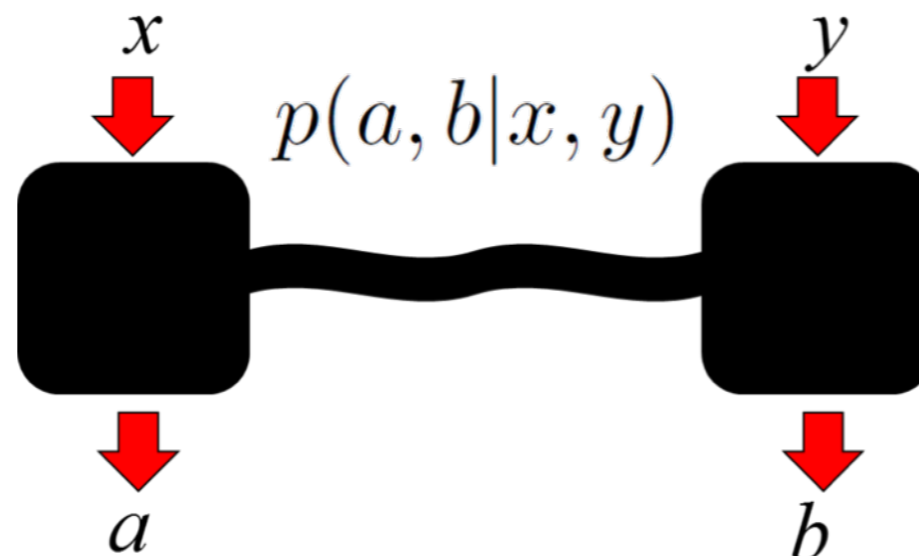


Black boxes in space and time: semi-device-independent information processing via representation theory

Markus P. Müller

Institute for Quantum Optics and Quantum Information (IQOQI), Vienna
Perimeter Institute for Theoretical Physics (PI), Waterloo, Canada

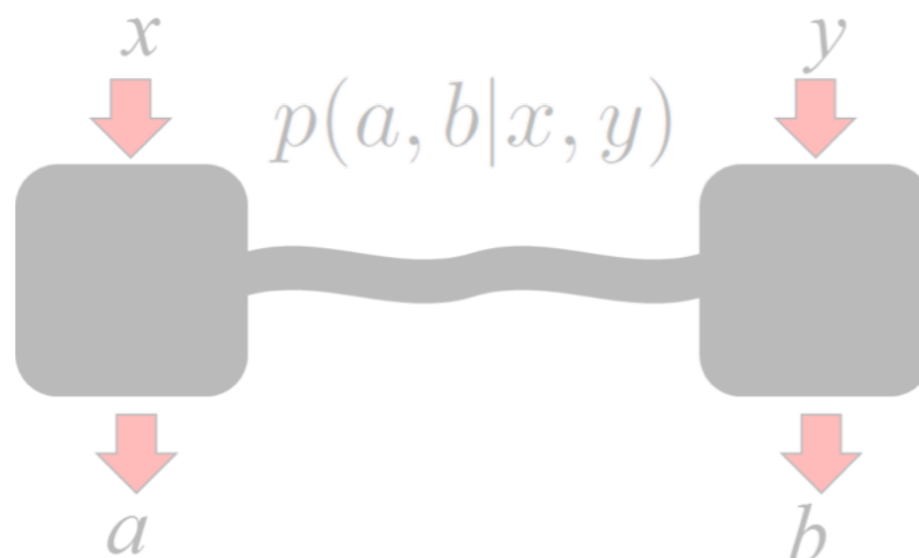


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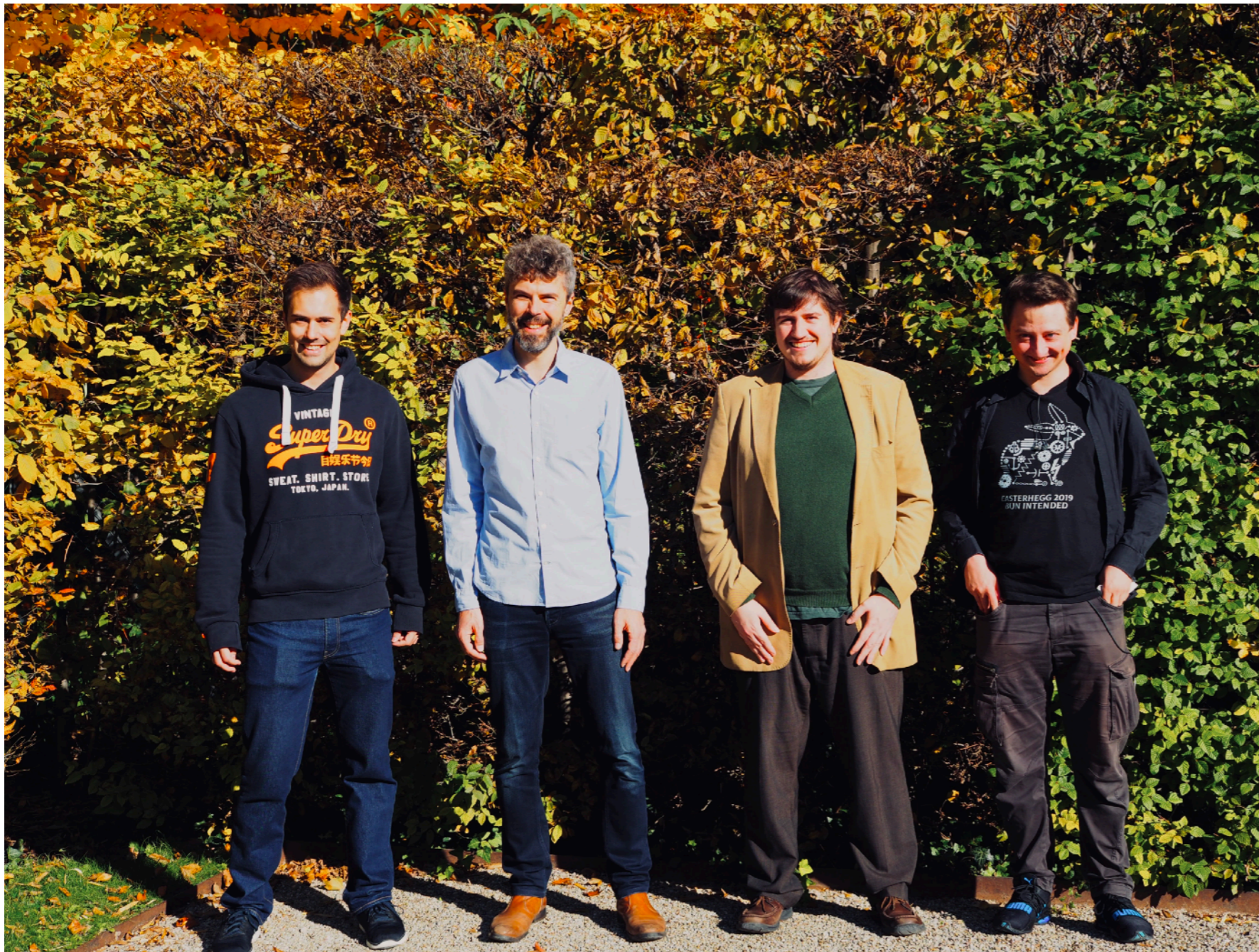
Work in progress!

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Our group at IQOQI



left to right:

Stefan Ludescher (PhD student), **Markus Müller** (group leader), **Andy Garner** (postdoc), **Marius Krumm** (PhD student).

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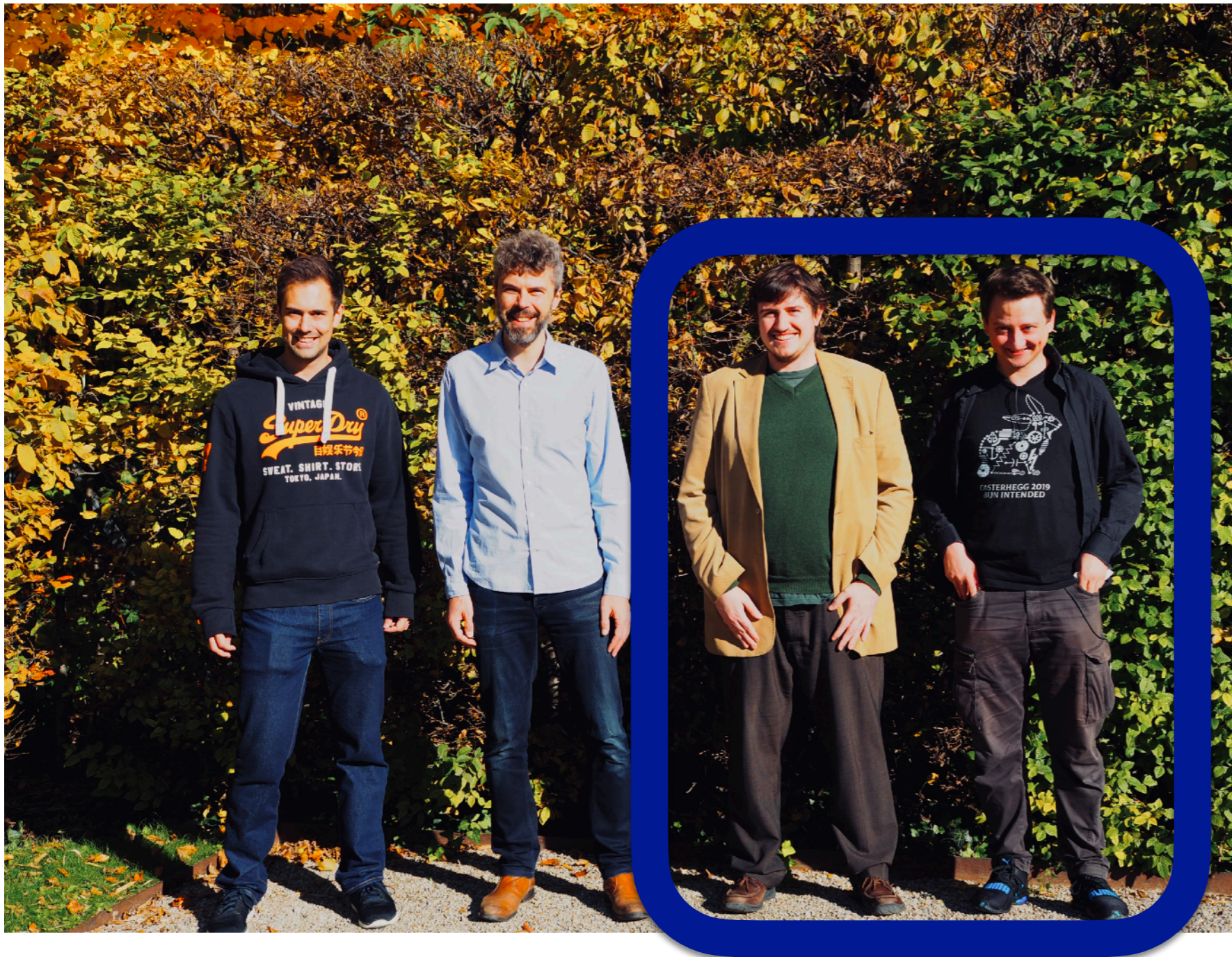


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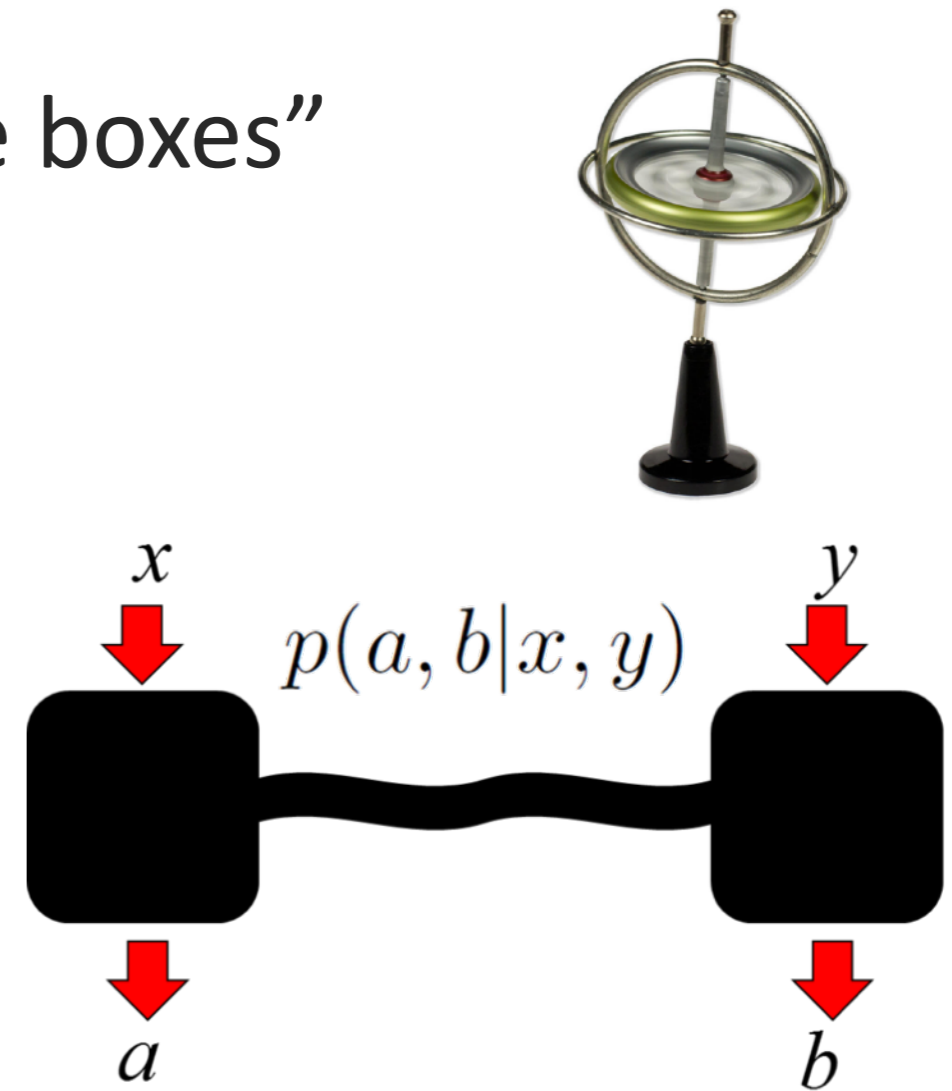
Overview

1. General framework of “spacetime boxes”

2. Foundational insights

3. Towards novel protocols...

4. ... and experimental tests of QT



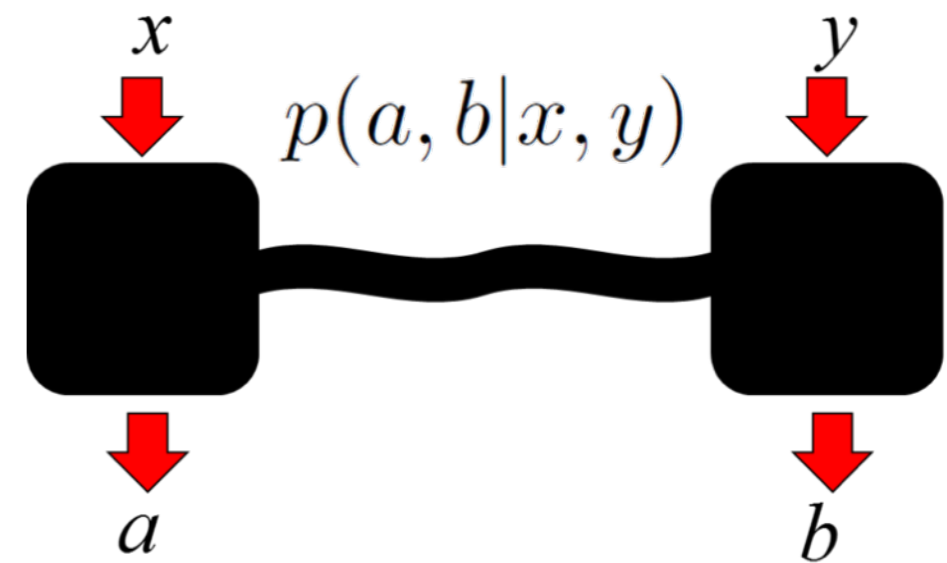
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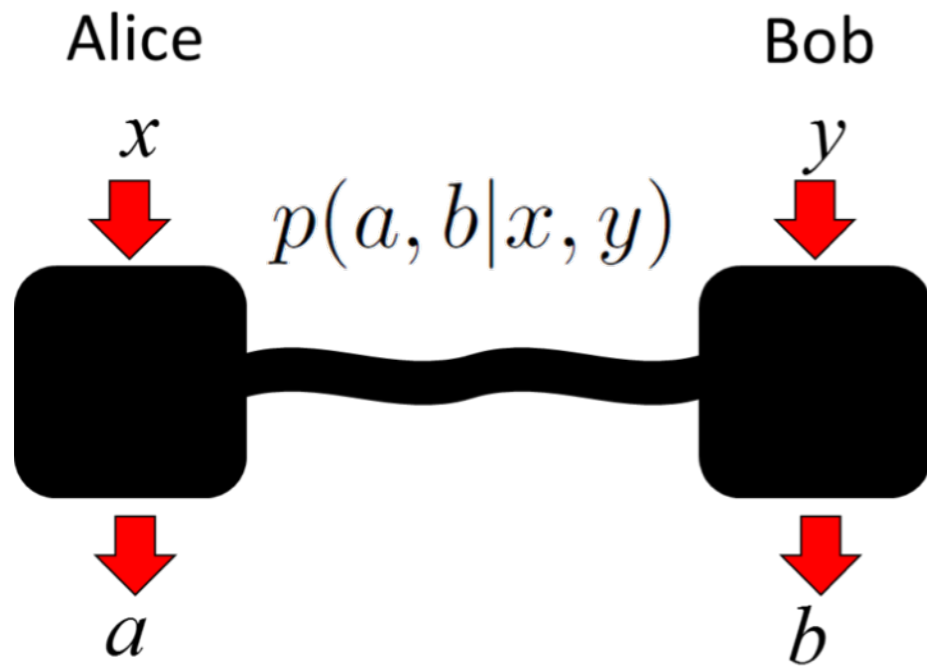
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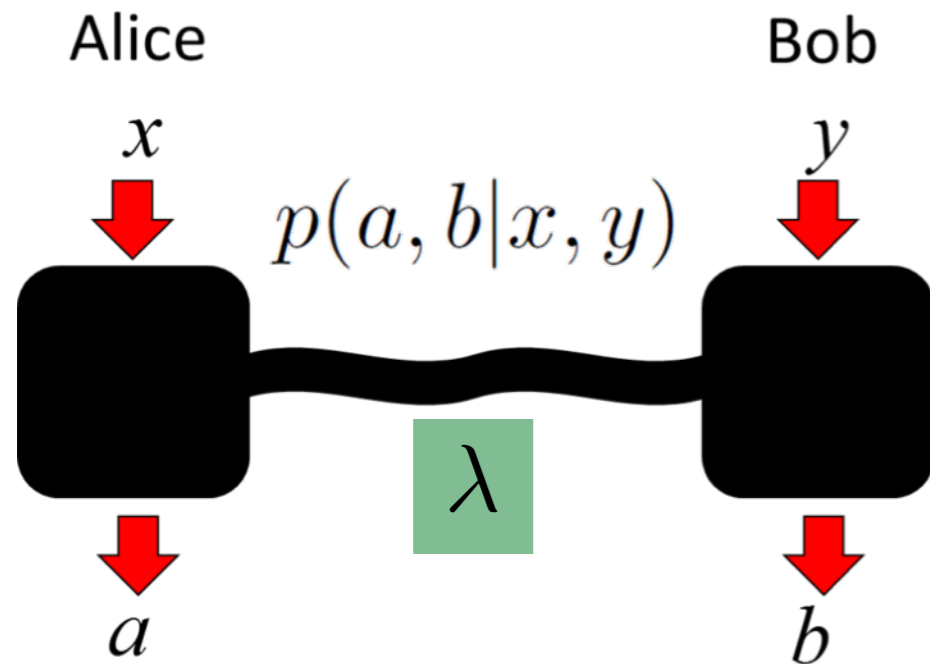


Black boxes and correlations

Black boxes and correlations



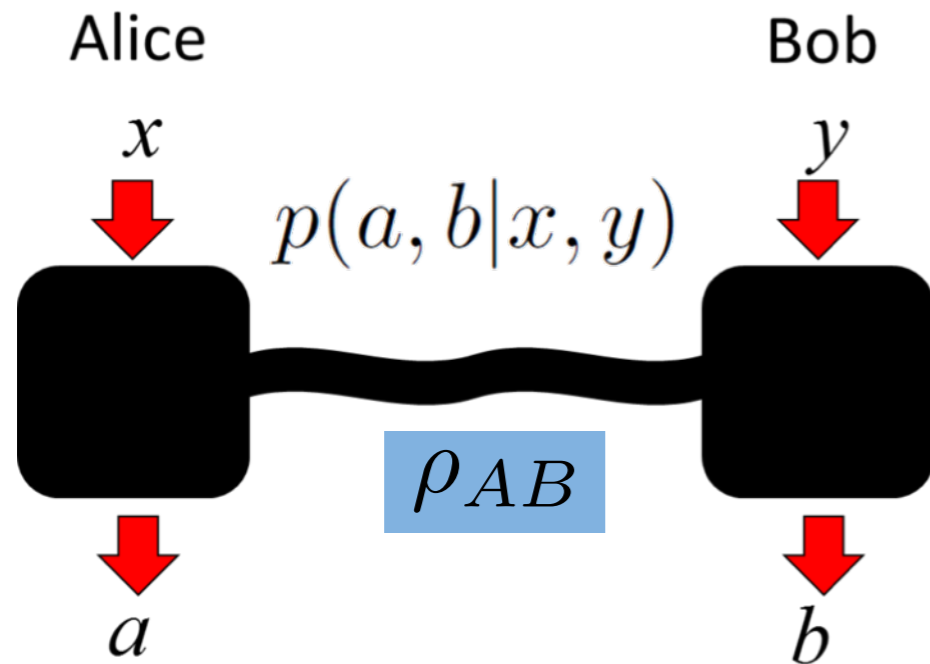
Black boxes and correlations



- In **classical** physics / prob. theory:

$$P(a, b|x, y) = \sum_{\lambda \in \Lambda} P_A(a|x, \lambda) P_B(b|y, \lambda) P_\Lambda(\lambda)$$

Black boxes and correlations



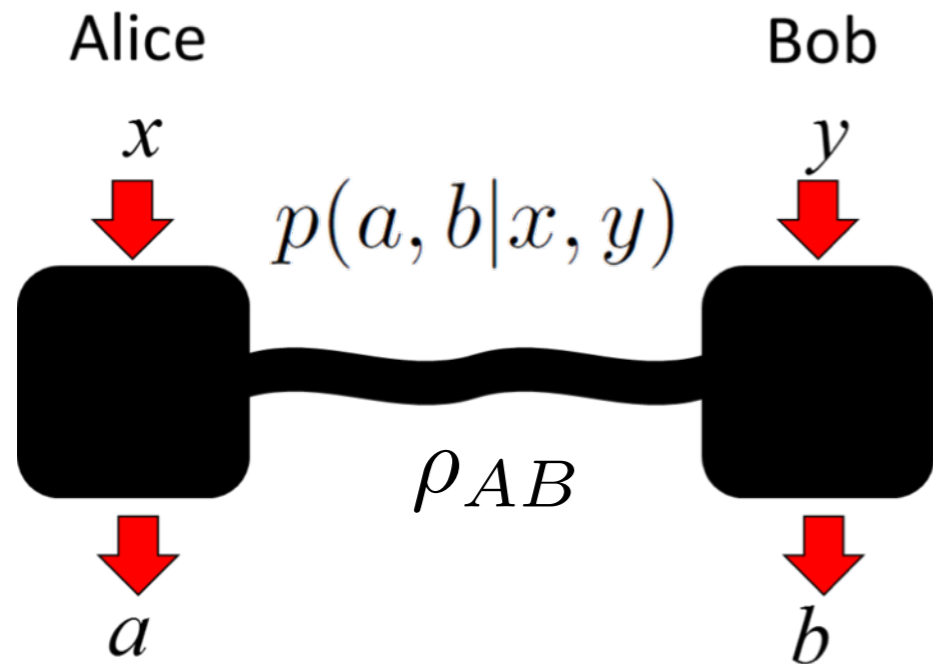
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Black boxes and correlations



No-signalling conditions:

$P(a|x, y)$ is independent of y ,
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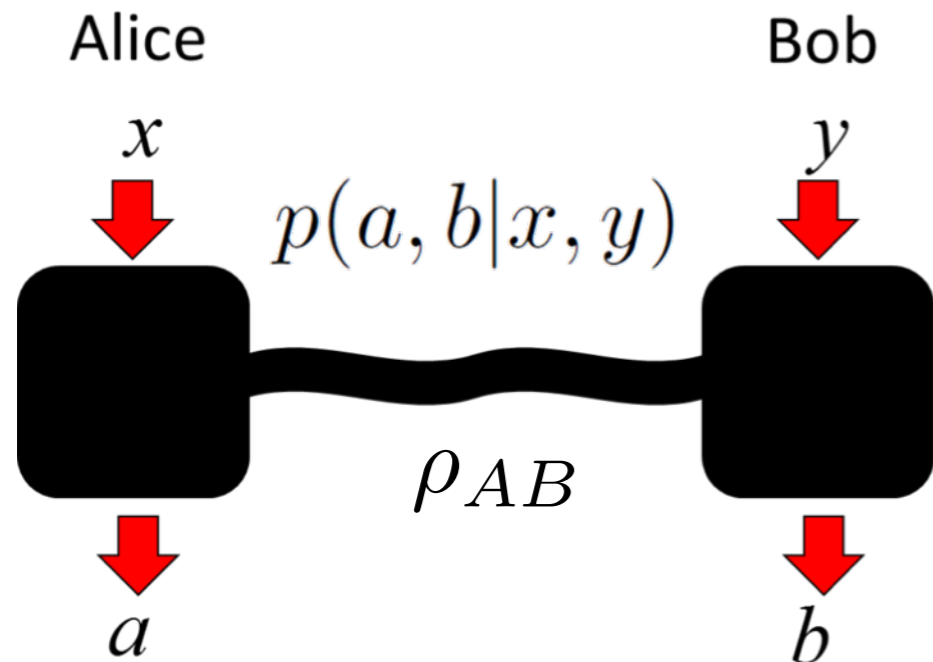
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Quantum admits more general P 's due to the **violation of Bell inequalities**.

The Bell-CHSH inequality

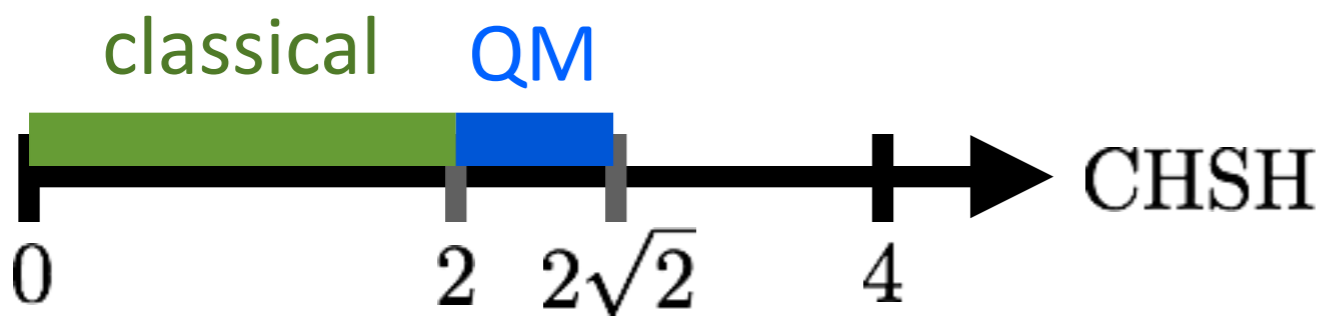
Classical probability distributions satisfy Bell inequality:

$$\text{CHSH} := |C_{00} + C_{01} + C_{10} - C_{11}| \leq 2 \quad \text{where} \quad C_{ab} := \mathbb{E}(x \cdot y | a, b) .$$

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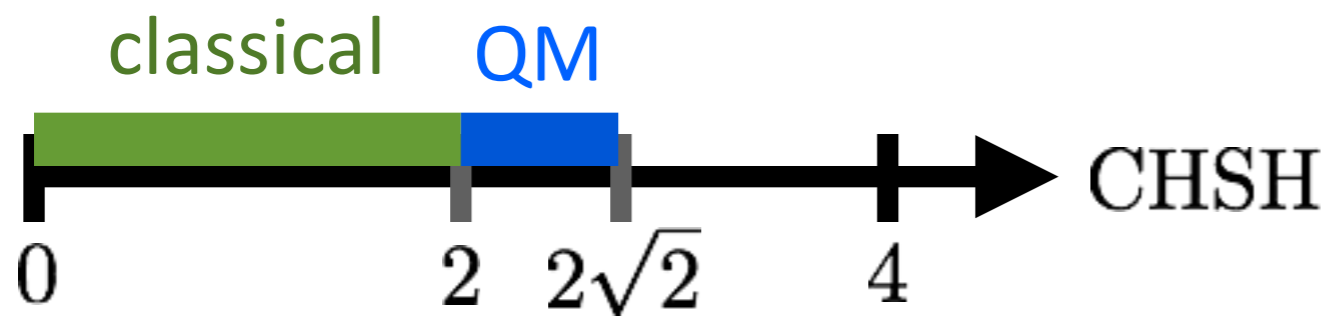
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$$\text{CHSH} \leq 2\sqrt{2}.$$

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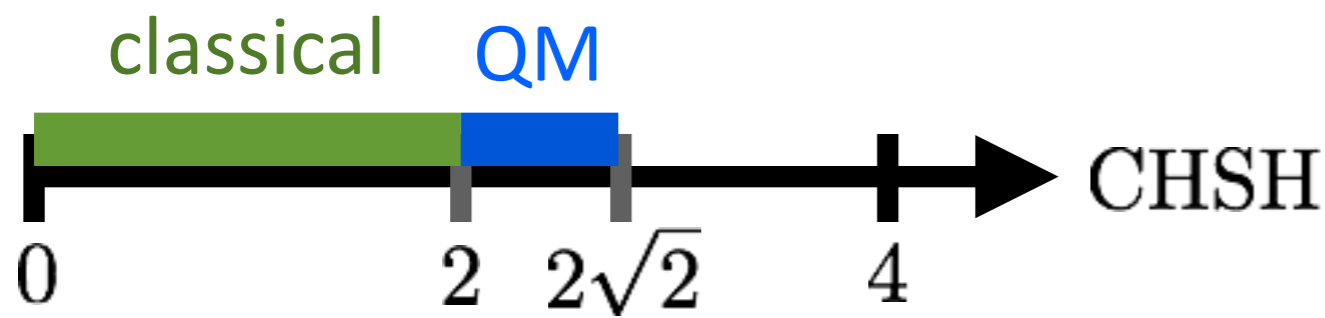
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Are **quantum** correlations the **most general** $P(a, b | x, y)$ that satisfy the no-signalling principle?

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$$p(+1, +1 | a, b) = p(-1, -1 | a, b) = \frac{1}{2} \\ \text{if } (a, b) \in \{(0, 0), (0, 1), (1, 0)\}$$

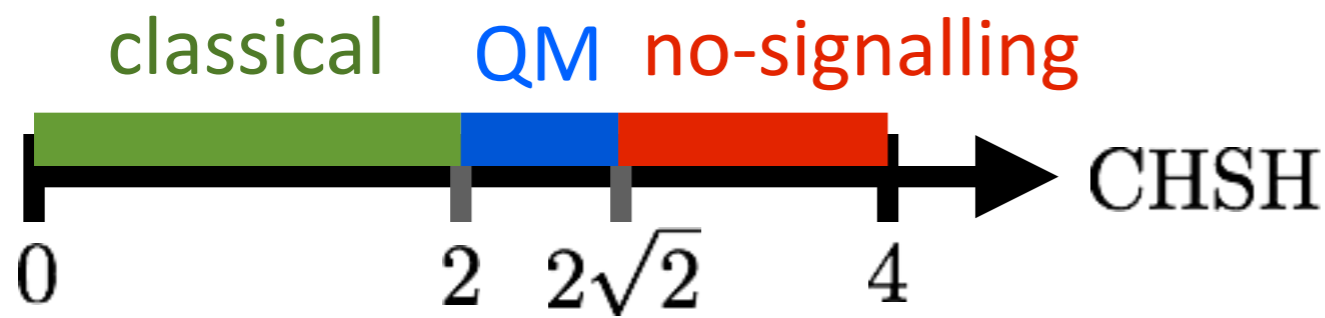
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CHSH=4

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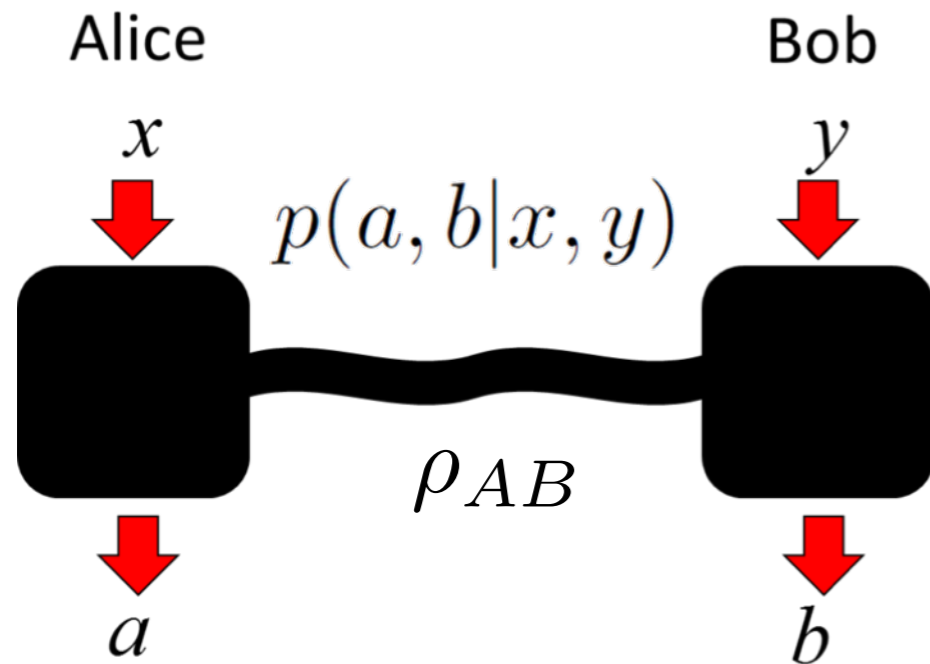
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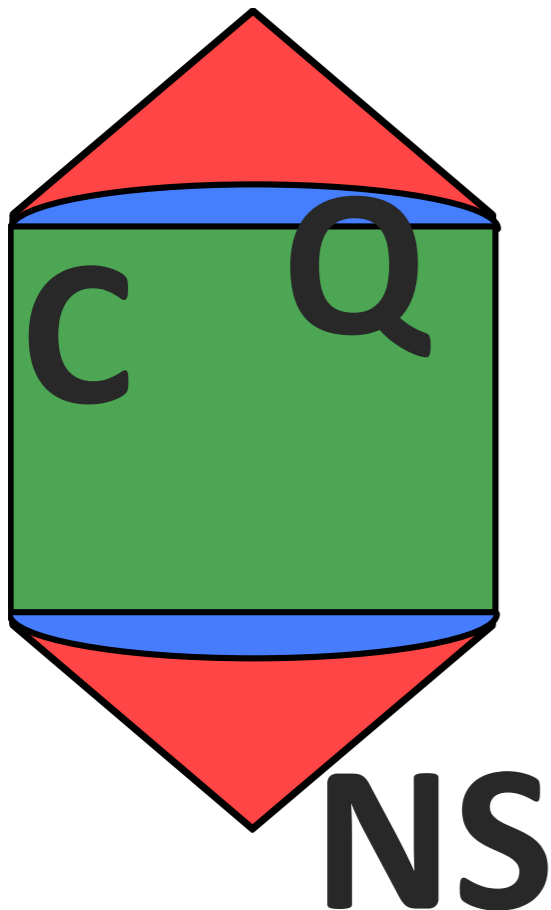
Black boxes and correlations



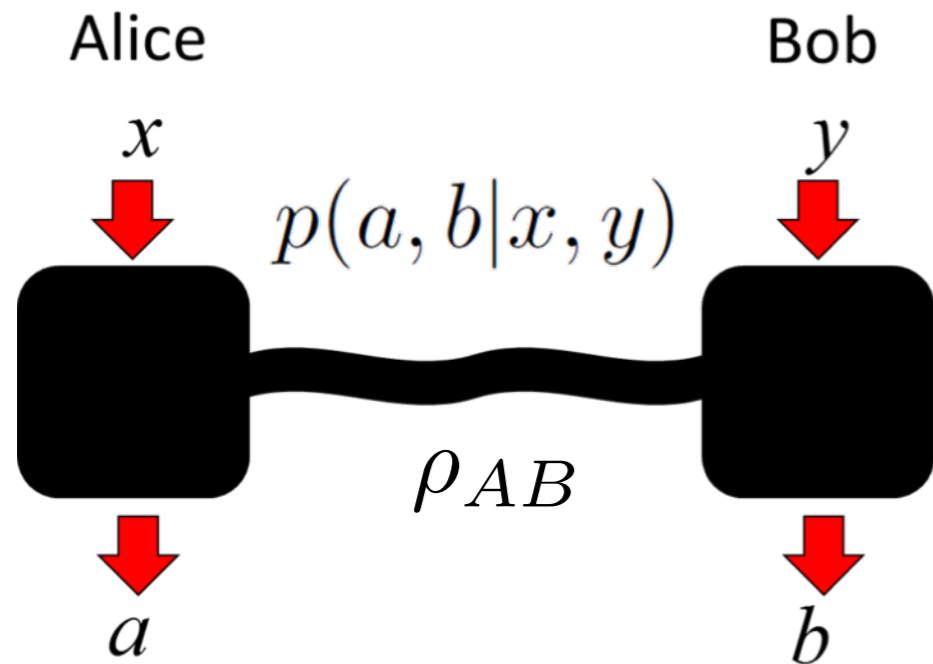
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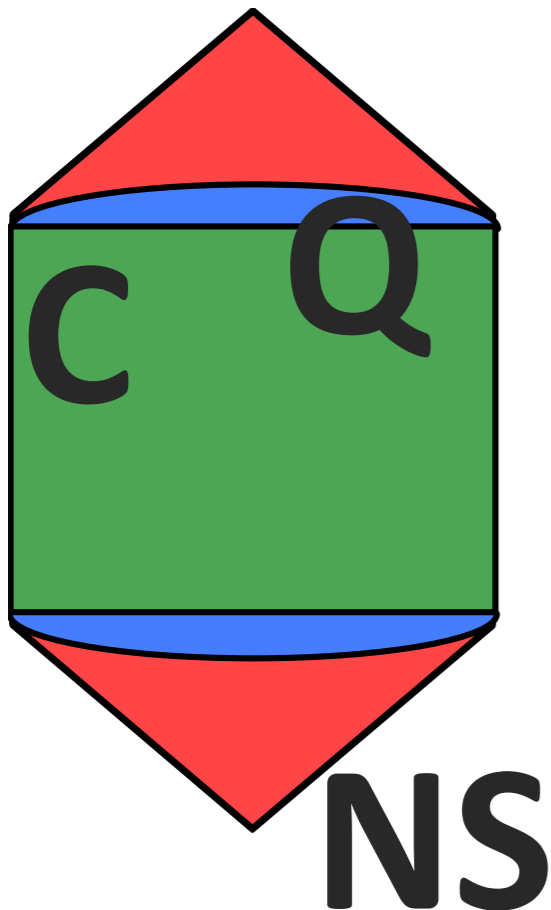


Black boxes and correlations



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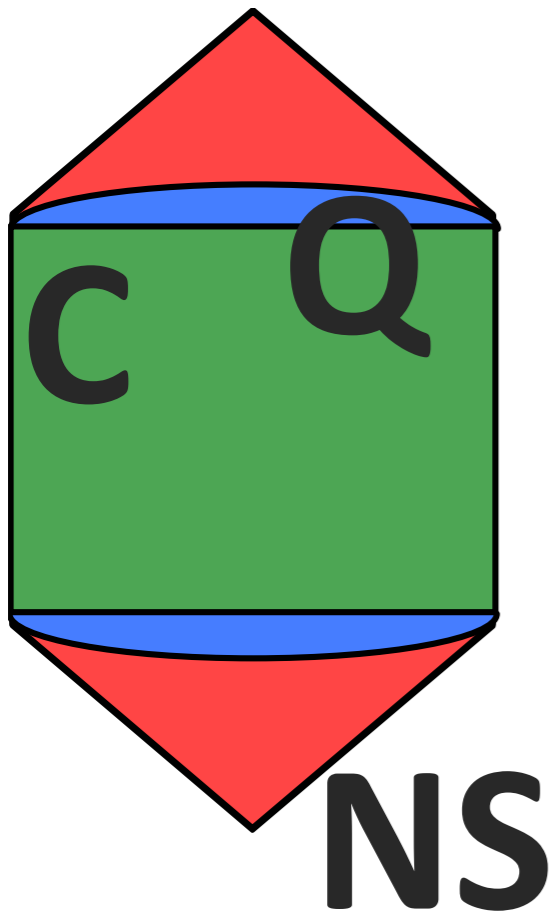
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Correlations in **C** come from **classical prob. theory**,
correlations in **Q** from **quantum theory**,
correlations in **NS** describe **alternative physics**.

Black boxes and correlations

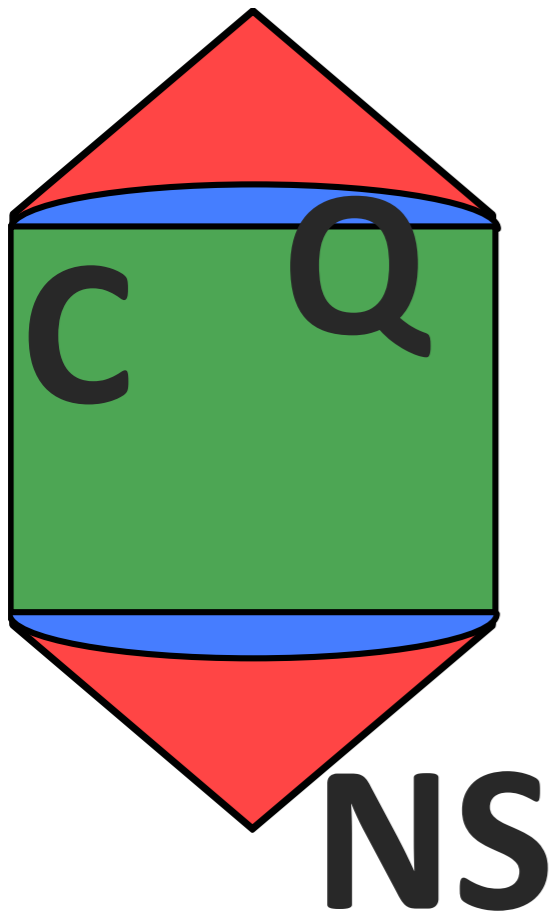
Why study such correlations?



Black boxes and correlations

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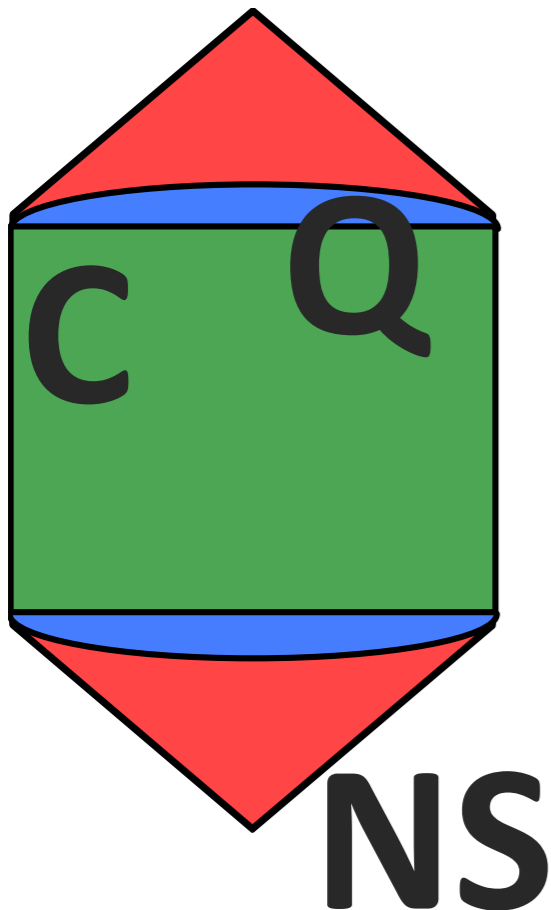
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Black boxes and correlations

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Experimental Realization of Device-Independent Quantum Randomness Expansion

Ming-Han Li, Xingjian Zhang, Wen-Zhao Liu, Si-Ran Zhao, Bing Bai, Yang Liu, Qi Zhao, Yuxiang Peng, Jun Zhang, Yanbao Zhang, W. J. Munro, Xiongfeng Ma, Qiang Zhang, Jingyun Fan, and Jian-Wei Pan
Phys. Rev. Lett. **126**, 050503 – Published 4 February 2021

Article

References

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ABSTRACT

Randomness expansion where one generates a longer sequence of random numbers from a short one is viable in quantum mechanics but not allowed classically. Device-independent quantum randomness

Black boxes and correlations

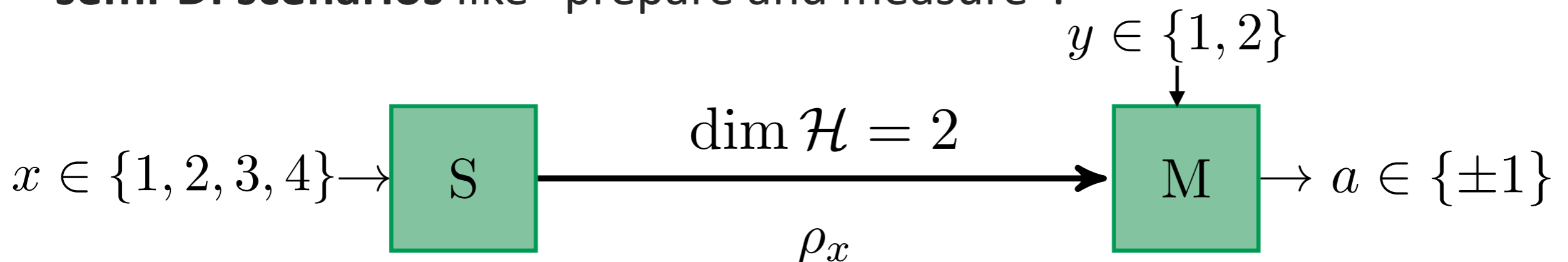
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not just Bell scenarios, but for example

semi-DI scenarios like “prepare and measure”:



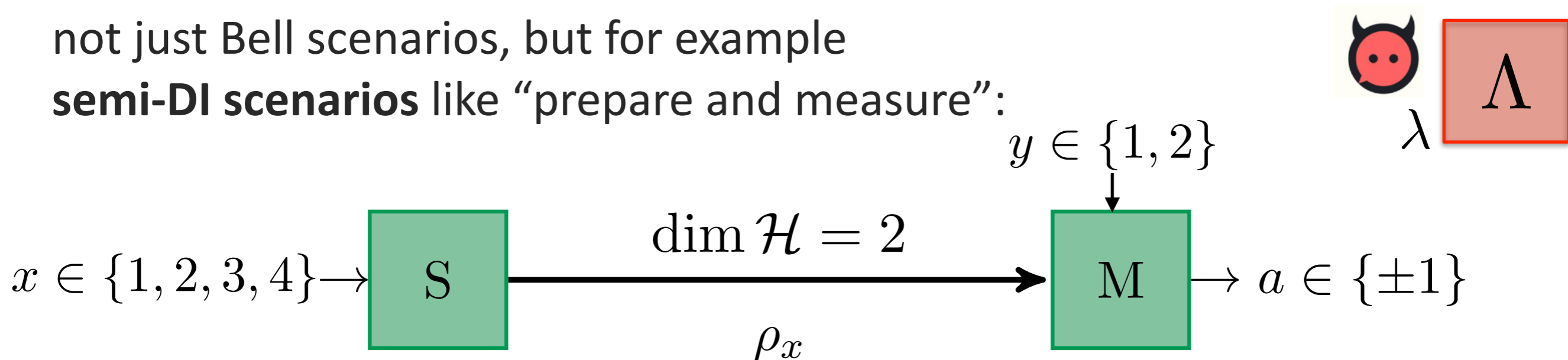
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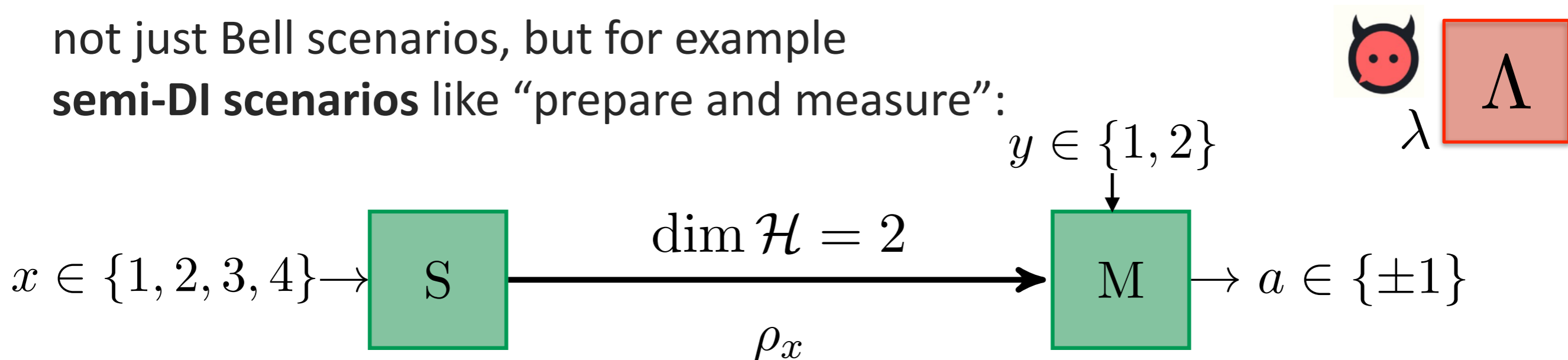
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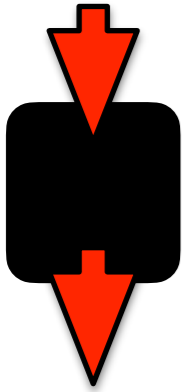
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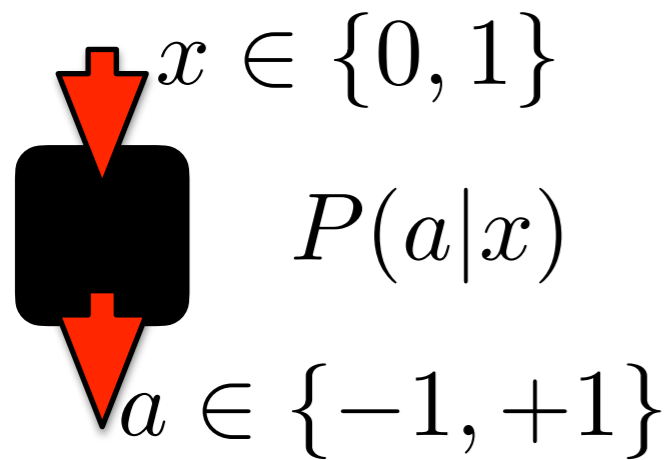
From the data table $p(a|x, y)$ and the assumption $\dim \mathcal{H} = 2$ **alone**, one can infer that $H(A|X, Y, \Lambda) \geq \dots > 0$.

Single black boxes



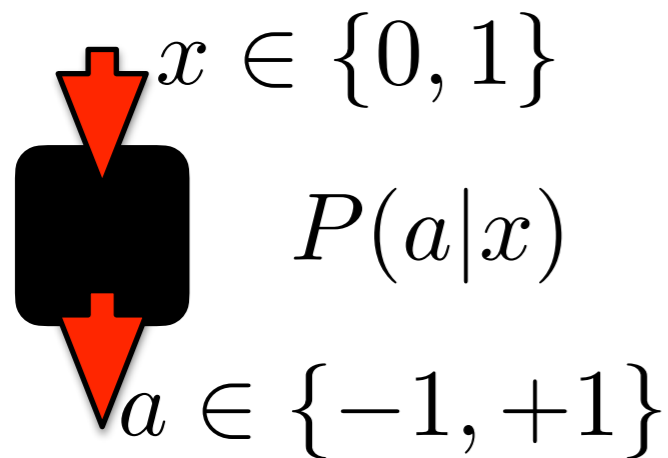
$$P(a|x)$$

Single black boxes



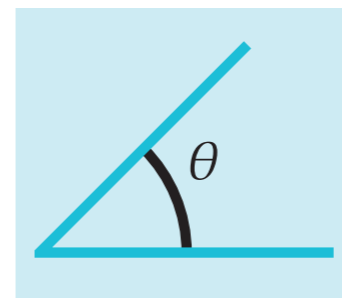
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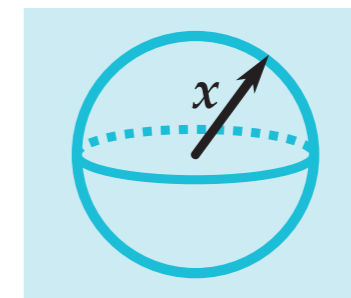


Inputs and outputs are typically taken as **abstract labels** (bits etc.)

What if inputs (and perhaps outputs) are **spatiotemporal quantities**?



ANGLES

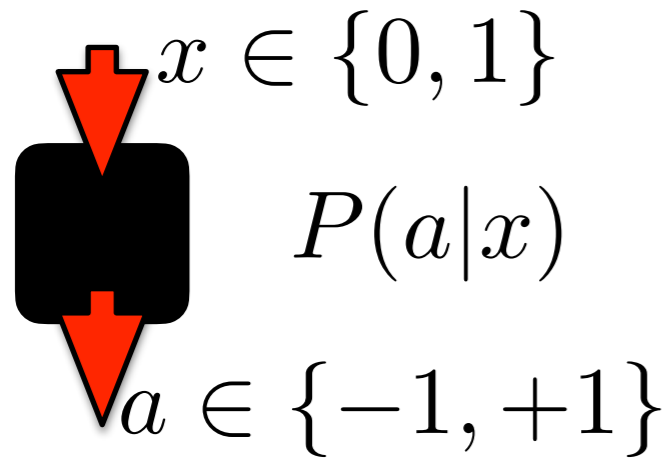


DIRECTIONS



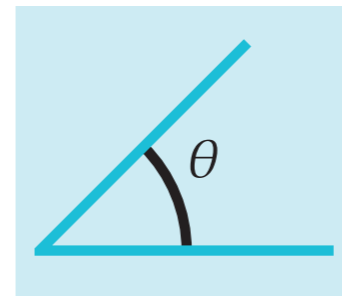
DURATIONS

Single black boxes

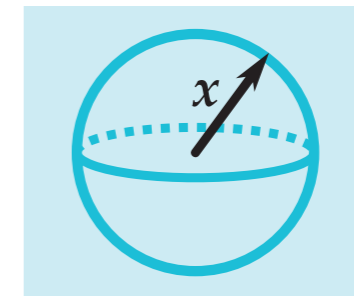


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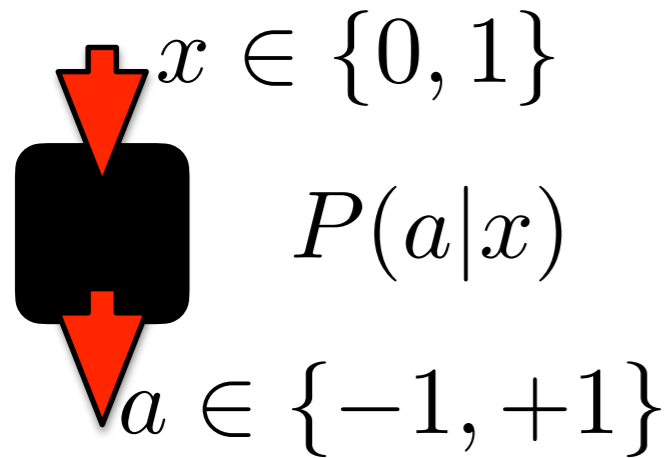
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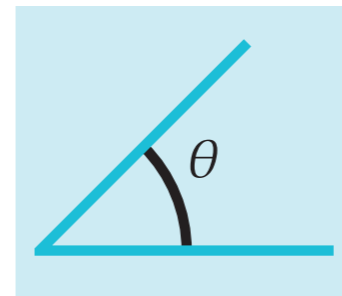
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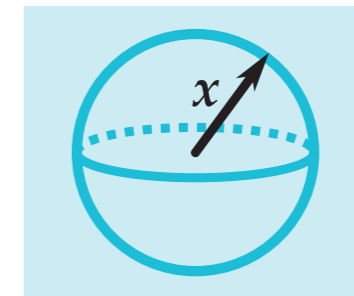


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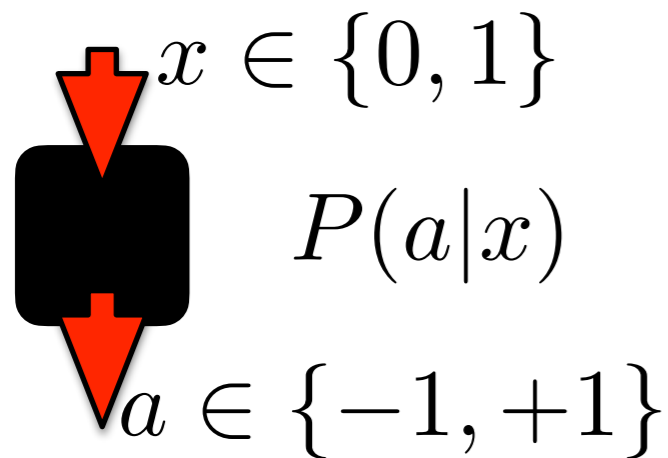
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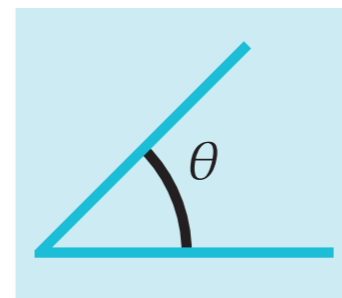
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- Study **interplay of probability, space and time** under minimal assumptions (even without assuming QT). Recall QFT!

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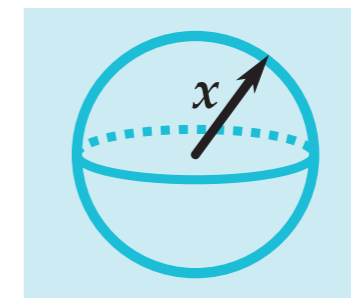


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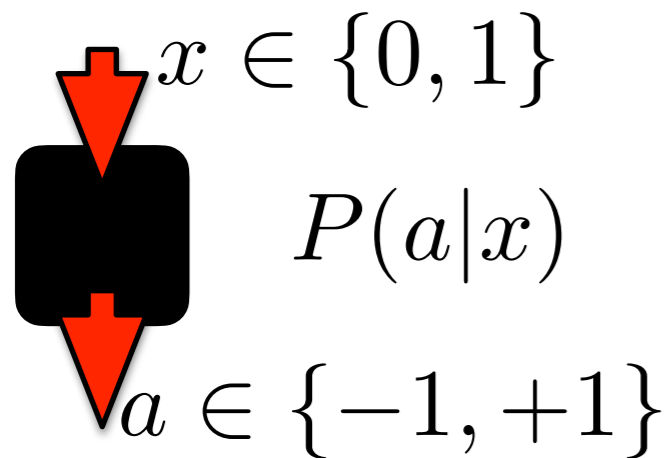
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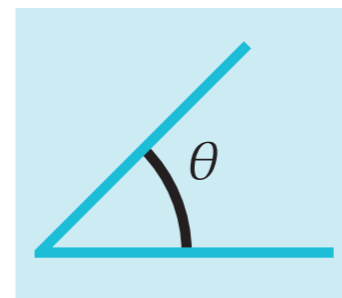
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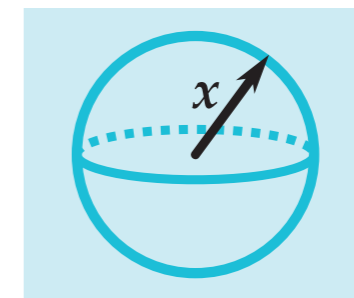


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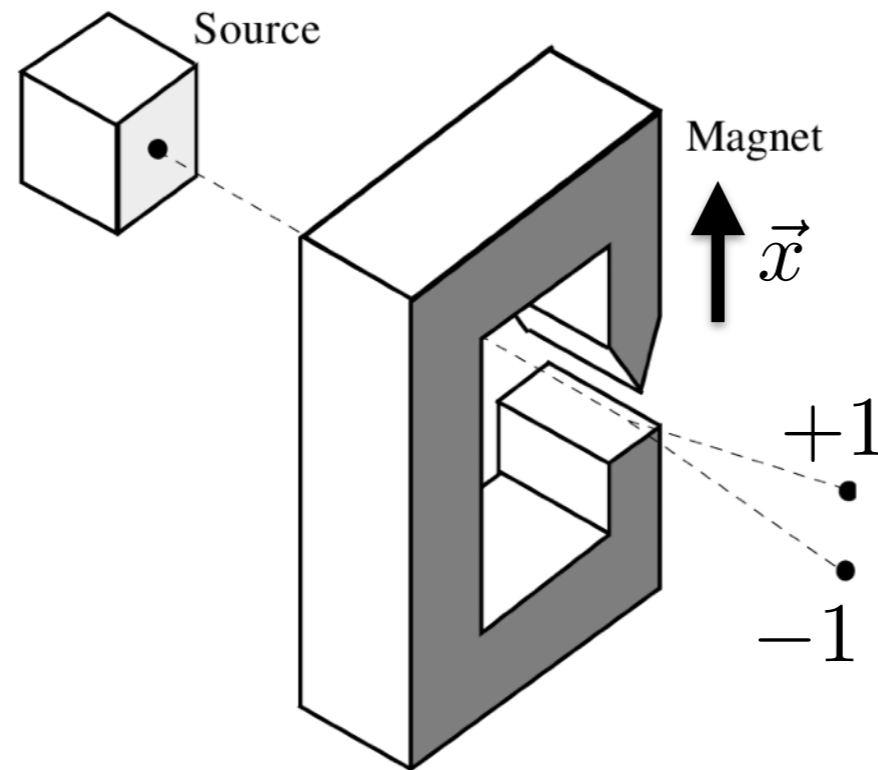


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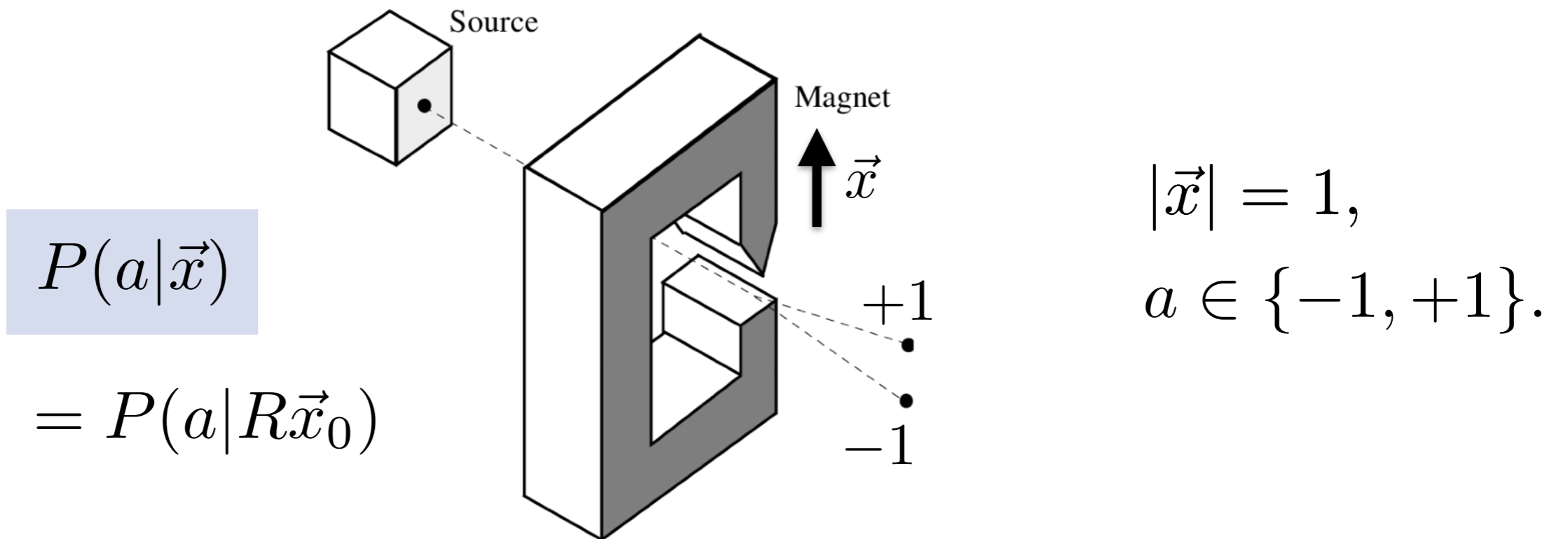
- This is the case in many **actual experimental settings**.
- Study **interplay of probability, space and time** under minimal assumptions (even without assuming QT). Recall QFT!
- Use spacetime symmetries in protocols?
- How could possible beyond-quantum physics fit into space and time?

Example: Stern-Gerlach experiment

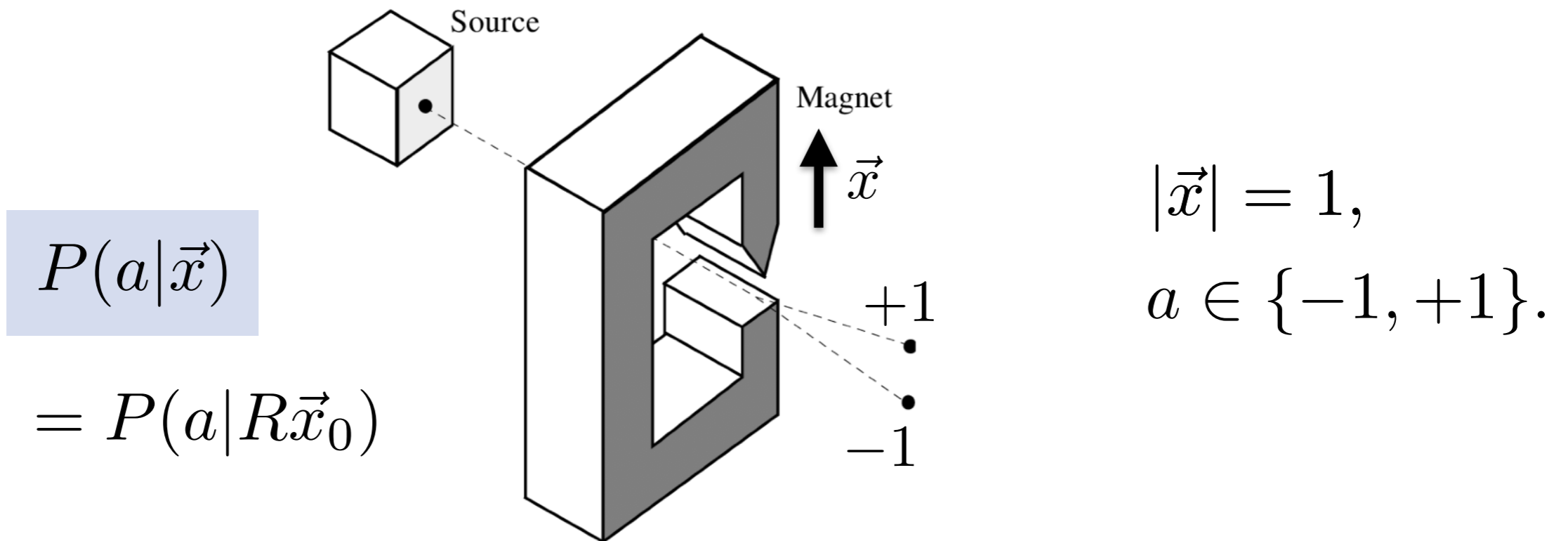
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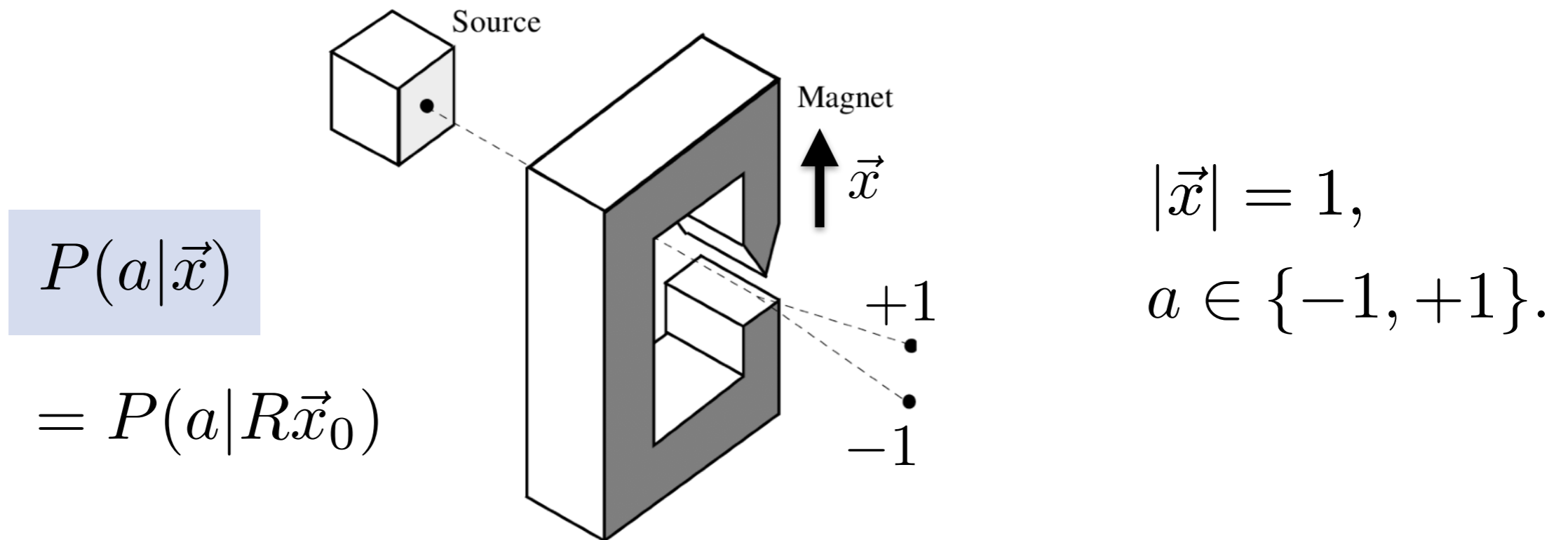


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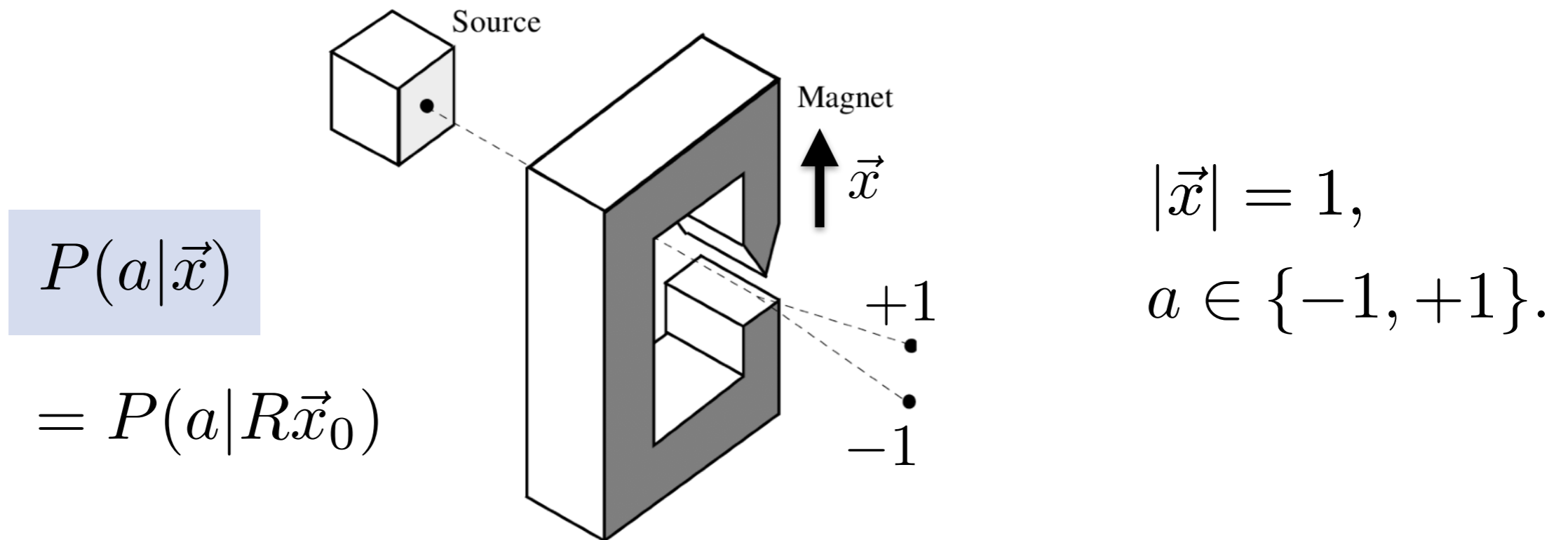
- Default direction of inhomogeneity of field: \vec{x}_0 .
- Spatial rotation applied to it: $R \in \mathcal{G} = \text{SO}(3)$.

Example: Stern-Gerlach experiment



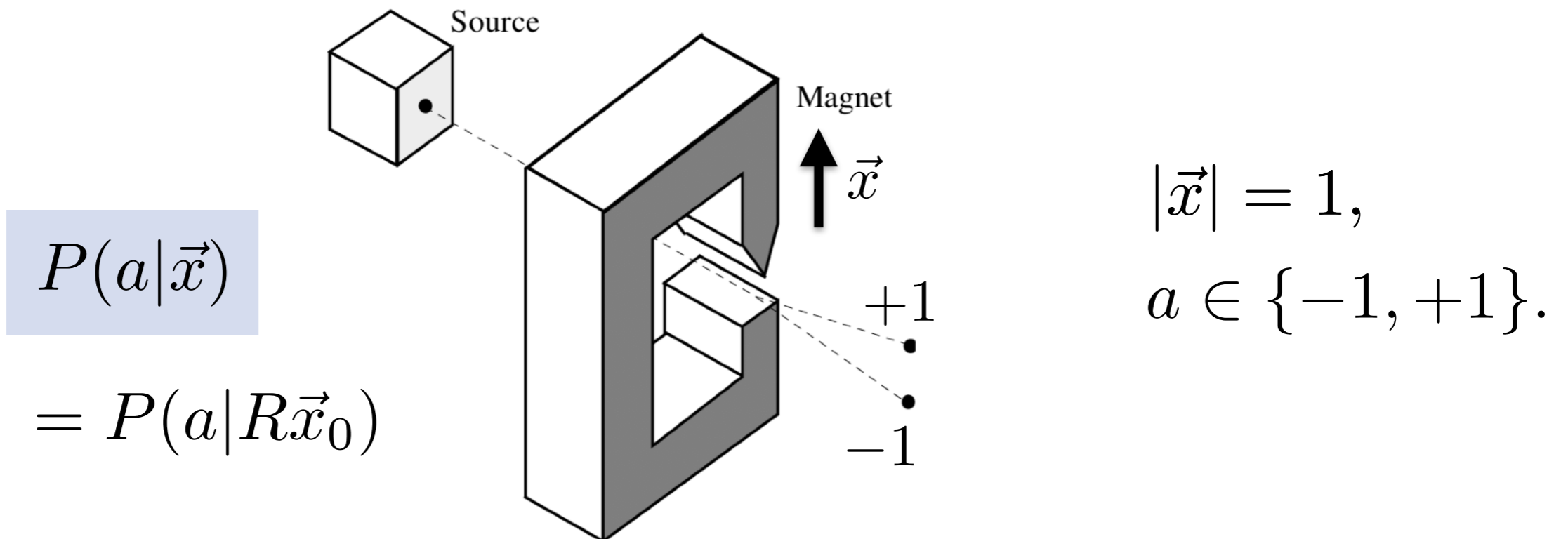
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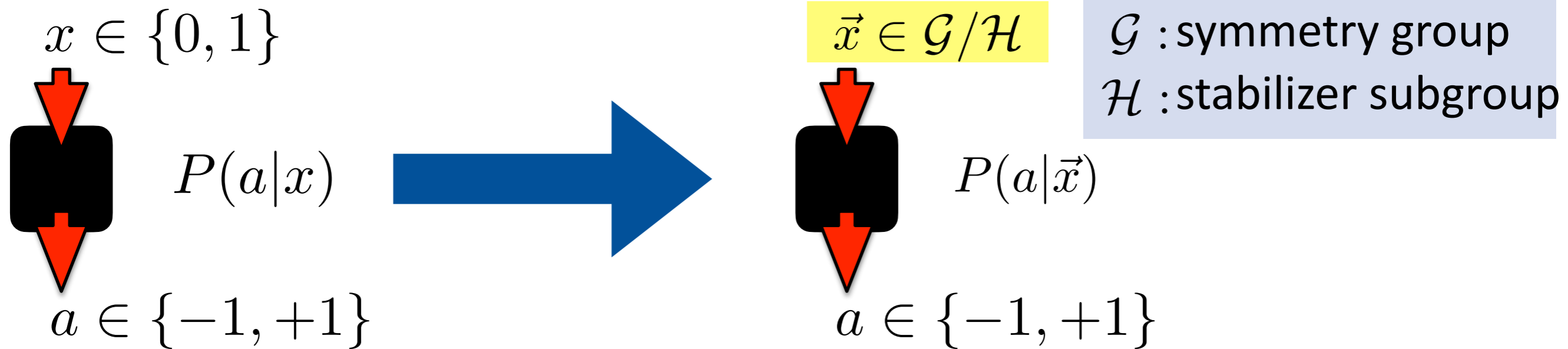
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- Manifold of inputs: the **unit sphere**, $S^2 = \text{SO}(3)/\text{SO}(2)$.
- In general, **inputs are elements of a homogeneous space**, \mathcal{G}/\mathcal{H} . Inputs are (partially) **symmetry-breaking DOFs**.

Spacetime boxes

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

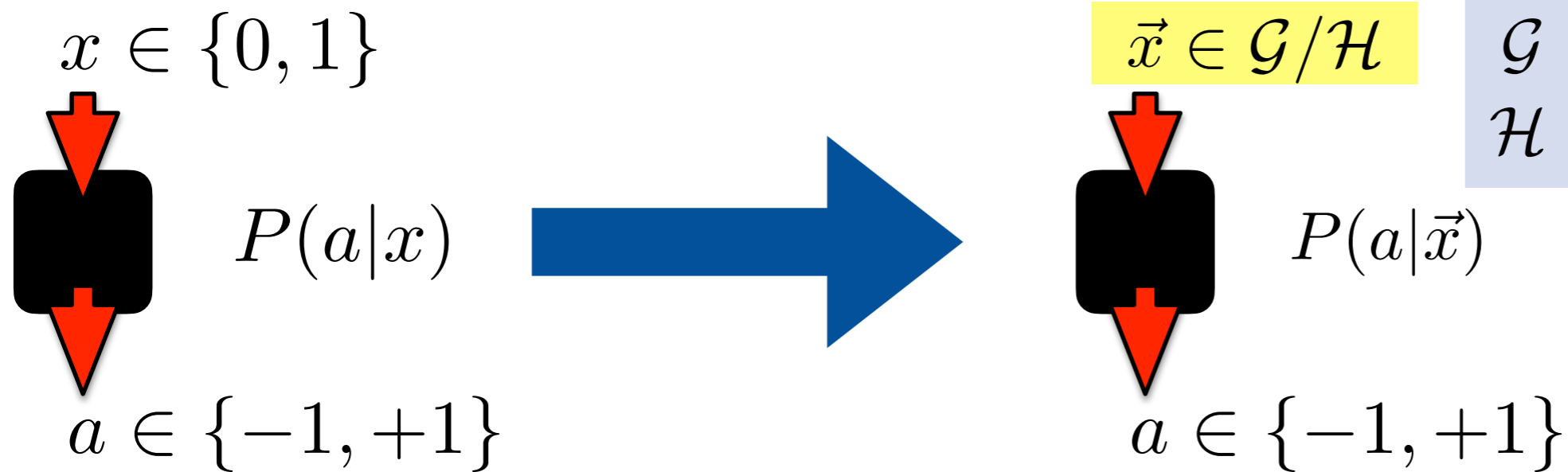
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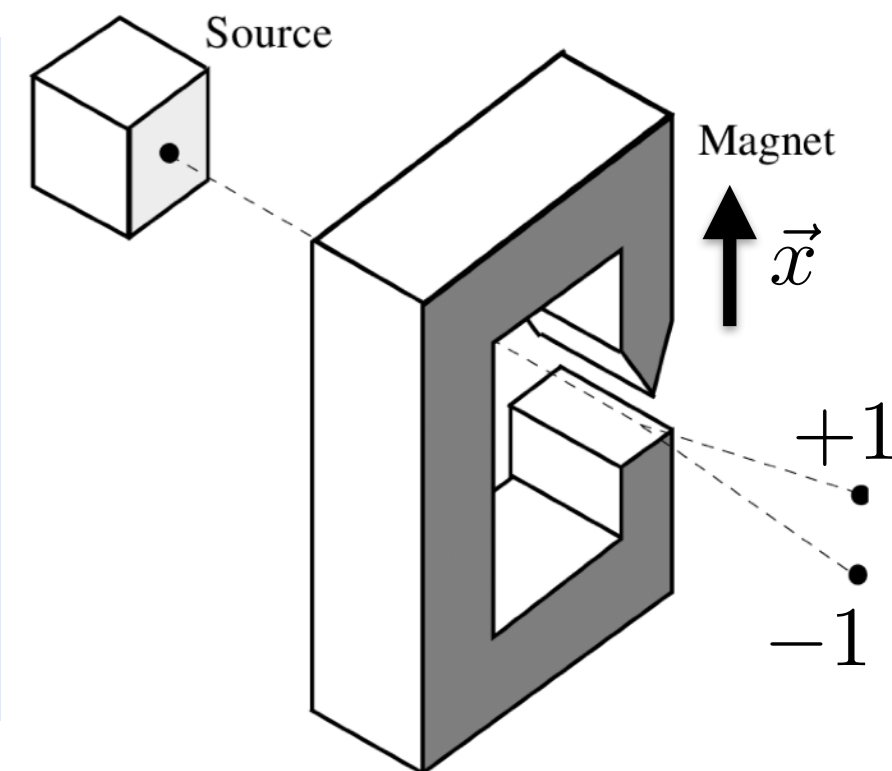


Example: Stern-Gerlach experiment

$\mathcal{G} = \text{SO}(3)$ (spatial rotations)

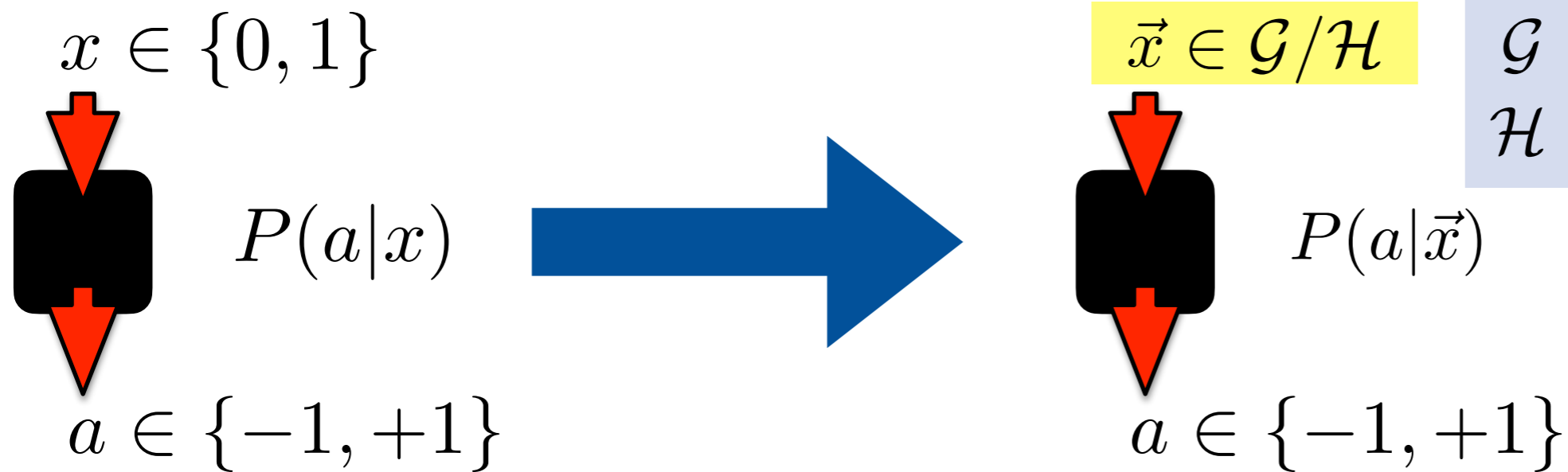
$\mathcal{H} = \text{SO}(2)$ (axial symmetry of magnetic field)

$\vec{x} \in \mathcal{G}/\mathcal{H} = S^2$ (unit vector: field direction)



Spacetime boxes

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).



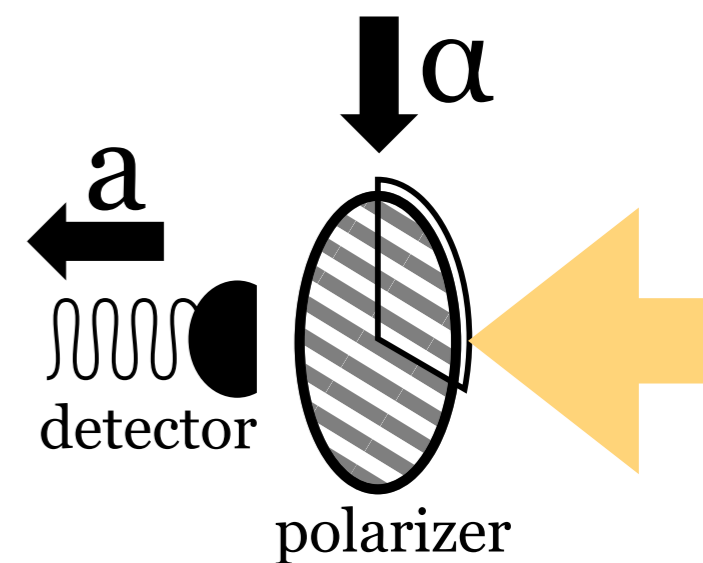
\mathcal{G} : symmetry group
 \mathcal{H} : stabilizer subgroup

Example: Polarizer, $P(a|\alpha)$.

$\mathcal{G} = \text{SO}(2)$ (rotations around beam axis)

$\mathcal{H} = \{1\}$ (no additional symmetry)

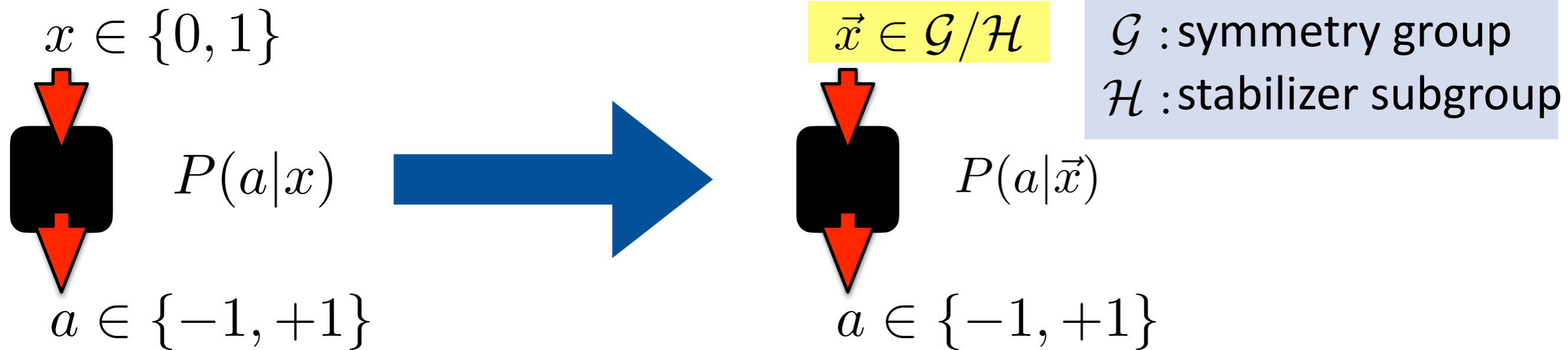
$\alpha \in \mathcal{G}/\mathcal{H} = \text{SO}(2)$.



click / no click: $a = \pm 1$.

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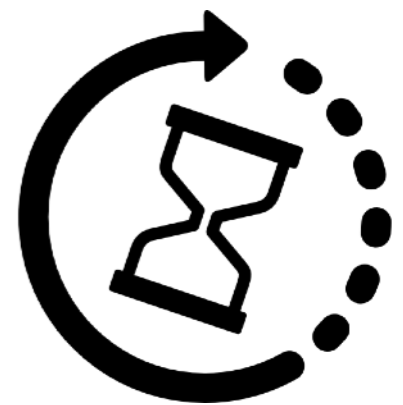


Example: Input is time t , $P(a|t)$.

$\mathcal{G} = (\mathbb{R}, +)$ (group of time translations)

$\mathcal{H} = \{1\}$ (no additional symmetry)

$\vec{x} = t \in \mathbb{R}$



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Spacetime boxes

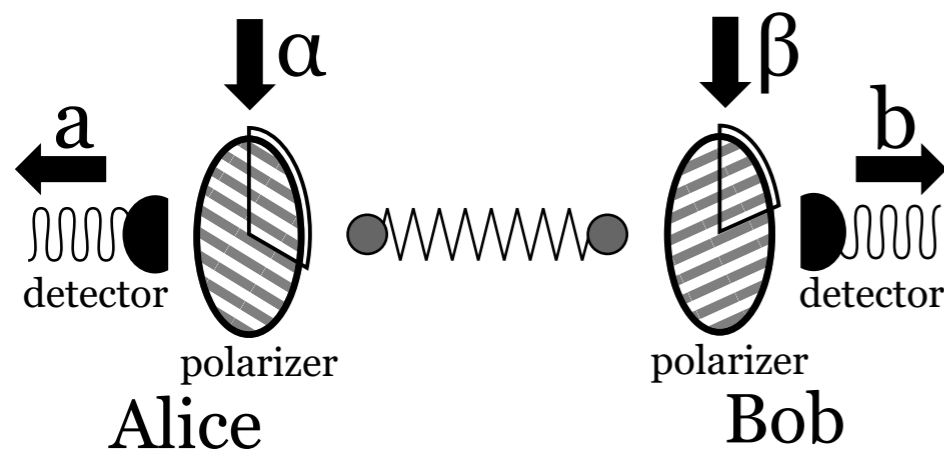
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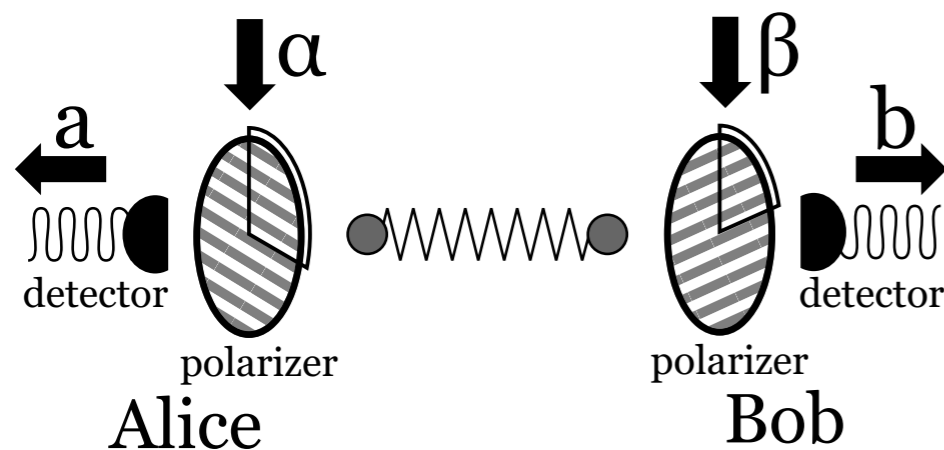


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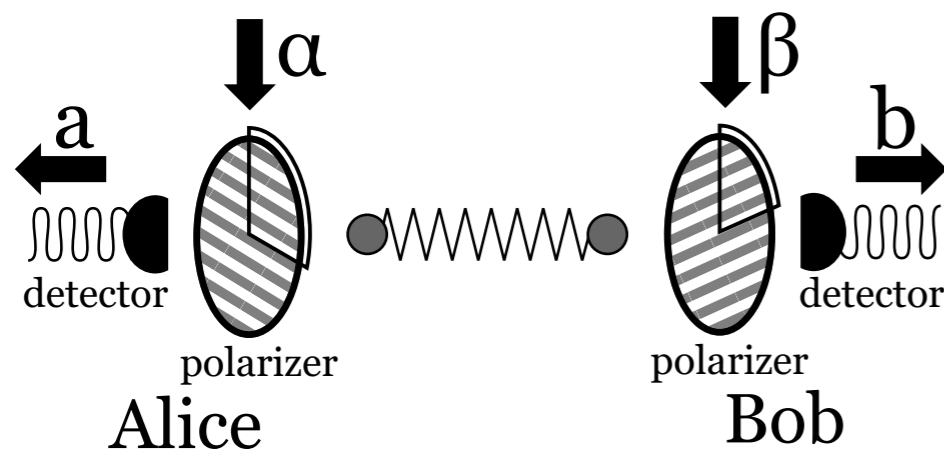
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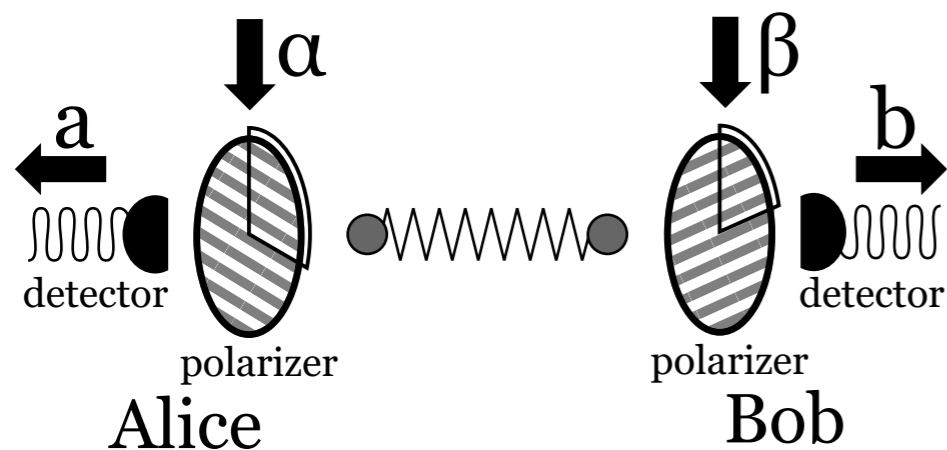
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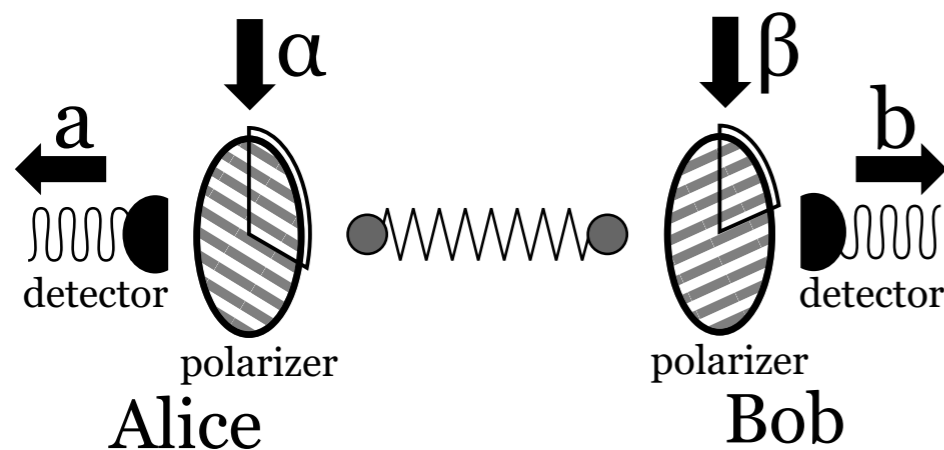
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Examples: $C(\alpha, \beta) := \sum_{a,b=-1}^{+1} abP(a, b|\alpha, \beta) \quad (a, b = \pm 1)$

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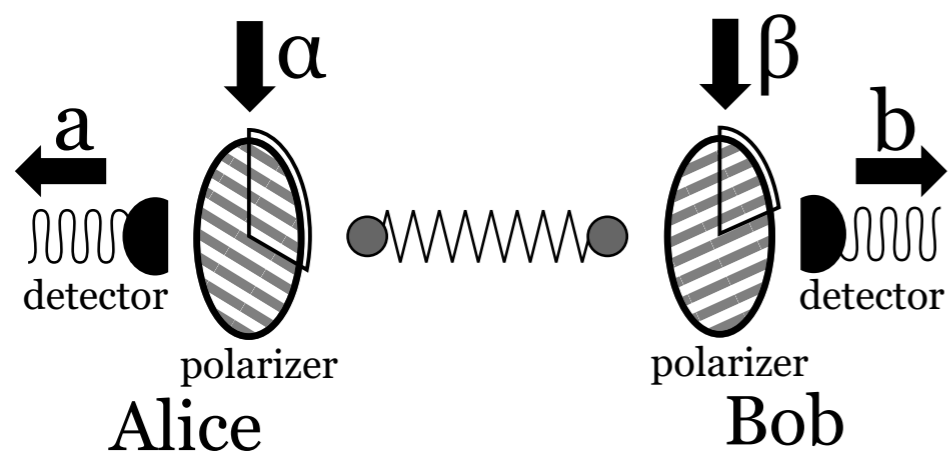
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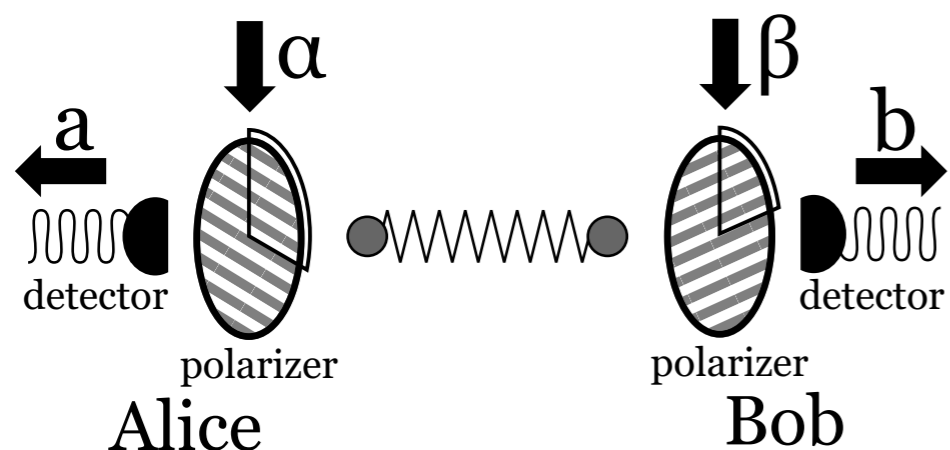
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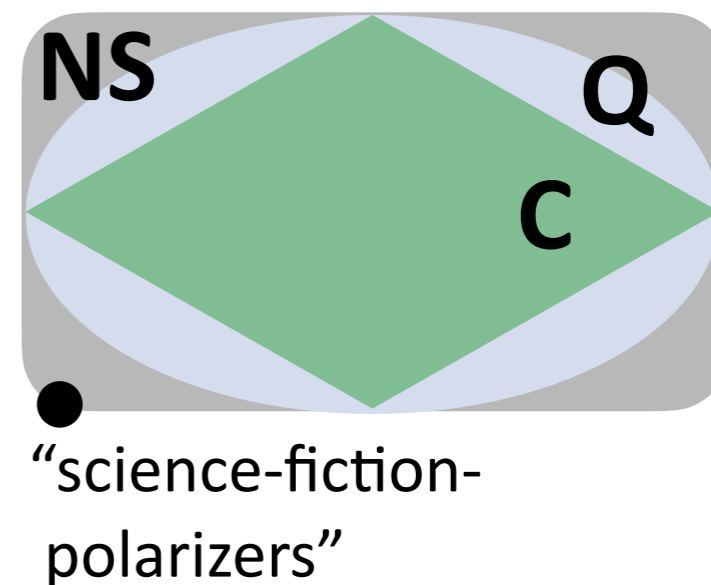
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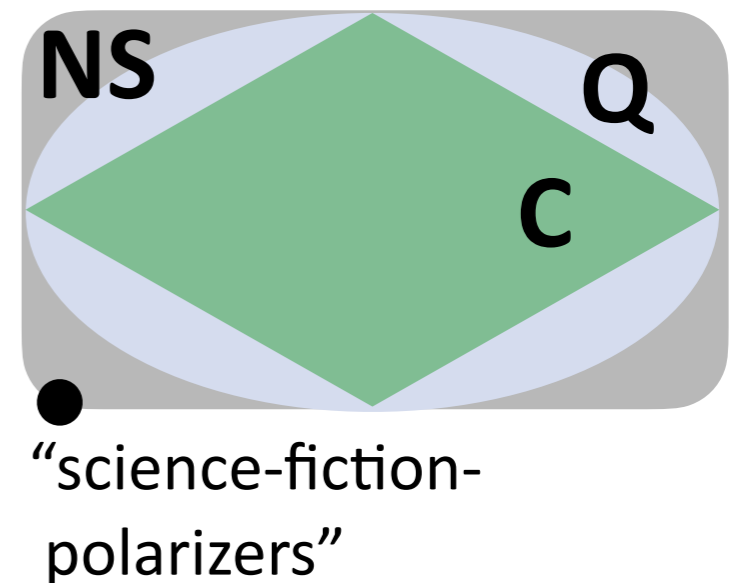
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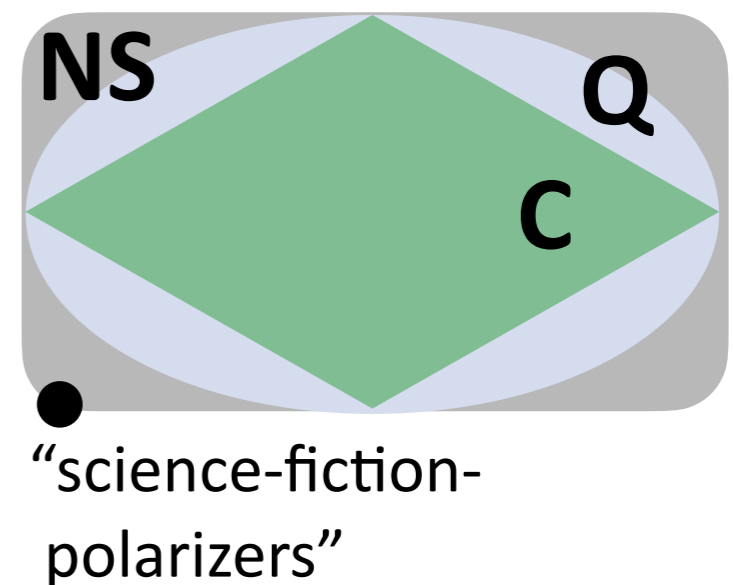
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Answer: No. If $\max_{\alpha, \beta} |C(\alpha, \beta)| \leq \sqrt{2}e^{-1}[4J(2J+1)]^{-3/2}$ then C admits of a local hidden-variable model. Likely true for other groups too.

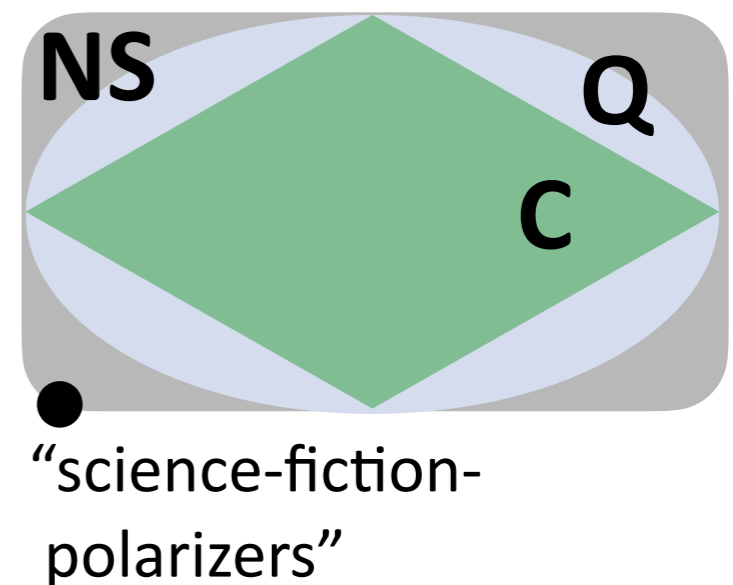
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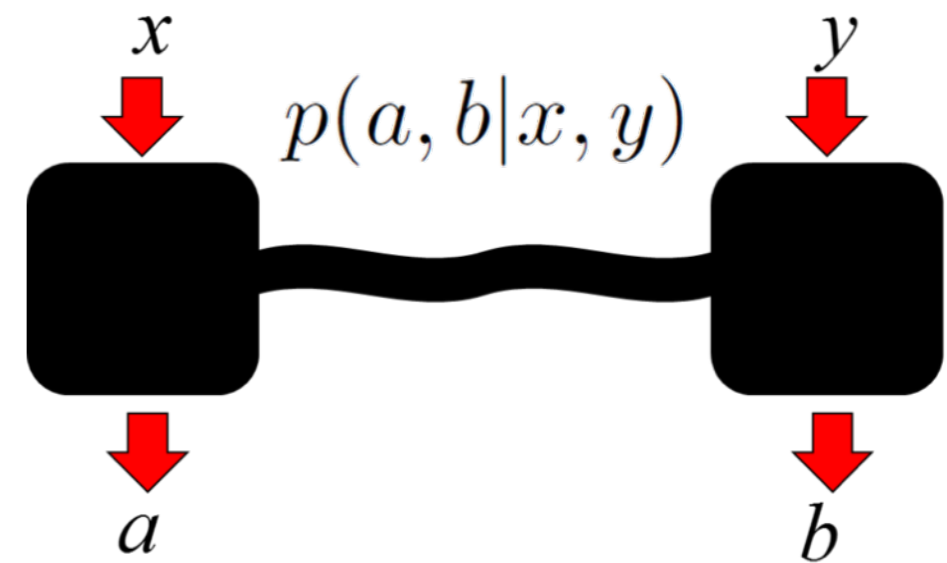
Overview

1. General framework of “spacetime boxes”

2. Foundational insights

3. Towards novel protocols...

4. ... and experimental tests of QT



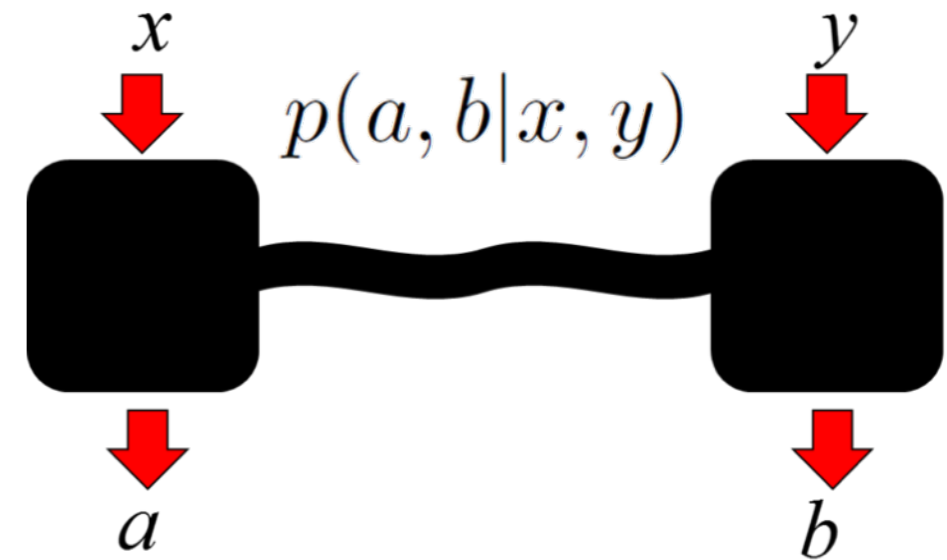
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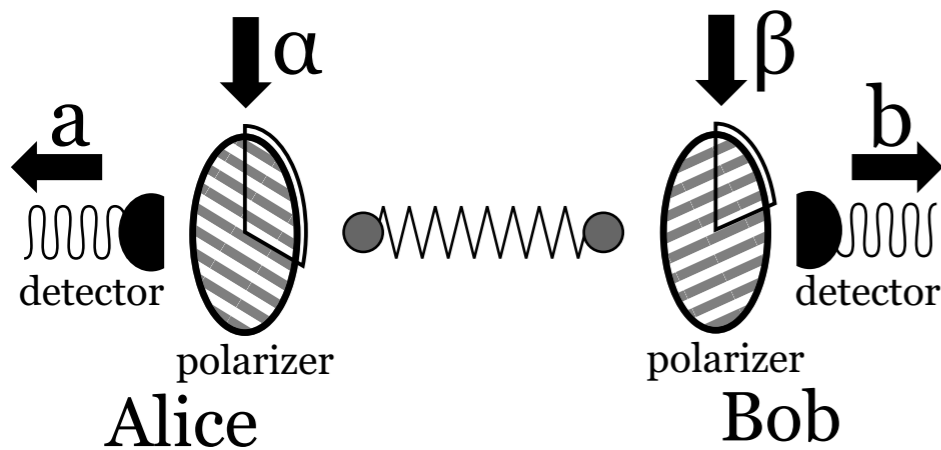


Foundational consequences

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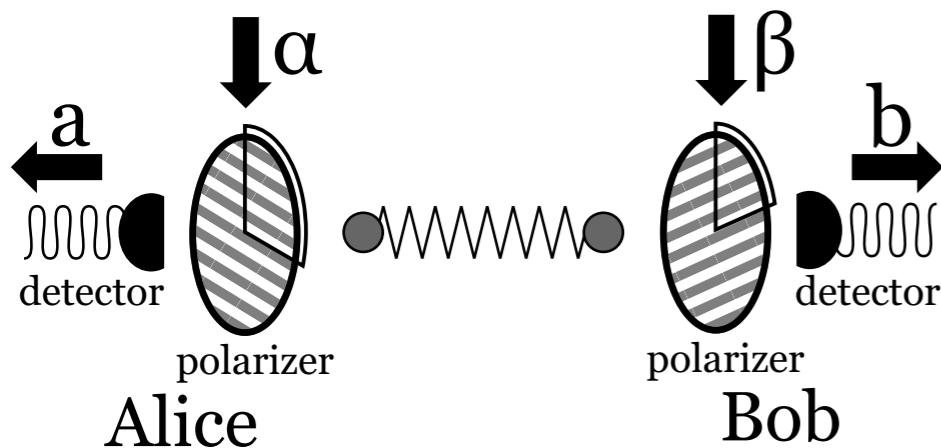
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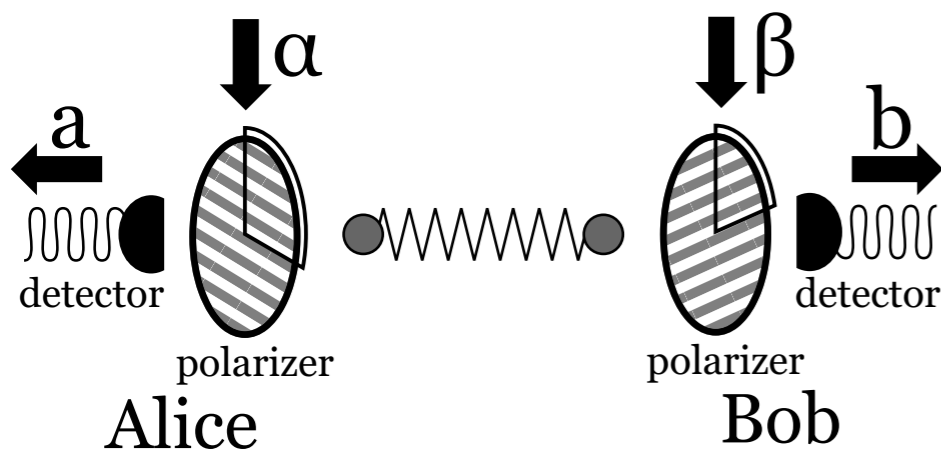
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1. Prob's transform **locally fundamentally**,
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More details:

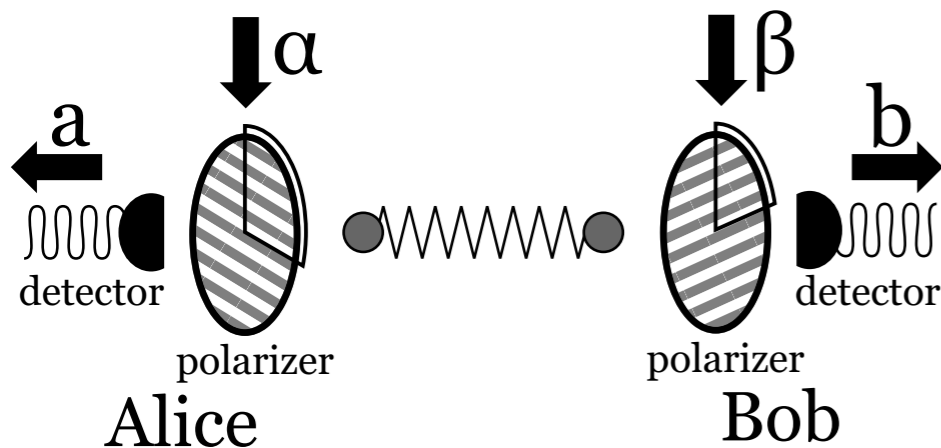
Bob inputs \vec{y} , obtains outcome b , and tells Alice this

→ conditional box $P_{b, \vec{y}}^A(a | \vec{x}) = \frac{P(a, b | \vec{x}, \vec{y})}{P_B(b | \vec{y})}$

transforms fundamentally (*“like a [co-]vector”*).

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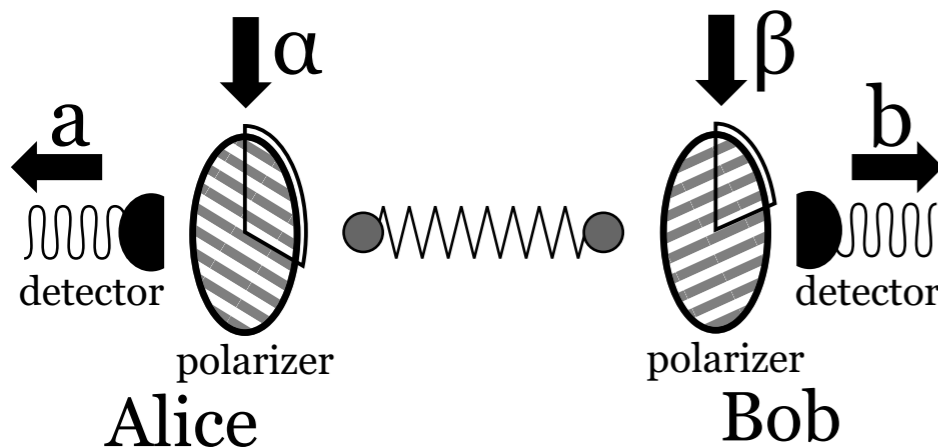
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both for $a = +1, a = -1$ (similarly for b).

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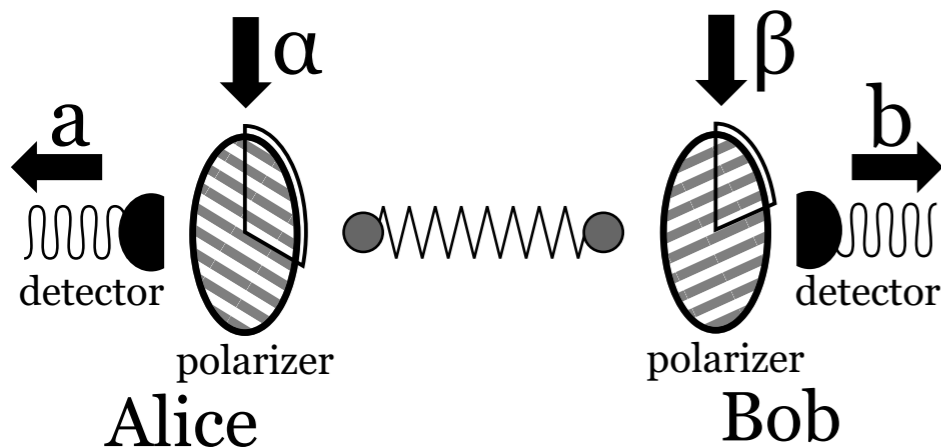
Suppose a has **geometric interpretation** as “parallel or antiparallel to \vec{x} ”

$$\Rightarrow P^A(-a|\vec{x}) = P^A(a|-\vec{x})$$

\Rightarrow Local unbiasedness holds automatically.

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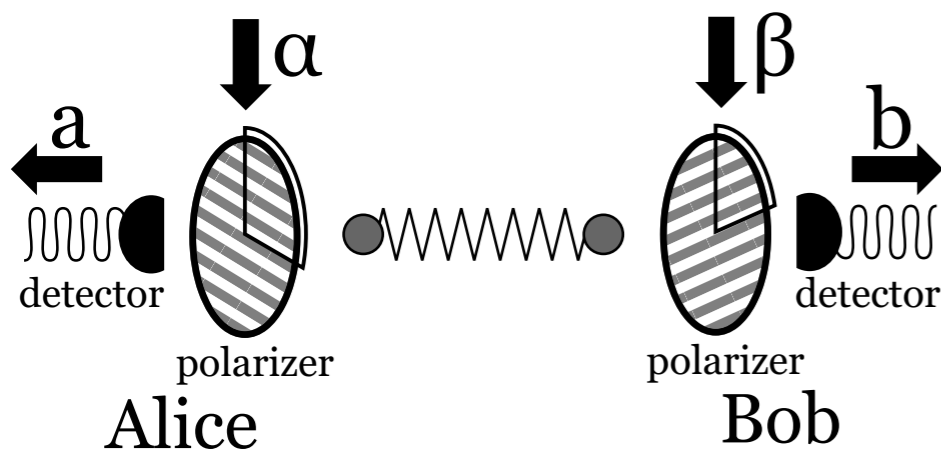
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Theorem. In any world where these assumptions hold (**not assuming QT!**), Alice and Bob see **quantum correlations** (i.e. in **Q**).

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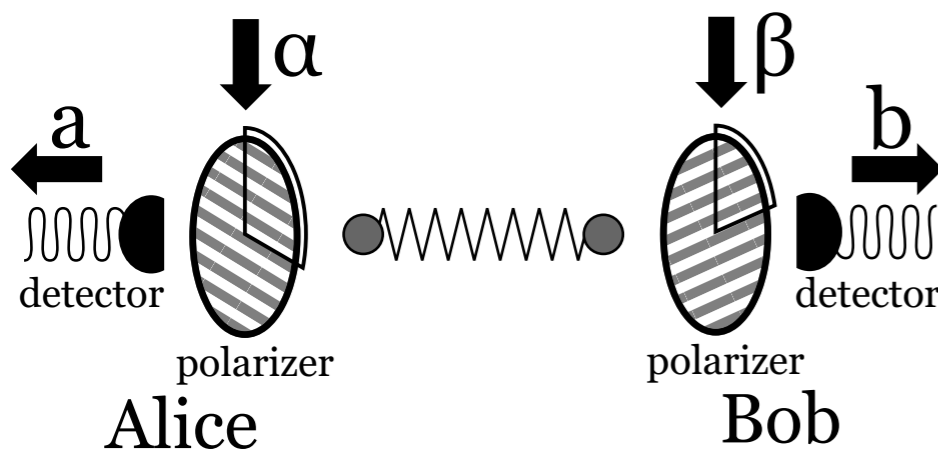
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Theorem: The quantum (2,2,2)-correlations **Q** are **exactly those** that can be obtained by $\text{SO}(d) \times \text{SO}(d)$ -boxes that transform locally fundamentally and are locally unbiased, restricted to two inputs per party, and supplemented by shared randomness.

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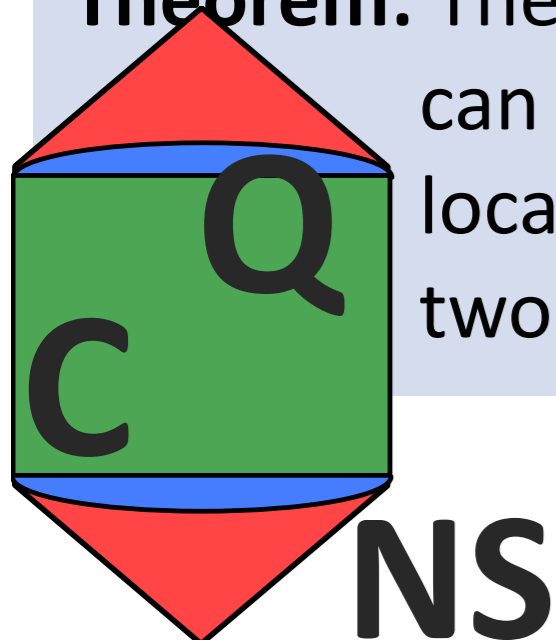
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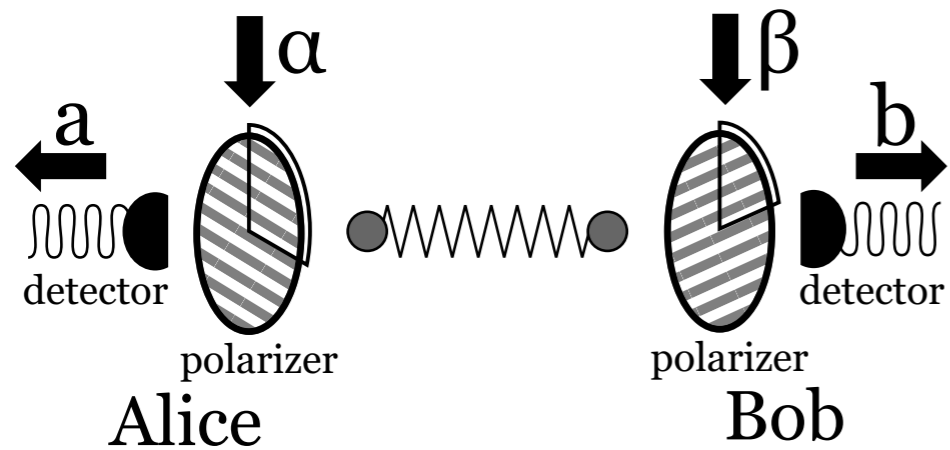
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Fundamental relation between QT and space(time)?

Recall the $d=2$ case

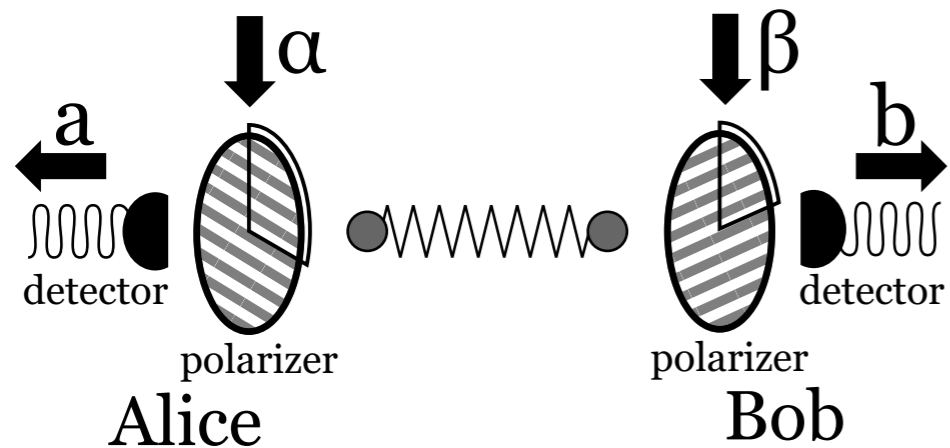
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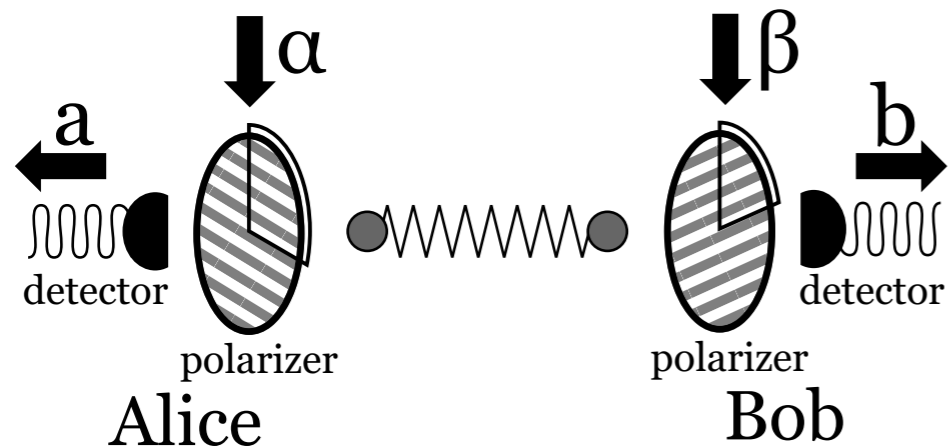


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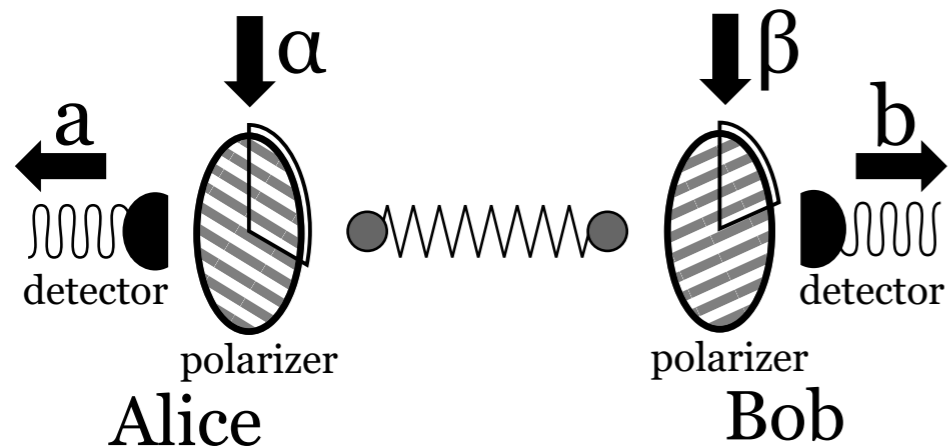
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Hence, **bounding the representation label can severely constrain the possible correlations.**

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This amounts to an assumption of “how the devices respond to spatiotemporal symmetry transformations”.

Idea: use this for **protocols**.

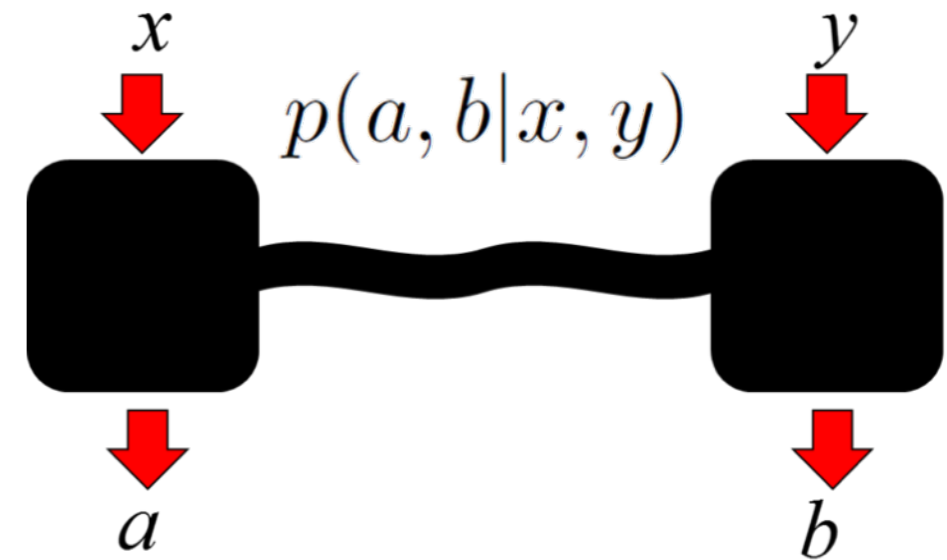
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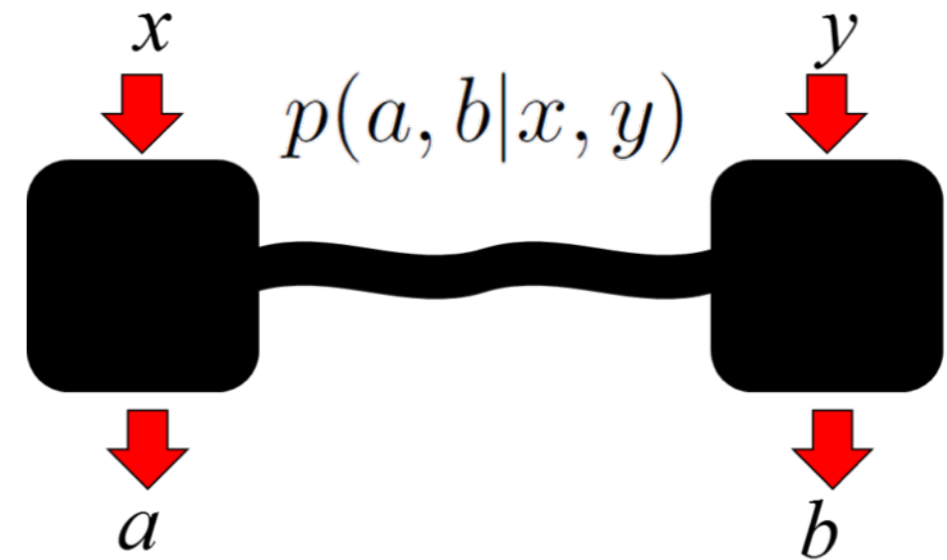
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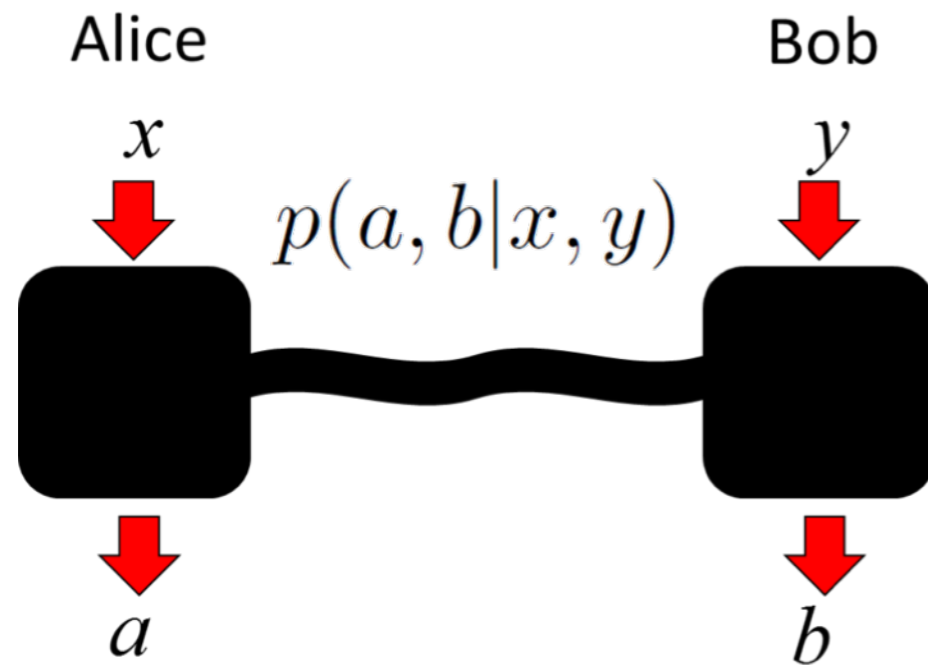
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Device-independent QIT:



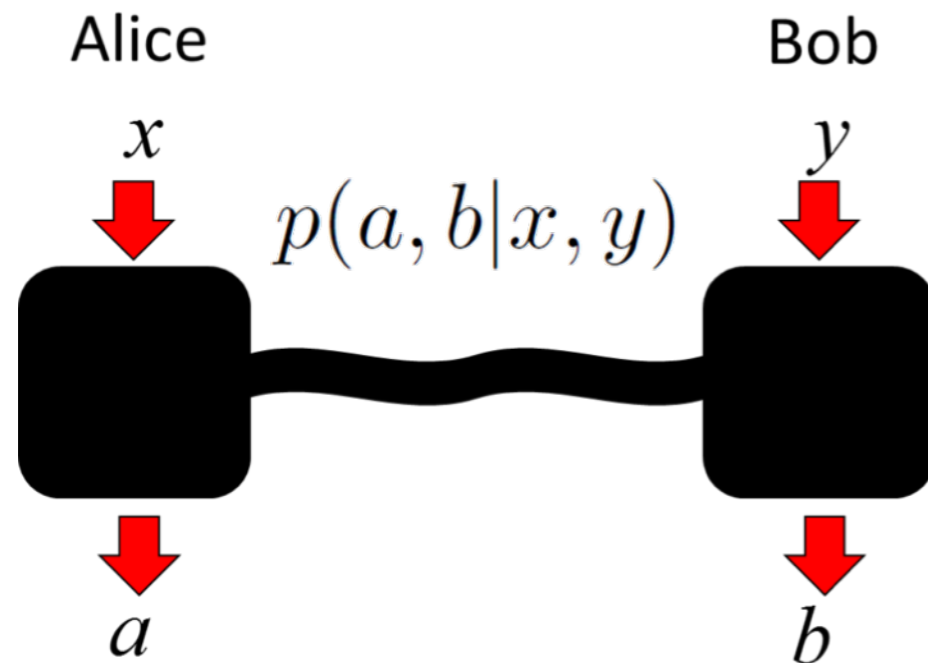
Violation of a Bell inequality admits

- randomness expansion
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even if **devices are untrusted**.

Towards protocols

Device-independent QIT:

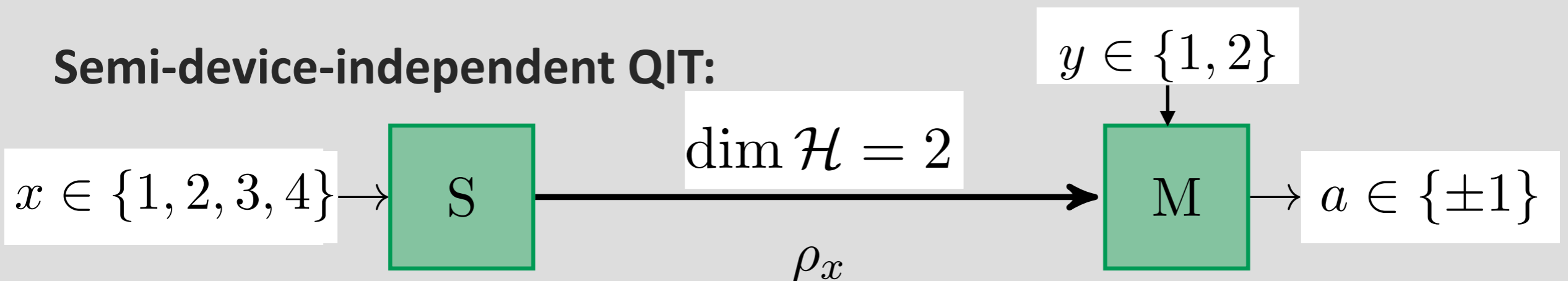


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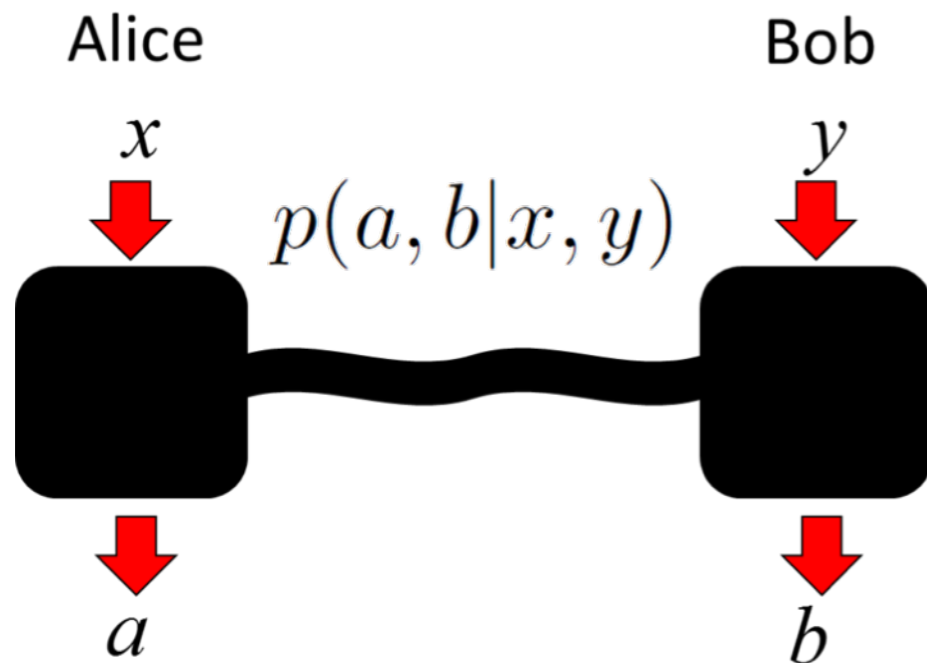
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Devices untrusted, but **some assumptions on transmitted states** have to be made.

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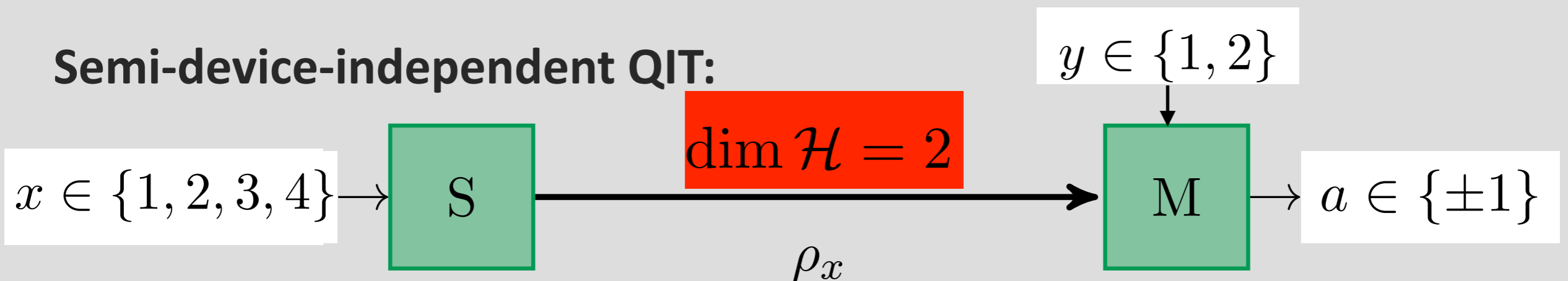


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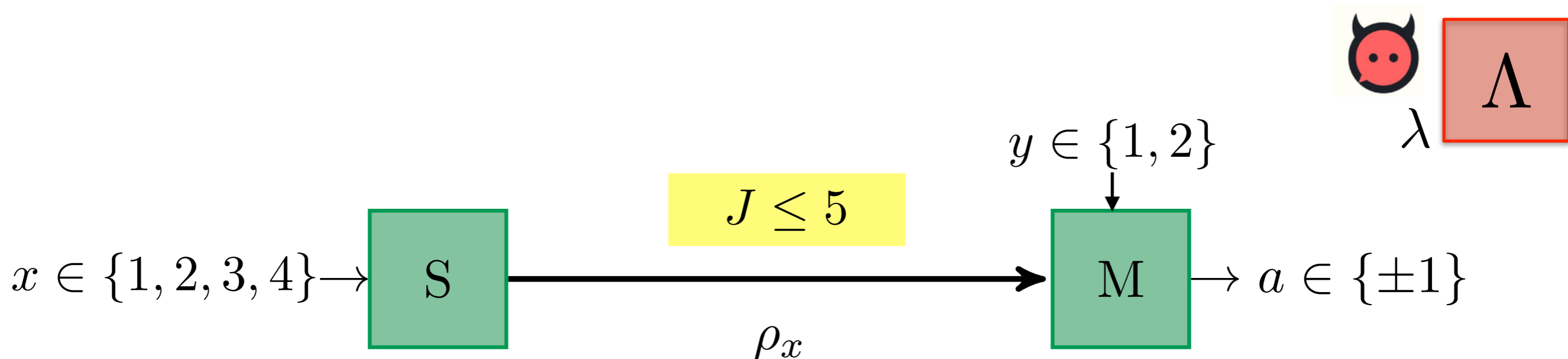
Physical motivation?

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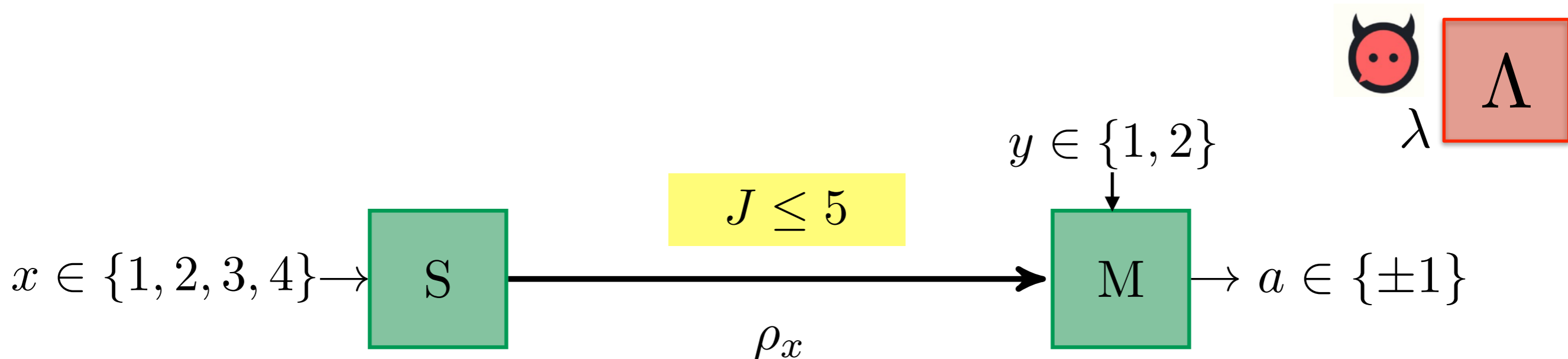
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Idea: For SDI protocols, replace dimension bounds by **physically better motivated assumptions** on how systems respond to symmetries.



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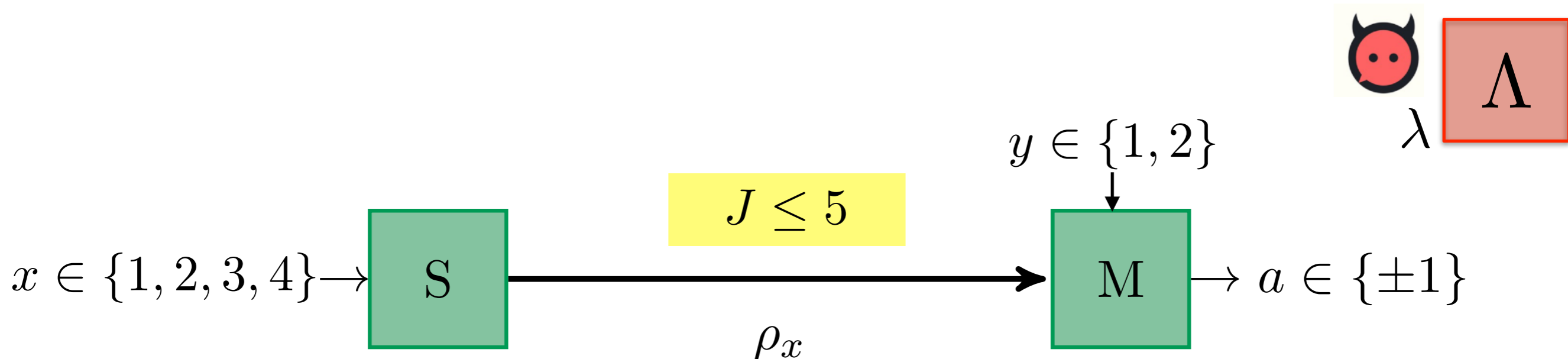
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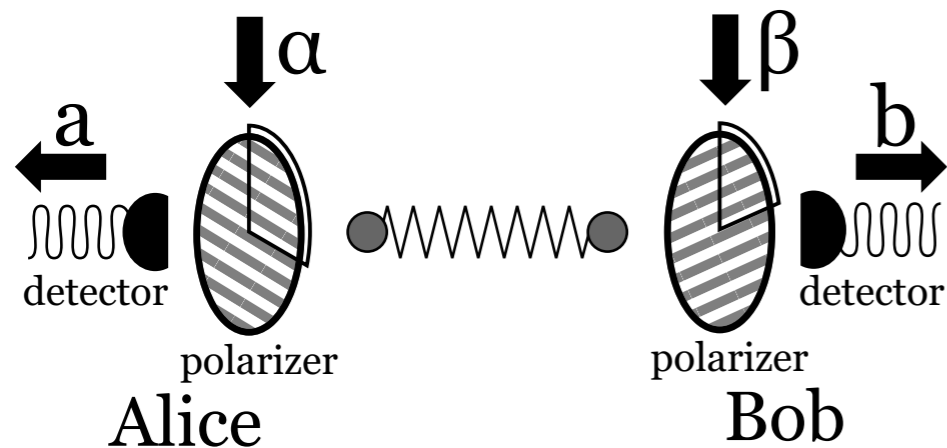
Also, closer to **particle physics intuition**: don't count dimensions, but representation labels (of the Poincaré group).

Proof of principle for Bell witnesses

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

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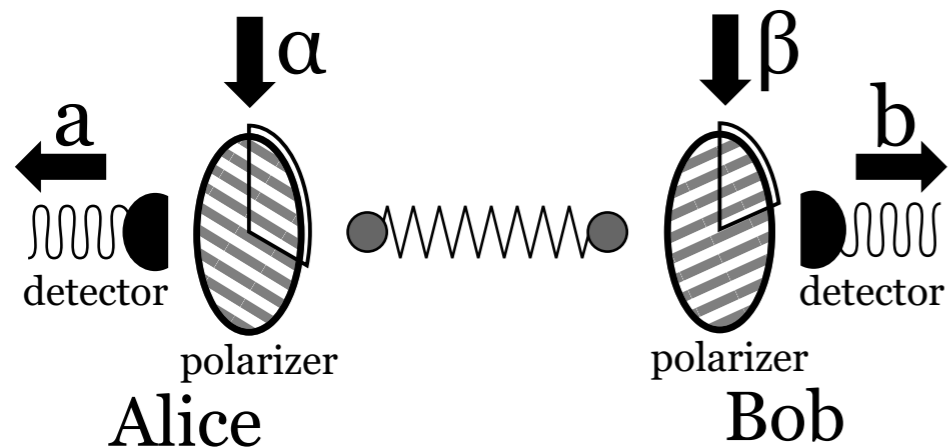
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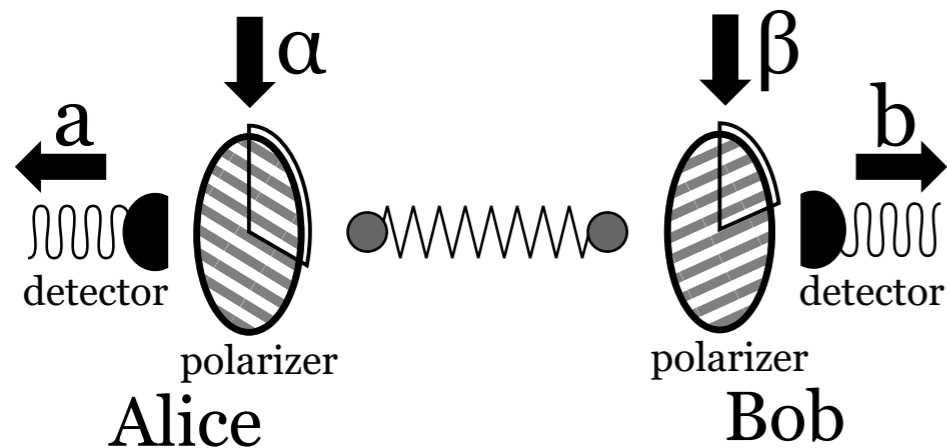


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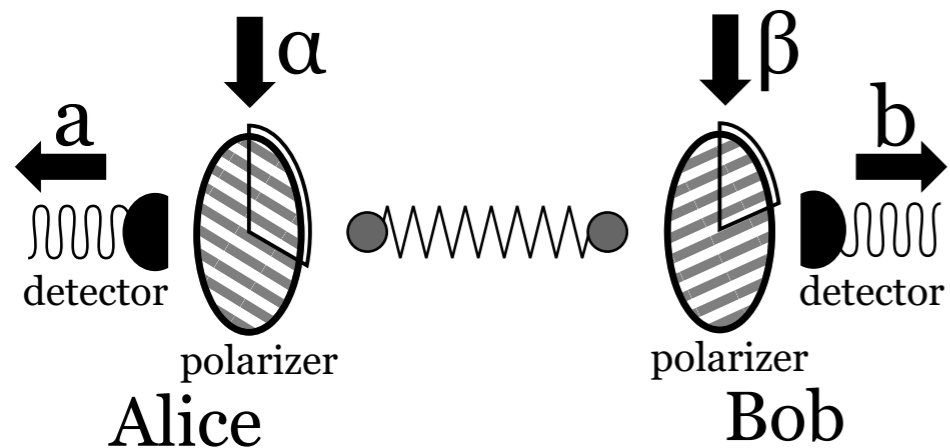
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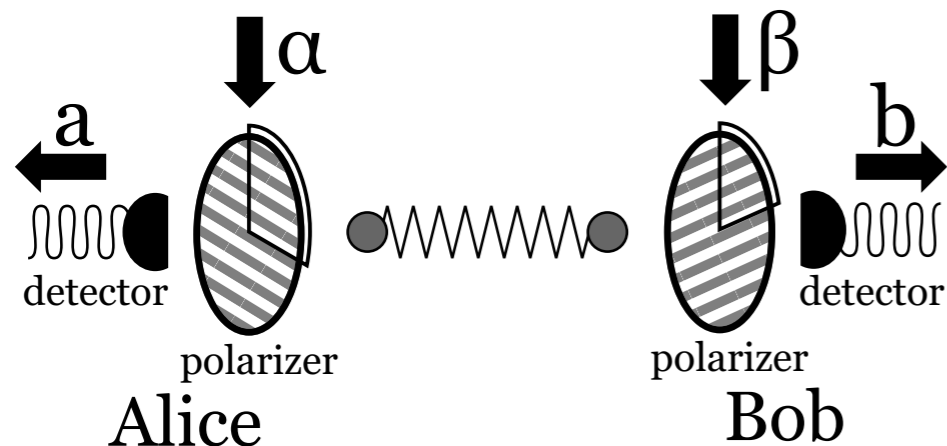
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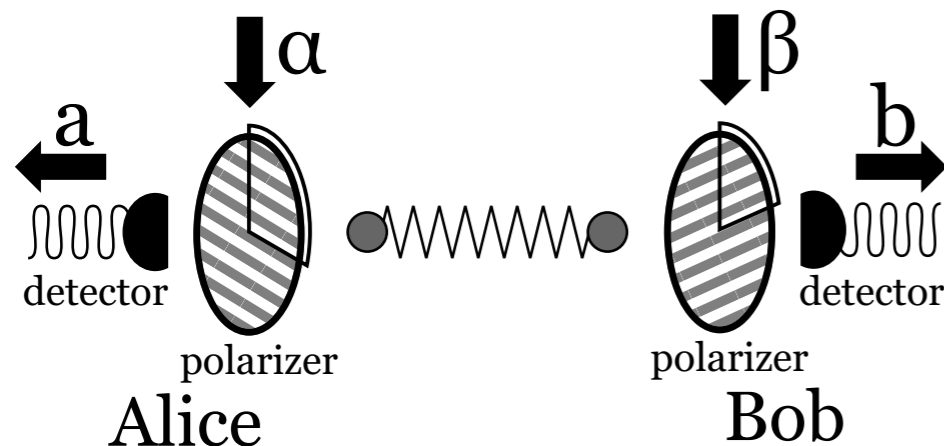
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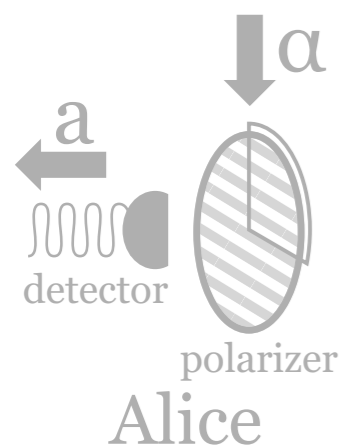
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**Completely useless,
don't try this at home!
But: proof of principle.**

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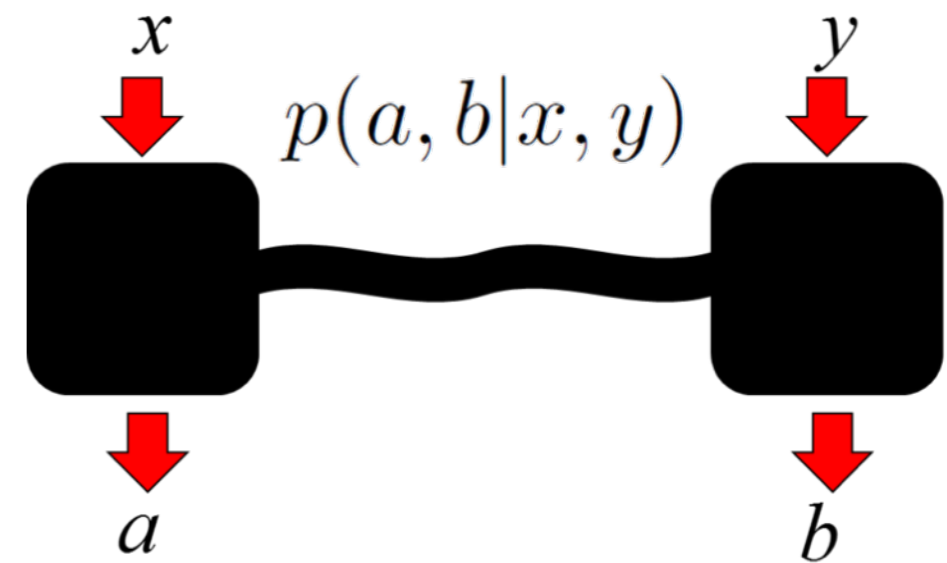
Overview

1. General framework of “spacetime boxes”

2. Foundational insights

3. Towards novel protocols...

4. ... and experimental tests of QT



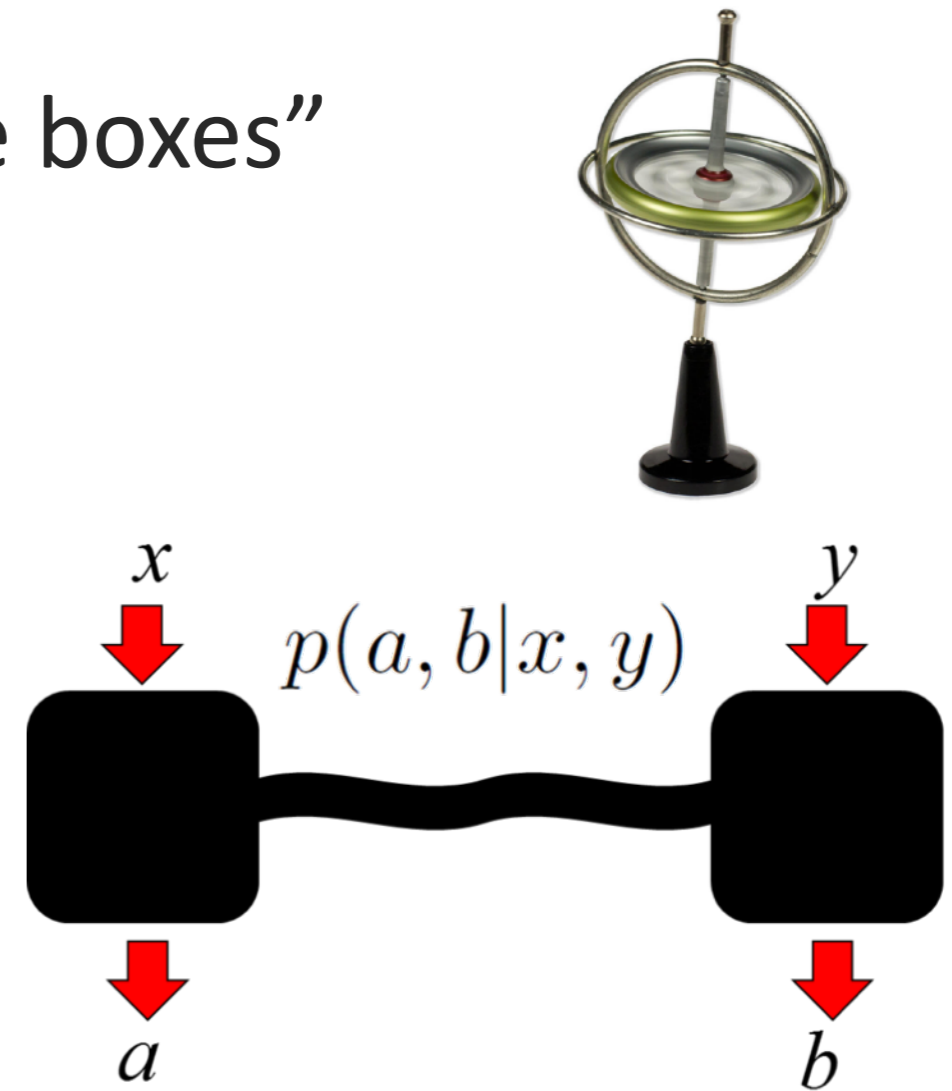
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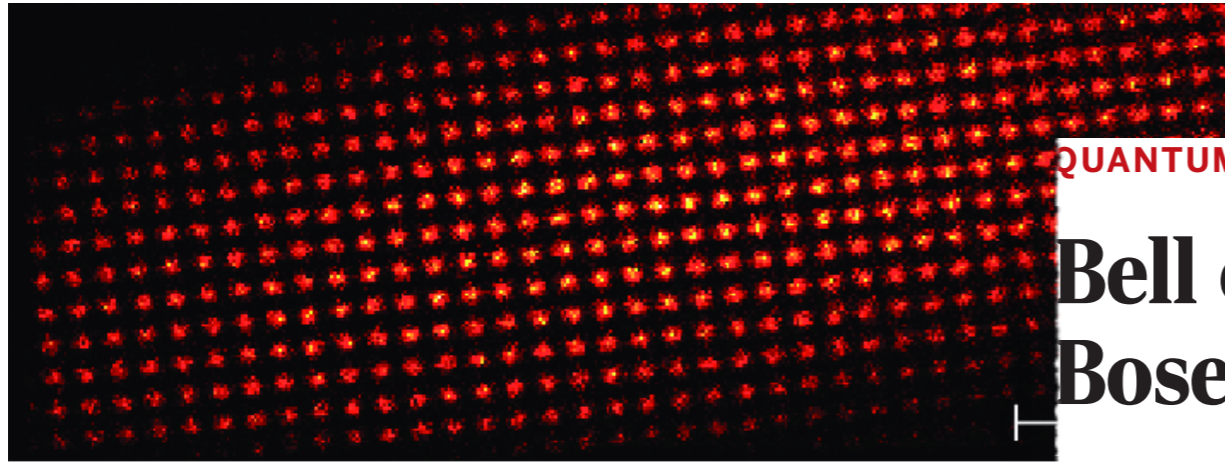
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Experiments as “black boxes”

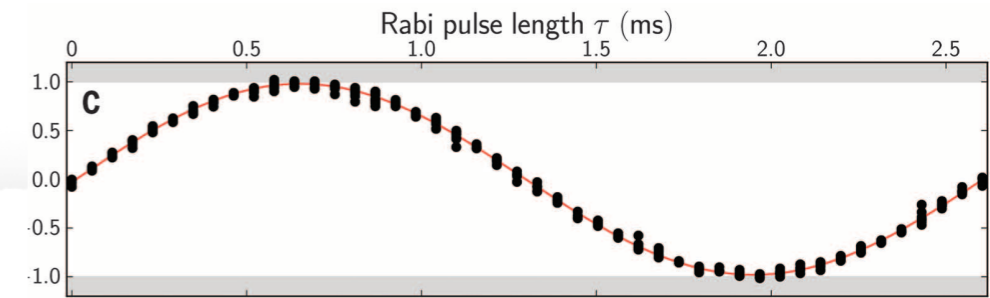


QUANTUM OPTICS

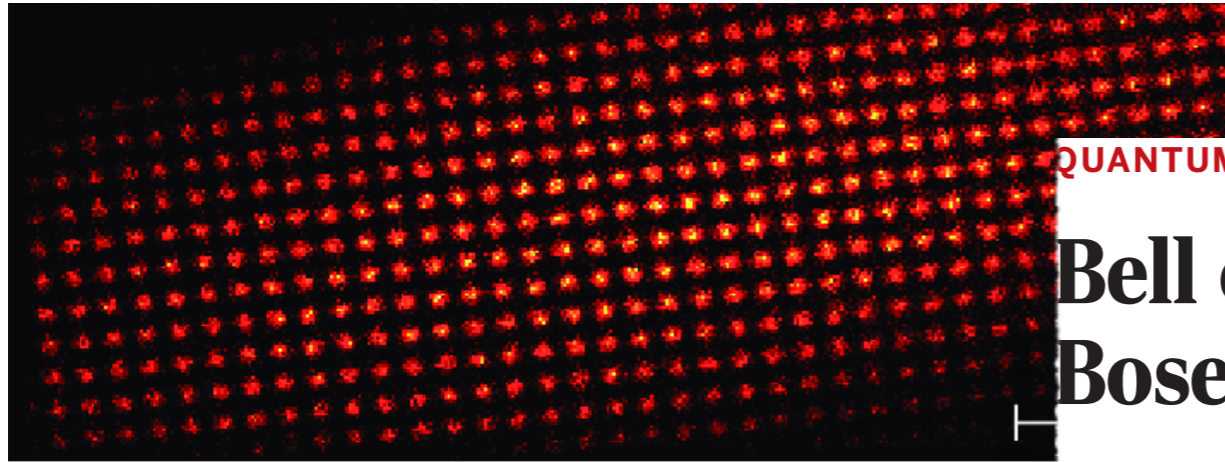
Bell correlations in a Bose-Einstein condensate

Roman Schmied,^{1*} Jean-Daniel Bancal,^{2,4*} Baptiste Allard,^{1*} Matteo Fadel,¹
Valerio Scarani,^{2,3} Philipp Treutlein,^{1†} Nicolas Sangouard^{4†}

Characterizing many-body systems through the quantum correlations between their constituent particles is a major goal of quantum physics. Although entanglement is routinely observed in many systems, we report here the detection of stronger correlations—Bell correlations—between the spins of about 480 atoms in a Bose-Einstein condensate. We derive a Bell correlation witness from a many-particle Bell inequality involving only one- and two-body correlation functions. Our measurement on a spin-squeezed state exceeds the threshold for Bell correlations by 3.8 standard deviations. Our work shows that the strongest possible nonclassical correlations are experimentally accessible in many-body systems and that they can be revealed by collective measurements.



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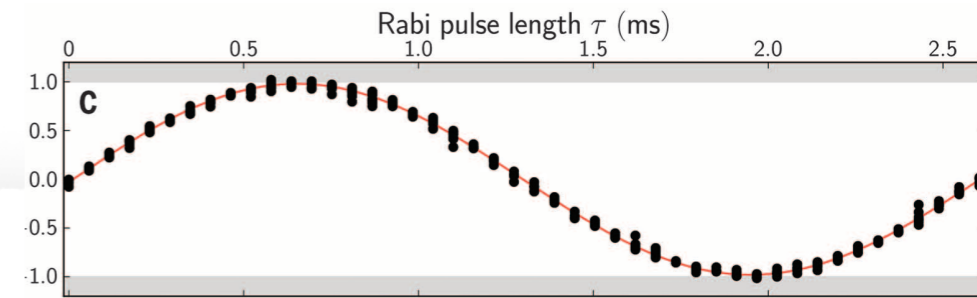


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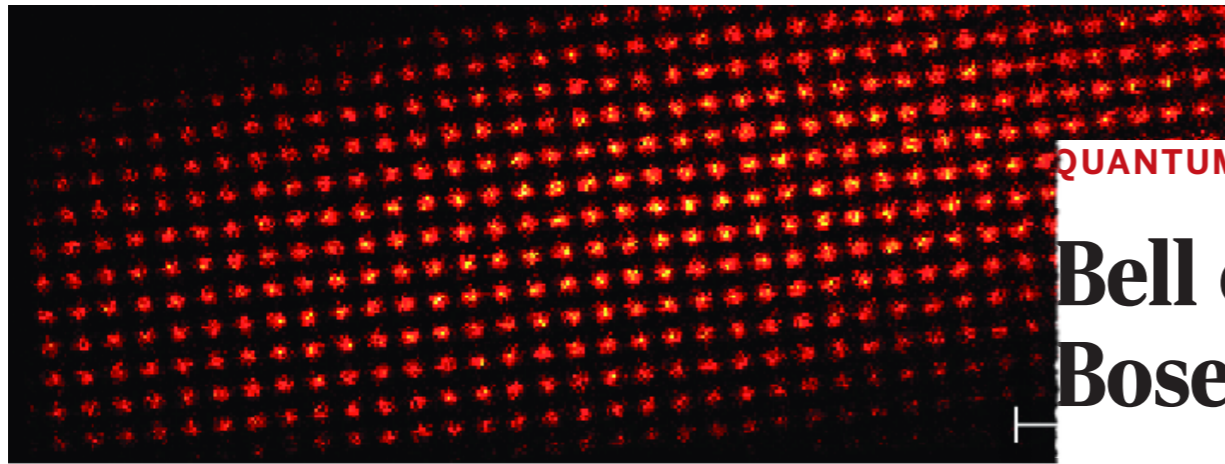
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What can we infer **from this alone?**
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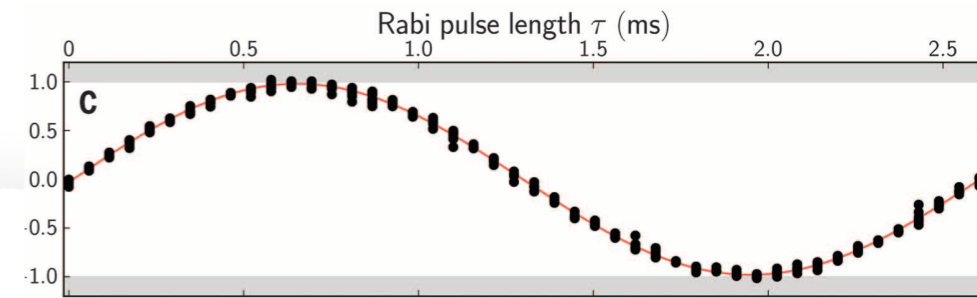


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Under what conditions could the result **falsify Quantum Theory?**

Conclusions

- “**Spacetime boxes**” via group representation theory.
- Foundational insights: study of **interplay probability vs. spacetime**, exact characterization of the **quantum (2,2,2)-correlations**.
- Towards protocols: **bounding representation labels** as a physically well-motivated assumption. “Proof of principle” nonlocality certification.
- Novel experimental tests of QT?

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