ÖAW AUSTRIAN ACADEMY OF SCIENCES

IQOQI – INSTITUTE FOR QUANTUM OPTICS AND QUANTUM INFORMATION VIENNA

PERIMETER INSTITUTE

# It From Qubit underground:

## how quantum theory and spacetime constrain each other


**Markus P. Müller**

IQOQI Vienna & Perimeter Institute


FWF Der Wissenschaftsfonds.

# Why “underground”?

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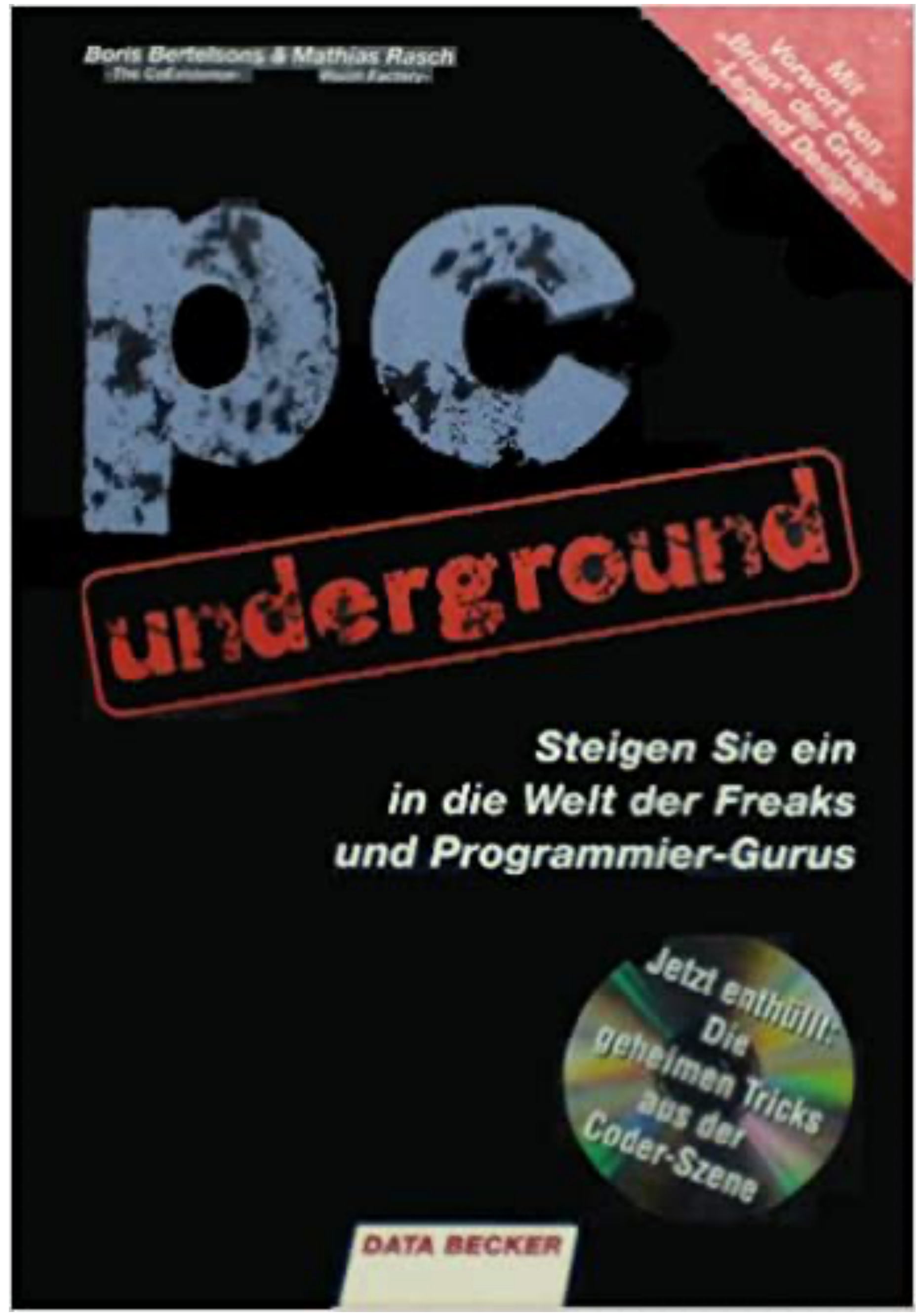

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Simons Collaboration on
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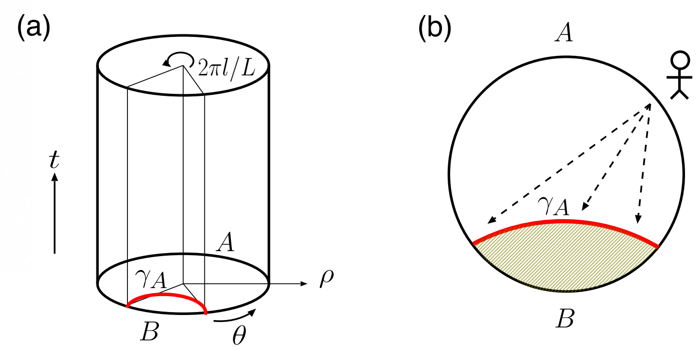


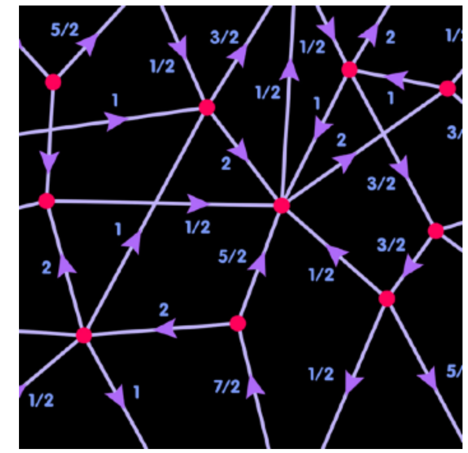


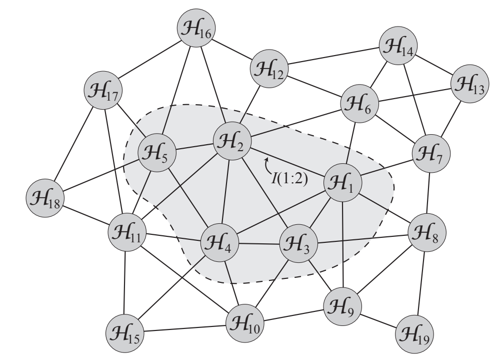

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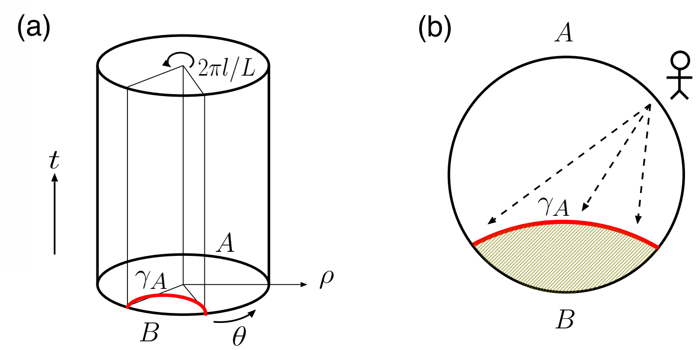


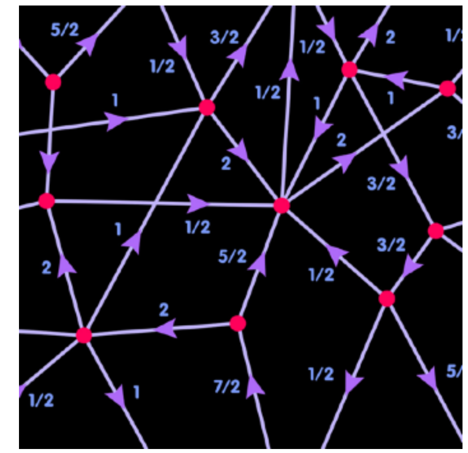

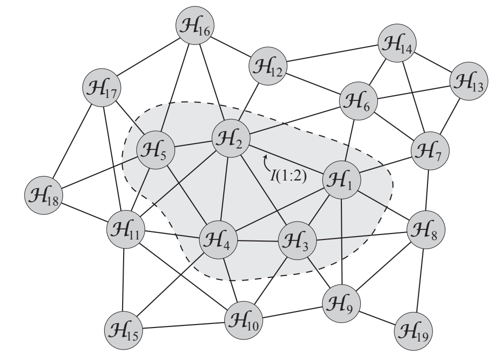


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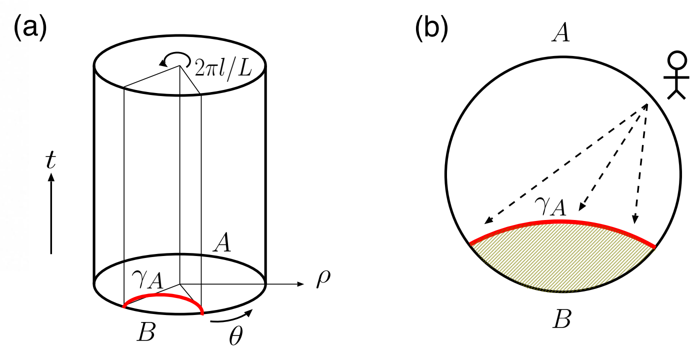


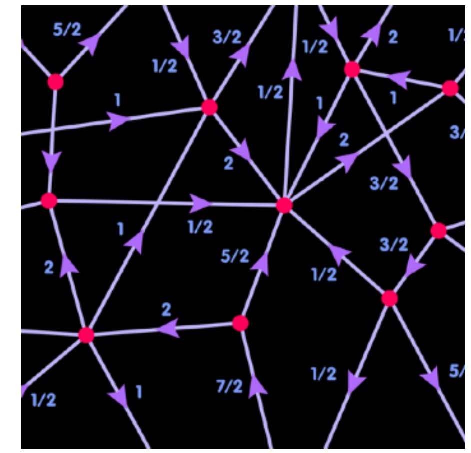

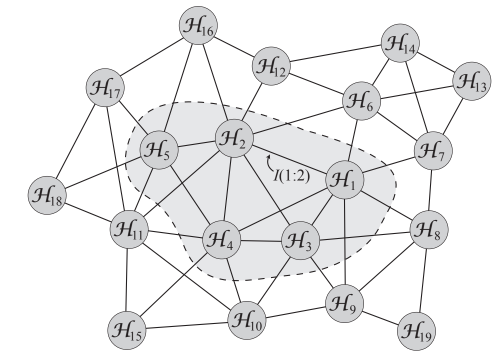

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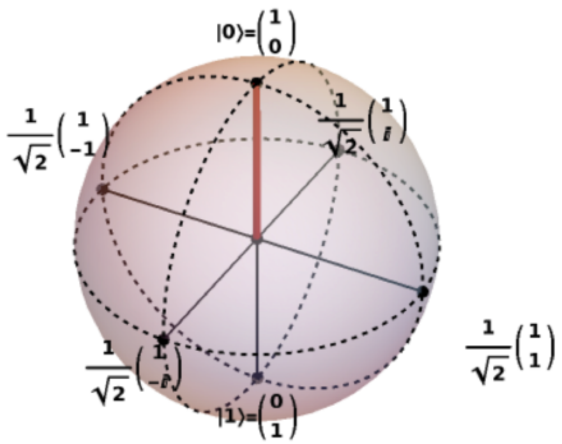
Most of this research:

structure of QT $\longleftarrow$ structure of spacetime
$\not\longrightarrow$

# Invitation: the qubit and spatial rotations

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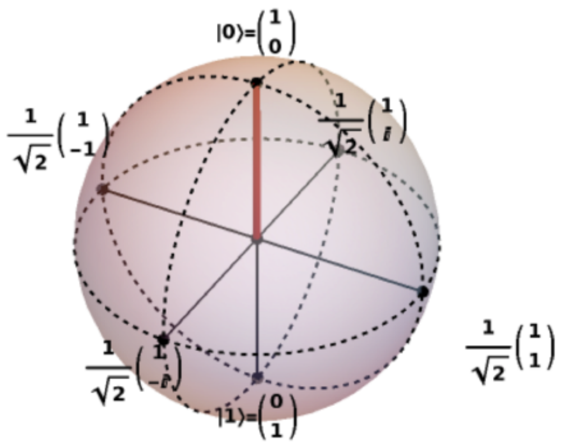

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$$\rho = \frac{1}{2}\mathbf{1} + \vec{r}\cdot\vec{\sigma} = \frac{1}{2}\begin{pmatrix} 1+z & x-iy \\ x+iy & 1-z \end{pmatrix}$$

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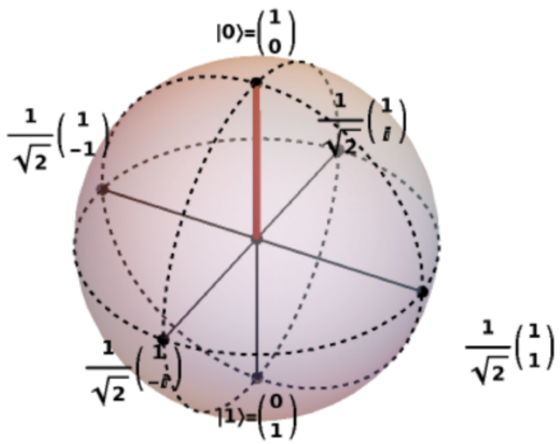

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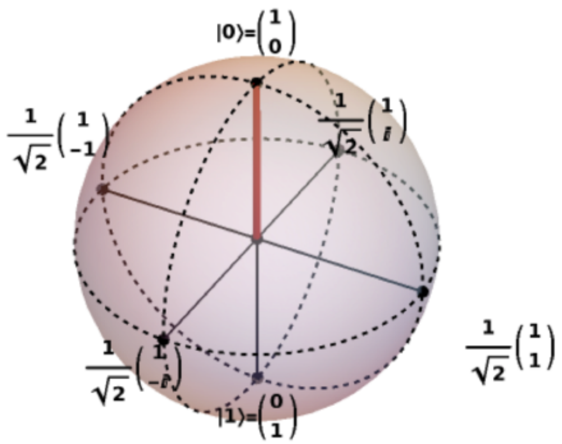

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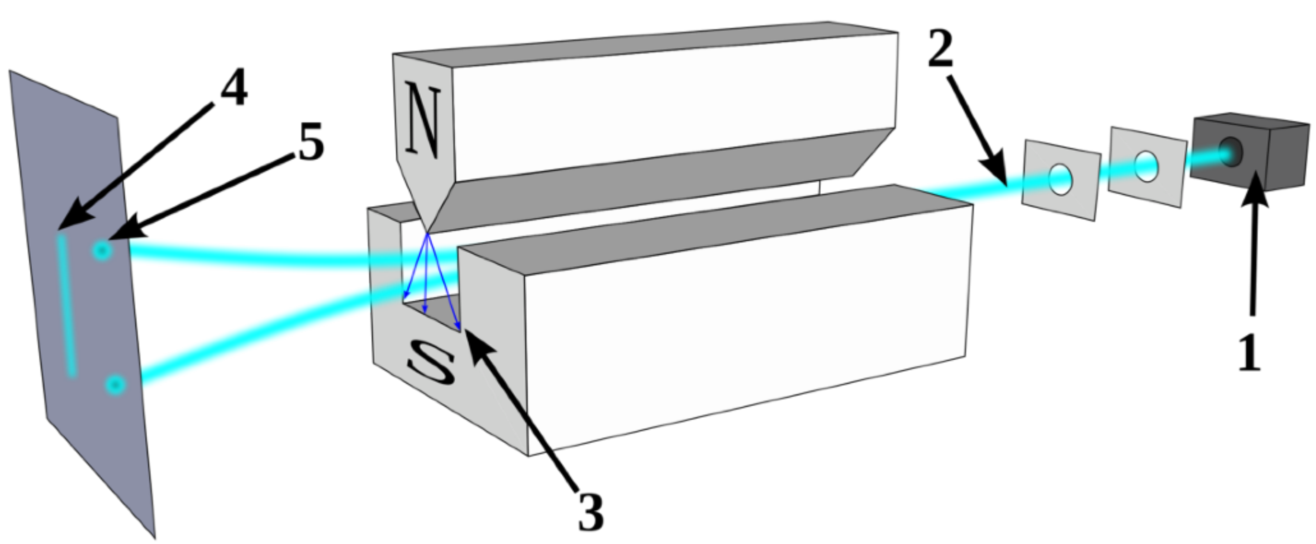

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1. Motivation

2. Some history: von Weizsäcker & Wootters

3. Relativity of simultaneity and the qubit

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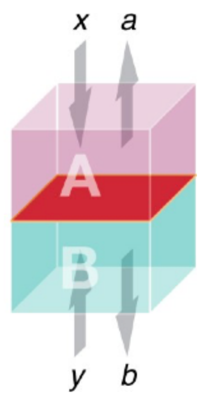


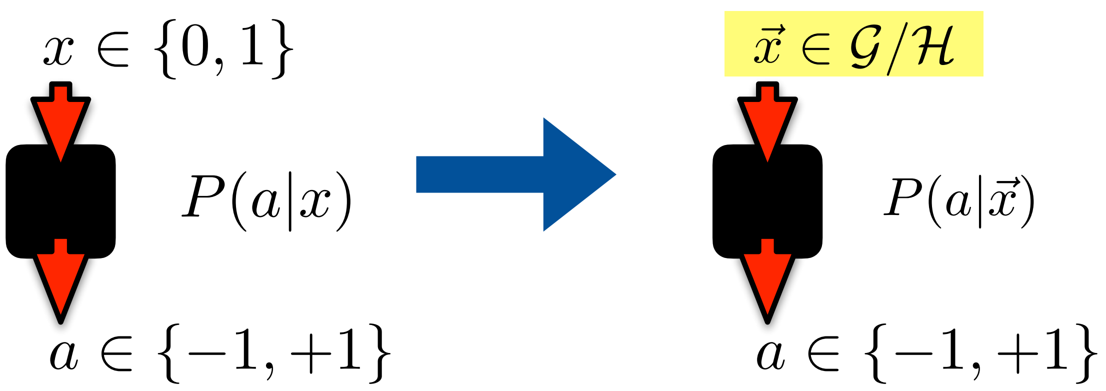

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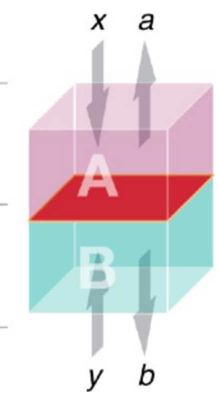


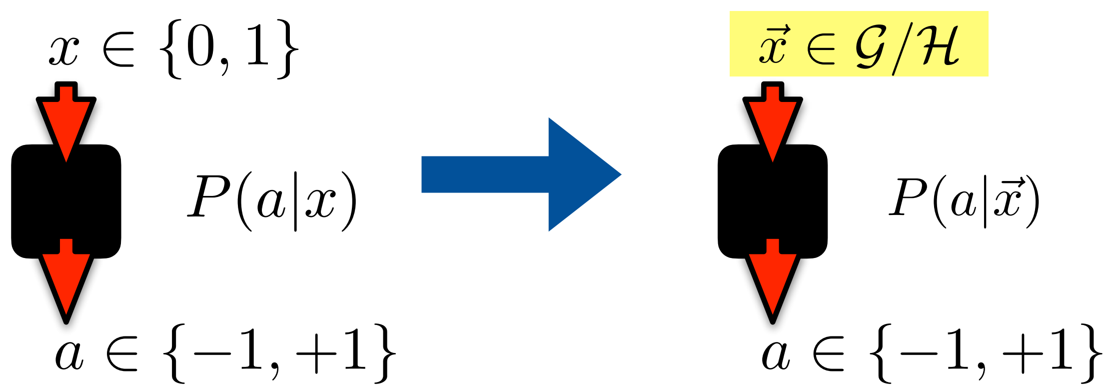


- Rigorous insights into the structural architecture of physics (geometry vs. probability)

  $\longrightarrow$ very modest steps towards QG?

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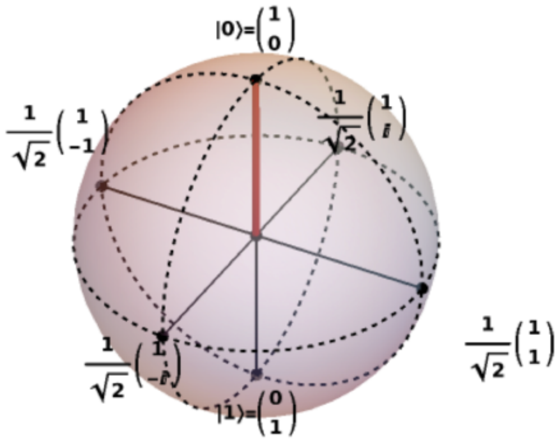

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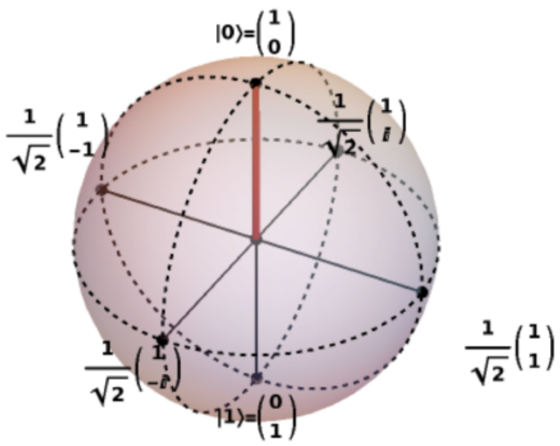

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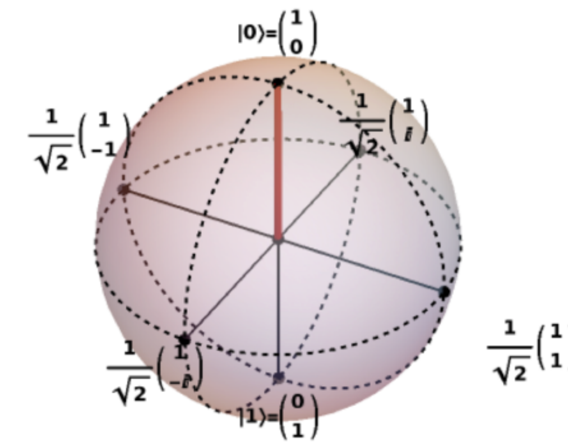

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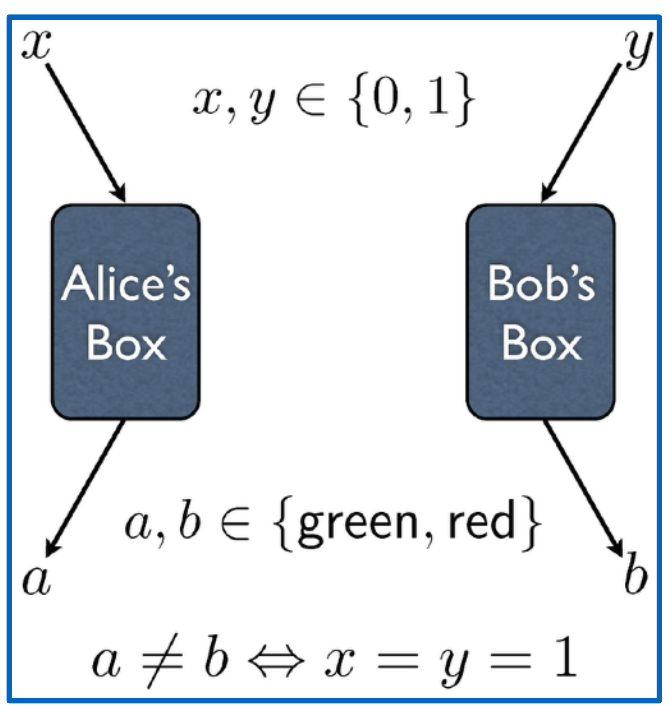
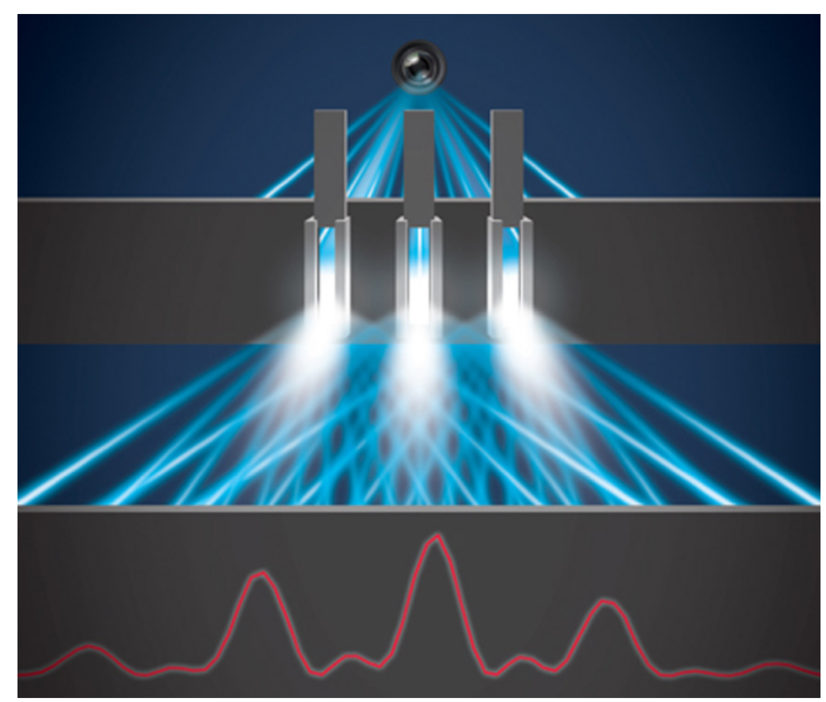



**superstrong nonlocality?**



**higher-order interference?**

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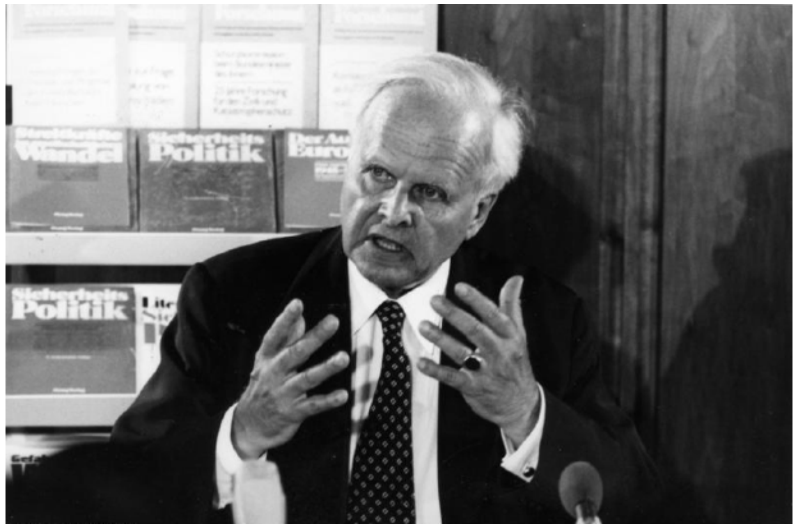
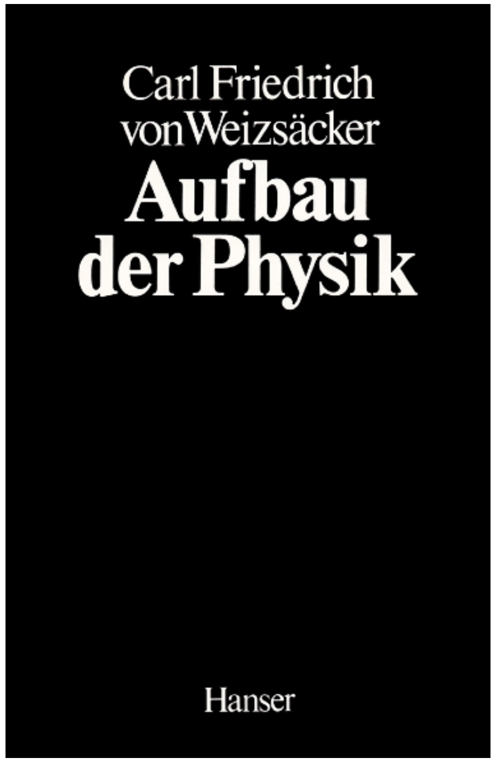


Carl-Friedrich von Weizsäcker
(1912-2007)

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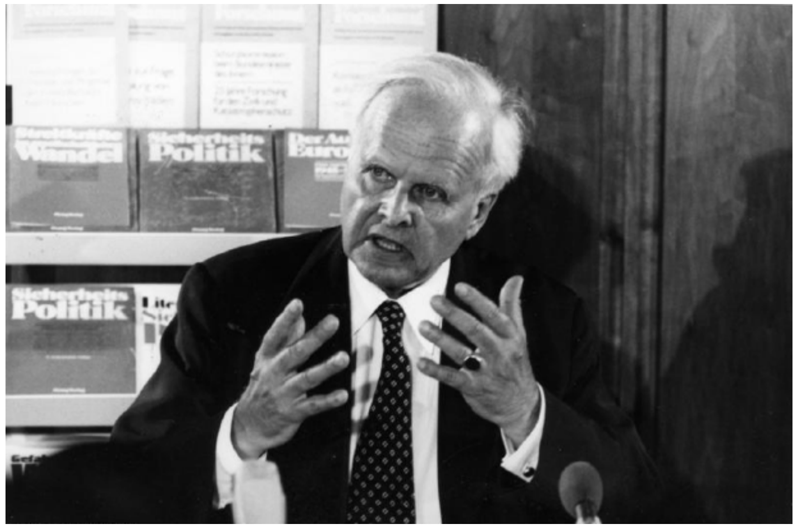

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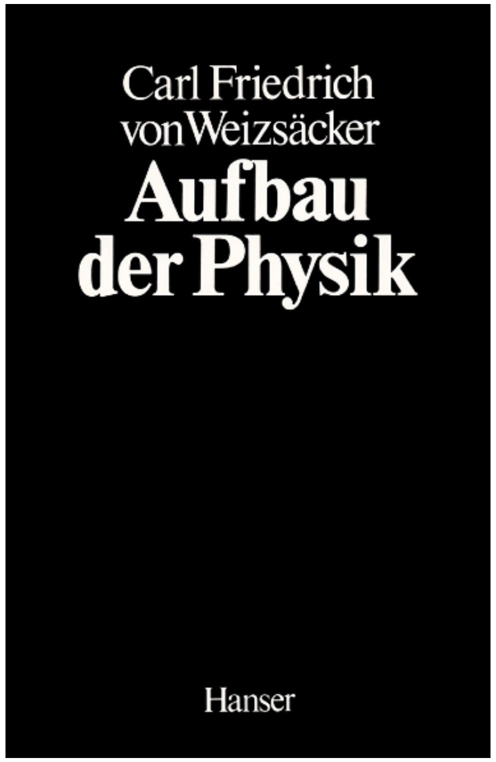


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Summary via lyre.de/urinfo.htm (imperfect English translation is mine):

Von Weizsäcker gives the following definition of the central notion of "ur-alternative":
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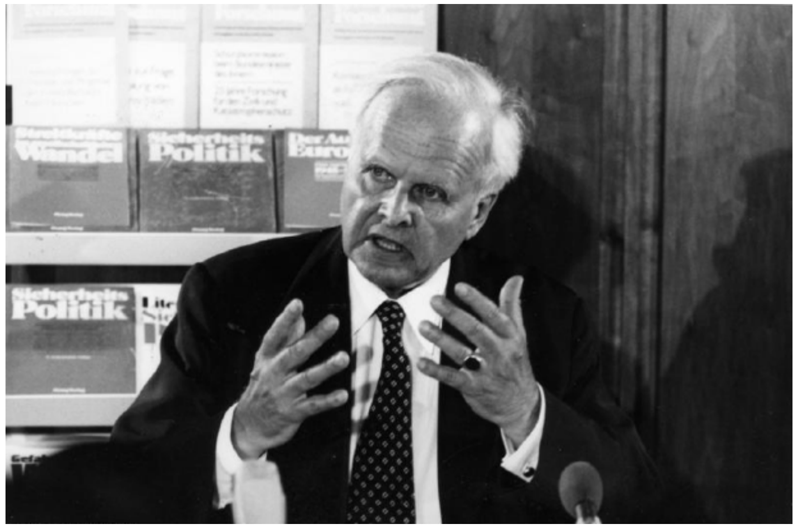

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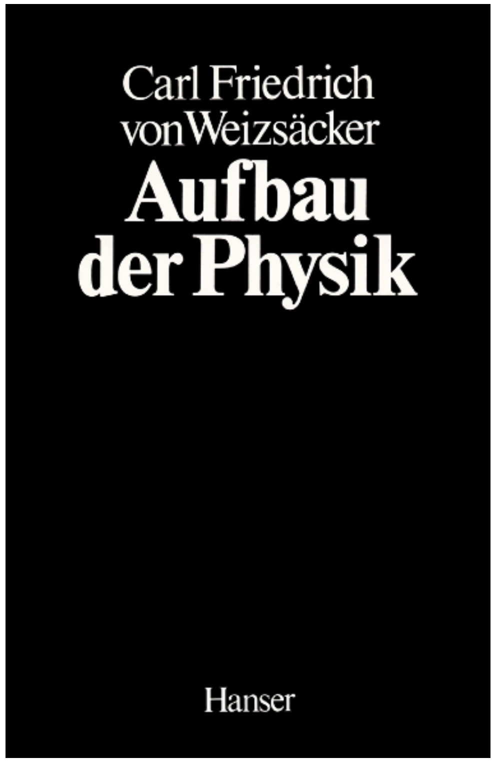


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1955 and 1958: Out of the three works by von Weizsäcker under the title *Complementarity and Logic*, the third one in particular contains the basic mathematical motive for ur theory: *"The special theory of relativity, insofar as it is a theory of space and time, is already the quantum theory of a deeper simple alternative. The Lorentz group is an (unfaithful) real representation of the group of complex linear transformations of the quantum-mechanical state space of that alternative."*

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This assumption can be motivated as follows: one can regard space as the parameter space of interaction strength. A suitable model for space would hence be the parameter space of SU(2) itself, i.e. SU(2) as a homogeneous space. This is topologically an S^3, i.e. the unit sphere of R^4, i.e. isomorphic to the spatial part of an Einstein cosmos. Thus, in ur theory, SU(2) is itself seen as an obvious, approximate model of the cosmos.

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Up to this point we have used urs as spinorial wavefunctions, i.e., we considered an ur as the first step of quantization of a simple alternative. The second quantization is done by the replacement $u_r \to \hat{a}_r$ and $u_r^* \to \hat{a}_r^+$ and the Bose commutation relations

$$[\hat{a}_r, \hat{a}_s^+] = \delta_{rs}, \qquad [\hat{a}_r, \hat{a}_s] = [\hat{a}_r^+, \hat{a}_s^+] = 0. \tag{26}$$

Thus, we get a quantum field theory of urs, i.e., a many-ur - theory with a variable number of urs. Consequently from (26) the quantization of the ur-tetrad (22) - (25) follows. We use a special choise of the components of the bispinorial ur $\binom{u}{u^*}$, i.e., $u_r$ with $r$=1...4 ($u^*$ denotes an

H. Lyre, *Quantum Space-Time and Tetrads*, Int. J. Theor. Phys. **37**, 393—400 (1998).

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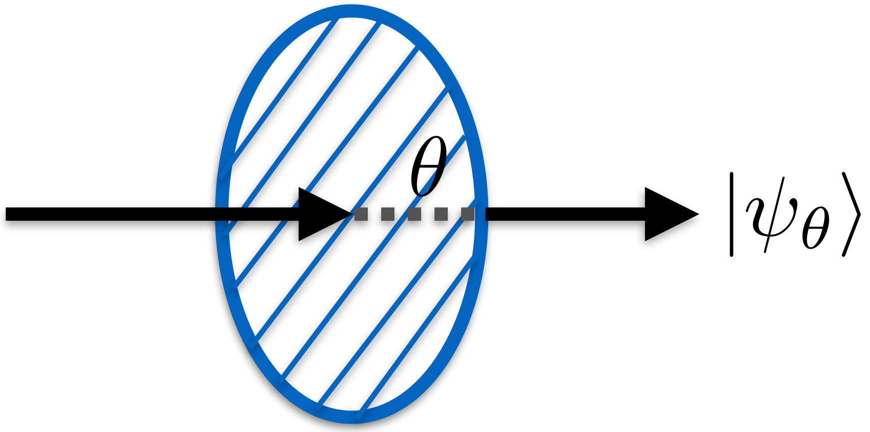

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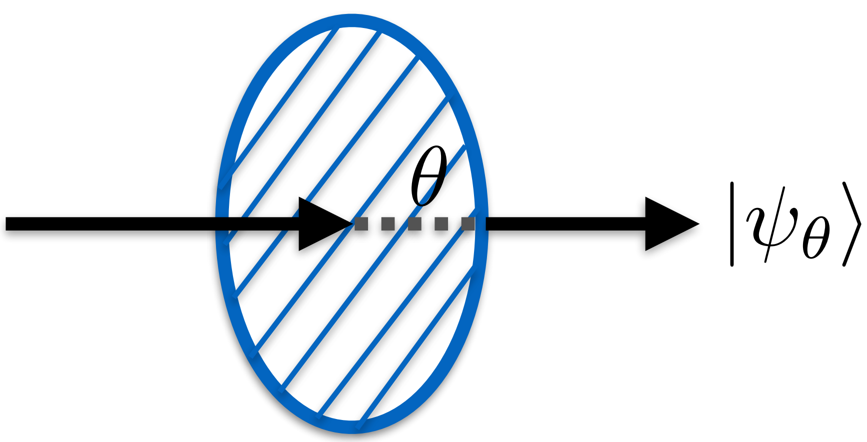


Shown **without** assuming QM:

$$\underbrace{d(\psi_\theta, \psi_{\theta'})}_{\text{statistical distance for yes-no measurement}} = c \cdot \underbrace{|\theta - \theta'|}_{\text{"actual" distance of angles}}$$

$$\Leftrightarrow p(\theta) = \cos^2 \frac{n}{2}(\theta - \theta_0)$$

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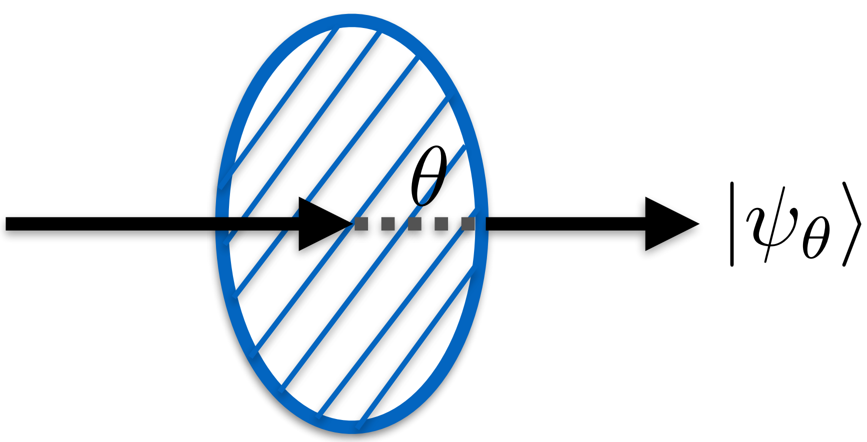


$$d(\theta_1,\theta_2)=\frac{1}{\sqrt{n}}\int_{\theta_1}^{\theta_2}\frac{d\theta}{2\Delta\theta}=\int_{\theta_1}^{\theta_2}d\theta\,\frac{|dp/d\theta|}{2[p(1-p)]^{1/2}}$$

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PHYSICAL REVIEW D VOLUME 23, NUMBER 2 15 JANUARY 1981

## Statistical distance and Hilbert space*

W. K. Wootters

*Center for Theoretical Physics, University of Texas, Austin, Texas 78712*

(Received 2 September 1980)

A concept of “statistical distance” is defined between different preparations of the same quantum system, or in other words, between different rays in the same Hilbert space. Statistical distance is determined entirely by the size of statistical fluctuations occurring in measurements designed to distinguish one state from another. It is not related, *a priori*, to the usual distance (or angle) between rays. One finds, however, that these two kinds of distance are in fact the same, a result which depends on certain peculiarities of quantum mechanics.

# Bill Wootters: “statistical distance equals actual distance”

PHYSICAL REVIEW D VOLUME 23, NUMBER 2 15 JANUARY 1981

## Statistical distance and Hilbert space*

W. K. Wootters
*Center for Theoretical Physics, University of Texas, Austin, Texas 78712*

(Received 2 September 1980)

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$$d(\theta_1, \theta_2) = \lim_{n \to \infty} \frac{1}{\sqrt{n}} \times [\text{maximum number of intermediate orientations each of which is distinguishable (in } n \text{ trials) from its neighbors}]. \tag{3}$$

Let us now suppose that the experimenter, in making his determination of the value of $\theta$, has only a limited number of photons to work with, so that precisely $n$ photons actually pass through the filter to be analyzed by the nicol prism. Then, because of the statistical fluctuations associated with a finite sample, the observed frequency of occurrence of yes is only an approximation to the actual probability of yes, the typical error being of the order of $n^{-1/2}$. More precisely, the experimenter's uncertainty (root-mean-square deviation) in the value of $p$ is[1,2]

$$\Delta p = \left[\frac{p(1-p)}{n}\right]^{1/2}$$

$$d(\theta_1, \theta_2) = \frac{1}{\sqrt{n}} \int_{\theta_1}^{\theta_2} \frac{d\theta}{2\Delta\theta} = \int_{\theta_1}^{\theta_2} d\theta \, \frac{|dp/d\theta|}{2[p(1-p)]^{1/2}}$$

# Today

1. Motivation

2. Some history: von Weizsäcker & Wootters

3. Relativity of simultaneity and the qubit

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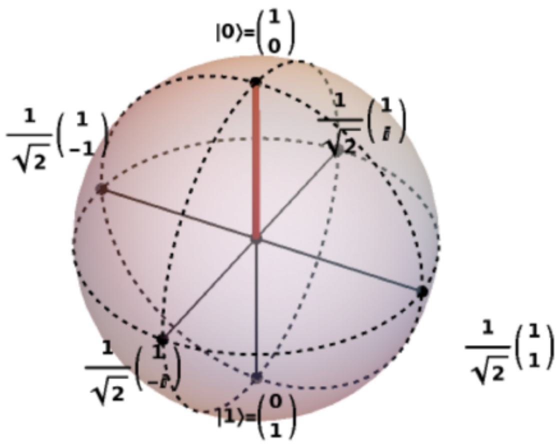
# Alternative bit state spaces



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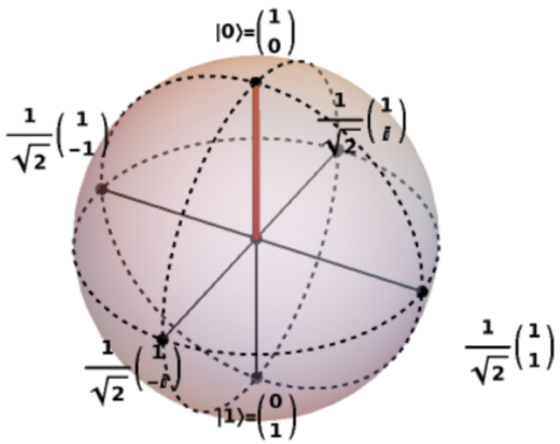

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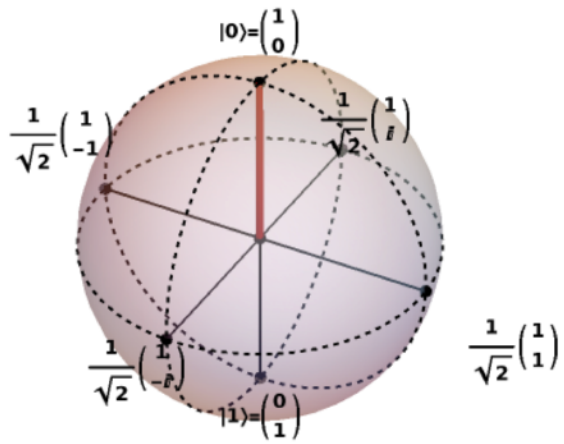


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$d = 1$ bit

$d = 2$ $\mathbb{R}-$qubit

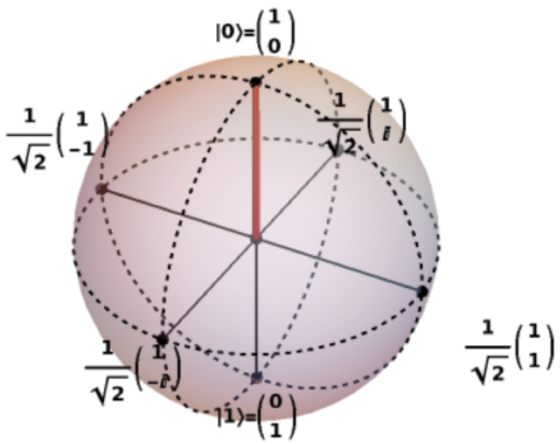
$d = 3$ $\mathbb{C}-$qubit

$d = 4$

$d = 5$ $\mathbb{H}-$qubit

...

# Alternative bit state spaces



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| $d=1$ | $d=2$ | $d=3$ | $d=4$ | $d=5$ | … |
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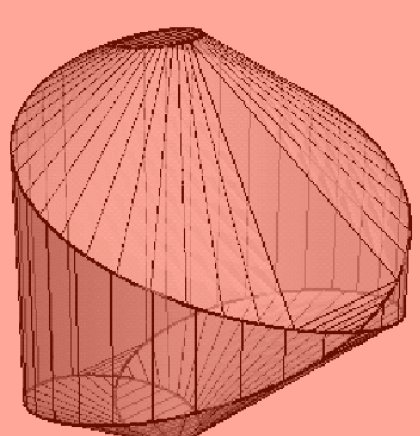
| bit | $\mathbb{R}-$qubit | $\mathbb{C}-$qubit | | $\mathbb{H}-$ qubit | |



Take care:
qutrit etc.
**not a ball!**

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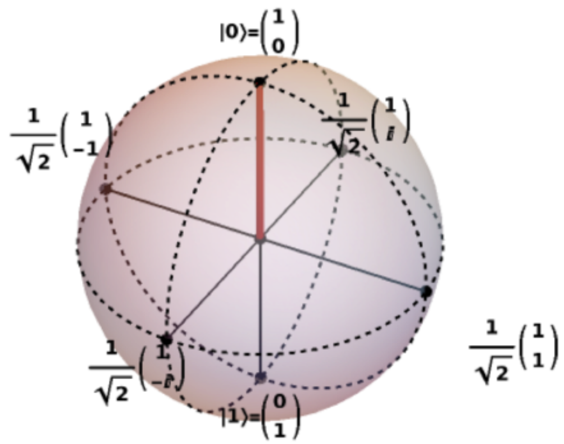

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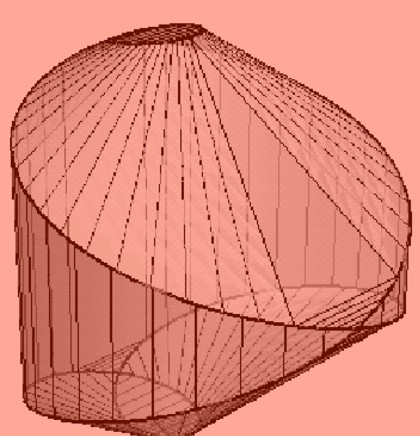

Take care:
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Does spacetime constrain *d?*

"Why" *d=3* ?

# Constraints from relativity

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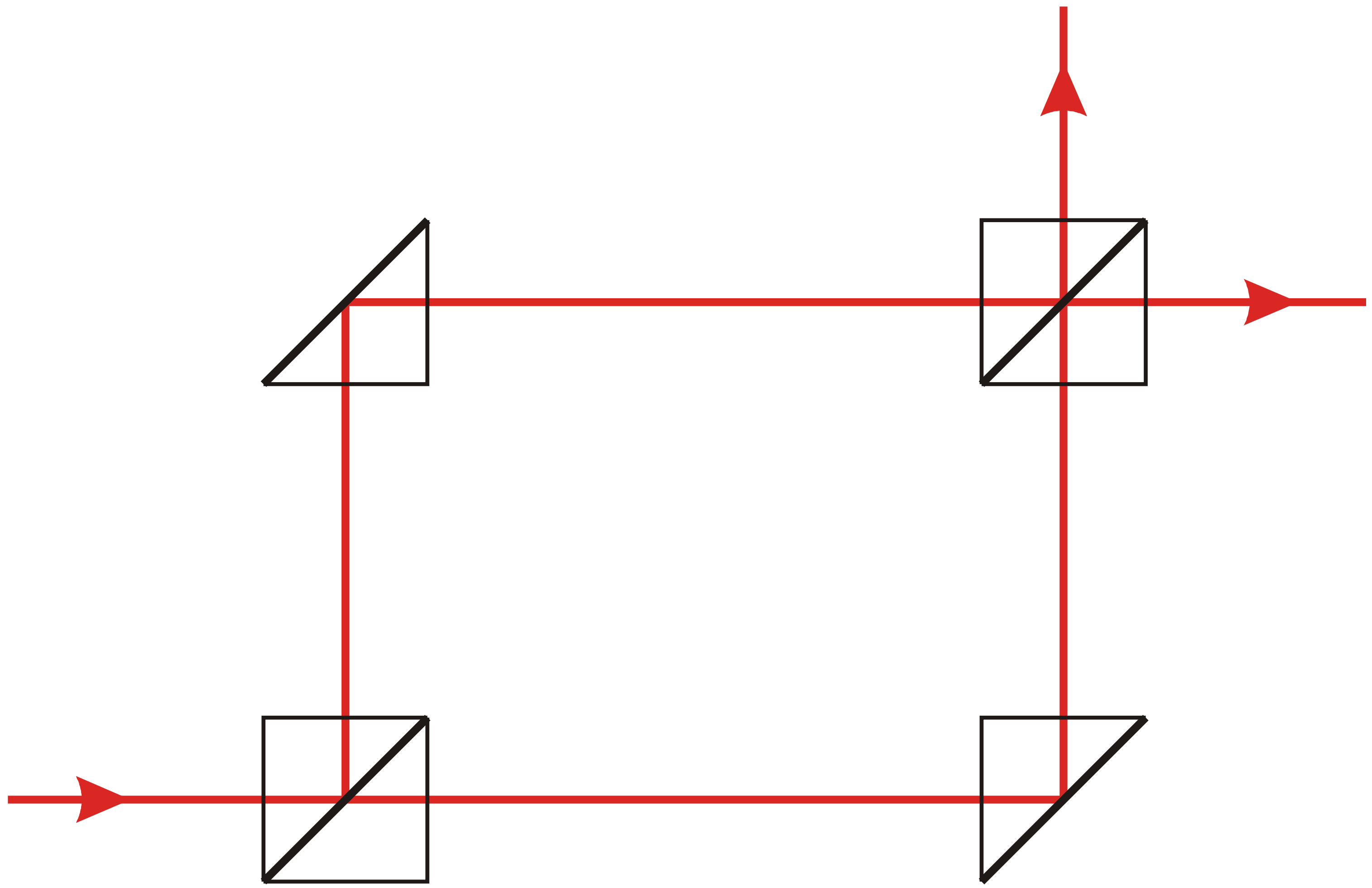

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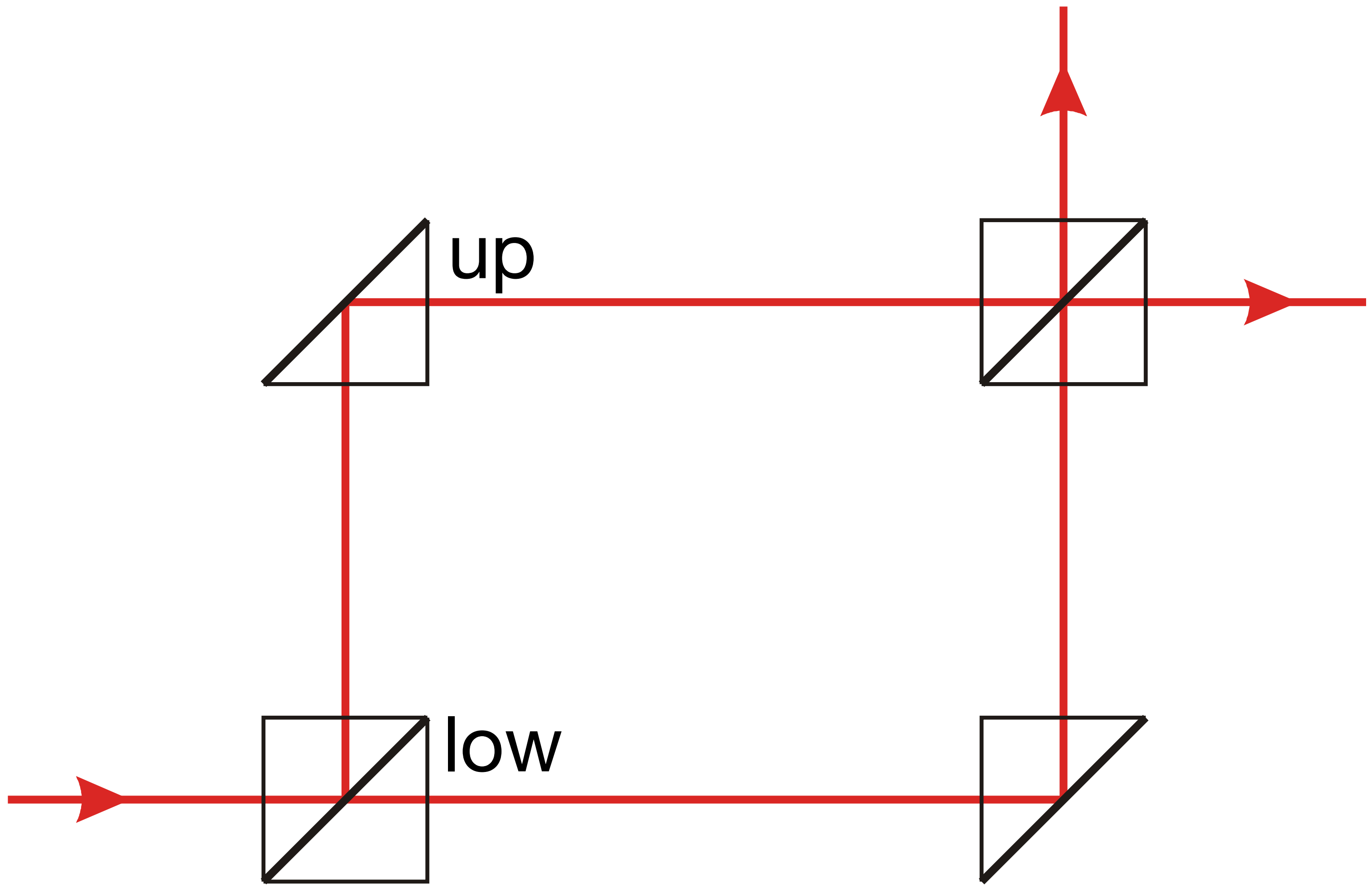

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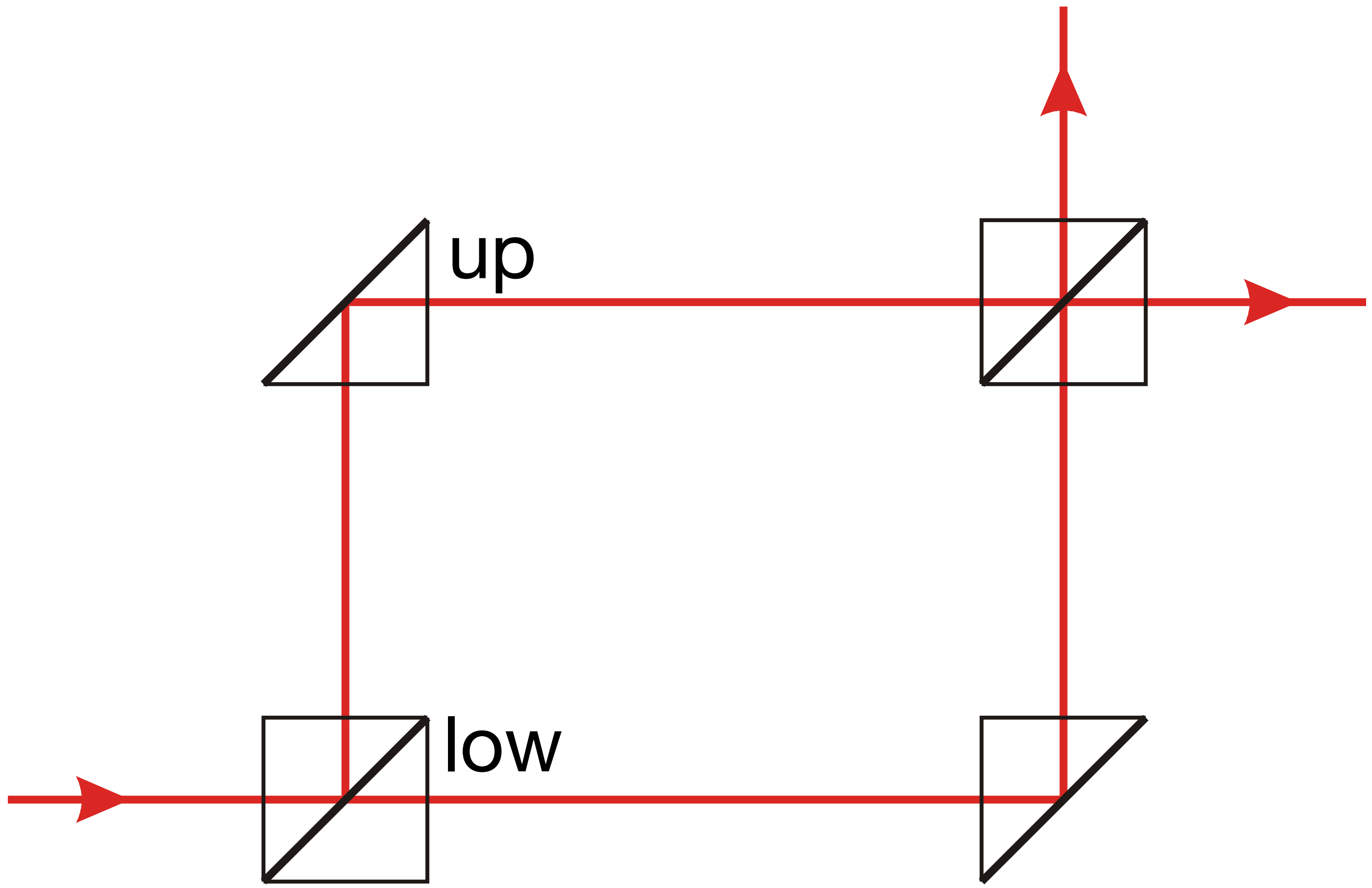


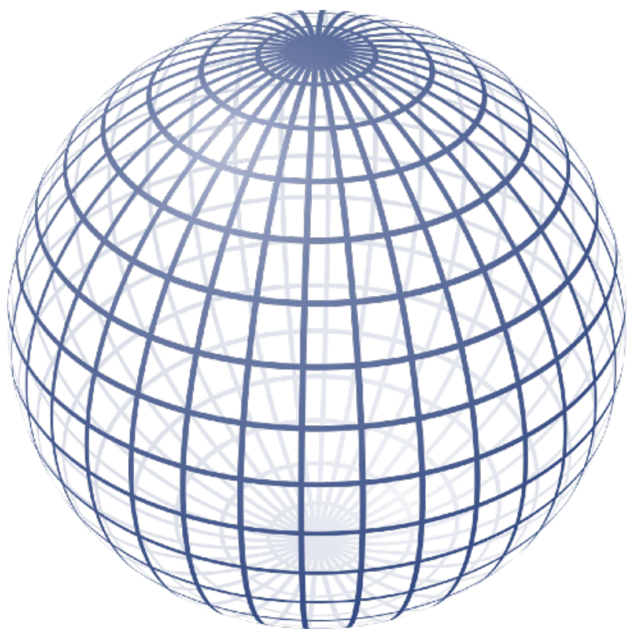

*d*-dim. "Bloch sphere"

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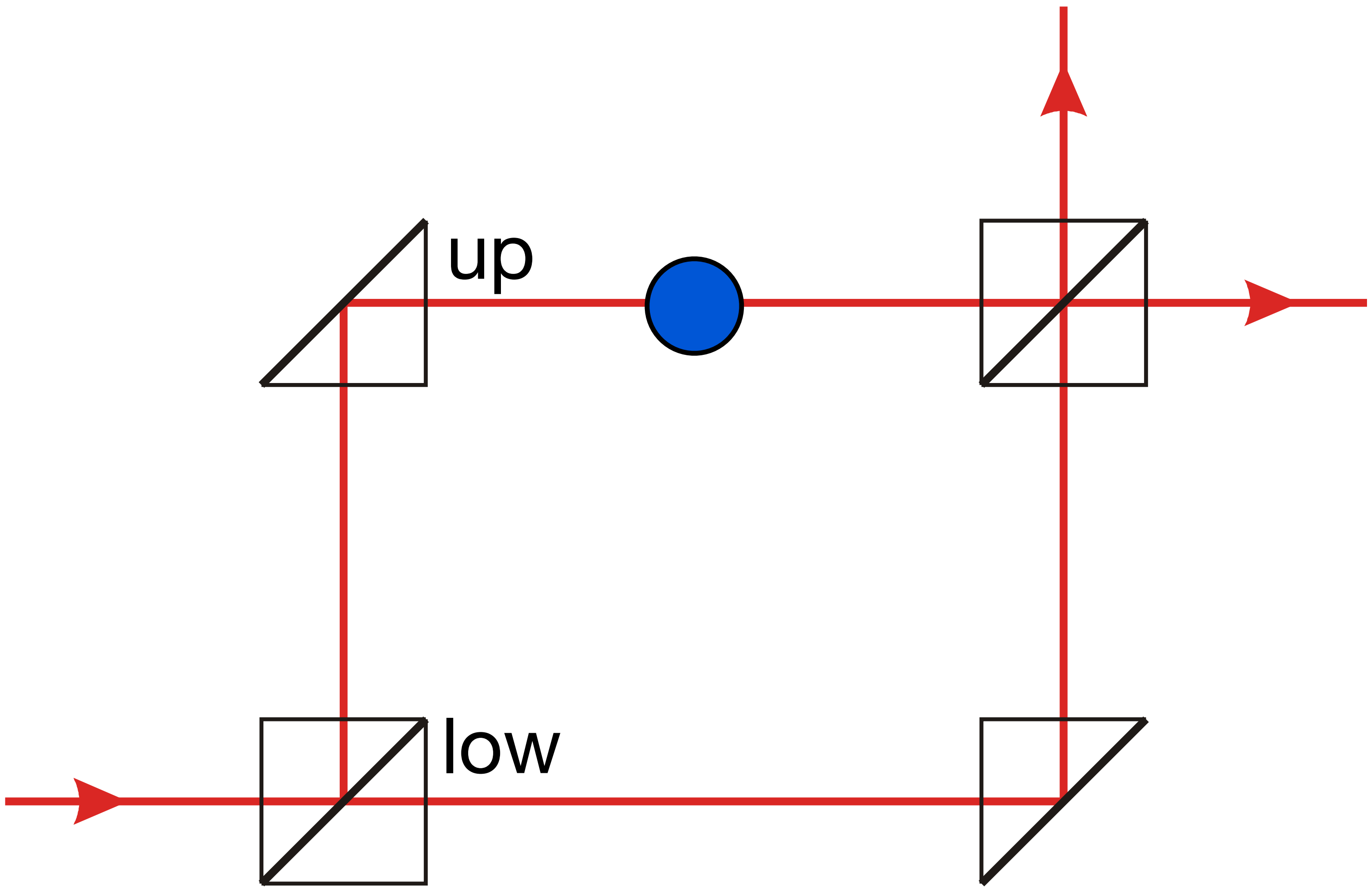


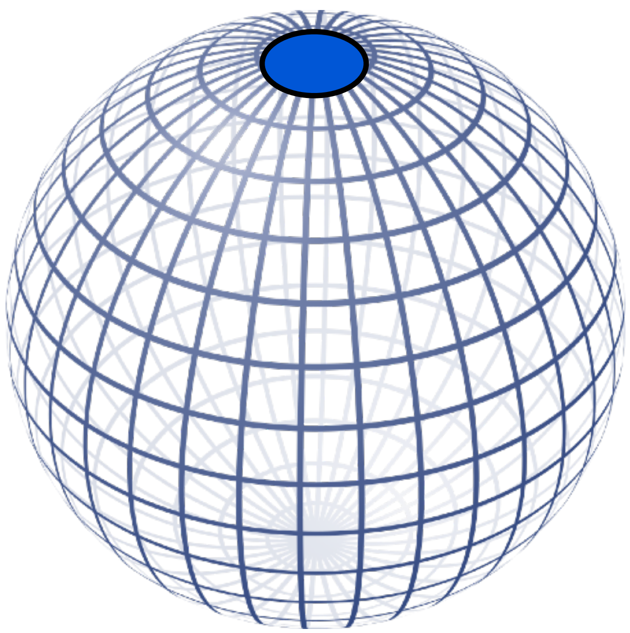

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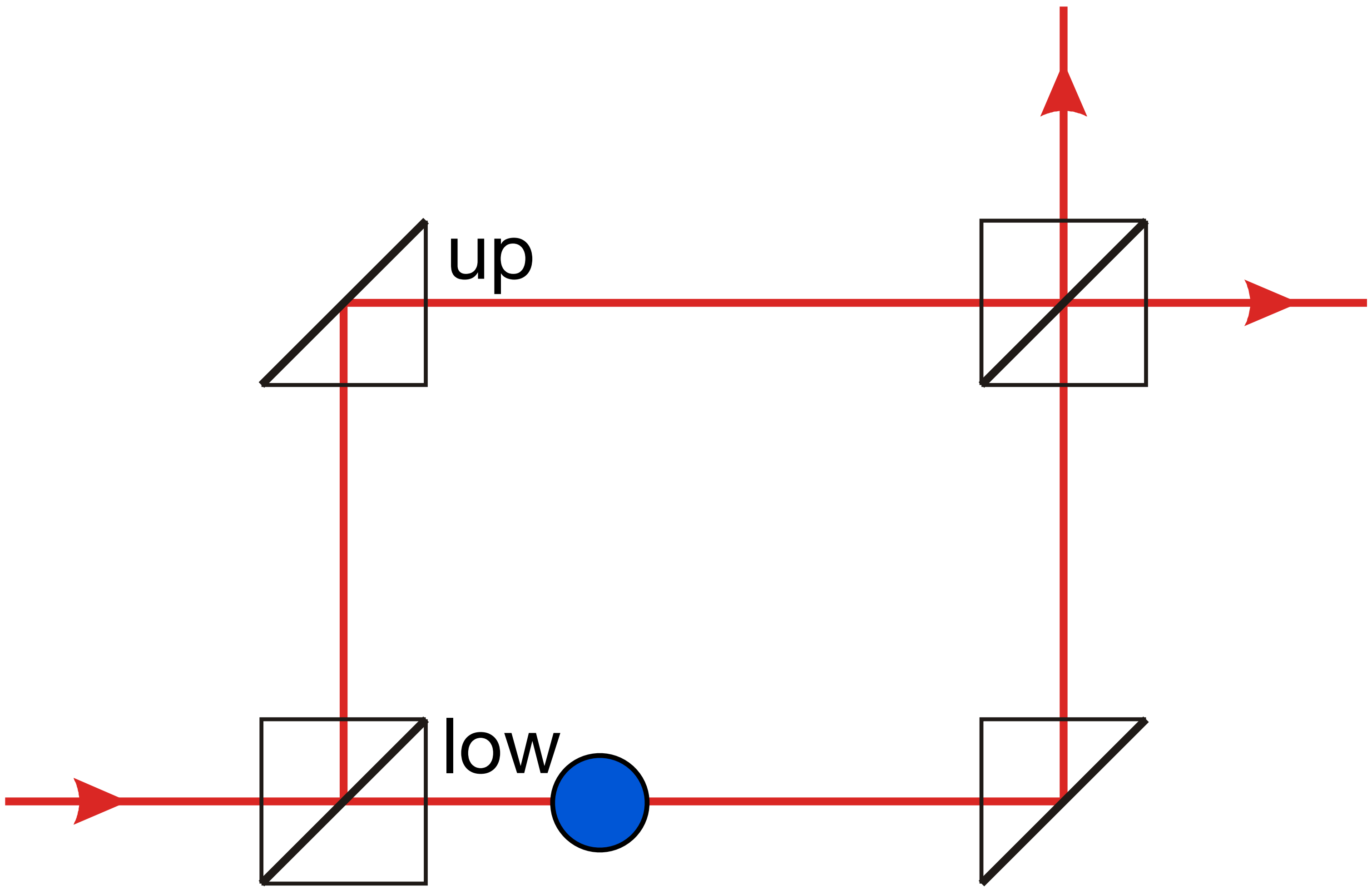


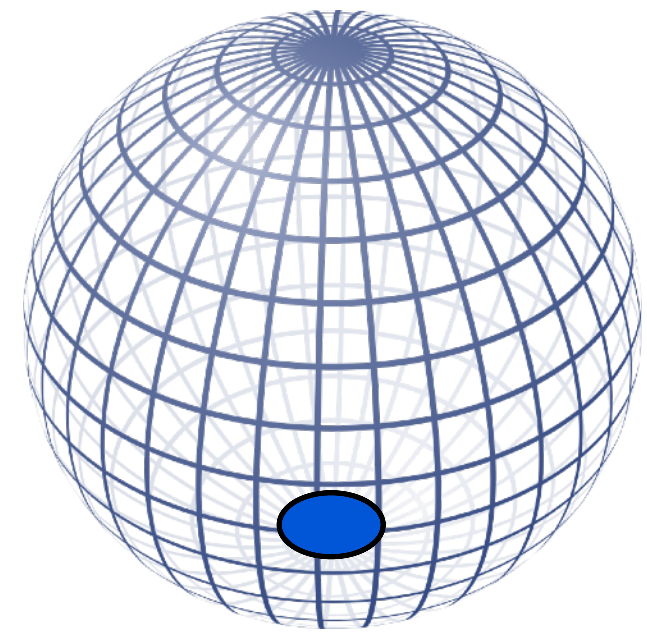

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South-pole state: particle definitely in lower branch.

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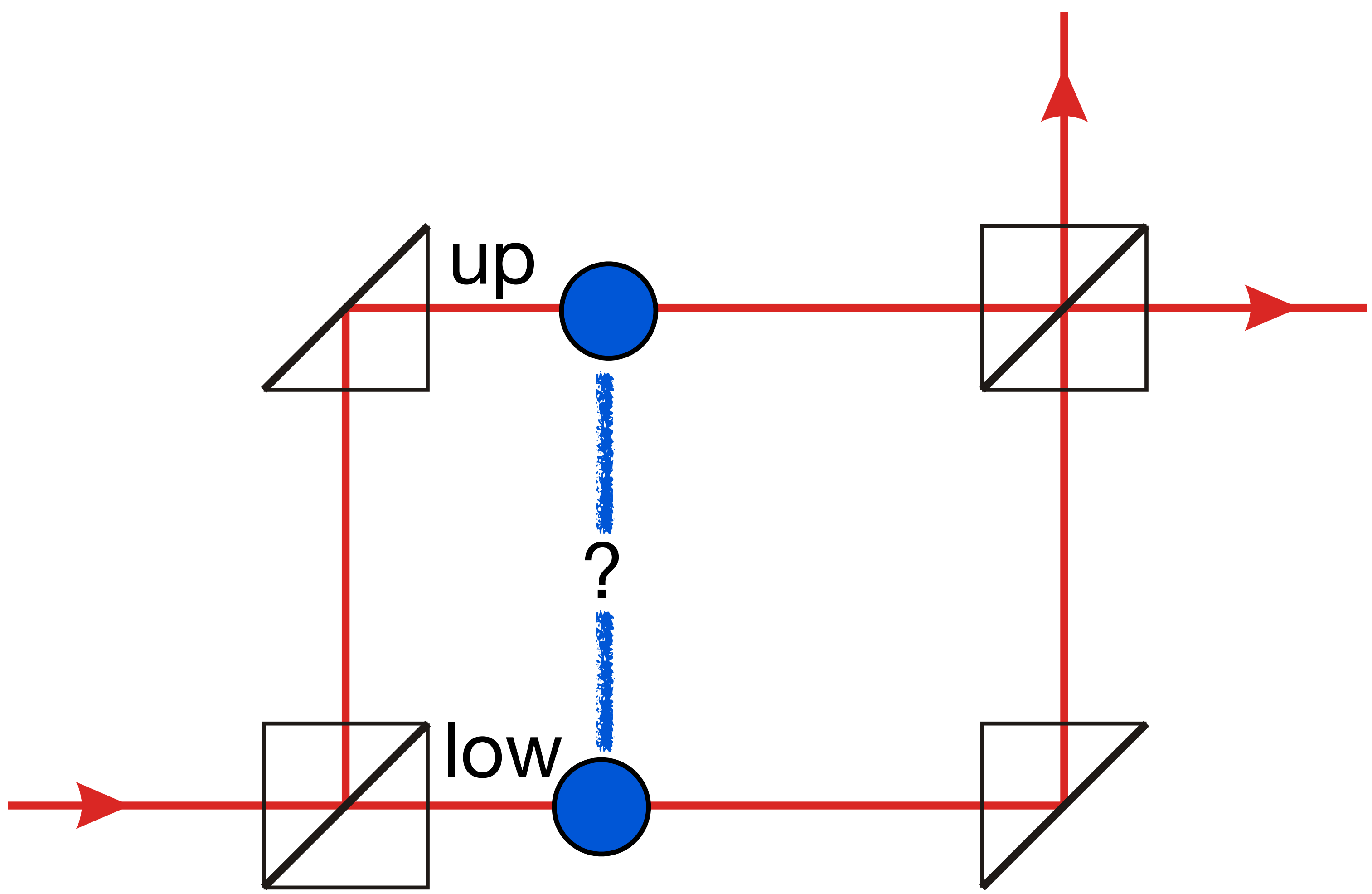


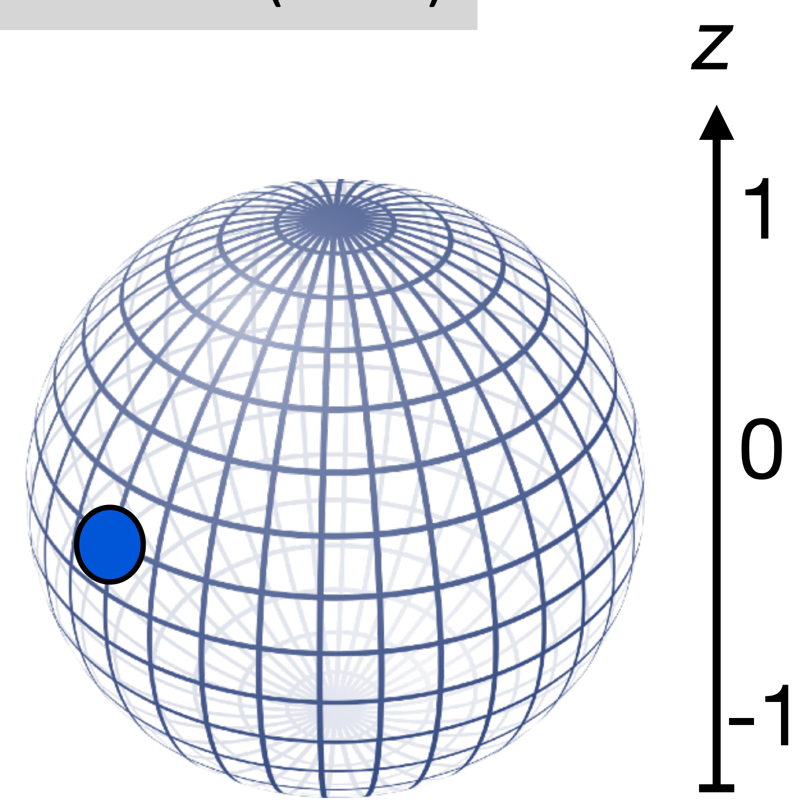


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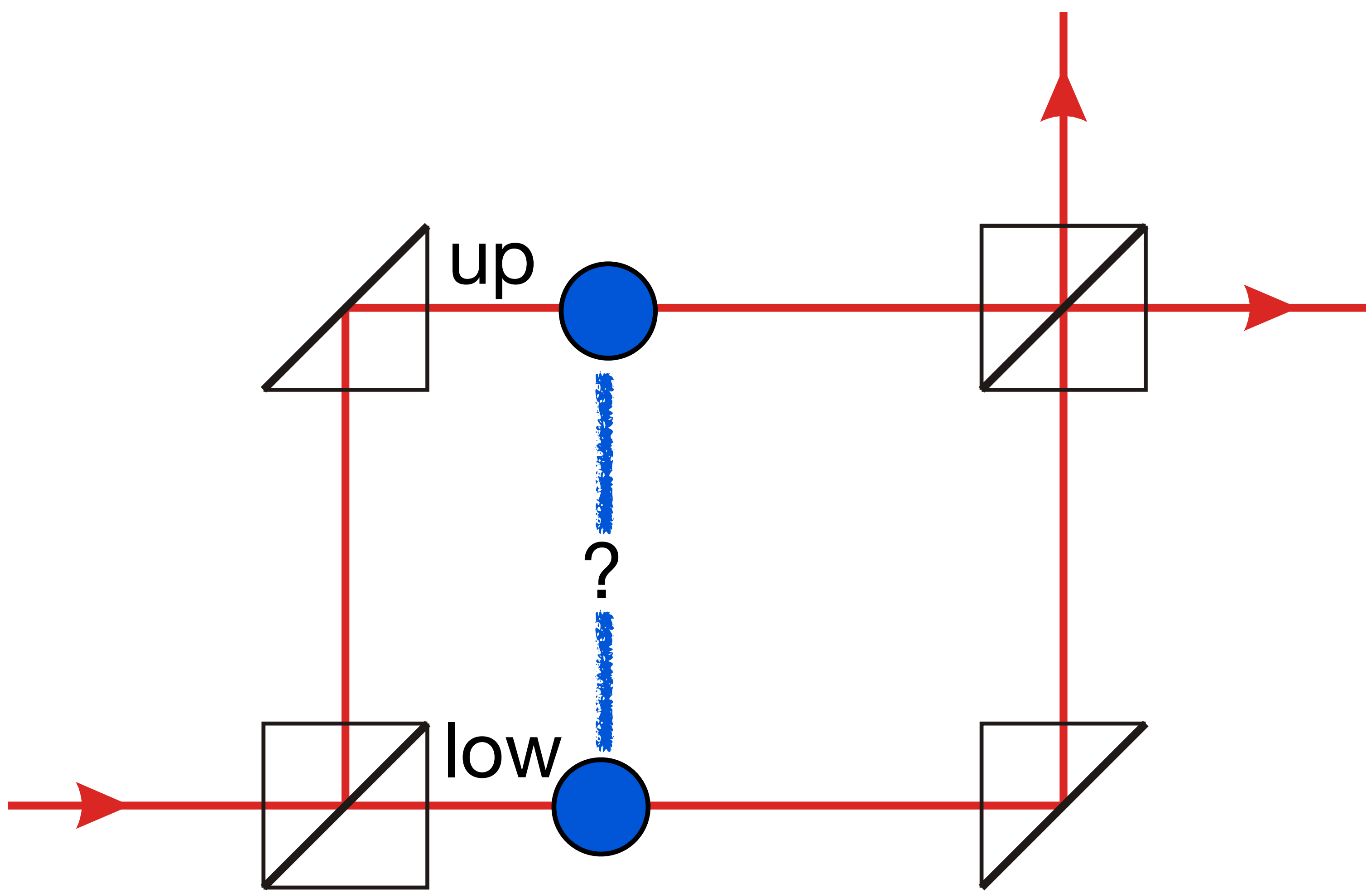


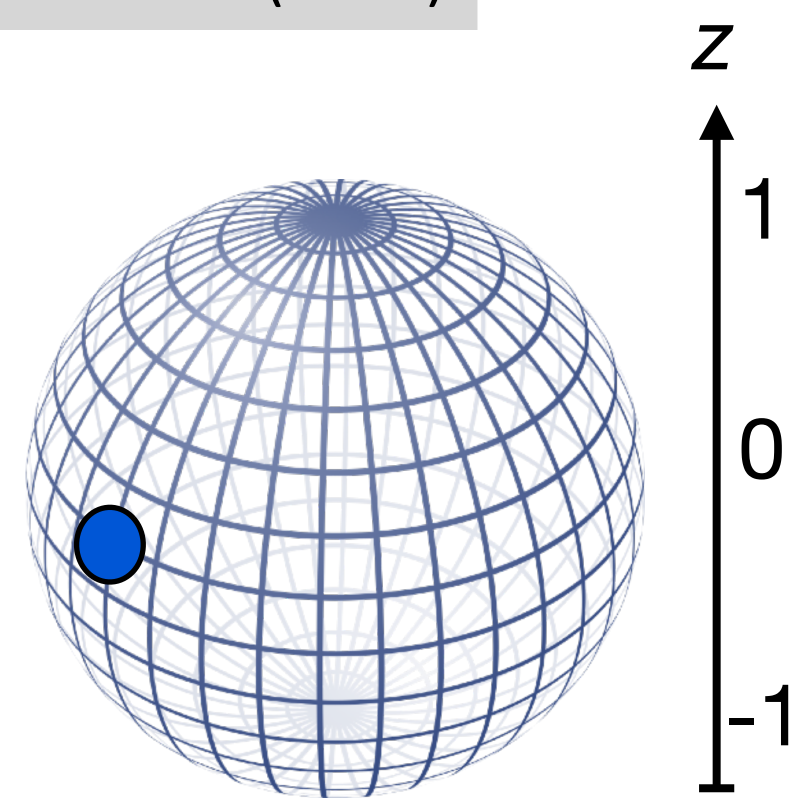


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$$p(\mathrm{up}) = \frac{1}{2}(z+1)$$

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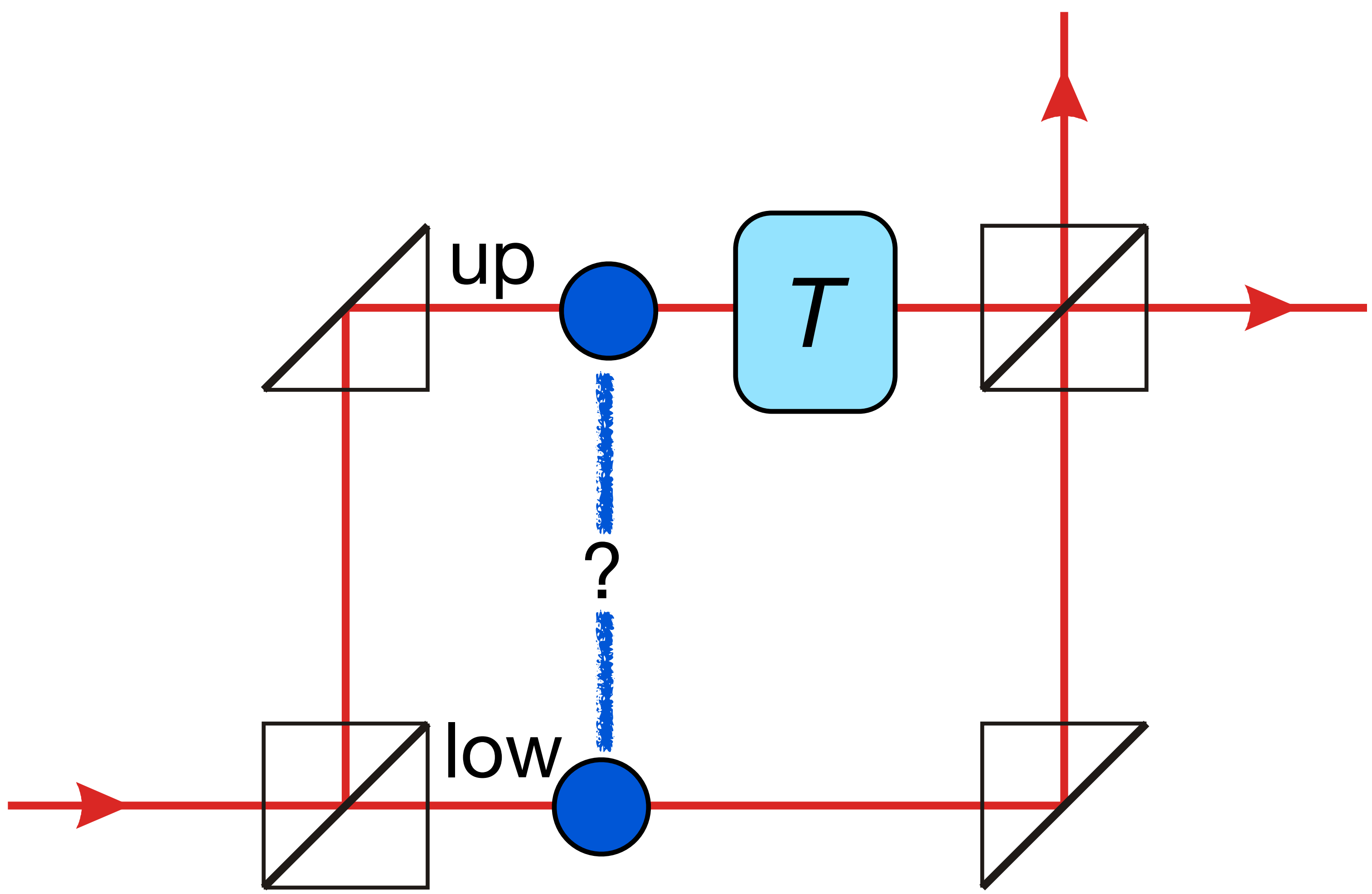


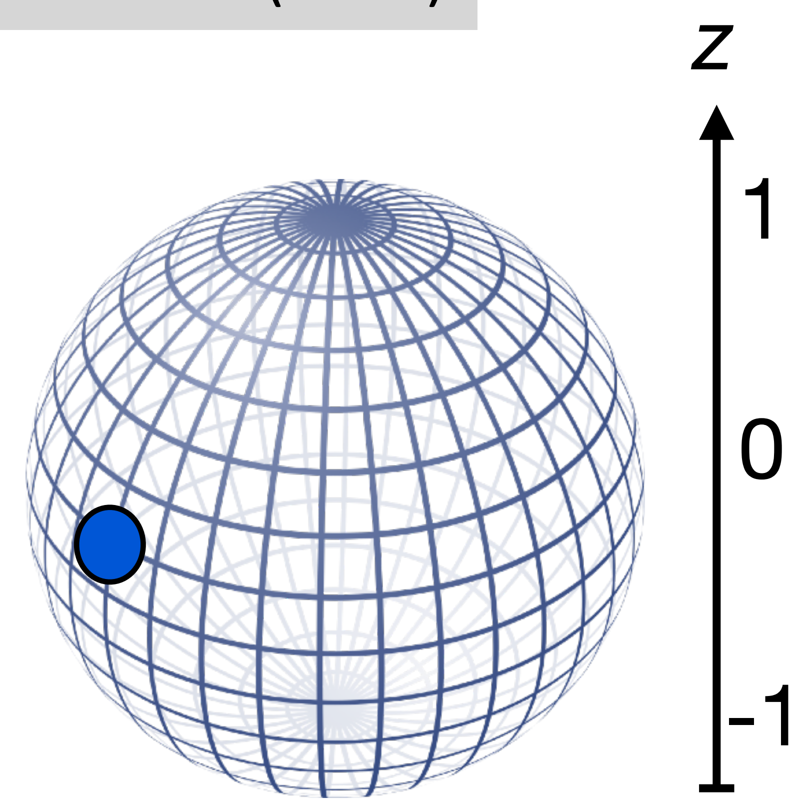


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What transformations *T* can we perform locally in one arm...
... reversibly, i.e. without any information loss?

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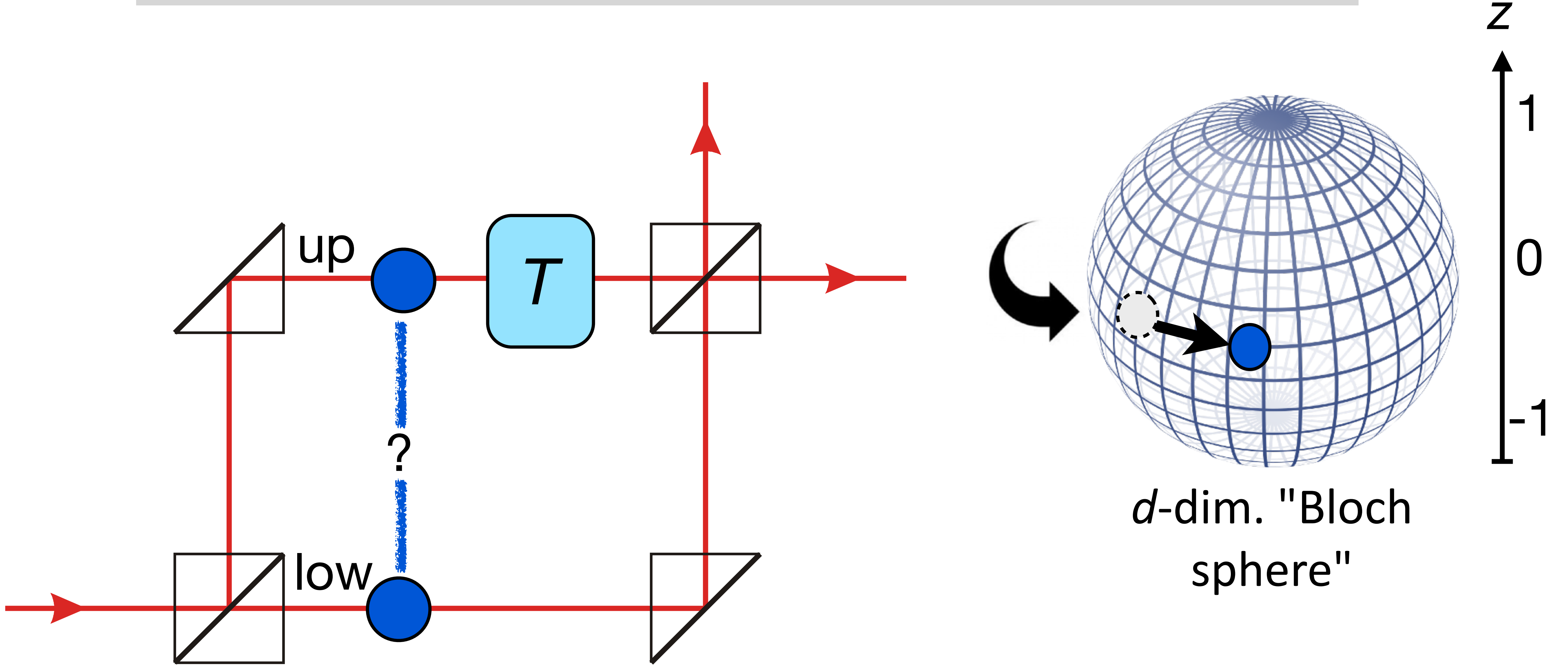


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*T* must be a rotation of the Bloch ball (reversible+linear)...
... and must preserve *p*(up), i.e. preserve the *z*-axis.

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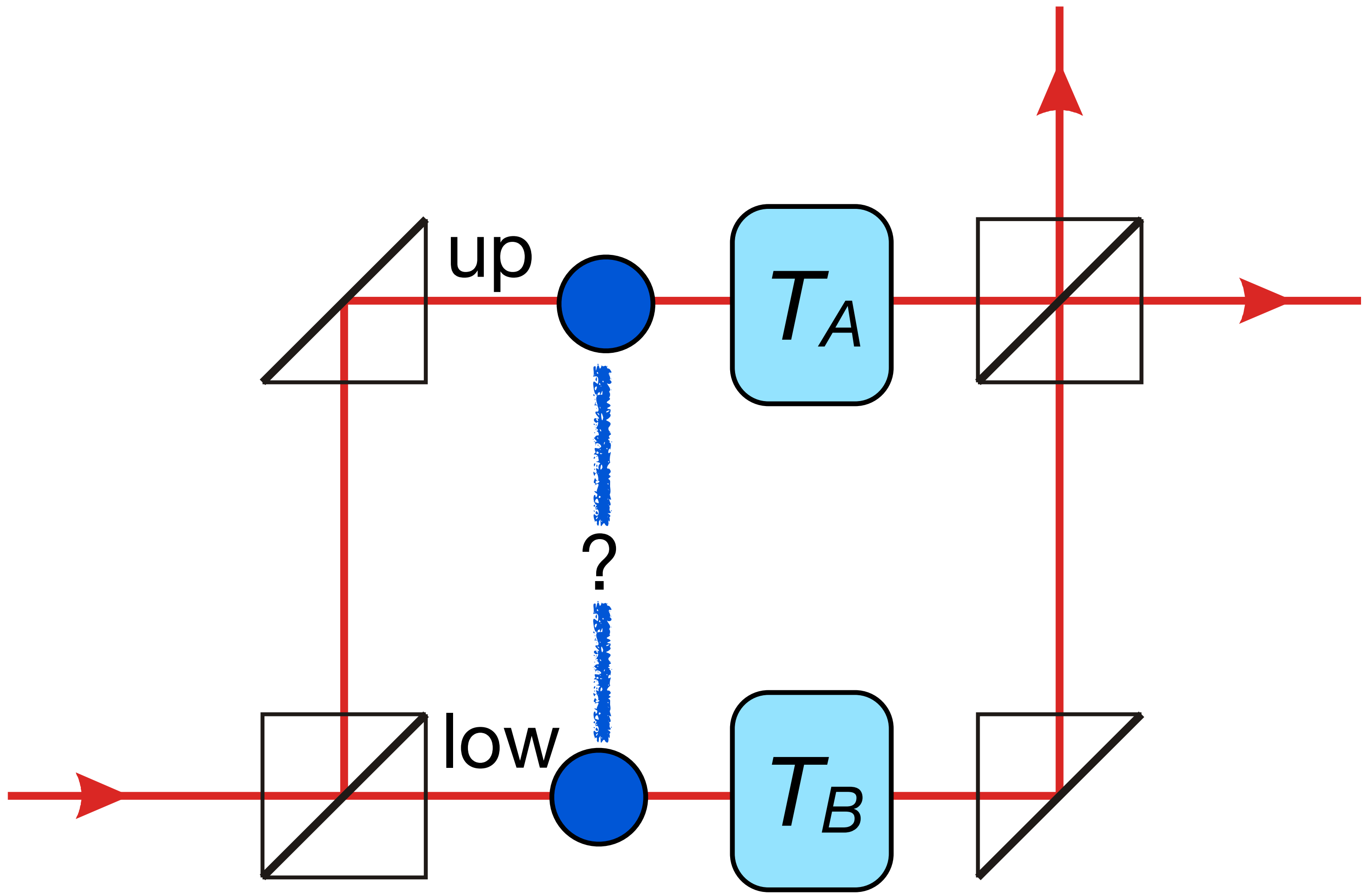


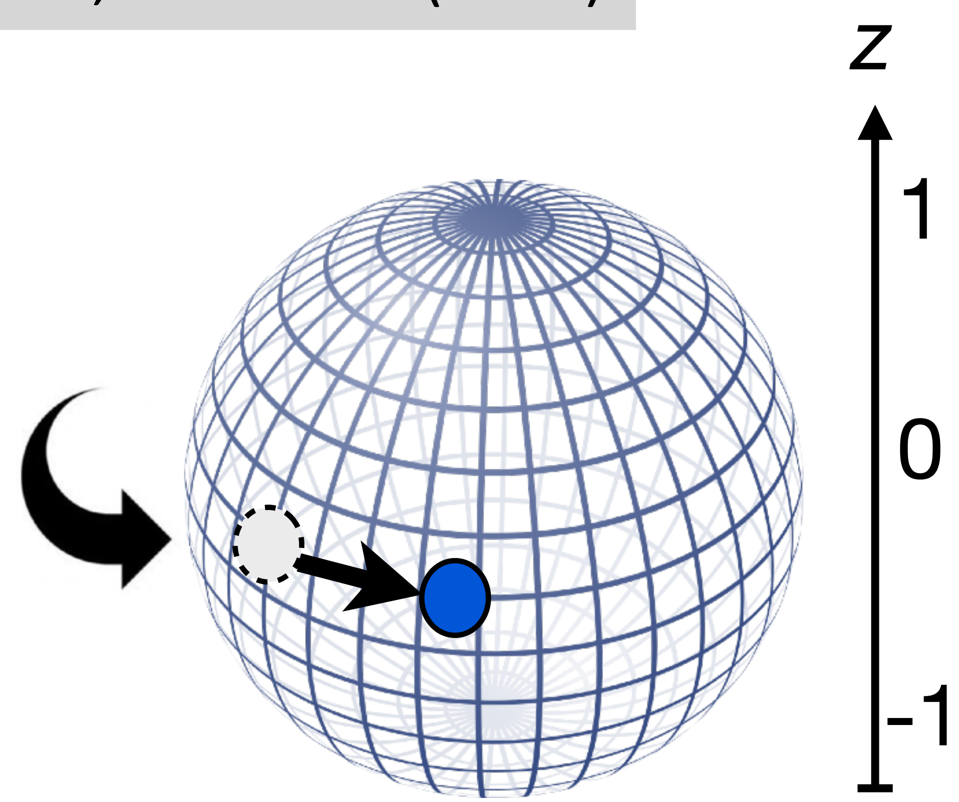


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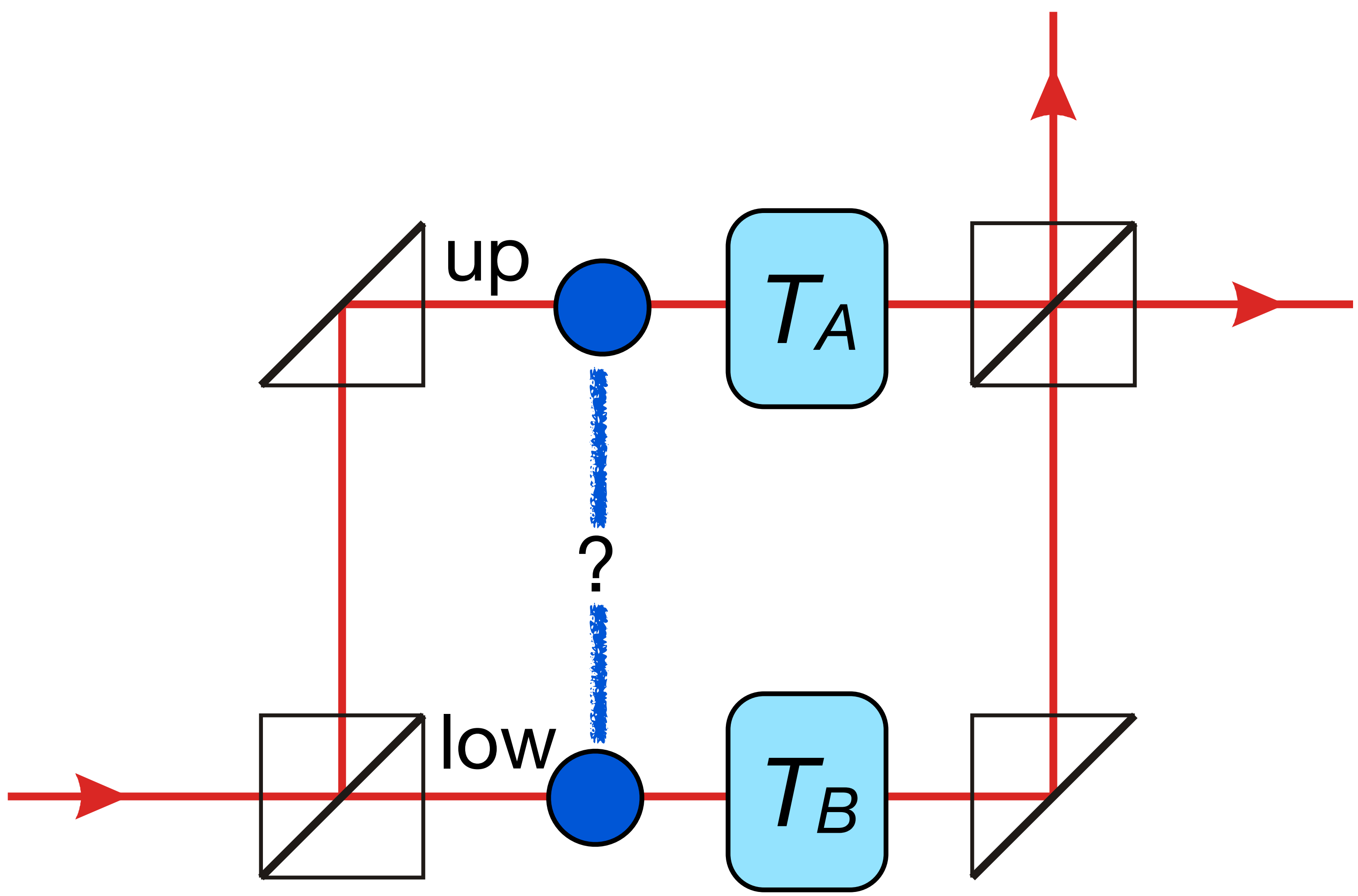


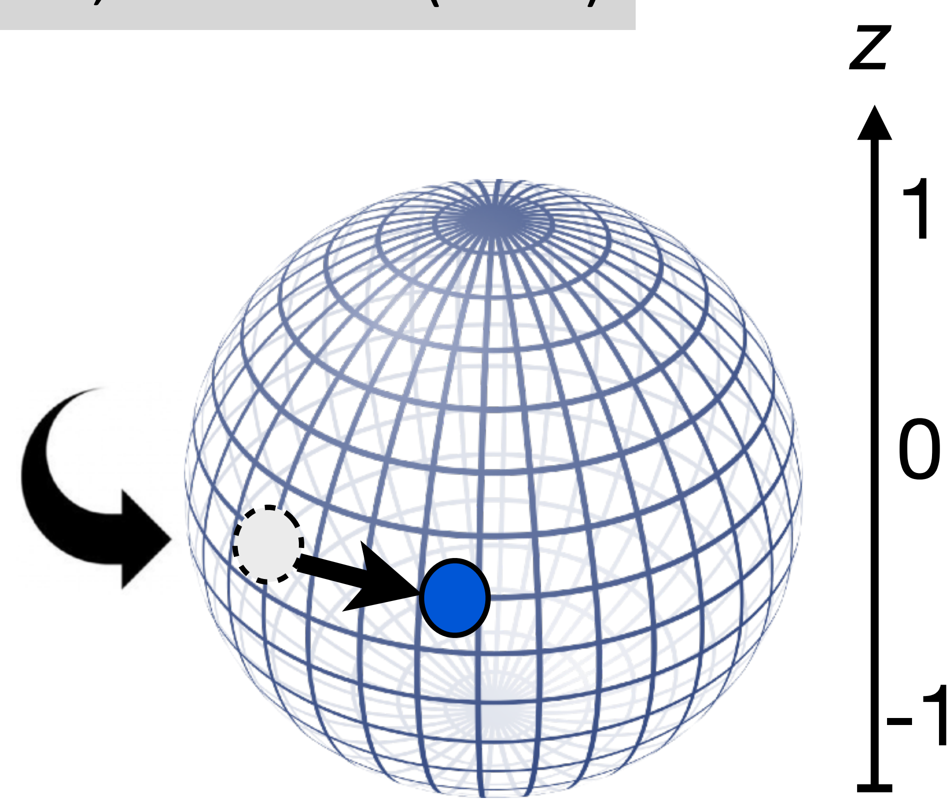


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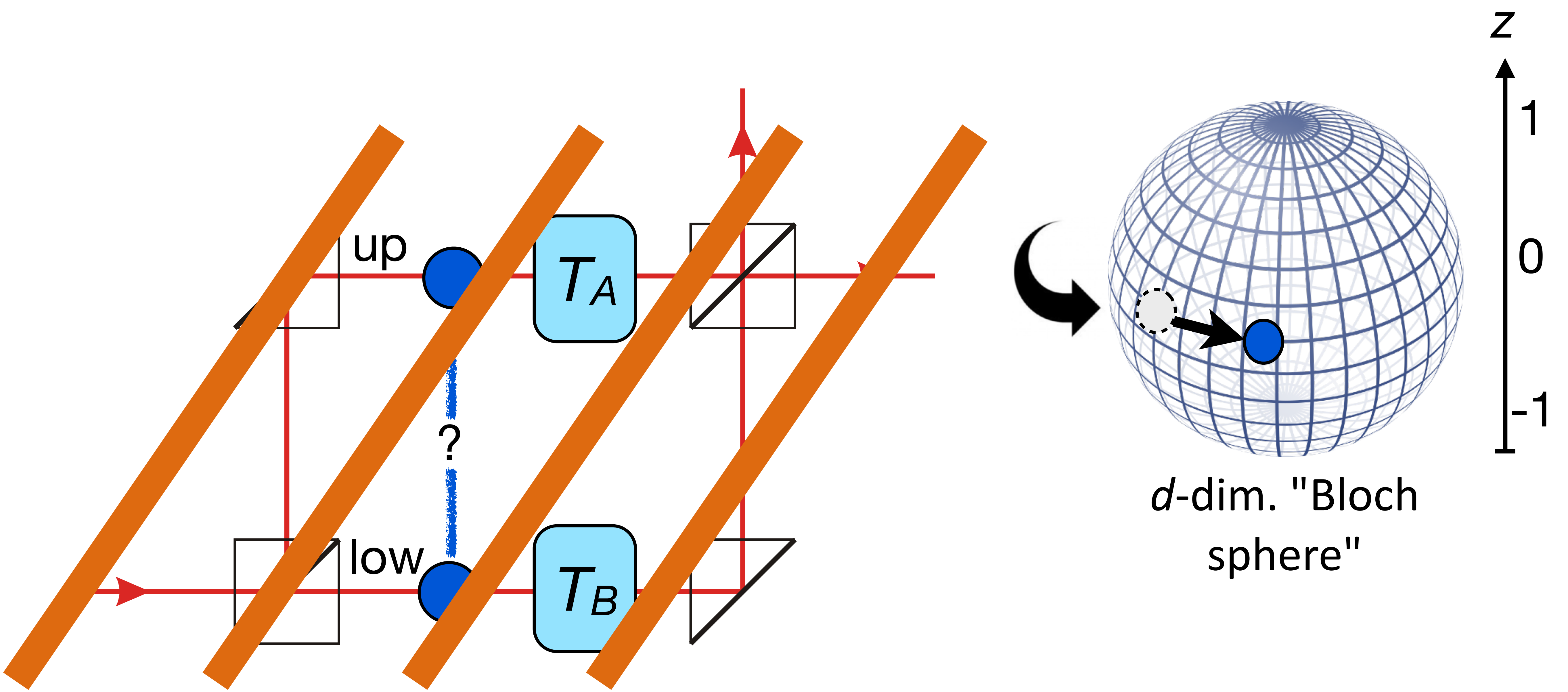


Relativity: there's a frame of reference in which $T_A$ happens before $T_B$...

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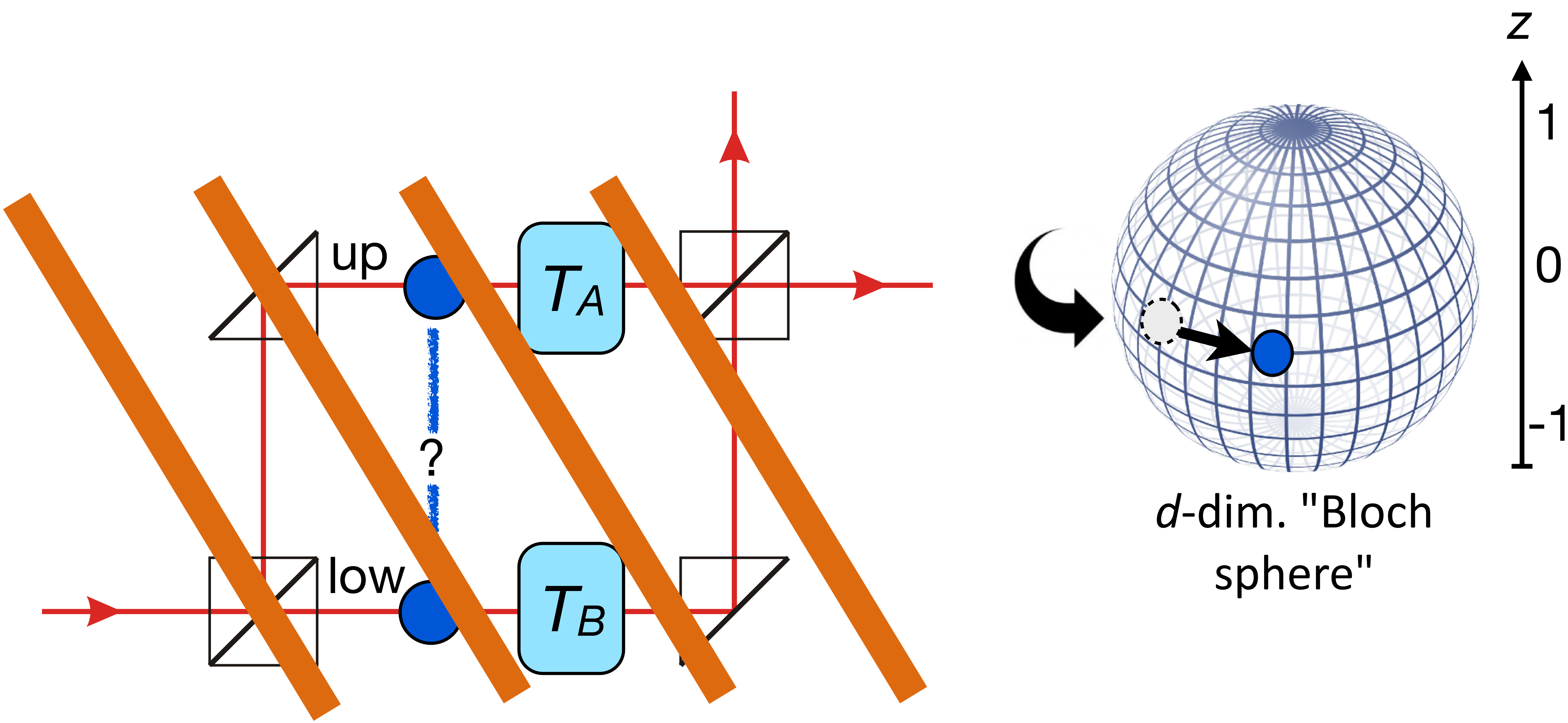


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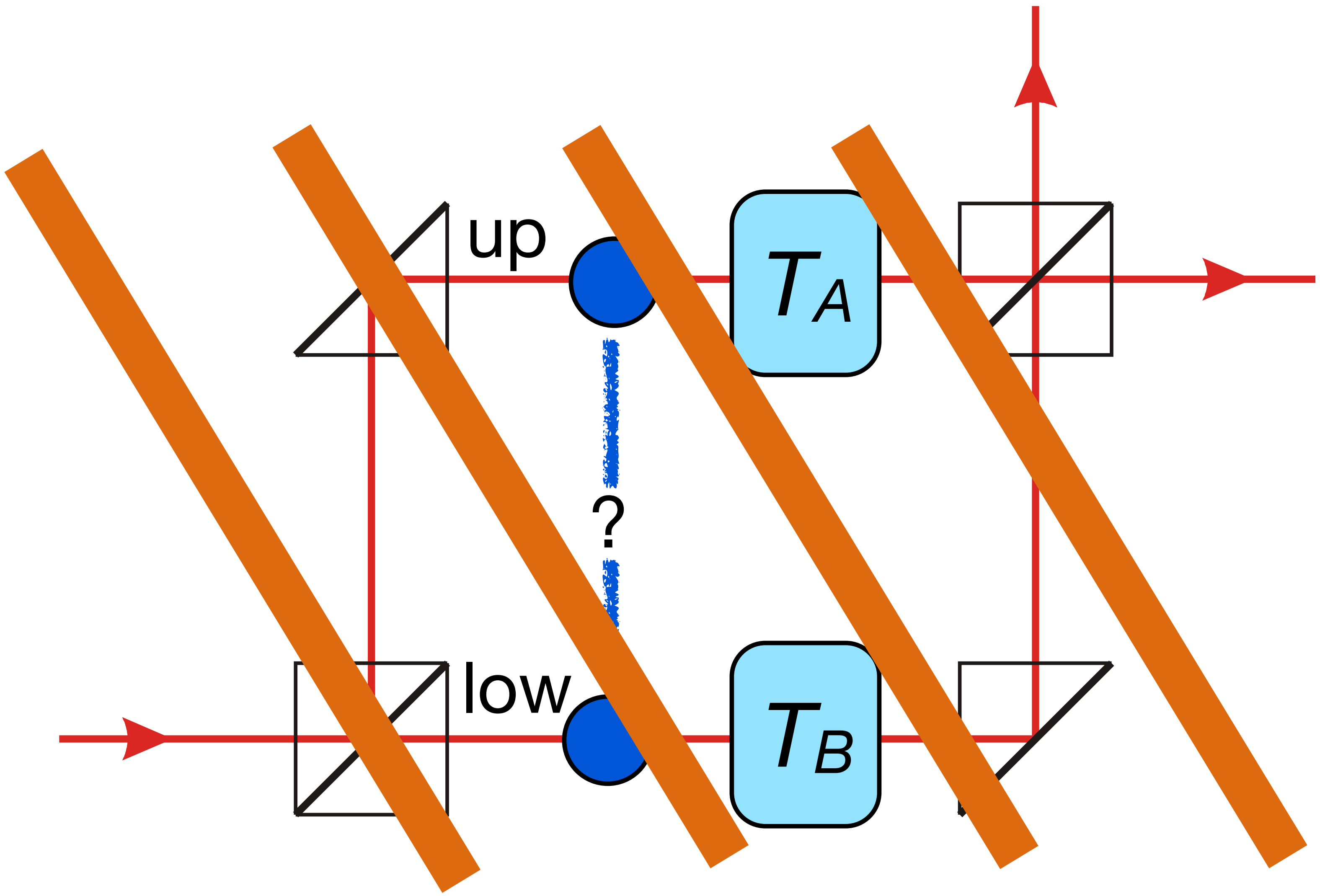
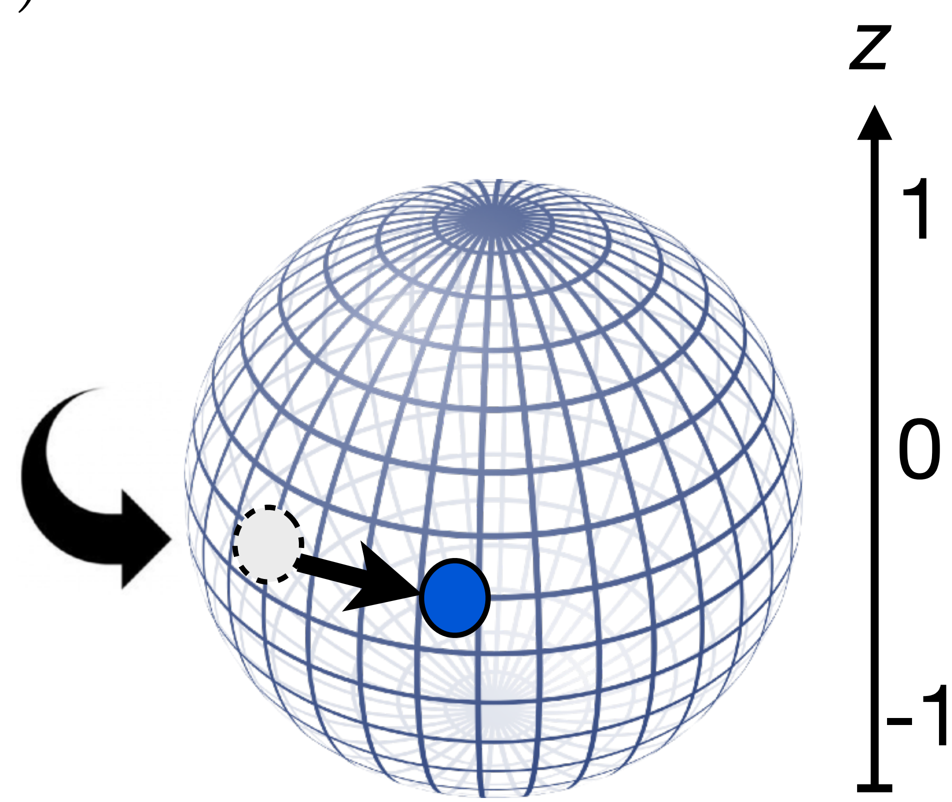
$$\Rightarrow\ T_A T_B = T_B T_A \text{ for all } T_A, T_B \in \mathrm{SO}(d-1).$$

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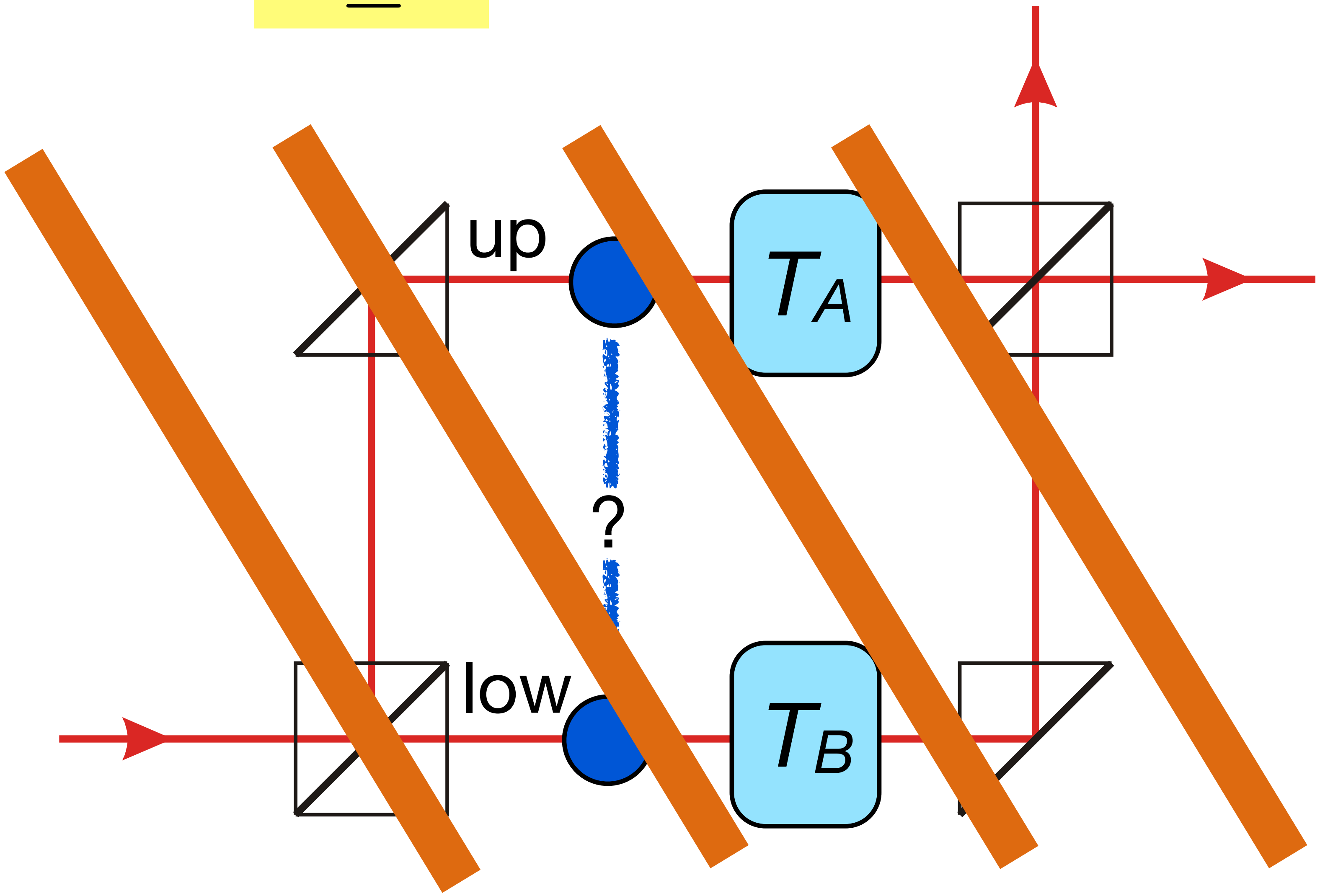
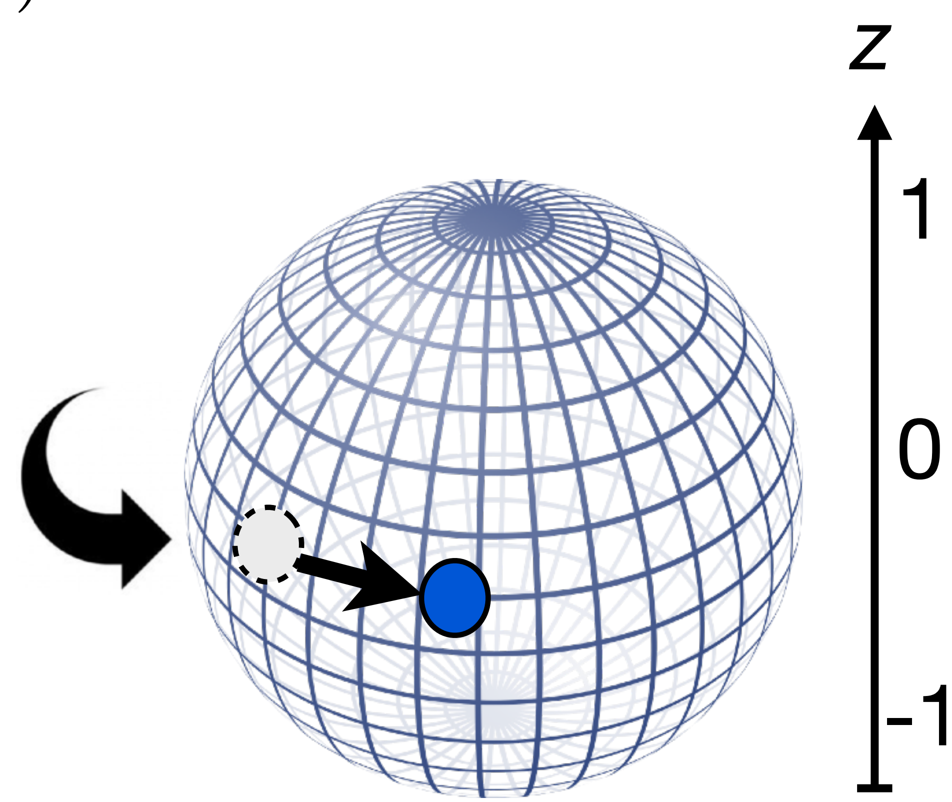
$\Rightarrow T_A T_B = T_B T_A$ for all $T_A, T_B \in \mathrm{SO}(d-1)$.

$\Rightarrow d \leq 3$.

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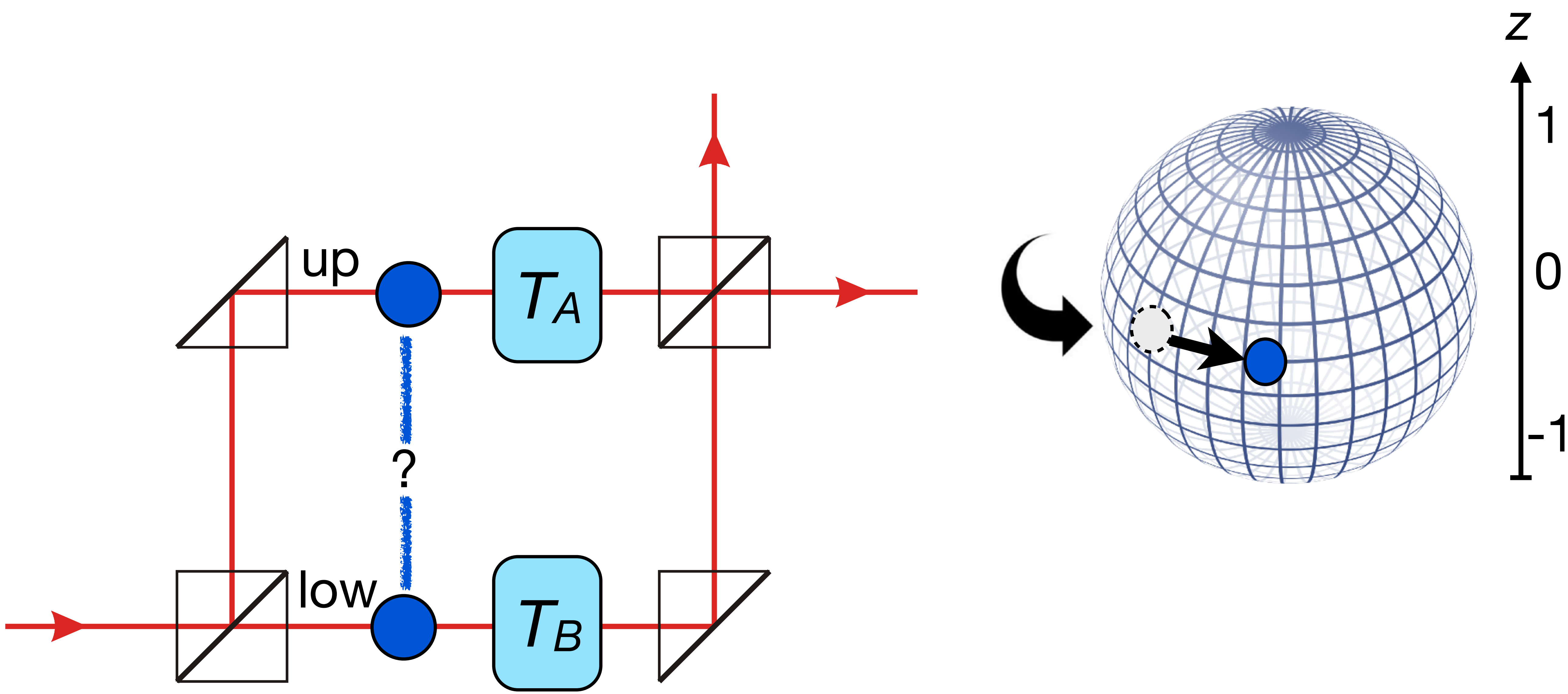


So far, we assumed: $\mathcal{G}_A = \mathcal{G}_B$. Assumption of **relationality.**

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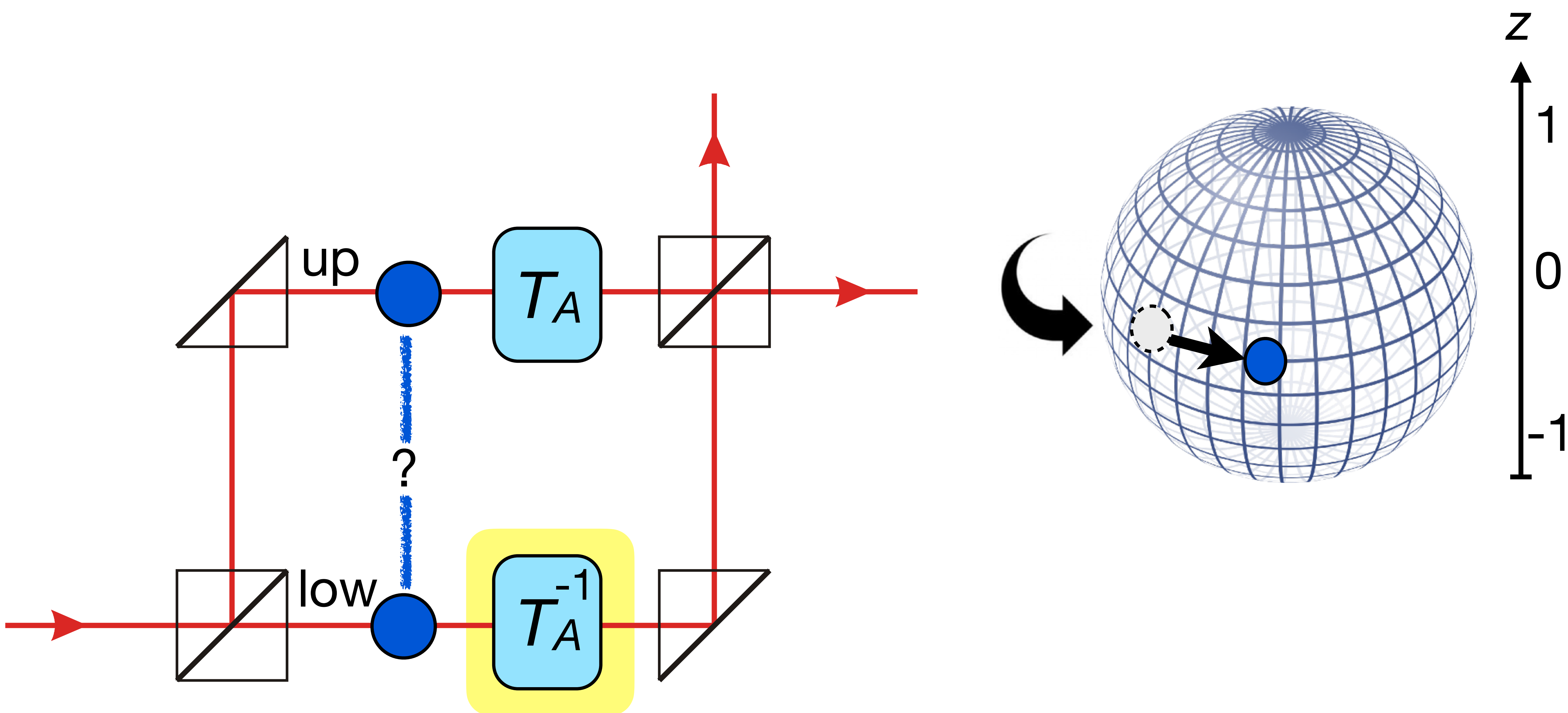


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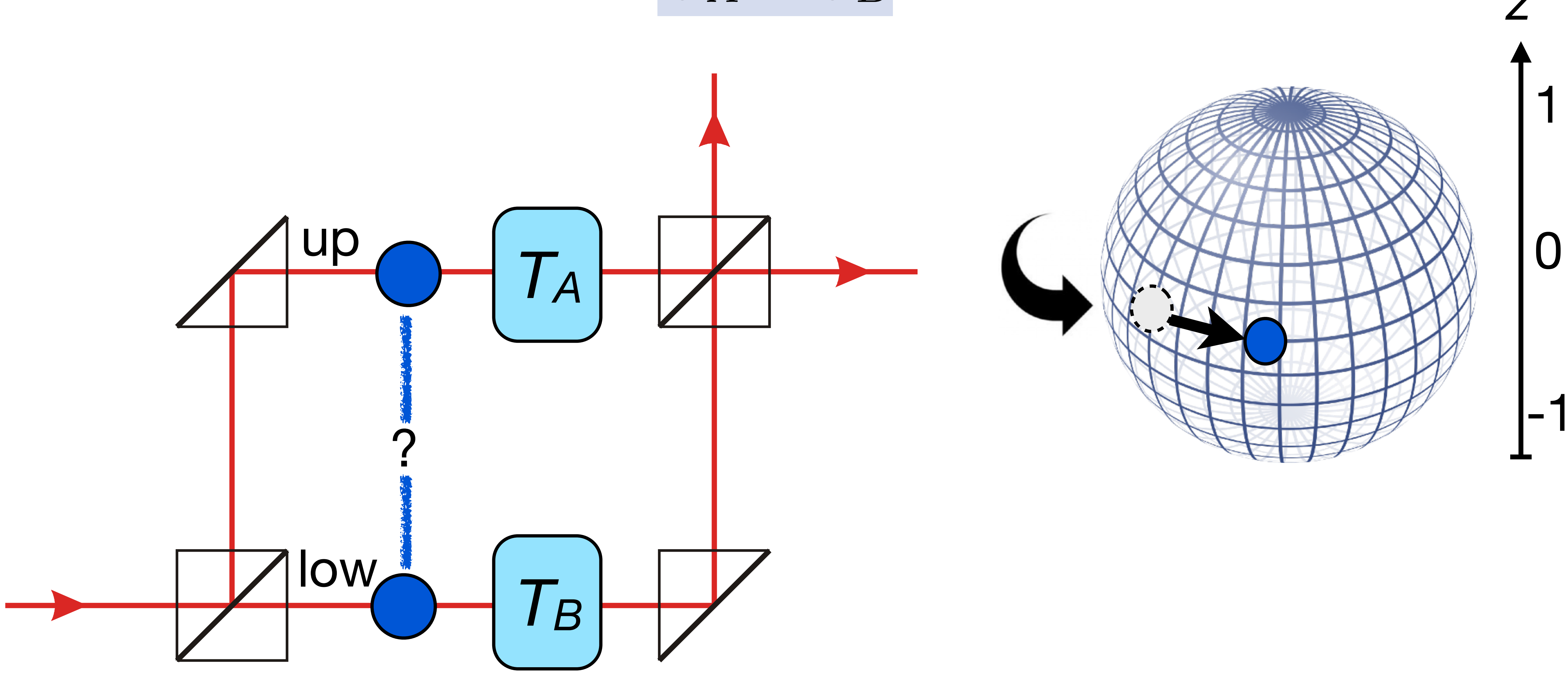
Let's **relax** this assumption to $\mathcal{G}_A \simeq \mathcal{G}_B$.

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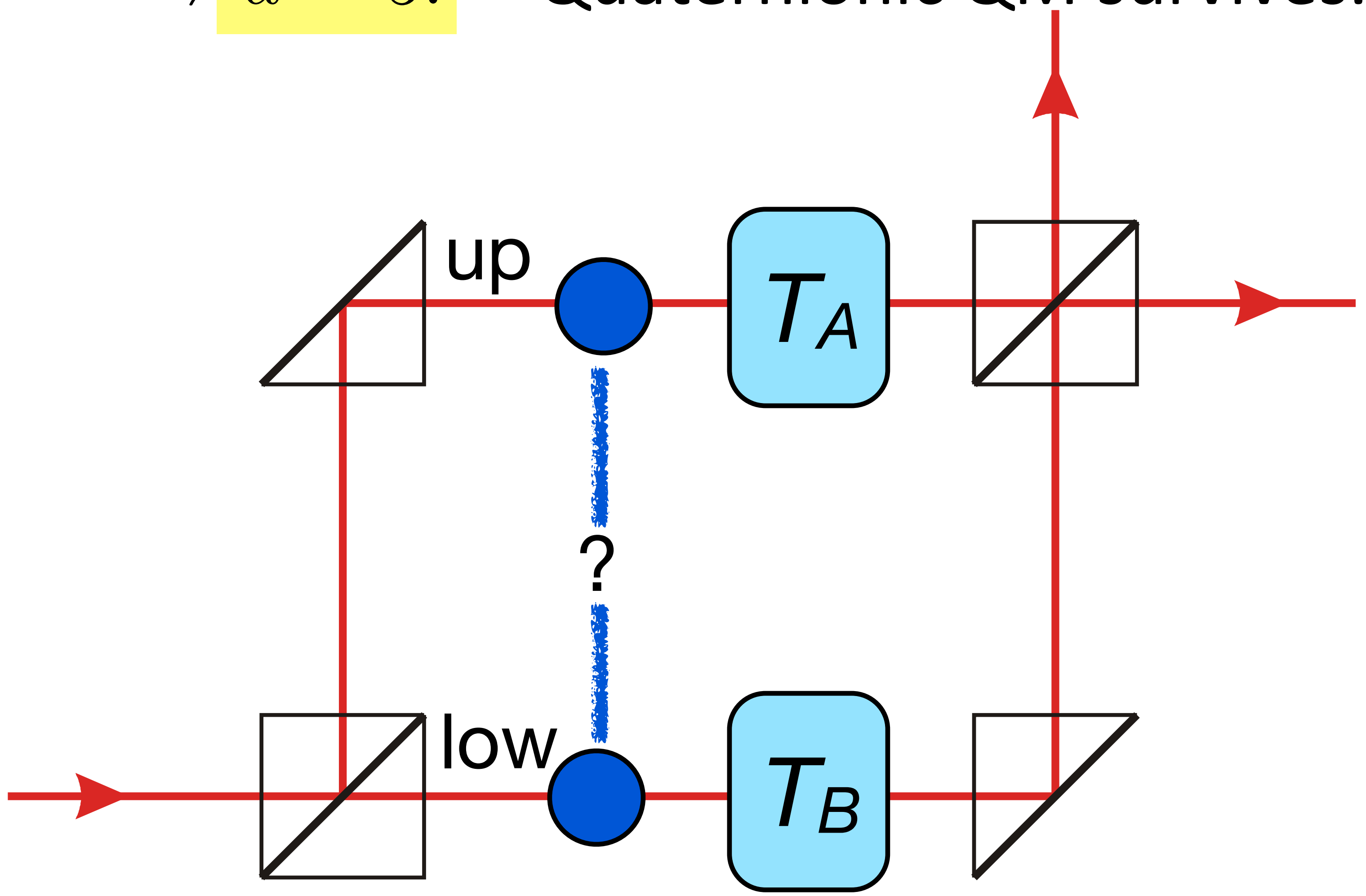
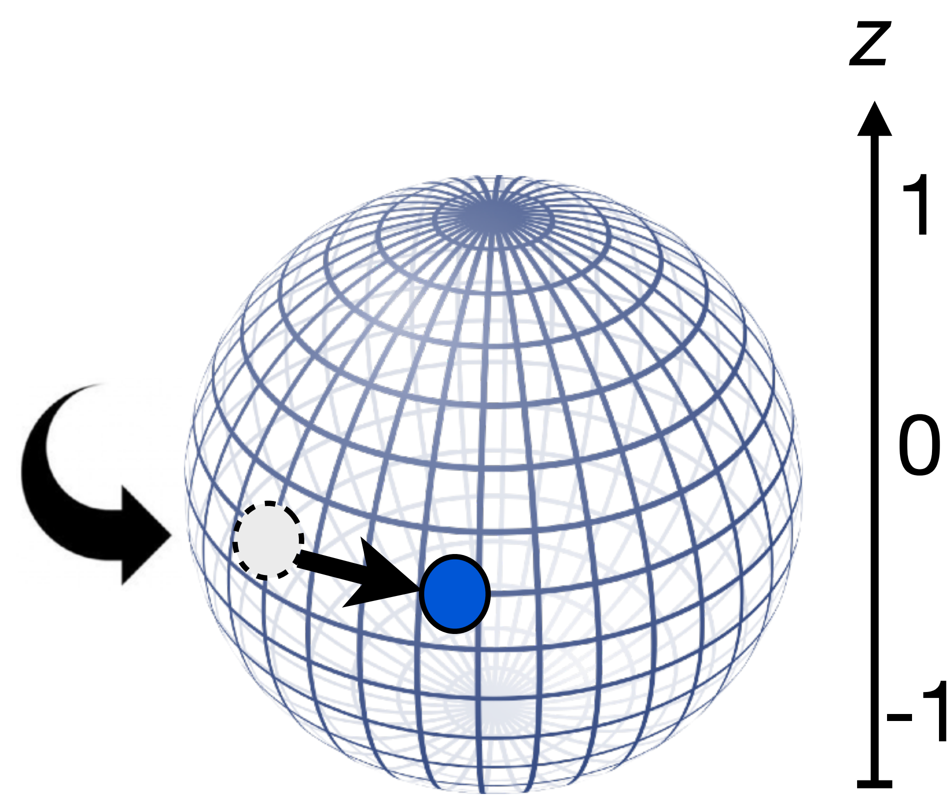
$\Rightarrow d = 5.$ Quaternionic QM survives.

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## Classification of possibilities

A1) Beam splitter can prepare **any upper-branch probability** $p$.

A2) Every pure state with the same $p$ can be reached by **reversible operations applied locally** on the two arms.

A3) The groups of operations of A and B are **isomorphic**.

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**Theorem 6.2.** *Under the assumptions* A1, A2, A3, *relativity of simultaneity* (*REL*) *allows for the following possibilities and not more*:

— $d = 1$ (*the classical bit*), *with* $\mathcal{G}_{\mathrm{A}} = \mathcal{G}_{\mathrm{B}} = \{\mathbf{1}\}$ (*i.e. without any non-trivial local transformations*),
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— $d = 3$ (*the standard quantum bit over the complex numbers*), *with* $\mathcal{G}_{\mathrm{A}} = \mathcal{G}_{\mathrm{B}} = \mathrm{SO}(2) = \mathrm{U}(1)$,
— $d = 5$ (*the quaternionic quantum bit*), *with* $\mathcal{G}_{\mathrm{AB}} = \mathrm{SO}(4)$, $\mathcal{G}_{\mathrm{A}}$ *the left- and* $\mathcal{G}_{\mathrm{B}}$ *the right-isoclinic rotations in* $\mathrm{SO}(4)$ (*or vice versa*) *which are both isomorphic to* $\mathrm{SU}(2)$, *and* $\mathcal{G}_{\mathrm{A}} \cap \mathcal{G}_{\mathrm{B}} = \{+\mathbb{I}, -\mathbb{I}\}$.

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Relativity of simultaneity singles out the **associative division algebras**.

## Day 2

1. Quantum (2,2,2)-Bell correlations from symmetry

2. **Open Research**: geometry from probability

3. Motivation: semi-device-independent QIT

4. Theory-independent randomness with spacetime symm.

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# Black boxes and correlations

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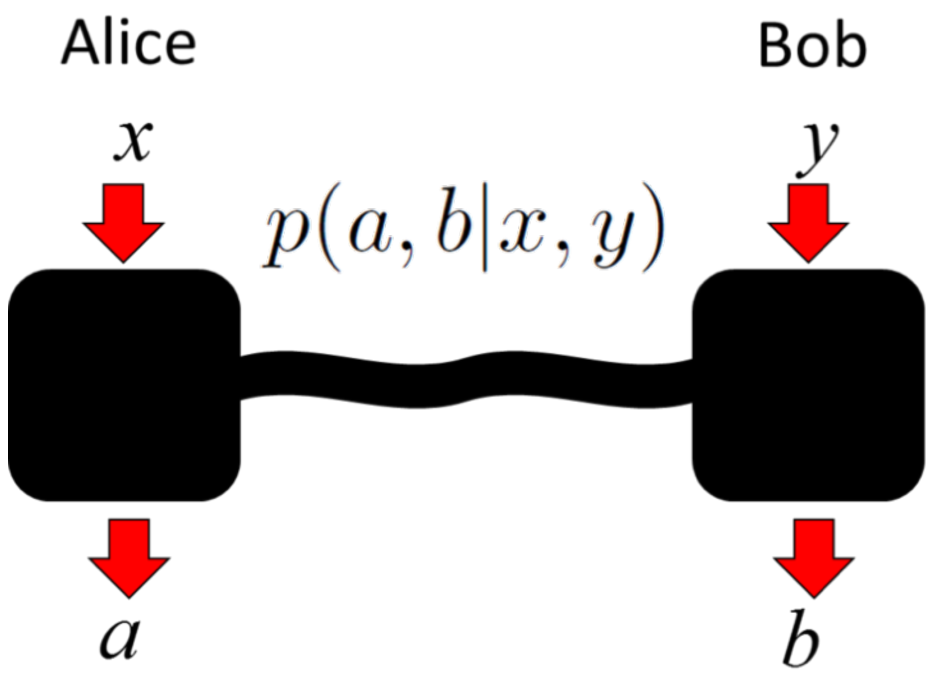

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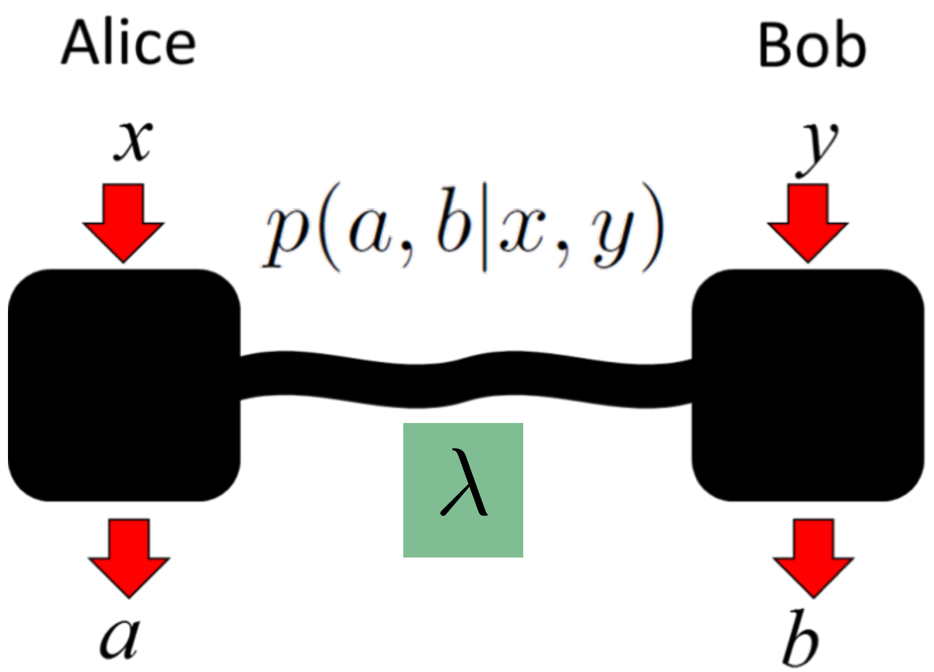


- In **classical** physics / prob. theory:

$$P(a,b|x,y) = \sum_{\lambda \in \Lambda} P_A(a|x,\lambda) P_B(b|y,\lambda) P_\Lambda(\lambda)$$

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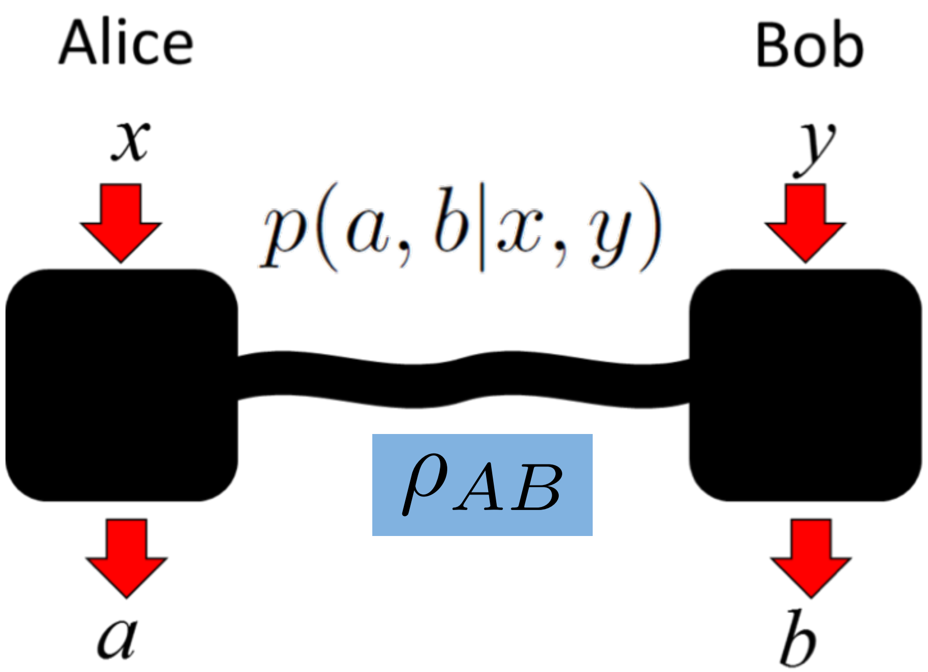


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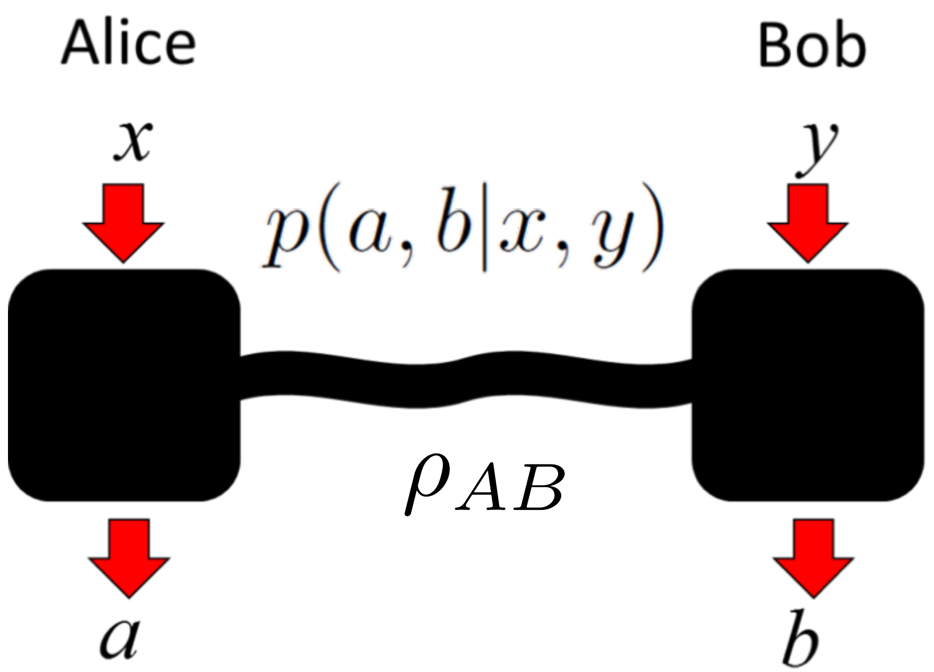
- In **quantum** physics:

$$P(a, b|x, y) = \mathrm{tr}\left[\rho_{AB}(E_x^a \otimes F_y^b)\right]$$

# Black boxes and correlations




**No-signalling** conditions:

$P(a|x,y)$ is independent of $y$,

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- In **classical** physics / prob. theory:

$$P(a,b|x,y) = \sum_{\lambda\in\Lambda} P_A(a|x,\lambda)P_B(b|y,\lambda)P_\Lambda(\lambda)$$

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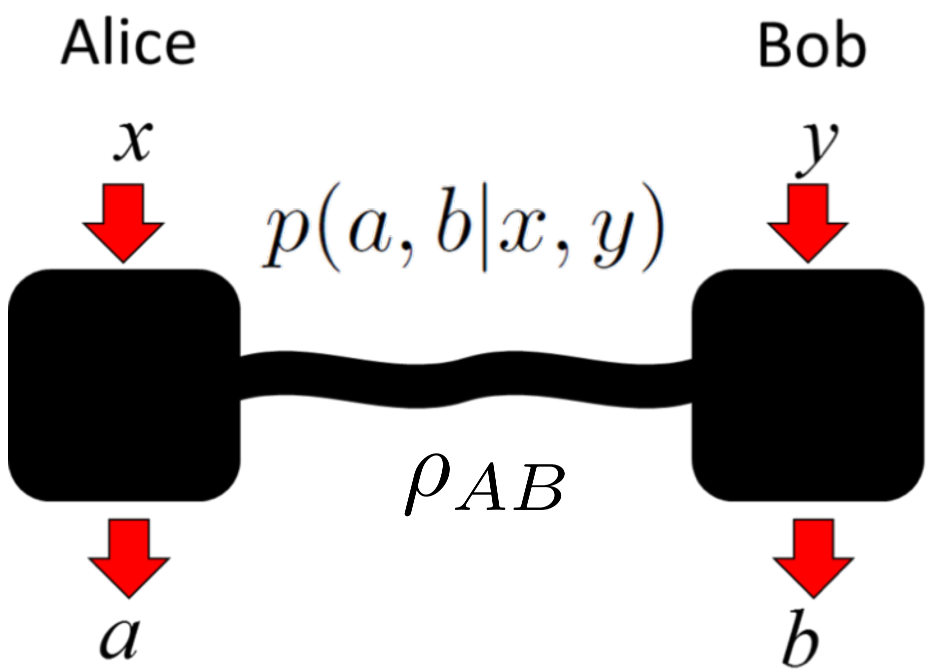
$$P(a,b|x,y) = \mathrm{tr}\left[\rho_{AB}(E_x^a \otimes F_y^b)\right]$$

# Black boxes and correlations

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Quantum admits more general *P*'s due to the **violation of Bell inequalities**.

# The Bell-CHSH inequality

Classical probability distributions satisfy Bell inequality:

$$\mathrm{CHSH} := |C_{00} + C_{01} + C_{10} - C_{11}| \leq 2 \quad \text{where} \quad C_{ab} := \mathbb{E}\left(x \cdot y | a, b\right).$$

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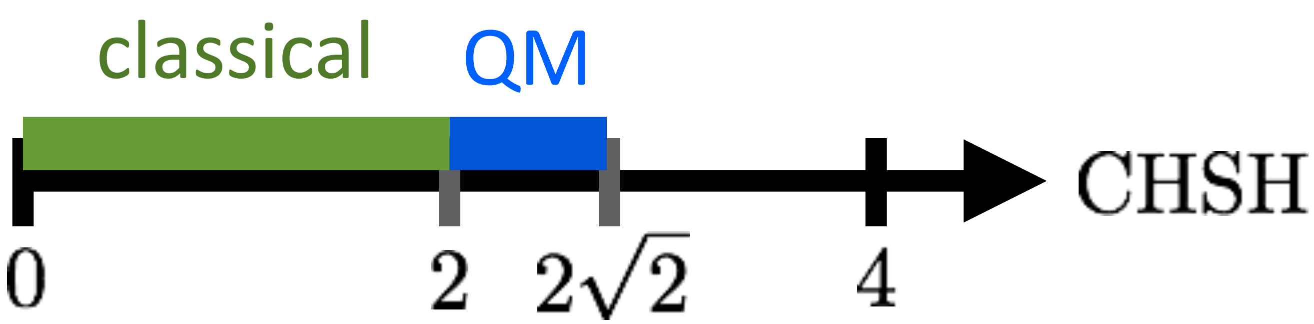


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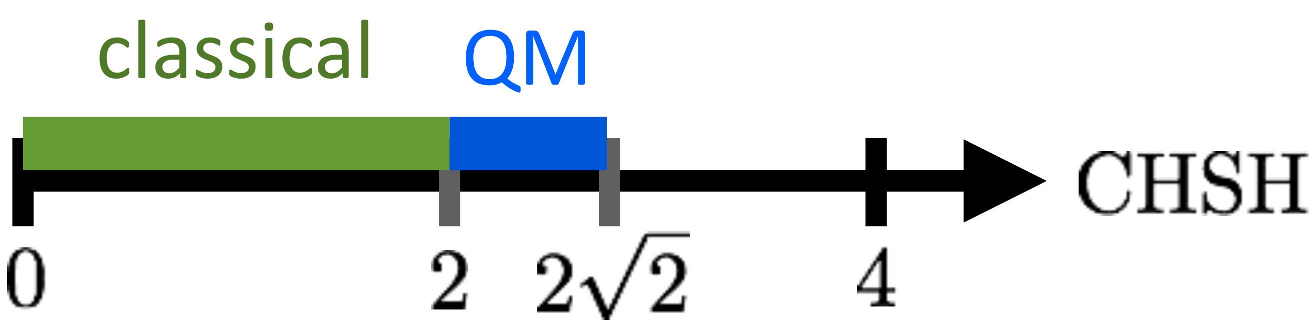


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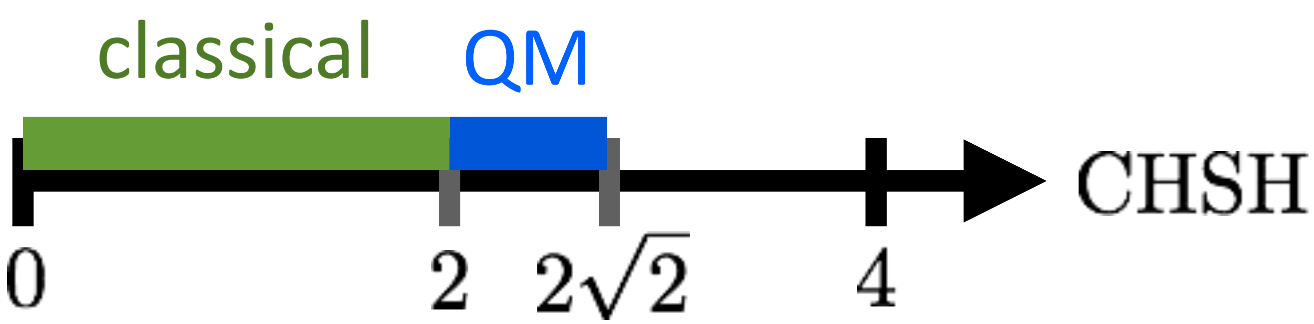


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No. Counterexample: the PR-box correlations

$$\begin{aligned} p(+1,+1|a,b) &= p(-1,-1|a,b) = \frac{1}{2} \\ &\quad \text{if } (a,b) \in \{(0,0),(0,1),(1,0)\} \\ p(+1,-1|1,1) &= p(-1,+1|1,1) = \frac{1}{2} \end{aligned}$$

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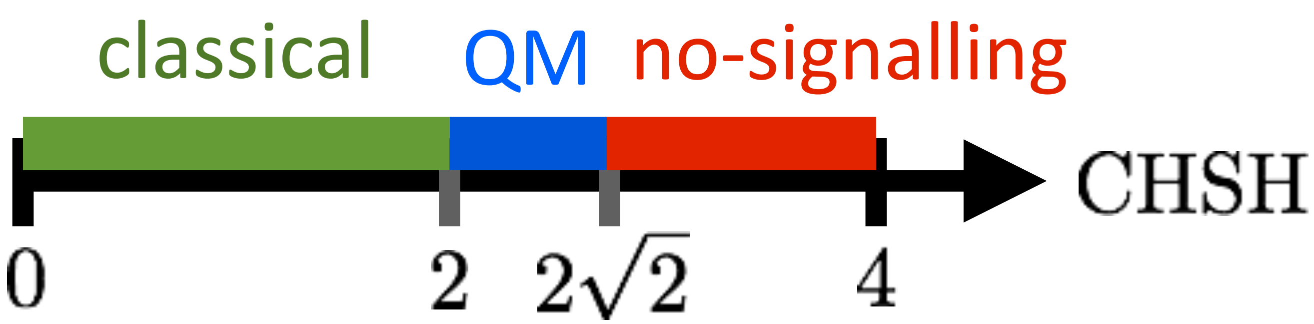


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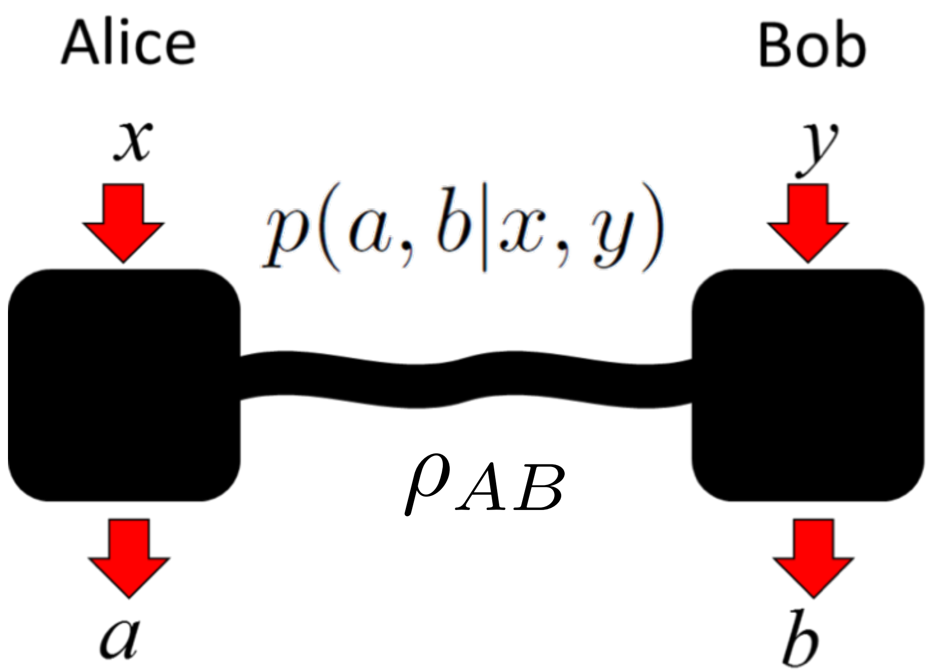


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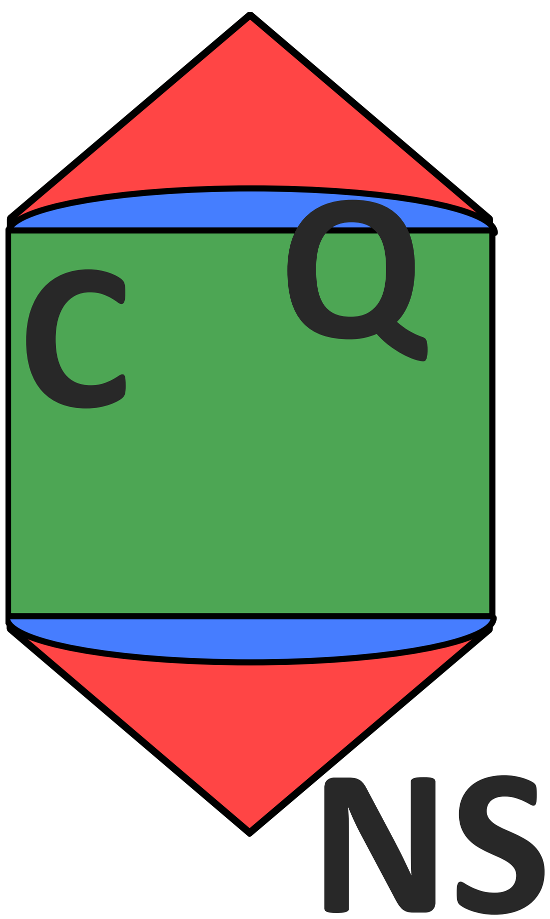

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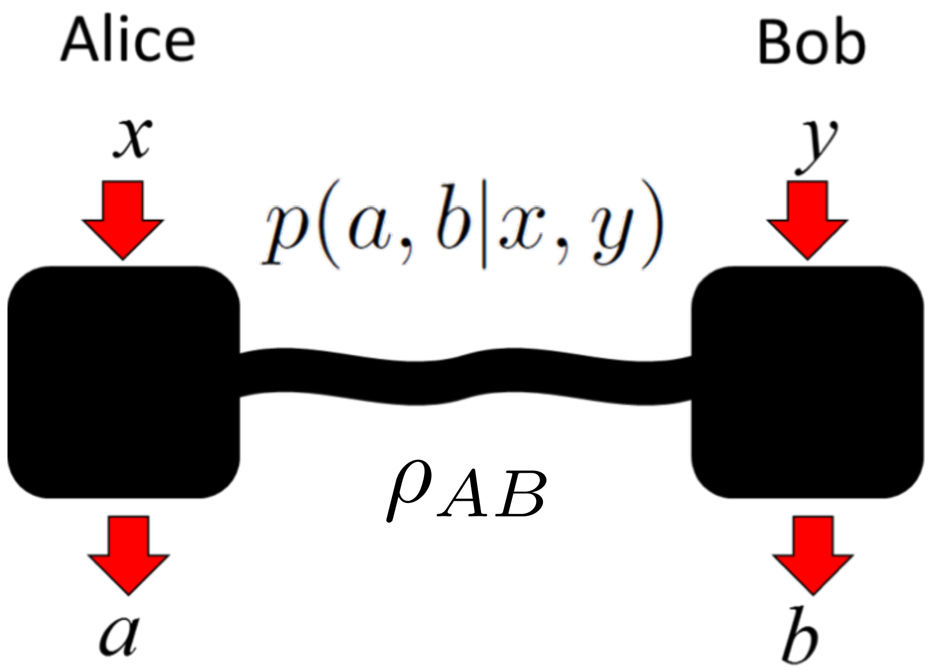


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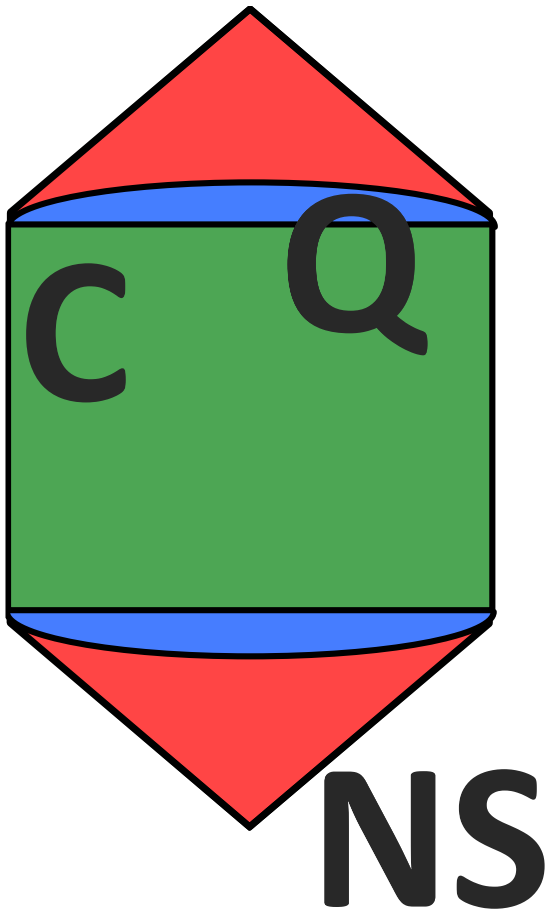
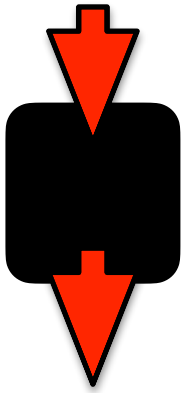



Why study such correlations?

- **Foundational:** "Why" does nature admit **Q** but not more? **Principles?**
- **Applications** in DI-independent protocols.

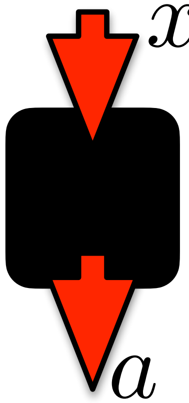
# Single black boxes



$$P(a|x)$$

# Single black boxes

$x \in \{0, 1\}$



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Inputs and outputs are typically taken as **abstract labels** (bits etc.)

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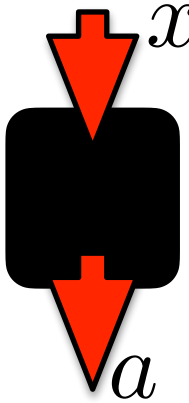

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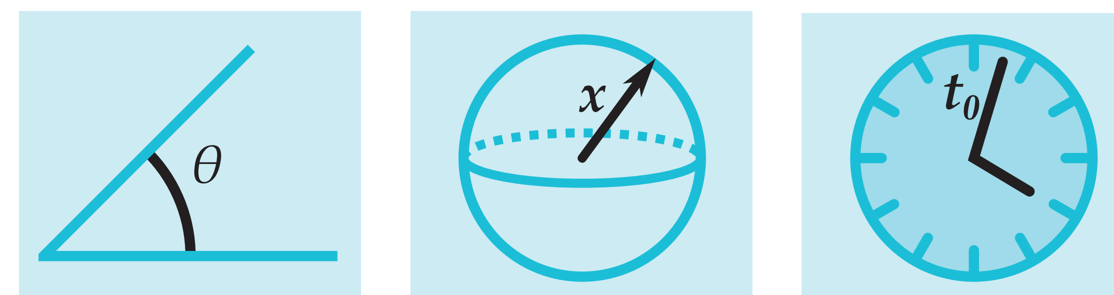


ANGLES DIRECTIONS DURATIONS

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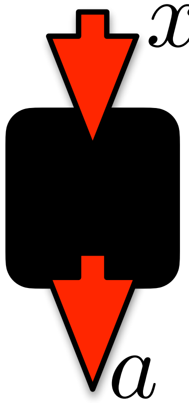

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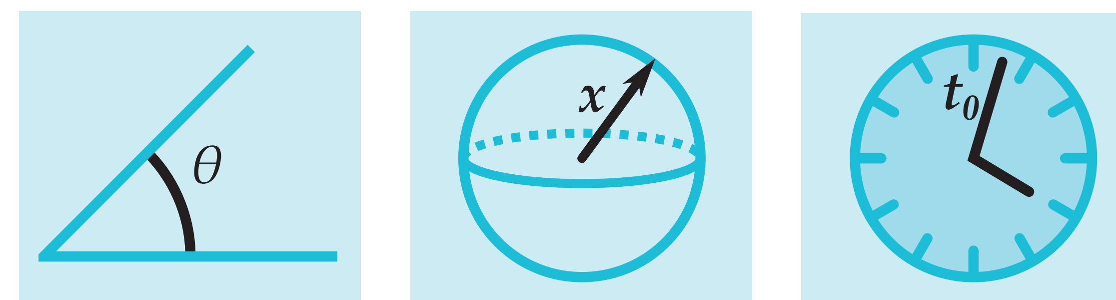


ANGLES DIRECTIONS DURATIONS

- This is the case in many **actual experimental settings**.

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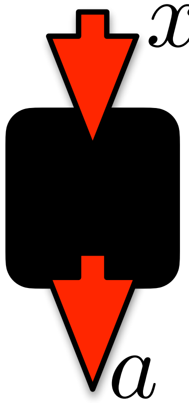

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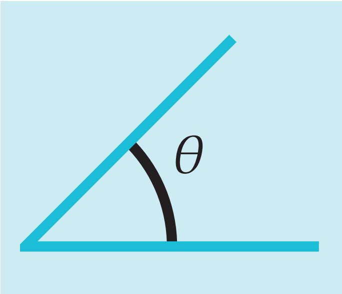
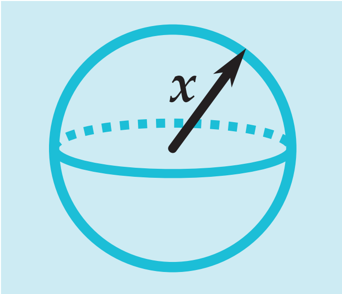



ANGLES




DIRECTIONS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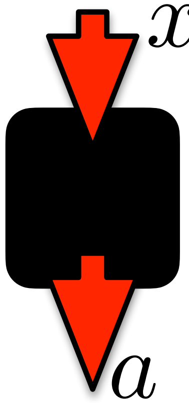

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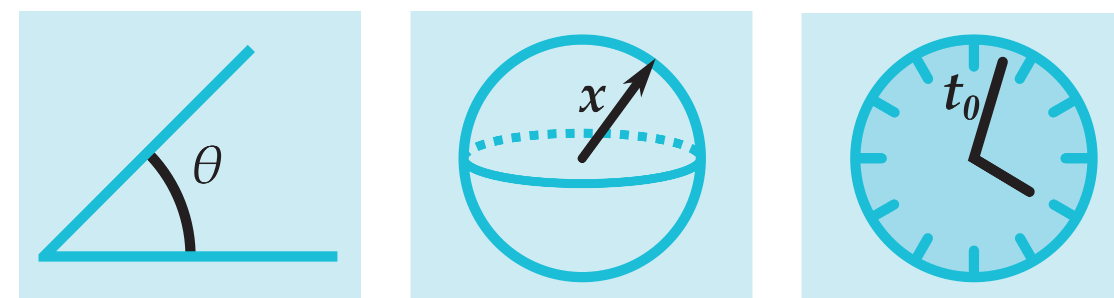


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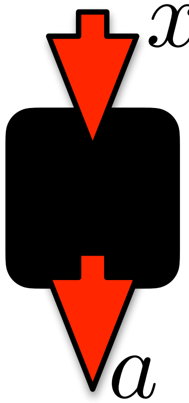

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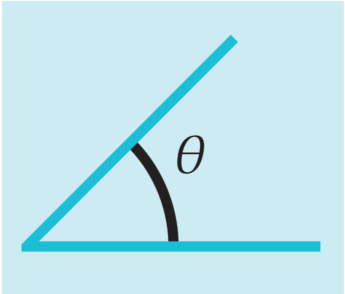


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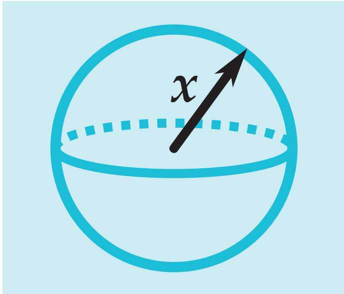


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- How could possible beyond-quantum physics fit into space and time?

# Example: Stern-Gerlach experiment

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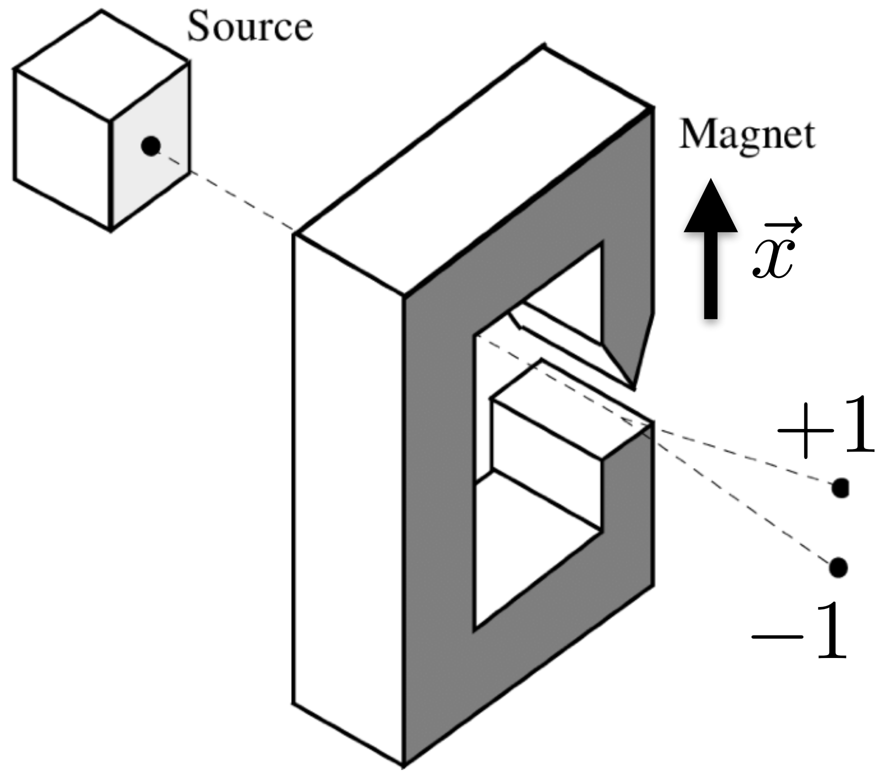

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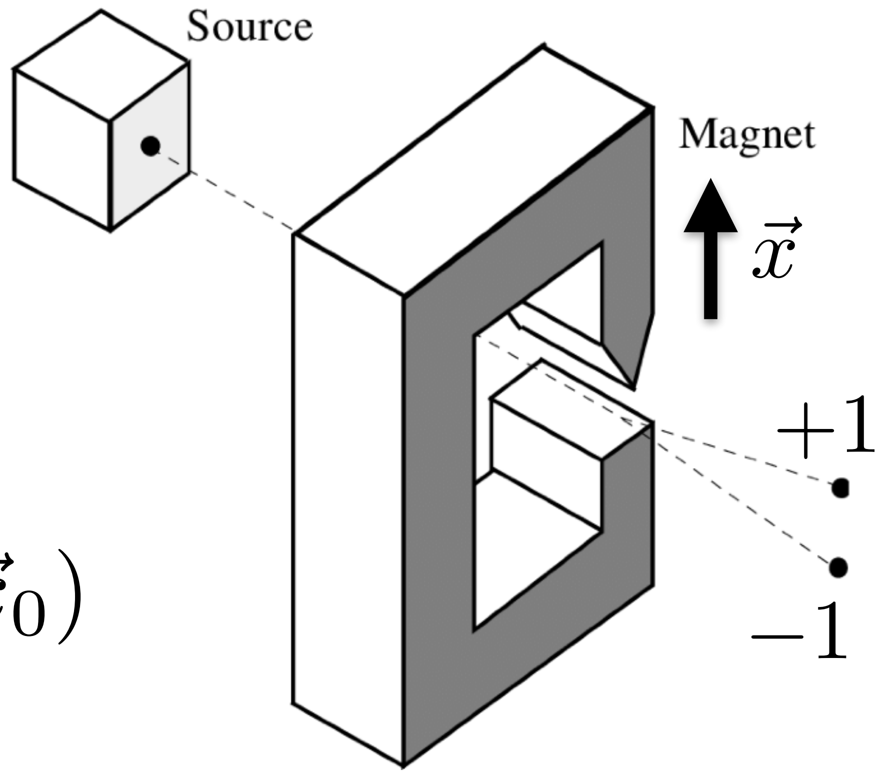


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$$= P(a|R\vec{x}_0)$$

$$|\vec{x}| = 1,$$

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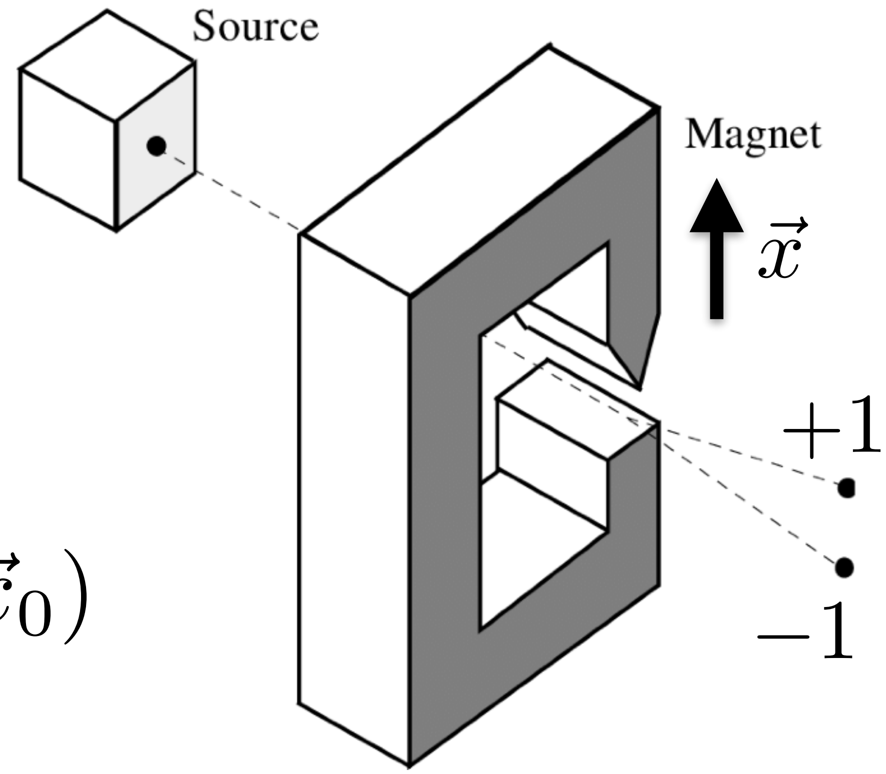


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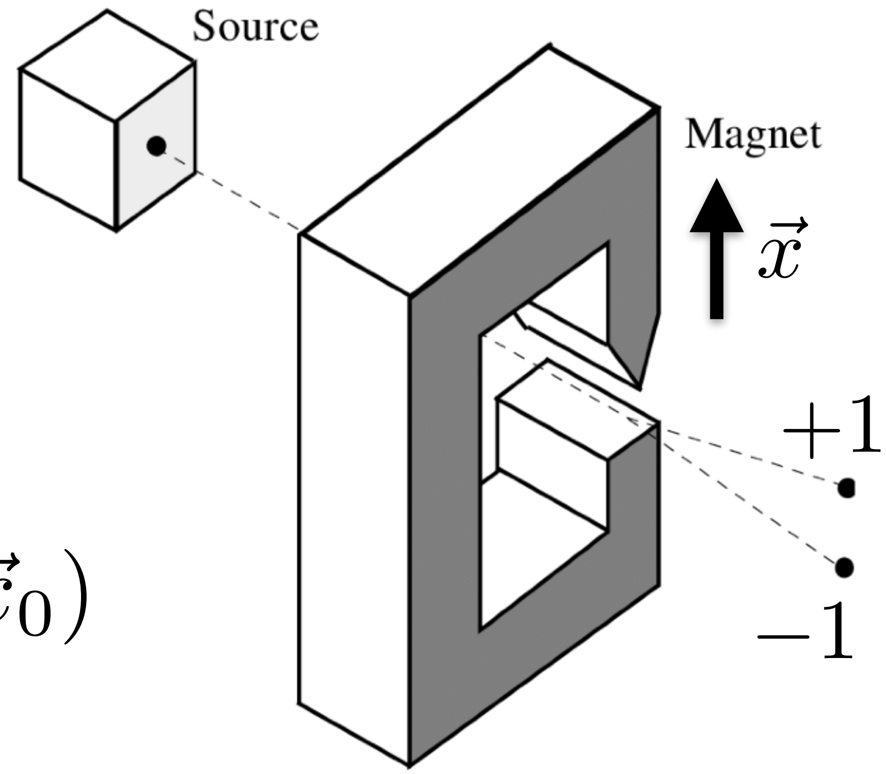


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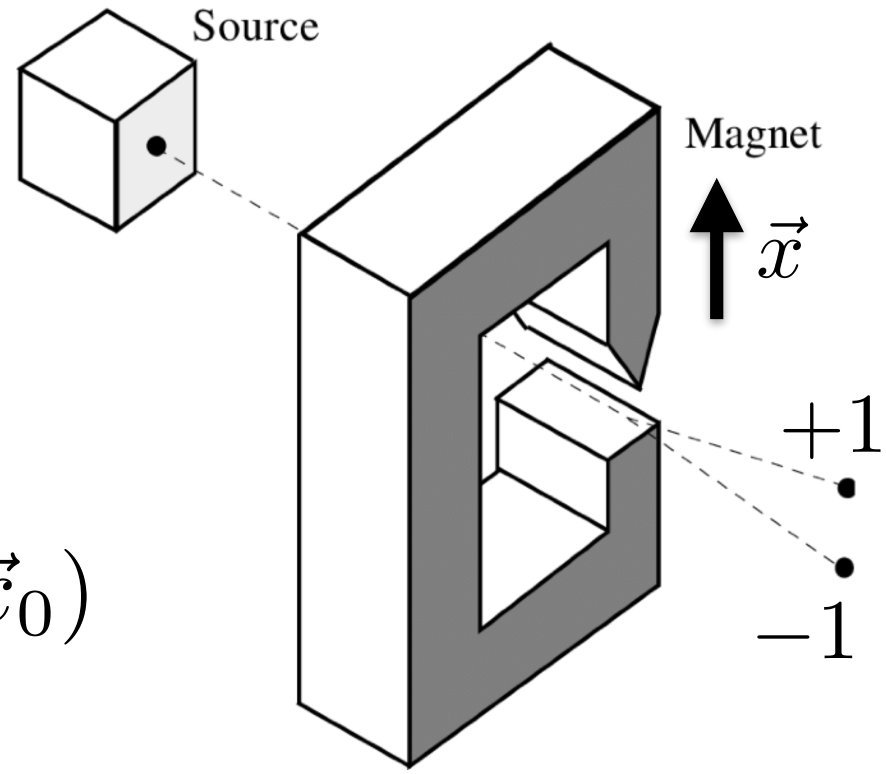


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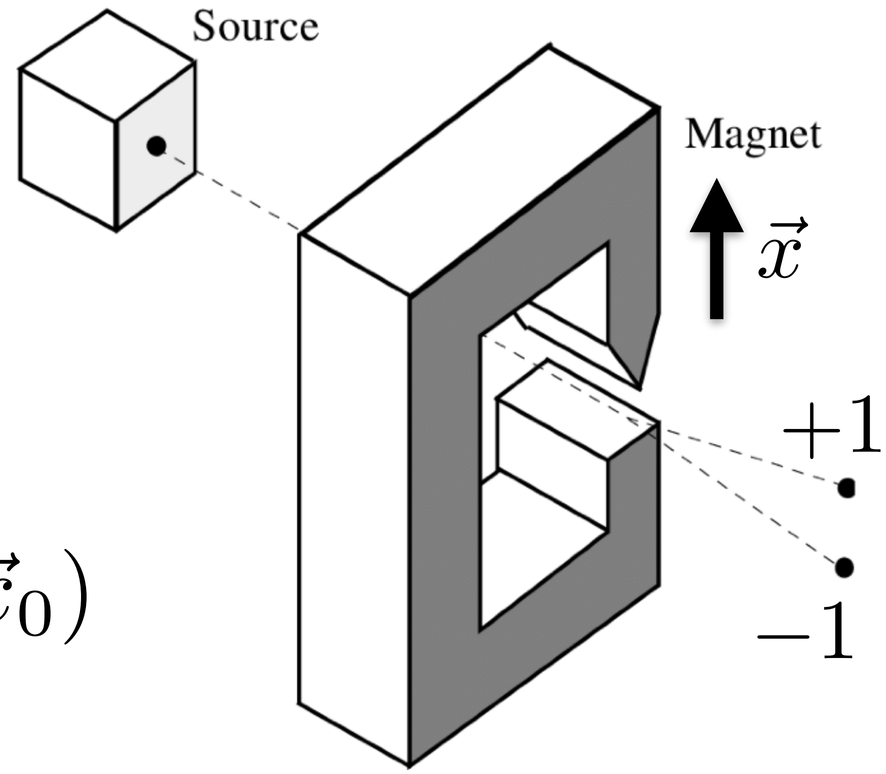


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- In general, **inputs are elements of a homogeneous space,** $\mathcal{G}/\mathcal{H}$.

# Spacetime boxes

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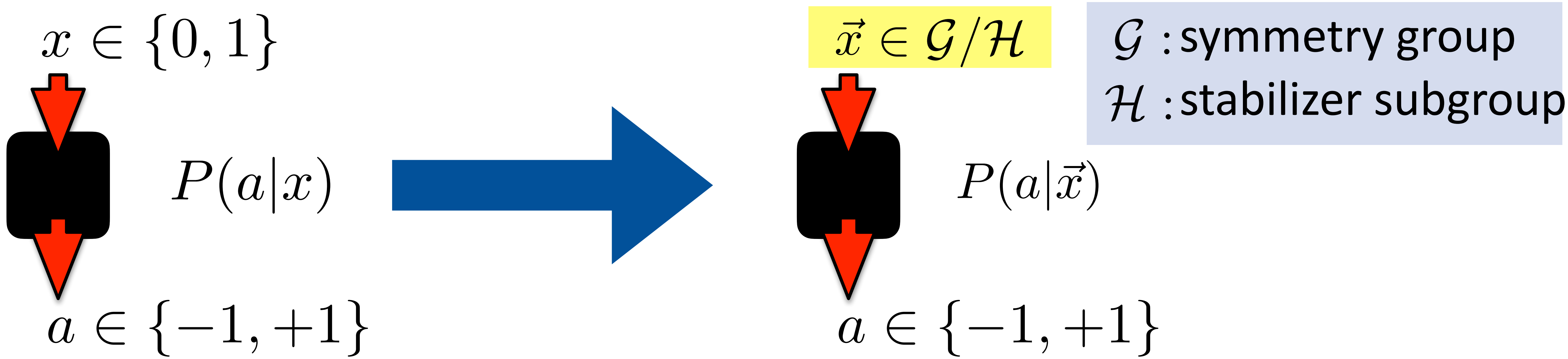

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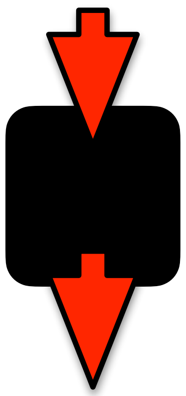

$P(a|x)$

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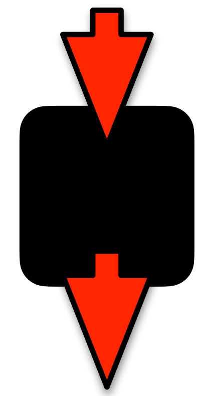
$\vec{x} \in \mathcal{G}/\mathcal{H}$

$\mathcal{G}$ : symmetry group

$\mathcal{H}$ : stabilizer subgroup



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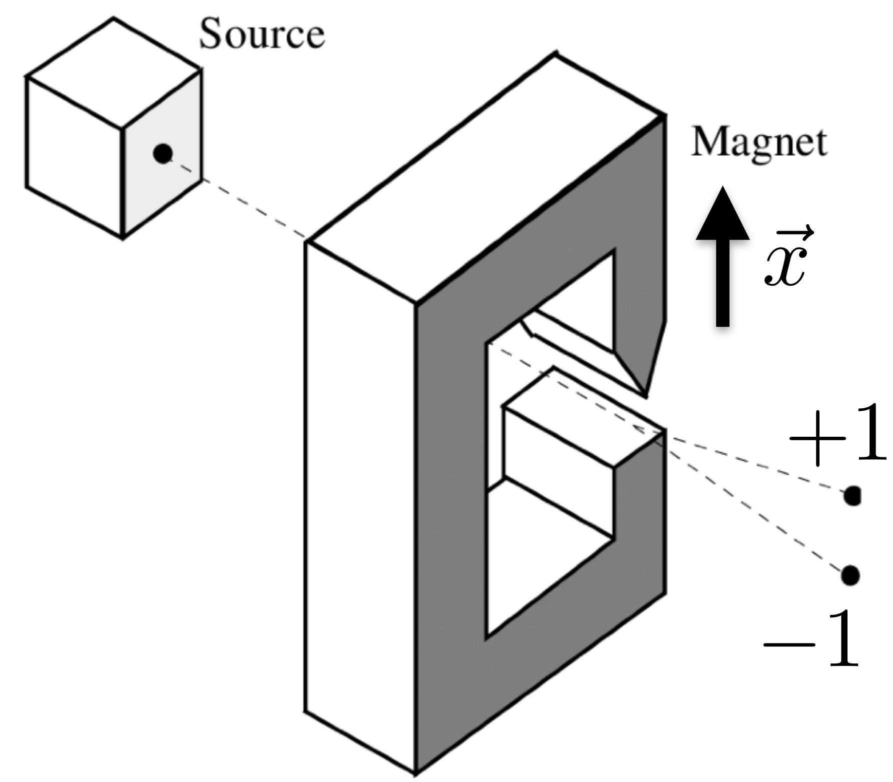
**Example:** Stern-Gerlach experiment

$\mathcal{G} = \mathrm{SO}(3)$ (spatial rotations)

$\mathcal{H} = \mathrm{SO}(2)$ (axial symmetry of magnetic field)

$\vec{x} \in \mathcal{G}/\mathcal{H} = S^2$ (unit vector: field direction)



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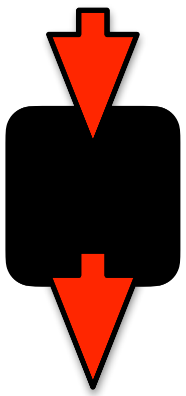

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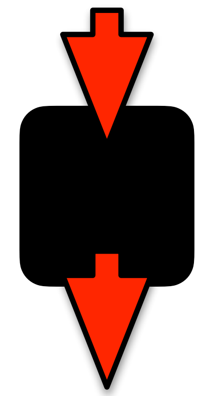

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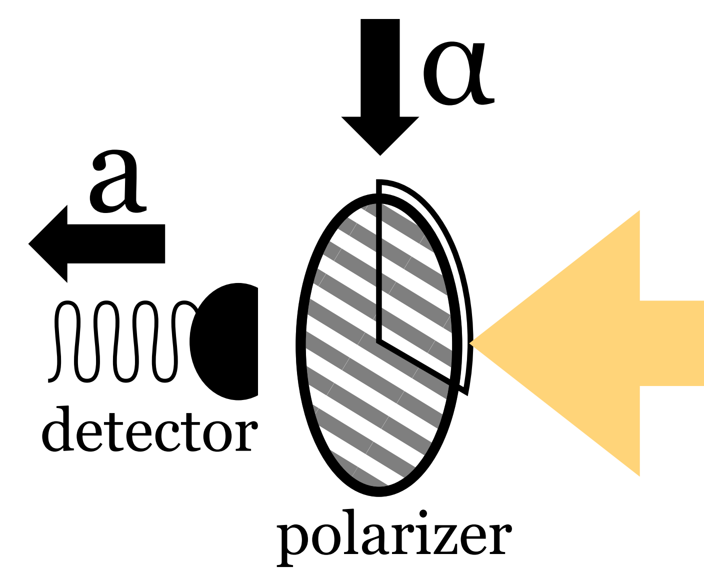
**Example:** Polarizer, $P(a|\alpha)$.

$\mathcal{G} = \mathrm{SO}(2)$ (rotations around beam axis)

$\mathcal{H} = \{\mathbf{1}\}$ (no additional symmetry)

$\alpha \in \mathcal{G}/\mathcal{H} = \mathrm{SO}(2)$.




click / no click: $a = \pm 1$.

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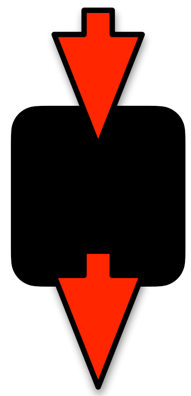

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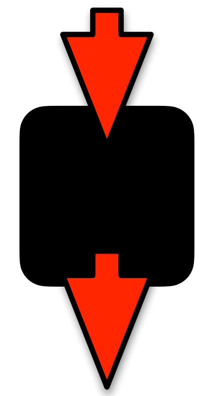

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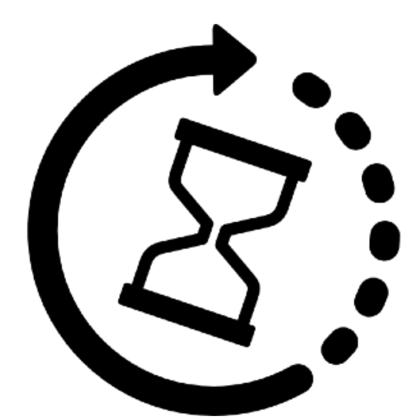
**Example:** Input is time *t*, $P(a|t)$.

$\mathcal{G} = (\mathbb{R}, +)$ (group of time translations)

$\mathcal{H} = \{\mathbf{1}\}$ (no additional symmetry)

$\vec{x} = t \in \mathbb{R}$



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**Theorem.** Probabilistic consistency implies that $P(a|\vec{x})$ is a linear combination of matrix entries of a real **group representation** of $\mathcal{G}$.

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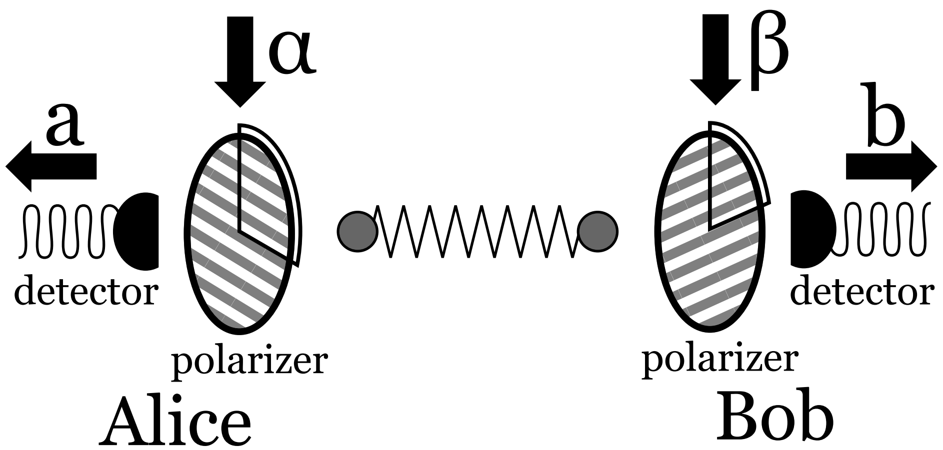


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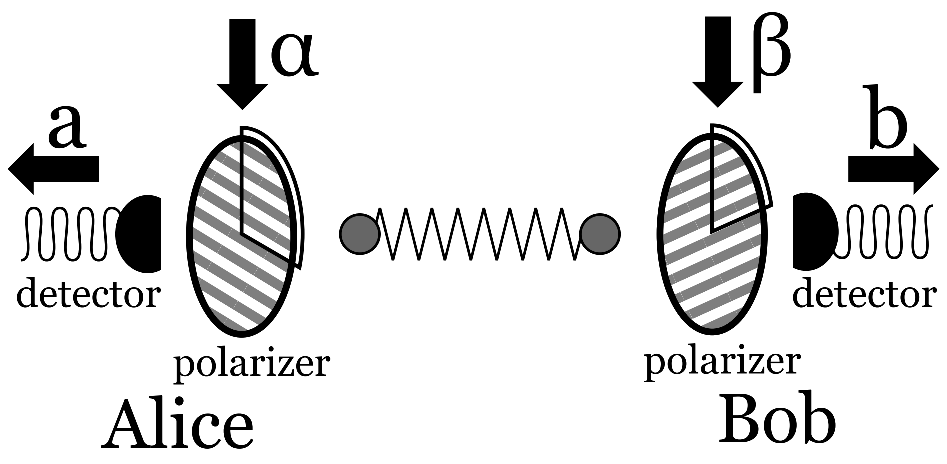


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$$T_{\alpha,\beta} = \bigoplus_{m,n} \begin{pmatrix} \cos(m\alpha - n\beta) & \sin(m\alpha - n\beta) \\ -\sin(m\alpha - n\beta) & \cos(m\alpha - n\beta) \end{pmatrix}.$$

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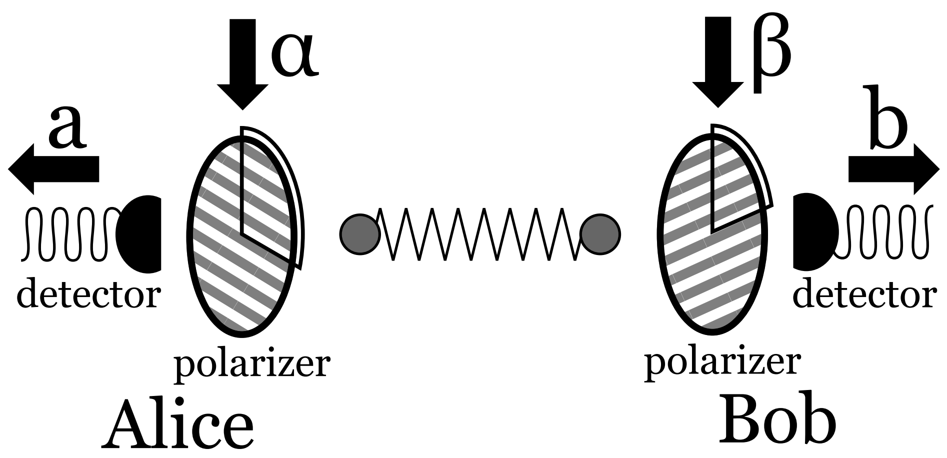


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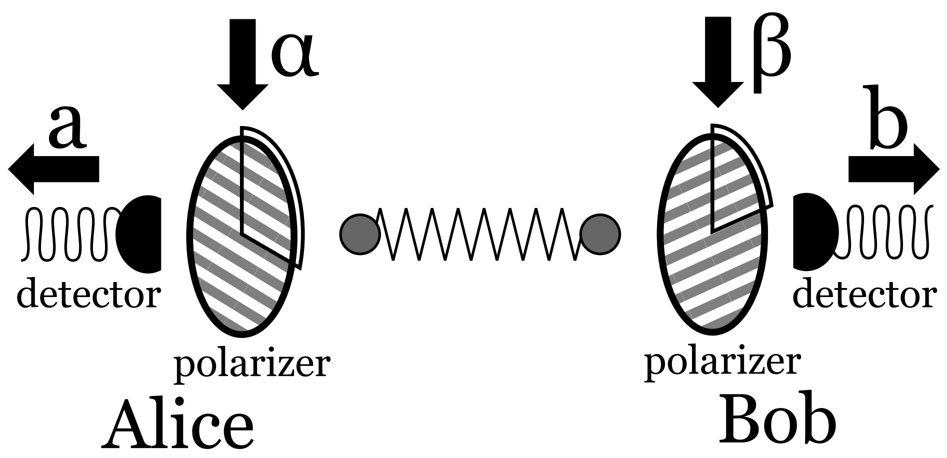


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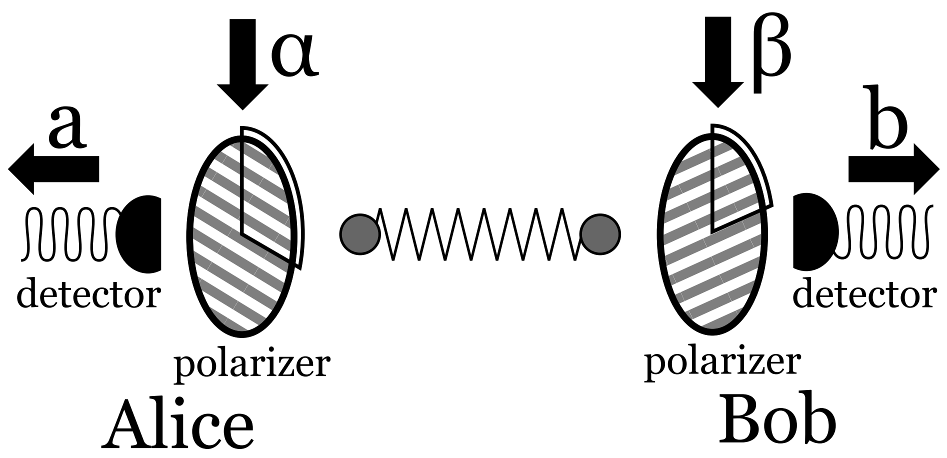


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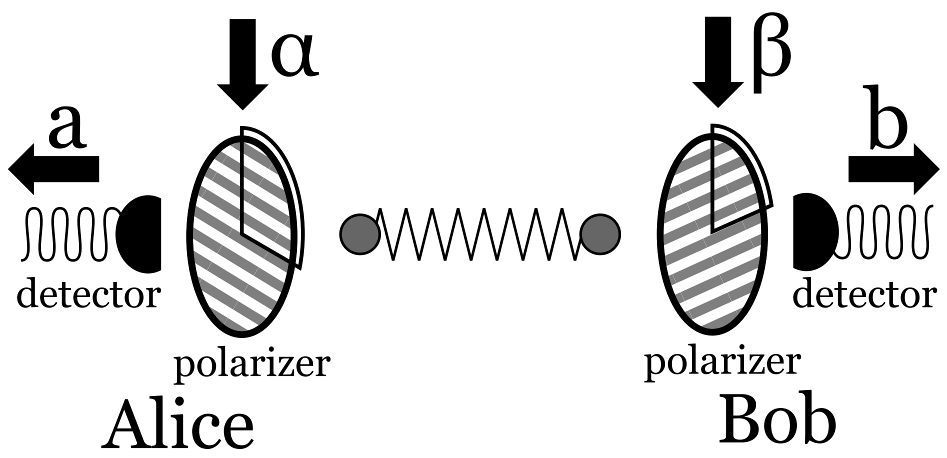


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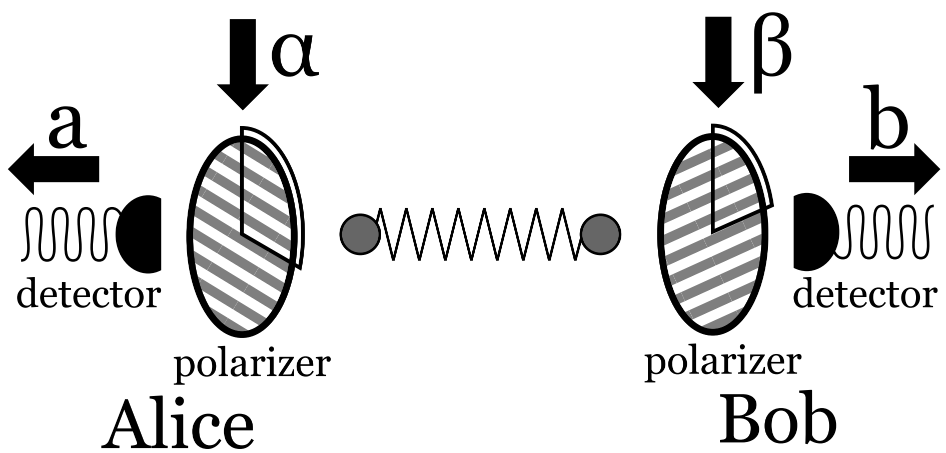


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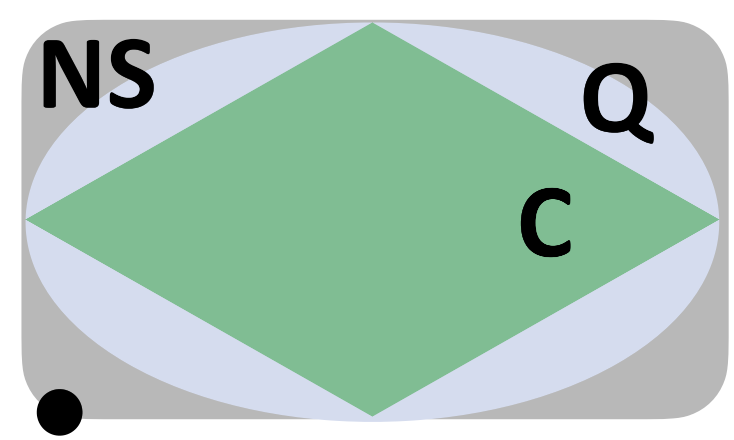


“science-fiction-polarizers”

# Foundational consequences

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

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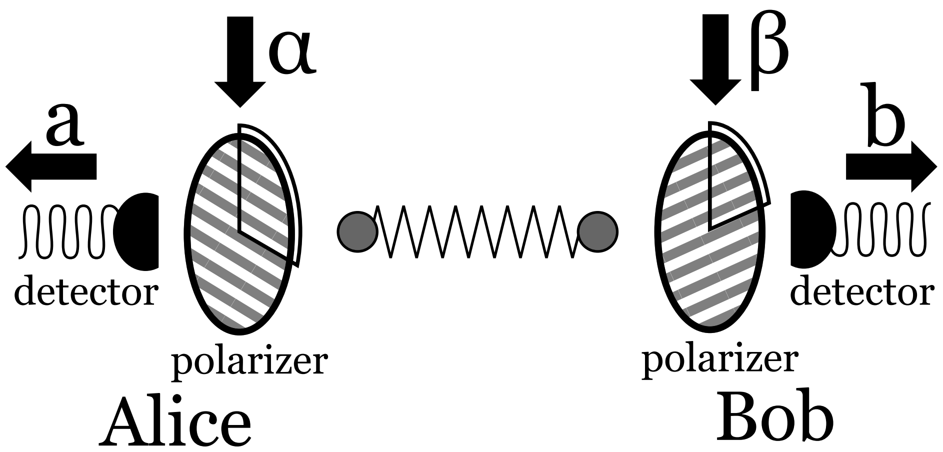


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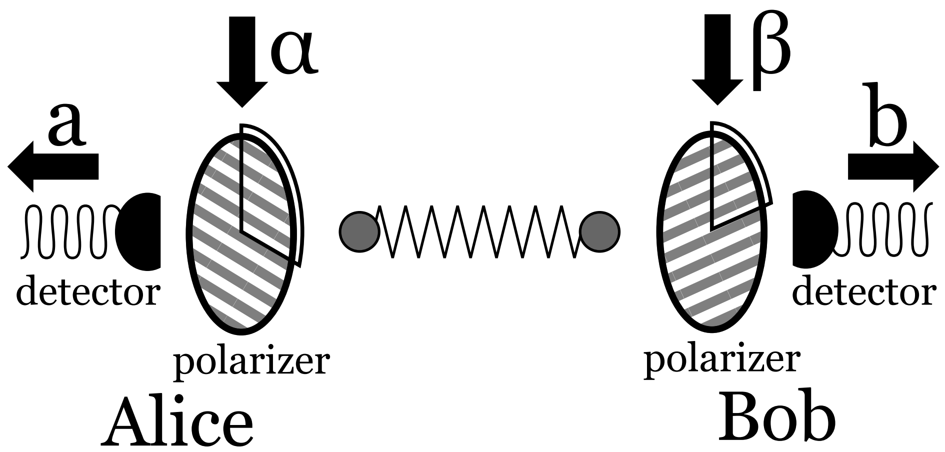


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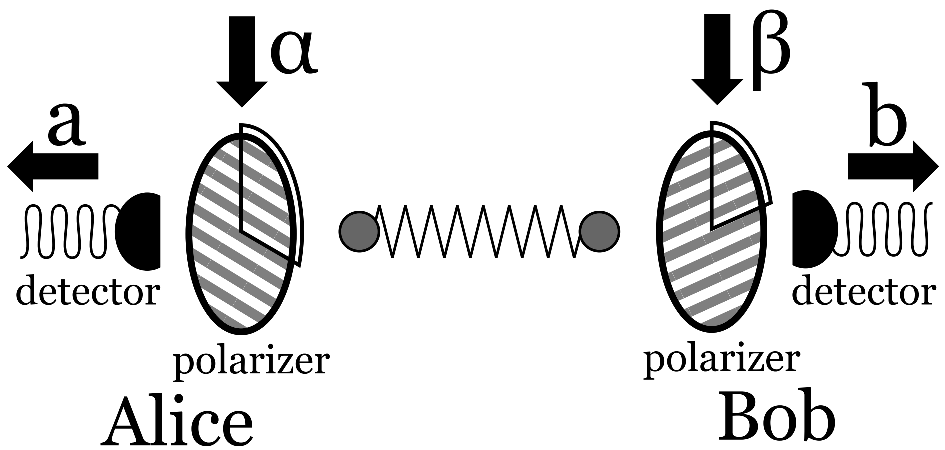


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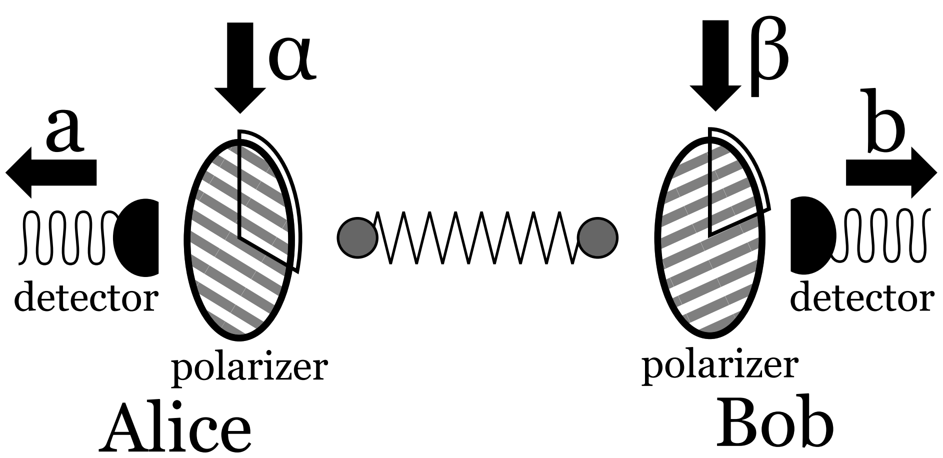


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**Theorem.** In any world where these assumptions hold (**not assuming QT!**), Alice and Bob see **quantum correlations** (i.e. in **Q**). (For arbitrarily many settings, 2 outcomes.)

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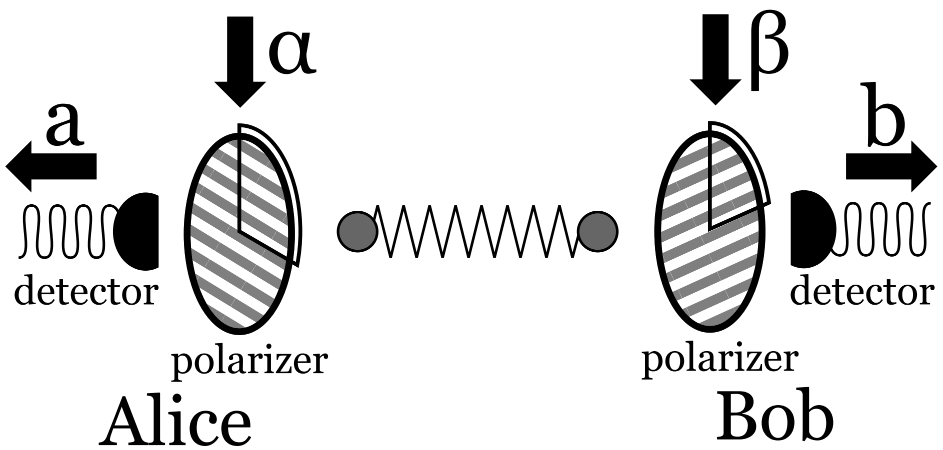


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**Theorem:** The quantum (2,2,2)-correlations **Q** are **exactly those** that can be obtained by $\mathrm{SO}(d) \times \mathrm{SO}(d)$-boxes that satisfy the assumptions above, restricted to 2 inputs per party, and supplemented with shared randomness.

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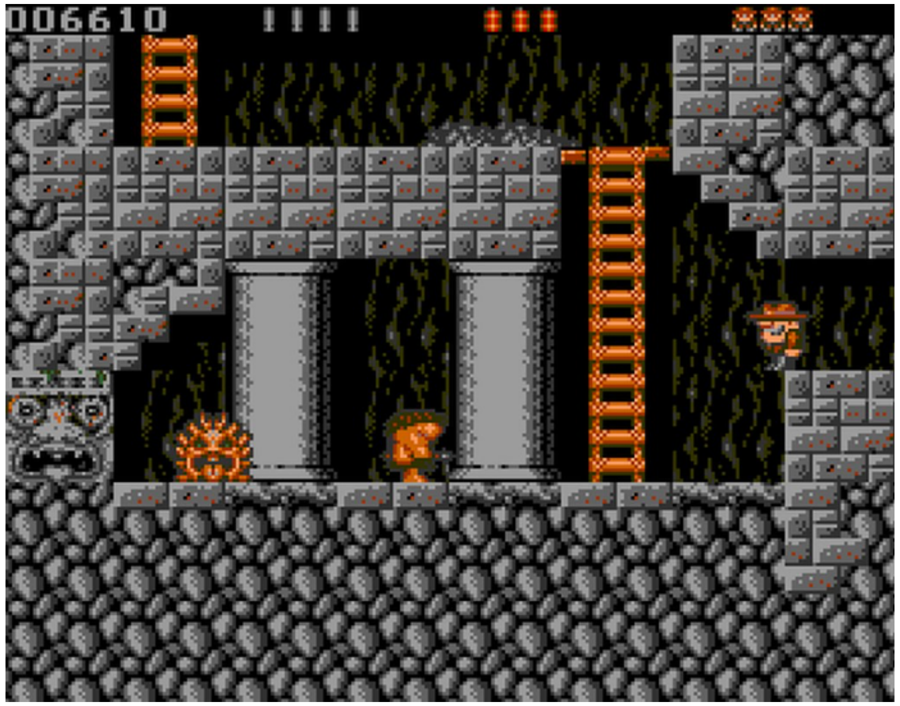


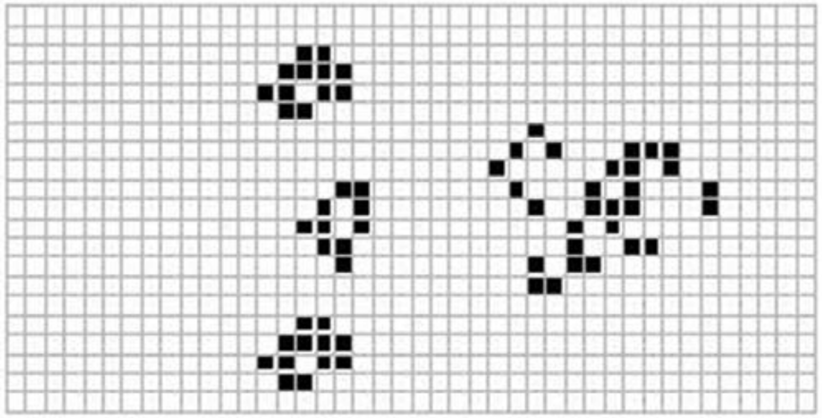

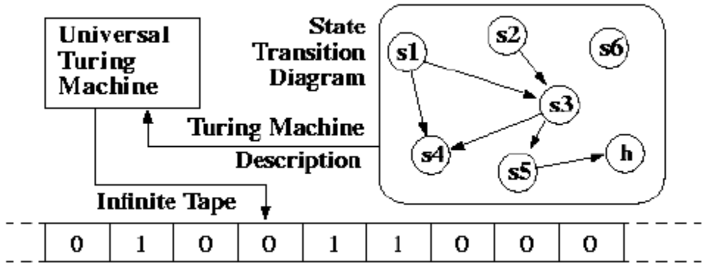

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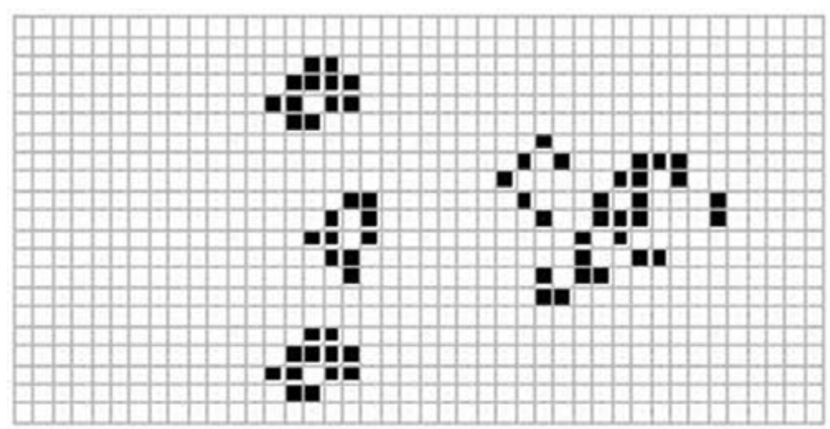

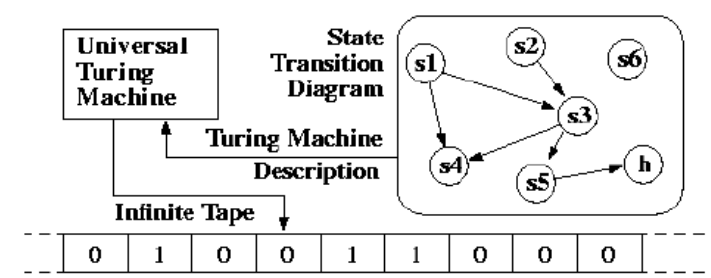


$$p(a)=\sum_b p(a|b)p(b).$$

These structures are very well-motivated in probability theory!

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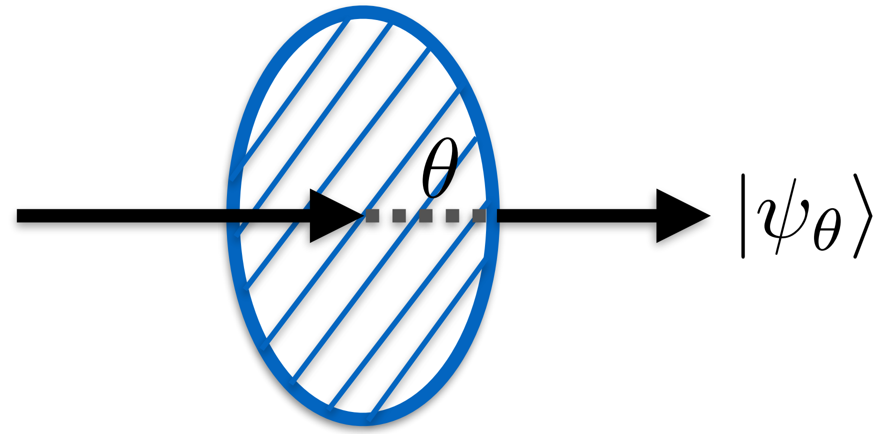
Bill Wootters, PhD thesis, 1980: "*The Acquisition of Information from Quantum Measurements*":



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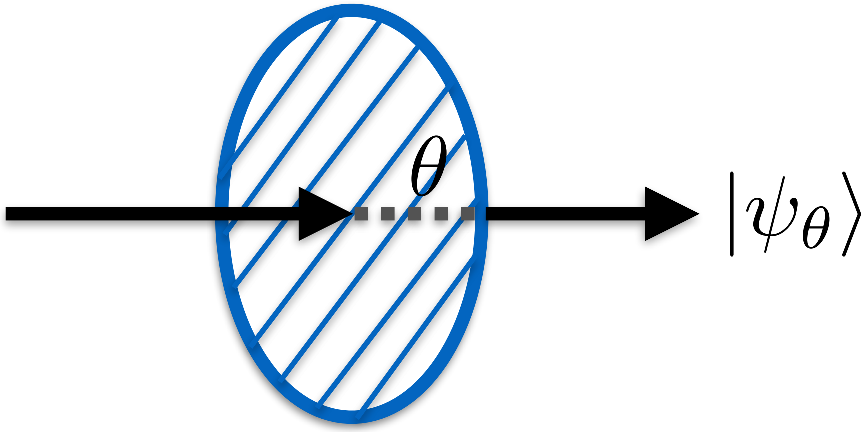
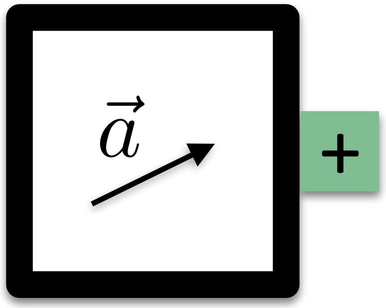
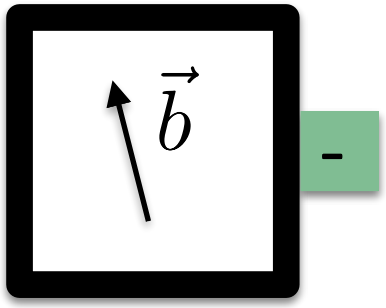



"But why should the statistical distance between two orientations be equal to the angle between them? The best answer we know is the one given at the end of Section A, namely, that the angles we observe in nature may ultimately be derived from the probabilities of the outcomes of spin measurements, which are more primary."

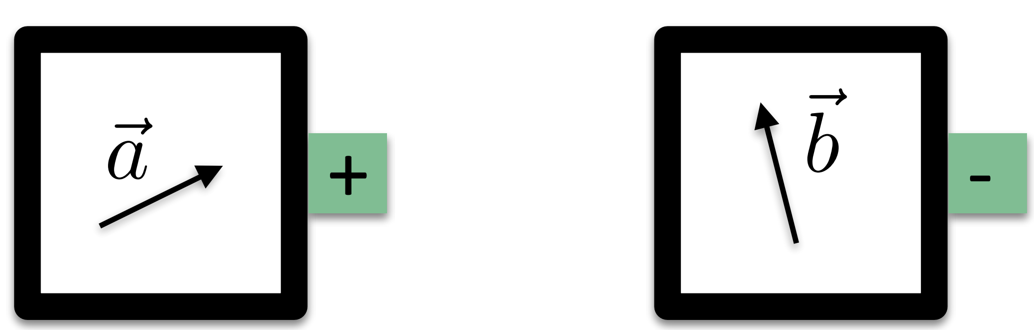
# Rules of the game



# Rules of the game

You are given two black boxes, for which the manufacturer promises that they implement projective Stern-Gerlach measurements along some directions a and b that you don't know.
The devices give the result (+ or -) on a digital display.
You find "devices" in your world that prepare spin-(1/2) systems in states.
You can press a button and prepare the state, but don't know which one it is. The universe gives a supply of many such devices (not necessarily uniformly distributed), exhausting the quantum state space.



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**Goal:** determine the angle $\angle(\vec{a}, \vec{b})$.
**Note:** you are only allowed to "press buttons" and record digital outcomes.
You don't know how to measure lengths, angles, add vectors…
However, you know the type of system that a device prepares.

# Rules of the game


**Sketch of solution:**

- Search the world for preparation devices until you find one such that, for the systems prepared by that state if fed into device A,

$$p_A(+) \approx 1.$$

- Feed the states of that device into measurement device B. Then:

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**Research questions:**

- What are the detailed operational assumptions in this scenario?
- Can we get rid of the assumption of a *projective* measurement?
- Can we show that this is (suitably generalized) *classically impossible*?

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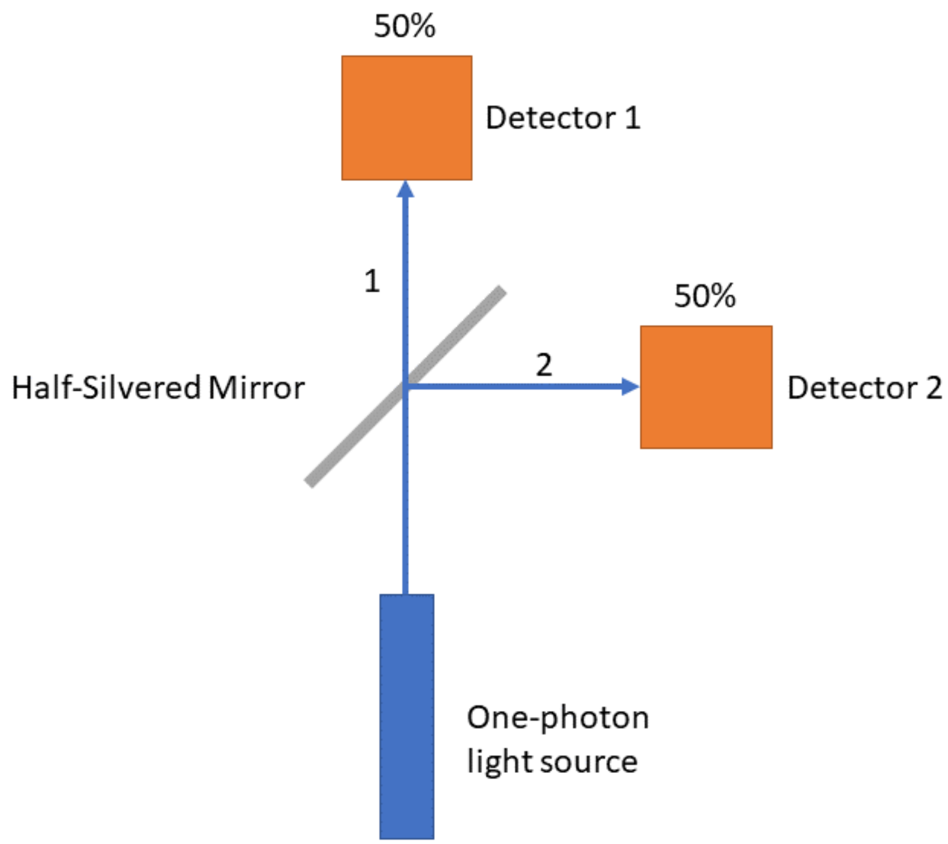

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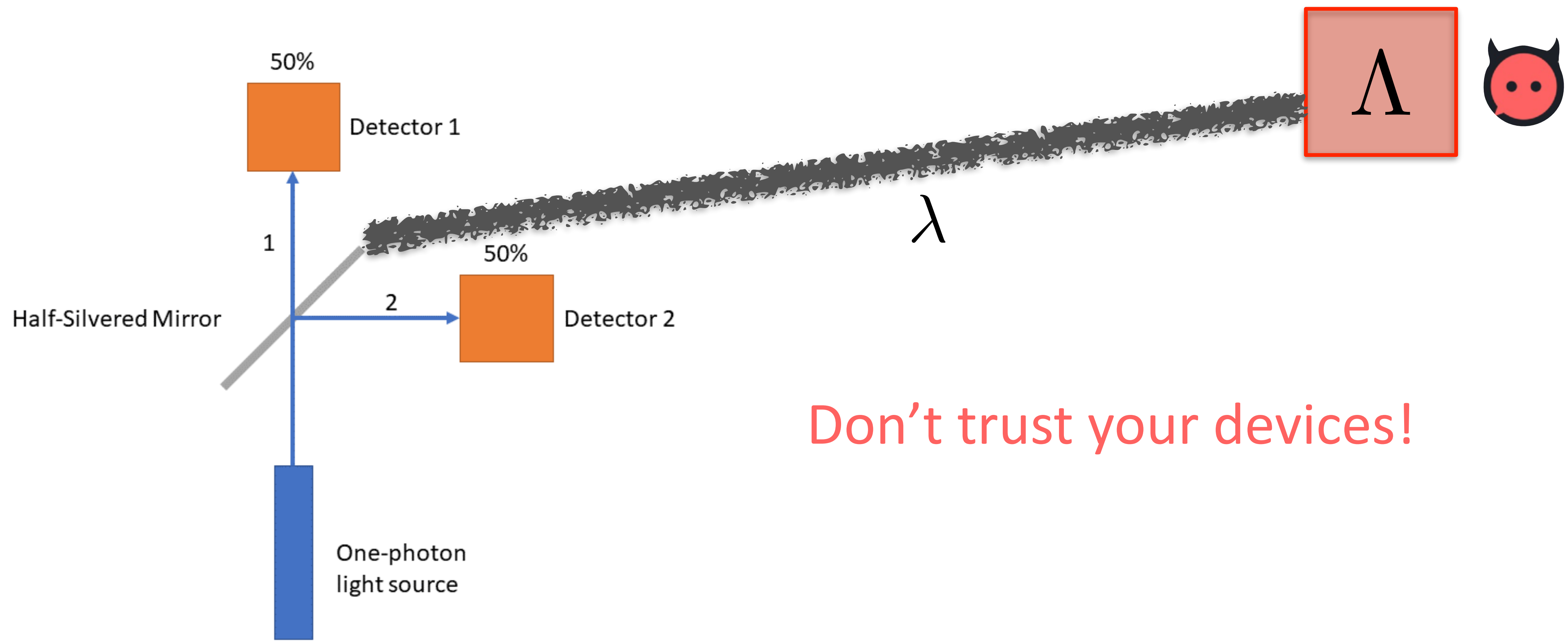


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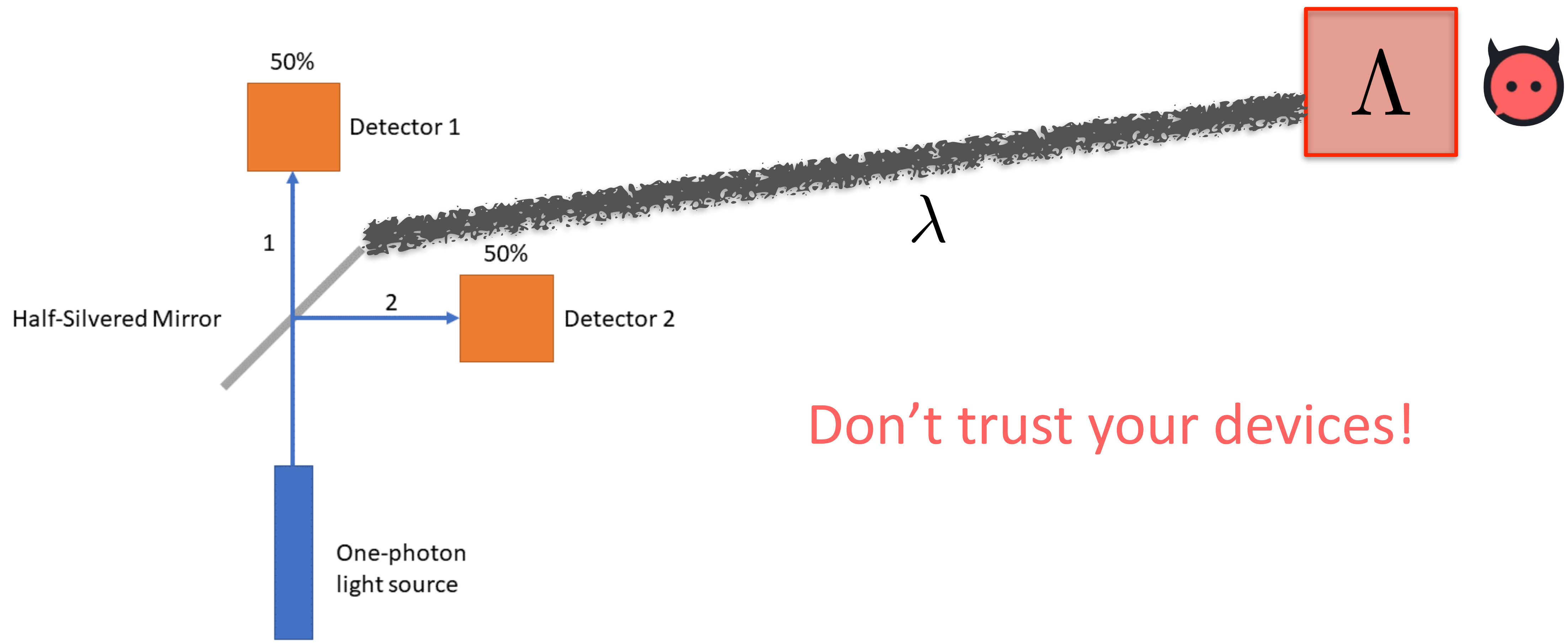


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**Device-independent** randomness expansion:

Violation of Bell inequality $\Rightarrow$ outcomes uncorrelated with rest of the world

**See e.g.:** A. Acín, *Randomness and quantum non-locality*, QCRYPT 2012 talk.
V. Scarani, *Bell nonlocality*, Oxford Graduate Texts (2019).

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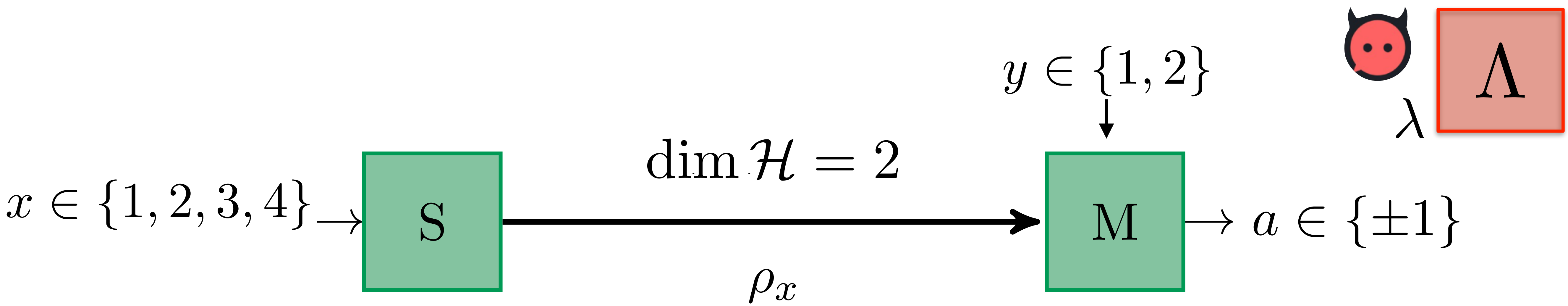

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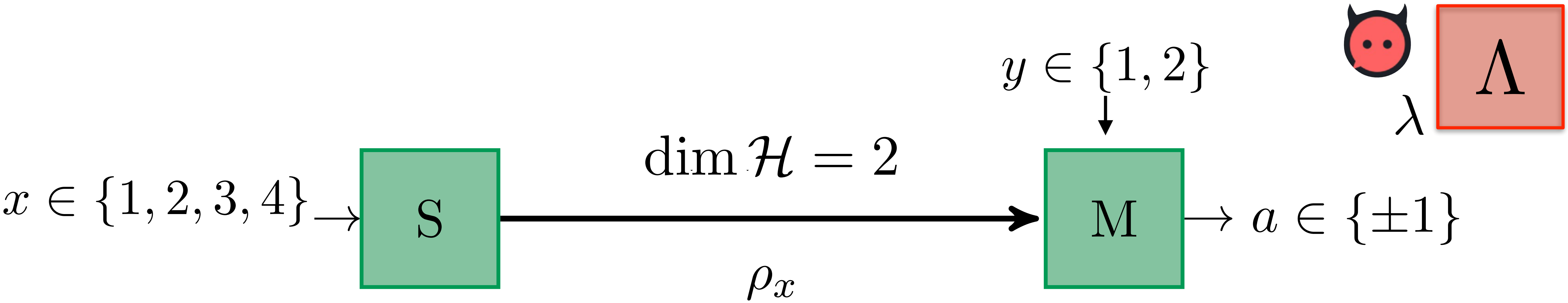


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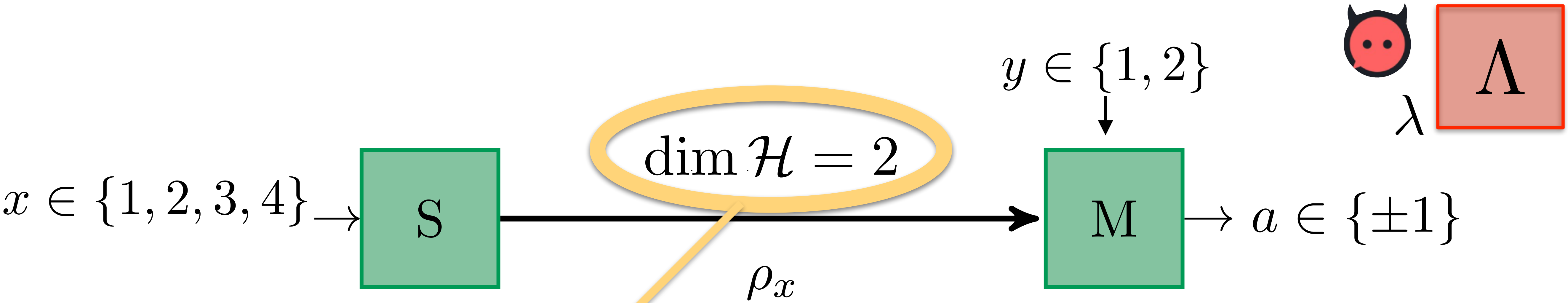


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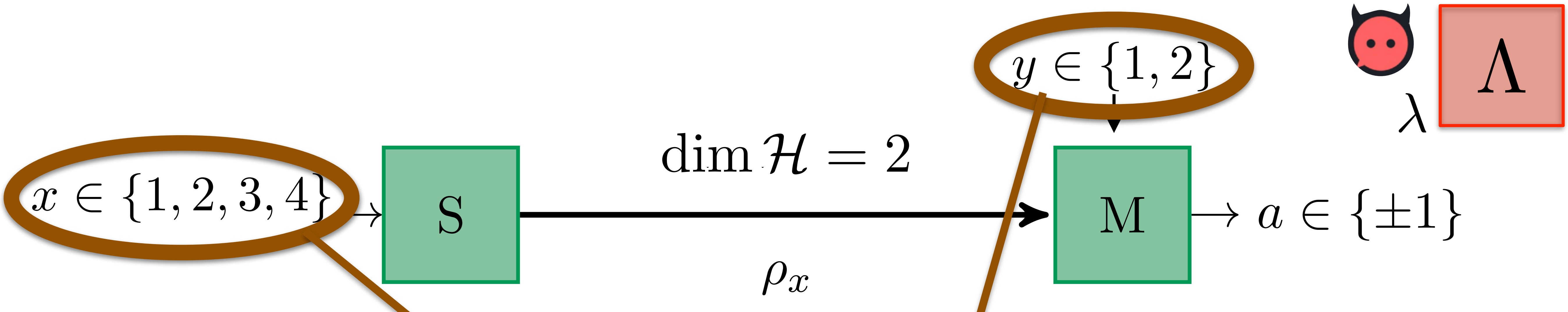


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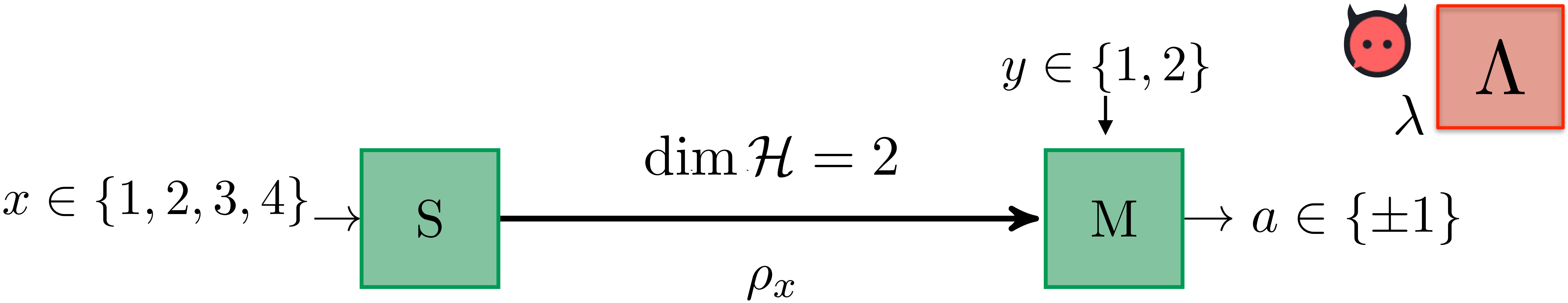


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**Idea:** reformulate in terms of spacetime symmetries, **w/o assuming QT.**
Can quantum phenomenology / functionality be reproduced?

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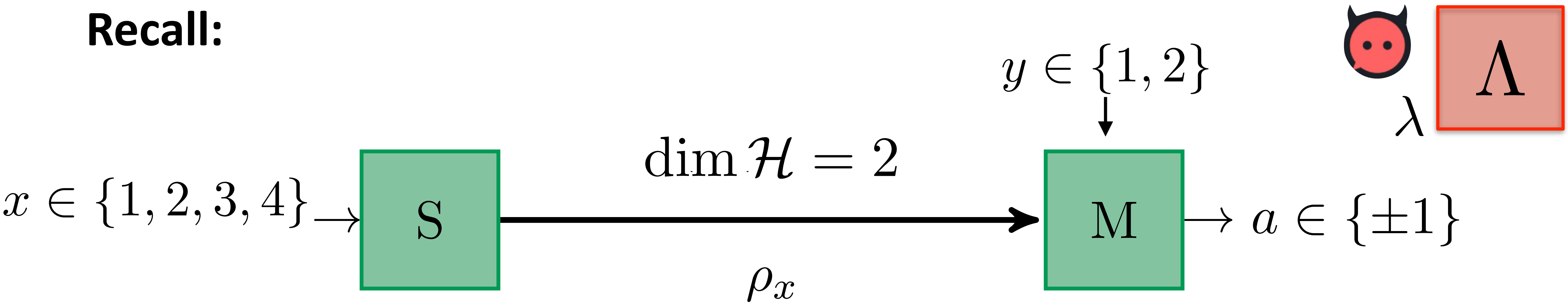
# Randomness generation: quantum analysis

**Recall:**



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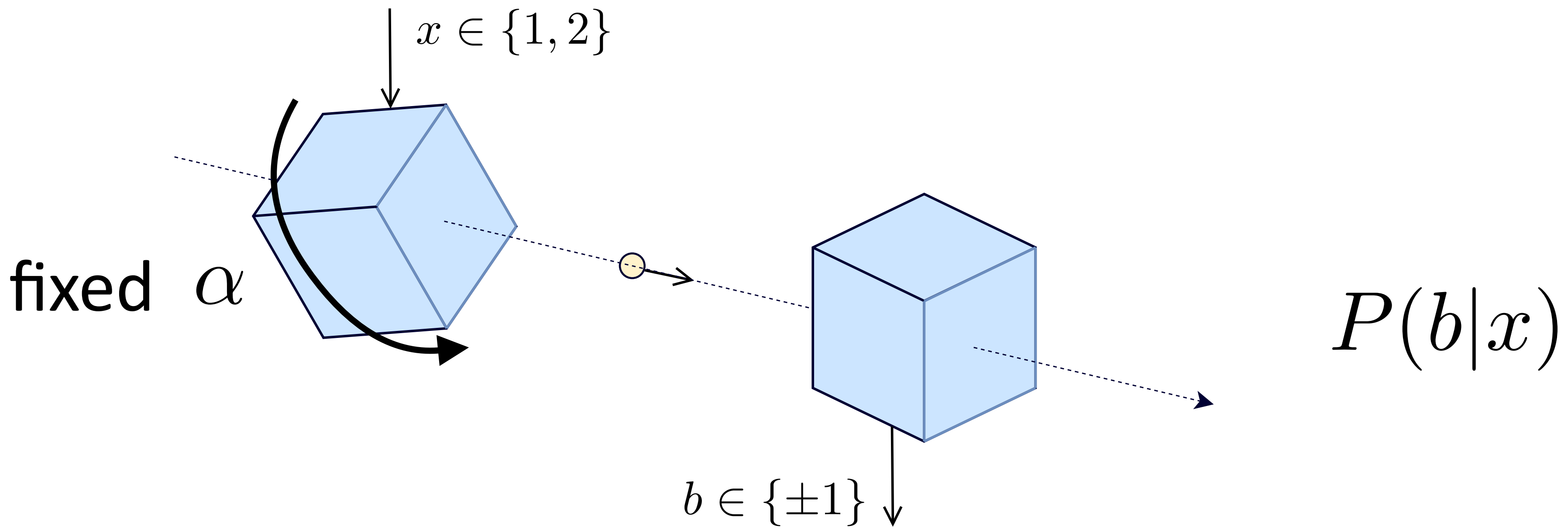

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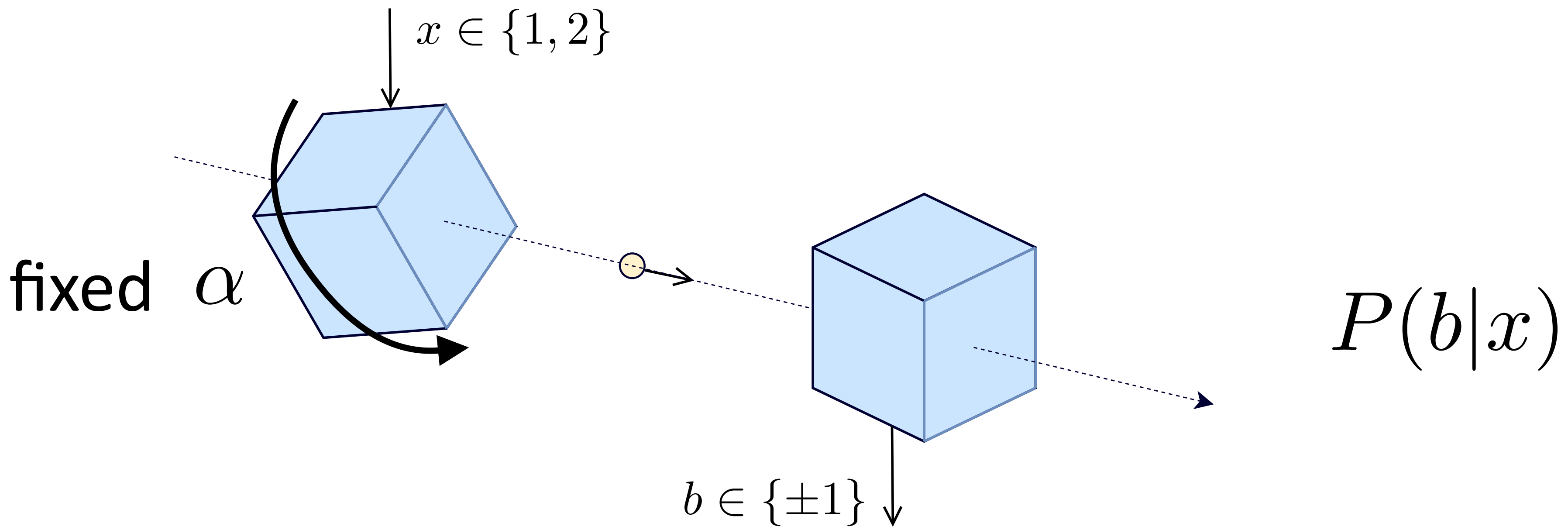


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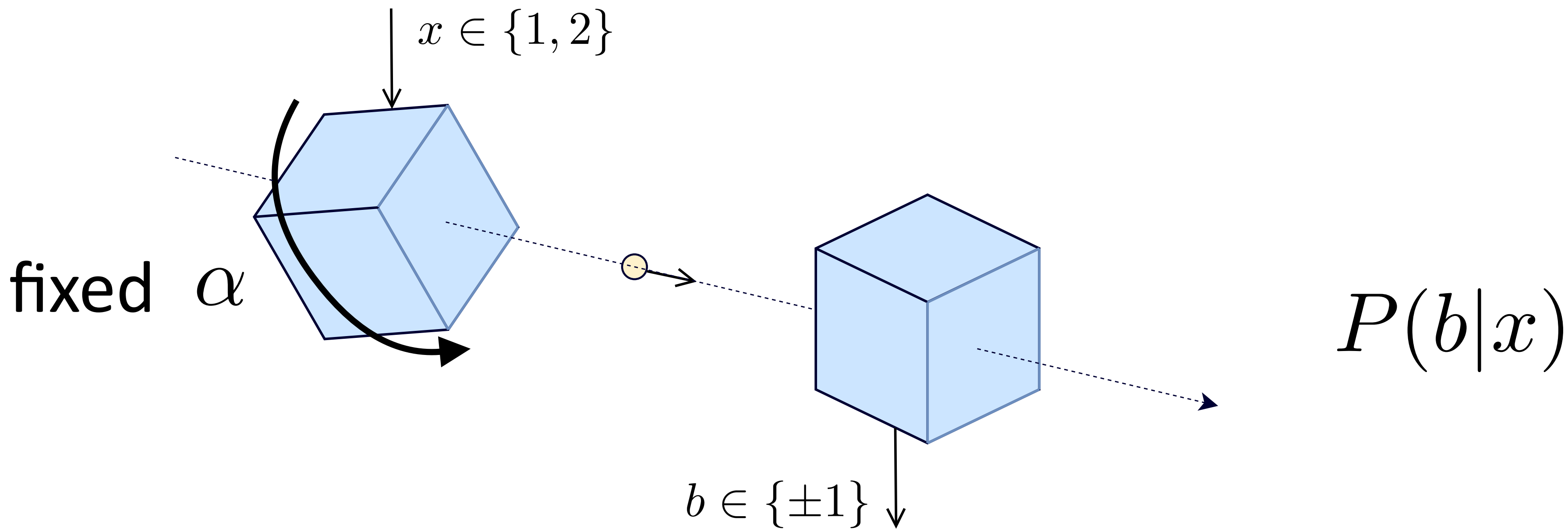


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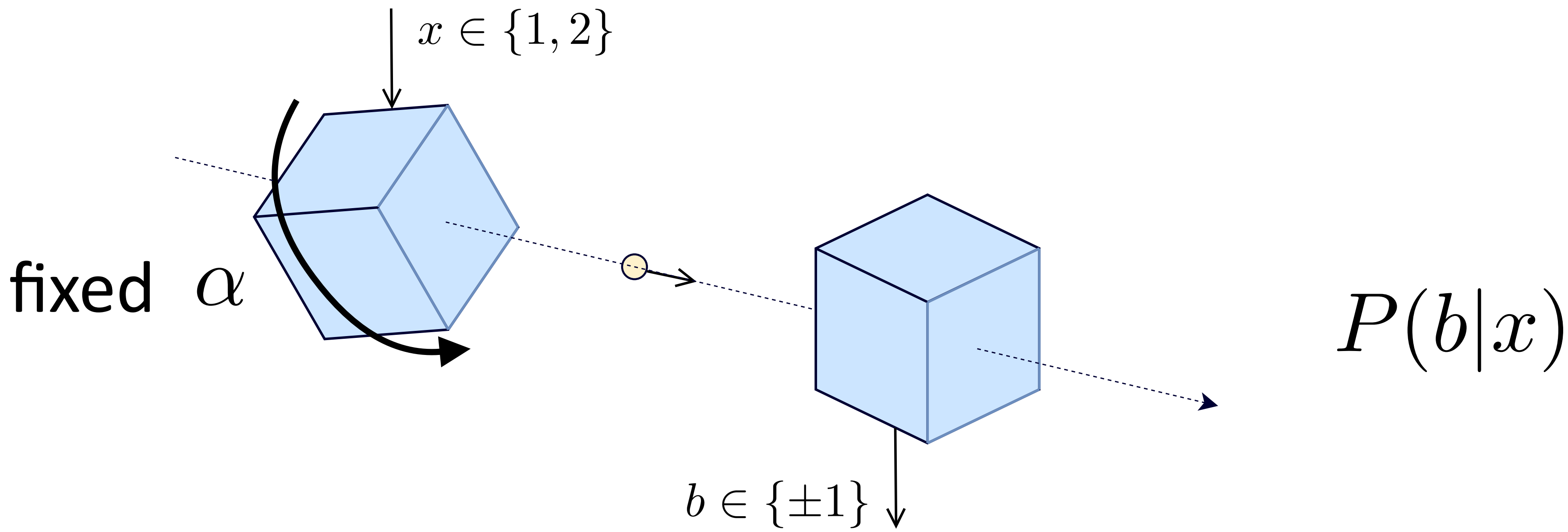


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Rotation described by (projective) unitary representation of SO(2):

$$U_\alpha = \bigoplus_{j=-J}^{J} n_j e^{ij\alpha}, \qquad P(b|\alpha) = \mathrm{tr}(M_b U_\alpha \rho U_\alpha^\dagger).$$

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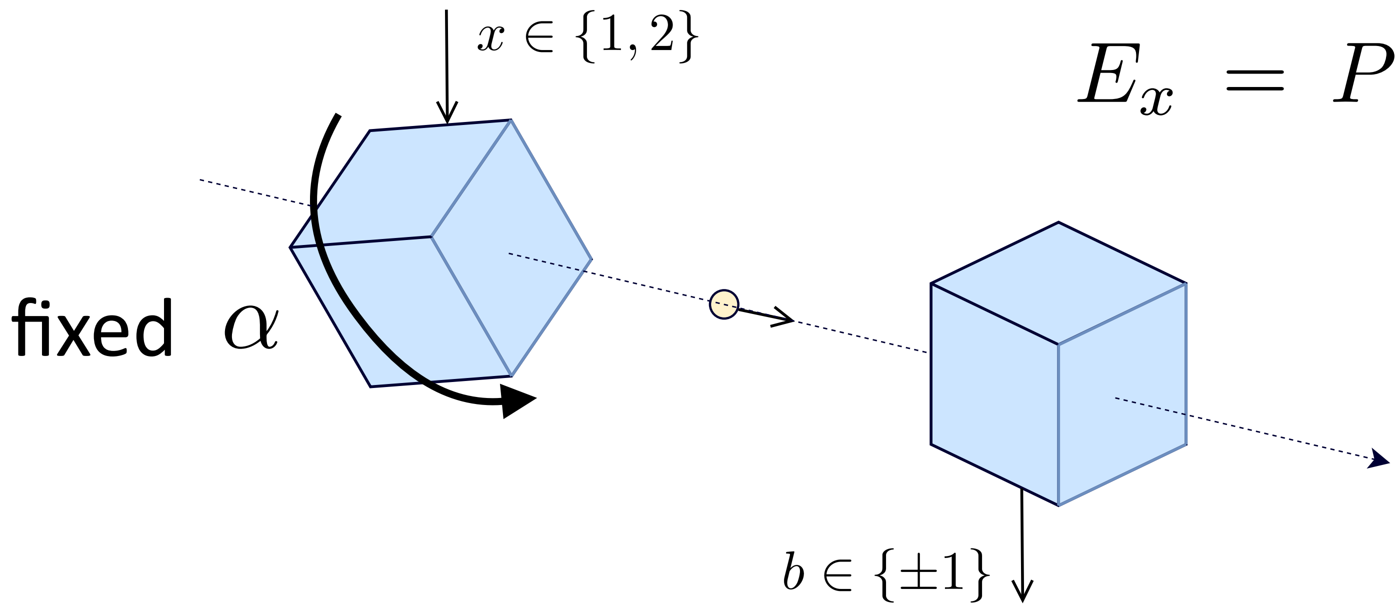


$$E_x = P(+1|x) - P(-1|x)$$

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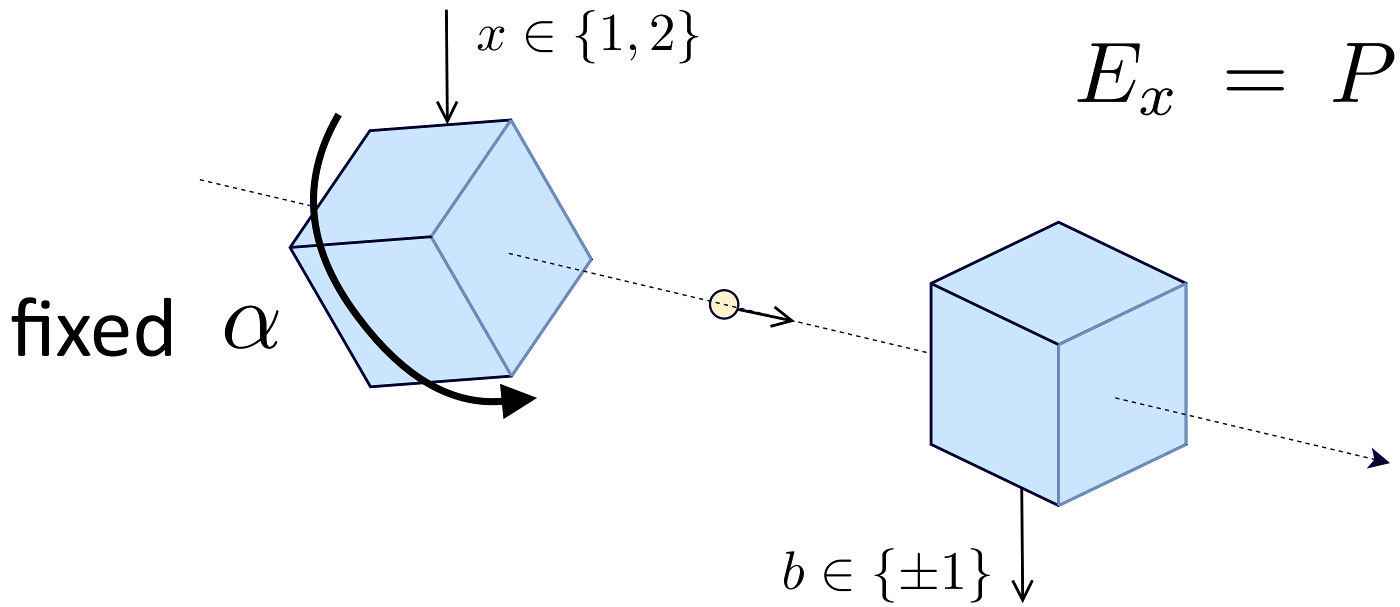


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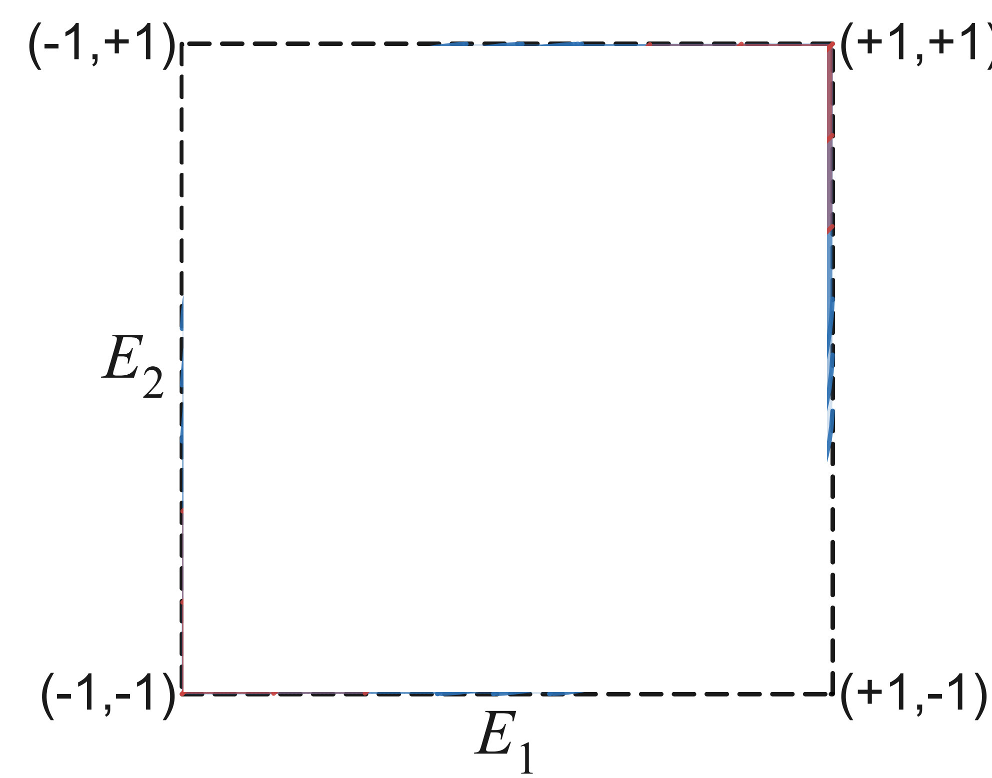

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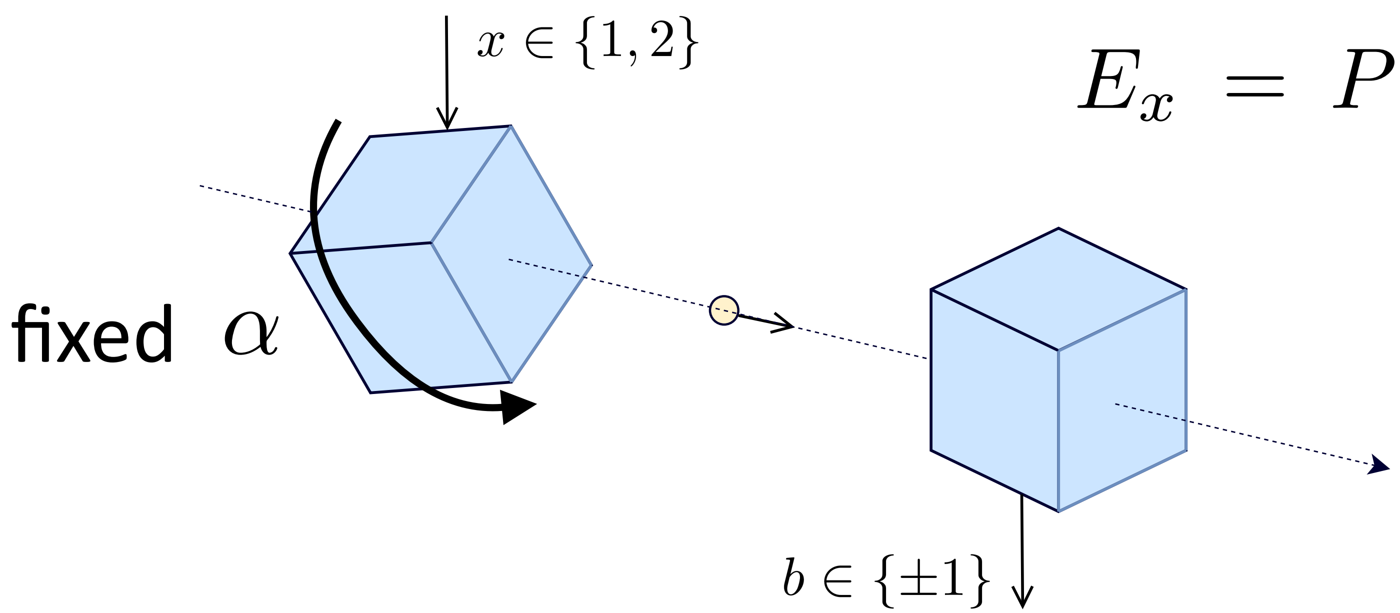


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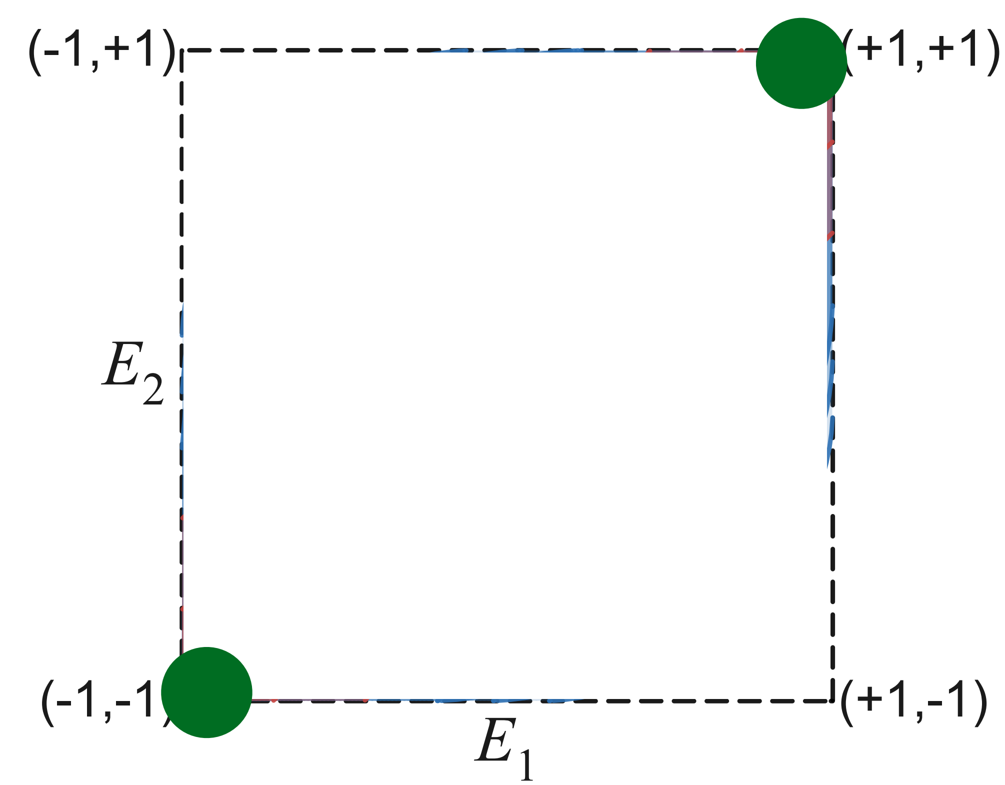

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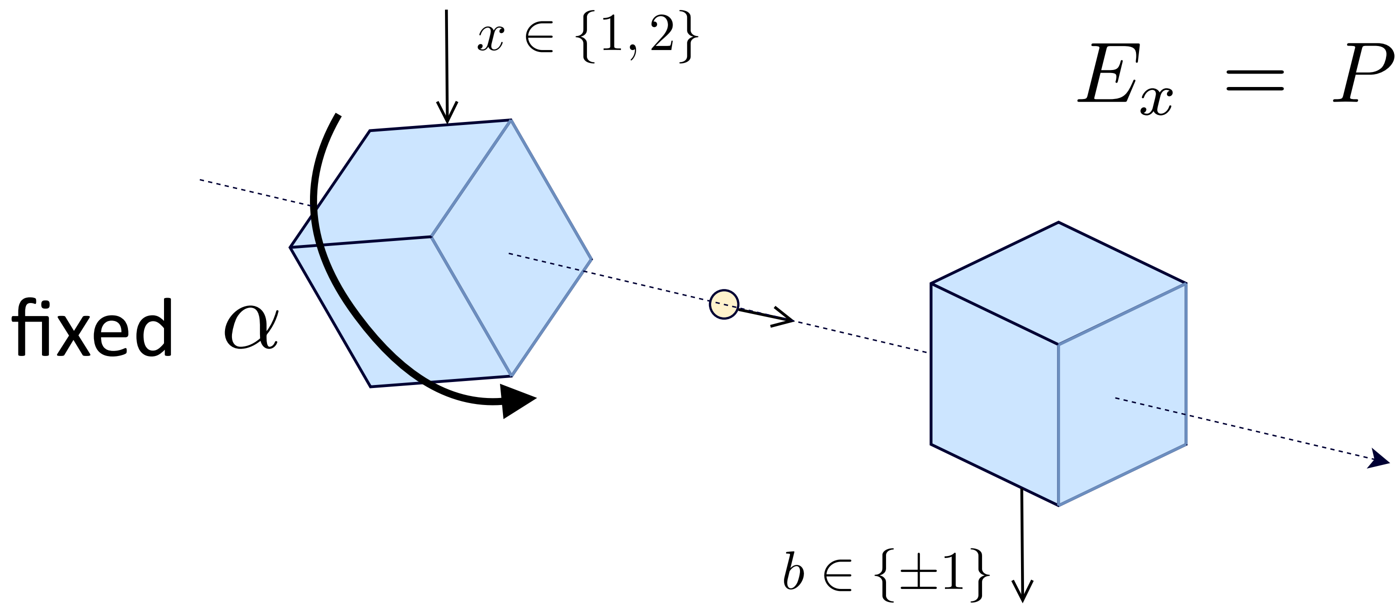


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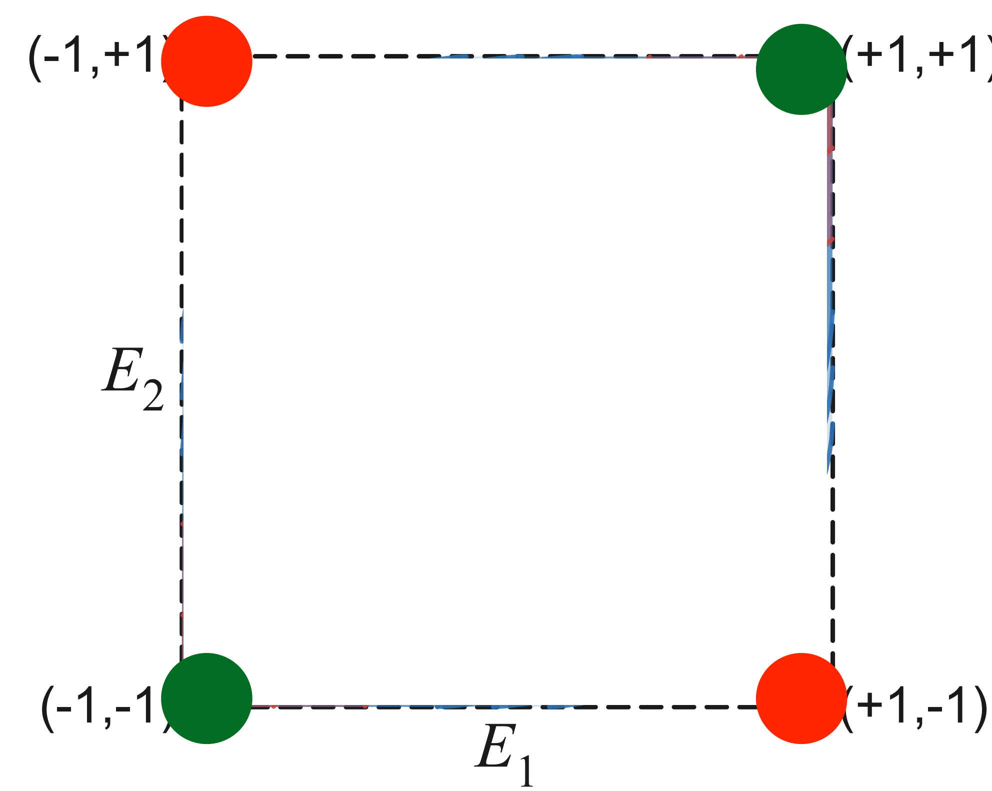

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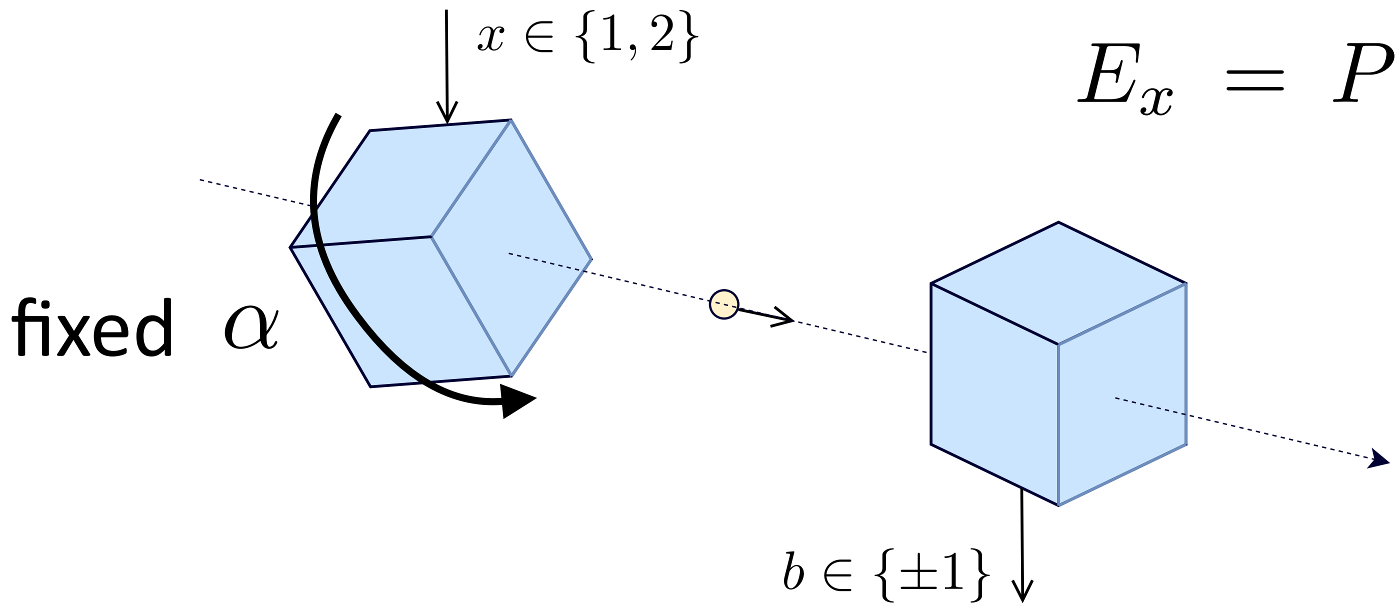


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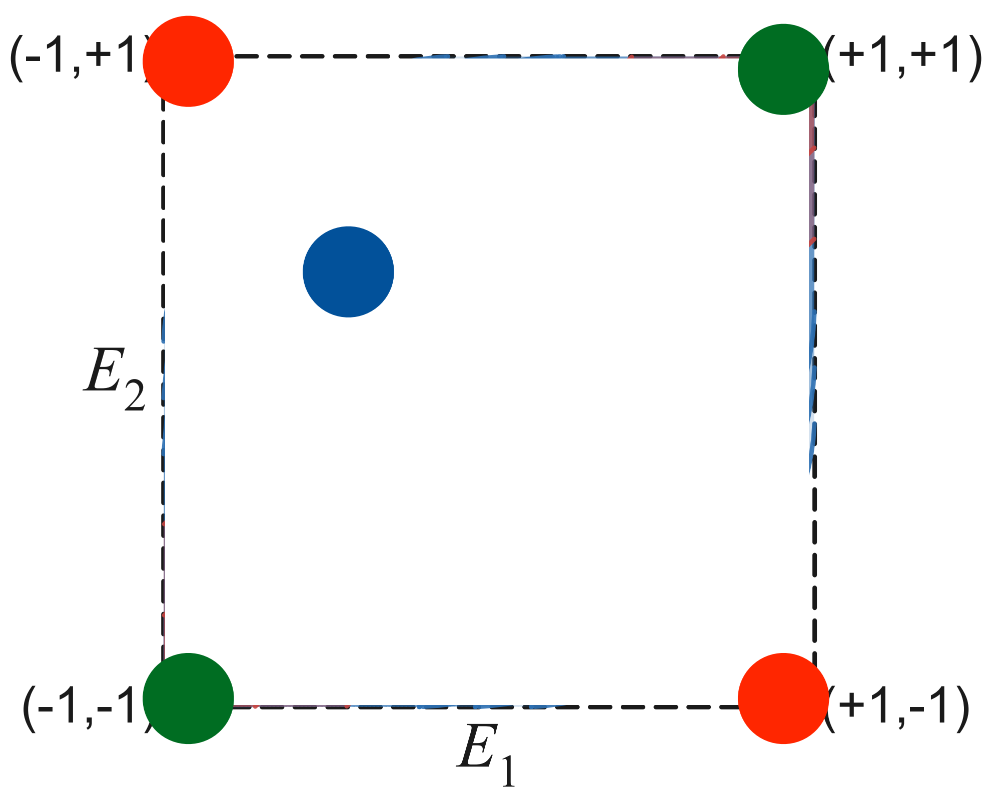


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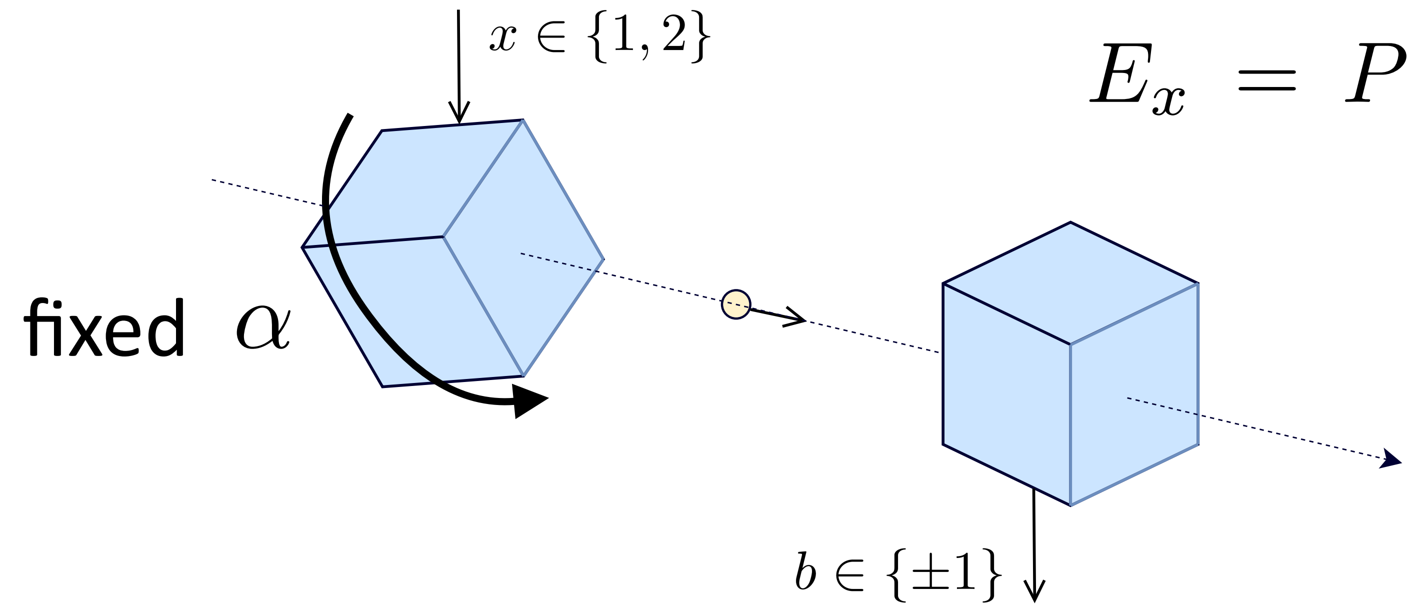


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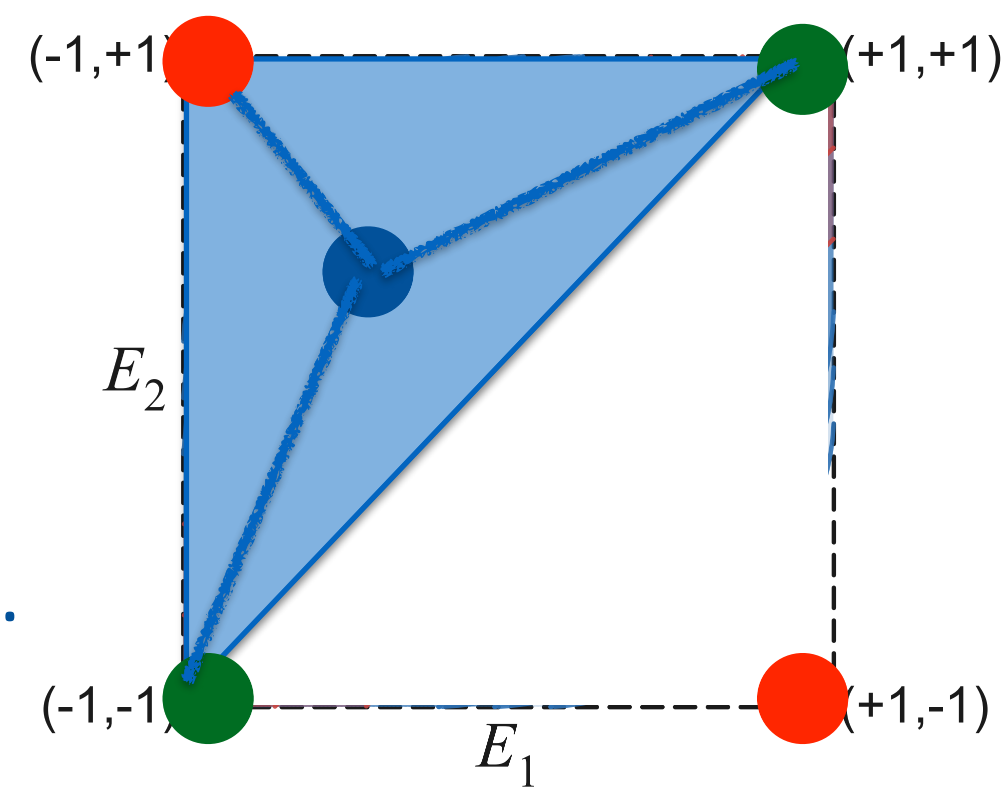


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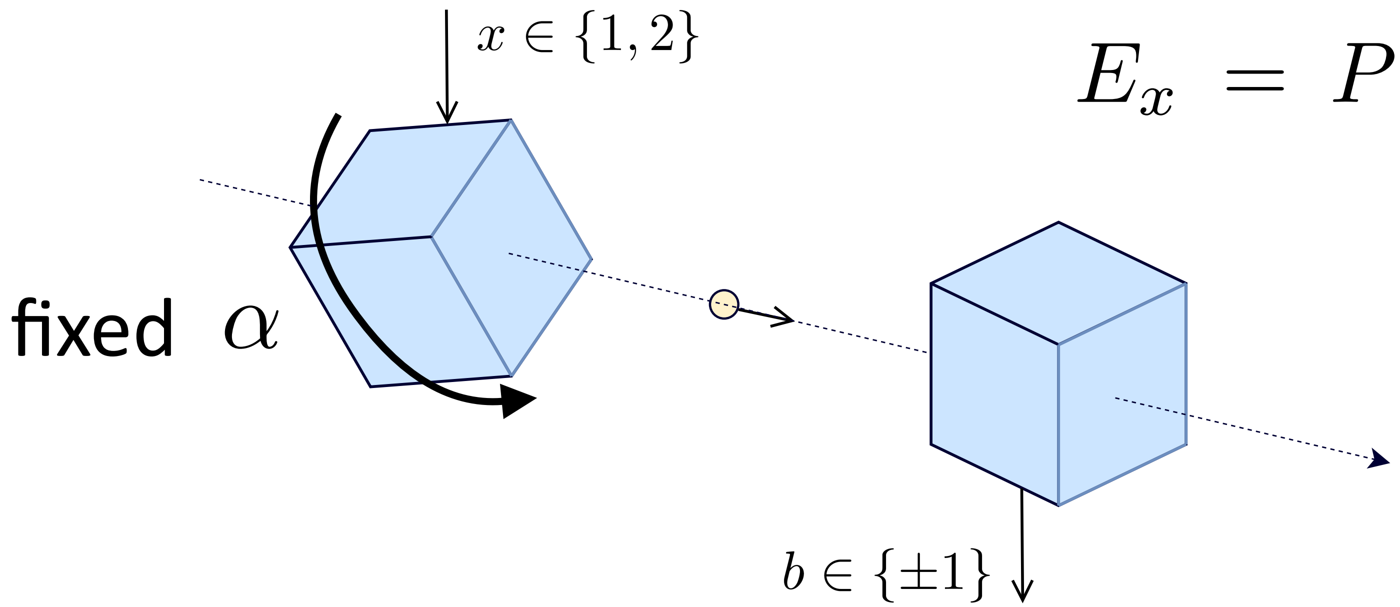


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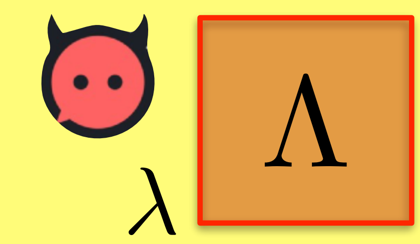


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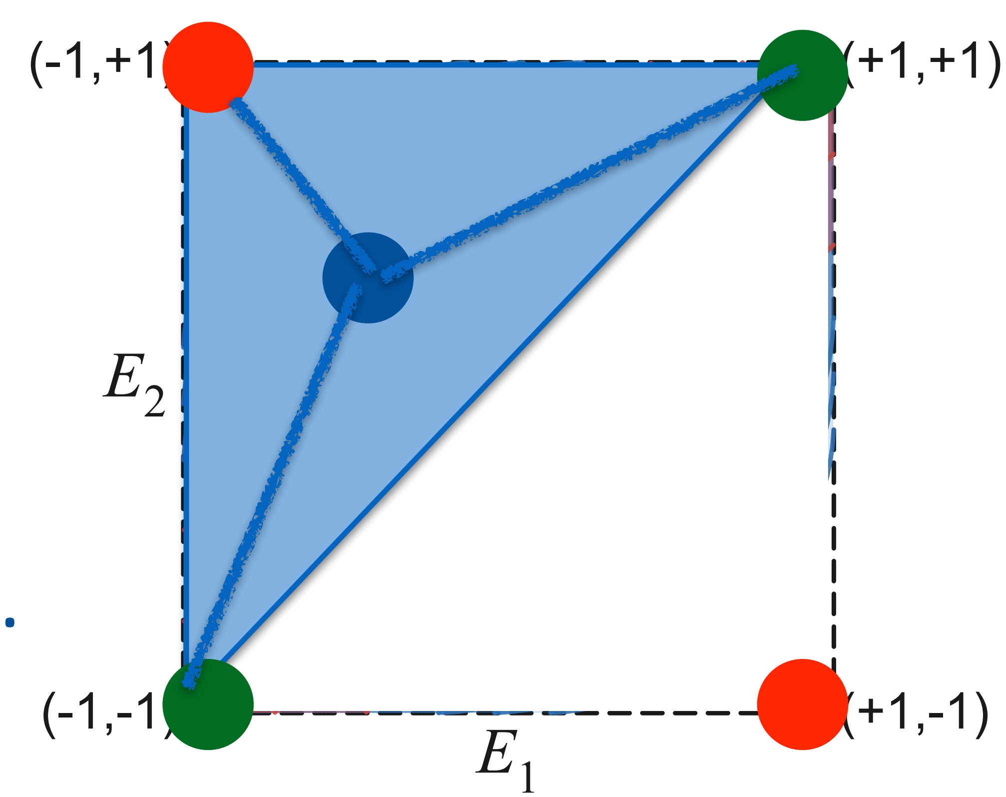


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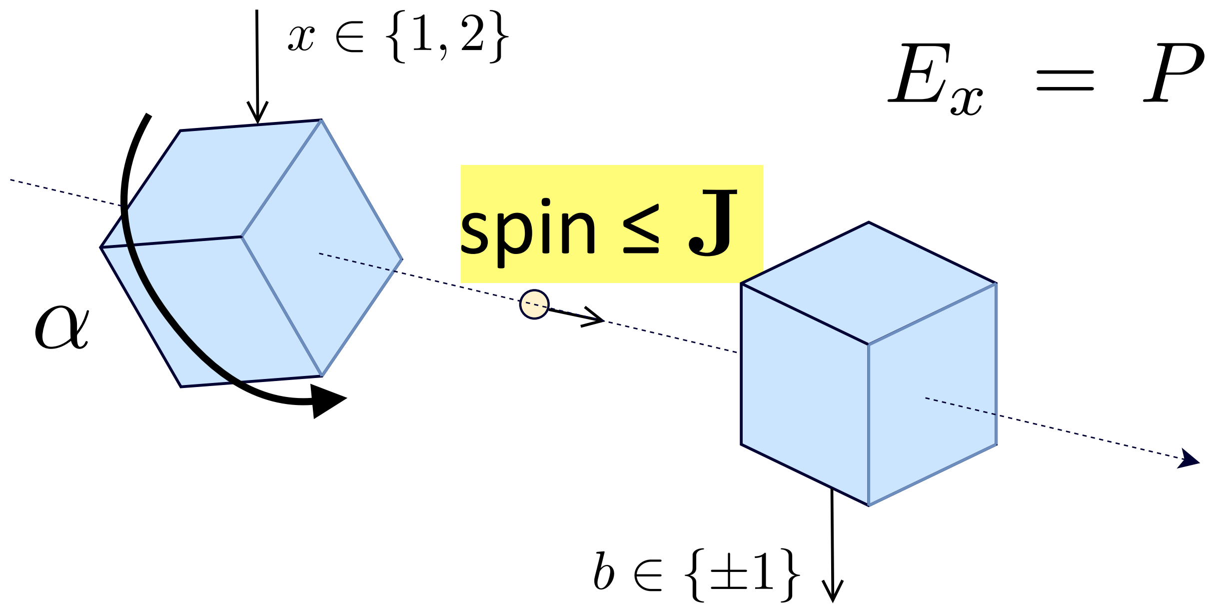


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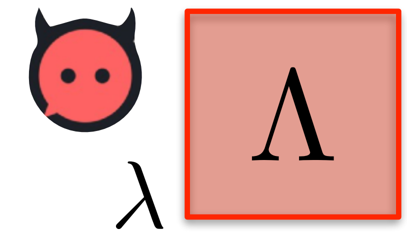


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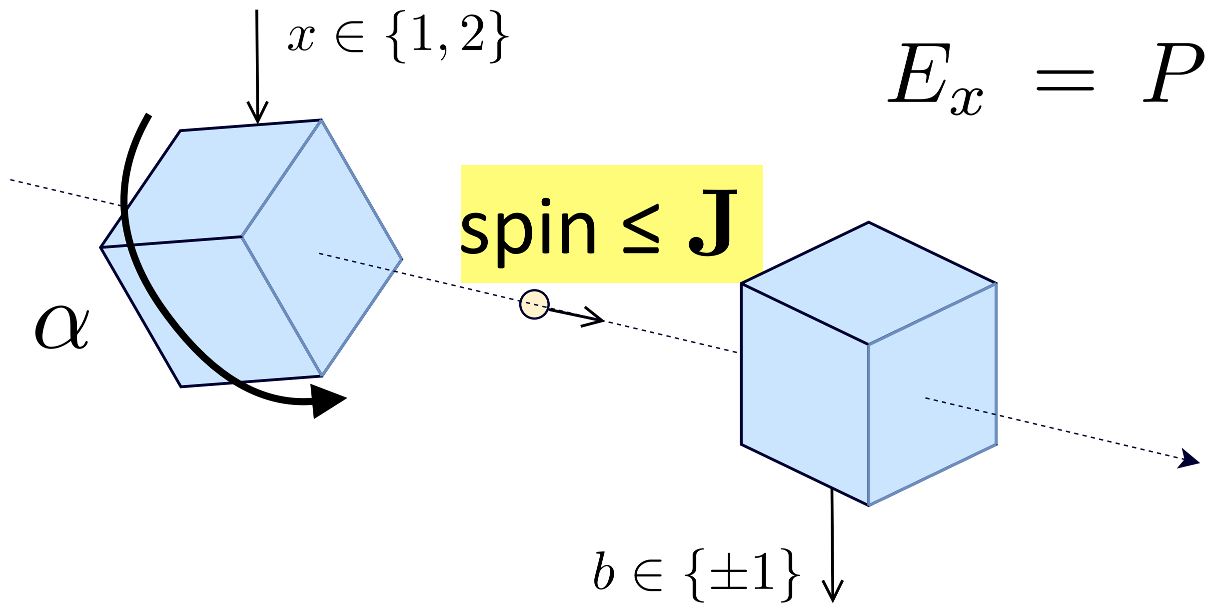


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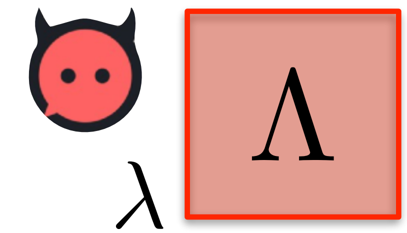


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C. L. Jones, S. L. Ludescher, A. Aloy, MM, arXiv:2210.14811

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T. Van Himbeeck, E. Woodhead, N. J. Cerf, R. García-Patrón, S. Pironio, Quantum **1**, 33 (2017).

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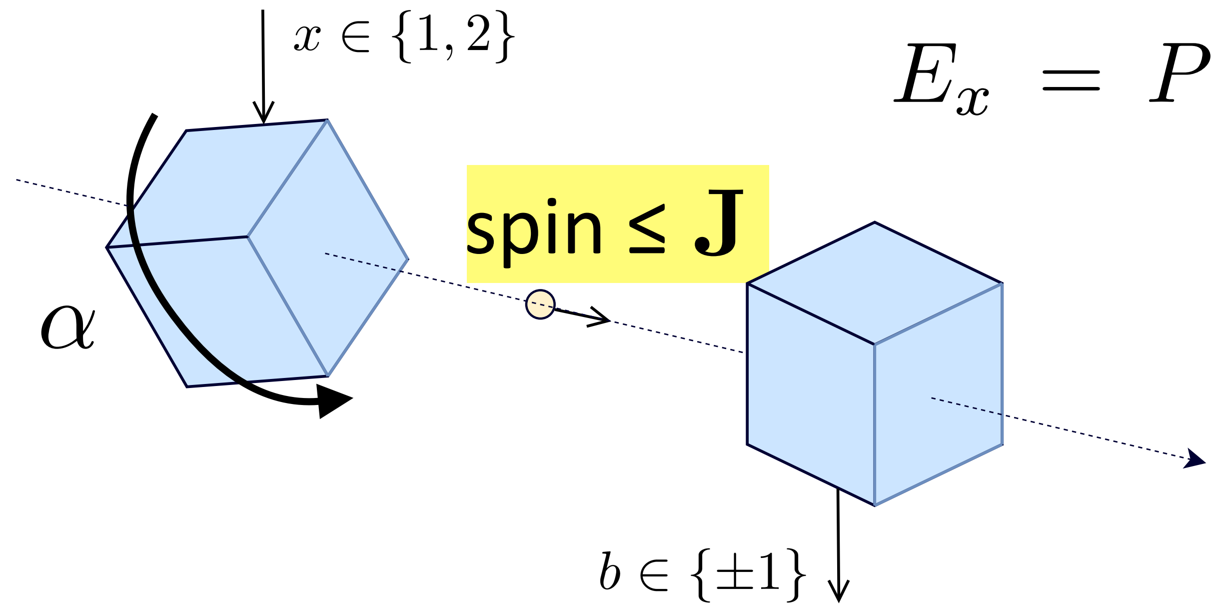


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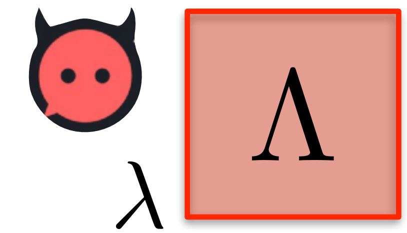


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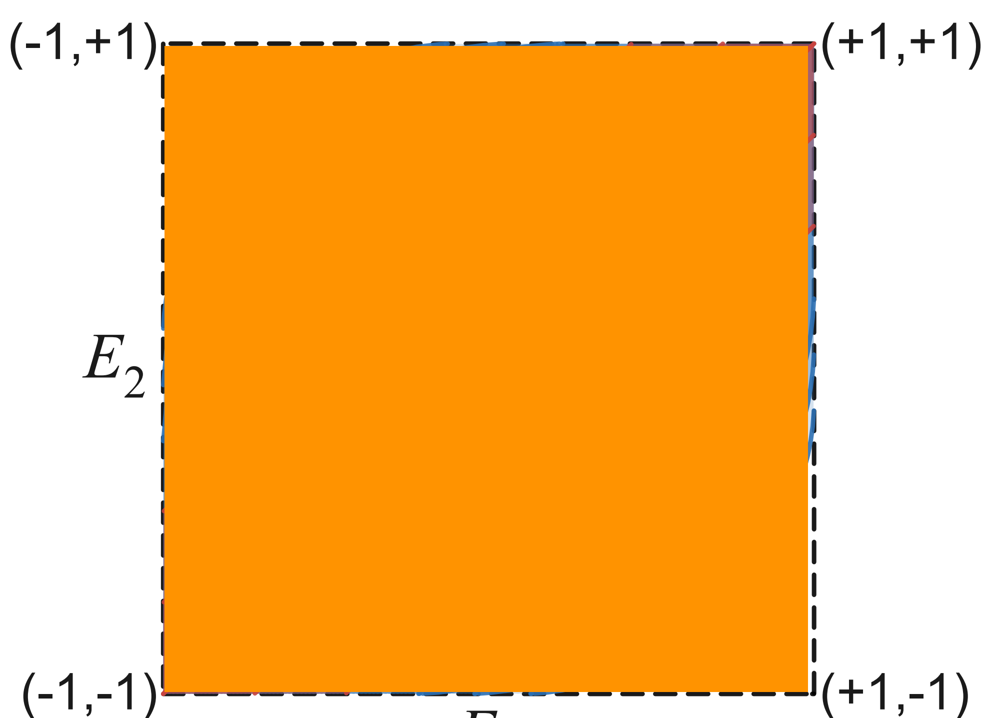


Angle $\alpha \geq \pi/(2J)$ :
no certifiable randomness.

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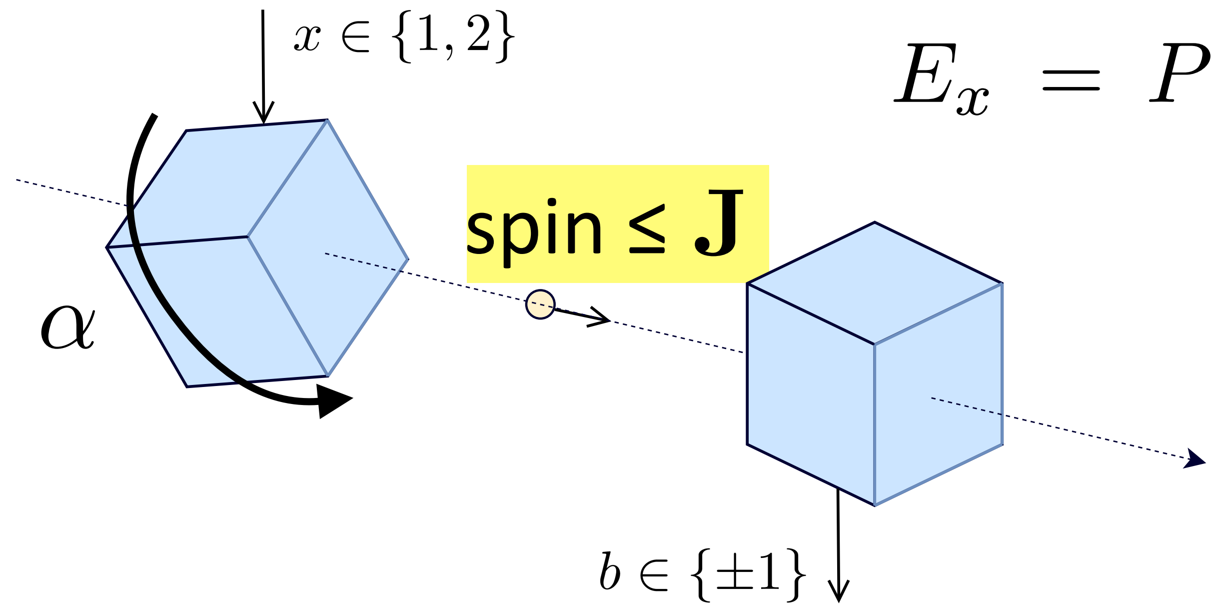


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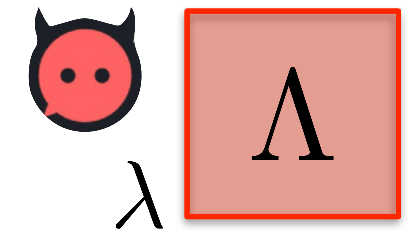


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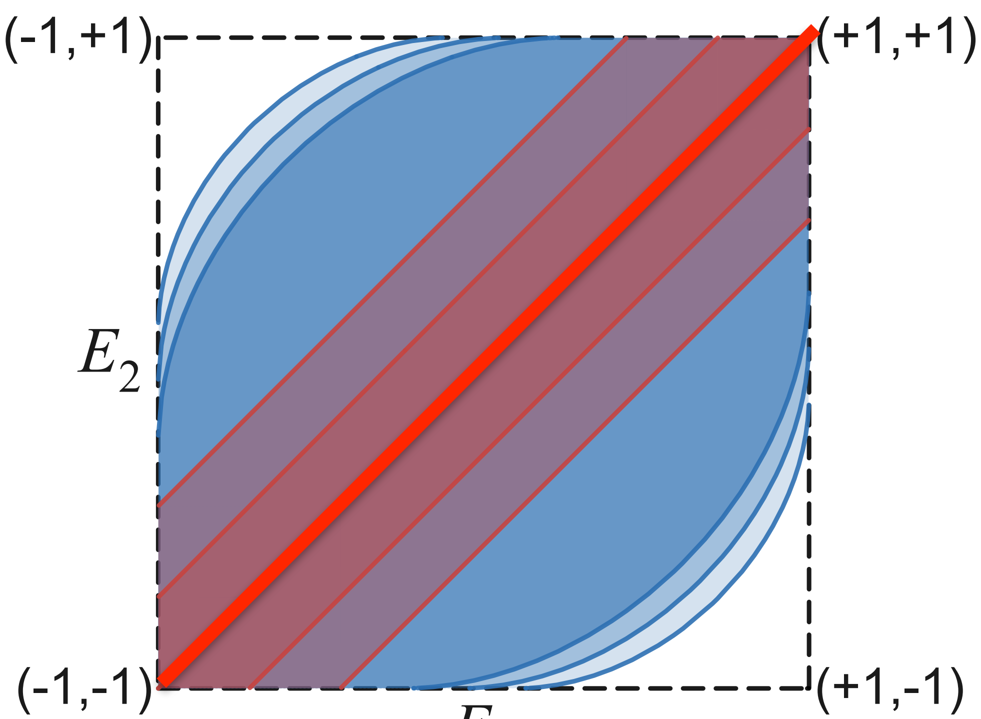


Blue curved set of correlations.

If observed correlation away from red line:

**certifiable private randomness**.

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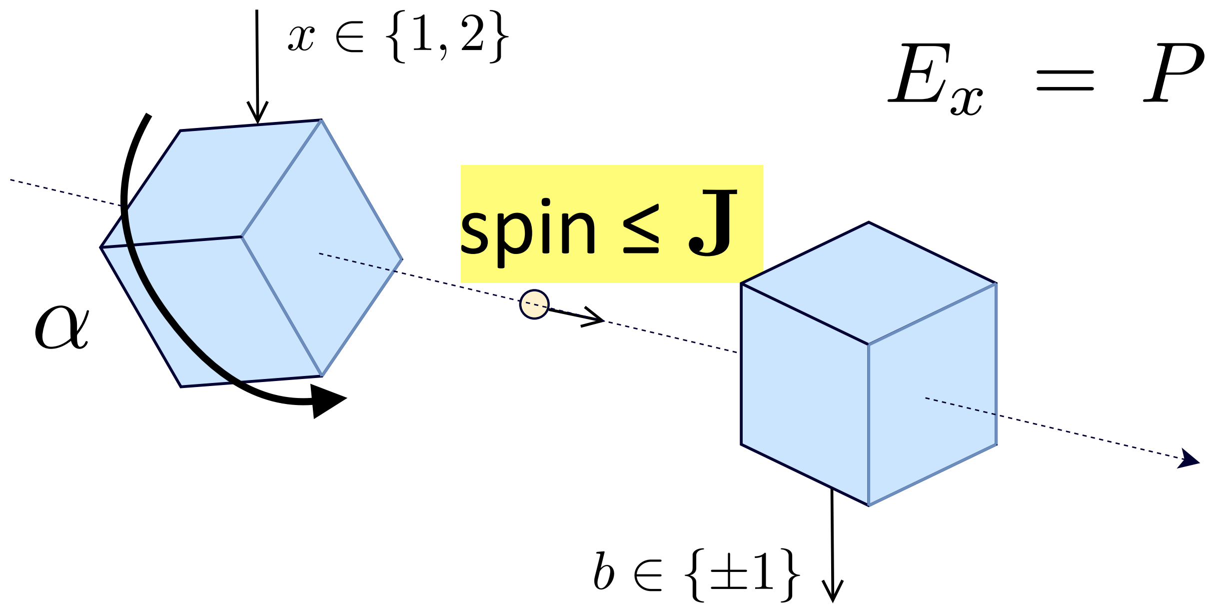


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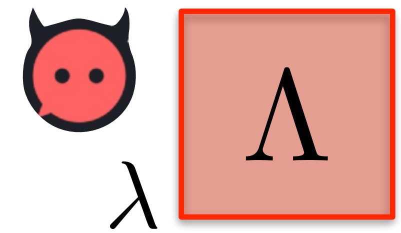

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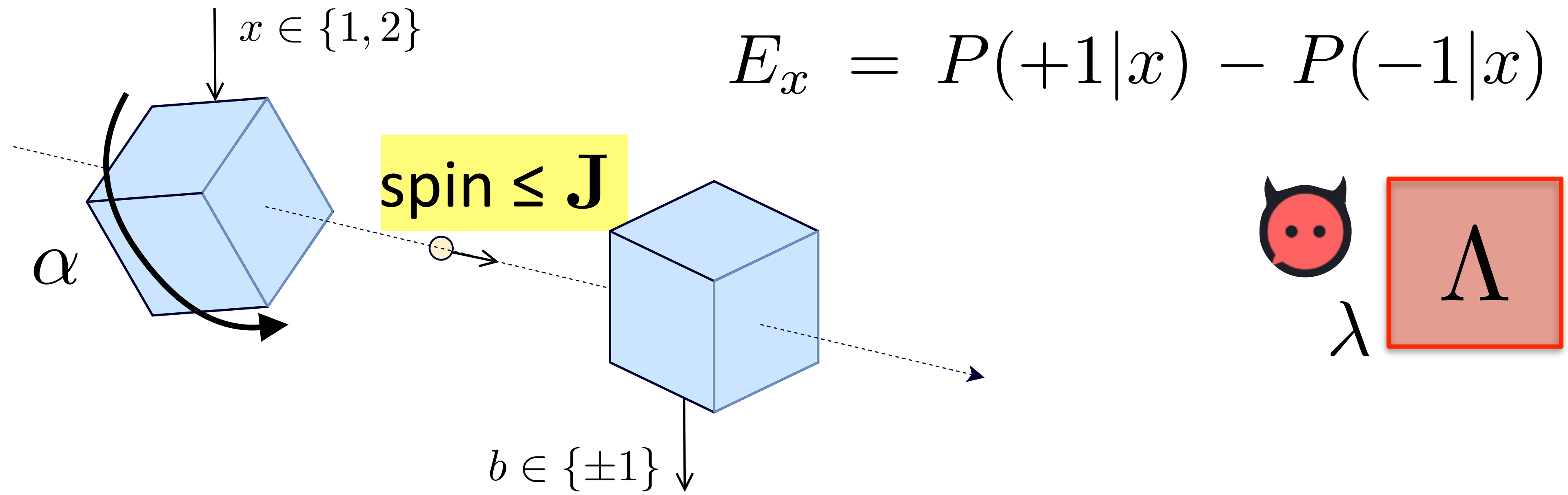


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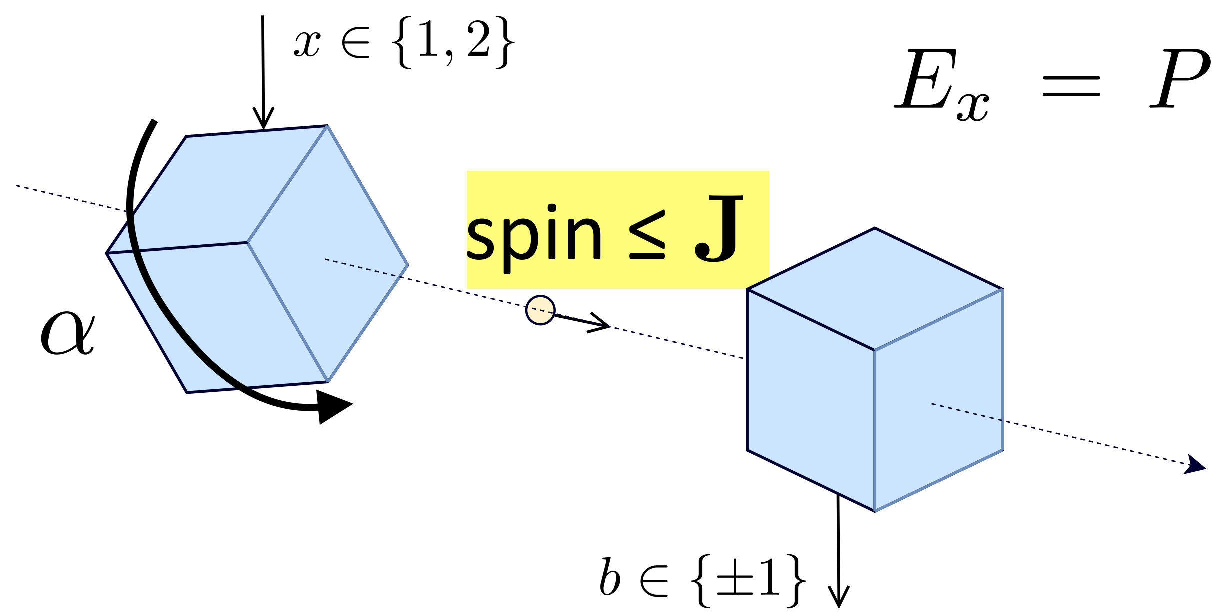


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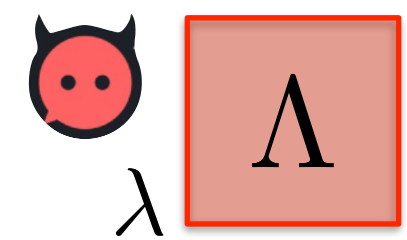


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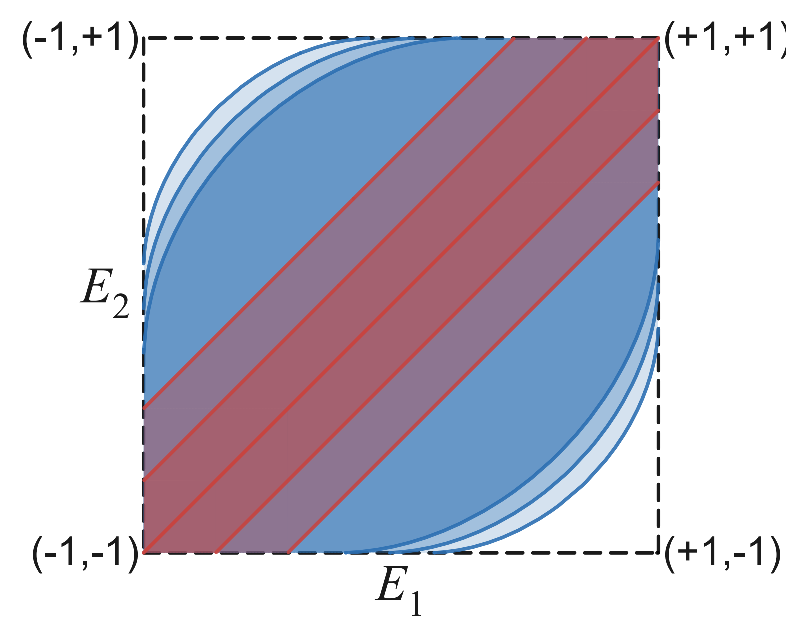

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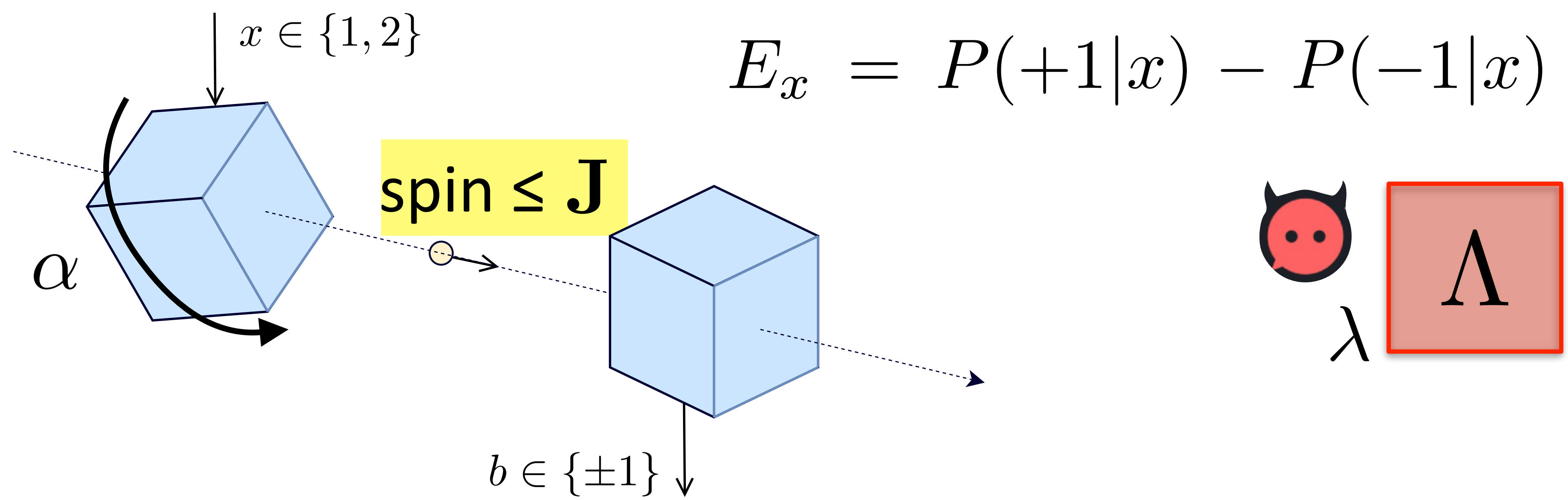


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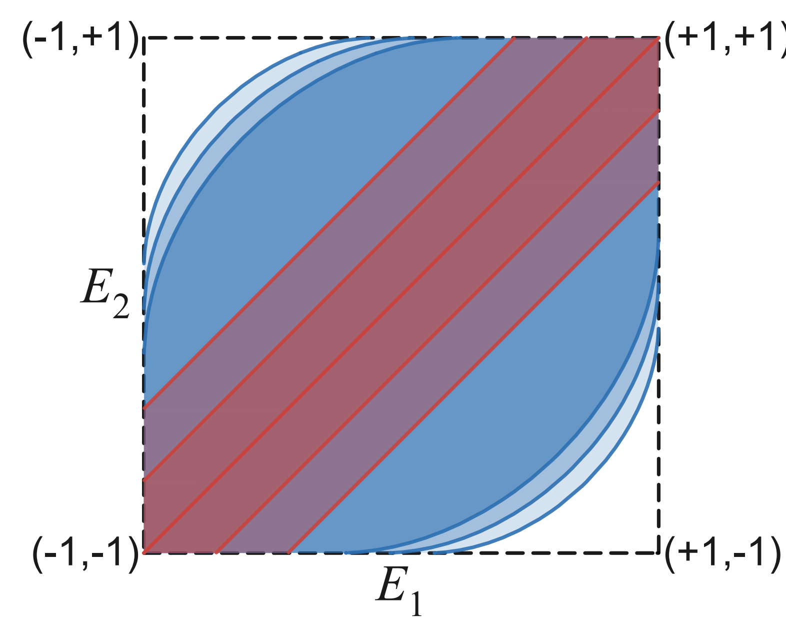

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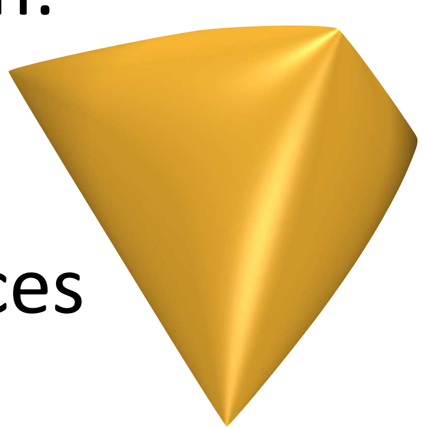
$\mathcal{R}_J$ from rep. of SO(2) on (non-quantum) "orbitope" state spaces



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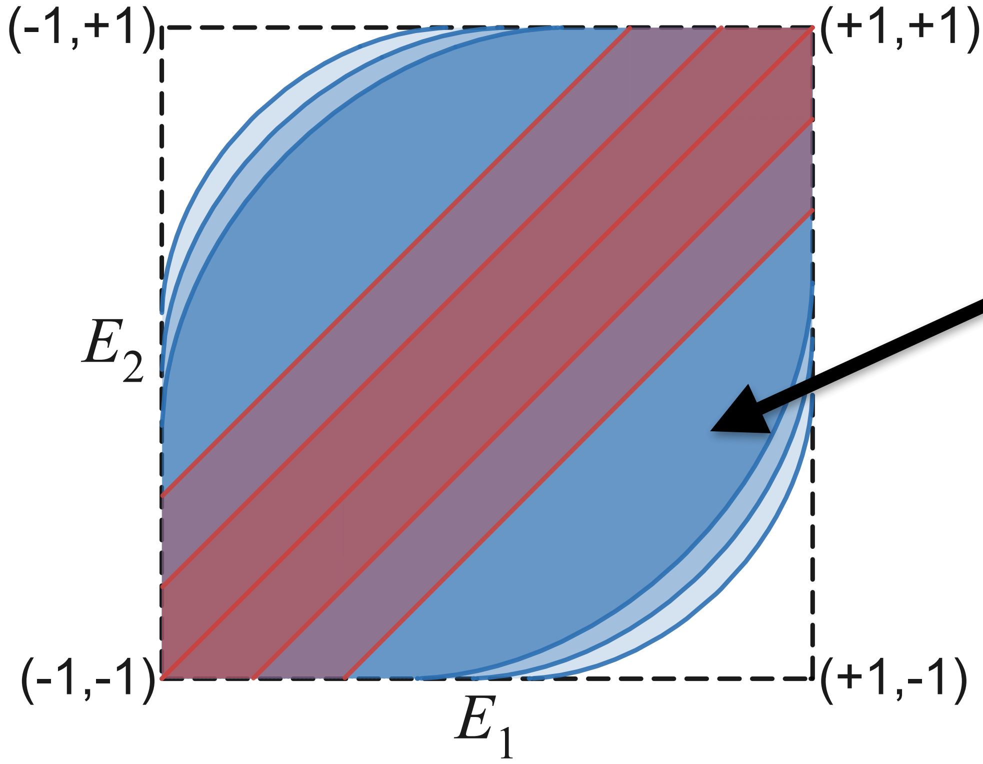


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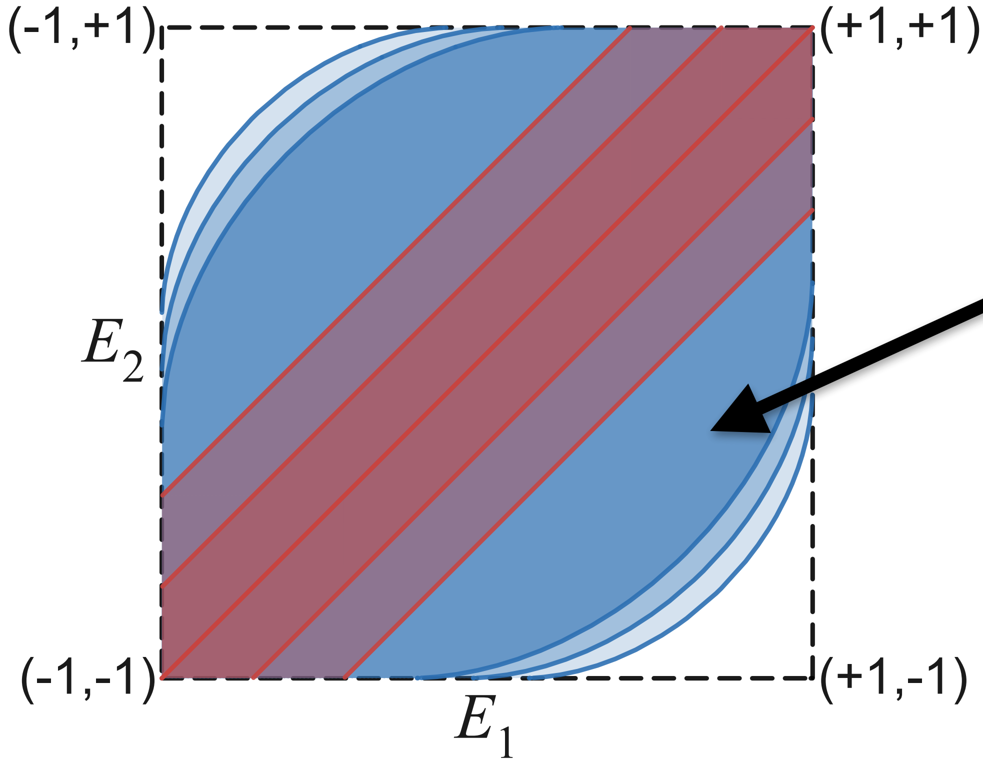


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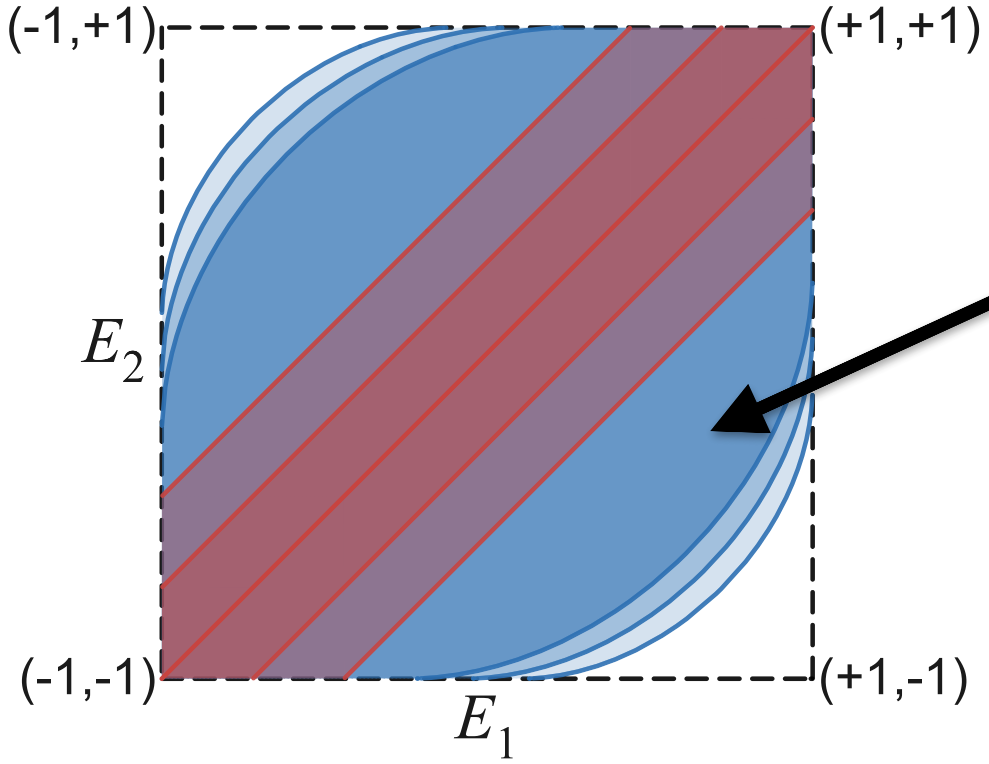


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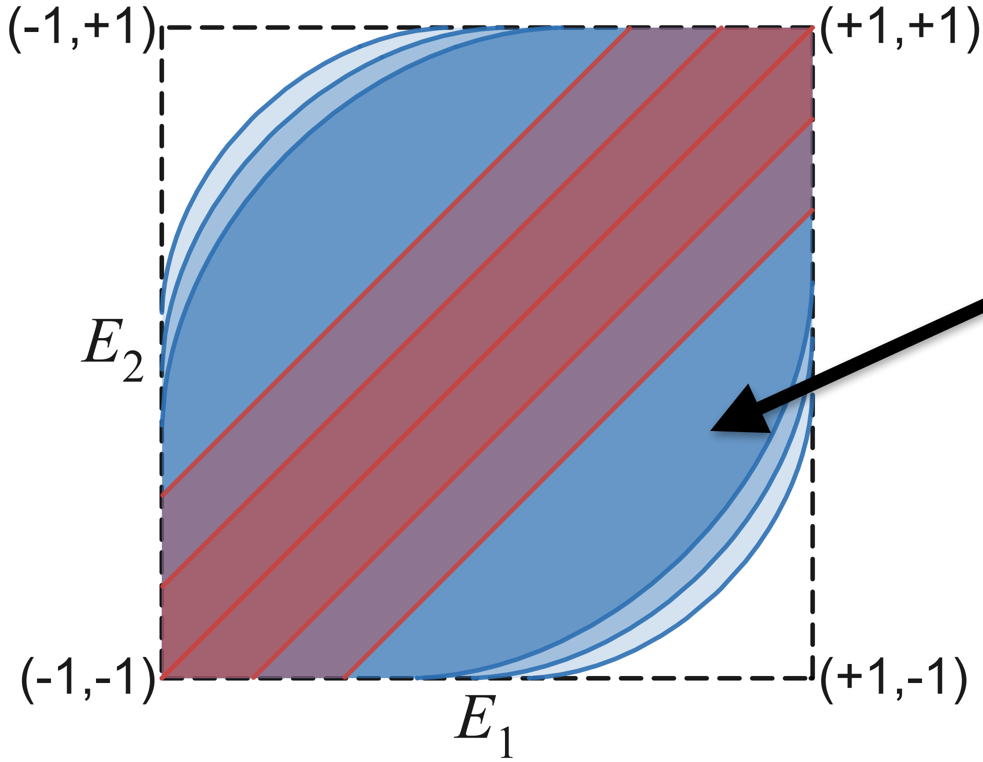


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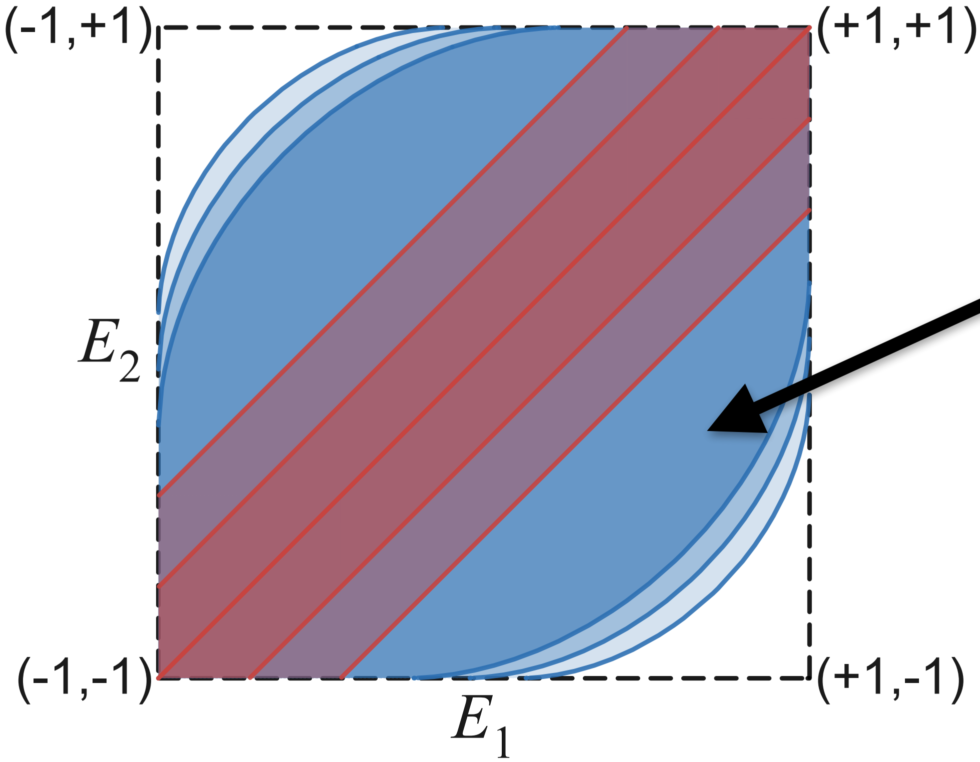


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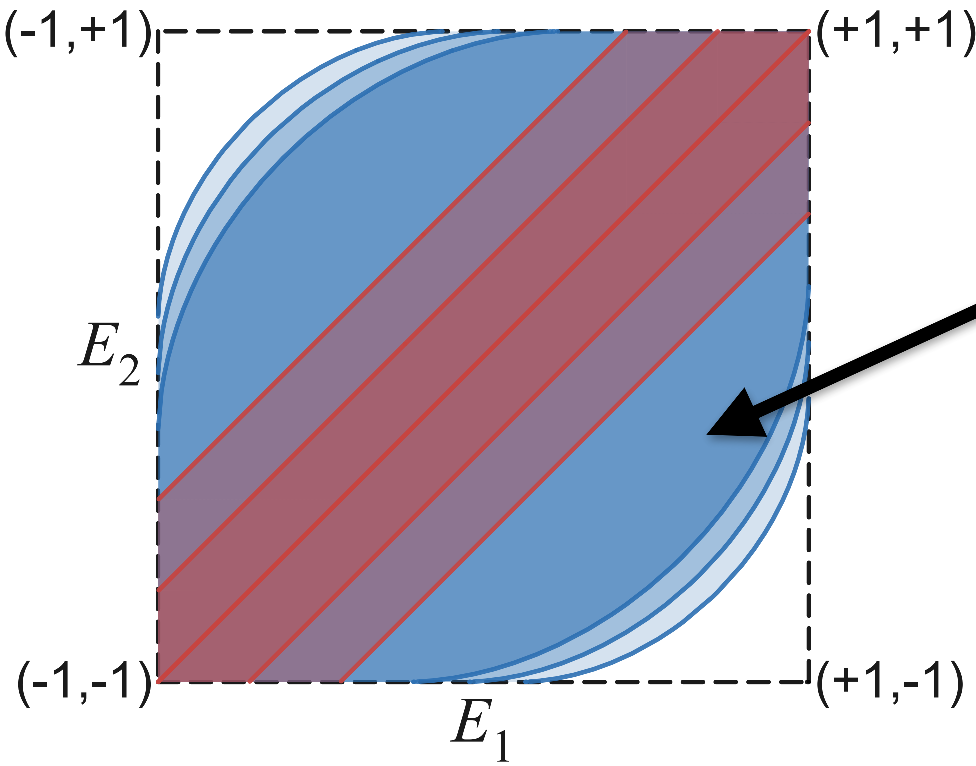


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**Theorem:** $\mathcal{Q}_{J,\alpha} = \mathcal{R}_{J,\alpha}$. C. L. Jones, S. L. Ludescher, A. Aloy, MM, arXiv:2210.14811

Can derive set of quantum correlations without assuming QT.

Even eavesdropper with classical side information about beyond-quantum physics cannot predict the outcomes.