IQOQI – INSTITUTE FOR QUANTUM OPTICS AND QUANTUM INFORMATION VIENNA

Testing quantum theory by generalizing noncontextuality

Markus P. Müller[1,2,3] and Andrew J. P. Garner[1,2]

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[3]Perimeter Institute for Theoretical Physics (PI), Waterloo, Canada

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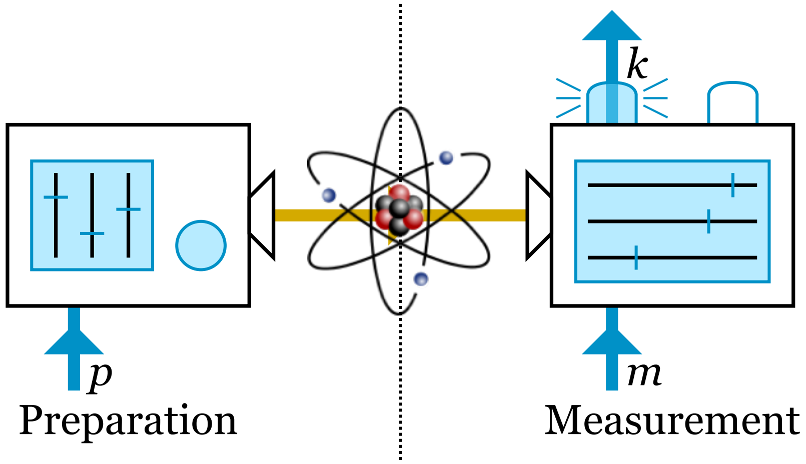
Two motivations

Suppose we **prepare** and **measure** a physical system in all ways accessible to us.



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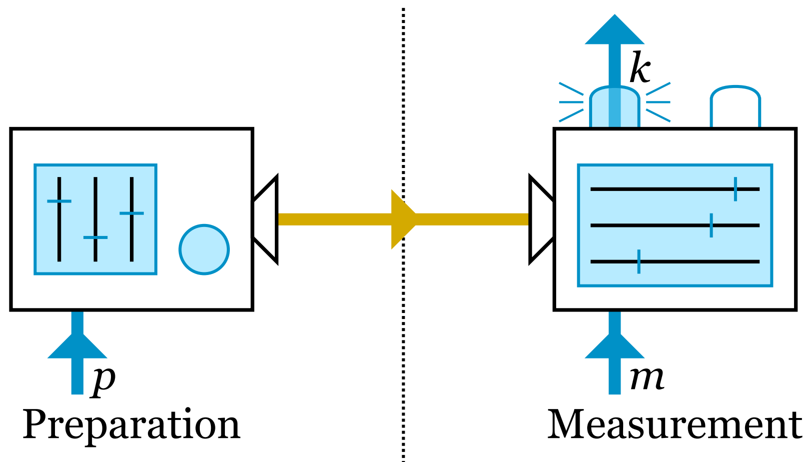

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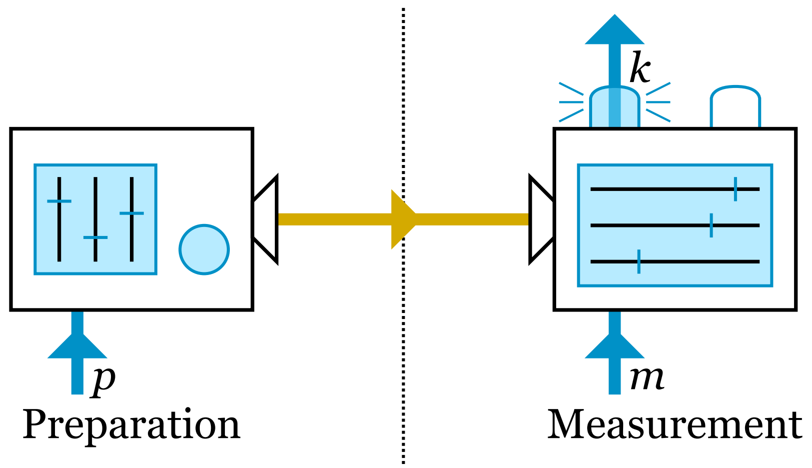

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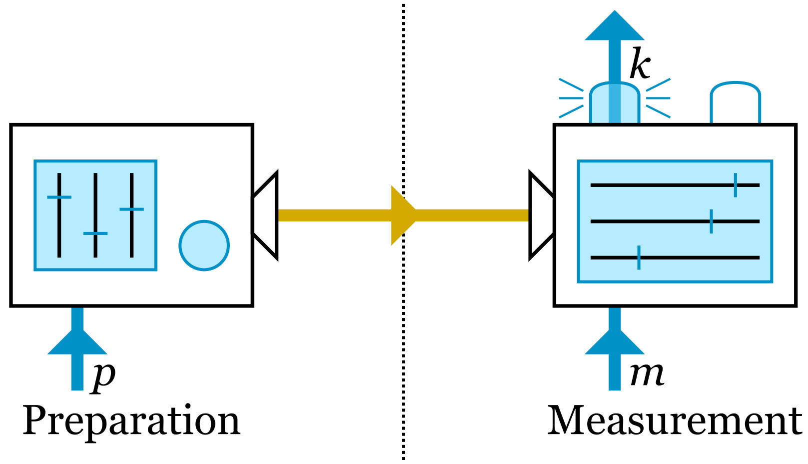

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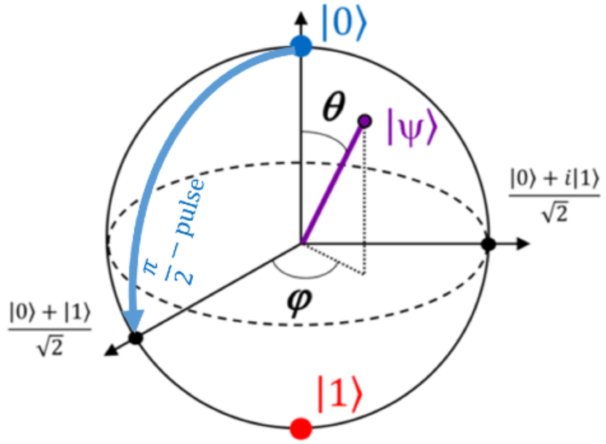

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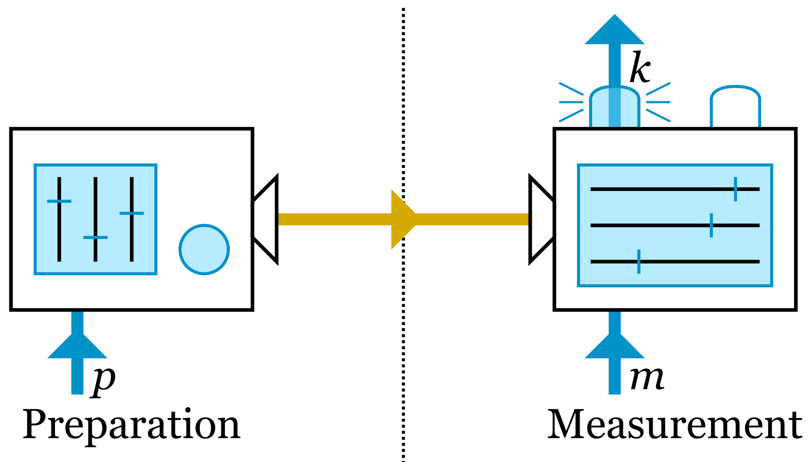

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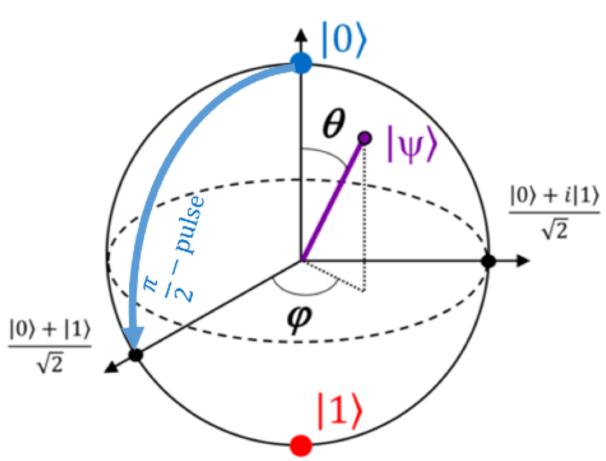

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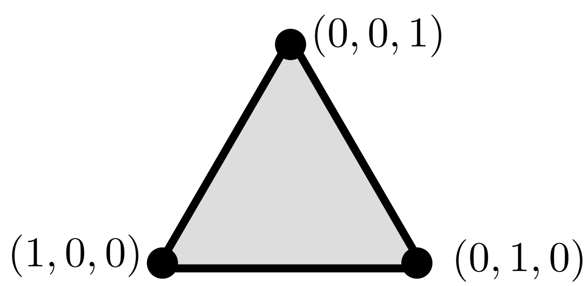

$$\Omega = \{p = (p_1, \ldots, p_n) \mid p_i \geq 0, \sum p_i = 1\}$$

- classical probability theory
- noisy qubits etc.
- QT *w/* superselection rules
- … **?**

Unambiguously testing / falsifying QT is **really hard**!

Science AAAS

Ruling Out Multi-Order Interference in Quantum Mechanics
Urbasi Sinha *et al.*
Science **329**, 418 (2010);
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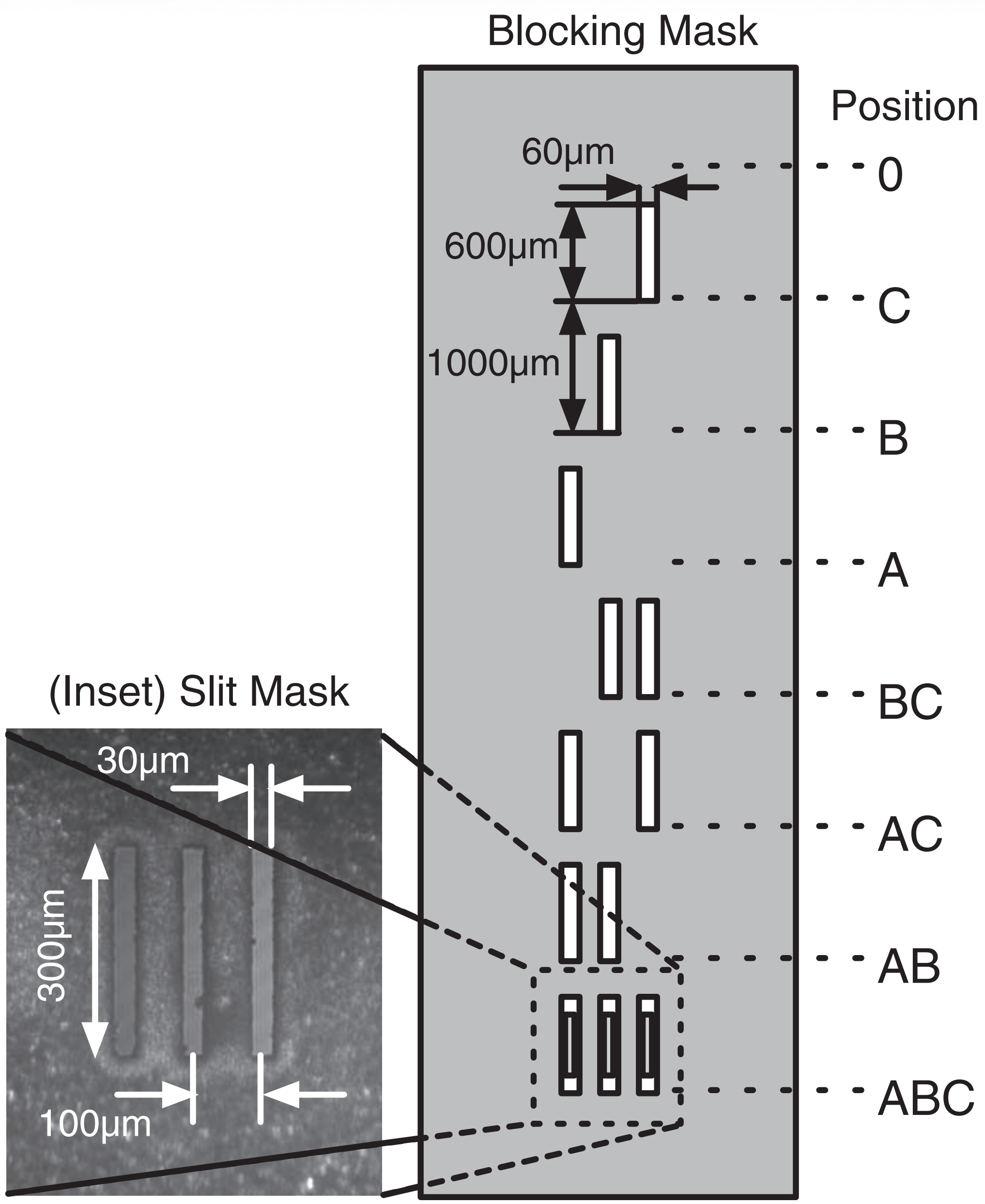

$$
\begin{aligned}
I_{ABC} := P_{ABC} - (P_A + P_B + P_C + I_{AB} + \\
I_{BC} + I_{AC}) \\
= P_{ABC} - P_{AB} - P_{BC} - P_{AC} + P_A + \\
P_B + P_C \qquad (5)
\end{aligned}
$$

In QT, only **pairs of paths** interfere (Sorkin 1994)

$$\Rightarrow I_{ABC} = 0.$$

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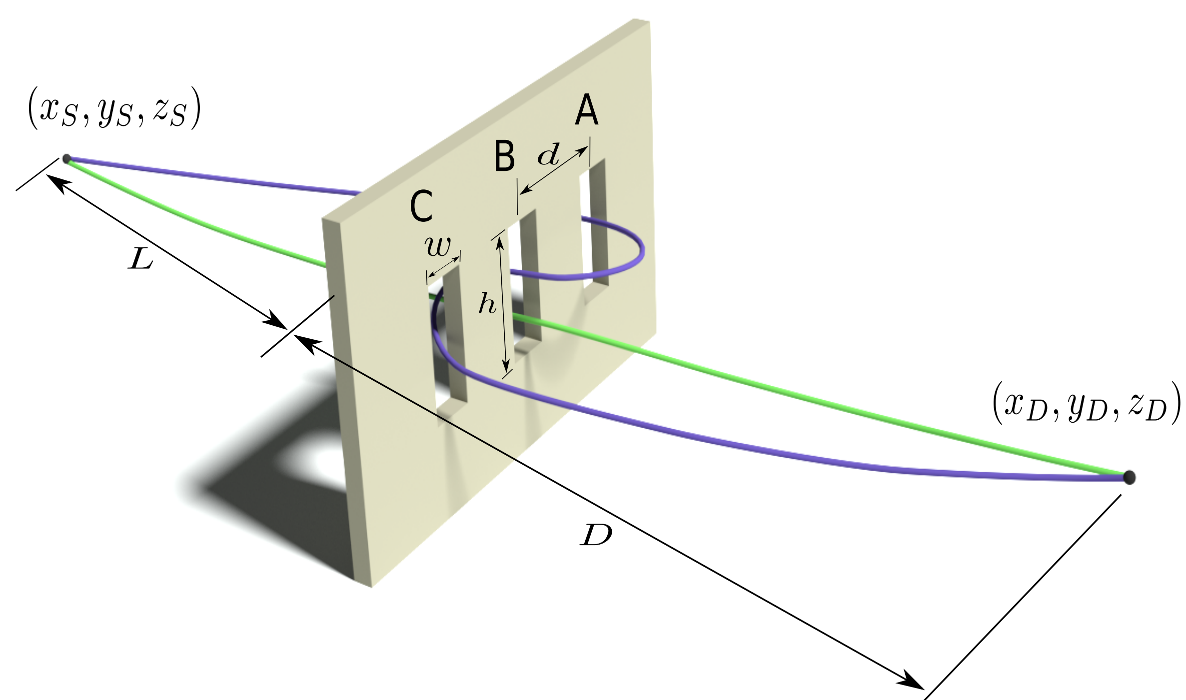
Non-classical paths in interference experiments

Rahul Sawant [1], Joseph Samuel [1], Aninda Sinha [2], Supurna Sinha [1] and Urbasi Sinha [1,3]
[1] *Raman Research Institute, Sadashivanagar, Bangalore, India.*
[2] *Centre for High Energy Physics, Indian Institute of Science, Bangalore, India.*
[3] *Institute for Quantum Computing, 200 University Avenue West, Waterloo, Ontario, Canada.*
* *To whom correspondence should be addressed; E-mail: usinha@rri.res.in.*



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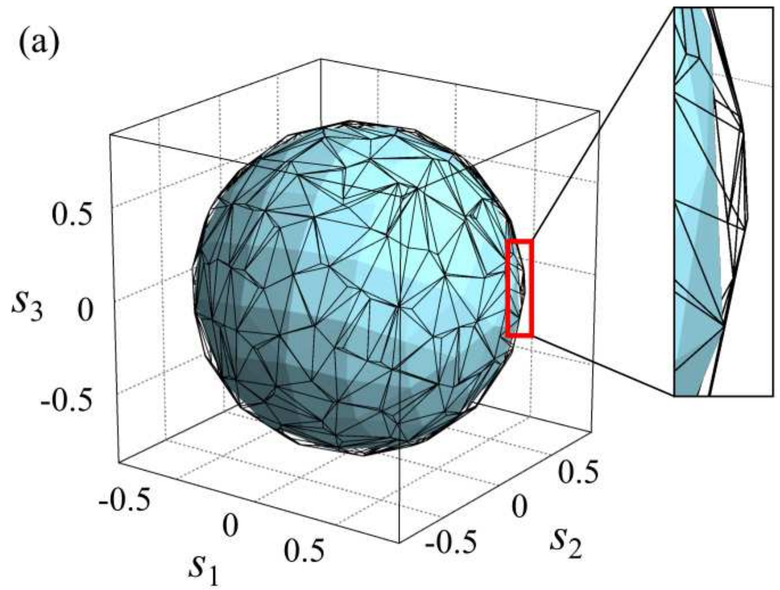

2. Contextuality, simulations, and embeddings

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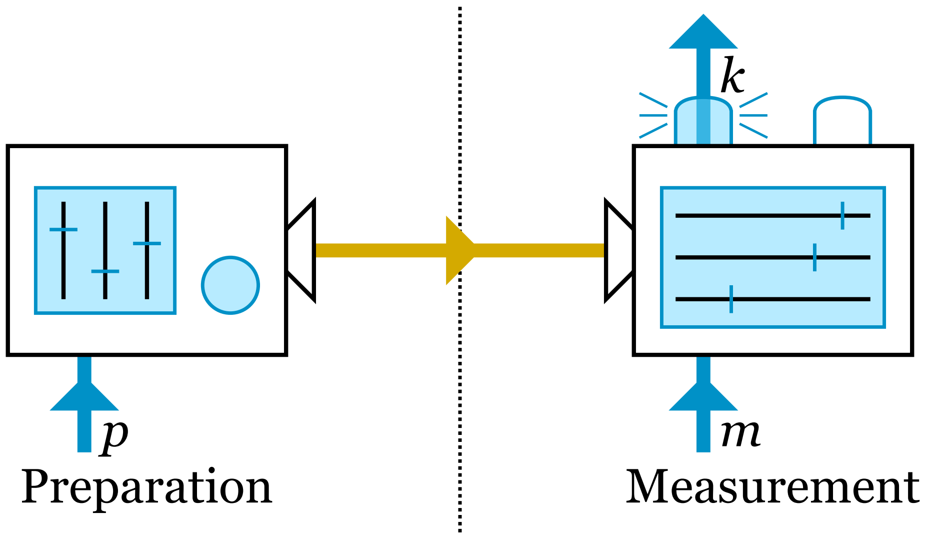

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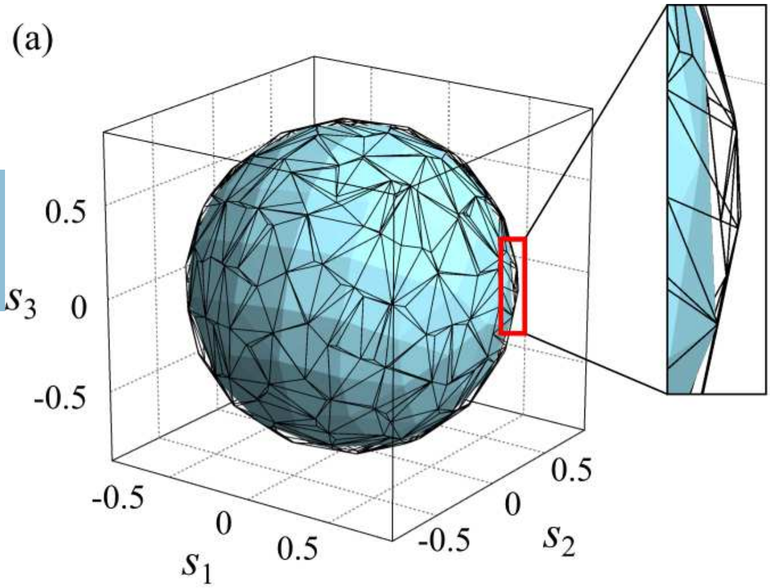

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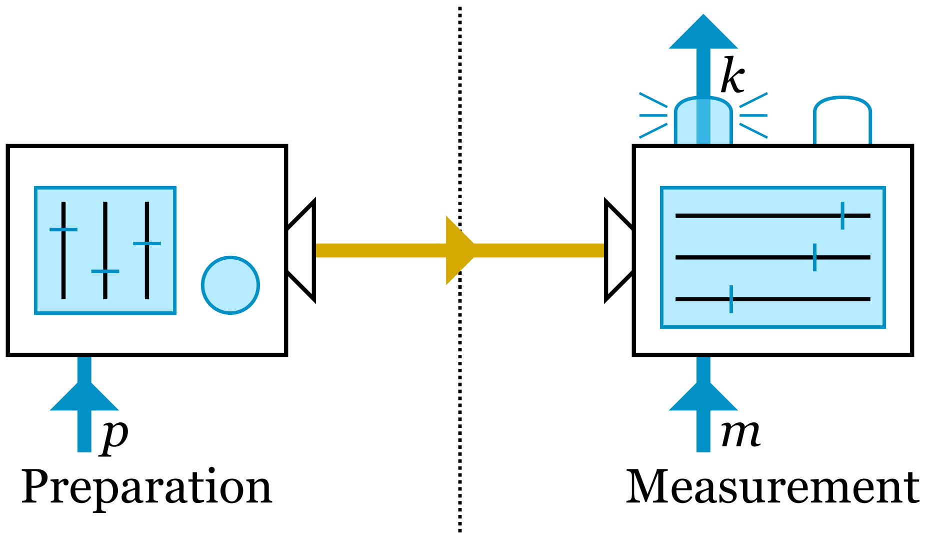

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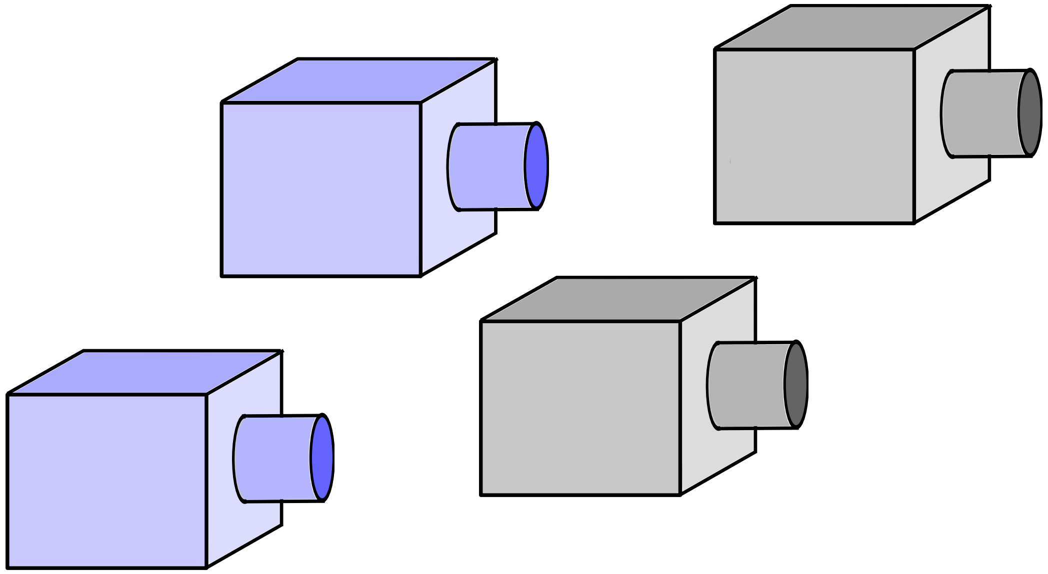

(all accessible preparation procedures)

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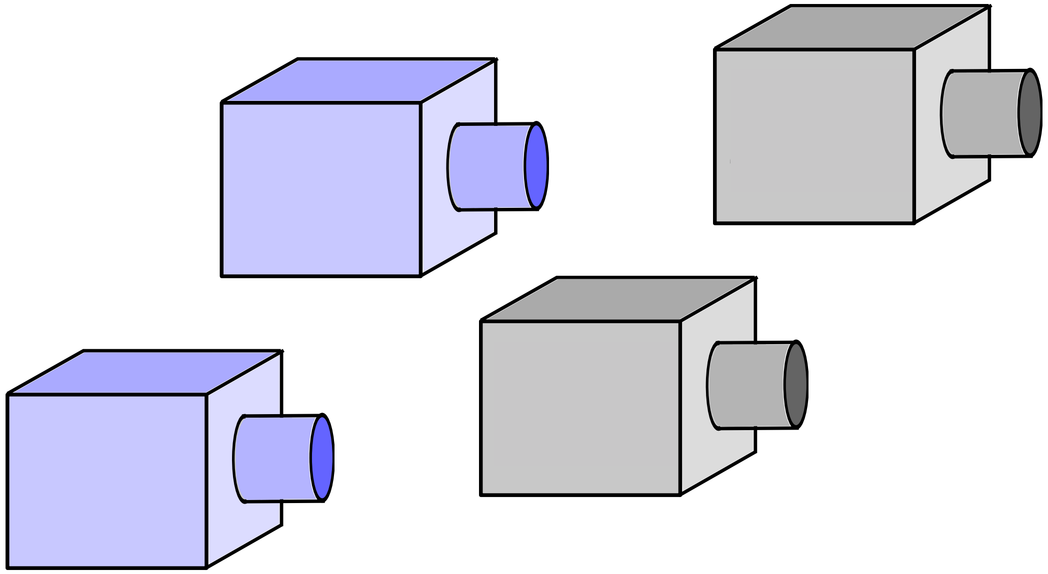

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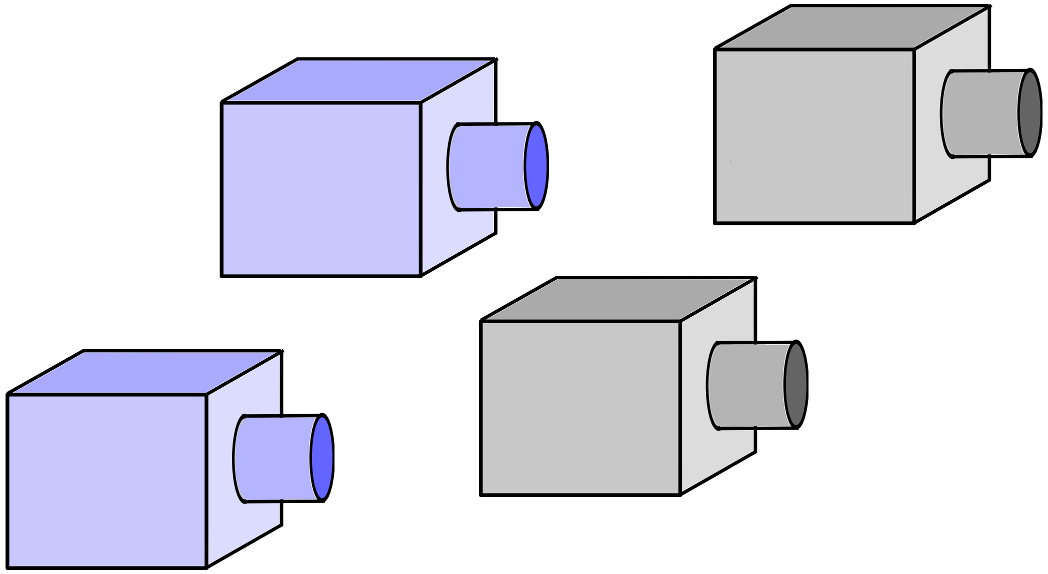

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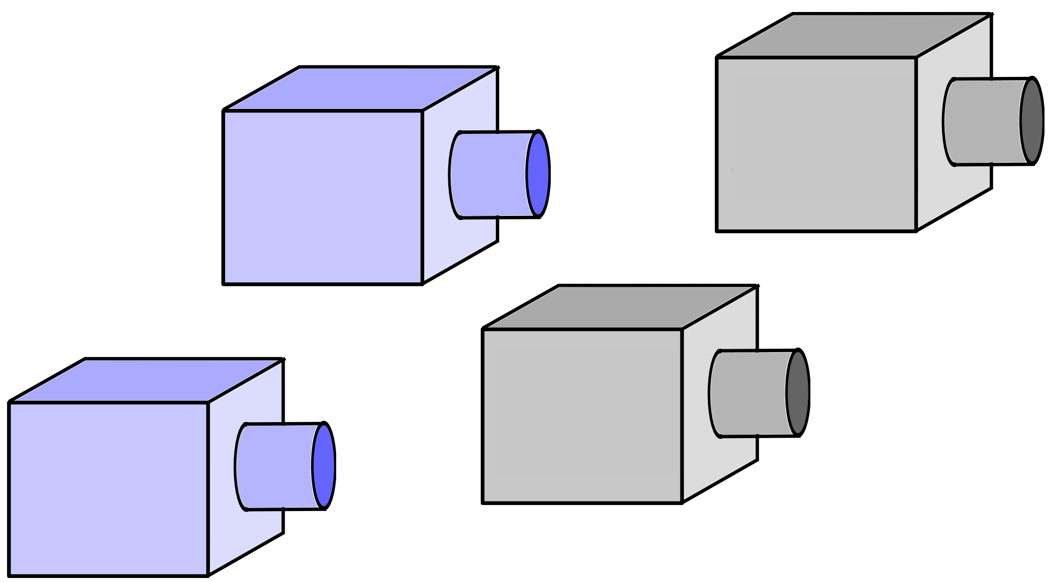

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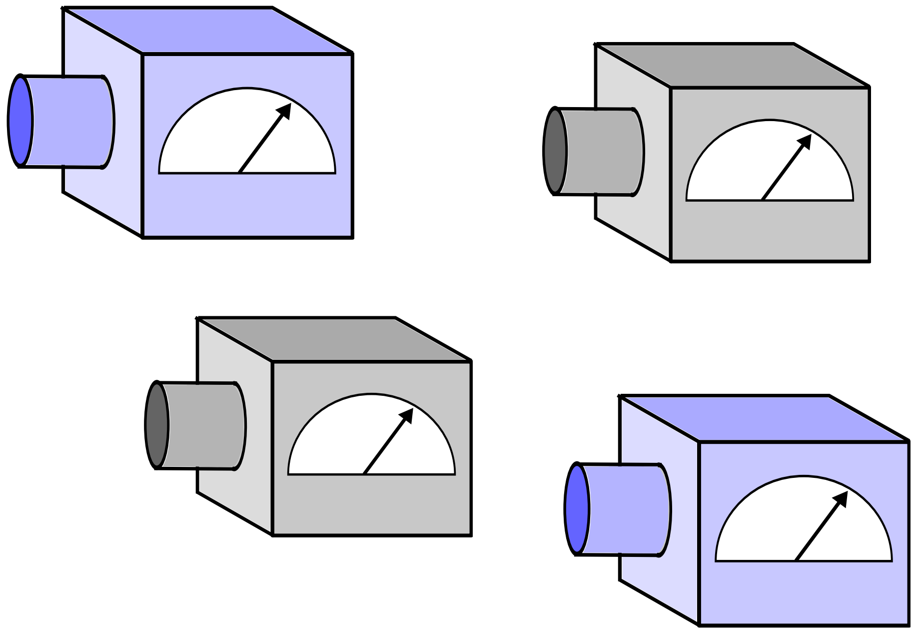

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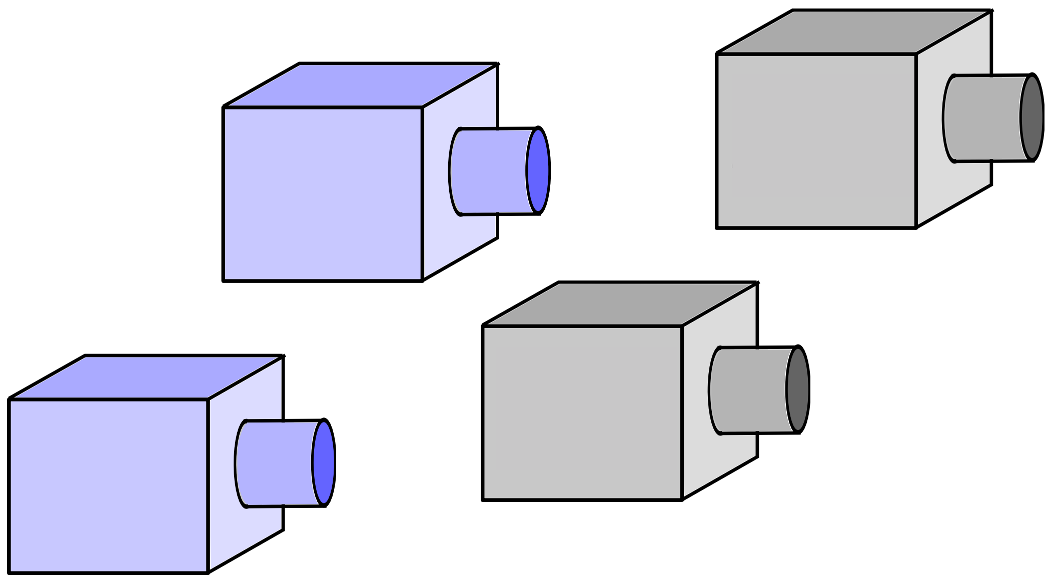

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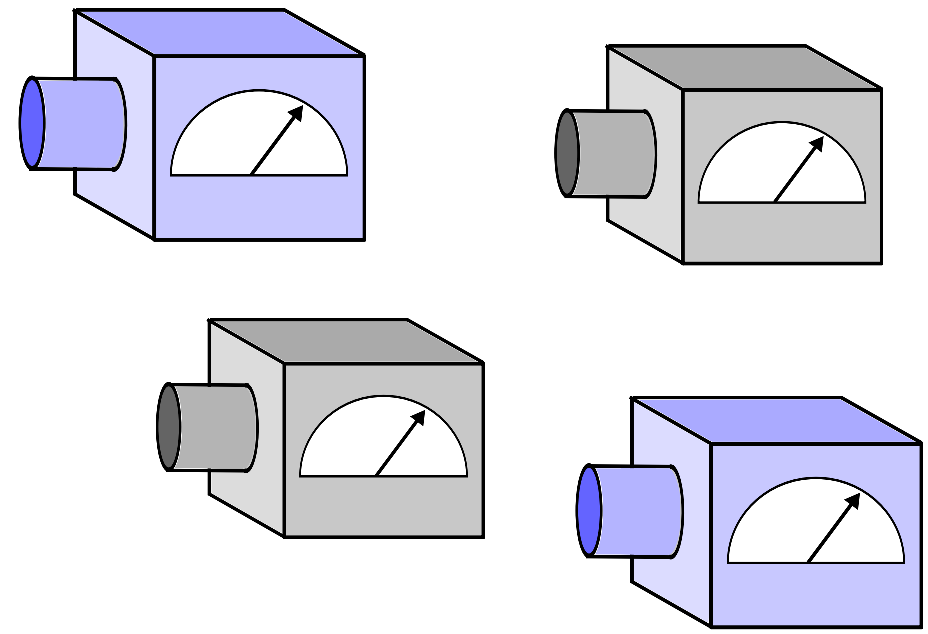

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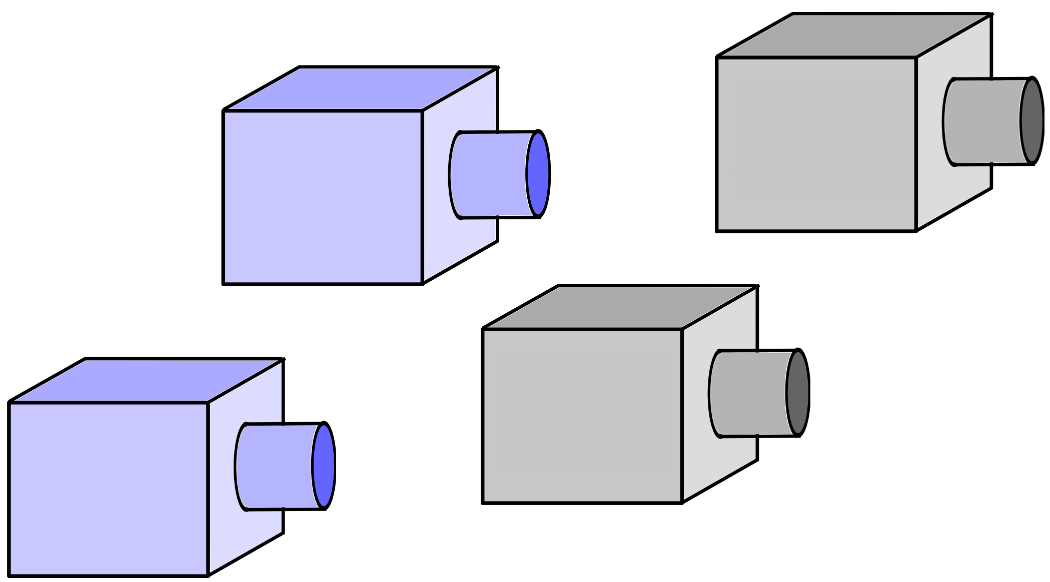

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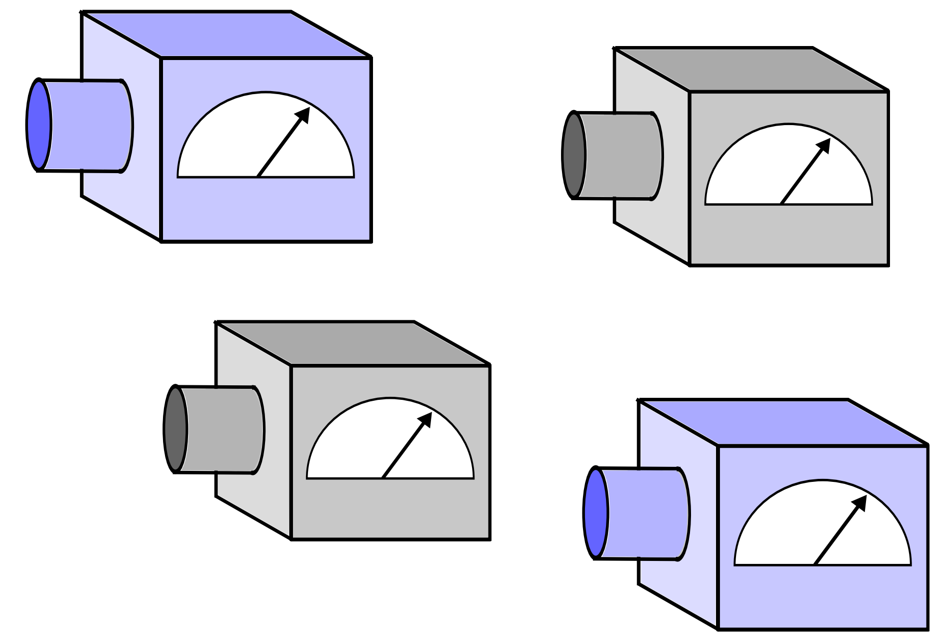

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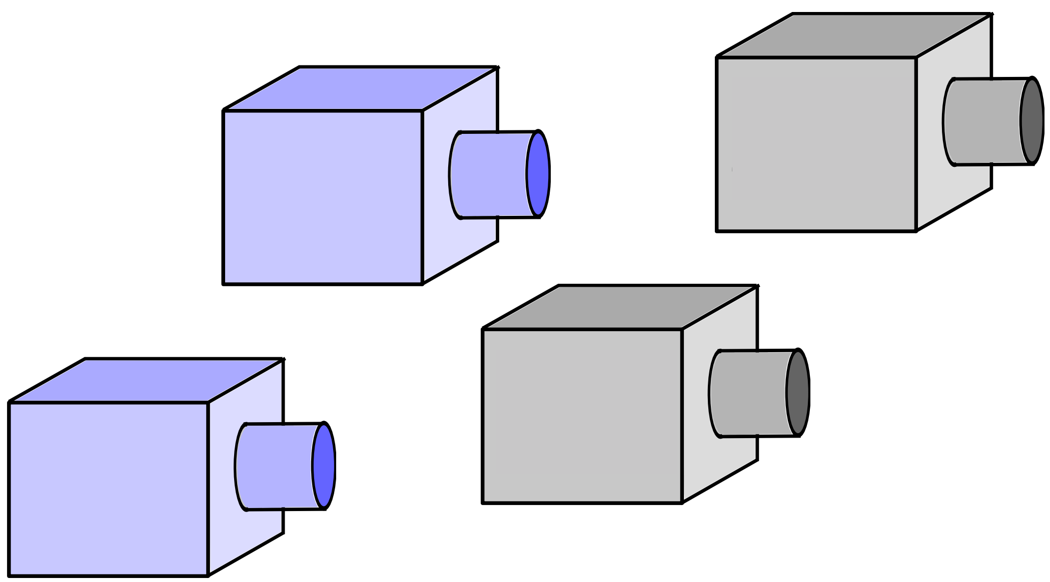

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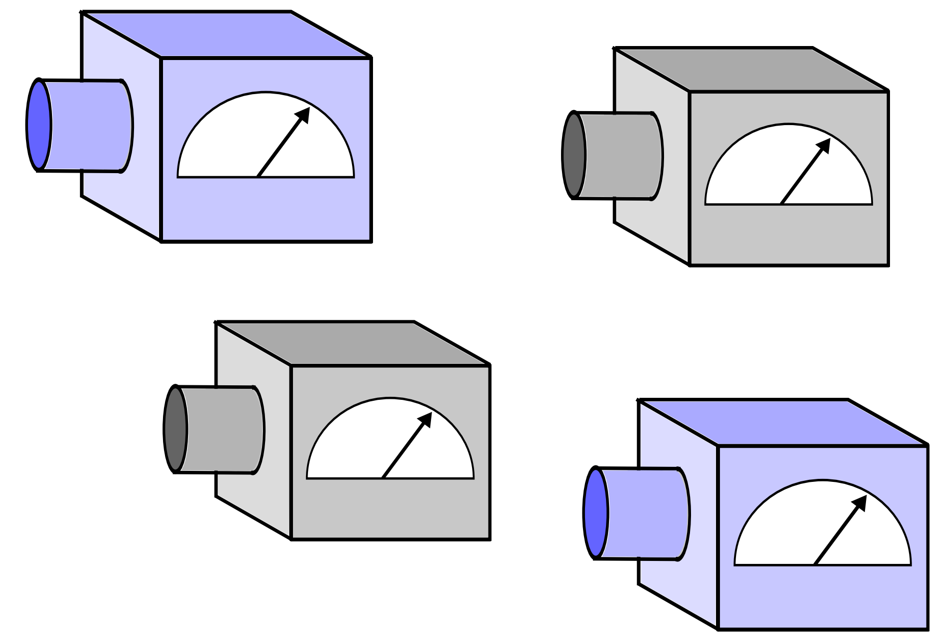

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General probabilistic theories

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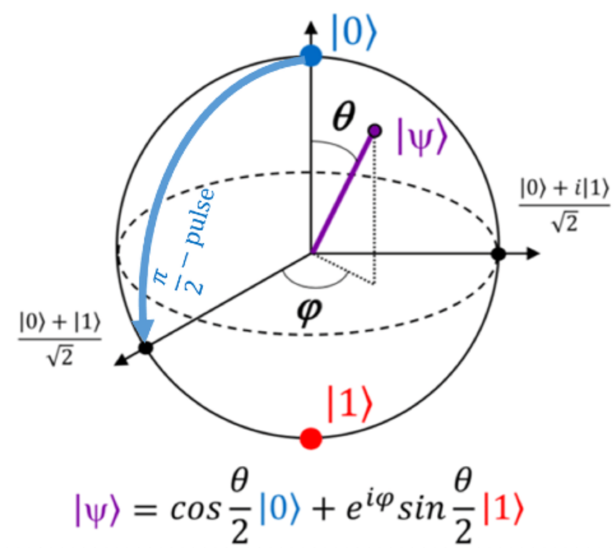

Quantum theory (QT): $\mathcal{Q}_n$

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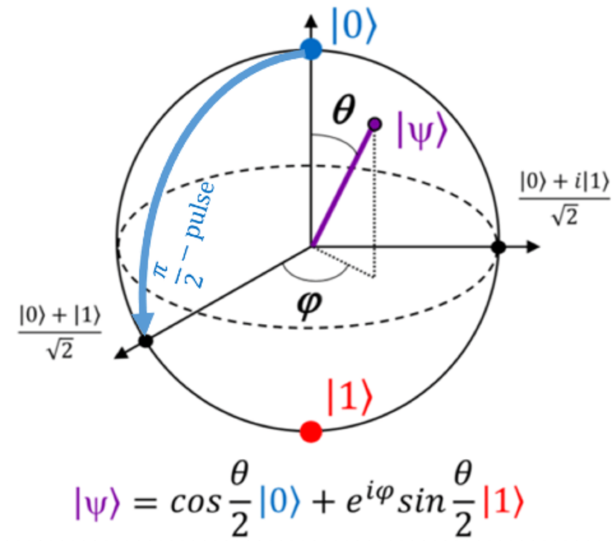

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Classical probability theory (QT): $\mathcal{C}_n$

$$A = \mathbb{R}^n \simeq A^*$$

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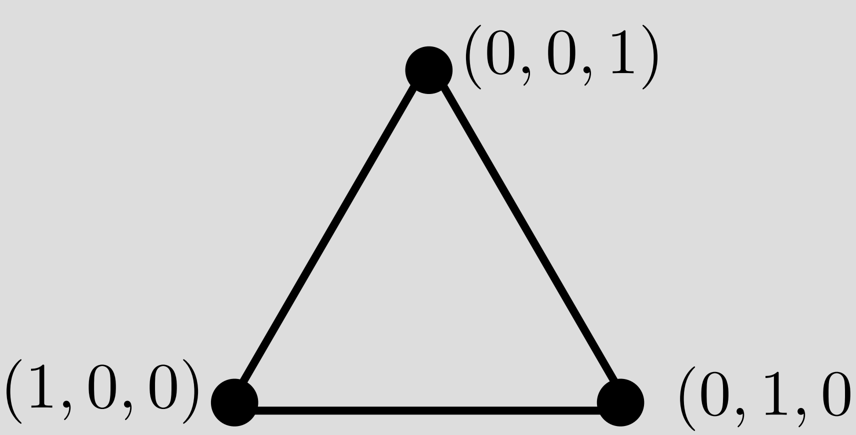

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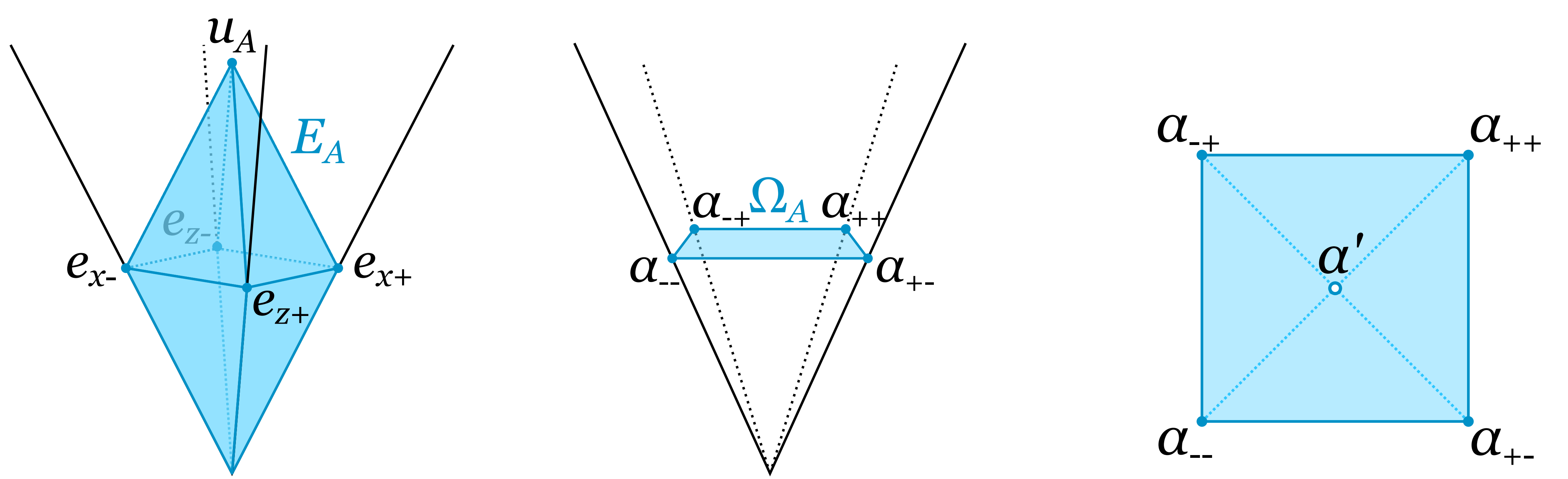

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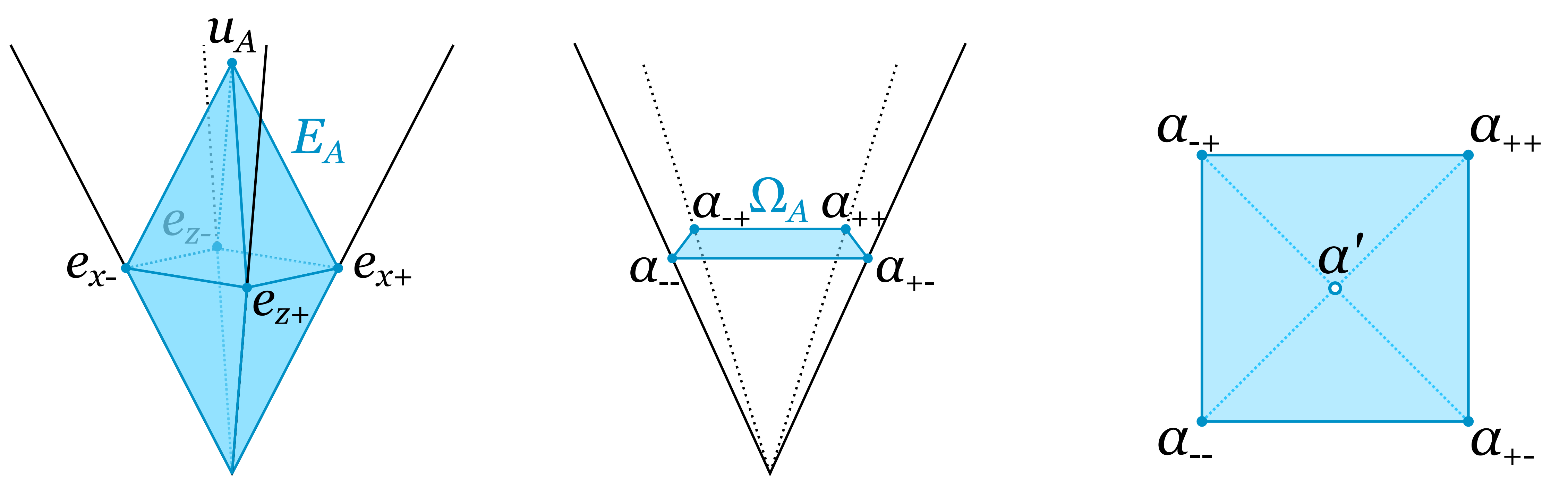

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The four pure states $\alpha_{\pm,\pm}$ are **pairwise** perfectly distinguishable, but **not jointly** $\Longrightarrow$ this cannot be a classical or quantum system.

Theory-agnostic tomography

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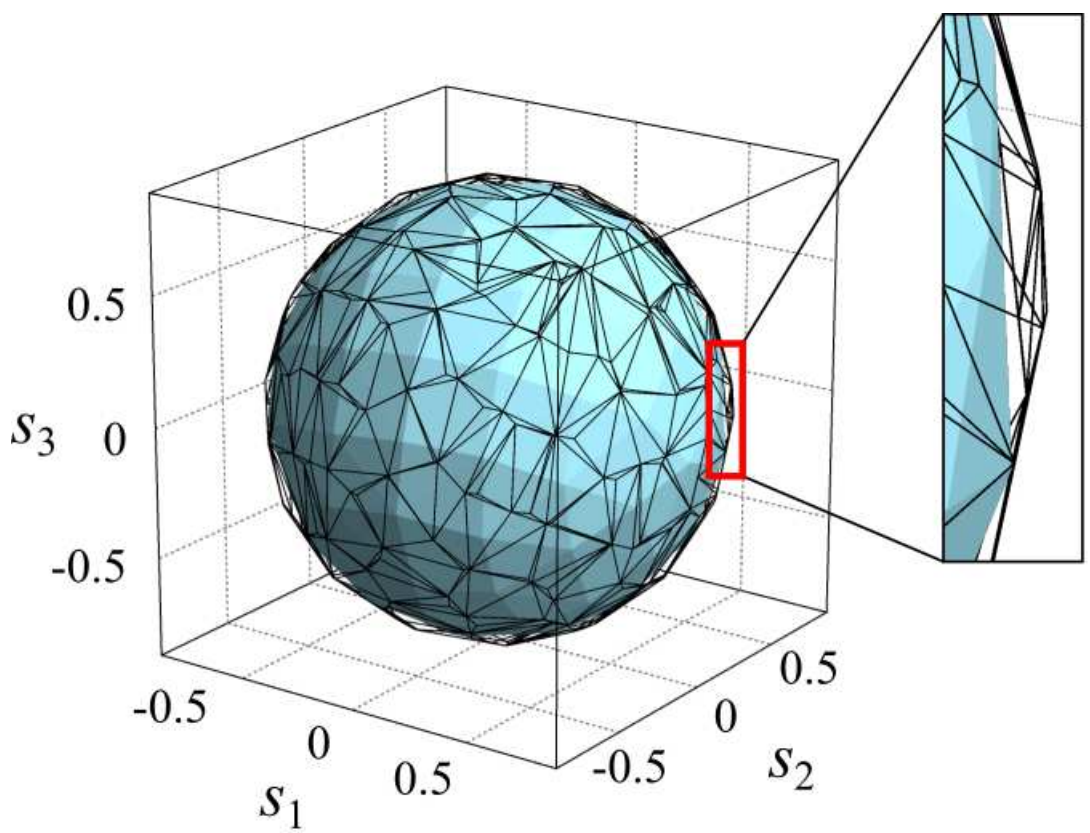

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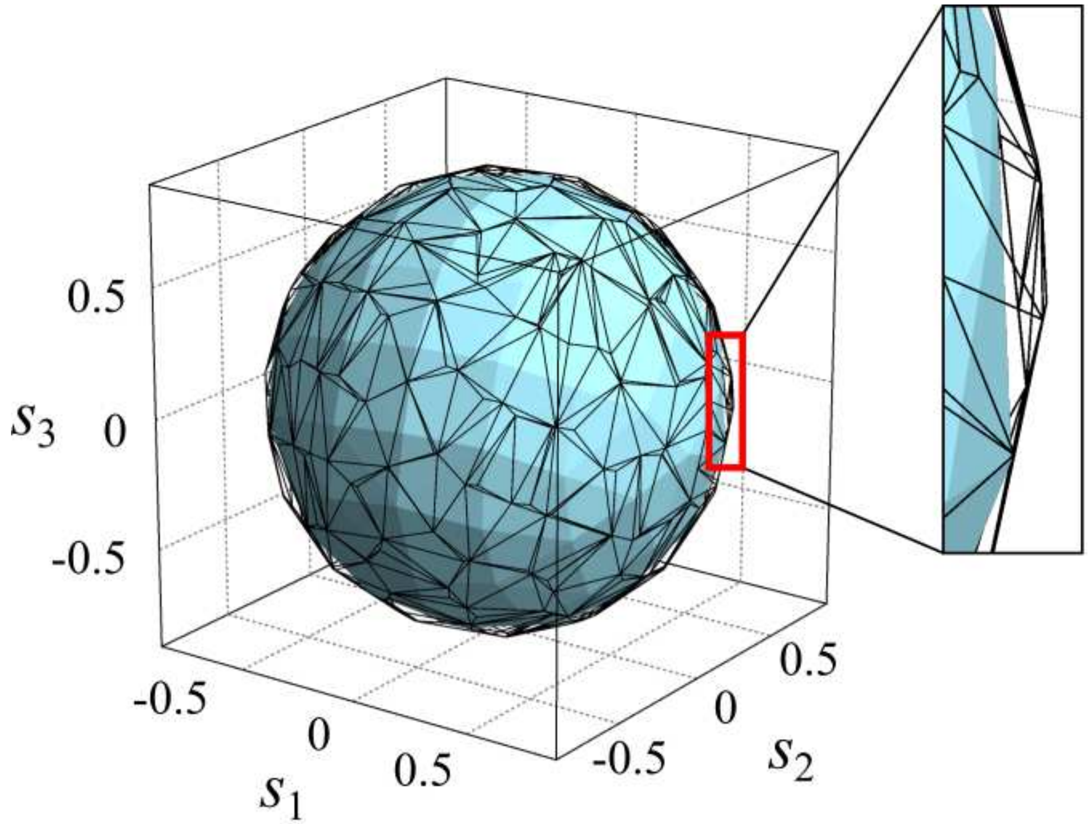

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Tomographic completeness loophole: can never be sure that we probed the system *completely*.

What if we just see a (low-dimensional) **“shadow”**?

Let’s **drop** the **tomographic completeness** assumption.

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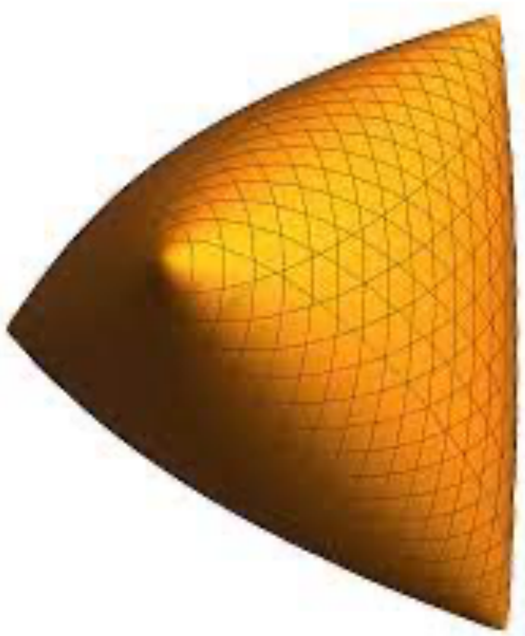

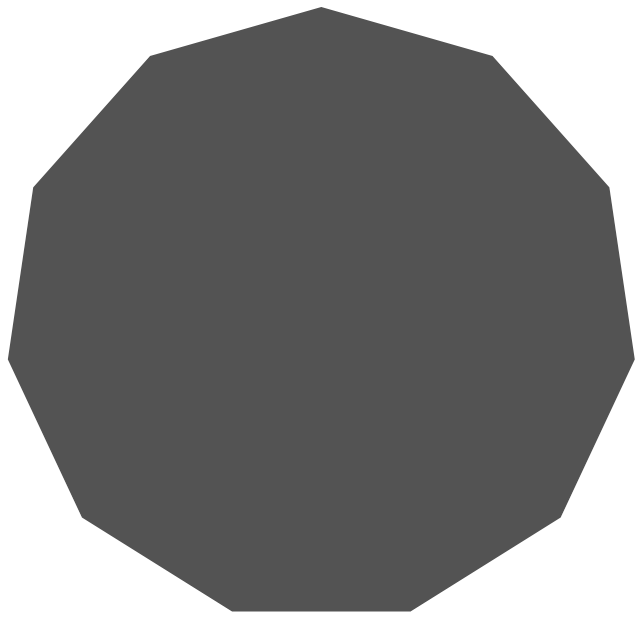

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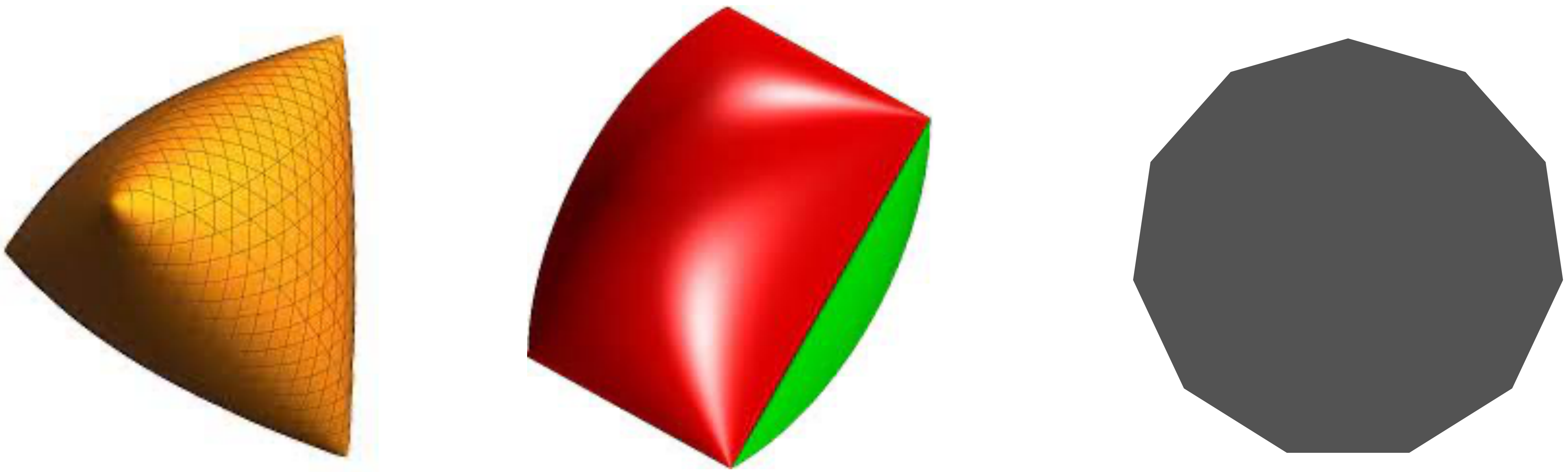

Is **fundamental QT** a plausible explanation of a given **effective GPT**?

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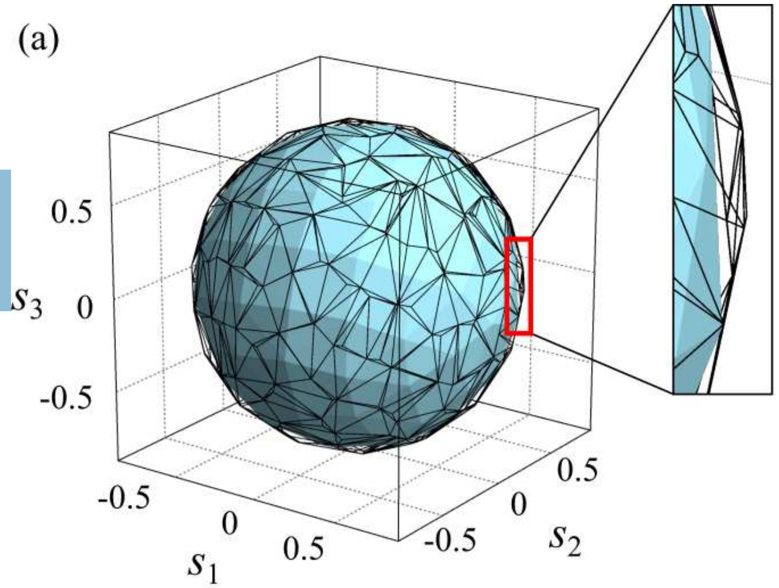

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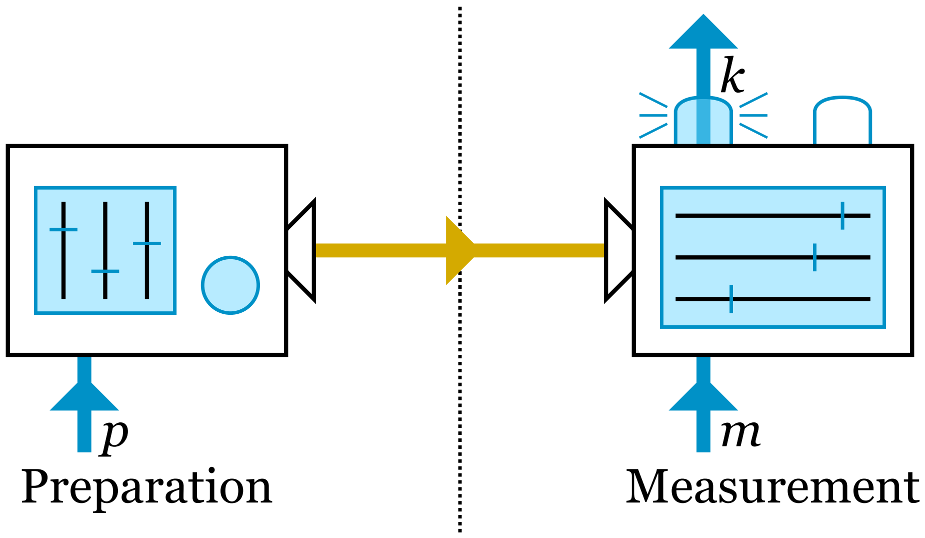

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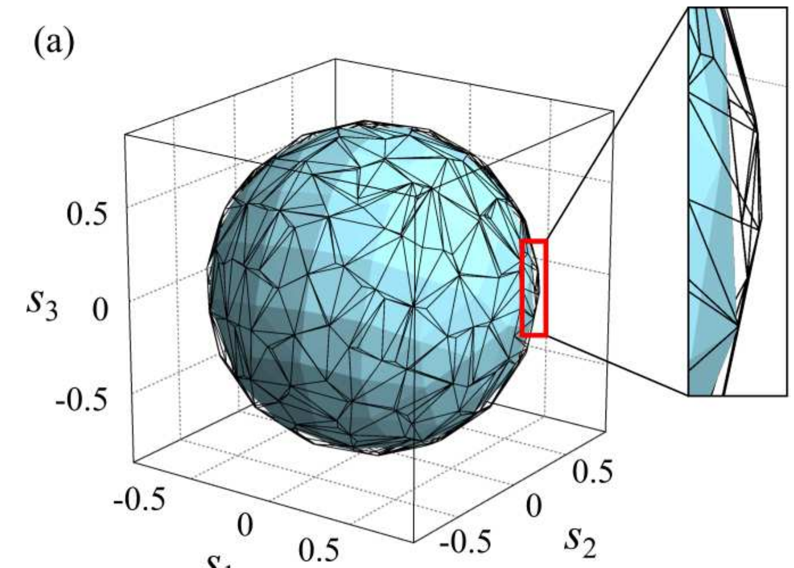

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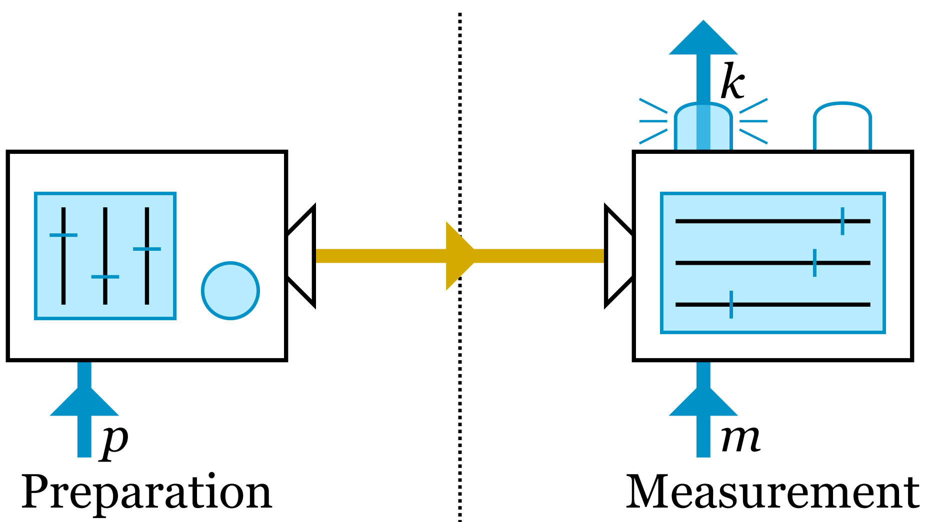

Spekkens' notion of noncontextuality: quick recap

Contextuality for preparations, transformations and unsharp measurements

R. W. Spekkens*
Perimeter Institute for Theoretical Physics, 35 King St. North, Waterloo, Ontario N2J 2W9, Canada
(Dated: Feb. 25, 2005)

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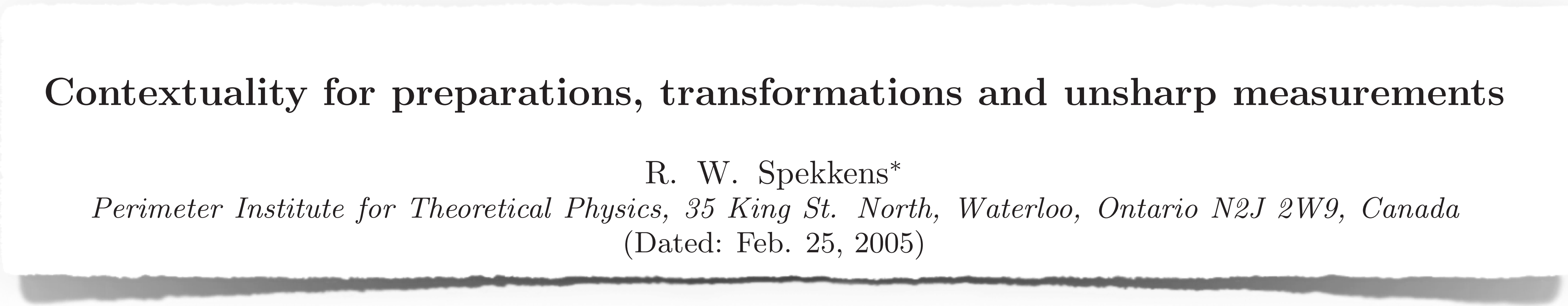
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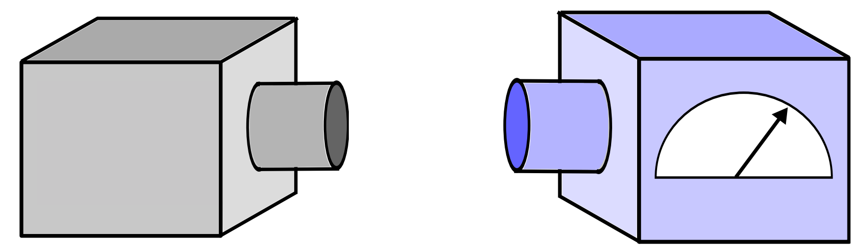

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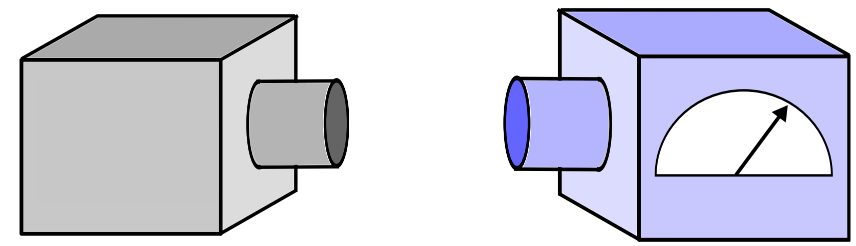

Ontological model of a system (e.g. of a qubit):

A set of classical variables Λ.

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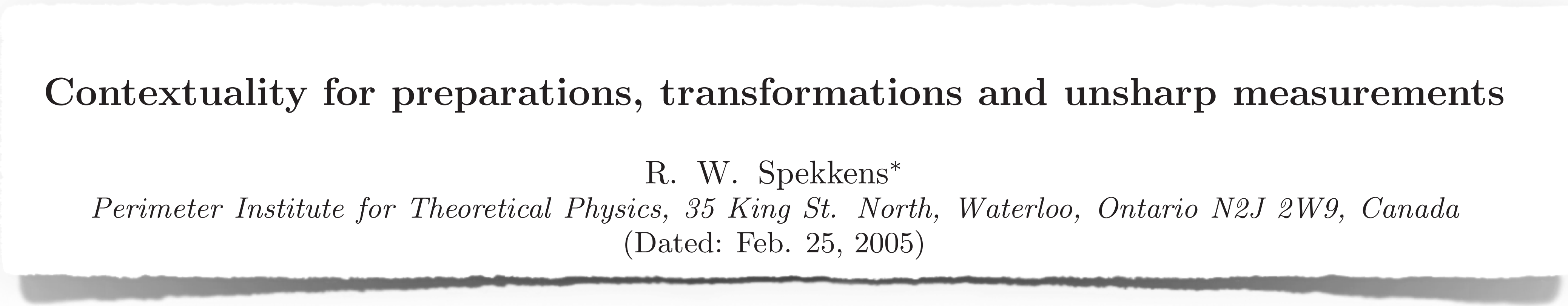

Contextuality for preparations, transformations and unsharp measurements

R. W. Spekkens*
Perimeter Institute for Theoretical Physics, 35 King St. North, Waterloo, Ontario N2J 2W9, Canada
(Dated: Feb. 25, 2005)

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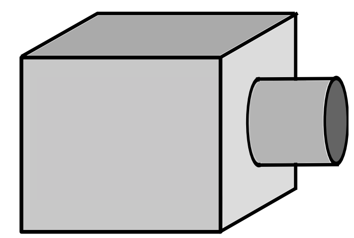

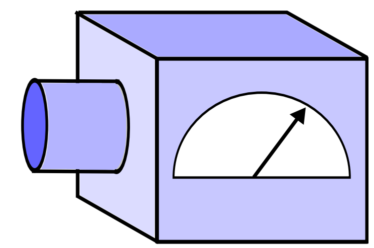

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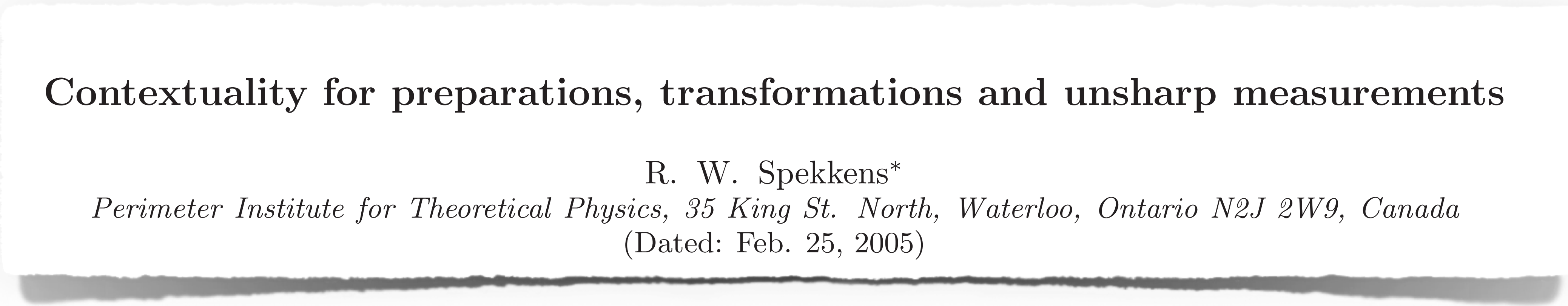
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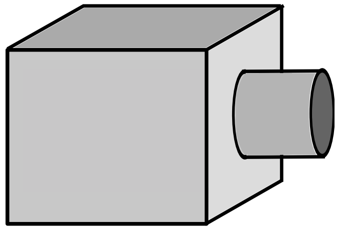

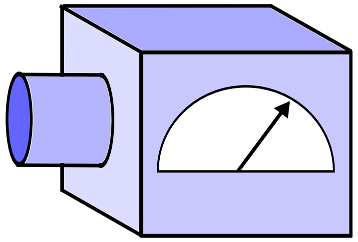

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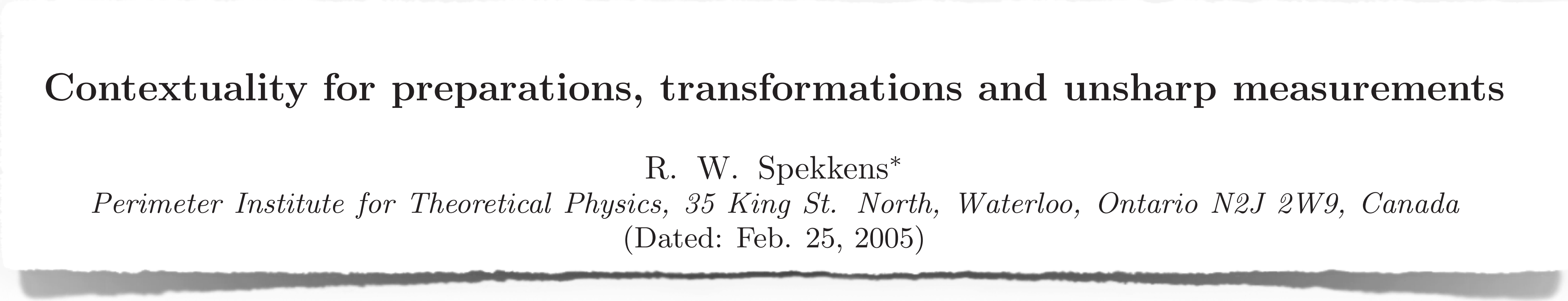

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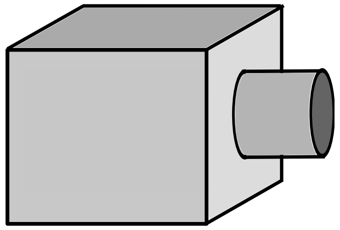

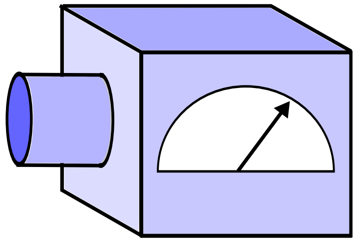

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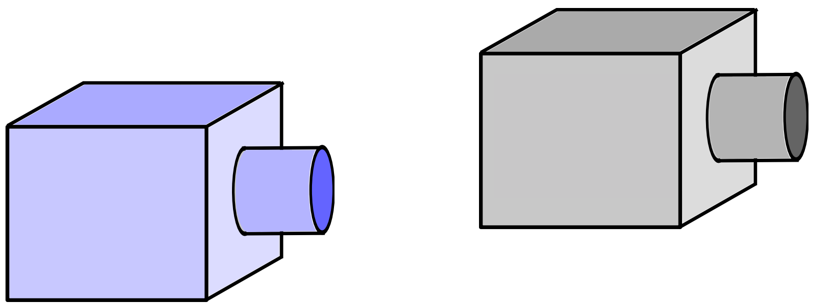

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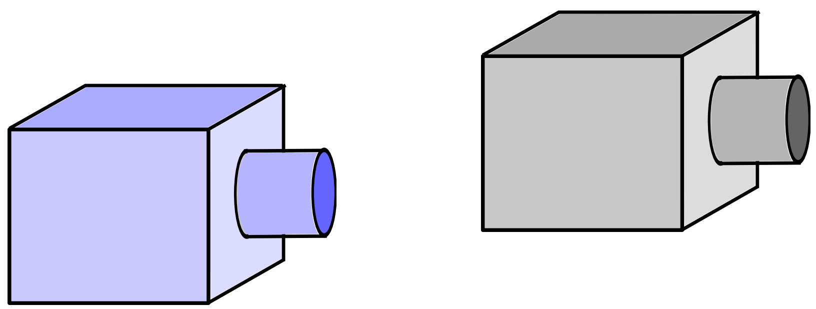

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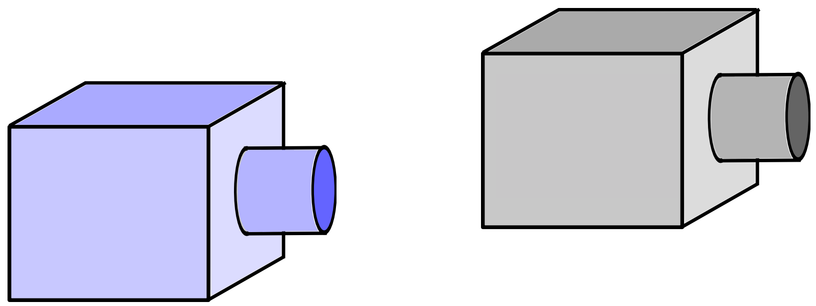

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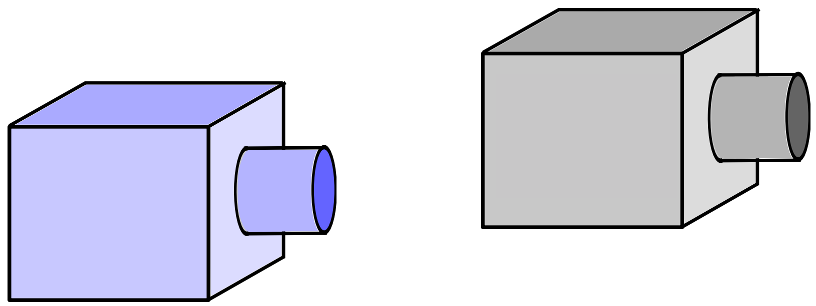

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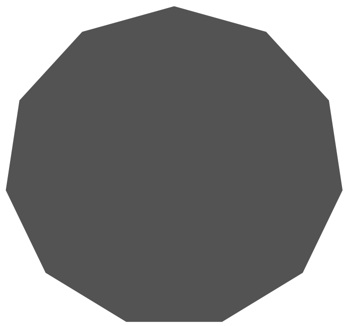
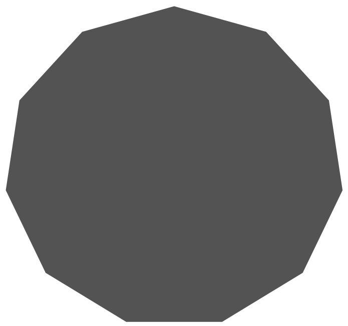
<u>Intuition</u>: Contextual models are implausible because they are **fine-tuned**: operationally, $P \sim P'$, but ontologically, $\mu_P \neq \mu_{P'}$.

An instance of Leibniz' principle of the "identity of the indiscernibles".

Simulations and embeddings



Effective GPT $\mathcal{A} = (A, \Omega_A, E_A)$ found in the lab



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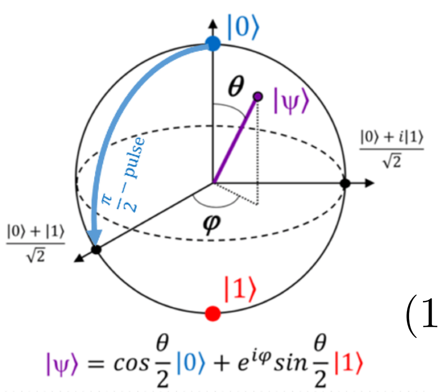

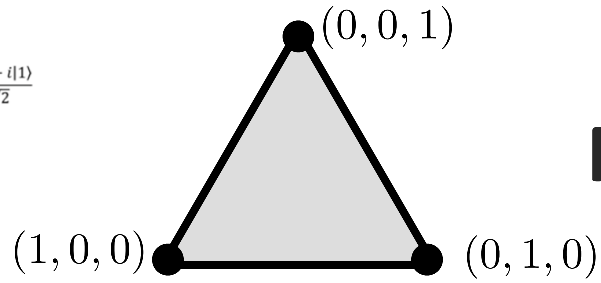

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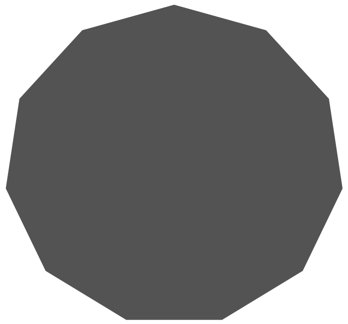

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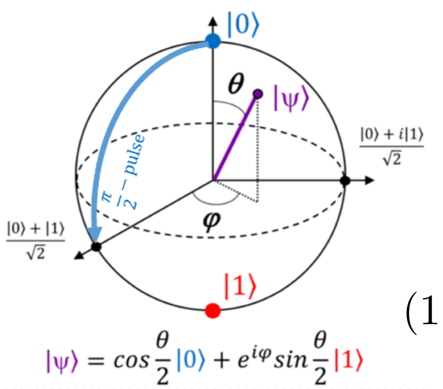

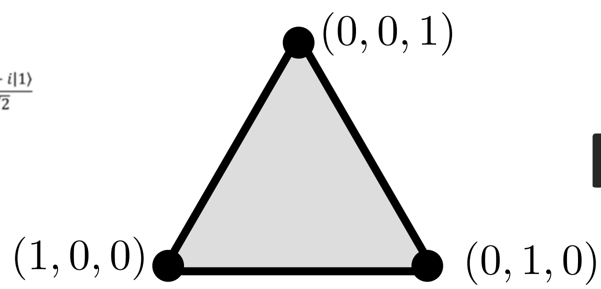

Fundamental GPT $\mathcal{B} = (B, \Omega_B, E_B)$

Effectively preparing state ω_A means **fundamentally** preparing some ω_B, but ω_B may depend on the preparation *procedure*, i.e. the *context*. Collect all those states into a set $\Omega_B(\omega_A) := \{\omega_B\}$.

Definition. An ε-**simulation** of effective GPT $\mathcal{A}$ by fundamental GPT $\mathcal{B}$:

Effective state ω_A $\longmapsto$ set of simulating states $\Omega_B(\omega_A)$,

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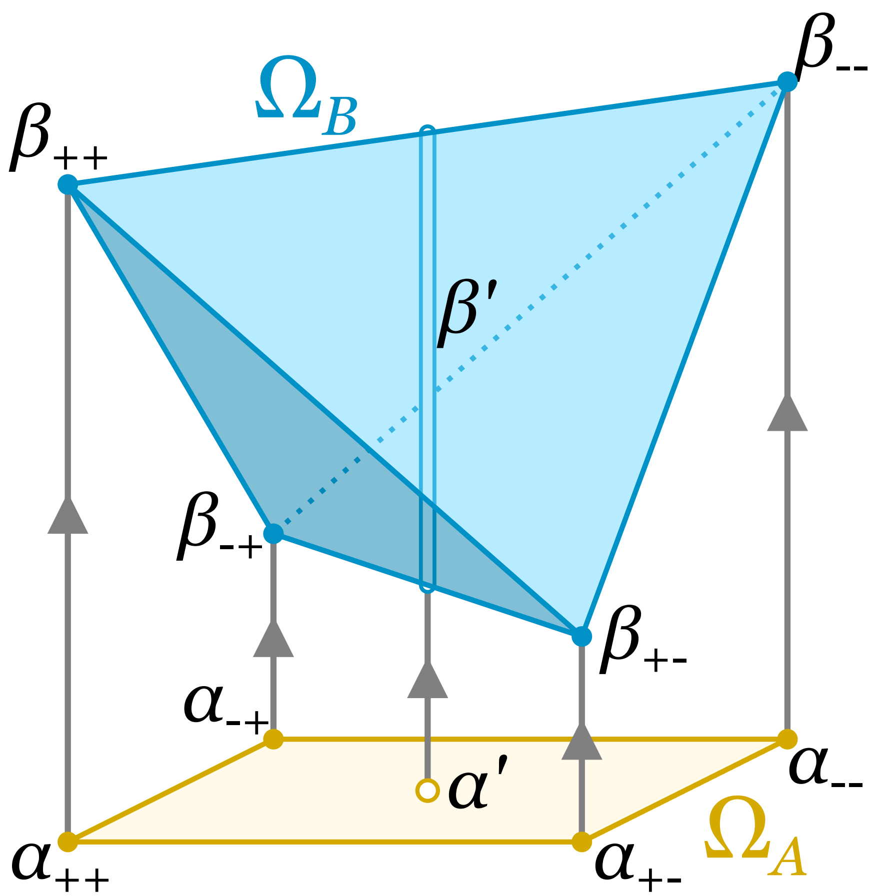
Special case $\mathcal{A}$ = QT, $\mathcal{B}$ = classical probability theory:

Simulations are **ontological models**, and univalence = **noncontextuality**.

Example ("Holevo projection"): simulating the gbit $\mathcal{A} = (\mathbb{R}^3, \Omega_A, E_A)$ with a classical 4-level system $\mathcal{C}_4$.



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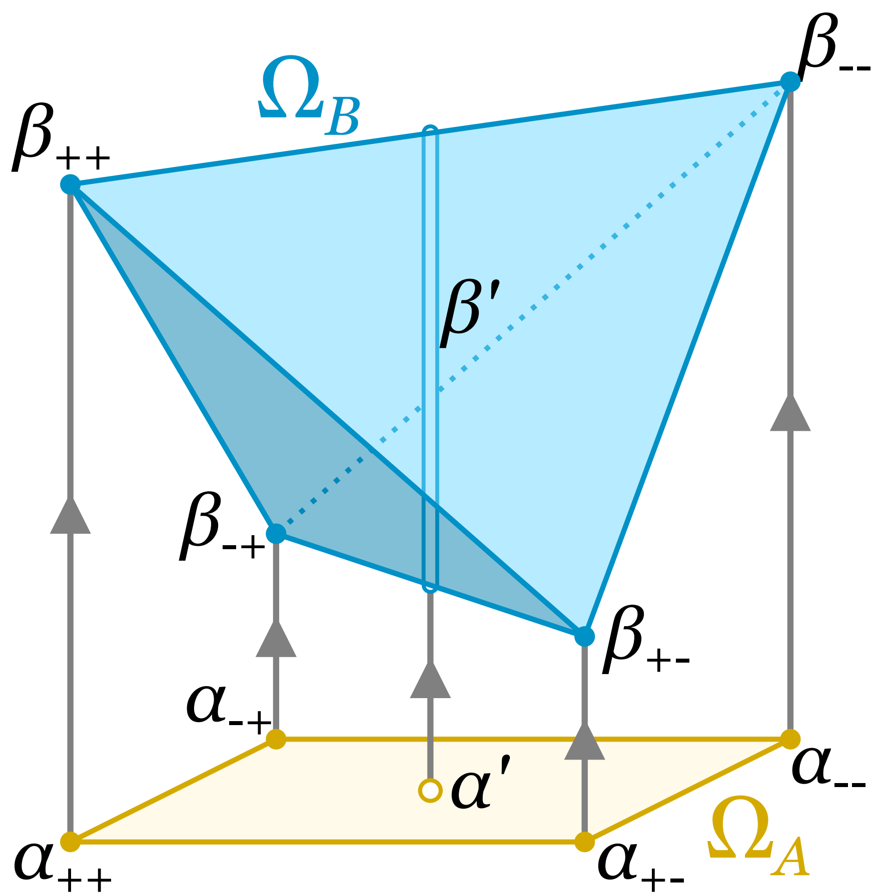

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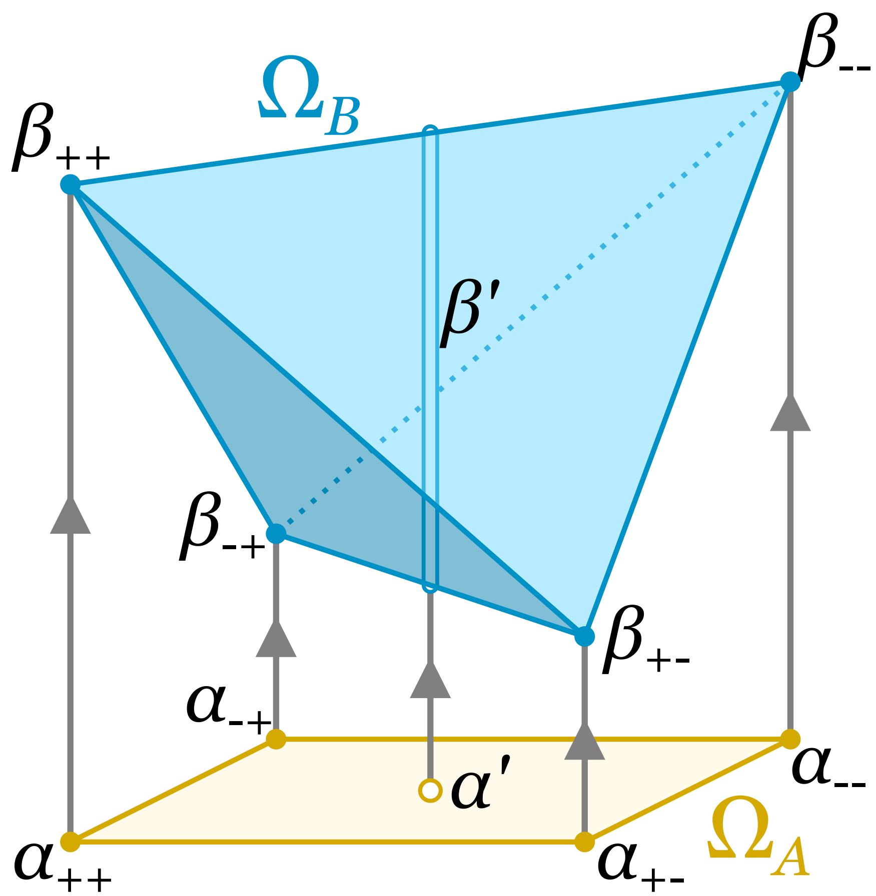

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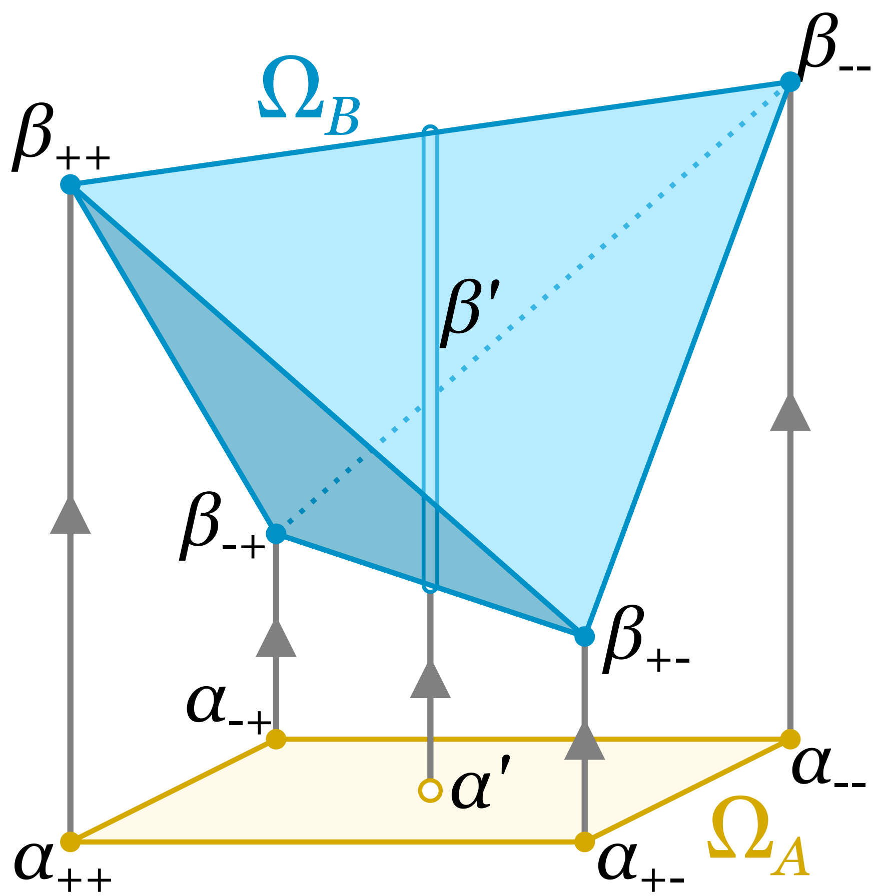

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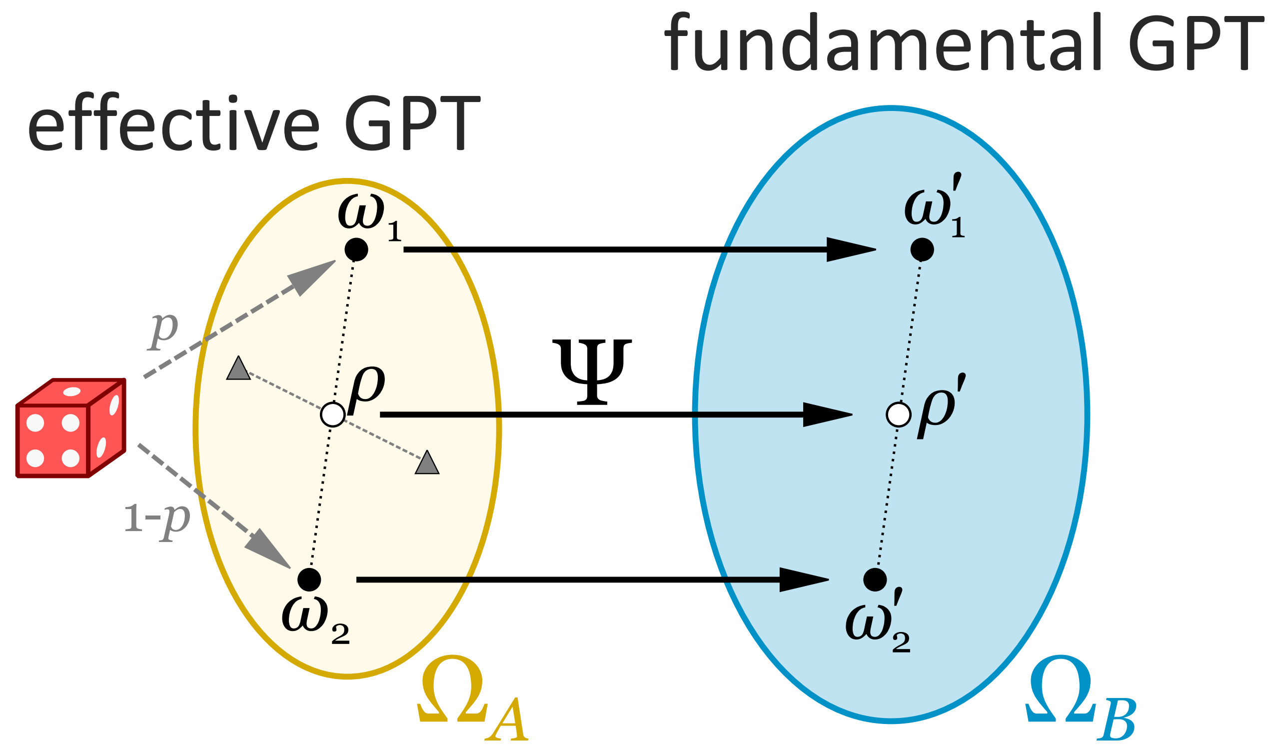
This is an instance of implausible fine-tuning:
the statistical differences among the fundamental states are miraculously *exactly "washed out"* on the effective level.

Univalent simulations are **embeddings**



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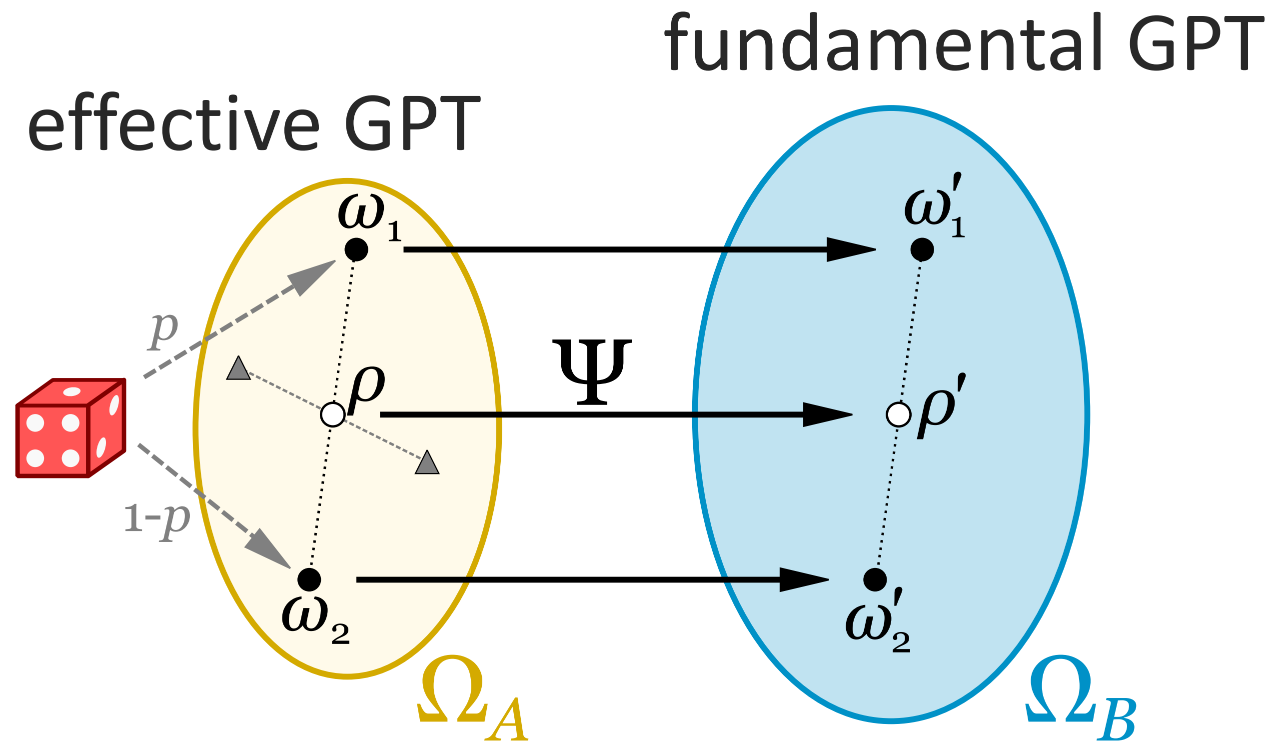

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An ε-**embedding** consists of two linear maps Ψ and Φ such that

- Ψ maps the normalized states of A into those of B,
- Φ maps the effects of A into those of B,
- outcome probabilities are preserved up to ε.

Summary of this part

Multivalent simulations (that cannot be made univalent) are **implausible** because they are **fine-tuned**, cf. Holevo projection.

Univalent simulation (of A by B) = **embedding** (of A into B).

Embeddable into CPT (a classical probability simplex) $\mathcal{C}_n$
= univalently simulatable by fundamental CPT
= **noncontextual** in the sense of **Spekkens**
= plausibly "classical".

Embeddable into QT (a positive semidefinite cone) $\mathcal{Q}_n$
= univalently simulatable by fundamental QT
= plausibly "quantum".

Noncontextual inequalities and approximate embeddings

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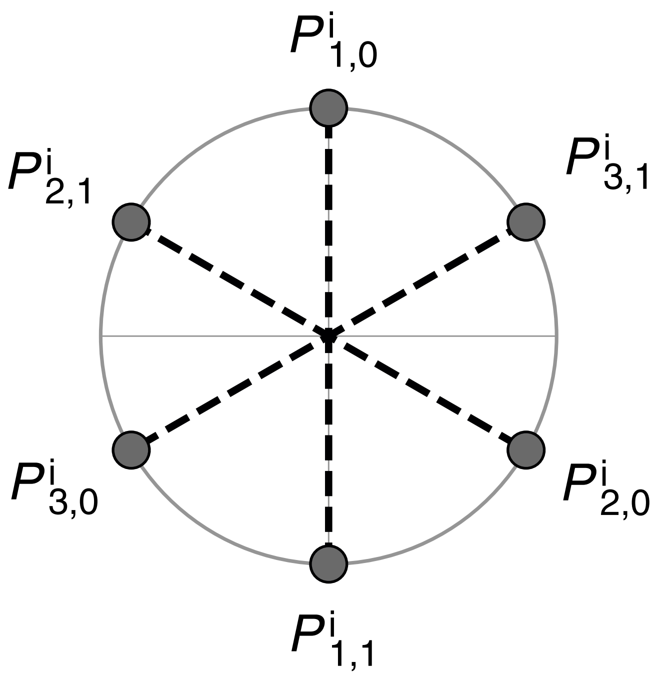

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Quantitative statement:

$$A := \frac{1}{6} \sum_{t\in\{1,2,3\}} \sum_{b\in\{0,1\}} P(b\,|\,p_{t,b}, m_t) \leq \frac{5}{6}.$$

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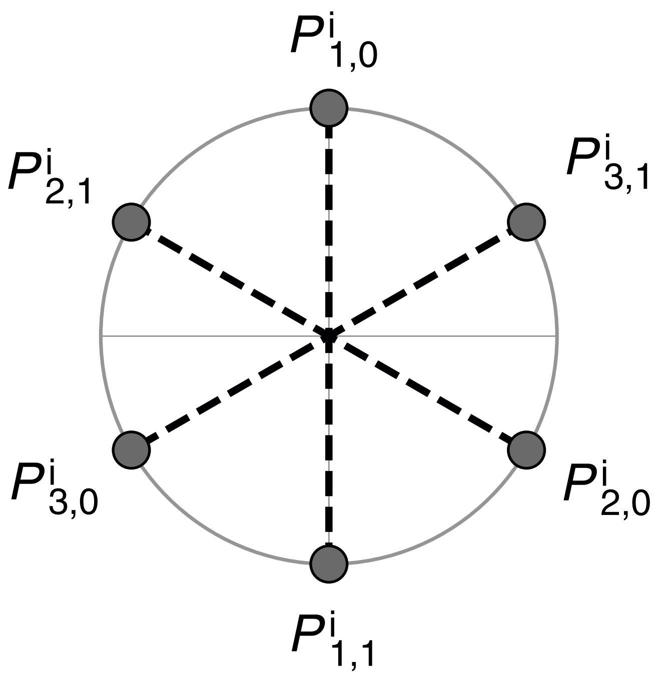

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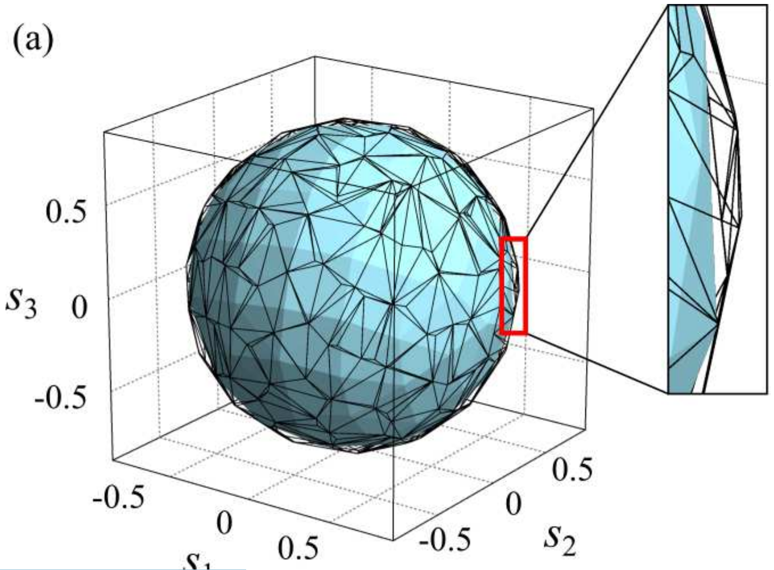
These imply bounds on the approximate embeddability into classical:

Example 1. *Let $\varepsilon < \frac{1}{6}$. Then the rebit (and thus, also the qubit) cannot be ε-embedded into any $\mathcal{C}_n$.*

Overview

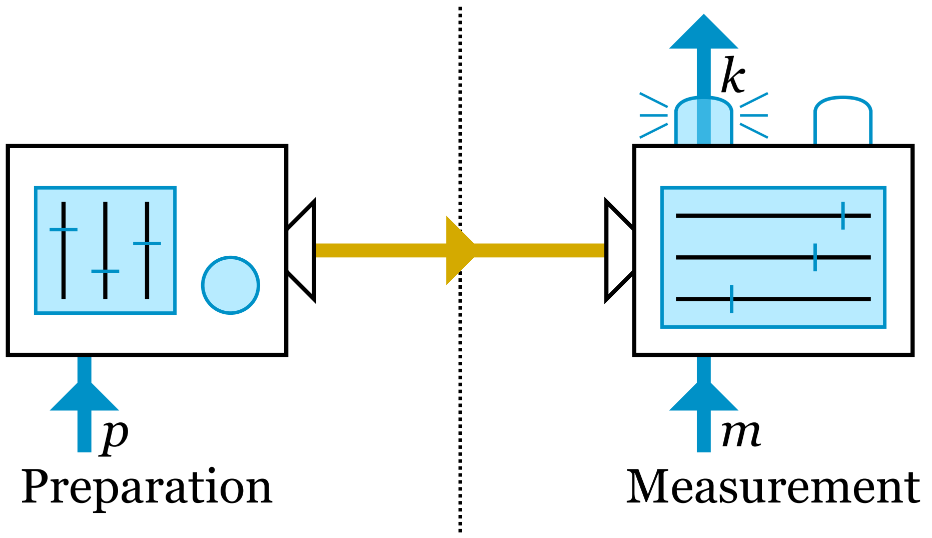
1. GPTs and theory-agnostic tomography



2. Contextuality, simulations, and embeddings

3. Exact embeddings into quantum theory

4. An experimental test of QT



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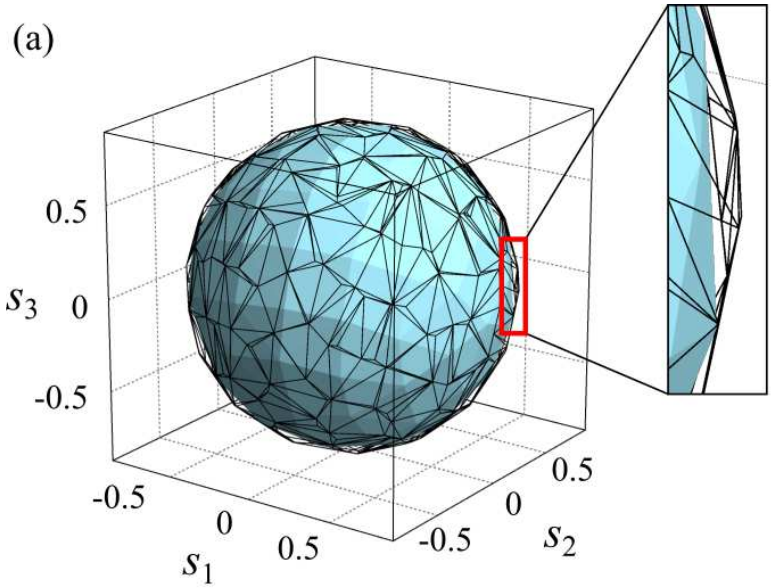

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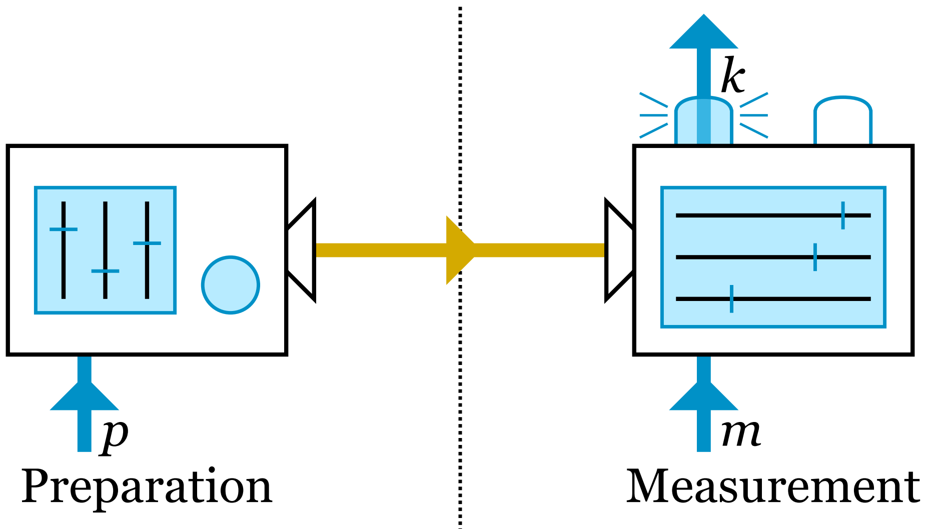

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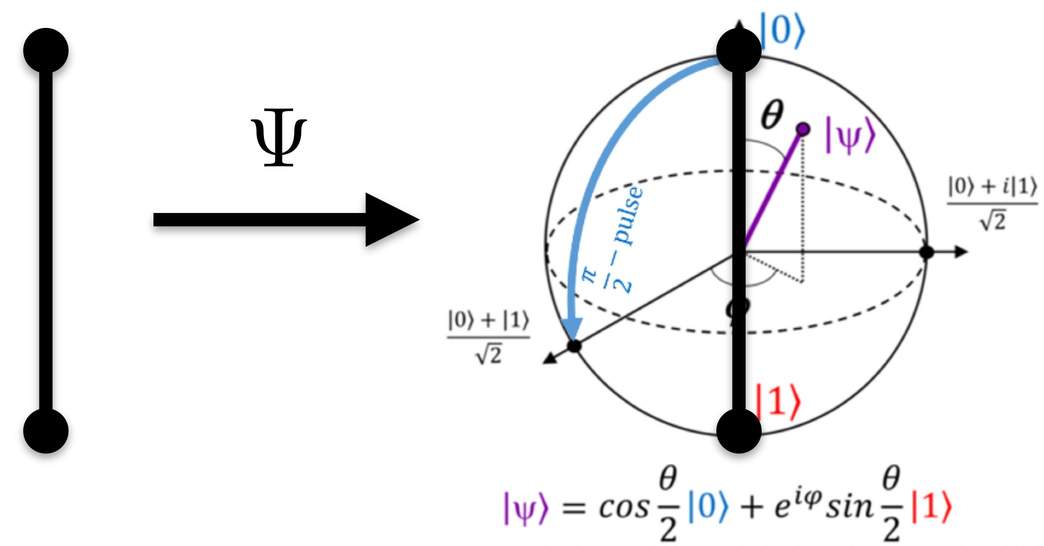

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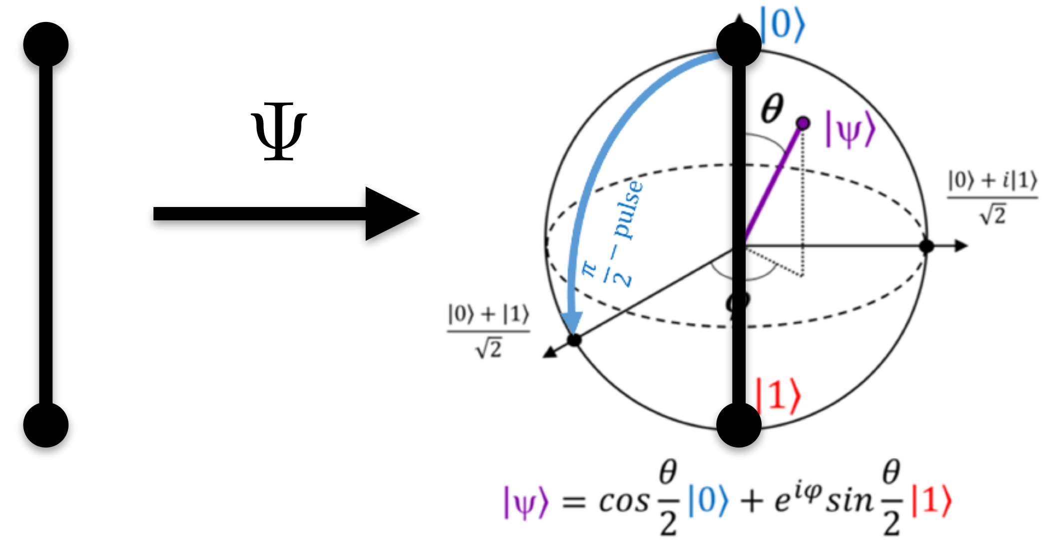
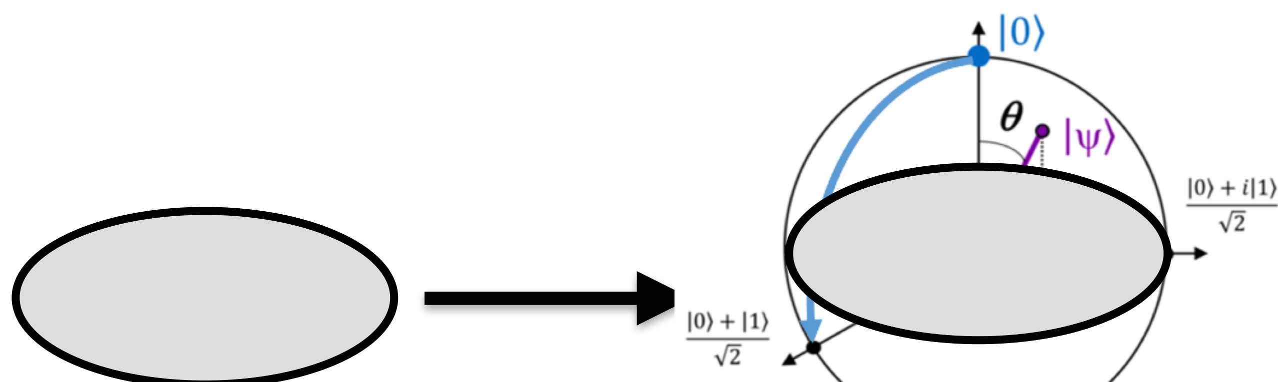


Similarly, **QT over the real numbers** can be embedded into QT.



3. Exact embeddings into quantum theory

Focus on the "**unrestricted** GPTs" where all vectors yielding valid probabilities on all states are effects: $\mathcal{A} = (A, \Omega_A, E_A)$

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These are the **only unrestricted GPTs** that are "plausibly quantum".

3. Exact embeddings into quantum theory

Focus on the "**unrestricted** GPTs" where all vectors yielding valid probabilities on all states are effects: $\mathcal{A} = (A, \Omega_A, E_A)$

$$E_A = \{e \in A \mid 0 \leq \langle \omega, e \rangle \leq 1 \text{ for all } \omega \in \Omega_A\}.$$

Theorem 2. *An unrestricted GPT can be exactly embedded into finite-dimensional quantum* [illegible] *only if it corresponds to a* special Euclidea[illegible]a.

Needs 2^d-dim. Hilbert space for simulation!

- QT over real numbers $\mathbb{R}$, complex nu[illegible]bers [illegible], **quaternions** $\mathbb{H}$,
- *d*-dimensional **Bloch ball** state spaces,
- direct sums of those, including **CPT** and QT with **superselection rules**.

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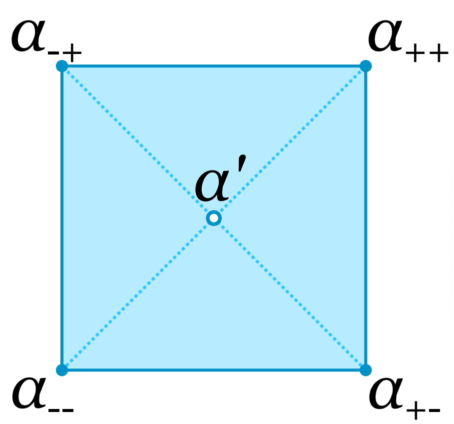
Non-exact embeddings into quantum theory

Example: the gbit



Example 2. *Let* $\varepsilon \leq 0.1014$. *Then the gbit cannot be* ε*-embedded into any* $\mathcal{Q}_n$ *or* $\mathcal{Q}_\infty$.

There is no better-than-10% univalent ("noncontextual") simulation by QT.

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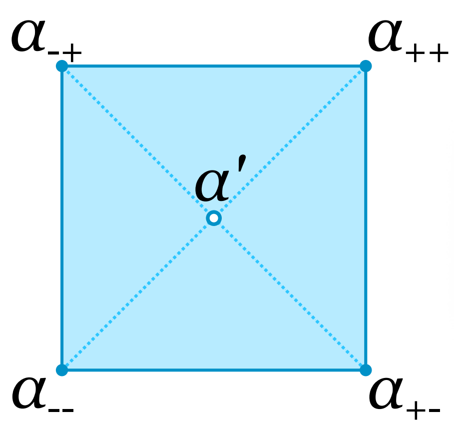

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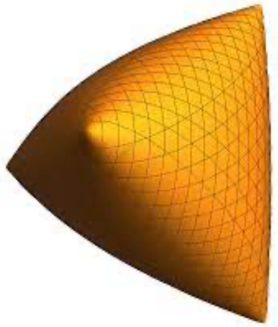
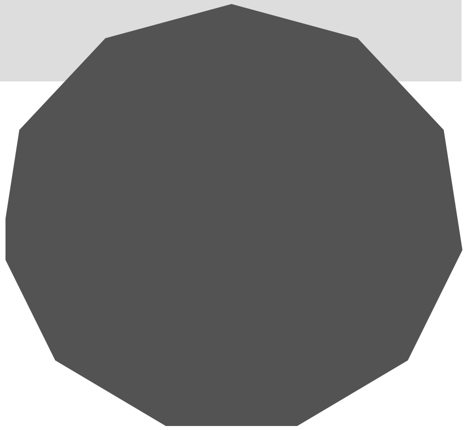
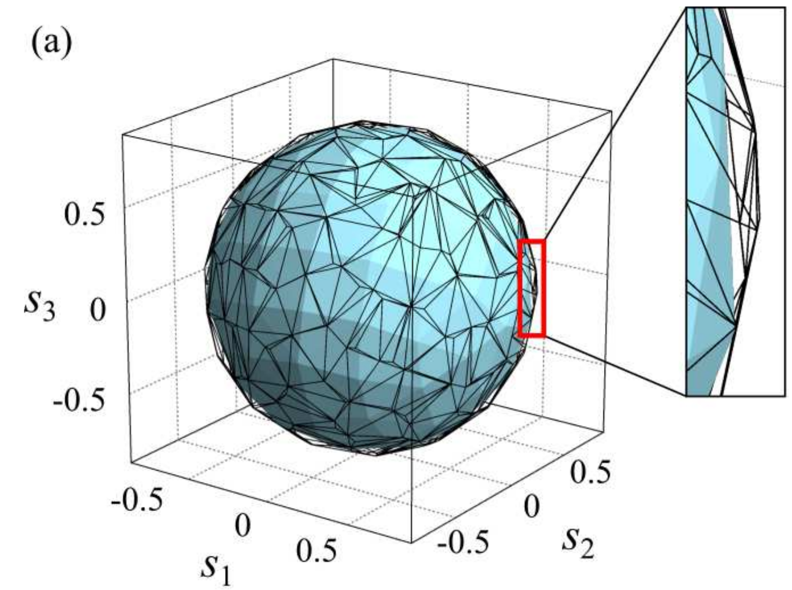
Also shown in our paper:
can **use known results on Bell inequalities** to certify nonembeddability.
Impractical and inefficient, but "proof of principle".





Overview

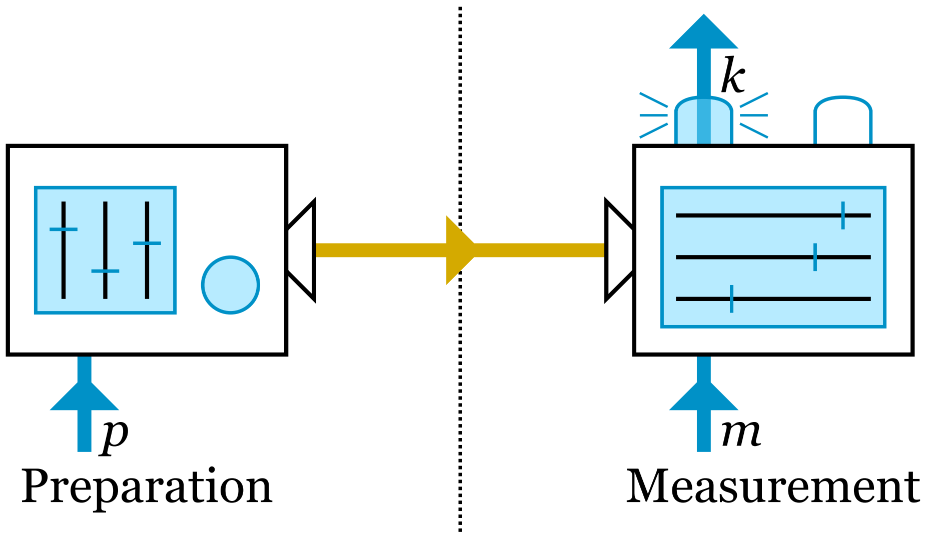
1. GPTs and theory-agnostic tomography



2. Contextuality, simulations, and embeddings

3. Exact embeddings into quantum theory

4. An experimental test of QT



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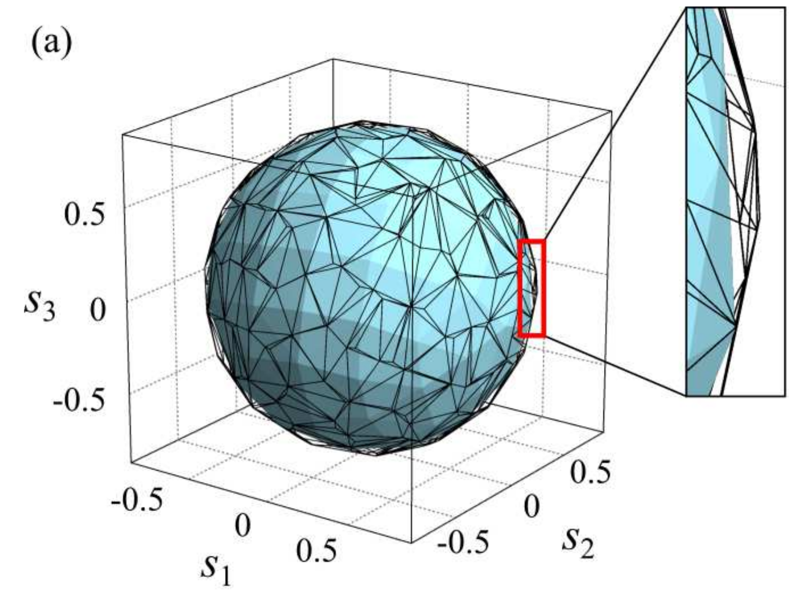

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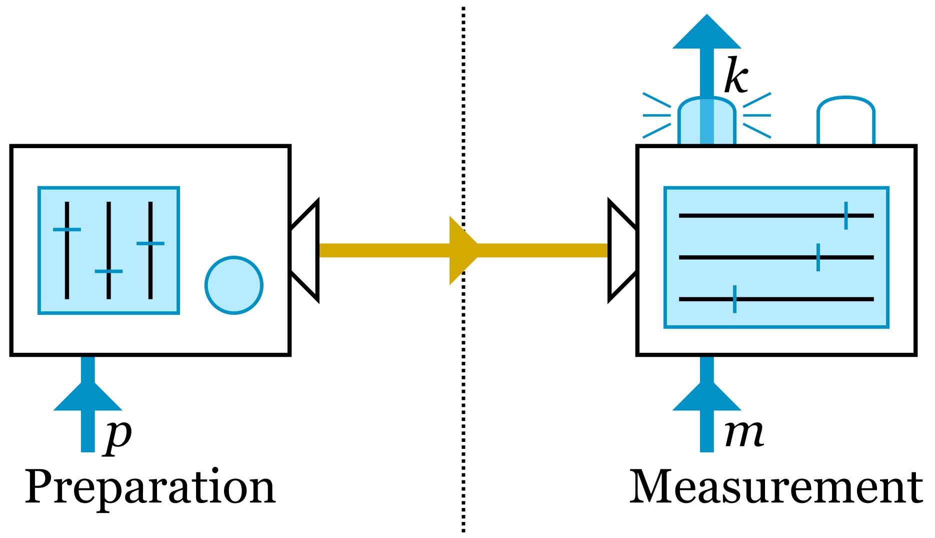

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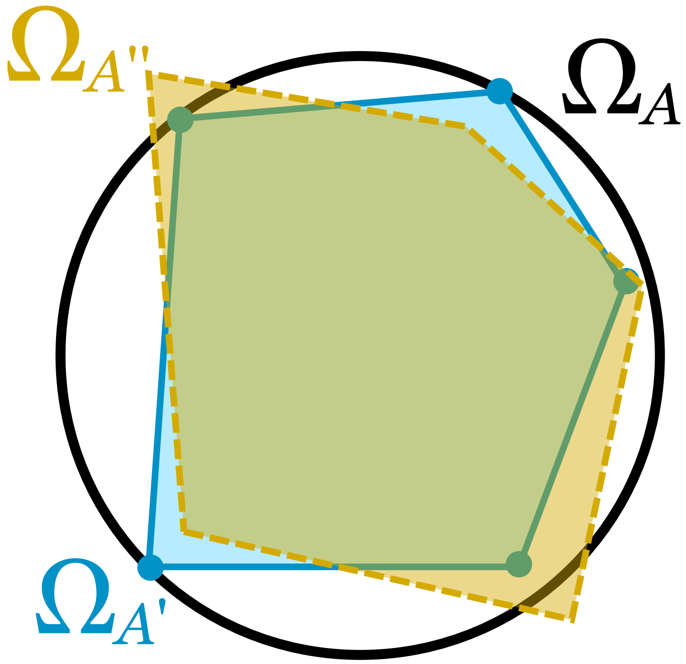
Perform theory-agnostic tomography on an **effective physical system** in your laboratory.

Test whether the resulting effective GPT is $\varepsilon-$ **embeddable into QT**, where ε is a function of the experimental uncertainty.

If, surprisingly, “no”, then this challenges QT.



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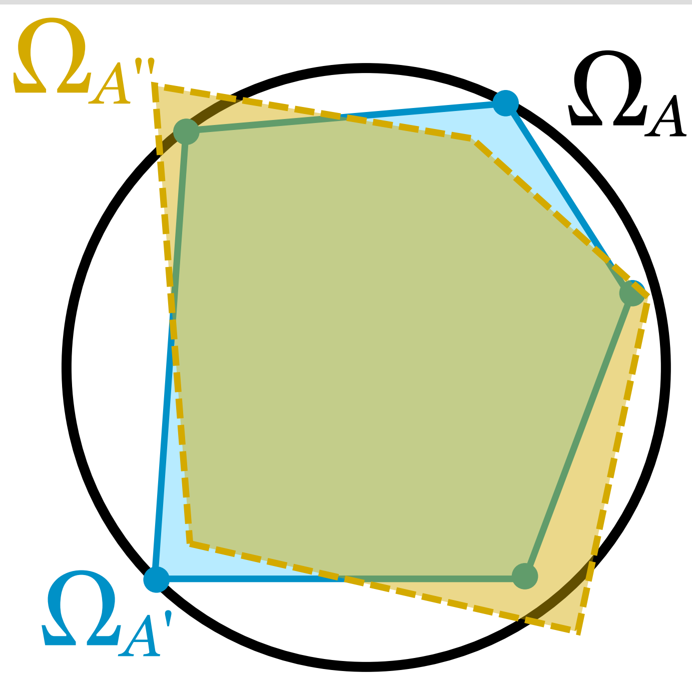
Typically a **restricted** GPT.



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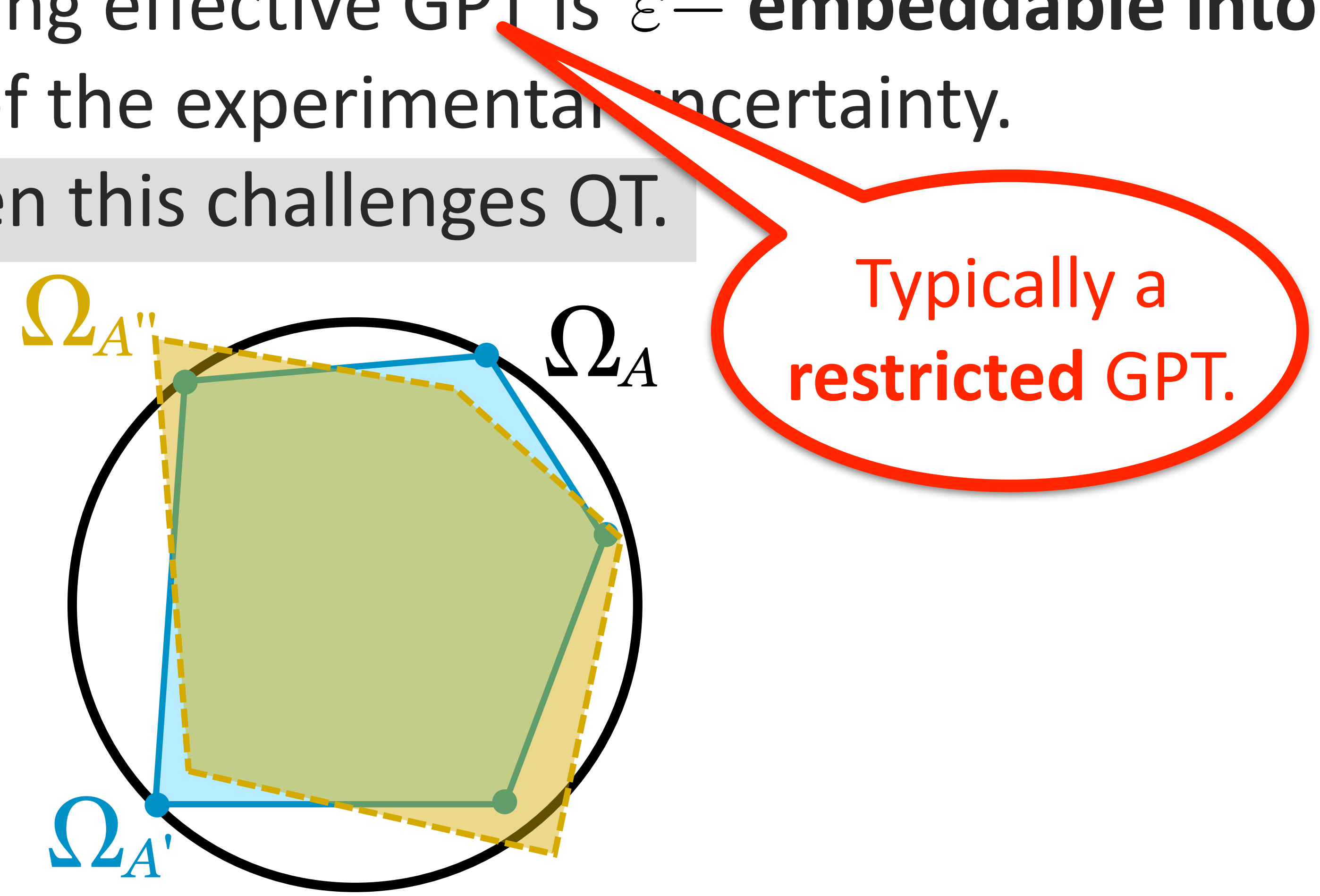

A quantum explanation of the result is then **similarly implausible as** a classical (contextual) explanation of the quantum state space.

Summary

- Have generalized Spekkens' notion of generalized noncontextuality:
  "Processes that are statistically indistinguishable in an effective theory should not require explanation by multiple distinguishable processes in a more fundamental theory."

- Results: Several structural insights, a new experimental test of QT, Jordan algebras are the only **unrestricted** GPTs embeddable into QT.

- Note: the experiments will **not** just test QT "against other probabilistic theories / GPTs", **but against arbitrary** modifications impacting prepare-and-measure-statistics. We use GPTs only as a tool to analyze the latter.

arXiv:2112.09719 (update soon),
to appear in Physical Review X.

Thank you!