ÖAW AUSTRIAN ACADEMY OF SCIENCES

IQOQI – INSTITUTE FOR QUANTUM OPTICS AND QUANTUM INFORMATION VIENNA

Theory-independent randomness generation with spacetime symmetries

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FWF Der Wissenschaftsfonds.

PERIMETER INSTITUTE

Overview

1. Motivations: QG and device-independent QIT

2. Our protocol, and its quantum analysis

3. Rotation boxes beyond quantum theory

4. Conclusions

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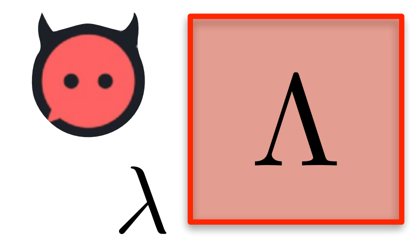
Motivation 1: SDI randomness expansion

Goal: Generate **certifiably random** bits, unpredictable even by eavesdroppers with arbitrary classical side information.



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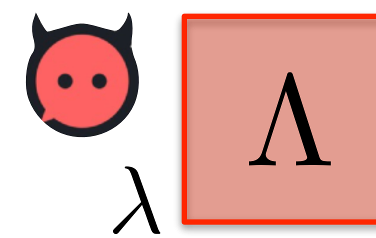

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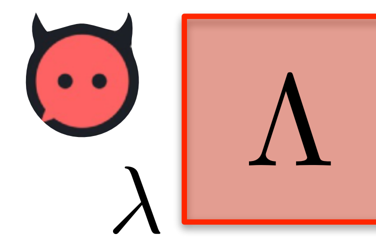
Make some (modest?!) assumption on the transmitted phys. system.



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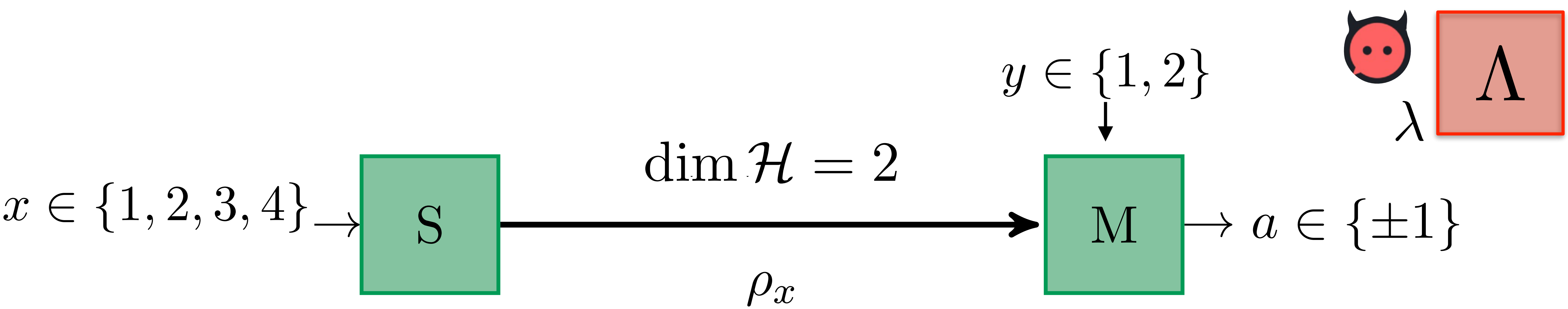

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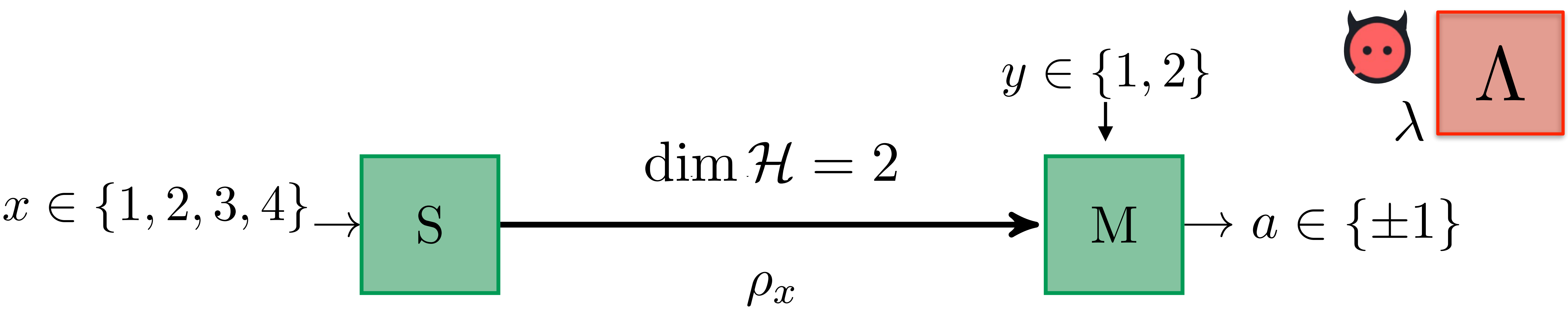

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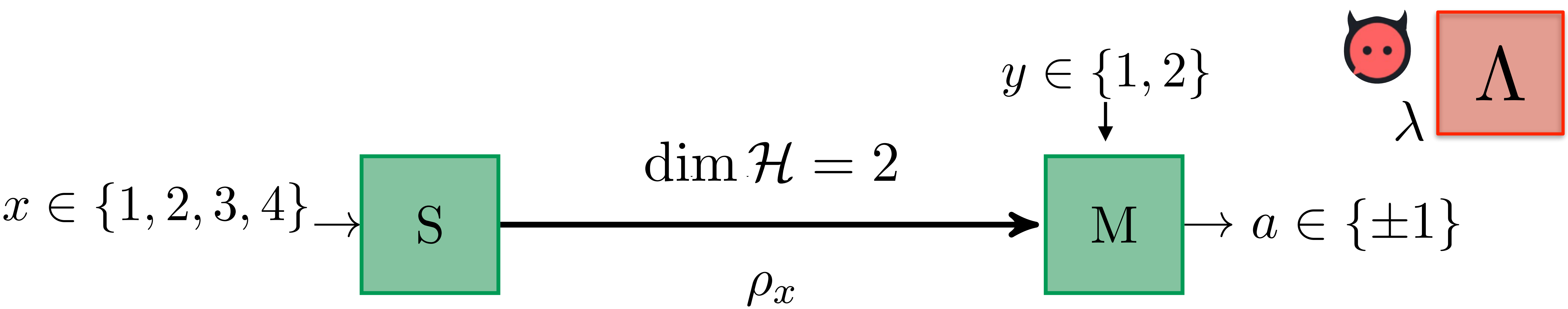

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Problems: assumption not very well motivated; assumes QT is correct.

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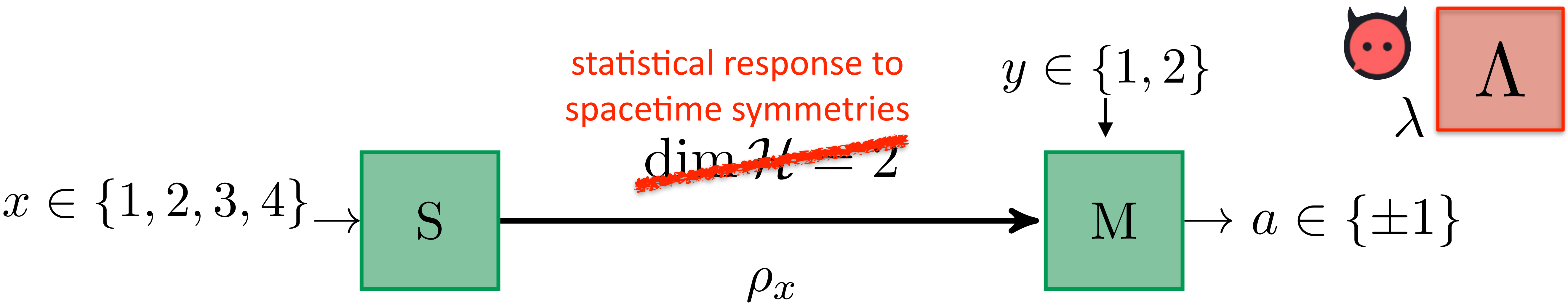
Our SDI assumption: essentially, a bound on how sensitive the system responds to spatial rotations (in QT: “spin quantum number”).
This turns out to make sense (and work) without assuming QT.



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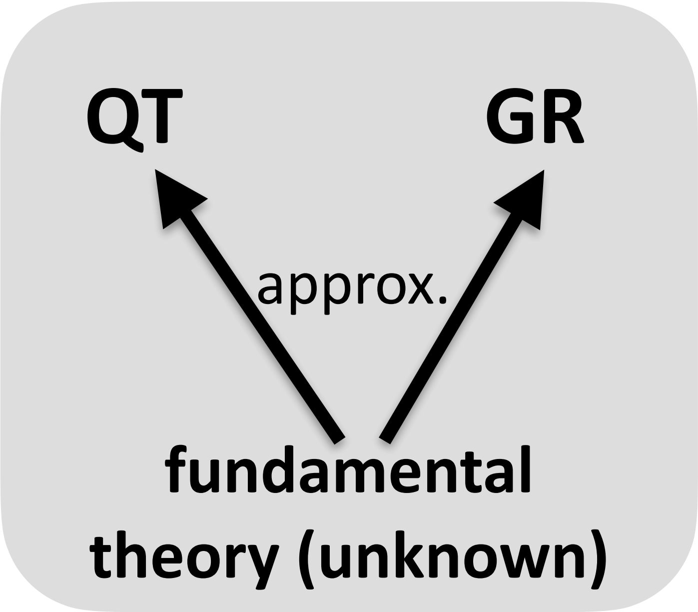

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study the **logical architecture of physics**:
how do QT and spacetime constrain each other?

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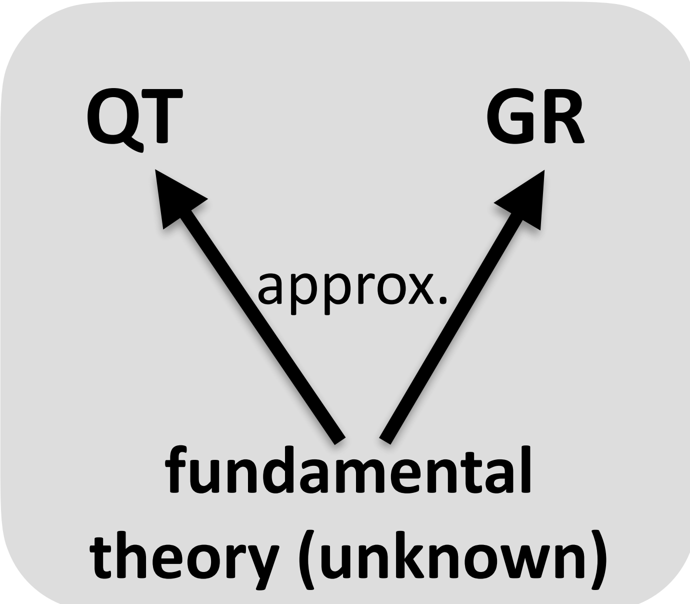

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Analogy: suppose we would like to find a detailed theory of (bio.) **evolution**.

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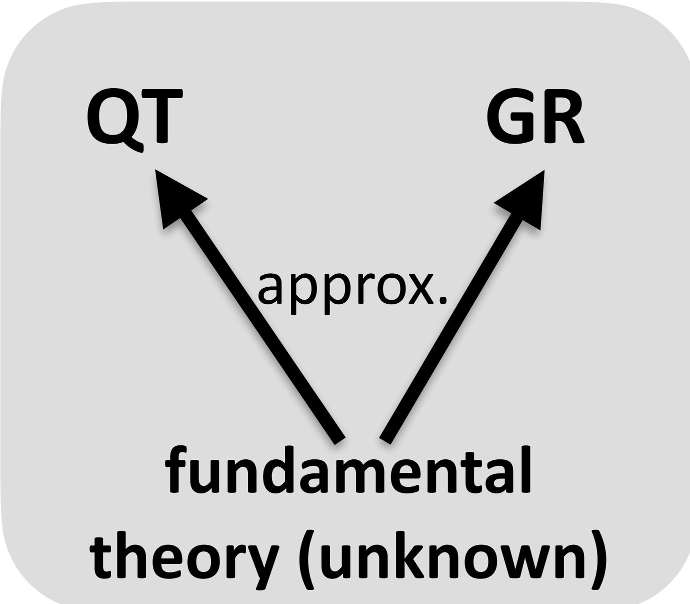

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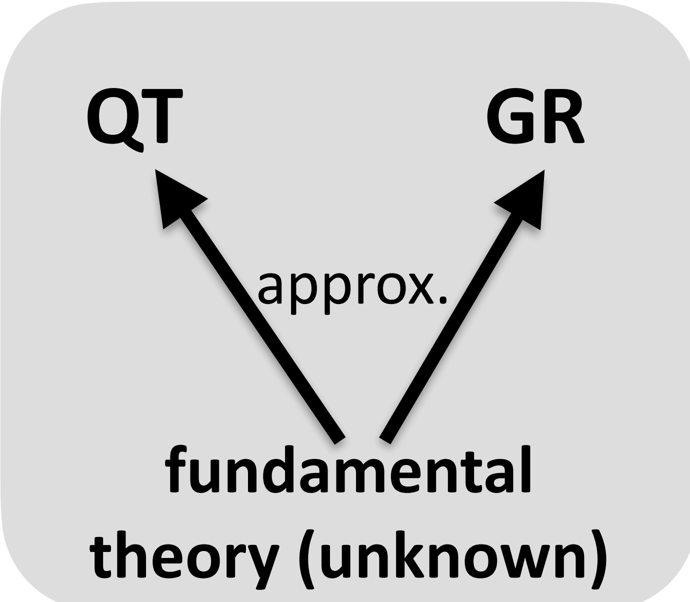

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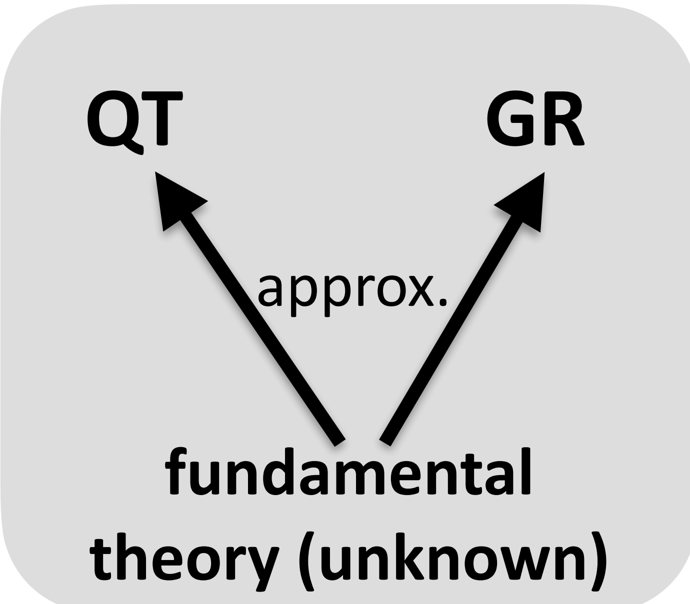

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Camel humps and thorns as a *consequence* of environment. What kind of life fits into a given environment *in principle?*

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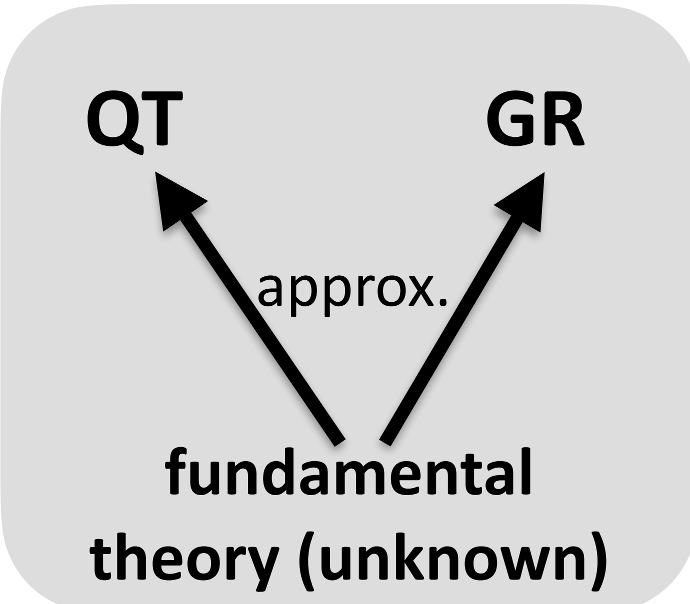

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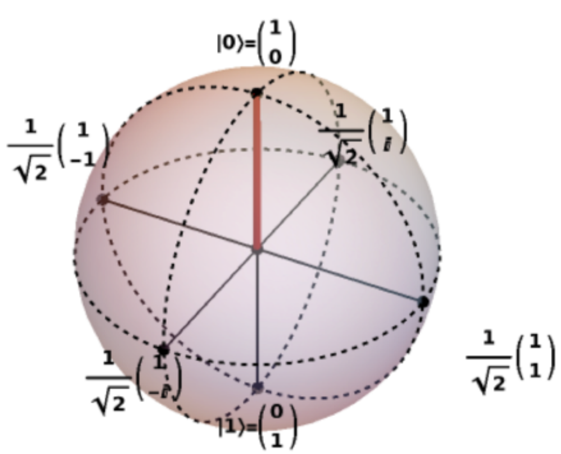

Qubit Bloch ball and quantum correlations as *consequences of* spacetime structure? Which detector click probabilities fit *in principle* into space and time?

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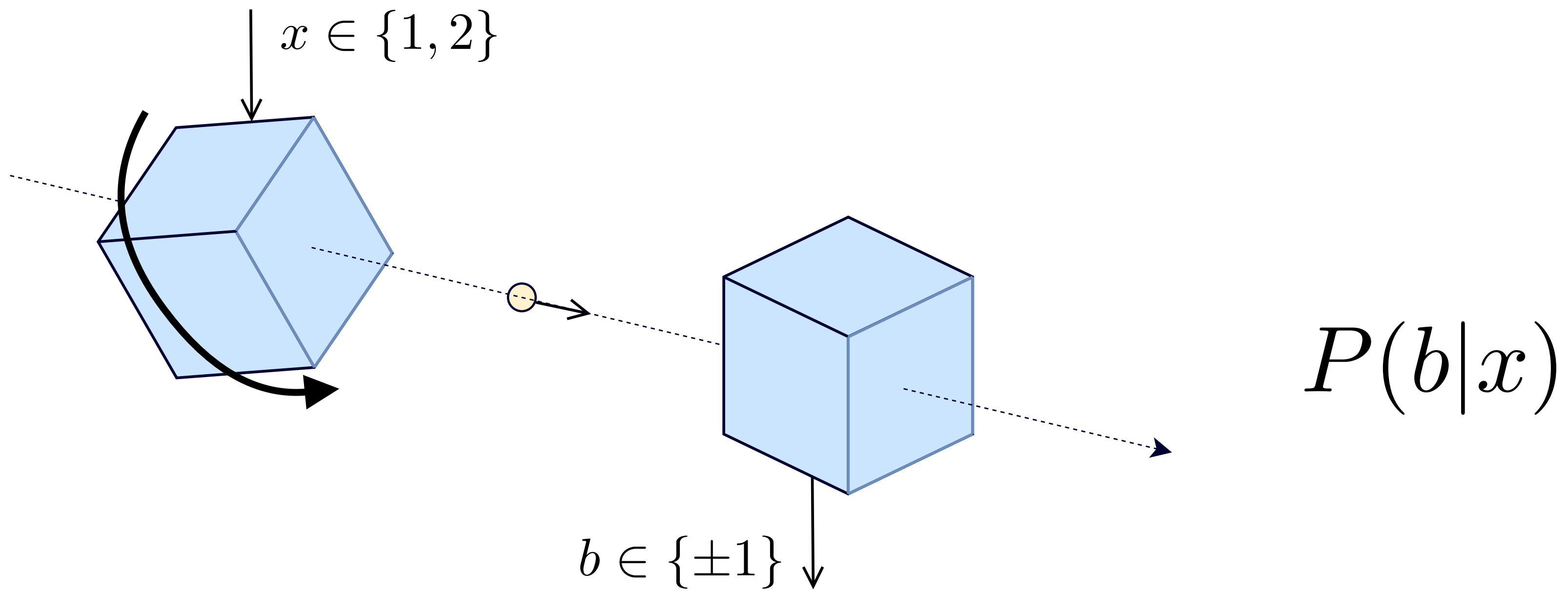

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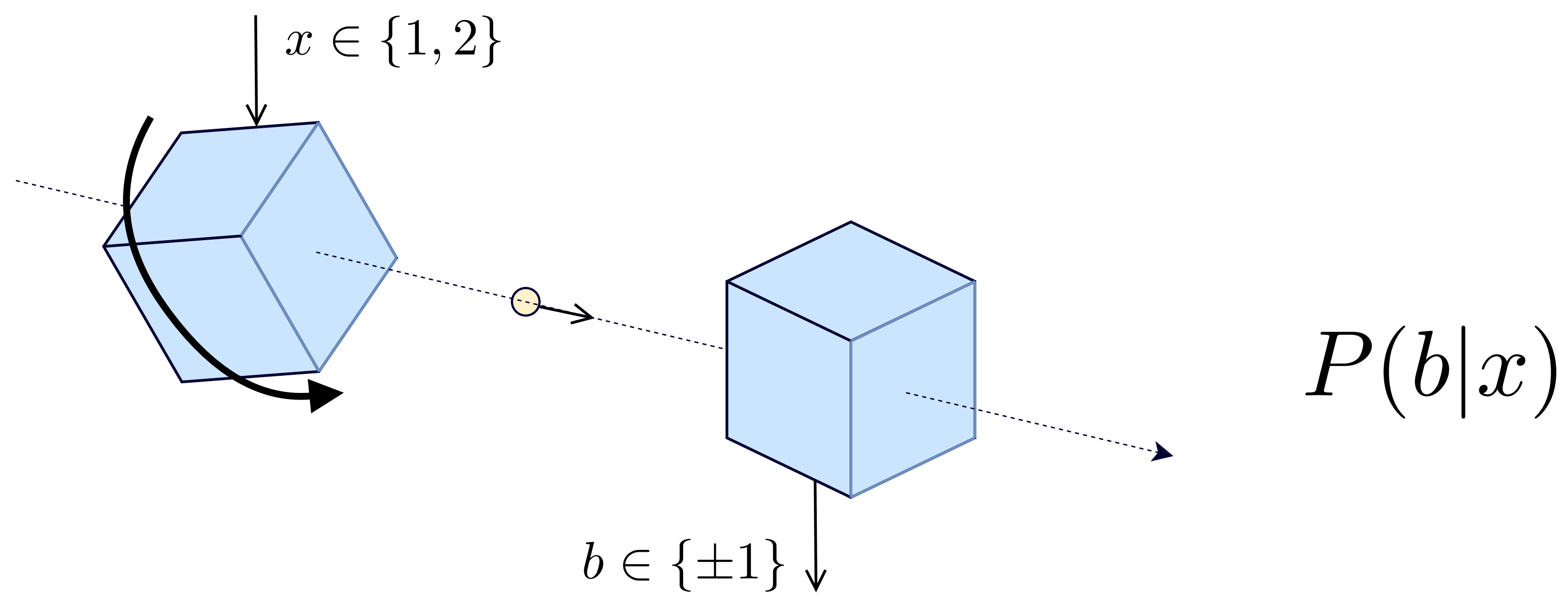

If input is x=1: do nothing to preparation device;

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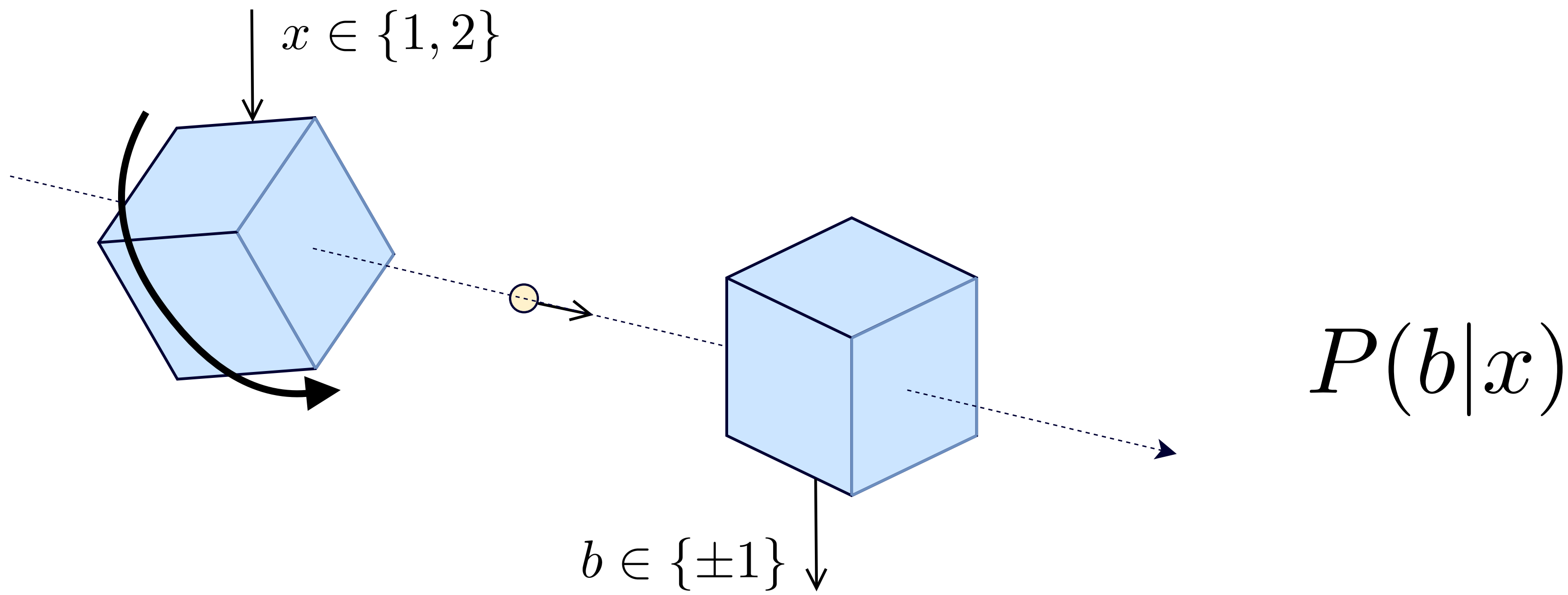

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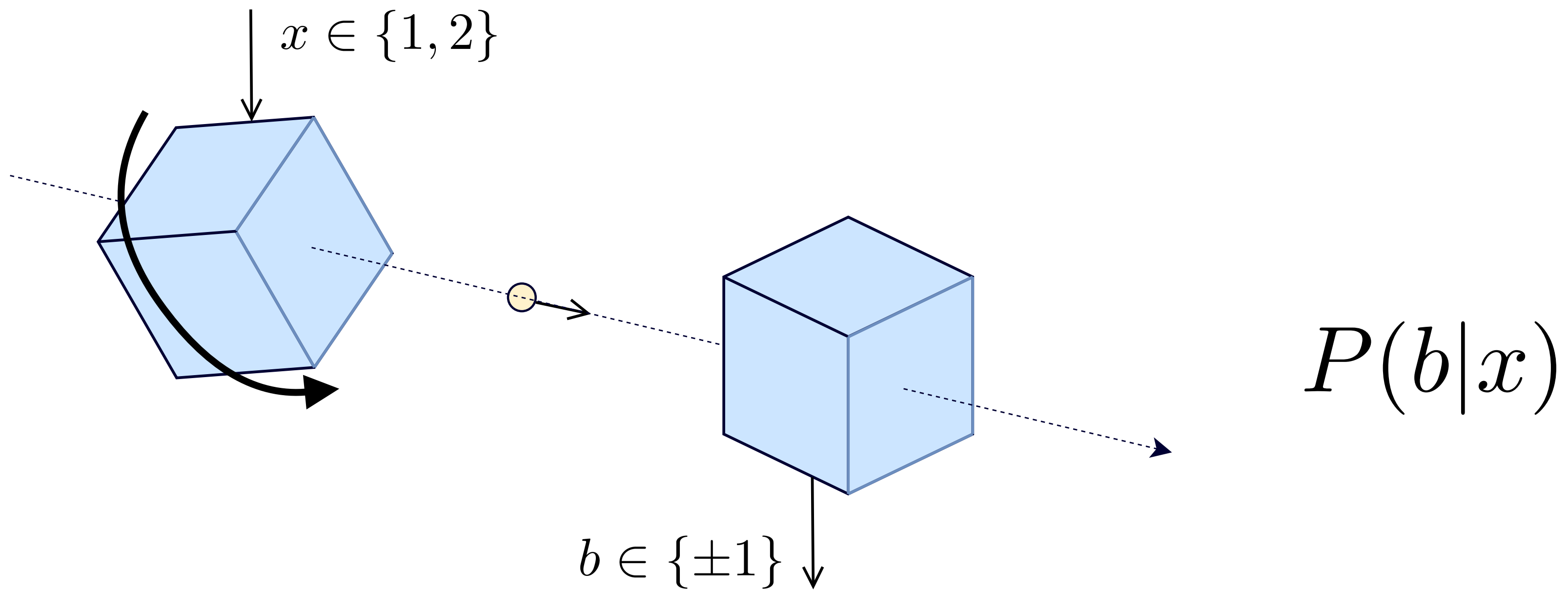

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Rotation described by (projective) unitary representation of SO(2):

$$U_\alpha = \bigoplus_{j=-J}^{J} n_j e^{ij\alpha}, \qquad P(b|\alpha) = \sum_\lambda p(\lambda)\mathrm{tr}(M_b(\lambda)U_\alpha \rho_1(\lambda)U_\alpha^\dagger)$$

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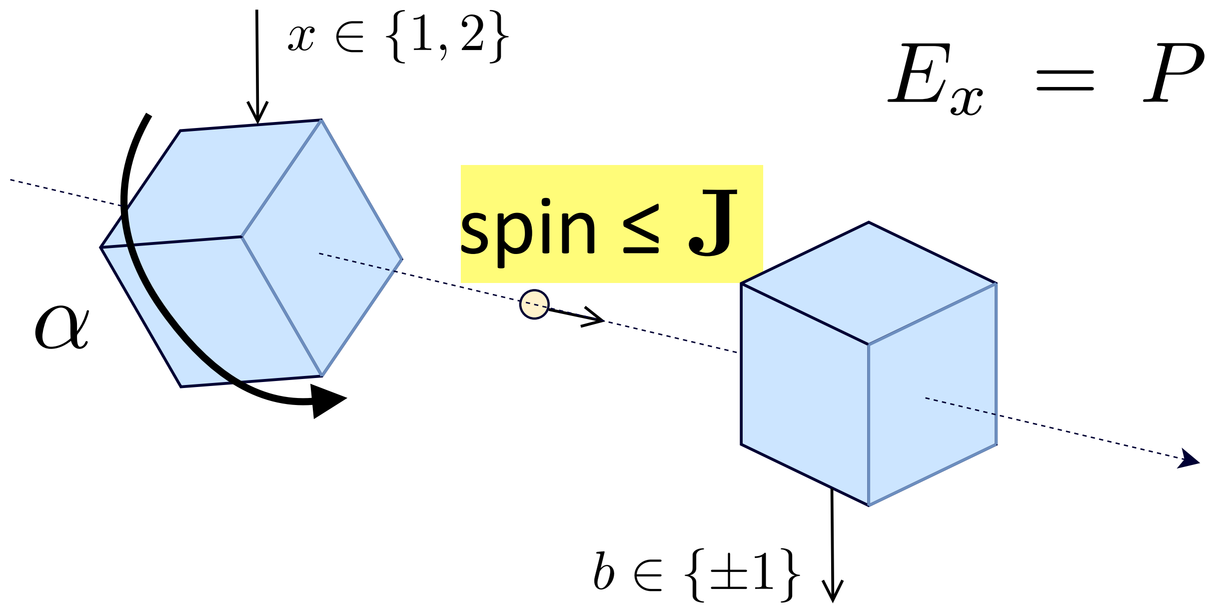

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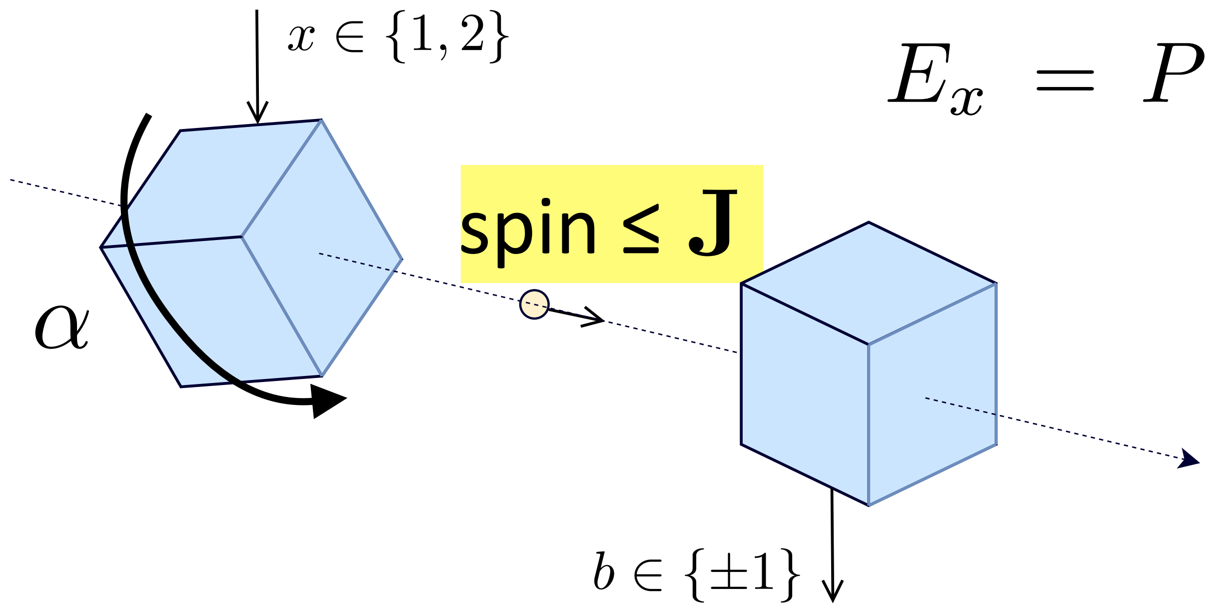

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Theorem. The following correlations are possible:

$$\frac{1}{2}\left(\sqrt{1+E_1}\sqrt{1+E_2}+\sqrt{1-E_1}\sqrt{1-E_2}\right) \geq \begin{cases} \cos(J\alpha) & \text{if } \ |J\alpha| < \frac{\pi}{2} \\ 0 & \text{if } \ |J\alpha| \geq \frac{\pi}{2} \end{cases}$$

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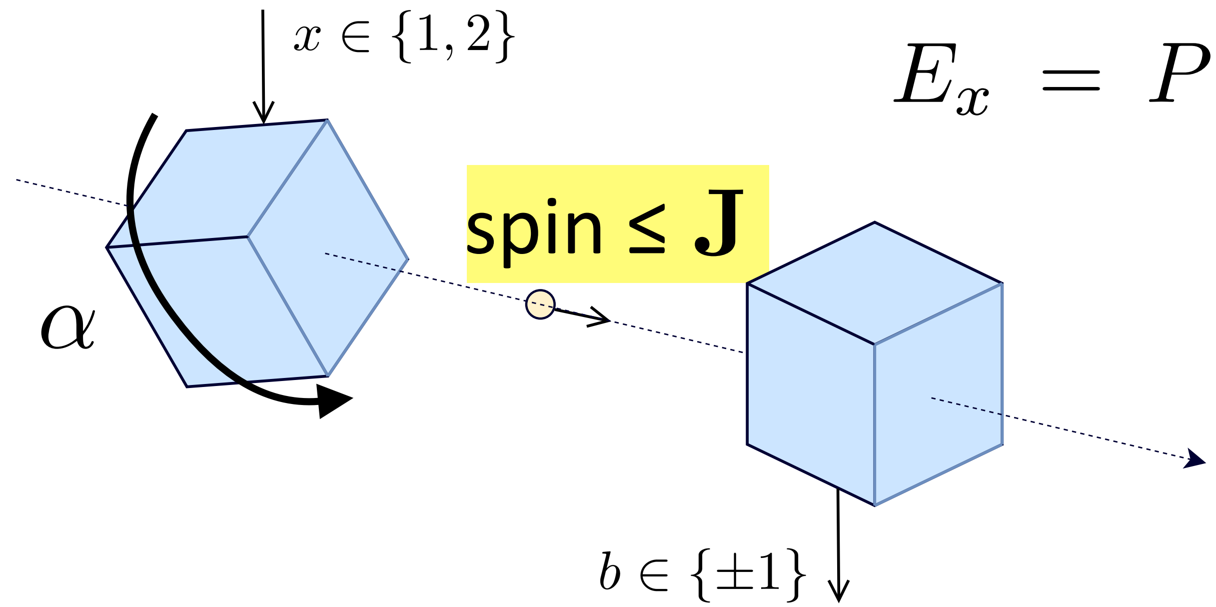

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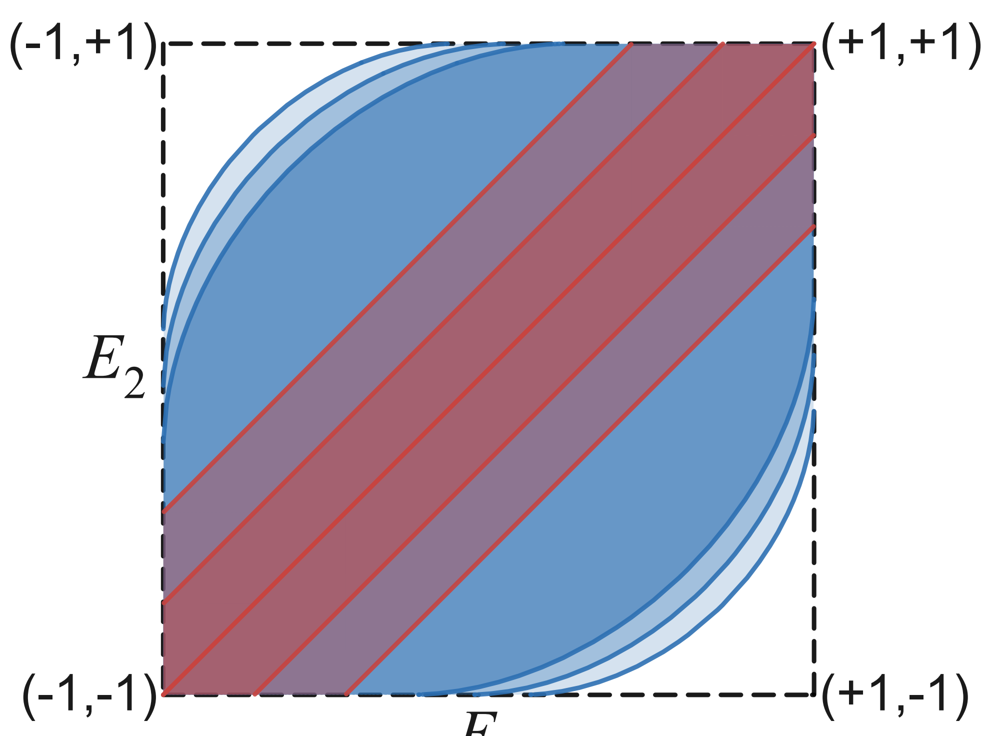

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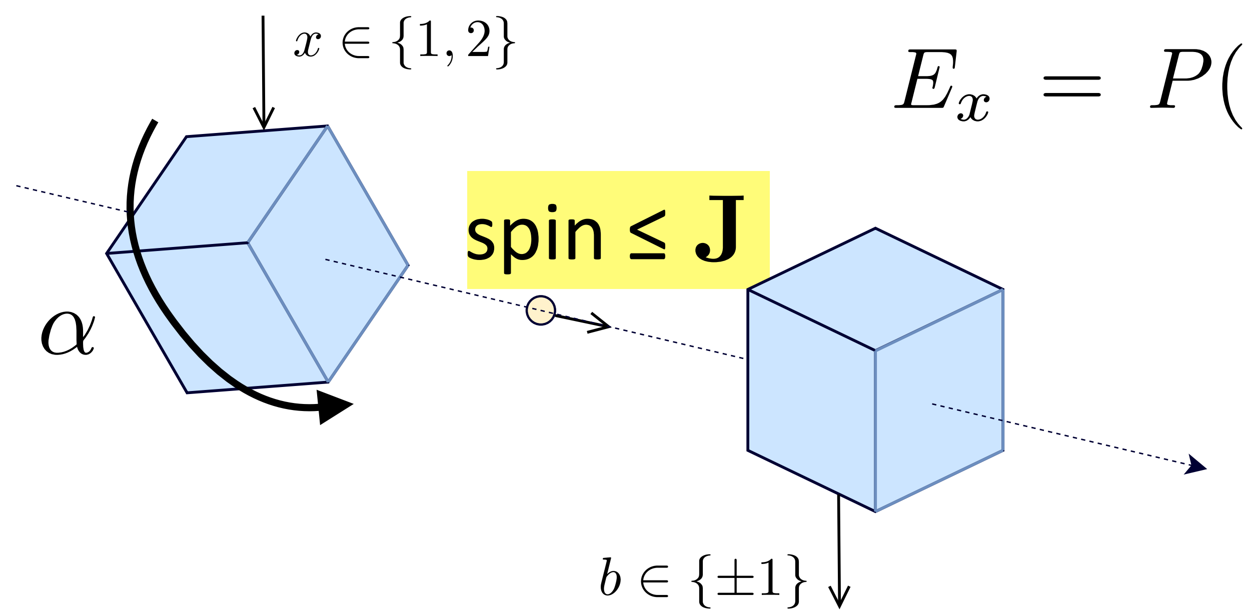

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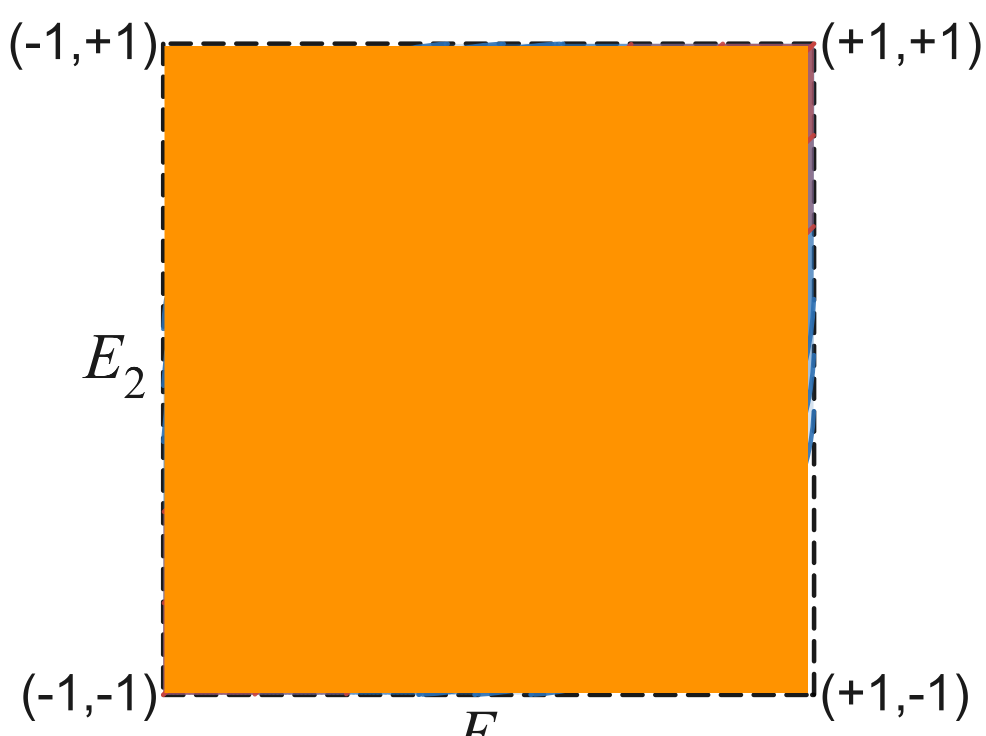

Angle $|J\alpha| \geq \pi/2$:
Rotated and unrotated states may be **orthogonal**; outcome *b* may carry perfect classical info on *x*, i.e. $(E_1, E_2) = (\pm 1, \mp 1)$
All correlations possible, **no certifiable randomness.**

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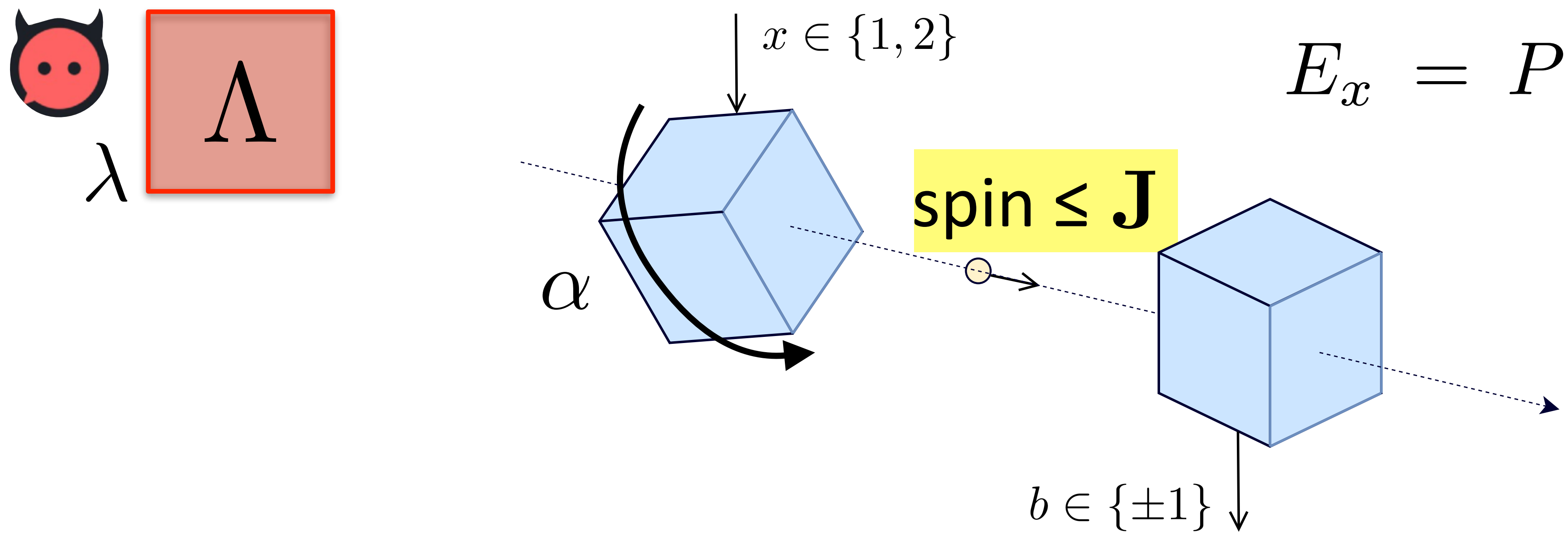

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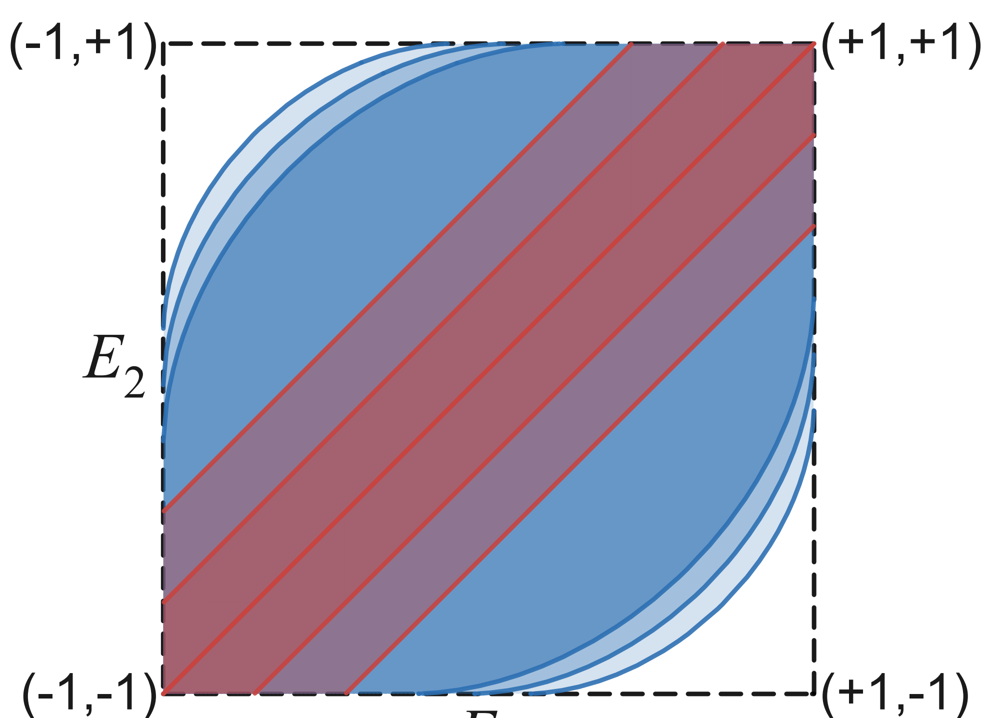
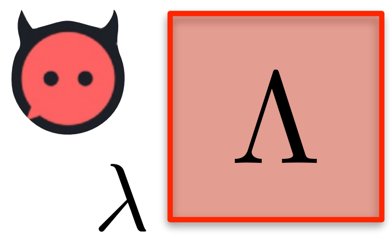


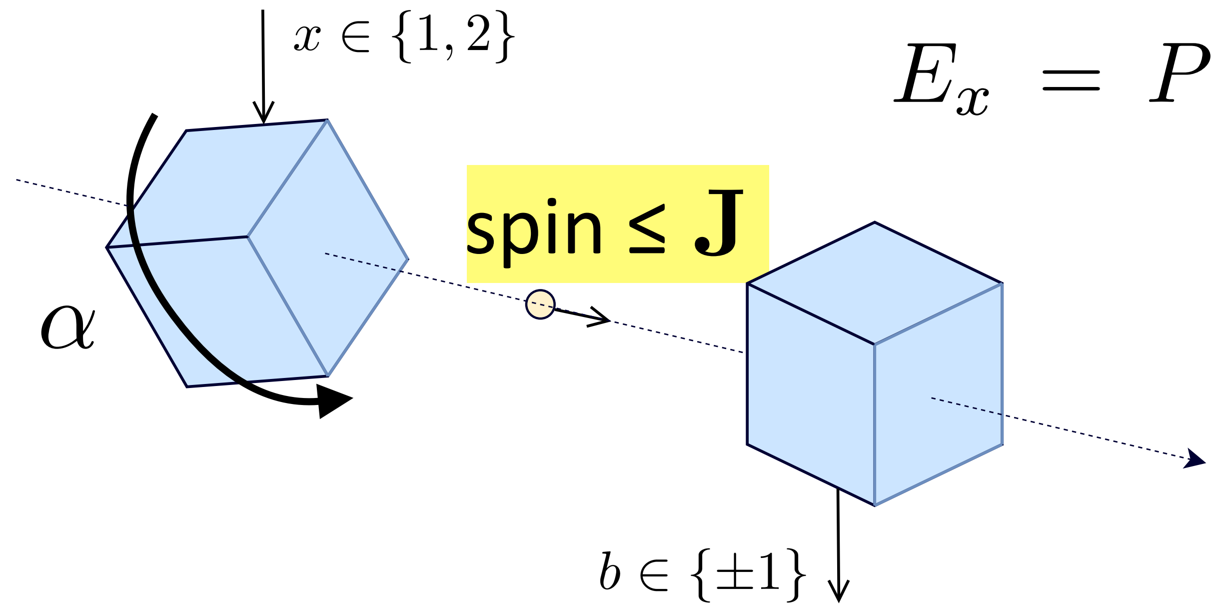
The curved set of correlations is possible.

b cannot carry full information on *x*, hence *b* must contain some randomness, even relative to classical side information λ, if *E* outside the red (“classical”) line:

non-zero amount of certified randomness.

A slide to scare the non-experts





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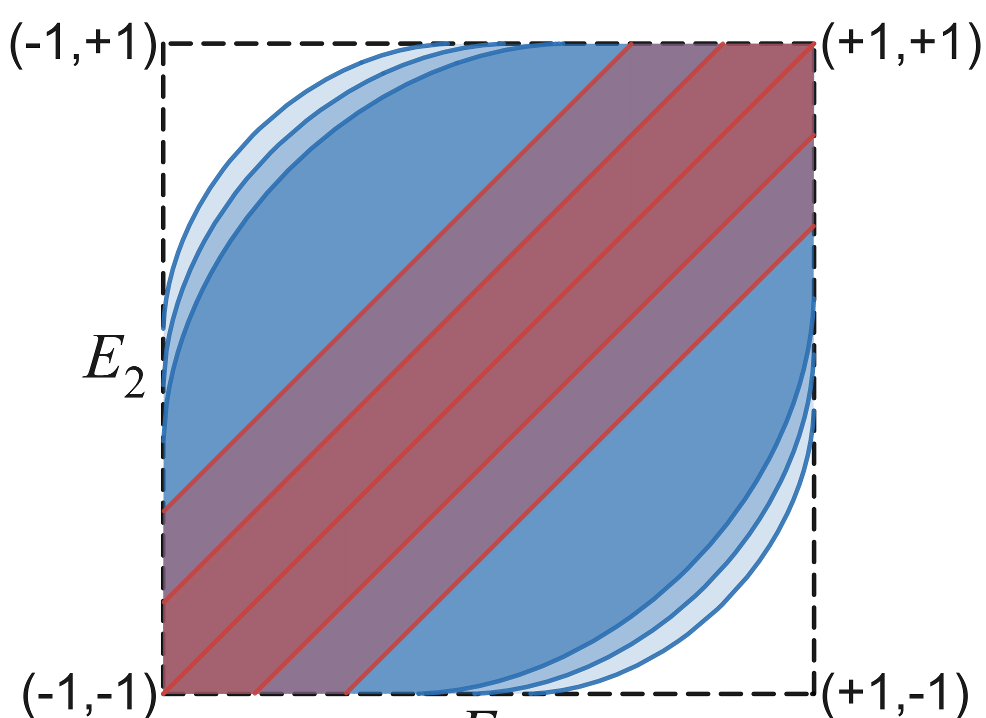
Up to prob. ϵ, all “hidden” systems satisfy spin bound approximately.

$$\mathbf{E} = (E_1, E_2) = \sum_\lambda p(\lambda)(E_1^\lambda, E_2^\lambda) = \sum_\lambda p(\lambda)\mathbf{E}^\lambda.$$



# of certified random bits: $H(B|X,\Lambda) \geq H^\star,$

$$H^\star = \min_{\{p(\lambda),\mathbf{E}^\lambda\}} \sum_\lambda p(\lambda) H(\mathbf{E}^\lambda)$$

$$\text{subject to} \sum_{\lambda:\mathbf{E}^\lambda \in \mathcal{Q}^\omega_{J,\alpha}} p(\lambda) \geq 1 - \varepsilon$$

$$\text{and} \sum_\lambda p(\lambda)\mathbf{E}^\lambda = \mathbf{E}.$$

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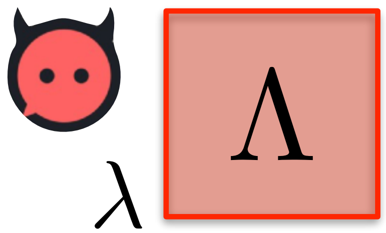

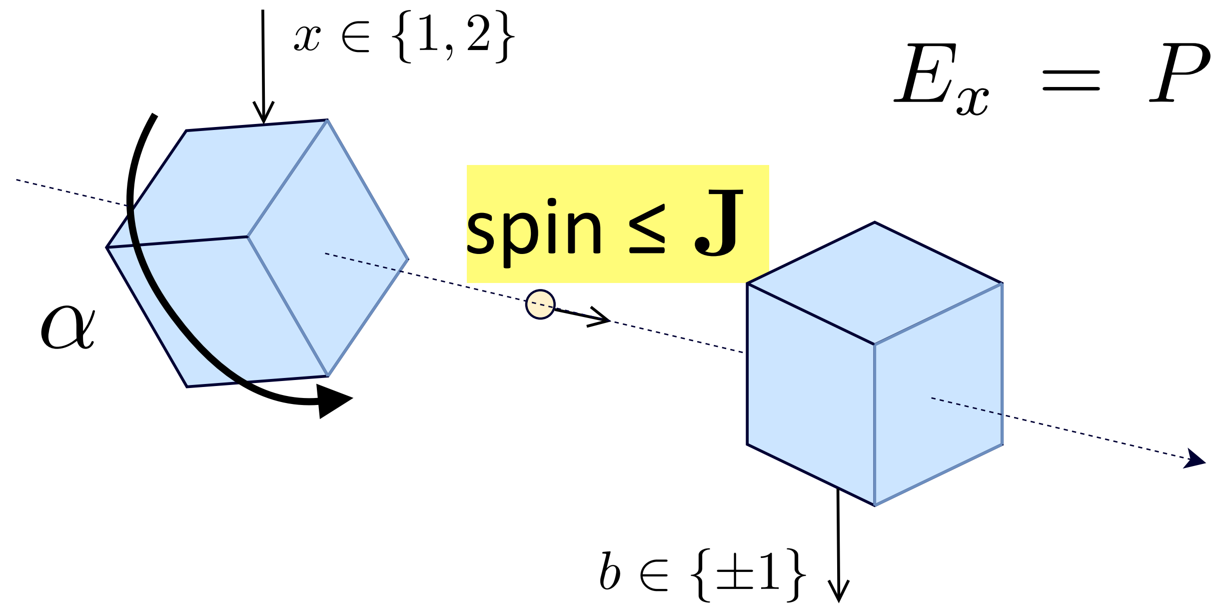

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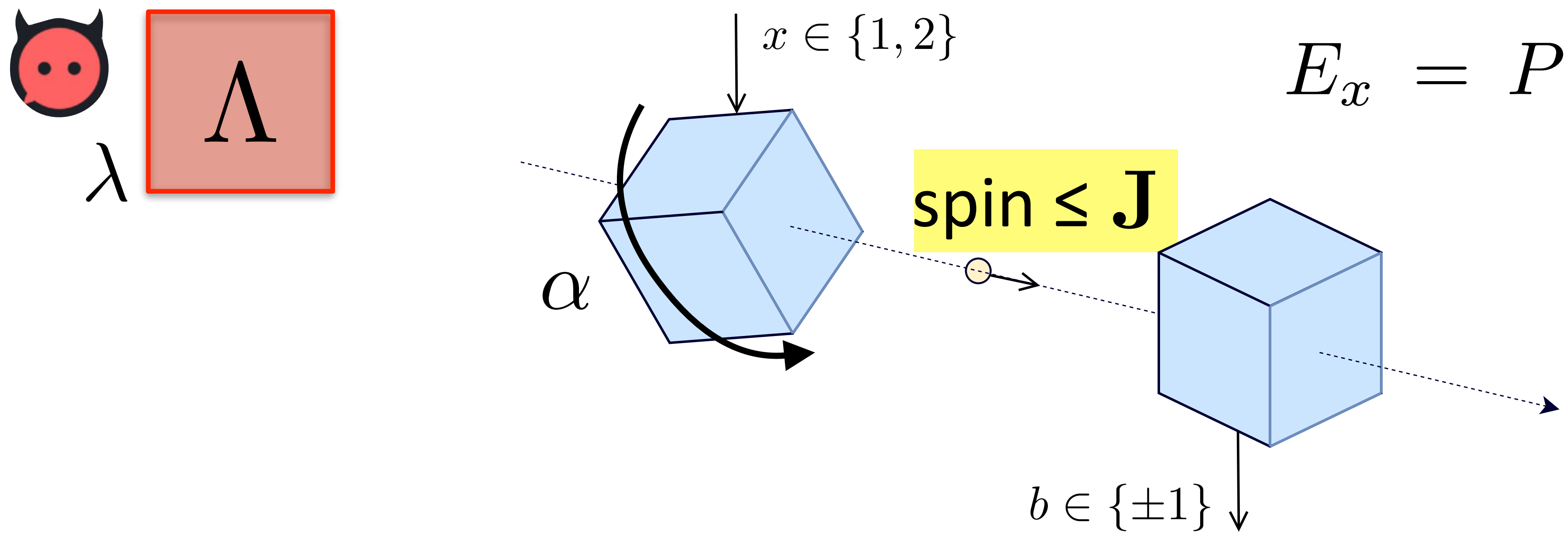

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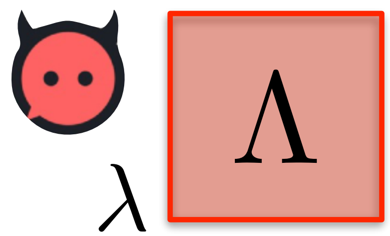

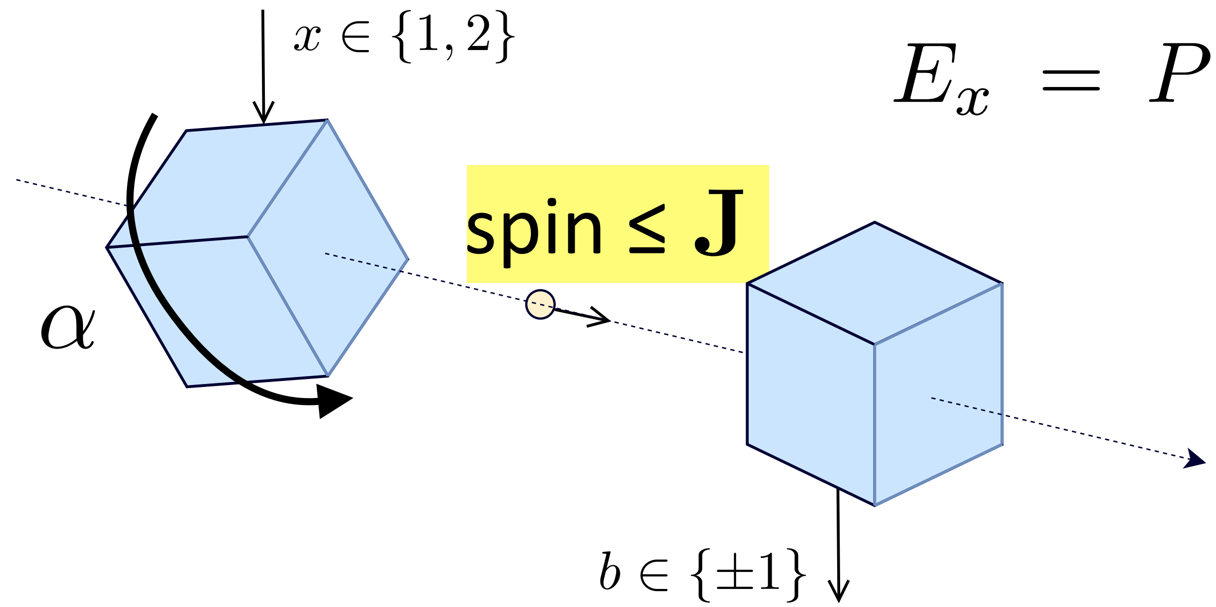

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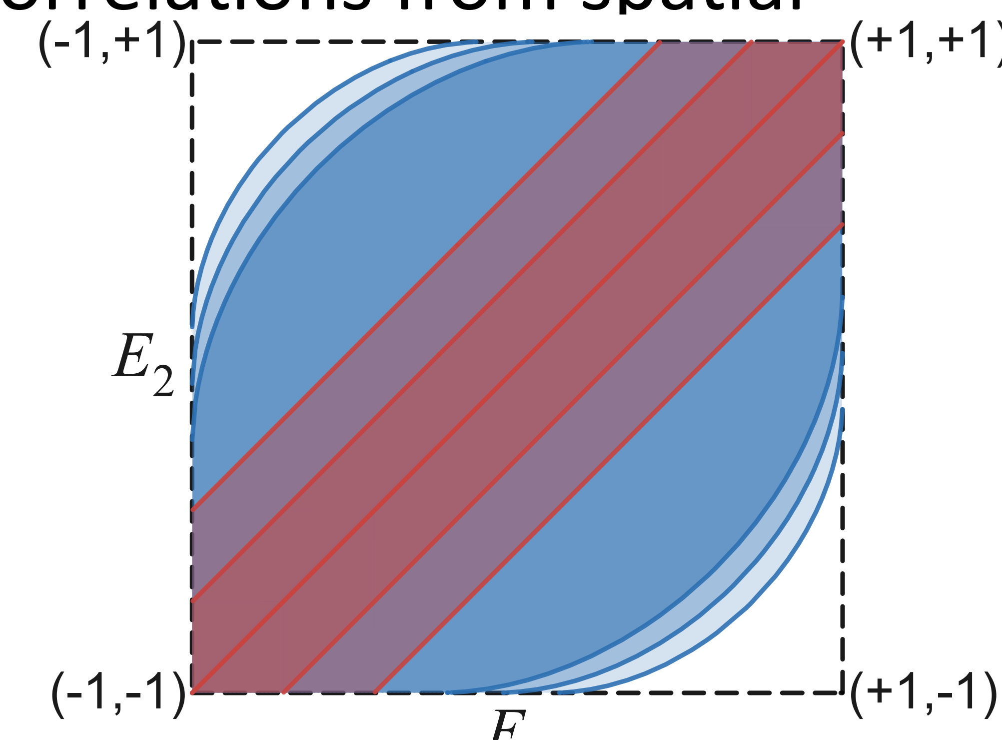

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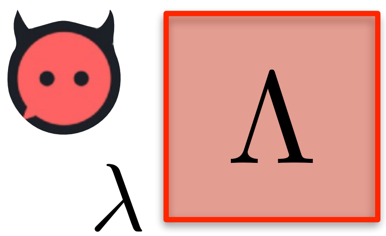

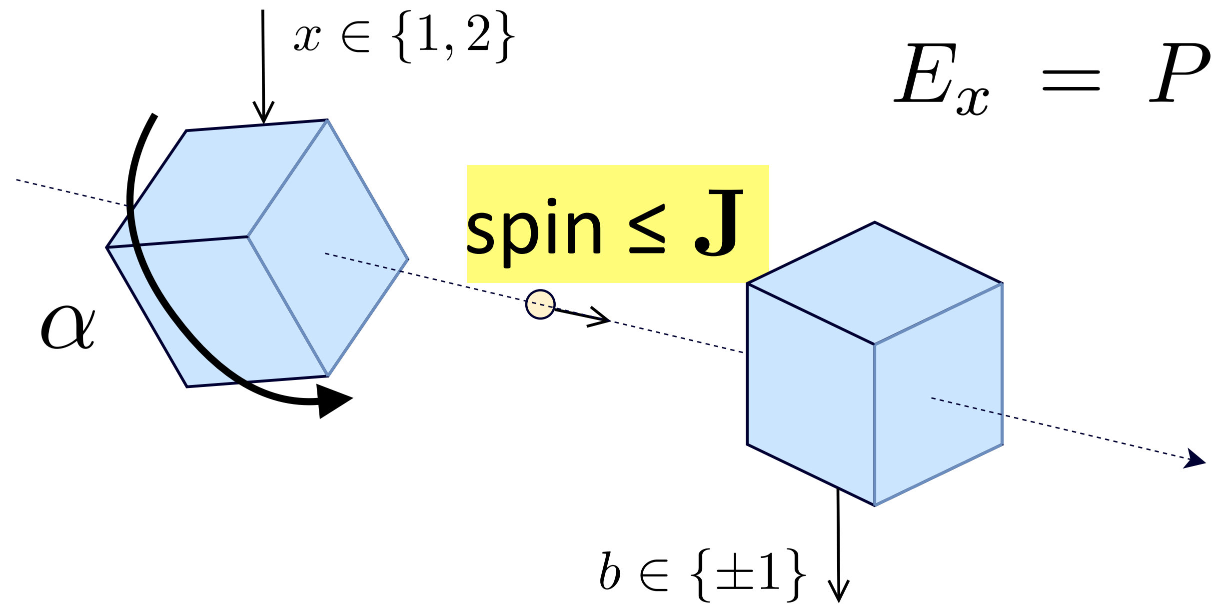

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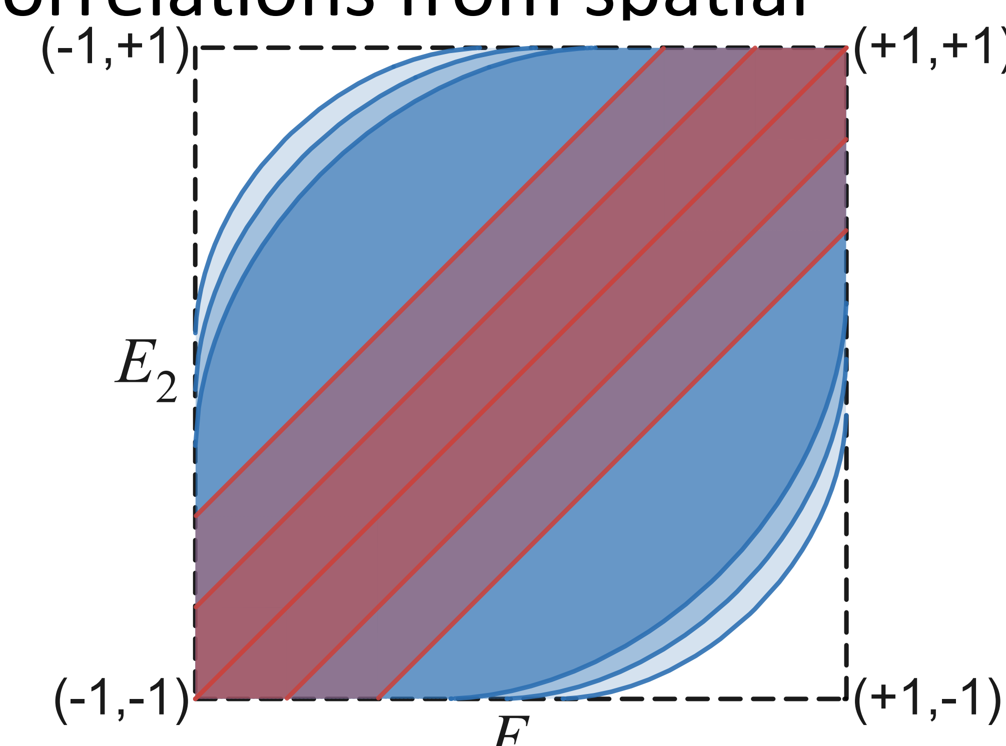

Yes we can!

Rotation boxes beyond quantum theory

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$$\mathcal{Q}_J := \left\{ \alpha \mapsto p(+1|\alpha) \mid p(b|\alpha) = \mathrm{tr}(E_b U_\alpha \rho U_\alpha^\dagger) \right\},$$

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$$U_\alpha = \bigoplus_{j=-J}^{J} n_j e^{ij\alpha},$$ with arbitrary multiplicities n_j.

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Consequence: every *p* is a trigonometric polynomial of degree 2**J**

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$$\mathcal{R}_J := \left\{ \alpha \mapsto p(+1|\alpha) = c_0 + \sum_{j=1}^{2J} c_j \cos(j\alpha) + s_j \sin(j\alpha) \right\},$$

$0 \leq p(+1|\alpha) \leq 1$ for all α.

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- Definition of (general) **spin-J rotation boxes**:

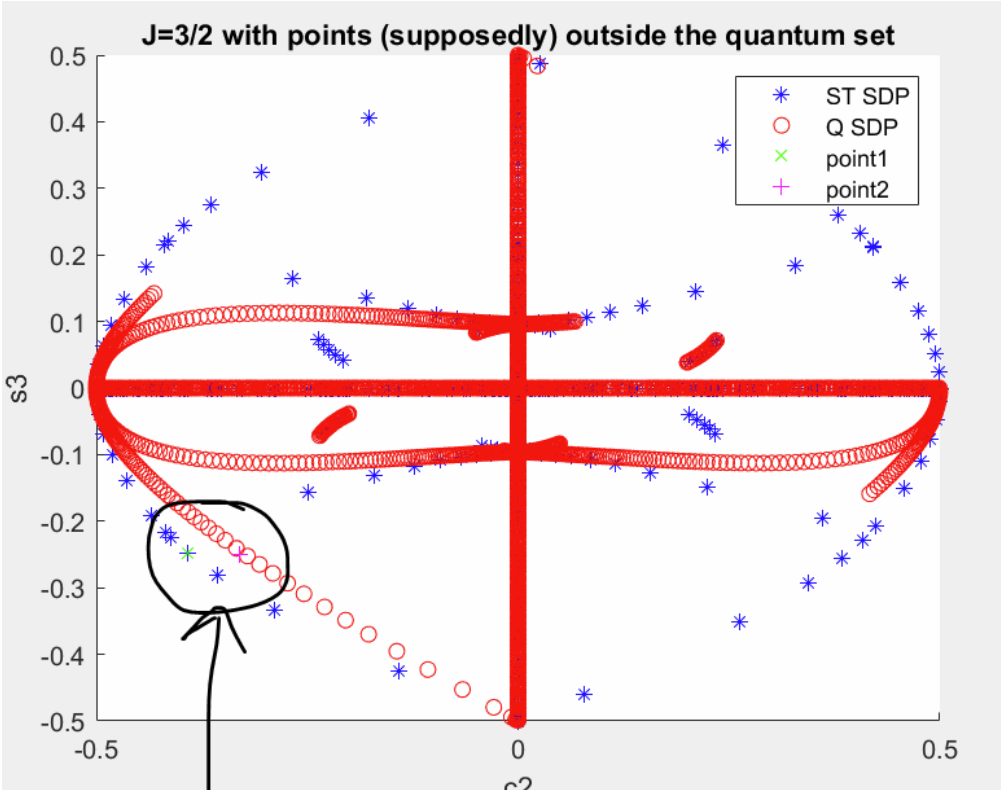
$$\mathcal{R}_J := \left\{\alpha \mapsto p(+1|\alpha) = c_0 + \sum_{j=1}^{2J} c_j \cos(j\alpha) + s_j \sin(j\alpha)\right\},$$

Clearly $\mathcal{Q}_J \subseteq \mathcal{R}_J$.

It can be shown directly that $\mathcal{Q}_{1/2} = \mathcal{R}_{1/2}$.

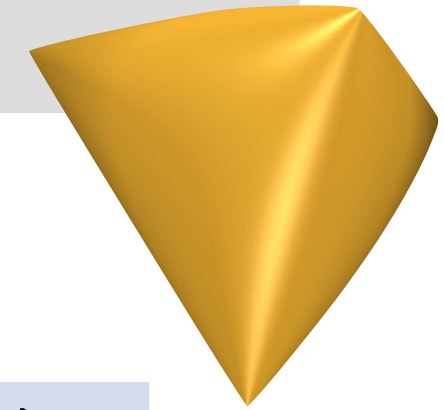
Upcoming paper (mid-2023): $\mathcal{Q}_{3/2} \subsetneq \mathcal{R}_{3/2}$.

We do not know whether $\mathcal{Q}_1 = \mathcal{R}_1$,
but numerics suggests equality!



Rotation boxes beyond quantum theory

Quantum boxes: real representation of SO(2) on the density matrices.
Rotation boxes: real rep. of SO(2) on "orbitope" state spaces.



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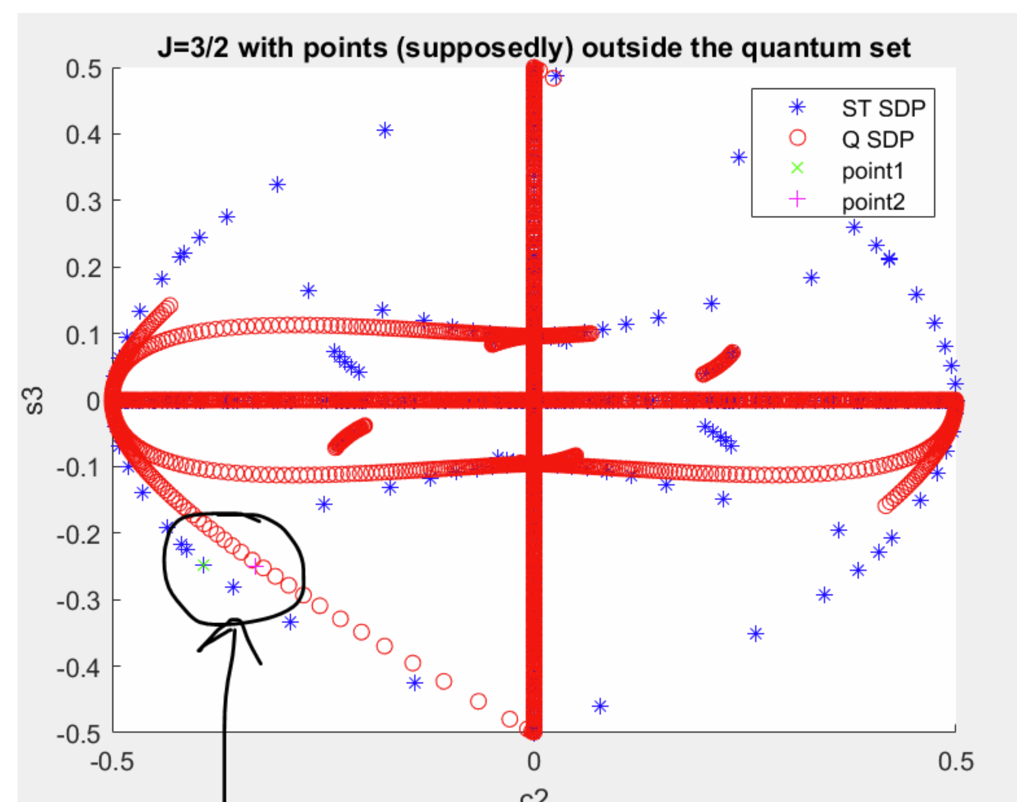

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$$\mathcal{Q}_{J,\alpha} = \{(E_1, E_2) \quad | \quad E_1 = P(+1|0) - P(-1|0), E_2 = P(+1|\alpha) - P(-1|\alpha),$$
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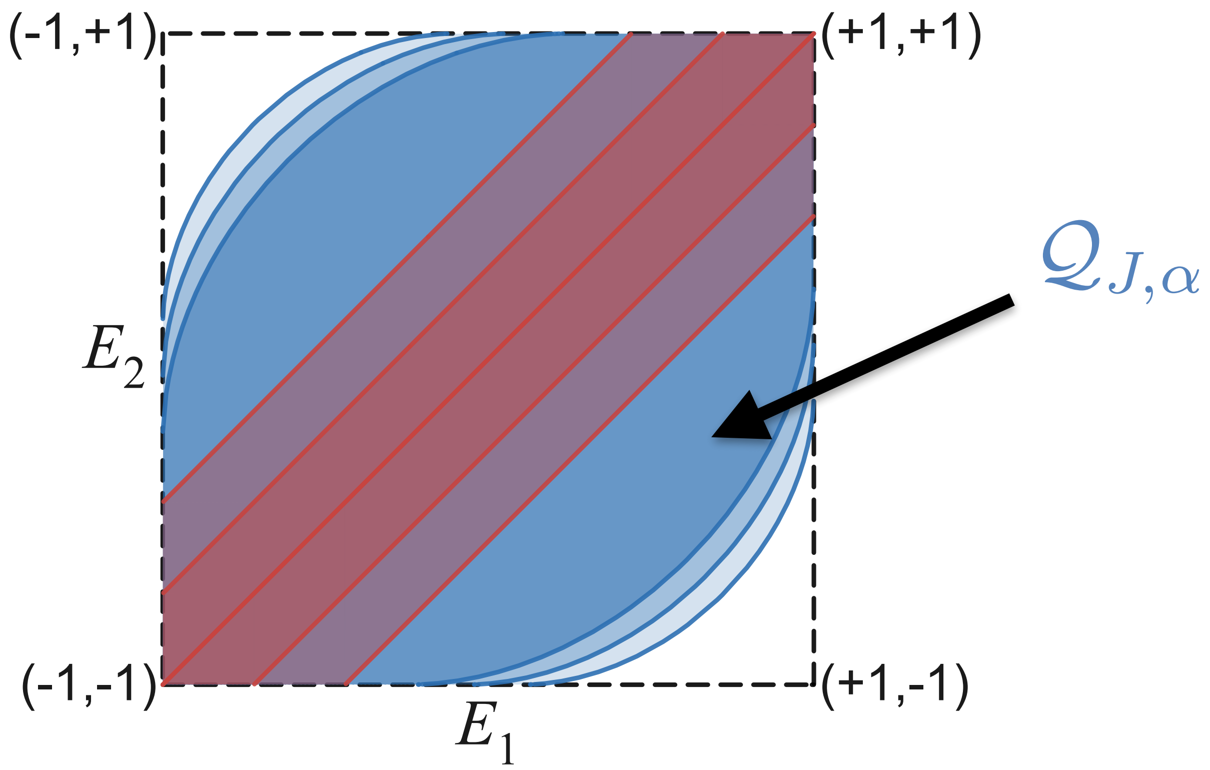

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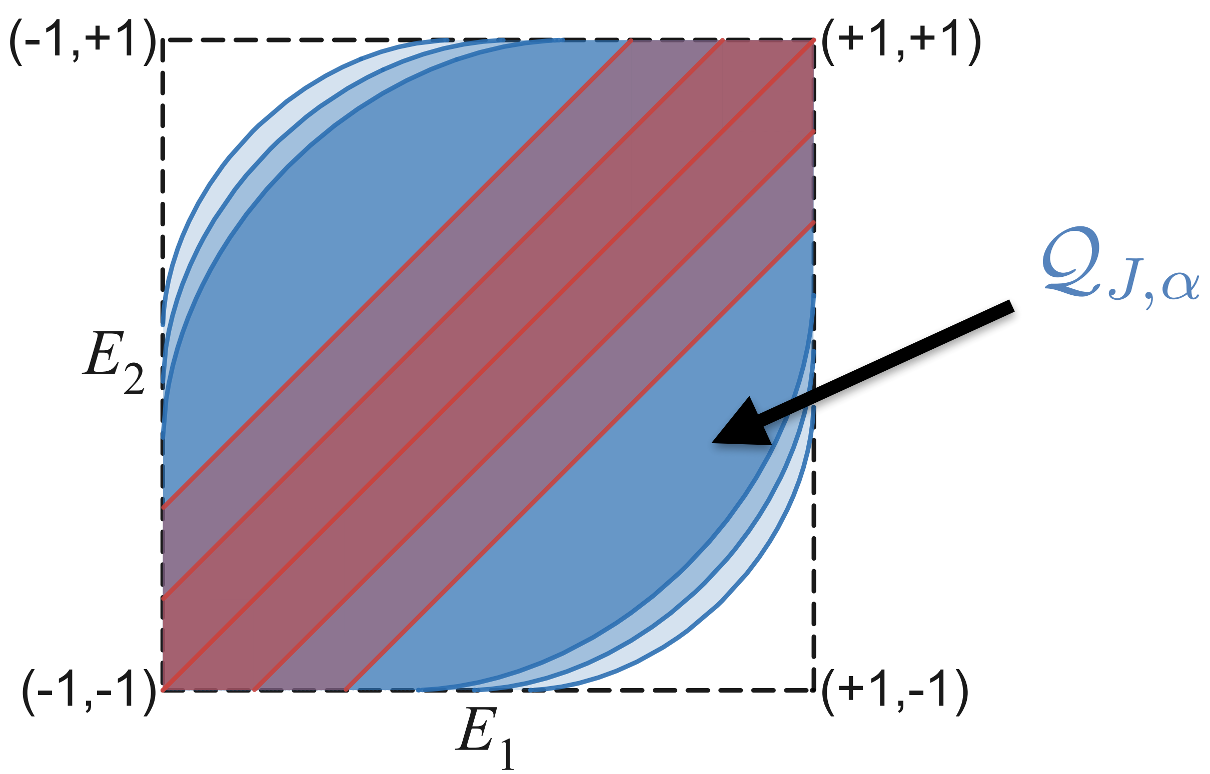

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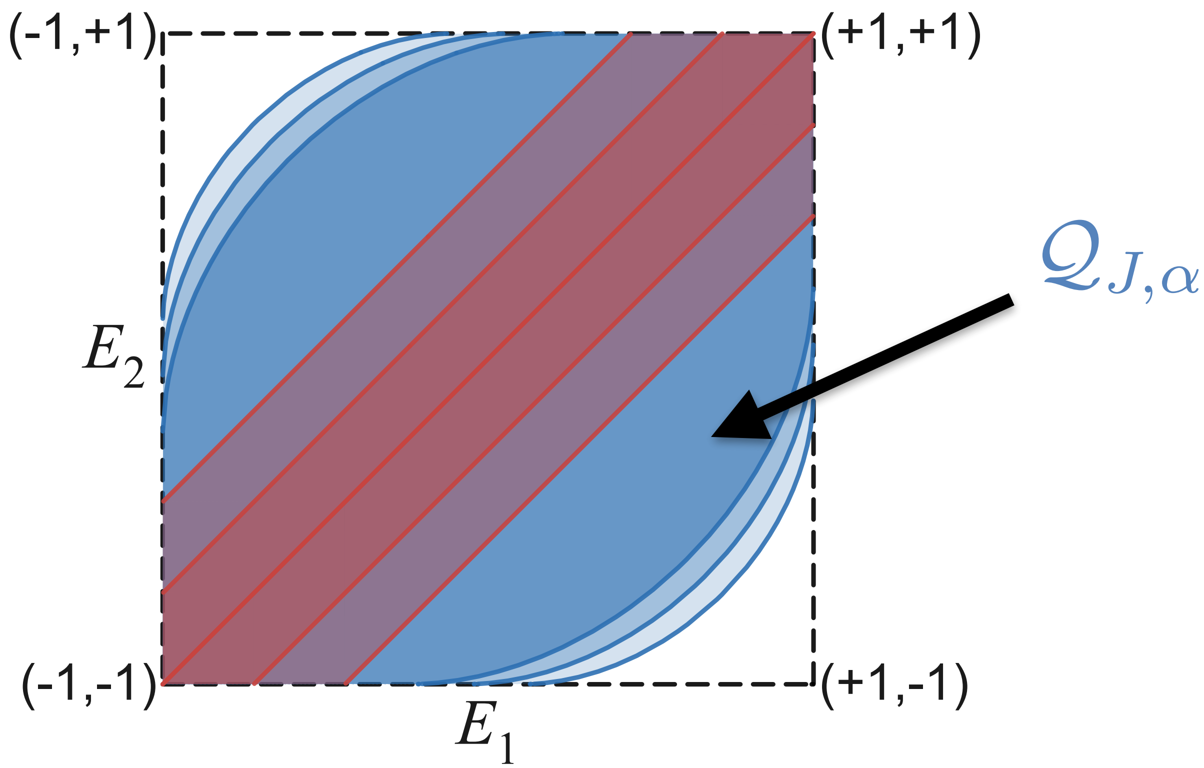

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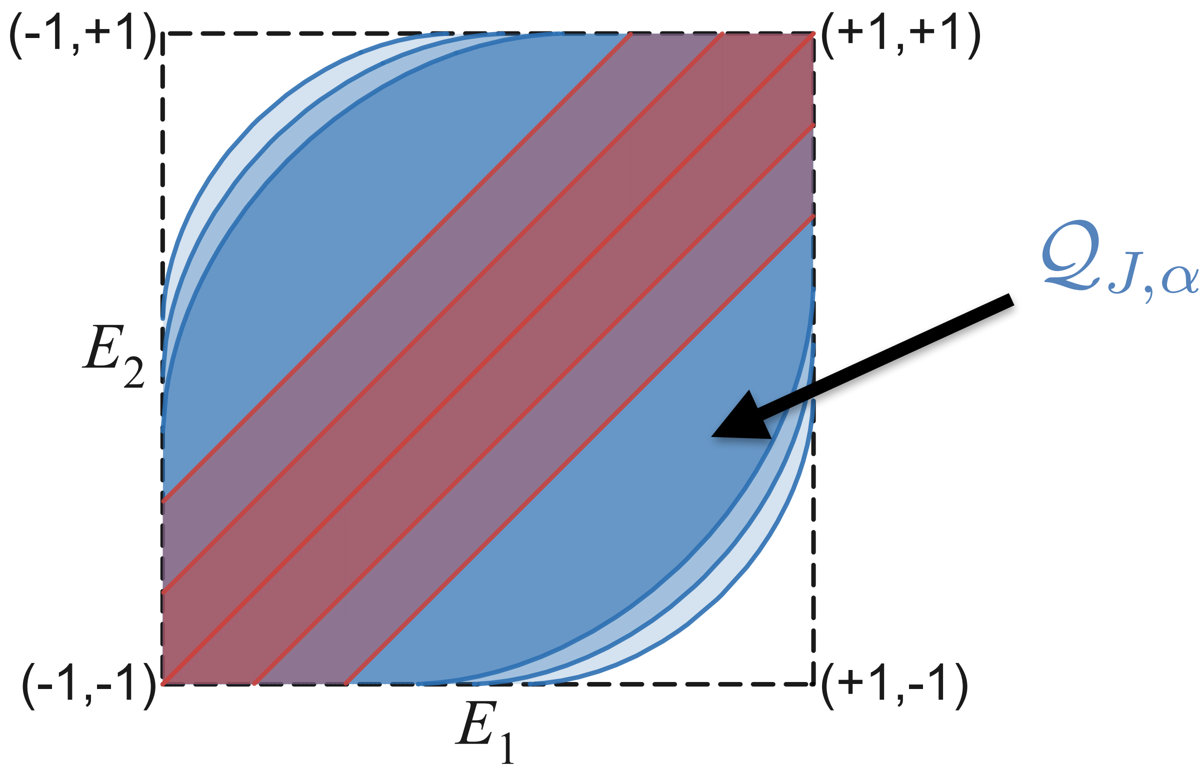

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We do not need to assume QT to derive the blue set of correlations!

Consequences of $\mathcal{Q}_{J,\alpha} = \mathcal{R}_{J,\alpha}$

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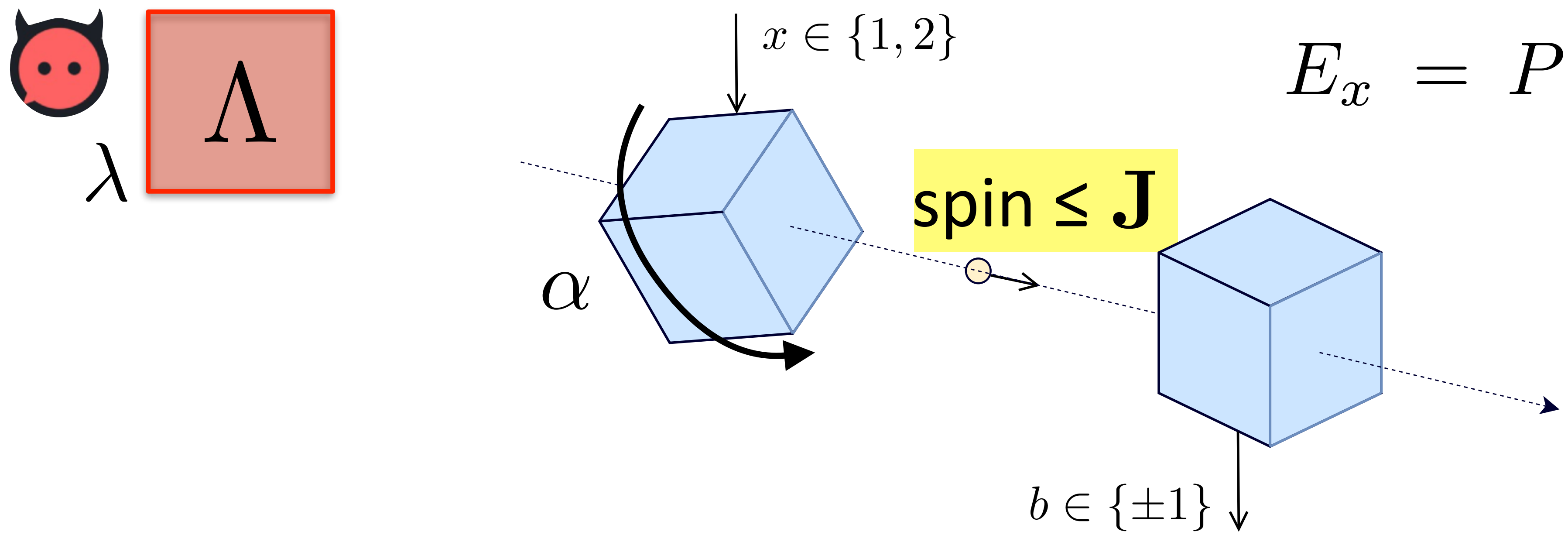

$$E_x = P(+1|x) - P(-1|x)$$

$$P(b|x)$$

Theorem. The following correlations are possible:

$$\frac{1}{2}\left(\sqrt{1+E_1}\sqrt{1+E_2}+\sqrt{1-E_1}\sqrt{1-E_2}\right) \geq \begin{cases} \cos(J\alpha) & \text{if } |J\alpha| < \frac{\pi}{2} \\ 0 & \text{if } |J\alpha| \geq \frac{\pi}{2} \end{cases}$$

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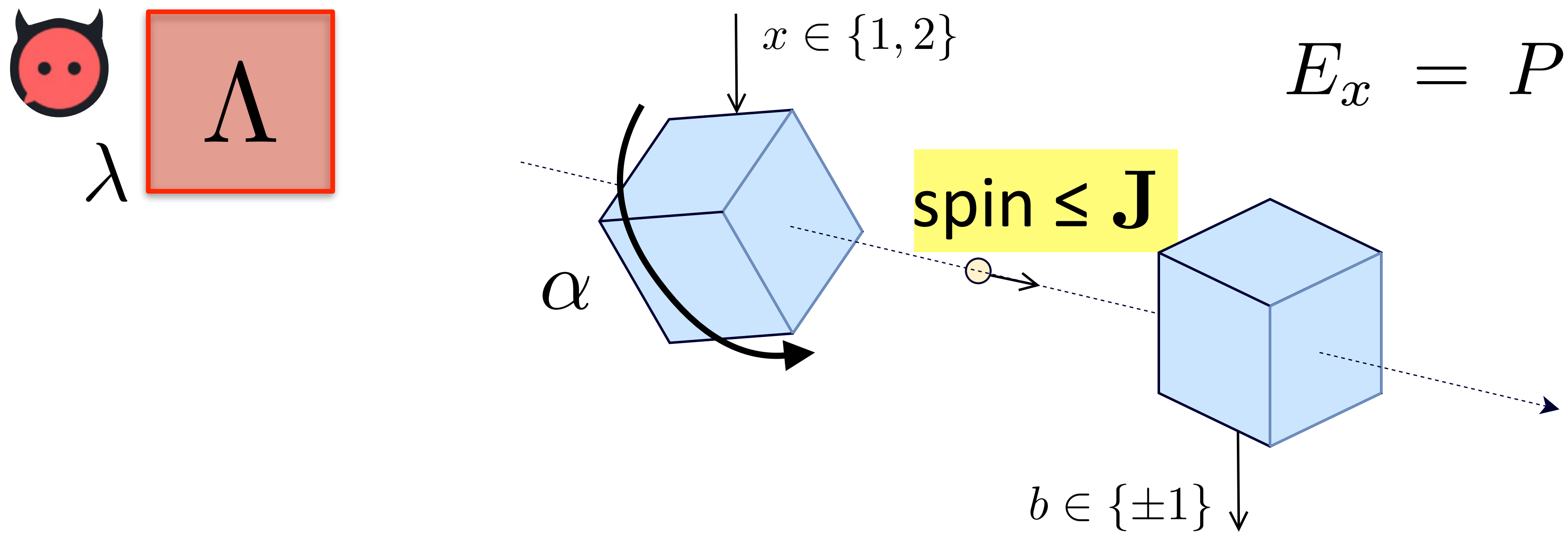

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All results for our protocol remain valid beyond QT:

- The set of correlations,
- the number of certifiable random bits,
- security against eavesdropper with classical side information…

… and this may include information about **beyond-quantum systems** that are sent between the devices (whose average is quantum).

Overview

1. Motivations: QG and device-independent QIT

2. Our protocol, and its quantum analysis

3. Rotation boxes beyond quantum theory

4. Conclusions

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Conclusions

- Modest approach complementing direct QG approaches: use SDI quantum information to study the **relation between spacetime and QT**.
- Simplest setup: rotations around fixed axis, but can study more general setups. **"Spacetime boxes"**.

$$\mathrm{SO}(2) \subset \mathrm{SO}(3) \subset \mathrm{SO}(3,1)$$

- Result: protocols can be formulated and analyzed without assuming QT. Sets of correlations agreed in our case!
  $\longrightarrow$ Many actual experiments work on spatiotemporal DOFs. Our approach may admit a **theory-agnostic analysis** and security proofs.
- Spacetime structure determines part of quantum correlations.

arXiv:2210.14811