Thermodynamics as a resource theory: versions of the second law(s)

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Outline

1. Resource-theoretic approach to thermodynamics

2. Single-shot interpretation of von Neumann entropy and free energy (block-diagonal states)

3. Beyond block-diagonal states: on coherence, clocks, and timing information

4. Conclusions

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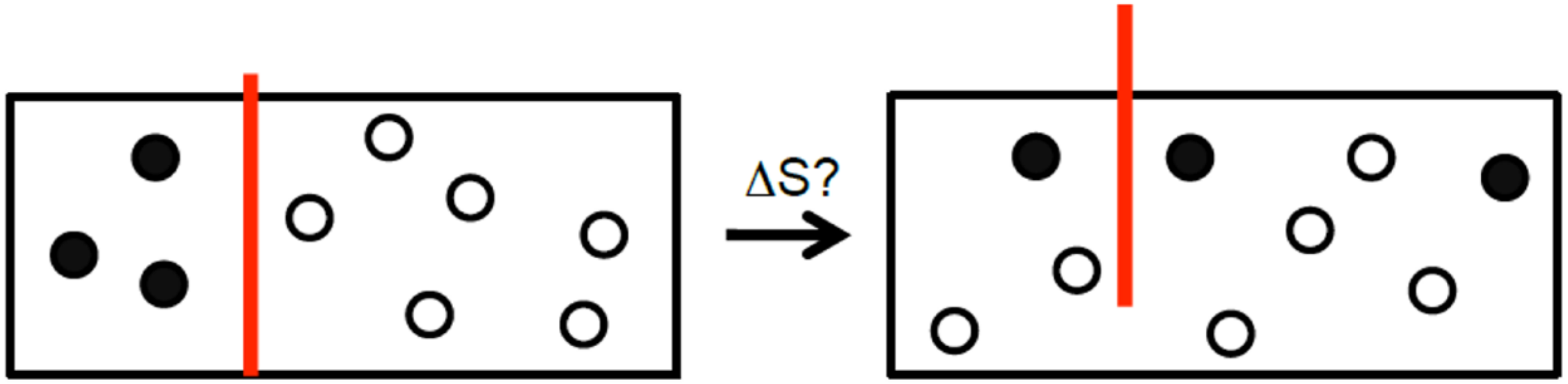
Standard view: thermodynamic limit



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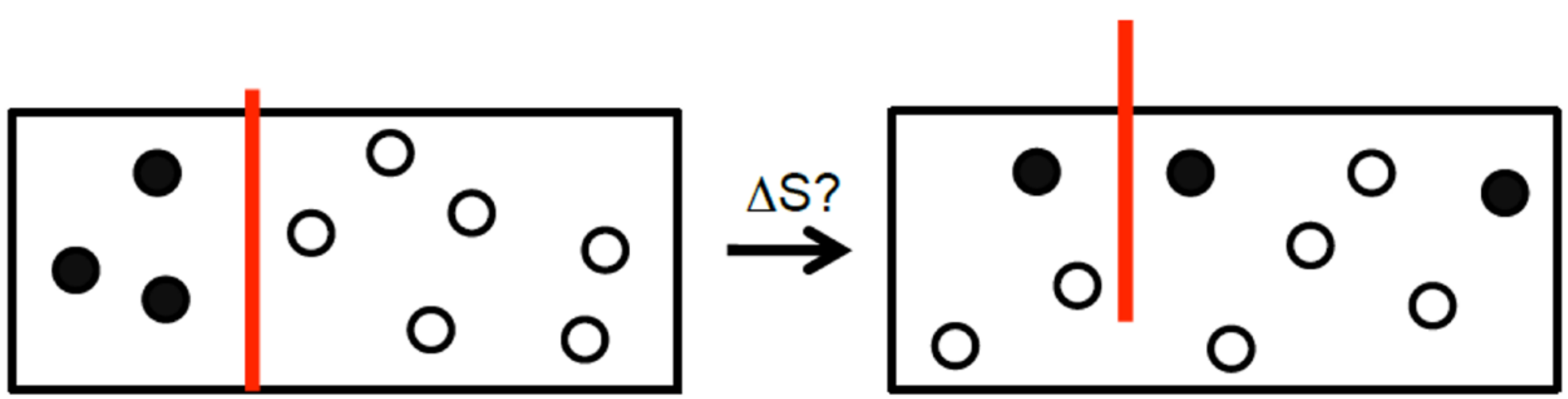

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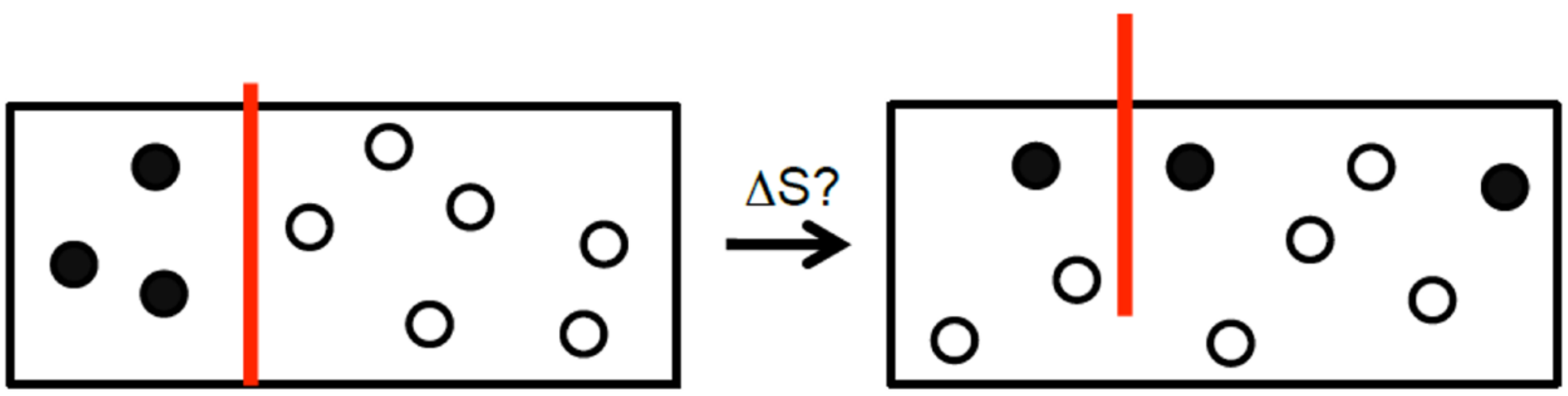
$$\Delta F \leq 0 \qquad \text{(2nd law)},$$

where $F = U - TS$.
If this is negative, then we can extract $|\Delta F|$ of work from the system.



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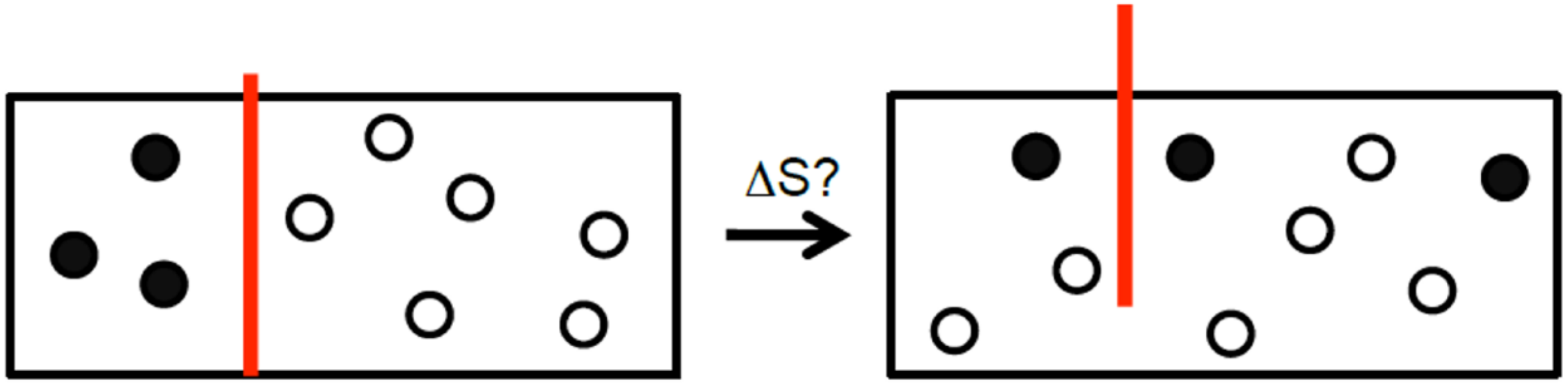
But this is a statement **on average**, since "work" is a random variable.



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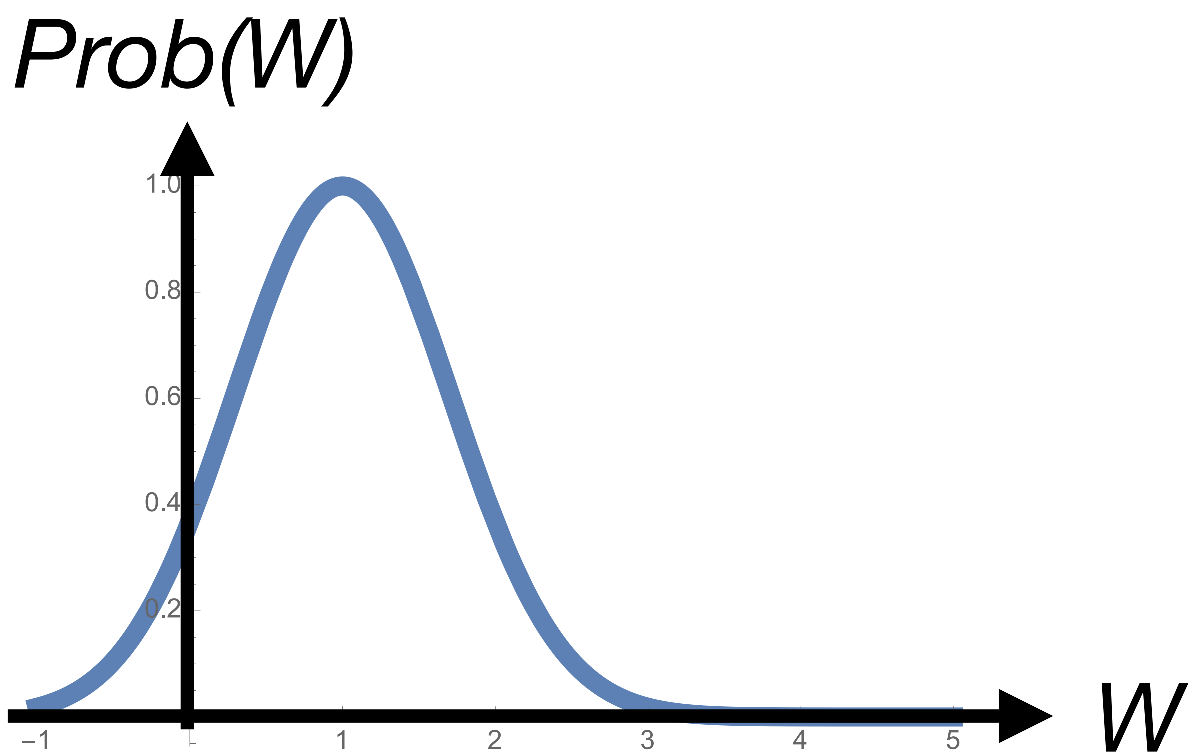

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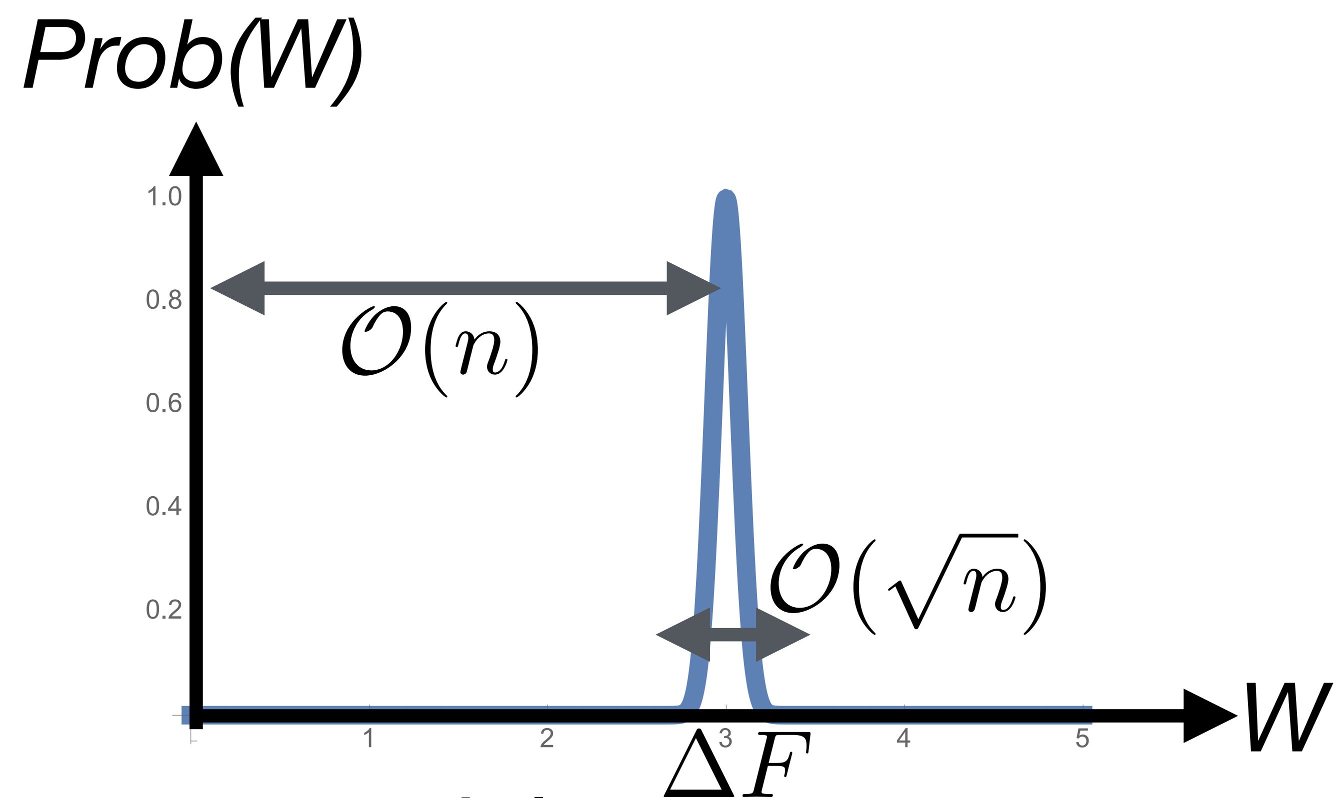

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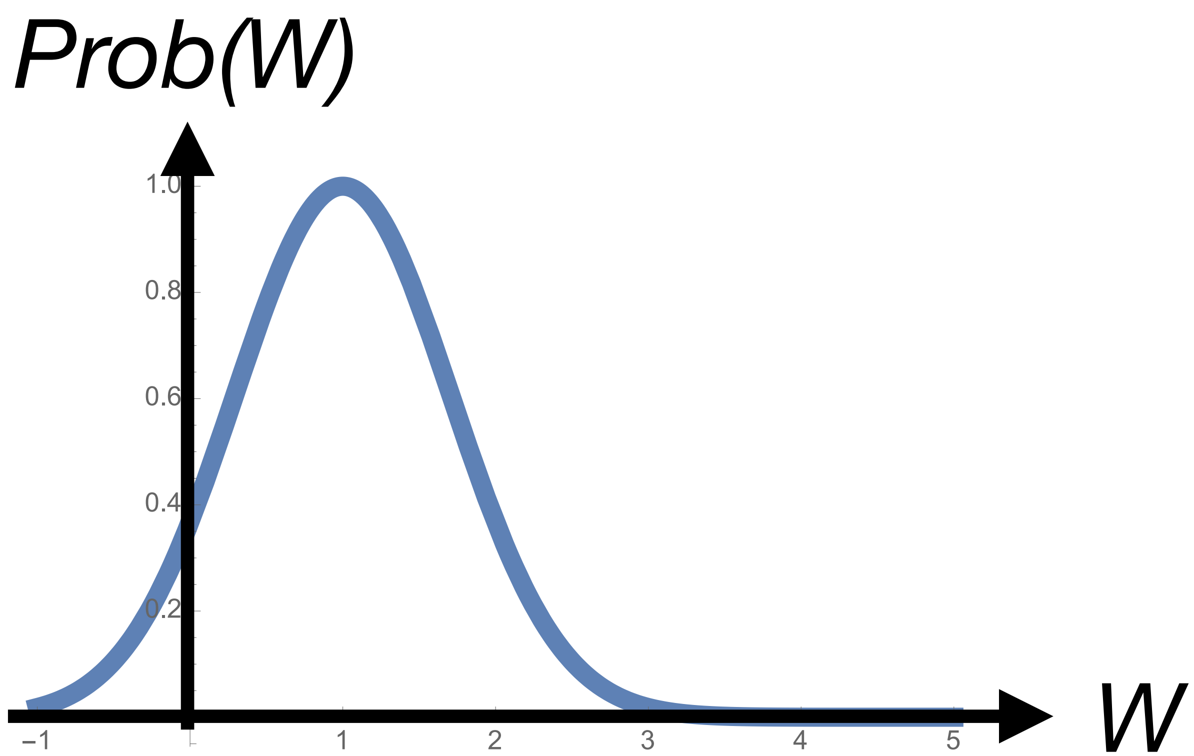

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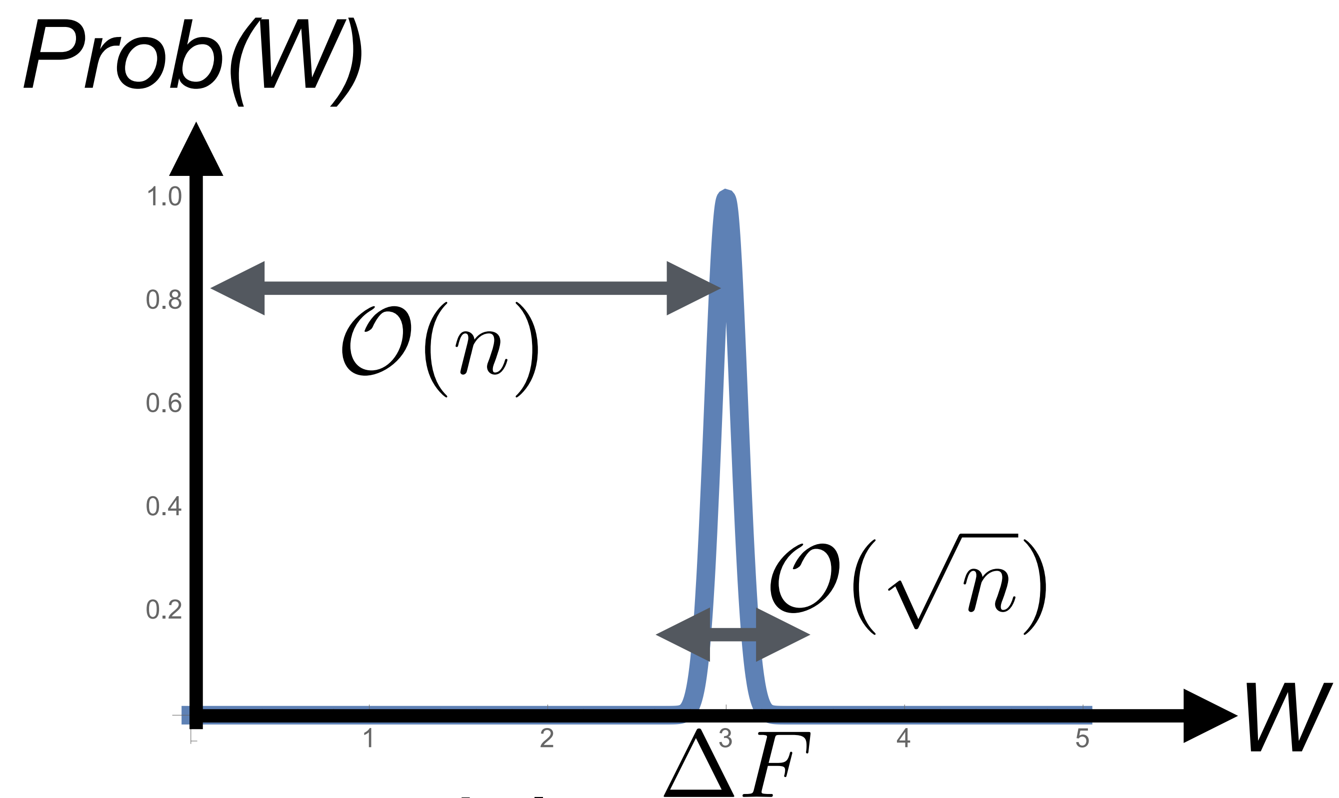

n particles

Extractable work “is” (optimally) ΔF :
only true in the thermodynamic limit $n \to \infty$
when fluctuations become irrelevant (law of large numbers).

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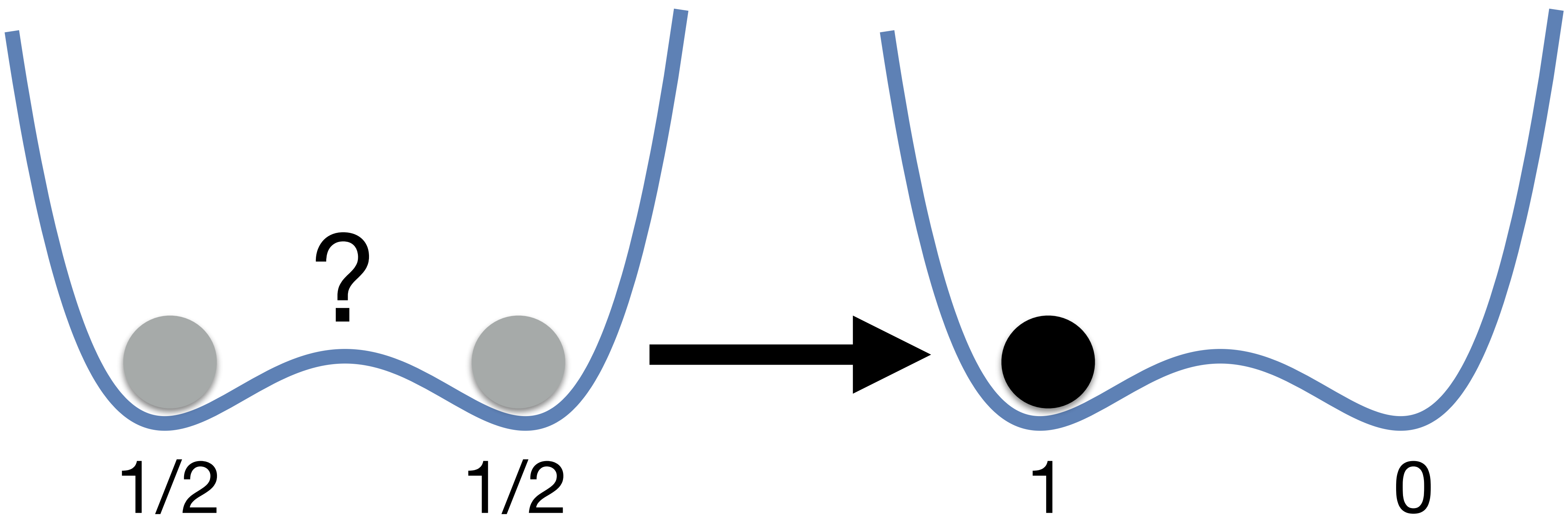

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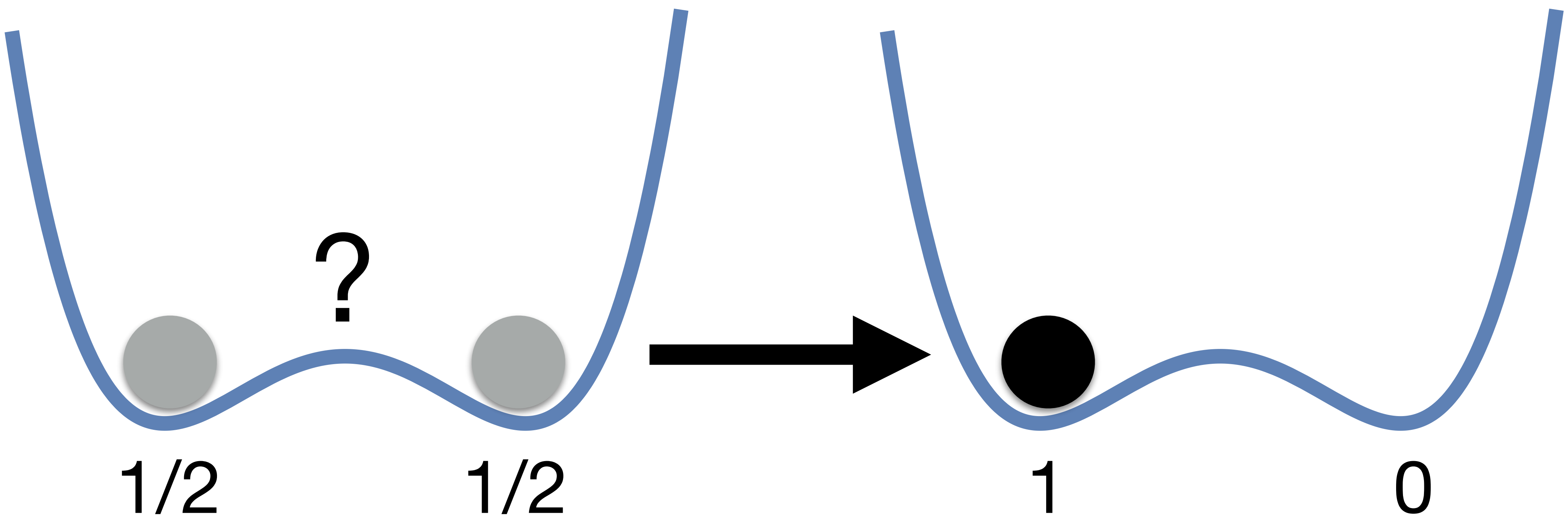

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But: Bennett's puzzle: $$\left(\frac{1}{2},\frac{1}{2},0,\ldots,0\right) \longrightarrow \left(1-\epsilon,\frac{\epsilon}{N},\frac{\epsilon}{N},\ldots,\frac{\epsilon}{N}\right)$$

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Landauer

e

Free energy *F* determines possibility of state transitions **only in the thermodynamic limit**.
For single systems, resource theory formulation gives **additional constraints** (and solves Bennett’s puzzle). More soon.

In

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Thermodynamics as a resource theory

Incomplete list of key references:

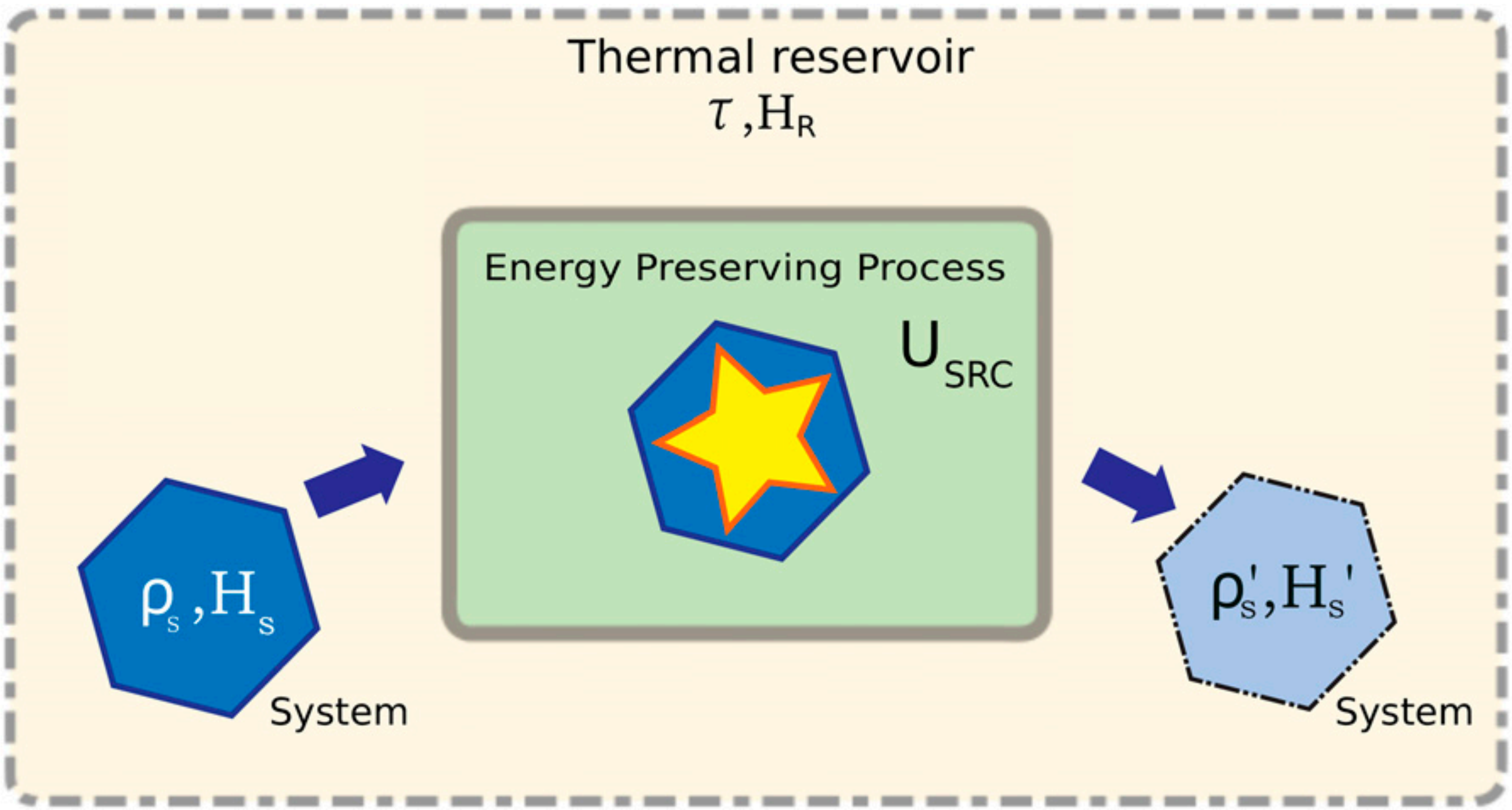
M. Horodecki and J. Oppenheim, *Fundamental limitations for quantum and nanoscale thermodynamics*, Nat. Commun. **4**, 2059 (2013).

F. Brandão, M. Horodecki, J. Oppenheim, J. M. Renes, and R. W. Spekkens, *Resource Theory of Quantum States Out of Thermal Equilibrium*, Phys. Rev. Lett. **111**, 250404 (2013).

F. Brandão, M. Horodecki, N. Ng, J. Oppenheim, and S. Wehner, *The second laws of quantum thermodynamics*, PNAS **112**, 3275 (2015).

The rules of the game:

- It is "free" to bring in any "bath" *B* in its thermal state $\gamma_B = \exp(-H_B/(k_B T))$,
- strictly energy-preserving unitaries are free,
- and it is free to trace over (ignore) systems.



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allowing *any other* state would *trivialize* the theory.

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Question: Which transitions (work extraction etc.) are possible via thermal operations?

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Theorem (Horodecki, Oppenheim, Nat. Comm. 4 (2013)):

For **block-diagonal** states, $\rho_A \mapsto \rho'_A$ is possible via some thermal operation iff ρ_A *thermo-majorizes* ρ'_A.

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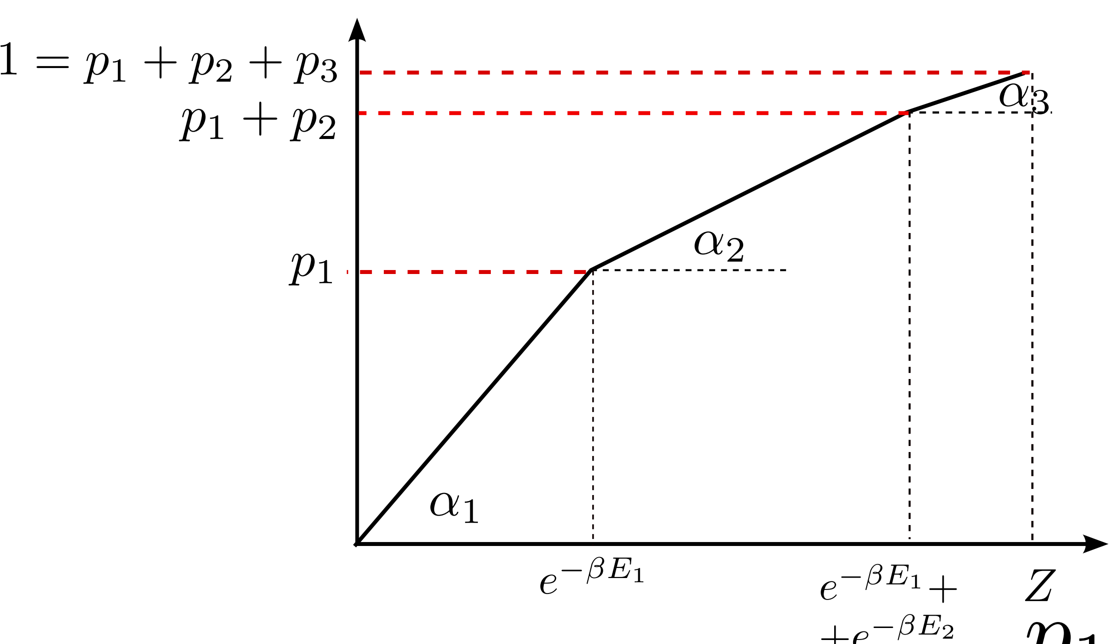
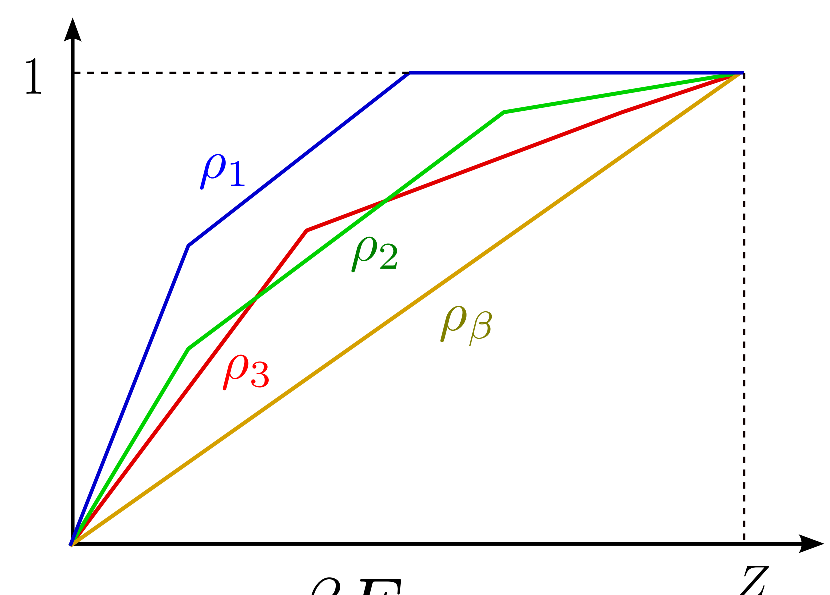
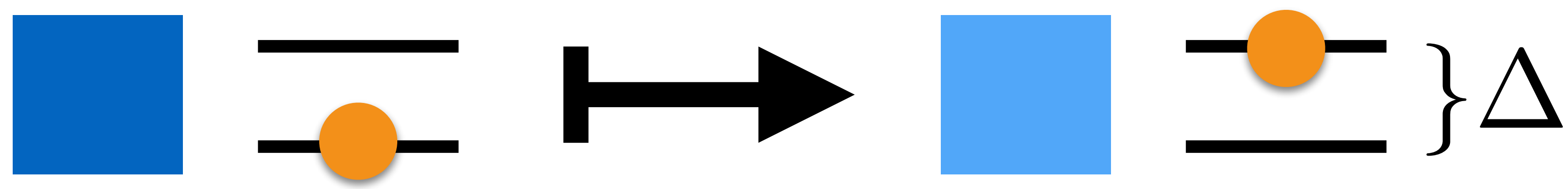


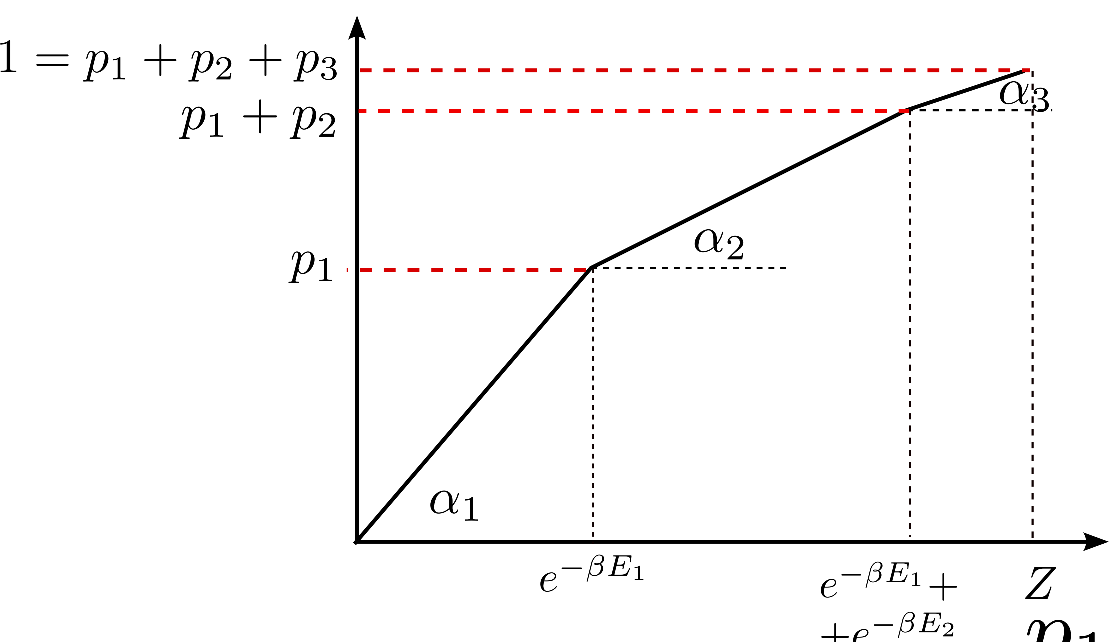


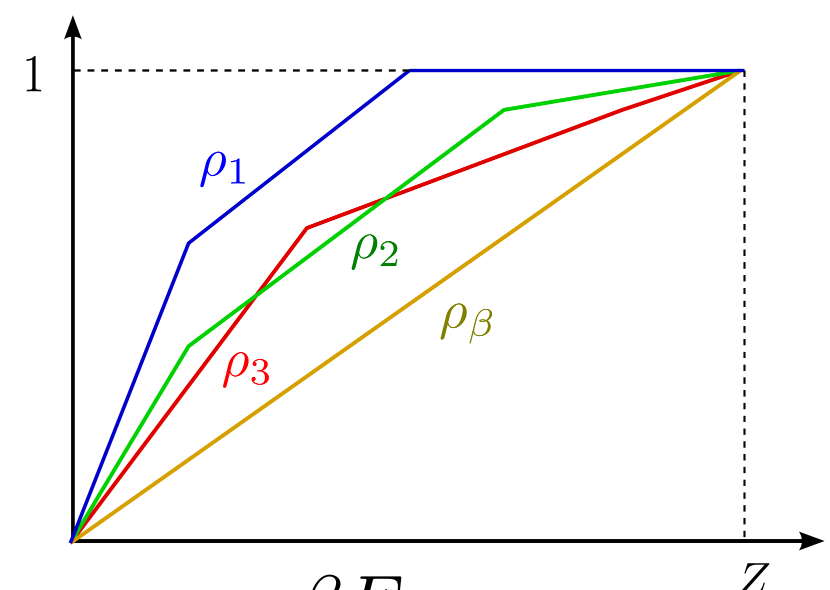
$$p_1 e^{\beta E_1} \geq p_2 e^{\beta E_2} \geq \ldots$$

Work extraction and work of formation

Work extraction: $\sigma_A \otimes |g\rangle\langle g|_W \mapsto \sigma'_A \otimes |e\rangle\langle e|_W$







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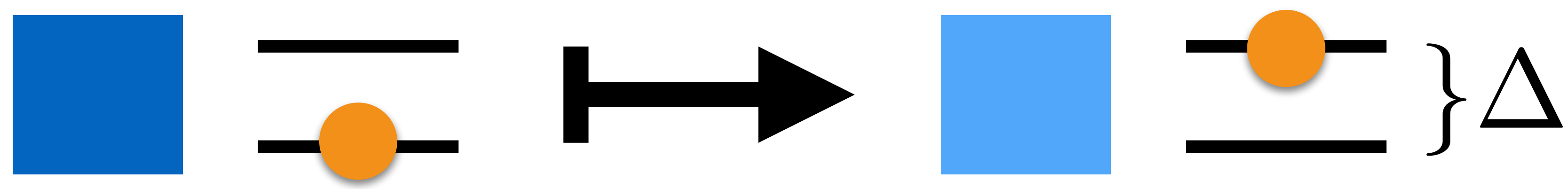

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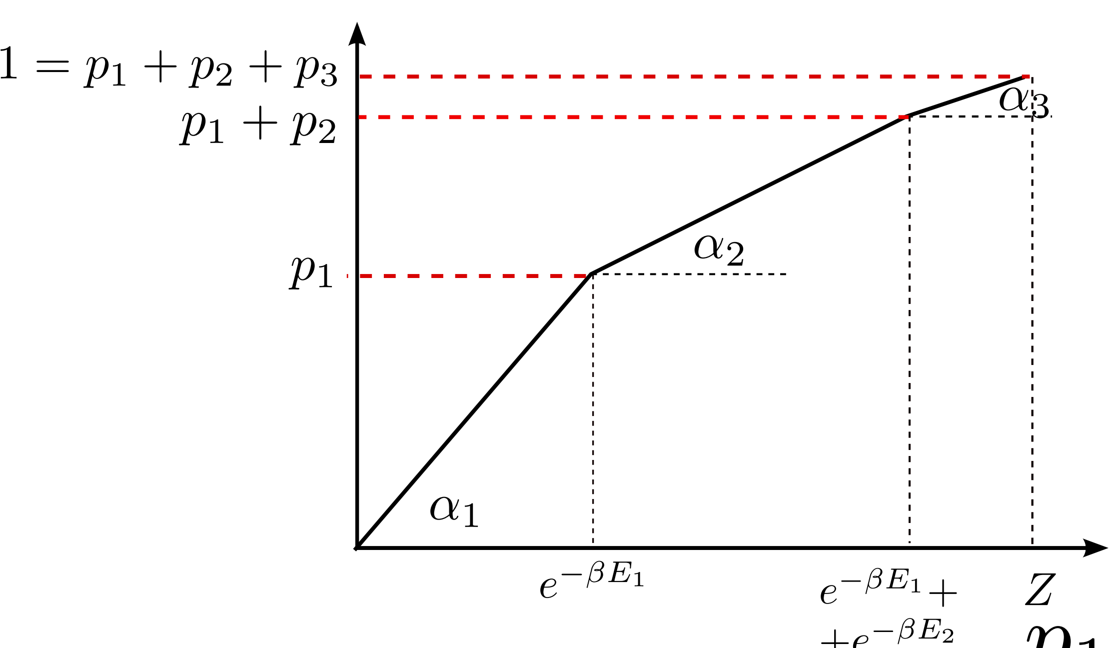

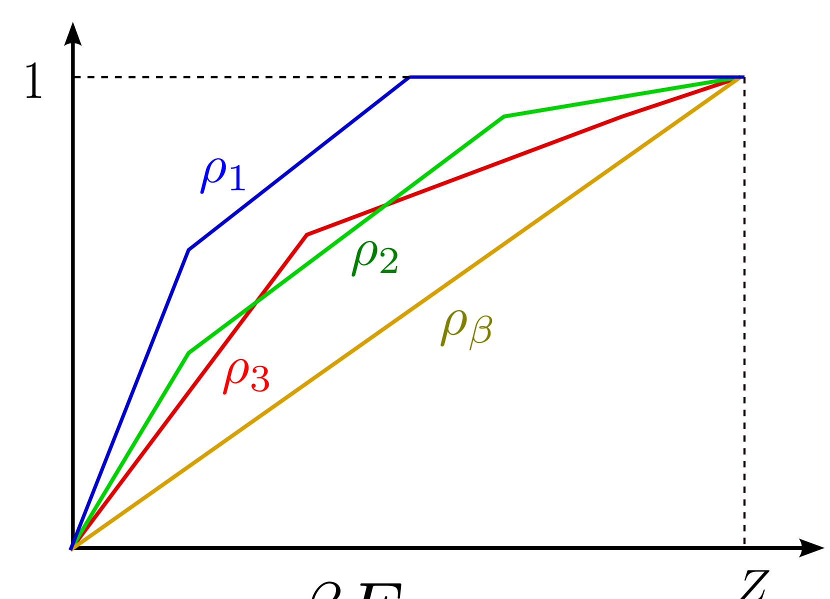

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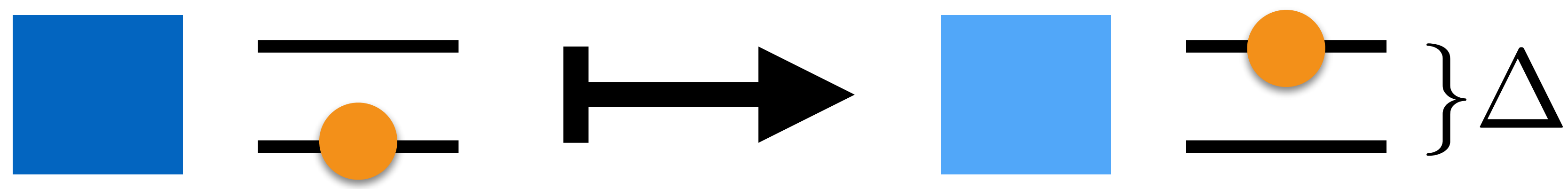

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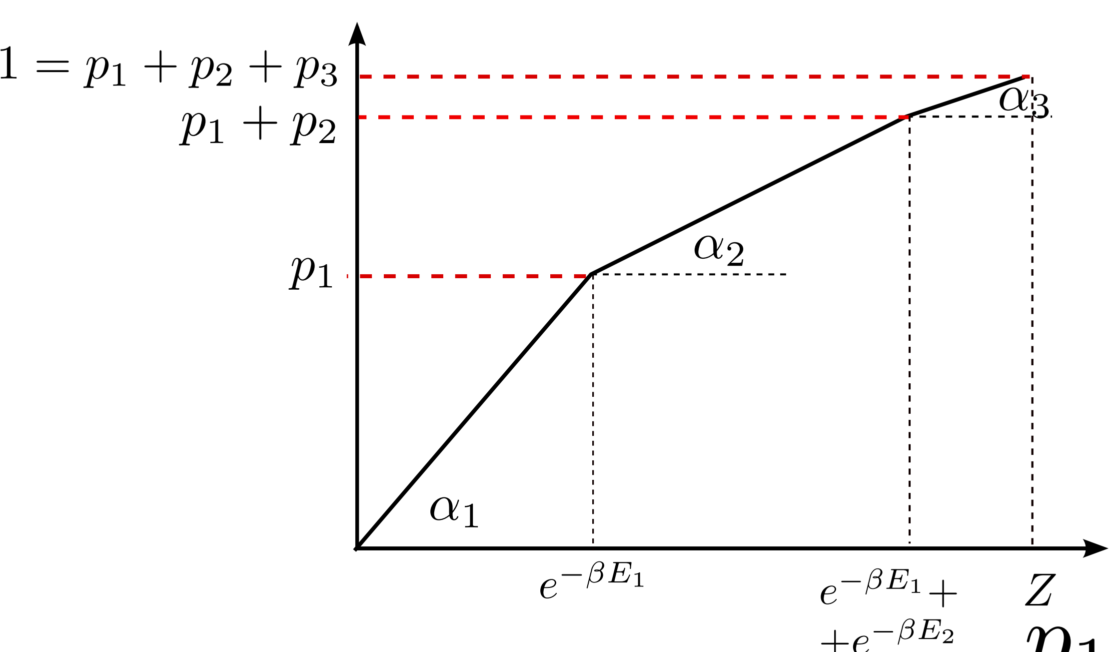
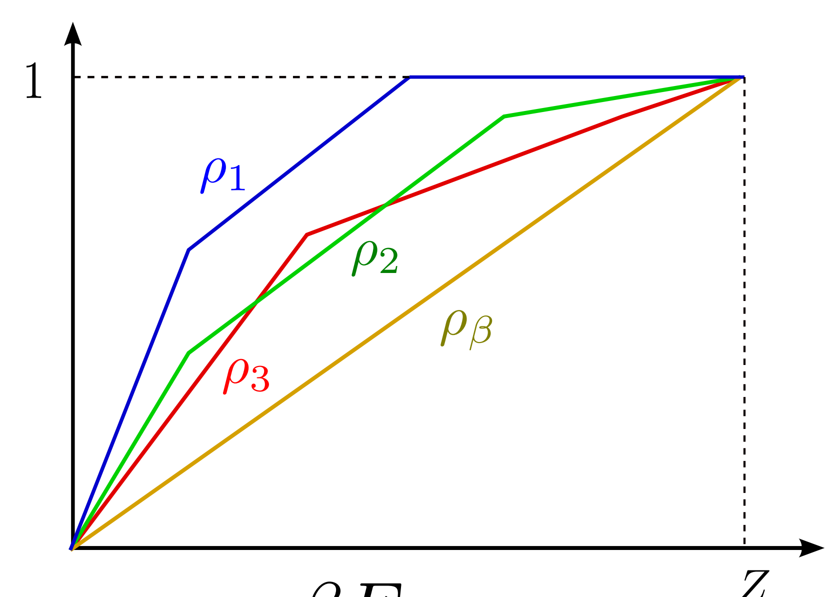
Easy to see: $\sigma'_A = \gamma_A$ (thermal state) gives largest Δ.





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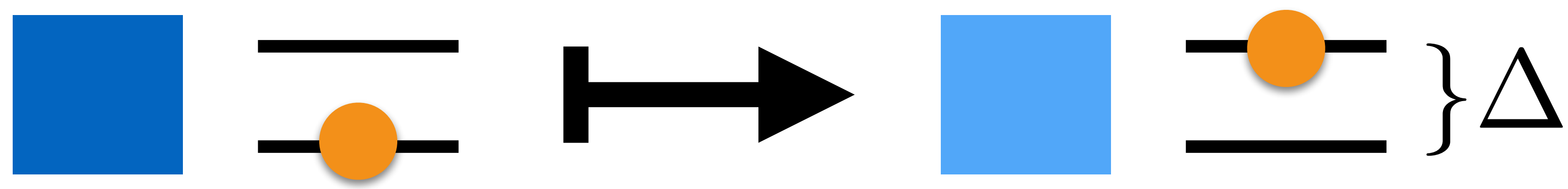

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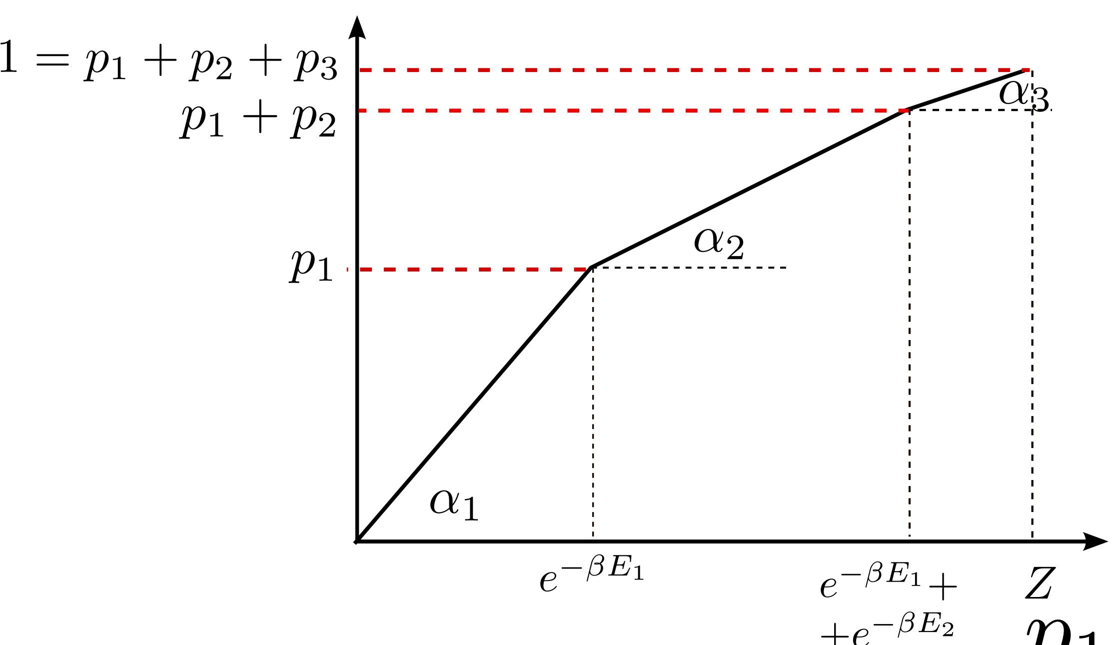

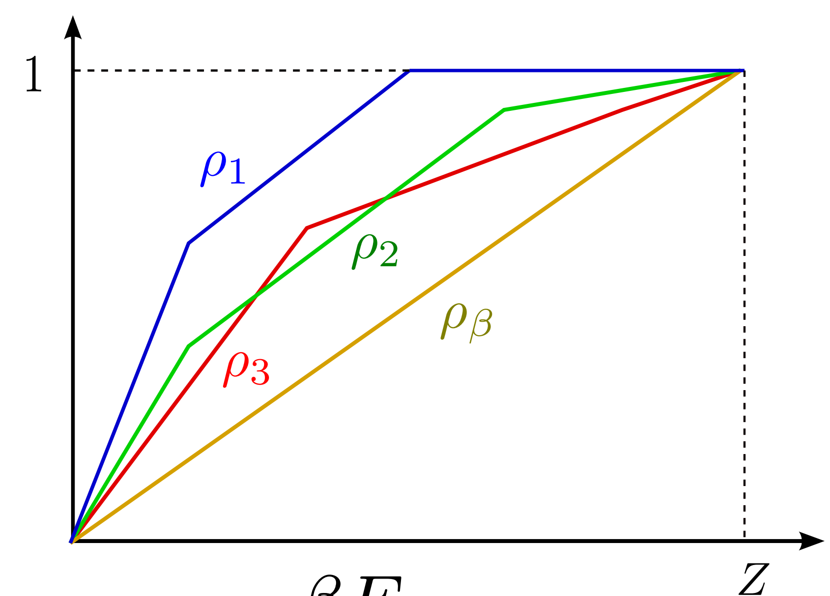

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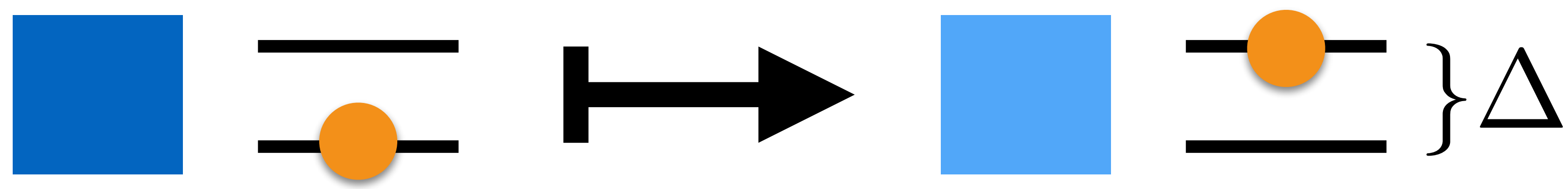

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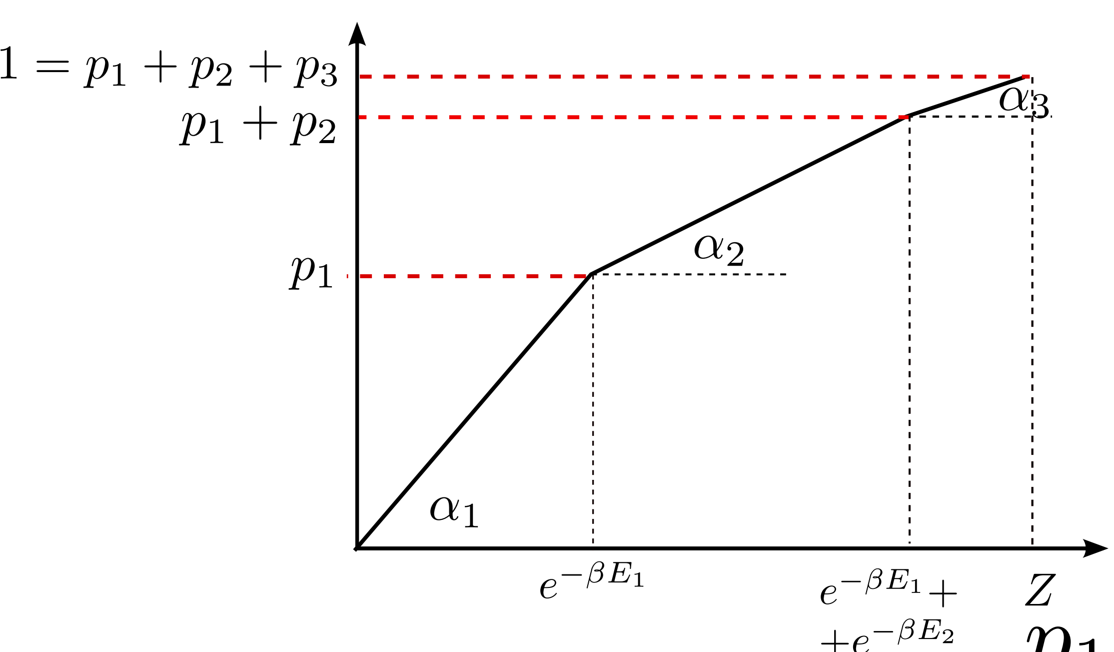

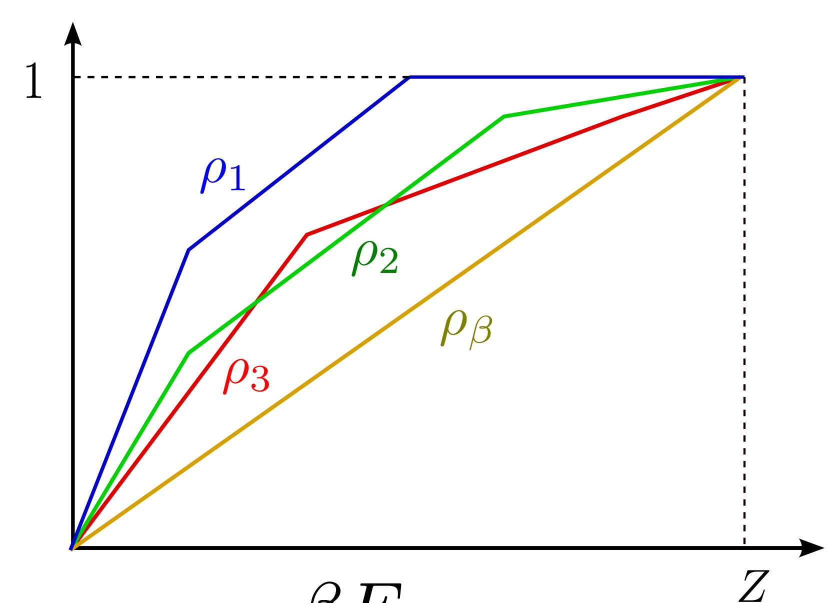

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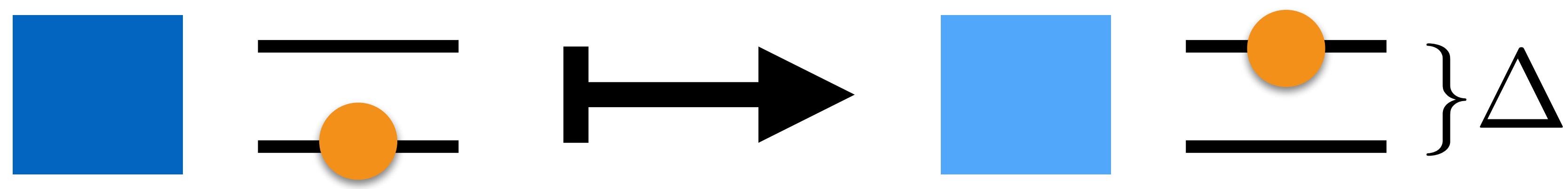

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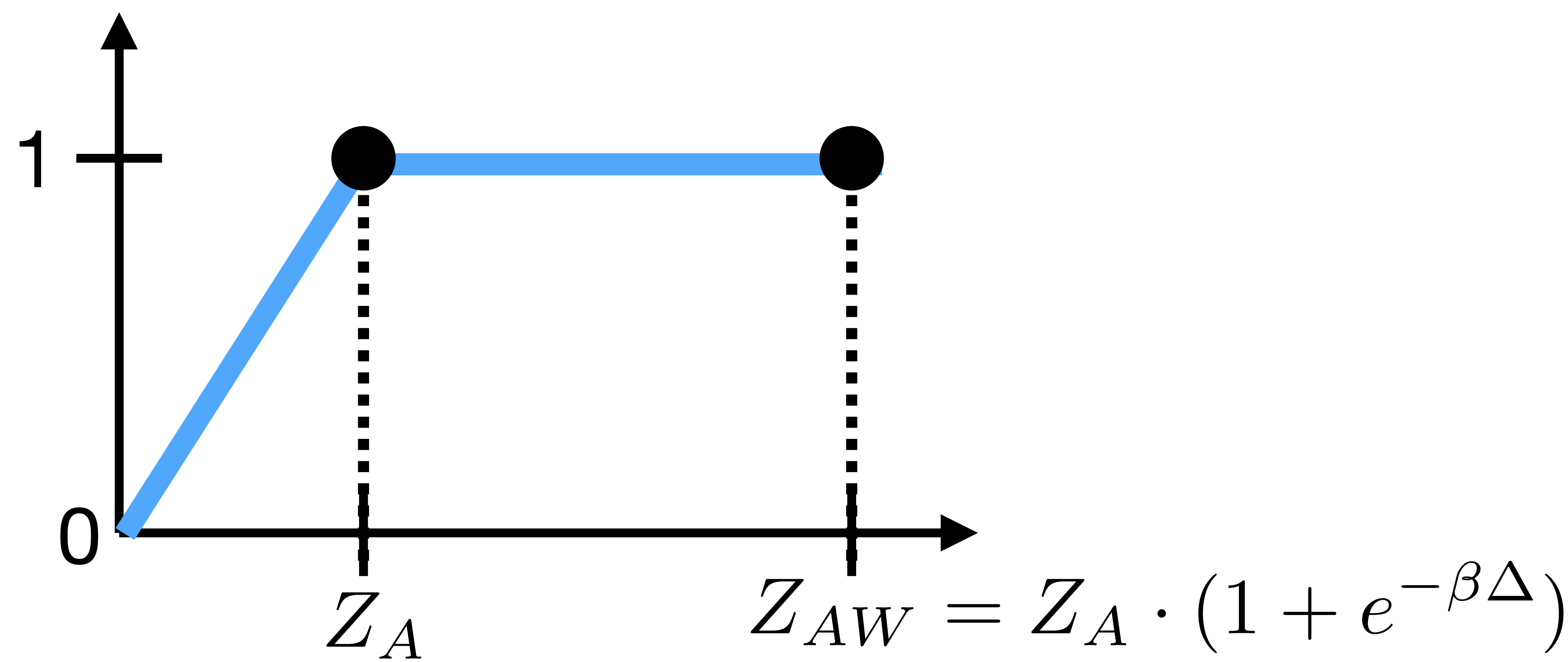

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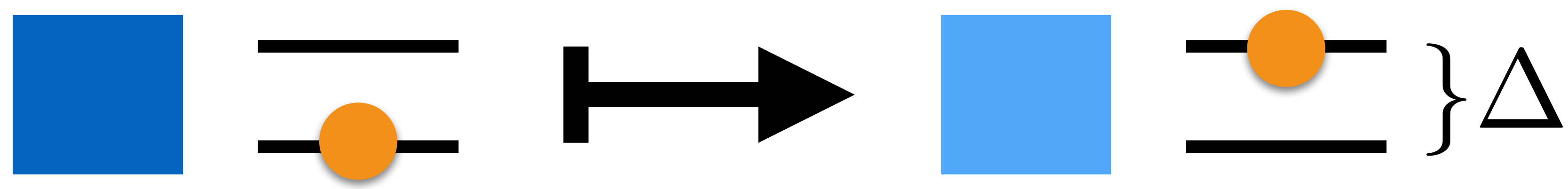

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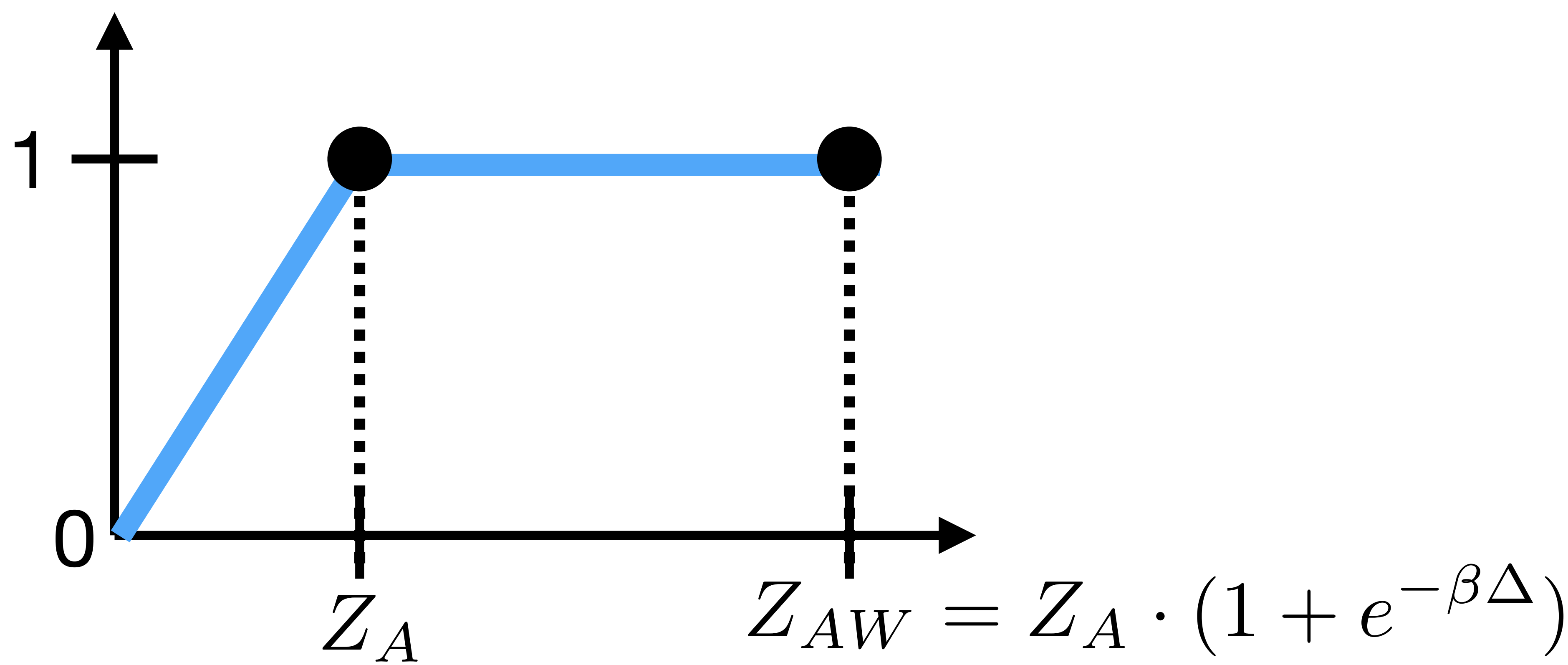

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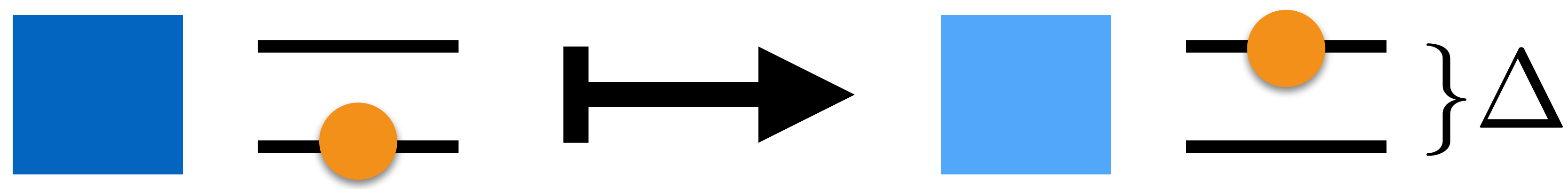

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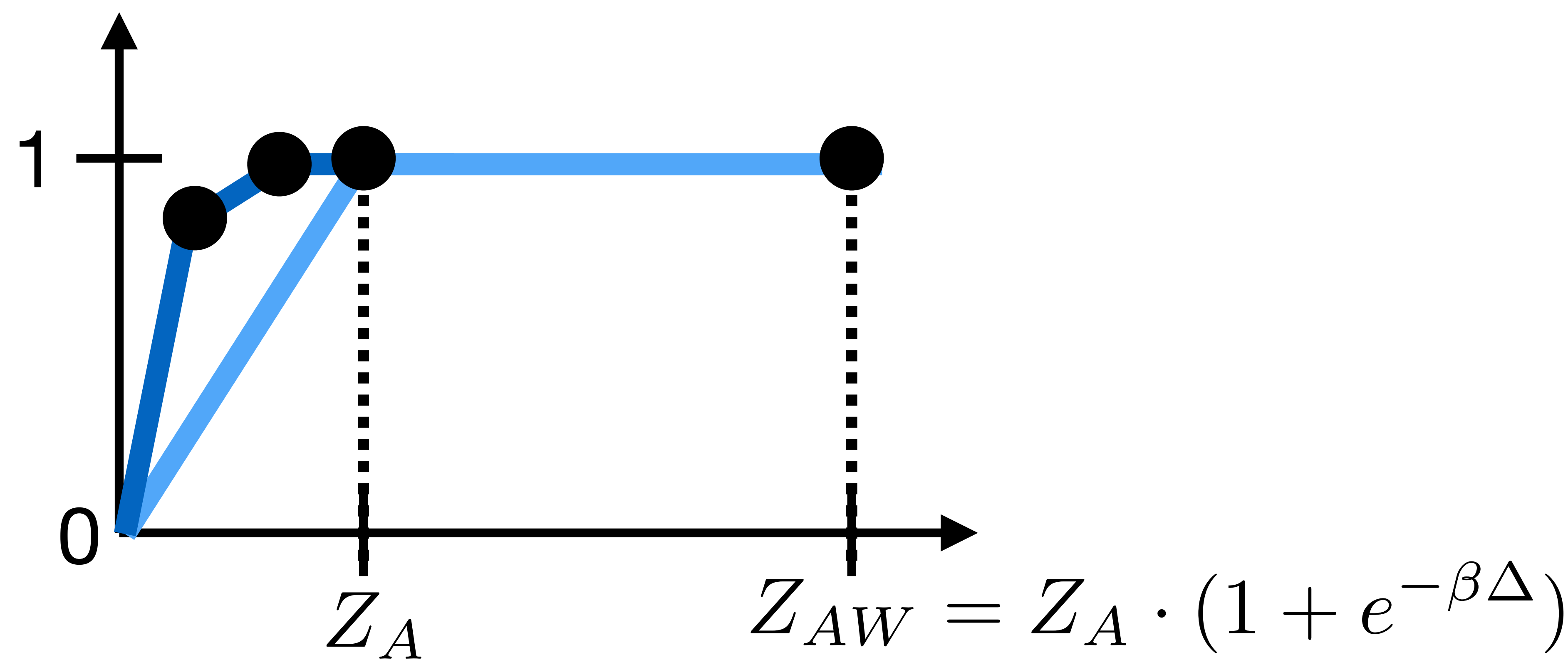

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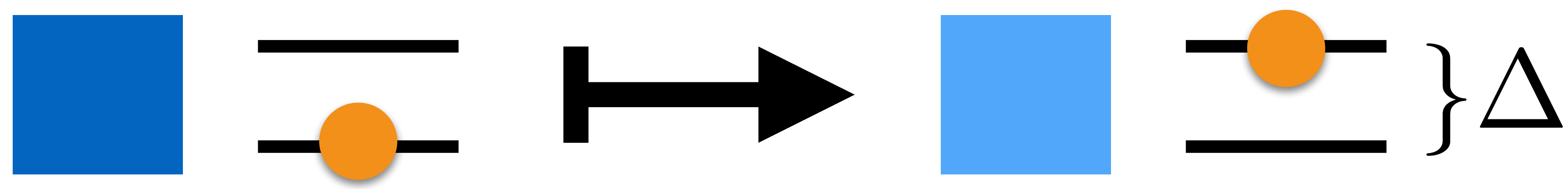

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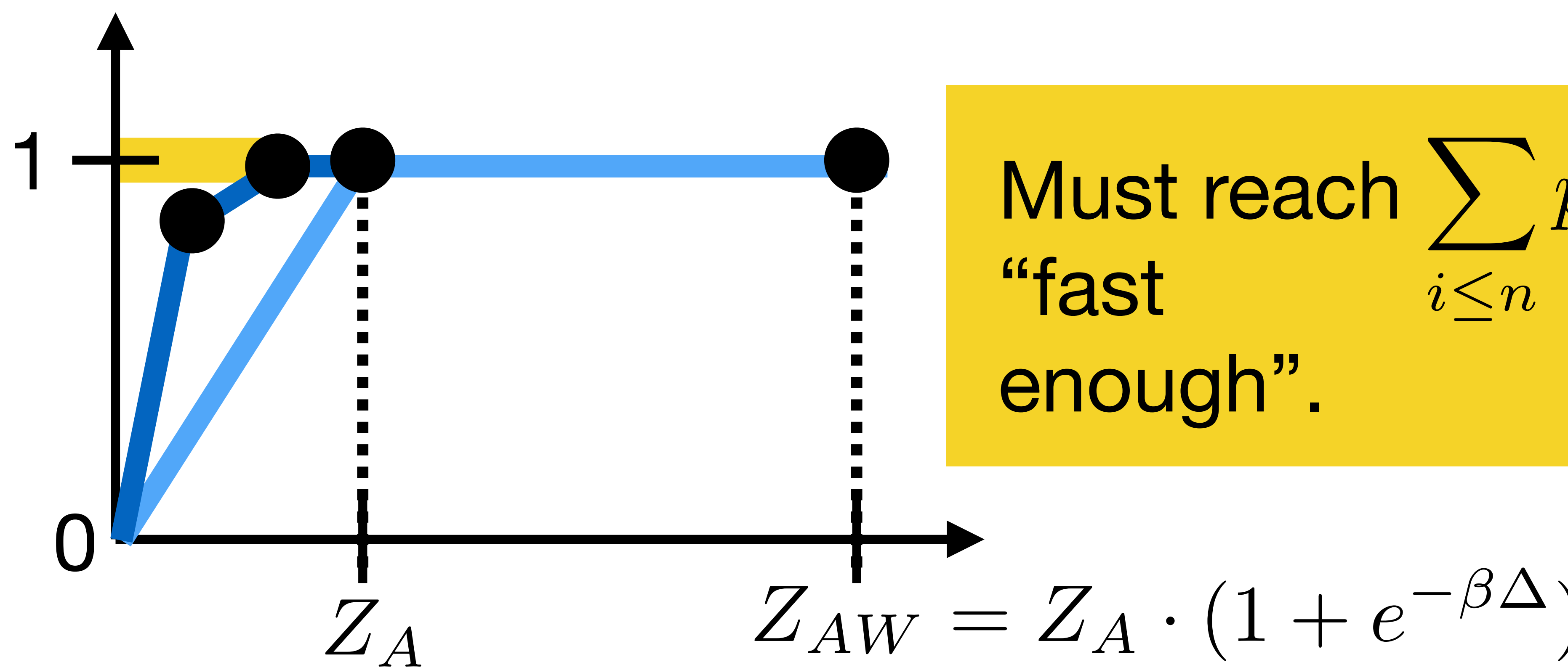

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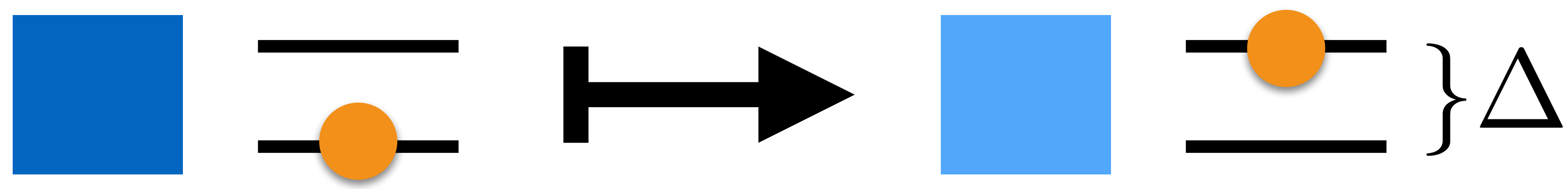

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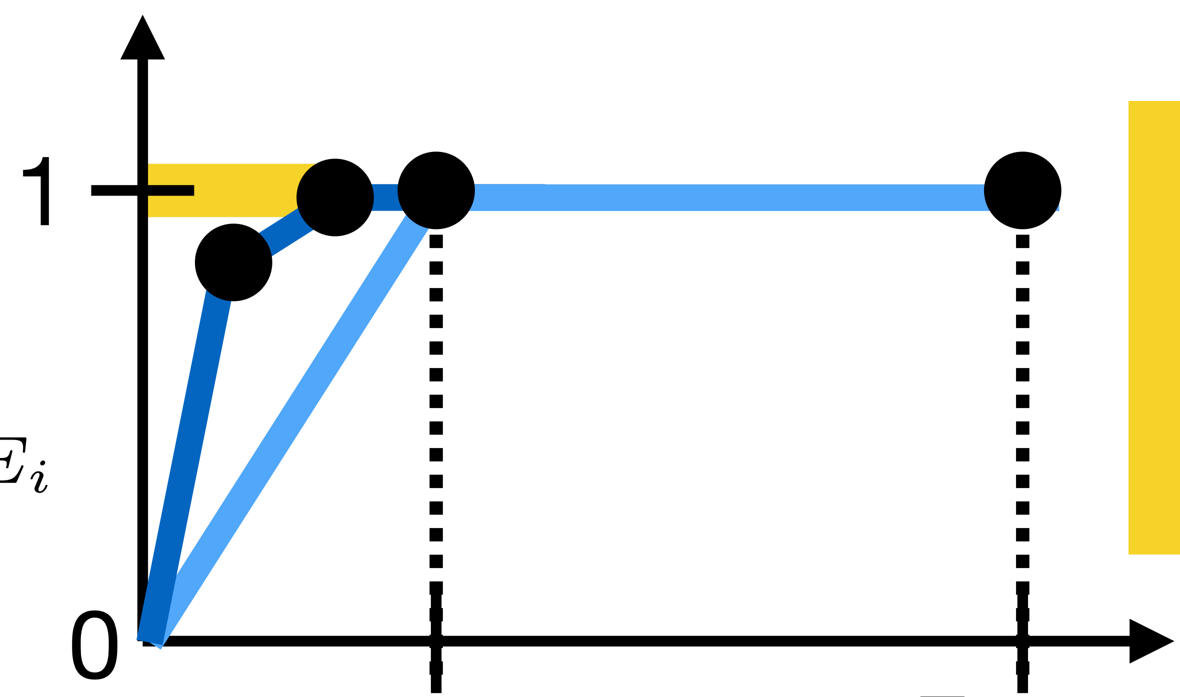

Z_A

$Z_{AW} = Z_A \cdot (1 + e^{-\beta\Delta})$

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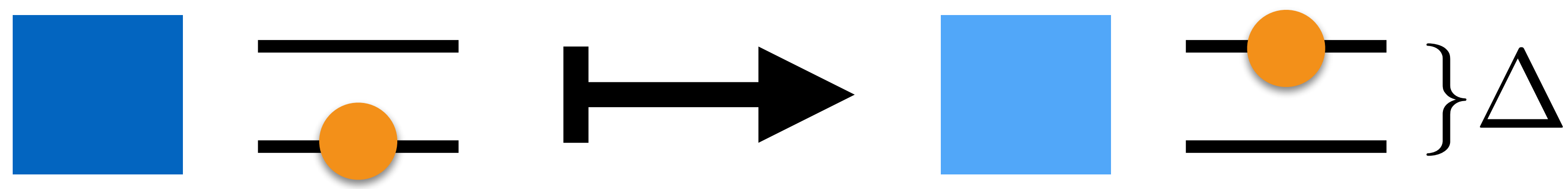

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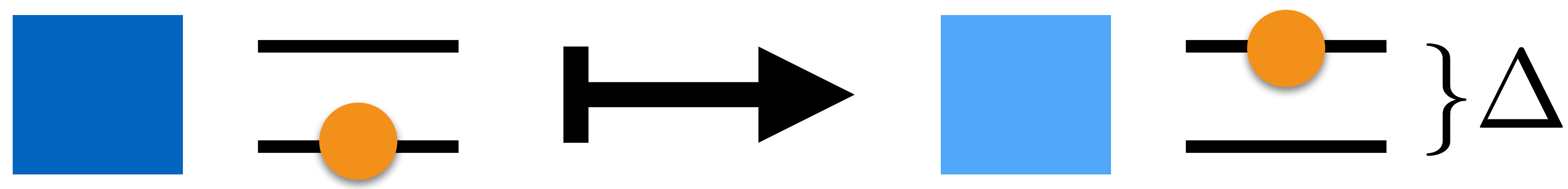

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LHS: Must lie everwhere **above** that curve (note: *concave*).

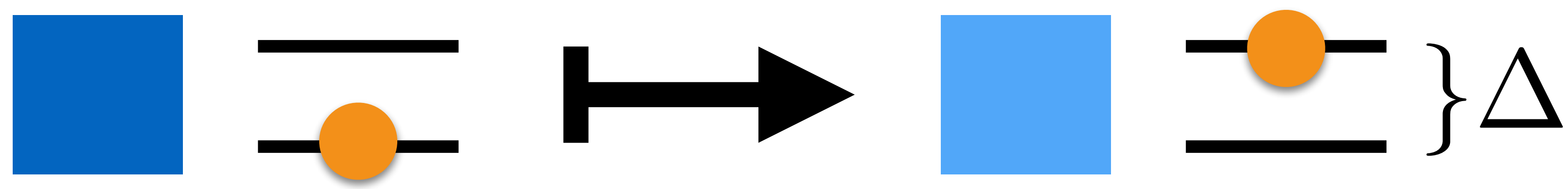
$$F_0(\sigma_A) \geq F(\gamma_A) + \Delta$$

$$\beta\, F_0(\sigma) := \log \sum_{p_i' \neq 0} e^{-\beta E_i}$$

Extractable work: $F_0(\sigma_A) + k_B T \log Z_A$.

Work extraction and work of formation

Work extraction: $\sigma_A \otimes |g\rangle\langle g|_W \mapsto \gamma_A \otimes |e\rangle\langle e|_W$



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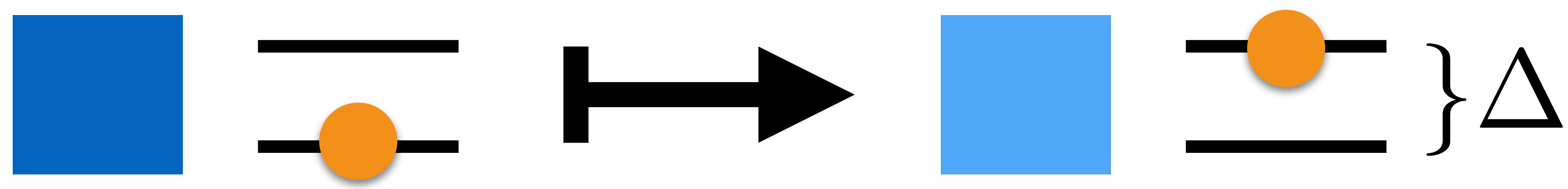

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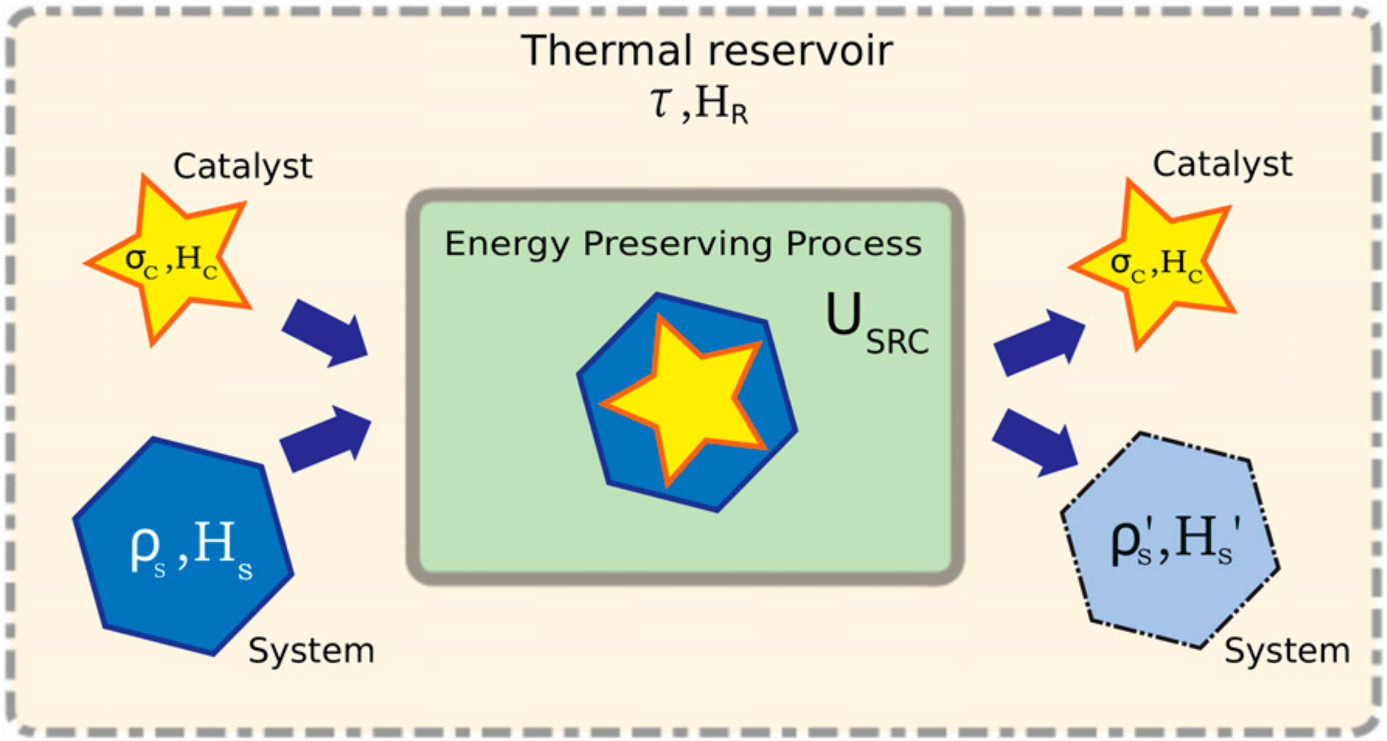
Fundamental irreversibility: $F_0 \ll F \ll F_\infty.$

General state transitions — with a catalyst

Allow for additional system *C* that is involved but doesn't change.

Brandão et al., *The second laws of quantum thermodynamics*, PNAS **112**, 3275 (2015).



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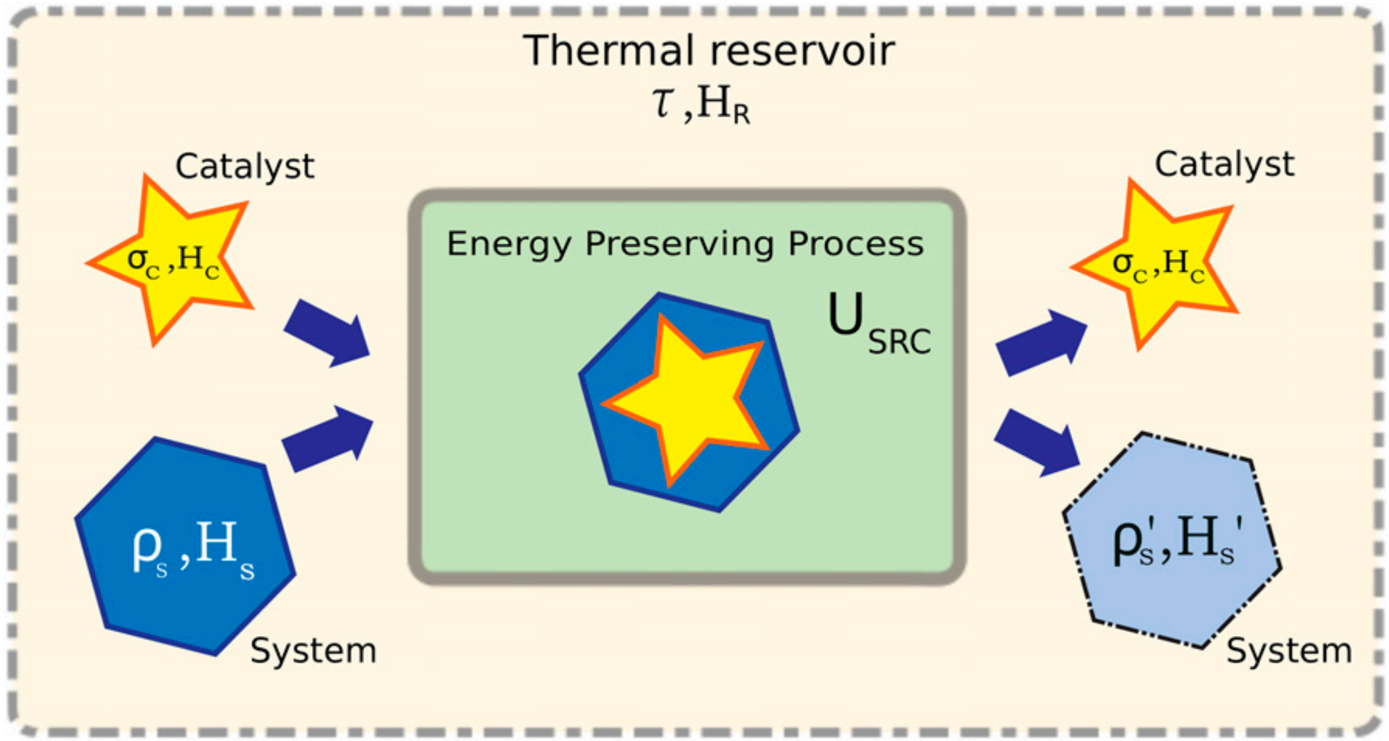

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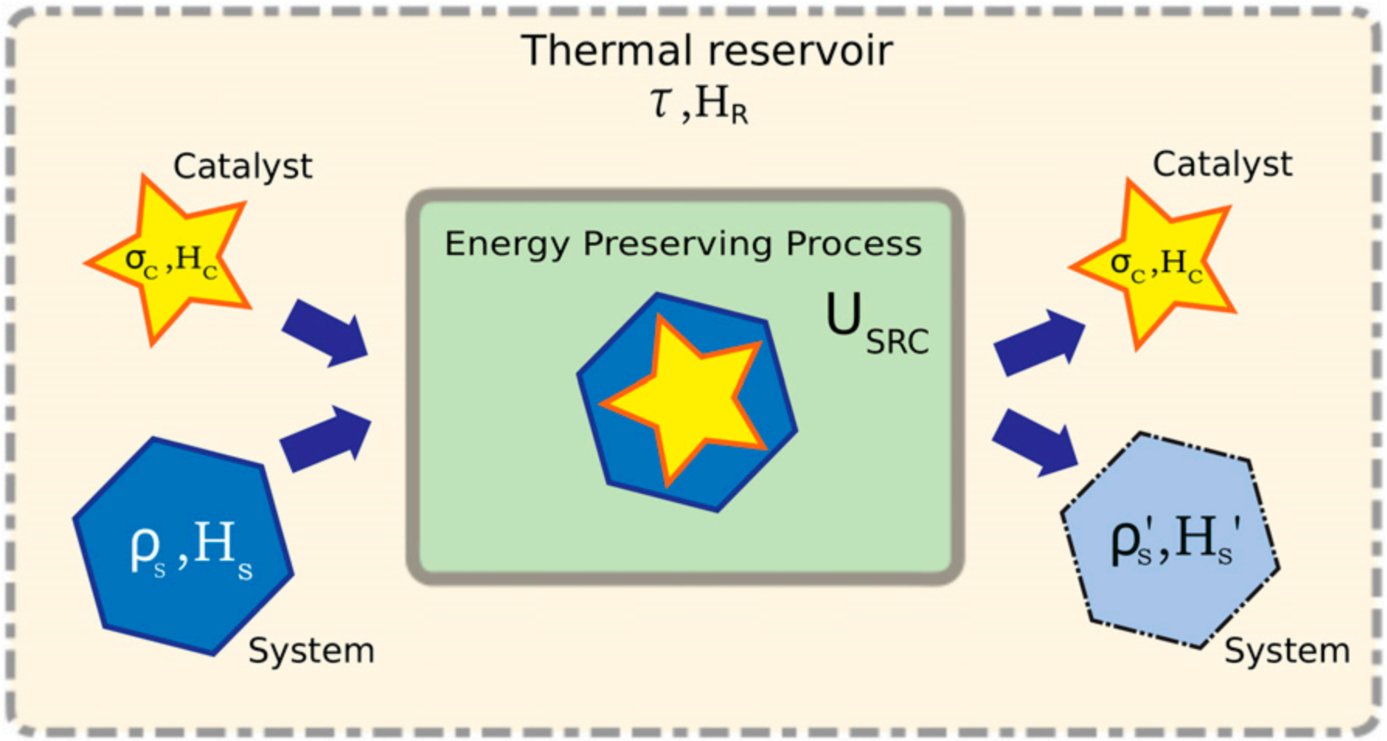

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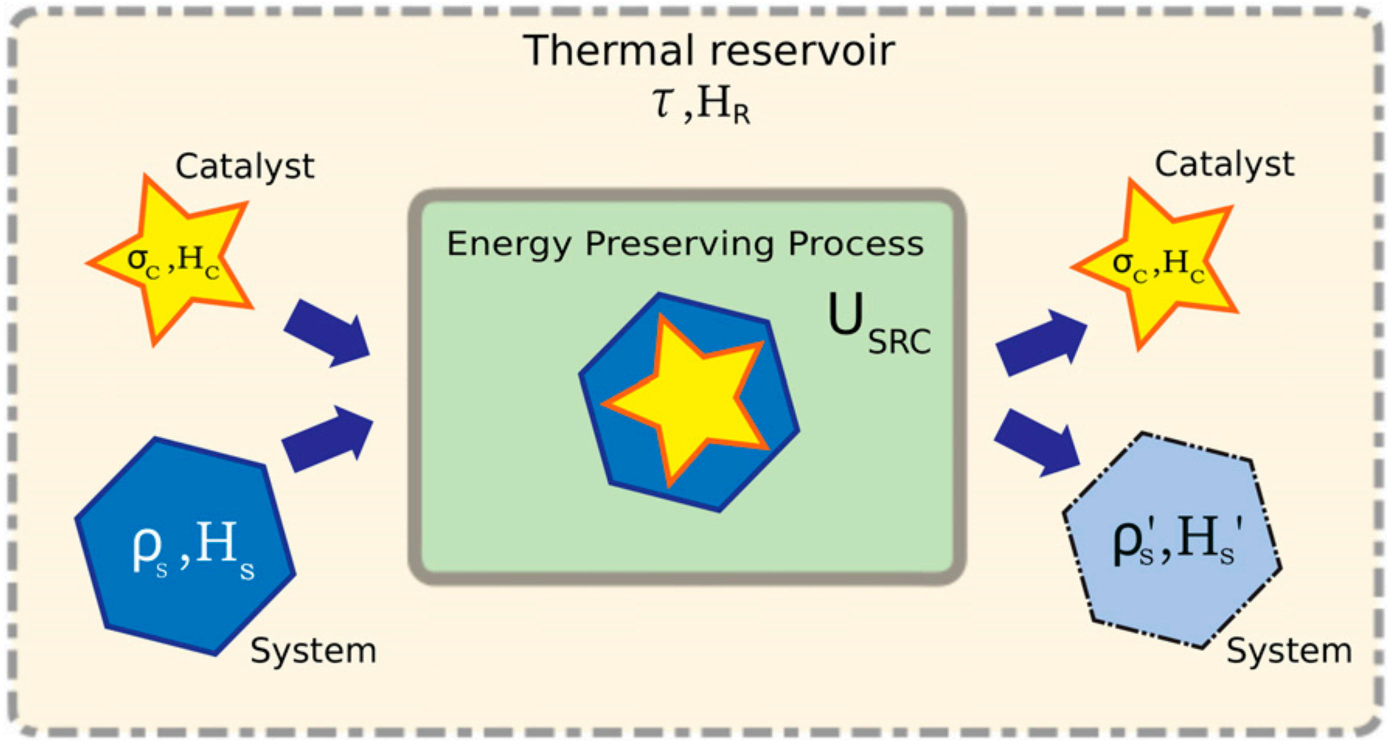

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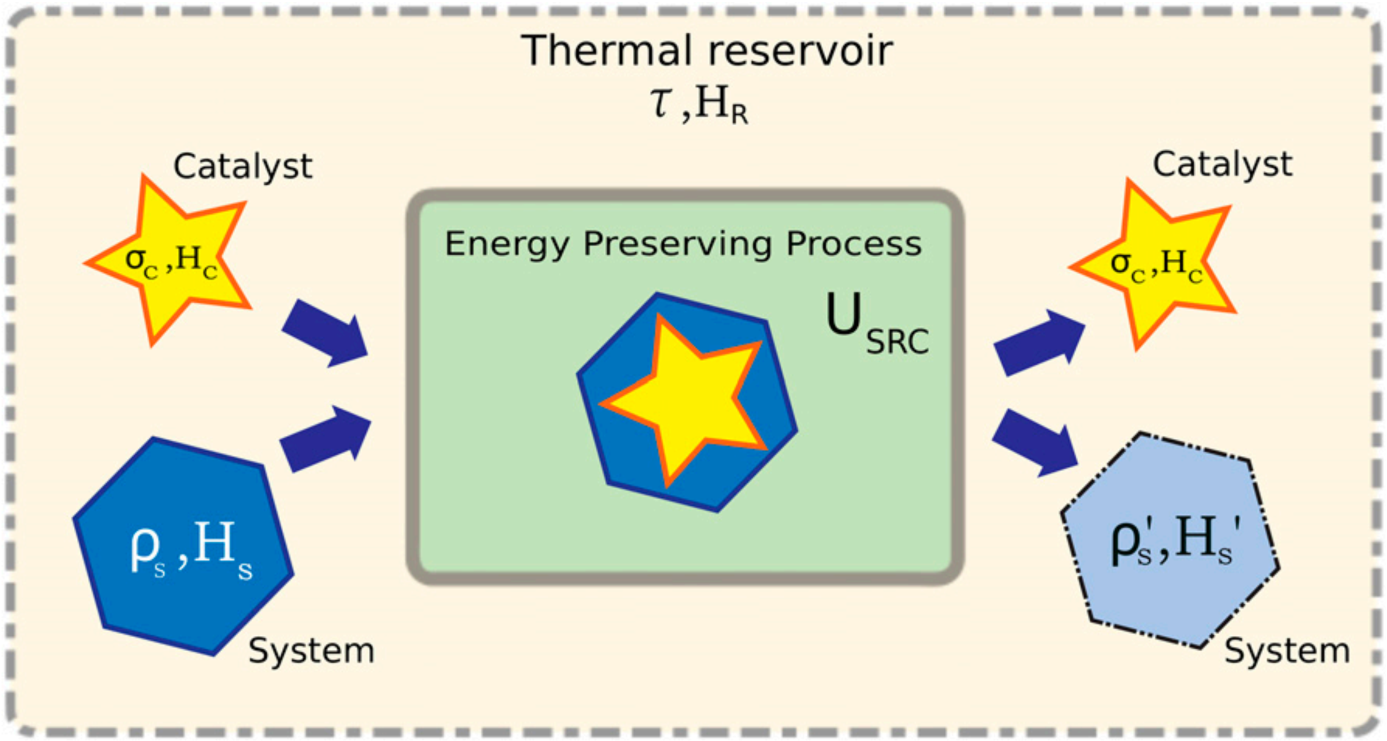

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$$\left(\frac{1}{2}, \frac{1}{2}, 0, \ldots, 0\right) \longrightarrow \left(1-\epsilon, \frac{\epsilon}{N}, \frac{\epsilon}{N}, \ldots, \frac{\epsilon}{N}\right)$$

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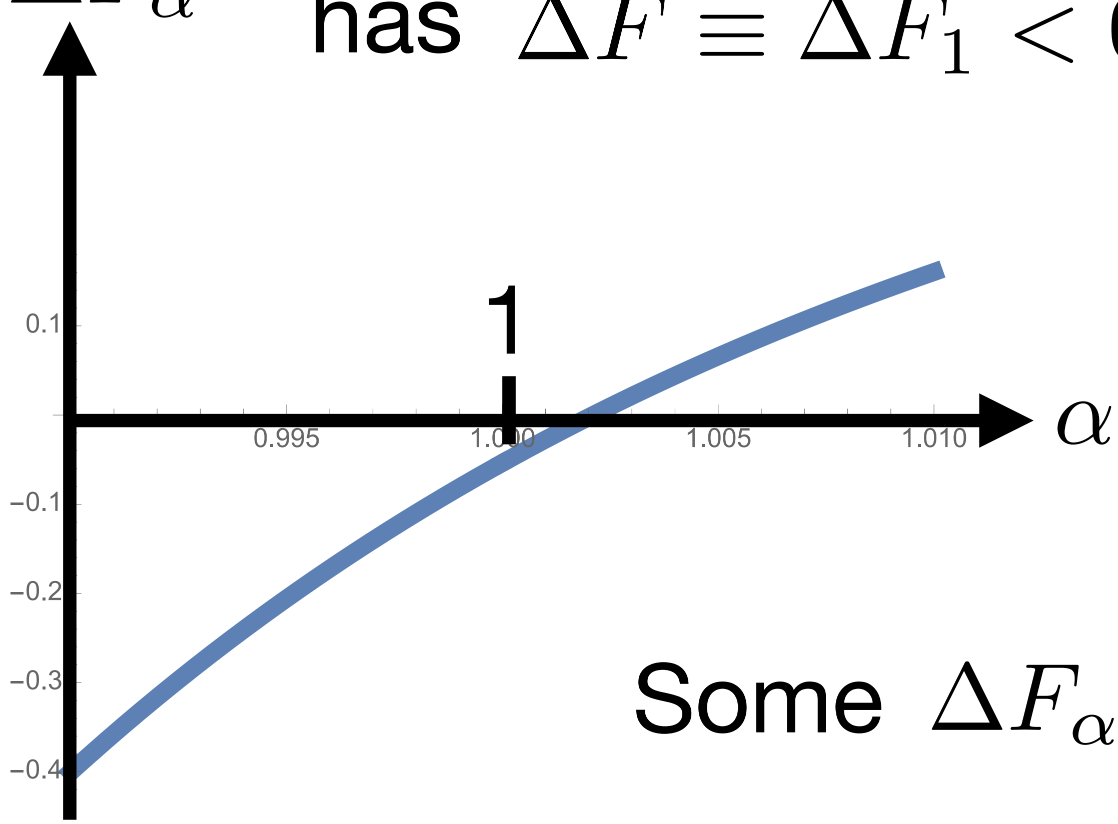

$$\epsilon = \frac{1}{100}, \quad N = 10^{30}.$$

Some $\Delta F_\alpha > 0$ hence **indeed impossible.**

How does the thermodynamic limit enter?

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(Rates of) work cost and extractable work become *F*.
Reversibility is restored in the thermodynamic limit!

Outline

1. **Resource-theoretic approach to thermodynamics**

2. Single-shot interpretation of von Neumann entropy and free energy (block-diagonal states)

3. Beyond block-diagonal states: on coherence, clocks, and timing information

4. Conclusions

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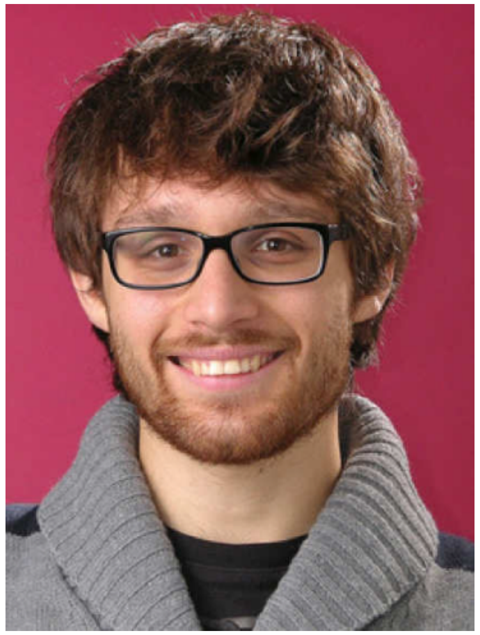
MM, *Correlating Thermal Machines and the Second Law at the Nanoscale*, Phys. Rev. X **8**, 041051 (2018)

Building on earlier work with my students Jakob Scharlau and Michele Pastena, and with Matteo Lostaglio.



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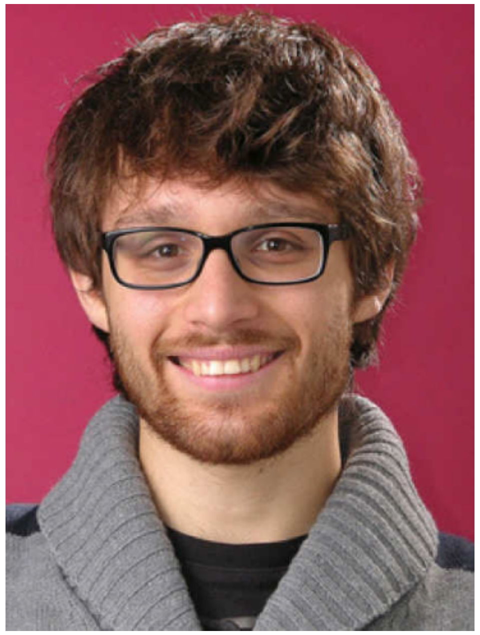

First, **recall the previous scenario:**

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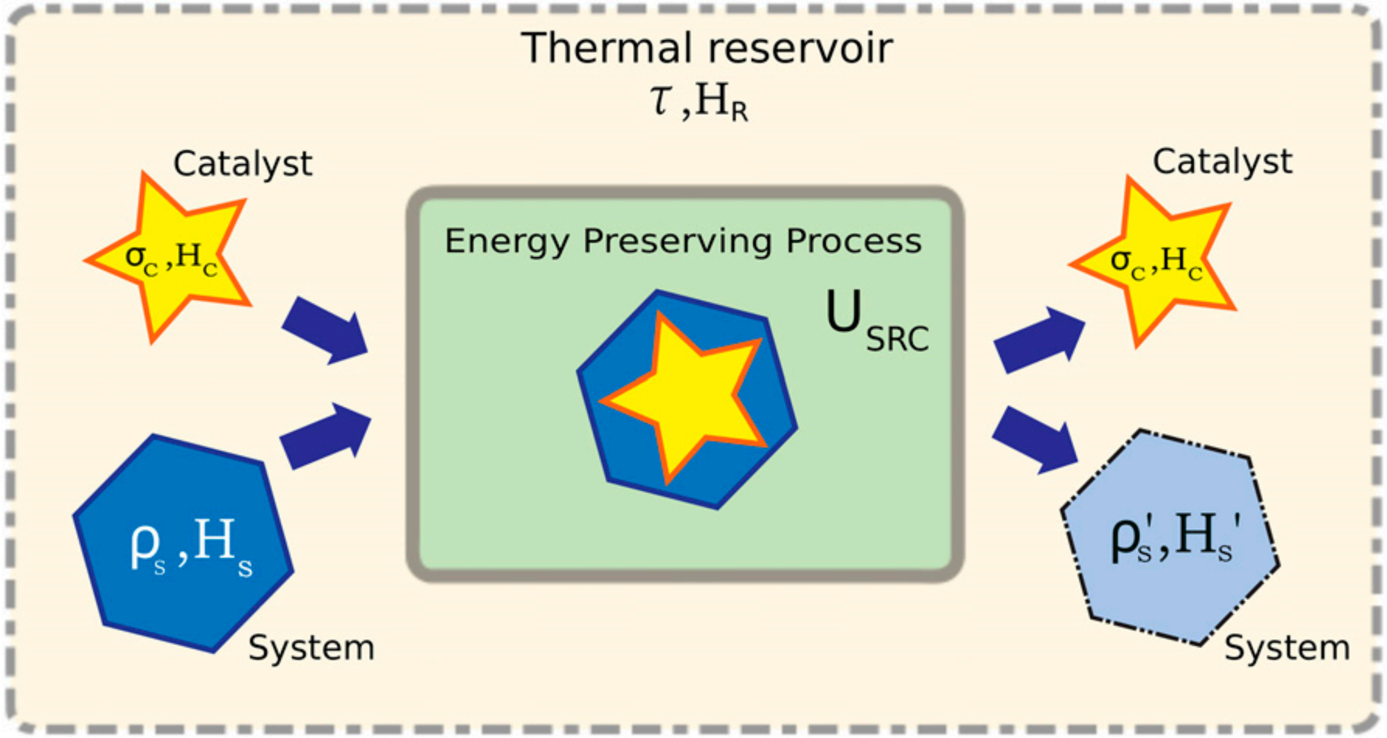

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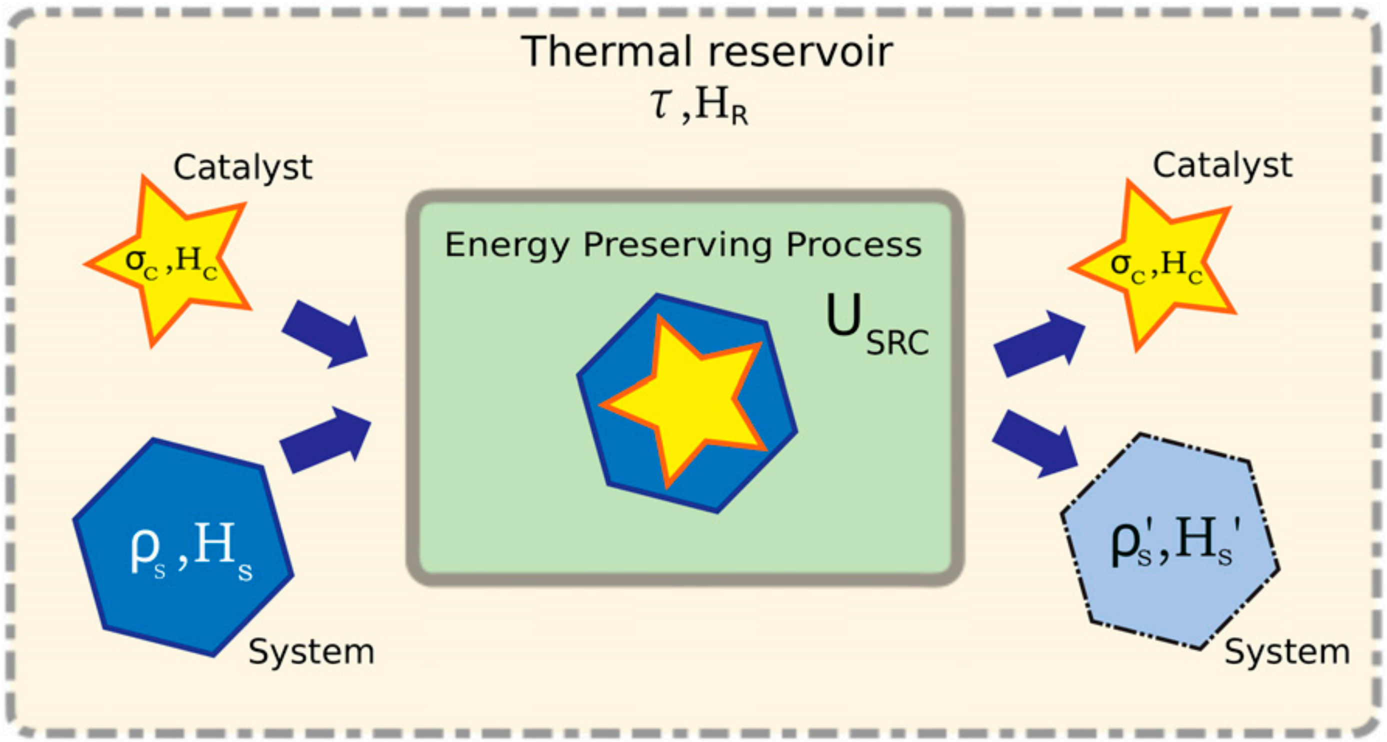

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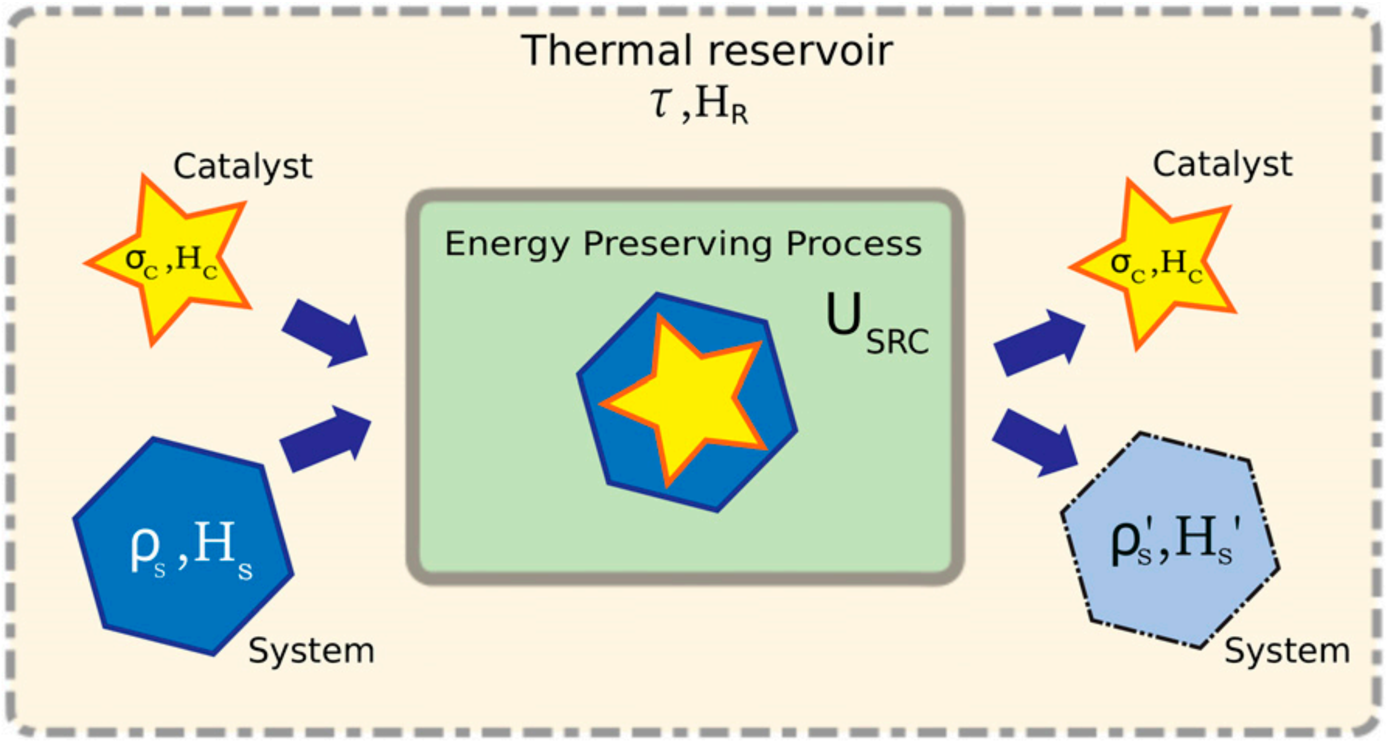

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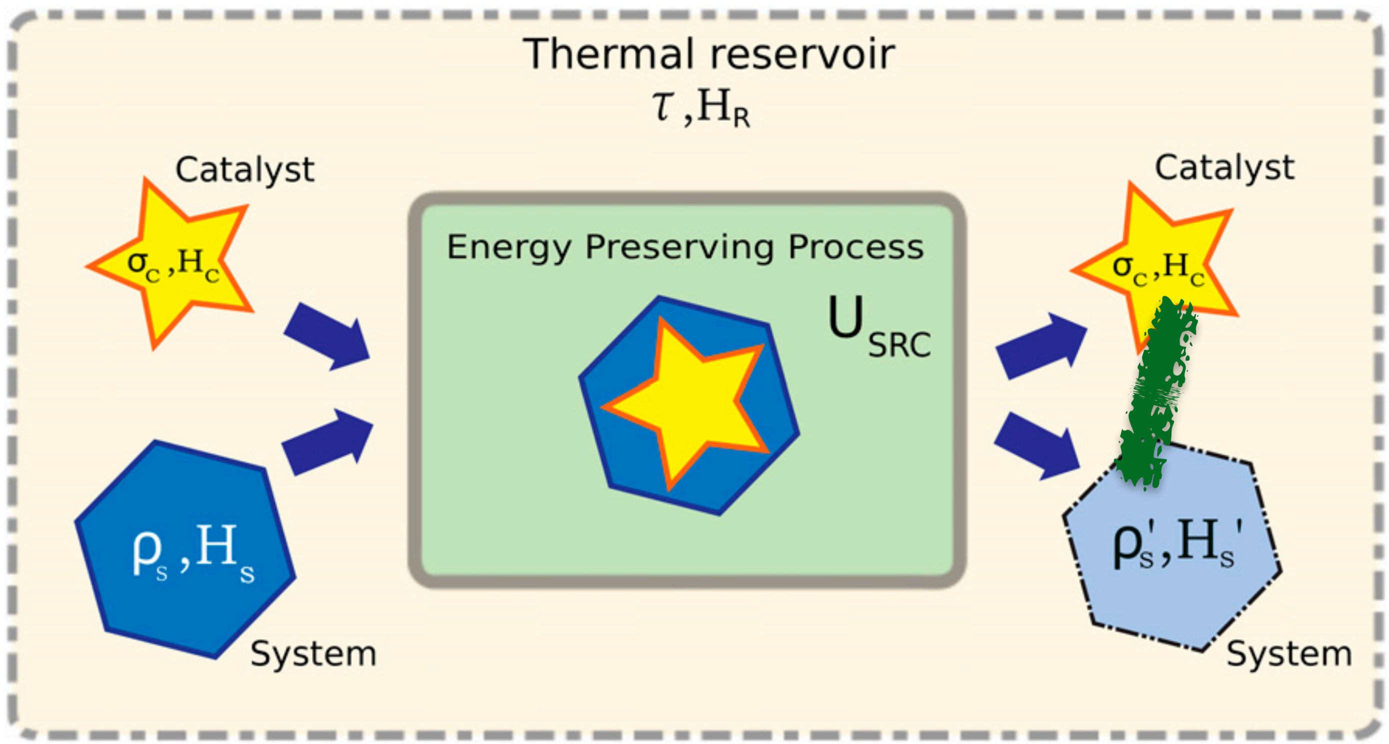

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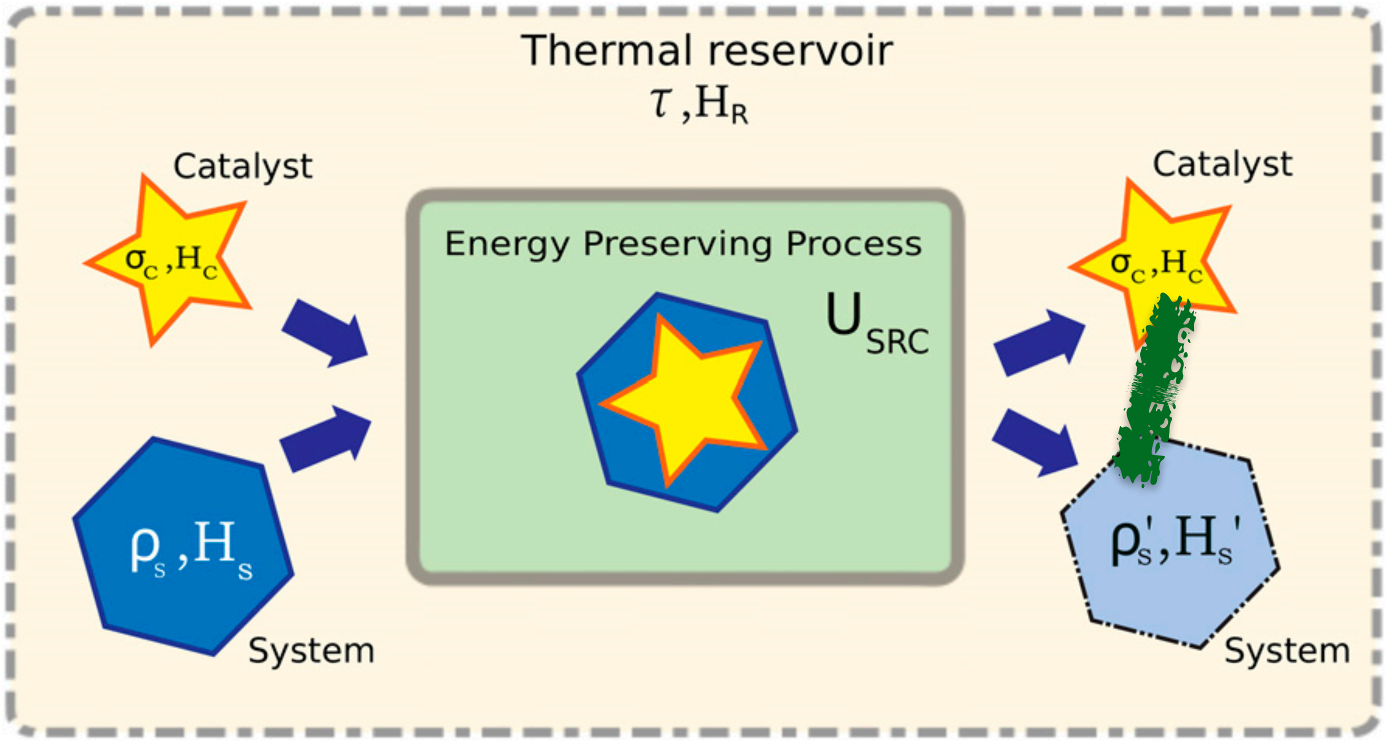

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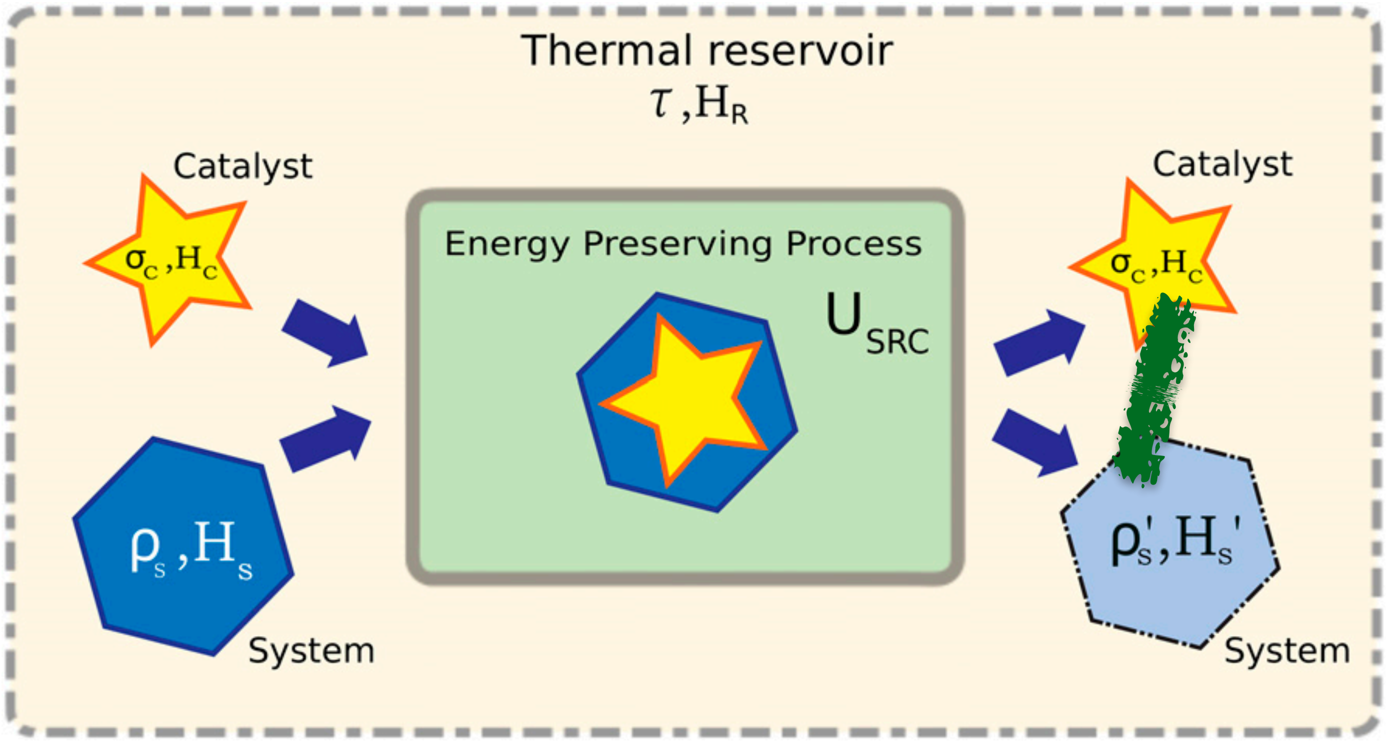

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One-shot interpretation of the free energy *F*.

New one-shot interpretation of free energy

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Theorem. Let ρ_A, ρ'_A be block-diagonal states. Then, for every $\varepsilon > 0$, there is a thermal operation $\mathcal{T}_\varepsilon$, a state $\rho'_A(\varepsilon)$ with $\|\rho'_A - \rho'_A(\varepsilon)\| < \varepsilon$ and a finite-dimensional catalyst σ_C such that

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Notation: $\omega_{AC} = \rho'_A \sigma_C$ means that

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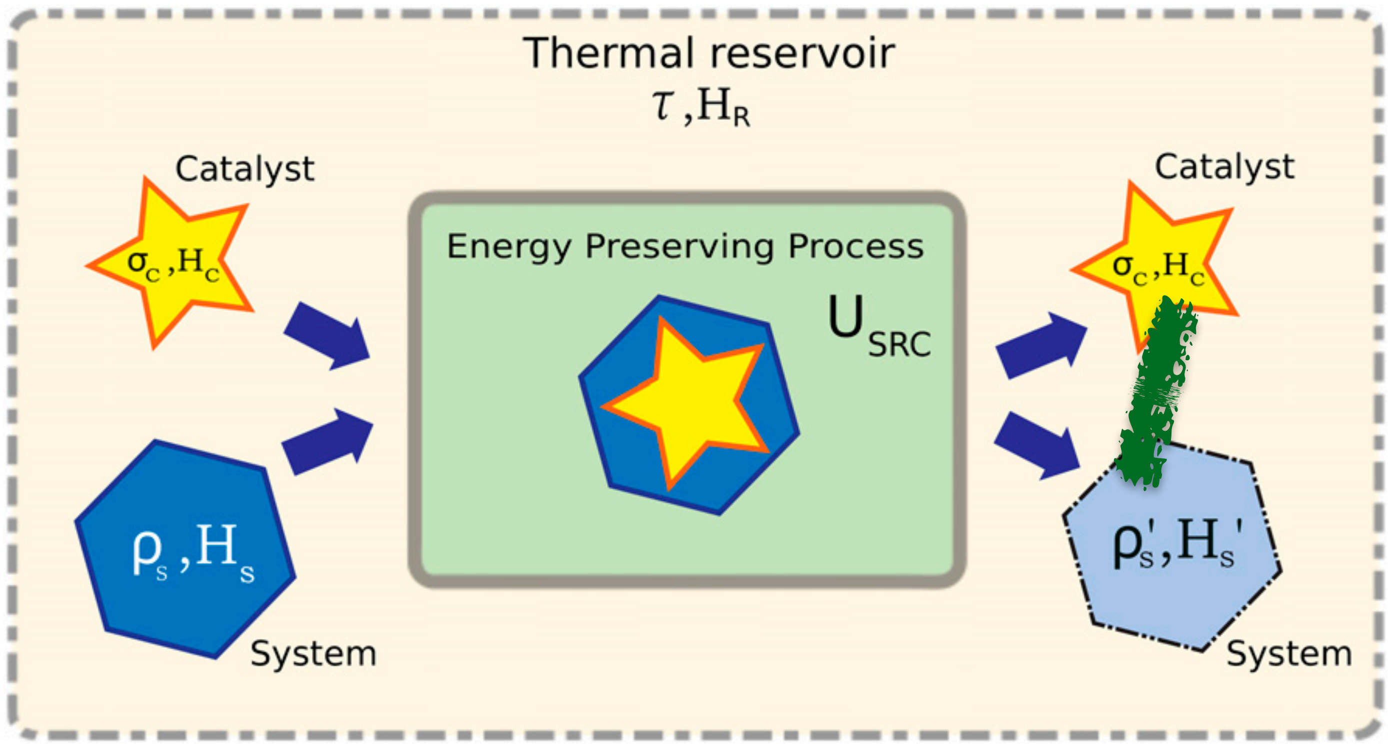

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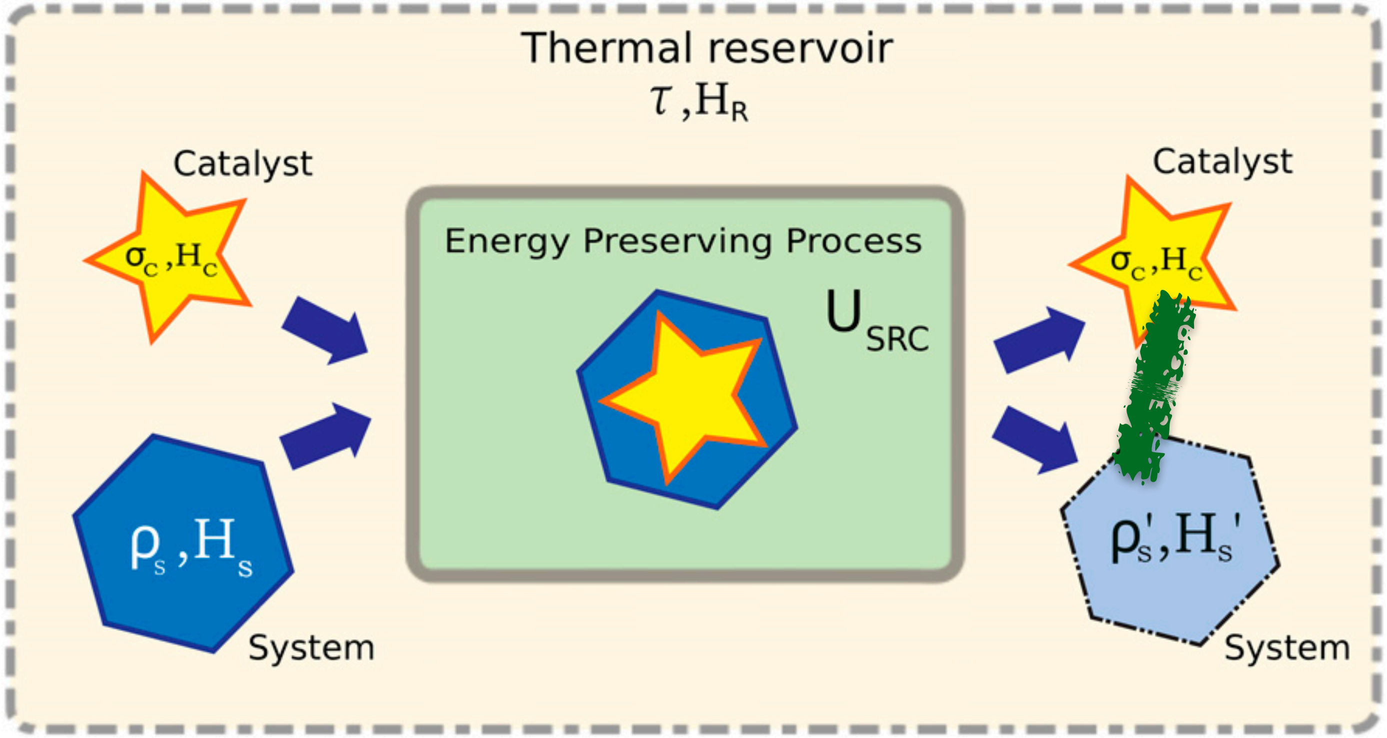

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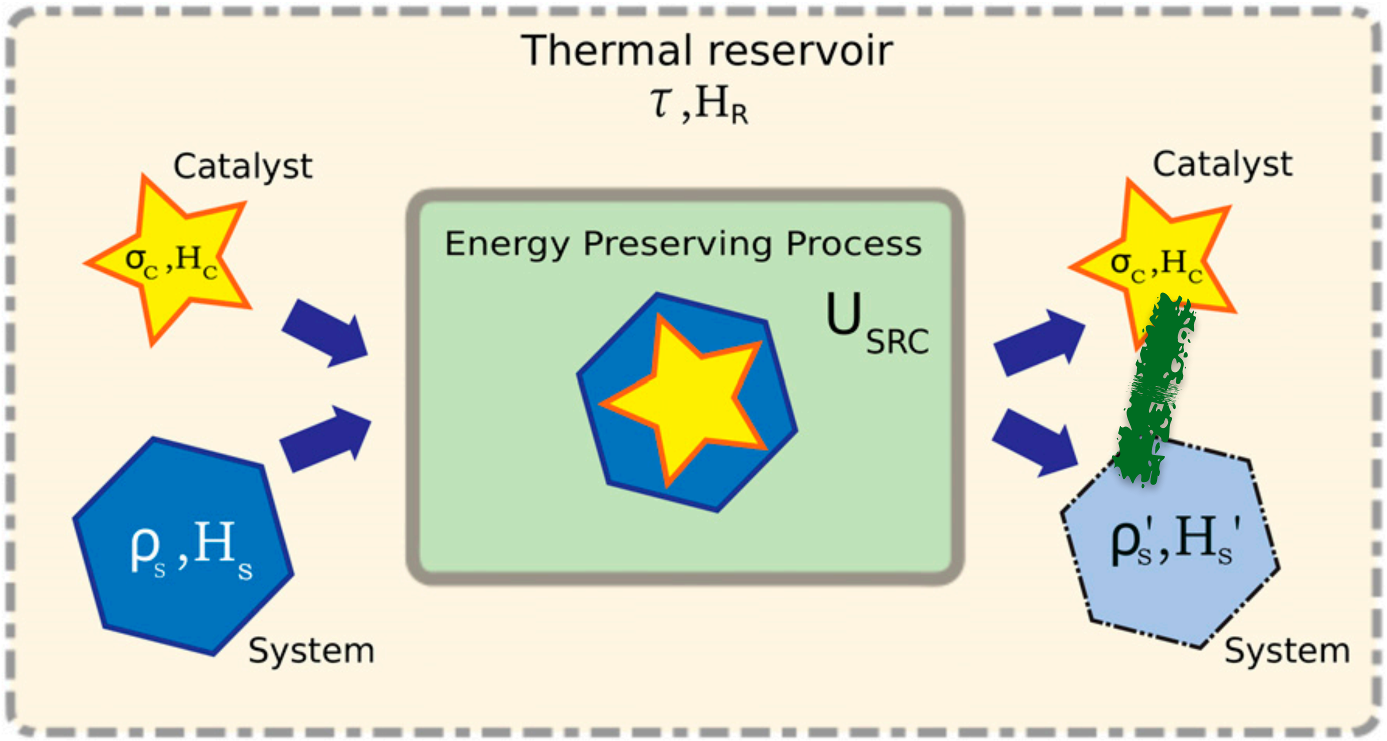

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- *Almost* fluct.-free **work extraction**: If $F(\rho_A) - F(\rho_A') > \Delta > 0$, then transition possible while work bit (for arbitrary $\delta > 0$)

$$|g\rangle\langle g|_W \mapsto (1-\delta)|e\rangle\langle e|_W + \delta\, \mathbf{1}/d.$$

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The "free energies" are related to the Rényi entropies:

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Therefore, correlations can "increase the α-disorder"
and lead to automatic satisfaction of the α-free energy conditions.

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Coherence

Thermal operations are **time-translation-covariant**:

If $\mathcal{U}(t) = \exp(-iHt) \bullet \exp(iHt)$ then $\mathcal{T} \circ \mathcal{U}(t) = \mathcal{U}(t) \circ \mathcal{T}$.

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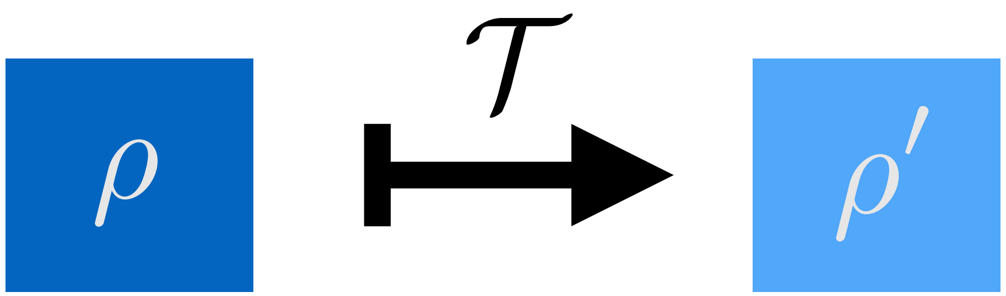

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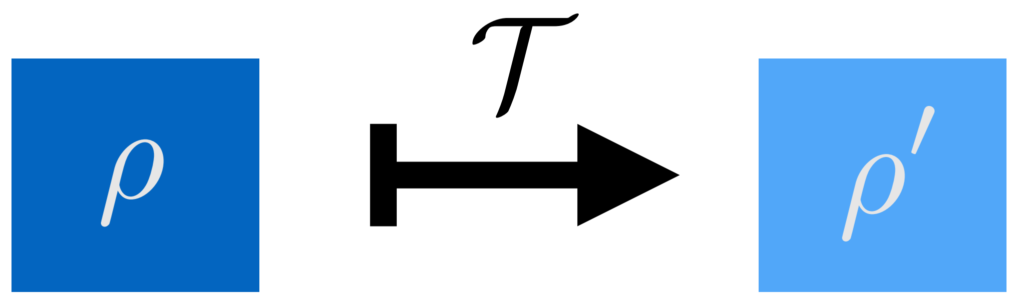

incoherent

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$$[H, \rho] = 0 \qquad \rho \overset{\mathcal{T}}{\longmapsto} \rho' \qquad [H, \rho'] = 0$$

incoherent → incoherent

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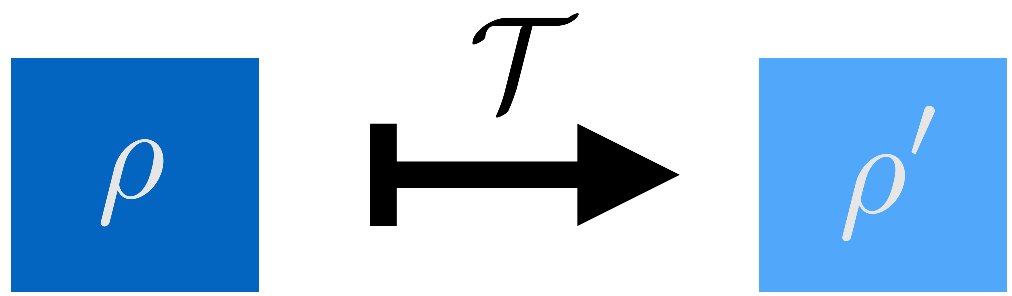

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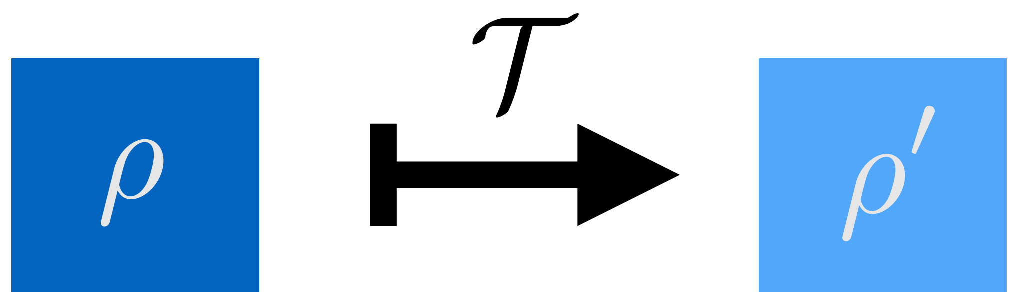

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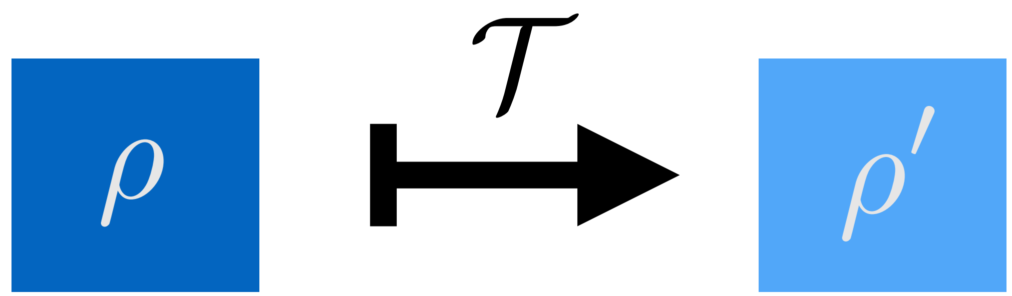

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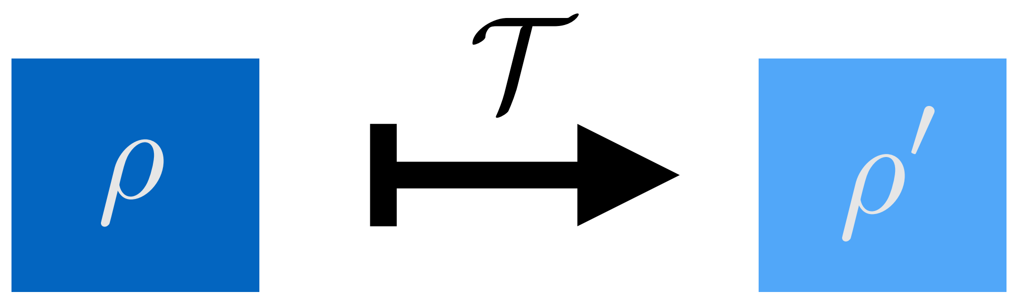

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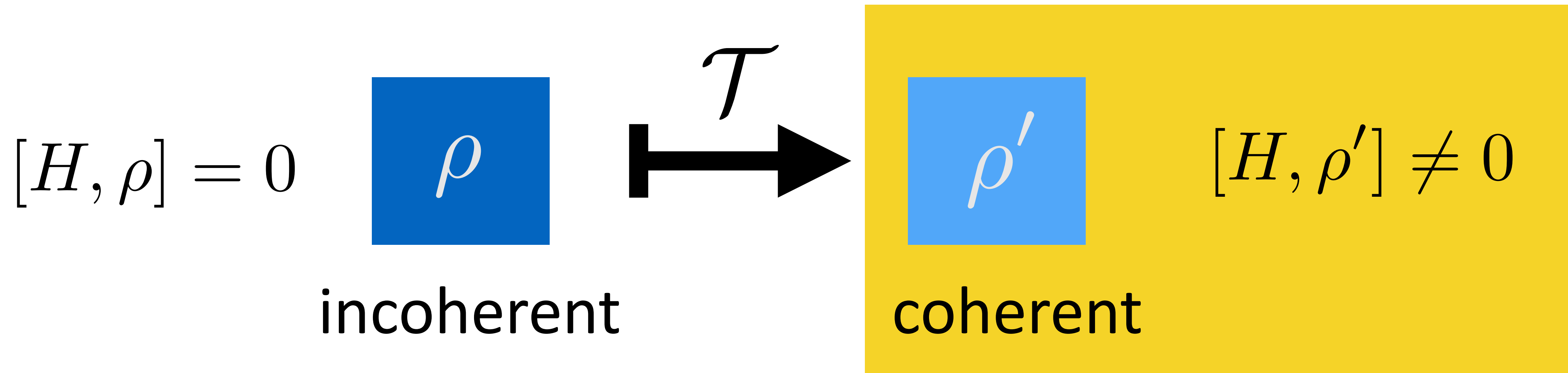

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Impossibility of above process:
Cannot generate timing information (coherence) "for free" **without** an **initial** timing reference (clock).

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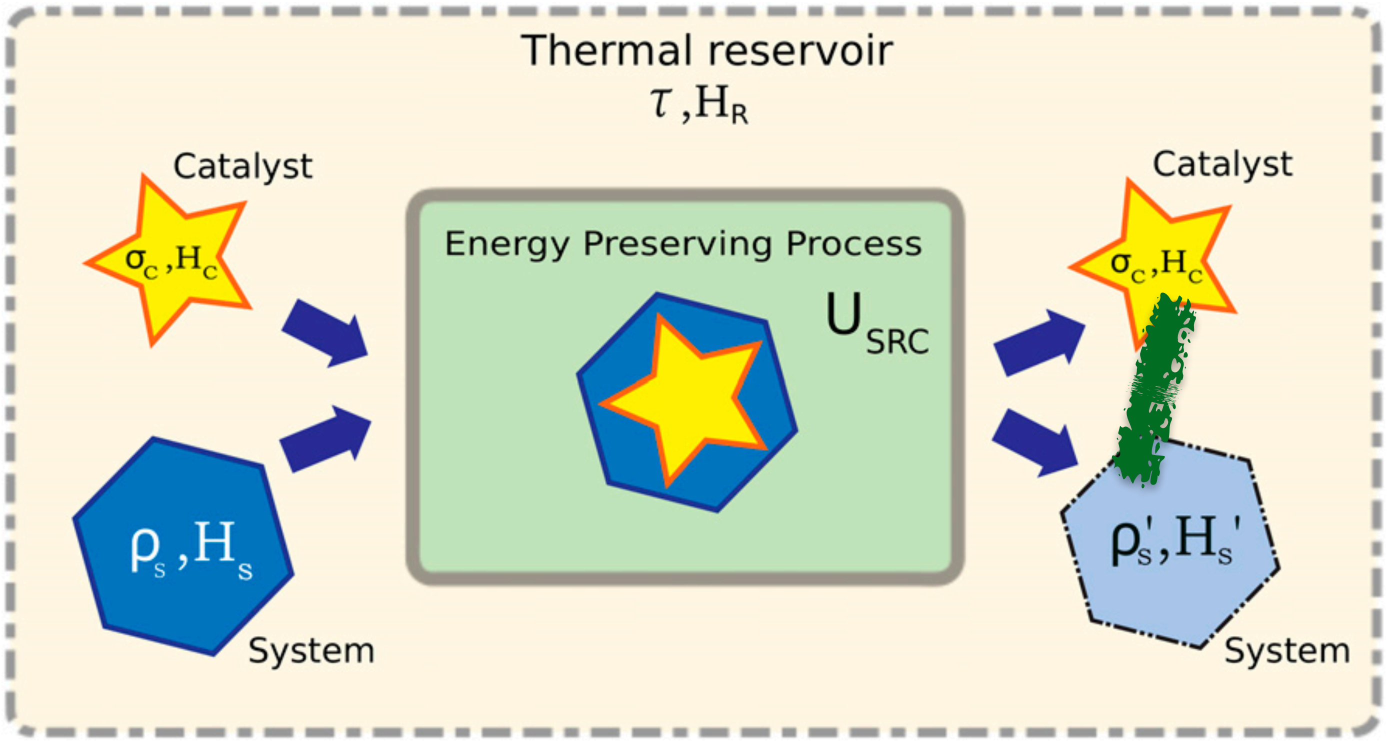

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When is a transition $\rho_S \to \rho'_S$ possible?

(Blockdiagonal states!)

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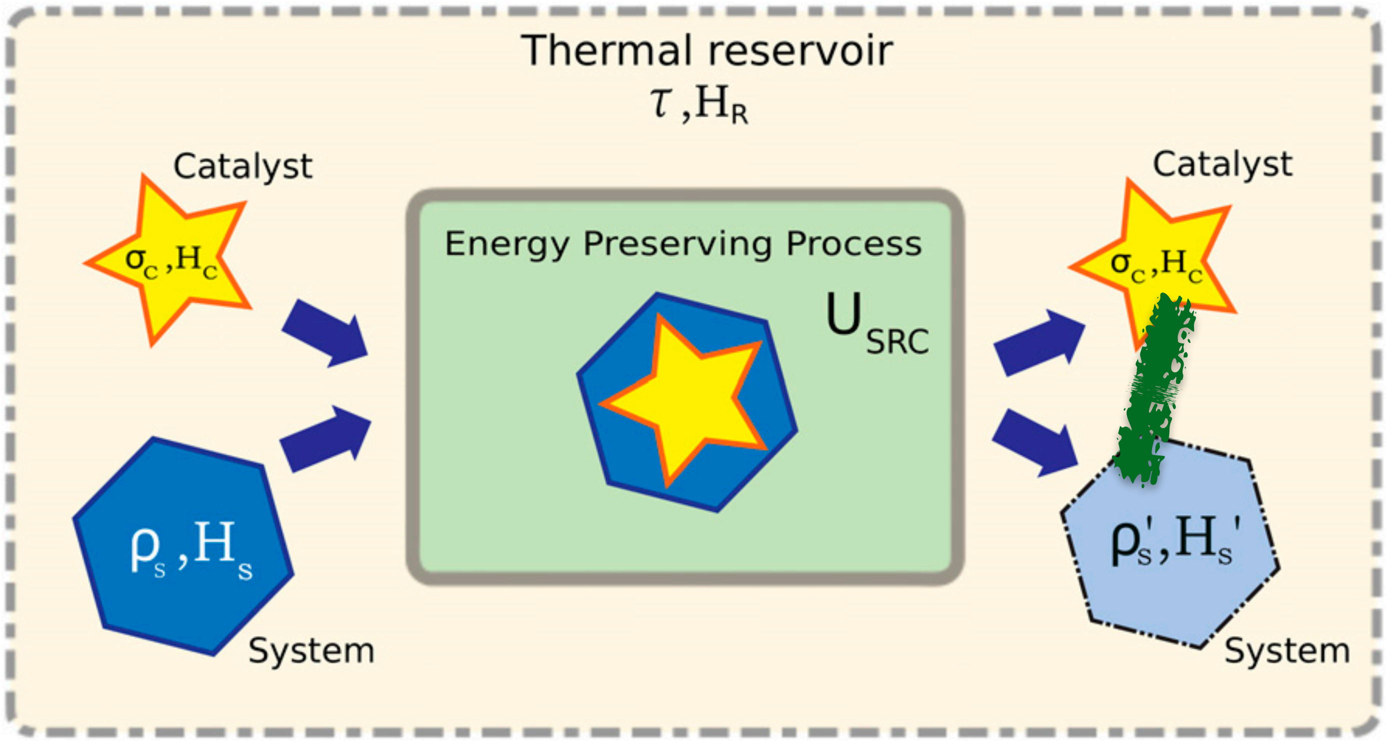

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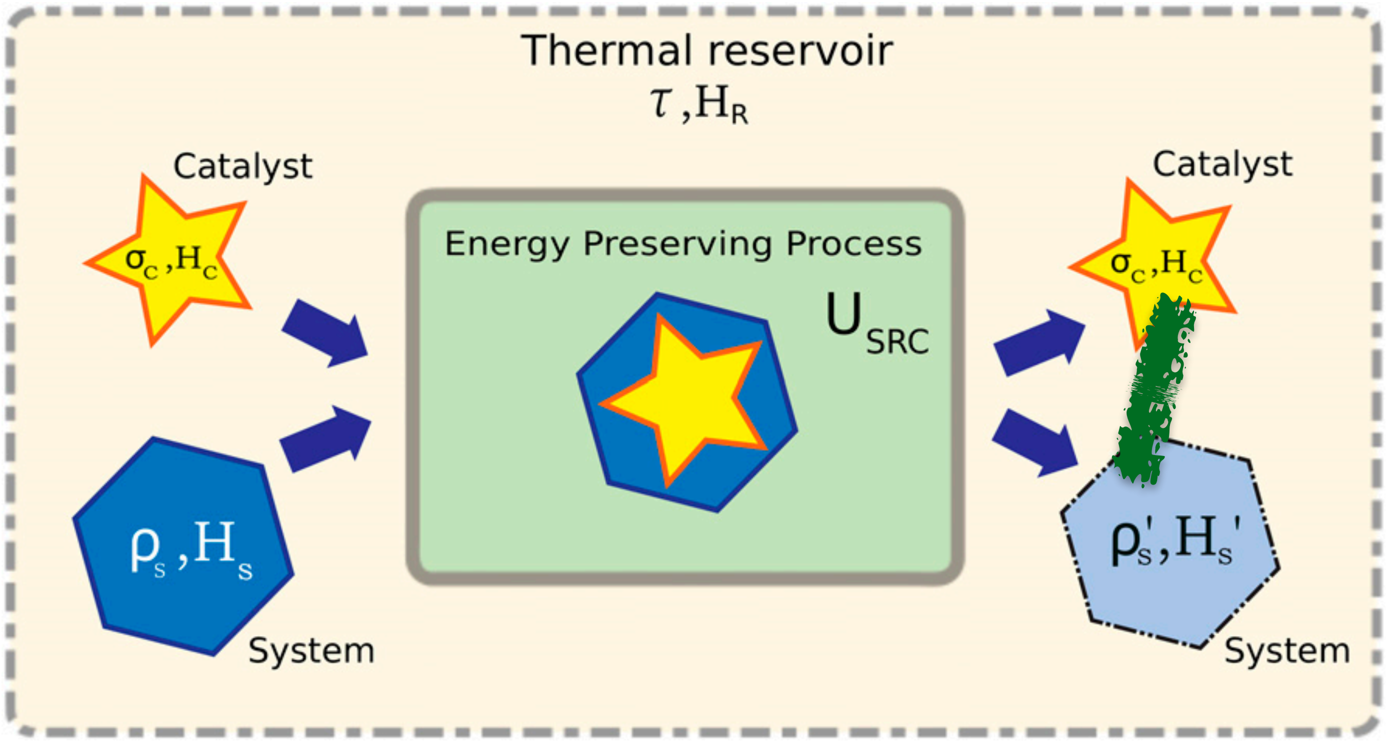

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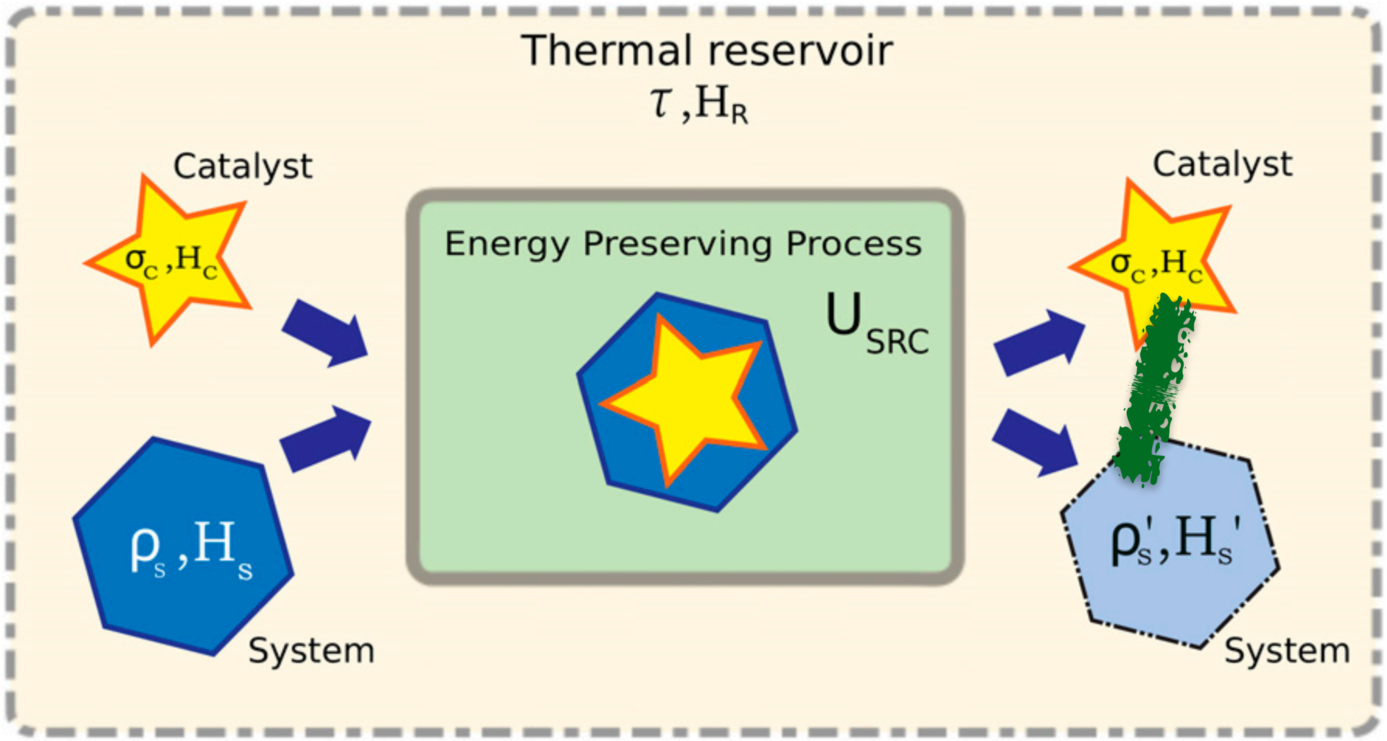

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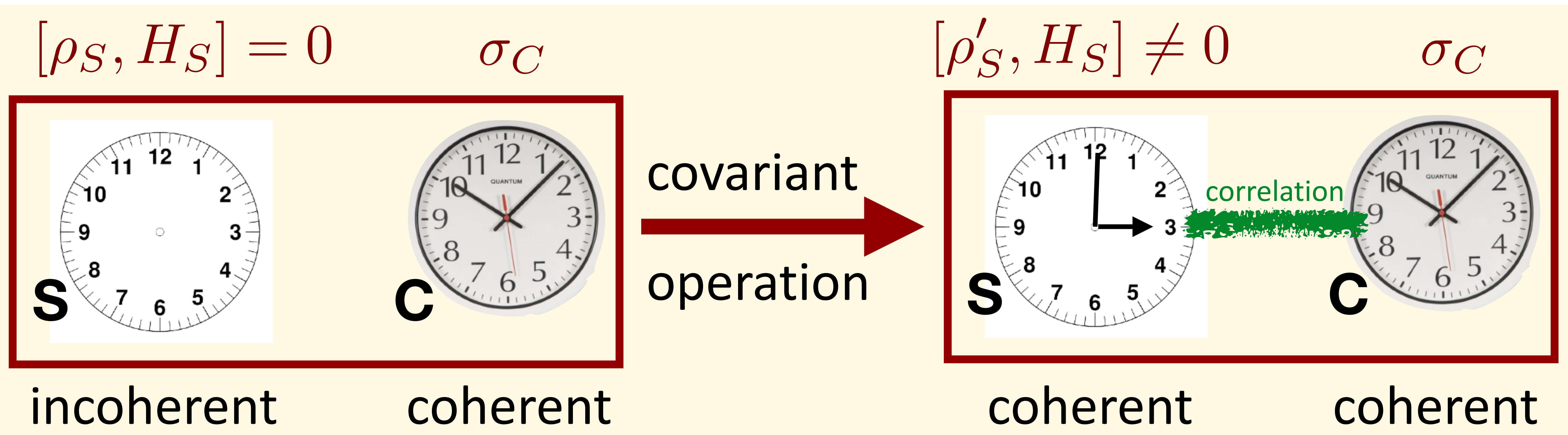

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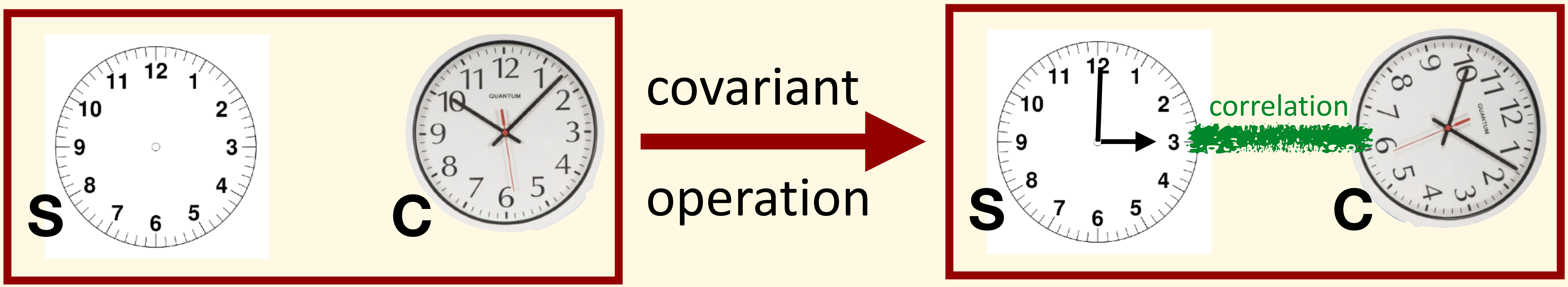

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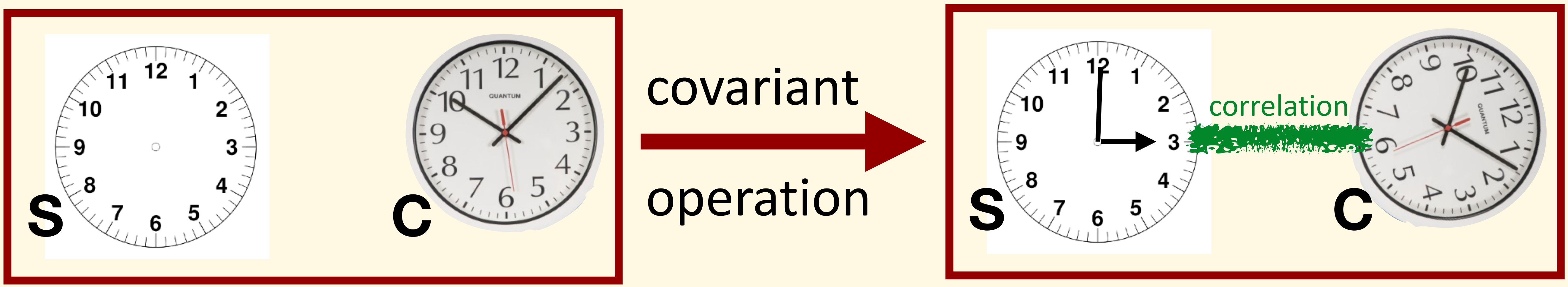

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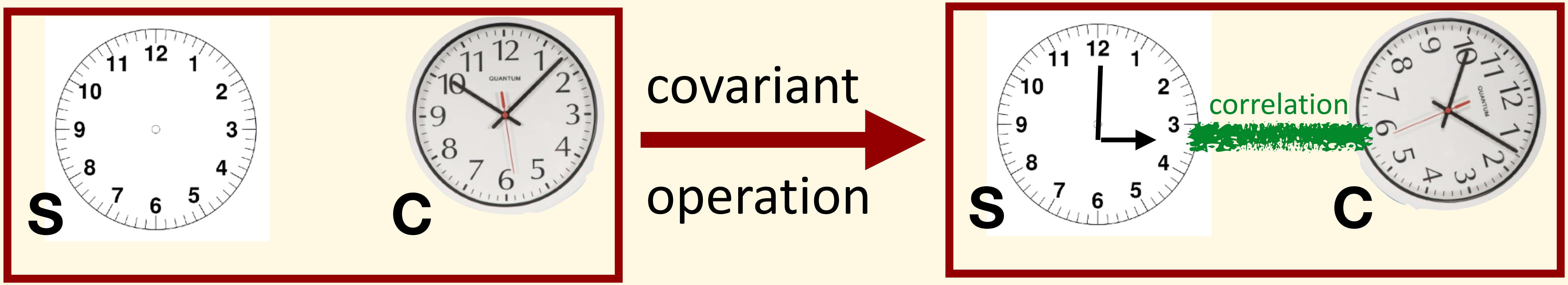

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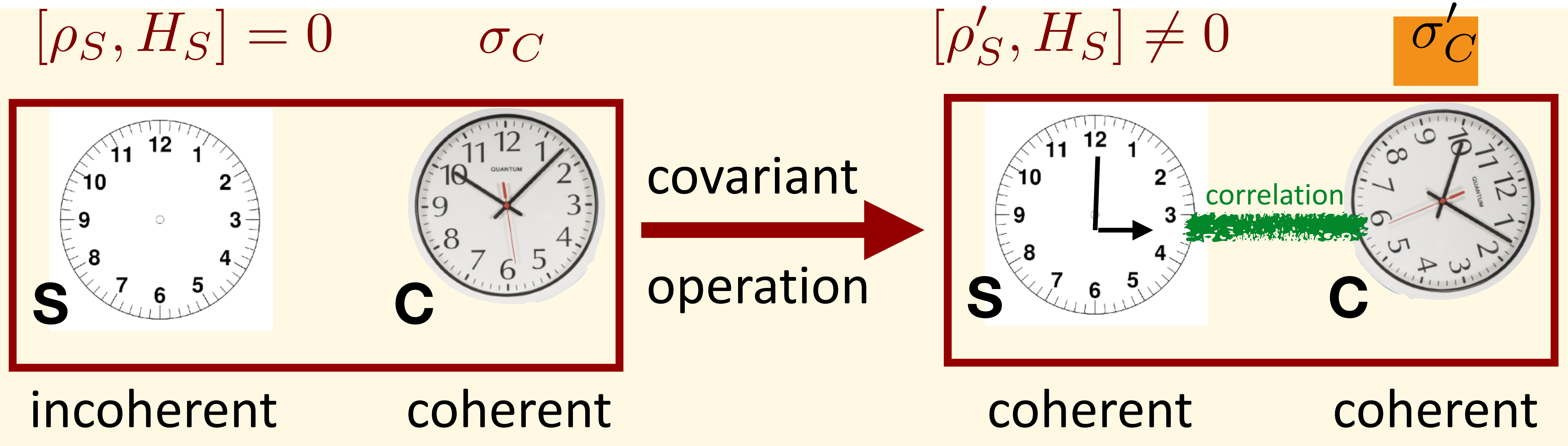

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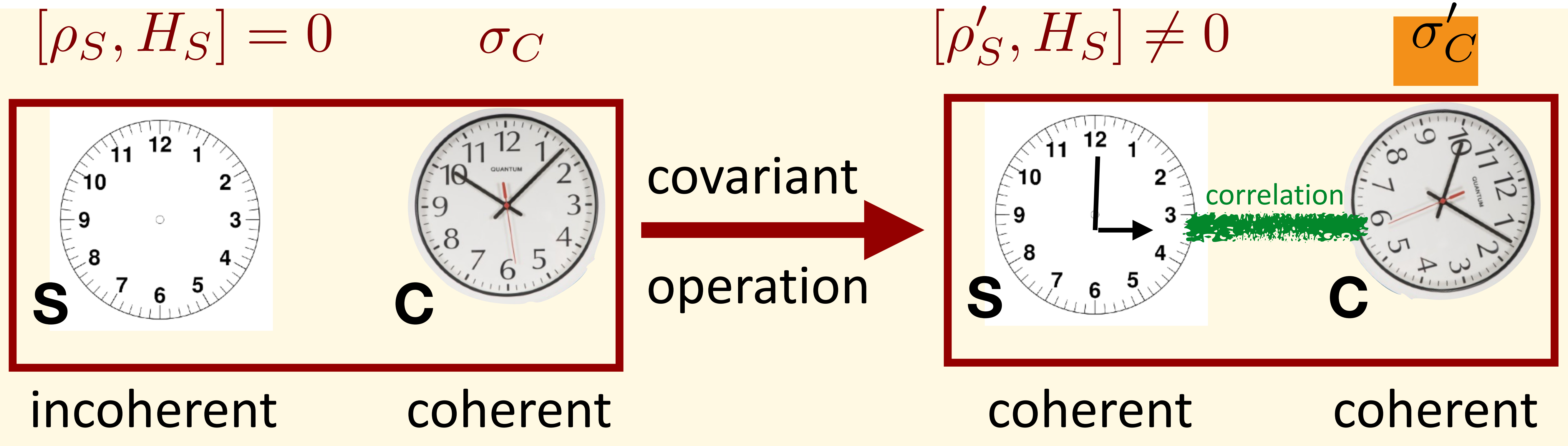

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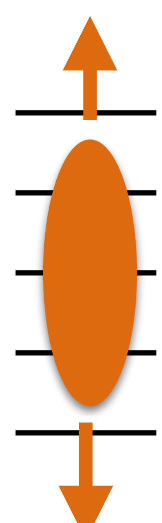

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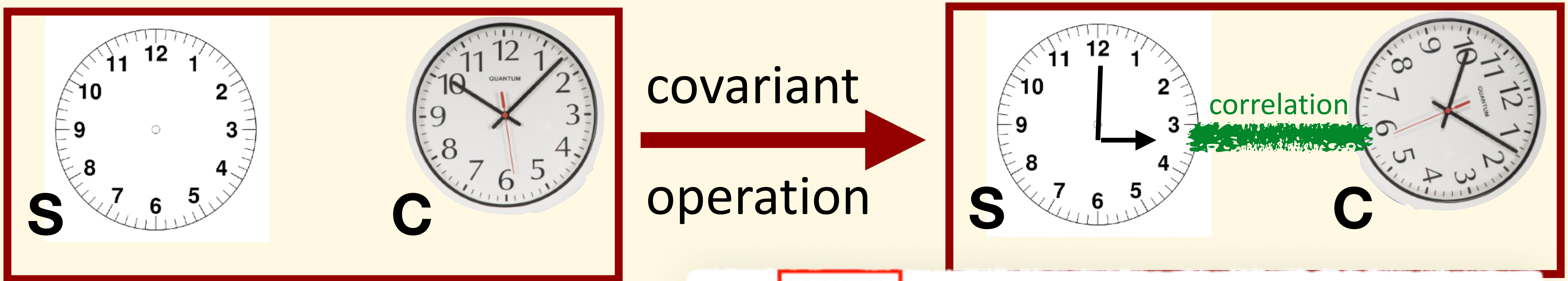

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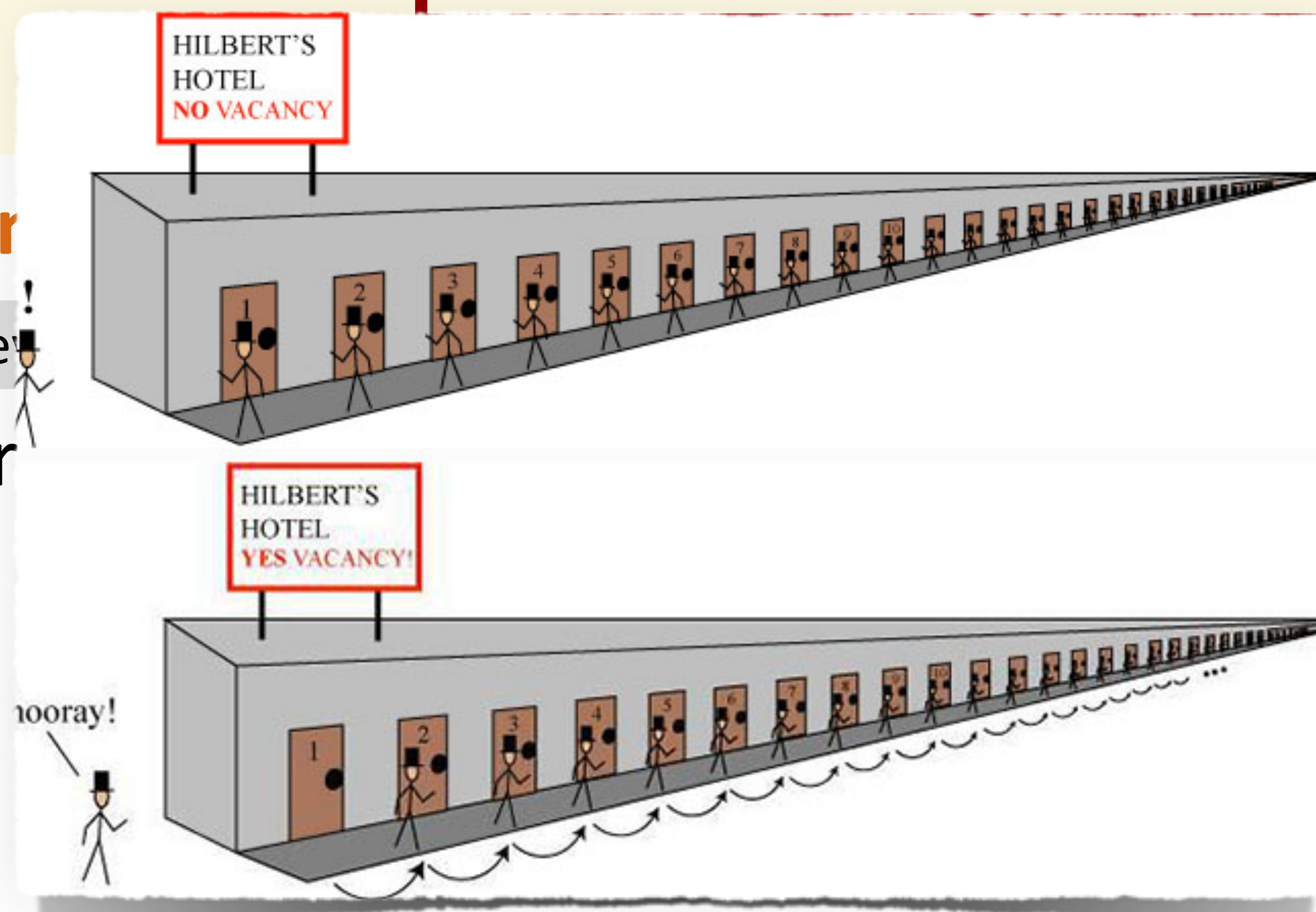

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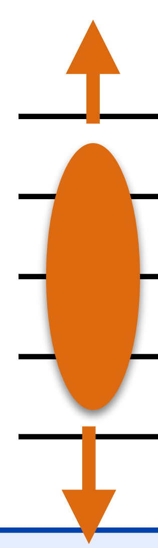

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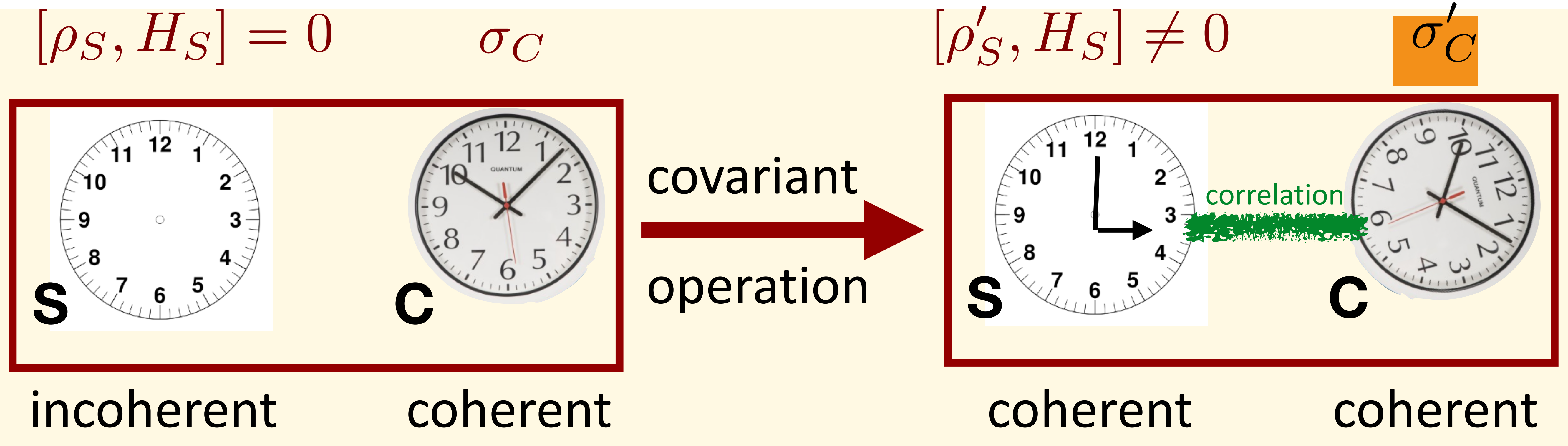

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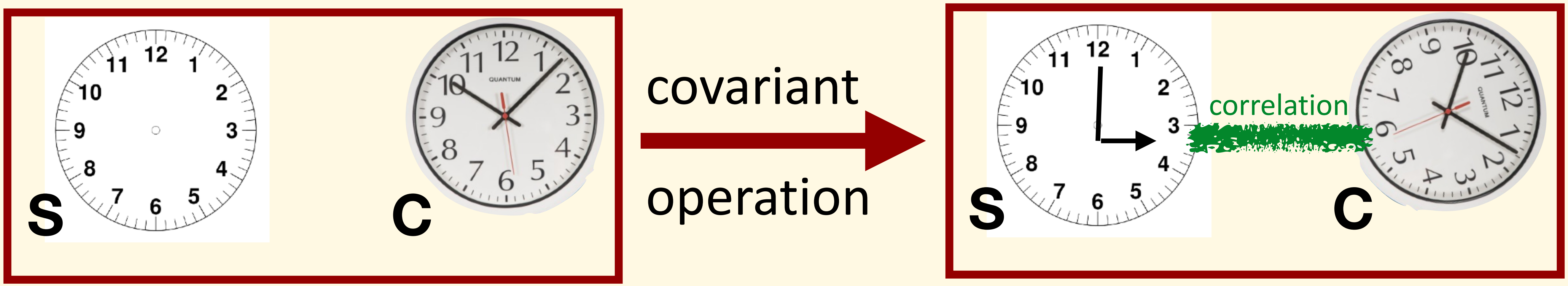

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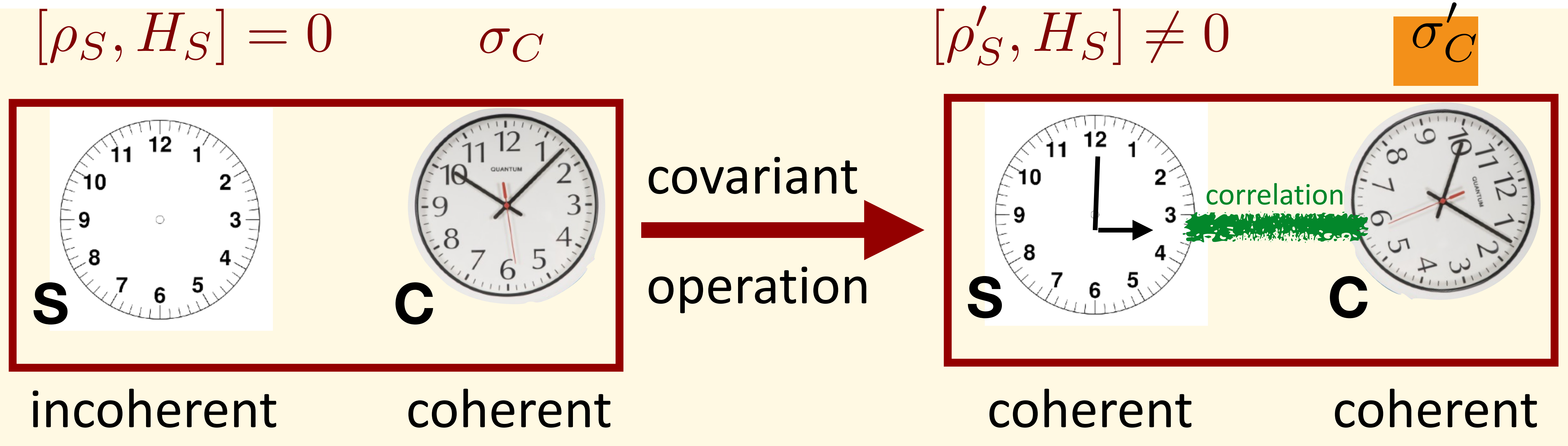

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More generally, (weak) broadcasting of *G*-asymmetry is impossible, for every connected Lie group *G*. (Time translations: $G = \mathbb{R}$)

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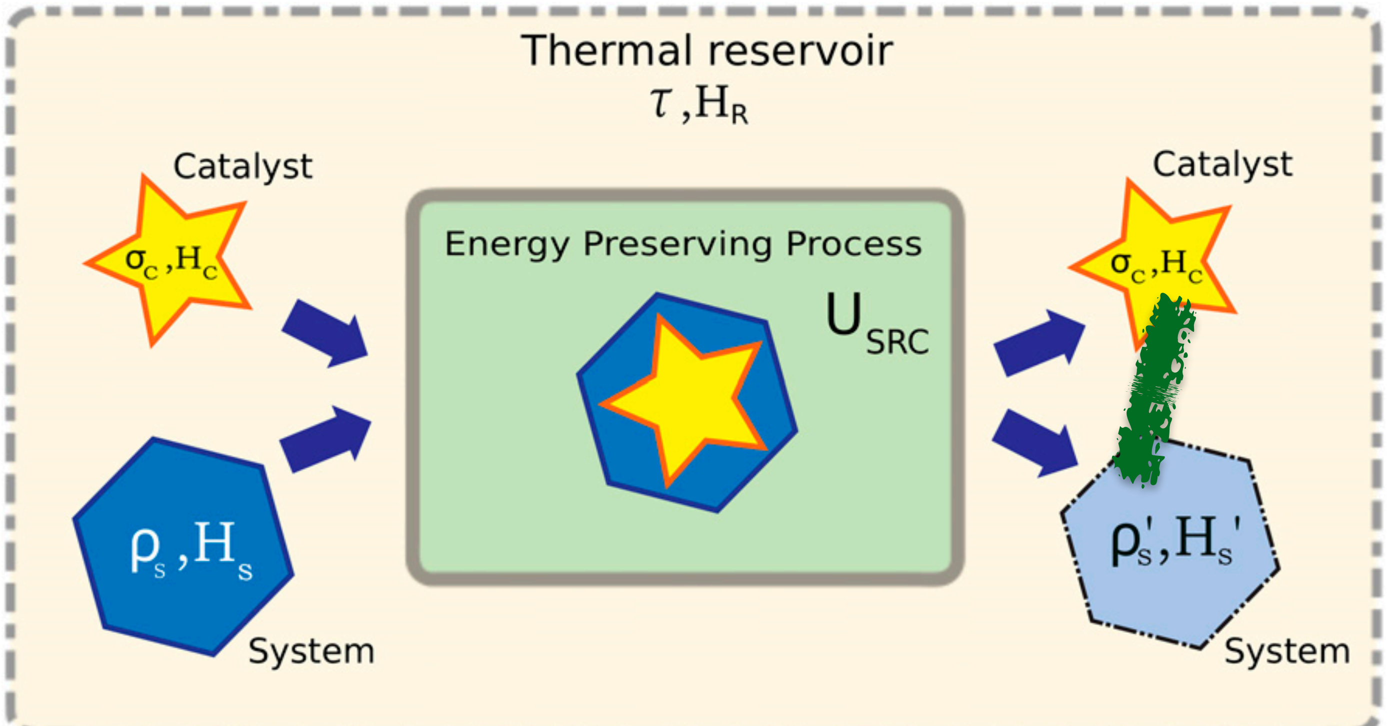

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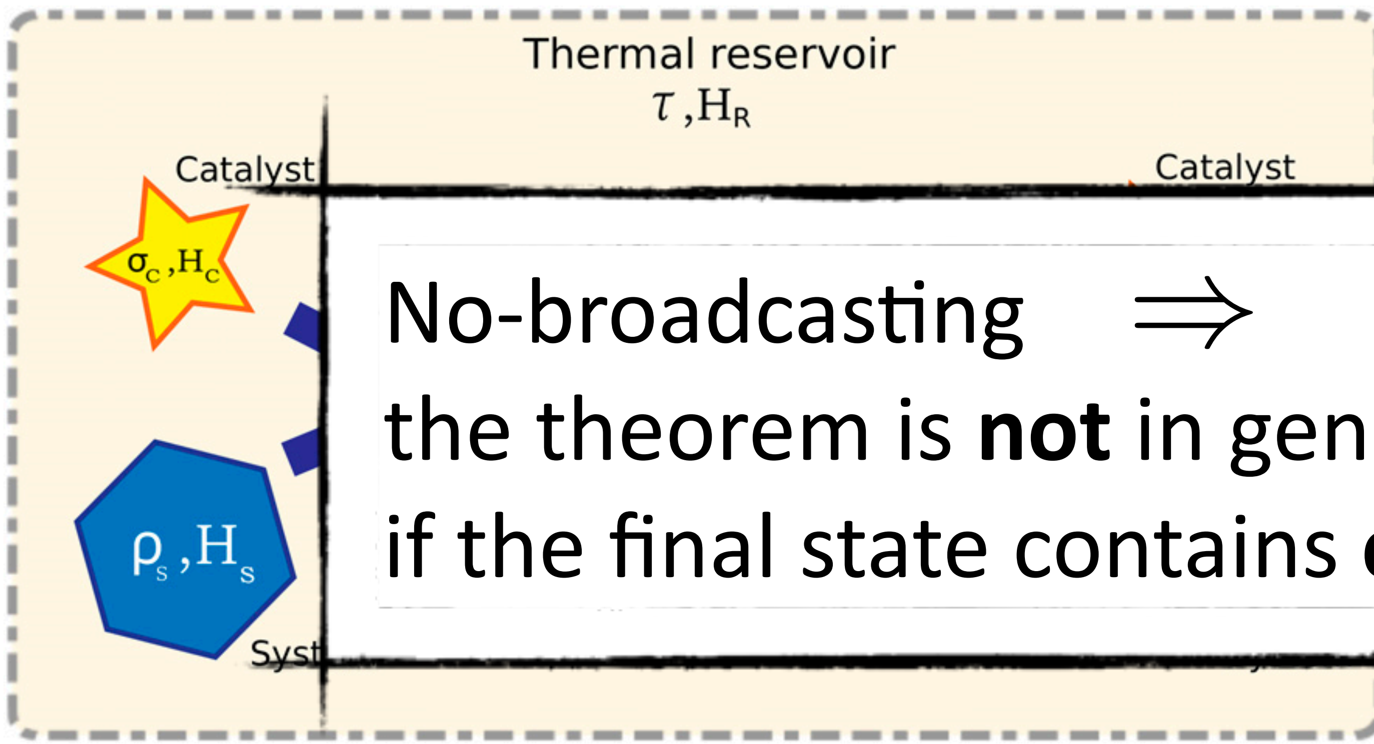

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No-broadcasting $\Rightarrow$
the theorem is **not** in general true
if the final state contains **coherence**.

…transition

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Conclusions

- Thermodynamics as a **resource theory**
- Fundamental irreversibility for work extraction/cost; "**second laws**". Correlations restore unique 2nd law.
- **Coherence** introduces additional constraints; related to reference frames for timing info ("clocks").

Own work:

- MM, *Correlating thermal machines and the second law at the nanoscale*, Phys. Rev. X **8**, 041051 (2018); arXiv:1707.03451.
- M. Lostaglio and MM, *Coherence and asymmetry cannot be broadcast,* Phys. Rev. Lett. **123**, 020403 (2019); arXiv:1812.08214.

Thank you!