Quantum correlations and generalized probabilistic theories: an introduction

Exercises to prepare for the oral exam in February 2019

(Dated: November 8, 2018)

I will list a few exercises here that you should solve as a preparation for the oral exam. These exercises are *not* meant as a kind of homework; you will not have to hand them in, and they will not be corrected or graded. Instead, you should sit down, solve them (either alone or in groups, as you prefer) and make sure that you understand what is going on, both mathematically and conceptually.

This document will grow over the semester, until it is about 2-3 pages long. So please have a look at the homepage from time to time, where this document will be hosted: `http://mpmueller.net/vl.html`

Exercise 1 (PR boxes, no signalling, and randomness)**.** *Consider a two-party behavior* $P(x,y|a,b)$ *with settings* $a,b\in\{0,1\}$ *and outcomes* $x,y\in\{-1,+1\}$*. Suppose that the outcomes are*

- *perfectly correlated if* $(a,b)\in\{(0,0),(0,1),(1,0)\}$*, and*
- *perfectly anticorrelated if* $(a,b)=(1,1)$.

Prove that no-signalling implies that both parties must locally see perfectly random outcomes for every setting. Conceptually, how does that relate to what we have learned in Lecture 4?

Exercise 2 (Filters in quantum mechanics)**.** *Suppose that an* $n\times n$ *density matrix* ρ *impinges on an* n*-slit arrangement with the* k*th slit blocked, where* $1\leq k\leq n$*. As a consequence, the state after the slits is described by a subnormalized density matrix* ρ' *with* $\langle k|\rho'|k\rangle=0$*, i.e. a (positive semidefinite) matrix with a zero at the* k*th diagonal entry.*

Prove that we must then have $\langle j|\rho'|k\rangle=0$ *for all* j*, i.e. the* k*th row and the* k*th column of* ρ' *must be identically equal to zero. Where did this appear in Lecture 6?*

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[1] *This is a placeholder for references that may come up in some of the exercises.*
J. Barrett, *Information processing in generalized probabilistic theories*, Phys. Rev. A **75**(3), 032304 (2007).