ÖAW AUSTRIAN ACADEMY OF SCIENCES

IQOQI – INSTITUTE FOR QUANTUM OPTICS AND QUANTUM INFORMATION VIENNA

Quantum theory from simple principles

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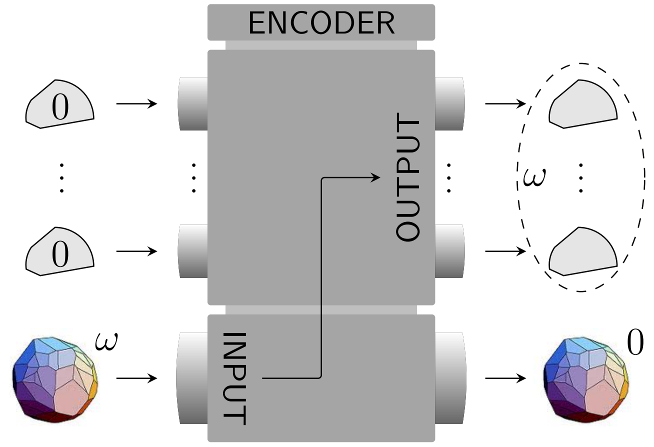
Perimeter Institute for Theoretical Physics (PI), Waterloo, Canada

PERIMETER INSTITUTE

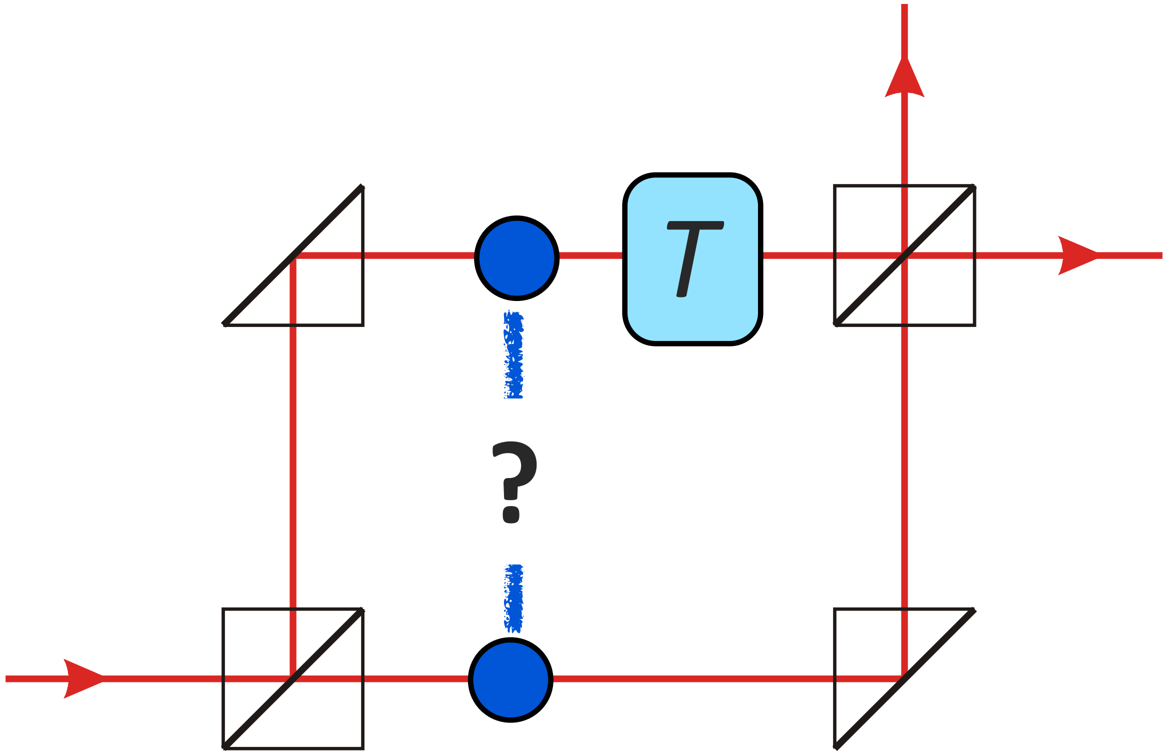
Overview

1. Probabilistic theories beyond quantum theory

2. Quantum theory from simple principles



3. The quest for higher-order interference



4. QT and spacetime

5. Conclusion

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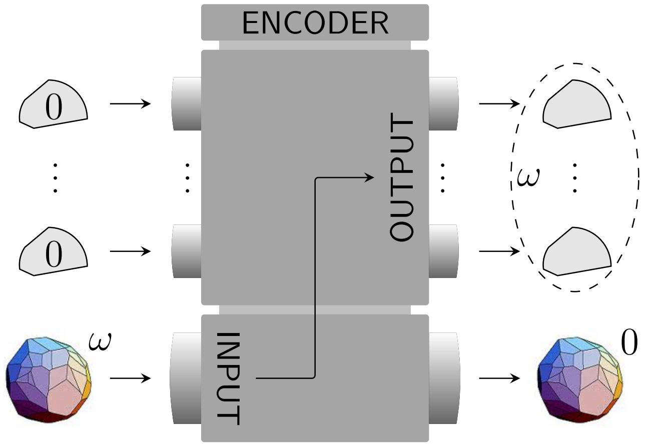

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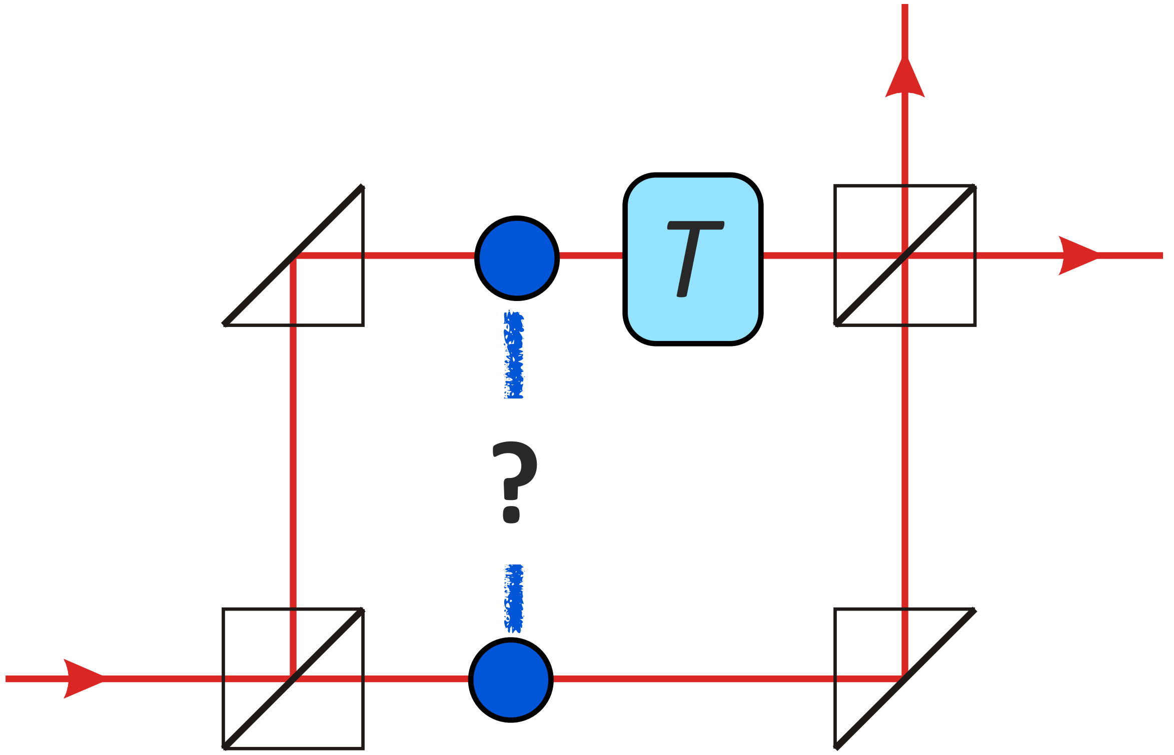

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More general than quantum?

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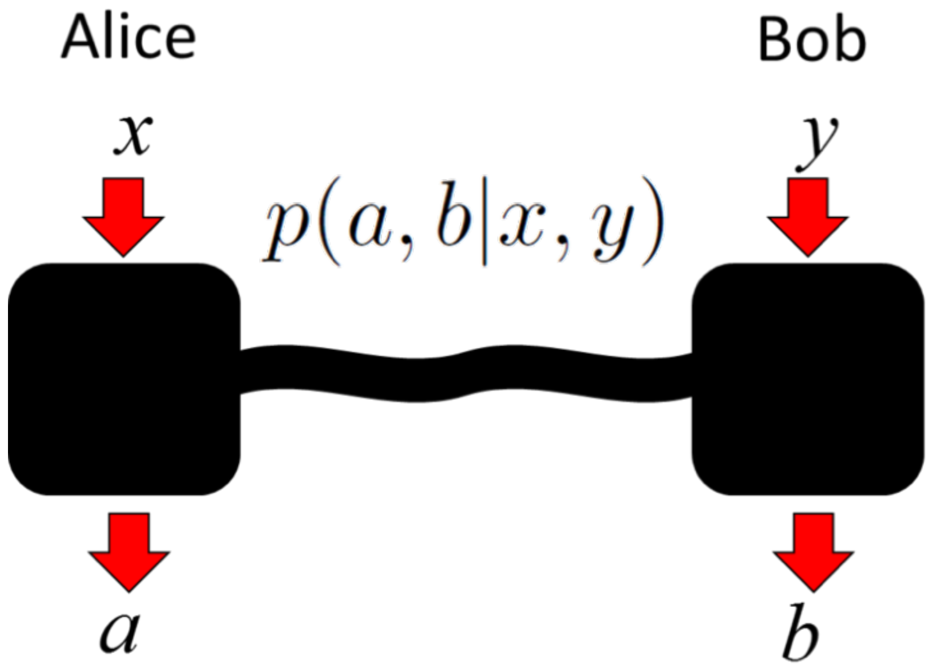
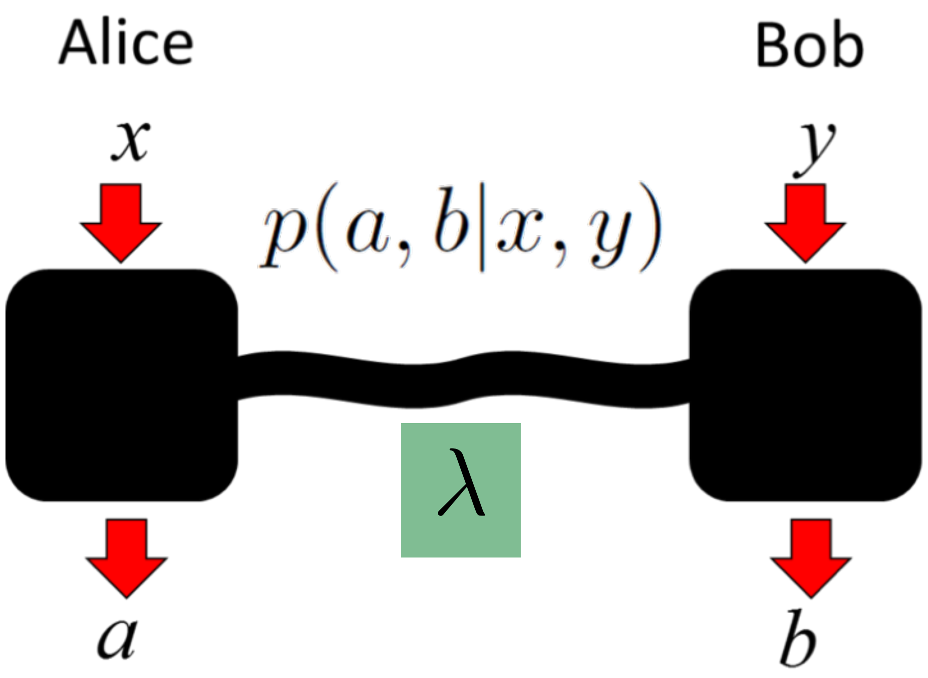


More general than quantum?



- In **classical** physics / prob. theory:

$$P(a,b|x,y) = \sum_{\lambda \in \Lambda} P_A(a|x,\lambda) P_B(b|y,\lambda) P_\Lambda(\lambda)$$

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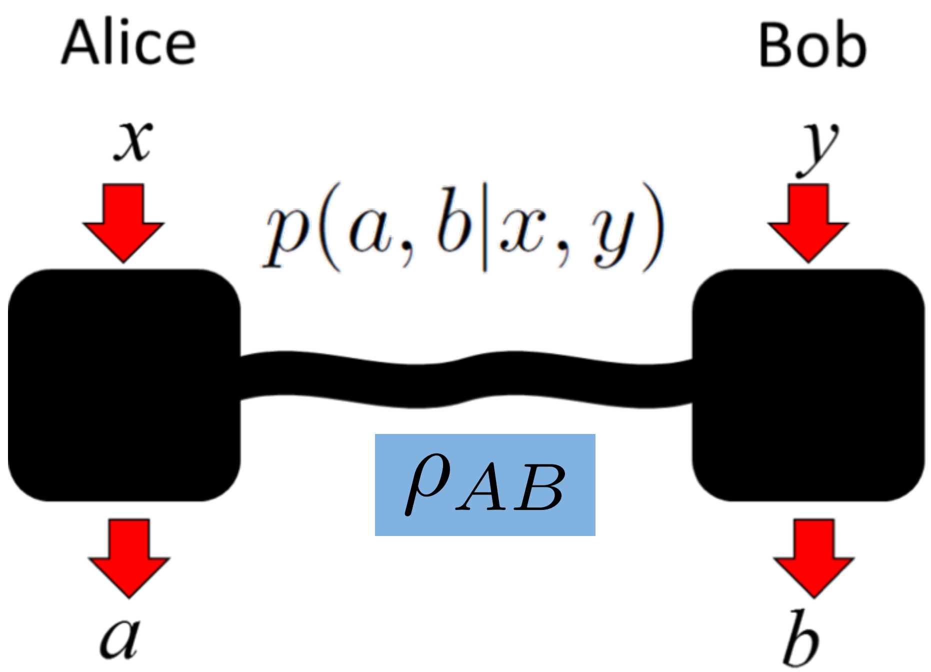

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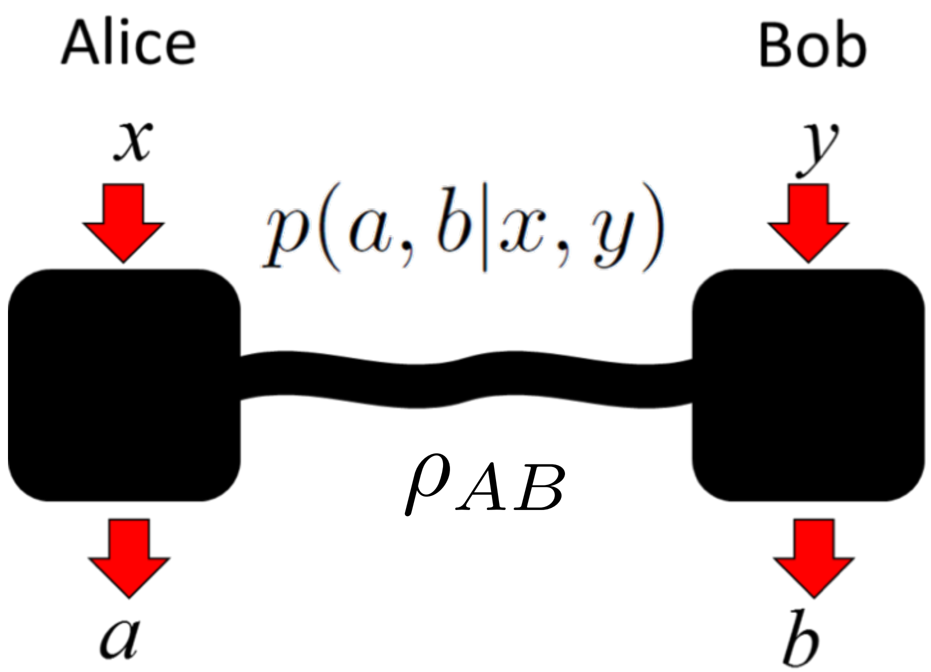

No-signalling conditions:

$P(a|x,y)$ is independent of y,

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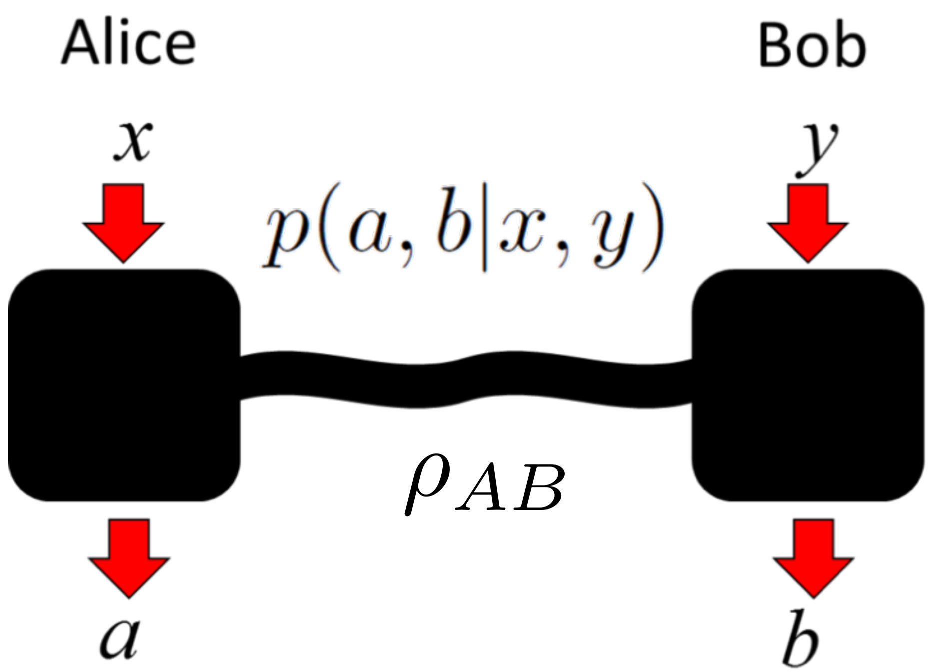

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Quantum admits more general *P*'s due to the **violation of Bell inequalities**.

The Bell-CHSH inequality

Classical probability distributions satisfy Bell inequality:

$$\mathrm{CHSH} := |C_{00} + C_{01} + C_{10} - C_{11}| \leq 2 \quad \text{where} \quad C_{xy} := \mathbb{E}(a \cdot b | x, y).$$

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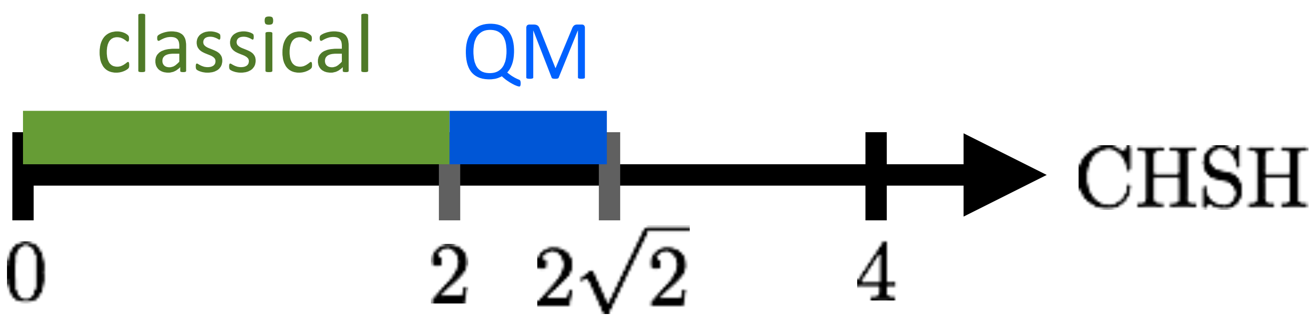

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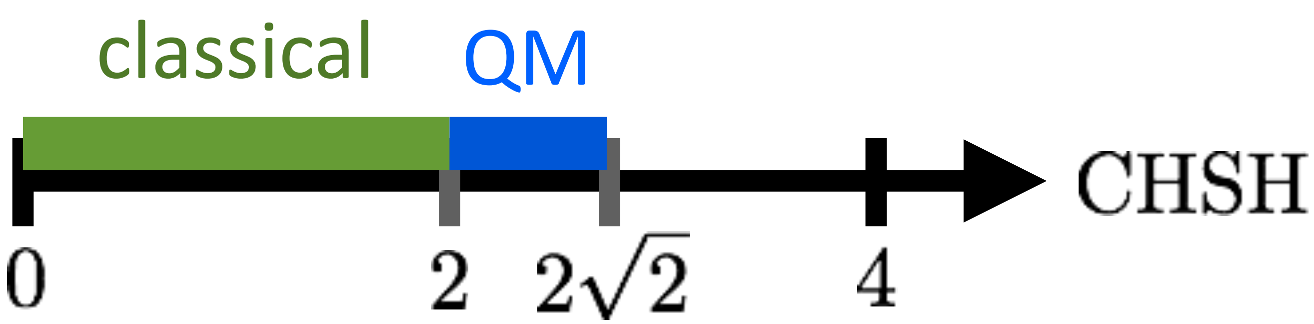

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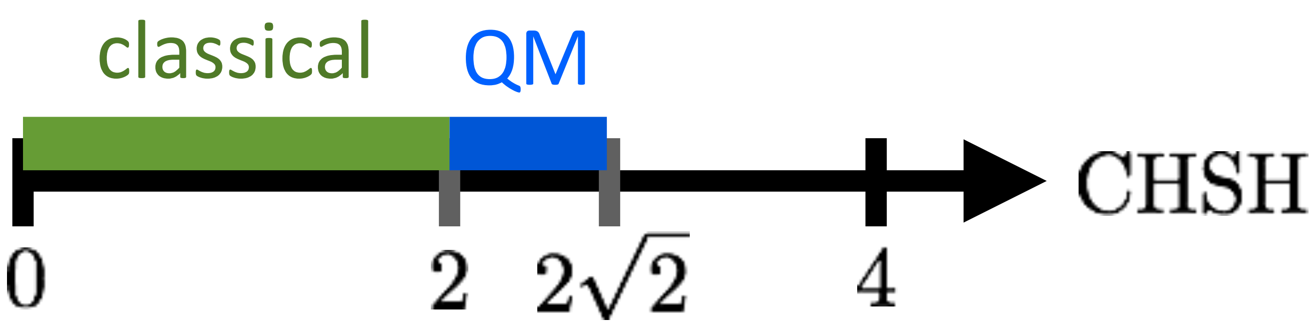

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No! Counterexample: the PR-box correlations

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$$\text{if } (x, y) \in \{(0,0), (0,1), (1,0)\}$$

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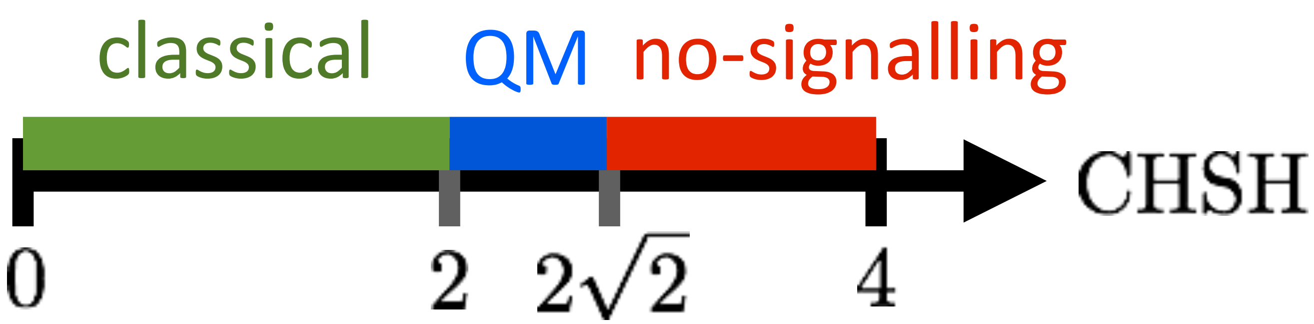

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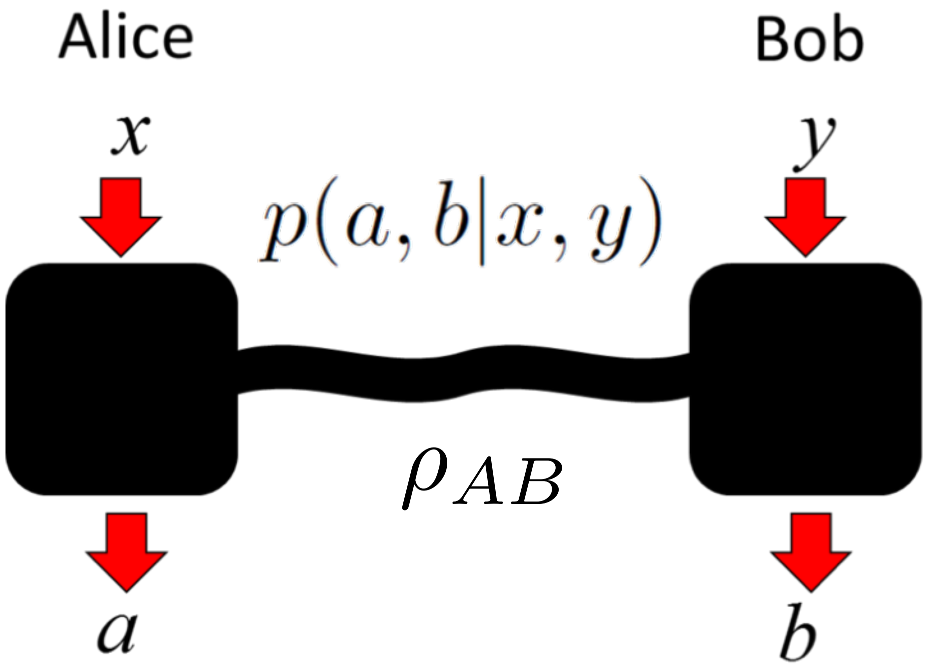

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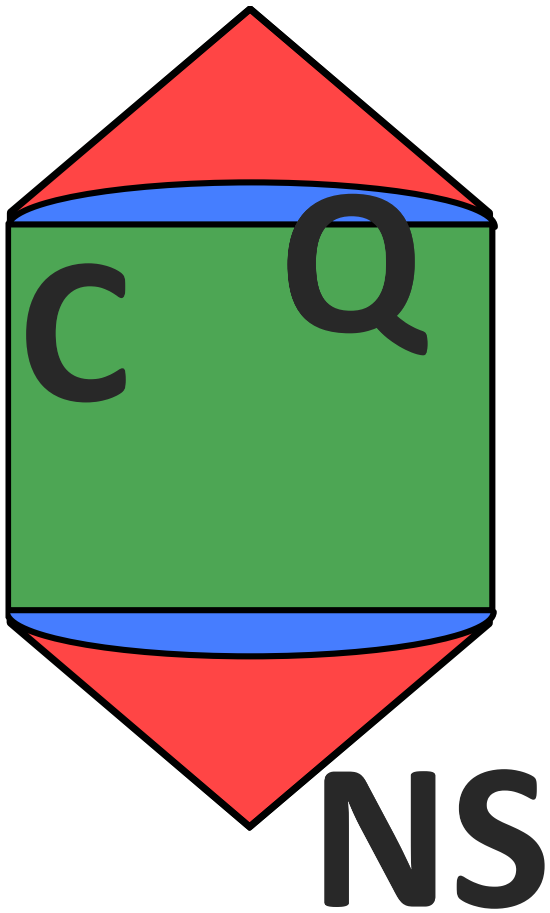

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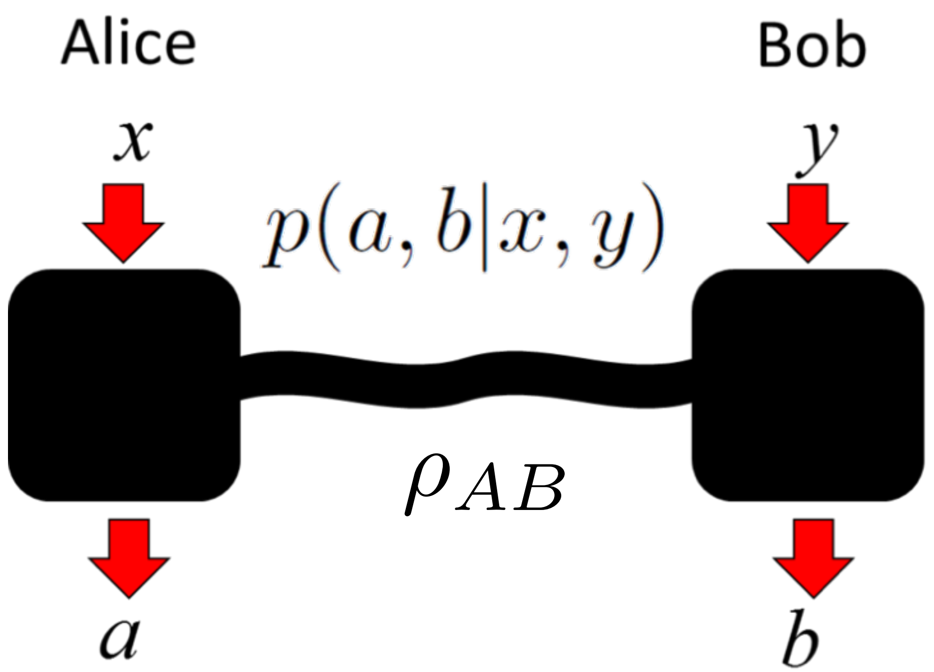

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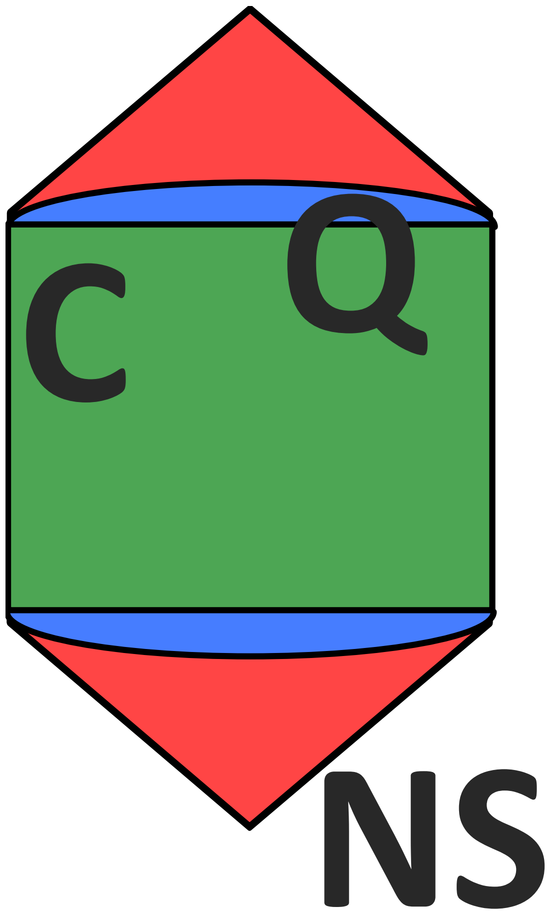

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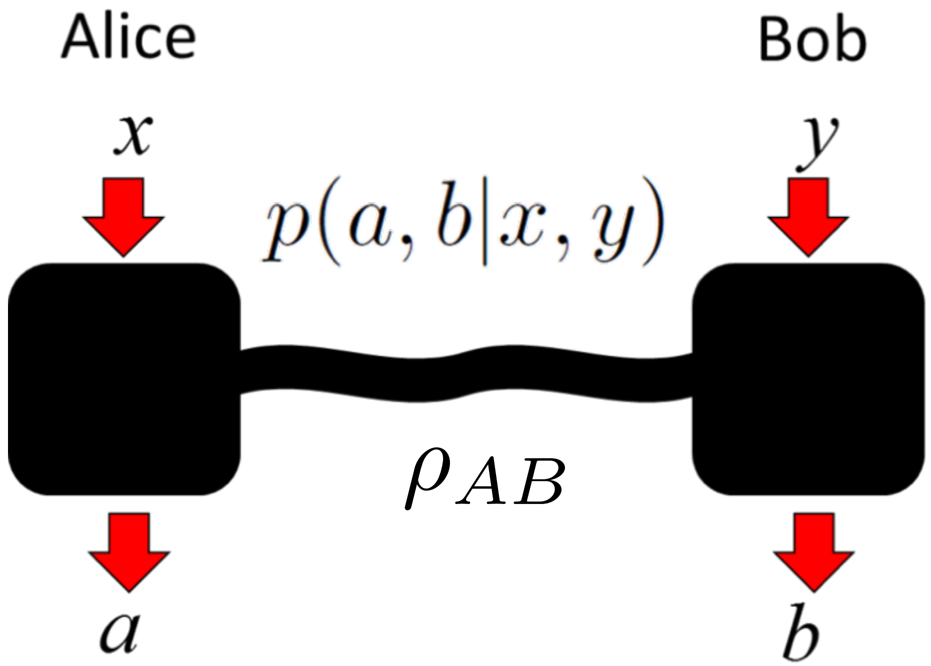

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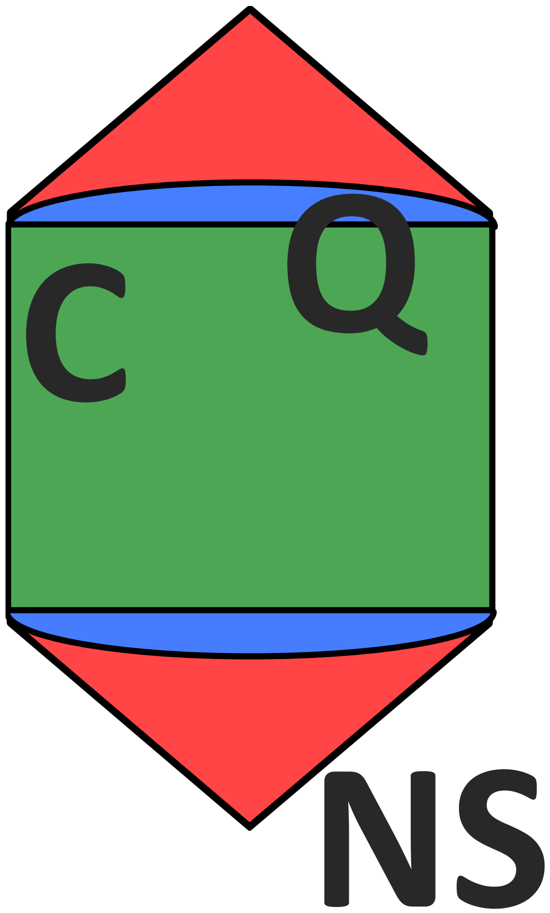

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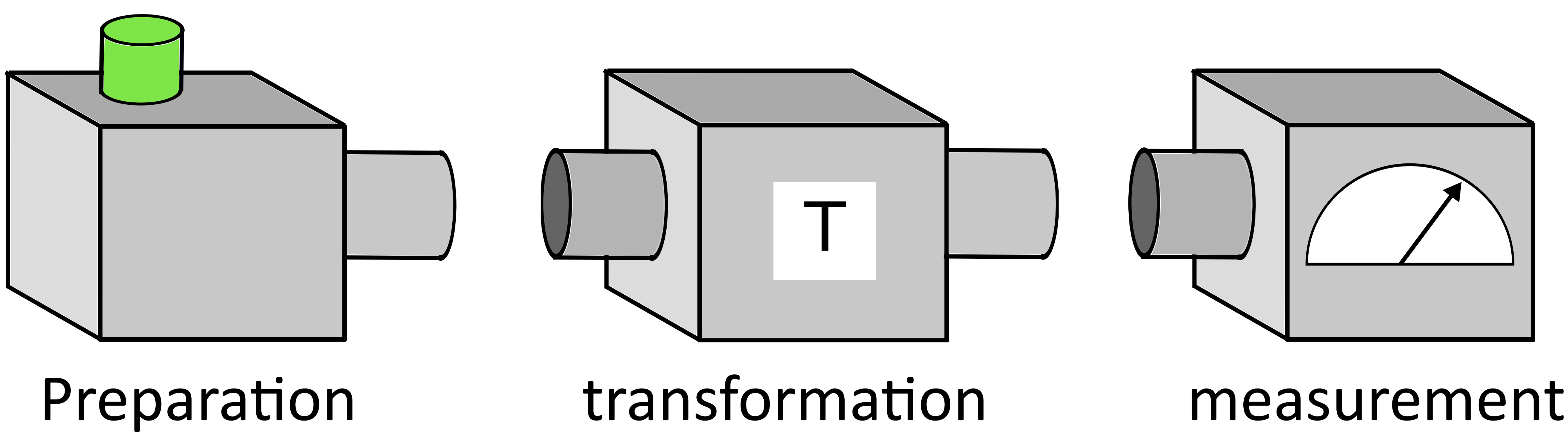
3 examples of a "**generalized probabilistic theory**".

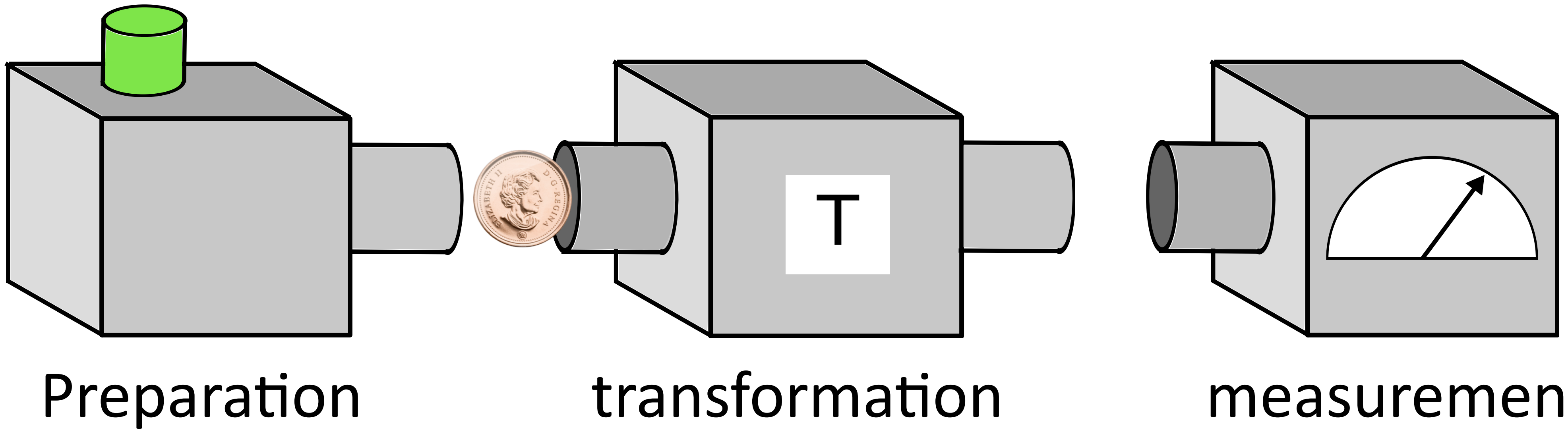
Generalized probabilistic theories



Generalized probabilistic theories

Example: classical coin toss.

- On every push of button, the preparation device performs a biased coin toss.



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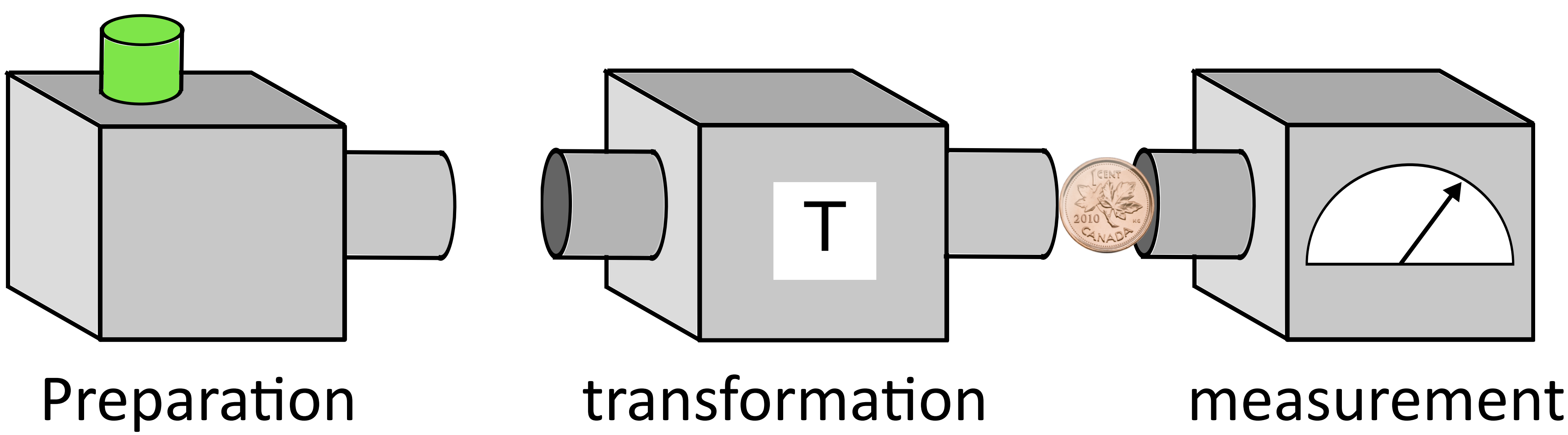

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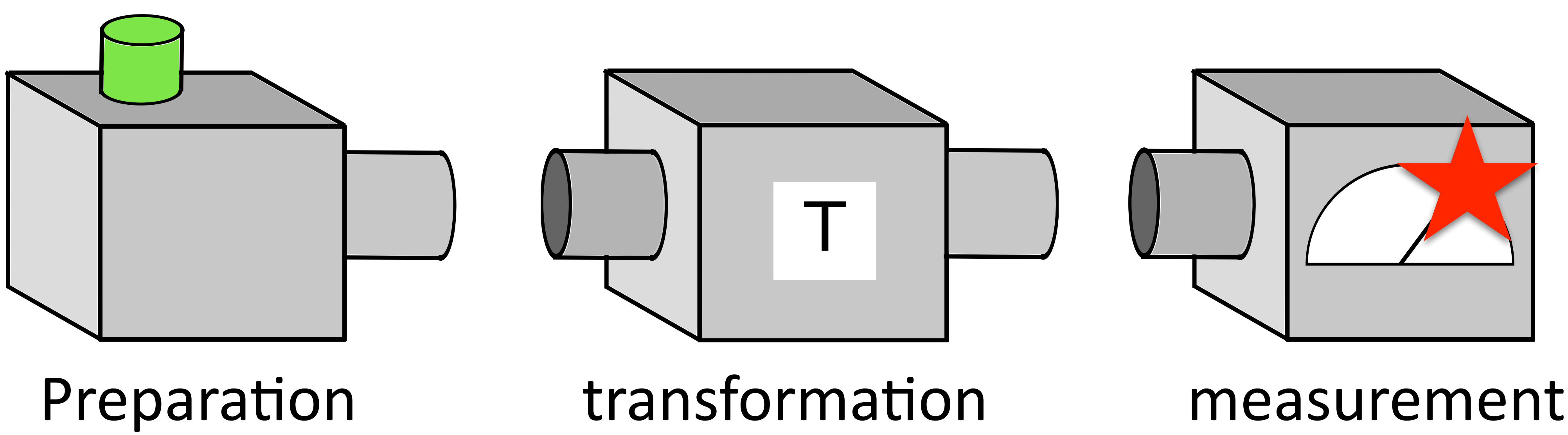
- The measurement outcome is "heads" or "tails".



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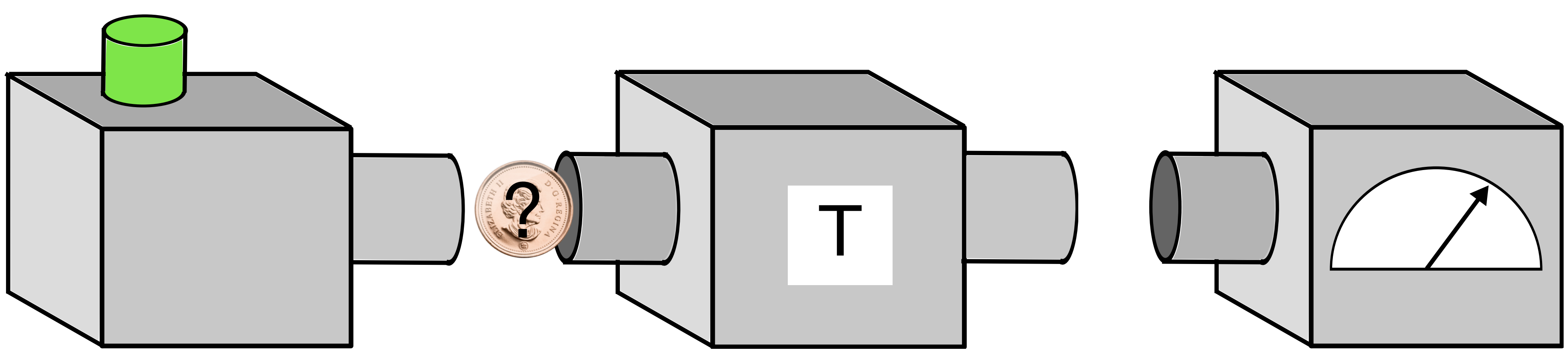
Example: classical coin toss.

- The preparation device prepares a physical system in a state ω. Here

$$\omega = \begin{pmatrix} \text{Prob(heads)} \\ \text{Prob(tails)} \end{pmatrix} = \begin{pmatrix} p \\ 1-p \end{pmatrix}.$$



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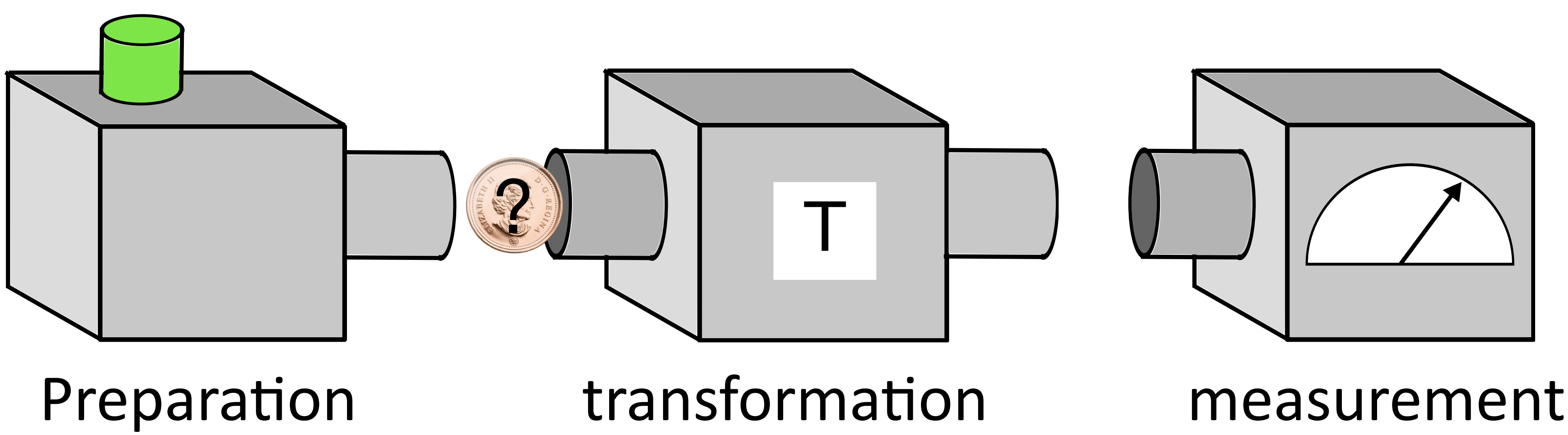

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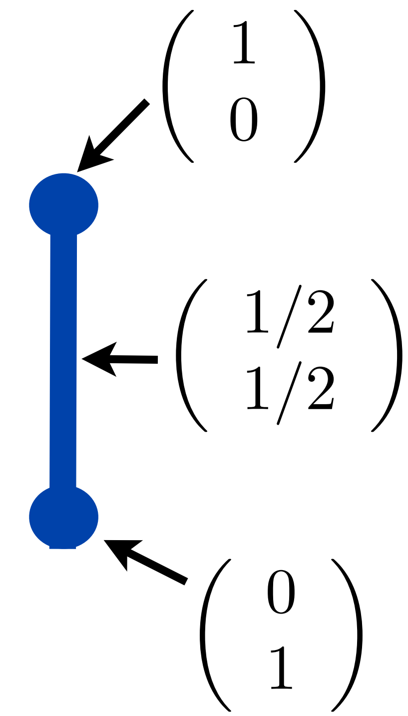

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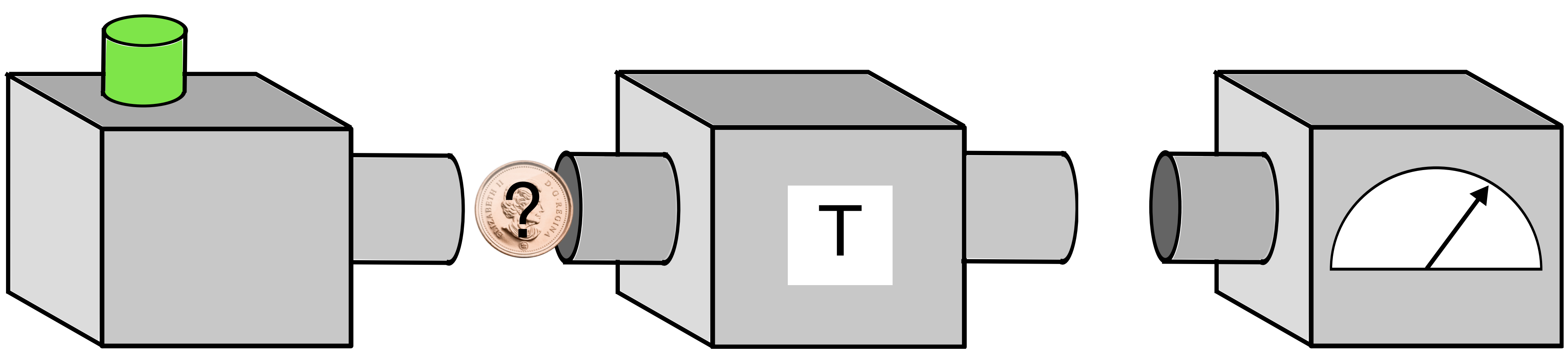

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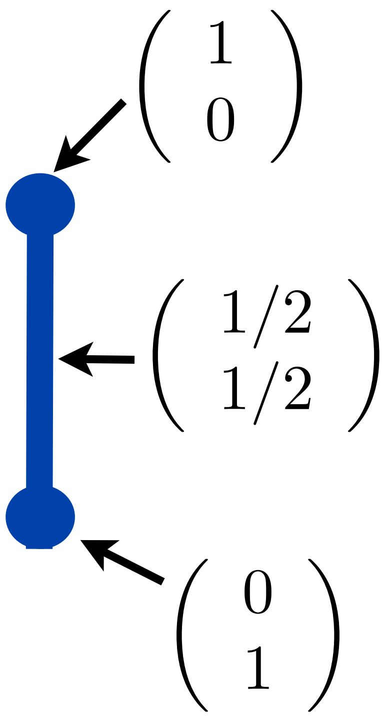

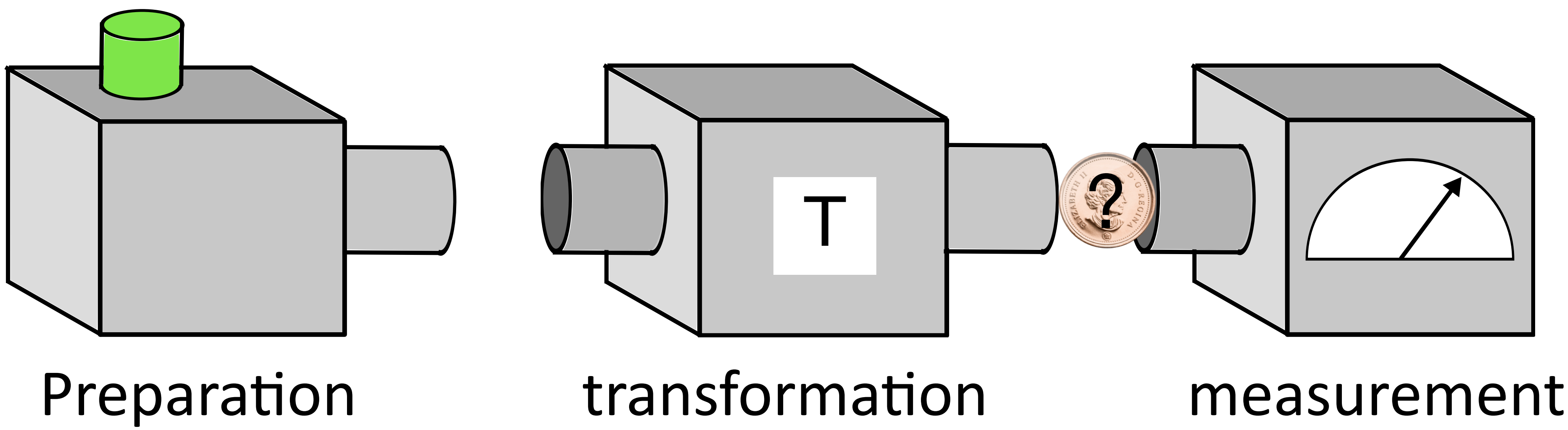

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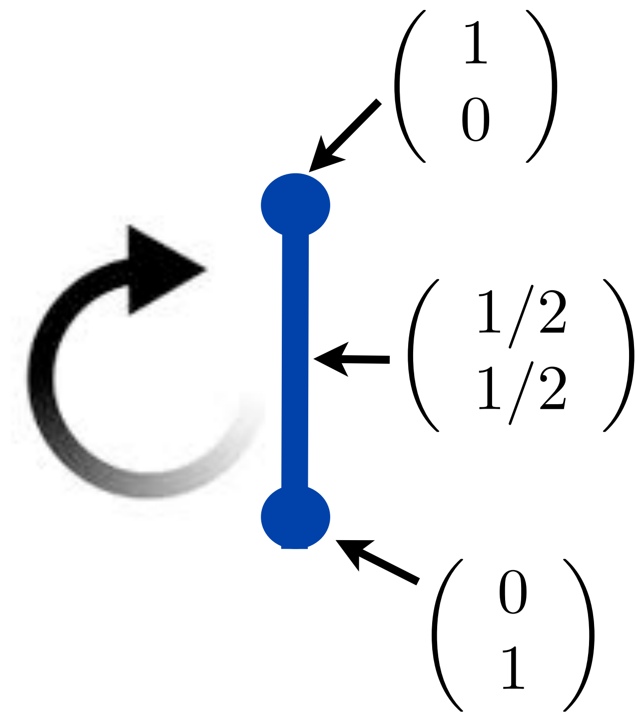
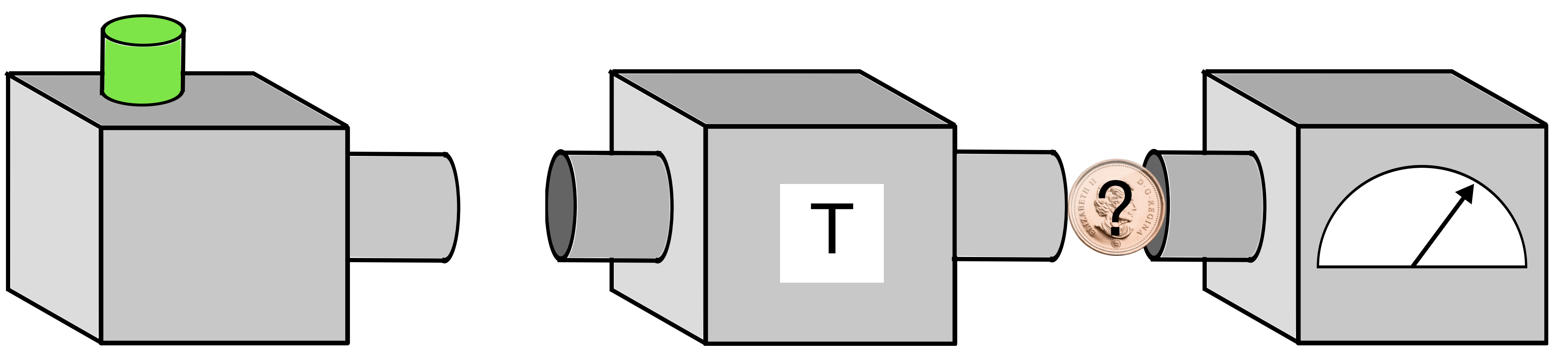
  Maps states to states and is linear.





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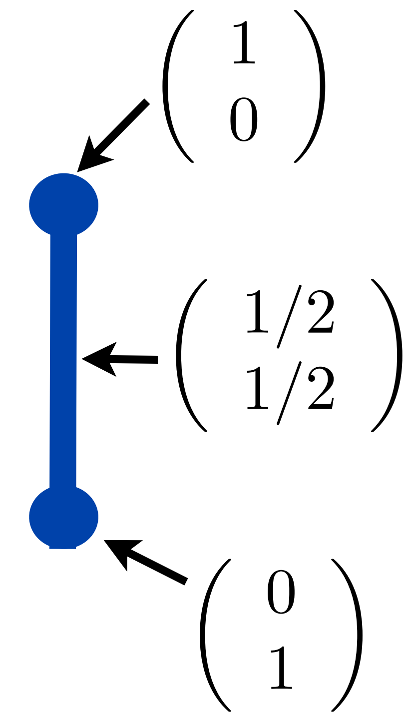

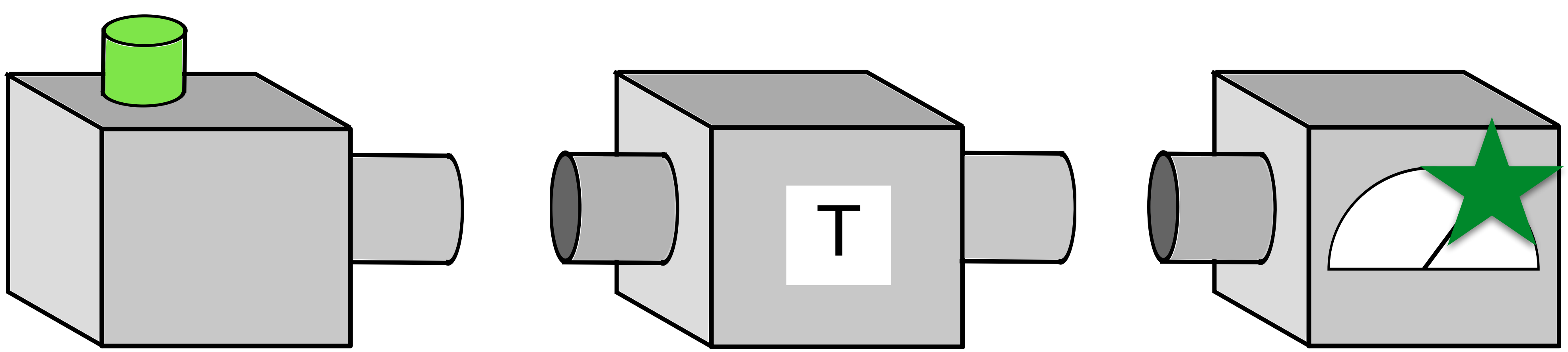

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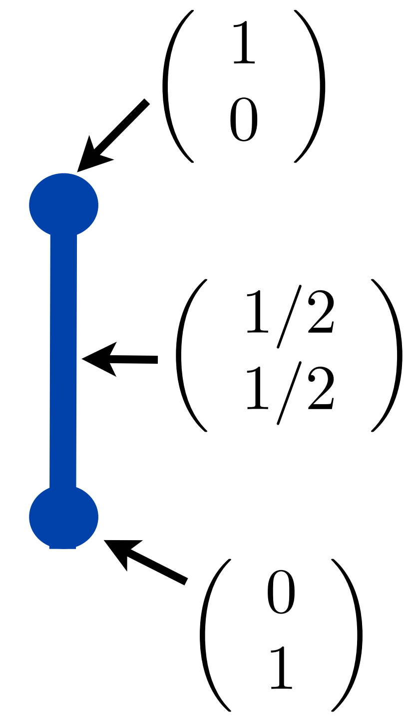
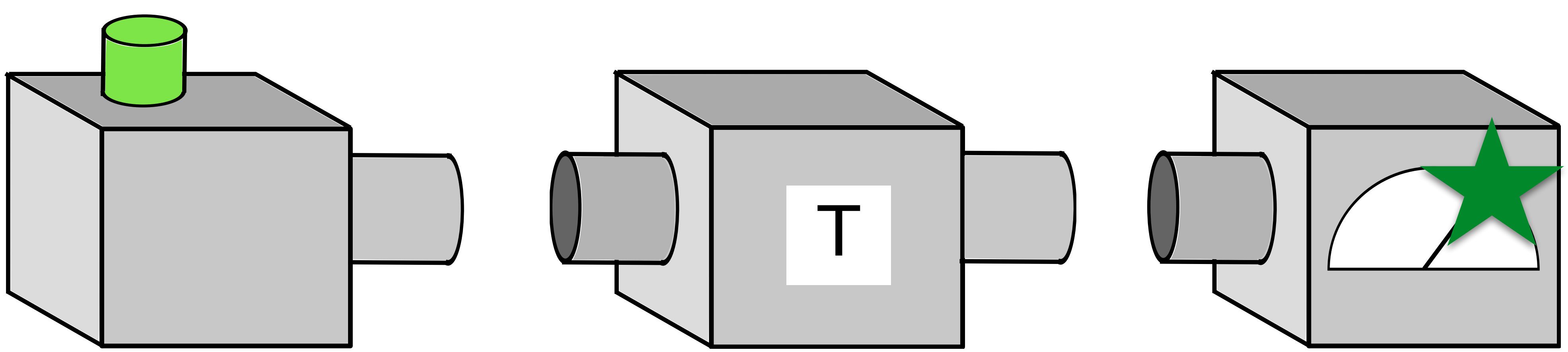
$$\mathrm{Prob}(\text{heads}|\omega) = p = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} p \\ 1-p \end{pmatrix} = e \cdot \omega.$$





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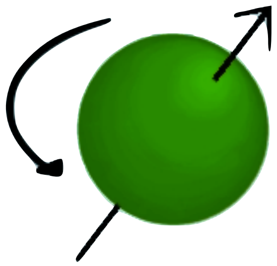
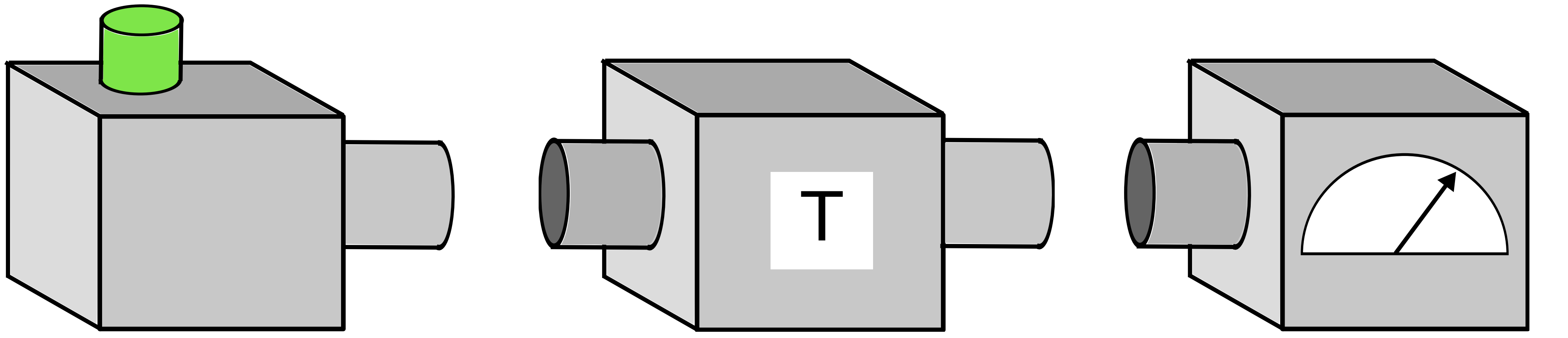
Generalized probabilistic theories

Example: quantum spin-1/2 particle.





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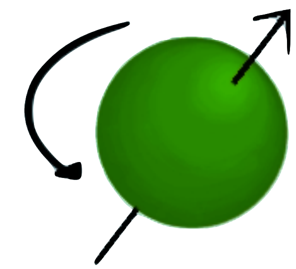
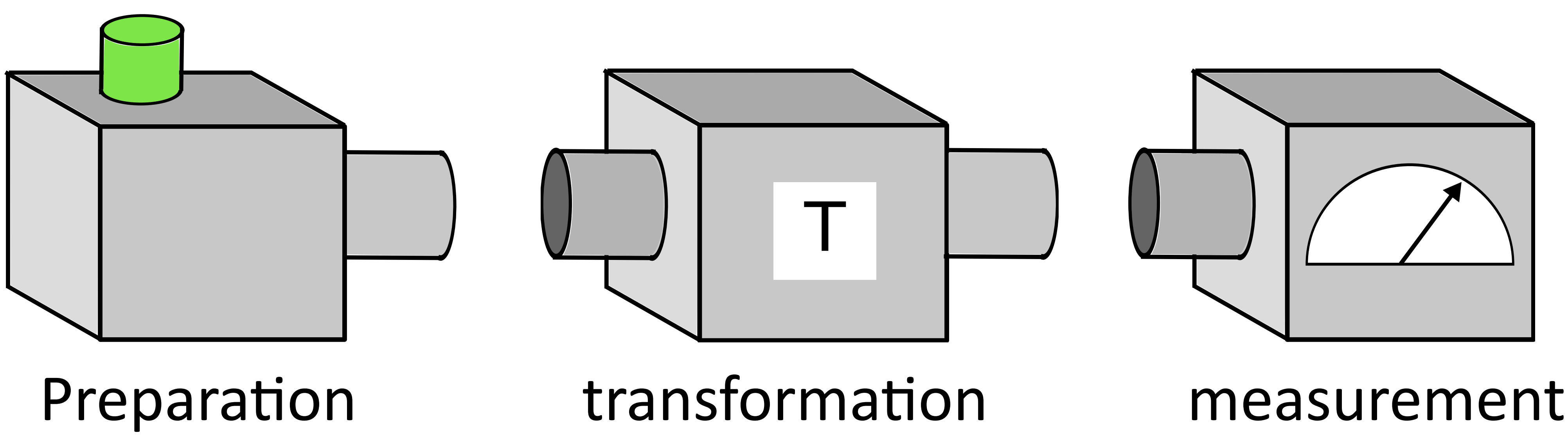


- The preparation device prepares a spin-1/2 particle in quantum state ω.

$$\alpha|\uparrow\rangle + \beta|\downarrow\rangle$$

More generally: ω is 2x2 density matrix.



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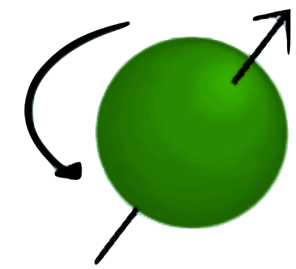

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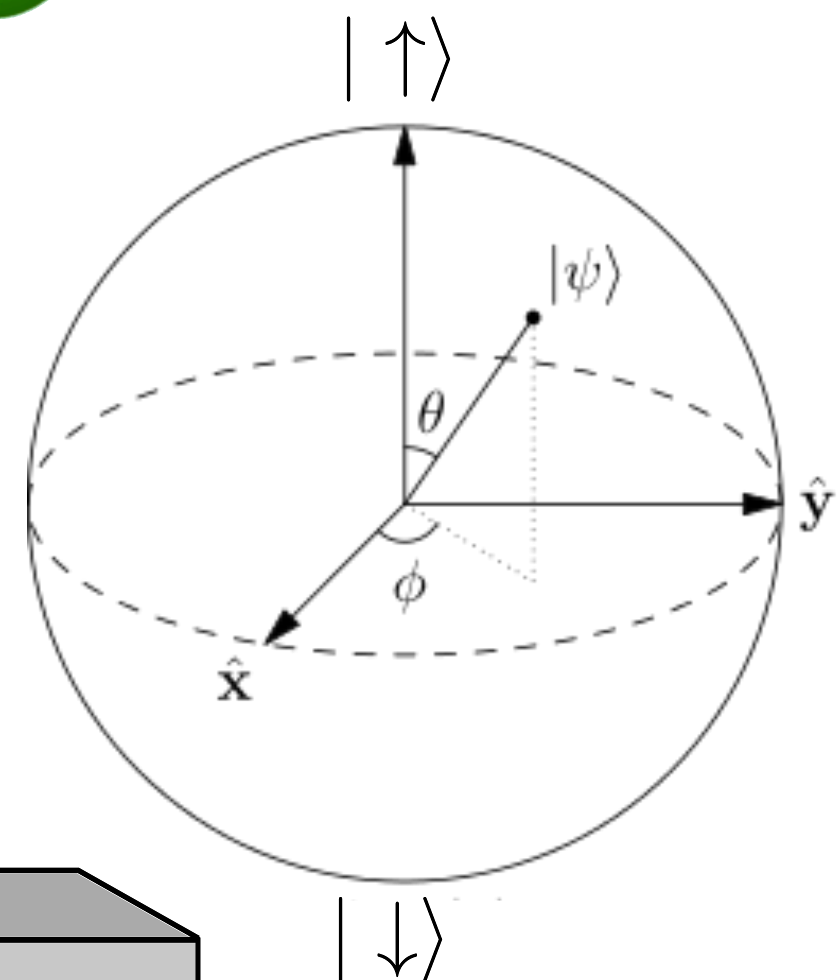

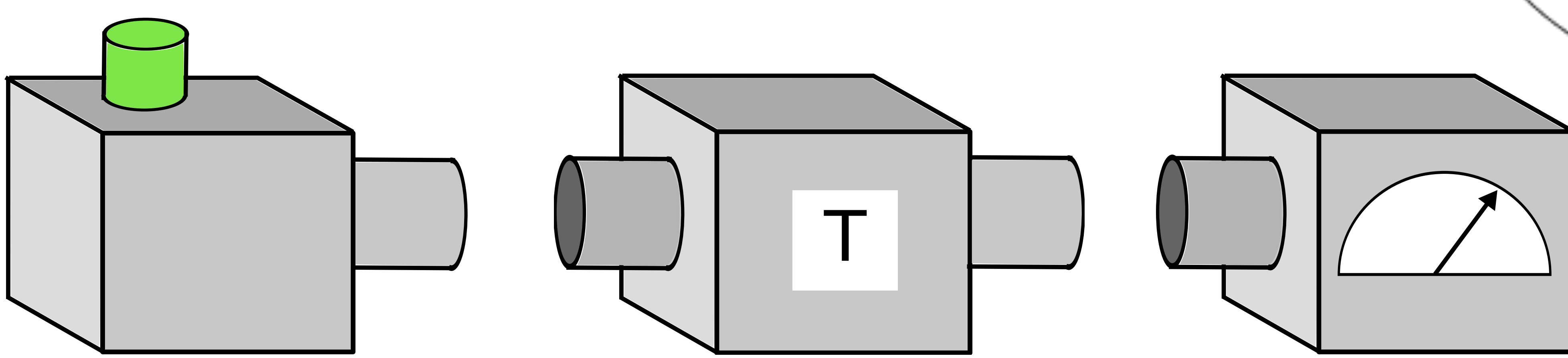

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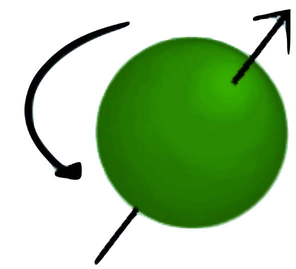
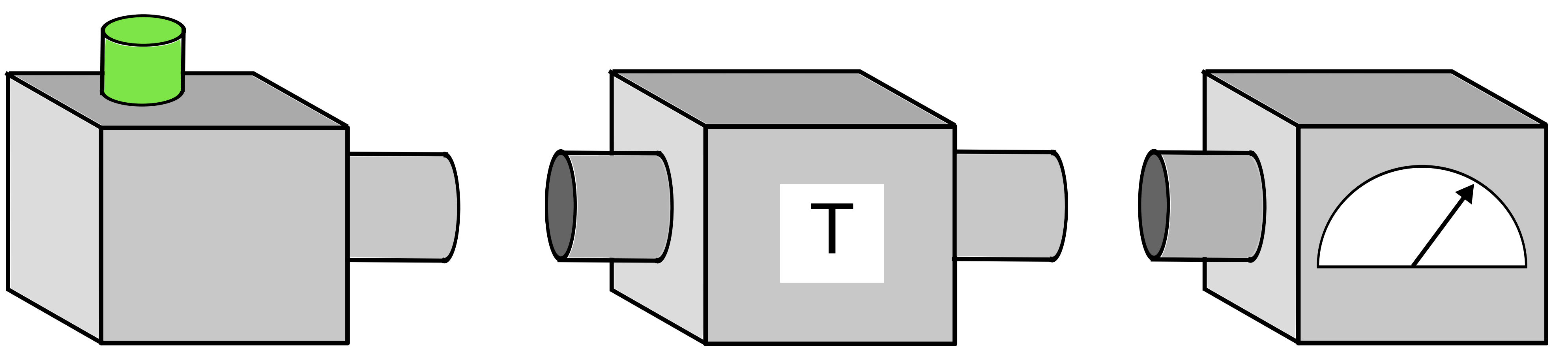


• Unitary transformation of the density matrix:

$$\omega \mapsto U\omega U^\dagger.$$



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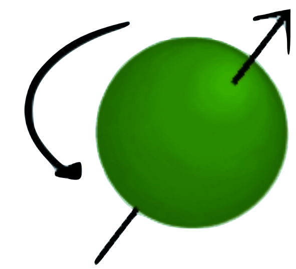

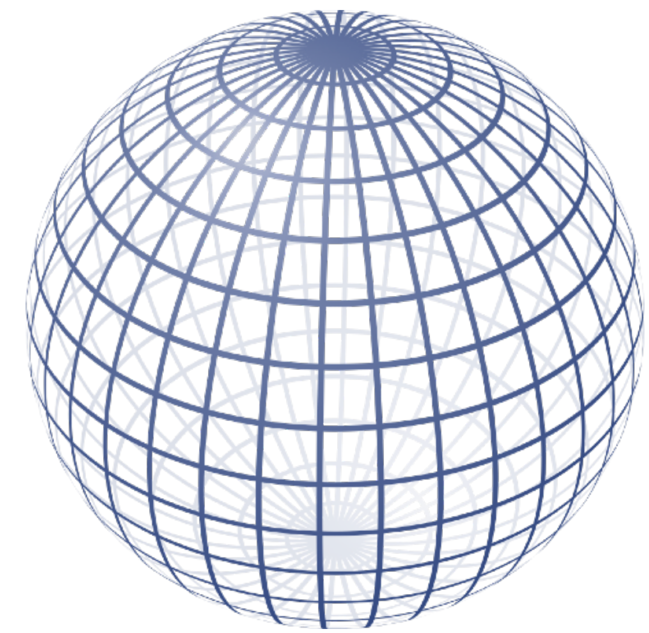

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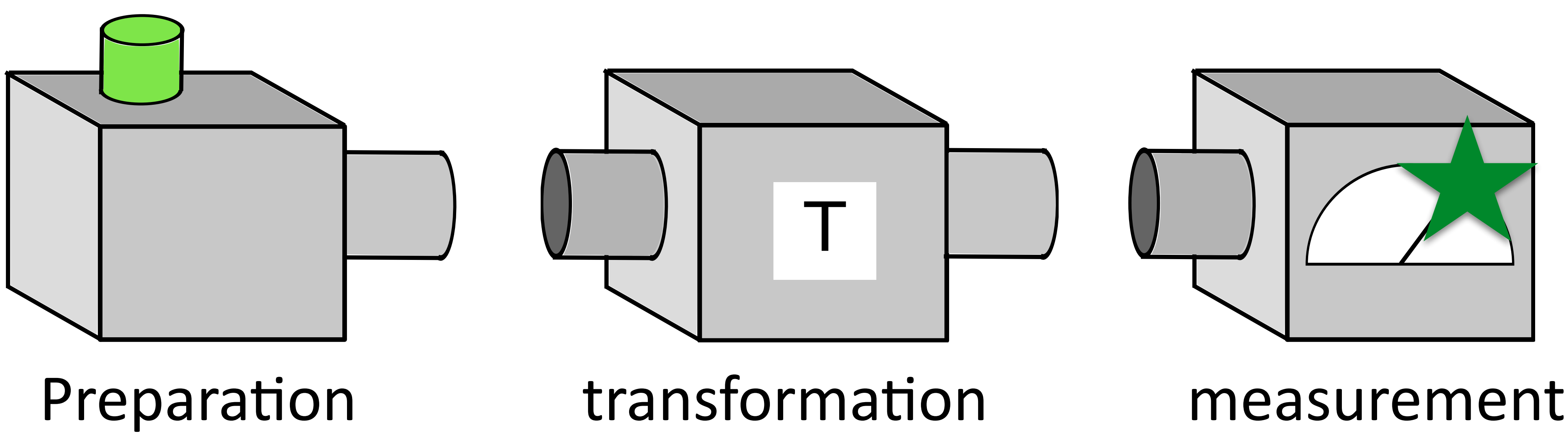
- Measurement in arbitrary spin direction *d*:

$$\text{Prob}(\uparrow \,|\, \omega) = \text{Tr}(P_d\, \omega)$$



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$$\text{even: } \omega \quad \text{odd: } \tau \longrightarrow \sigma = \frac{1}{2}\omega + \frac{1}{2}\tau$$

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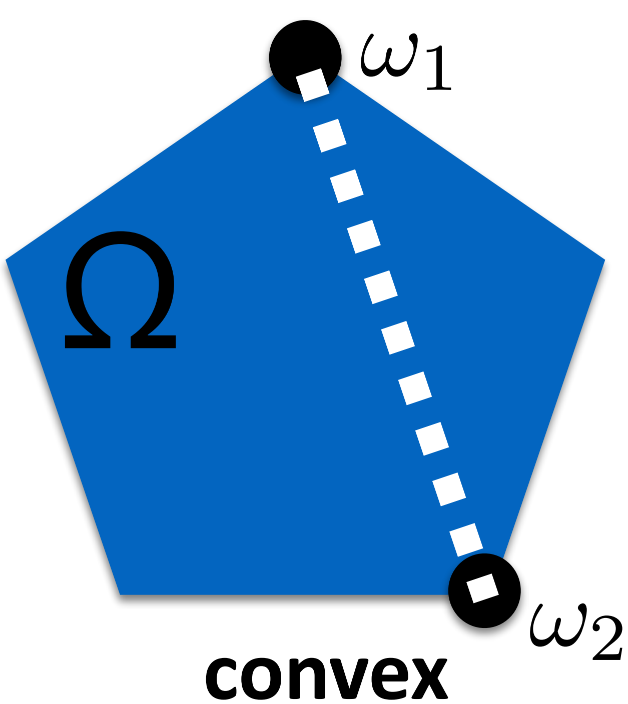
Measure whether spin is up or down: $P(\text{up}) = \text{tr}(\rho|\uparrow\rangle\langle\uparrow|)$.

- What is a **state space**?
  It is the collection of all states that a system could possibly be in, **closed under statistical mixtures**.

even: ω
odd: τ $\longrightarrow$ $\sigma = \frac{1}{2}\omega + \frac{1}{2}\tau$



Generalized probabilistic theories

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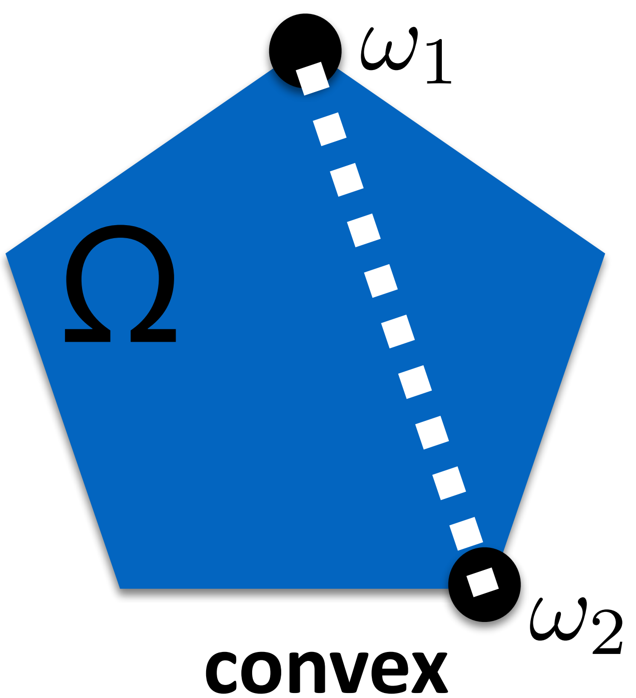

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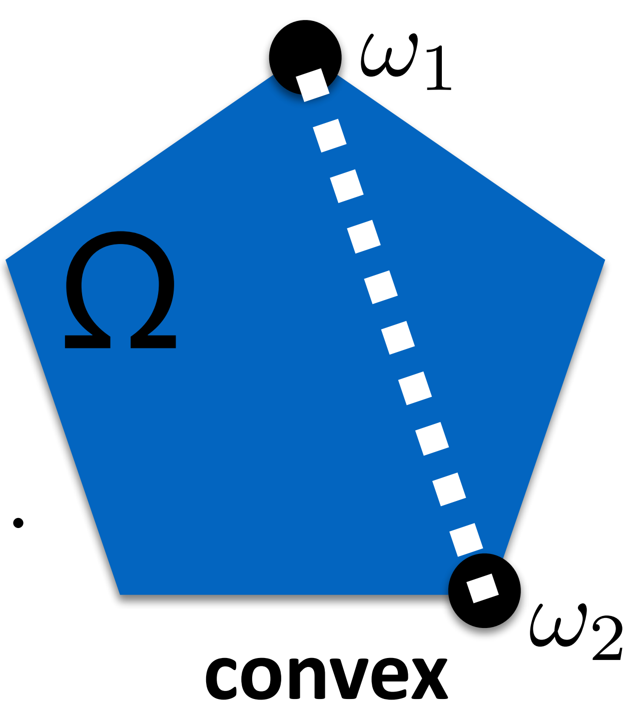
CPT: $\Omega = \{(p_1, \ldots, p_N) \mid p_i \geq 0,\ \sum_i p_i = 1\}$.



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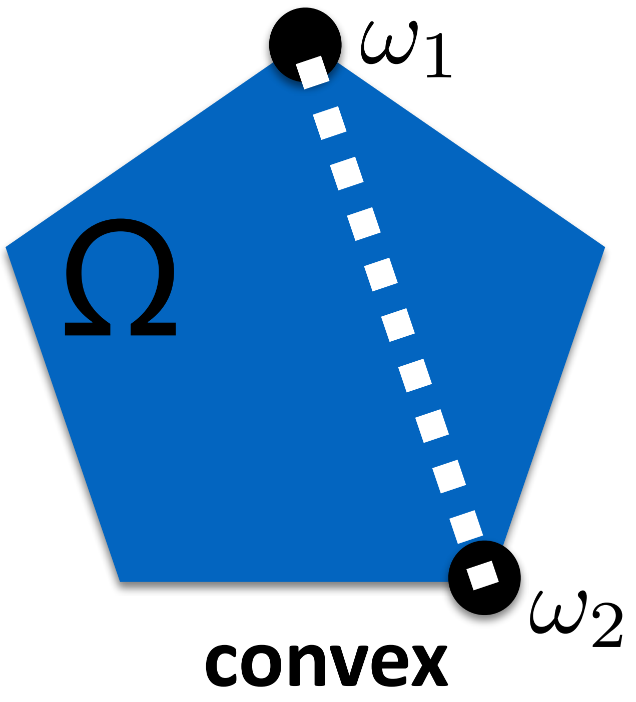
  T is **reversible** if T^{-1} is also a transformation.



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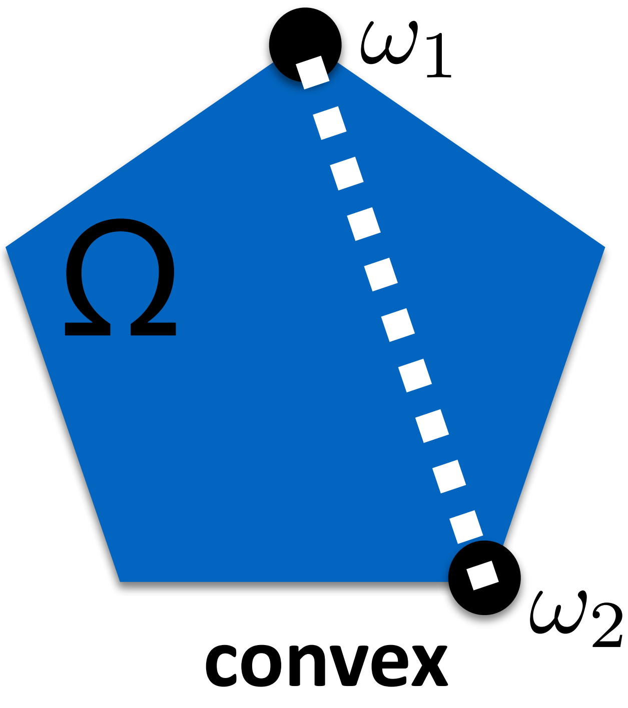

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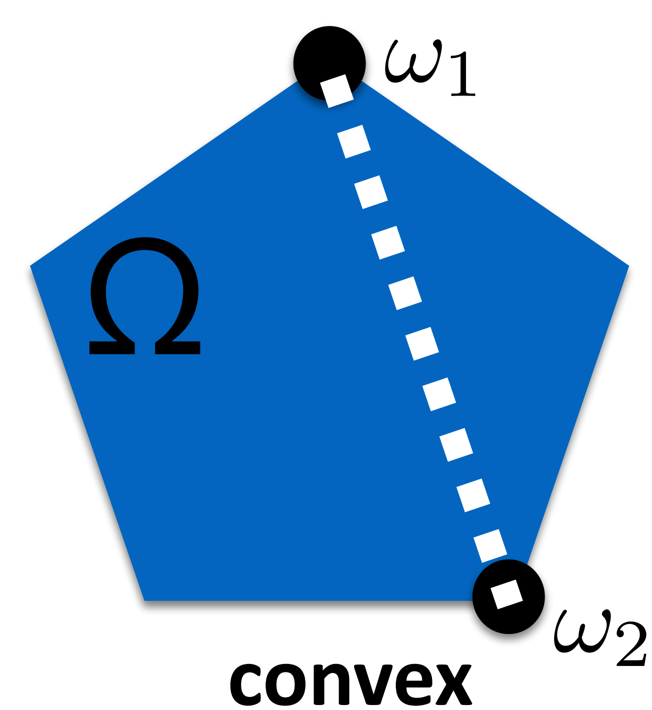

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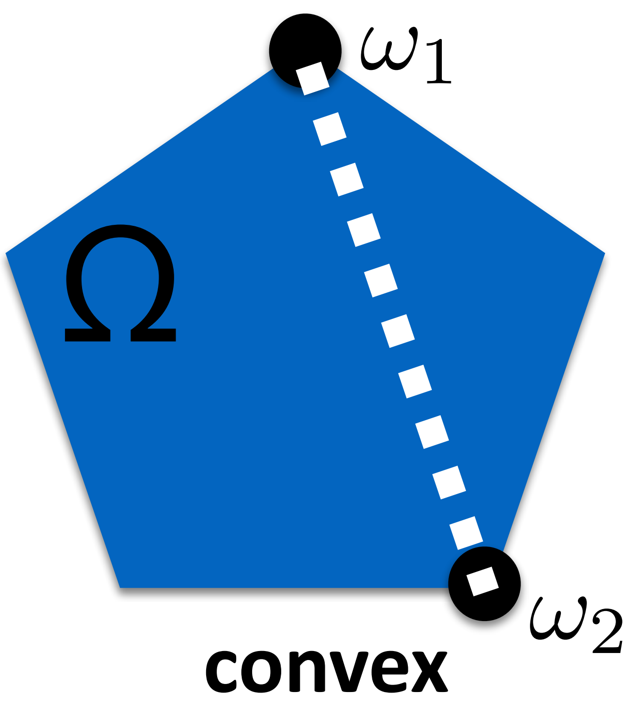
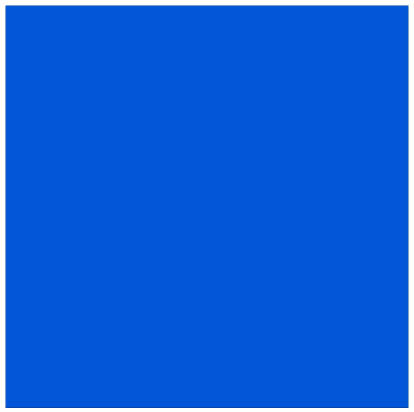
QT: POVMs (positive operator-valued measures),
$e_i(\omega)=\mathrm{tr}(E_i\omega)$.



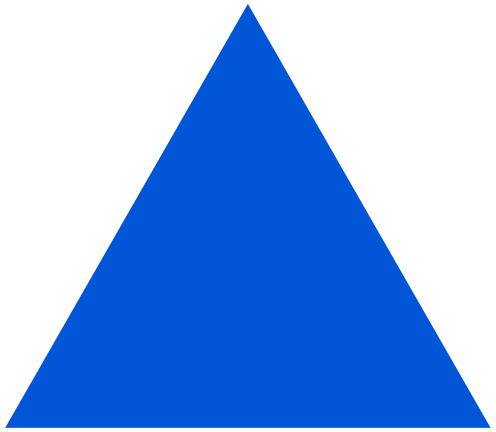
Generalized probabilistic theories

classical
bit

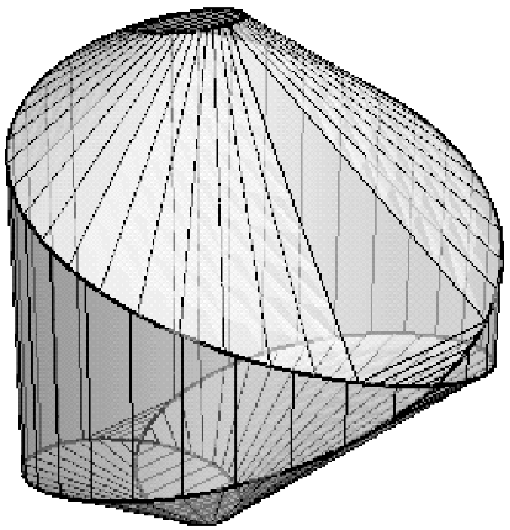
quantum
bit



"gbit"



Classical trit
(3-level-system)

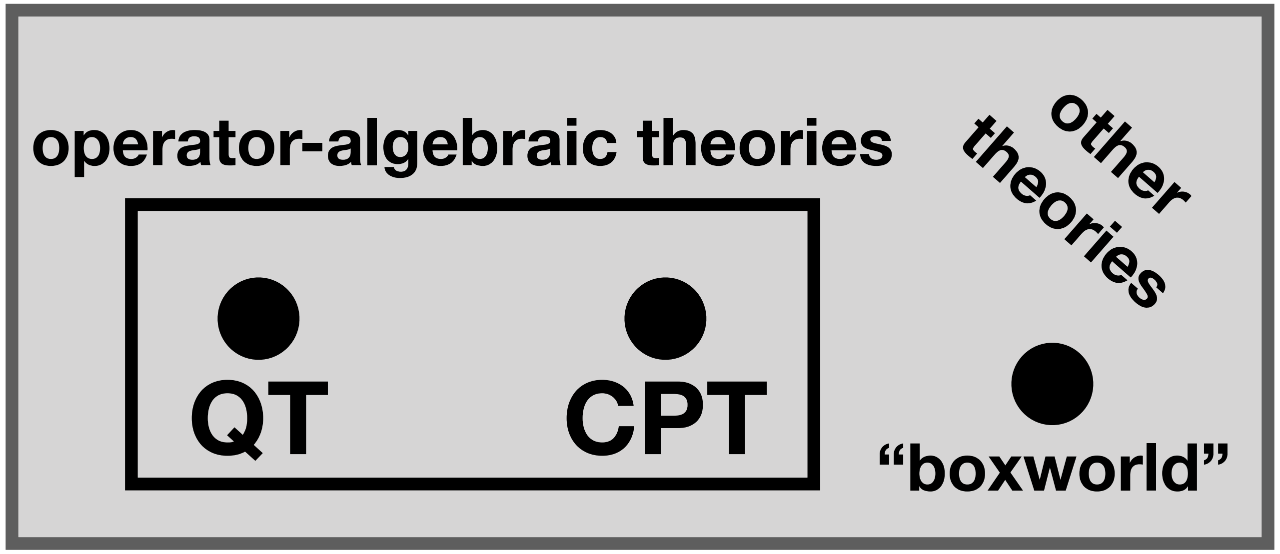


Quantum trit:
8D and complicated!

Arbitrary convex
state space

Generalized probabilistic theories

There is a large landscape of state spaces, or *theories* (collections of allowed state spaces):



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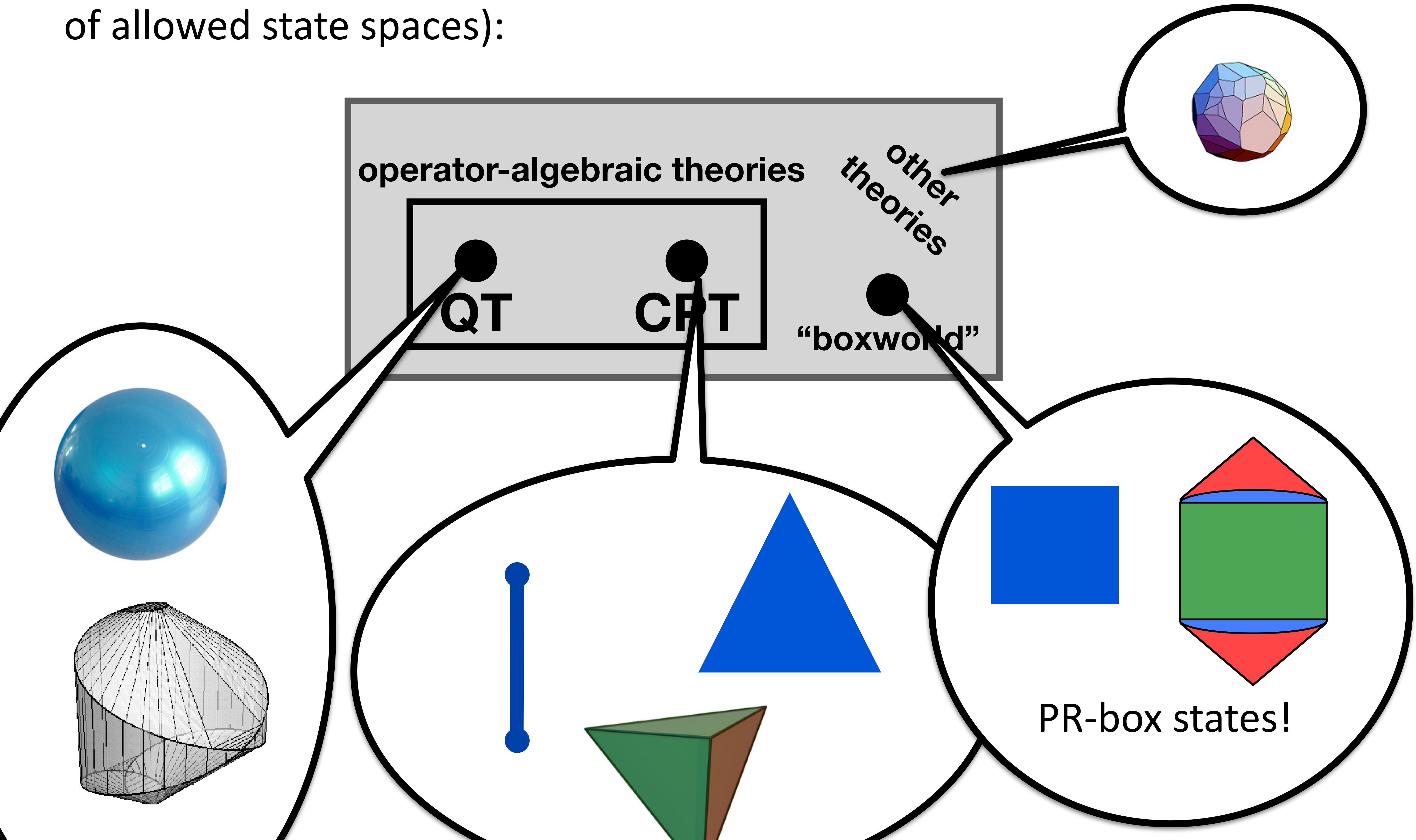

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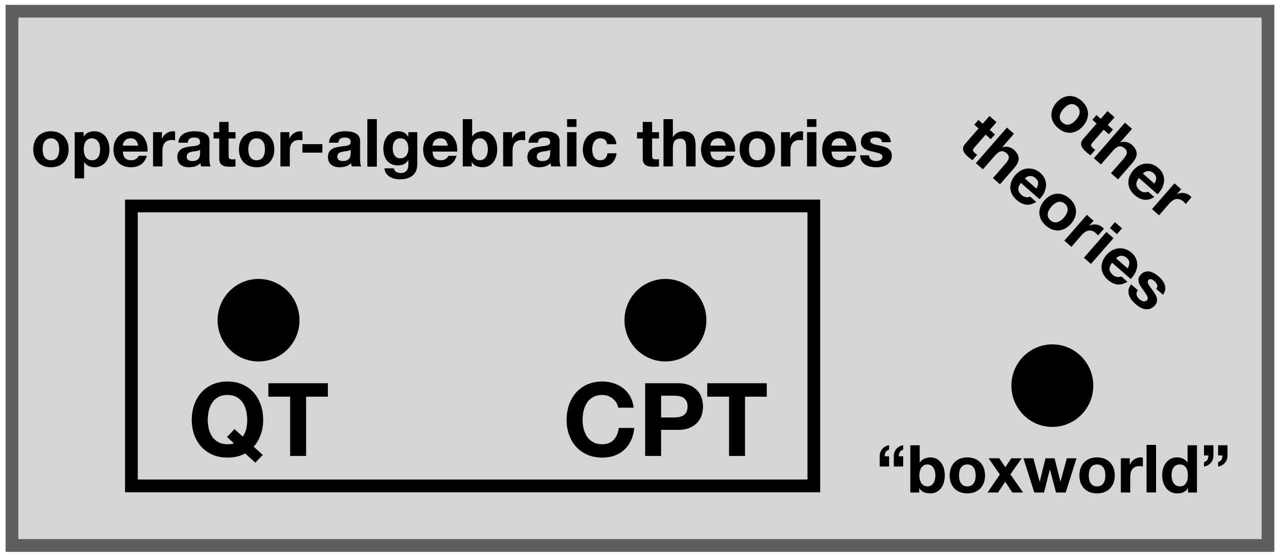

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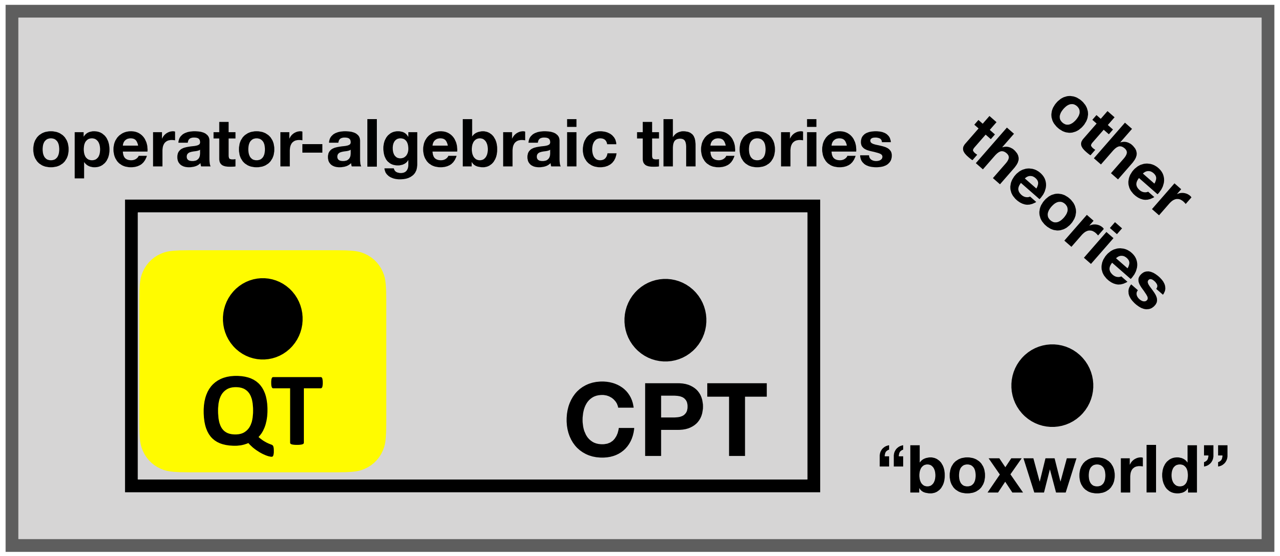

Goal: Find a small set of simple physical / information-theoretic **principles** that singles out QT uniquely.

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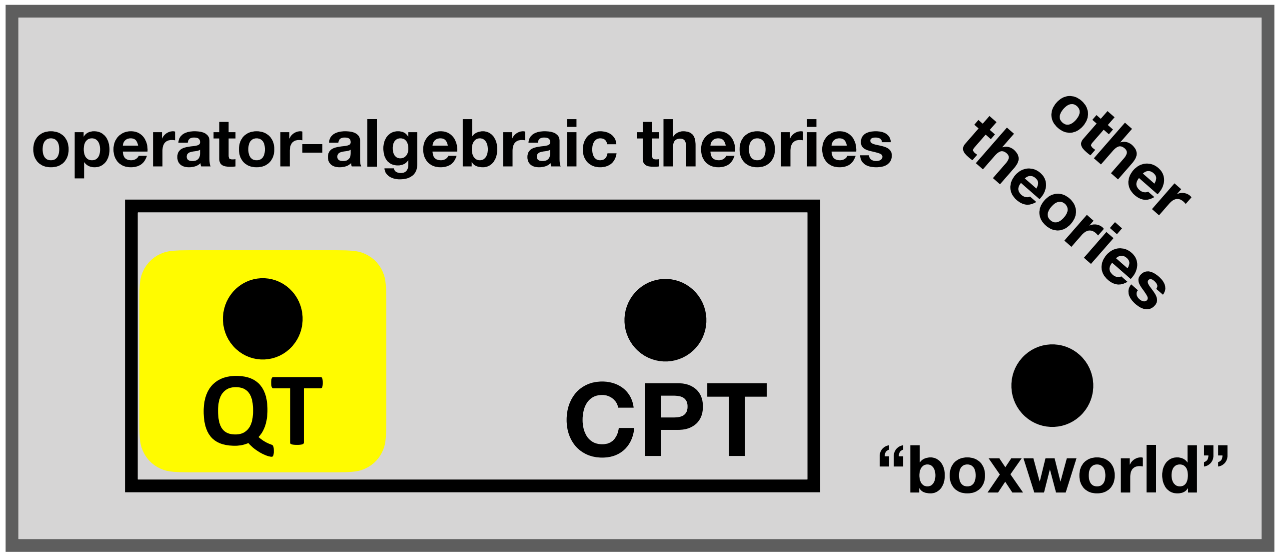
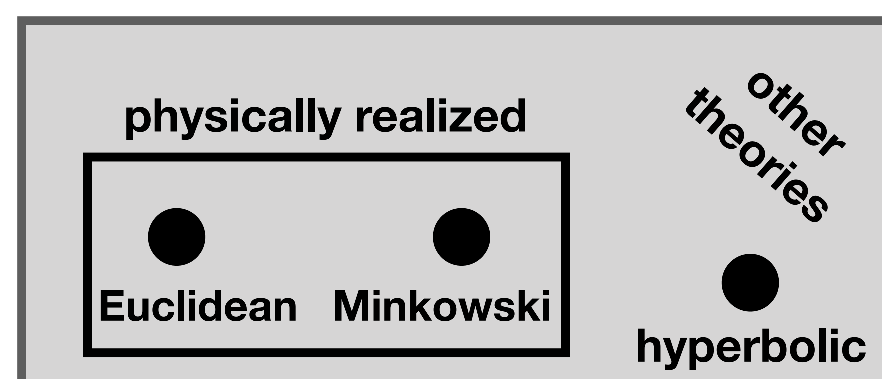


Goal: Find a small set of simple physical / information-theoretic **principles** that singles out QT uniquely.

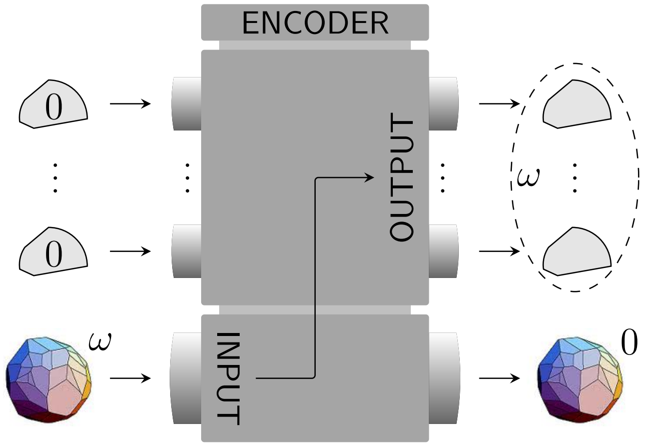
Role model: Einstein's **Relativity Principle** and **Light Postulate** determine Minkowski spacetime.



Overview

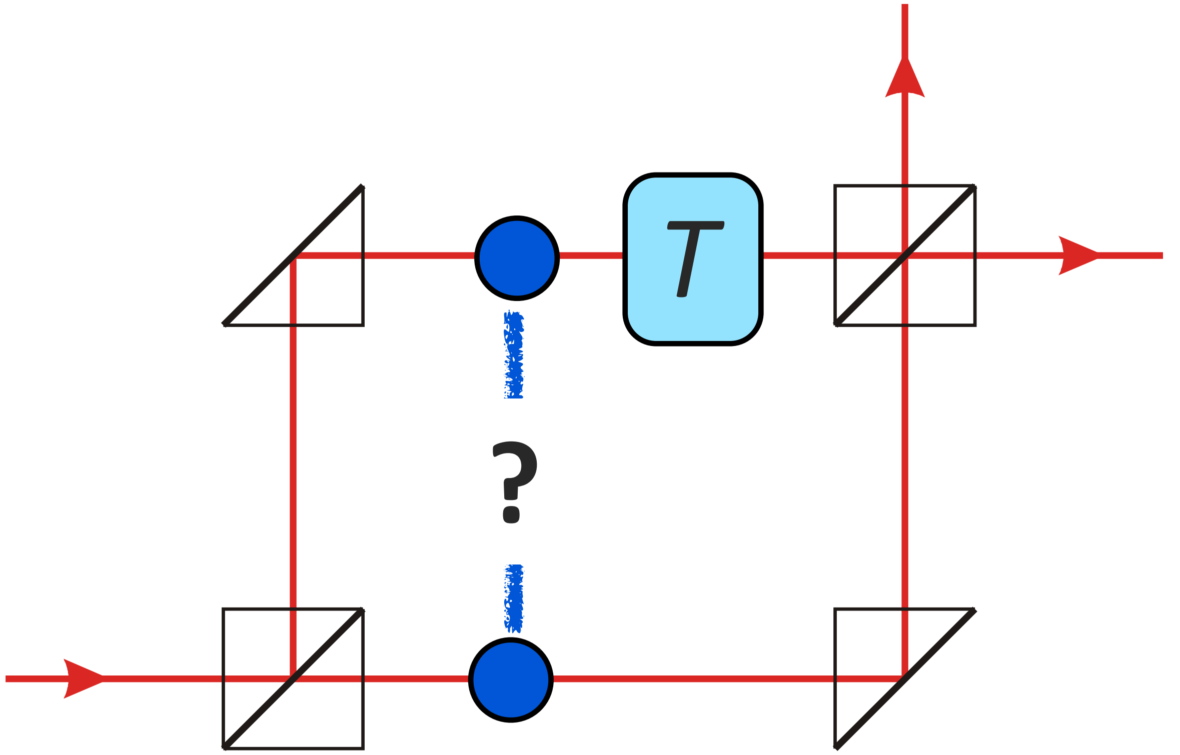
1. Probabilistic theories beyond quantum theory

2. Quantum theory from simple principles



3. The quest for higher-order interference

4. QT and spacetime



5. Conclusion

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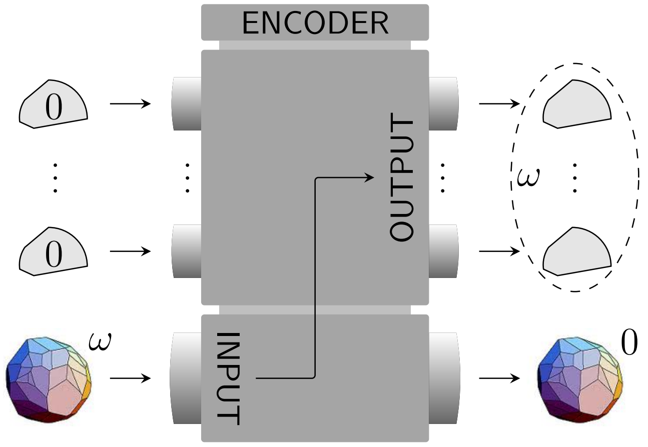

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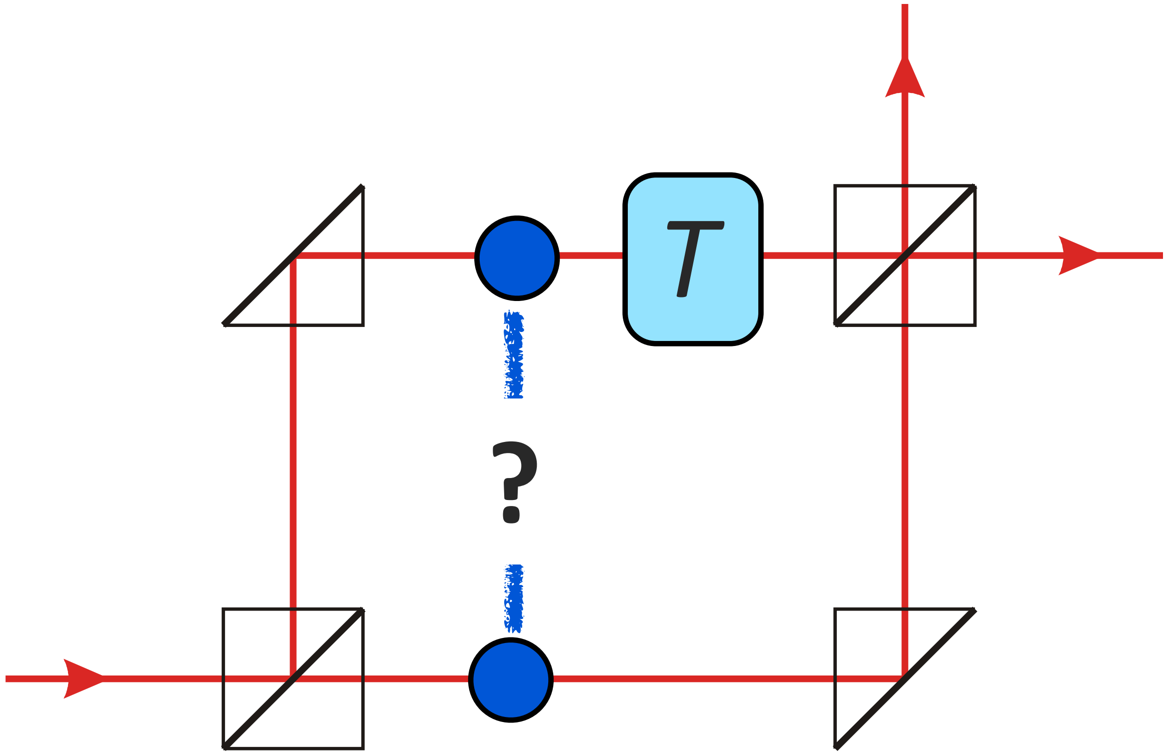

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5. Conclusion

A reconstruction of quantum theory

- Prehistory:

  Birkhoff & von Neumann (1936); quantum logic (Piron, ...), Ludwig (1954); Alfsen&Shultz (≈1980); .....

- Quantum information revolution:

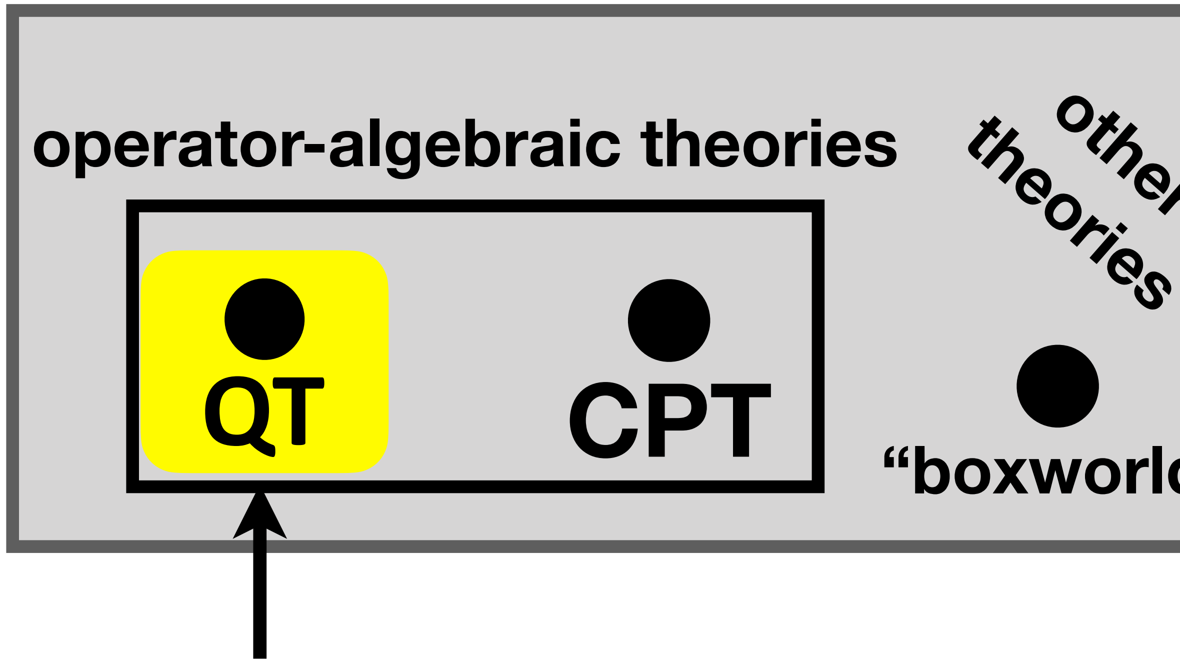
  L. Hardy 2001: Quantum Theory From Five Reasonable Axioms. But needs "simplicity axiom"...

- Clifton, Bub, and Halvorson 2002.

  But assumed C*-algebras.

Dakić+Brukner 2009; Masanes+MM 2009
Chiribella, d'Ariano, Perinotti 2010; Hardy 2011
the one I'll present now 2013;
Barnum, MM, Ududec 2014; Hoehn 2015;
Wilce 2016, ...

A reconstruction of quantum theory



Which principles
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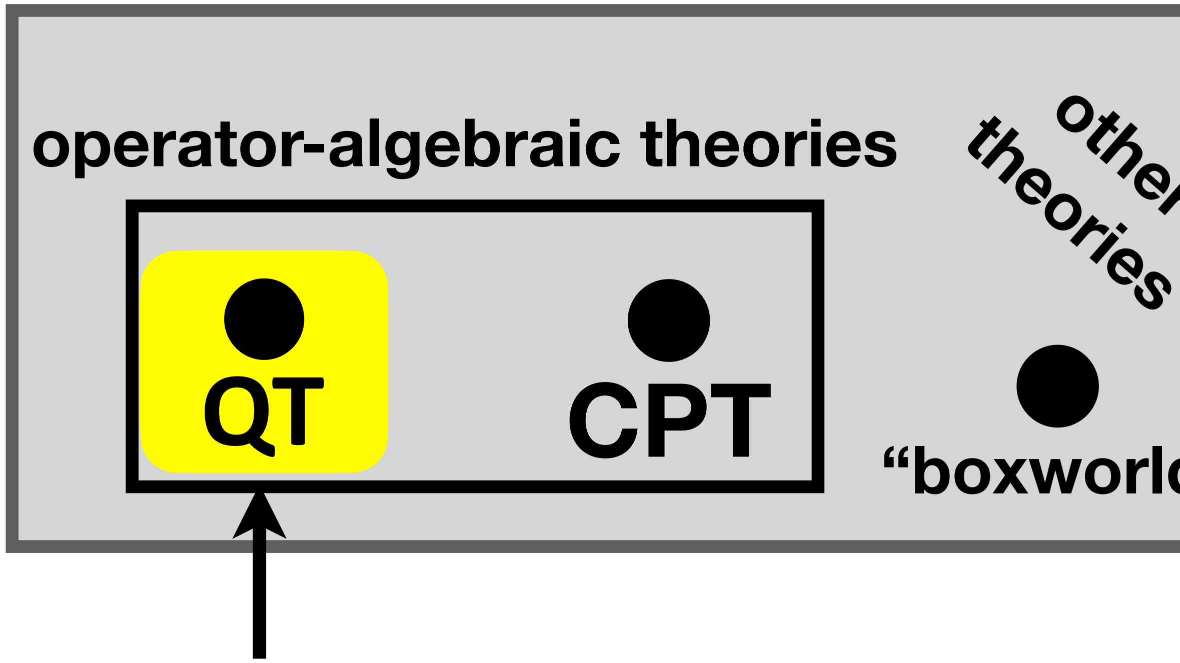
Ll. Masanes, **MM**, R. Augusiak, and D. Pérez-García, PNAS **110**(4), 16373 (2013).



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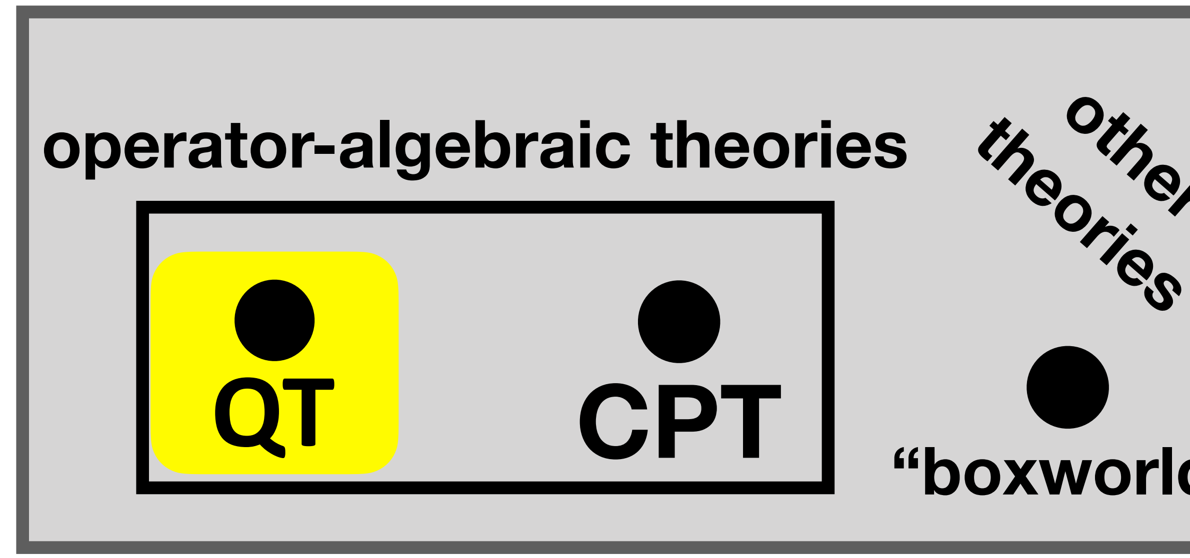
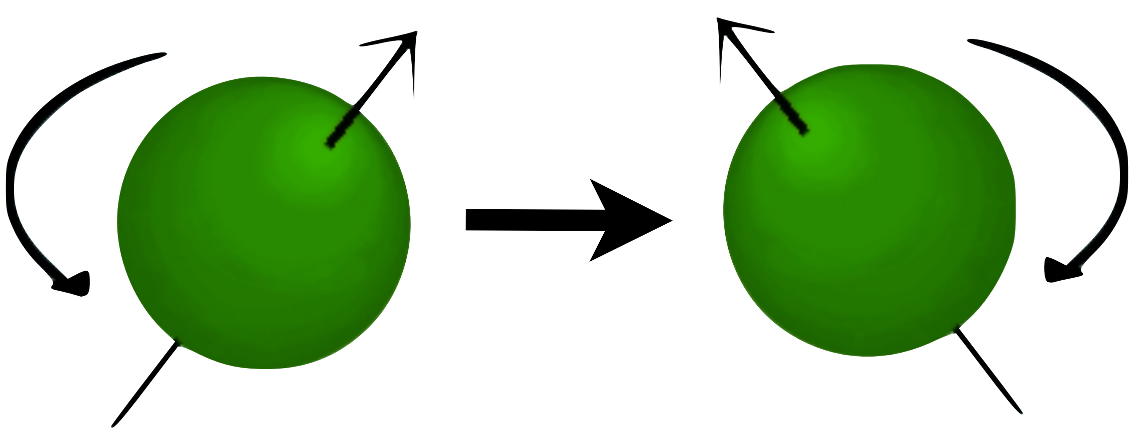


> *Reversible transformations can (in principle) map every pure state continuously to every other.*



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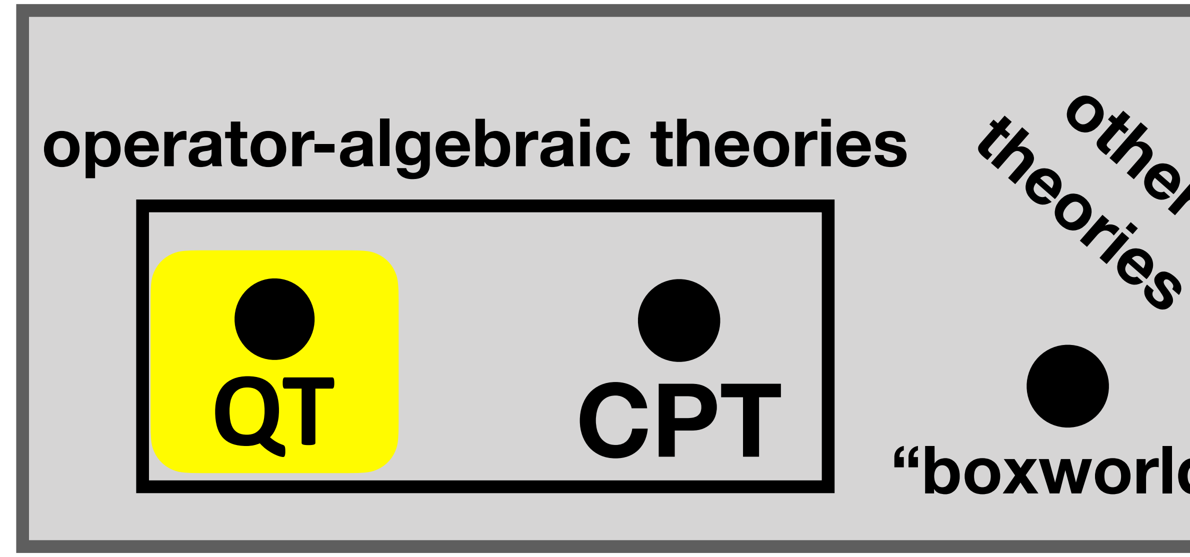
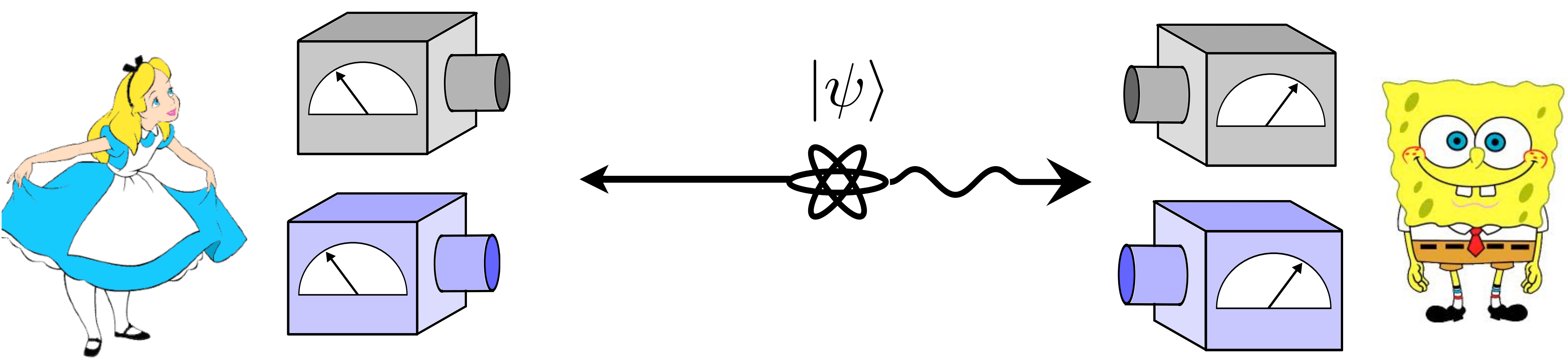


The state of a composite system is completely characterized by the correlations of measurements on the individual components.



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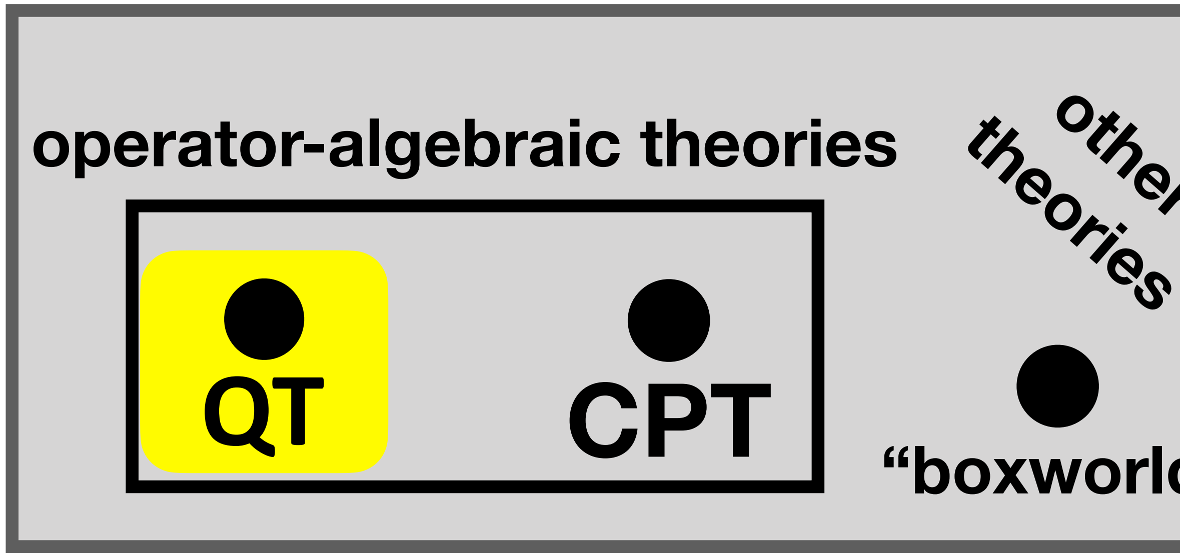

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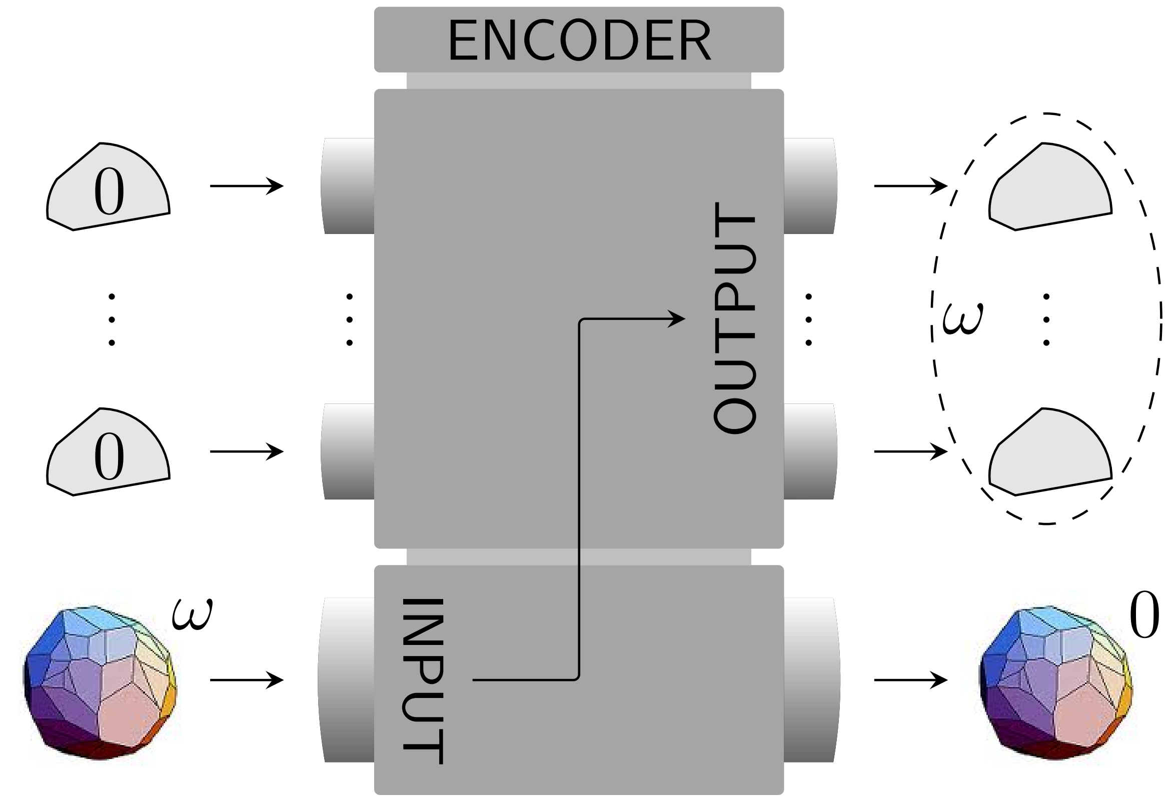

There is a type of system (the "ubit") such that every system can be encoded into a sufficiently large number of ubits.

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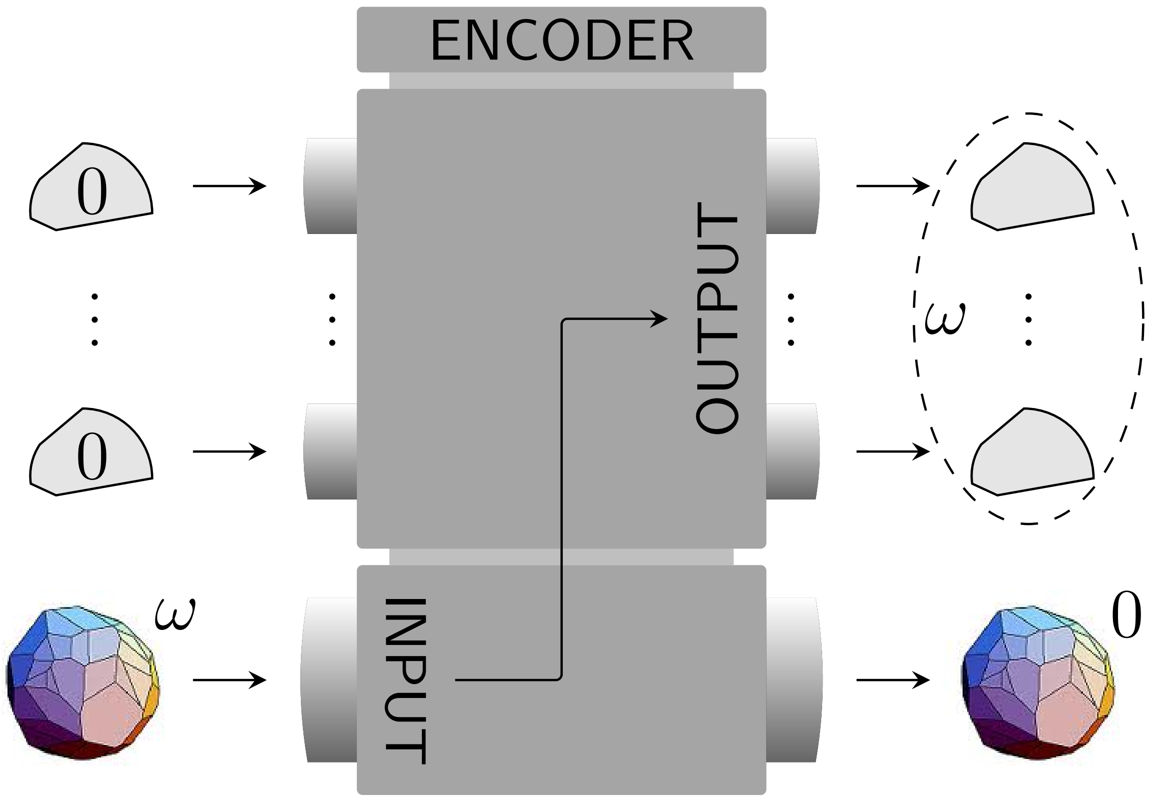
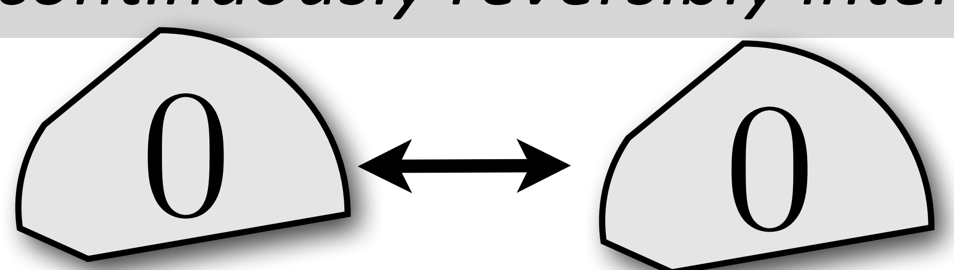


There is a type of system (the "ubit") such that every system can be encoded into a sufficiently large number of ubits. Pairs of ubits can continuously reversibly interact.



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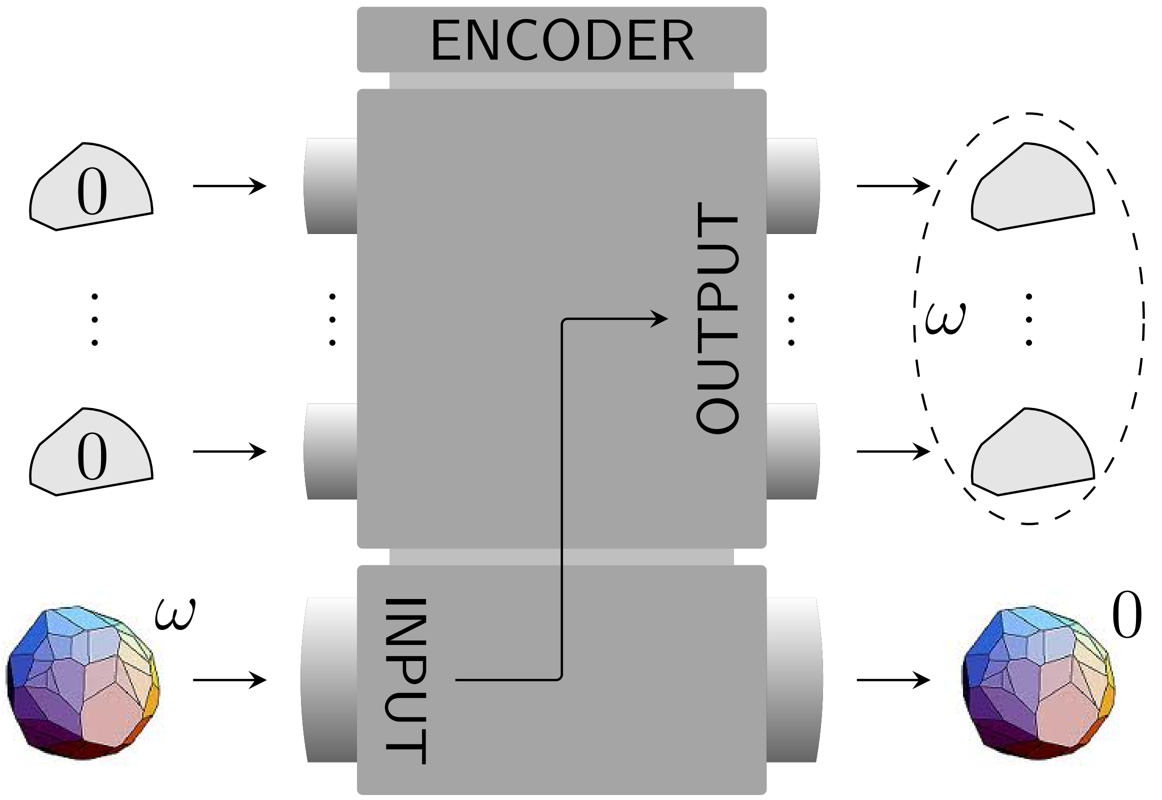

If a ubit is used to perfectly encode one classical bit, it cannot simultaneously encode any further information.

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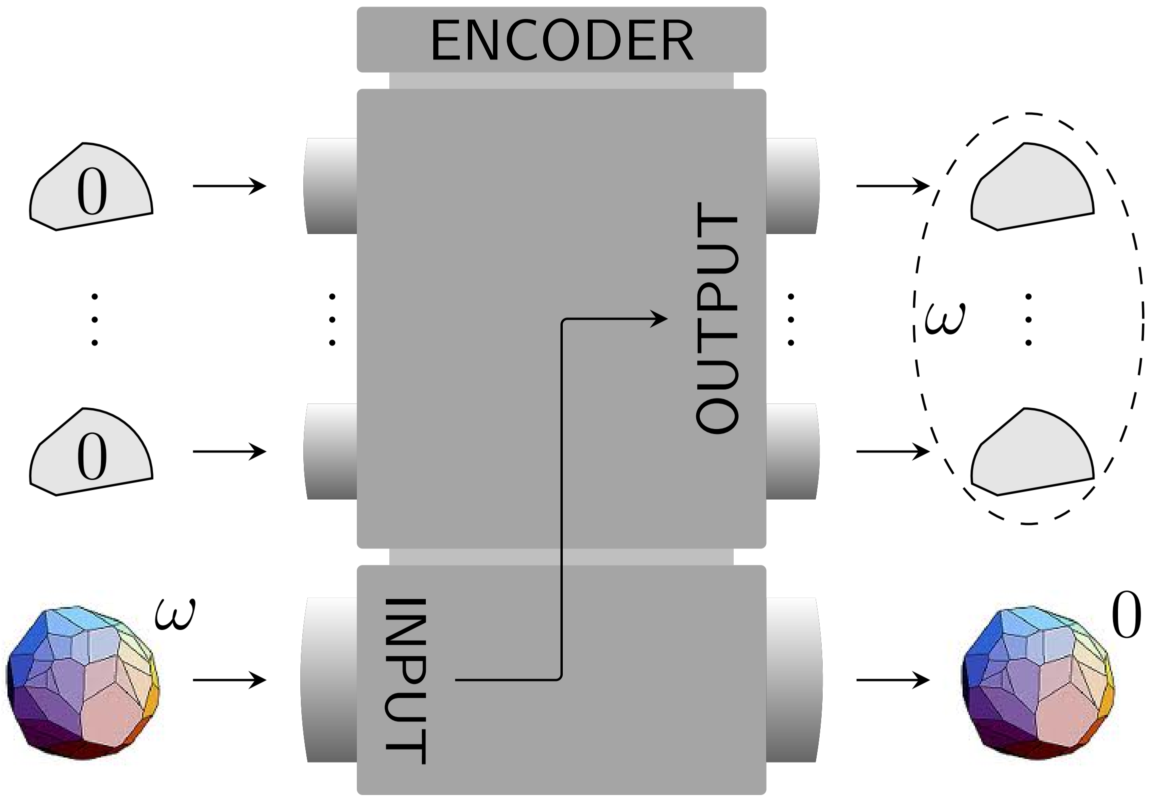

Theorem. If Postulates 1-4 hold, then the state space of *n* ubits is

$$\Omega = \{\rho \in \mathbf{H}_{2^n}(\mathbb{C}) \mid \mathrm{tr}(\rho) = 1, \rho \geq 0\},$$

and the reversible transformations are the unitaries, $\rho \mapsto U\rho U^\dagger$.

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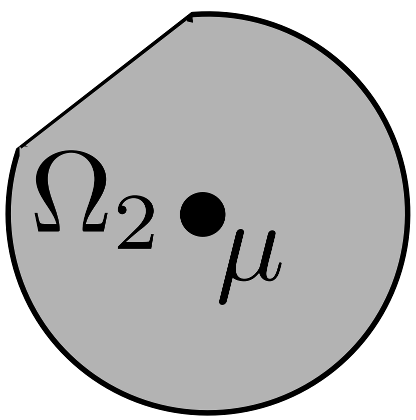

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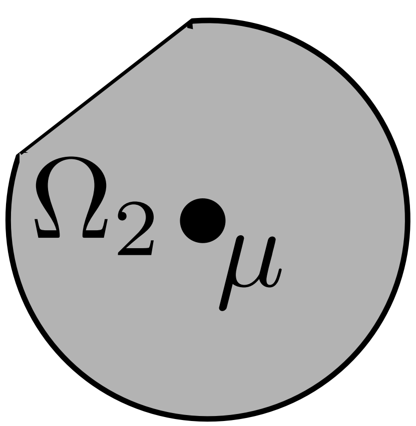
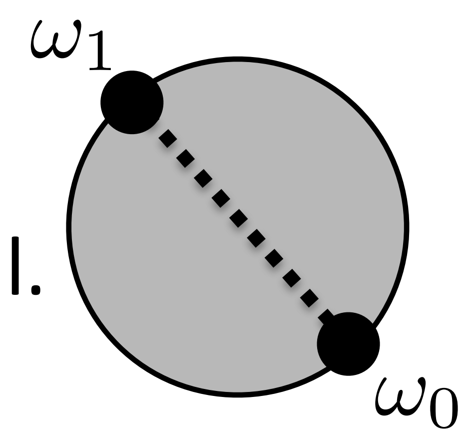


If **full** ball: can encode one bit by preparing state or antipodal state. That's all.



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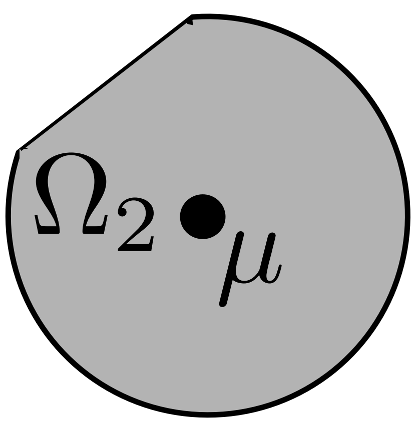

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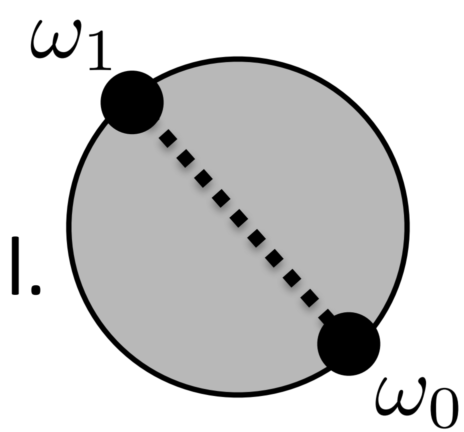
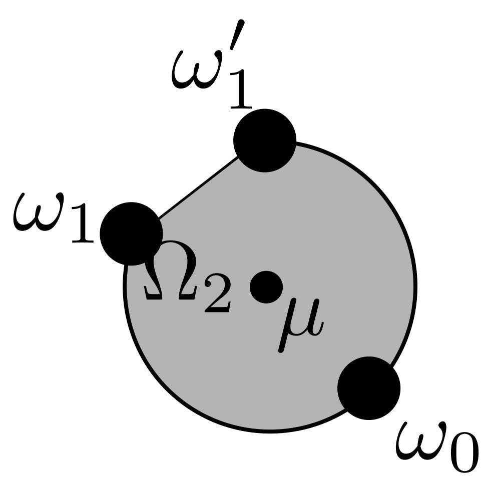


If **not** full ball: can encode one bit **and a little more** by preparing state or **one of** antipodal states.



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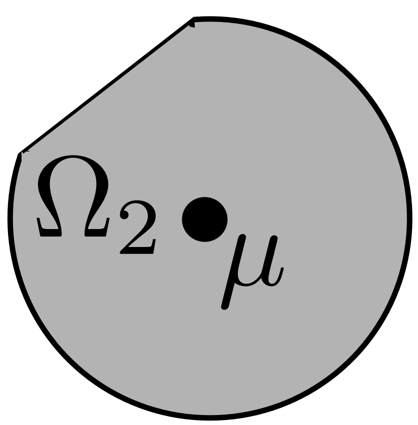

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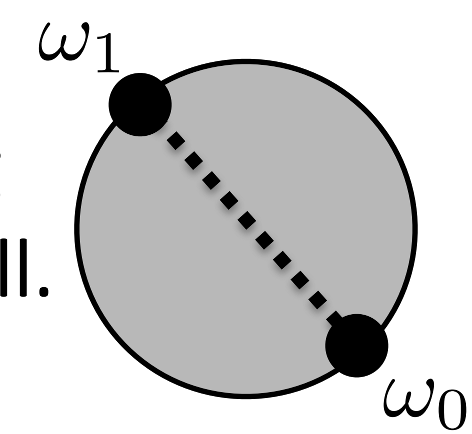

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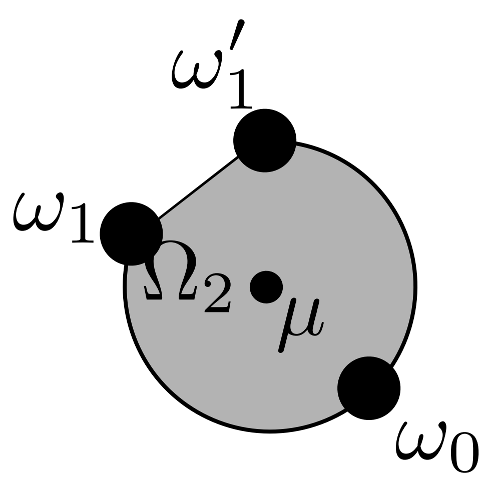

Violates Postulate 4.

Why is the ubit “Bloch ball” 3-dimensional?

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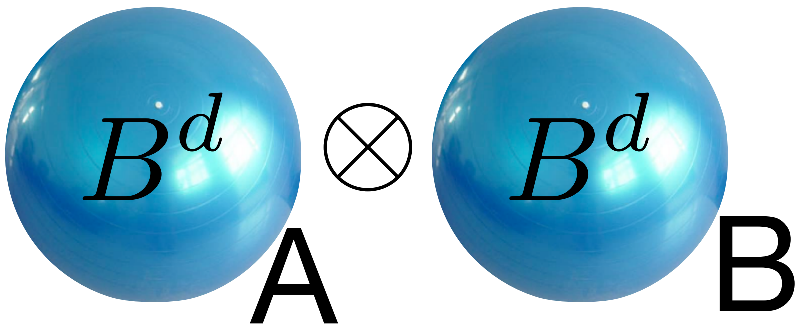

Two ubits: *some* composite state space of two *d*-balls, $\mathcal{G}_A = \mathcal{G}_B$ transitive on ∂B^d.

Tomographic locality $\Leftrightarrow d_{AB} = d^2 + 2d$

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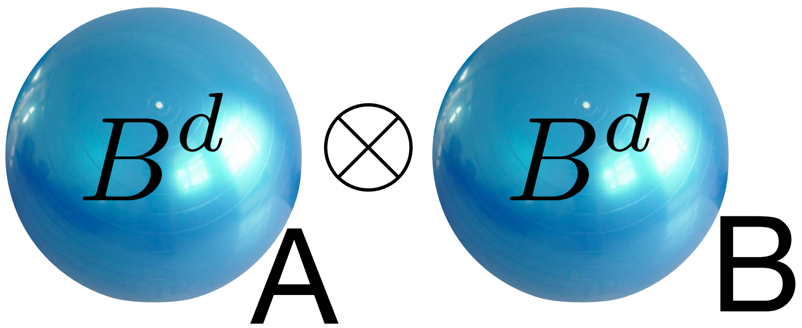

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Theorem. Among all dimensions d and all groups $\mathcal{G}_A$, there are only the following possibilities:

- The trivial solution: $\mathcal{G}_{AB} = \mathcal{G}_A \otimes \mathcal{G}_B$.
- $d = 3$, $\mathcal{G}_A = \mathrm{SO}(3)$ (i.e. the quantum bit), $\mathcal{G}_{AB} \simeq \mathrm{PU}(4)$, and Ω_{AB} is equivalent to the two-qubit quantum state space.

In particular, **continuous reversible interaction** is only possible for $d = 3$, in standard complex two-qubit quantum theory.

Ll. Masanes, **MM**, R. Augusiak, and D. Pérez-García, J. Math. Phys. **55**, 122203 (2014).

Proof idea

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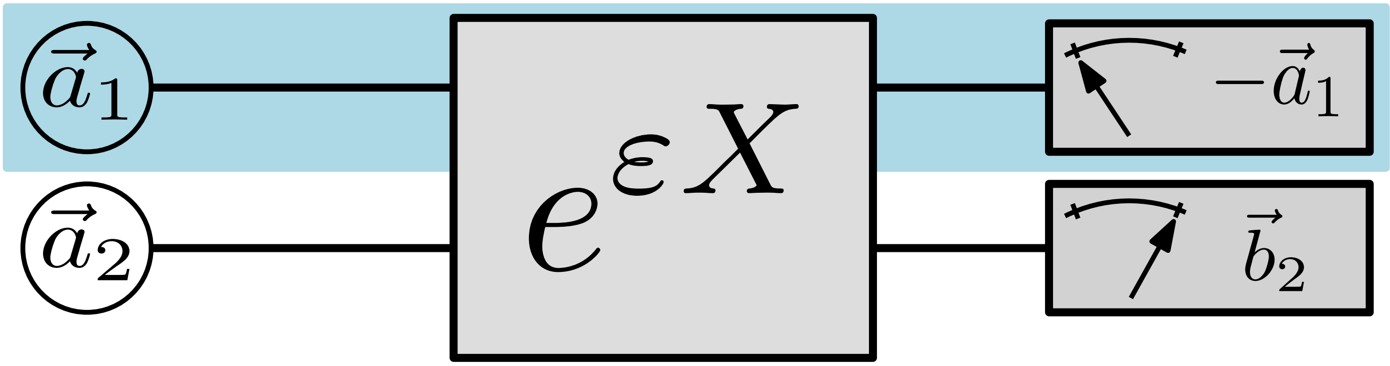
Generator *X* of global reversible transformation (no idea what it is…)



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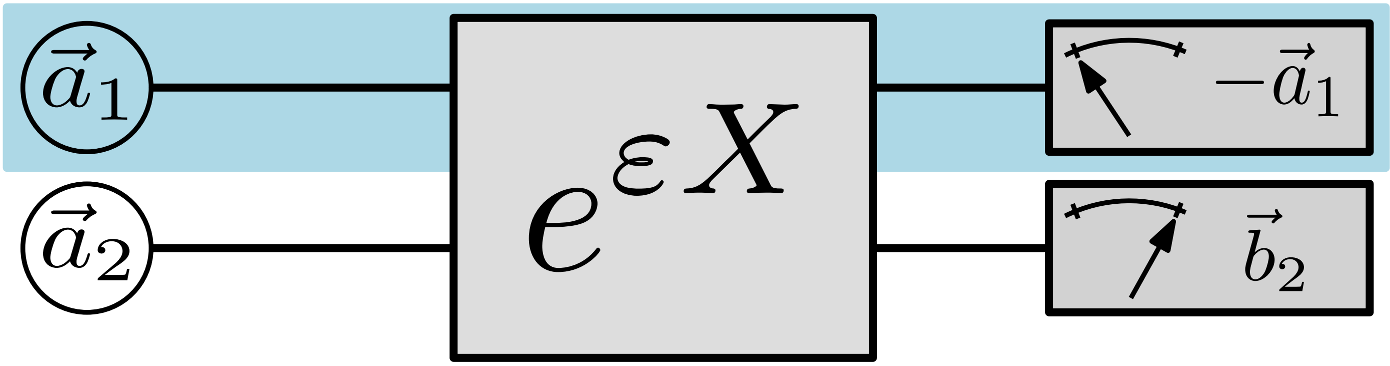

We must obtain **valid probabilities**. For example,

$$0 \leq (e_{-\vec{a}_1} \otimes e_{\vec{b}_2}) e^{\varepsilon X} (\omega_{\vec{a}_1} \otimes \omega_{\vec{a}_2}) \leq 1.$$

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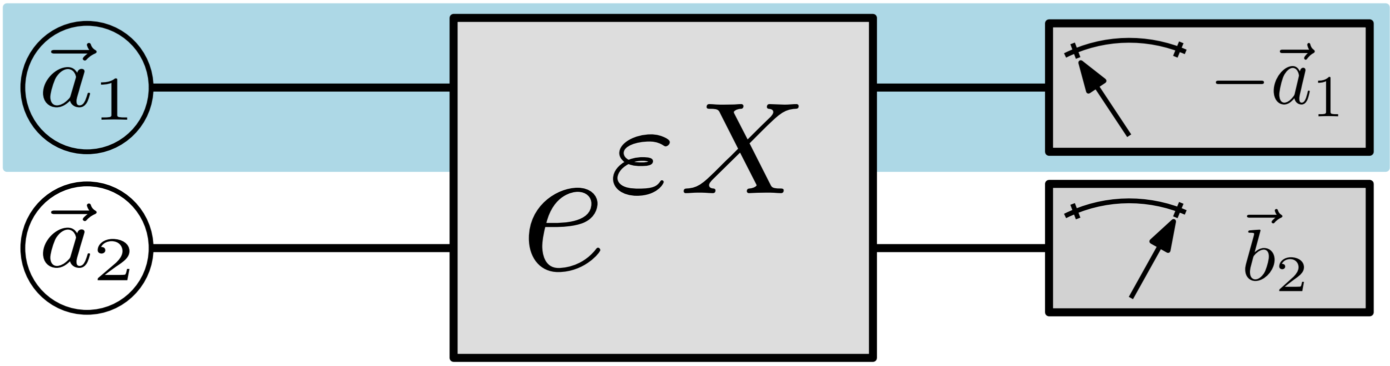

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$$0 \leq (e_{-\vec{a}_1} \otimes e_{\vec{b}_2}) e^{\varepsilon X} (\omega_{\vec{a}_1} \otimes \omega_{\vec{a}_2}) \leq 1.$$

For $\varepsilon = 0$ this gives probability zero, which must be a local minimum.

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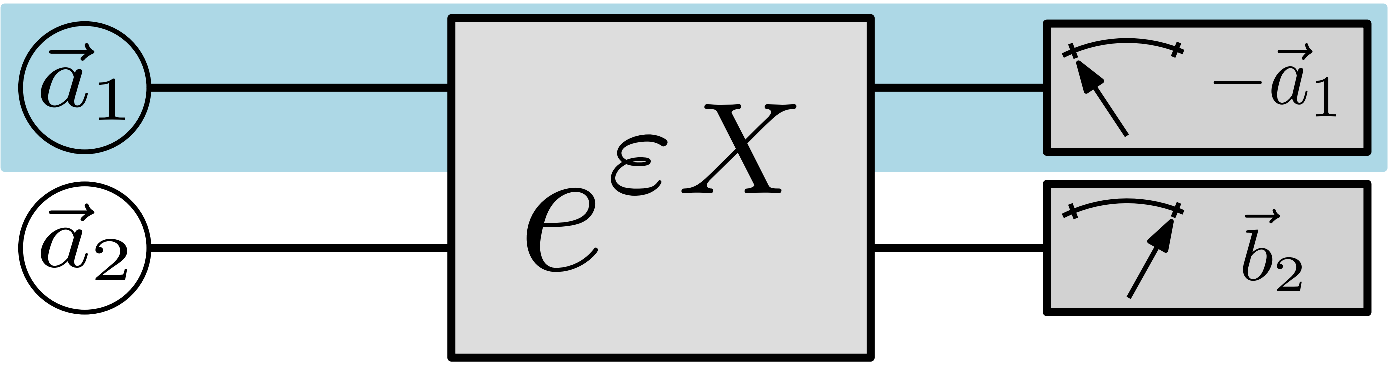

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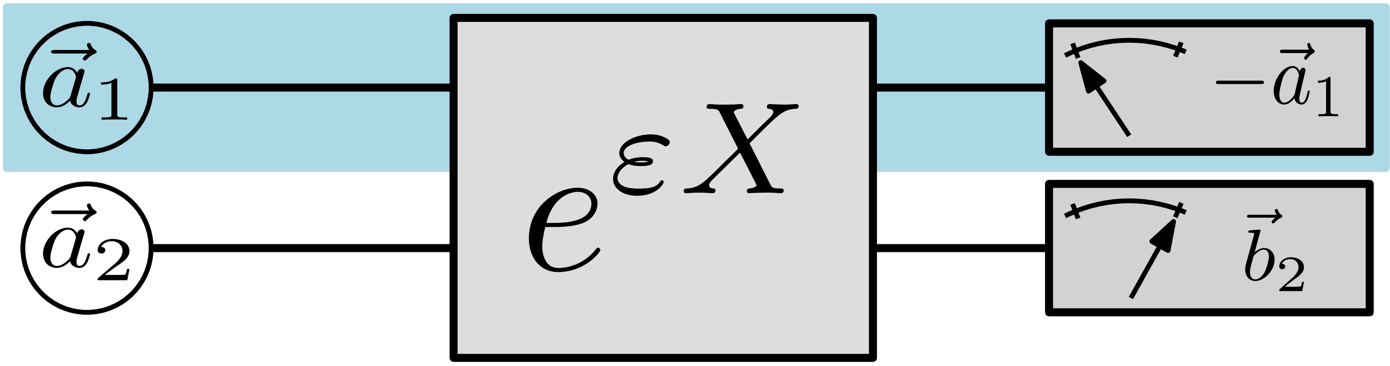

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Main reason: $\mathrm{SO}(d-1)$ is only non-trivial and **commutative** for $d = 3$.

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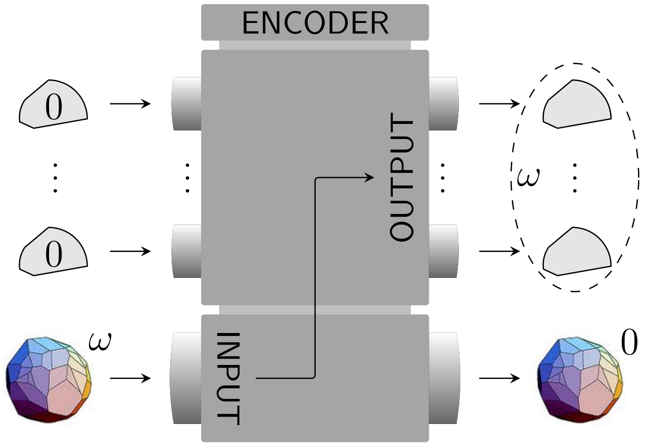
1. Probabilistic theories beyond quantum theory

2. Quantum theory from simple principles



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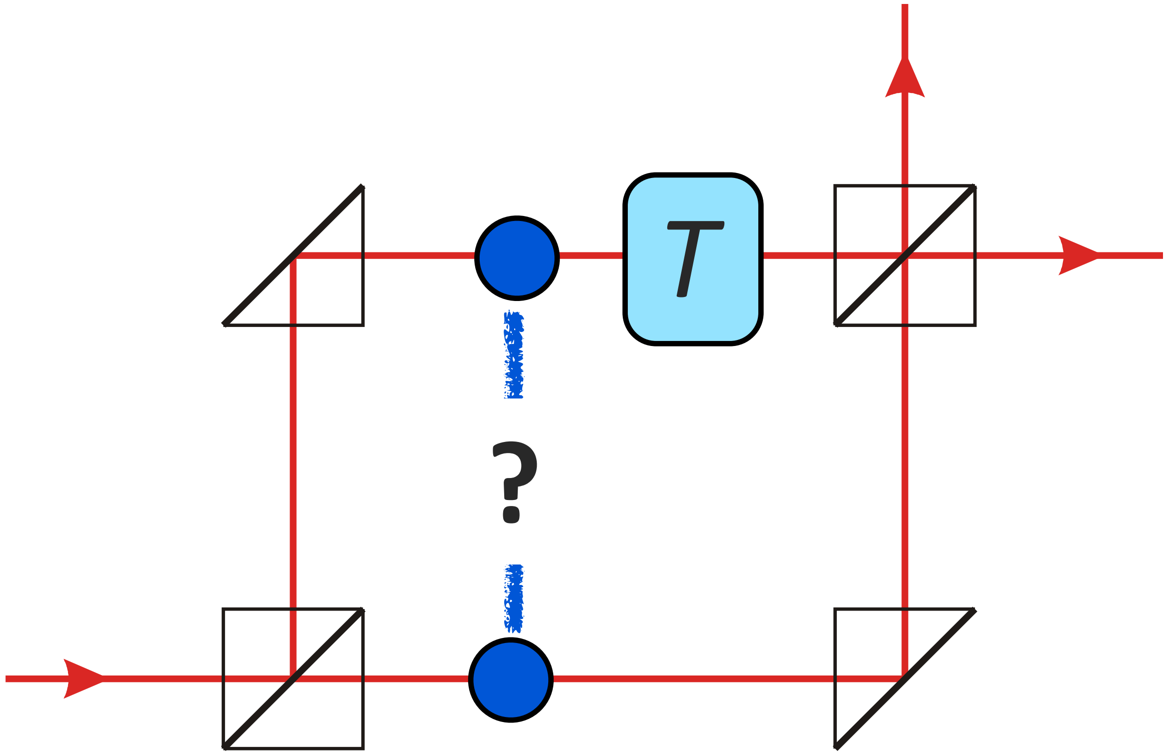

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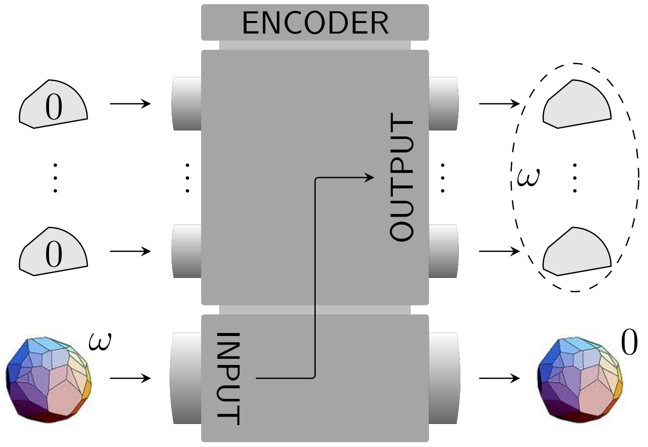

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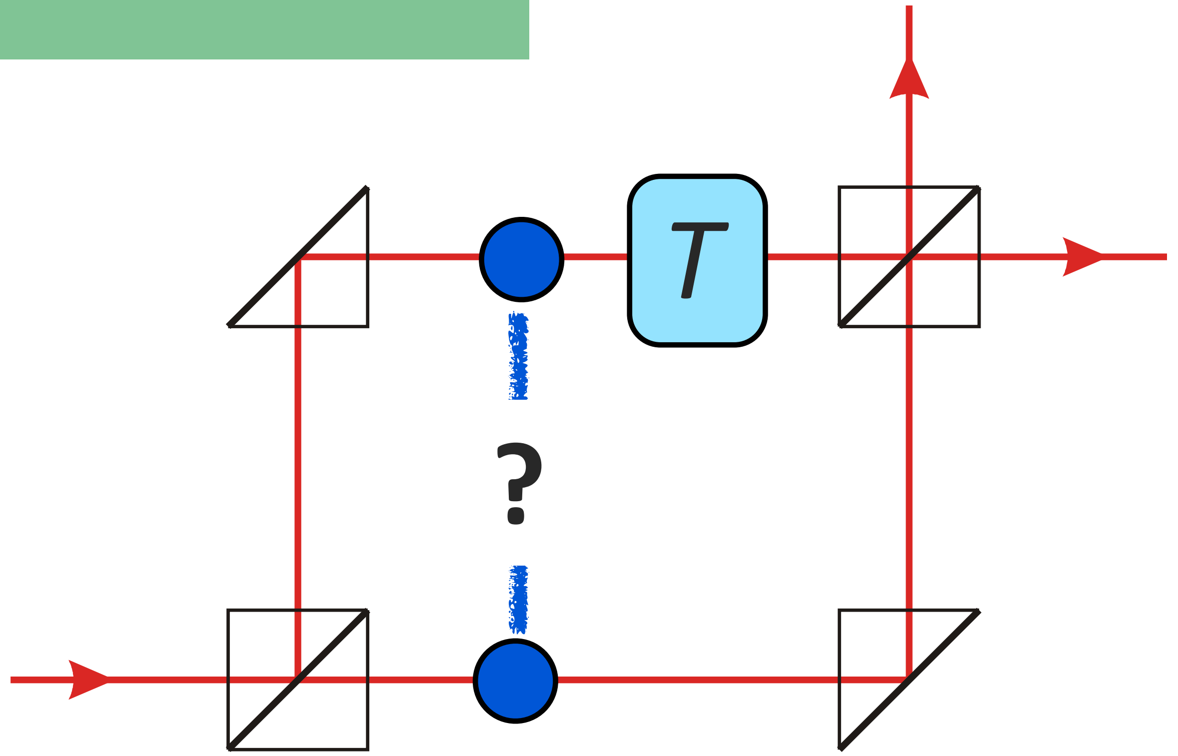

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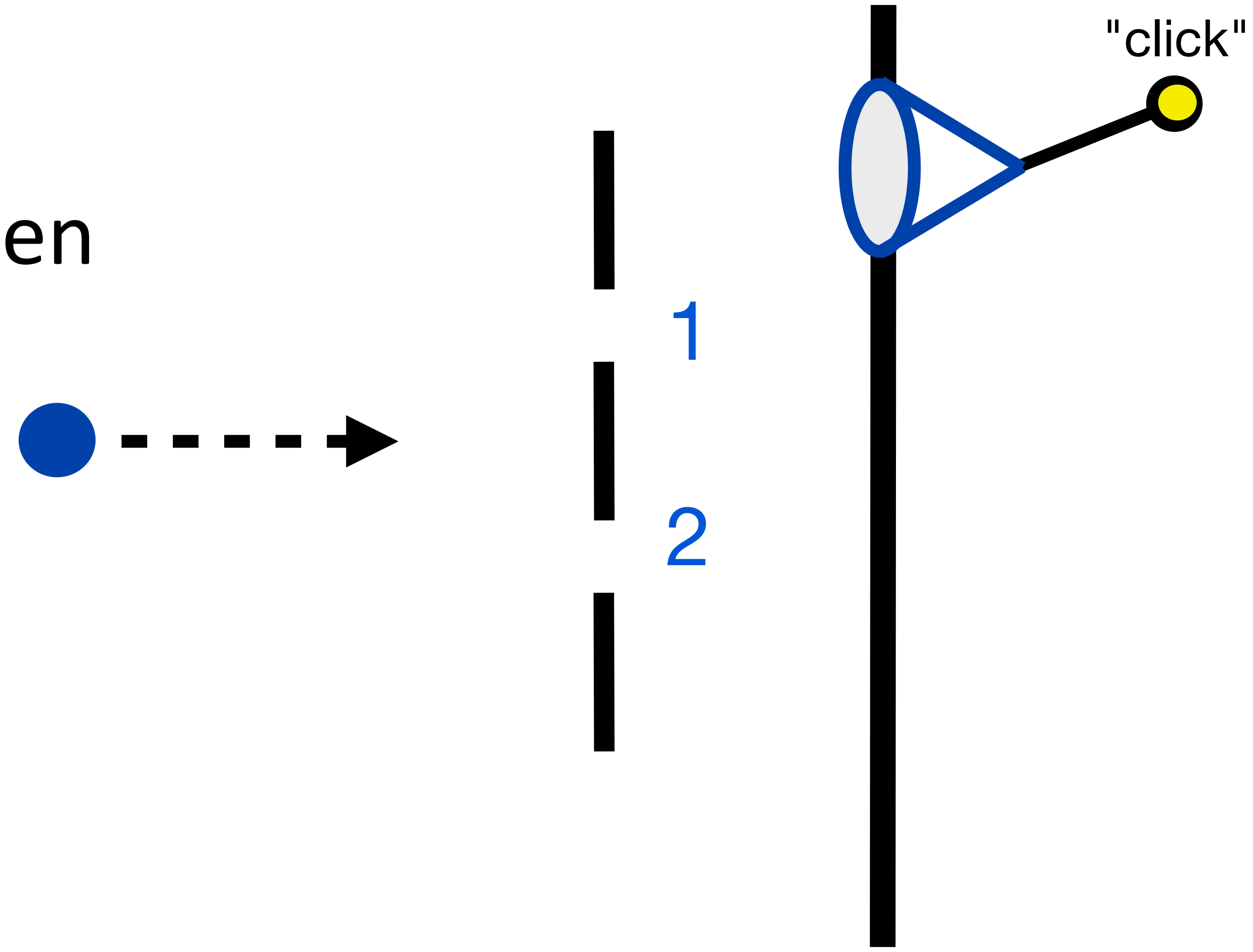

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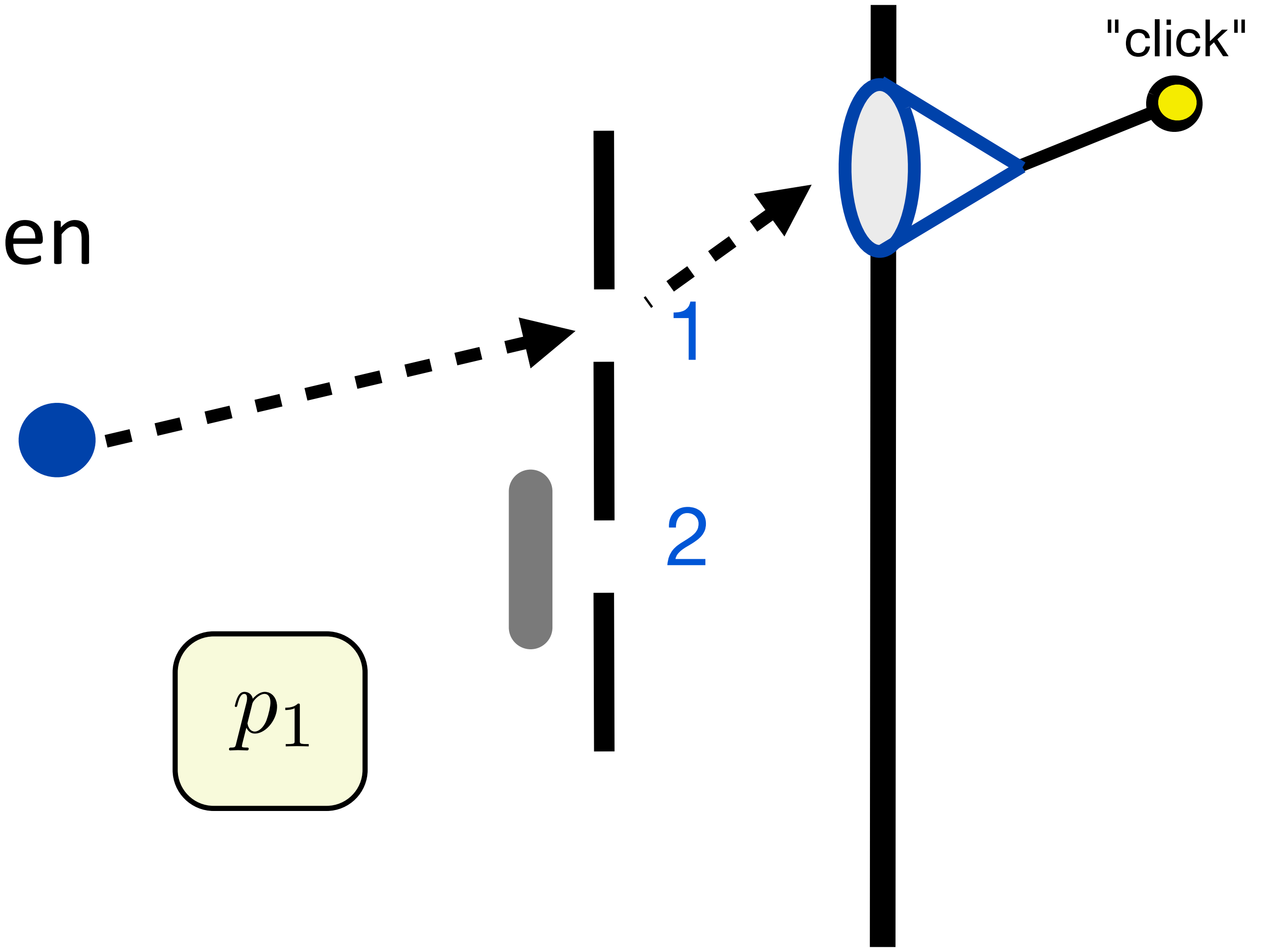

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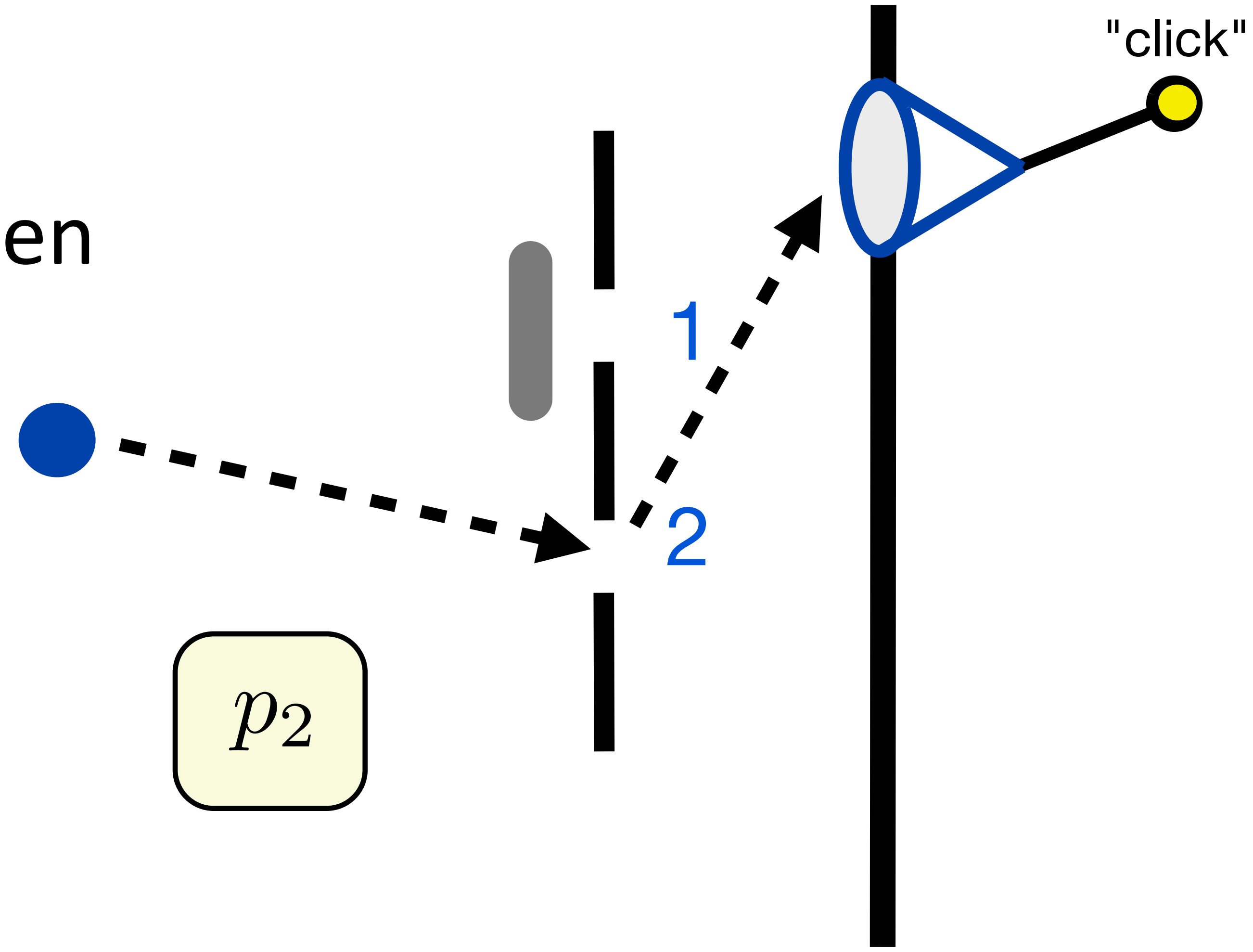

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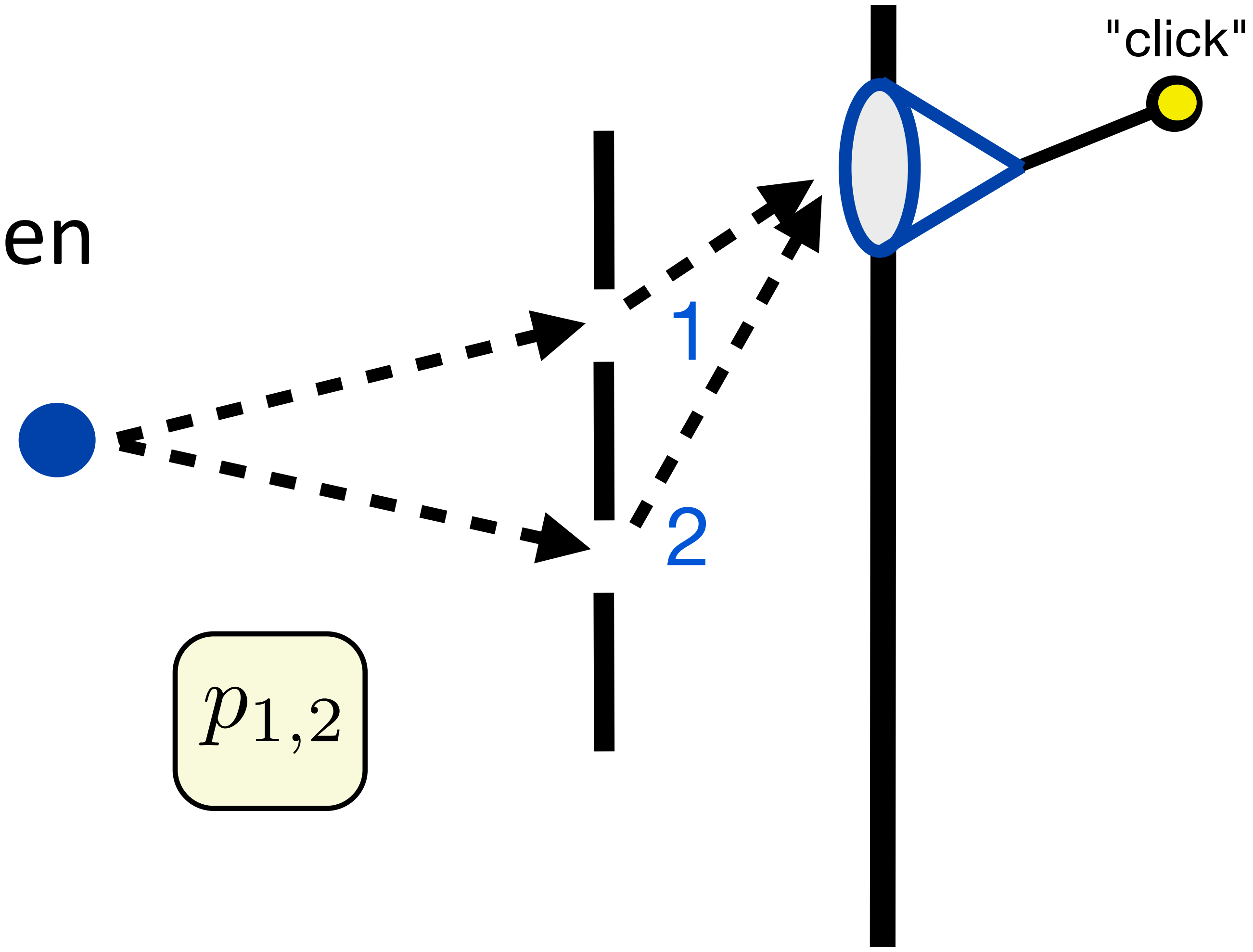

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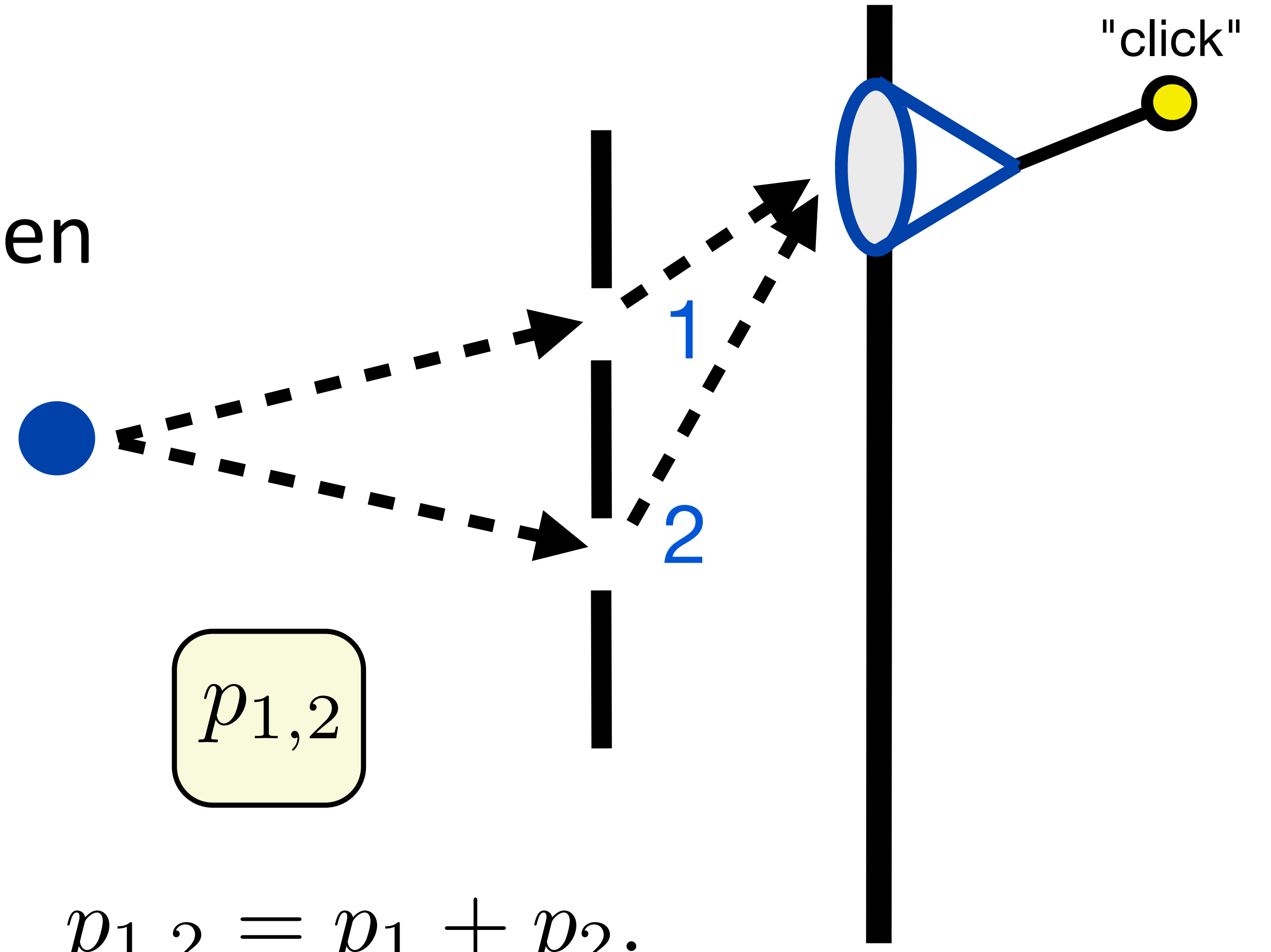

Classical probability theory: $p_{1,2} = p_1 + p_2.$

Quantum theory: $p_{1,2} \neq p_1 + p_2.$ (2nd order) **interference**!

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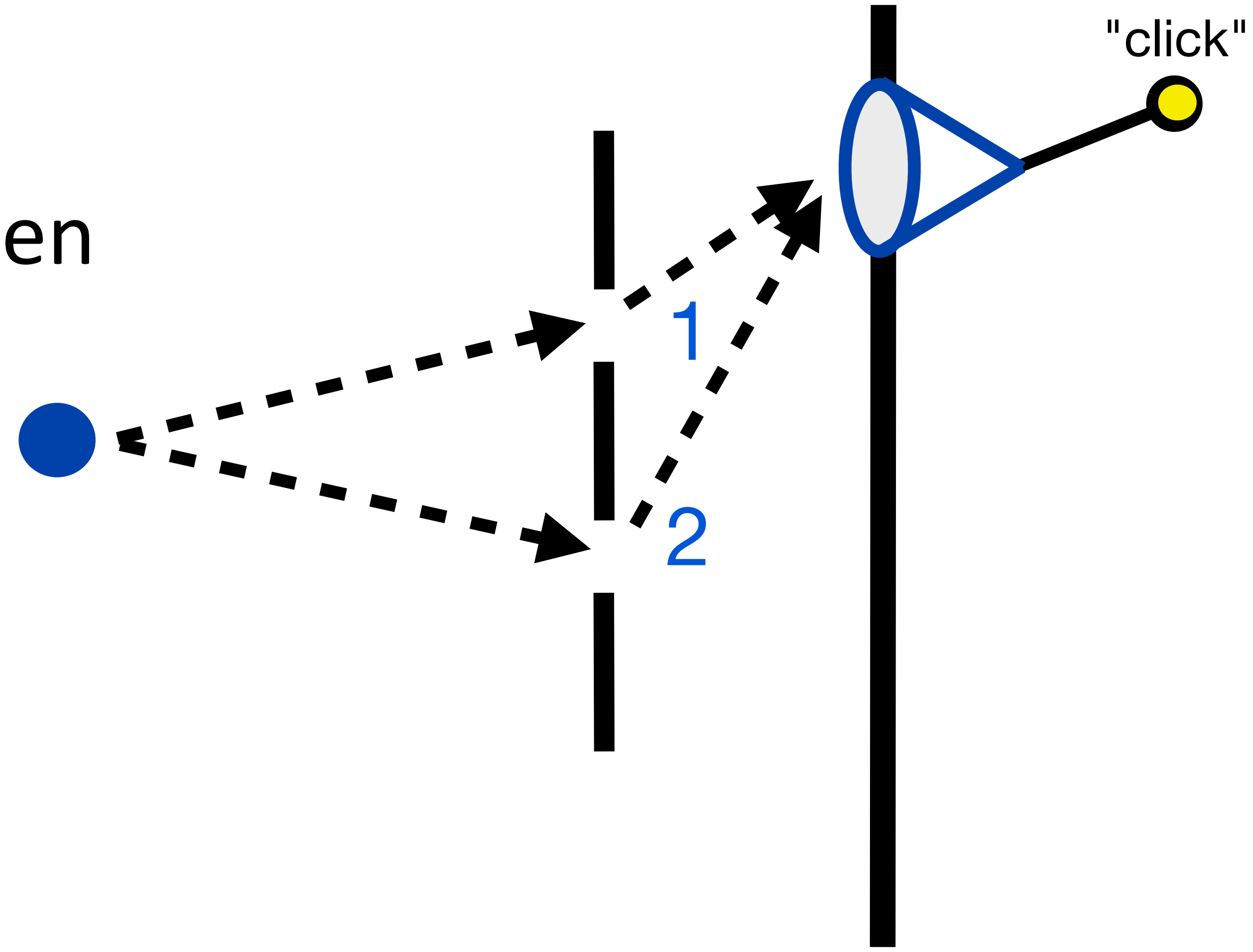

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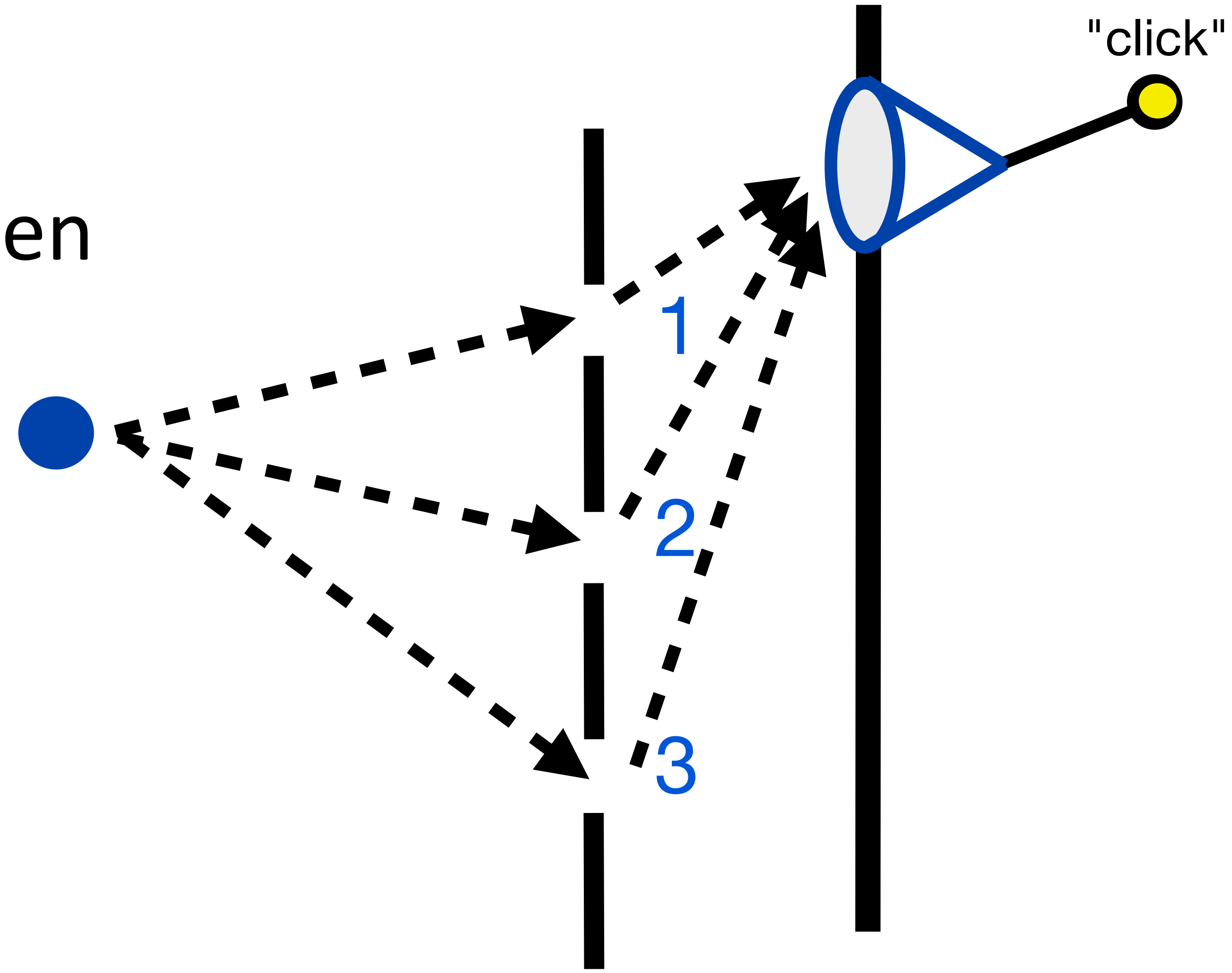

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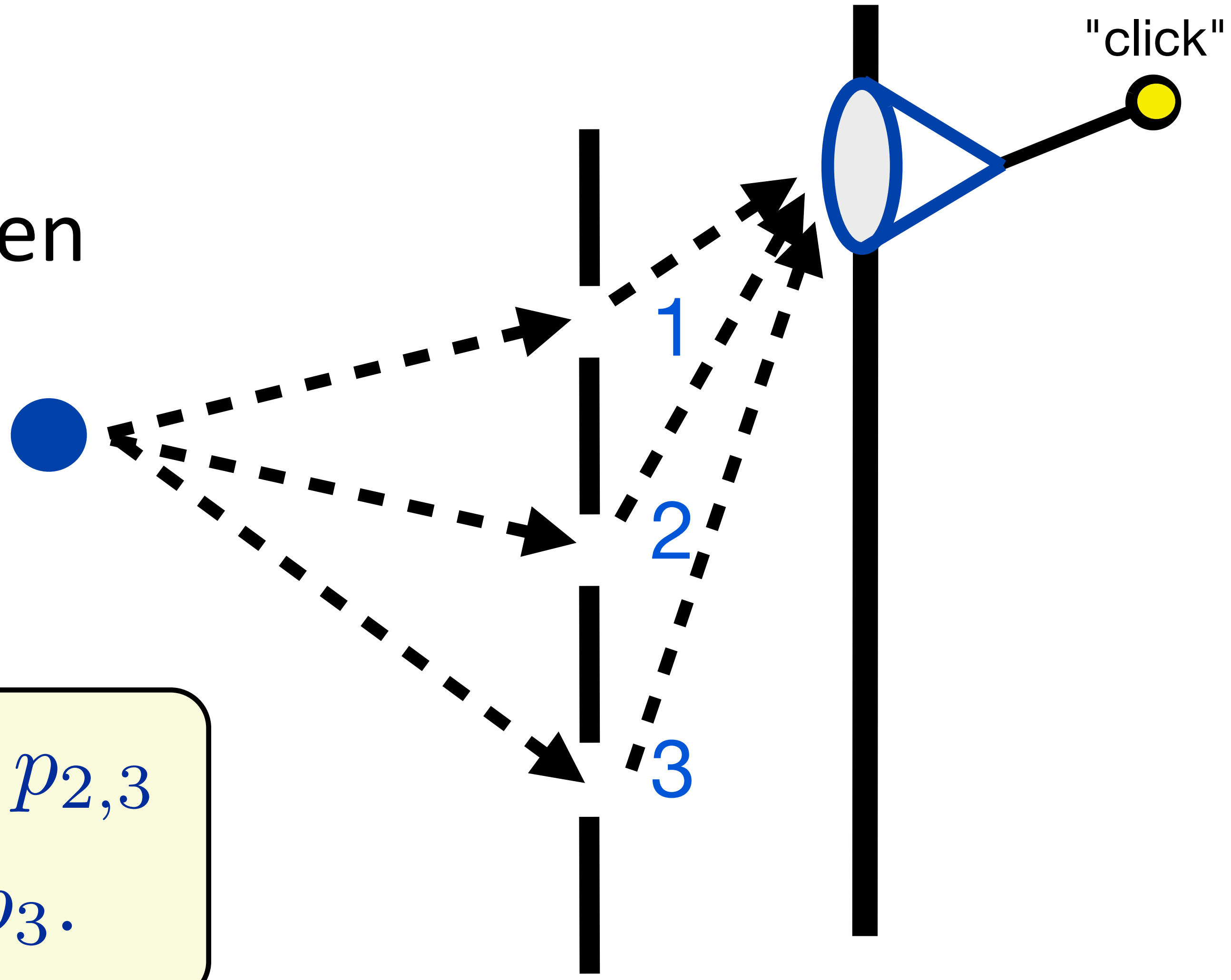

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$$p_{1,2,3} = p_{1,2} + p_{1,3} + p_{2,3} - p_1 - p_2 - p_3.$$

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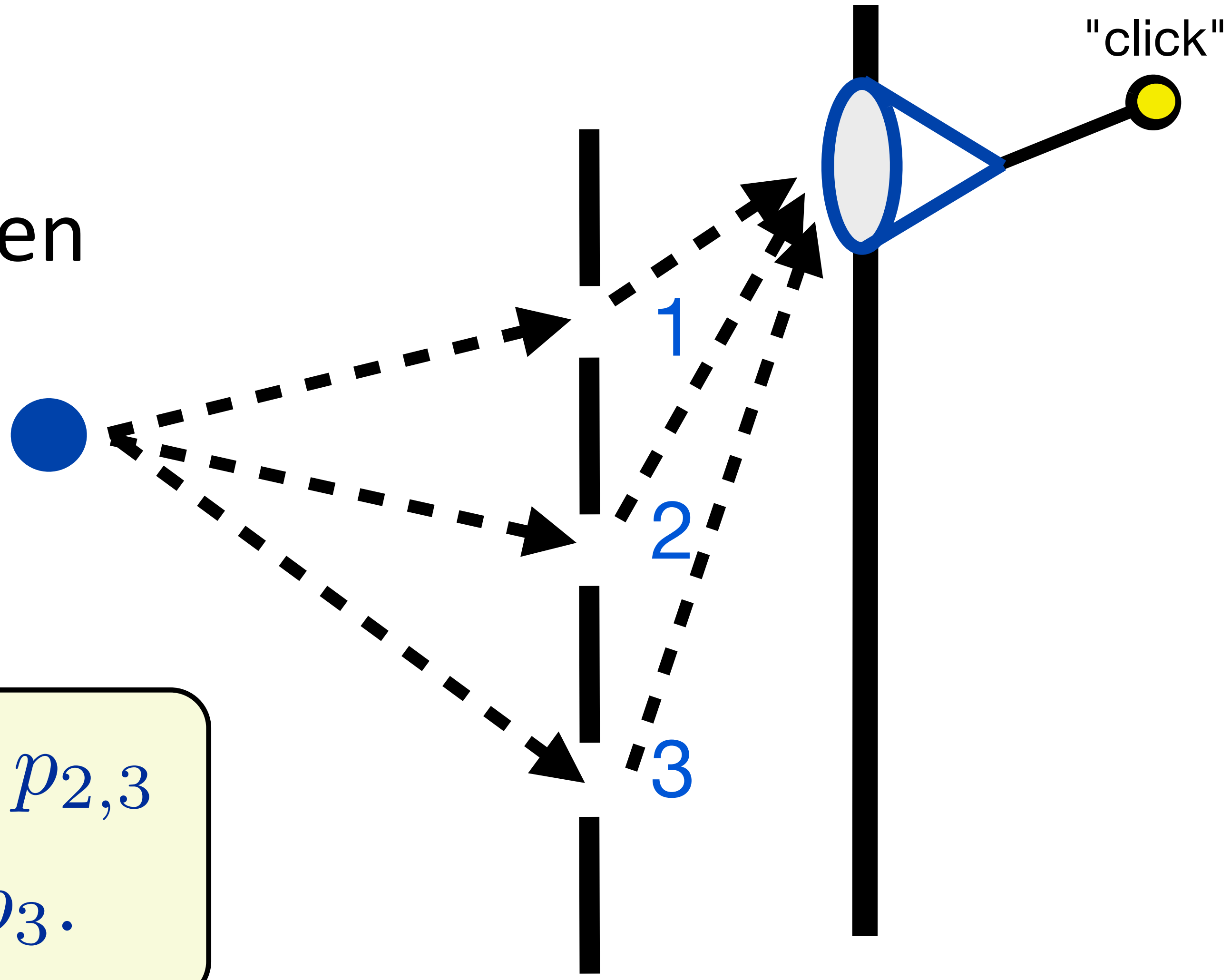

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No 3rd-order interference in QT.

Why does QT not have 3rd-order interference?

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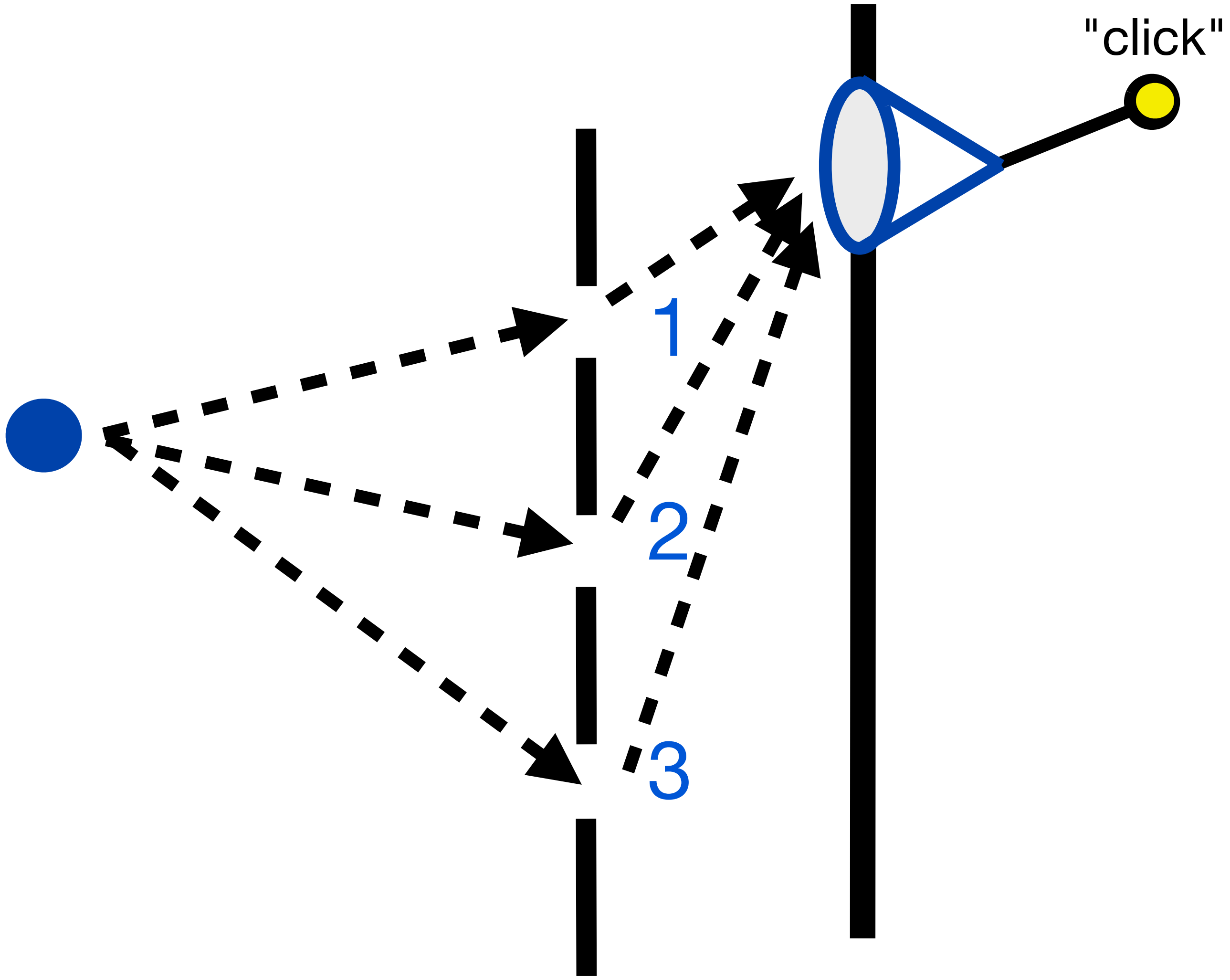

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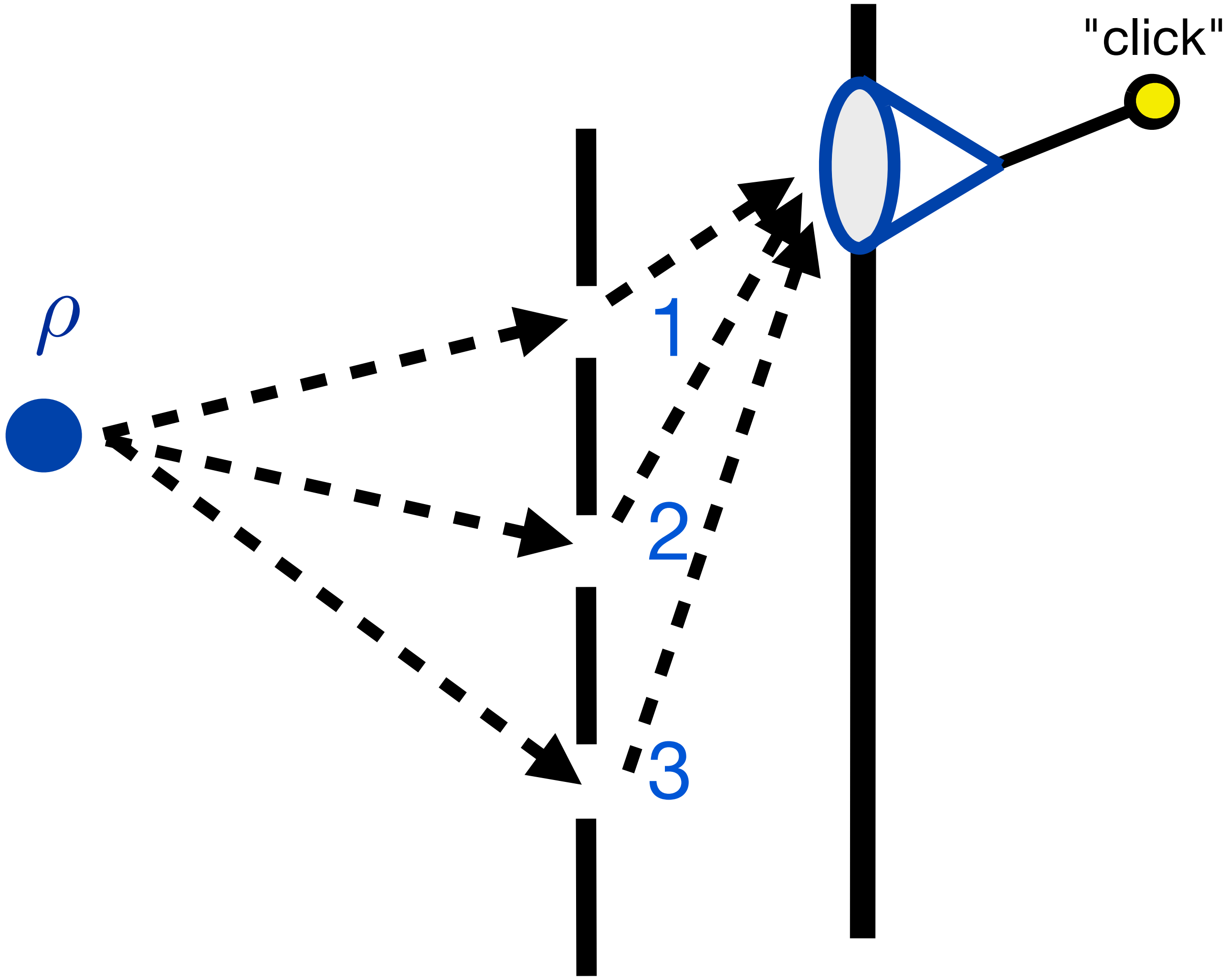
$$\rho = \begin{pmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{pmatrix}$$



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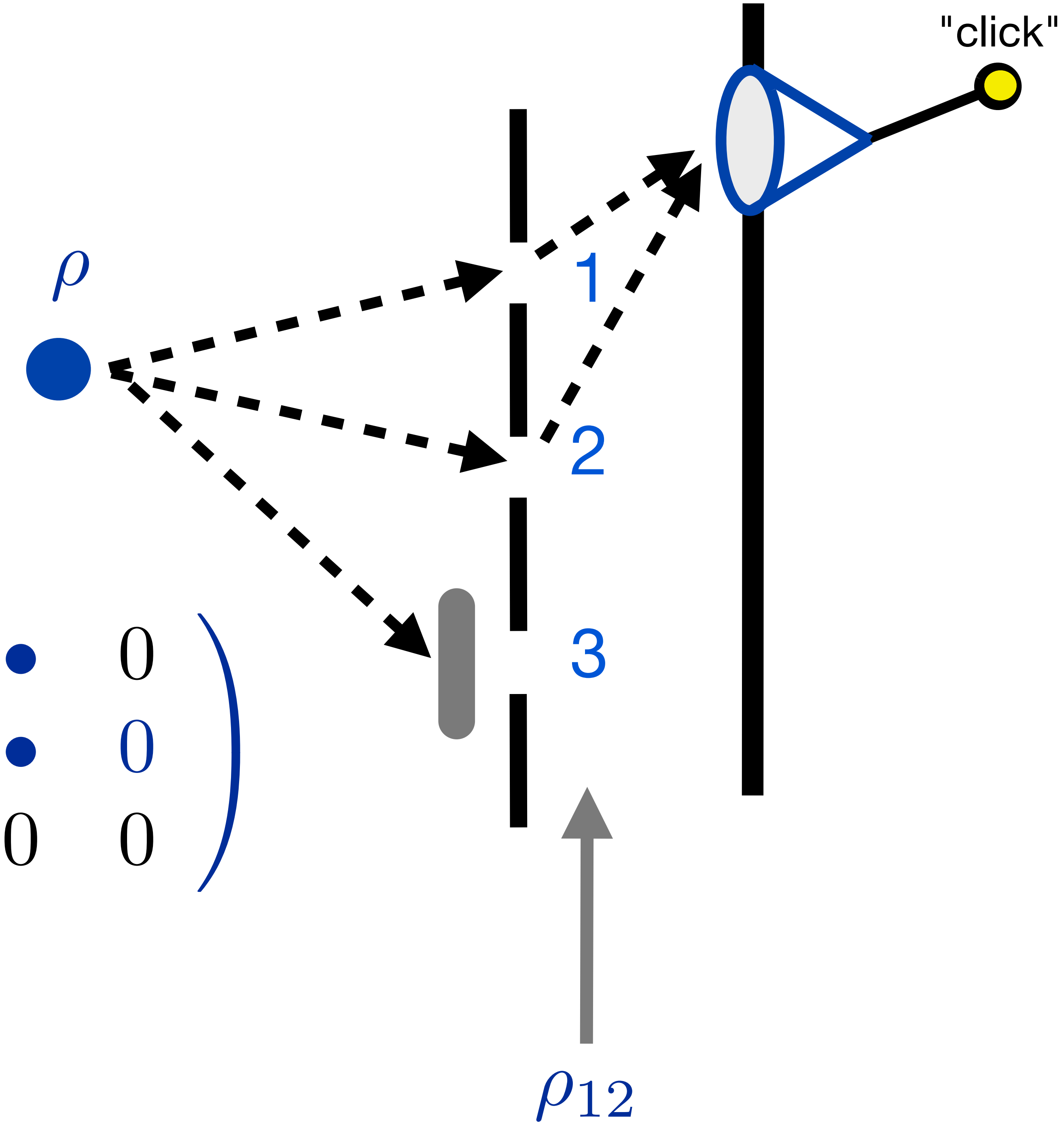

$$\rho \mapsto P_{12}\rho P_{12} =: \rho_{12} = \begin{pmatrix} \bullet & \bullet & 0 \\ \bullet & \bullet & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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CPT:

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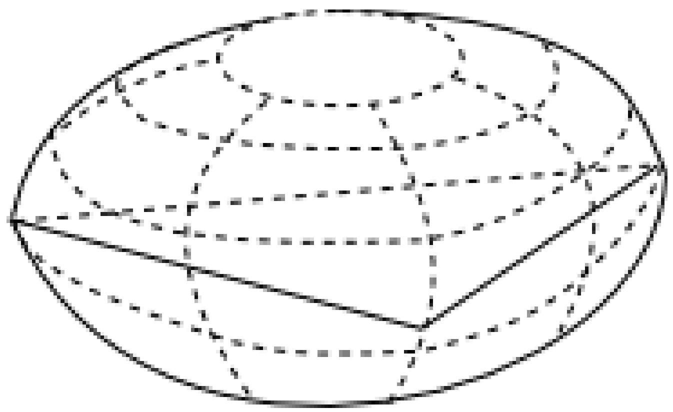
Why does CPT not have 2nd-order interference?

Some "artificial" GPTs exhibit order-3 interference:



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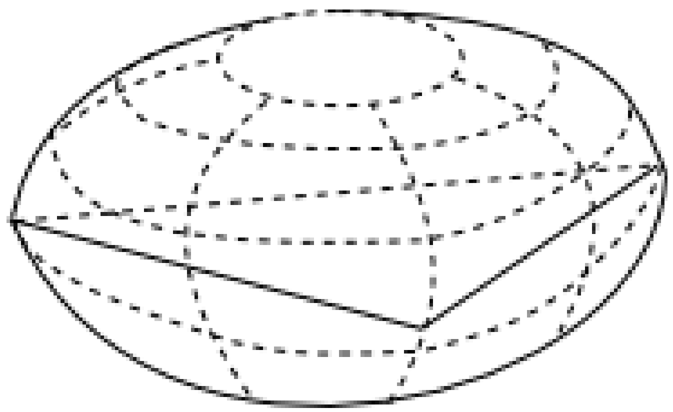

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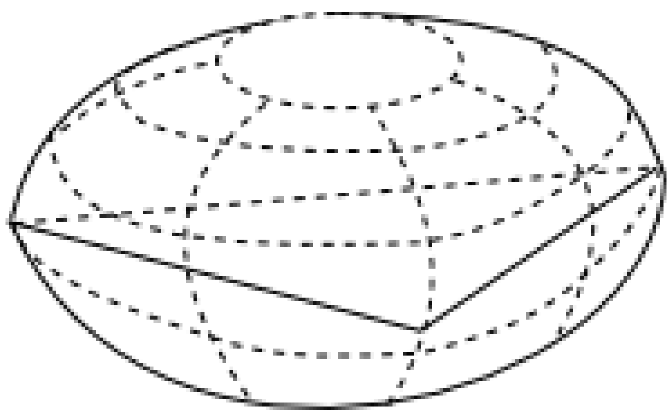

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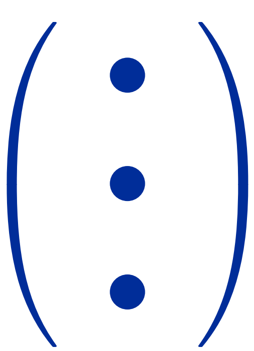

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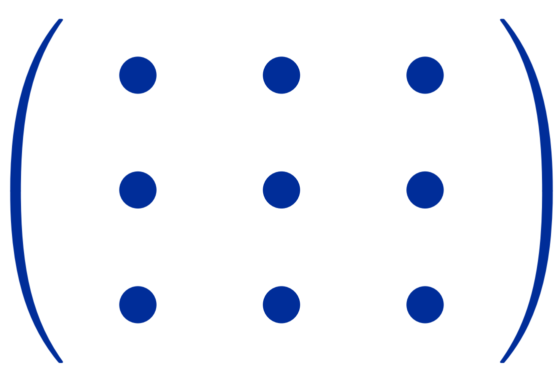

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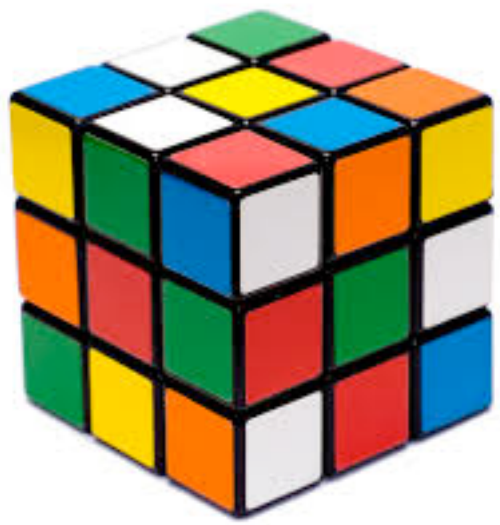

3rd-order interference?

A quantum detective story

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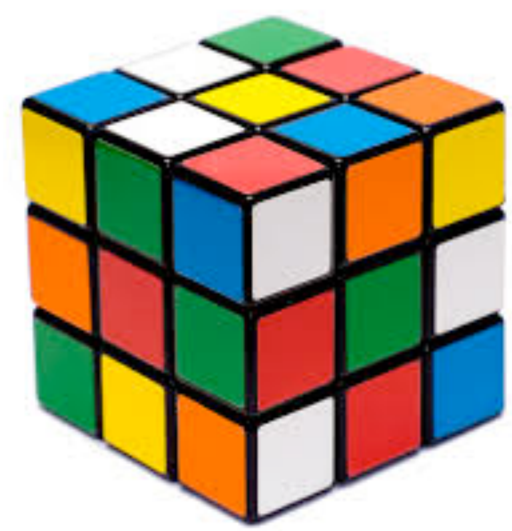

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- These would predict higher-order interference.
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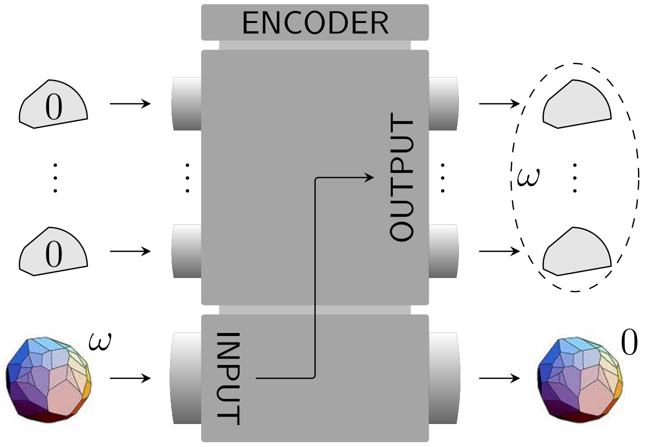

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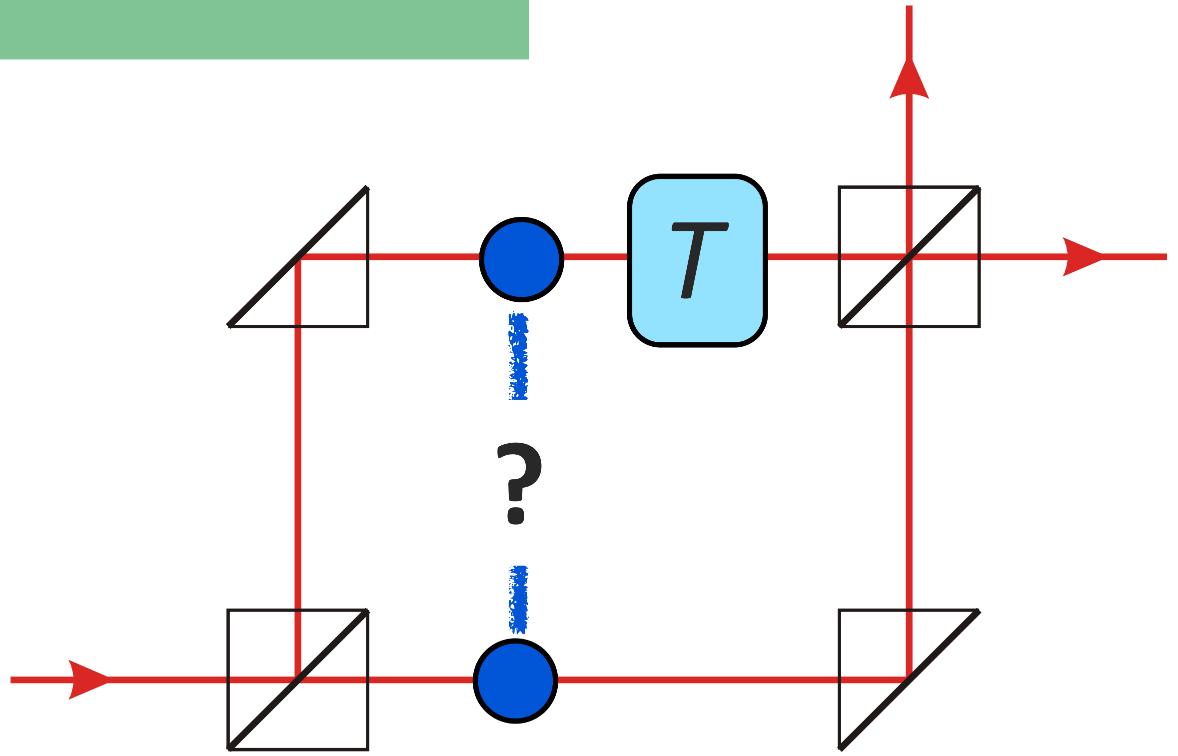

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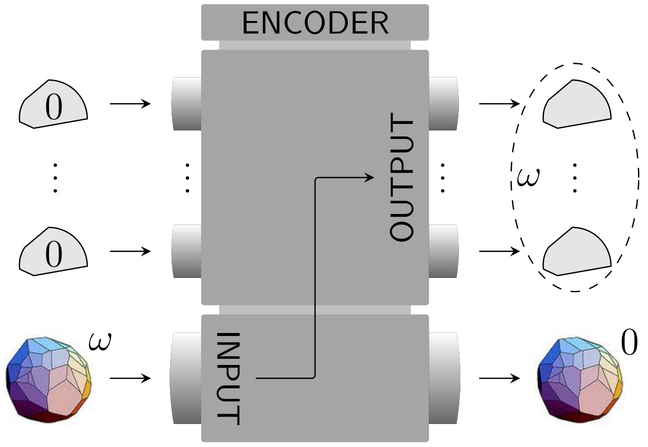

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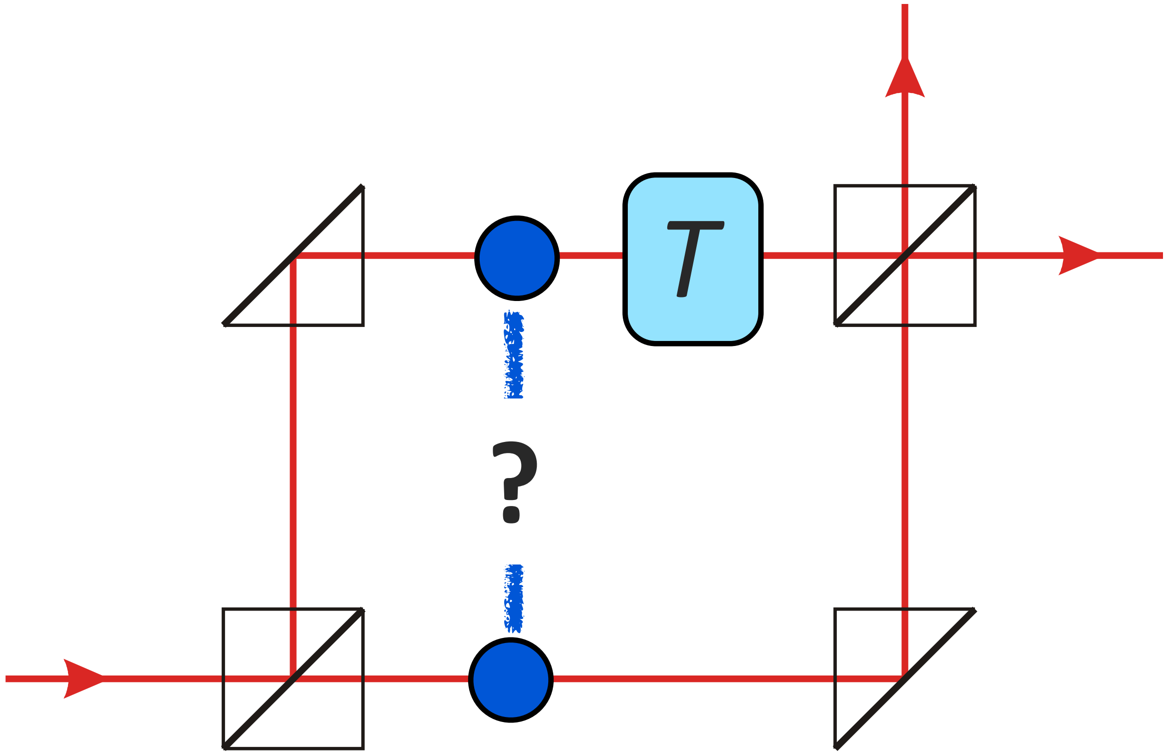

5. Conclusion

The qubit revisited

We have seen: simple assumptions tell us that a **bit** should have a **Euclidean ball** state space.

$d = 1$ classical bit

$d = 2$

$d = 3$ quantum bit

$d = 4$

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$d=1$ classical bit

$d=2$

$d=3$ quantum bit

$d=4$

$\mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}$ -qubits would have $d=2,3,5,9.$

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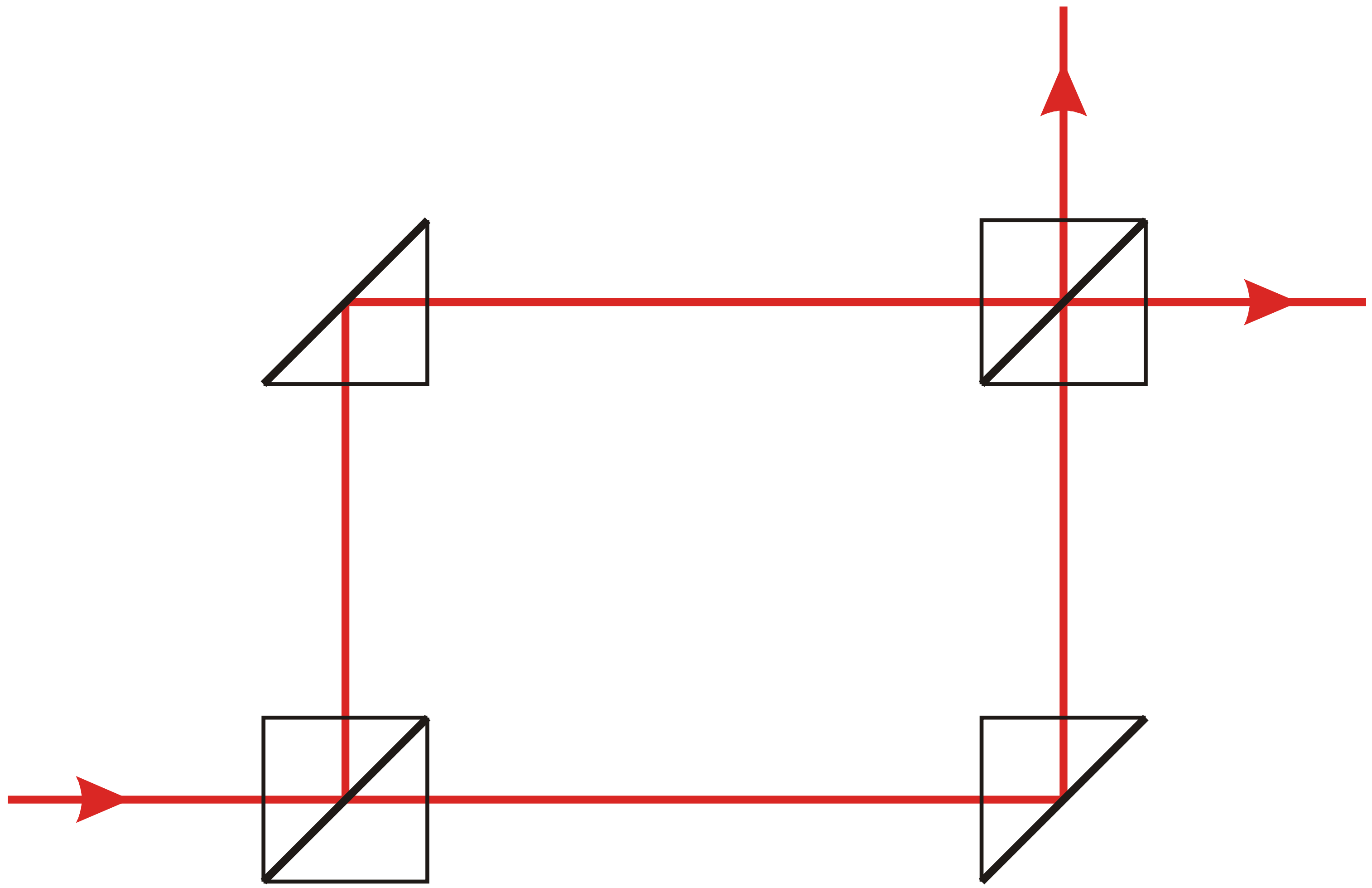
We have already seen an **information-theoretic** reason.
But there is also a "spacetime" reason!

Constraints from relativity

A. Garner, **MM**, O. C. O. Dahlsten, Proc. R. Soc. A **473**, 20170596 (2017).



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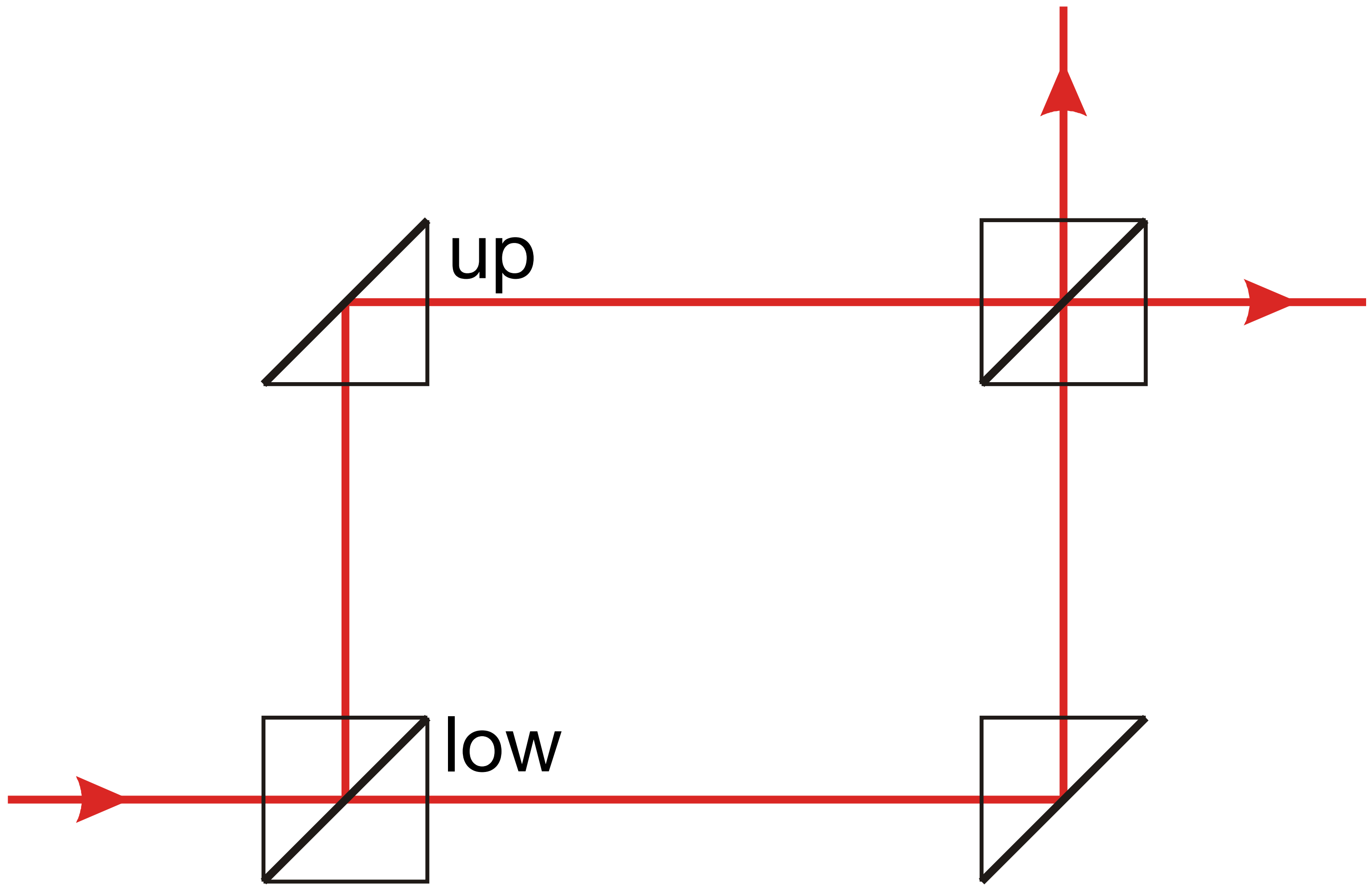

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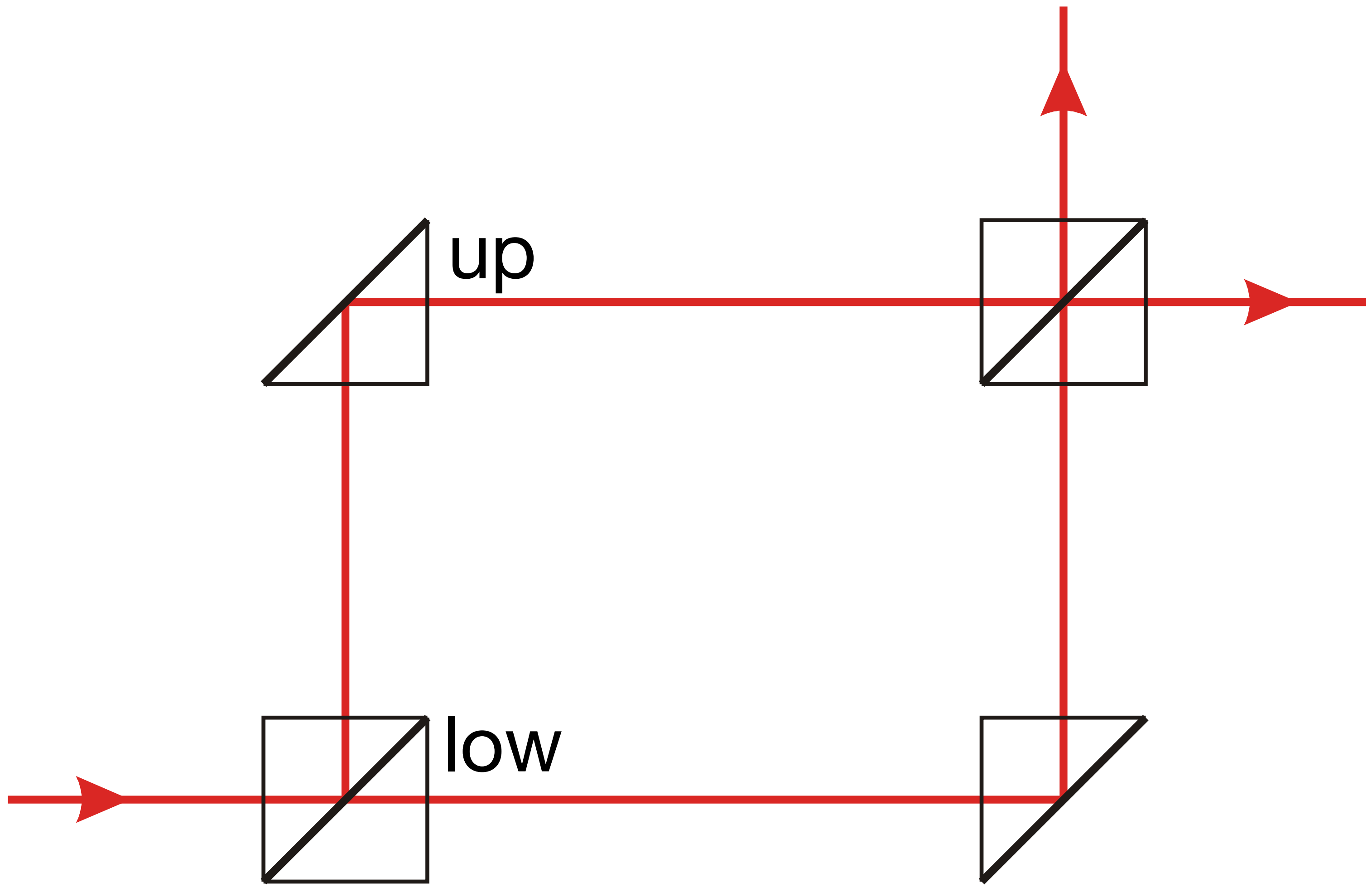

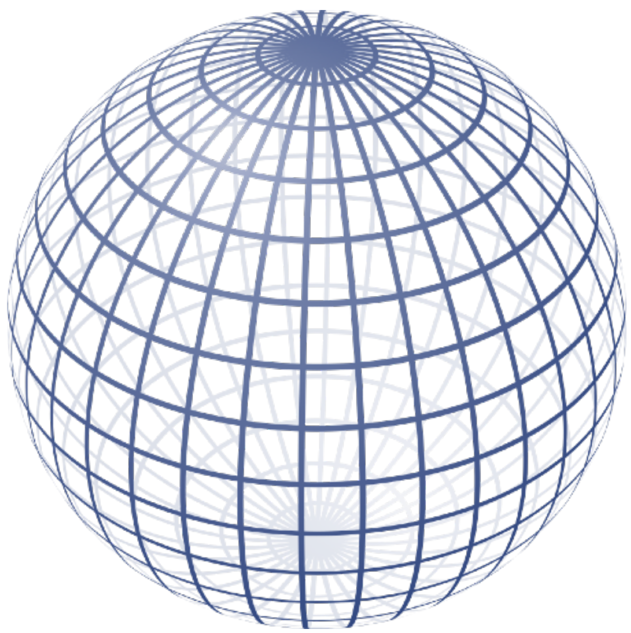

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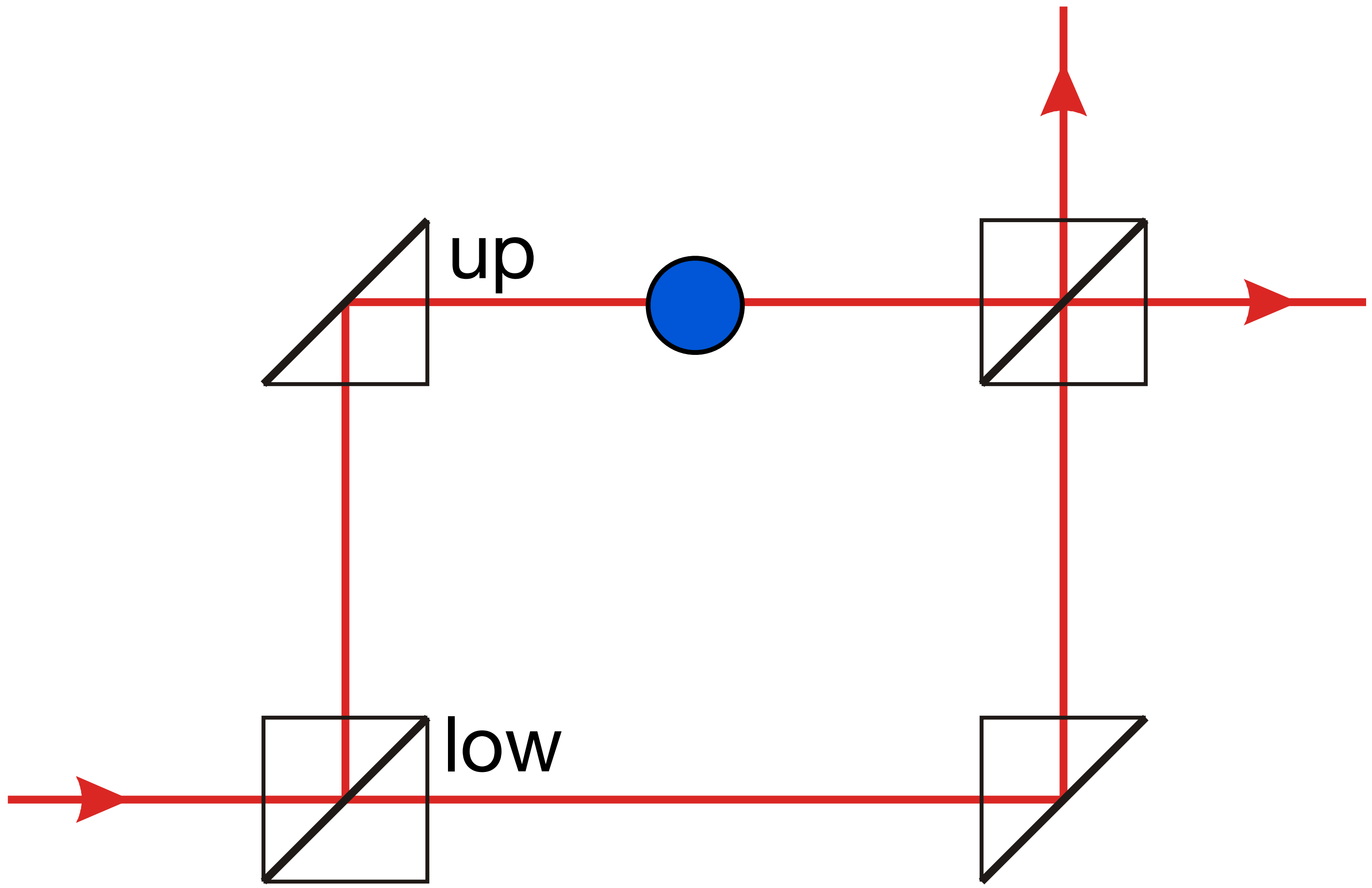

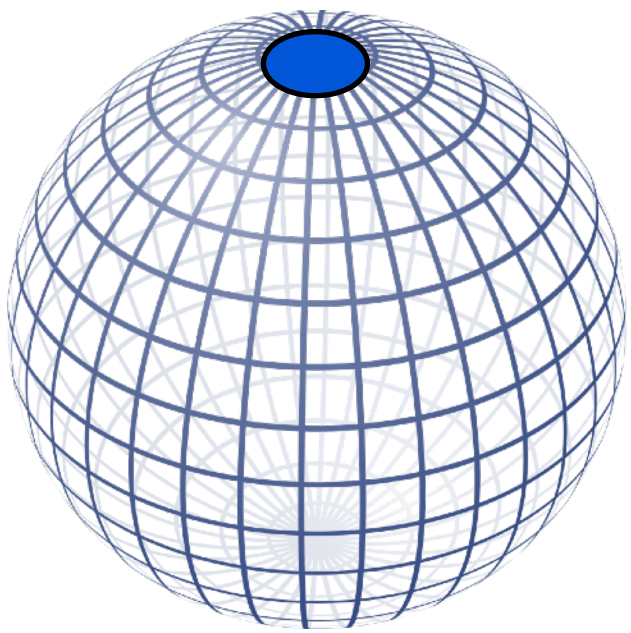

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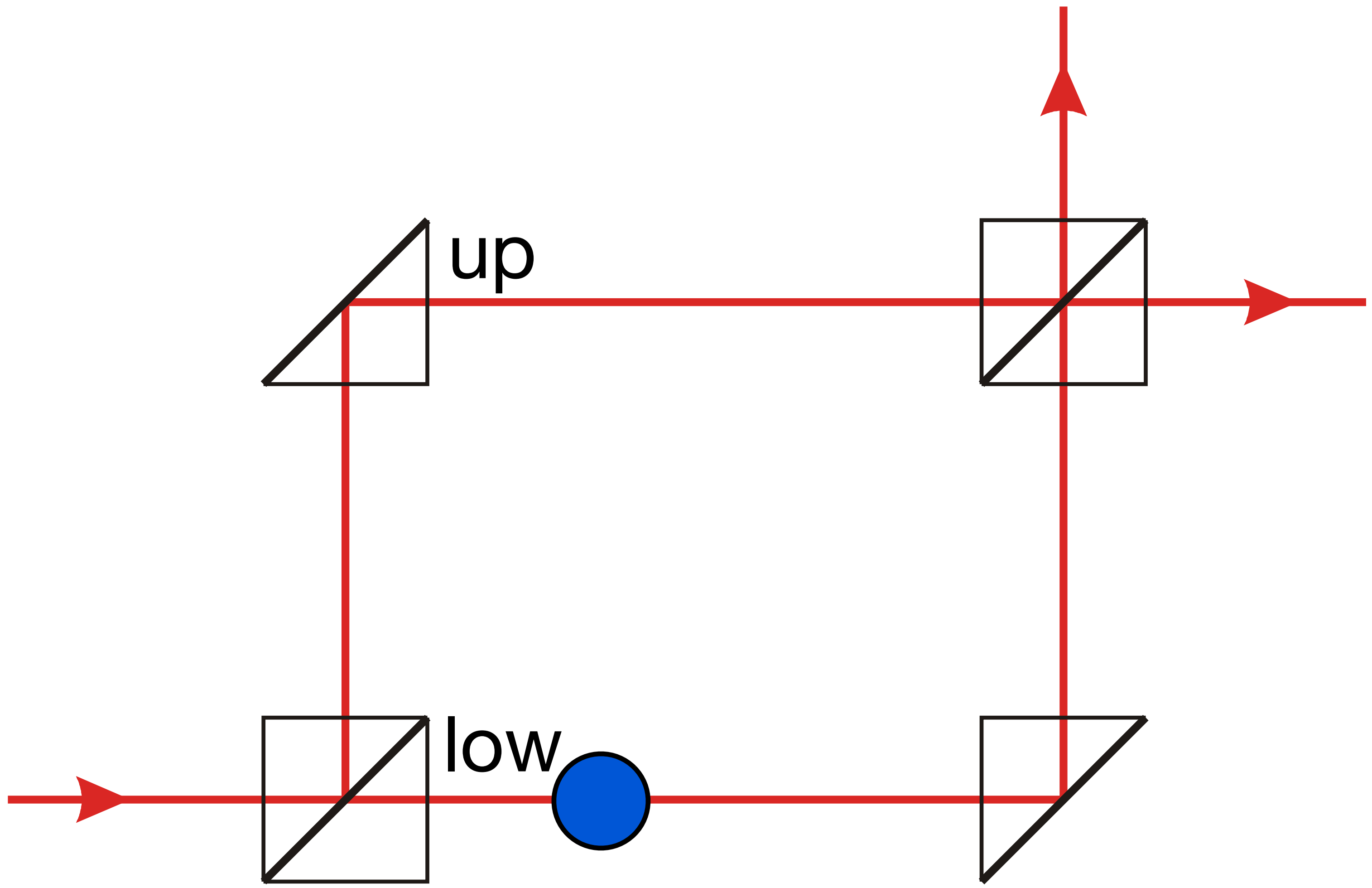

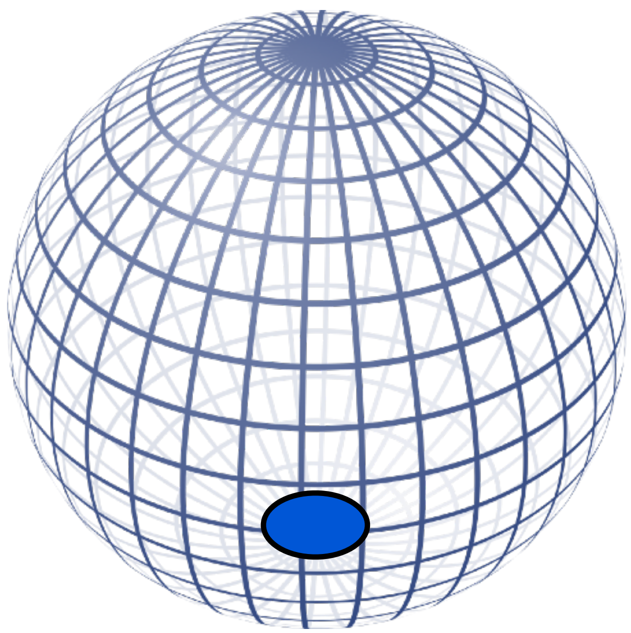

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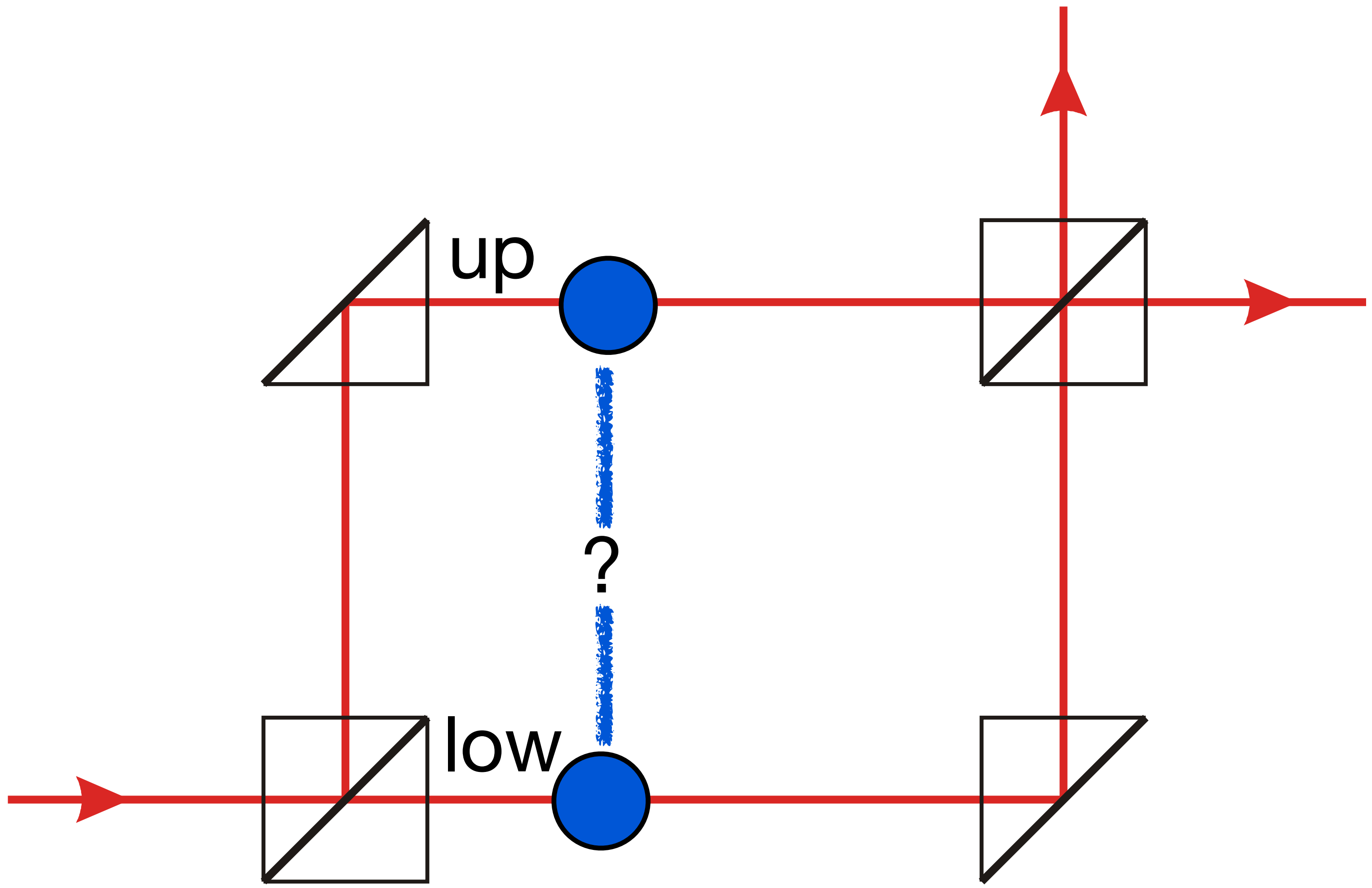

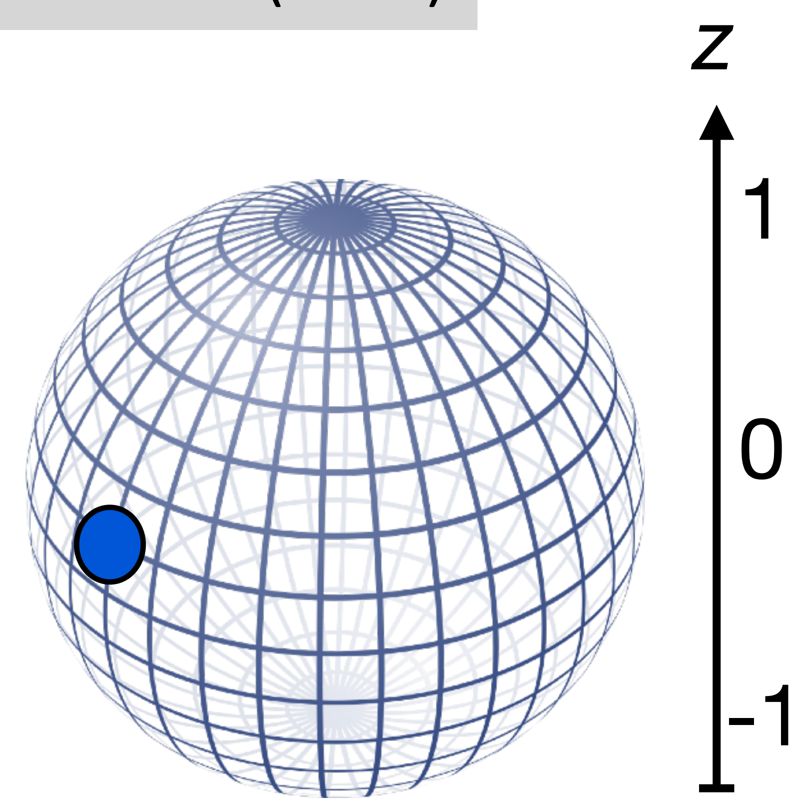

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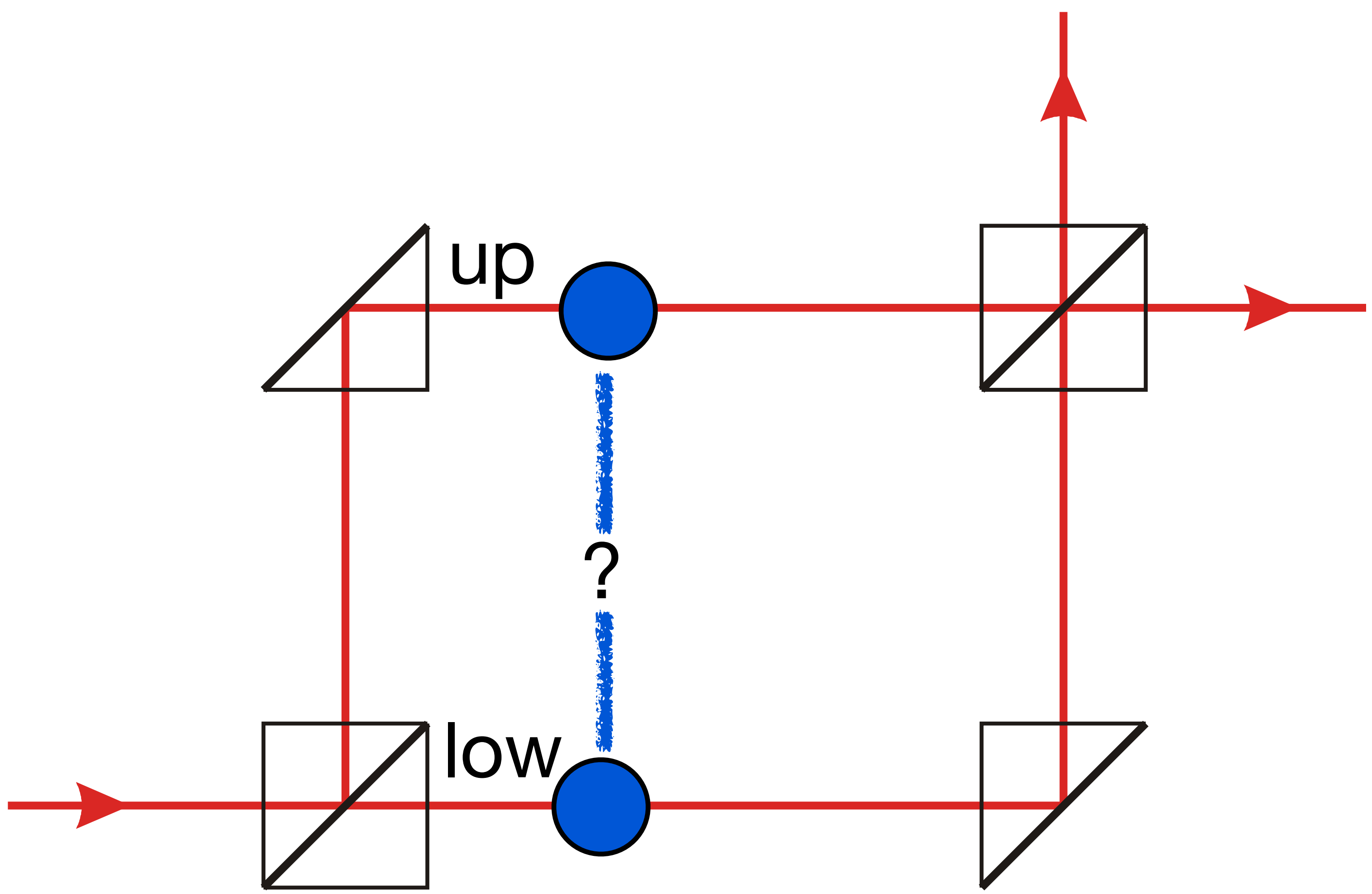

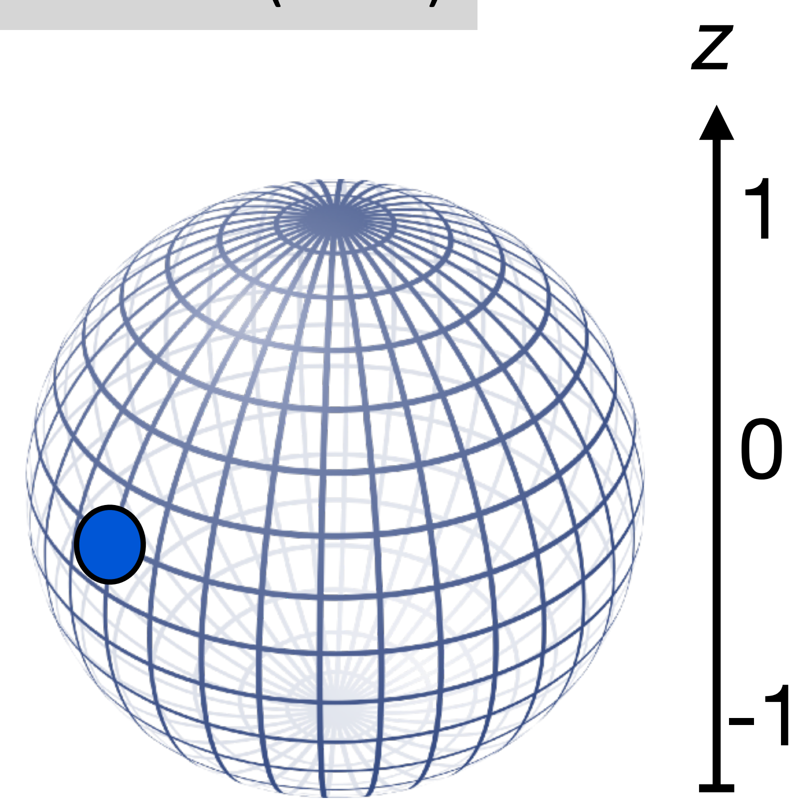

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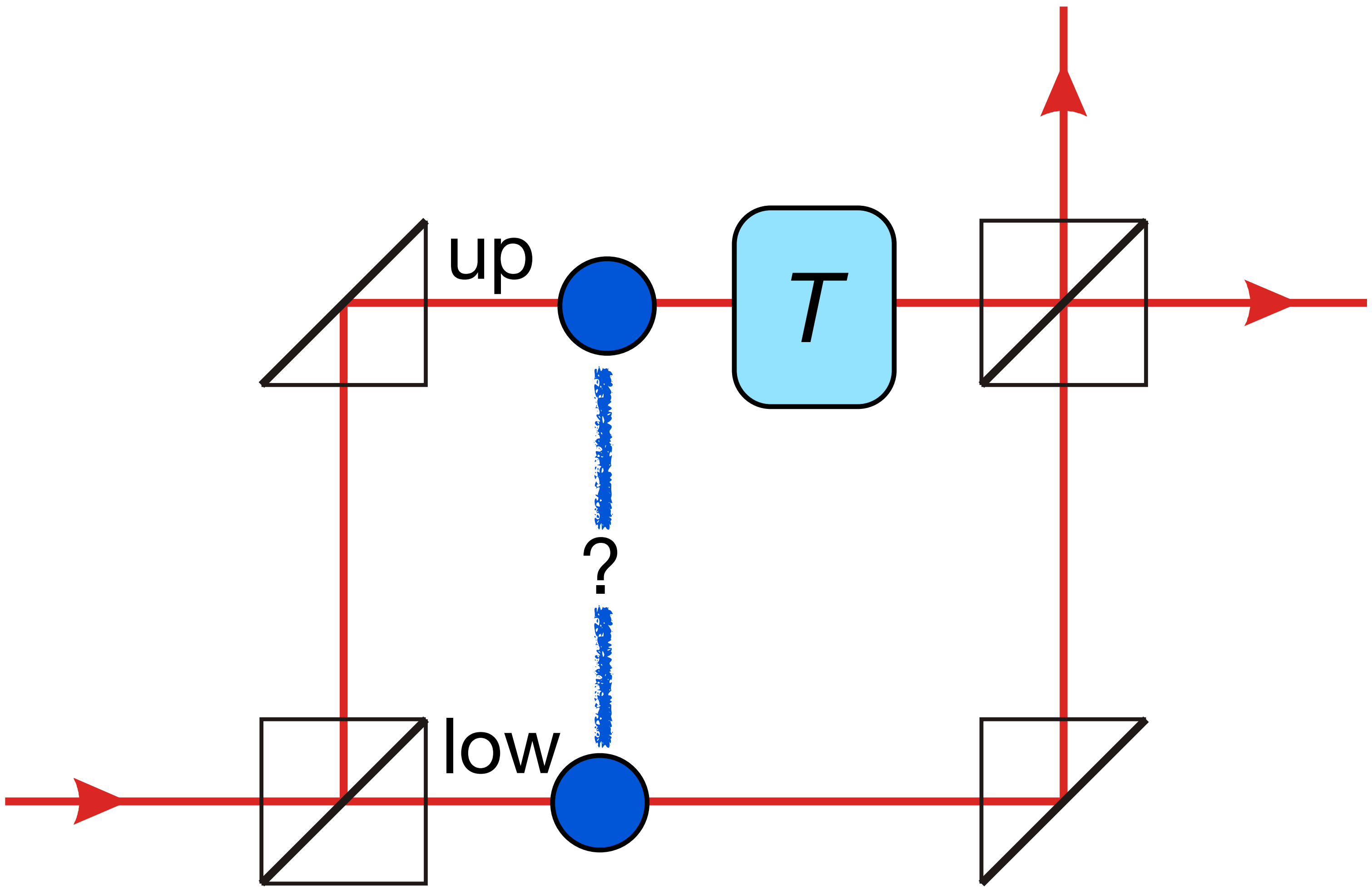

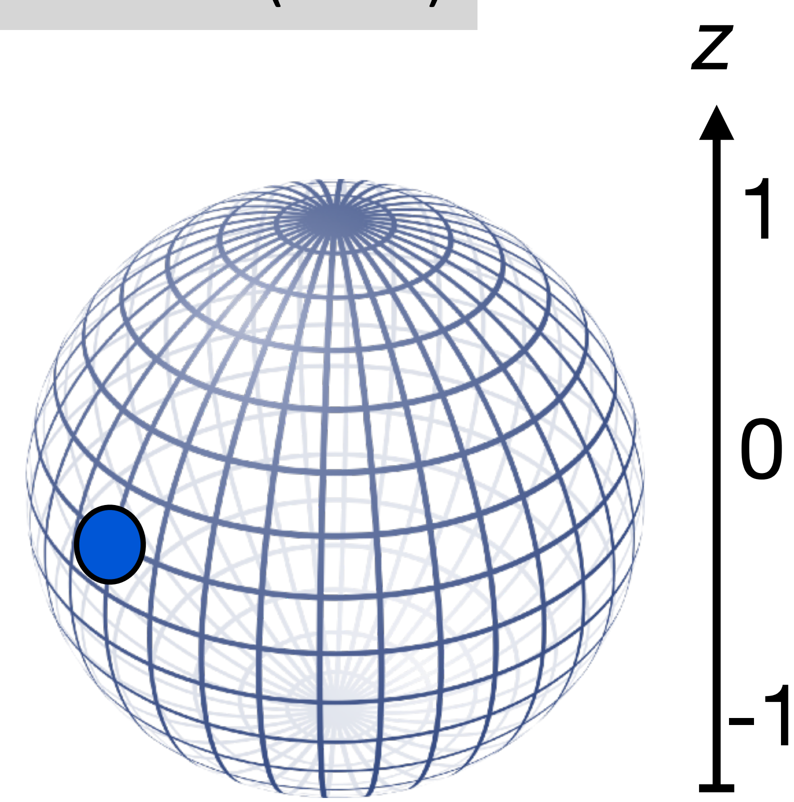

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What transformations *T* can we perform locally in one arm...
... reversibly, i.e. without any information loss?

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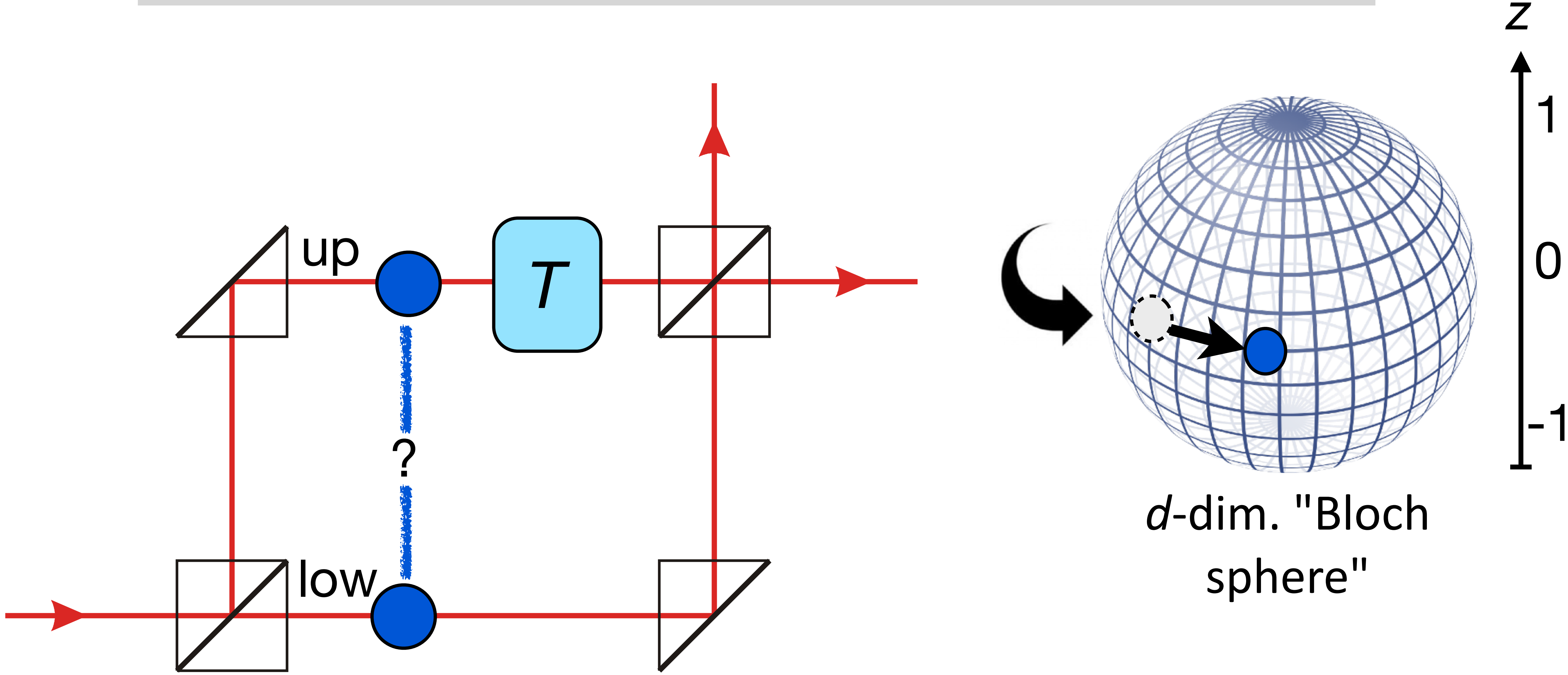

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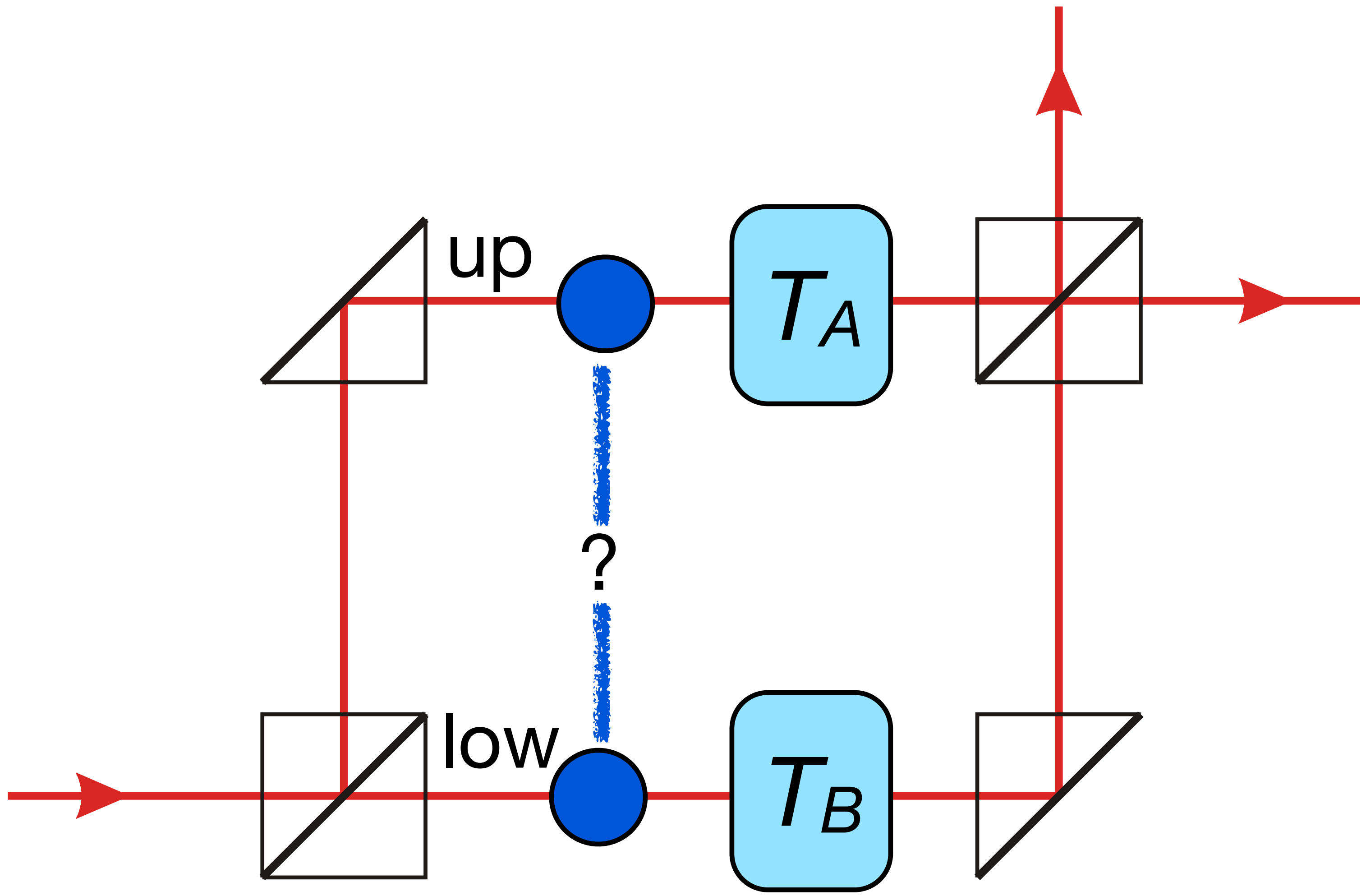

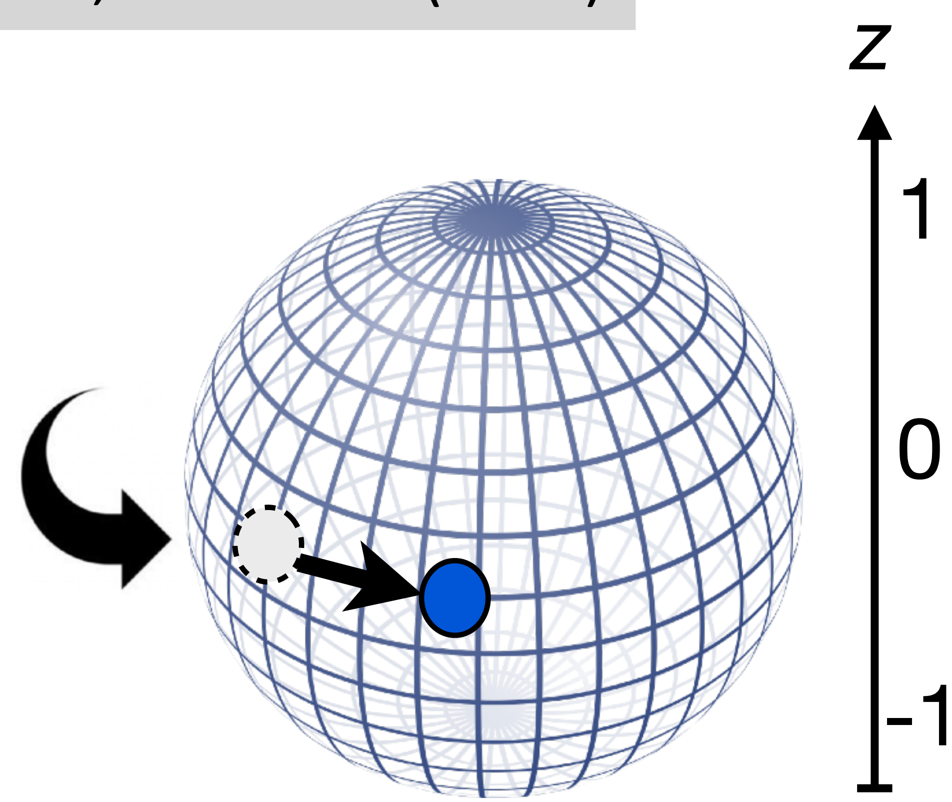

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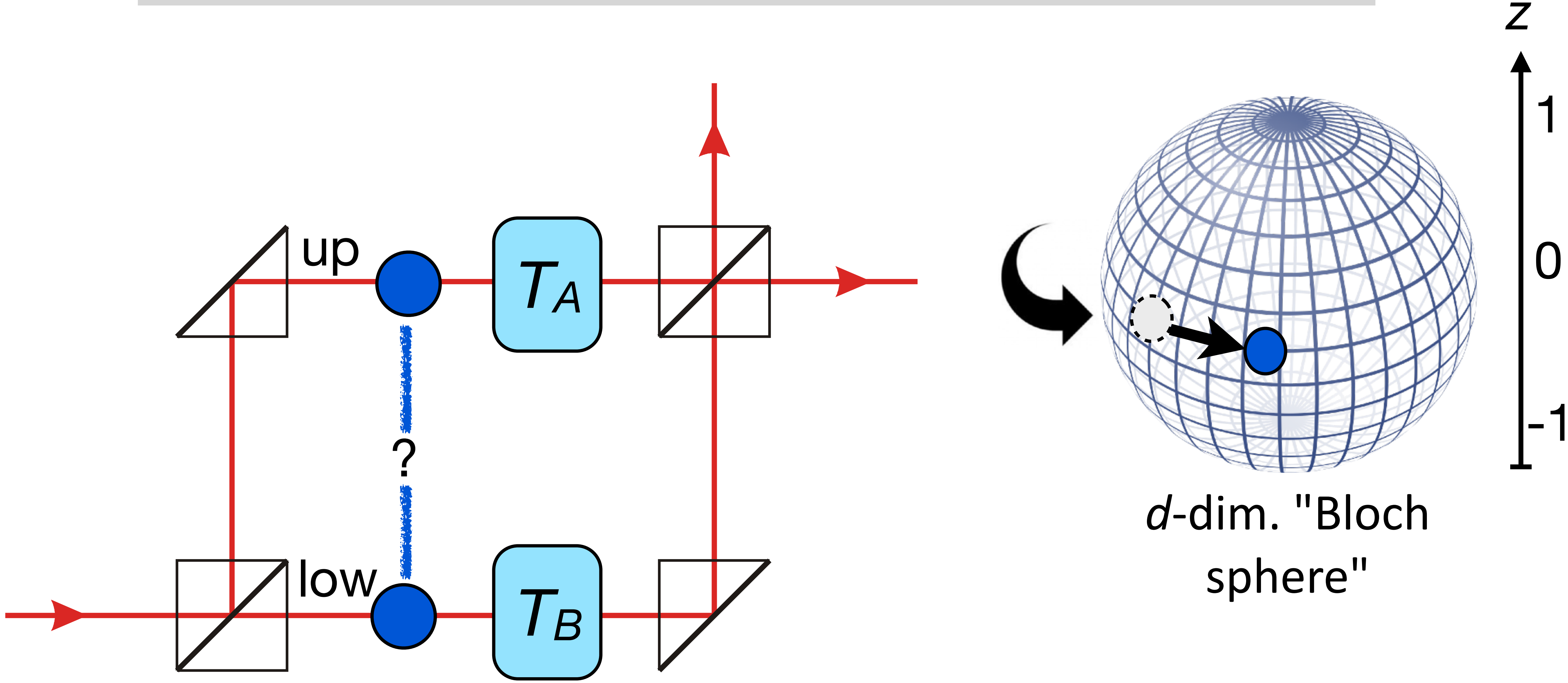

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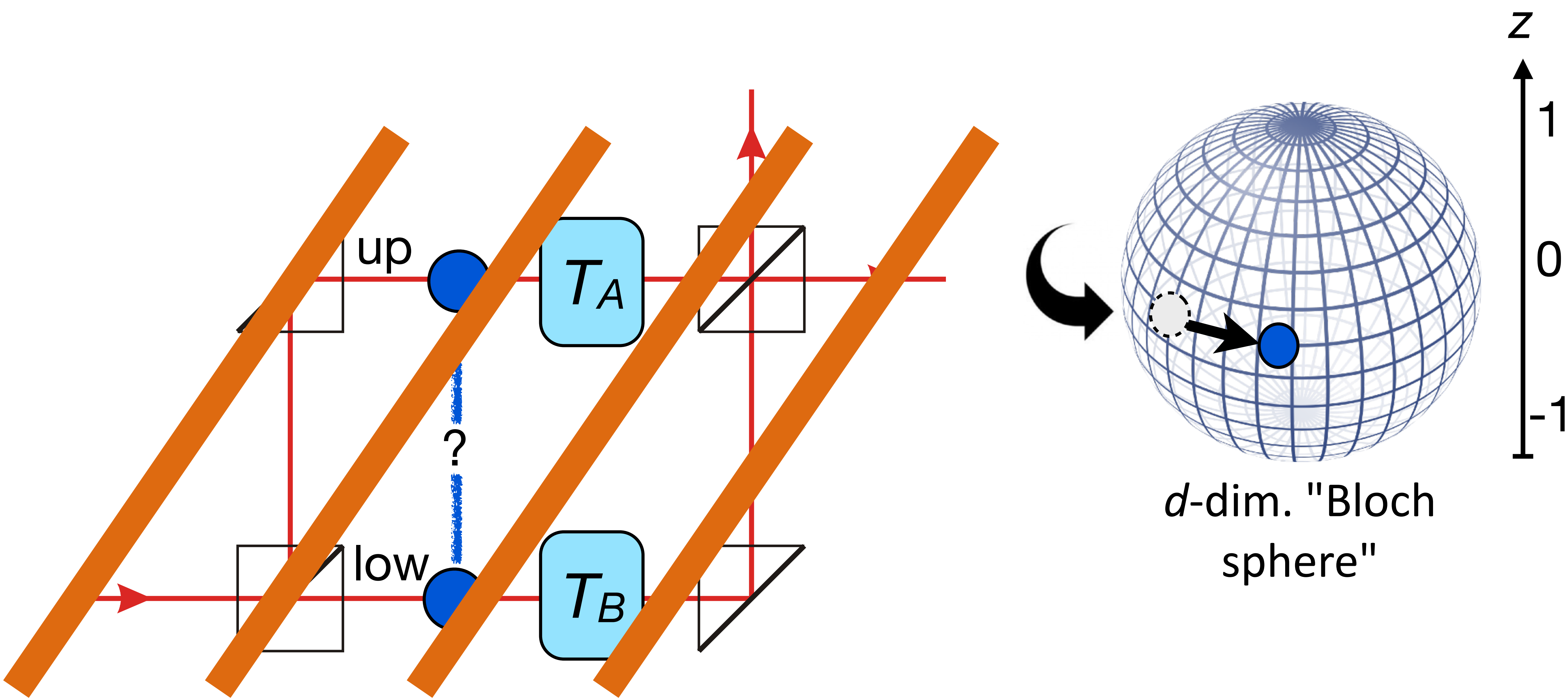

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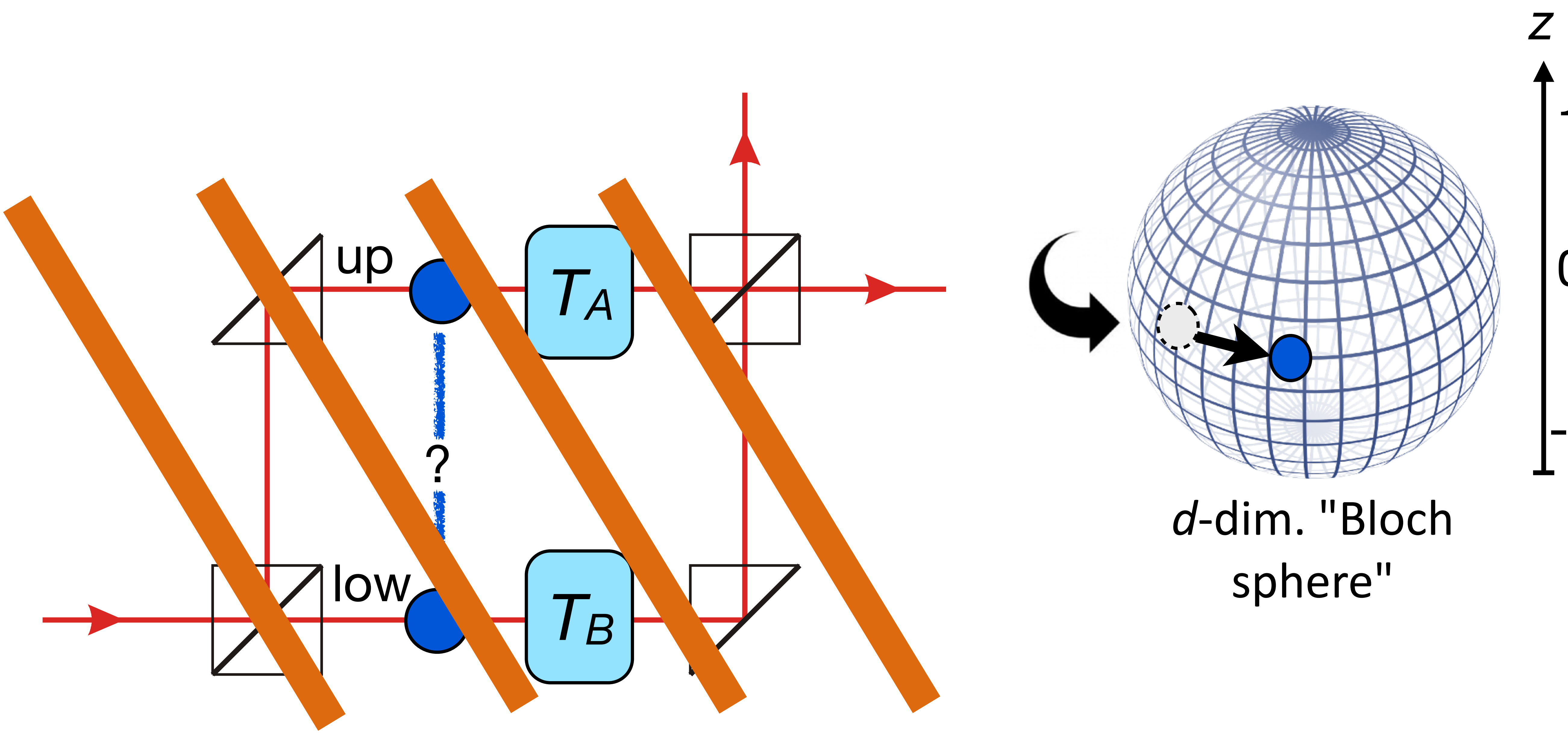

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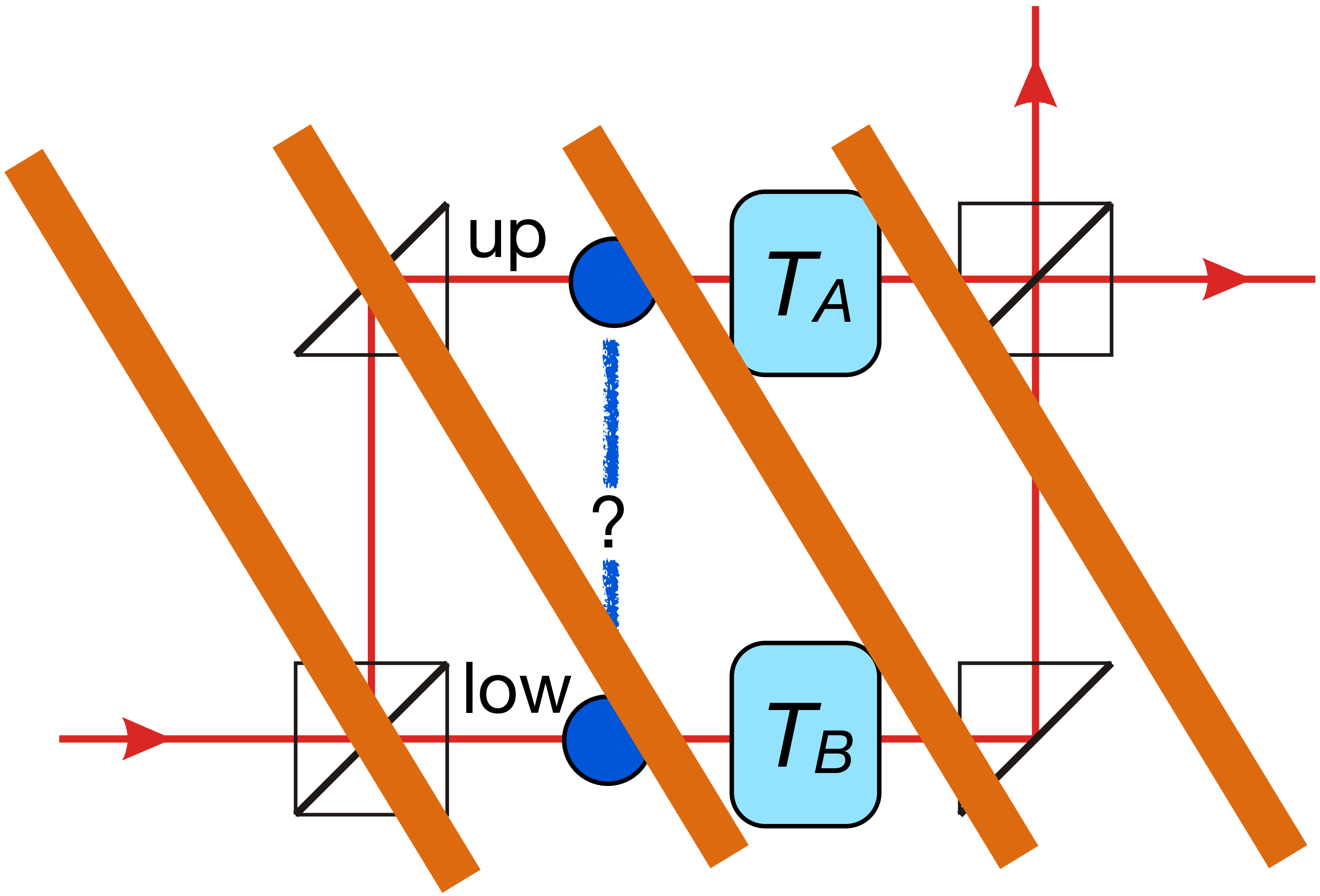

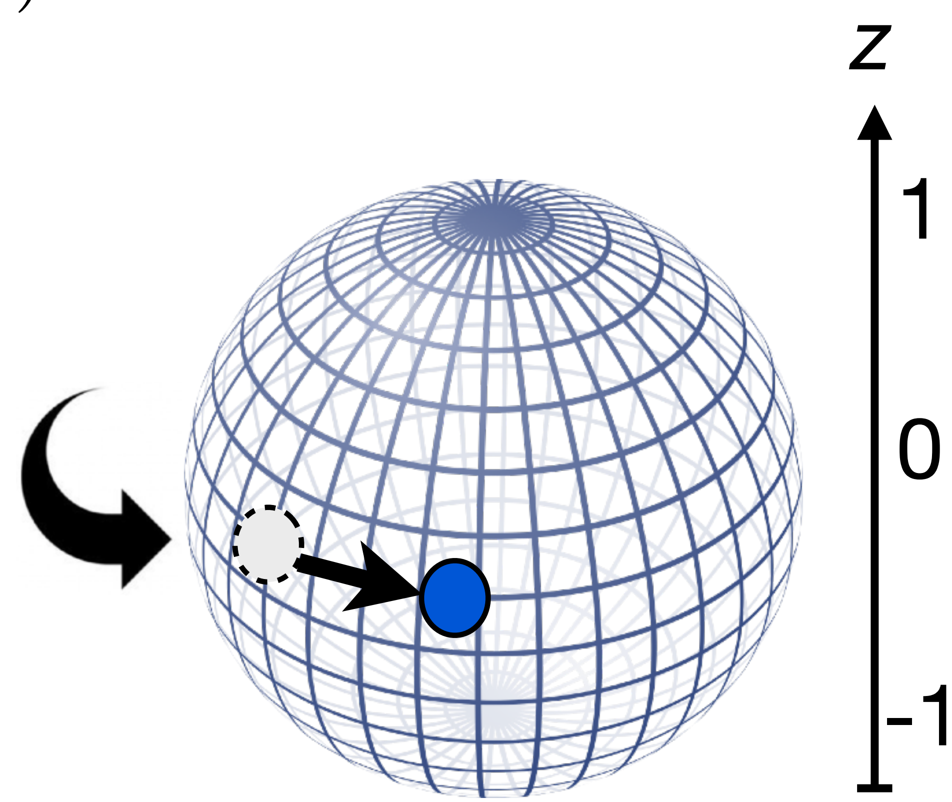

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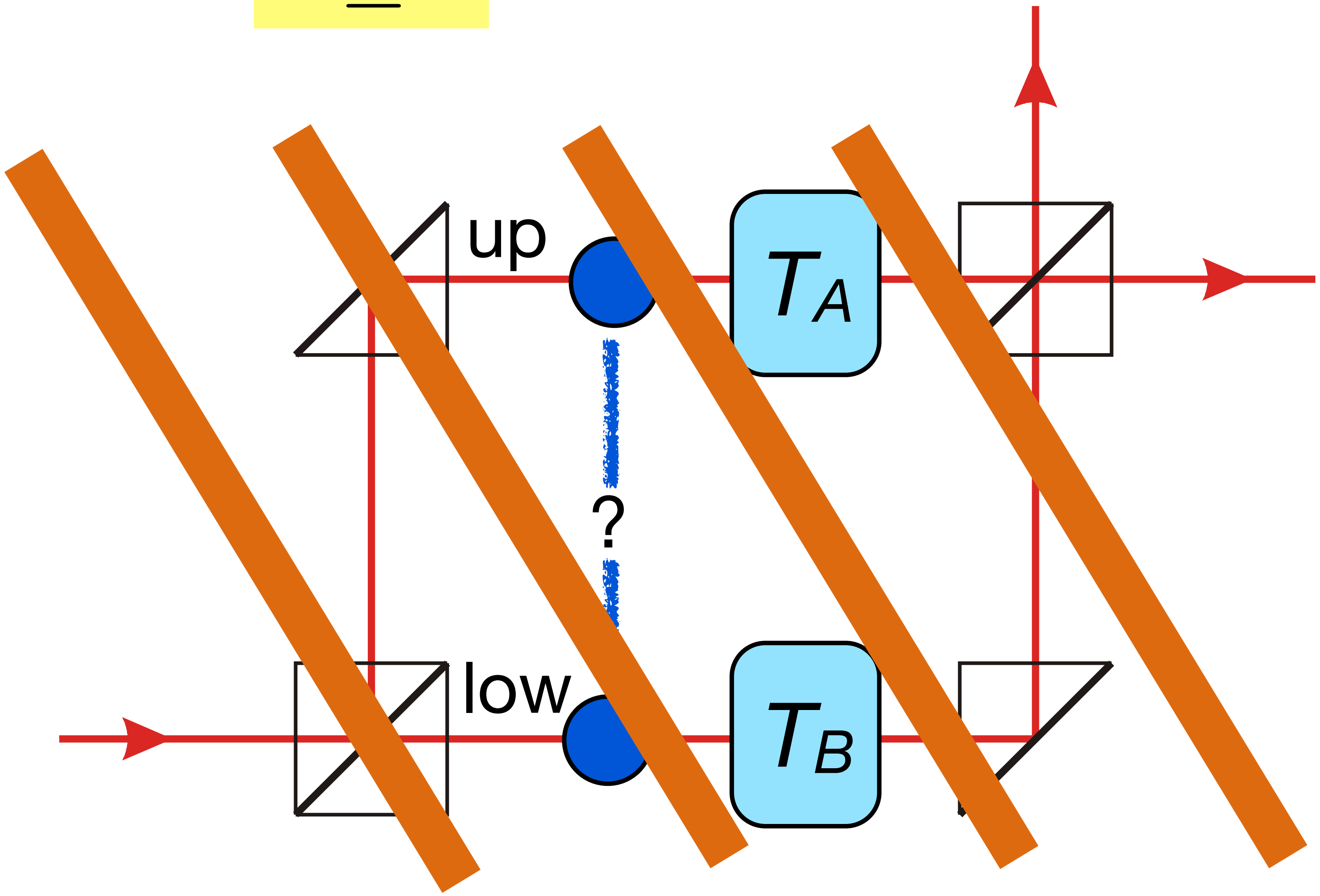

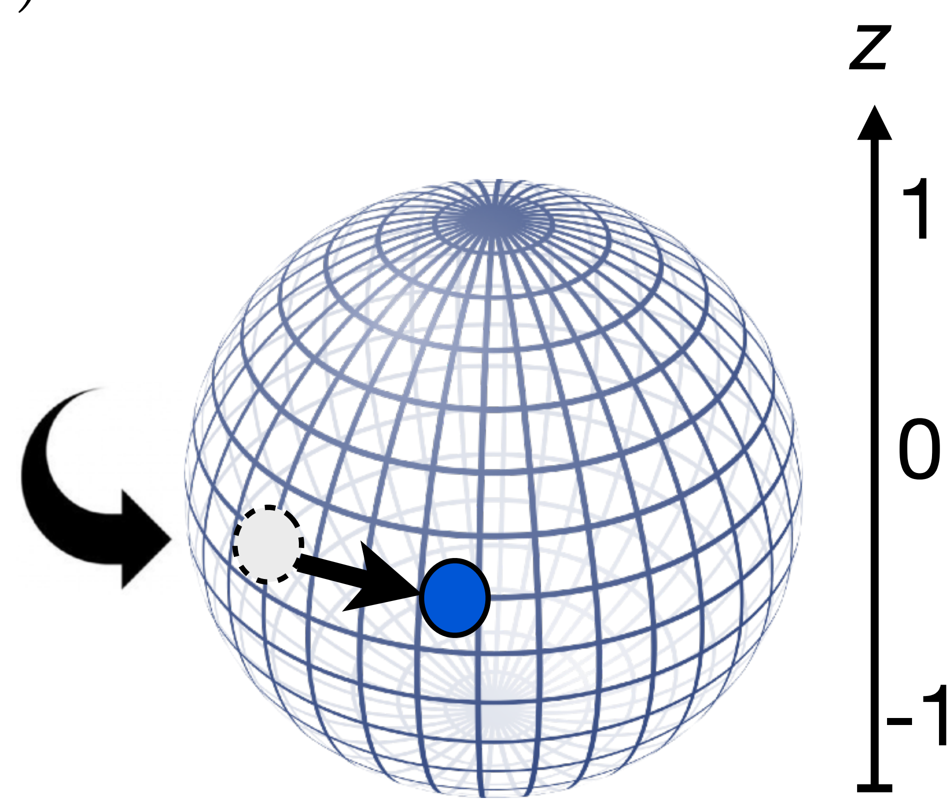

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Information theory

↕

spacetime

?!

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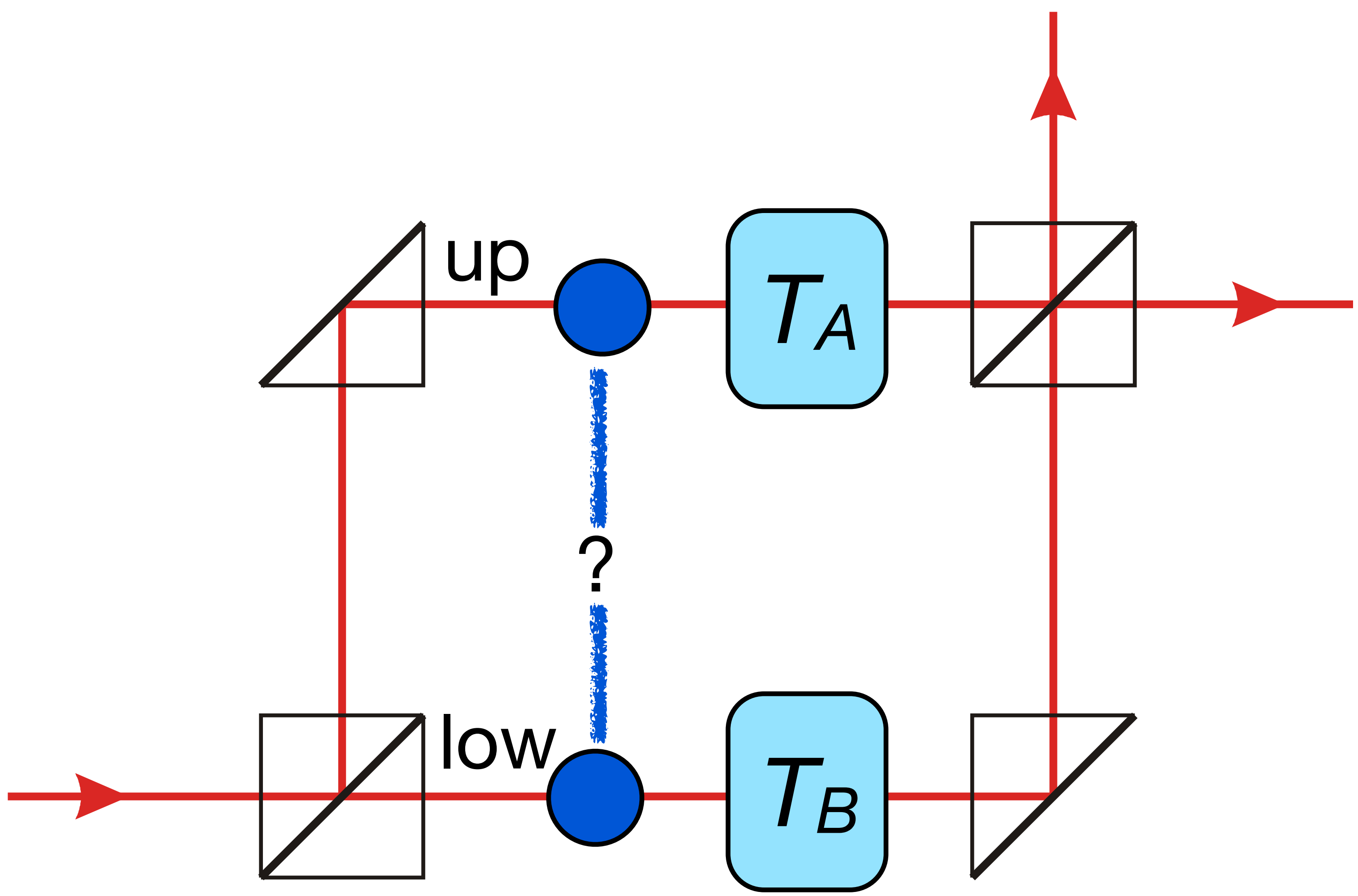

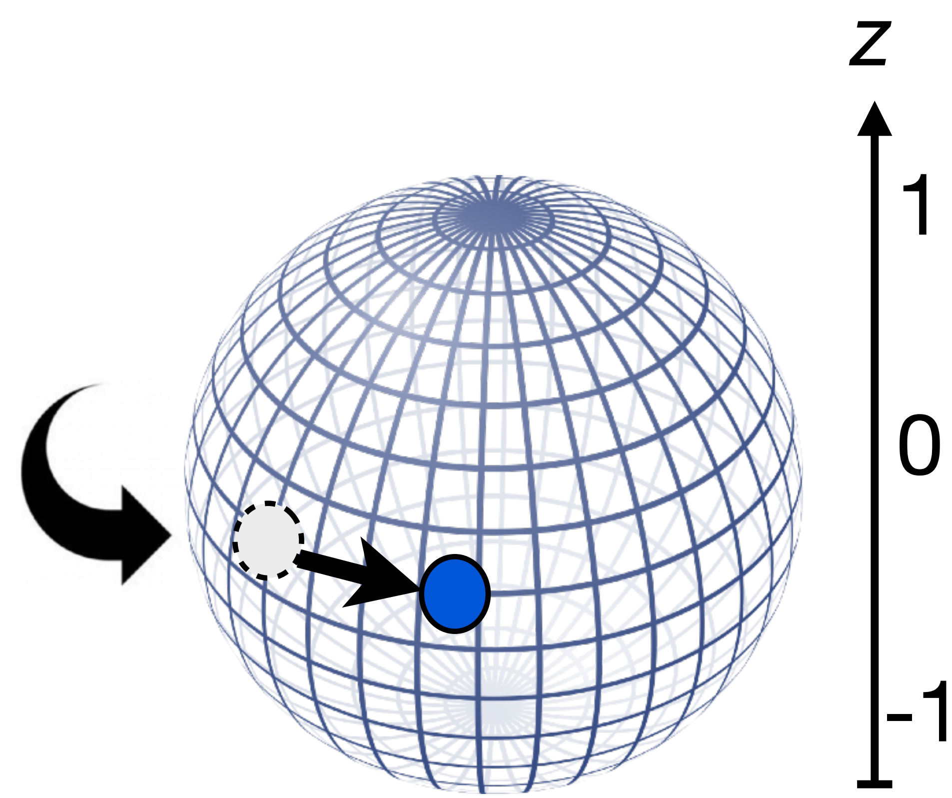

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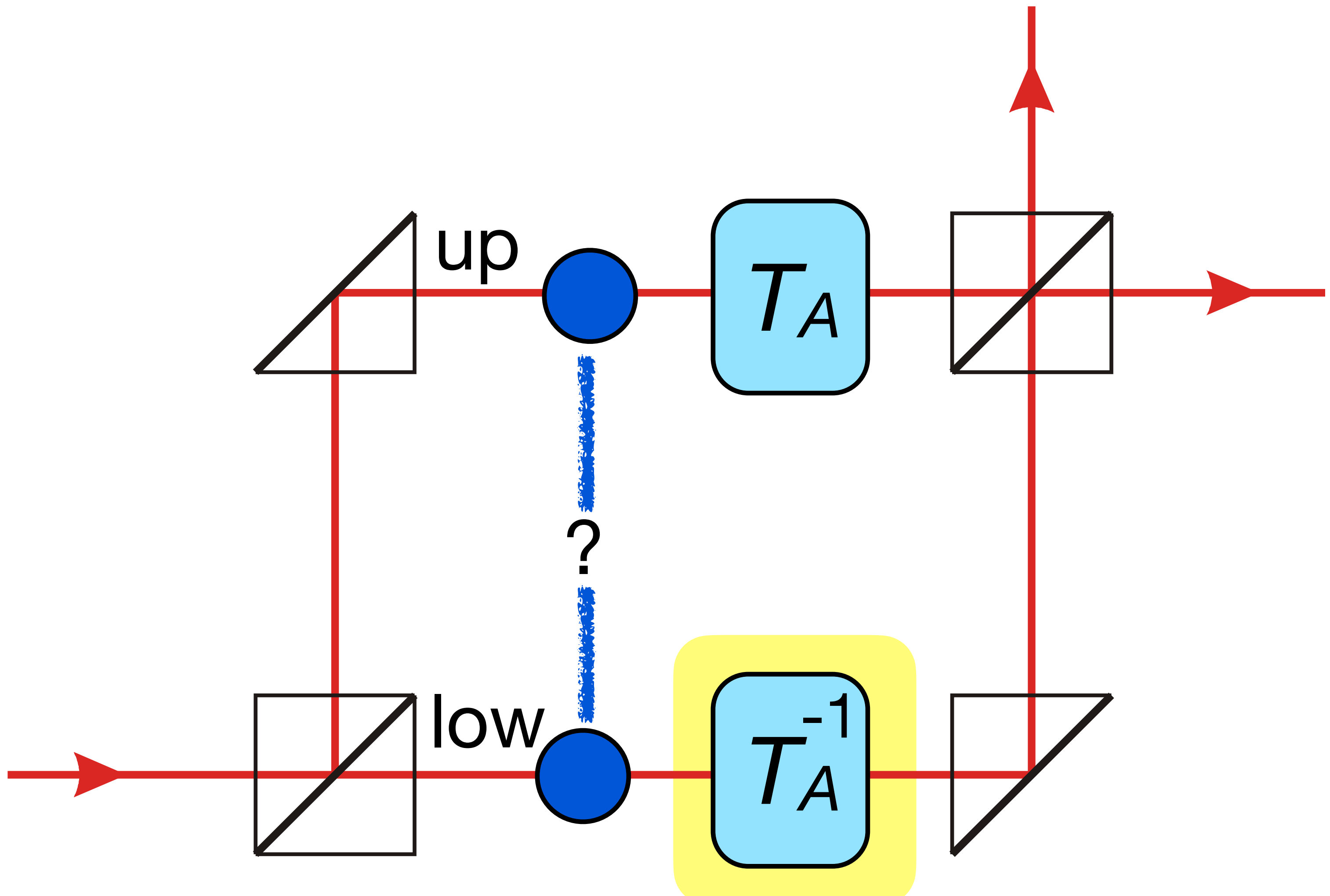

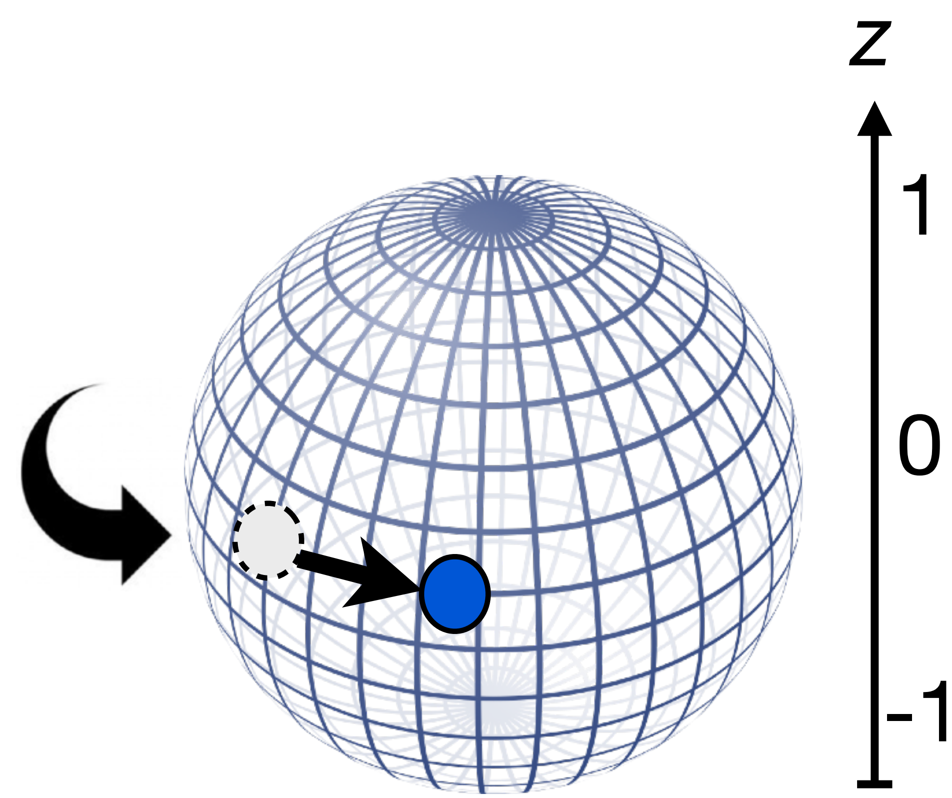

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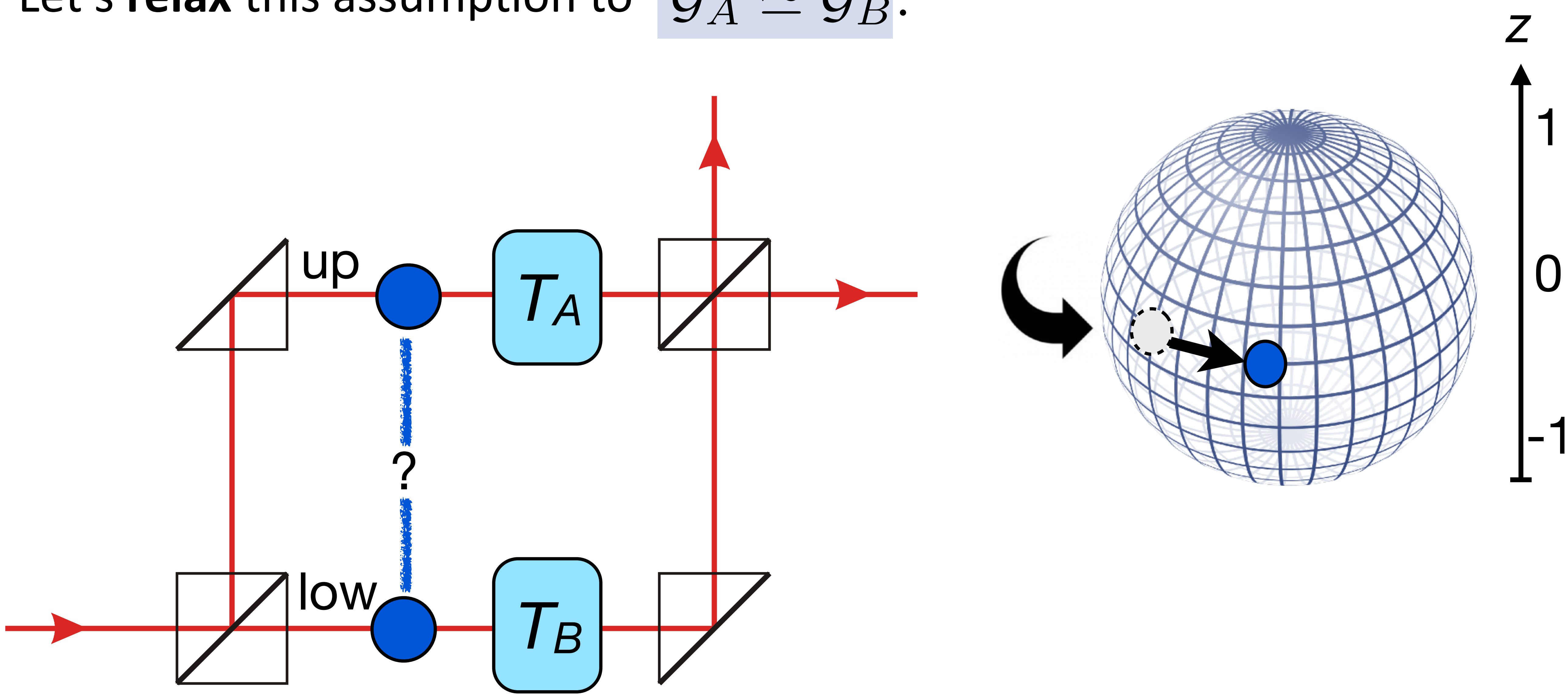

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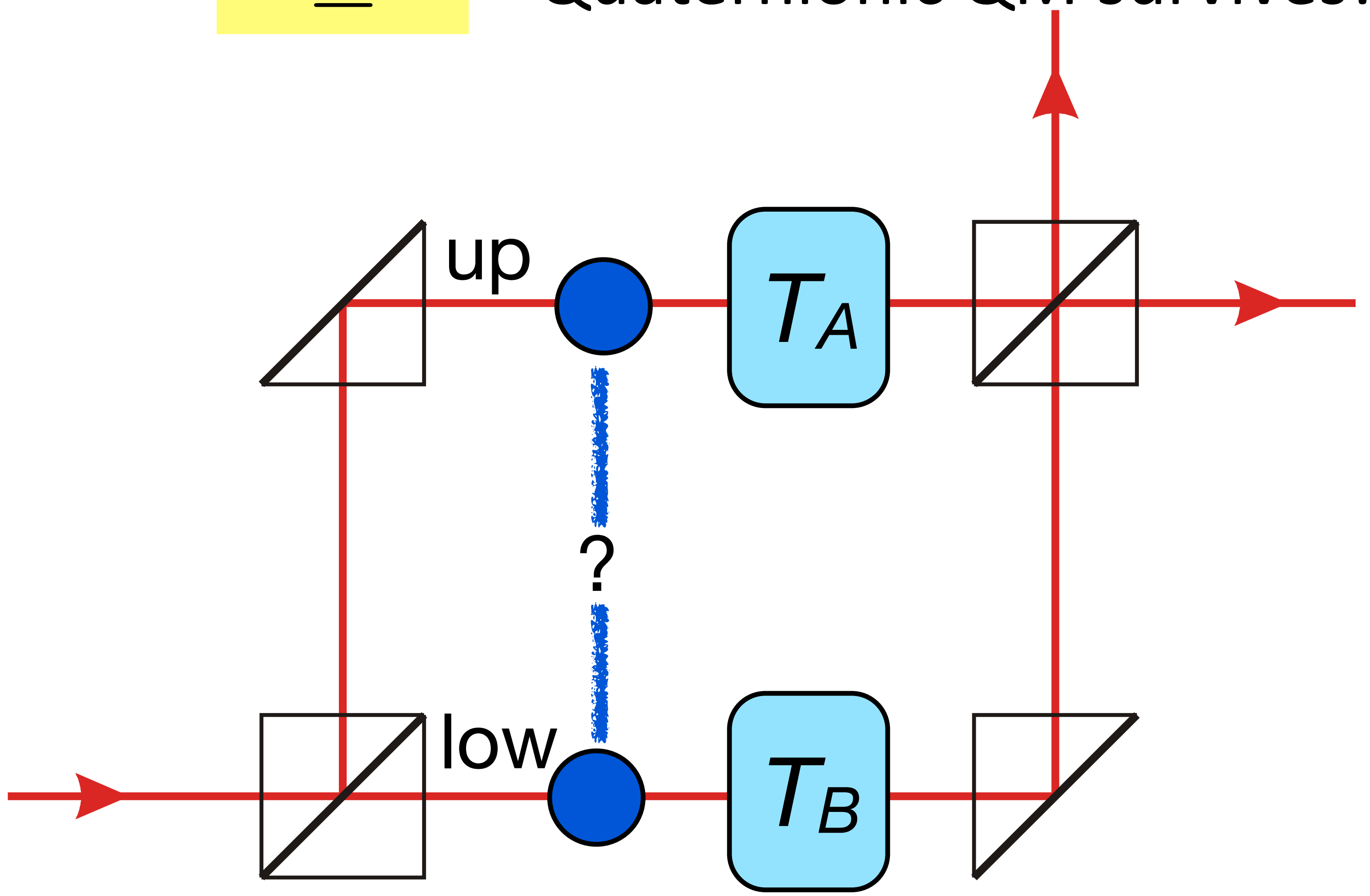
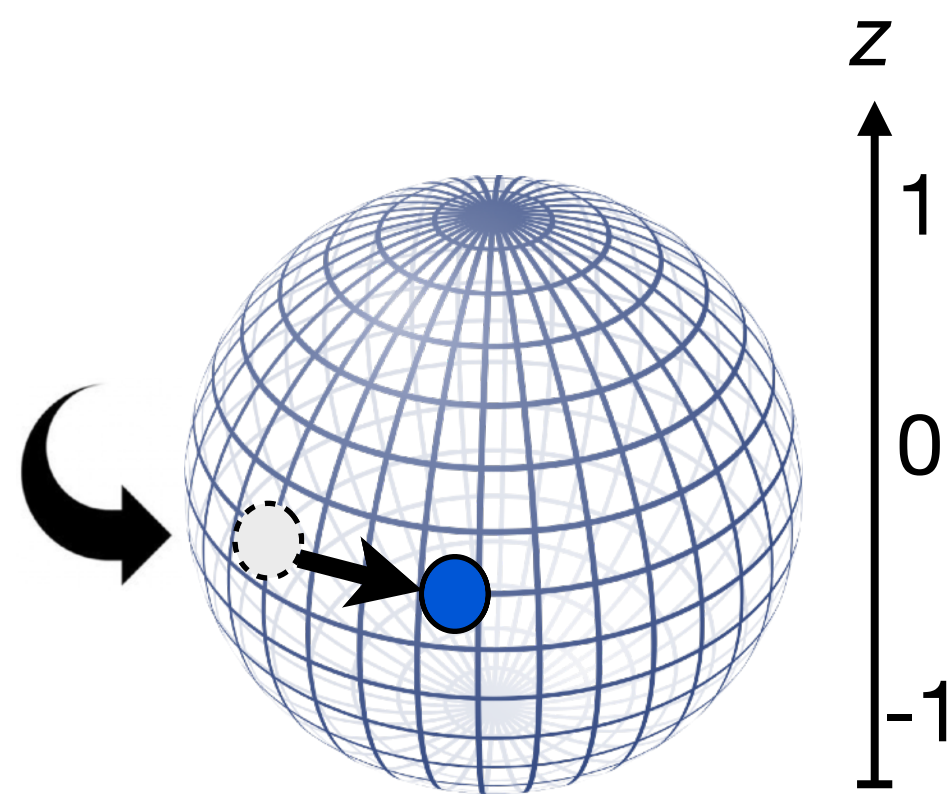
$\Rightarrow d \leq 5$. Quaternionic QM survives!





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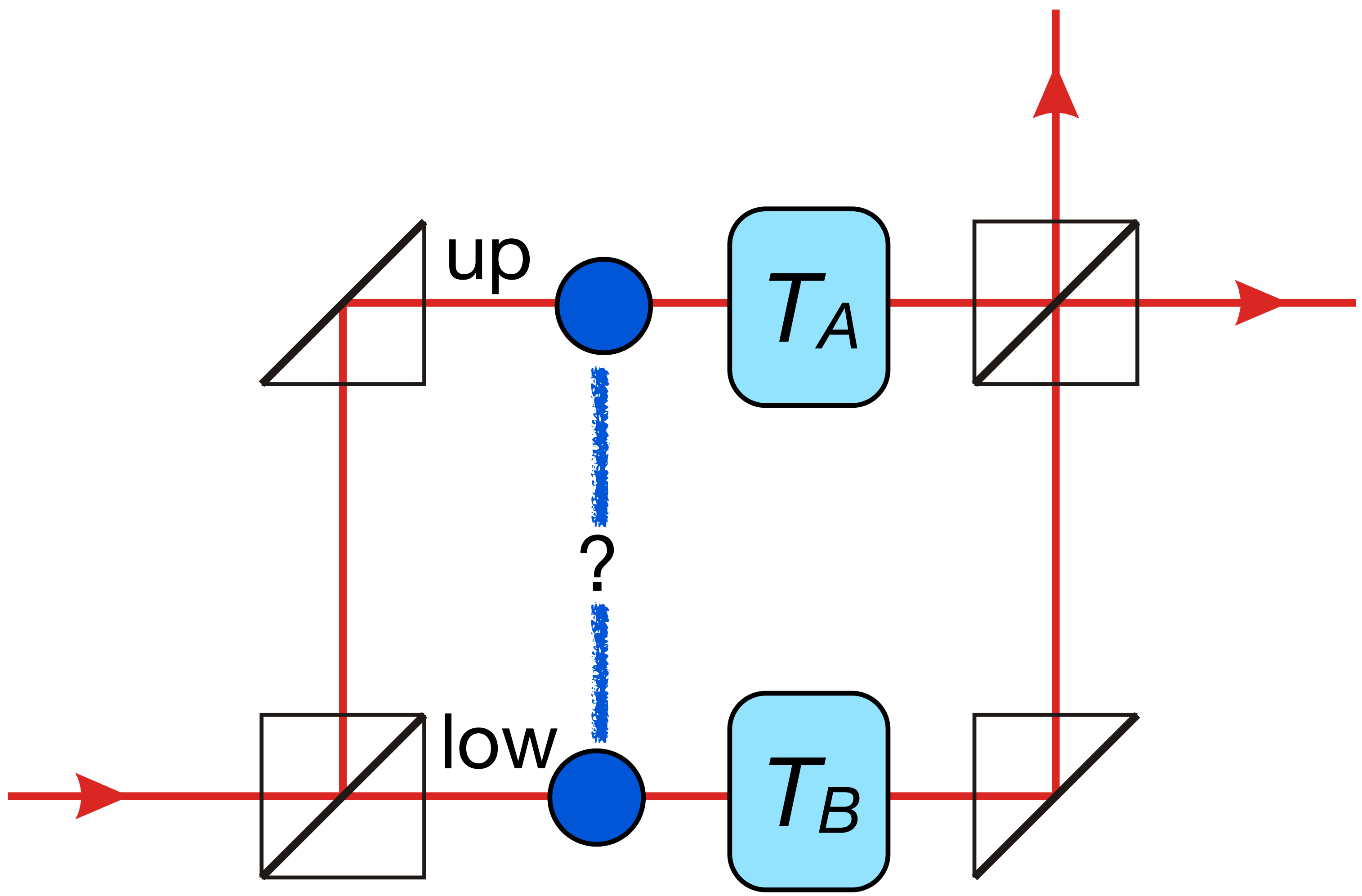

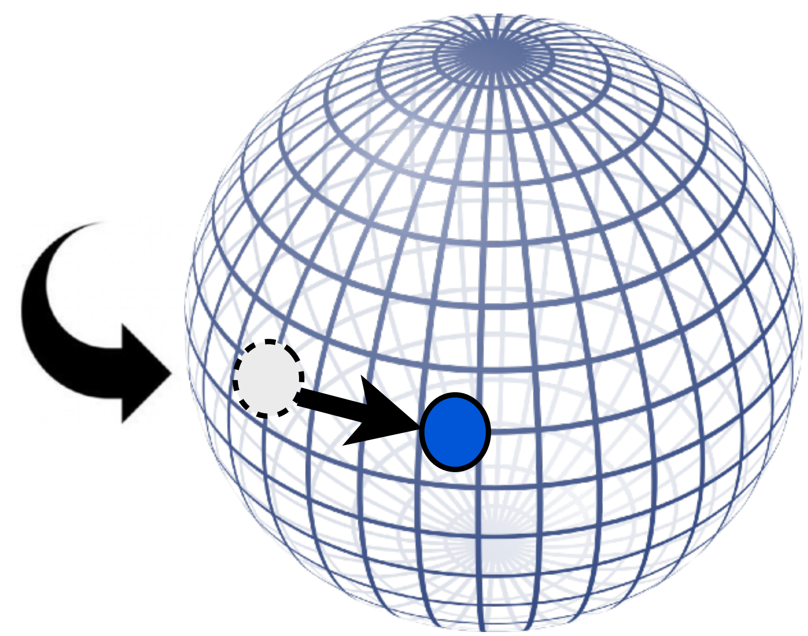

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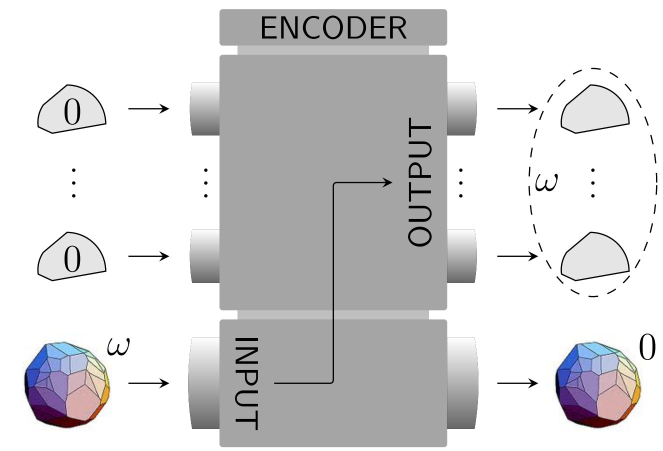
Relativity constrains the state space to $d = 1, 2, 3, 5$!

Overview

1. Probabilistic theories beyond quantum theory

2. Quantum theory from simple principles



3. The quest for higher-order interference

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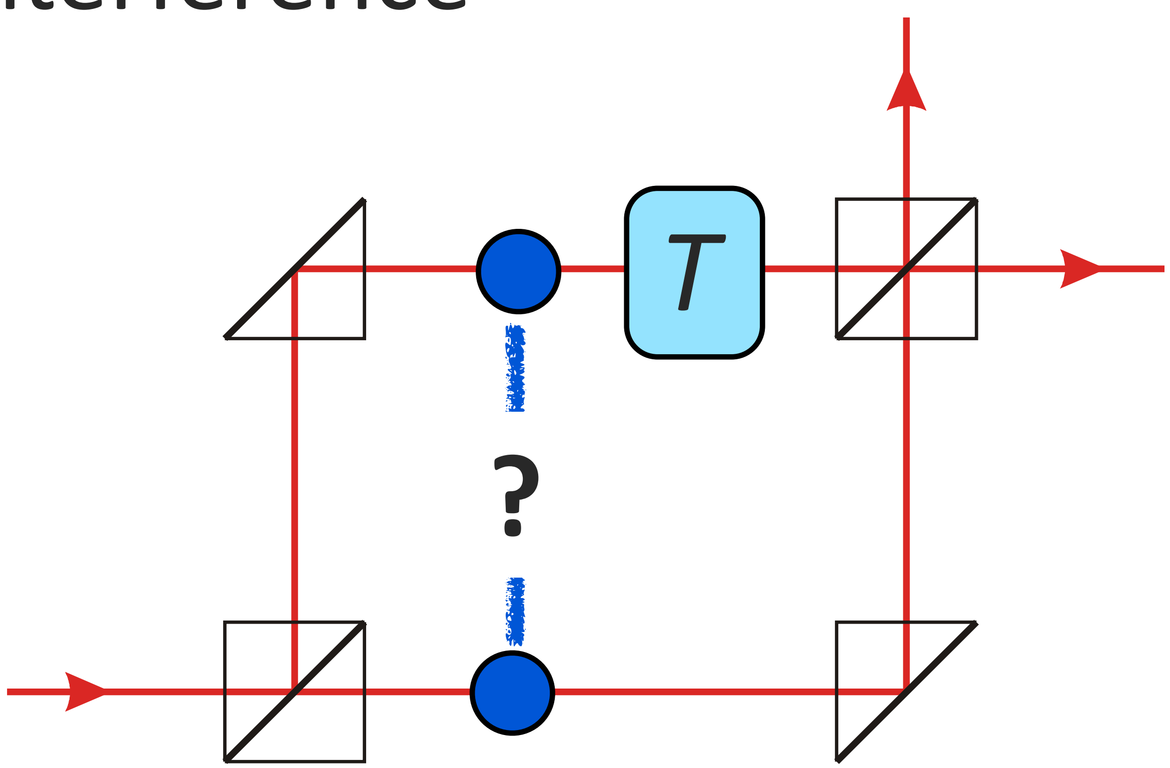

5. Conclusion

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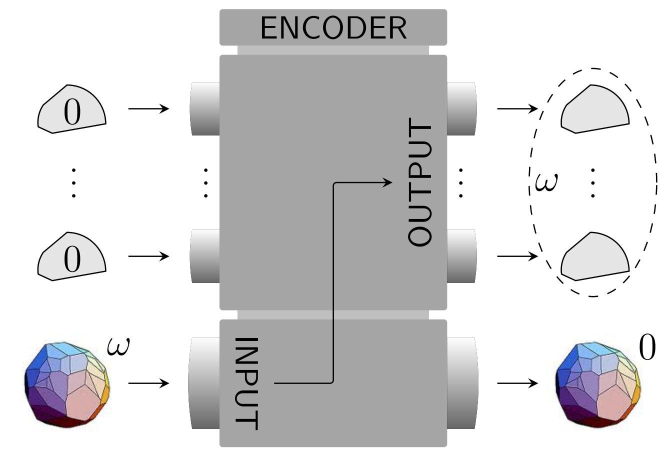

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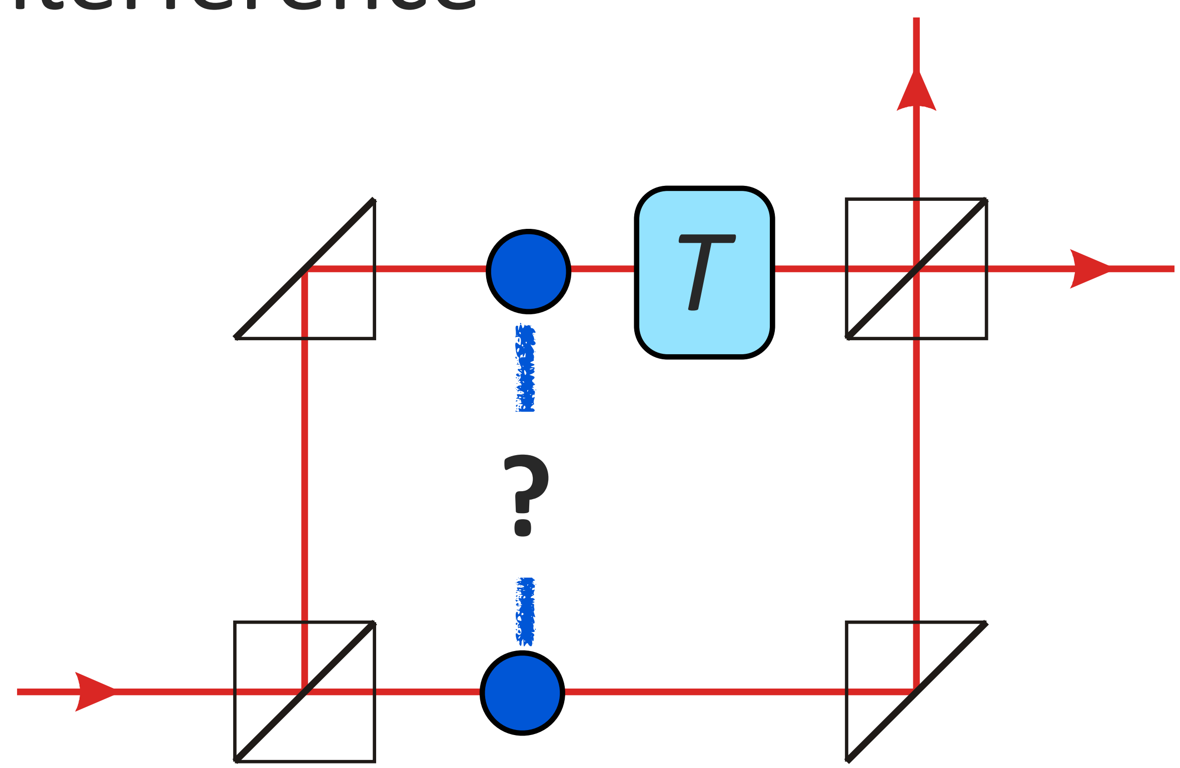

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- **Unitary evolution**: correlation with idealized clock variables.
- **Superposition principle**: not a principle, but a mathematical accident

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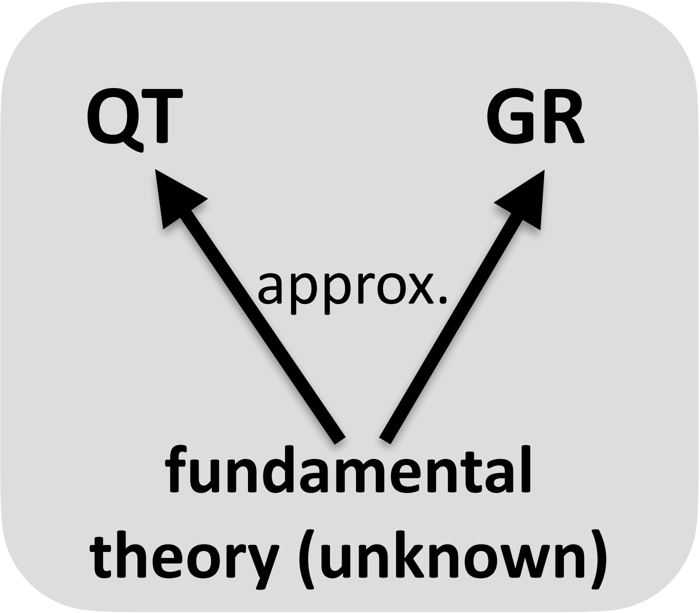
Challenge to Everettians: start with a landscape of "theories of many worlds", write down a few simple principles of some kind, and prove that QT is the unique many-worlds-like theory that satisfies those.

A. Koberinski and **MM**, arXiv:1707.05602

Outlook



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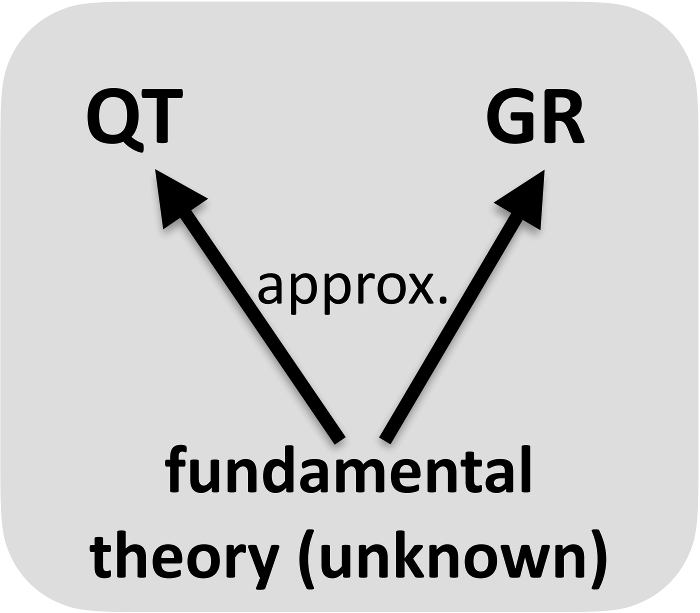

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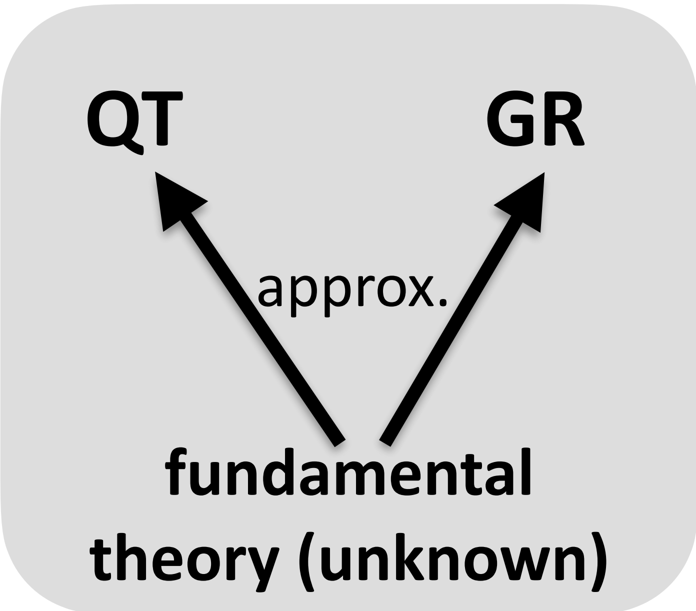

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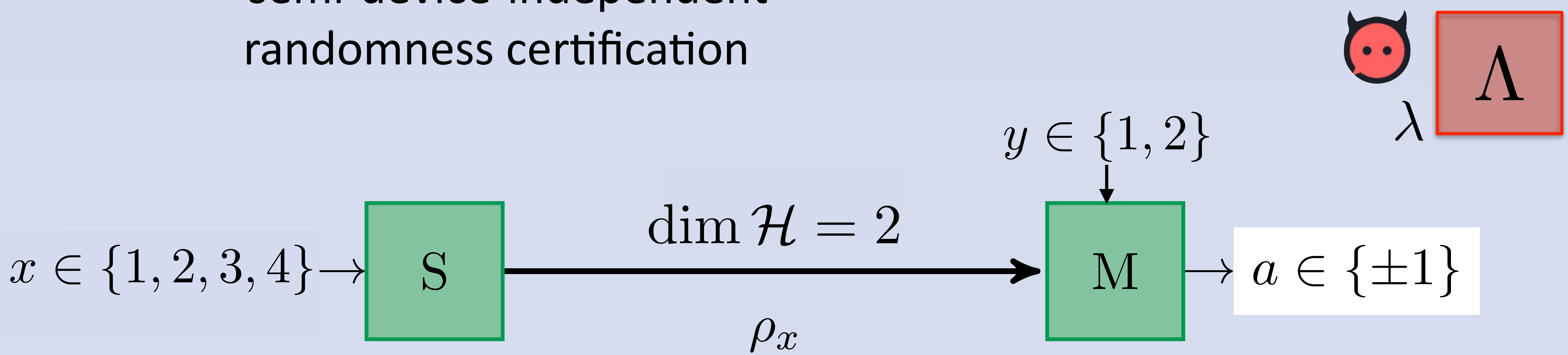
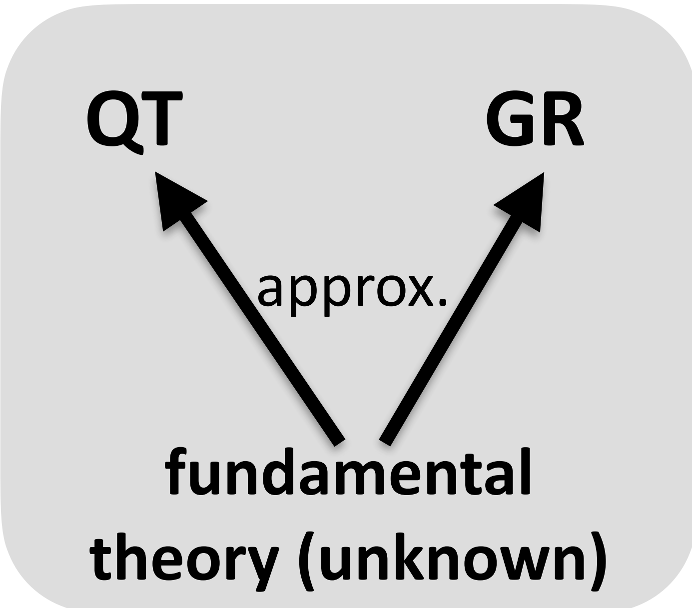
semi-device-independent
randomness certification



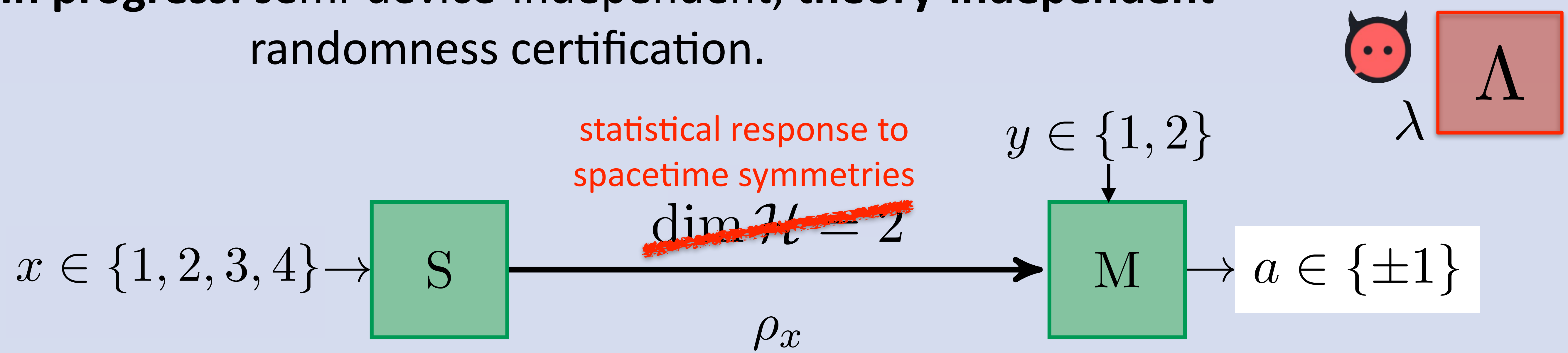
From data table $p(a|x,y)$ and this assumption, one can infer that $H(A|X,Y,\Lambda) \geq \ldots > 0$.

Outlook



Instead of jumping directly to Quantum Gravity, study the **logical architecture of physics**: how do QT and spacetime constrain each other?

In progress: semi-device-independent, **theory-independent** randomness certification.



From data table $p(a|x,y)$ and this assumption, one can infer that $H(A|X,Y,\Lambda) \geq \ldots > 0$.

Summary

Quantum theory can be **derived from simple principles**,
and this improves our understanding of its structure in several ways.

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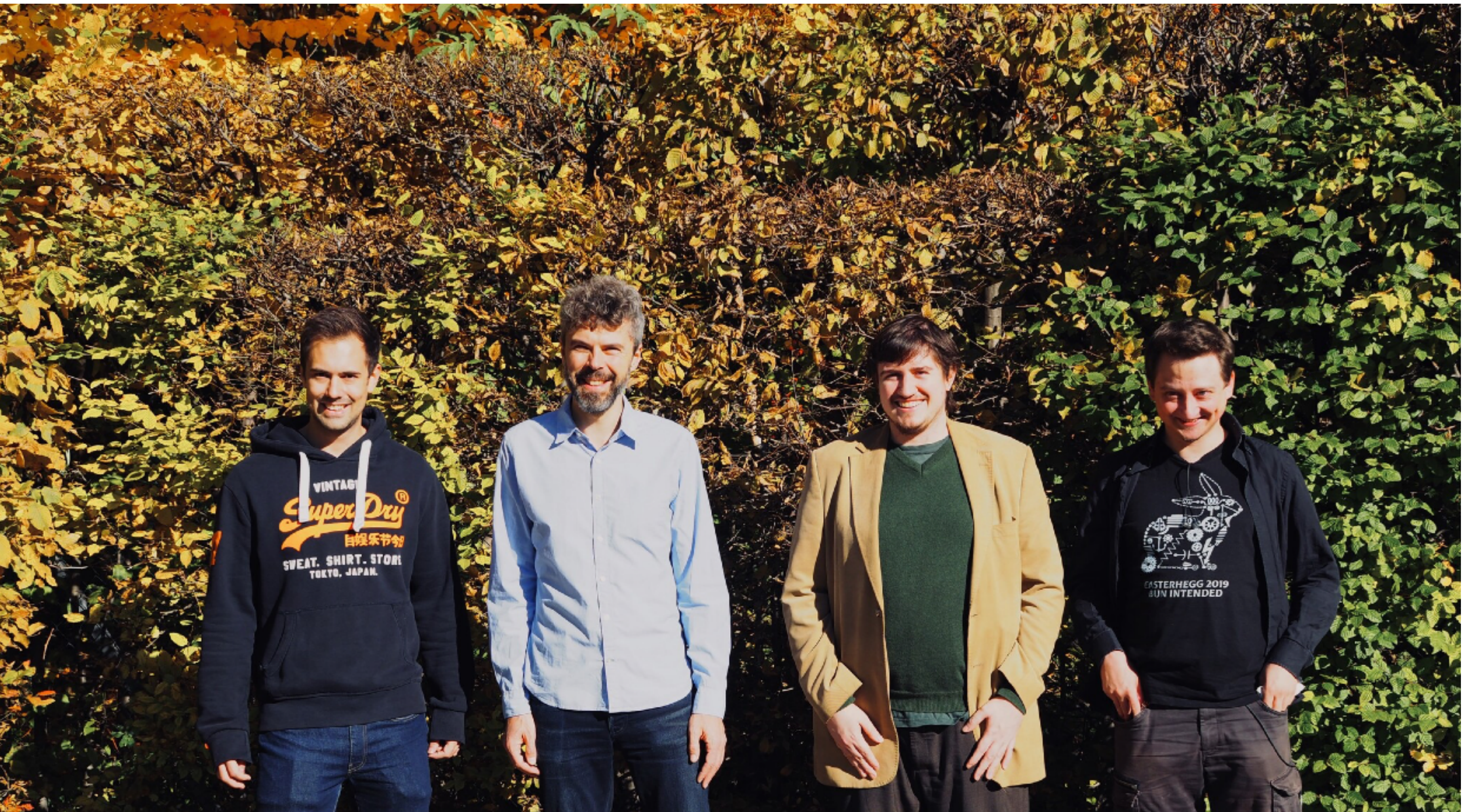
- my collaborators, in particular Lluís Masanes,



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