ÖAW AUSTRIAN ACADEMY OF SCIENCES

IQOQI – INSTITUTE FOR QUANTUM OPTICS AND QUANTUM INFORMATION VIENNA

PERIMETER INSTITUTE

How spacetime constrains the structure of quantum theory

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Andrew J. P. Garner, Oscar C. O. Dahlsten, **Markus P. Müller**

IQOQI Vienna & Perimeter Institute

FWF

Der Wissenschaftsfonds.

Overview

1. Motivations: QG and device-independent QIT

2. Relativity of simultaneity and the qubit

3. Randomness generation via rotational symmetry

4. Conclusions

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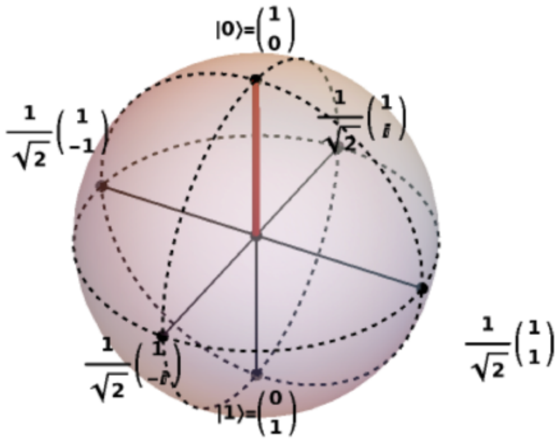

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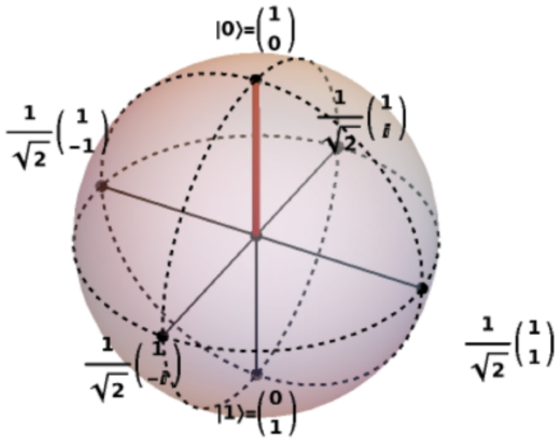

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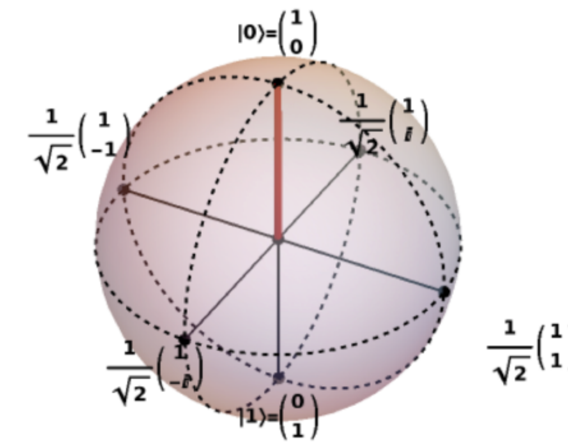

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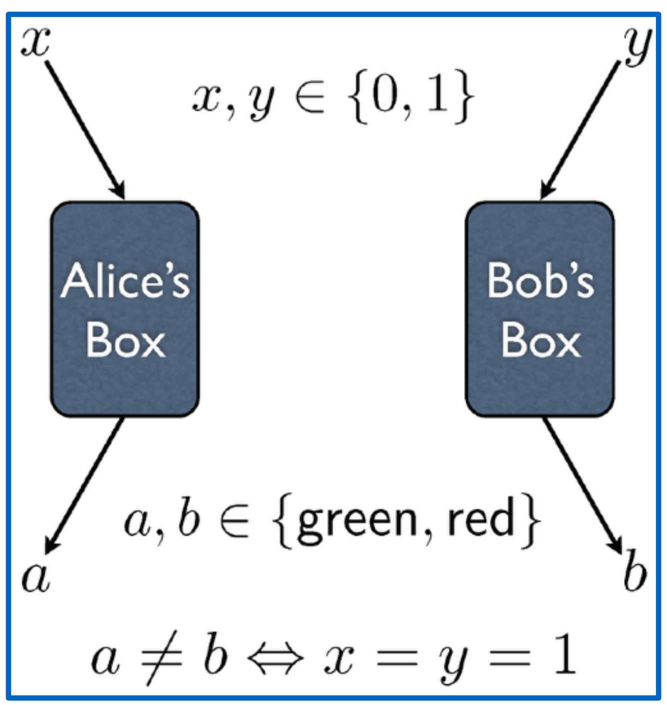
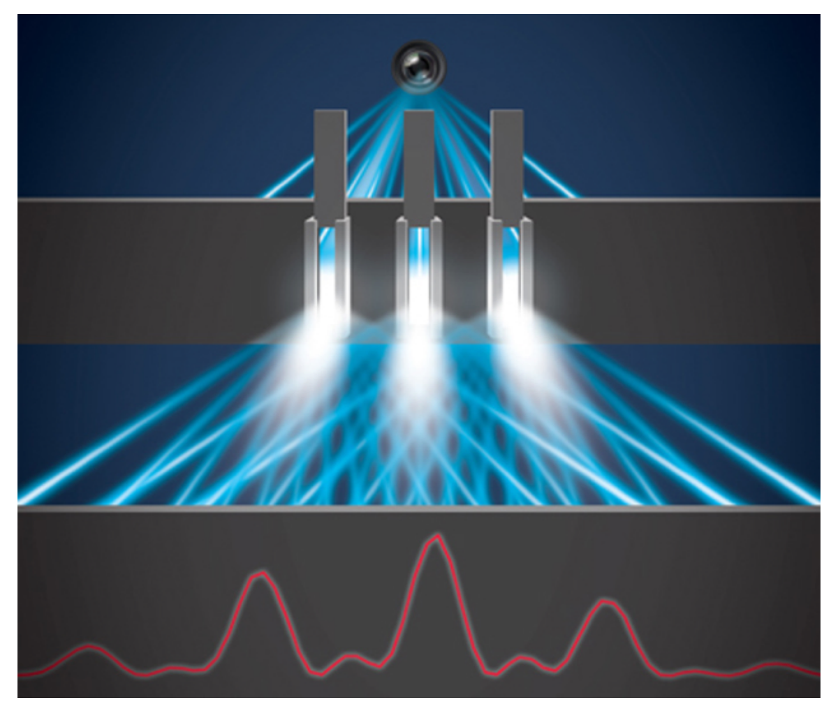


superstrong nonlocality?



higher-order interference?

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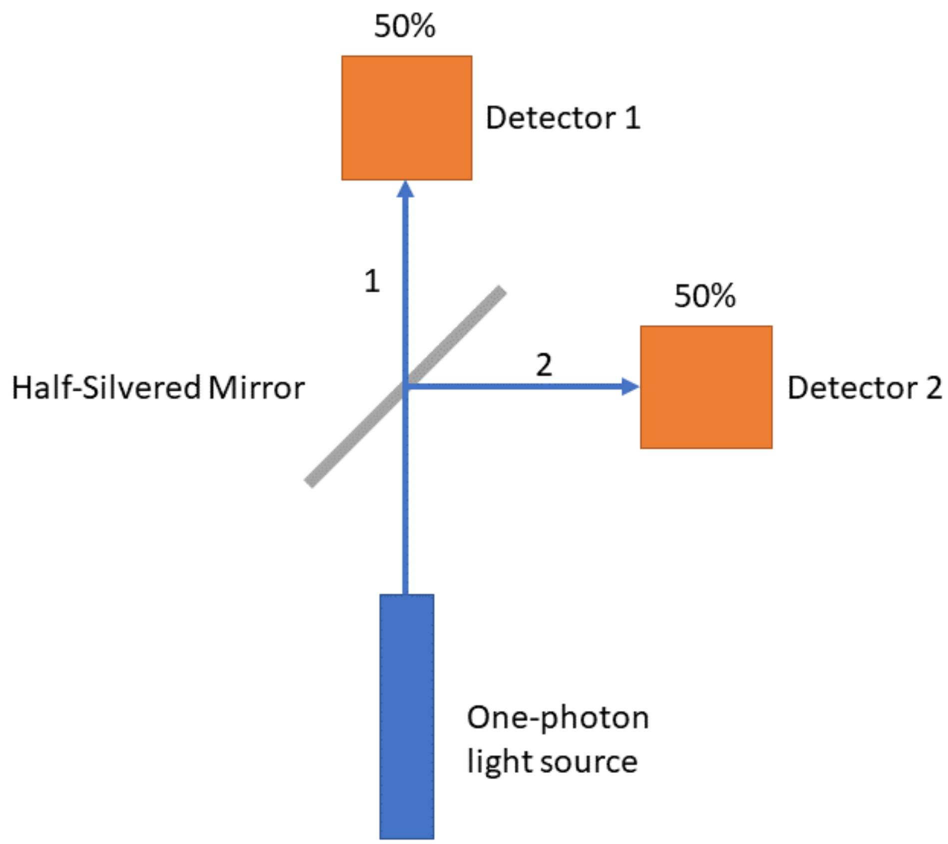

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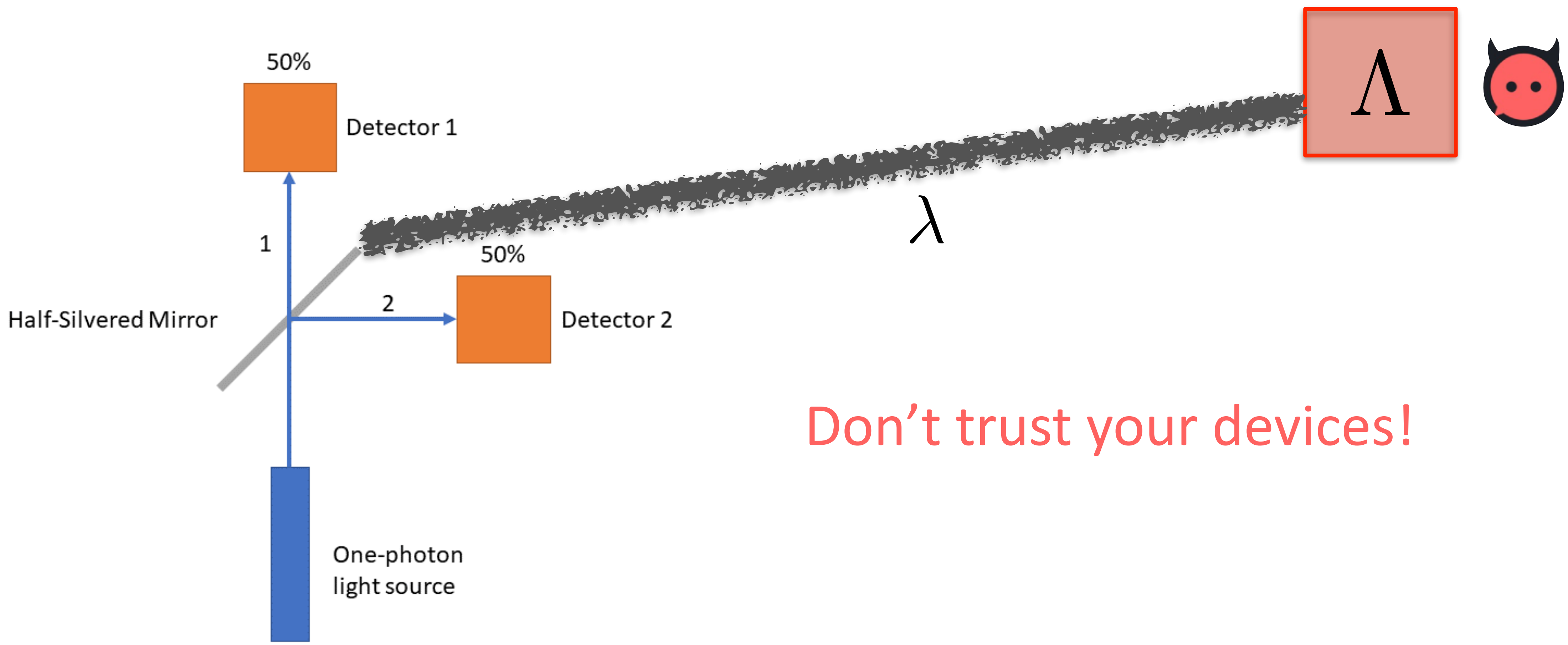

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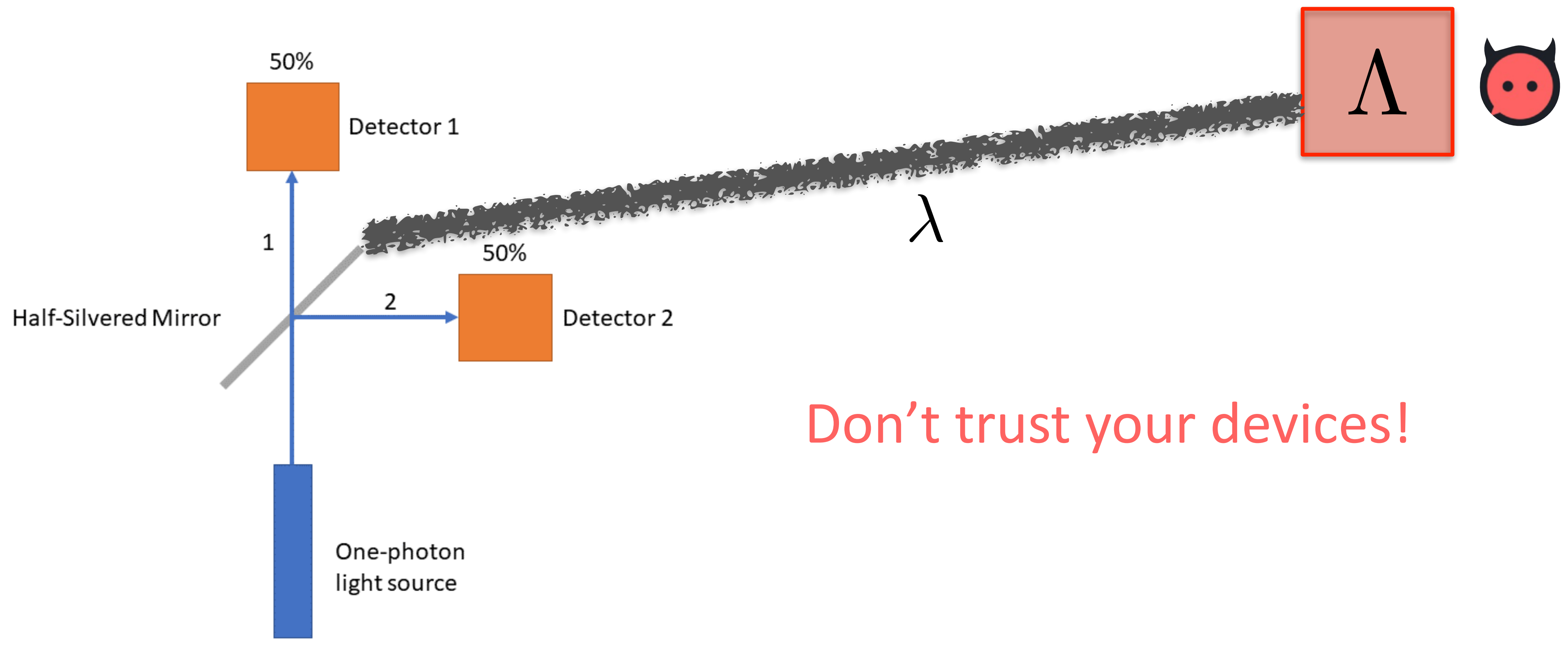

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Device-independent randomness expansion:

Violation of Bell inequality $\Rightarrow$ outcomes uncorrelated with rest of the world

See e.g.: A. Acín, *Randomness and quantum non-locality*, QCRYPT 2012 talk.
V. Scarani, *Bell nonlocality*, Oxford Graduate Texts (2019).

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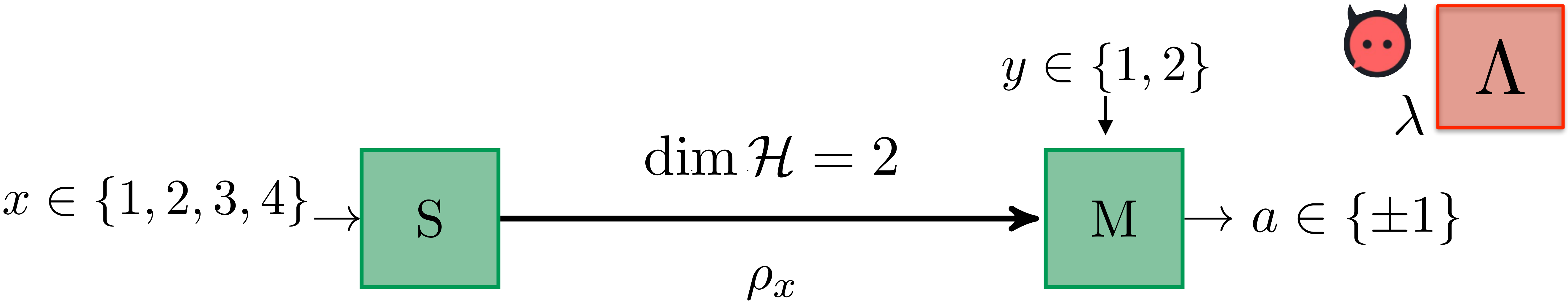

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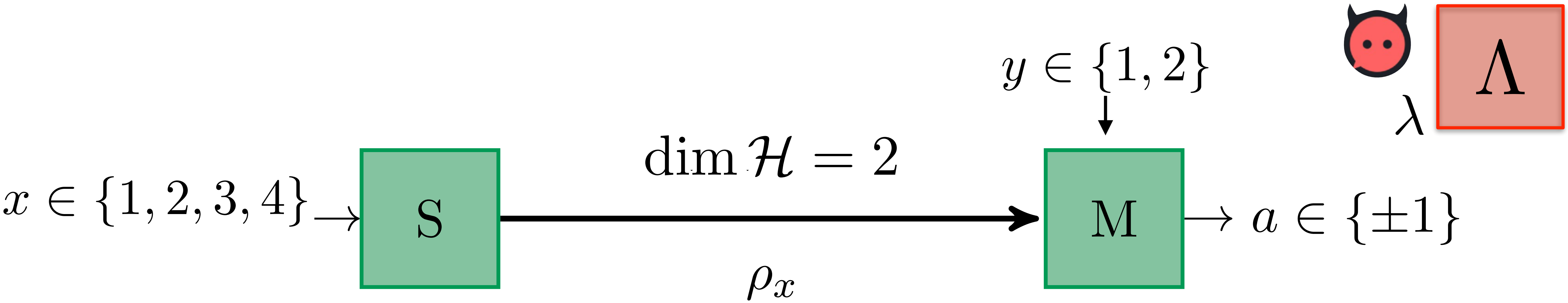

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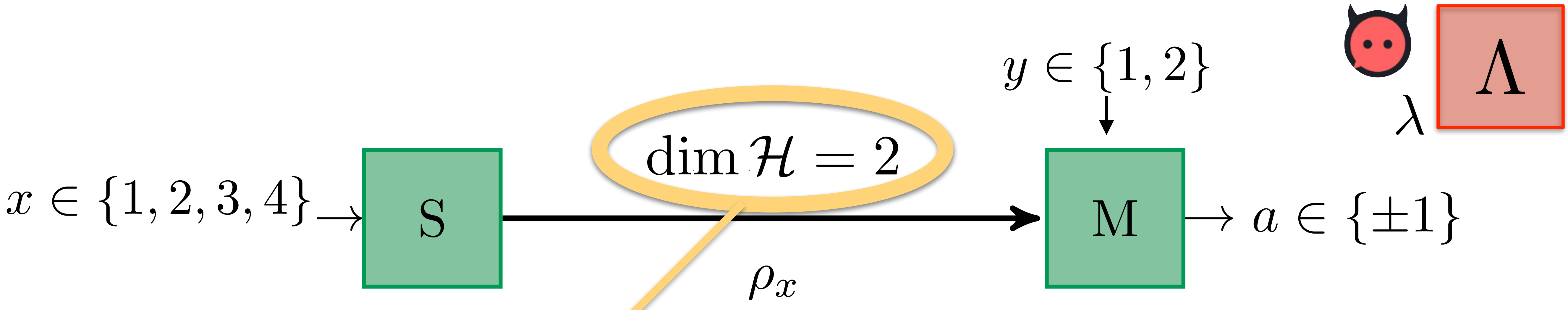

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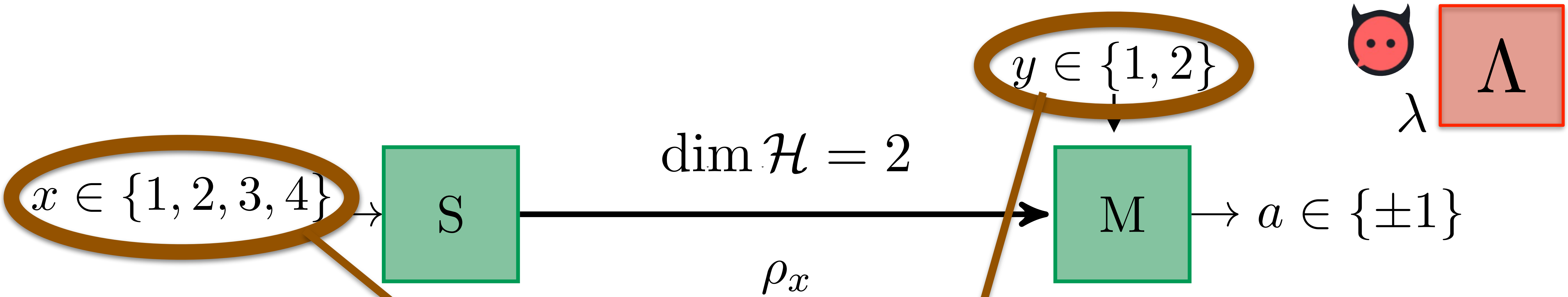

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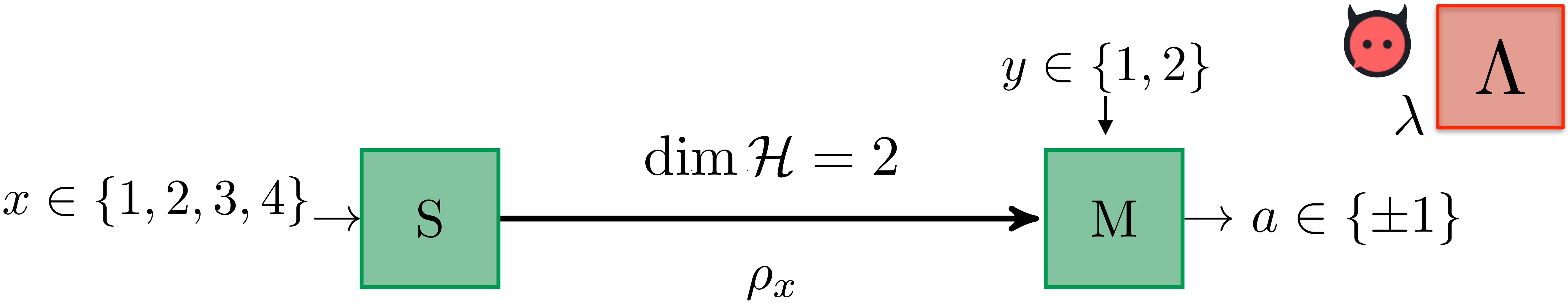

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Idea: reformulate in terms of spacetime symmetries, **w/o assuming QT.**
Can quantum phenomenology / functionality be reproduced?

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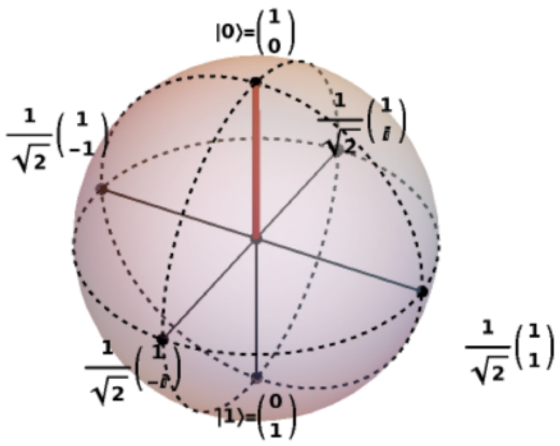

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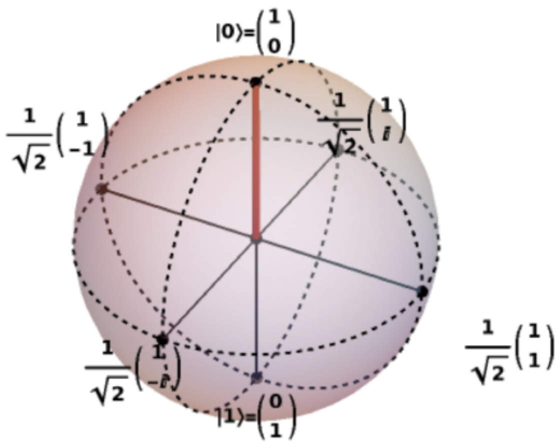

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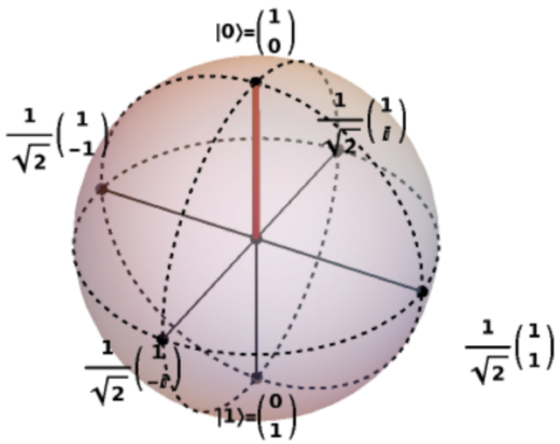

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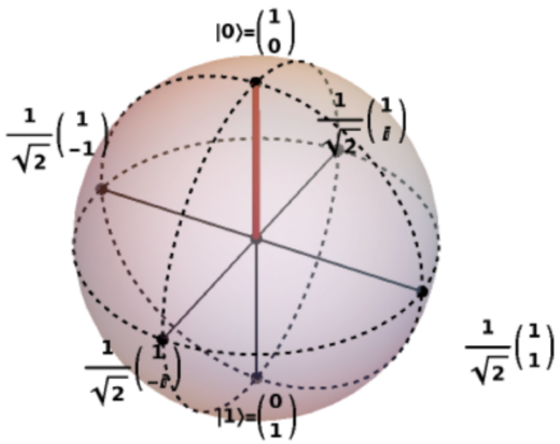

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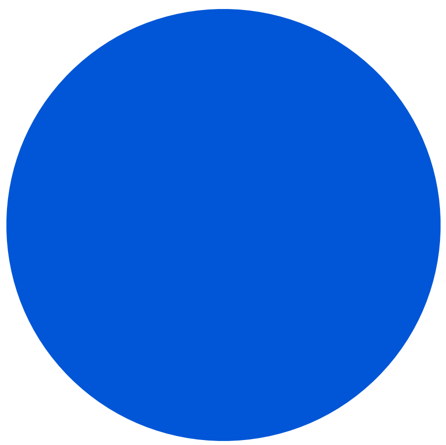

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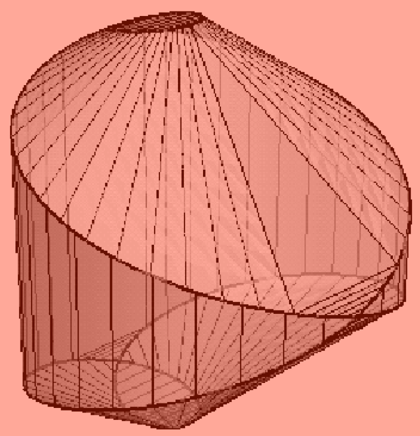

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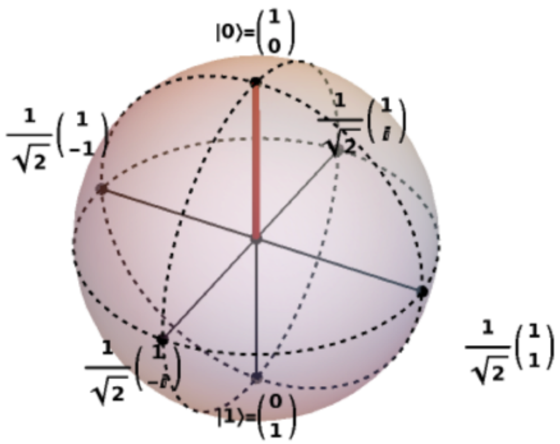

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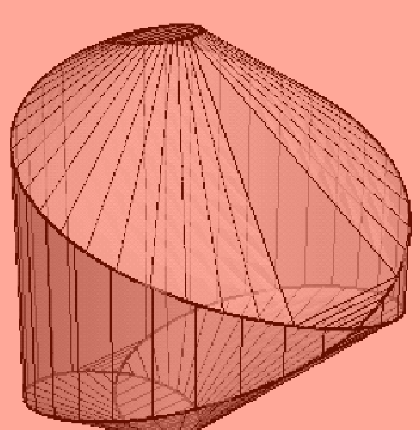

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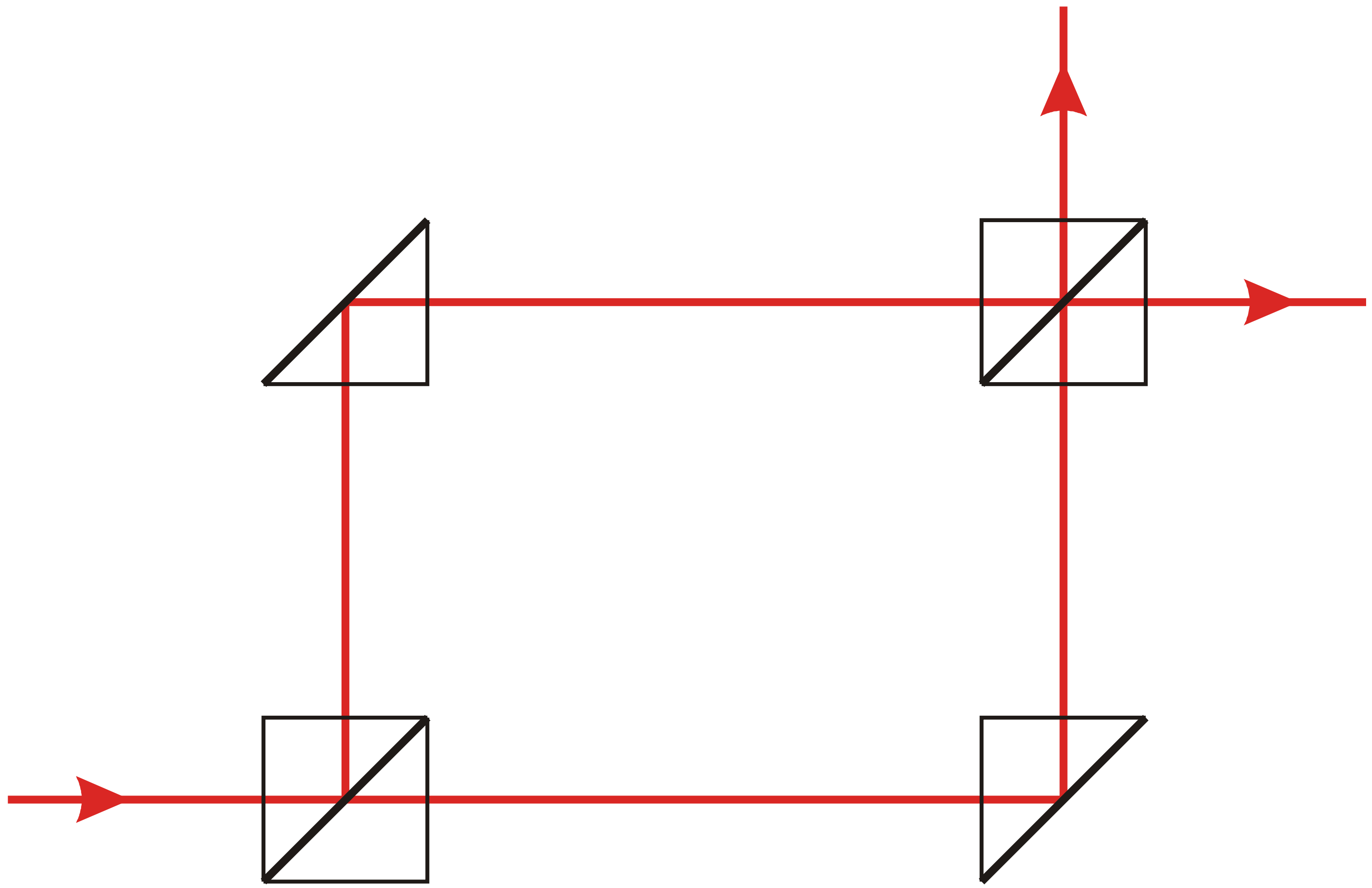
Does spacetime constrain *d?*

"Why" *d=3* ?

Constraints from relativity

A. Garner, **MM**, O. C. O. Dahlsten, Proc. R. Soc. A **473**, 20170596 (2017).



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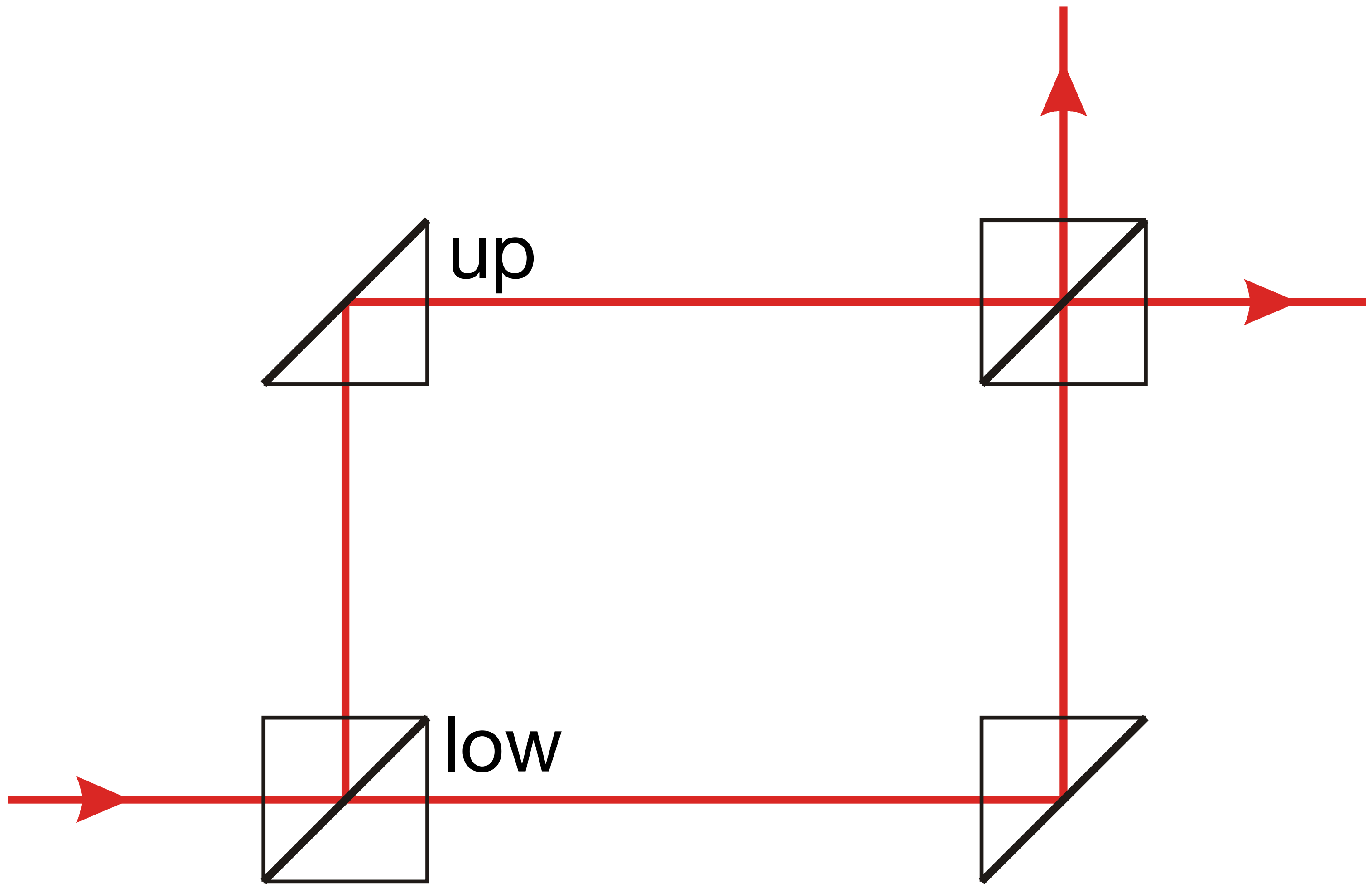

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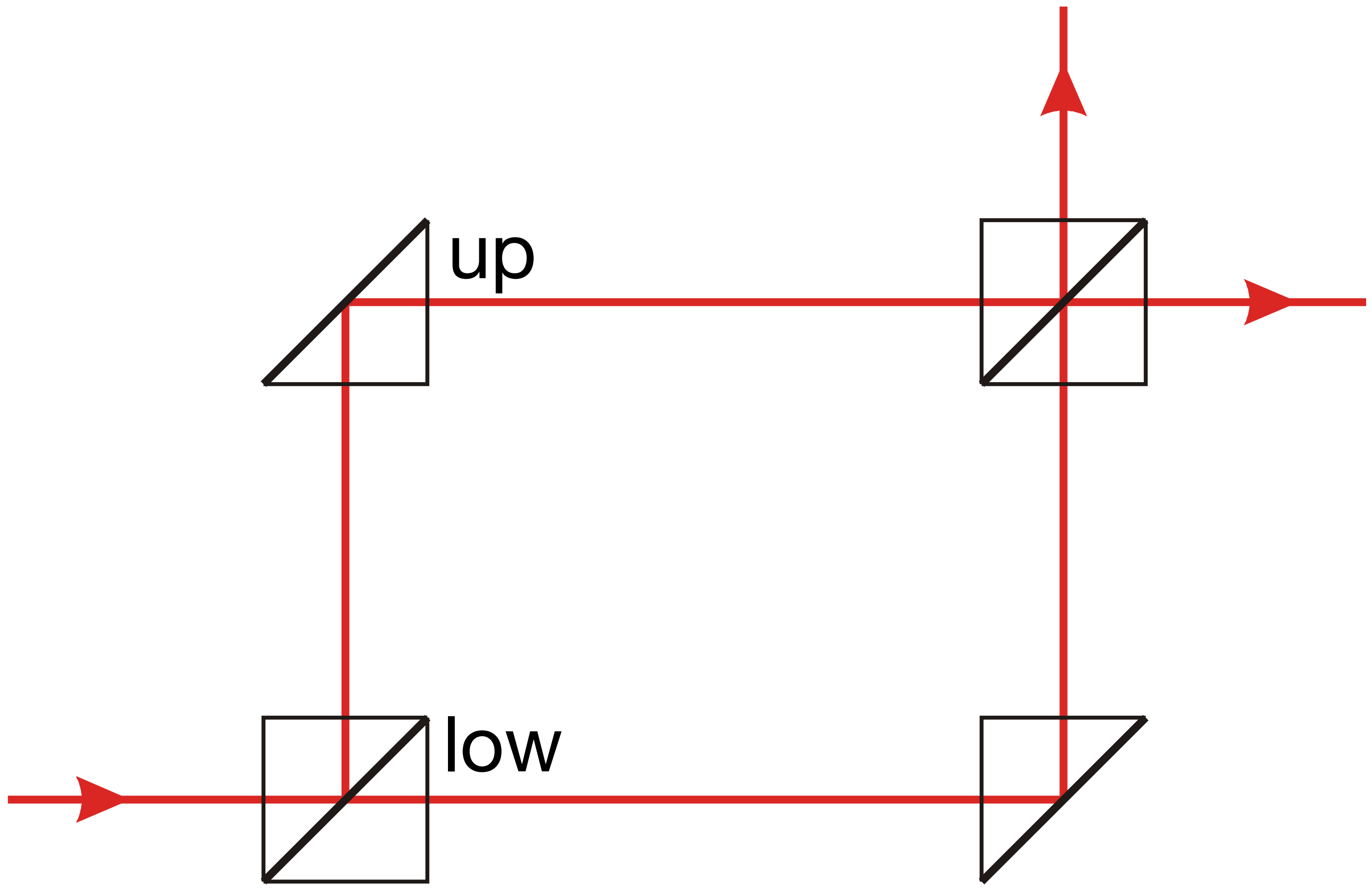

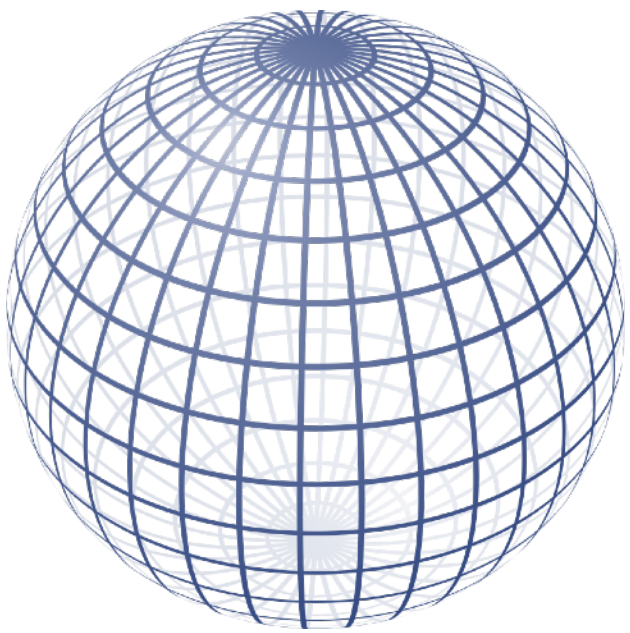

d-dim. "Bloch sphere"

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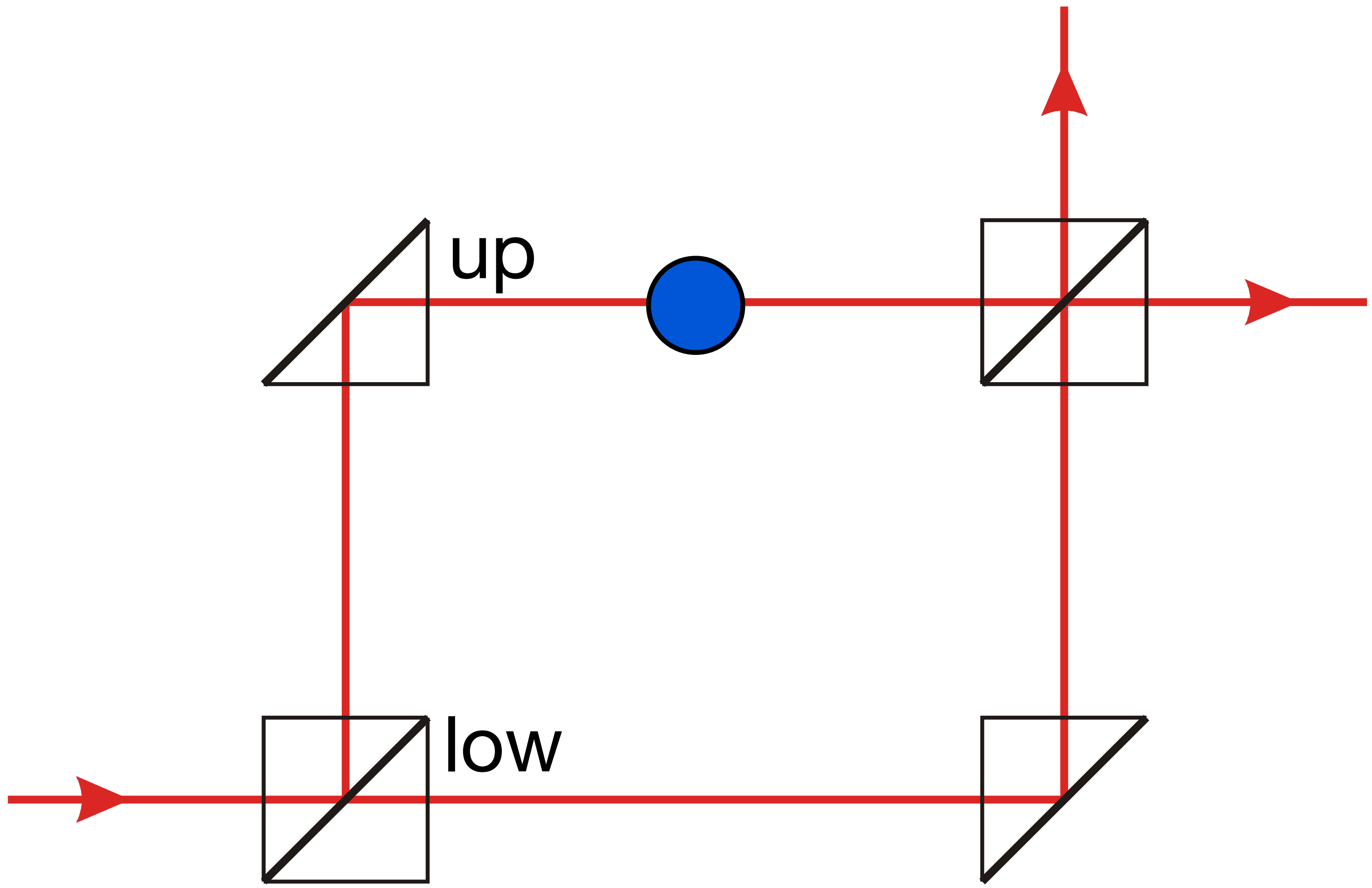

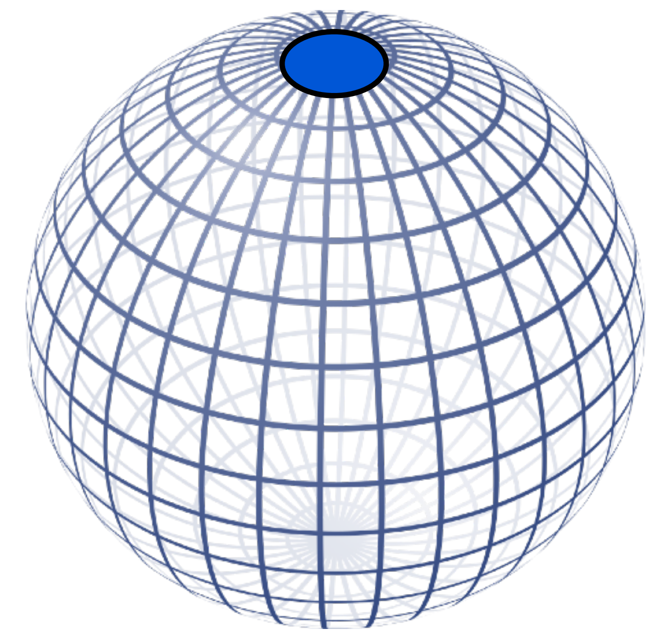

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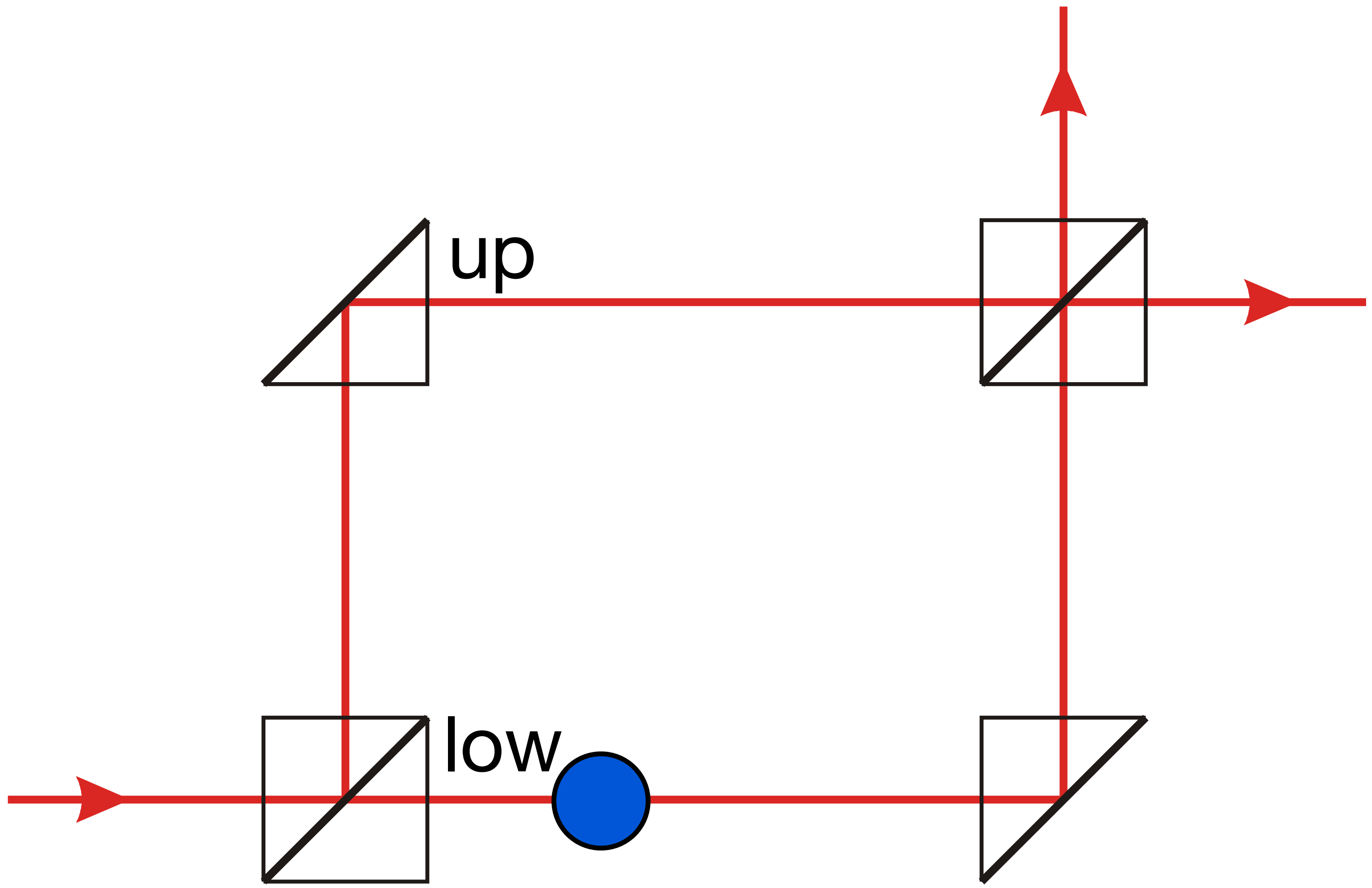

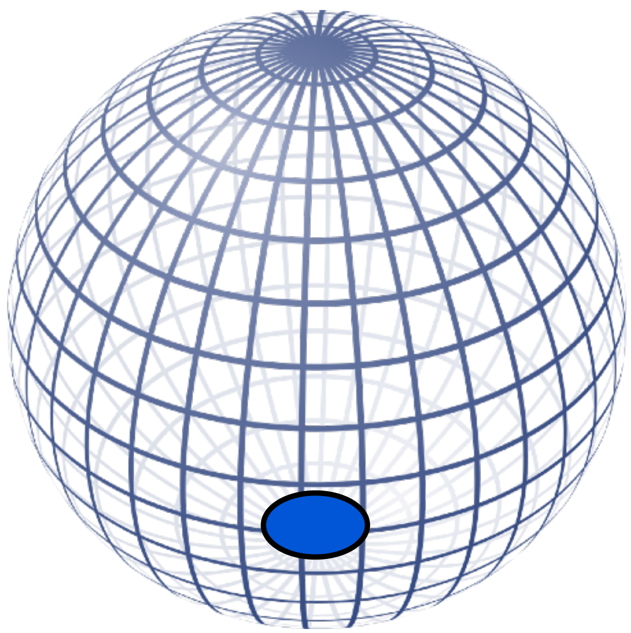

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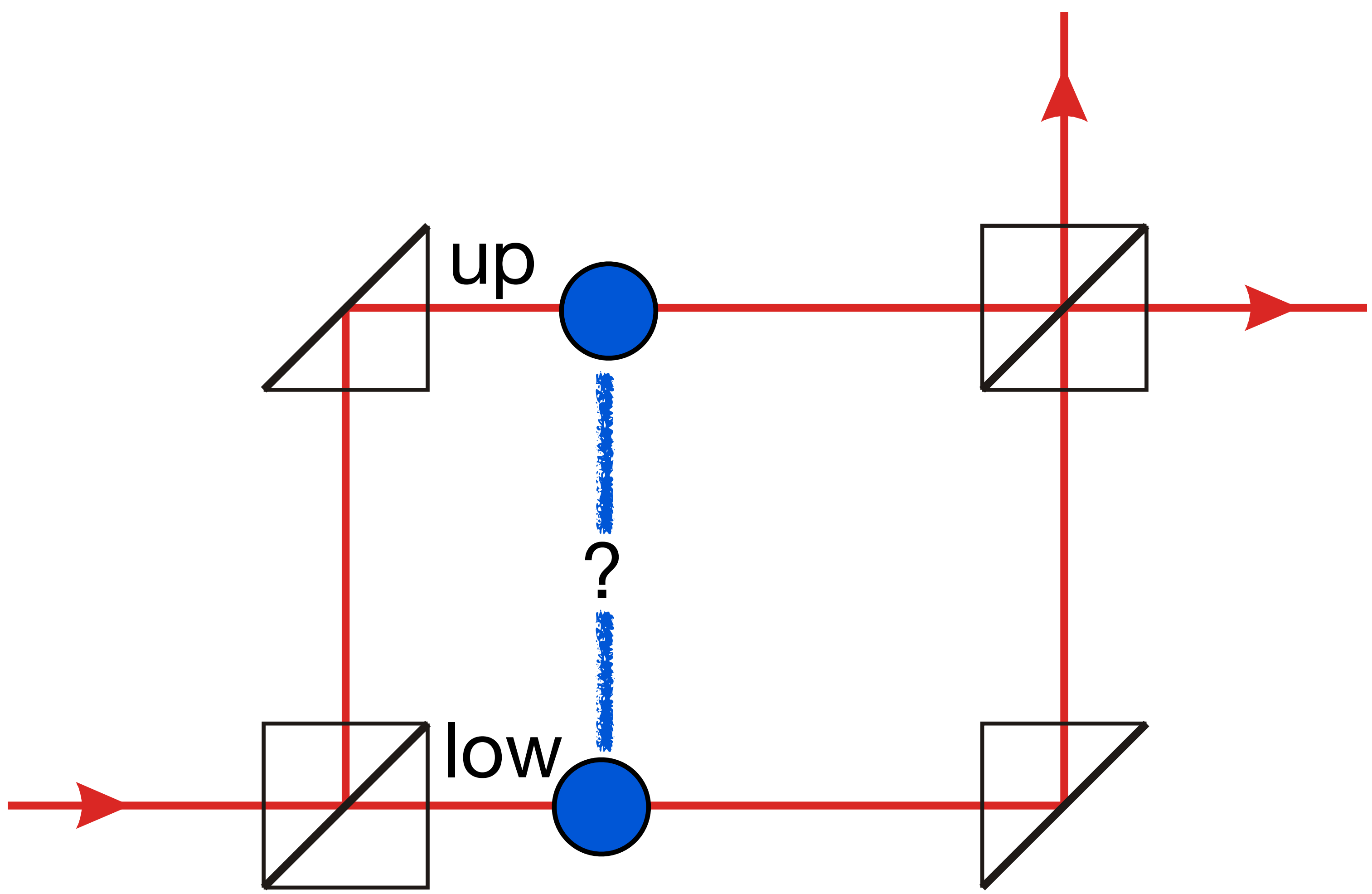

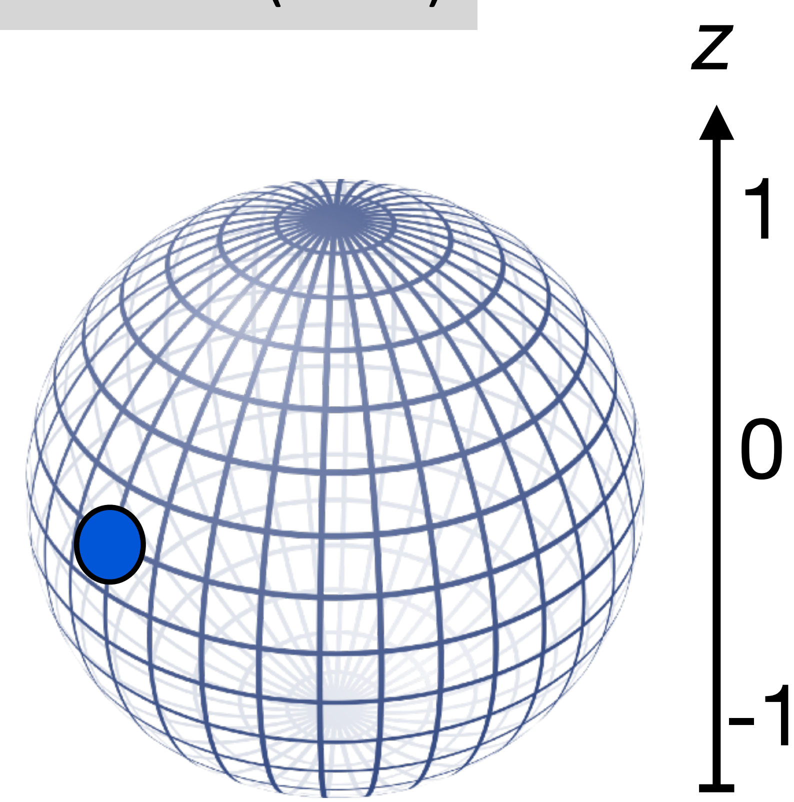

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State on equator *z=0*: probability 1/2 for each.

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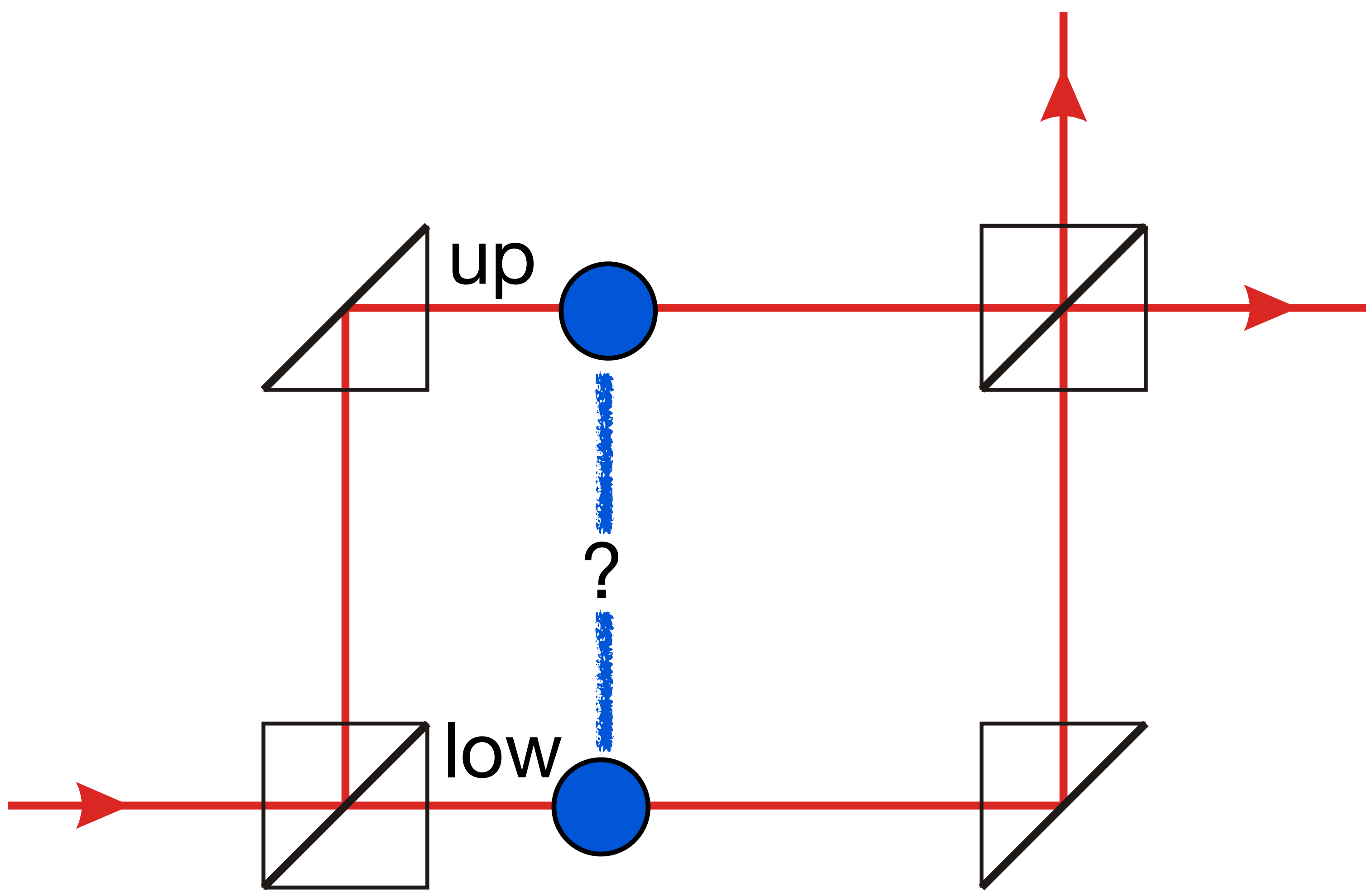

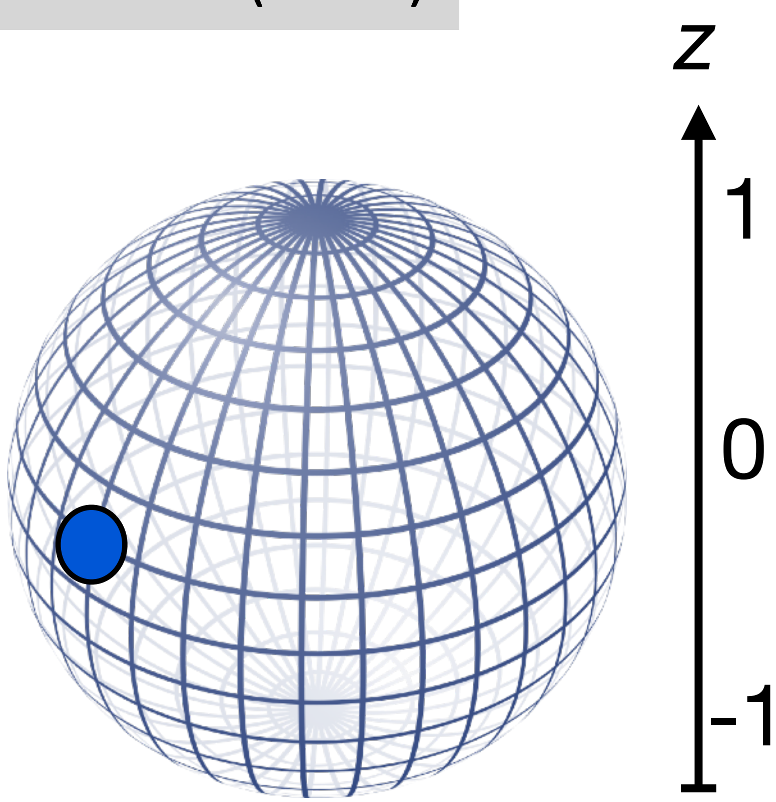

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$$p(\mathrm{up}) = \frac{1}{2}(z+1)$$

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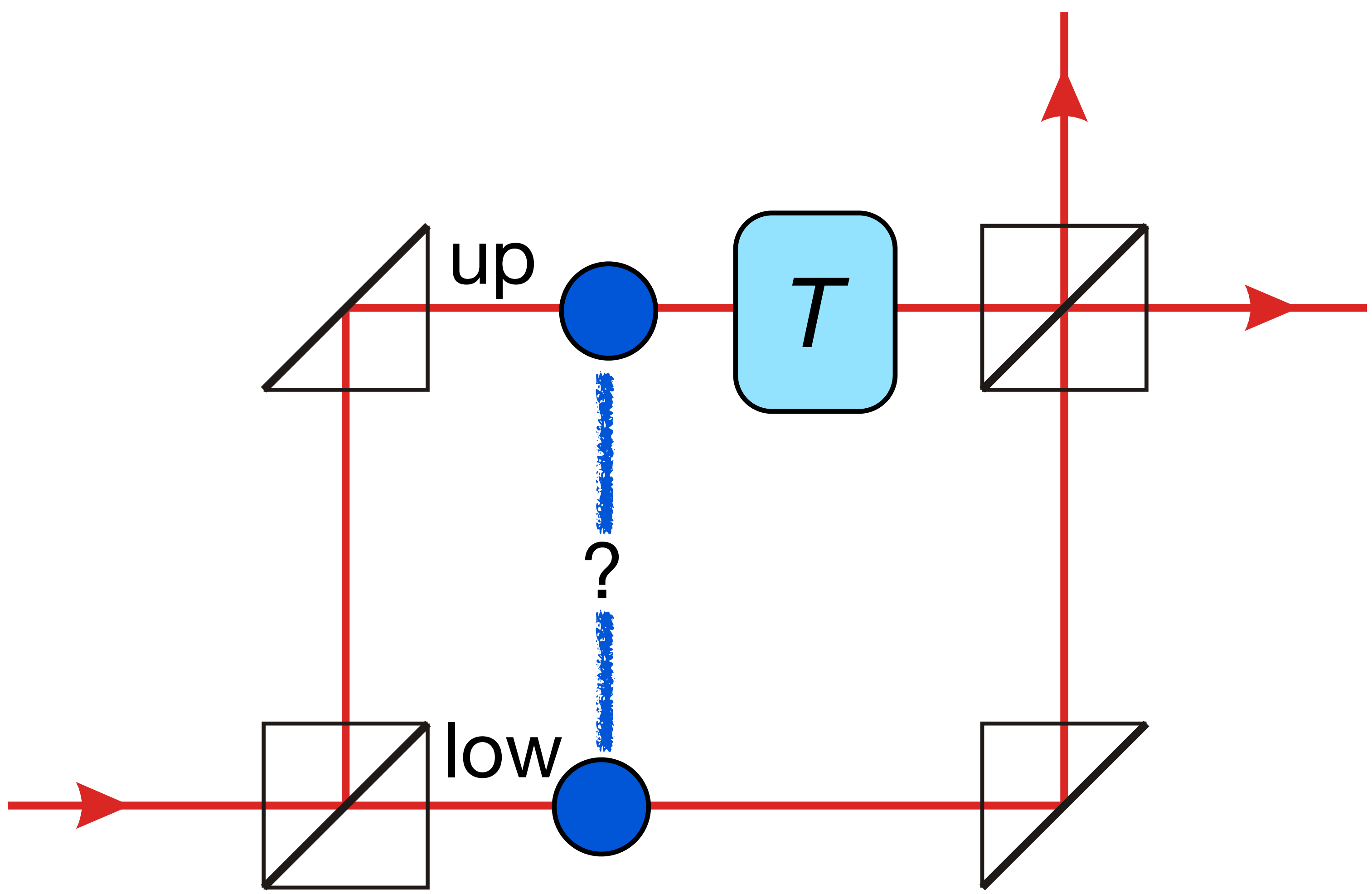

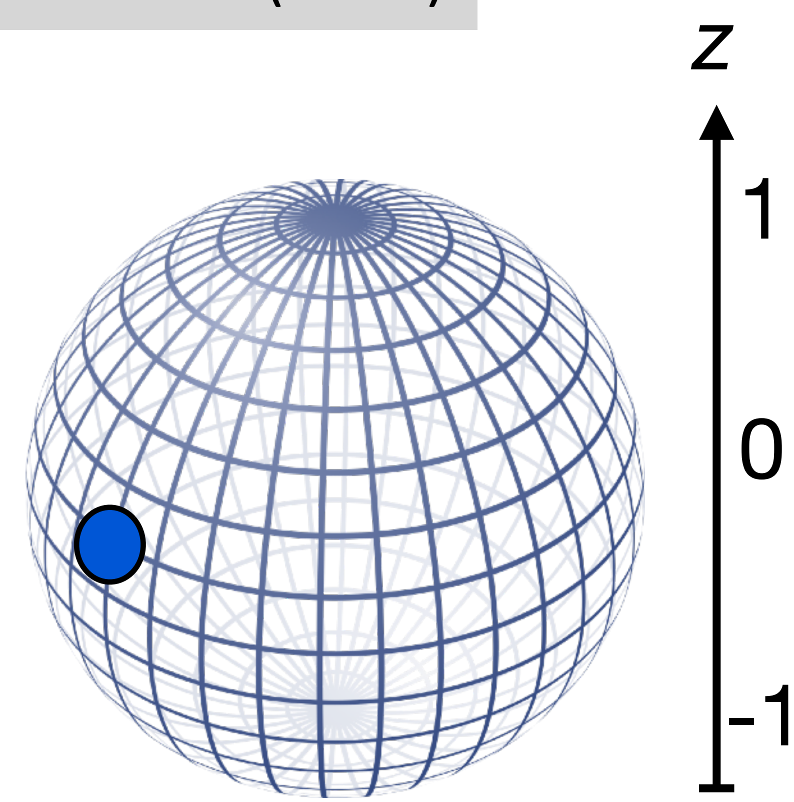

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What transformations *T* can we perform locally in one arm...
... reversibly, i.e. without any information loss?

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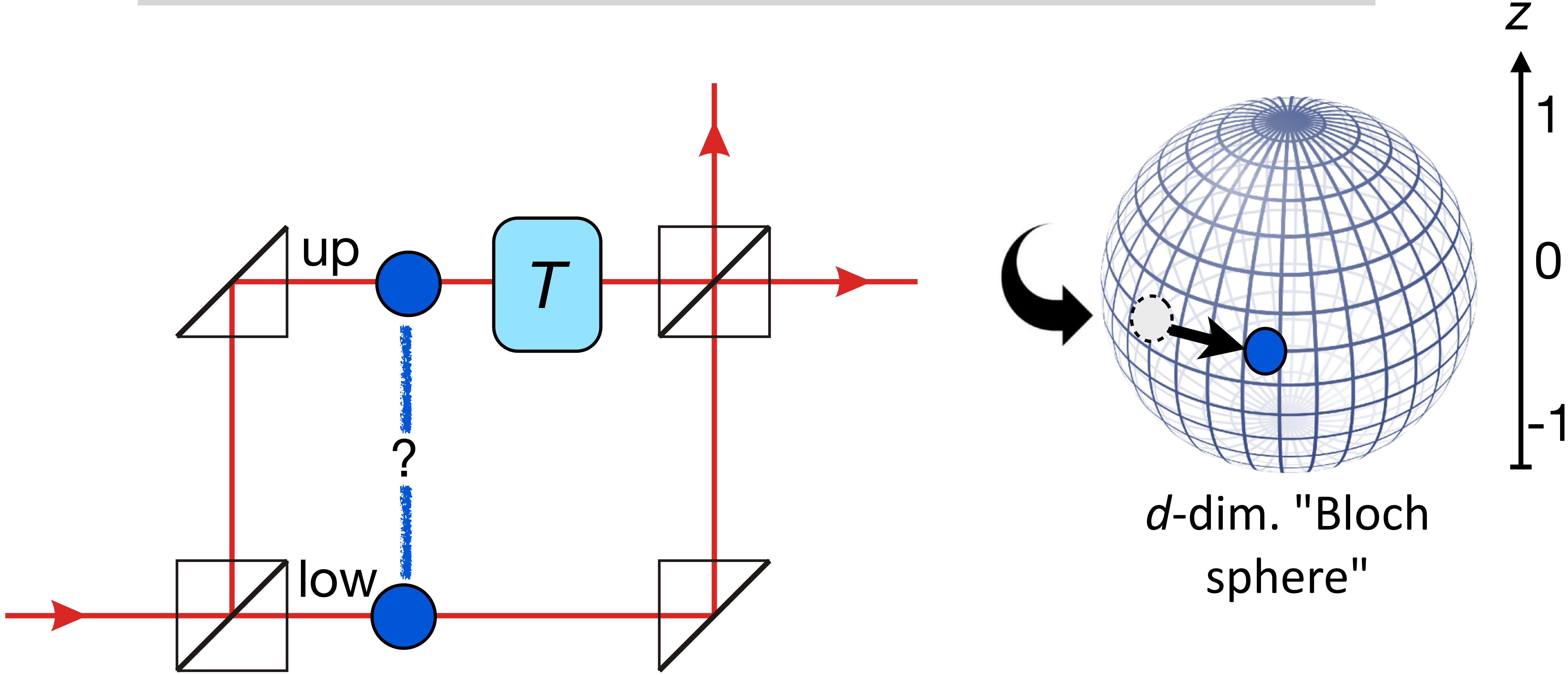

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T must be a rotation of the Bloch ball (reversible+linear)...

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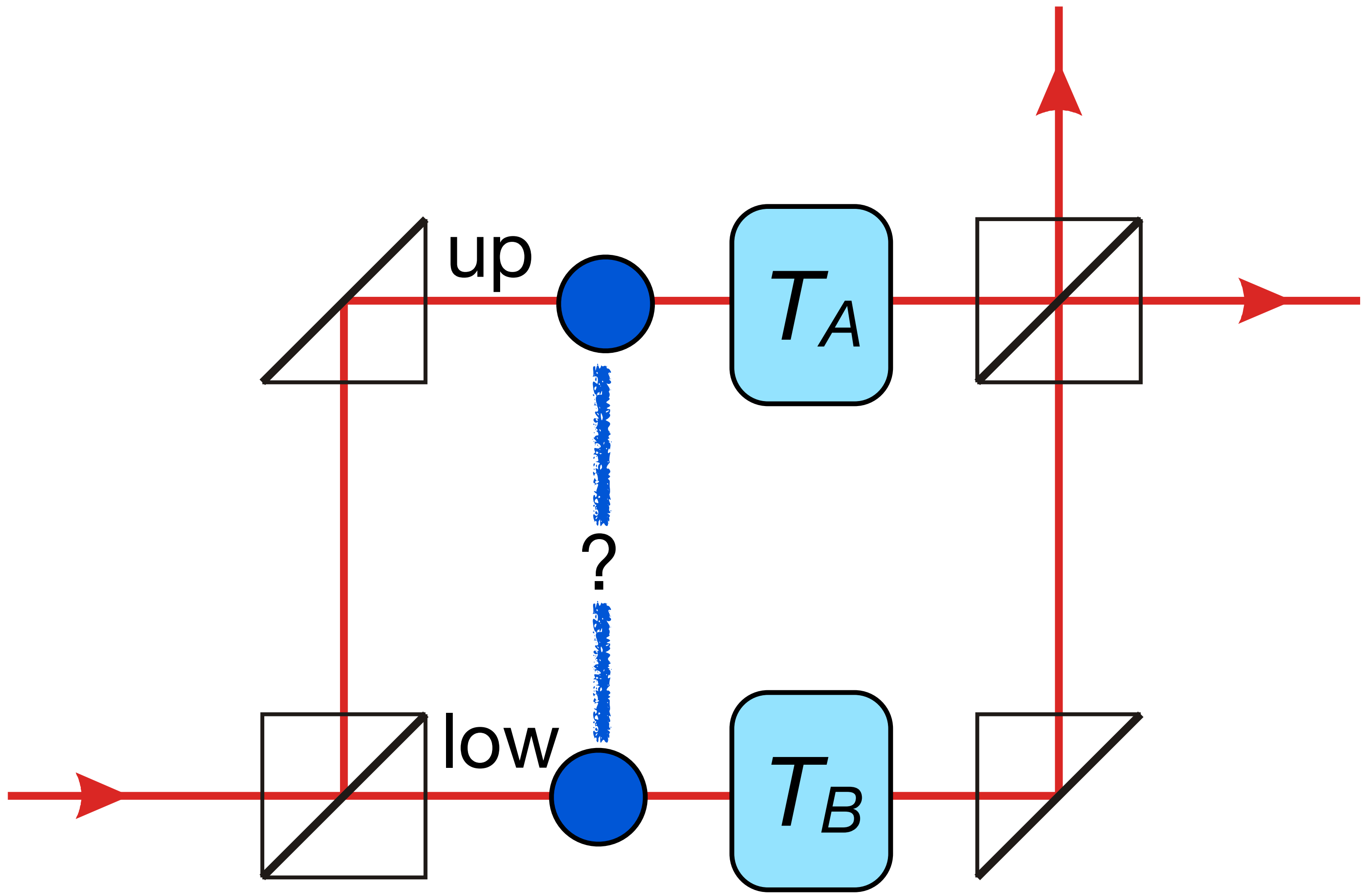

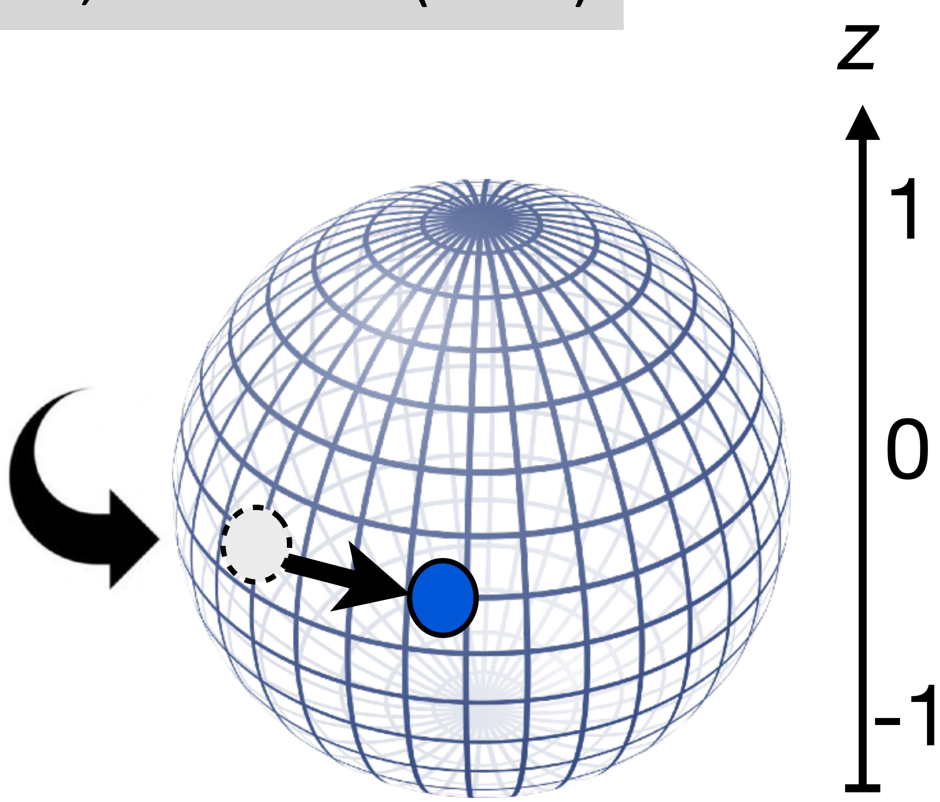

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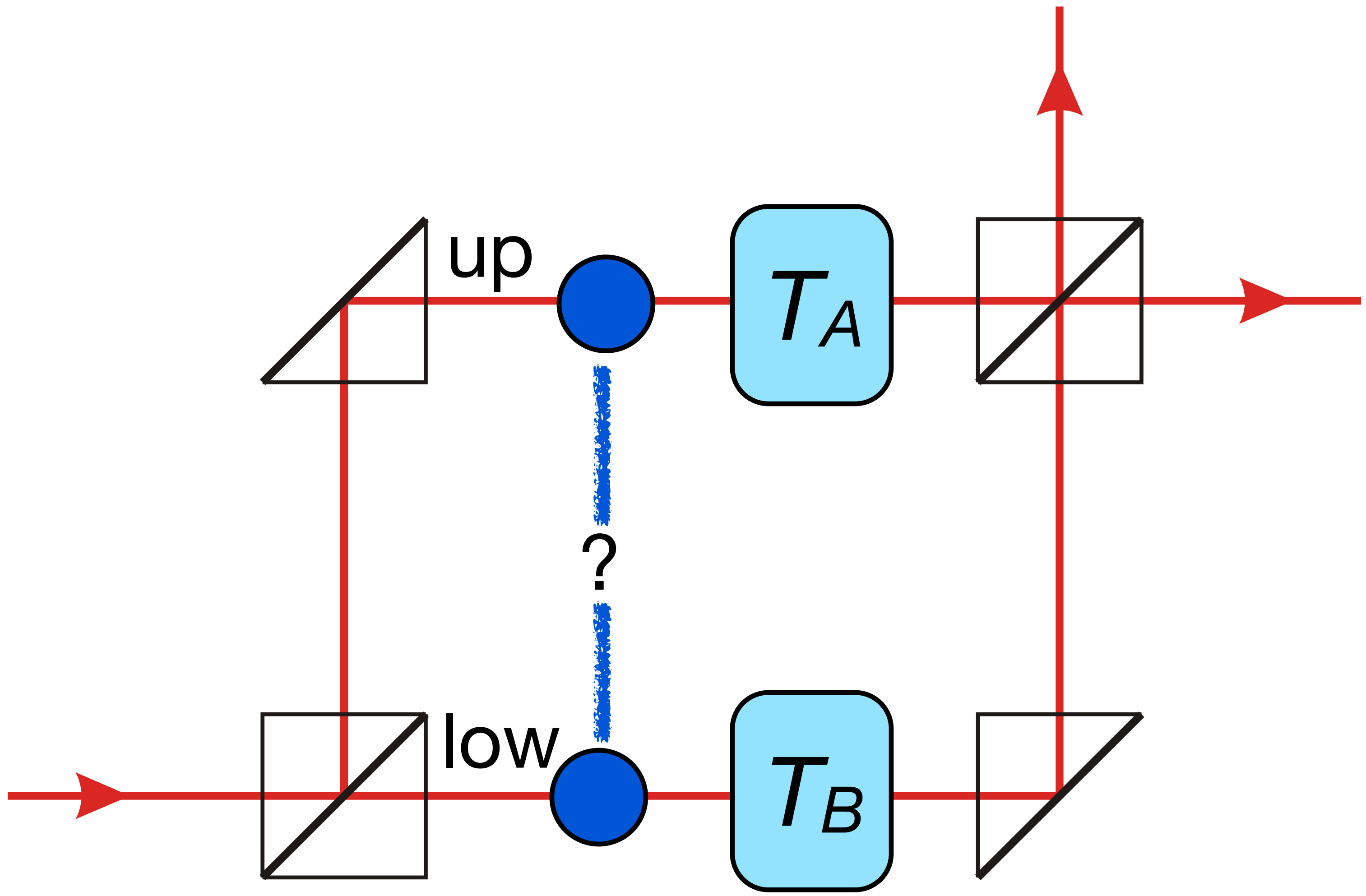

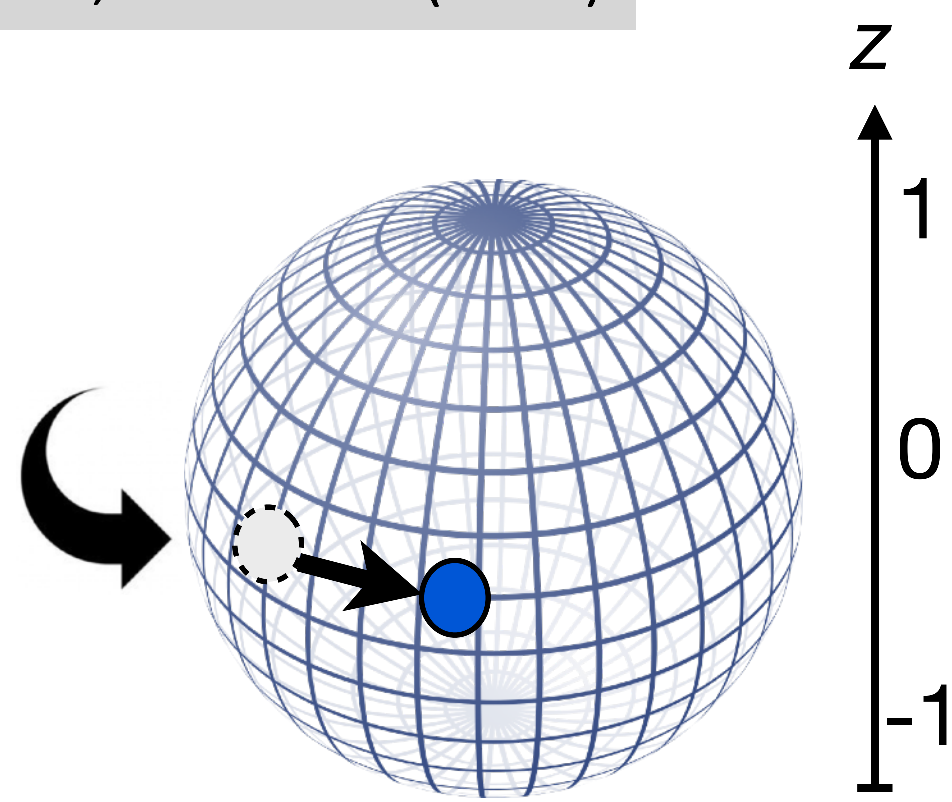

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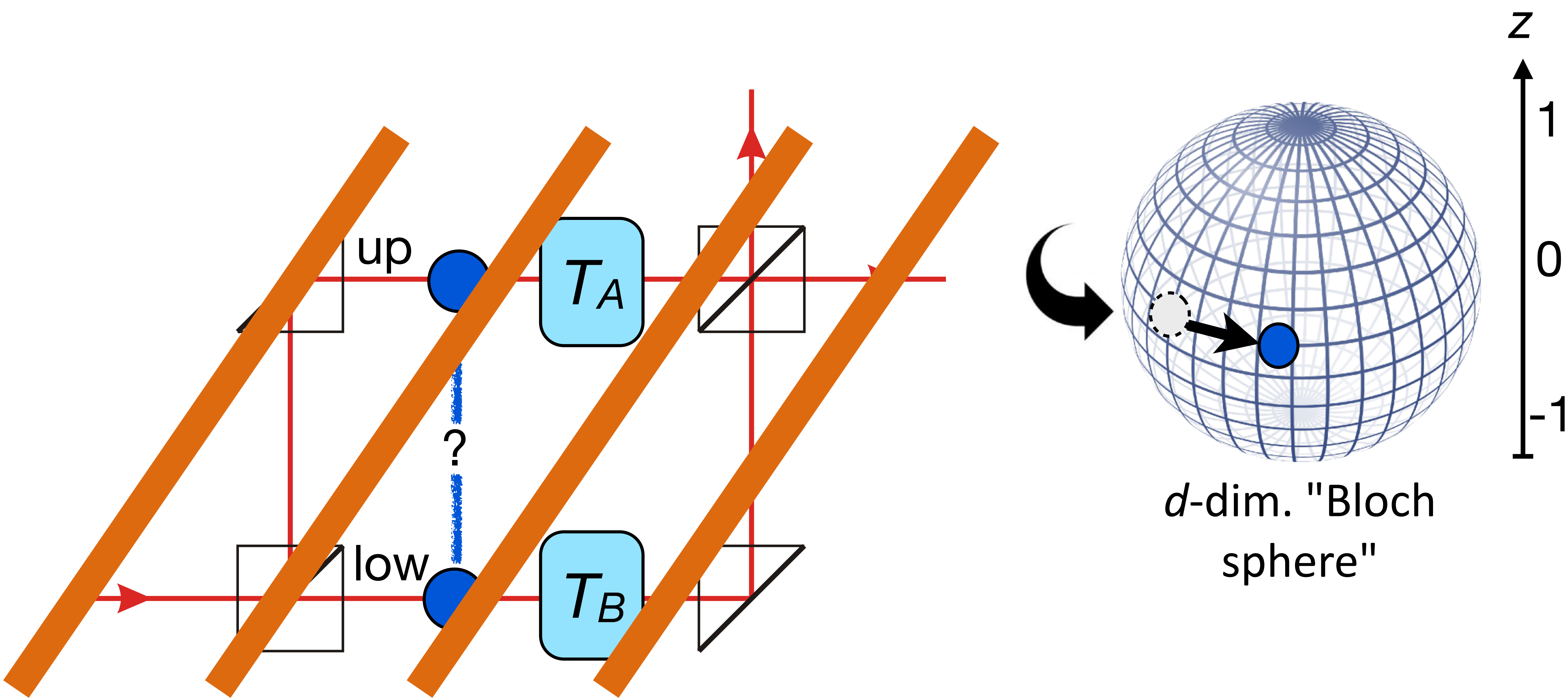

Relativity: there's a frame of reference in which T_A happens before T_B...

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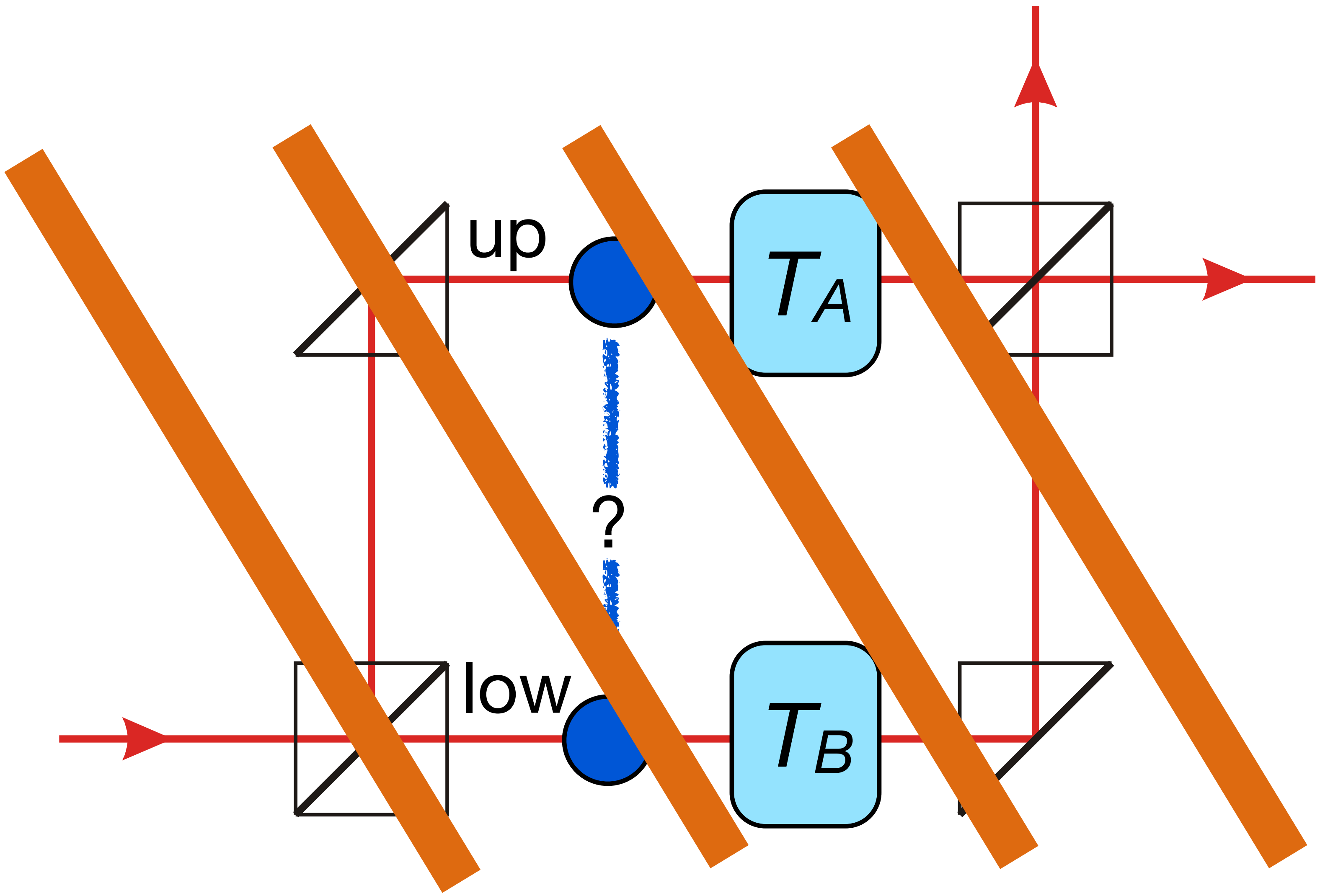

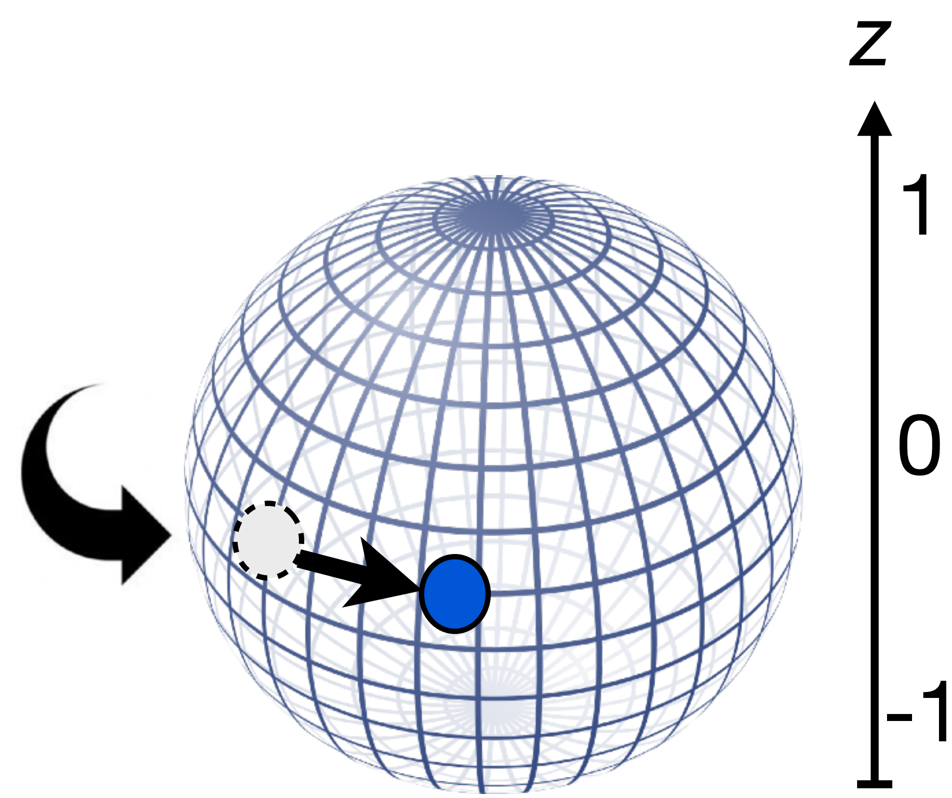

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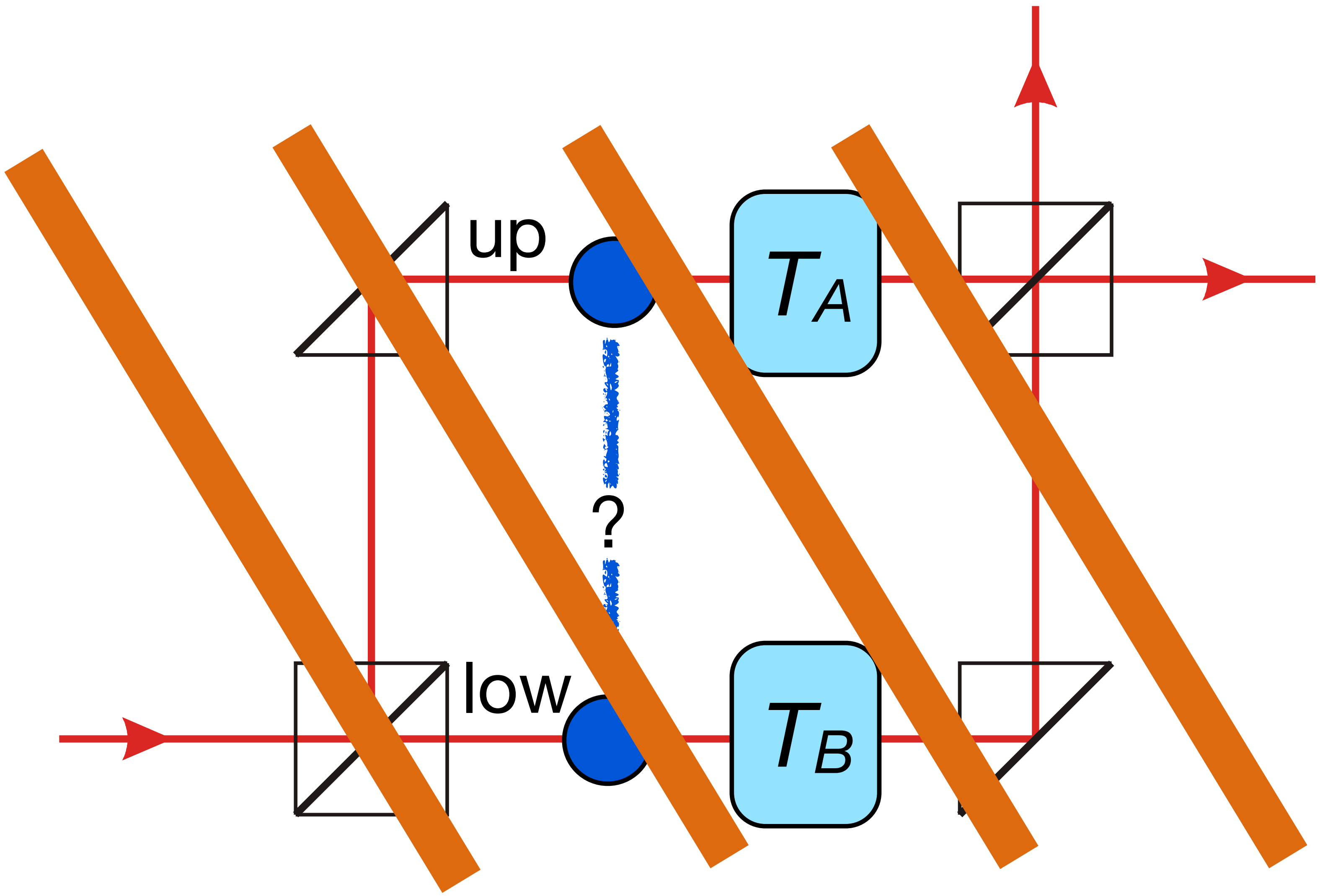

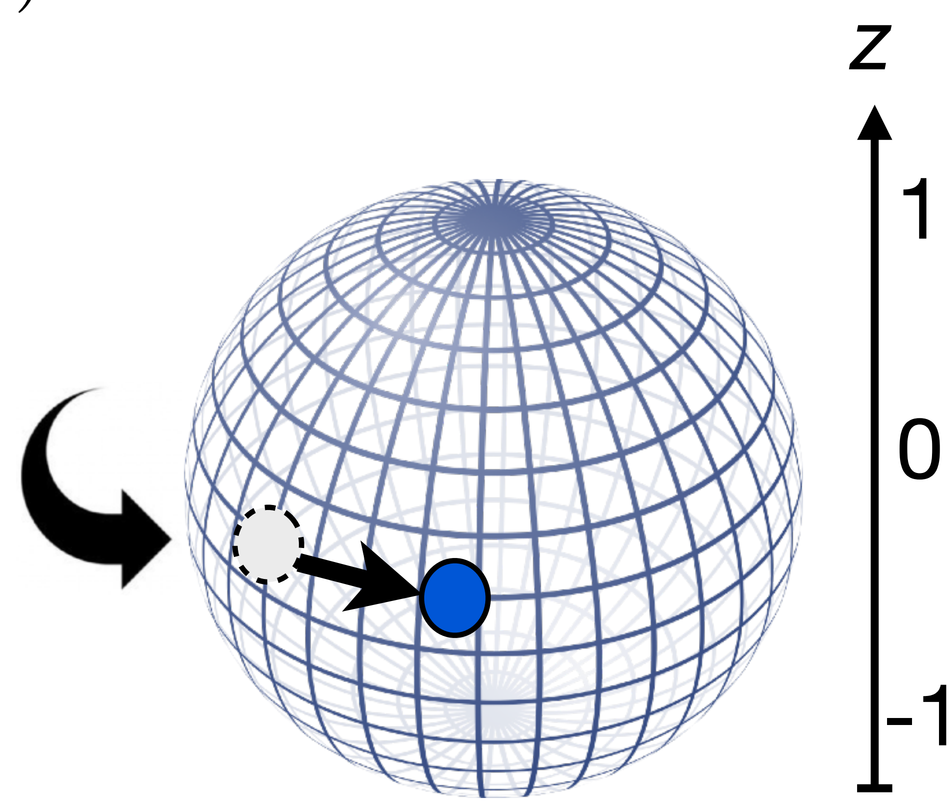

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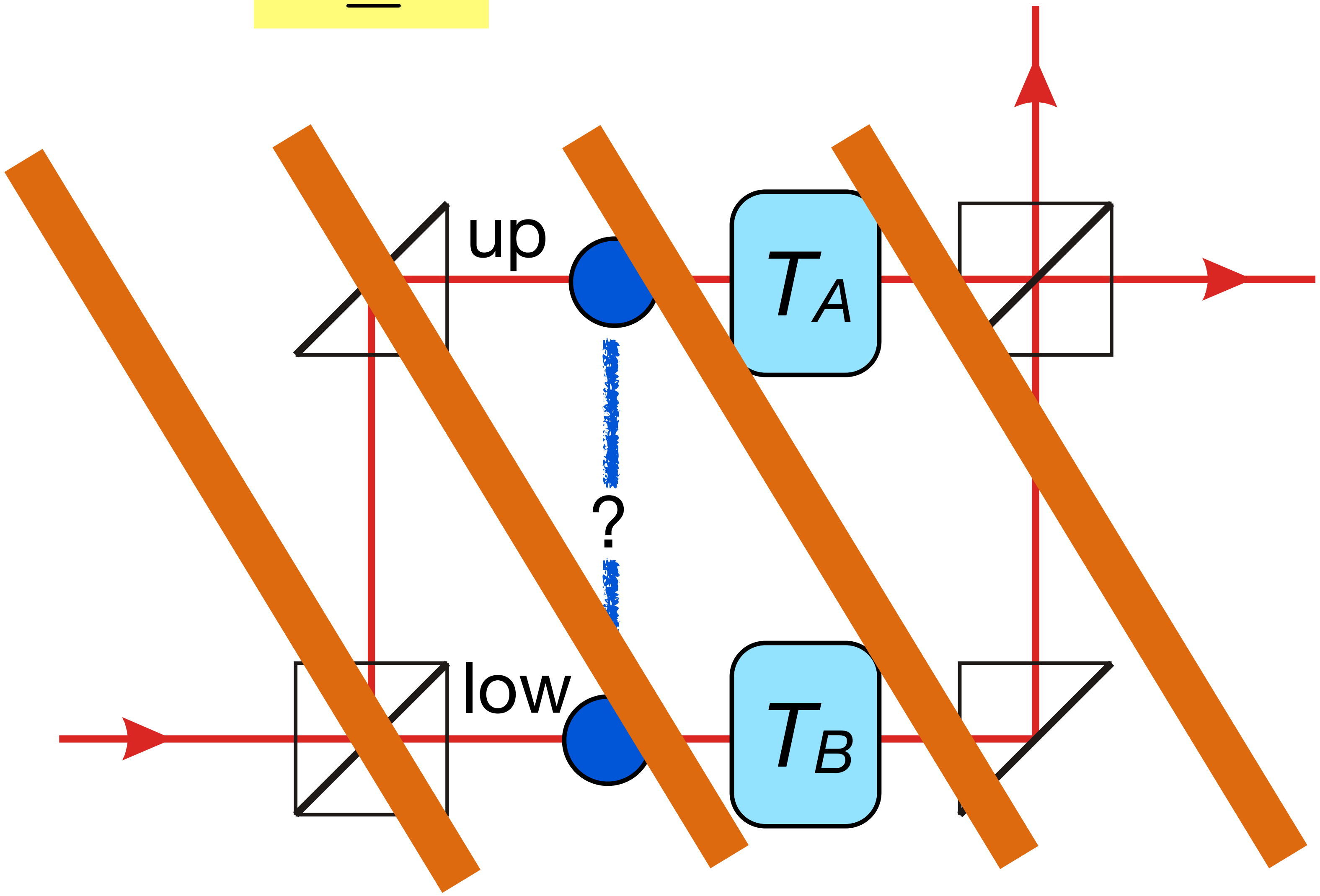
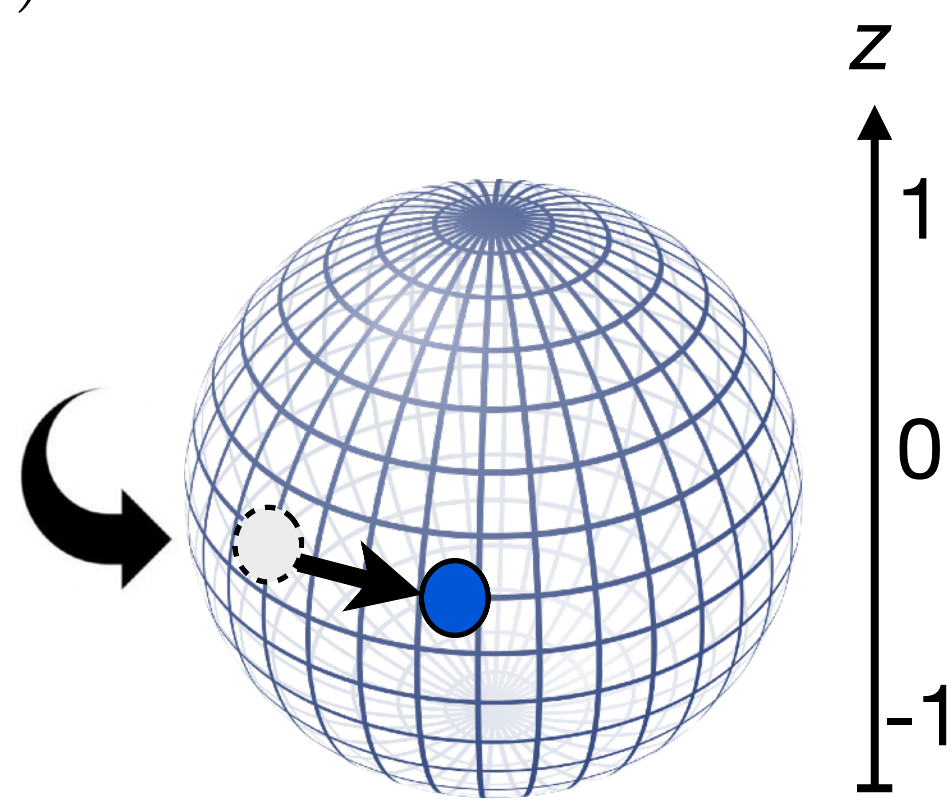
$\Rightarrow d \leq 3$.





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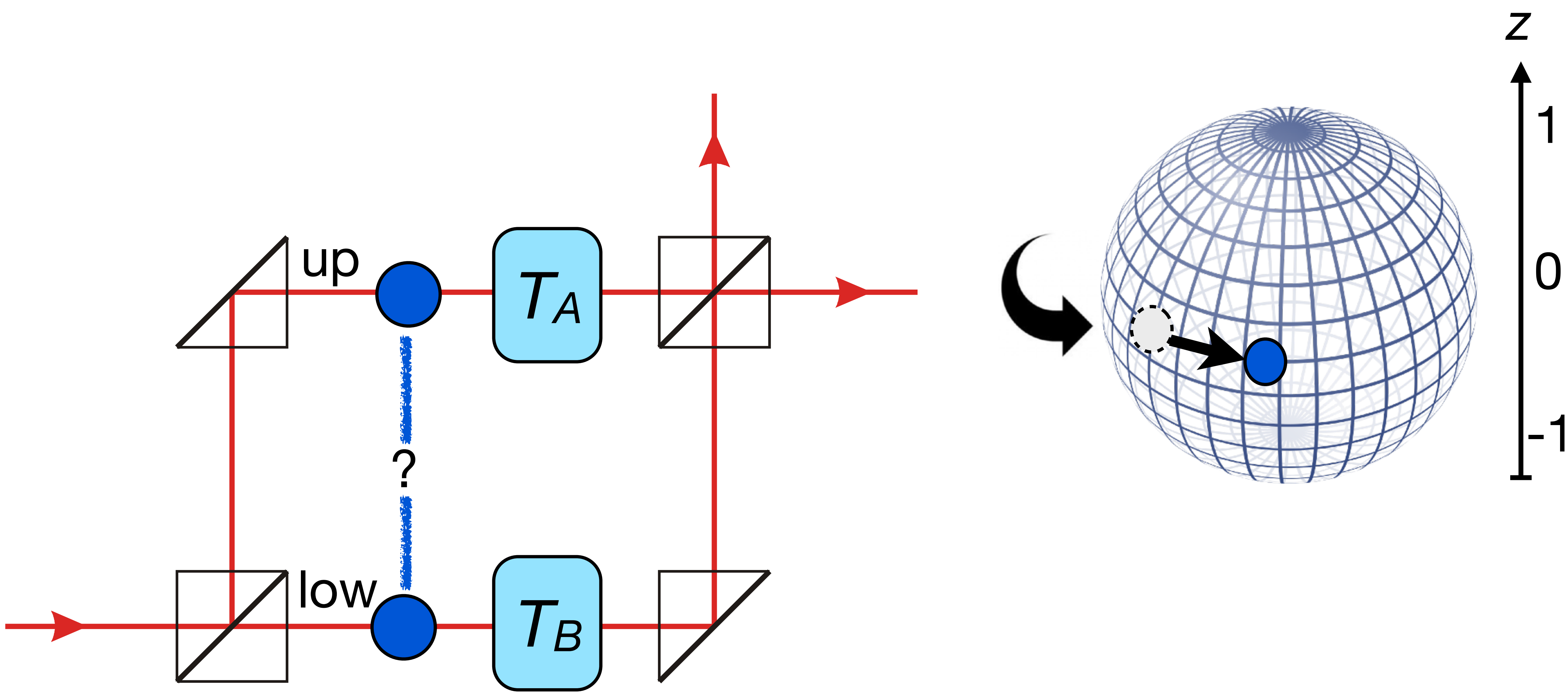

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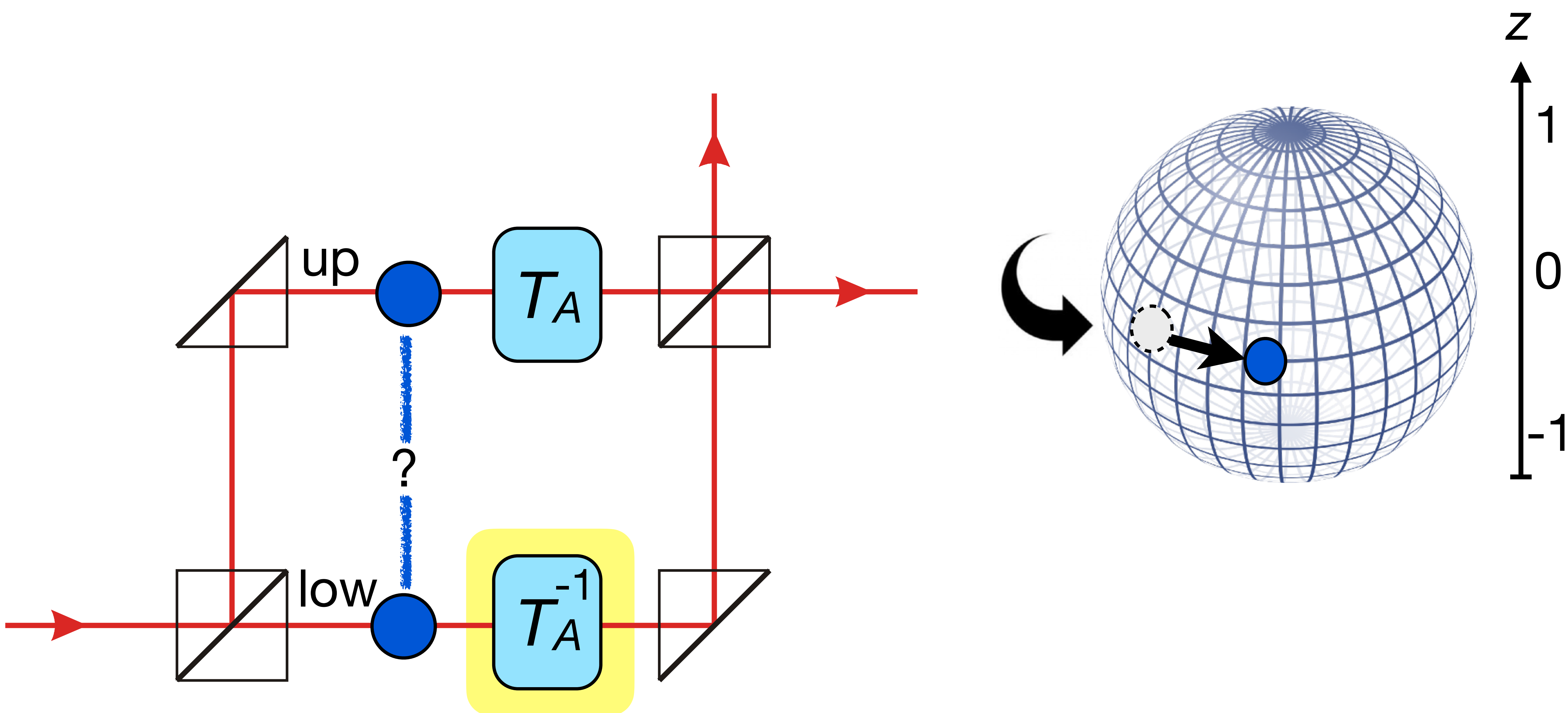

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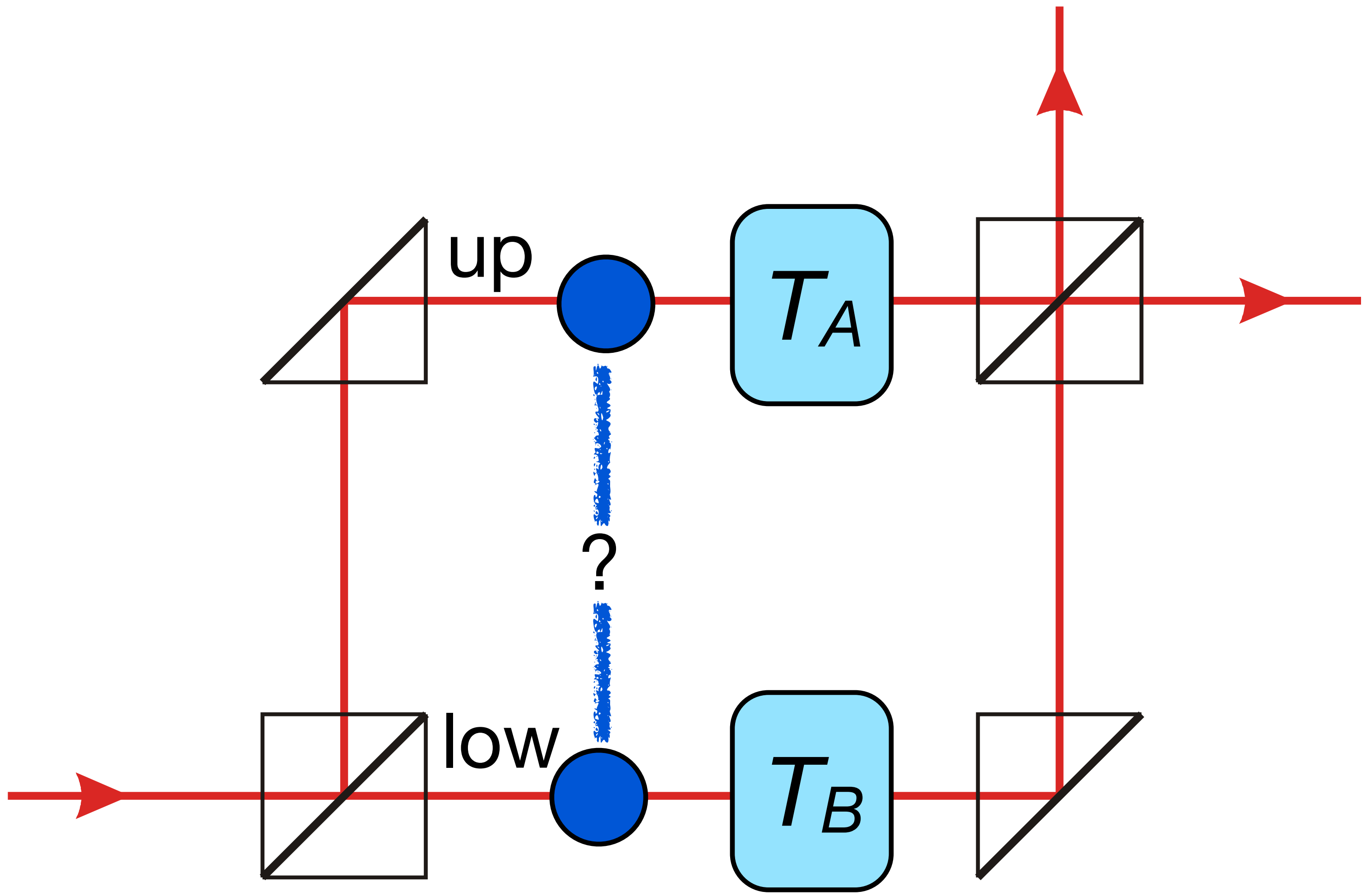

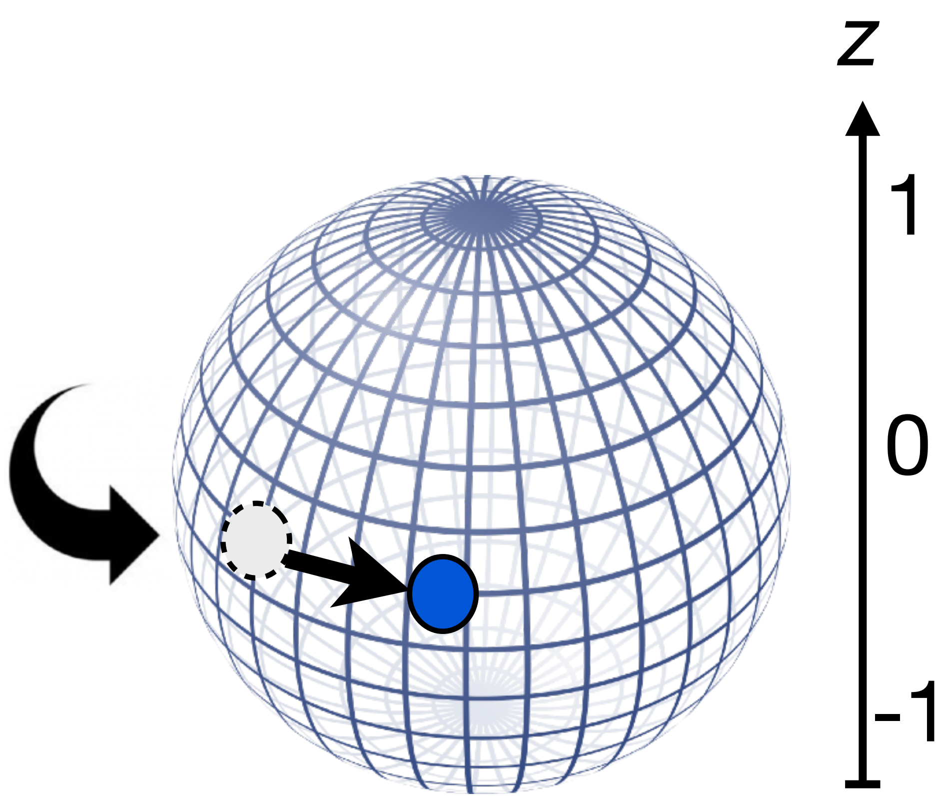

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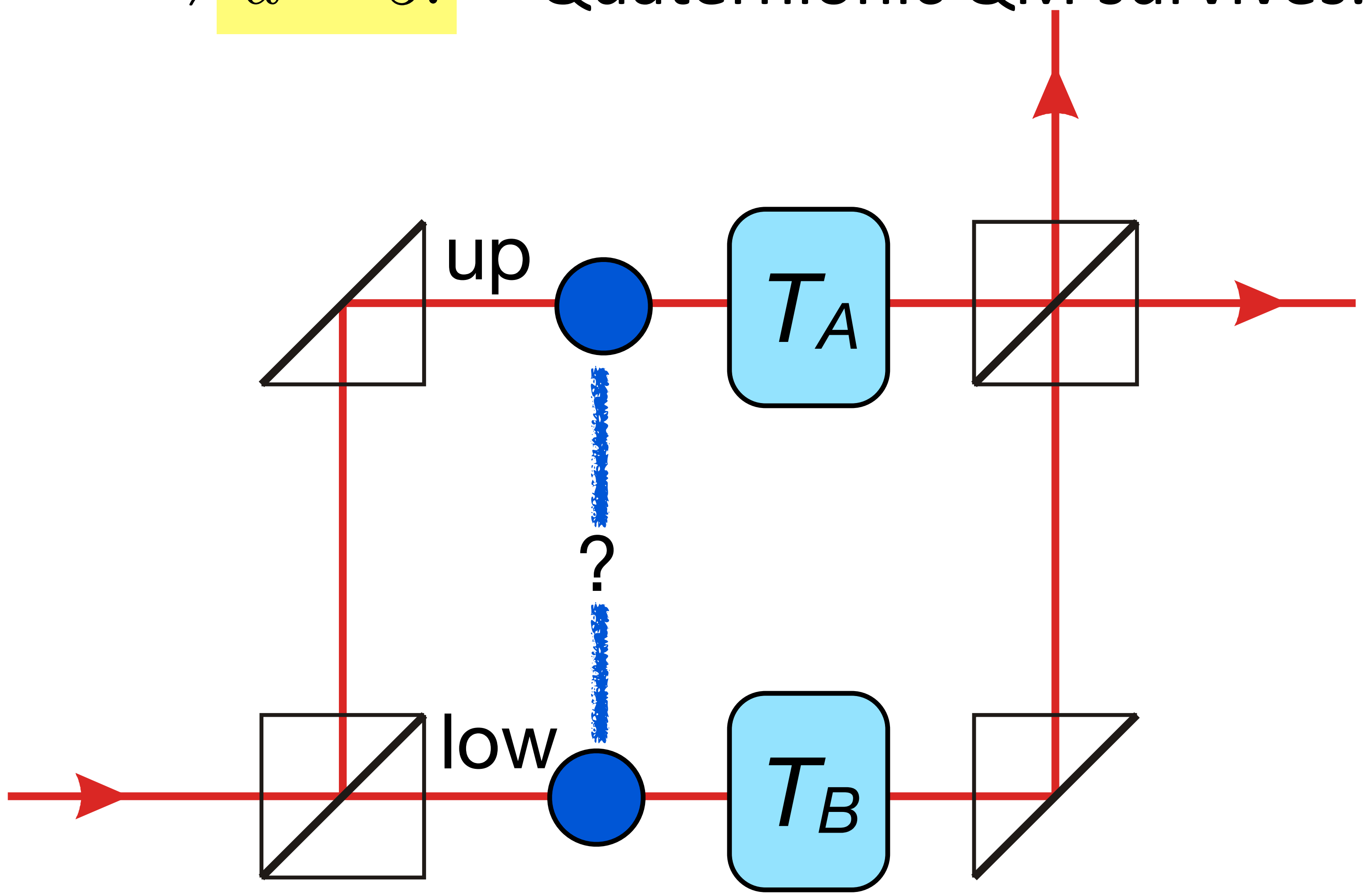
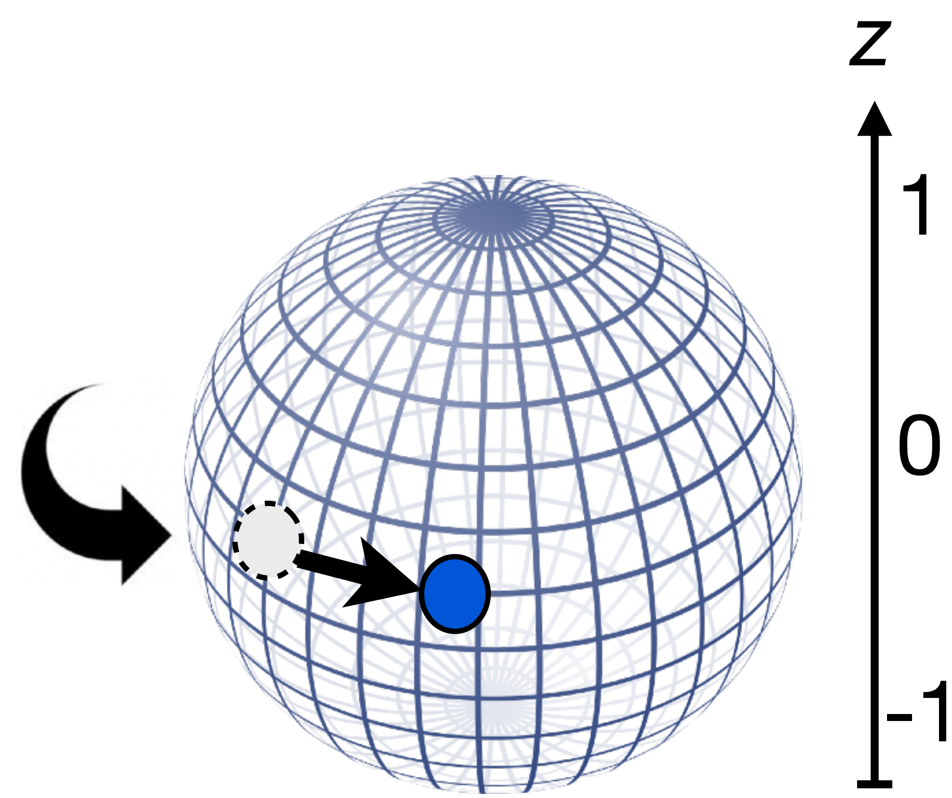
$\Rightarrow d = 5.$ Quaternionic QM survives.





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Classification of possibilities

A1) Beam splitter can prepare **any upper-branch probability** *p*.

A2) Every pure state with the same *p* can be prepared by **reversible operations applied locally** on the two arms.

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Theorem 6.2. *Under the assumptions* A1, A2, A3, *relativity of simultaneity* (*REL*) *allows for the following possibilities and not more*:

— $d = 1$ (*the classical bit*), *with* $\mathcal{G}_{\mathrm{A}} = \mathcal{G}_{\mathrm{B}} = \{\mathbf{1}\}$ (*i.e. without any non-trivial local transformations*),
— $d = 2$ (*the quantum bit over the real numbers*), *with* $\mathcal{G}_{\mathrm{A}} = \mathcal{G}_{\mathrm{B}} = \mathbb{Z}_2$,
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Relativity of simultaneity singles out the **associative division algebras**.

Overview

1. Motivations: QG and device-independent QIT

2. Relativity of simultaneity and the qubit

3. Randomness generation via rotational symmetry

4. Conclusions

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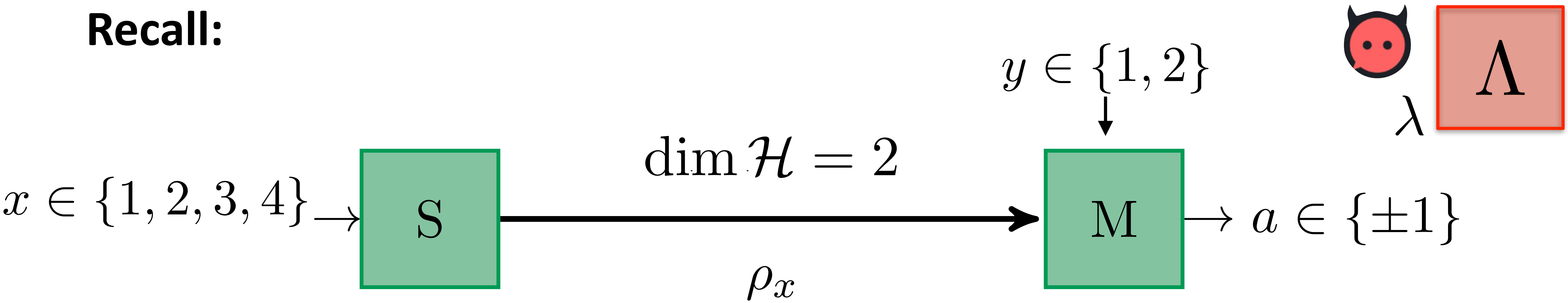
Randomness generation: quantum analysis

Recall:



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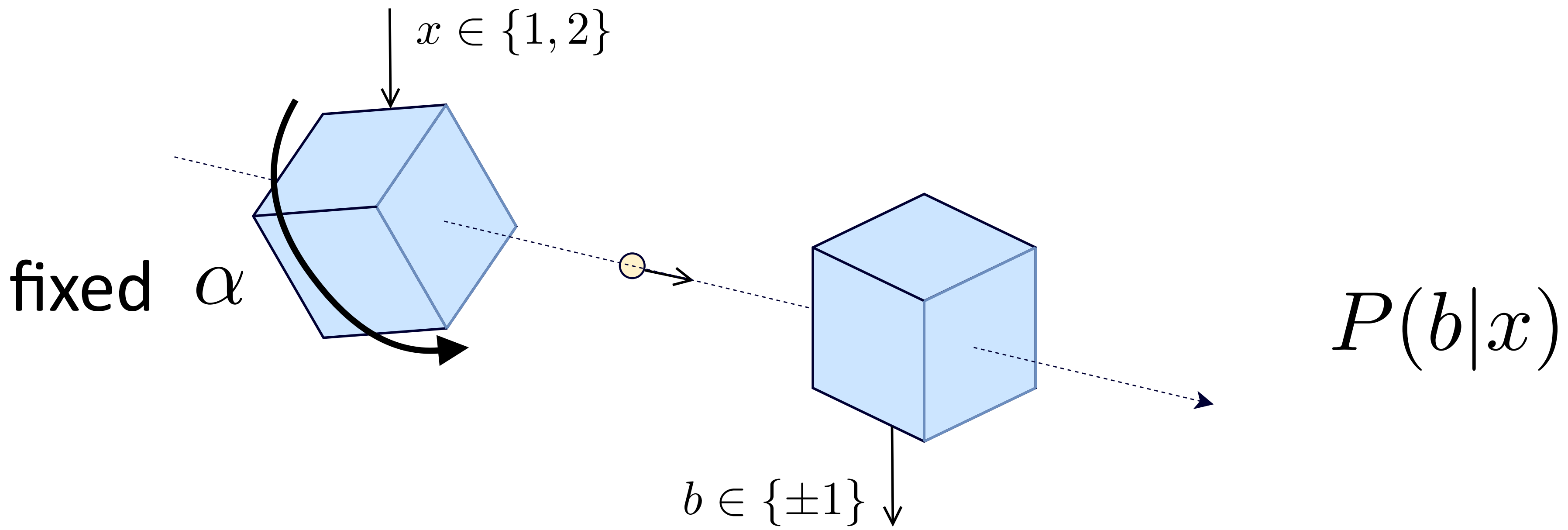

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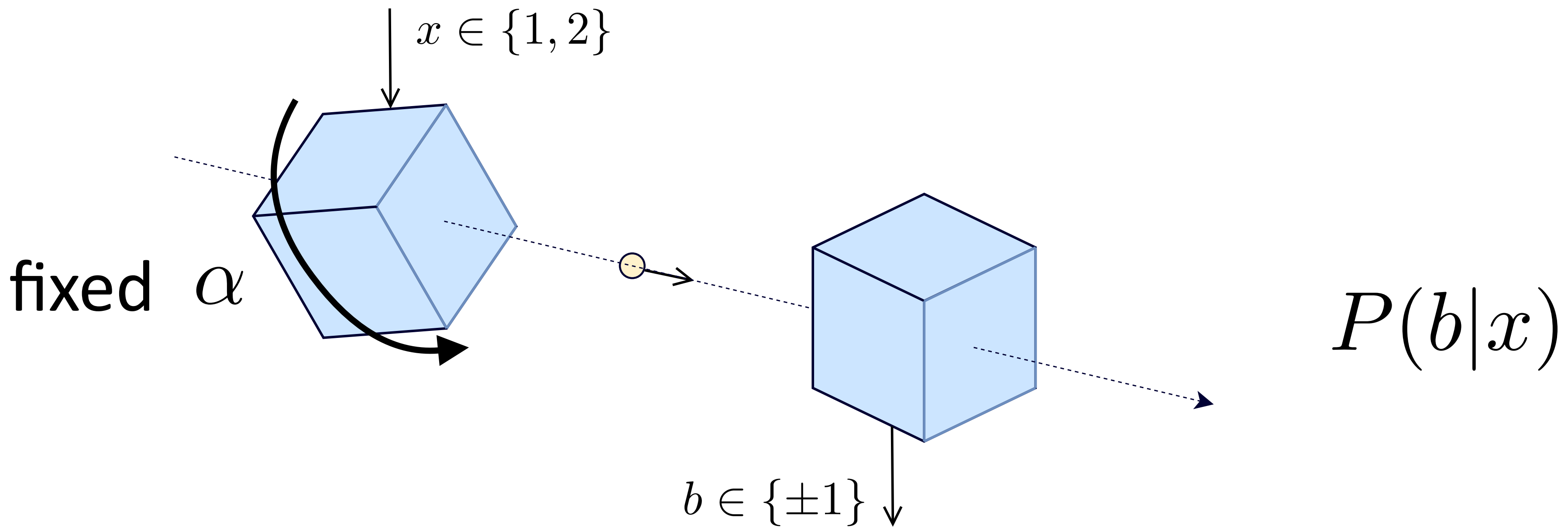

If input is x=1: do nothing to preparation device;
if x=2: **rotate it** (relative to measurement device) **by angle α**.

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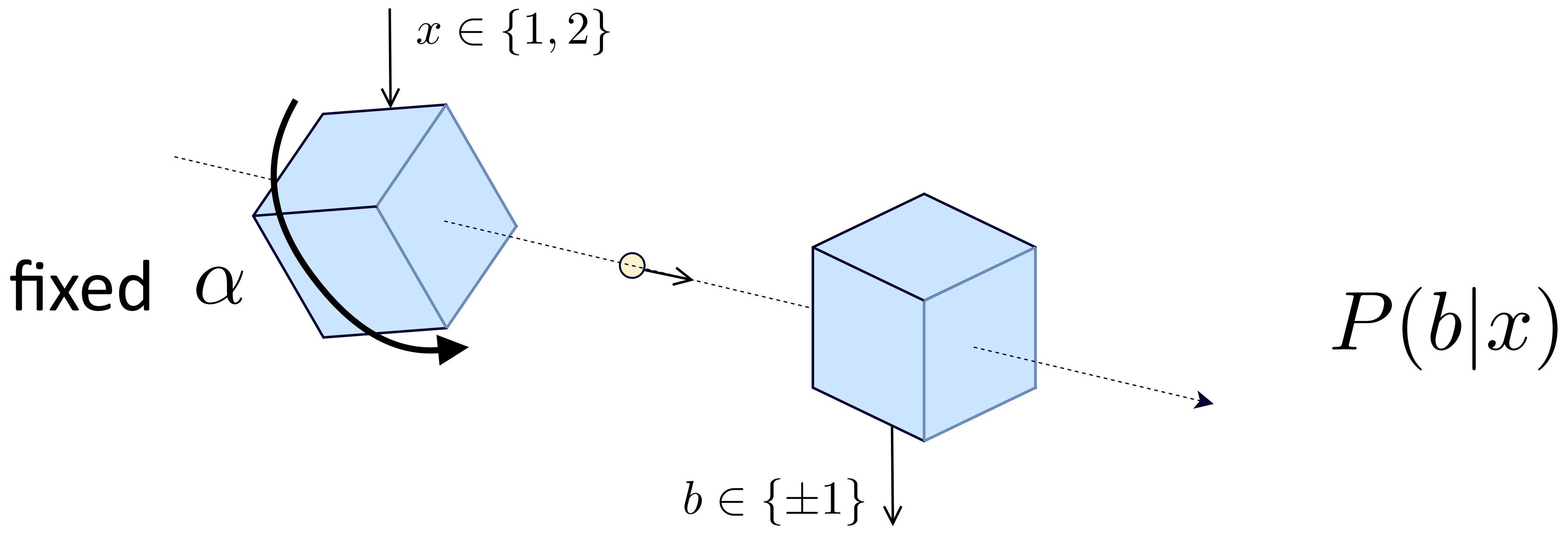

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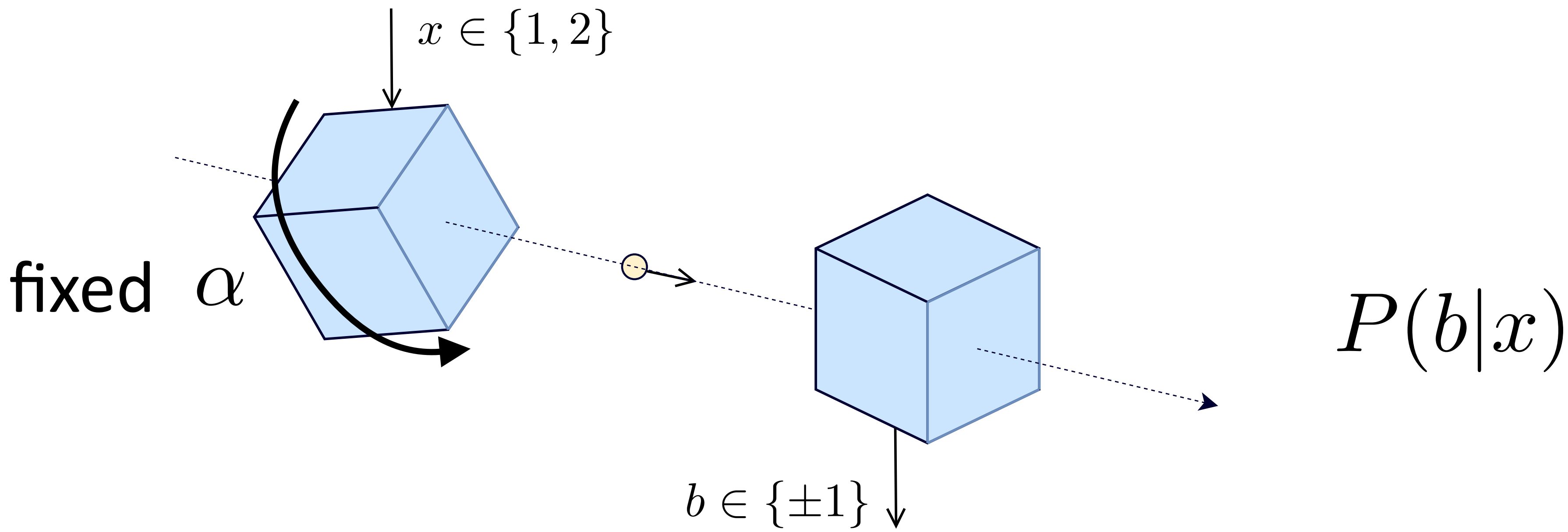

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Rotation described by (projective) unitary representation of SO(2):

$$U_\alpha = \bigoplus_{j=-J}^{J} n_j e^{ij\alpha}, \qquad P(b|\alpha) = \mathrm{tr}(M_b U_\alpha \rho U_\alpha^\dagger).$$

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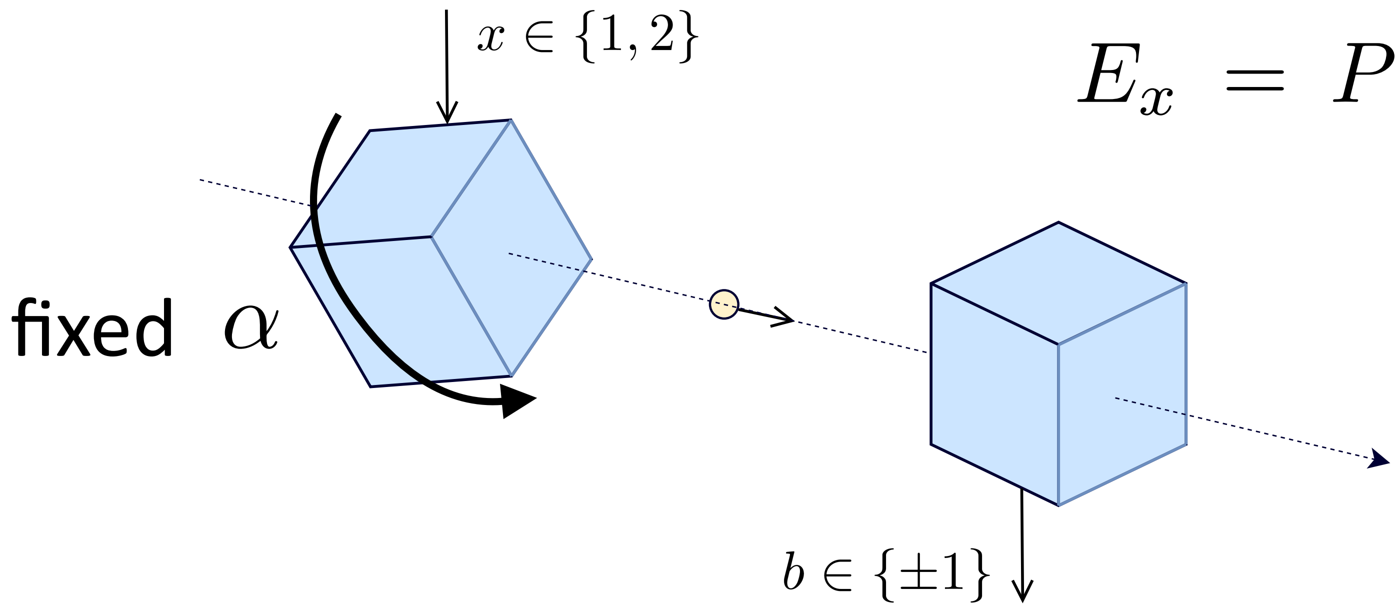

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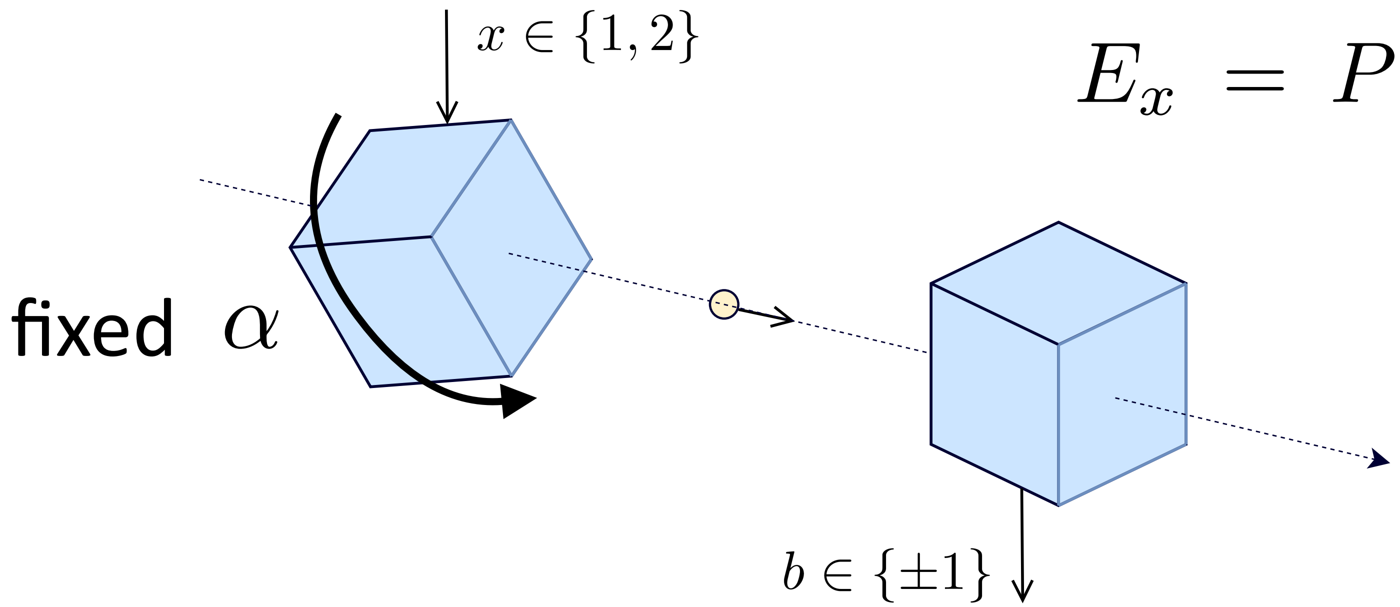

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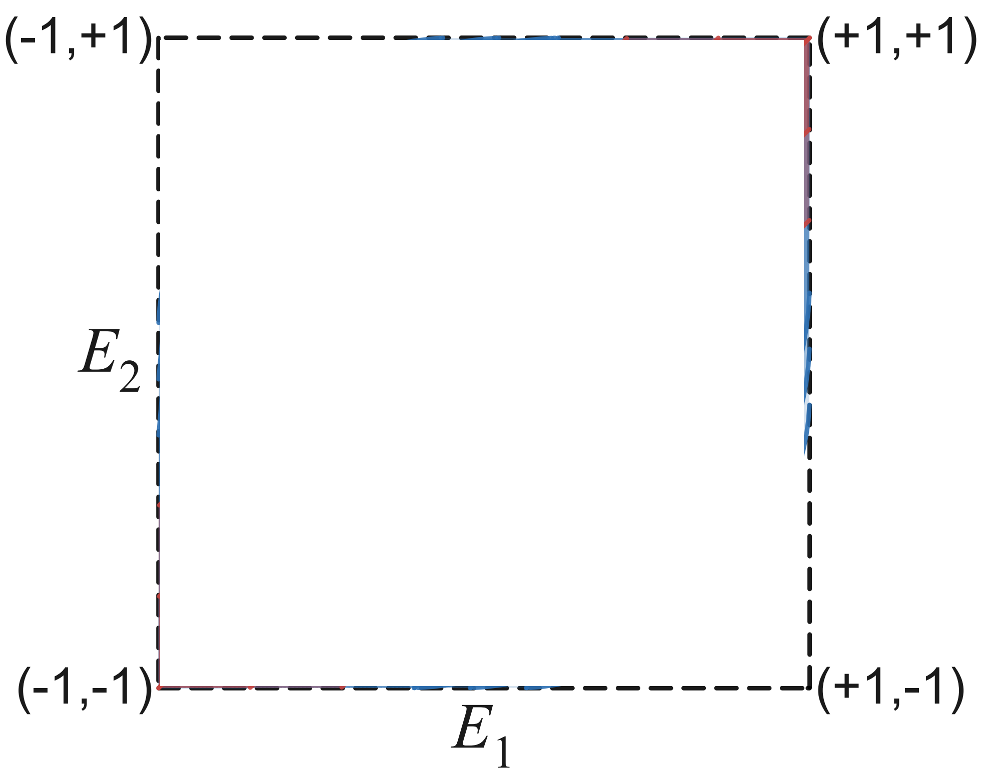

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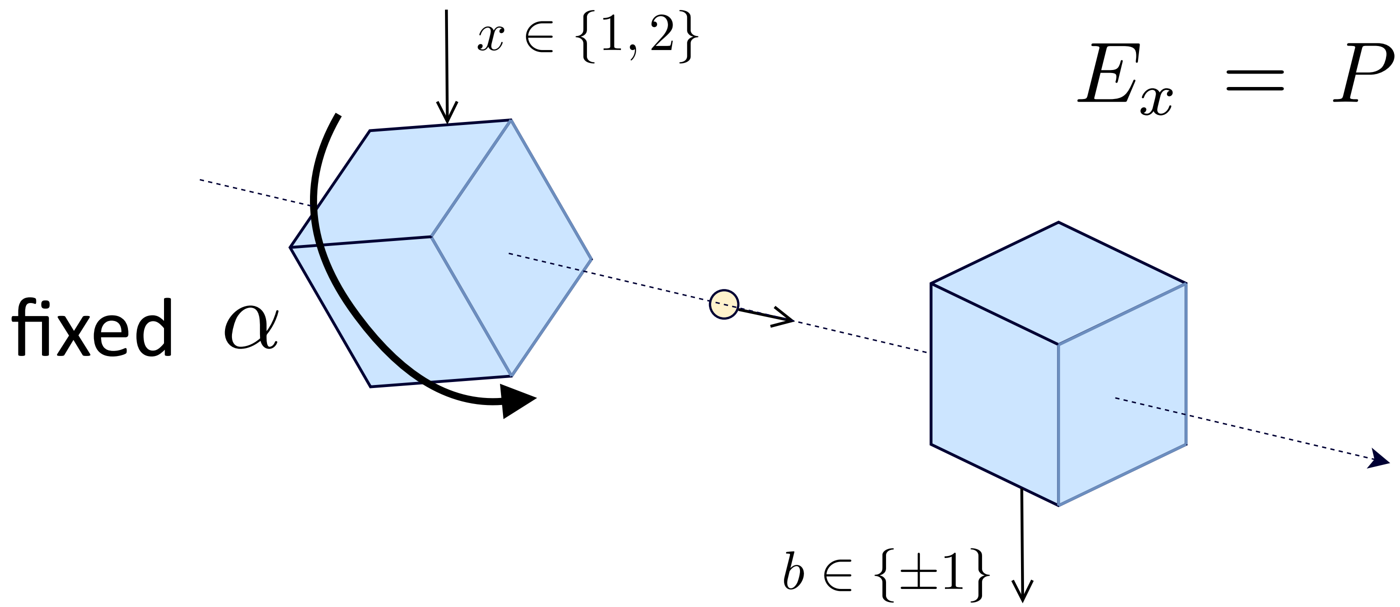

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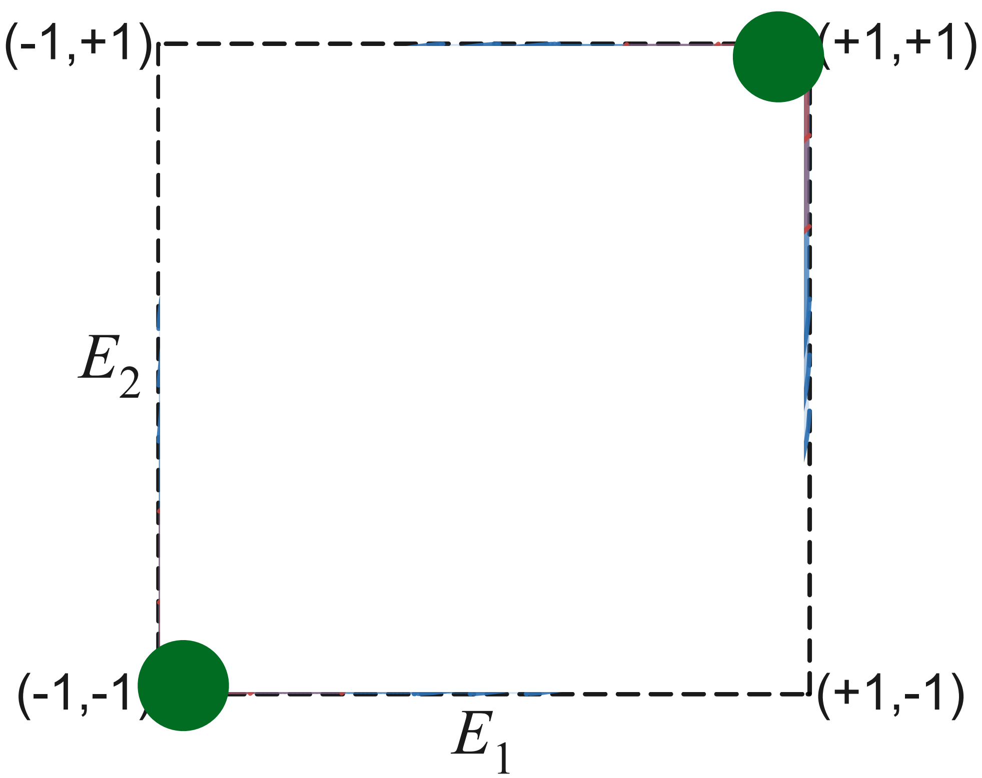

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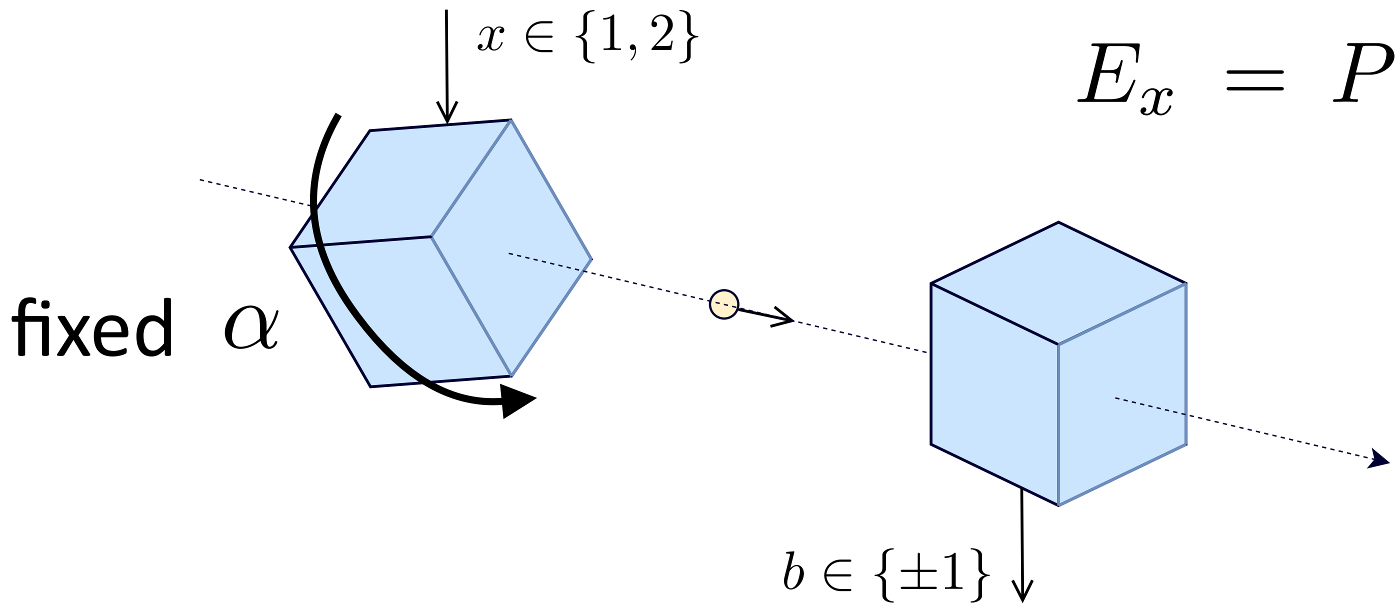

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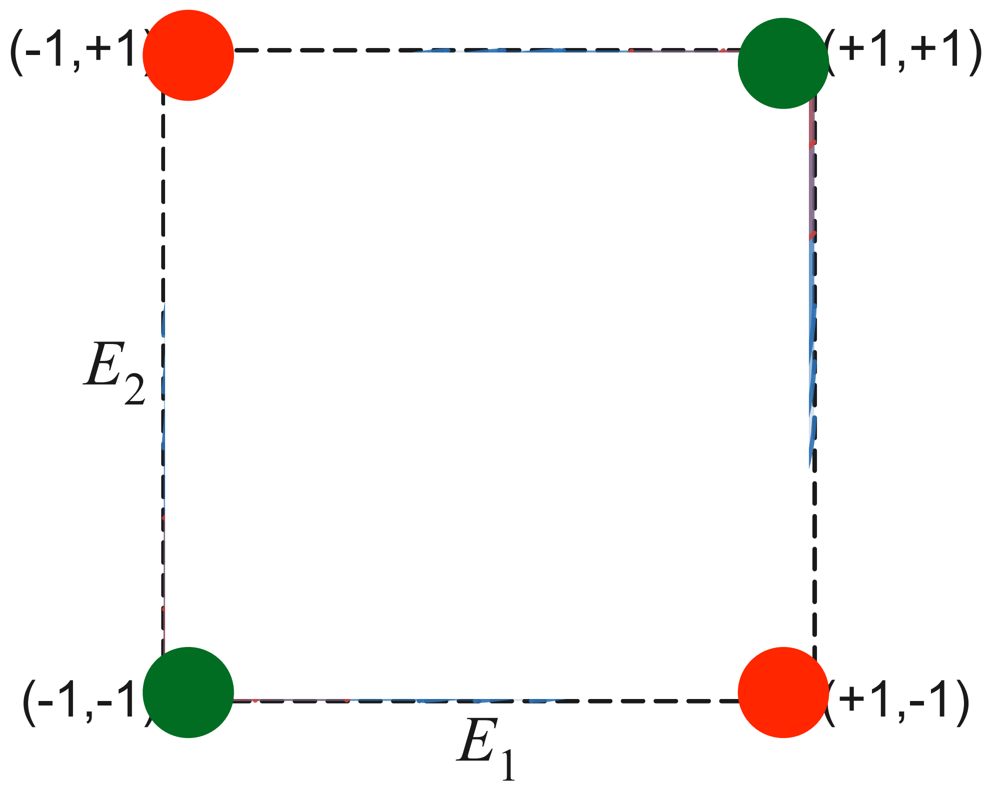

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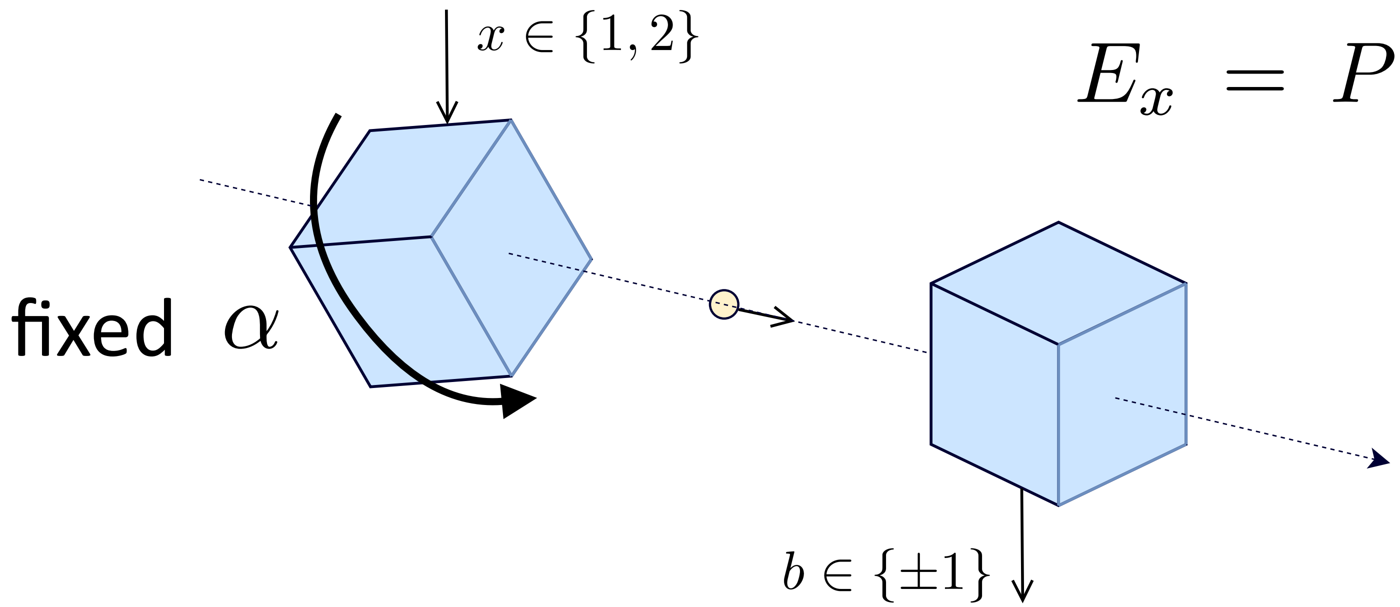

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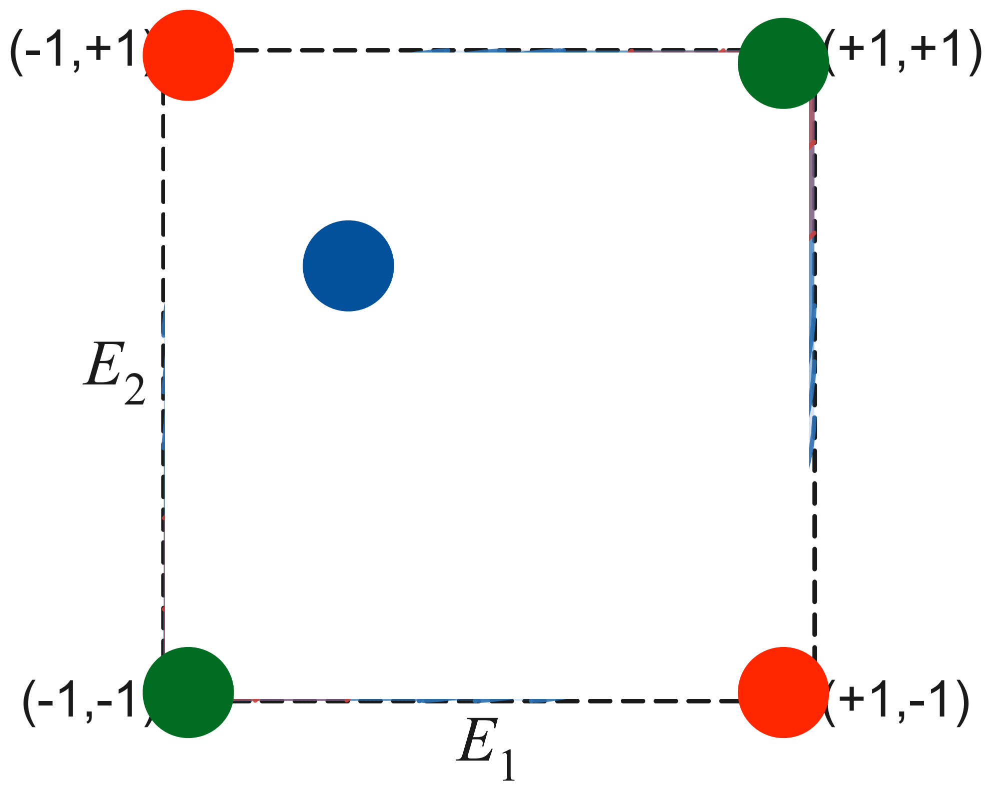

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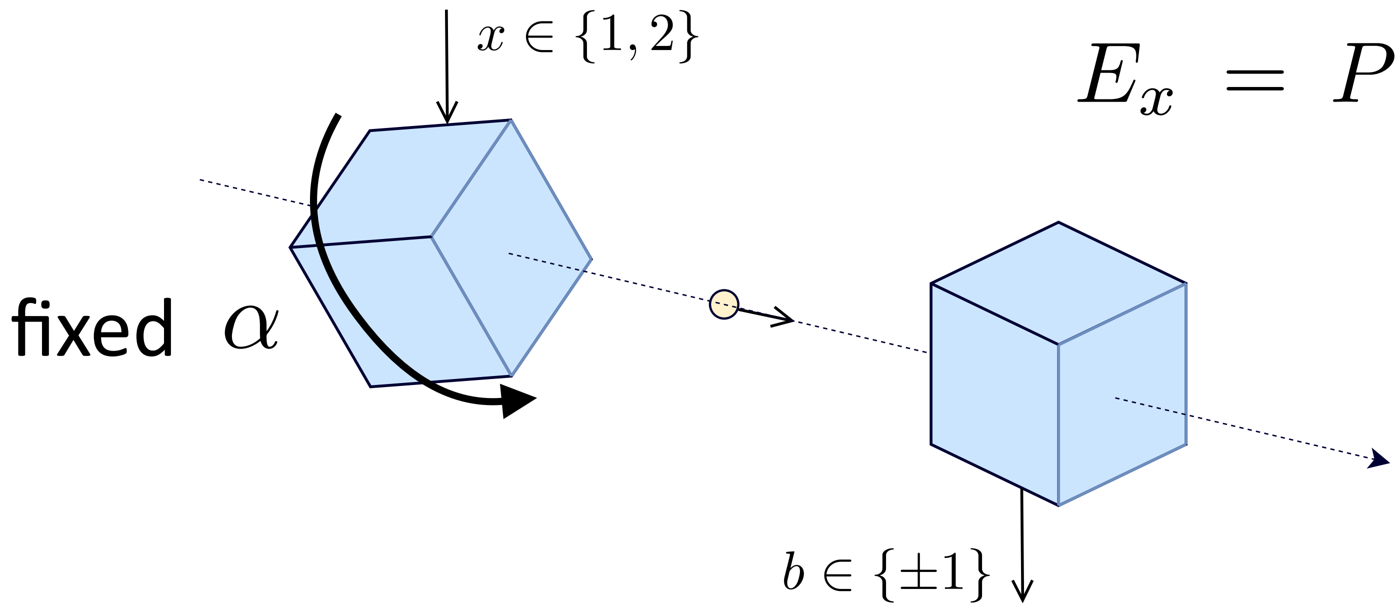
Randomness generation: quantum analysis



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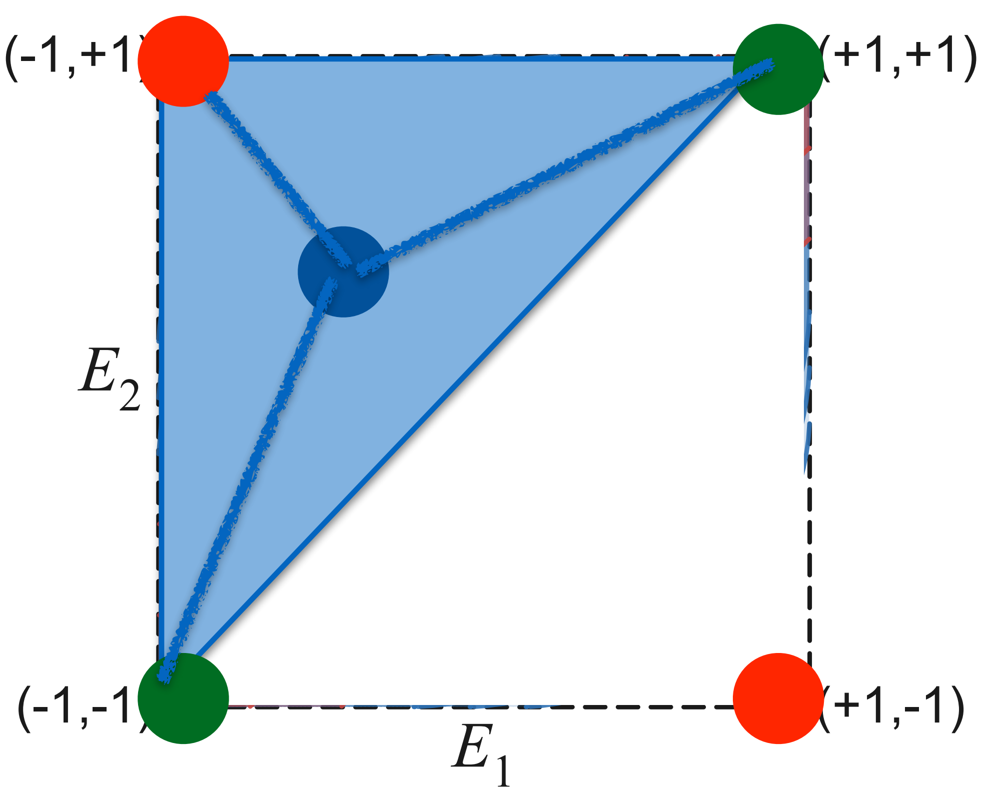

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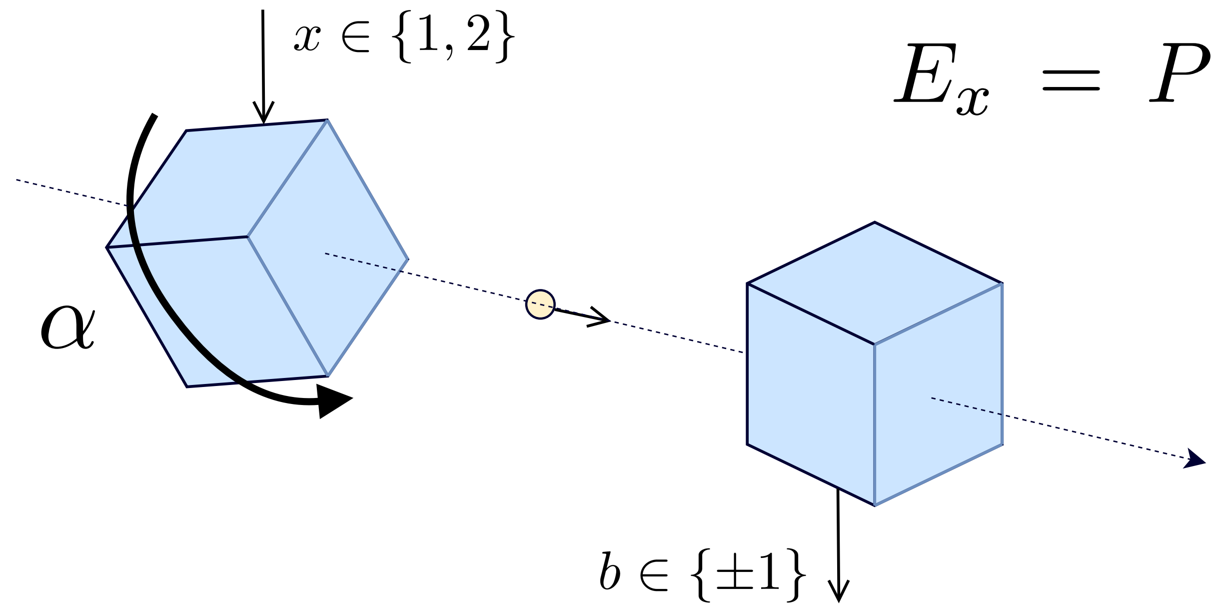

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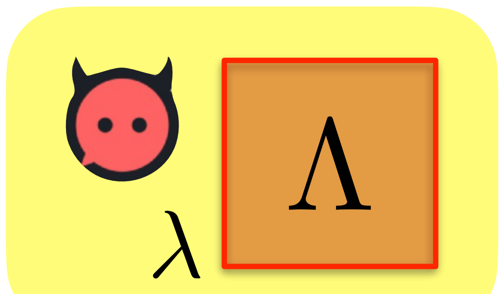

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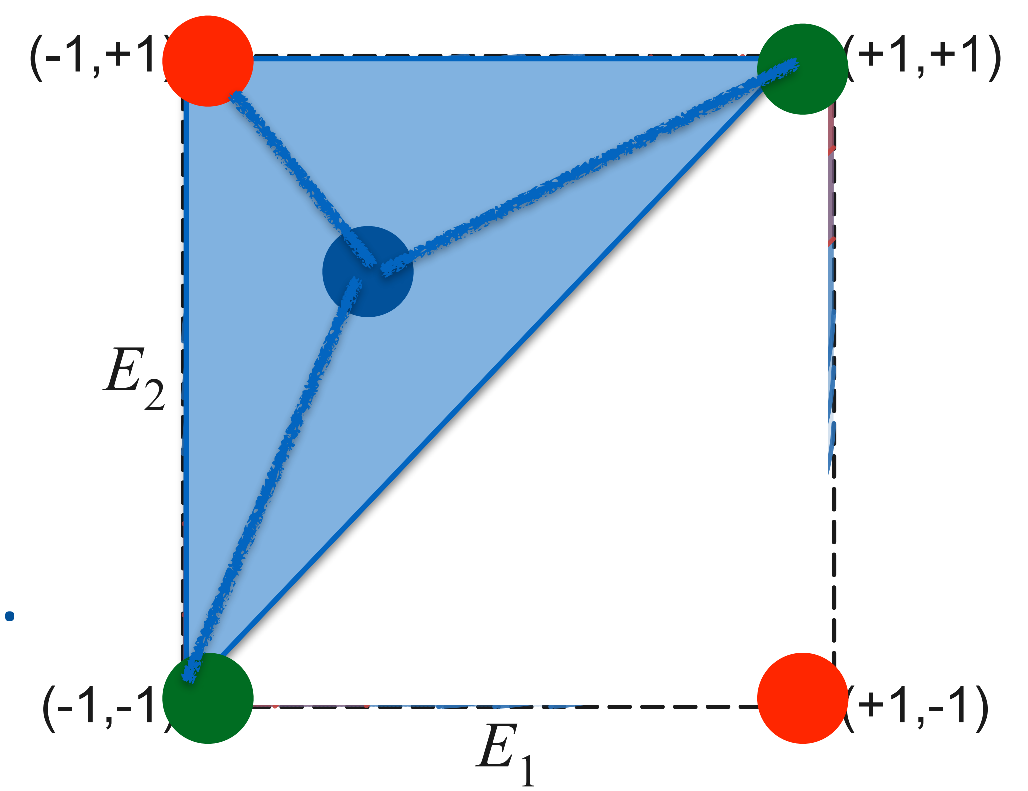

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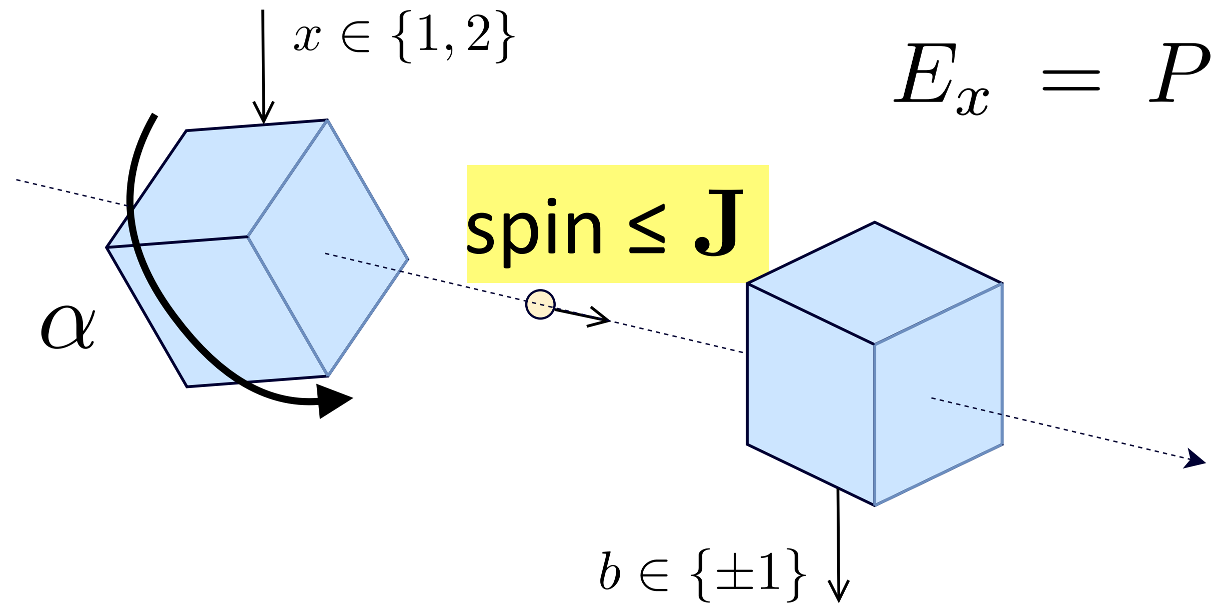

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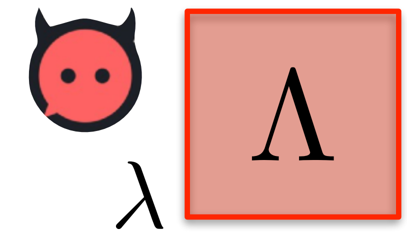
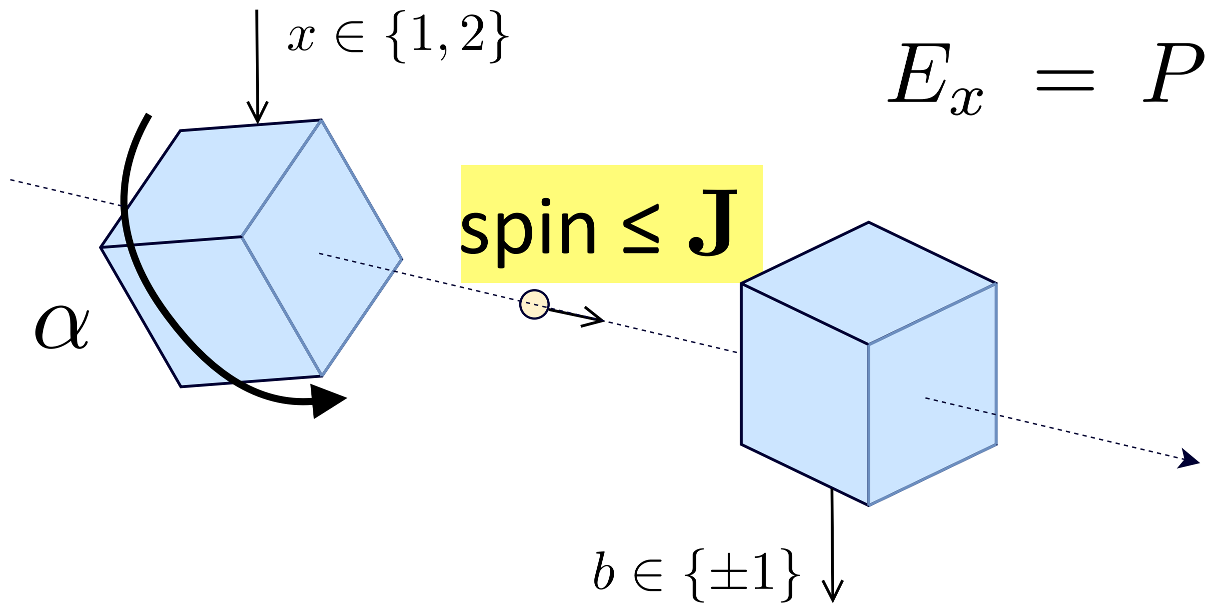


Which correlations are possible?

Randomness generation: quantum analysis



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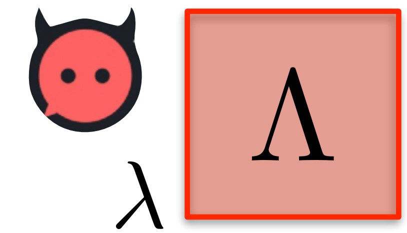


Which correlations are possible? **Theorem:** exactly those:

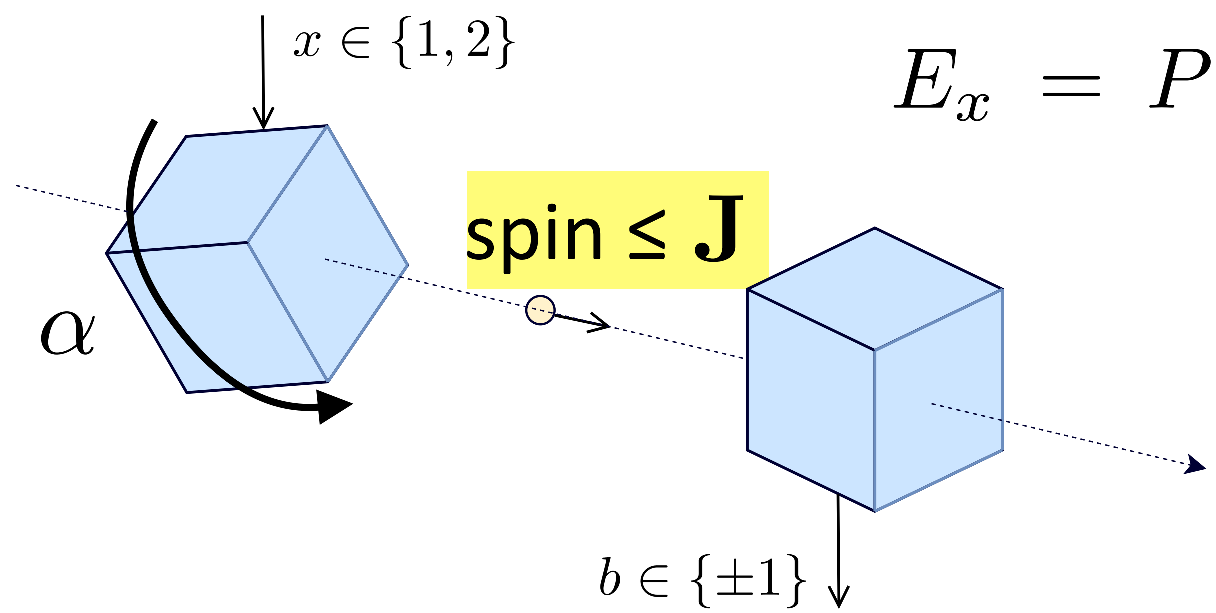
$$\frac{1}{2}\left(\sqrt{1+E_1}\sqrt{1+E_2}+\sqrt{1-E_1}\sqrt{1-E_2}\right) \geq \begin{cases} \cos(J\alpha) & \text{if } \ |J\alpha| < \frac{\pi}{2} \\ 0 & \text{if } \ |J\alpha| \geq \frac{\pi}{2} \end{cases}$$

C. L. Jones, S. L. Ludescher, A. Aloy, MM, arXiv:2210.14811

using results of

T. Van Himbeeck, E. Woodhead, N. J. Cerf, R. García-Patrón, S. Pironio, Quantum **1**, 33 (2017).

Randomness generation: quantum analysis



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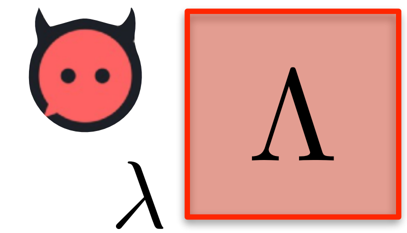

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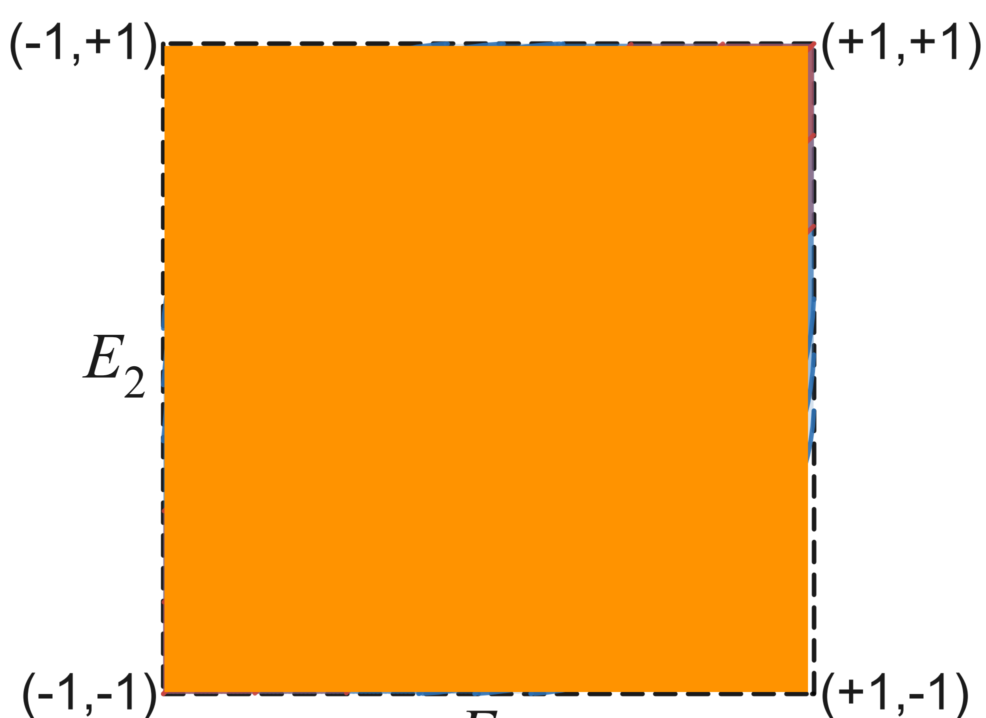
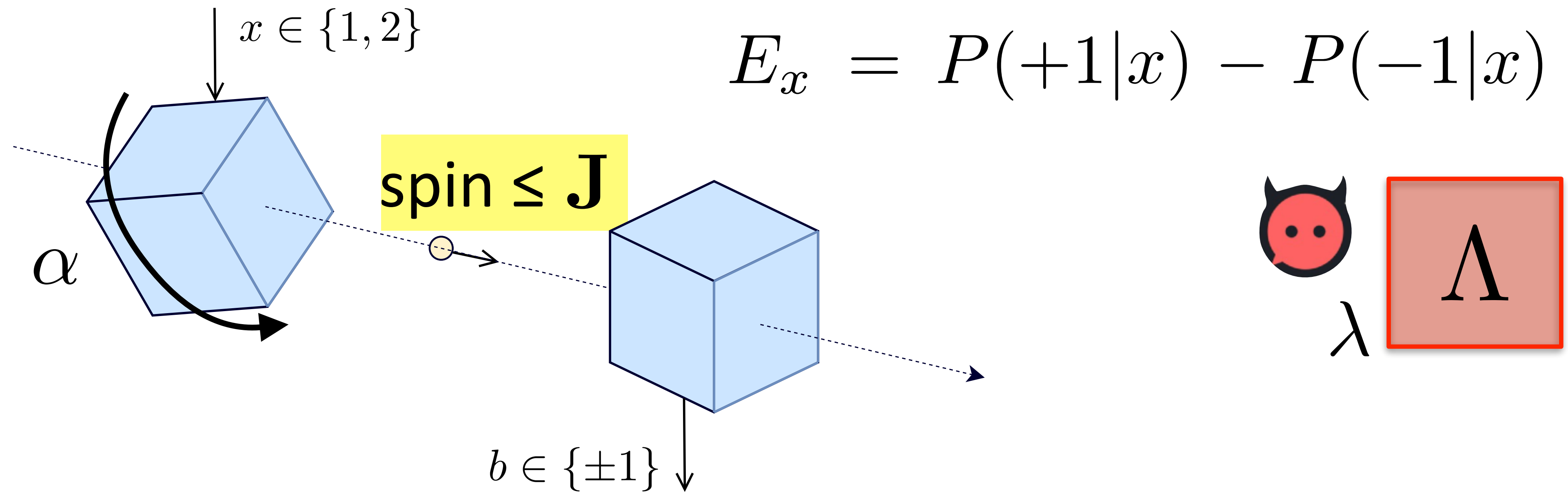


Angle $\alpha \geq \pi/(2J)$:
no certifiable randomness.

Randomness generation: quantum analysis



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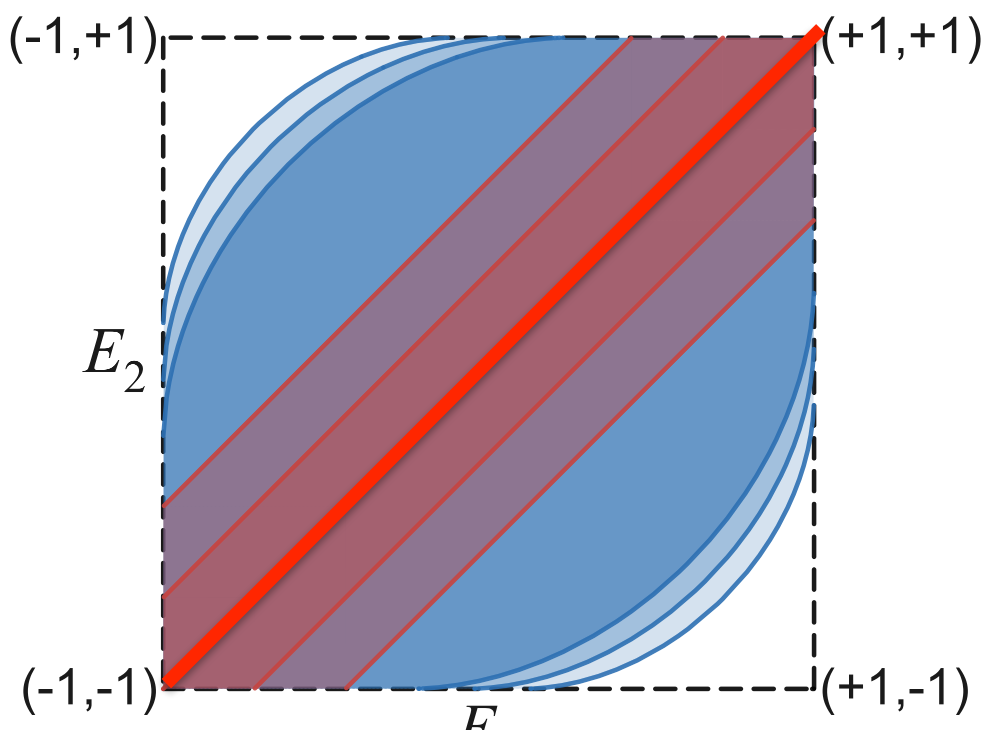

Blue curved set of correlations.

If observed correlation away from red line:

certifiable private randomness.

Quantum theory is actually not needed

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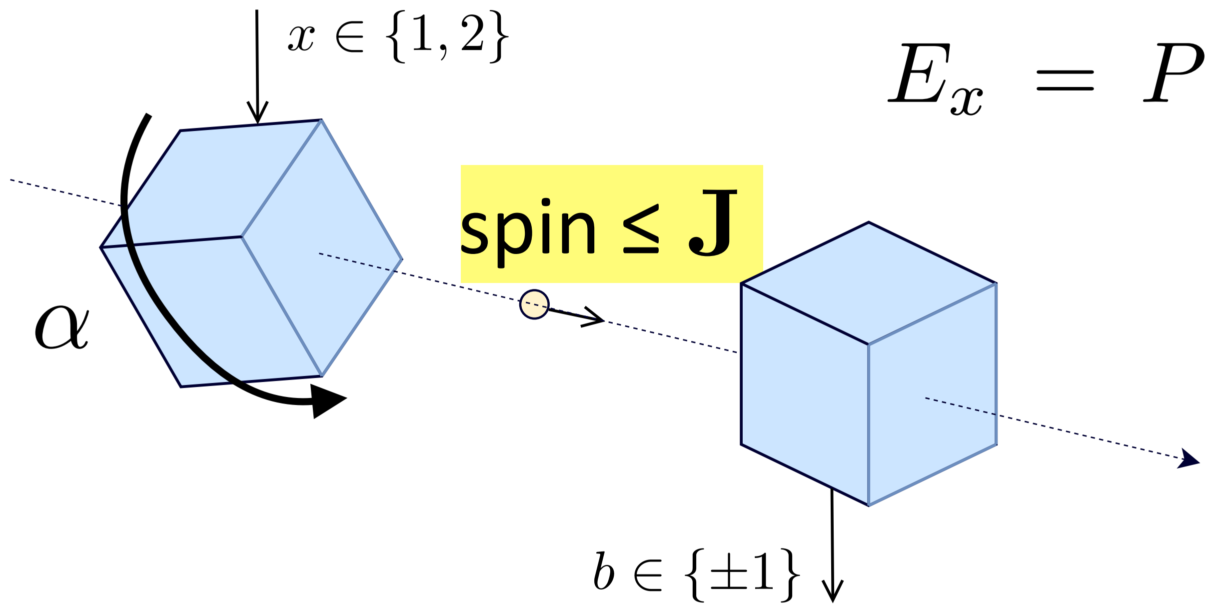

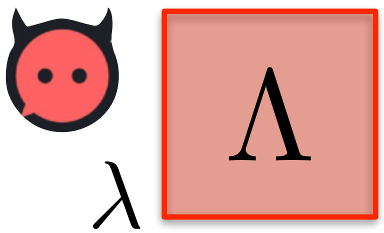

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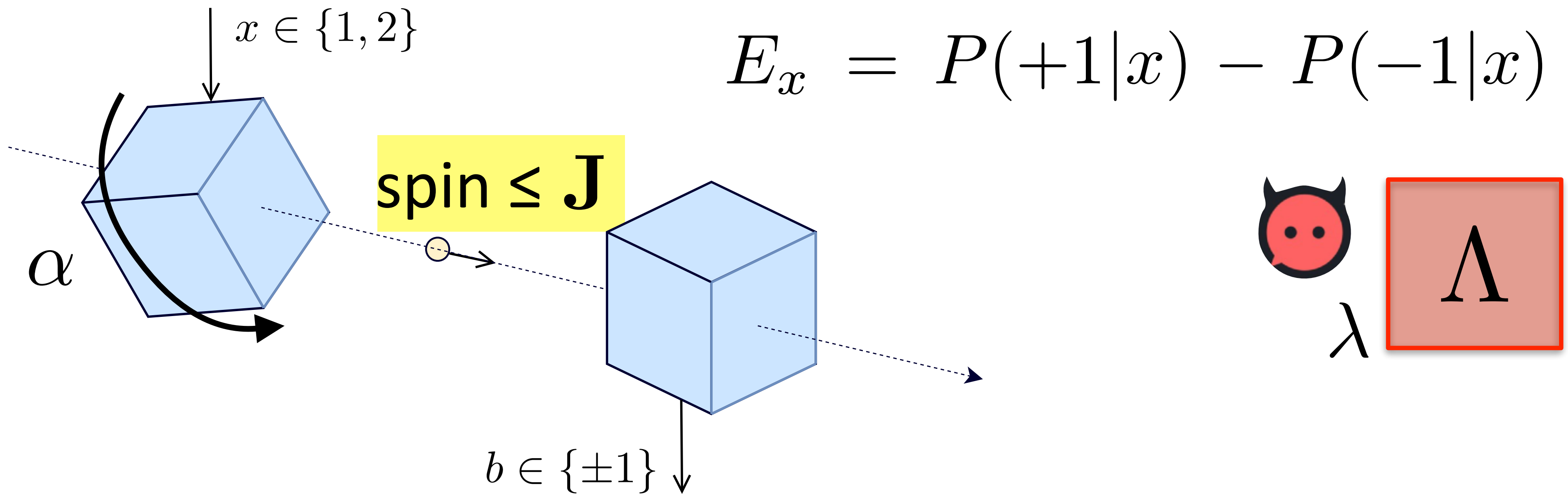

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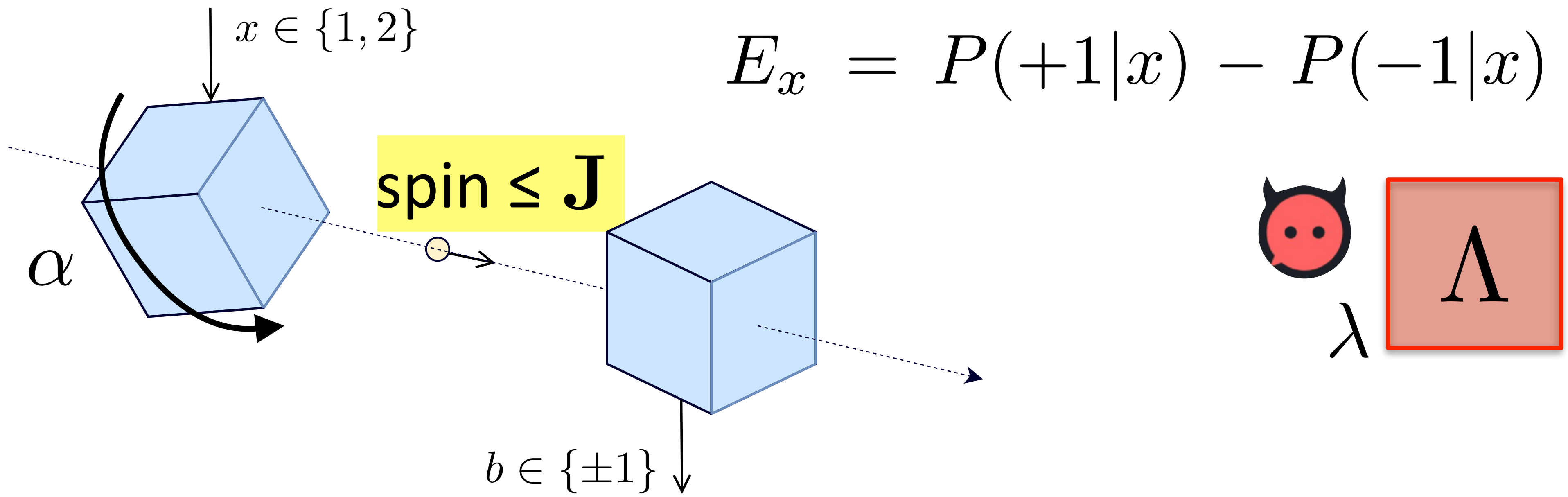

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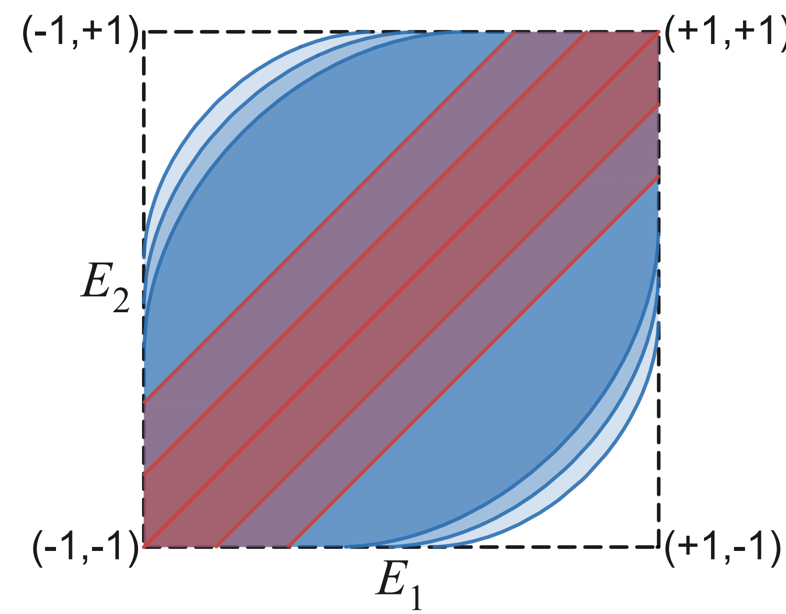

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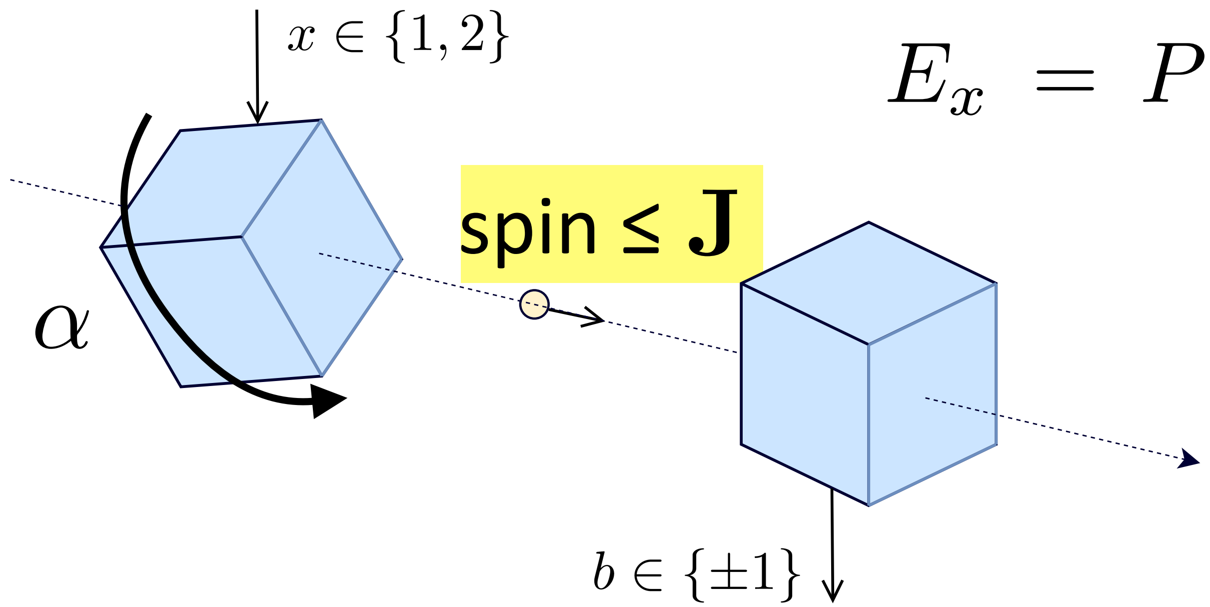

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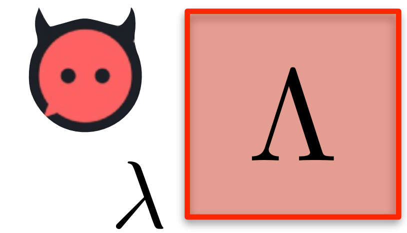

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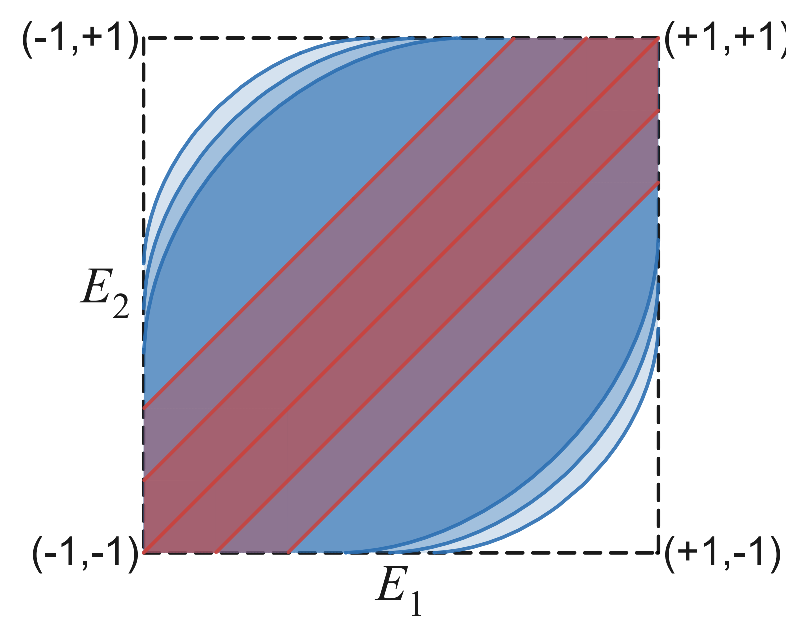
Yes we can!



Rotation boxes beyond quantum theory

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$$\mathcal{Q}_J := \left\{ \alpha \mapsto p(+1|\alpha) \mid p(b|\alpha) = \mathrm{tr}(E_b U_\alpha \rho U_\alpha^\dagger) \right\},$$

$\{E_b\}$ some POVM, ρ some density matrix,

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$0 \le p(+1|\alpha) \le 1$ for all α.

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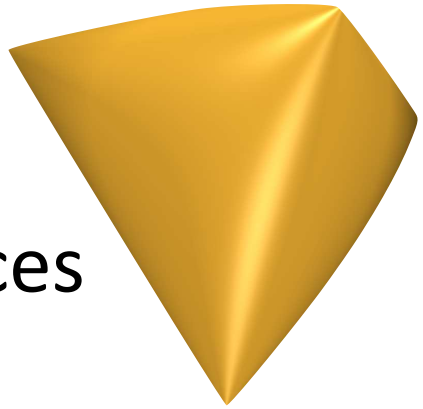

$\mathcal{R}_J$ from rep. of SO(2) on (non-quantum) “orbitope” state spaces

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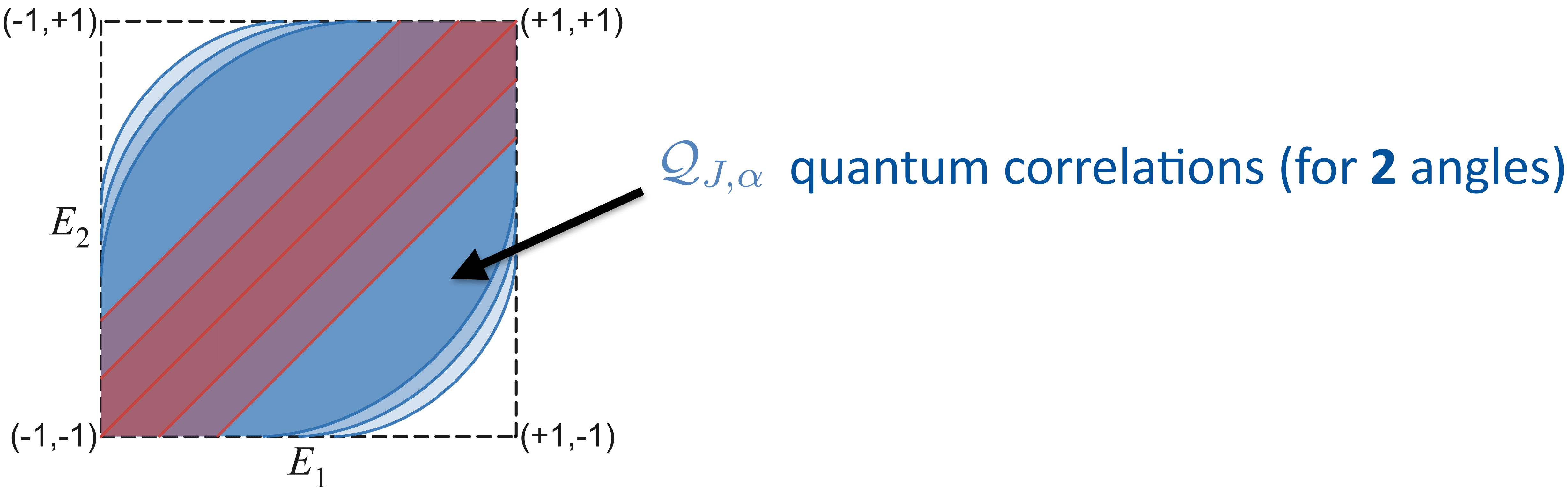

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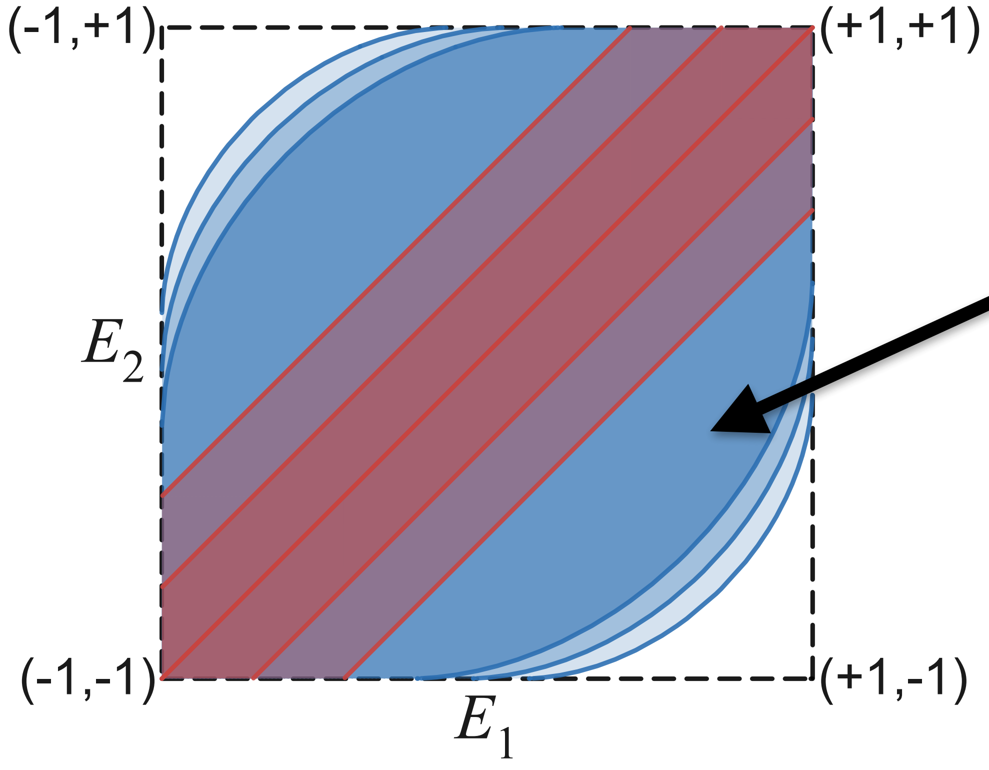

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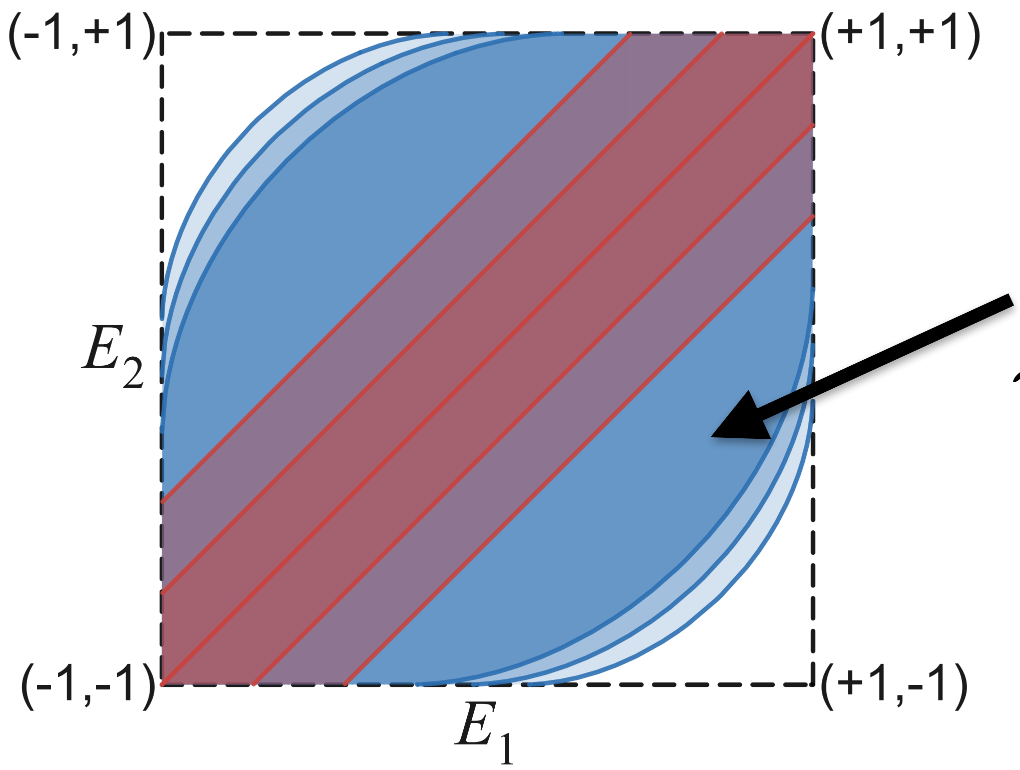

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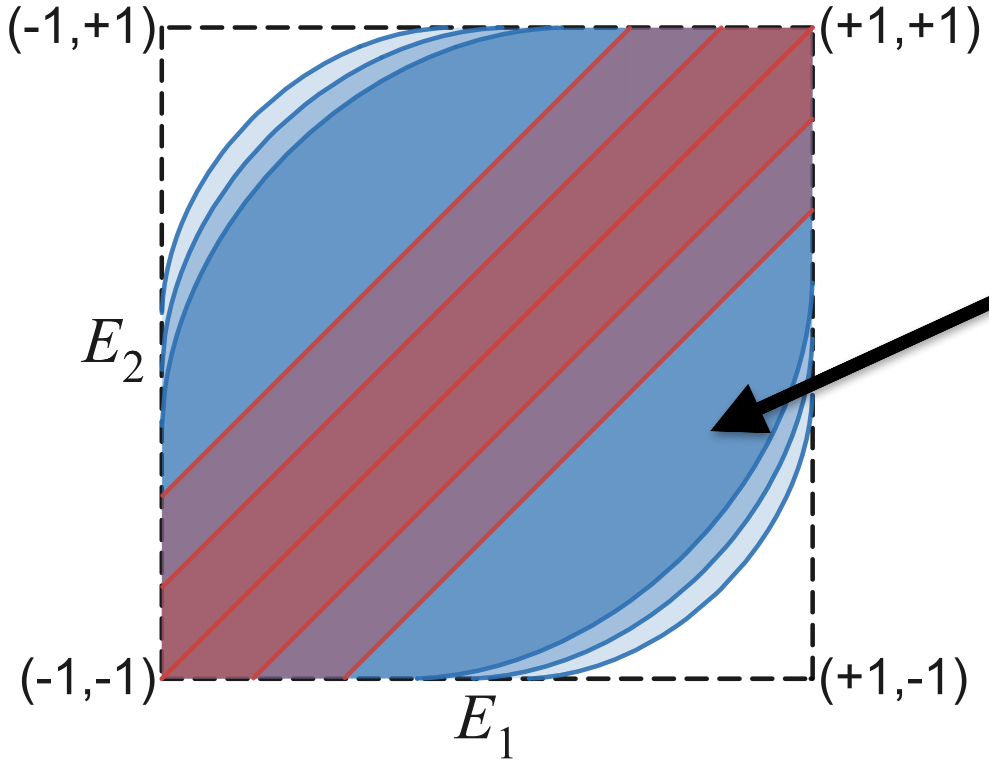

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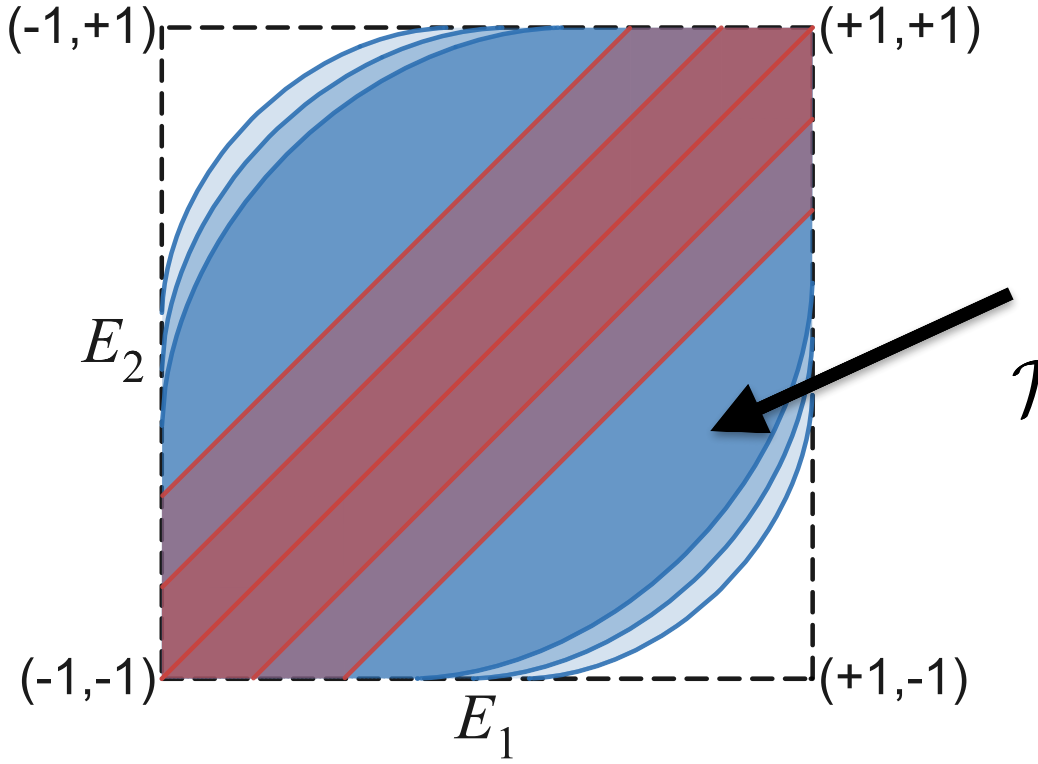

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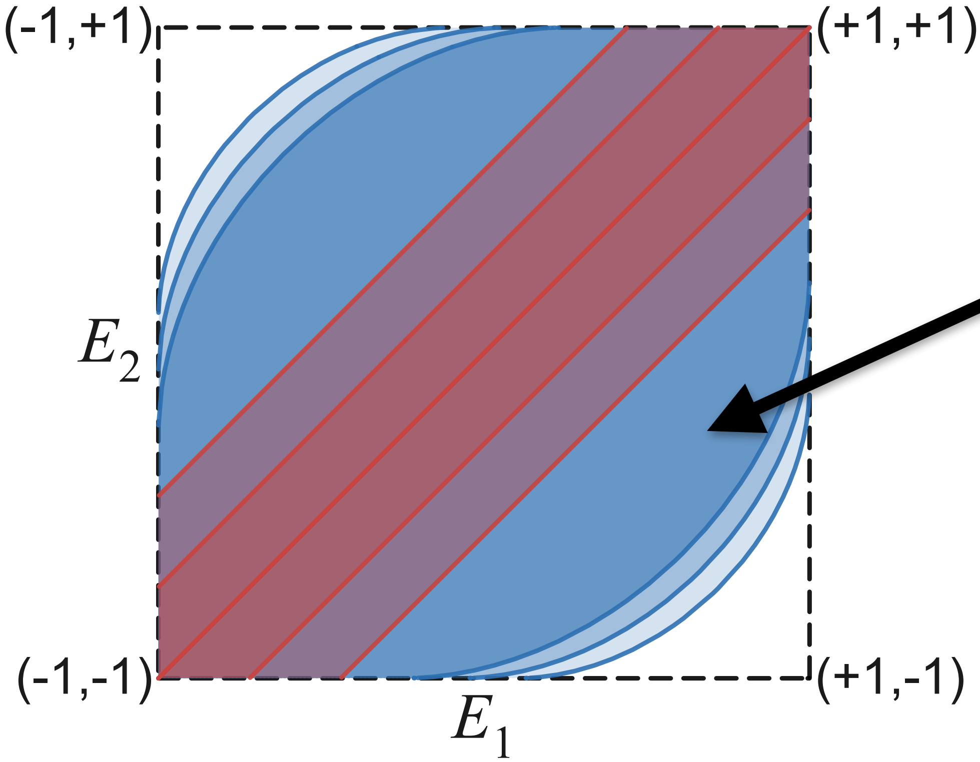

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Even eavesdropper with classical side information about beyond-quantum physics cannot predict the outcomes.

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1. Motivations: QG and device-independent QIT

2. Relativity of simultaneity and the qubit

3. Randomness generation via rotational symmetry

4. Conclusions

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- Modest approach complementing direct QG approaches: study the **constraints of spacetime on QT** in simple scenarios.
- Relativity of simultaneity constrains the dimensionality of the qubit.
- Rotational symmetry determines the set of quantum correlations and the security of a SDI randomness generation protocol.
- Goal: theory-agnostic analysis of experiments in space and time.
- Speculation: is this (weak) evidence that QT might be modified in other regimes of space and time?

qubit: arXiv:1412.7112

randomness: arXiv:2210.14811