ÖAW AUSTRIAN ACADEMY OF SCIENCES

IQOQI – INSTITUTE FOR QUANTUM OPTICS AND QUANTUM INFORMATION VIENNA

Testing quantum theory by generalizing noncontextuality

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[3]Perimeter Institute for Theoretical Physics (PI), Waterloo, Canada

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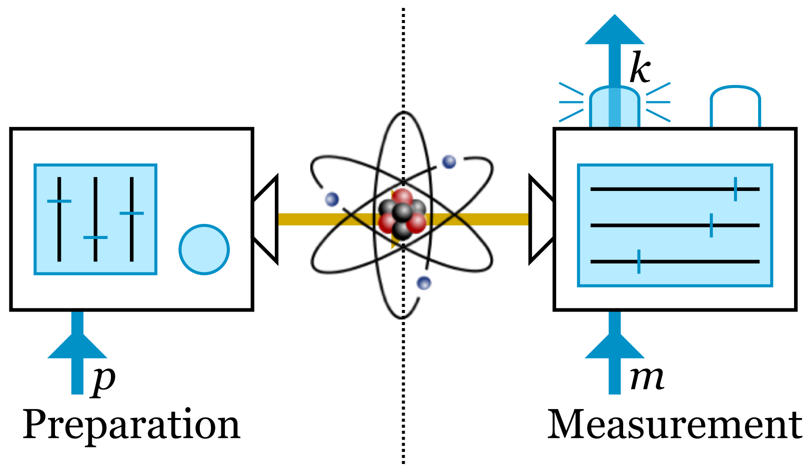
Two motivations

Suppose we **prepare** and **measure** a physical system in all ways accessible to us.



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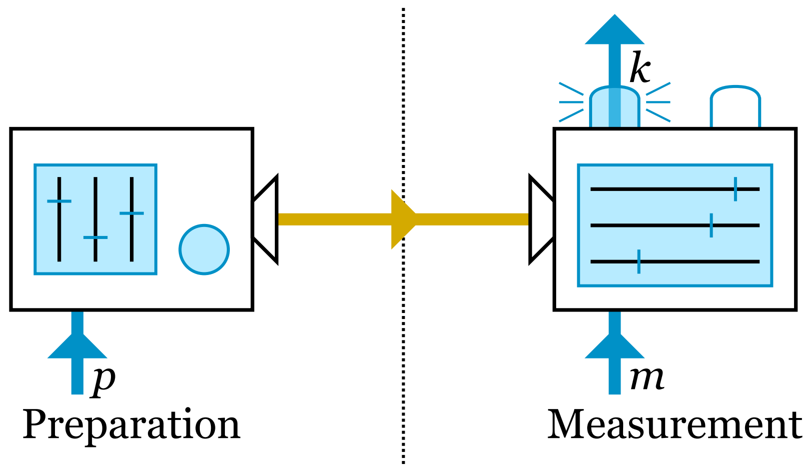

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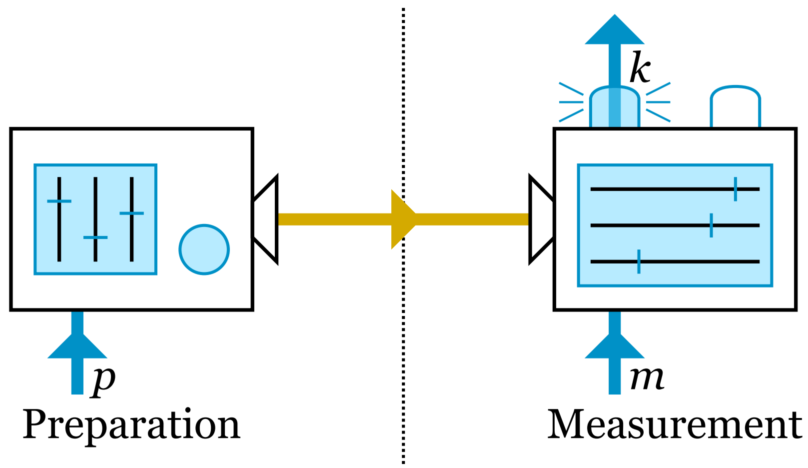

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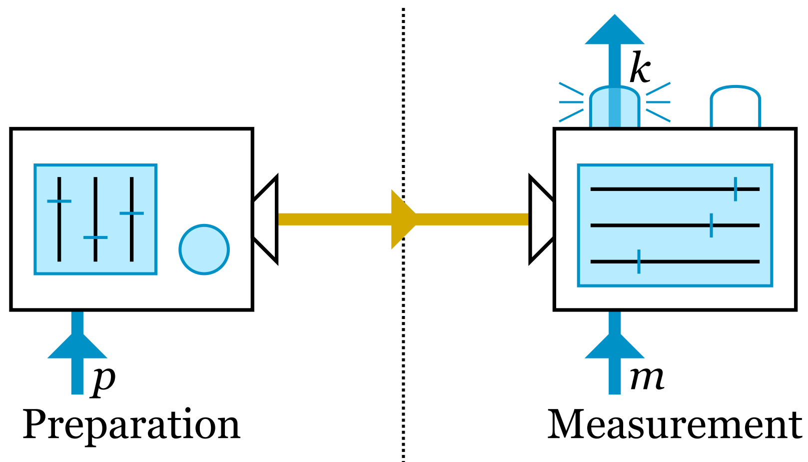

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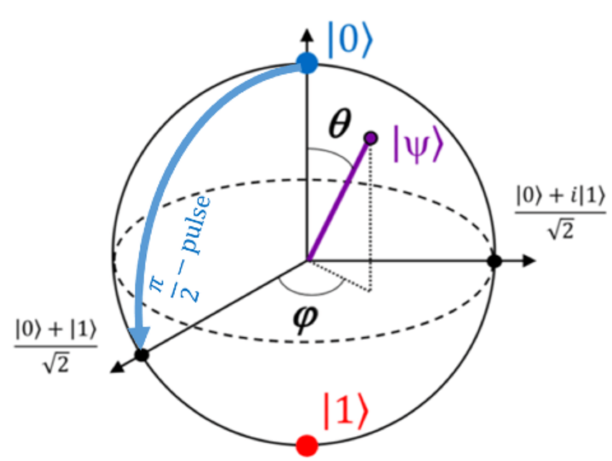

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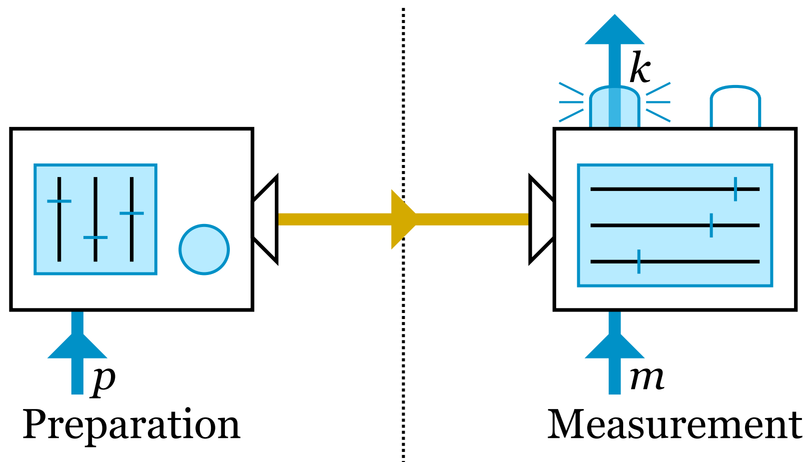

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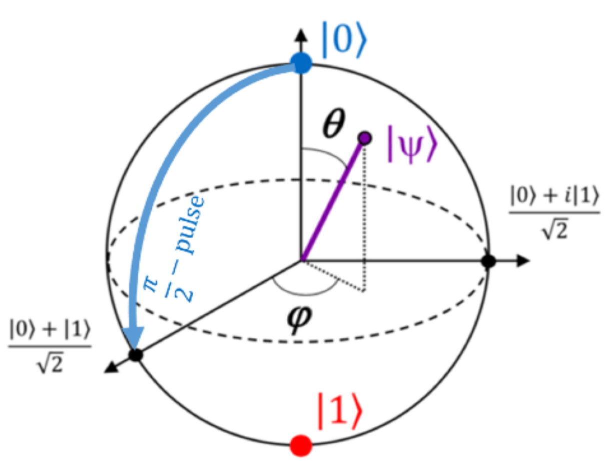

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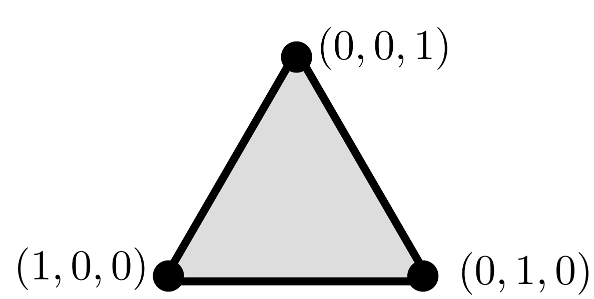
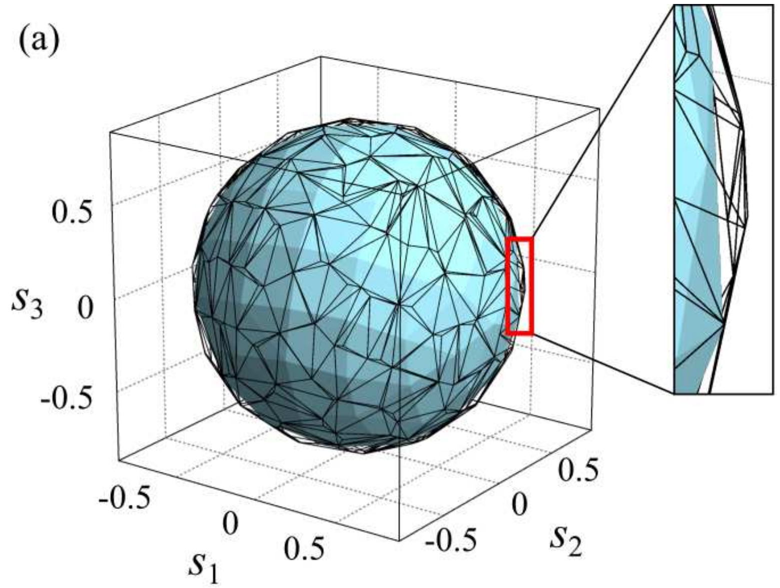


$\Omega = \{p = (p_1, \ldots, p_n) \mid p_i \geq 0, \sum p_i = 1\}$

- classical probability theory
- noisy qubits etc.
- QT *w/* superselection rules
- … **?**

Overview

1. GPTs and theory-agnostic tomography



2. Contextuality, simulations, and embeddings

3. Exact embeddings into quantum theory

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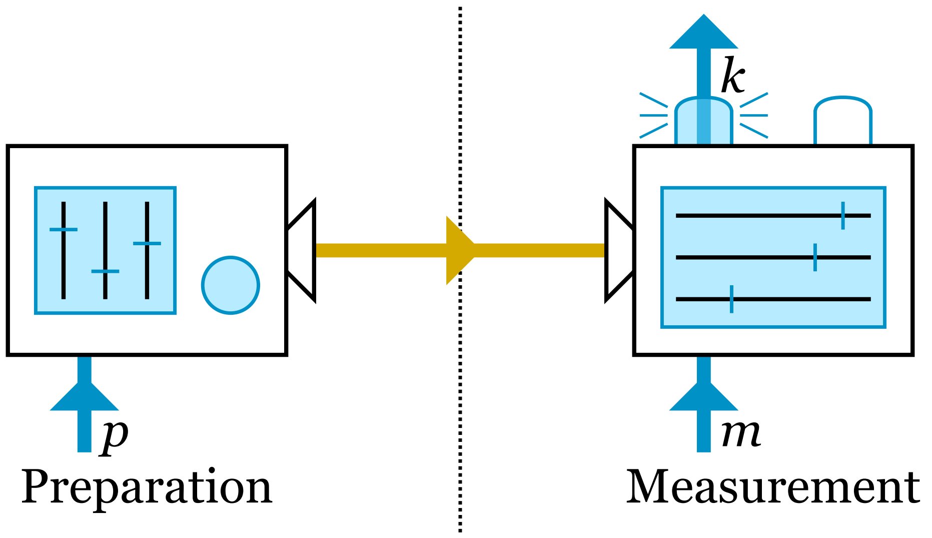

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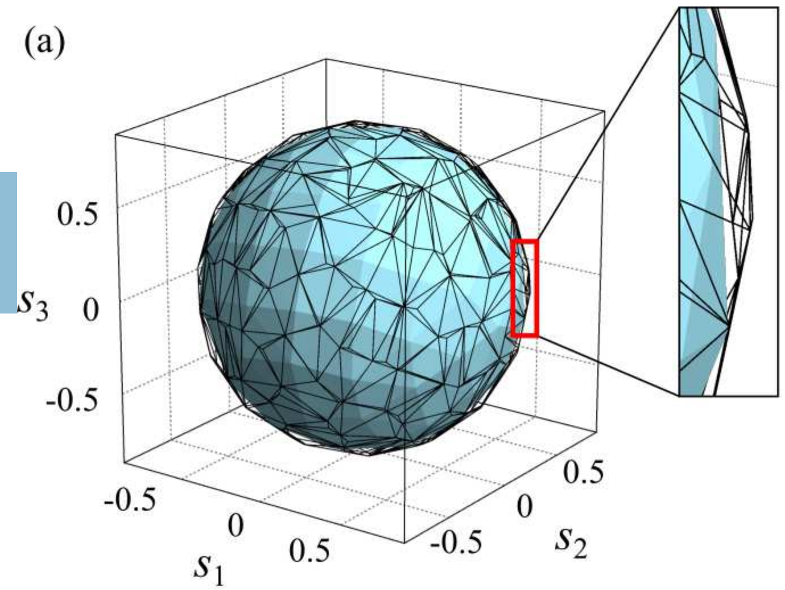

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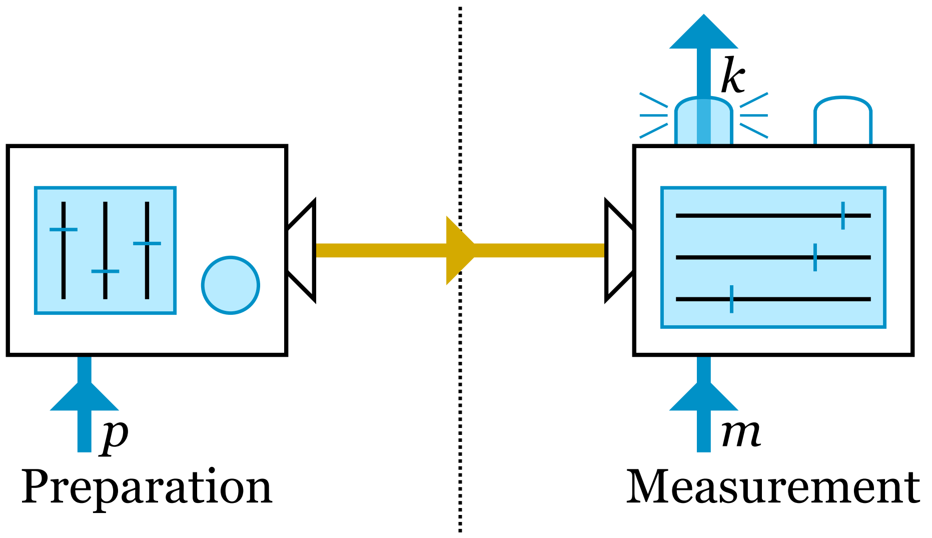
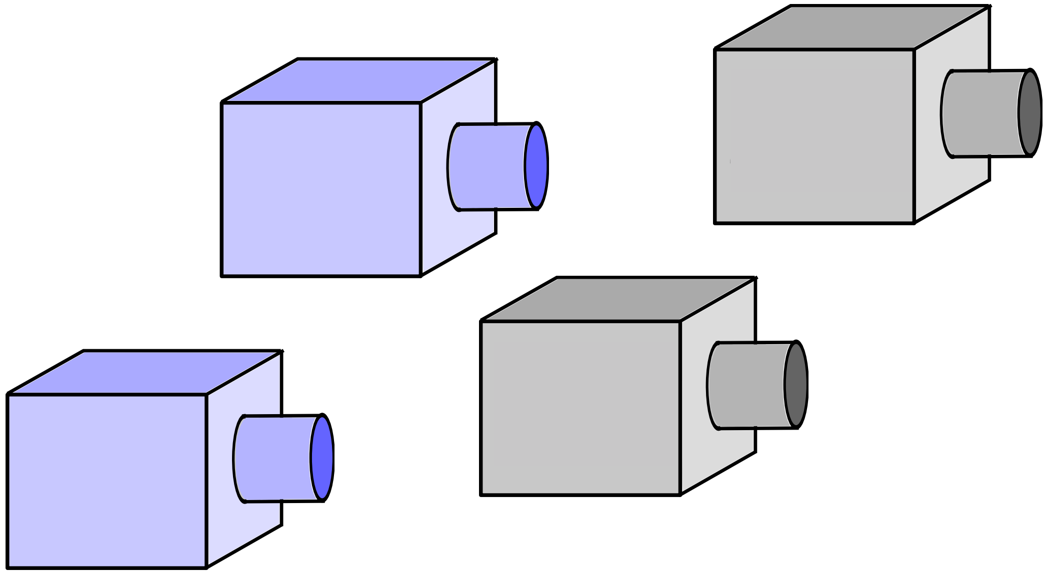


Operational theories



(all accessible preparation procedures)

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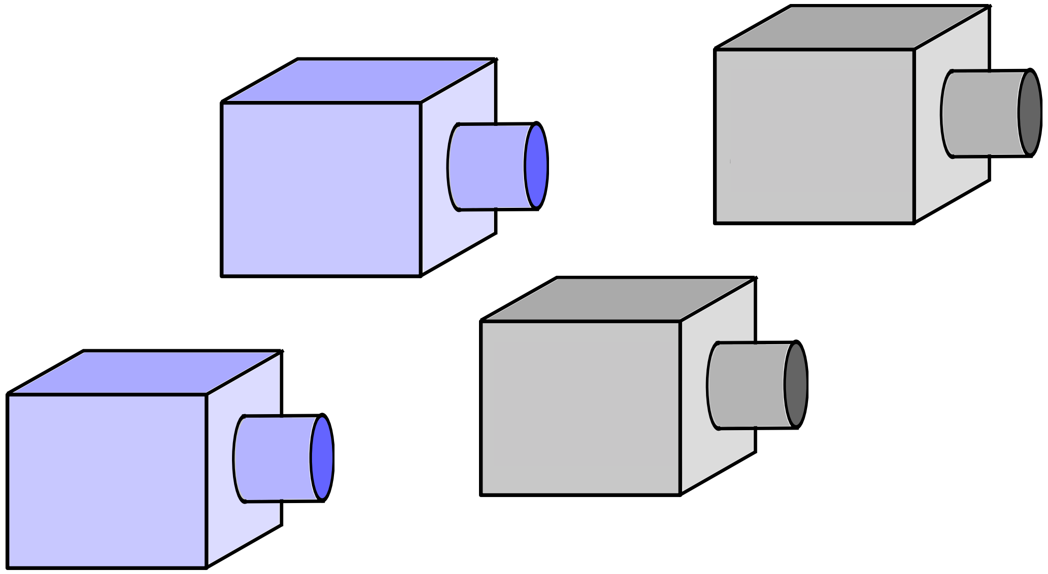

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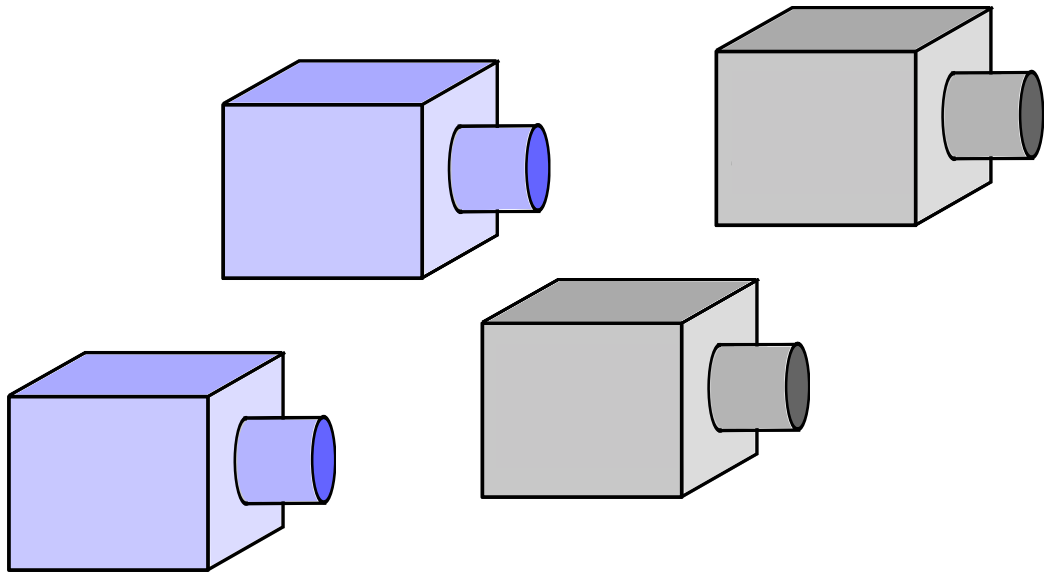

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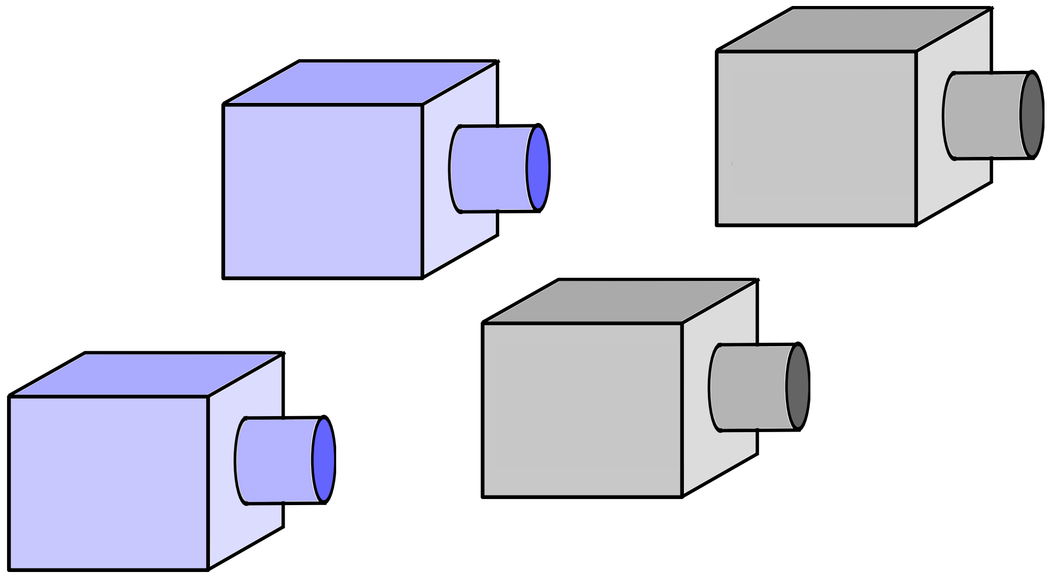

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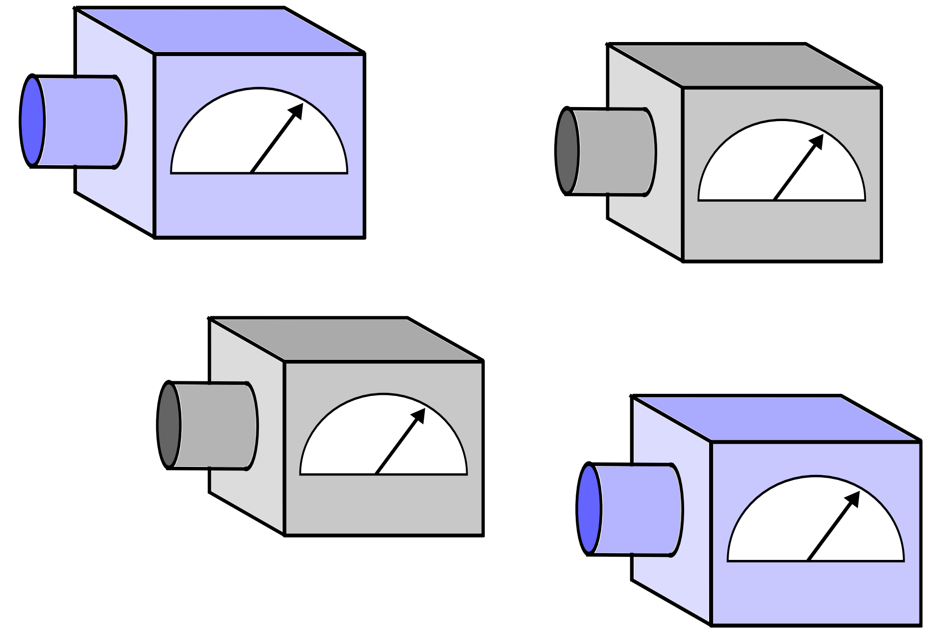

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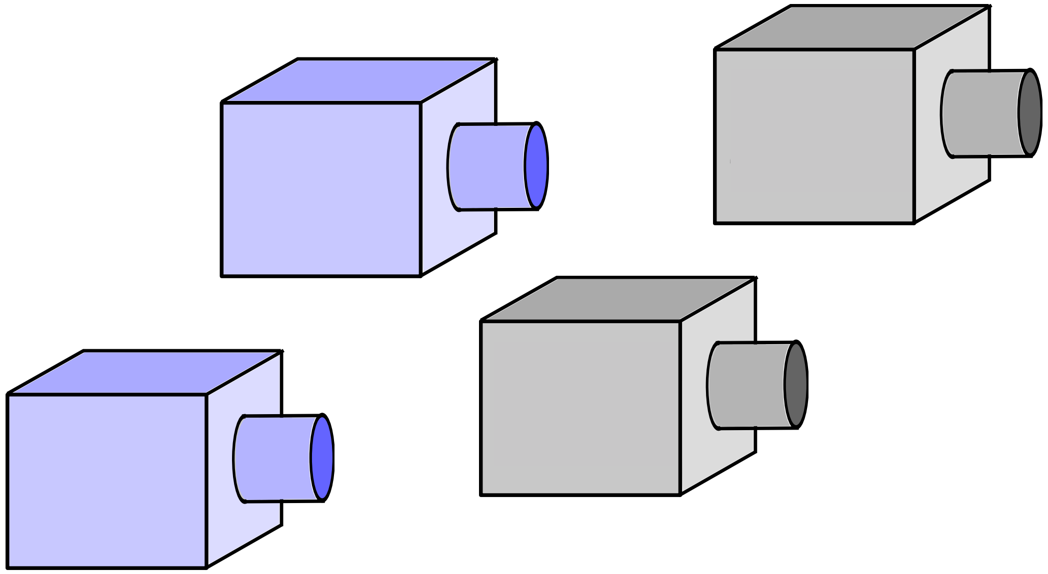

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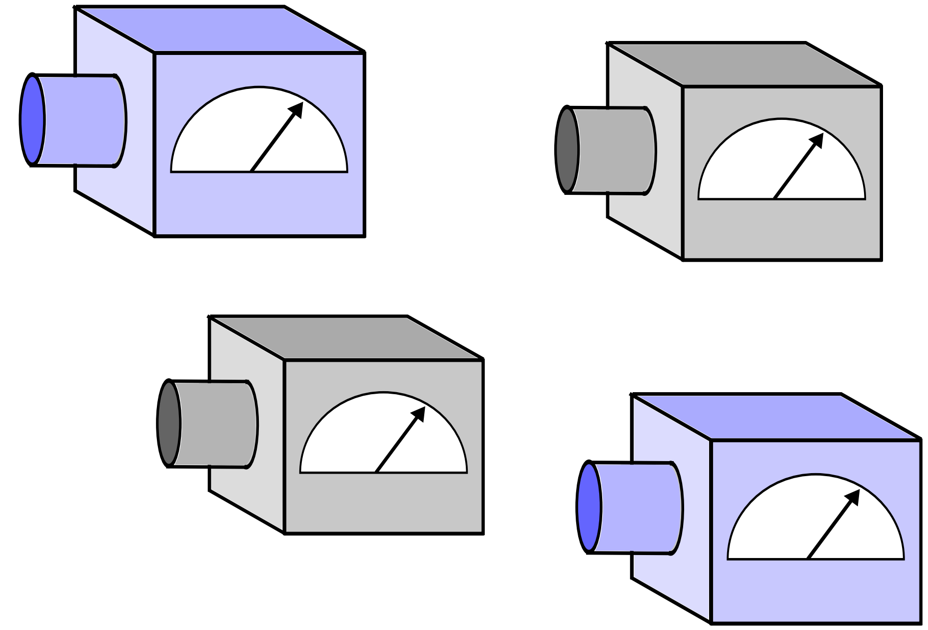

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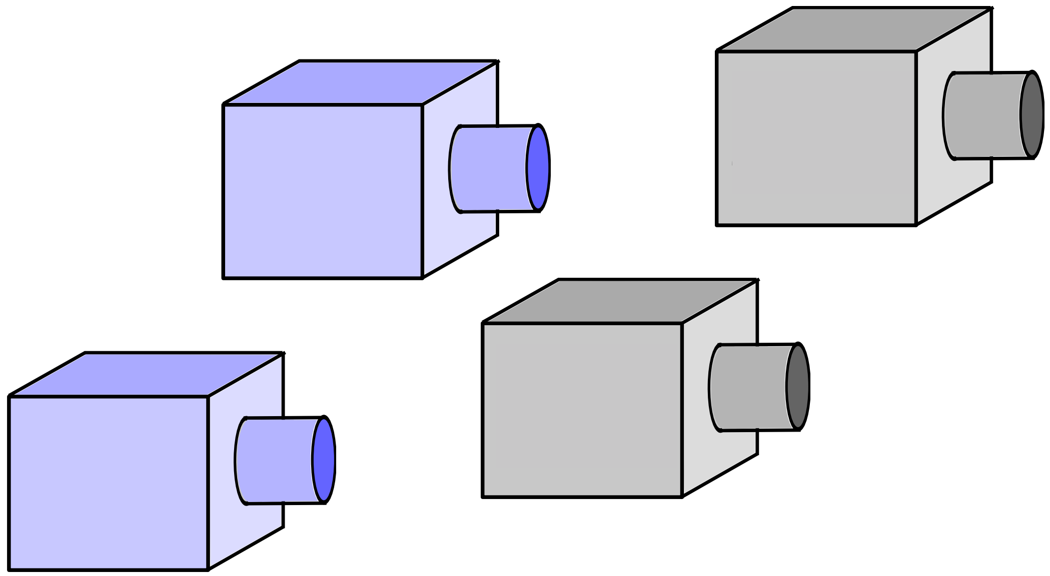

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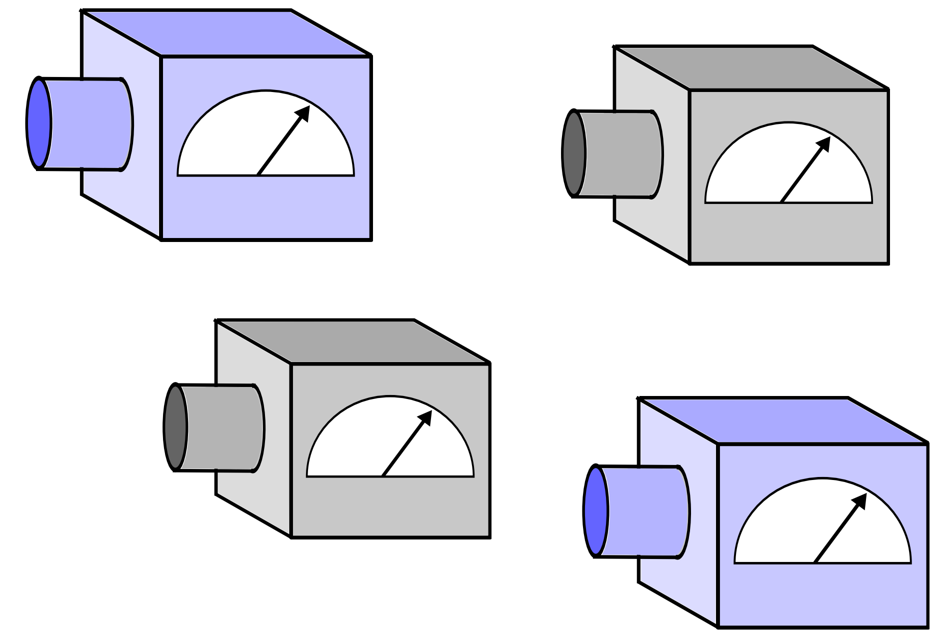

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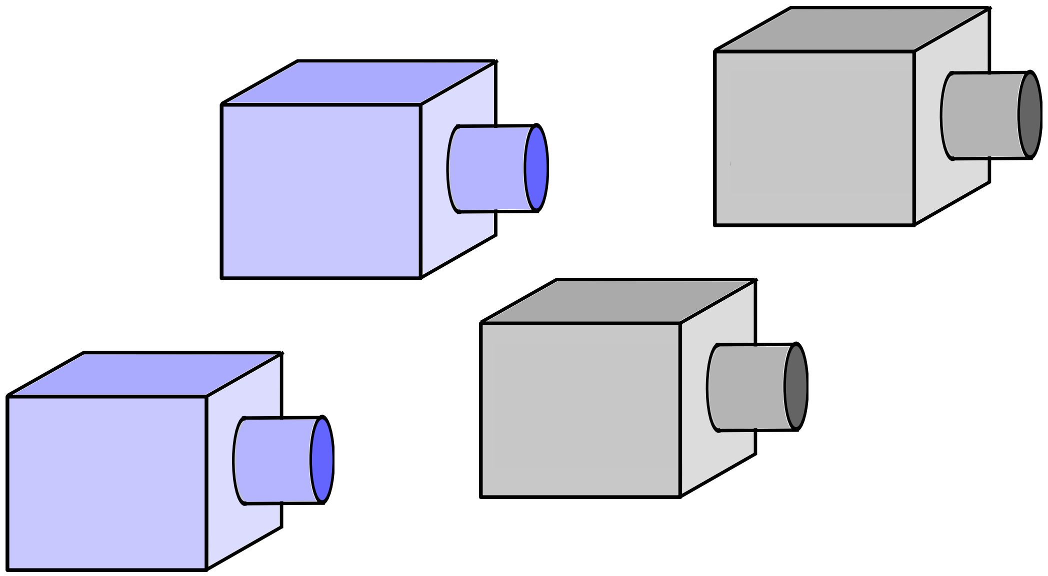

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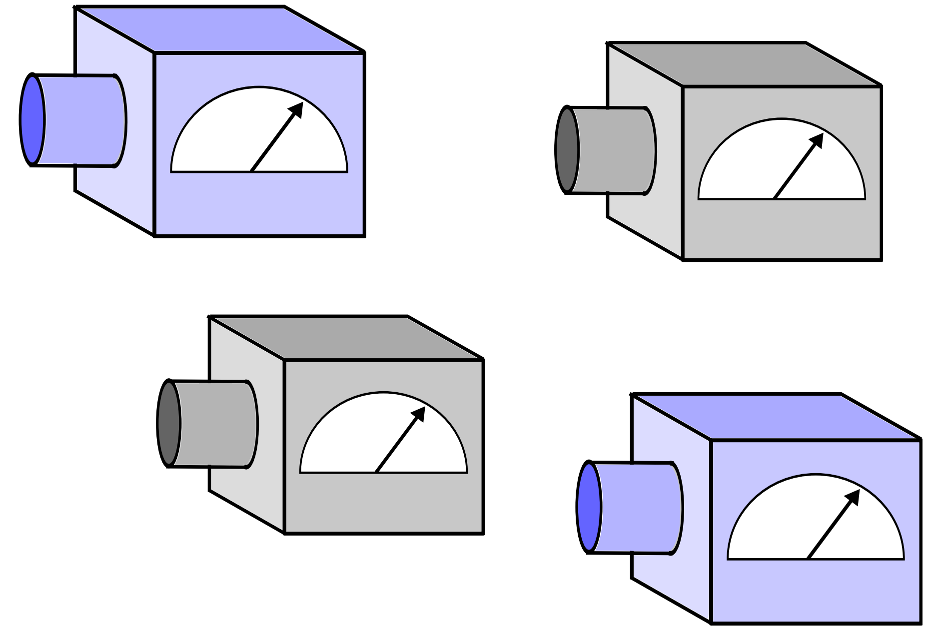

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General probabilistic theories

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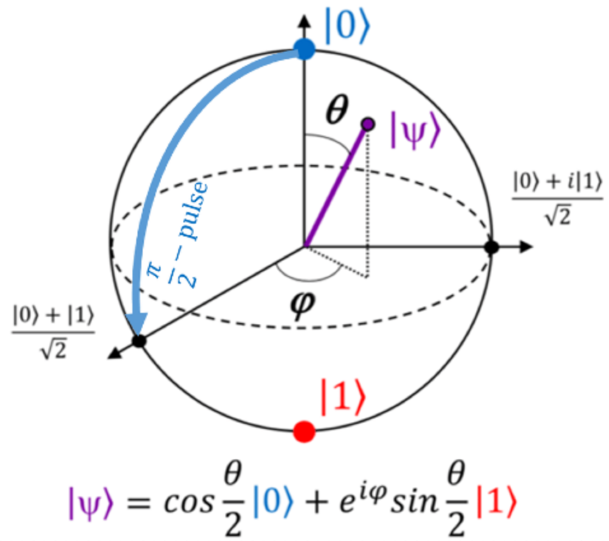

Quantum theory (QT): $\mathcal{Q}_n$

$A = \mathbb{H}_n(\mathbb{C})$ (complex Hermitian $n \times n$ matrices)

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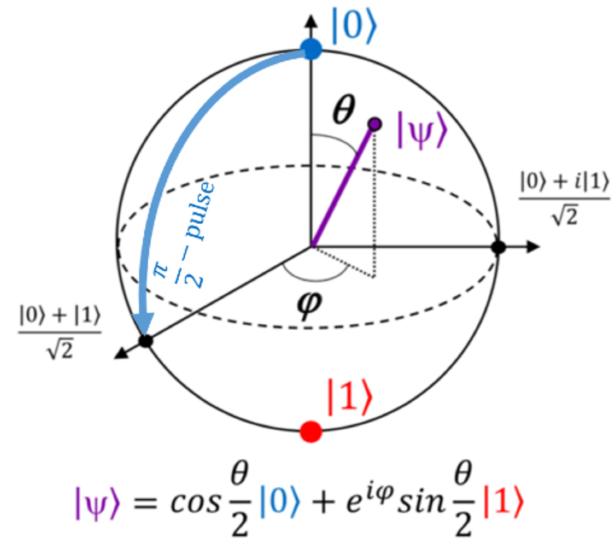

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Classical probability theory (QT): $\mathcal{C}_n$

$$A = \mathbb{R}^n \simeq A^*$$

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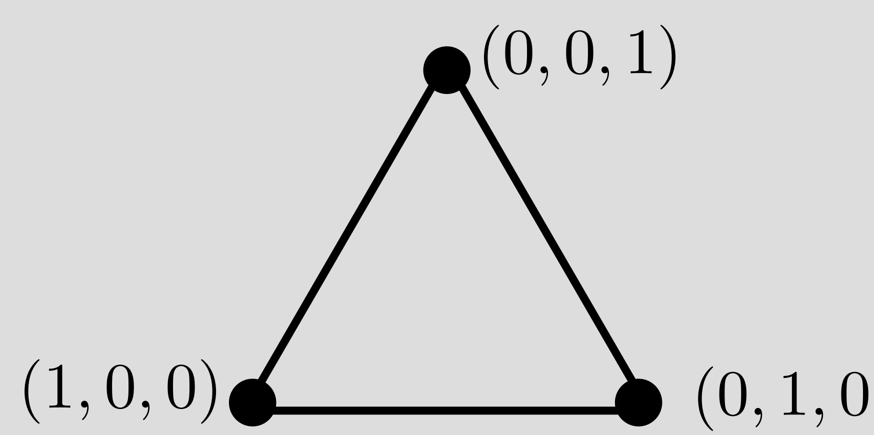

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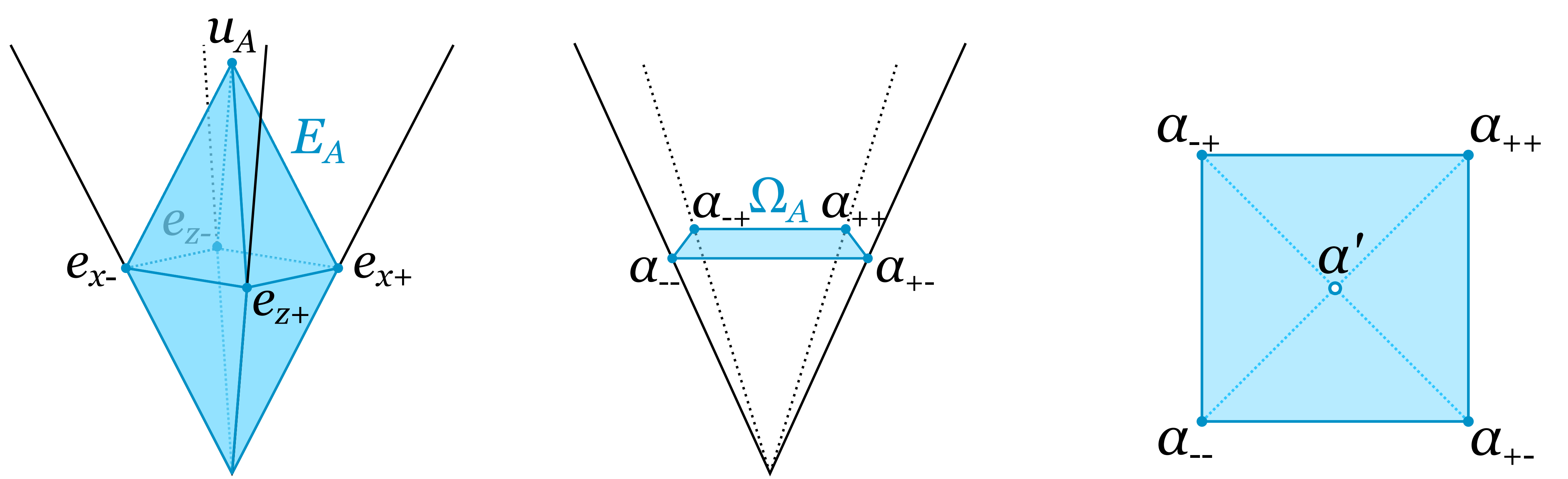

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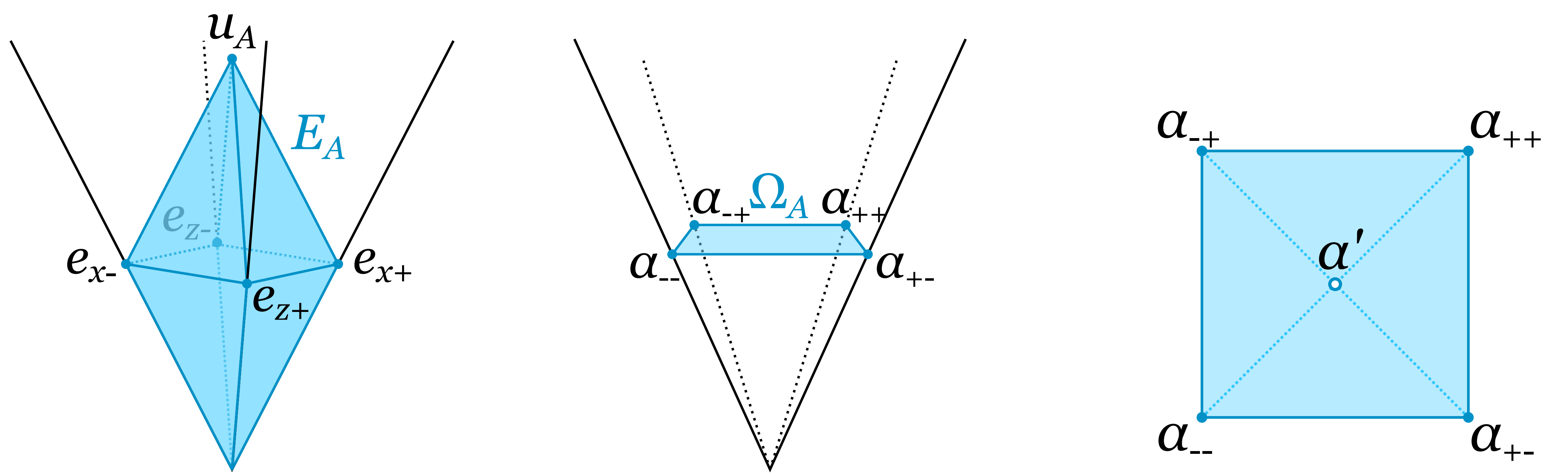

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The four pure states $\alpha_{\pm,\pm}$ are **pairwise** perfectly distinguishable, but **not jointly** $\Longrightarrow$ this cannot be a classical or quantum system.

Theory-agnostic tomography

Idea: Identify a **physical system**. Perform as many preparations and measurements as possible; **fit a GPT to the data**; compare with $\mathcal{Q}_n$.

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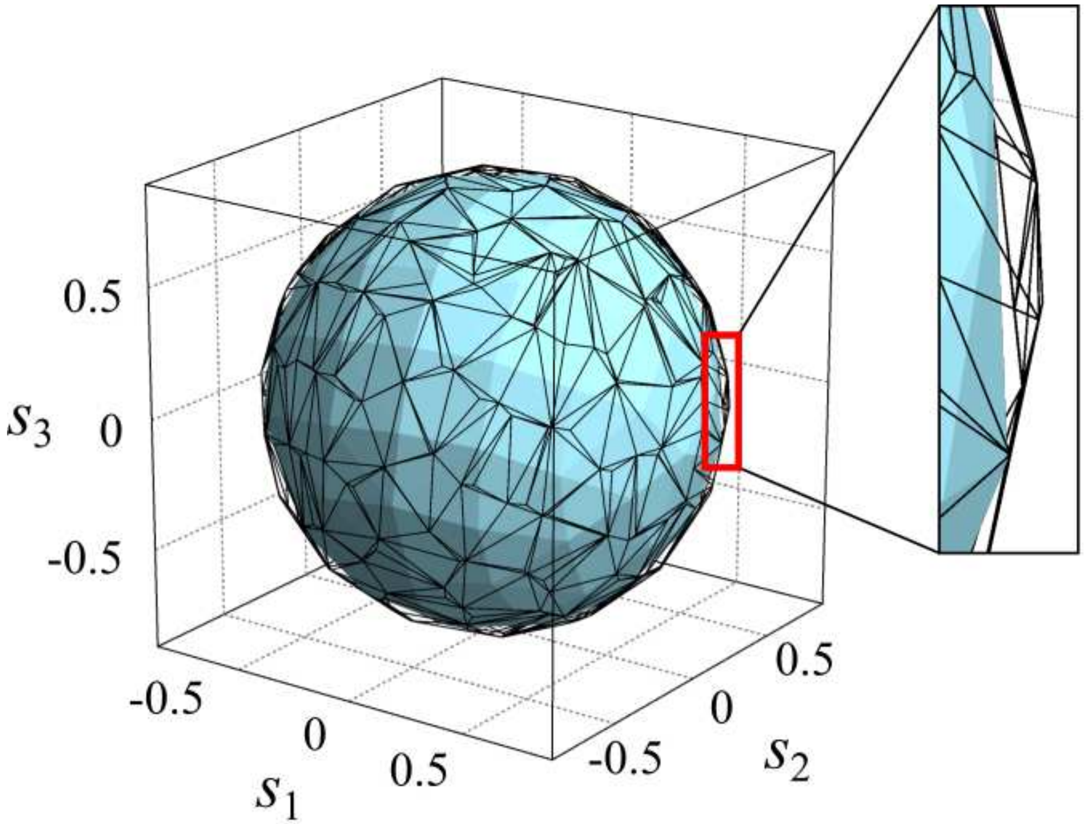

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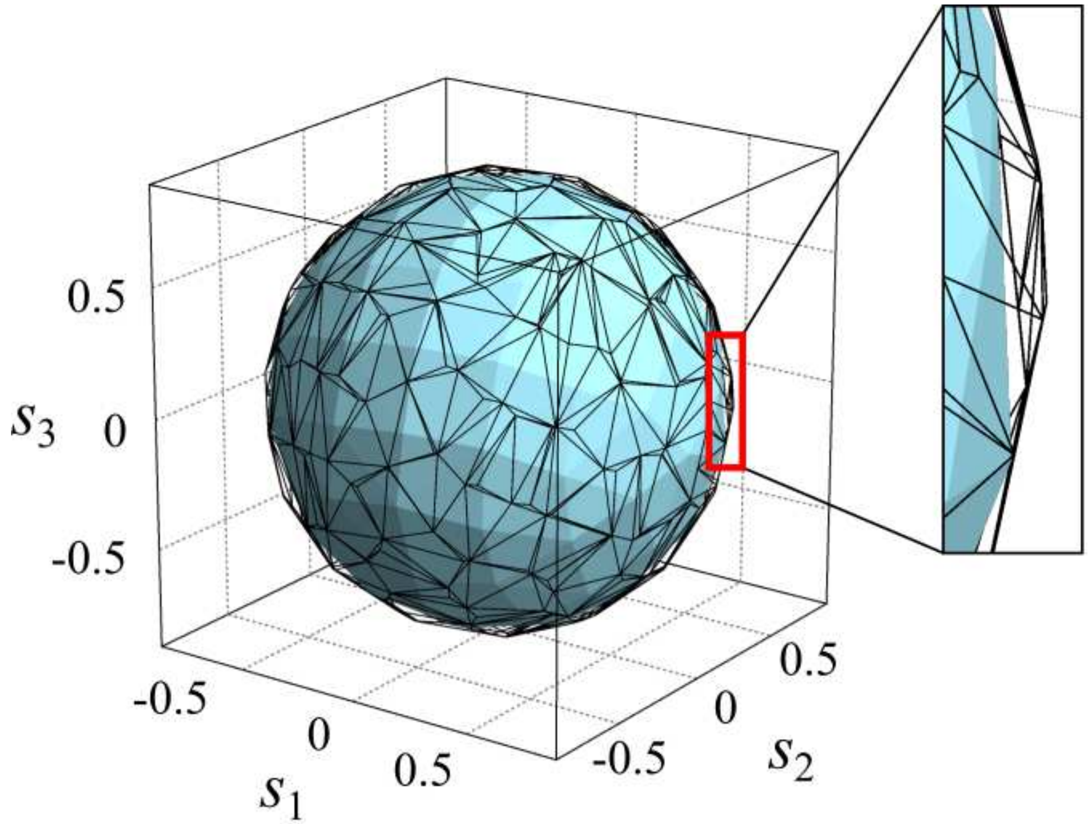

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Tomographic completeness loophole: can never be sure that we probed the system *completely*.

What if we just see a (low-dimensional) **"shadow"**?

Let's **drop** the **tomographic completeness** assumption.

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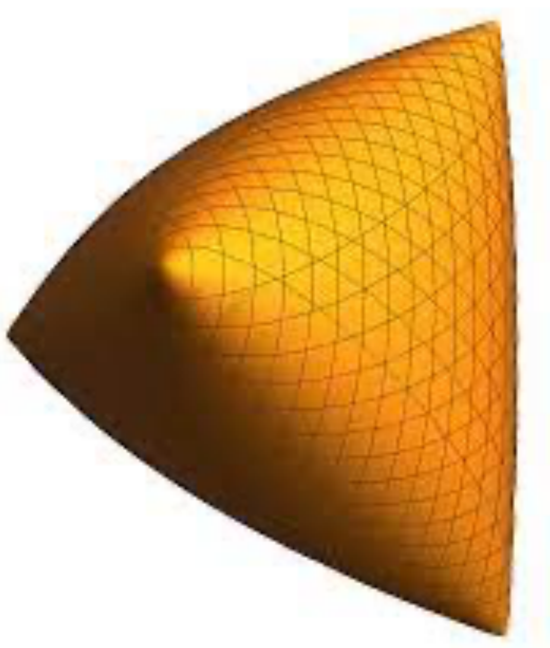

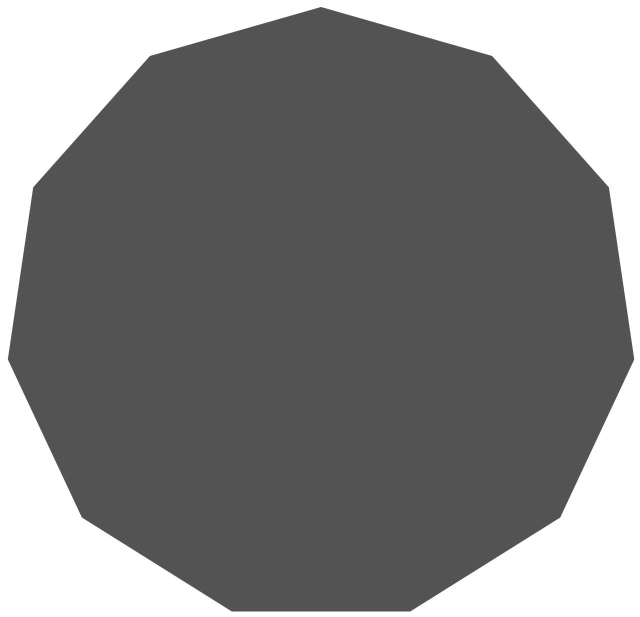

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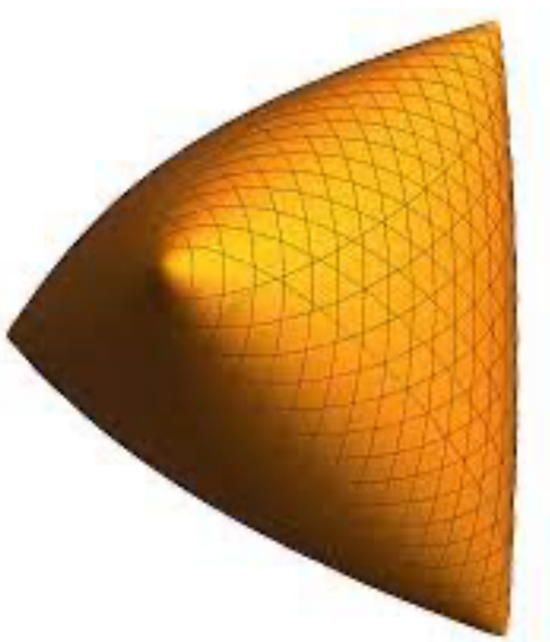

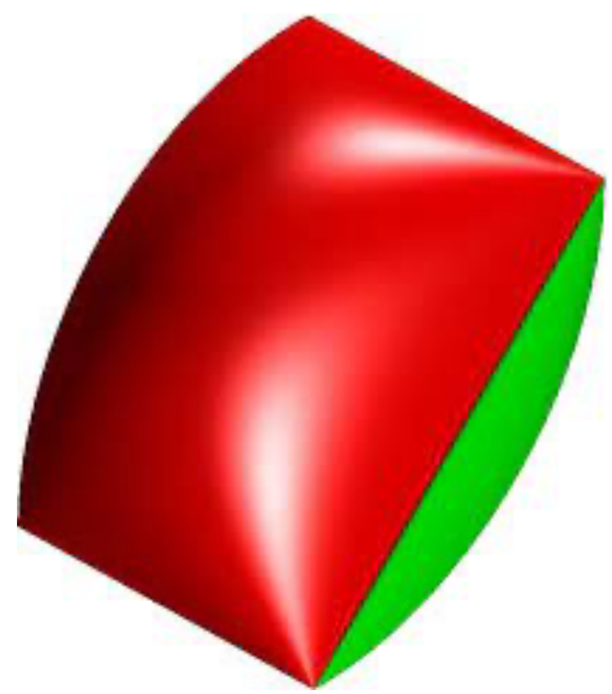

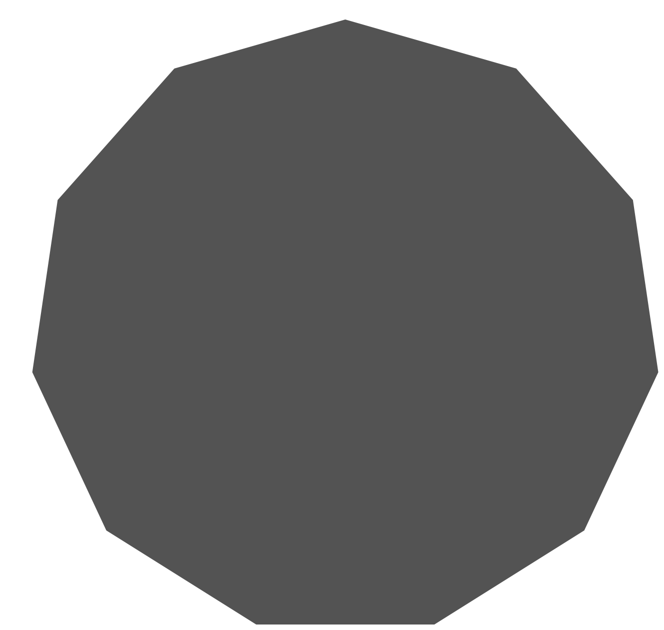

Is **fundamental QT** a plausible explanation of a given **effective GPT**?

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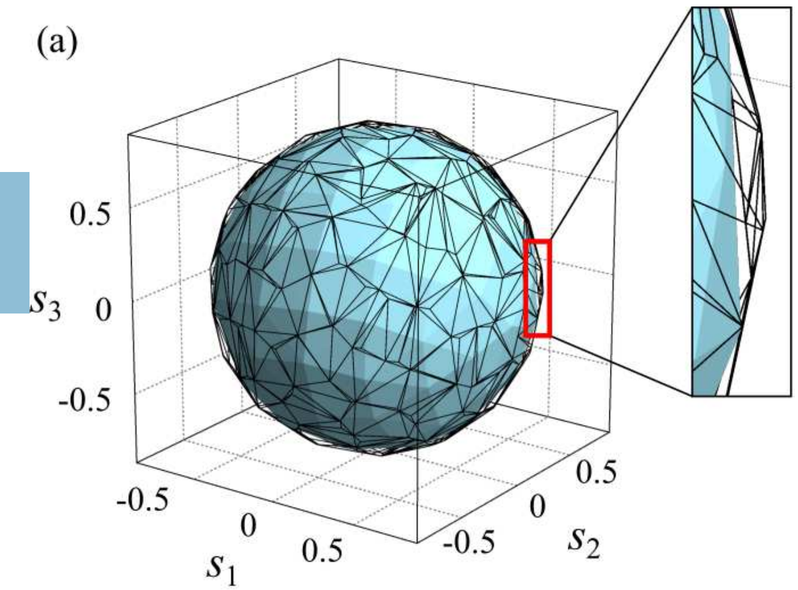

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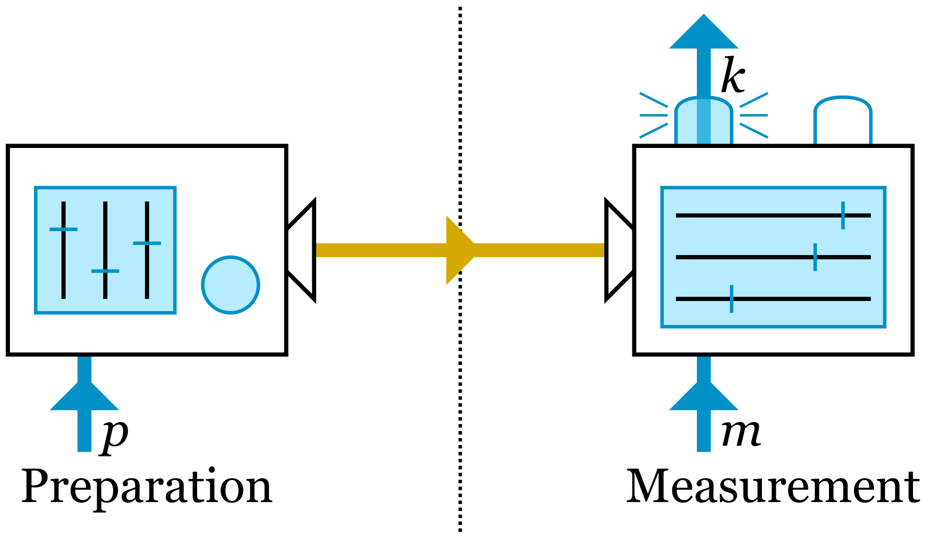

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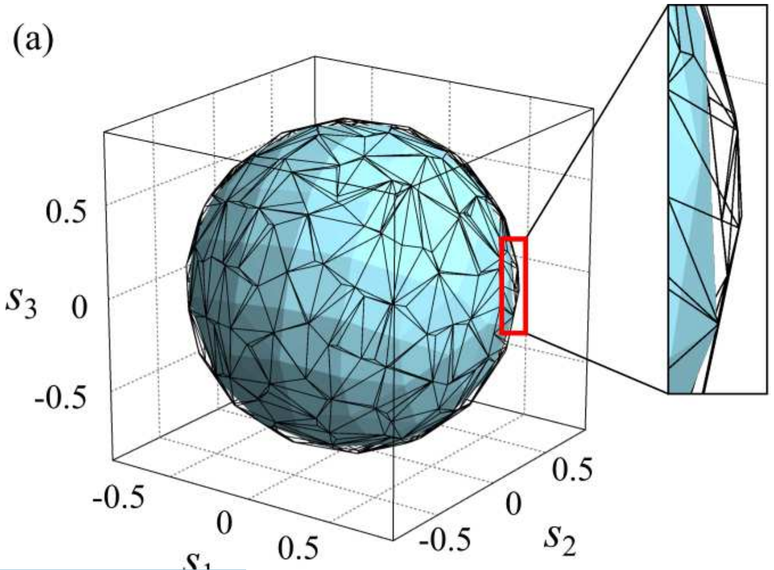

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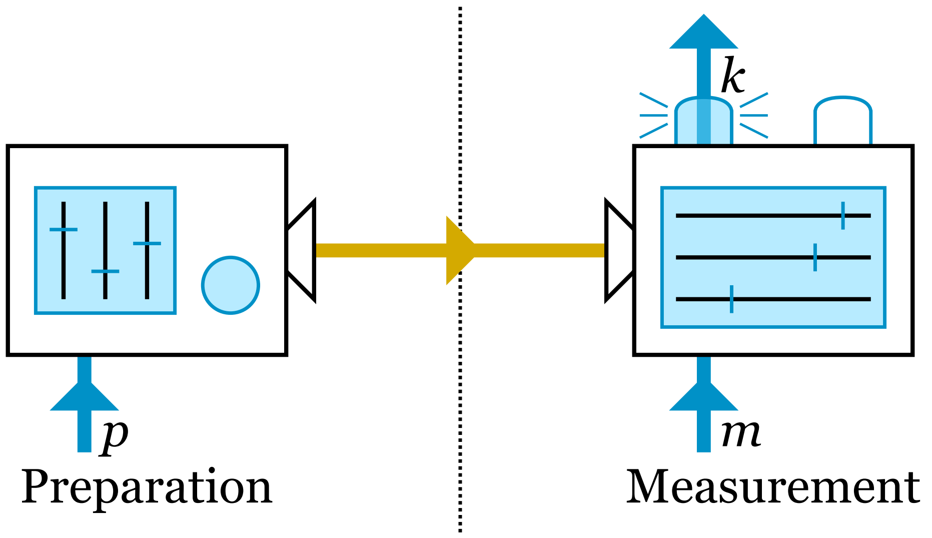

Spekkens’ notion of noncontextuality: quick recap

Contextuality for preparations, transformations and unsharp measurements

R. W. Spekkens*
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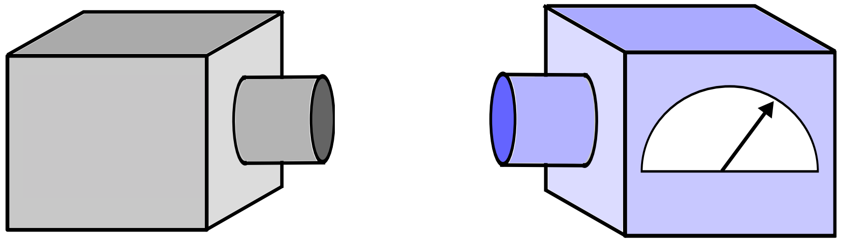

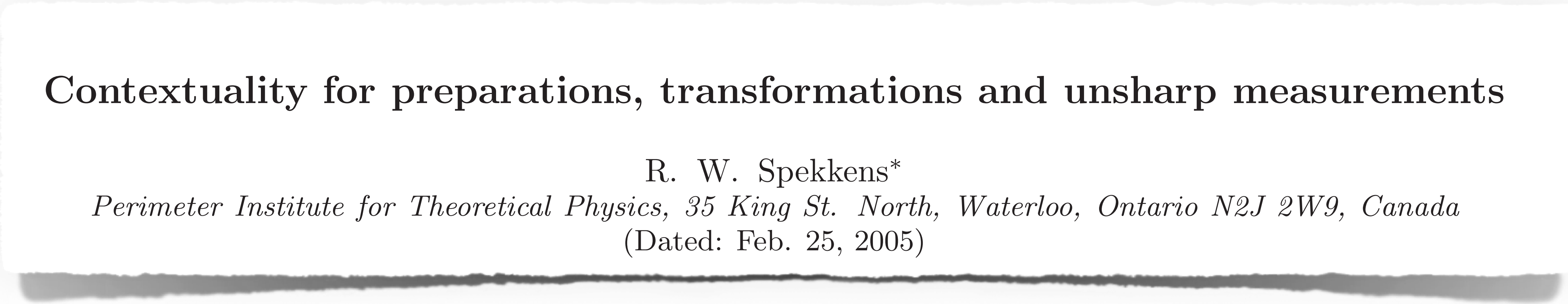
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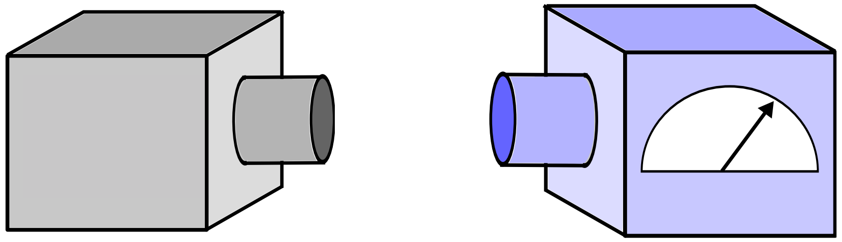

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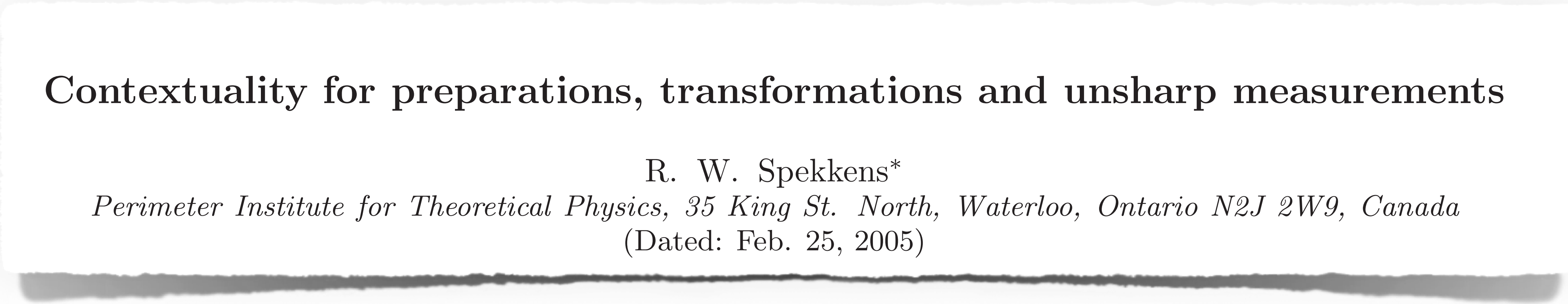
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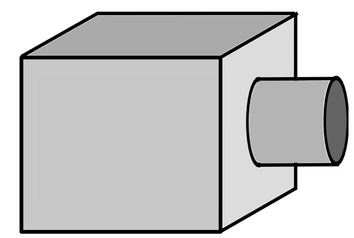

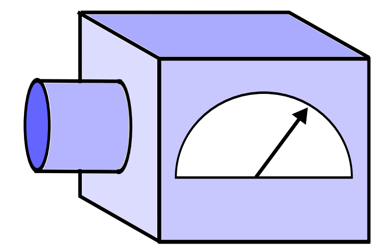

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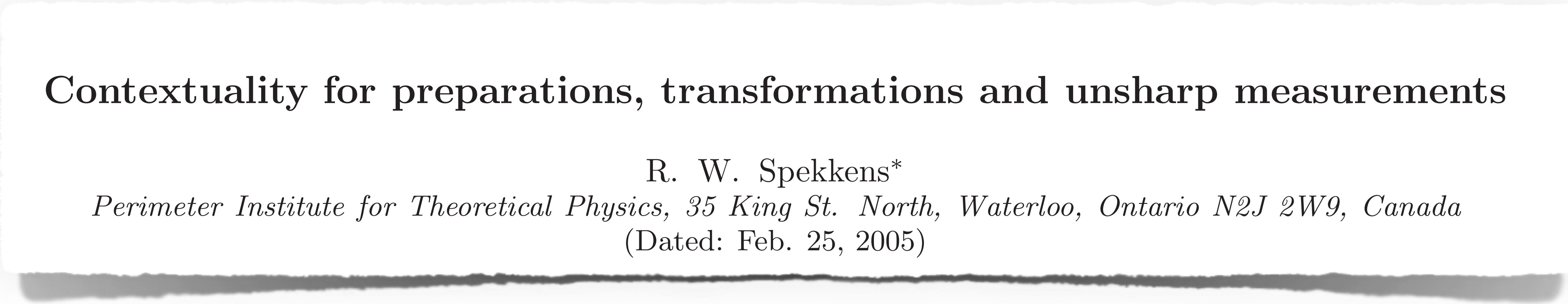
Contextuality for preparations, transformations and unsharp measurements

R. W. Spekkens*
Perimeter Institute for Theoretical Physics, 35 King St. North, Waterloo, Ontario N2J 2W9, Canada
(Dated: Feb. 25, 2005)

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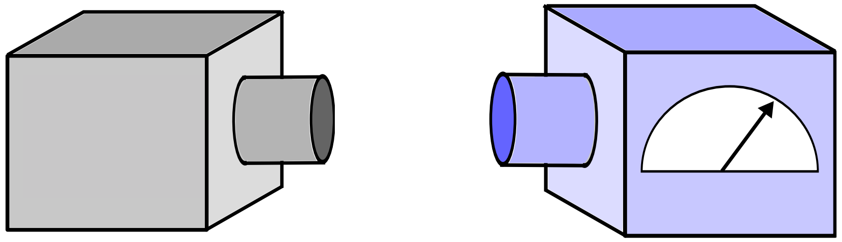

Ontological model of a system (e.g. of a qubit):

A set of classical variables Λ.

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Outcome k of measurement M ⟷ response function $\chi_{M,k}(\lambda)$

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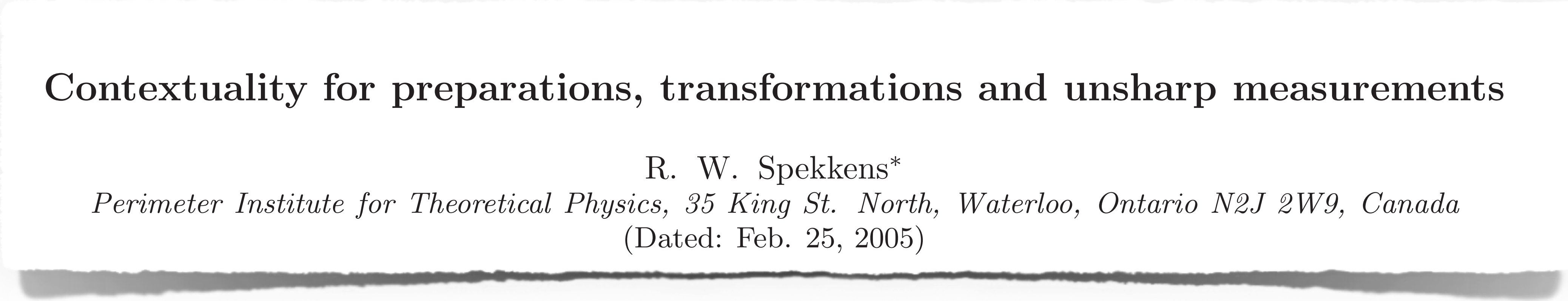
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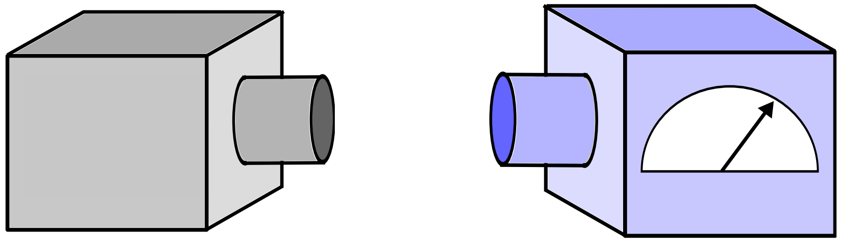

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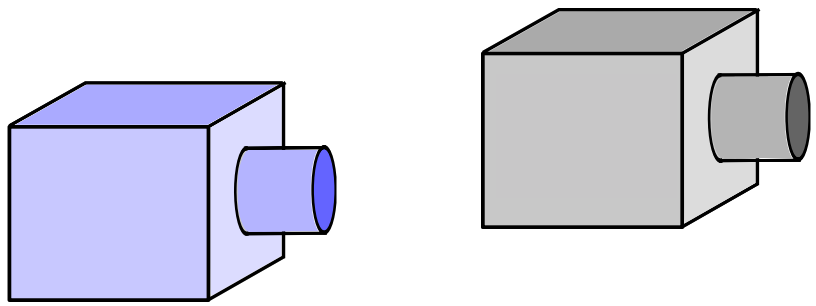

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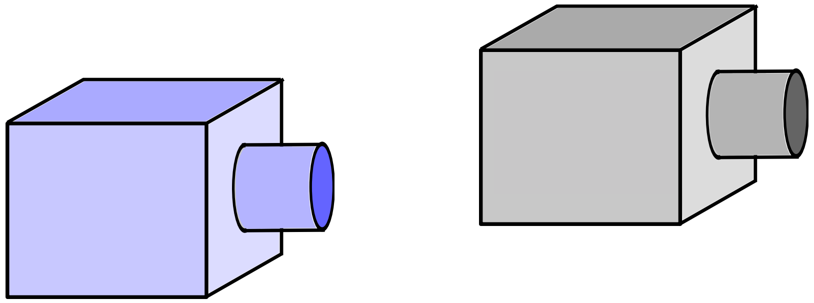

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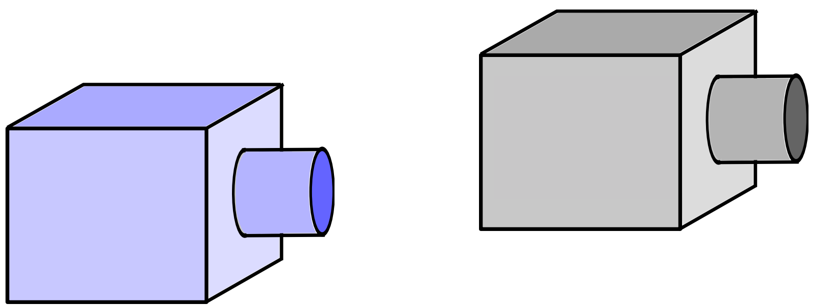

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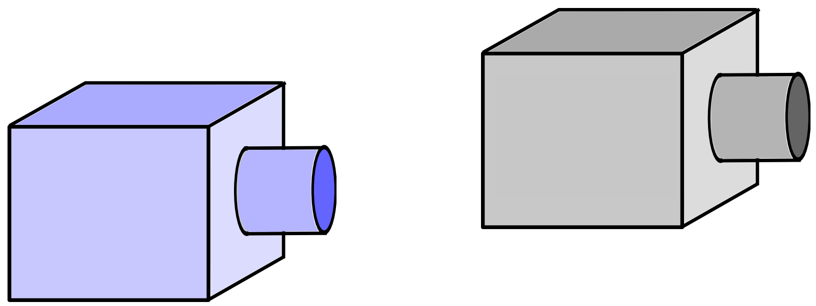

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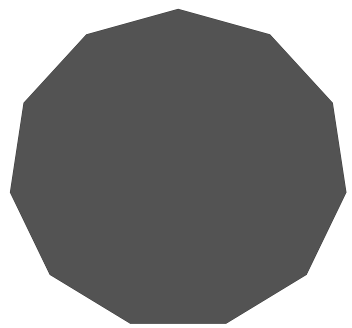
<u>Intuition</u>: Contextual models are implausible because they are **fine-tuned**: operationally, $P \sim P'$, but ontologically, $\mu_P \neq \mu_{P'}$.

An instance of Leibniz' principle of the "identity of the indiscernibles".

Simulations and embeddings



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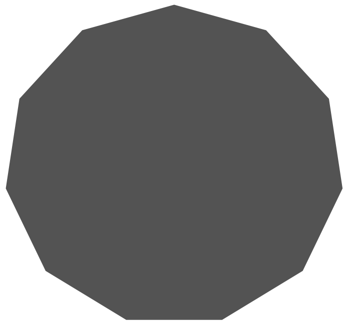

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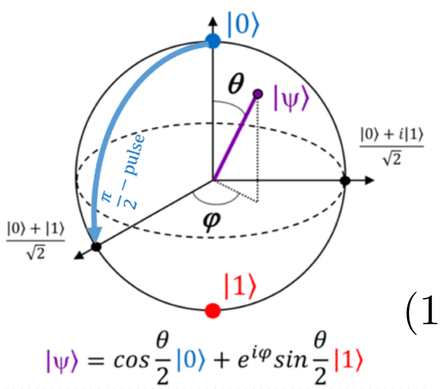

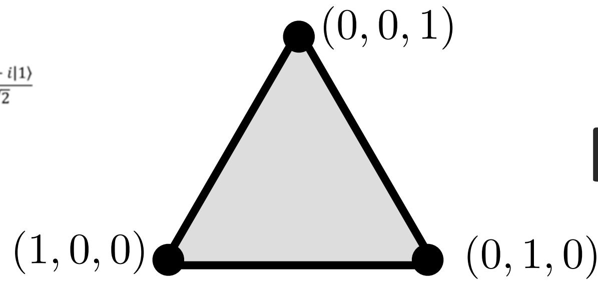

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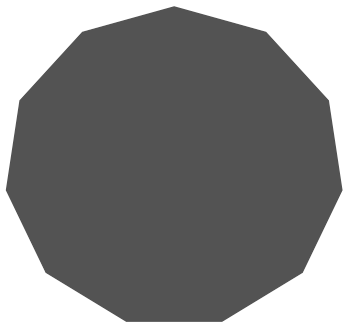

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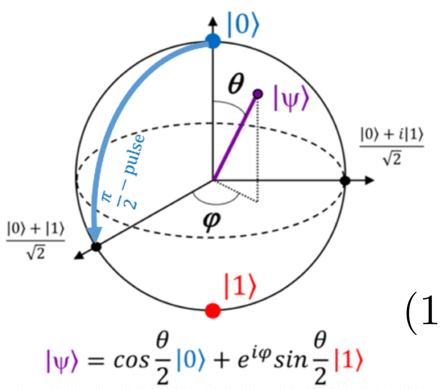

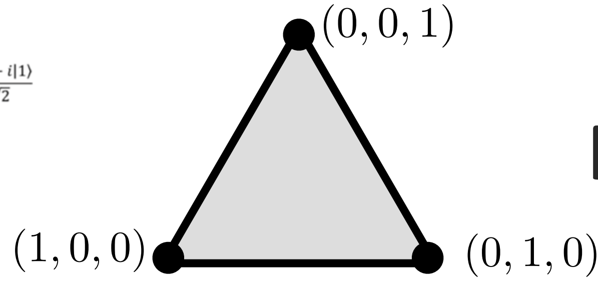

Fundamental GPT $\mathcal{B} = (B, \Omega_B, E_B)$

Effectively preparing state ω_A means **fundamentally** preparing some ω_B, but ω_B may depend on the preparation *procedure*, i.e. the *context*. Collect all those states into a set $\Omega_B(\omega_A) := \{\omega_B\}$.

Simulations and embeddings

Definition. An ε -**simulation** of effective GPT $\mathcal{A}$ by fundamental GPT $\mathcal{B}$:

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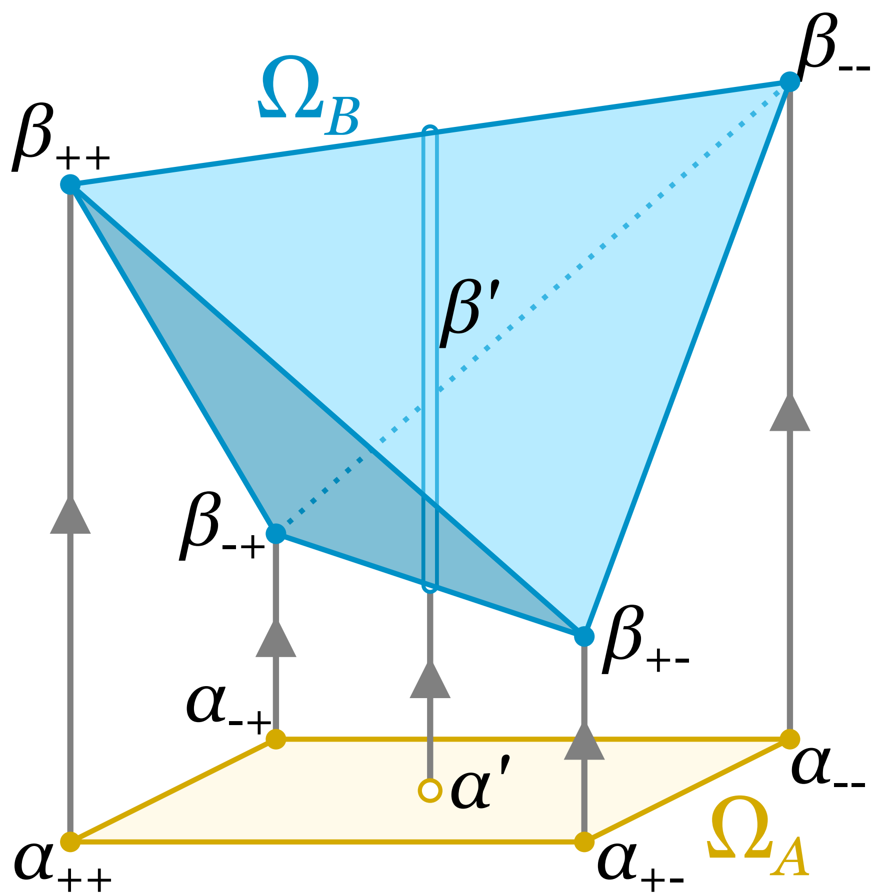
Simulation is **univalent** if all $\Omega_B(\omega_A)$, $E_B(e_A)$ contain **one** element.

Special case $\mathcal{A}$ = QT, $\mathcal{B}$ = classical probability theory:

Simulations are **ontological models**, and univalence = **noncontextuality**.

Example ("Holevo projection"): simulating the gbit $\mathcal{A} = (\mathbb{R}^3, \Omega_A, E_A)$ with a classical 4-level system $\mathcal{C}_4$.



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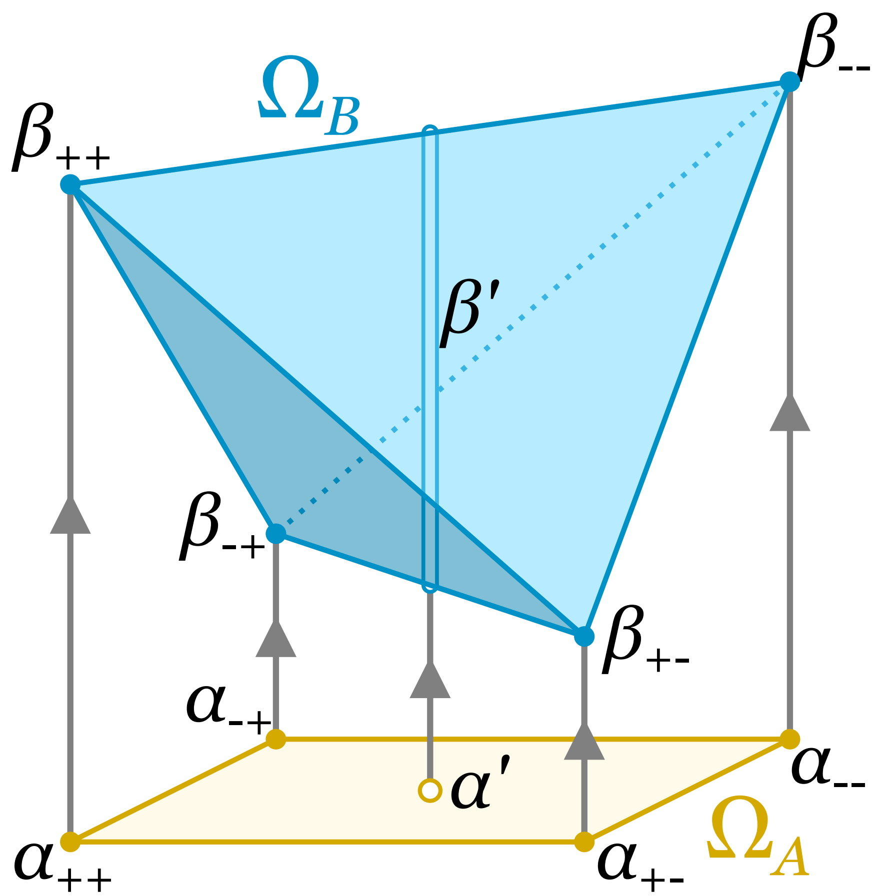

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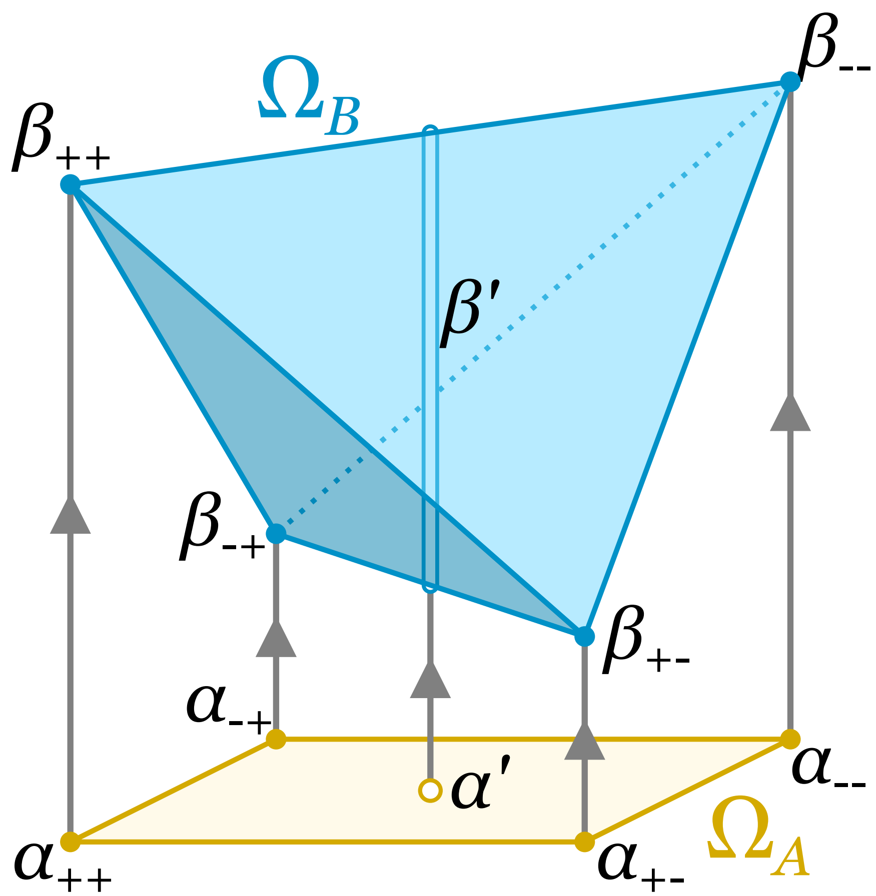

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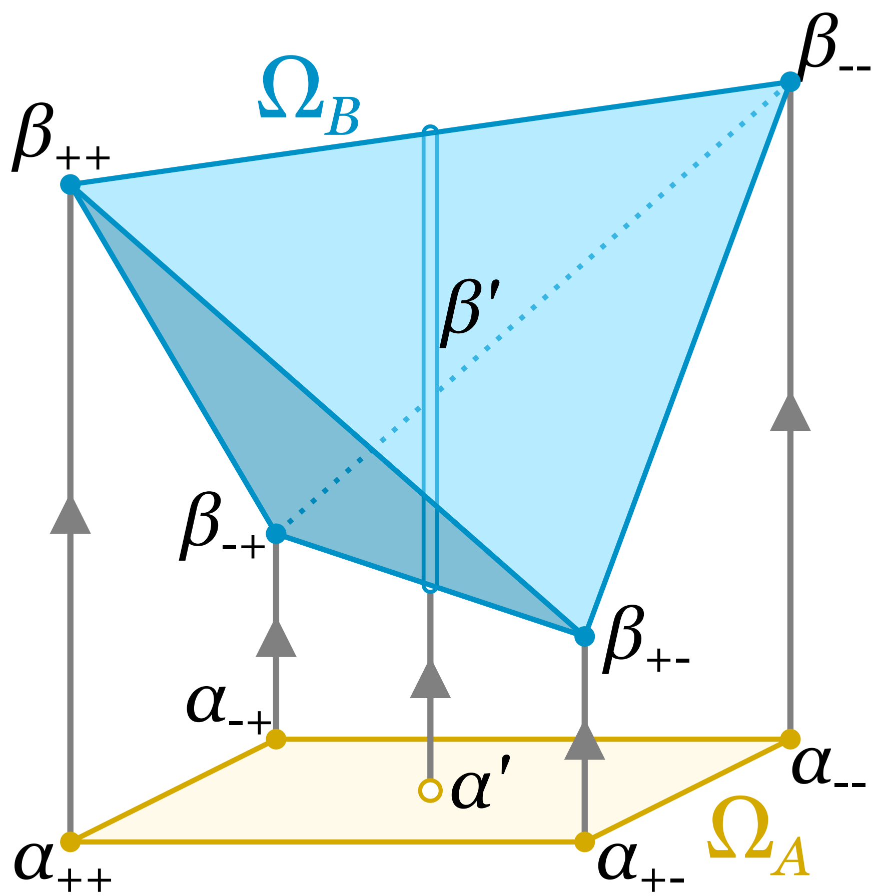

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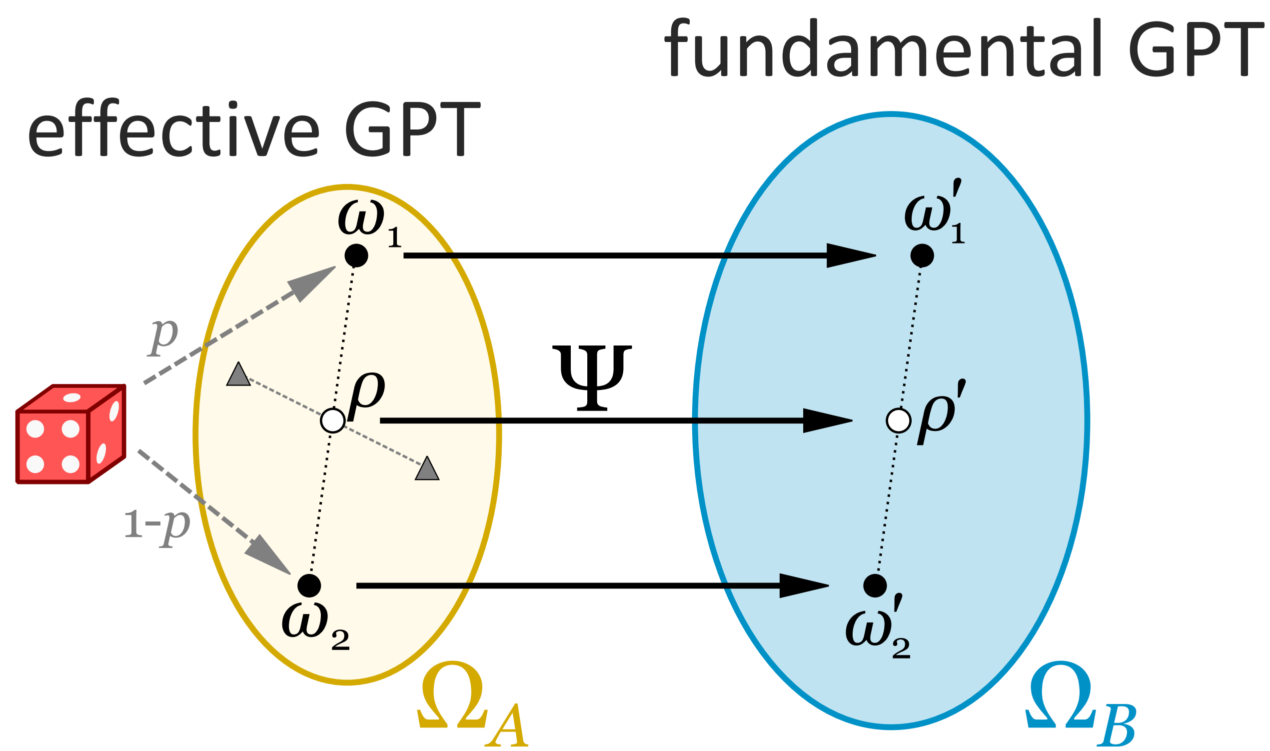
This is an instance of implausible fine-tuning:
the statistical differences among the fundamental states are miraculously *exactly "washed out"* on the effective level.

Univalent simulations are **embeddings**



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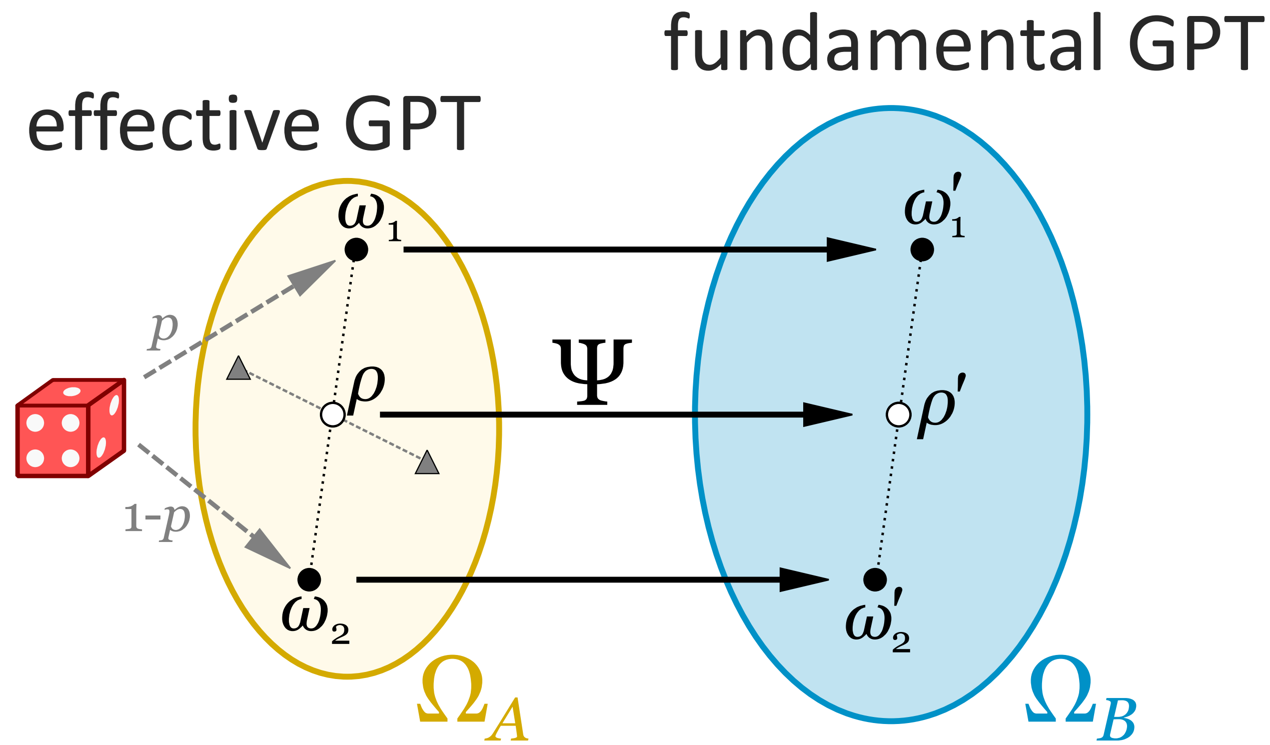

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An ε-**embedding** consists of two linear maps Ψ and Φ such that

- Ψ maps the normalized states of A into those of B,
- Φ maps the effects of A into those of B,
- outcome probabilities are preserved up to ε.

Noncontextual inequalities and approximate embeddings

[4] M. D. Mazurek et al., *An experimental test of noncontextuality without unphysical idealizations*, Nat. Comm. **7**, 11780 (2016).

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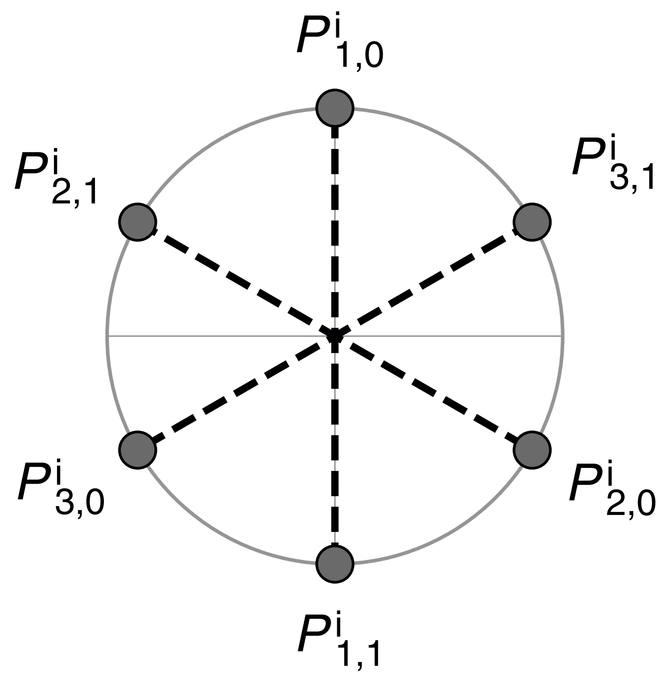

The qubit (actually, rebit) does not have a noncontextual ontological model.

Quantitative statement:

$$A := \frac{1}{6} \sum_{t\in\{1,2,3\}} \sum_{b\in\{0,1\}} P(b\,|\,p_{t,b}, m_t) \leq \frac{5}{6}.$$

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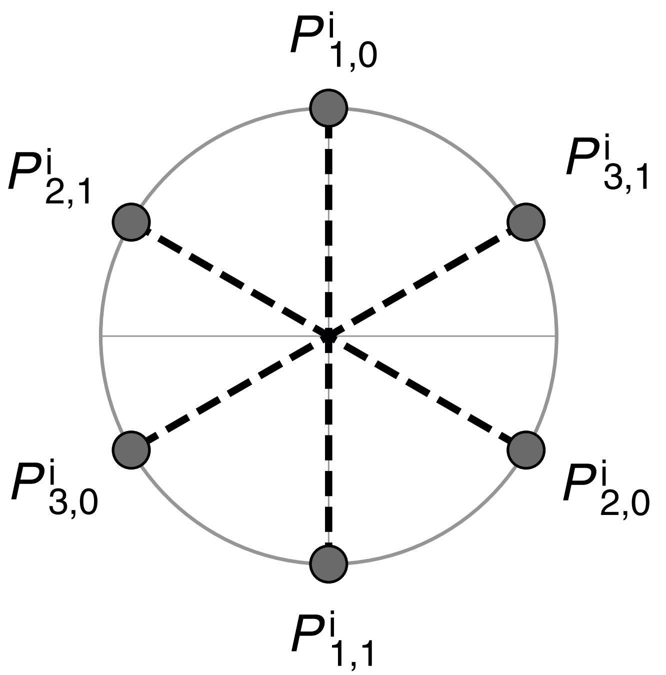

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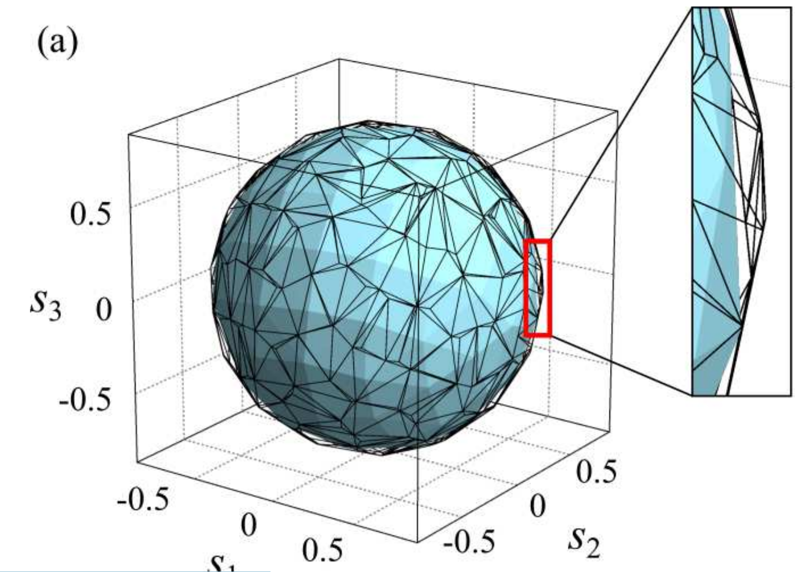
These imply bounds on the approximate embeddability into classical:

Example 1. *Let $\varepsilon < \frac{1}{6}$. Then the rebit (and thus, also the qubit) cannot be ε-embedded into any $\mathcal{C}_n$.*

Overview

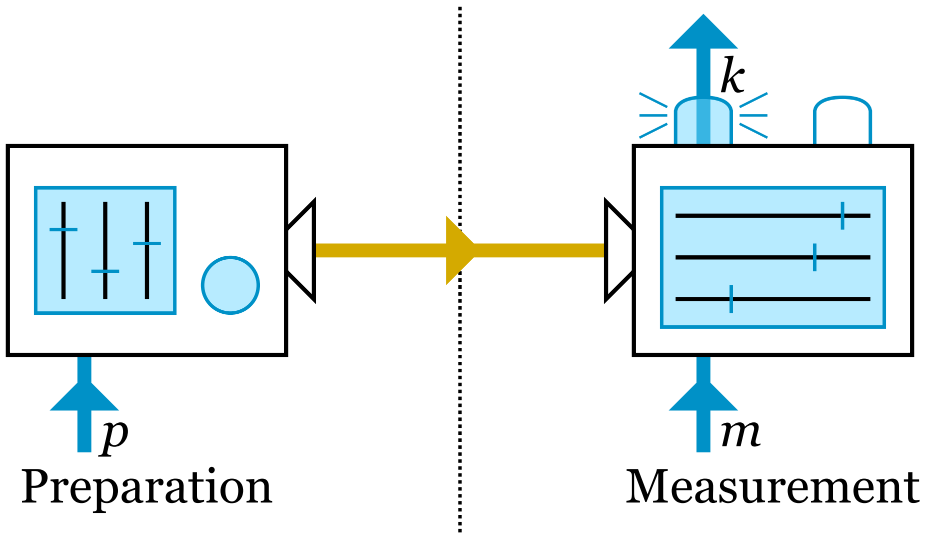
1. GPTs and theory-agnostic tomography



2. Contextuality, simulations, and embeddings

3. Exact embeddings into quantum theory

4. An experimental test of QT



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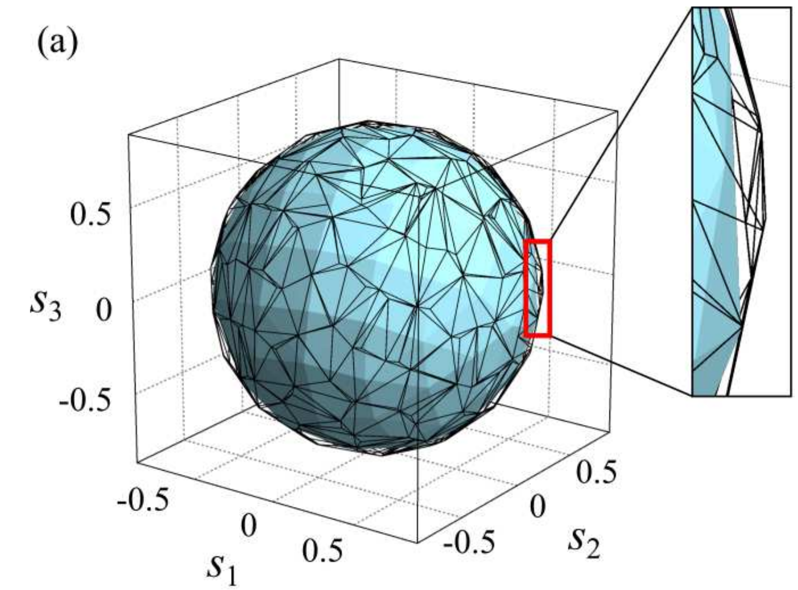

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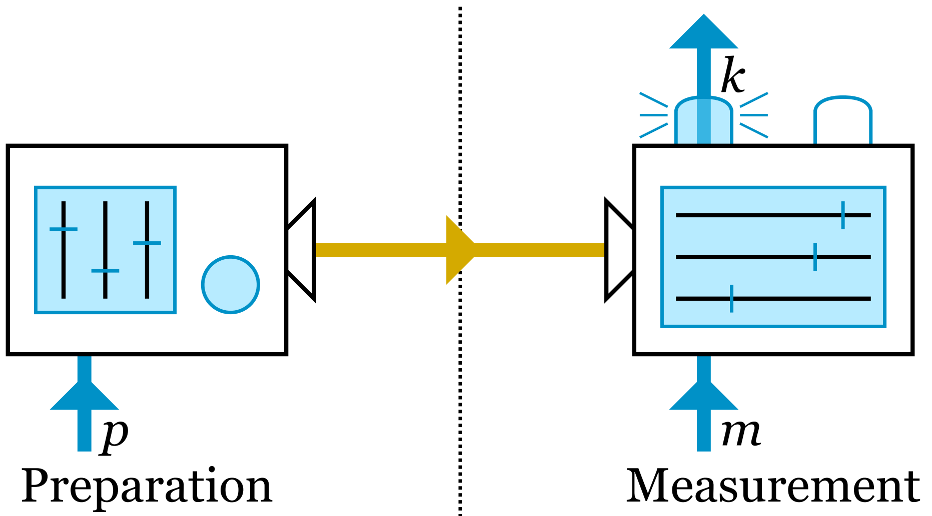

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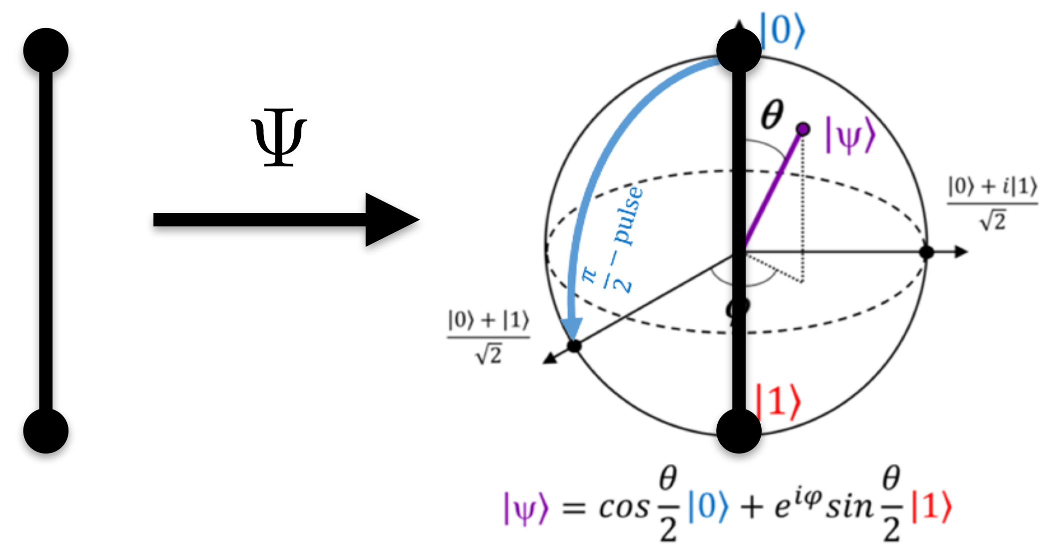

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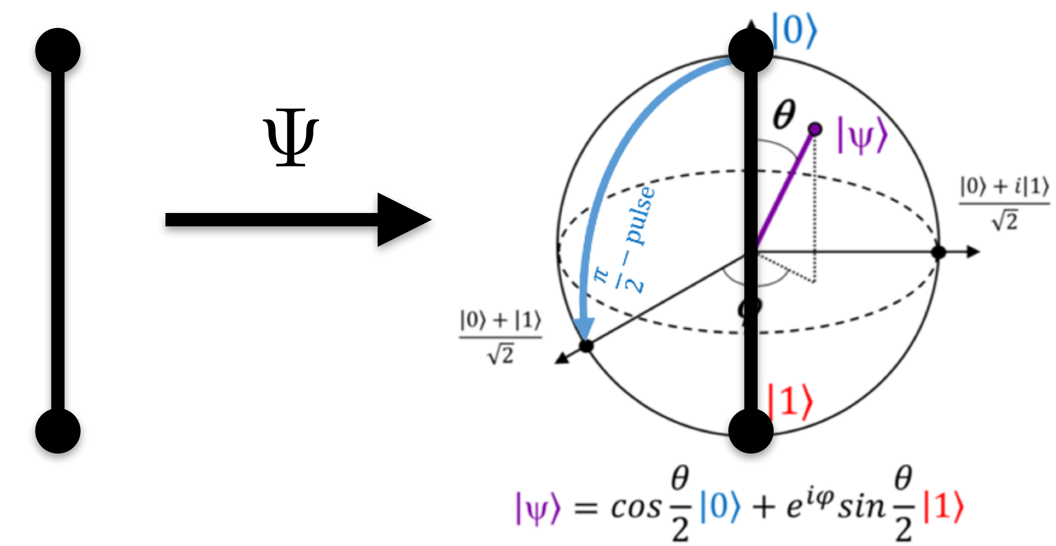
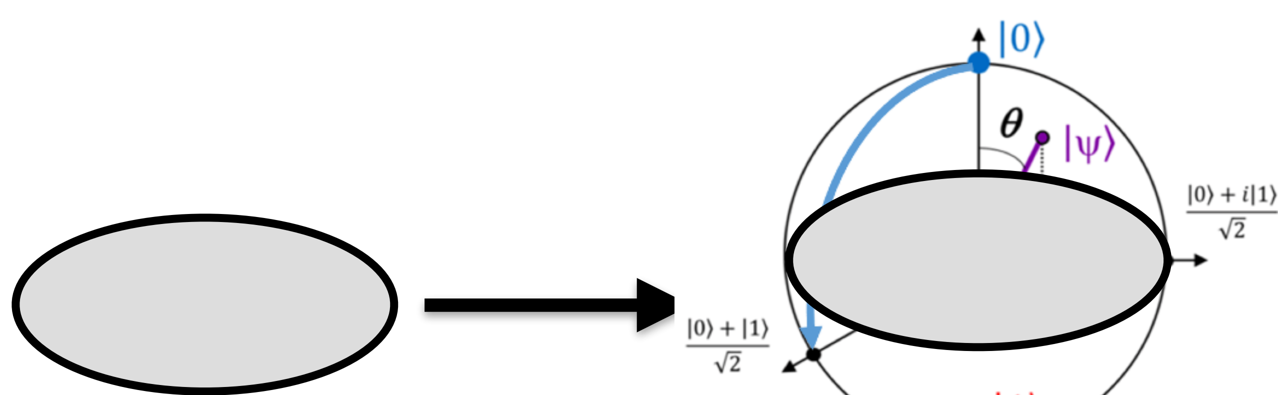


Similarly, **QT over the real numbers** can be embedded into QT.



3. Exact embeddings into quantum theory

Focus on the "**unrestricted** GPTs" where all vectors yielding valid probabilities on all states are effects: $\mathcal{A} = (A, \Omega_A, E_A)$

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- direct sums of those, including **CPT** and QT with **superselection rules**.

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Needs 2^d-dim. Hilbert space for simulation!

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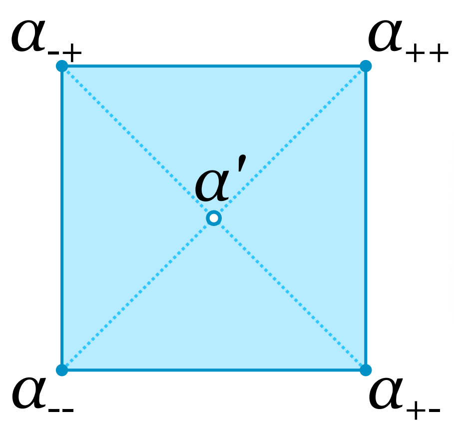
Non-exact embeddings into quantum theory

Example: the gbit



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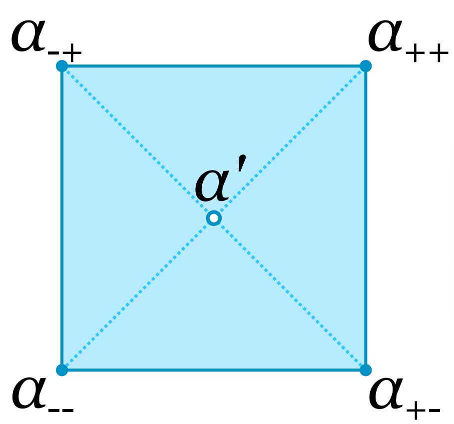

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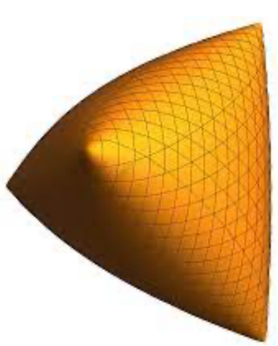
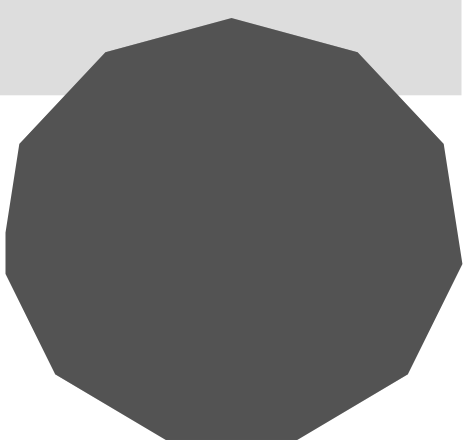
Also shown in our paper:
can **use known results on Bell inequalities** to certify nonembeddability.
Impractical and inefficient, but "proof of principle".





Multivalence / nonembeddability / contextuality is **hard** to obtain

We prove a couple of results:

- If A arises from B via **coarsegraining, noise, or generalized decoherence**, then A can be exactly embedded into B.
- Embeddability is **transitive**: If $A \mapsto B \mapsto C$ then $A \mapsto C$.

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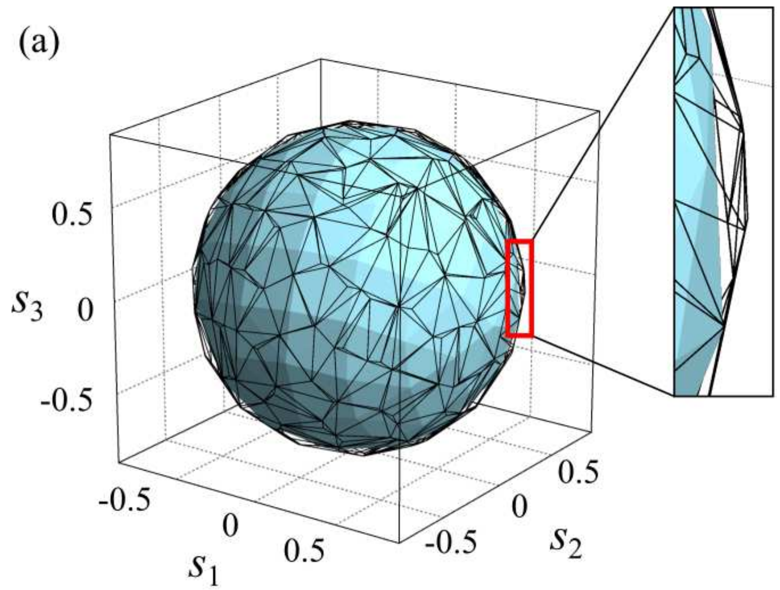
Example consequences:

- Any **future physical theory** that decoheres to QT must **also be contextual**.
- "Statistical-mechanics-like" fundamental theories correspond to **univalent** ("noncontextual") simulations.

Overview

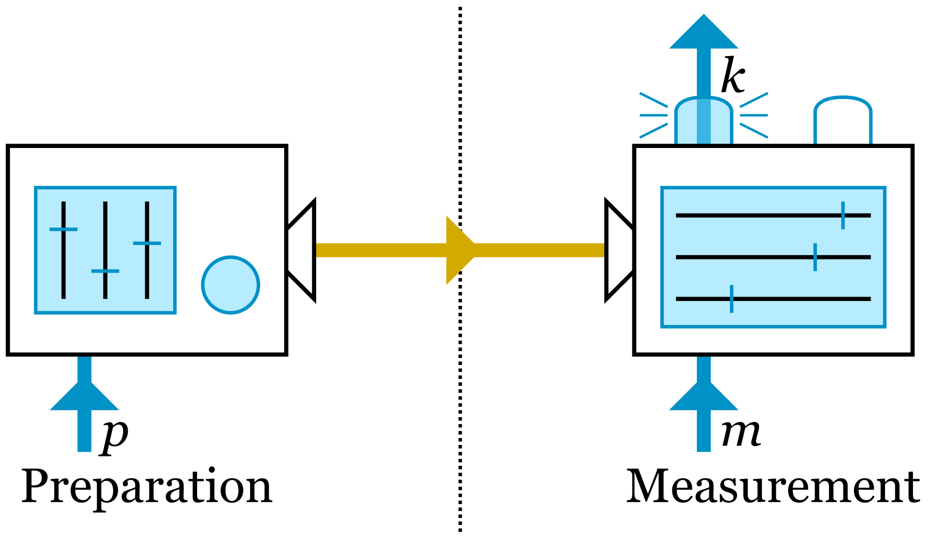
1. GPTs and theory-agnostic tomography



2. Contextuality, simulations, and embeddings

3. Exact embeddings into quantum theory

4. An experimental test of QT



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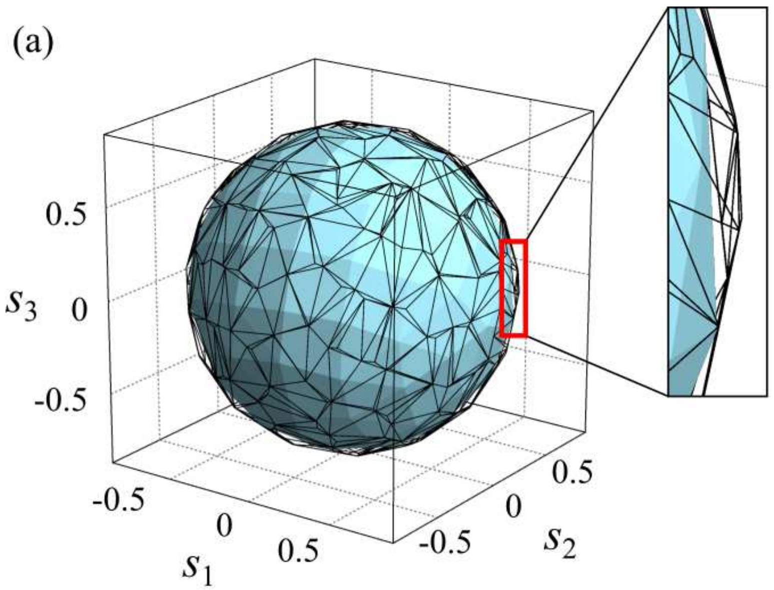

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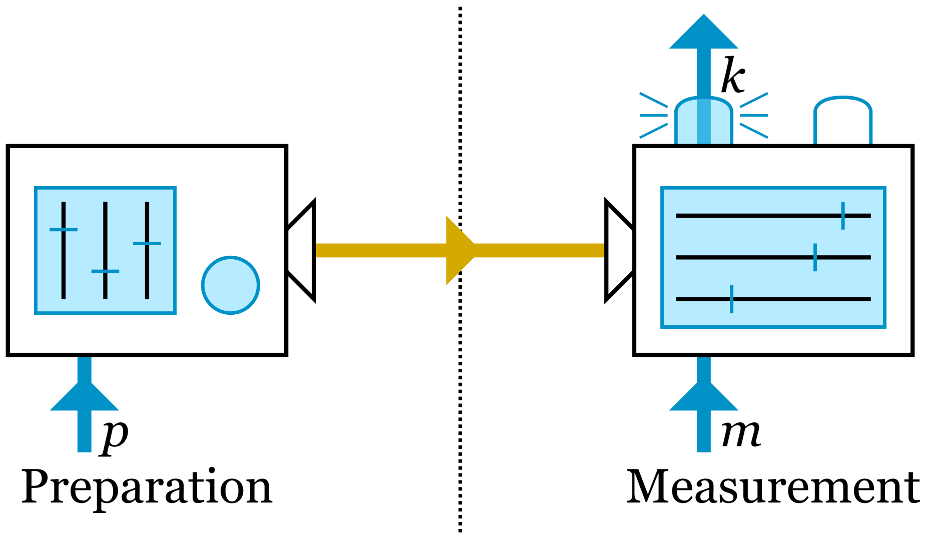

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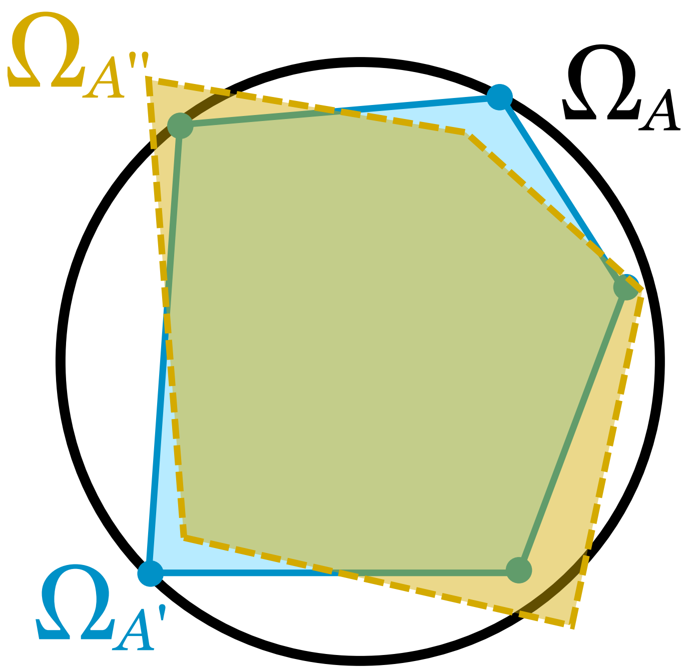
If, surprisingly, “no”, then this challenges QT.



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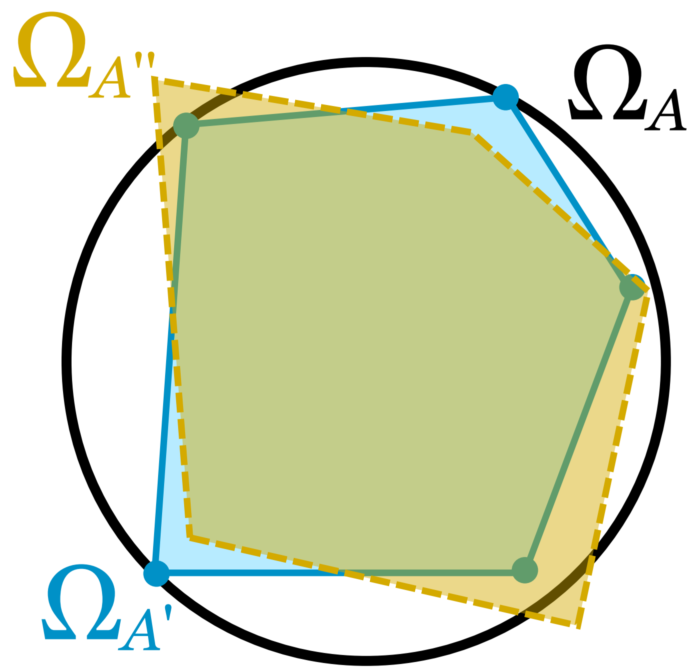

A quantum explanation of the result is then **similarly implausible as** a classical (contextual) explanation of the quantum state space.

What is a physical system?

Similarly as

V. Gitton and M. P. Woods, arXiv:2209.04469

we think that, for our purpose, we should not think of physical systems as "spatially localized objects in the world", but rather as being **defined by an experimental scenario**.

Example: collective preparations and measurements of a BEC.

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The implications of this for **contextuality** are interesting and to be debated (thanks to Rob Spekkens, David Schmid, and others so far).

Summary

- Have generalized Spekkens' notion of generalized noncontextuality:
  "Processes that are statistically indistinguishable in an effective theory should not require explanation by multiple distinguishable processes in a more fundamental theory."

- Result: Several mathematical insights, a new experimental test of QT, a conceptual discussion of the notion of a "physical system".
  Looking forward to discussions and comments.

arXiv:2112.09719 (update soon)

Thank you!