k}}{d_p N} \leq \frac{k}{d_p}$$

$$\Rightarrow I_0^\varepsilon(p\otimes\gamma_N) \geq I_0^\varepsilon(p)$$

Since adding γ_N is a free operation

$p \xrightarrow{\text{noisy}} p\otimes\gamma_N$ But this increases I_0^ε

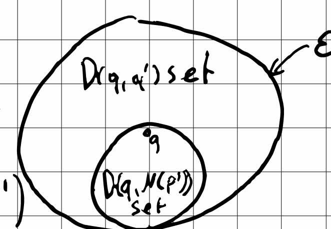
Thus I_0^ε is not a nonuniformity monotone.

ii) Show that I_∞^ε is a non uniformity monotone

the set $\{q' : D(q,q') \leq \varepsilon\}$ is a convex set, Its border is given by all q' st. $D(q,q') = \varepsilon$

Since $\varepsilon \geq D(p,p') \geq D(N(p), N(p')) = D(q, N(p'))$ (with $N(p) = q$), the border of $D(q, N(p')) \leq \varepsilon$

Due to the convexity of $\{q' : D(q,q') \leq \varepsilon\}$ all elements $N(p')$ with $D(p,p') \leq \varepsilon$ must lie in this set.



Show $I_\infty^\varepsilon(N(p)) \leq I_\infty^\varepsilon(p)$

Less elements to minimize

$$I_\infty^\varepsilon(q) = \min_{q' : D(q,q') \leq \varepsilon} I_\infty(q') \leq \min_{q' \in \{N(p') : D(p,p') \leq \varepsilon\}} I_\infty(q')$$

And $I_\infty(p') \geq I_\infty(N(p') = q')$

Therefore $I_\infty^\varepsilon(N(p)) \leq I_\infty^\varepsilon(p)$
