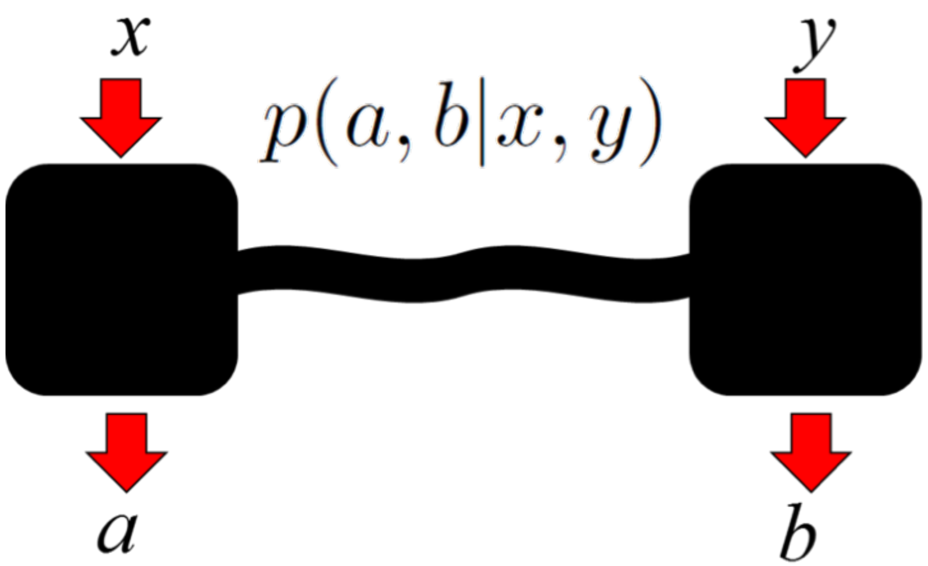
AUSTRIAN ACADEMY OF SCIENCES
IQOQI – INSTITUTE FOR QUANTUM OPTICS AND QUANTUM INFORMATION VIENNA

Black boxes in space and time:
semi-device-independent information processing via representation theory

Markus P. Müller
Institute for Quantum Optics and Quantum Information (IQOQI), Vienna
Perimeter Institute for Theoretical Physics (PI), Waterloo, Canada



PERIMETER INSTITUTE

ÖAW AUSTRIAN ACADEMY OF SCIENCES

IQOQI – INSTITUTE FOR QUANTUM OPTICS AND QUANTUM INFORMATION VIENNA

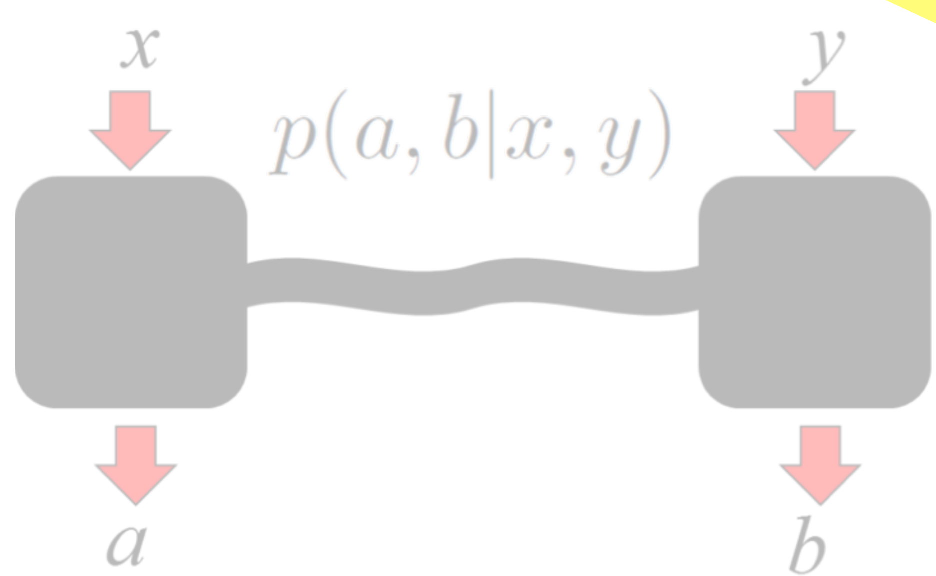
…ack boxes in space and time:

…dependent information processing
…ntation theory

Work in progress!

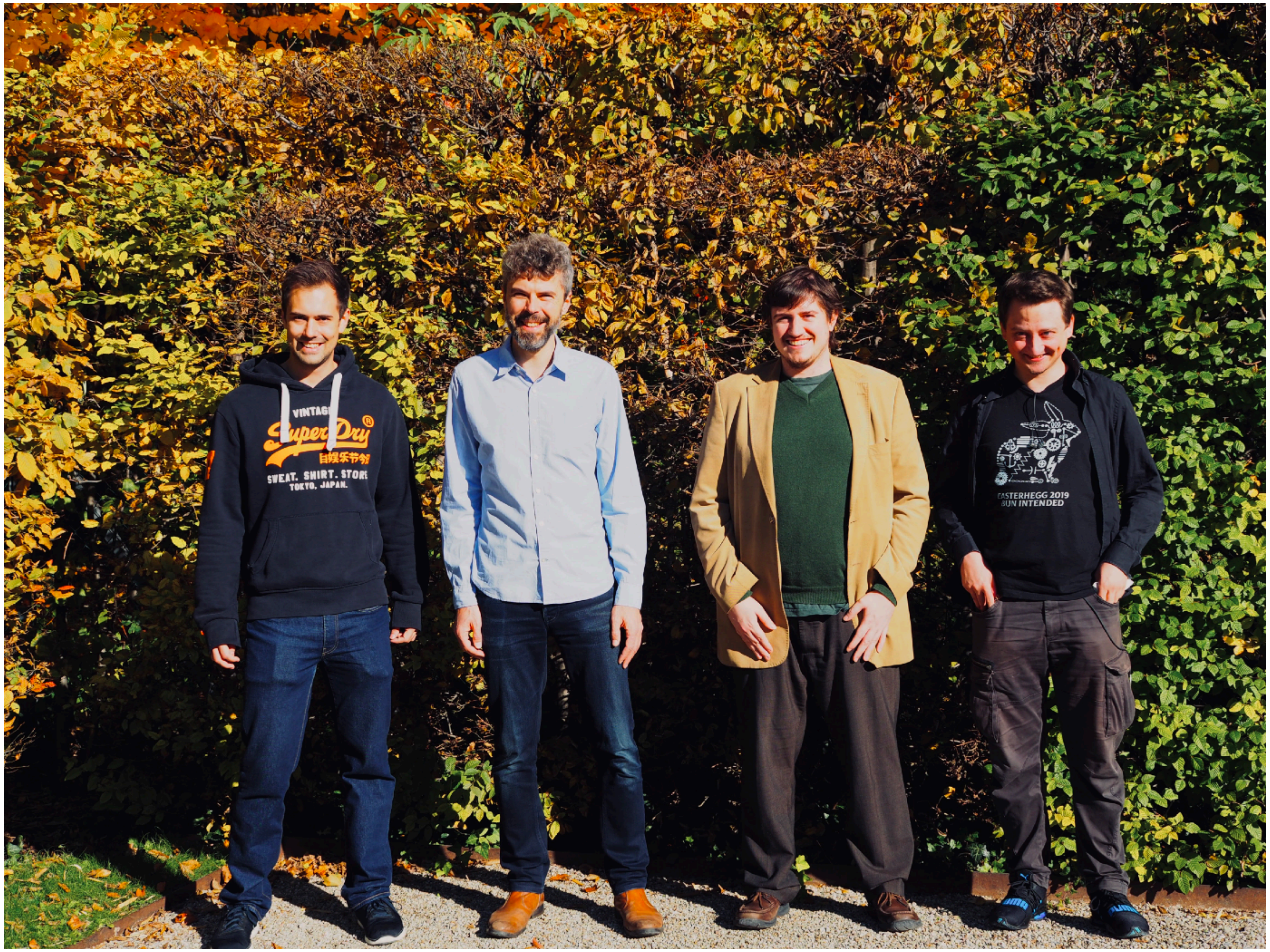
3-year FWF project just started NOW.

M…
Institute for Quantum Optics and Qu… …QOI), Vienna
Perimeter Institute for Theoretical Phys…



PERIMETER INSTITUTE

Müller group at IQOQI



left to right:

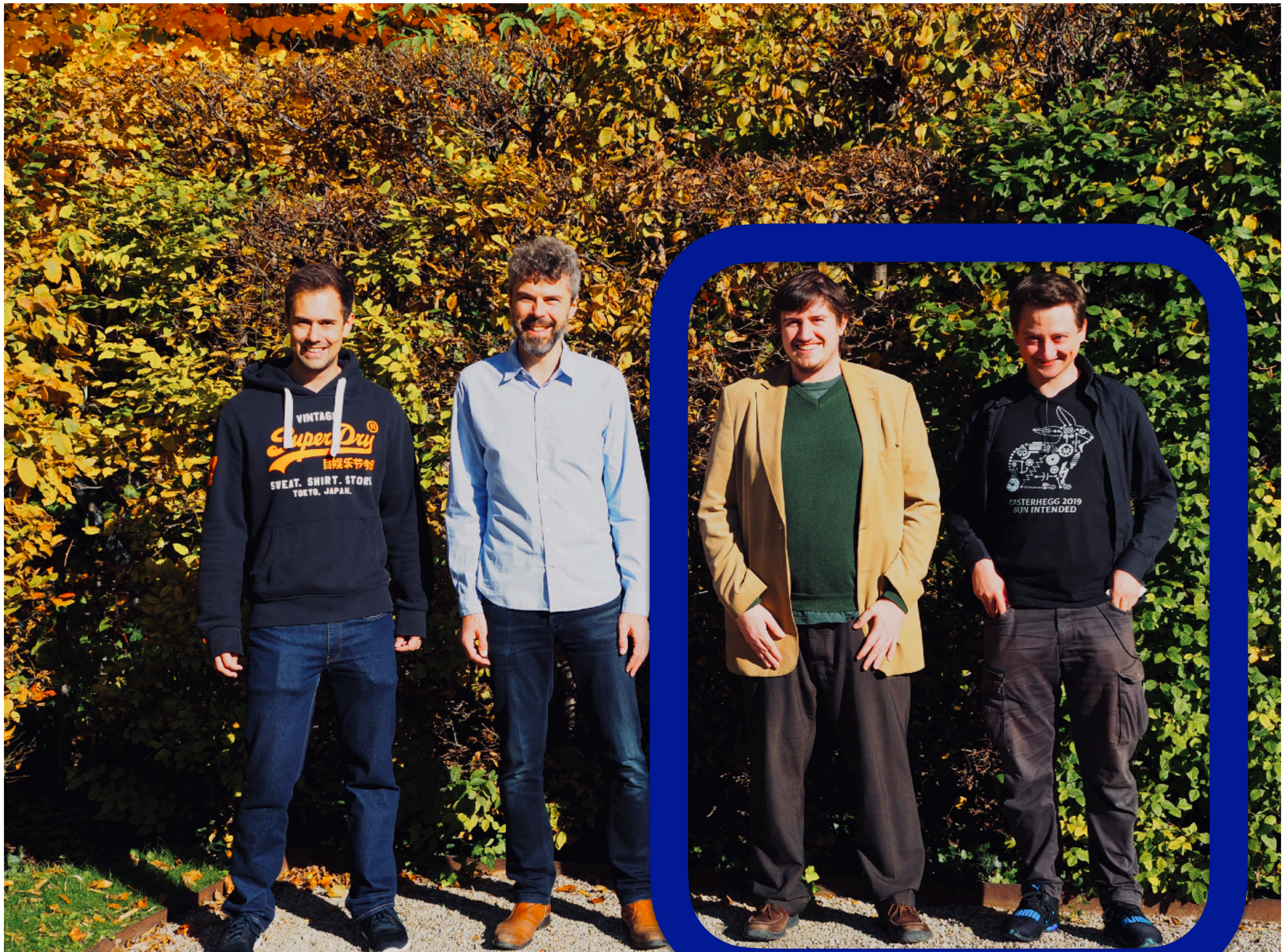
Stefan Ludescher (PhD student), **Markus Müller** (group leader), **Andy Garner** (postdoc), **Marius Krumm** (PhD student).

coming soon:

Caroline Jones
(PhD student)

Albert Aloy (postdoc)

Müller group at IQOQI



left to right:

Stefan Ludescher (PhD student), **Markus Müller** (group leader),
Andy Garner (postdoc), **Marius Krumm** (PhD student)

coming soon:

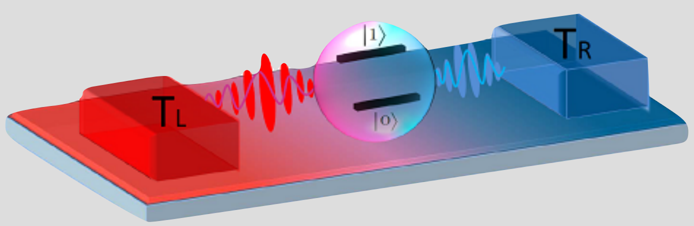
Caroline Jones
(PhD student)

Albert Aloy (postdoc)

Müller group at IQOQI

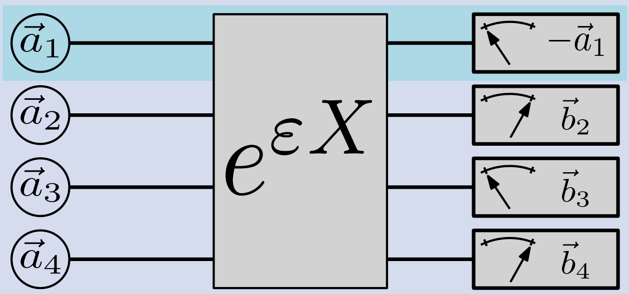
Spacetime and Relational **Quantum Information**

Resource-theoretic approach to **Quantum Thermodynamics**



Mathematical Q.I.T.

Quantum Foundations and reconstructions of QM



Foundations of Physics

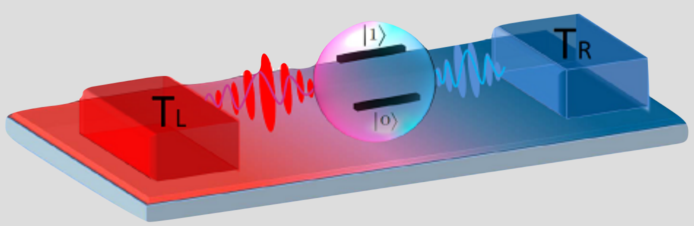
Müller group at IQOQI

Spacetime and Relational **Quantum Information**

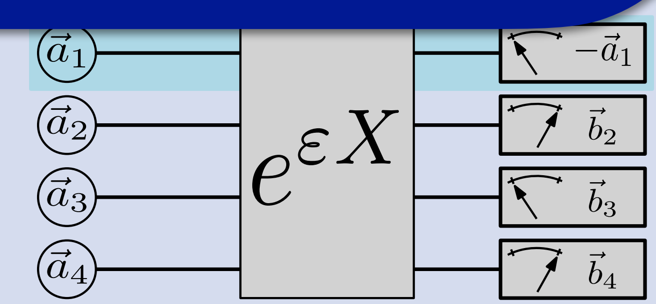
Mathematical Q.I.T.

Foundations of Physics

Resource-theoretic approach to **Quantum Thermodynamics**



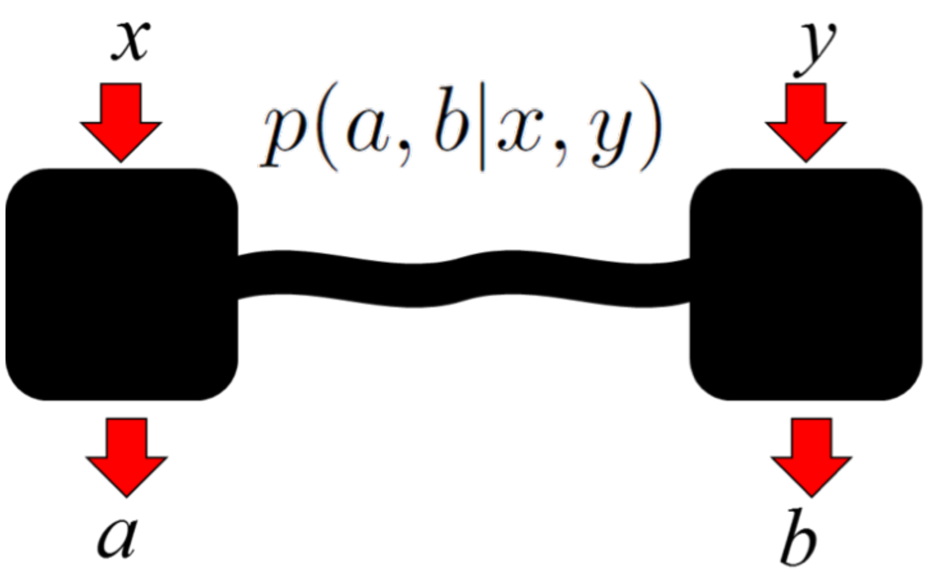
Quantum Foundations and reconstructions of QM



Overview

1. General framework of “spacetime boxes”

2. Foundational insights



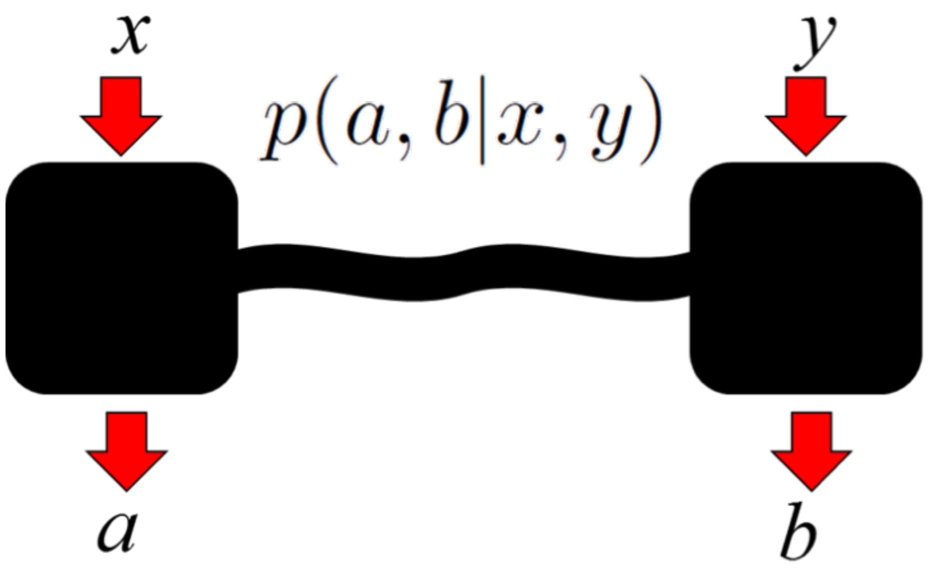
3. Towards novel protocols…

4. … and experimental tests of QT

Overview

1. General framework of “spacetime boxes”

2. Foundational insights

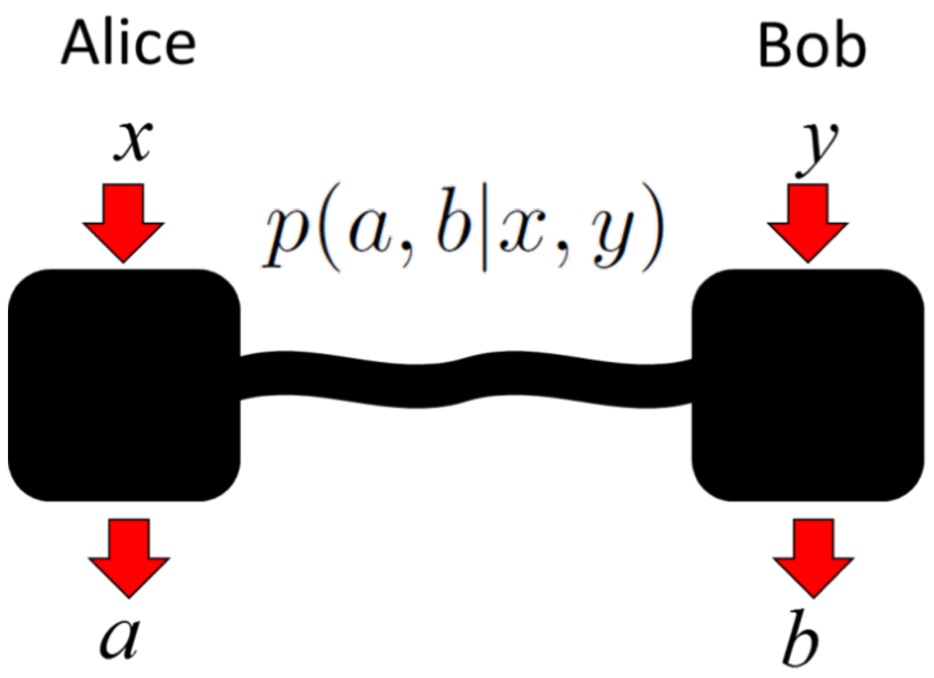


3. Towards novel protocols…

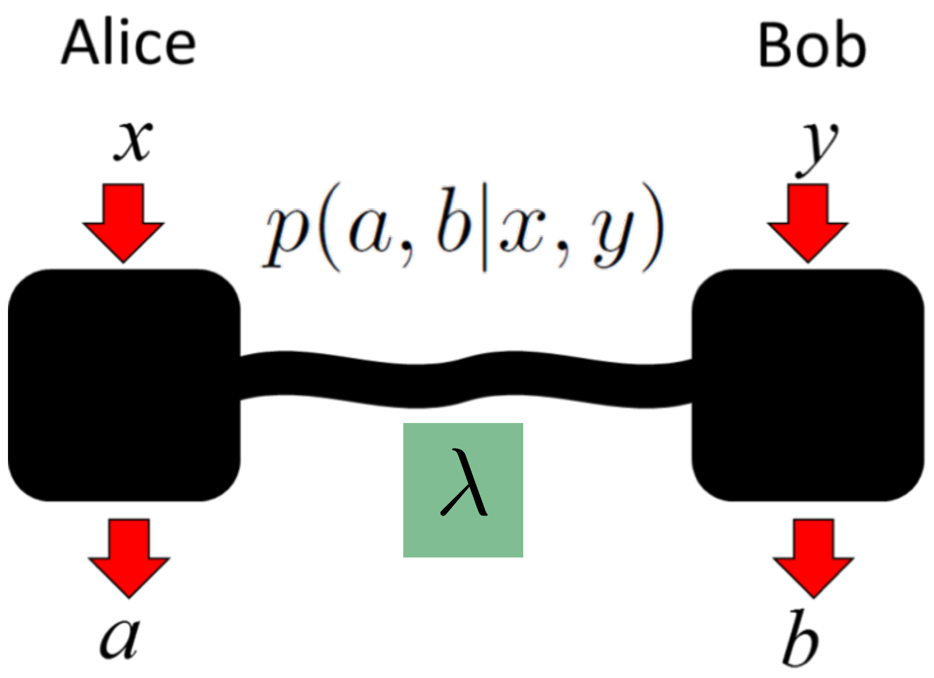
4. … and experimental tests of QT

Black boxes and correlations

Black boxes and correlations



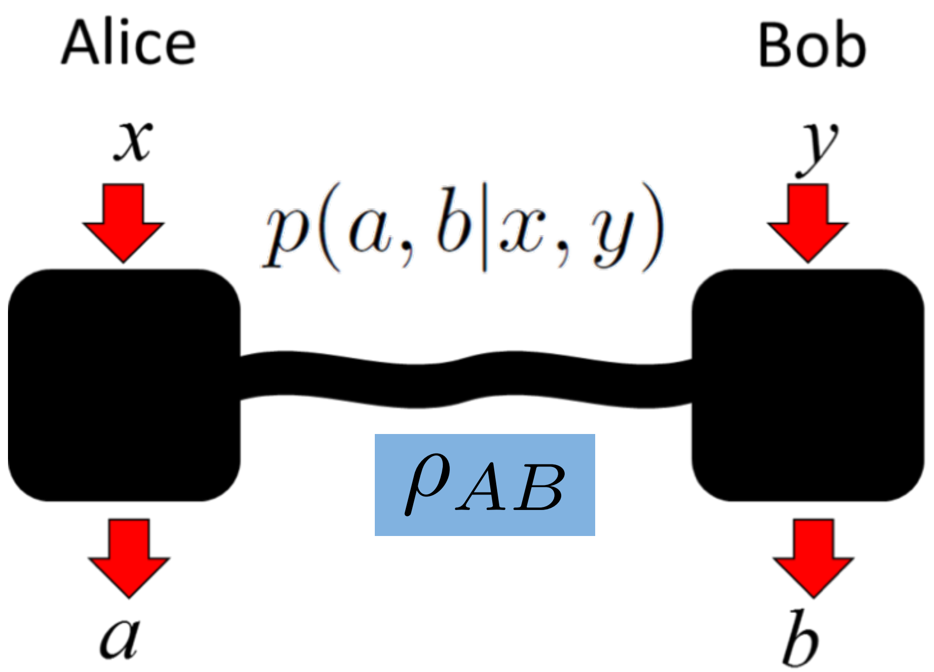
Black boxes and correlations



- In **classical** physics / prob. theory:

$$P(a,b|x,y) = \sum_{\lambda\in\Lambda} P_A(a|x,\lambda) P_B(b|y,\lambda) P_\Lambda(\lambda)$$

Black boxes and correlations



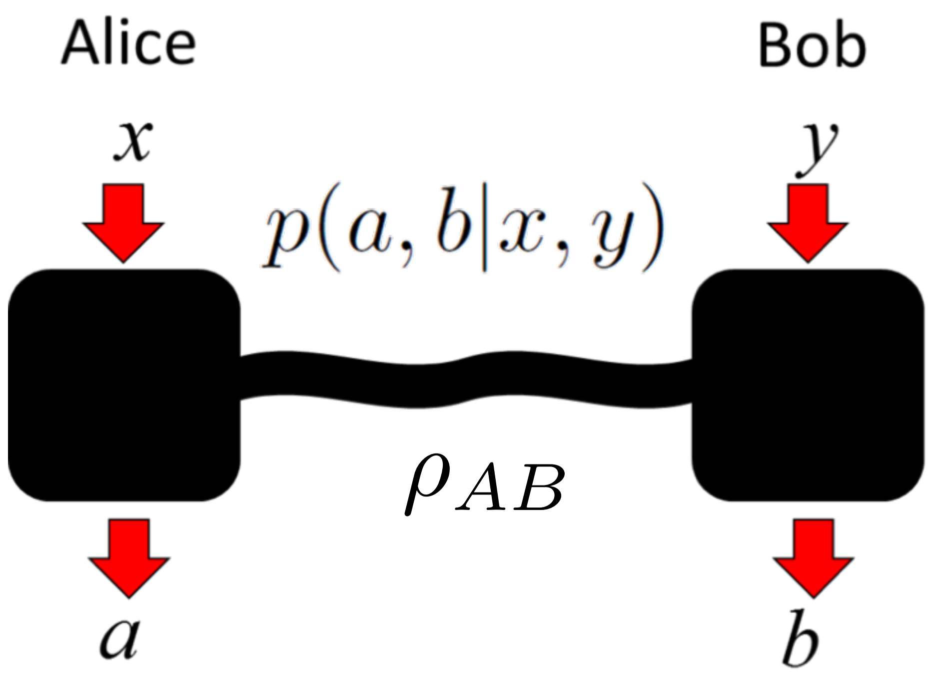
- In **classical** physics / prob. theory:

$$P(a,b|x,y) = \sum_{\lambda\in\Lambda} P_A(a|x,\lambda)P_B(b|y,\lambda)P_\Lambda(\lambda)$$

- In **quantum** physics:

$$P(a,b|x,y) = \mathrm{tr}\left[\rho_{AB}(E_x^a \otimes F_y^b)\right]$$

Black boxes and correlations



No-signalling conditions:

$P(a|x,y)$ is independent of y,

$P(b|x,y)$ is independent of x.

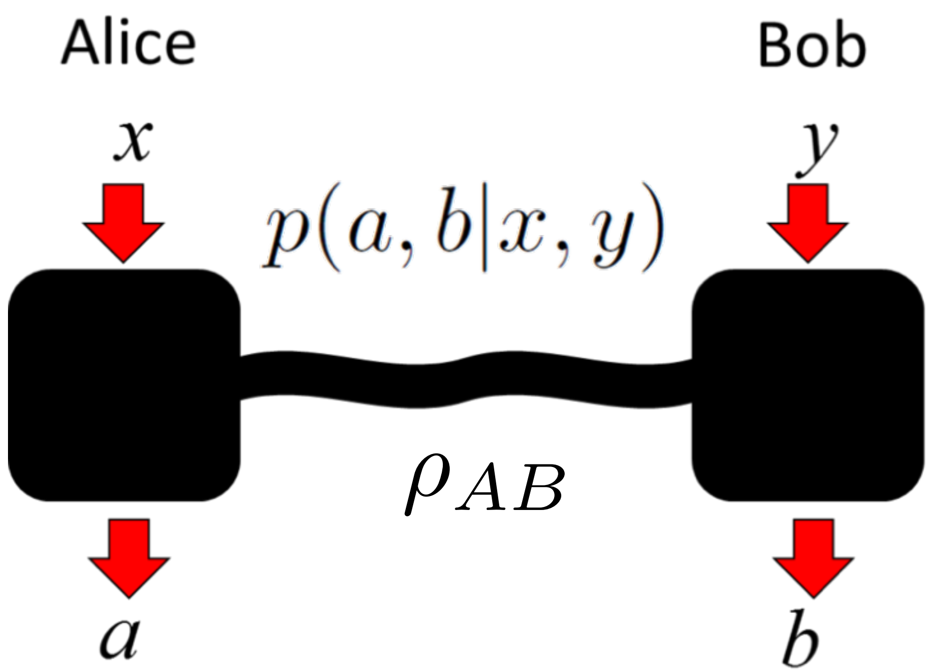
- In **classical** physics / prob. theory:

$$P(a,b|x,y) = \sum_{\lambda\in\Lambda} P_A(a|x,\lambda)P_B(b|y,\lambda)P_\Lambda(\lambda)$$

- In **quantum** physics:

$$P(a,b|x,y) = \mathrm{tr}\left[\rho_{AB}(E_x^a \otimes F_y^b)\right]$$

Black boxes and correlations



No-signalling conditions:

$P(a|x, y)$ is independent of y,

$P(b|x, y)$ is independent of x.

- In **classical** physics / prob. theory:

$$P(a, b|x, y) = \sum_{\lambda \in \Lambda} P_A(a|x, \lambda) P_B(b|y, \lambda) P_\Lambda(\lambda)$$

- In **quantum** physics:

$$P(a, b|x, y) = \mathrm{tr}\left[\rho_{AB}(E_x^a \otimes F_y^b)\right]$$

Quantum admits more general *P*'s due to the **violation of Bell inequalities**.

The Bell-CHSH inequality

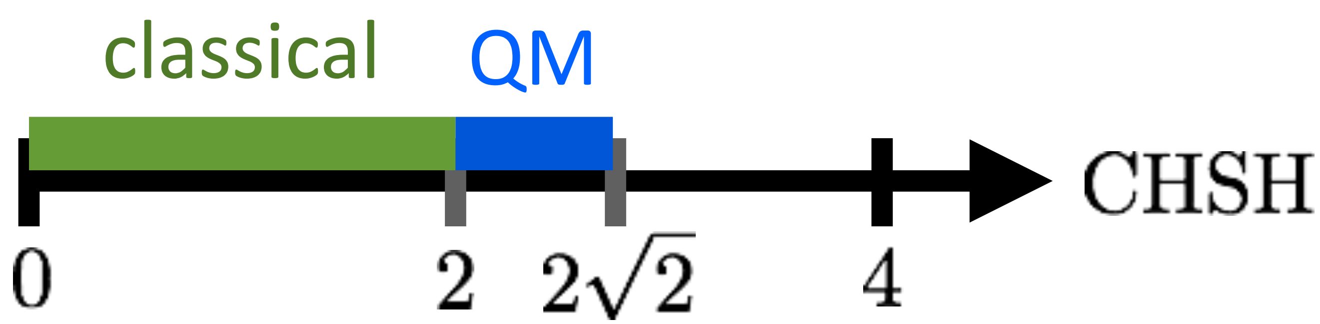
Classical probability distributions satisfy Bell inequality:

$$\mathrm{CHSH} := |C_{00} + C_{01} + C_{10} - C_{11}| \leq 2 \quad \text{where} \quad C_{ab} := \mathbb{E}\left(x \cdot y | a, b\right).$$

The Bell-CHSH inequality

Classical probability distributions satisfy Bell inequality:

$$\mathrm{CHSH} := |C_{00} + C_{01} + C_{10} - C_{11}| \leq 2 \quad \text{where} \quad C_{ab} := \mathbb{E}\left(x \cdot y | a, b\right).$$



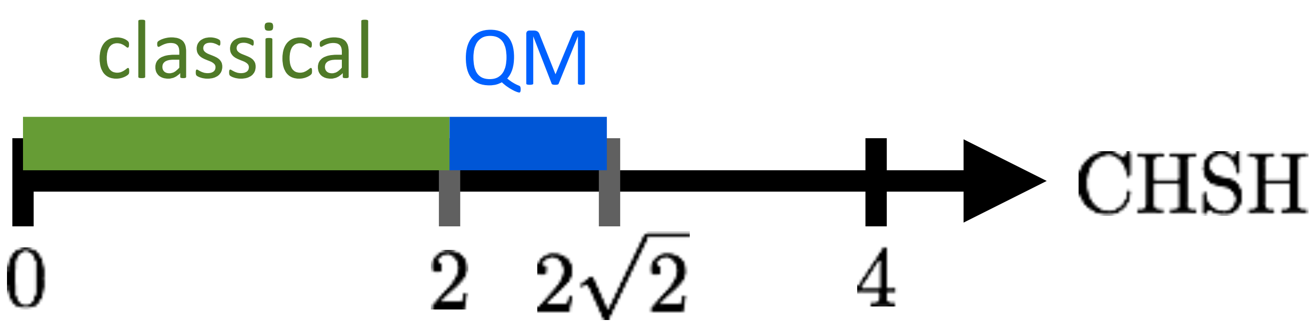
Quantum: Bell inequality violation.

$$\mathrm{CHSH} \leq 2\sqrt{2}.$$

The Bell-CHSH inequality

Classical probability distributions satisfy Bell inequality:

$$\mathrm{CHSH} := |C_{00} + C_{01} + C_{10} - C_{11}| \leq 2 \quad \text{where} \quad C_{ab} := \mathbb{E}\left(x \cdot y | a, b\right).$$



Quantum: Bell inequality violation.

$$\mathrm{CHSH} \leq 2\sqrt{2}.$$

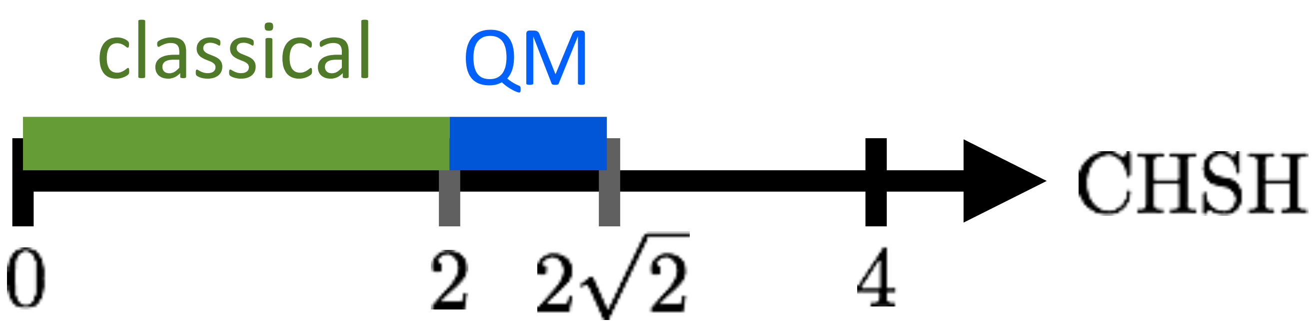
S. Popescu and D. Rohrlich, Found. Phys. **24**, 379 (1994):

Are **quantum** correlations the **most general** $P(a, b|x, y)$ that satisfy the no-signalling principle?

The Bell-CHSH inequality

Classical probability distributions satisfy Bell inequality:

$$\mathrm{CHSH} := |C_{00} + C_{01} + C_{10} - C_{11}| \leq 2 \quad \text{where} \quad C_{ab} := \mathbb{E}\,(x \cdot y|a,b)\,.$$



Quantum: Bell inequality violation.

$$\mathrm{CHSH} \leq 2\sqrt{2}.$$

S. Popescu and D. Rohrlich, Found. Phys. **24**, 379 (1994):

Are **quantum** correlations the **most general** $P(a,b|x,y)$ that satisfy the no-signalling principle?

No! Counterexample: the PR-box correlations

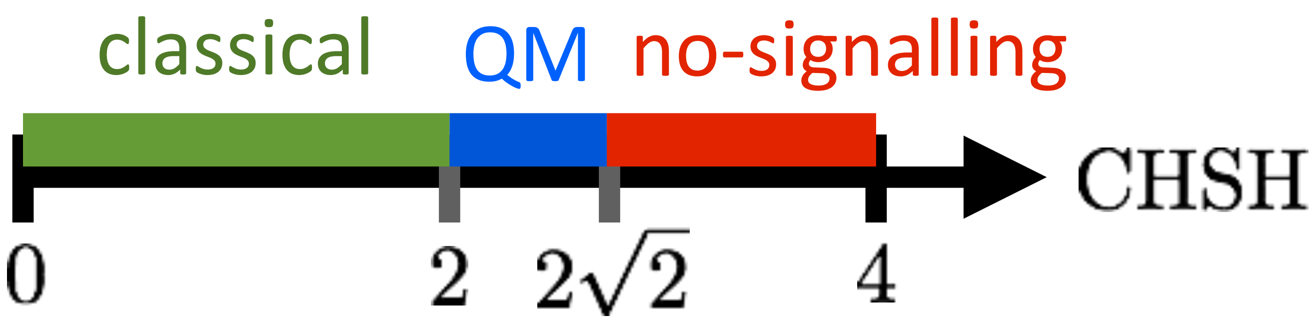
$$\begin{aligned} p(+1,+1|a,b) &= p(-1,-1|a,b) = \frac{1}{2} \\ &\quad \text{if } (a,b) \in \{(0,0),(0,1),(1,0)\} \\ p(+1,-1|1,1) &= p(-1,+1|1,1) = \frac{1}{2} \end{aligned}$$

CHSH=4

The Bell-CHSH inequality

Classical probability distributions satisfy Bell inequality:

$$\mathrm{CHSH} := |C_{00} + C_{01} + C_{10} - C_{11}| \leq 2 \quad \text{where} \quad C_{ab} := \mathbb{E}\,(x \cdot y | a, b)\,.$$



Quantum: Bell inequality violation.

$$\mathrm{CHSH} \leq 2\sqrt{2}.$$

S. Popescu and D. Rohrlich, Found. Phys. **24**, 379 (1994):

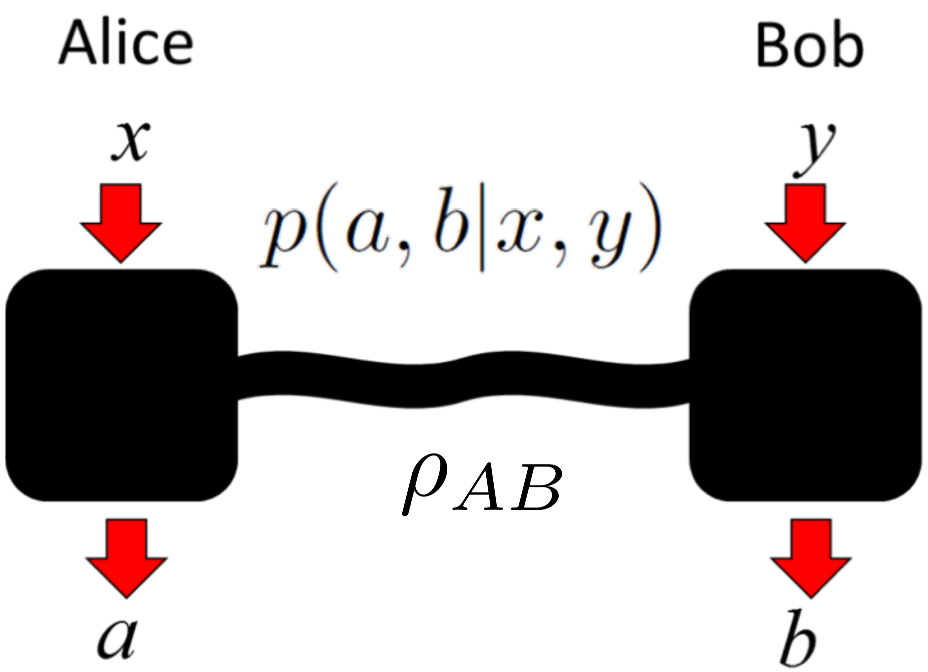
Are **quantum** correlations the **most general** $P(a, b|x, y)$ that satisfy the no-signalling principle?

No! Counterexample: the PR-box correlations

$$\begin{aligned} p(+1,+1|a,b) &= p(-1,-1|a,b) = \frac{1}{2} \\ &\quad \text{if } (a,b) \in \{(0,0),(0,1),(1,0)\} \\ p(+1,-1|1,1) &= p(-1,+1|1,1) = \frac{1}{2} \end{aligned}$$

CHSH=4

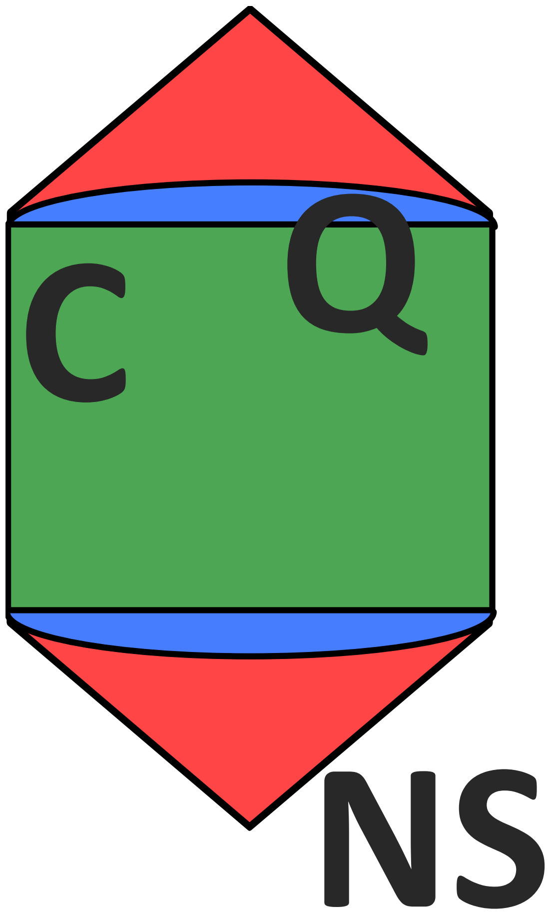
Black boxes and correlations



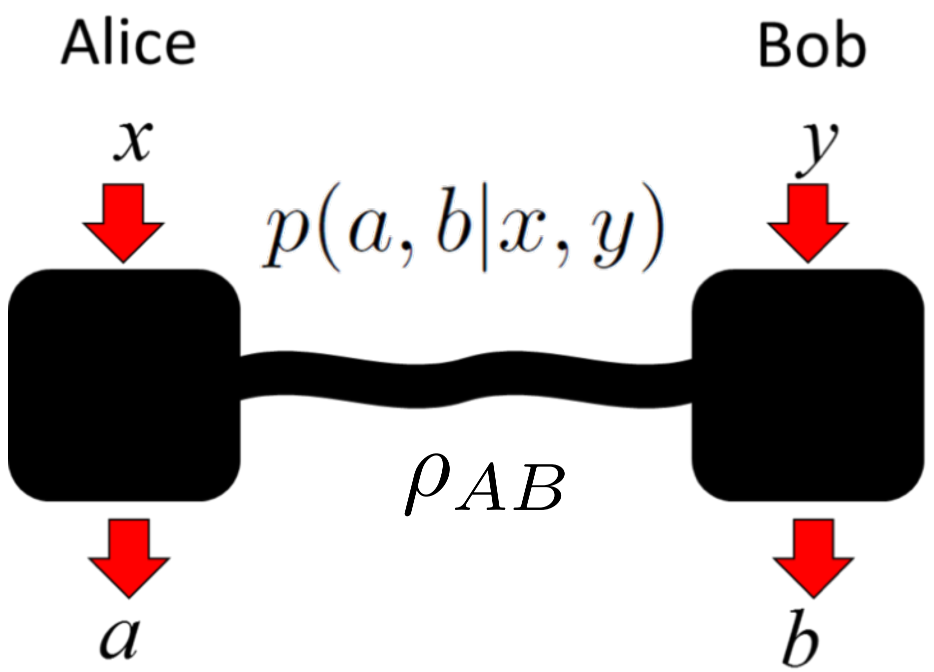
No-signalling conditions:

$P(a|x,y)$ is independent of y,

$P(b|x,y)$ is independent of x.



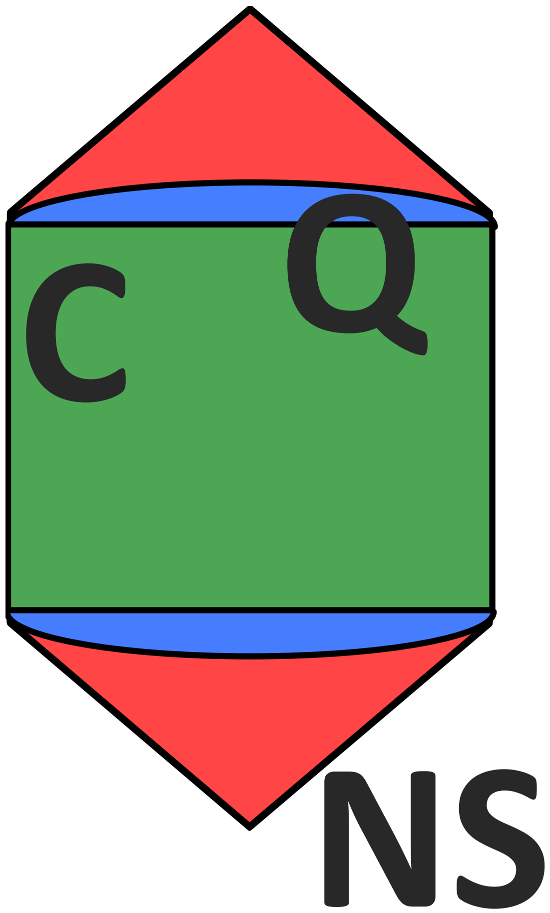
Black boxes and correlations



No-signalling conditions:

$P(a|x,y)$ is independent of y,

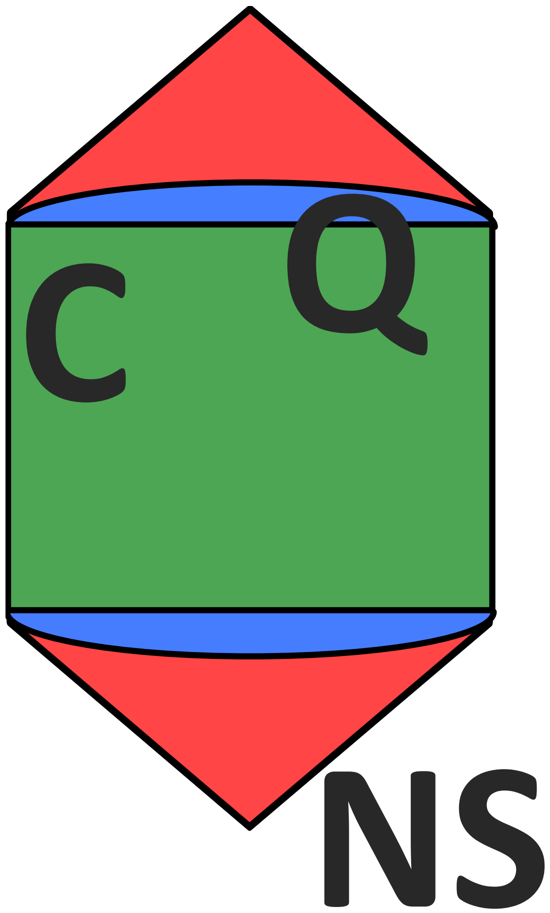
$P(b|x,y)$ is independent of x.



Correlations in **C** come from **classical prob. theory**,
correlations in **Q** from **quantum theory**,
correlations in **NS** describe **alternative physics.**

Black boxes and correlations

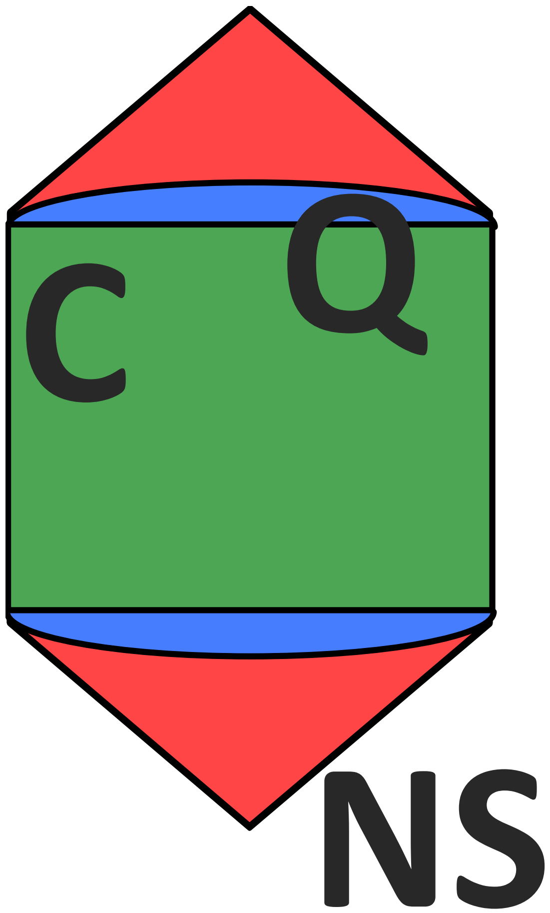
Why study such correlations?



Black boxes and correlations

Why study such correlations?

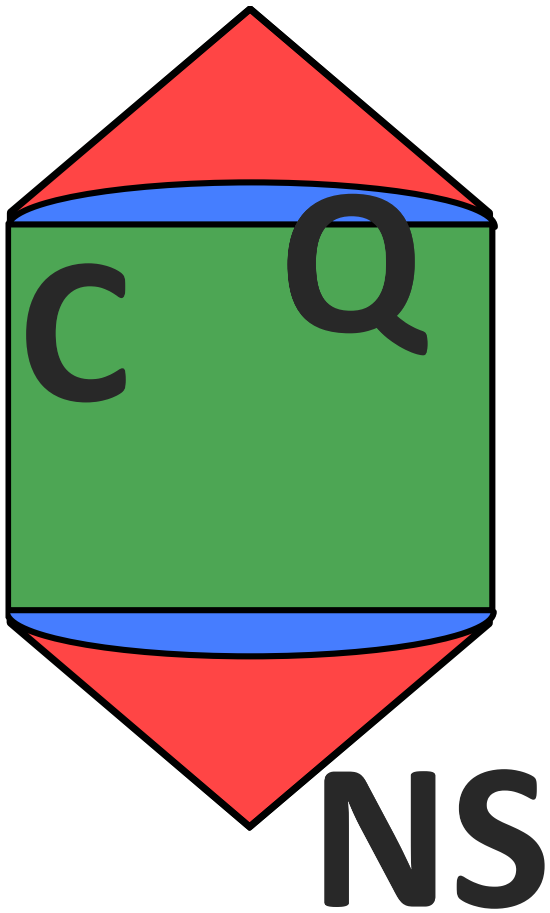
- **Foundational:** "Why" does nature admit **Q** but not more? **Principles?**



Black boxes and correlations

Why study such correlations?

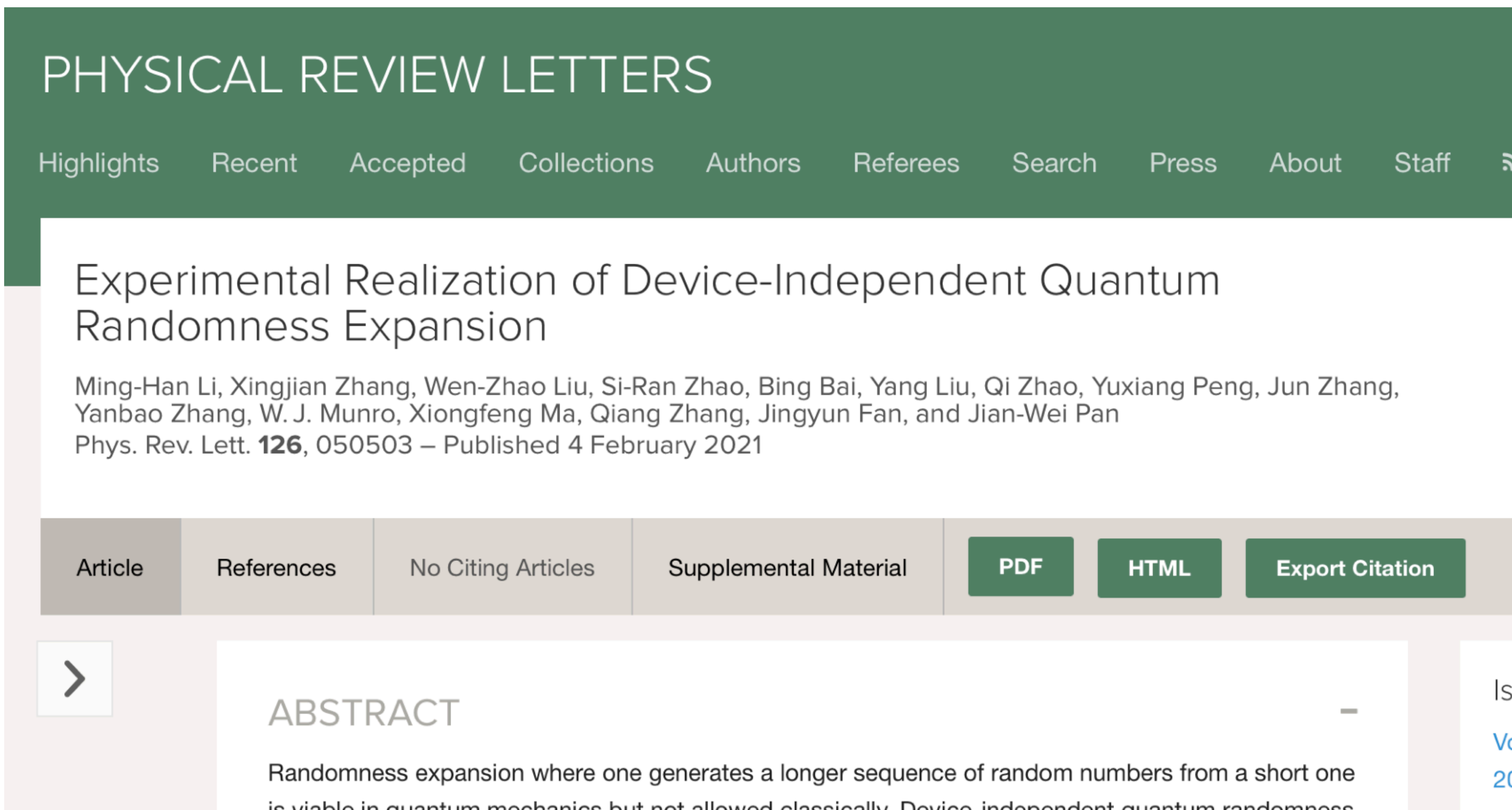
- **Foundational:** "Why" does nature admit **Q** but not more? **Principles?**
- **Applications:** Randomness amplification or cryptographic security from no-signalling principle & violation of Bell inequality alone. **Device-independent protocols**.



Black boxes and correlations

Why study such correlations?

- **Foundational:** “Why” does nature admit **Q** but not more? **Principles?**
- **Applications:** Randomness amplification or cryptographic security from no-signalling principle & violation of Bell inequality alone. **Device-independent protocols**.



PHYSICAL REVIEW LETTERS

Highlights Recent Accepted Collections Authors Referees Search Press About Staff

Experimental Realization of Device-Independent Quantum Randomness Expansion

Ming-Han Li, Xingjian Zhang, Wen-Zhao Liu, Si-Ran Zhao, Bing Bai, Yang Liu, Qi Zhao, Yuxiang Peng, Jun Zhang, Yanbao Zhang, W. J. Munro, Xiongfeng Ma, Qiang Zhang, Jingyun Fan, and Jian-Wei Pan

Phys. Rev. Lett. **126**, 050503 – Published 4 February 2021

Article | References | No Citing Articles | Supplemental Material | PDF | HTML | Export Citation

ABSTRACT

Randomness expansion where one generates a longer sequence of random numbers from a short one

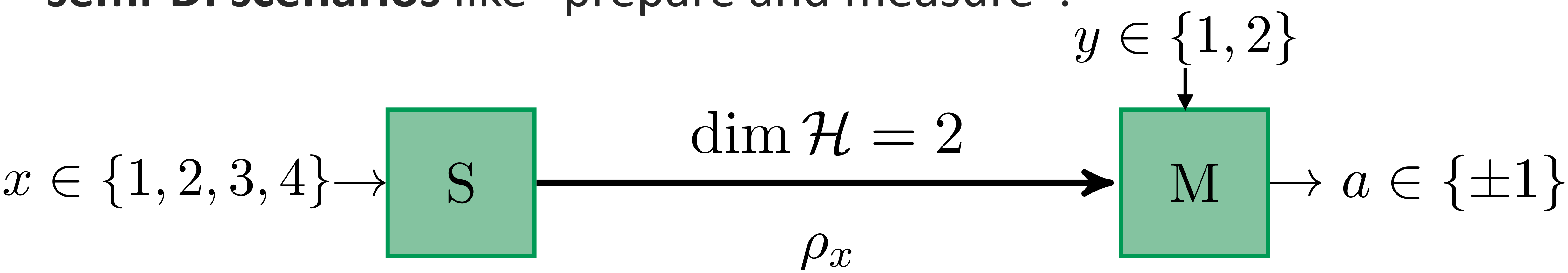
Black boxes and correlations

Why study such correlations?

- **Foundational:** "Why" does nature admit **Q** but not more? **Principles?**
- **Applications:** Randomness amplification or cryptographic security from no-signalling principle & violation of Bell inequality alone. **Device-independent protocols**.

Other scenarios:

not just Bell scenarios, but for example
semi-DI scenarios like "prepare and measure":



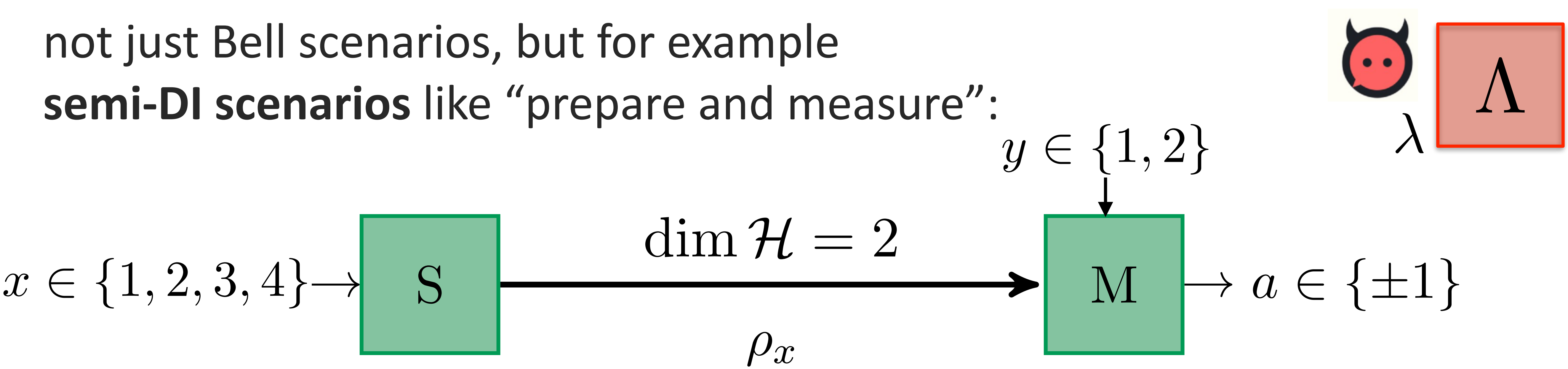
Black boxes and correlations

Why study such correlations?

- **Foundational:** "Why" does nature admit **Q** but not more? **Principles?**
- **Applications:** Randomness amplification or cryptographic security from no-signalling principle & violation of Bell inequality alone. **Device-independent protocols**.

Other scenarios:

not just Bell scenarios, but for example
semi-DI scenarios like "prepare and measure":



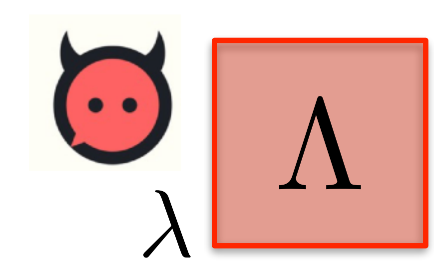
Black boxes and correlations

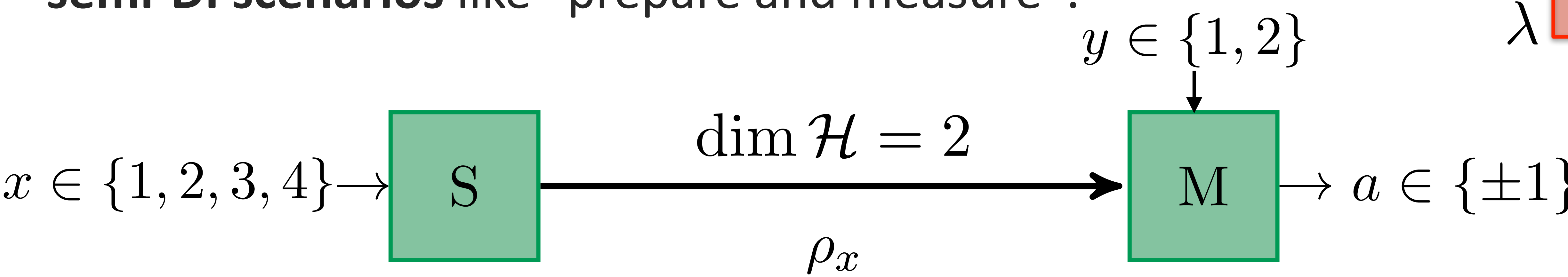
Why study such correlations?

- **Foundational:** "Why" does nature admit **Q** but not more? **Principles?**
- **Applications:** Randomness amplification or cryptographic security from no-signalling principle & violation of Bell inequality alone. **Device-independent protocols**.

Other scenarios:

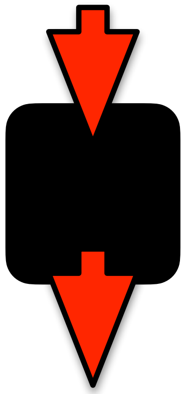
not just Bell scenarios, but for example
semi-DI scenarios like "prepare and measure":





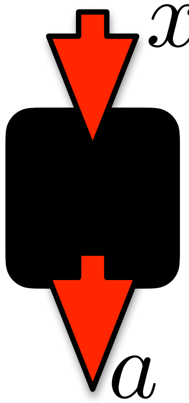
From the data table $p(a|x,y)$ and the assumption $\dim \mathcal{H} = 2$ **alone**, one can infer that $H(A|X,Y,\Lambda) \geq \ldots > 0$.

Single black boxes



$$P(a|x)$$

Single black boxes



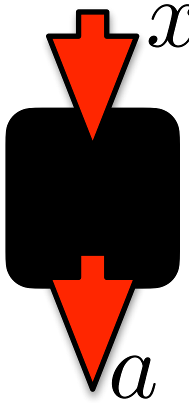
$x \in \{0, 1\}$

$P(a|x)$

$a \in \{-1, +1\}$

Inputs and outputs are typically taken as **abstract labels** (bits etc.)

Single black boxes



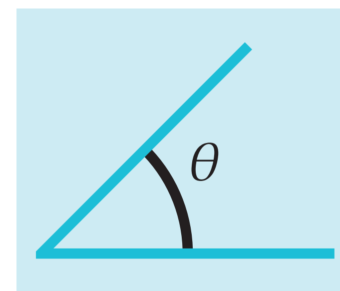
$x \in \{0, 1\}$

$P(a|x)$

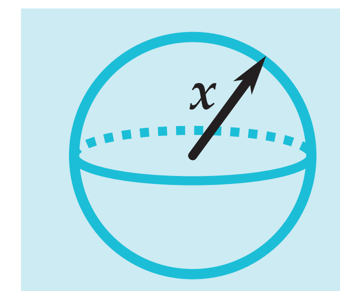
$a \in \{-1, +1\}$

Inputs and outputs are typically taken as **abstract labels** (bits etc.)

What if inputs (and perhaps outputs) are **spatiotemporal quantities**?



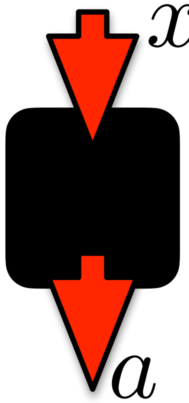
ANGLES



DIRECTIONS

DURATIONS

Single black boxes



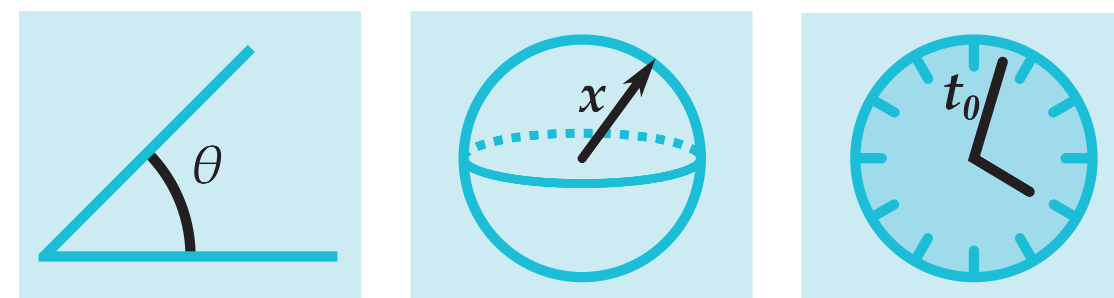
$x \in \{0, 1\}$

$P(a|x)$

$a \in \{-1, +1\}$

Inputs and outputs are typically taken as **abstract labels** (bits etc.)

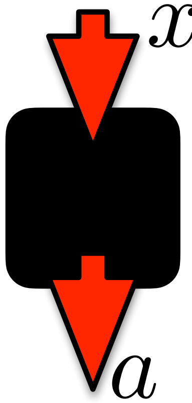
What if inputs (and perhaps outputs) are **spatiotemporal quantities**?



ANGLES DIRECTIONS DURATIONS

- This is the case in many **actual experimental settings**.

Single black boxes



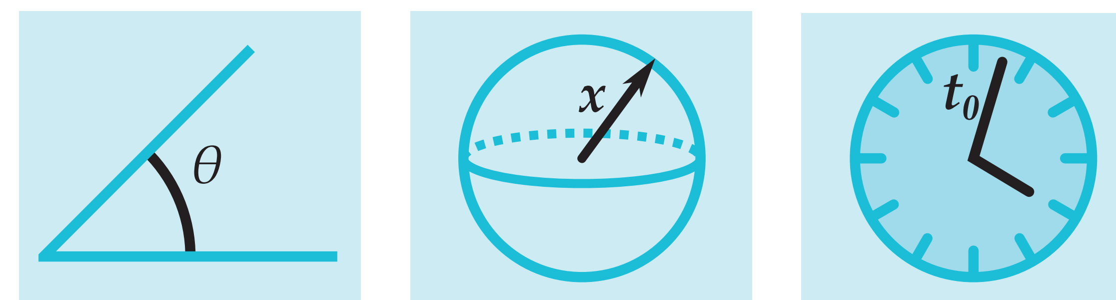
$x \in \{0, 1\}$

$P(a|x)$

$a \in \{-1, +1\}$

Inputs and outputs are typically taken as **abstract labels** (bits etc.)

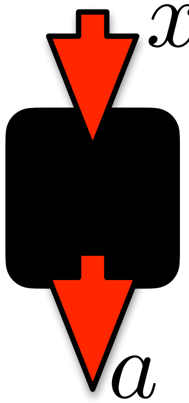
What if inputs (and perhaps outputs) are **spatiotemporal quantities**?



ANGLES DIRECTIONS DURATIONS

- This is the case in many **actual experimental settings**.
- Study **interplay of probability, space and time** under minimal assumptions (even without assuming QT). Recall QFT!

Single black boxes



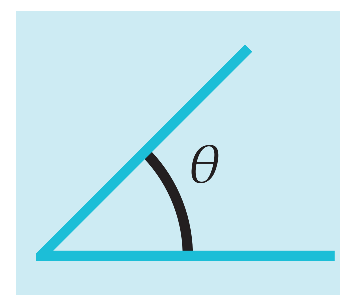
$x \in \{0, 1\}$

$P(a|x)$

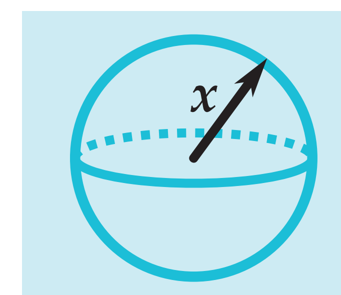
$a \in \{-1, +1\}$

Inputs and outputs are typically taken as **abstract labels** (bits etc.)

What if inputs (and perhaps outputs) are **spatiotemporal quantities**?



ANGLES

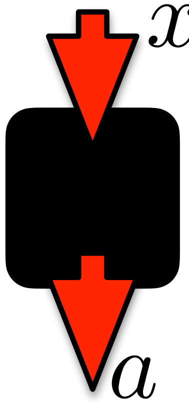


DIRECTIONS

DURATIONS

- This is the case in many **actual experimental settings**.
- Study **interplay of probability, space and time** under minimal assumptions (even without assuming QT). Recall QFT!
- Use spacetime symmetries in protocols?

Single black boxes



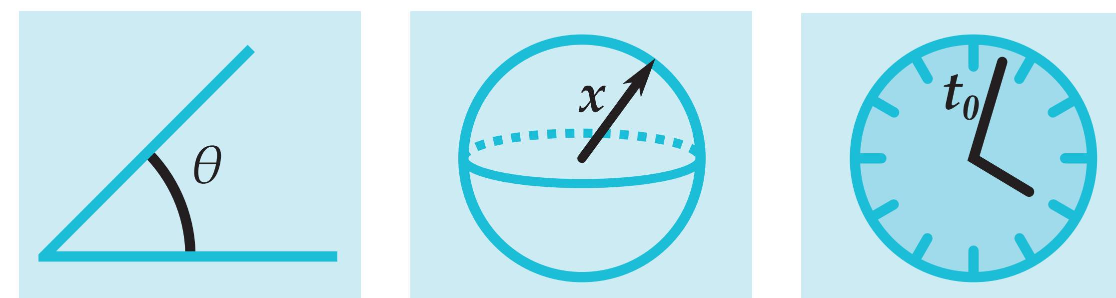
$x \in \{0, 1\}$

$P(a|x)$

$a \in \{-1, +1\}$

Inputs and outputs are typically taken as **abstract labels** (bits etc.)

What if inputs (and perhaps outputs) are **spatiotemporal quantities**?

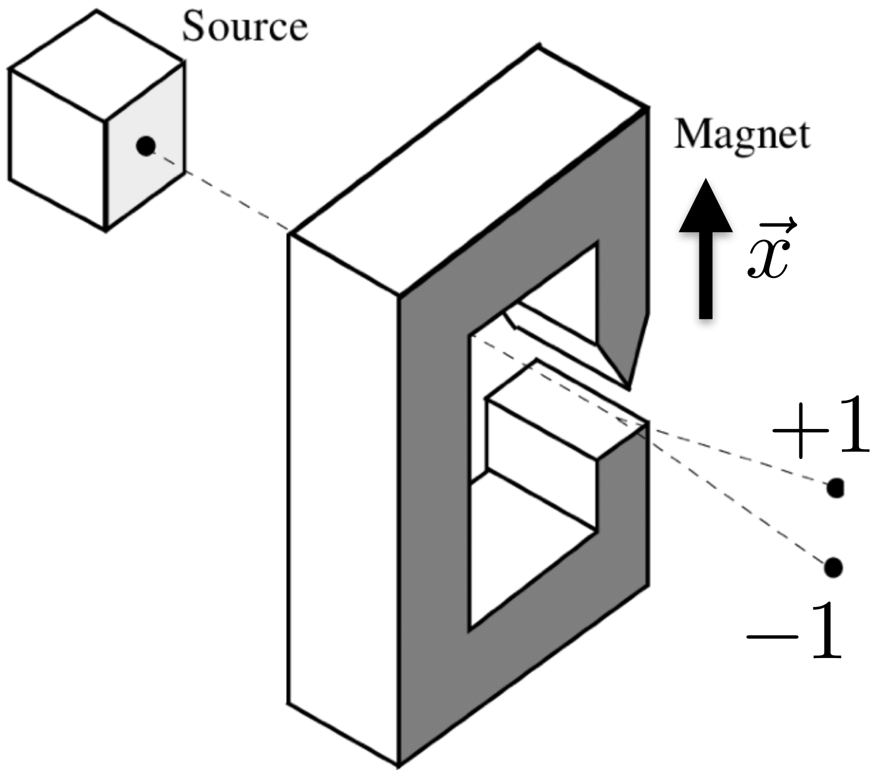


ANGLES DIRECTIONS DURATIONS

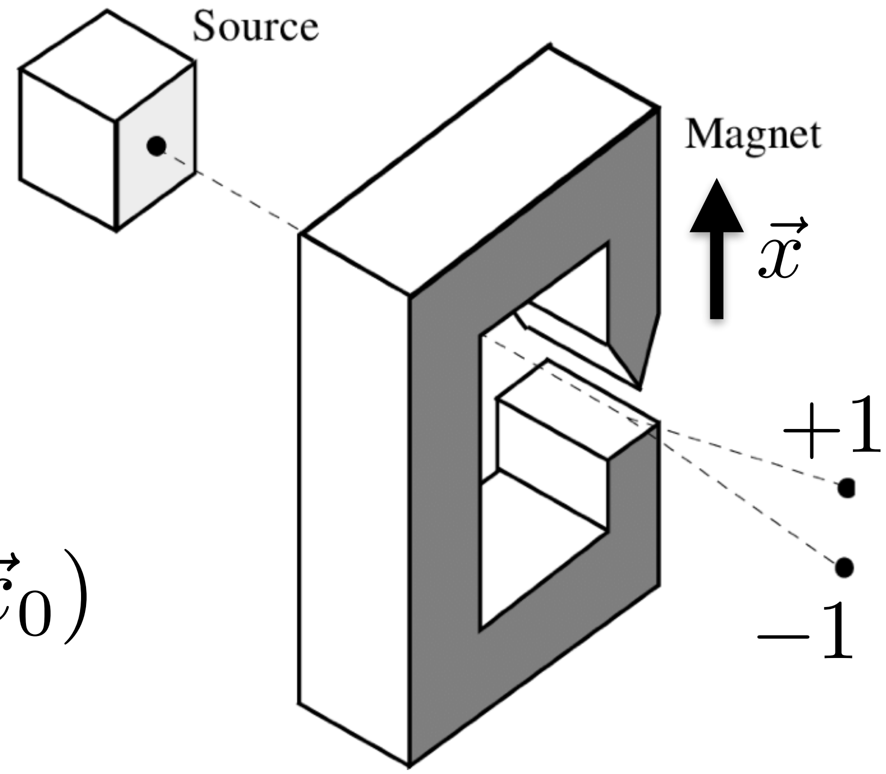
- This is the case in many **actual experimental settings**.
- Study **interplay of probability, space and time** under minimal assumptions (even without assuming QT). Recall QFT!
- Use spacetime symmetries in protocols?
- How could possible beyond-quantum physics fit into space and time?

Example: Stern-Gerlach experiment

Example: Stern-Gerlach experiment



Example: Stern-Gerlach experiment



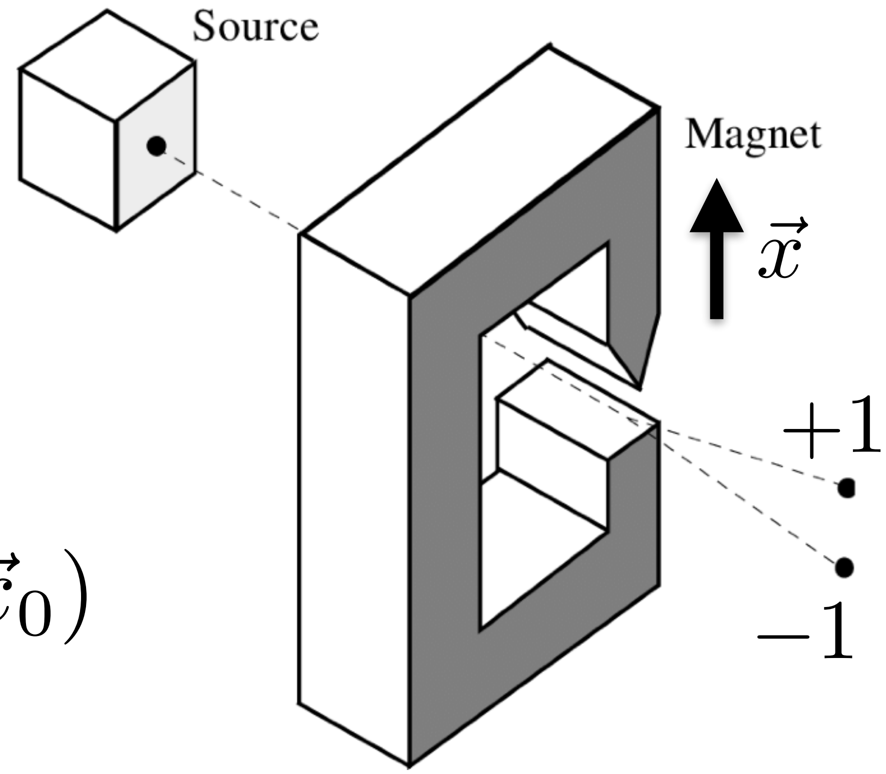
$$P(a|\vec{x})$$

$$= P(a|R\vec{x}_0)$$

$$|\vec{x}| = 1,$$

$$a \in \{-1, +1\}.$$

Example: Stern-Gerlach experiment



$$P(a|\vec{x})$$

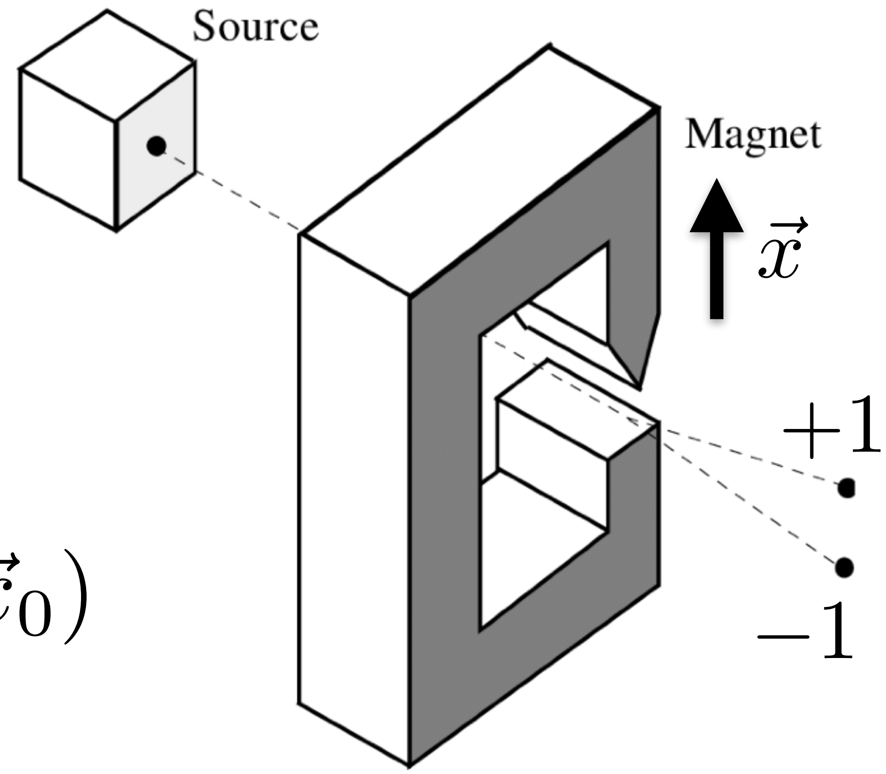
$$= P(a|R\vec{x}_0)$$

$$|\vec{x}| = 1,$$

$$a \in \{-1, +1\}.$$

- Default direction of inhomogeneity of field: $\vec{x}_0$.
- Spatial rotation applied to it: $R \in \mathcal{G} = \mathrm{SO}(3)$.

Example: Stern-Gerlach experiment



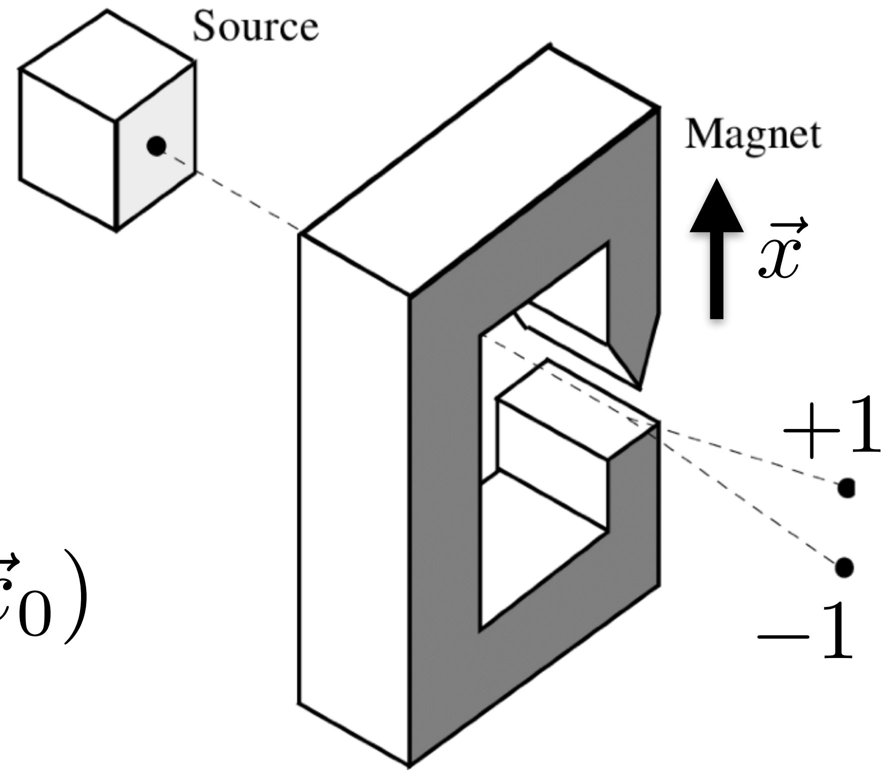
$$P(a|\vec{x})$$

$$= P(a|R\vec{x}_0)$$

$$|\vec{x}| = 1,$$
$$a \in \{-1, +1\}.$$

- Default direction of inhomogeneity of field: $\vec{x}_0$.
- Spatial rotation applied to it: $R \in \mathcal{G} = \mathrm{SO}(3)$.
- Stabilizer subgroup $\mathcal{H} \simeq \mathrm{SO}(2)$, i.e. $R\vec{x}_0 = \vec{x}_0$ for $R \in \mathcal{H}$.

Example: Stern-Gerlach experiment



$$P(a|\vec{x})$$

$$= P(a|R\vec{x}_0)$$

$$|\vec{x}| = 1,$$

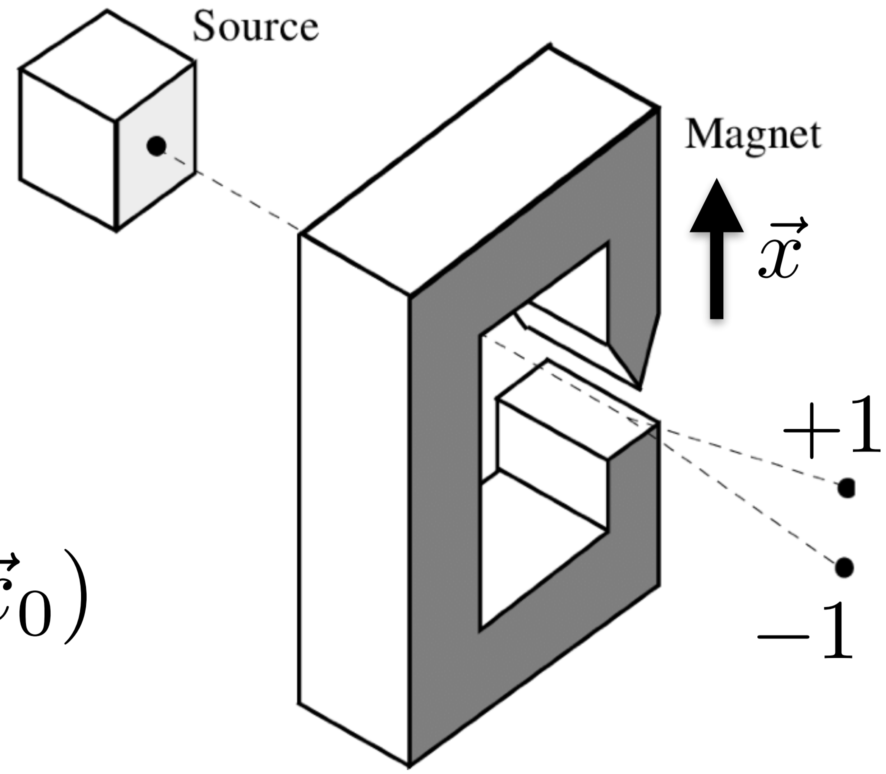
$$a \in \{-1, +1\}.$$

- Default direction of inhomogeneity of field: $\vec{x}_0$.
- Spatial rotation applied to it: $R \in \mathcal{G} = \mathrm{SO}(3)$.
- Stabilizer subgroup $\mathcal{H} \simeq \mathrm{SO}(2)$, i.e. $R\vec{x}_0 = \vec{x}_0$ for $R \in \mathcal{H}$.
- Manifold of inputs: the **unit sphere**, $S^2 = \mathrm{SO}(3)/\mathrm{SO}(2)$.

Example: Stern-Gerlach experiment

$$P(a|\vec{x})$$

$$= P(a|R\vec{x}_0)$$



$$|\vec{x}| = 1,$$
$$a \in \{-1, +1\}.$$

- Default direction of inhomogeneity of field: $\vec{x}_0$.
- Spatial rotation applied to it: $R \in \mathcal{G} = \mathrm{SO}(3)$.
- Stabilizer subgroup $\mathcal{H} \simeq \mathrm{SO}(2)$, i.e. $R\vec{x}_0 = \vec{x}_0$ for $R \in \mathcal{H}$.
- Manifold of inputs: the **unit sphere**, $S^2 = \mathrm{SO}(3)/\mathrm{SO}(2)$.
- In general, **inputs are elements of a homogeneous space,** $\mathcal{G}/\mathcal{H}$.

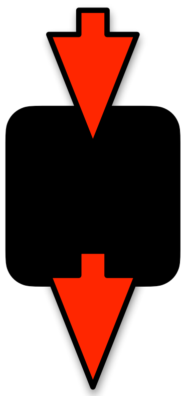
Spacetime boxes

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

Spacetime boxes

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

$x \in \{0, 1\}$



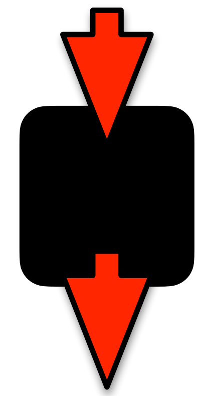
$P(a|x)$

$a \in \{-1, +1\}$

$\vec{x} \in \mathcal{G}/\mathcal{H}$

$\mathcal{G}$: symmetry group

$\mathcal{H}$: stabilizer subgroup



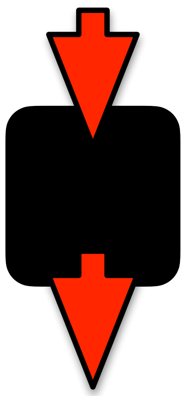
$P(a|\vec{x})$

$a \in \{-1, +1\}$

Spacetime boxes

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

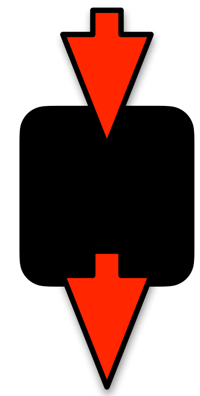
$x \in \{0, 1\}$



$P(a|x)$

$a \in \{-1, +1\}$

$\vec{x} \in \mathcal{G}/\mathcal{H}$



$P(a|\vec{x})$

$a \in \{-1, +1\}$

$\mathcal{G}$: symmetry group

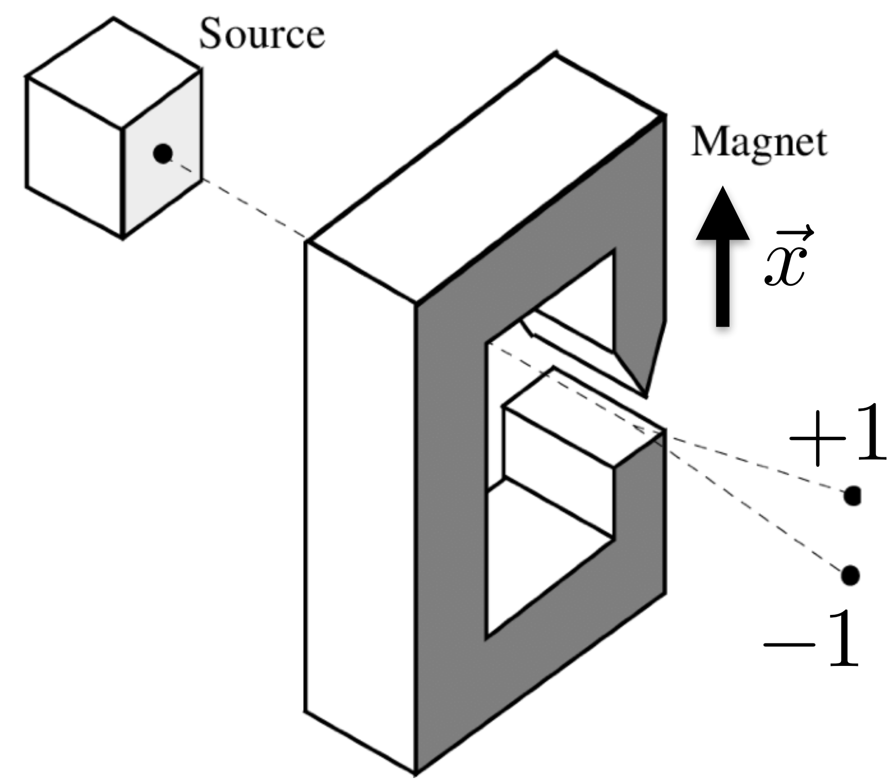
$\mathcal{H}$: stabilizer subgroup

Example: Stern-Gerlach experiment

$\mathcal{G} = \mathrm{SO}(3)$ (spatial rotations)

$\mathcal{H} = \mathrm{SO}(2)$ (axial symmetry of magnetic field)

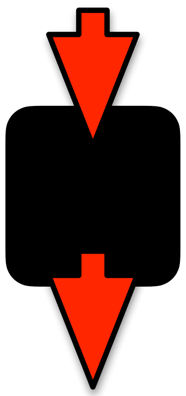
$\vec{x} \in \mathcal{G}/\mathcal{H} = S^2$ (unit vector: field direction)



Spacetime boxes

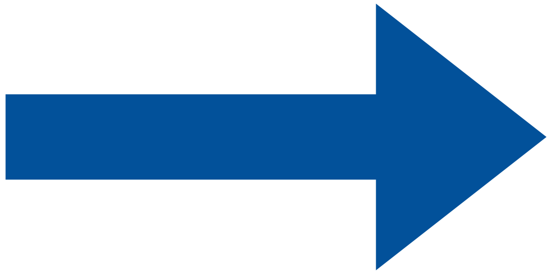
A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

$x \in \{0,1\}$

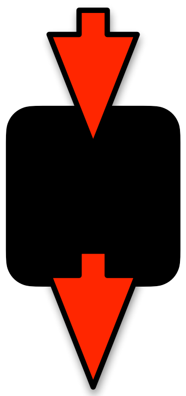


$P(a|x)$

$a \in \{-1,+1\}$



$\vec{x} \in \mathcal{G}/\mathcal{H}$



$P(a|\vec{x})$

$a \in \{-1,+1\}$

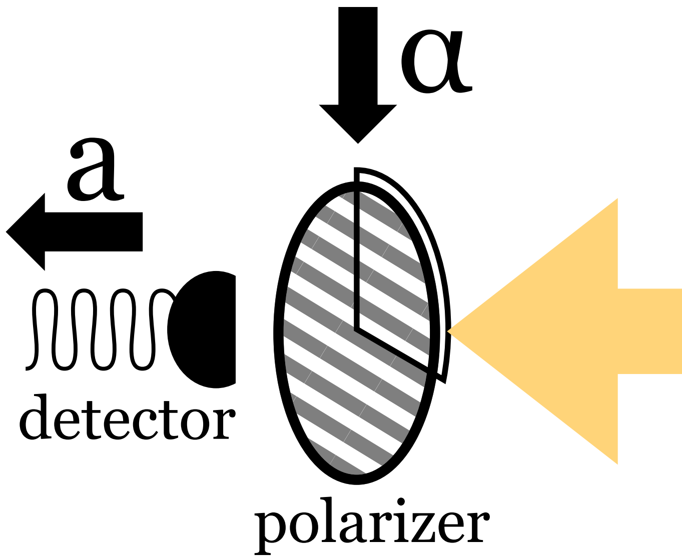
$\mathcal{G}$: symmetry group
$\mathcal{H}$: stabilizer subgroup

Example: Polarizer, $P(a|\alpha)$.

$\mathcal{G} = \mathrm{SO}(2)$ (rotations around beam axis)

$\mathcal{H} = \{\mathbf{1}\}$ (no additional symmetry)

$\alpha \in \mathcal{G}/\mathcal{H} = \mathrm{SO}(2)$.

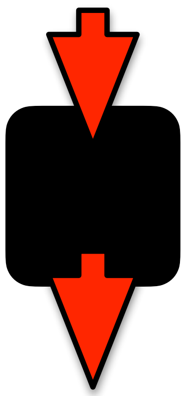


click / no click: $a = \pm 1$.

Spacetime boxes

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

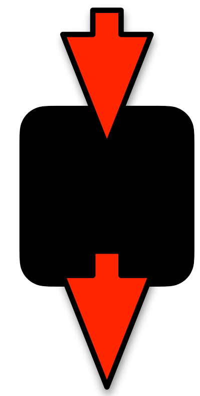
$x \in \{0, 1\}$



$P(a|x)$

$a \in \{-1, +1\}$

$\vec{x} \in \mathcal{G}/\mathcal{H}$



$P(a|\vec{x})$

$a \in \{-1, +1\}$

$\mathcal{G}$: symmetry group

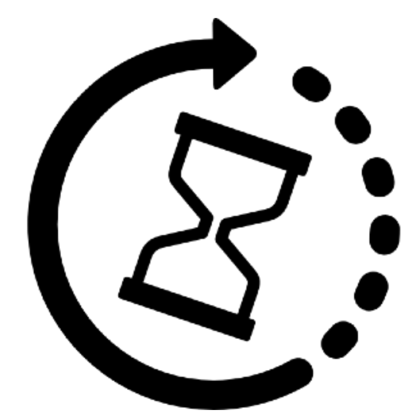
$\mathcal{H}$: stabilizer subgroup

Example: Input is time *t*, $P(a|t)$.

$\mathcal{G} = (\mathbb{R}, +)$ (group of time translations)

$\mathcal{H} = \{\mathbf{1}\}$ (no additional symmetry)

$\vec{x} = t \in \mathbb{R}$



Spacetime boxes

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

Spacetime boxes

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

Theorem. Probabilistic consistency implies that $P(a|\vec{x})$ is a linear combination of matrix entries of a real **group representation** of $\mathcal{G}$.

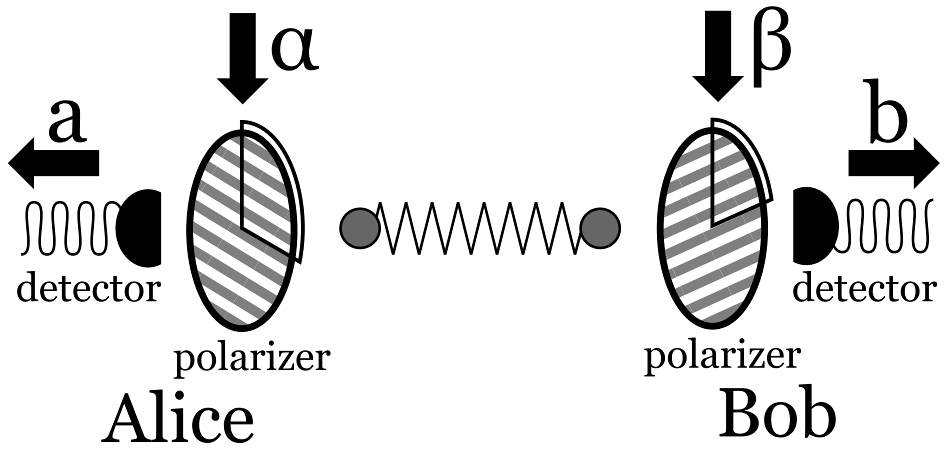
This must be true even if we do not assume that QT holds.

Spacetime boxes

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

Theorem. Probabilistic consistency implies that $P(a|\vec{x})$ is a linear combination of matrix entries of a real **group representation** of $\mathcal{G}$.

This must be true even if we do not assume that QT holds.



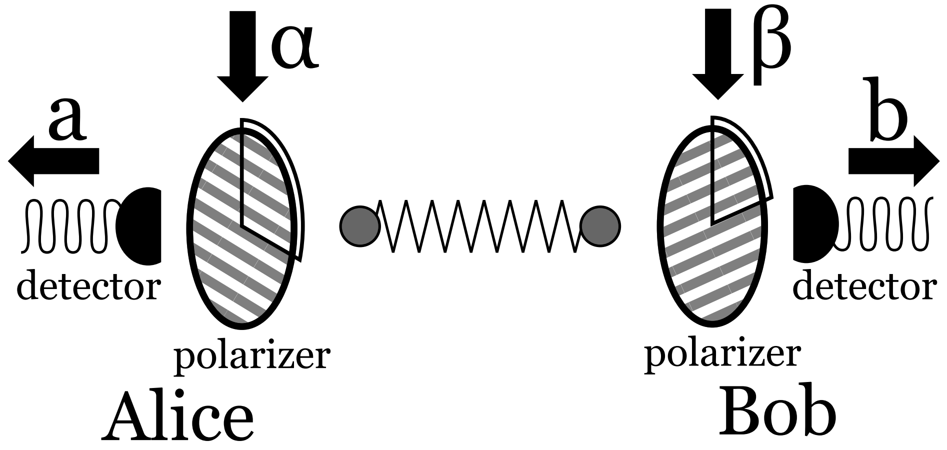
$$\mathcal{G} = \mathrm{SO}(2) \times \mathrm{SO}(2)$$

Spacetime boxes

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

Theorem. Probabilistic consistency implies that $P(a|\vec{x})$ is a linear combination of matrix entries of a real **group representation** of $\mathcal{G}$.

This must be true even if we do not assume that QT holds.



$$\mathcal{G} = \mathrm{SO}(2) \times \mathrm{SO}(2)$$

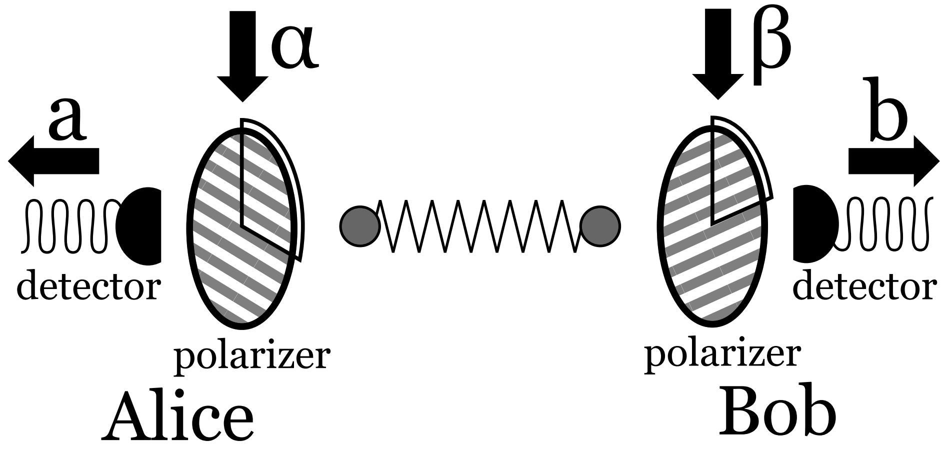
$$T_{\alpha,\beta} = \bigoplus_{m,n} \left(\begin{array}{cc} \cos(m\alpha - n\beta) & \sin(m\alpha - n\beta) \\ -\sin(m\alpha - n\beta) & \cos(m\alpha - n\beta) \end{array} \right).$$

Spacetime boxes

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

Theorem. Probabilistic consistency implies that $P(a|\vec{x})$ is a linear combination of matrix entries of a real **group representation** of $\mathcal{G}$.

This must be true even if we do not assume that QT holds.



$$P(a,b|\alpha,\beta) := \sum_{m=0}^{2J}\sum_{n=-2J}^{2J} c_{mn}^{ab}\cos(m\alpha - n\beta) + s_{mn}^{ab}\sin(m\alpha - n\beta),$$

$$\mathcal{G} = \mathrm{SO}(2)\times\mathrm{SO}(2)$$

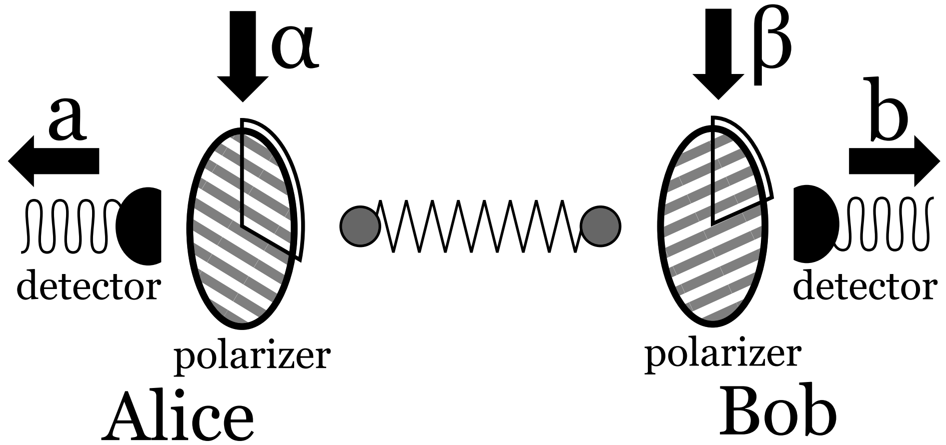
$$T_{\alpha,\beta} = \bigoplus_{m,n}\begin{pmatrix}\cos(m\alpha - n\beta) & \sin(m\alpha - n\beta)\\ -\sin(m\alpha - n\beta) & \cos(m\alpha - n\beta)\end{pmatrix}.$$

Spacetime boxes

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

Theorem. Probabilistic consistency implies that $P(a|\vec{x})$ is a linear combination of matrix entries of a real **group representation** of $\mathcal{G}$.

This must be true even if we do not assume that QT holds.



$$\mathcal{G} = \mathrm{SO}(2) \times \mathrm{SO}(2)$$

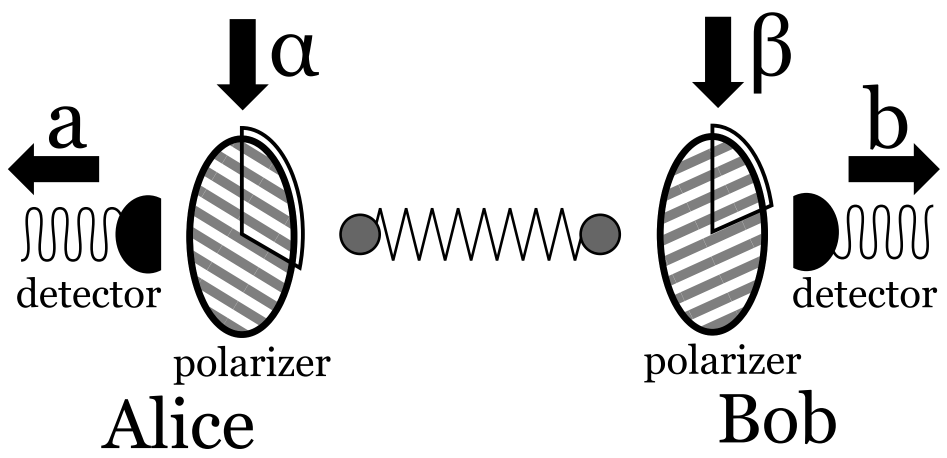
$$P(a, b|\alpha, \beta) := \sum_{m=0}^{2J} \sum_{n=-2J}^{2J} c_{mn}^{ab} \cos(m\alpha - n\beta) + s_{mn}^{ab} \sin(m\alpha - n\beta),$$

Examples: $C(\alpha, \beta) := \sum_{a,b=-1}^{+1} abP(a, b|\alpha, \beta)$ $\quad (a, b = \pm 1)$

Spacetime boxes

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

Theorem. Probabilistic consistency implies that $P(a|\vec{x})$ is a linear combination of matrix entries of a real **group representation** of $\mathcal{G}$.
This must be true even if we do not assume that QT holds.



$$\mathcal{G} = \mathrm{SO}(2) \times \mathrm{SO}(2)$$

$$P(a,b|\alpha,\beta) := \sum_{m=0}^{2J} \sum_{n=-2J}^{2J} c_{mn}^{ab} \cos(m\alpha - n\beta) + s_{mn}^{ab} \sin(m\alpha - n\beta),$$

Examples: $C(\alpha,\beta) := \sum_{a,b=-1}^{+1} ab P(a,b|\alpha,\beta)$ $\qquad (a, b = \pm 1)$

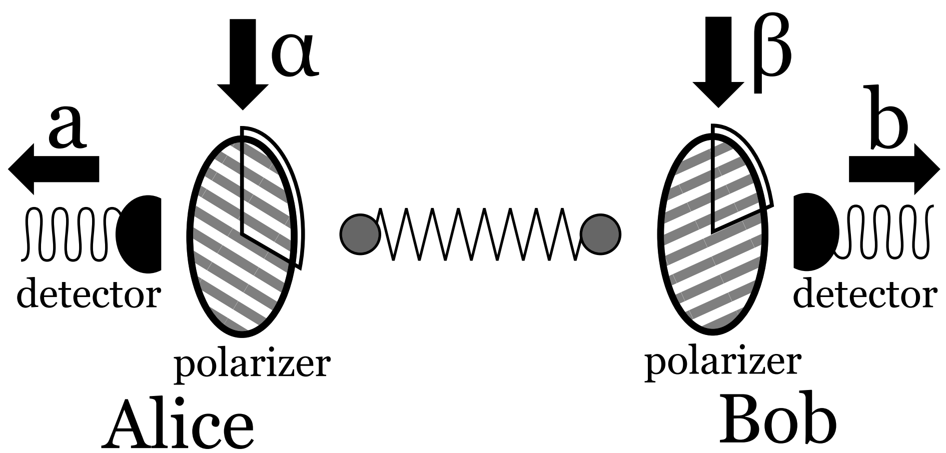
- Two-photon singlet state:
  $C(\alpha,\beta) = -\cos[2(\alpha - \beta)]$.

Spacetime boxes

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

Theorem. Probabilistic consistency implies that $P(a|\vec{x})$ is a linear combination of matrix entries of a real **group representation** of $\mathcal{G}$.

This must be true even if we do not assume that QT holds.



$$\mathcal{G} = \mathrm{SO}(2) \times \mathrm{SO}(2)$$

$$P(a,b|\alpha,\beta) := \sum_{m=0}^{2J} \sum_{n=-2J}^{2J} c_{mn}^{ab} \cos(m\alpha - n\beta) + s_{mn}^{ab} \sin(m\alpha - n\beta),$$

Examples: $C(\alpha,\beta) := \sum_{a,b=-1}^{+1} ab P(a,b|\alpha,\beta)$ $\quad (a,b = \pm 1)$

- Two-photon singlet state:

  $$C(\alpha,\beta) = -\cos[2(\alpha-\beta)].$$

- Science-fiction polarizers:

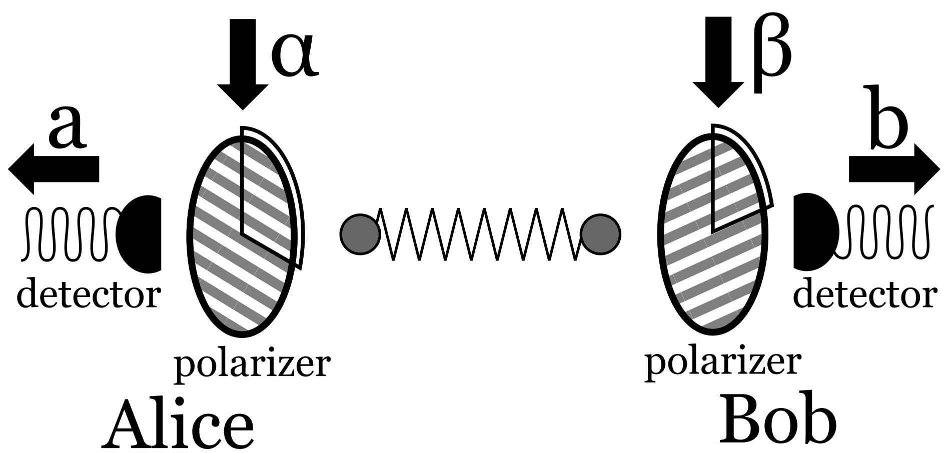
  $$C(\alpha,\beta) = -\frac{2}{7}\cos[3(\alpha-\beta)] - \cos(\alpha-\beta).$$

Spacetime boxes

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

Theorem. Probabilistic consistency implies that $P(a|\vec{x})$ is a linear combination of matrix entries of a real **group representation** of $\mathcal{G}$.

This must be true even if we do not assume that QT holds.



$$\mathcal{G} = \mathrm{SO}(2) \times \mathrm{SO}(2)$$

$$P(a,b|\alpha,\beta) := \sum_{m=0}^{2J} \sum_{n=-2J}^{2J} c_{mn}^{ab} \cos(m\alpha - n\beta) + s_{mn}^{ab} \sin(m\alpha - n\beta),$$

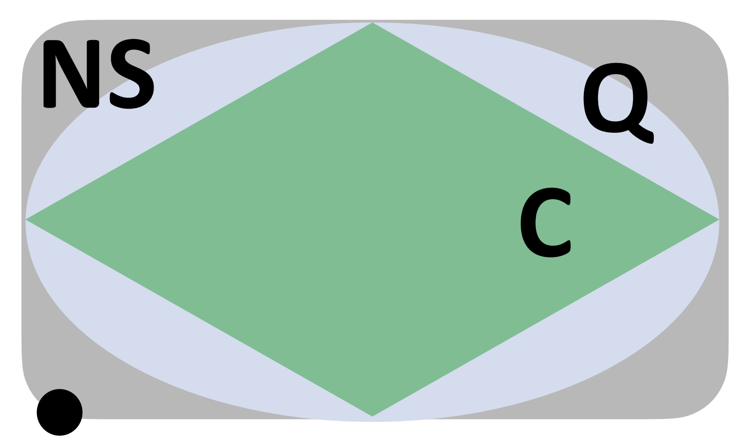
Examples: $C(\alpha,\beta) := \sum_{a,b=-1}^{+1} ab P(a,b|\alpha,\beta)$

- Two-photon singlet state:

  $C(\alpha,\beta) = -\cos[2(\alpha-\beta)].$

- Science-fiction polarizers:

  $C(\alpha,\beta) = -\frac{2}{7}\cos[3(\alpha-\beta)] - \cos(\alpha-\beta).$



"science-fiction-polarizers"

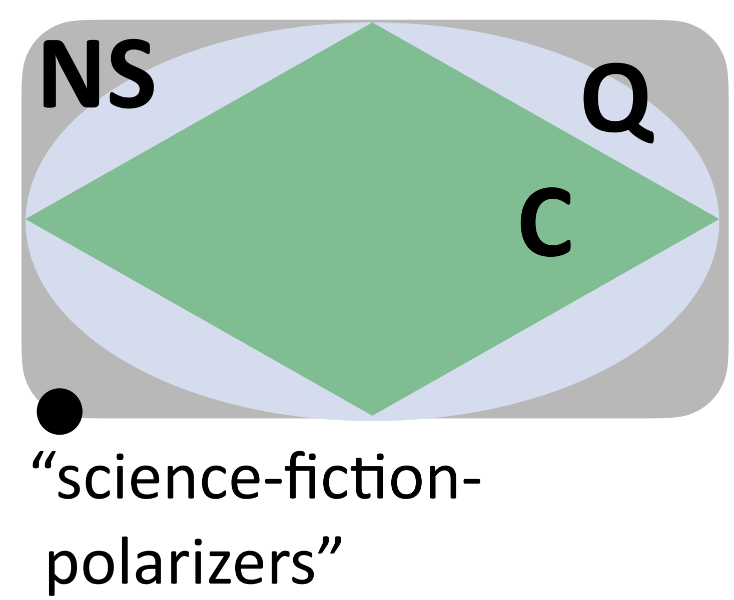
Spacetime boxes

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

Question: Could the **form** of the correlations (in contrast to the strength) already imply non-classicality?

Examples: $C(\alpha,\beta) := \sum_{a,b=-1}^{+1} ab P(a,b|\alpha,\beta)$

- Two-photon singlet state:
  $C(\alpha,\beta) = -\cos[2(\alpha-\beta)]$.
- Science-fiction polarizers:
  $C(\alpha,\beta) = -\frac{2}{7}\cos[3(\alpha-\beta)] - \cos(\alpha-\beta)$.



Spacetime boxes

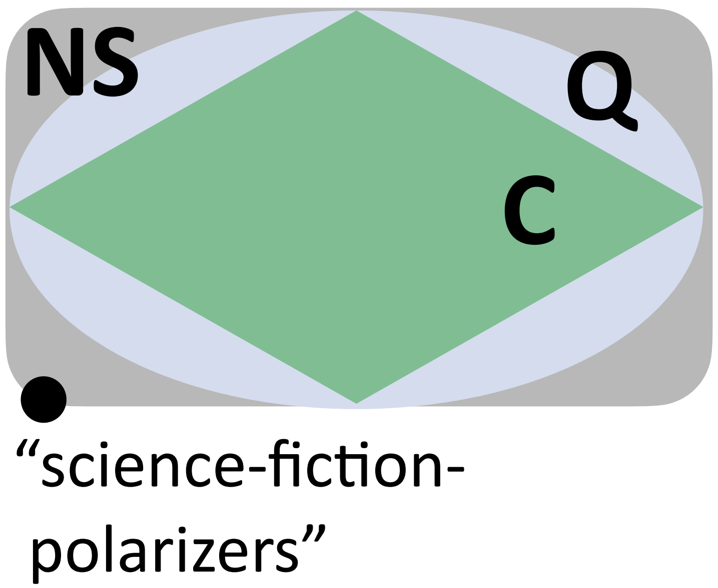
A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

Question: Could the **form** of the correlations (in contrast to the strength) already imply non-classicality? (Say, $c_{00}=0$ for simplicity)

$$C(\alpha,\beta)=\sum_{m=0}^{2J}\sum_{n=-2J}^{2J} c_{mn}\cos(m\alpha-n\beta)+s_{mn}\sin(m\alpha-n\beta)$$

Examples: $C(\alpha,\beta):=\sum_{a,b=-1}^{+1} abP(a,b|\alpha,\beta)$

- Two-photon singlet state:
  $C(\alpha,\beta)=-\cos[2(\alpha-\beta)].$
- Science-fiction polarizers:
  $C(\alpha,\beta)=-\frac{2}{7}\cos[3(\alpha-\beta)]-\cos(\alpha-\beta).$



Spacetime boxes

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

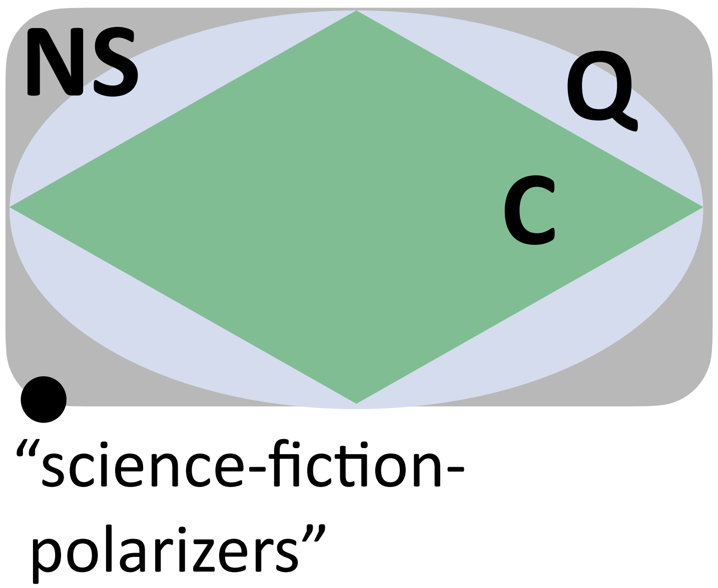
Question: Could the **form** of the correlations (in contrast to the strength) already imply non-classicality? (Say, $c_{00}=0$ for simplicity)

$$C(\alpha,\beta)=\sum_{m=0}^{2J}\sum_{n=-2J}^{2J} c_{mn}\cos(m\alpha-n\beta)+s_{mn}\sin(m\alpha-n\beta)$$

Answer: No. If $\max_{\alpha,\beta}|C(\alpha,\beta)|\leq\sqrt{2}e^{-1}[4J(2J+1)]^{-3/2}$ then *C* admits of a local hidden-variable model. Likely true for other groups too.

Examples: $C(\alpha,\beta):=\sum_{a,b=-1}^{+1} abP(a,b|\alpha,\beta)$

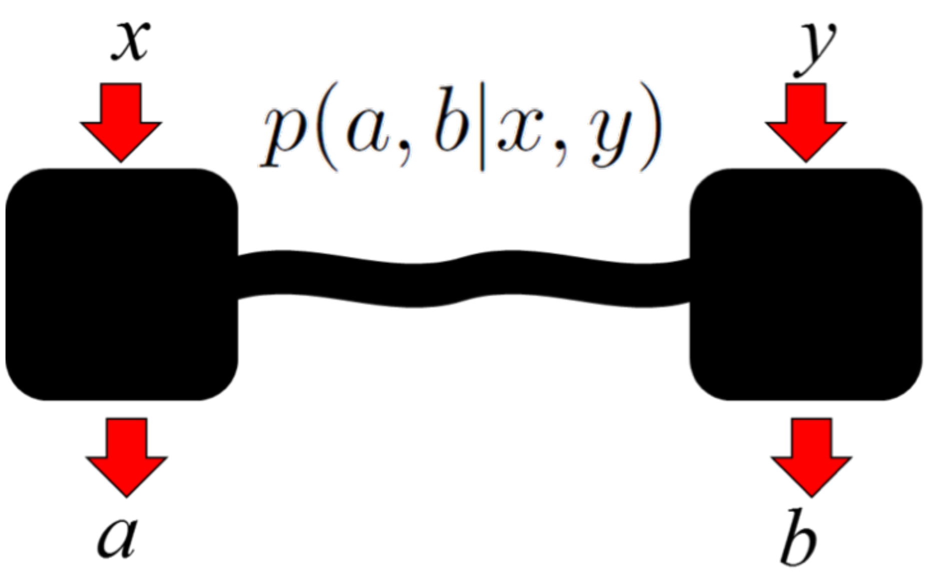
- Two-photon singlet state:
  $C(\alpha,\beta)=-\cos[2(\alpha-\beta)]$.
- Science-fiction polarizers:
  $C(\alpha,\beta)=-\frac{2}{7}\cos[3(\alpha-\beta)]-\cos(\alpha-\beta)$.



Overview

1. General framework of “spacetime boxes”

2. Foundational insights



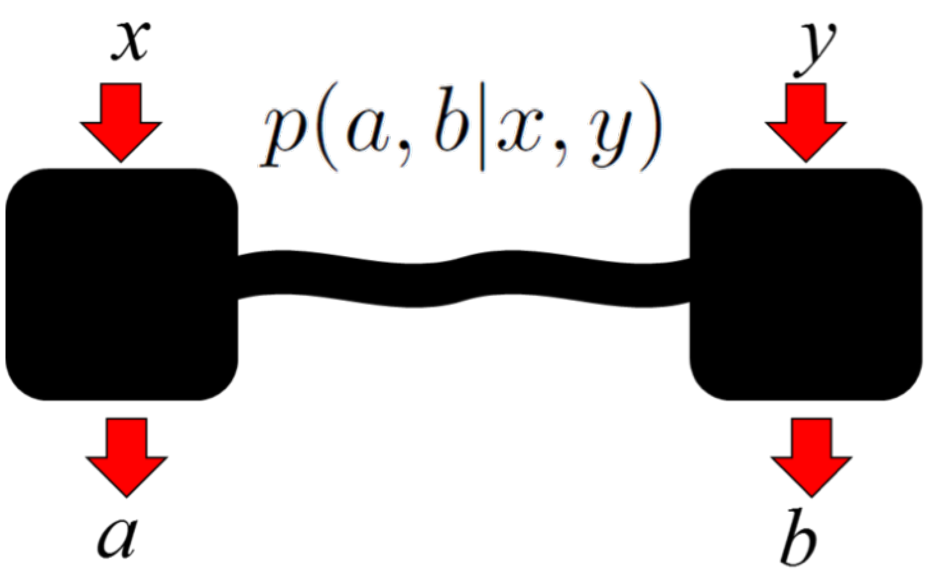
3. Towards novel protocols…

4. … and experimental tests of QT

Overview

1. General framework of "spacetime boxes"

2. Foundational insights



3. Towards novel protocols…

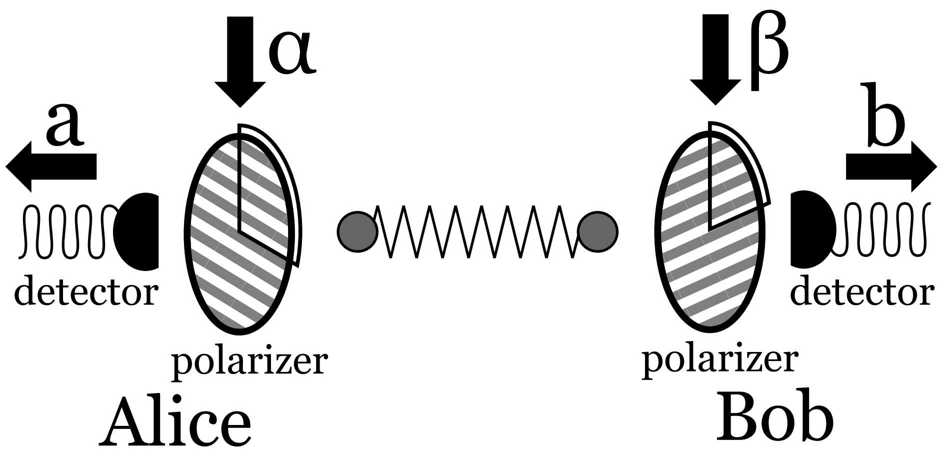
4. … and experimental tests of QT

Foundational consequences

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

Foundational consequences

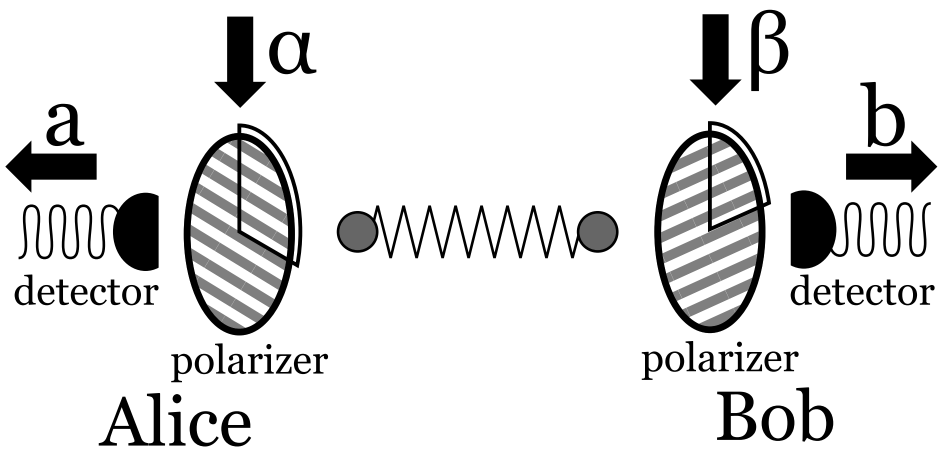
A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).



$$\mathcal{G} = \mathrm{SO}(d) \times \mathrm{SO}(d)$$

Foundational consequences

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

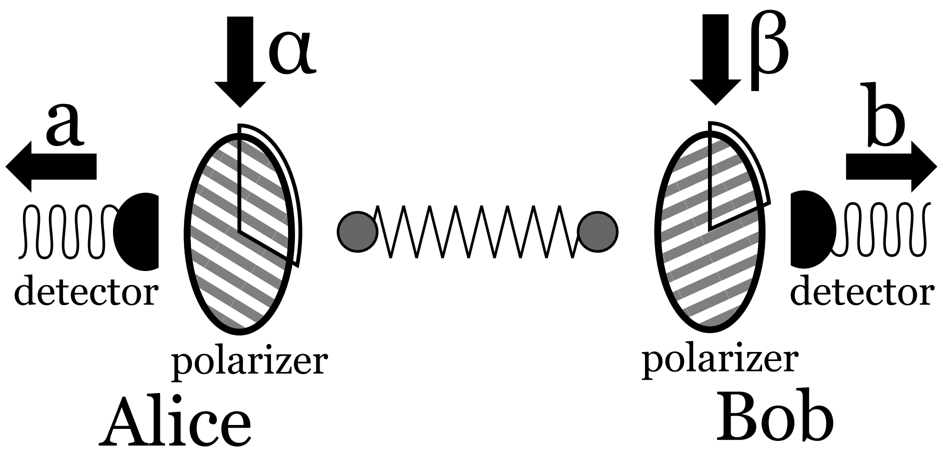


$$\mathcal{G} = \mathrm{SO}(d) \times \mathrm{SO}(d)$$

Assumptions for now:
Probabilities transform **locally fundamentally,** i.e. $P(a, b|R\vec{x}_0, S\vec{y}_0)$ is linear in the rotation matrices R, S.

Foundational consequences

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).



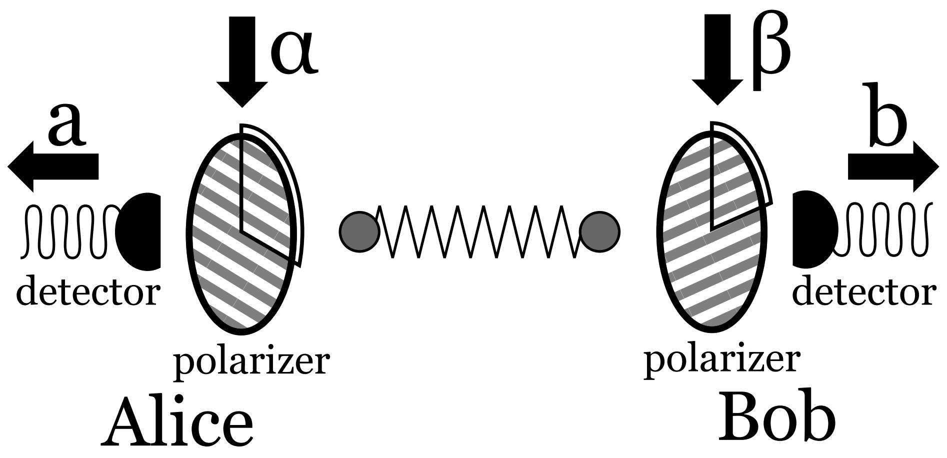
$$\mathcal{G} = \mathrm{SO}(d) \times \mathrm{SO}(d)$$

Assumptions for now:
Probabilities transform **locally fundamentally,** i.e. $P(a, b | R\vec{x}_0, S\vec{y}_0)$ is linear in the rotation matrices R, S.

Local unbiasedness: $a, b \in \{-1, +1\}$, averaging over all *R* gives $\langle a \rangle = 0$.

Foundational consequences

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).



$$\mathcal{G} = \mathrm{SO}(d) \times \mathrm{SO}(d)$$

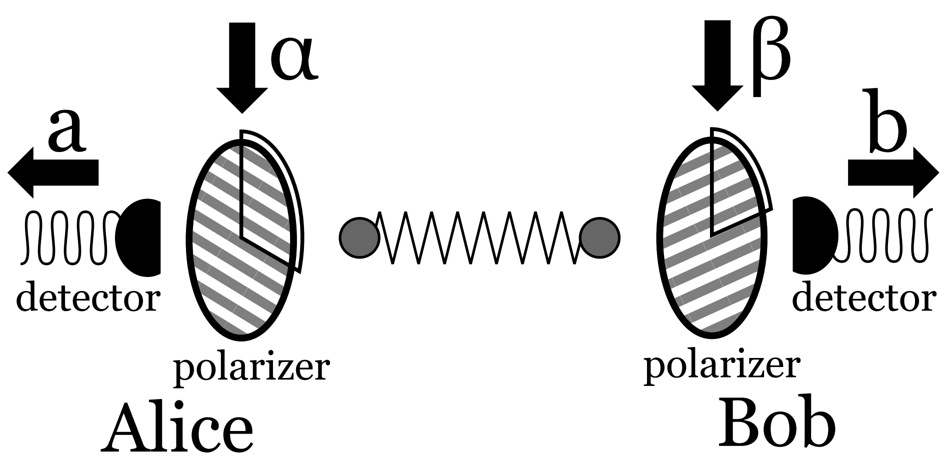
Assumptions for now:
Probabilities transform **locally fundamentally,** i.e. $P(a, b|R\vec{x}_0, S\vec{y}_0)$ is linear in the rotation matrices R, S.

Local unbiasedness: $a, b \in \{-1, +1\}$, averaging over all *R* gives $\langle a \rangle = 0$.

Theorem. In any world where these assumptions hold (**not assuming QT!**), Alice and Bob see **quantum correlations** (i.e. in **Q**).

Foundational consequences

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).



$$\mathcal{G} = \mathrm{SO}(d) \times \mathrm{SO}(d)$$

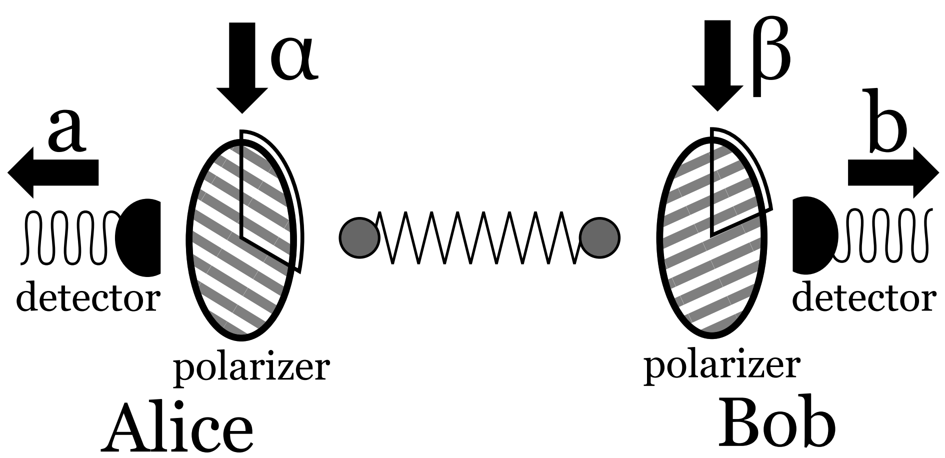
Assumptions for now:
Probabilities transform **locally fundamentally,**
i.e. $P(a,b|R\vec{x}_0, S\vec{y}_0)$ is linear in the rotation matrices R, S.

Local unbiasedness: $a, b \in \{-1, +1\}$,
averaging over all *R* gives $\langle a \rangle = 0$.

Theorem: The quantum (2,2,2)-correlations **Q** are **exactly those** that can be obtained by $\mathrm{SO}(d) \times \mathrm{SO}(d)$-boxes that transform locally fundamentally and are locally unbiased, restricted to two inputs per party, and supplemented by shared randomness.

Foundational consequences

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

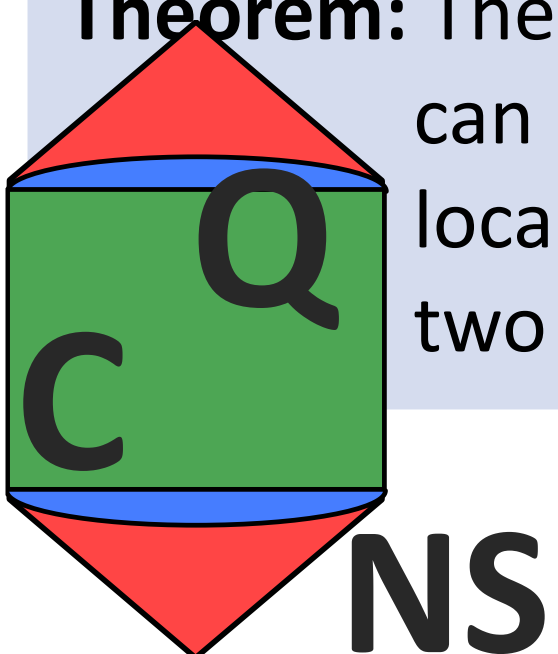


$$\mathcal{G} = \mathrm{SO}(d) \times \mathrm{SO}(d)$$

Assumptions for now:
Probabilities transform **locally fundamentally,** i.e. $P(a,b|R\vec{x}_0, S\vec{y}_0)$ is linear in the rotation matrices R, S.

Local unbiasedness: $a, b \in \{-1,+1\}$, averaging over all *R* gives $\langle a \rangle = 0$.

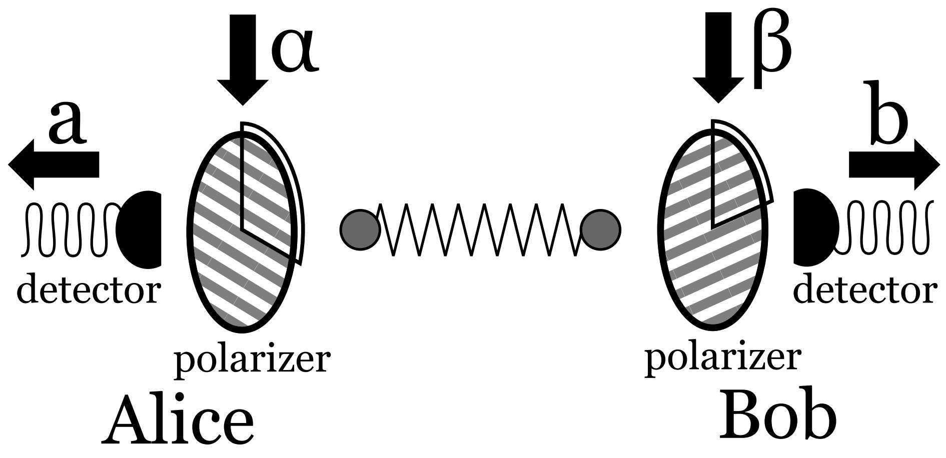
Theorem: The quantum (2,2,2)-correlations **Q** are **exactly those** that can be obtained by $\mathrm{SO}(d) \times \mathrm{SO}(d)$-boxes that transform locally fundamentally and are locally unbiased, restricted to two inputs per party, and supplemented by shared randomness.



Fundamental relation between QT and space(time)?

Recall the *d=2* case

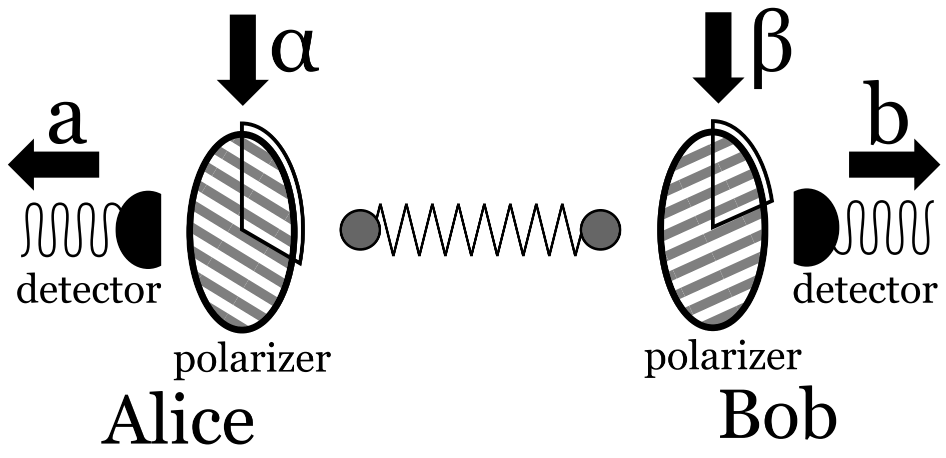
Recall the *d=2* case



$$\mathcal{G} = \mathrm{SO}(2) \times \mathrm{SO}(2)$$

$$P(a, b|\alpha, \beta) := \sum_{m=0}^{2J} \sum_{n=-2J}^{2J} c_{mn}^{ab} \cos(m\alpha - n\beta) + s_{mn}^{ab} \sin(m\alpha - n\beta),$$

Recall the *d=2* case

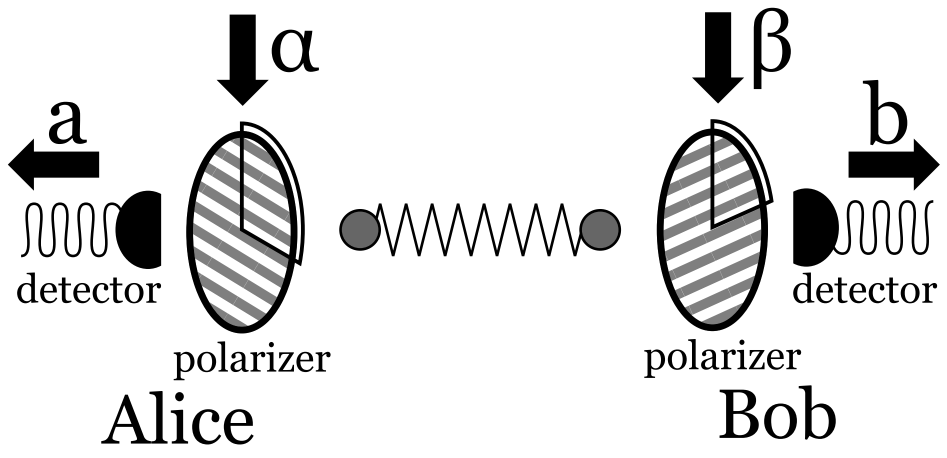


$$\mathcal{G} = \mathrm{SO}(2) \times \mathrm{SO}(2)$$

$$P(a, b|\alpha, \beta) := \sum_{m=0}^{2J} \sum_{n=-2J}^{2J} c_{mn}^{ab} \cos(m\alpha - n\beta) + s_{mn}^{ab} \sin(m\alpha - n\beta),$$

“Transforming locally fundamentally” basically means $J \leq \frac{1}{2}$.

Recall the *d=2* case



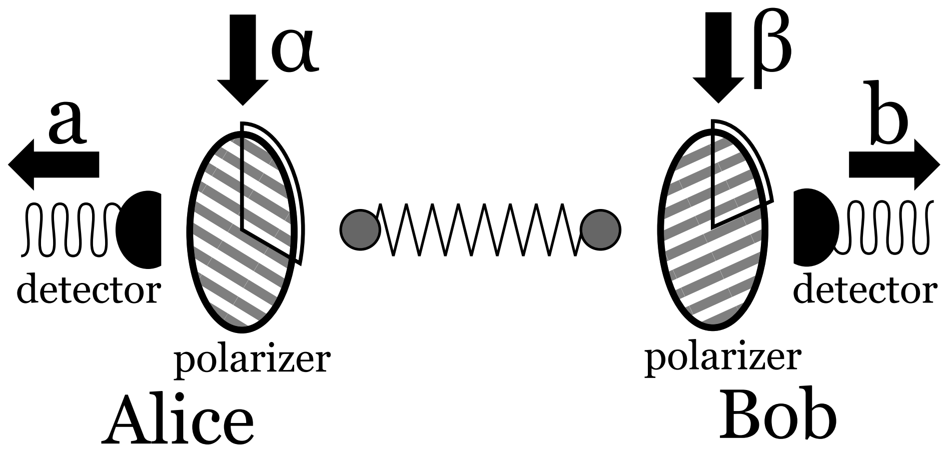
$$\mathcal{G} = \mathrm{SO}(2) \times \mathrm{SO}(2)$$

$$P(a, b|\alpha, \beta) := \sum_{m=0}^{2J} \sum_{n=-2J}^{2J} c_{mn}^{ab} \cos(m\alpha - n\beta) + s_{mn}^{ab} \sin(m\alpha - n\beta),$$

"Transforming locally fundamentally" basically means $J \leq \frac{1}{2}$.

Hence, **bounding the representation label can severely constrain the possible correlations.**

Recall the *d=2* case



$$\mathcal{G} = \mathrm{SO}(2) \times \mathrm{SO}(2)$$

$$P(a,b|\alpha,\beta) := \sum_{m=0}^{2J} \sum_{n=-2J}^{2J} c_{mn}^{ab} \cos(m\alpha - n\beta) + s_{mn}^{ab} \sin(m\alpha - n\beta),$$

"Transforming locally fundamentally" basically means $J \leq \frac{1}{2}$.

Hence, **bounding the representation label can severely constrain the possible correlations.**

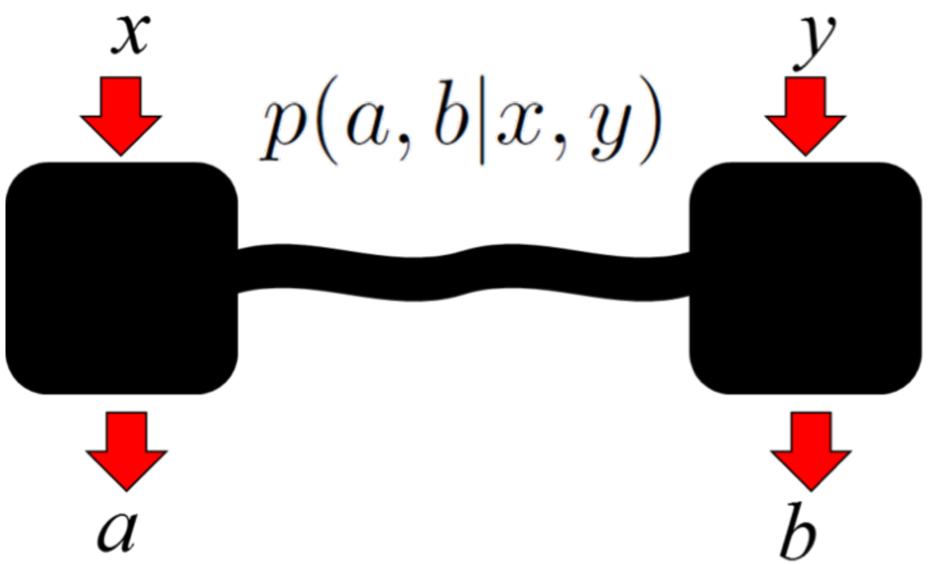
This amounts to an assumption of "how the devices respond to spatiotemporal symmetry transformations".

Idea: use this for **protocols**.

Overview

1. General framework of "spacetime boxes"

2. Foundational insights



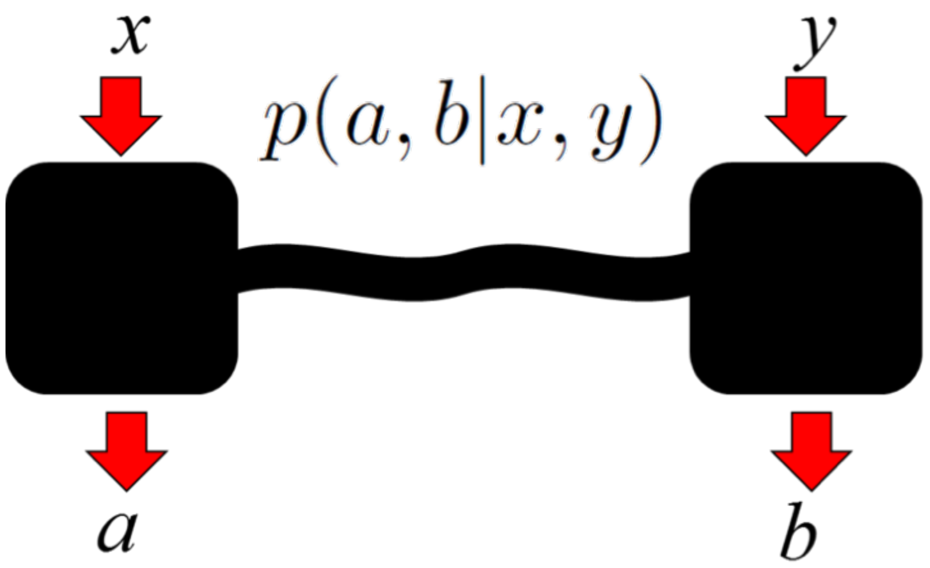
3. Towards novel protocols…

4. … and experimental tests of QT

Overview

1. General framework of "spacetime boxes"

2. Foundational insights

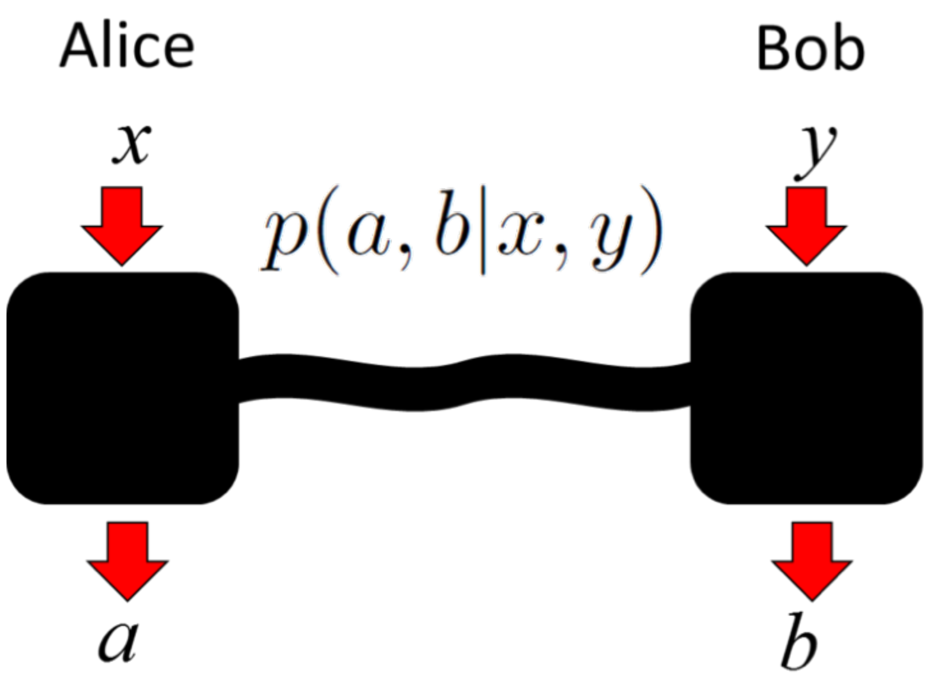


3. Towards novel protocols…

4. … and experimental tests of QT

Towards protocols

Device-independent QIT:



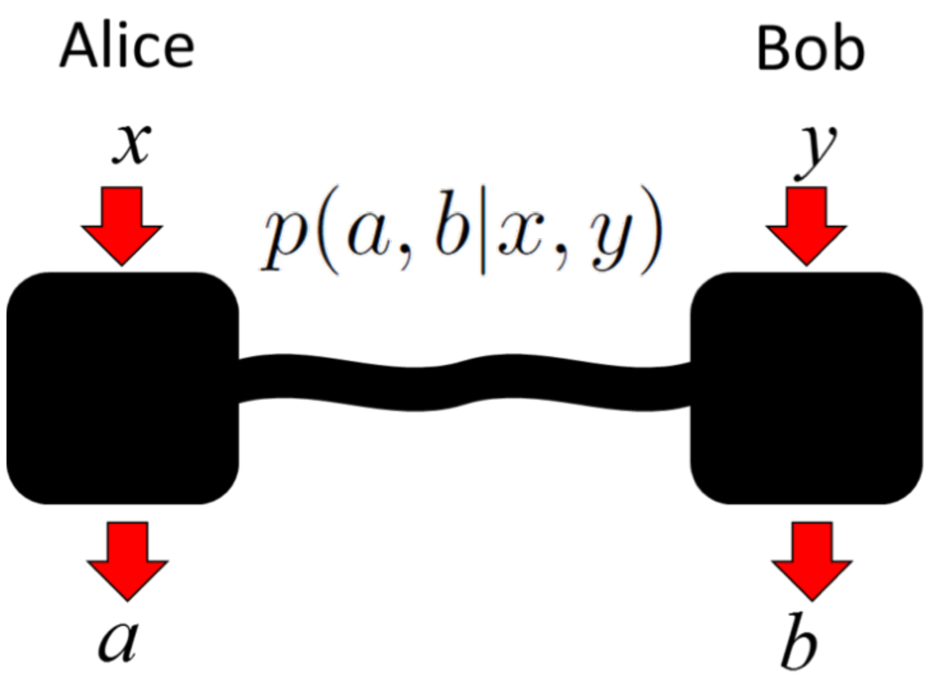
Violation of a Bell inequality admits

- randomness expansion
- cryptography

even if **devices are untrusted**.

Towards protocols

Device-independent QIT:

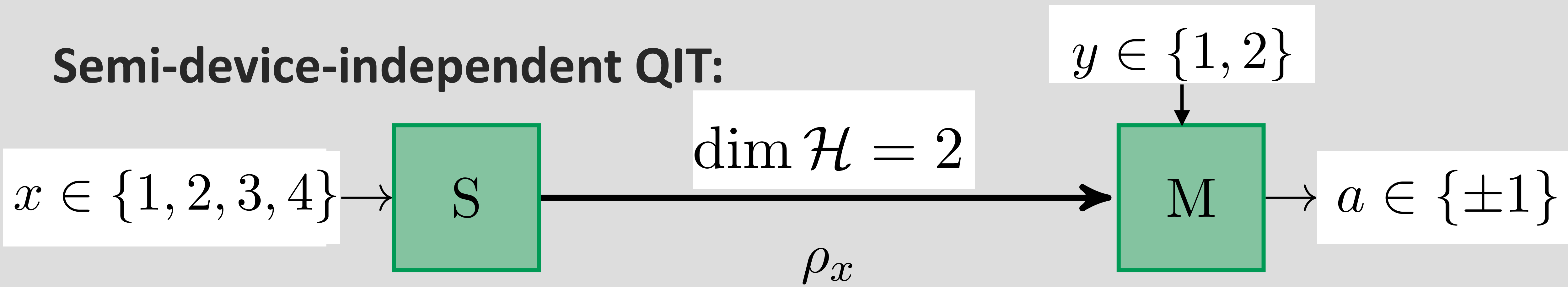


Violation of a Bell inequality admits

- randomness expansion
- cryptography

even if **devices are untrusted**.

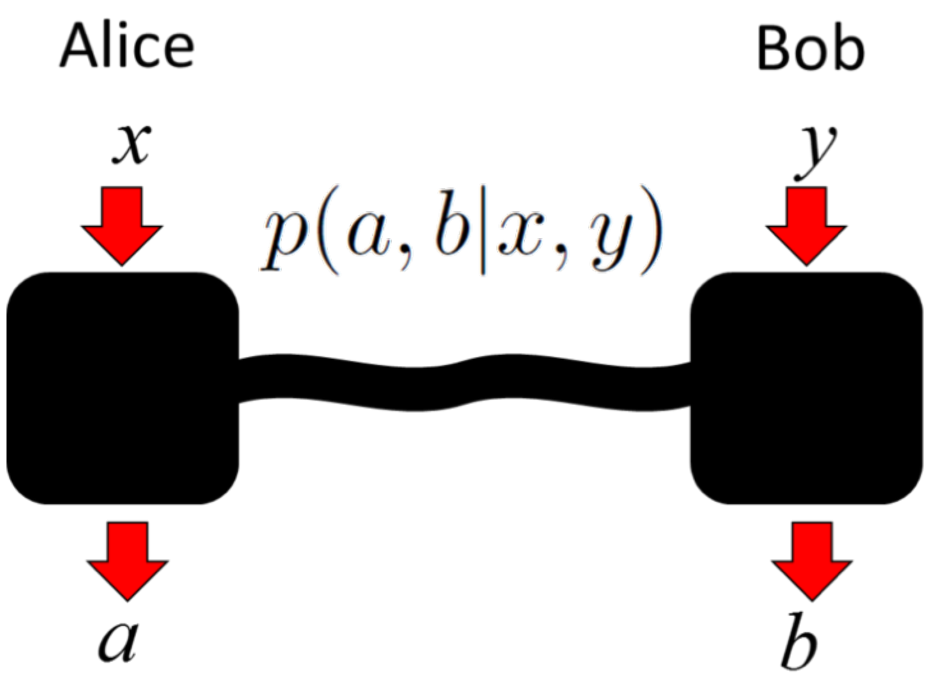
Semi-device-independent QIT:



Devices untrusted, but **some assumptions on transmitted states** have to be made.

Towards protocols

Device-independent QIT:

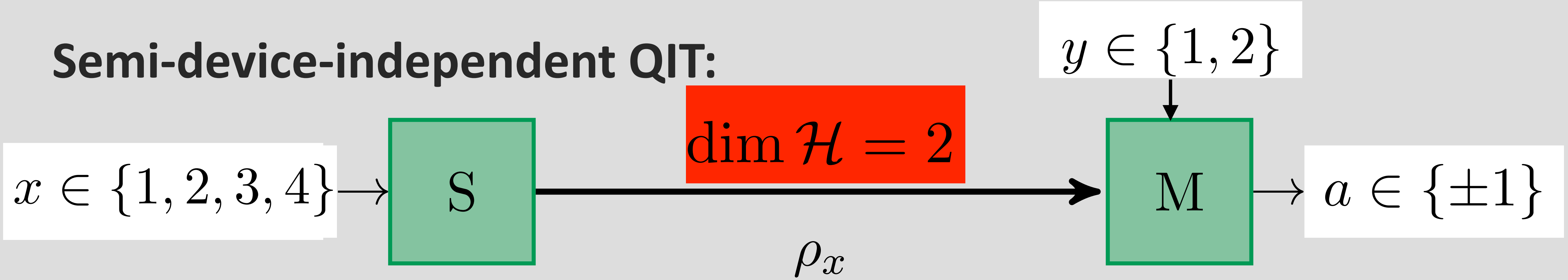


Violation of a Bell inequality admits

- randomness expansion
- cryptography

even if **devices are untrusted**.

Semi-device-independent QIT:



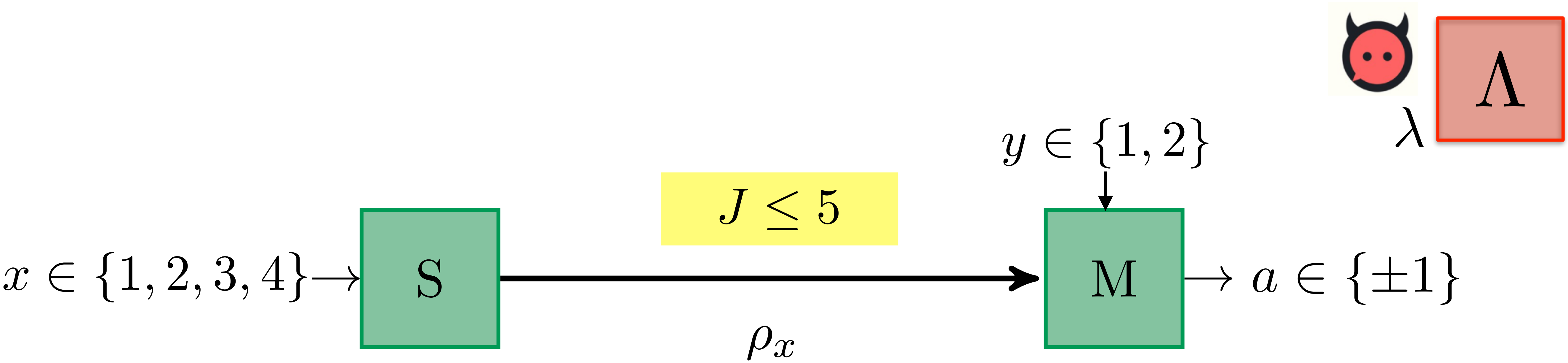
Physical motivation?

Devices untrusted, but **some assumptions on transmitted states** have to be made.

Towards protocols

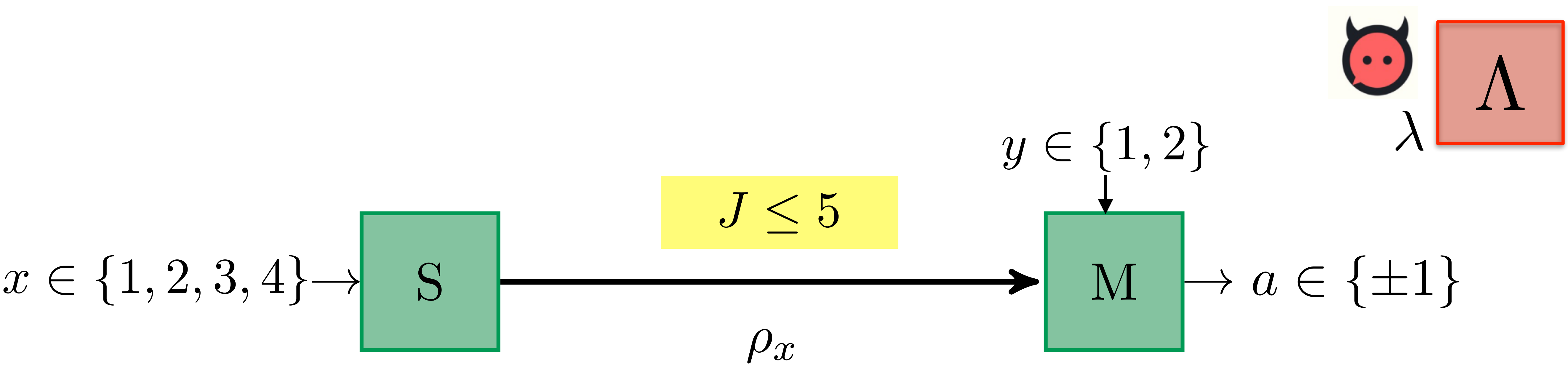
Towards protocols

Idea: For SDI protocols, replace dimension bounds by **physically better motivated assumptions** on how systems respond to symmetries.



Towards protocols

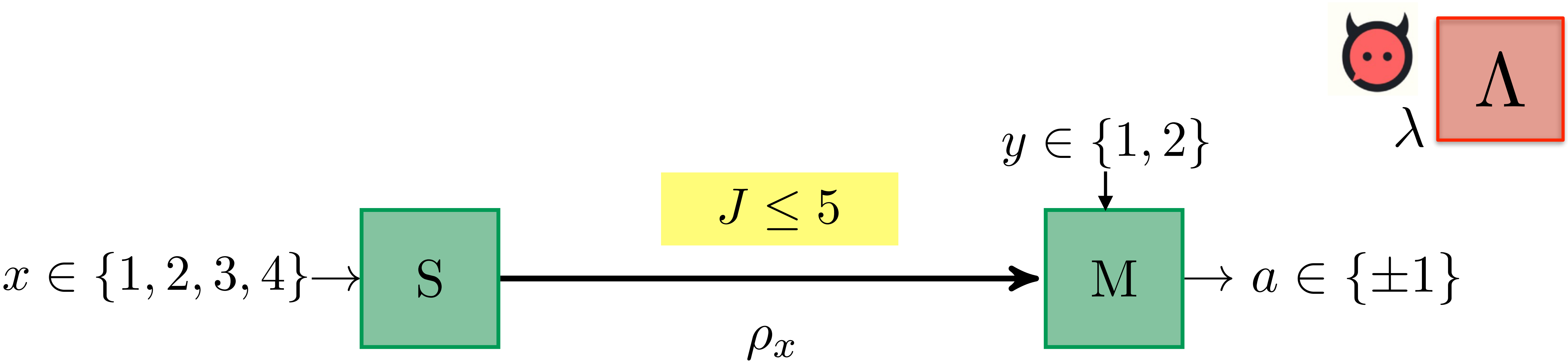
Idea: For SDI protocols, replace dimension bounds by **physically better motivated assumptions** on how systems respond to symmetries.



For $\mathcal{G}$=time translations, this corresponds to **energy upper bounds**.

Towards protocols

Idea: For SDI protocols, replace dimension bounds by **physically better motivated assumptions** on how systems respond to symmetries.



For $\mathcal{G}$=time translations, this corresponds to **energy upper bounds**.

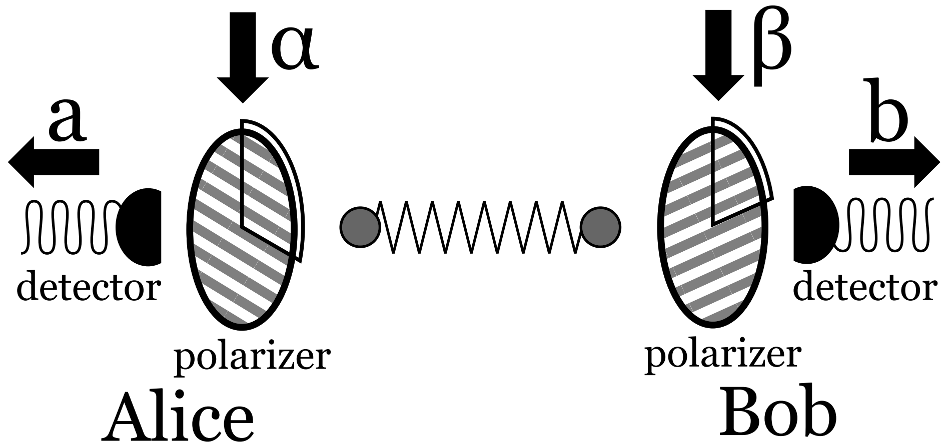
Also, closer to **particle physics intuition**: don't count dimensions, but representation labels (of the Poincaré group).

A proof of principle

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

A proof of principle

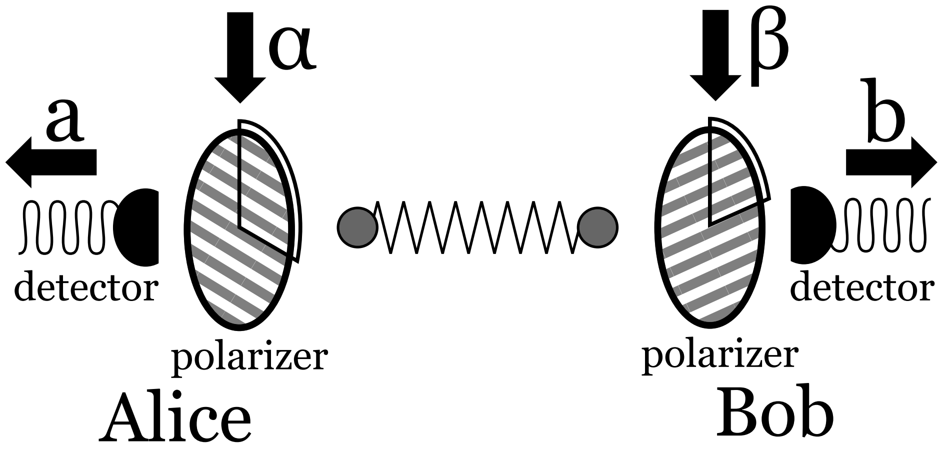
A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).



$$P(a,b|\alpha,\beta) := \sum_{m=0}^{2J}\sum_{n=-2J}^{2J} c_{mn}^{ab}\cos(m\alpha - n\beta) + s_{mn}^{ab}\sin(m\alpha - n\beta),$$

A proof of principle

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

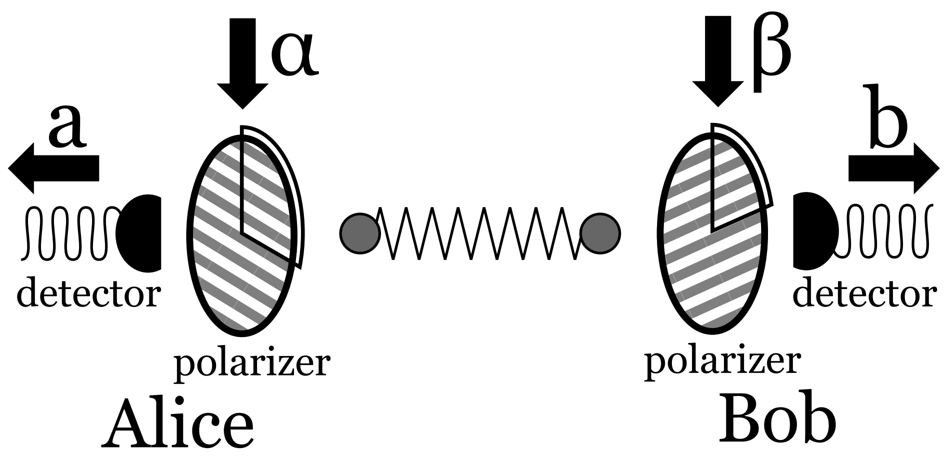


$$P(a,b|\alpha,\beta) := \sum_{m=0}^{2J} \sum_{n=-2J}^{2J} c_{mn}^{ab} \cos(m\alpha - n\beta) + s_{mn}^{ab} \sin(m\alpha - n\beta),$$

Suppose we have a physically well-motivated belief that $J \leq J_0 < \infty$.

A proof of principle

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).



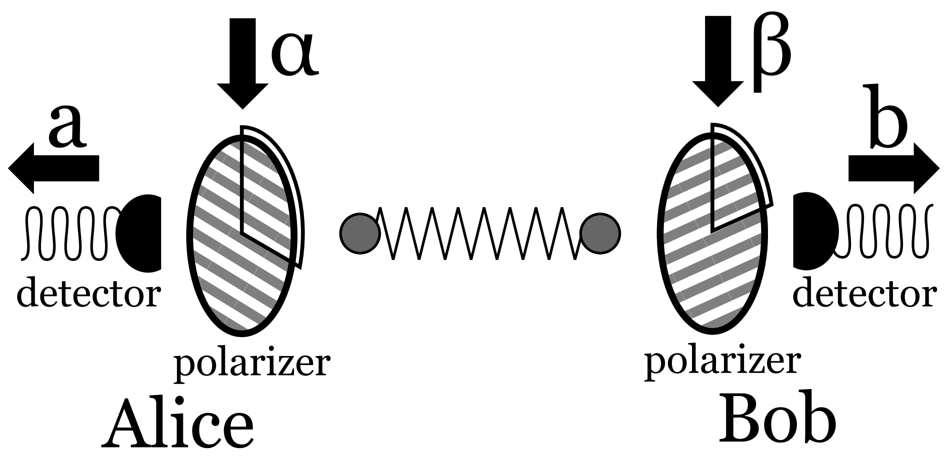
$$P(a,b|\alpha,\beta) := \sum_{m=0}^{2J}\sum_{n=-2J}^{2J} c_{mn}^{ab}\cos(m\alpha - n\beta) + s_{mn}^{ab}\sin(m\alpha - n\beta),$$

Suppose we have a physically well-motivated belief that $J \leq J_0 < \infty$.

Protocol: • Alice and Bob share random angles, uniform in $\lambda \in [0, 2\pi)$.

A proof of principle

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).



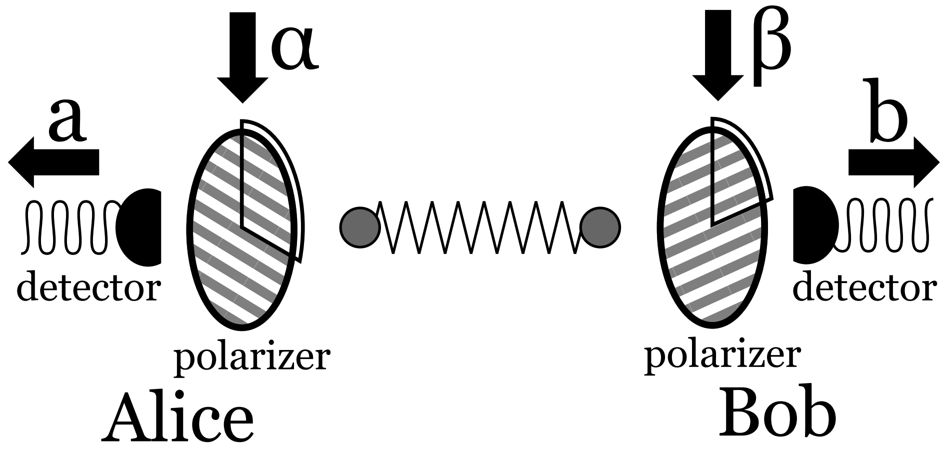
$$P(a,b|\alpha,\beta) := \sum_{m=0}^{2J}\sum_{n=-2J}^{2J} c_{mn}^{ab}\cos(m\alpha - n\beta) + s_{mn}^{ab}\sin(m\alpha - n\beta),$$

Suppose we have a physically well-motivated belief that $J \leq J_0 < \infty$.

Protocol:
- Alice and Bob share random angles, uniform in $\lambda \in [0, 2\pi)$.
- Alice chooses freely between $\alpha \in \{\Theta_+, \Theta_-\}$.

A proof of principle

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).



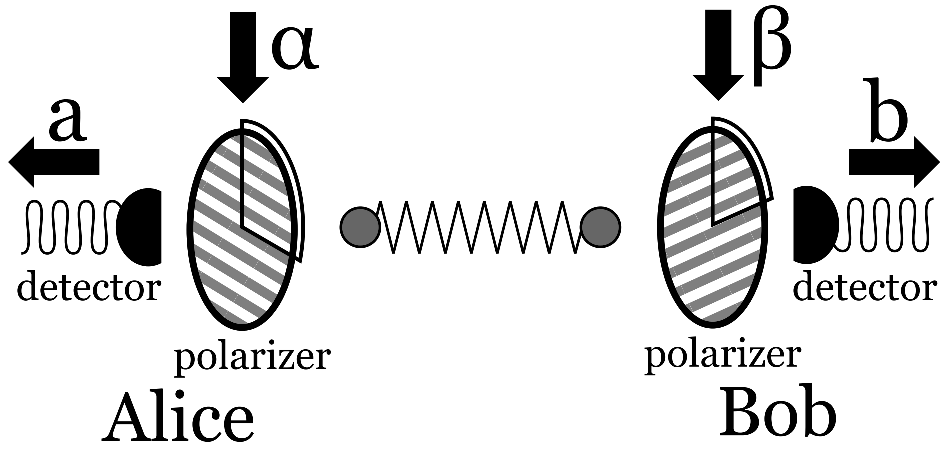
$$P(a,b|\alpha,\beta) := \sum_{m=0}^{2J} \sum_{n=-2J}^{2J} c_{mn}^{ab} \cos(m\alpha - n\beta) + s_{mn}^{ab} \sin(m\alpha - n\beta),$$

Suppose we have a physically well-motivated belief that $J \leq J_0 < \infty$.

Protocol:
- Alice and Bob share random angles, uniform in $\lambda \in [0, 2\pi)$.
- Alice chooses freely between $\alpha \in \{\Theta_+, \Theta_-\}$.
- Alice inputs $\alpha + \lambda$ into her box, while Bob inputs λ.

A proof of principle

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).



$$P(a,b|\alpha,\beta) := \sum_{m=0}^{2J} \sum_{n=-2J}^{2J} c_{mn}^{ab} \cos(m\alpha - n\beta) + s_{mn}^{ab} \sin(m\alpha - n\beta),$$

Suppose we have a physically well-motivated belief that $J \leq J_0 < \infty$.

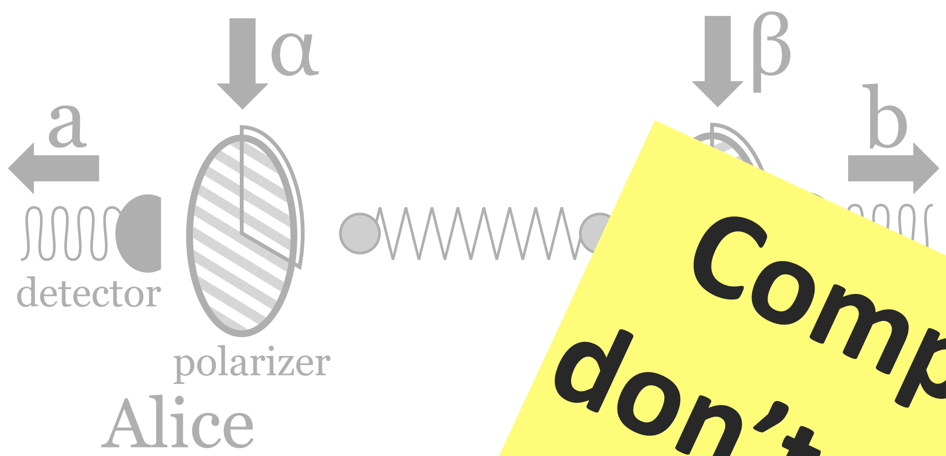
Protocol:
- Alice and Bob share random angles, uniform in $\lambda \in [0, 2\pi)$.
- Alice chooses freely between $\alpha \in \{\Theta_+, \Theta_-\}$.
- Alice inputs $\alpha + \lambda$ into her box, while Bob inputs λ.
- If on repeating many times, they find the correlations

  $$C(\Theta_+) \gtrapprox 1 - J_0^{-3}, \quad C(\Theta_-) \lessapprox -1 - J_0^{-3}$$

  then **nonlocality is certified** with "one-sided free choice" only.

A proof of principle

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).



$$P(a,b|\alpha,\beta) := \sum_{m=0}^{2J}\sum_{n=-2J}^{2J} c_{mn}^{ab}\cos(m\alpha-n\beta) + s_{mn}^{ab}\sin(m\alpha-n\beta),$$

Completely useless, don't try this at home!
But: proof of principle.

Suppose we hav ... belief that $J \leq J_0 < \infty$.

Protocol:
- Alice and Bob sha ... niform in $\lambda \in [0,2\pi)$.
- Alice chooses freely betw ... $\Theta_+,\Theta_-\}$.
- Alice inputs $\alpha+\lambda$ into her box, while Bob inputs λ.
- If on repeating many times, they find the correlations

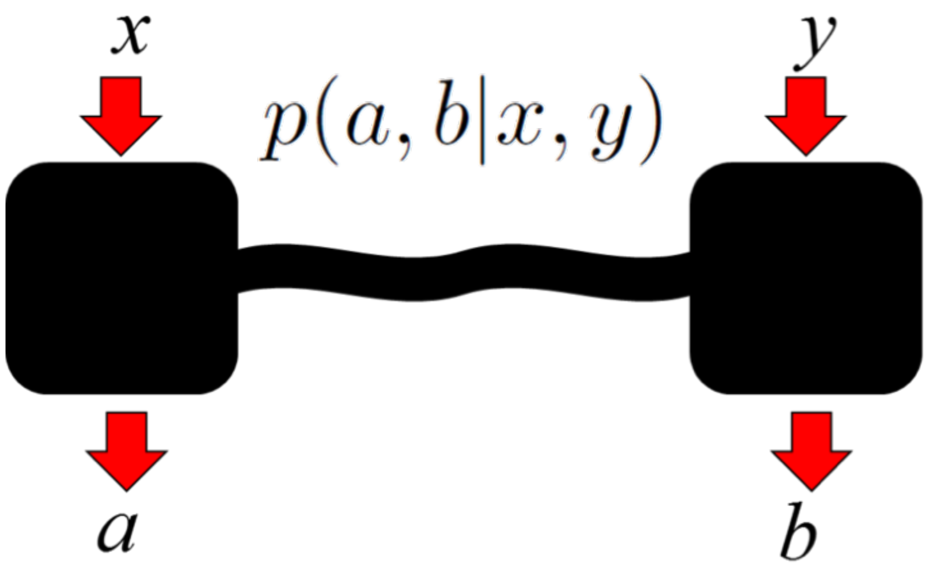
$$C(\Theta_+) \gtrapprox 1 - J_0^{-3}, \quad C(\Theta_-) \lessapprox -1 - J_0^{-3}$$

then **nonlocality is certified** with "one-sided free choice" only.

Overview

1. General framework of “spacetime boxes”

2. Foundational insights



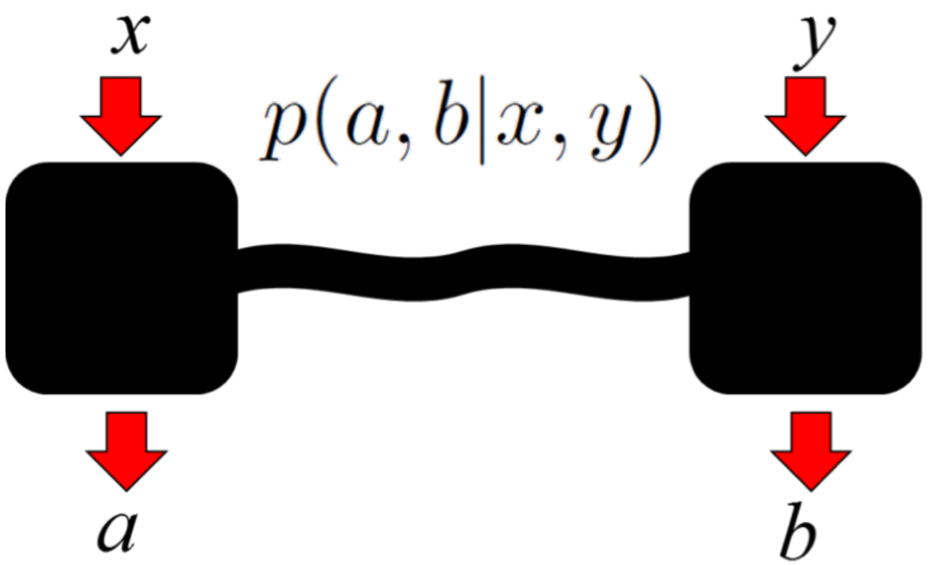
3. Towards novel protocols…

4. … and experimental tests of QT

Overview

1. General framework of “spacetime boxes”

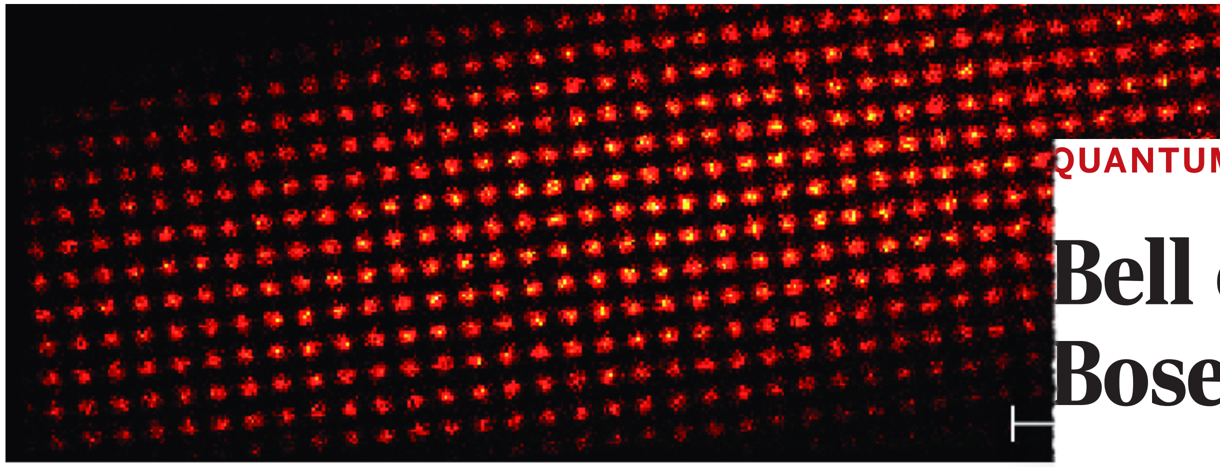
2. Foundational insights

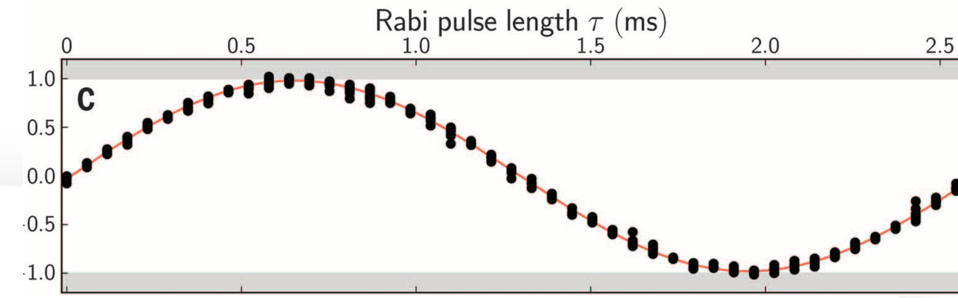


3. Towards novel protocols…

4. … and experimental tests of QT

Experiments as “black boxes”





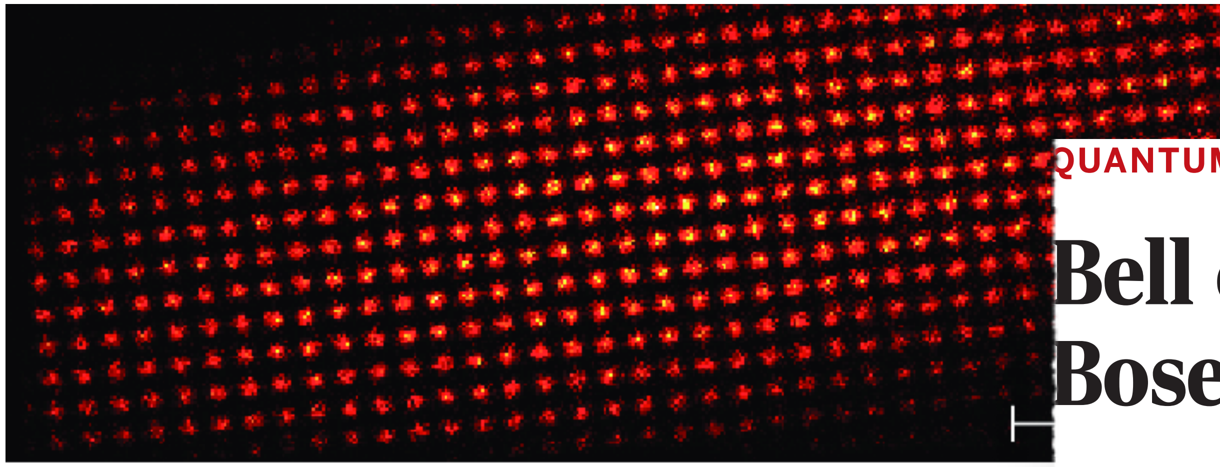
QUANTUM OPTICS

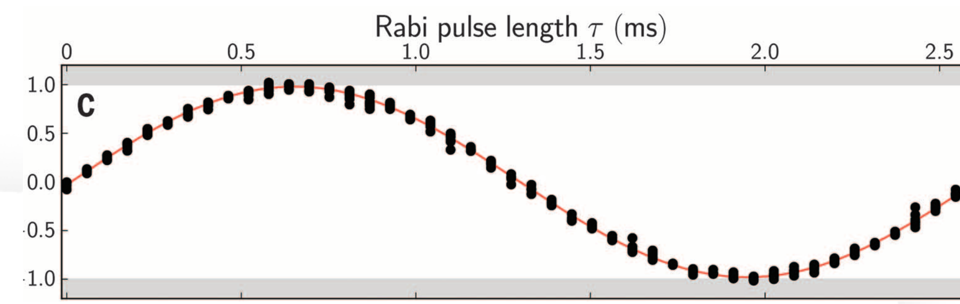
Bell correlations in a Bose-Einstein condensate

Roman Schmied,[1*] Jean-Daniel Bancal,[2,4*] Baptiste Allard,[1*] Matteo Fadel,[1] Valerio Scarani,[2,3] Philipp Treutlein,[1†] Nicolas Sangouard[4†]

Characterizing many-body systems through the quantum correlations between their constituent particles is a major goal of quantum physics. Although entanglement is routinely observed in many systems, we report here the detection of stronger correlations—Bell correlations—between the spins of about 480 atoms in a Bose-Einstein condensate. We derive a Bell correlation witness from a many-particle Bell inequality involving only one- and two-body correlation functions. Our measurement on a spin-squeezed state exceeds the threshold for Bell correlations by 3.8 standard deviations. Our work shows that the strongest possible nonclassical correlations are experimentally accessible in many-body systems and that they can be revealed by collective measurements.

Experiments as "black boxes"





QUANTUM OPTICS

Bell correlations in a Bose-Einstein condensate

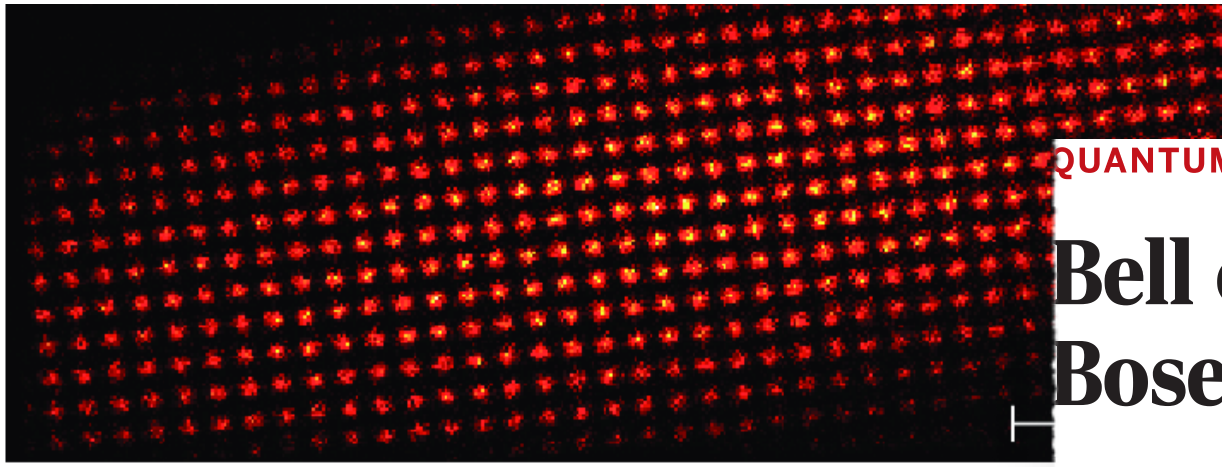
Roman Schmied,[1*] Jean-Daniel Bancal,[2,4*] Baptiste Allard,[1*] Matteo Fadel,[1]
Valerio Scarani,[2,3] Philipp Treutlein,[1†] Nicolas Sangouard[4†]

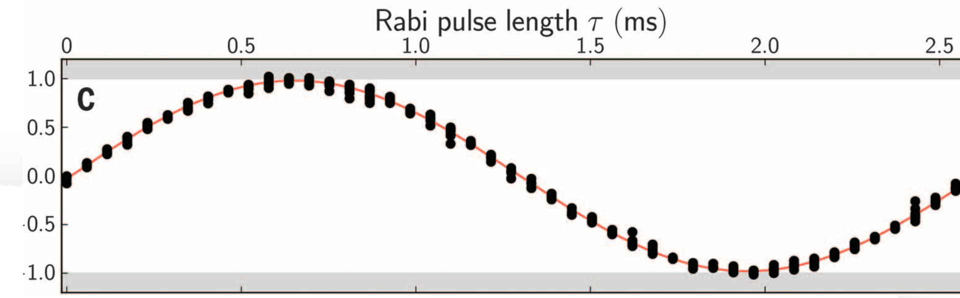
Characterizing many-body systems through the quantum correlations between their constituent particles is a major goal of quantum physics. Although entanglement is routinely observed in many systems, we report here the detection of stronger correlations—Bell correlations—between the spins of about 480 atoms in a Bose-Einstein condensate. We derive a Bell correlation witness from a many-particle Bell inequality involving only one- and two-body correlation functions. Our measurement on a spin-squeezed state exceeds the threshold for Bell correlations by 3.8 standard deviations. Our work shows that the strongest possible nonclassical correlations are experimentally accessible in many-body systems and that they can be revealed by collective measurements.

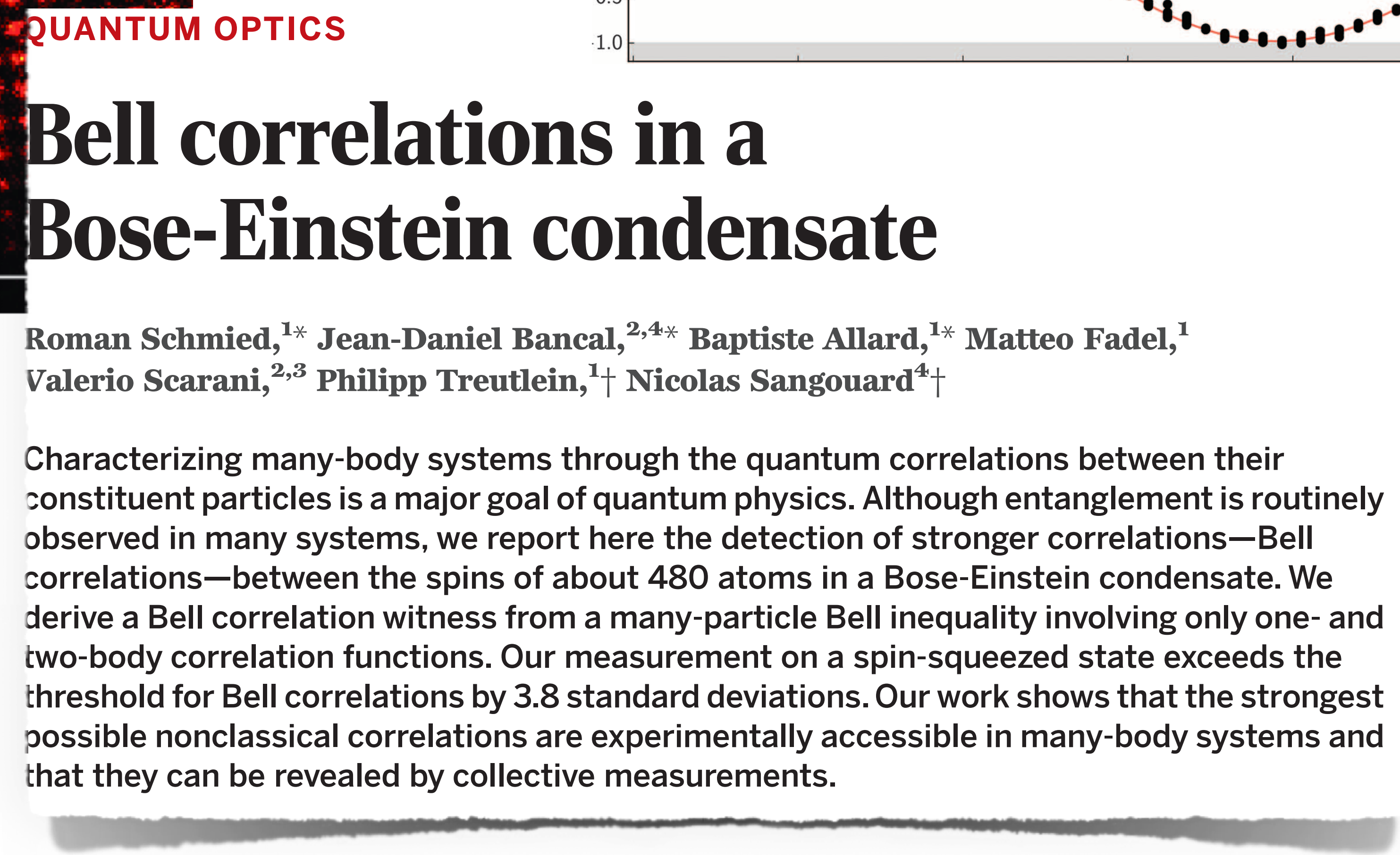
Sometimes, **all we know for sure** is that we've sent a pulse of a certain duration (or some other S.T.-quantity) and recorded an outcome.

What can we infer **from this alone?**
Or from **very few additional assumptions, incl. (or not) QT?**

Experiments as "black boxes"






QUANTUM OPTICS

Bell correlations in a Bose-Einstein condensate

Roman Schmied,[1*] Jean-Daniel Bancal,[2,4*] Baptiste Allard,[1*] Matteo Fadel,[1] Valerio Scarani,[2,3] Philipp Treutlein,[1†] Nicolas Sangouard[4†]

Characterizing many-body systems through the quantum correlations between their constituent particles is a major goal of quantum physics. Although entanglement is routinely observed in many systems, we report here the detection of stronger correlations—Bell correlations—between the spins of about 480 atoms in a Bose-Einstein condensate. We derive a Bell correlation witness from a many-particle Bell inequality involving only one- and two-body correlation functions. Our measurement on a spin-squeezed state exceeds the threshold for Bell correlations by 3.8 standard deviations. Our work shows that the strongest possible nonclassical correlations are experimentally accessible in many-body systems and that they can be revealed by collective measurements.

Sometimes, **all we know for sure** is that we've sent a pulse of a certain duration (or some other S.T.-quantity) and recorded an outcome.

What can we infer **from this alone?**
Or from **very few additional assumptions, incl. (or not) QT?**

Under what conditions could the result **falsify Quantum Theory**?

Conclusions

- “**Spacetime boxes**” via group representation theory.
- Foundational insights: study of **interplay probability vs. spacetime**, exact characterization of the **quantum (2,2,2)-correlations**.
- Towards protocols: **bounding representation labels** as a physically well-motivated assumption. “Proof of principle” nonlocality certification.
- Novel experimental tests of QT?

A. J. P. Garner, M. Krumm, and M. P. Müller, *Semi-device-independent information processing with spatiotemporal degrees of freedom*, Phys. Rev. Research **2**, 013112 (2020)

arXiv:1907.09274.

Conclusions

- "**Spacetime boxes**" via group representation theory.
- Foundational insights: study of **interplay probability vs. spacetime**, exact characterization of the **quantum (2,2,2)-correlations**.
- Towards protocols: **bounding representation labels** as a physically well-motivated assumption. "Proof of principle" nonlocality certification.
- Novel experimental tests of QT?

A. J. P. Garner, M. Krumm, and M. P. Müller, *Semi-device-independent information processing with spatiotemporal degrees of freedom*, Phys. Rev. Research **2**, 013112 (2020)
arXiv:1907.09274.

Thank you!