Exact operational interpretation of entropy and free energy without the thermodynamic limit

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IQI Vienna

Outline

1. Resource-theoretic approach to thermodynamics

2. Single-shot interpretation of von Neumann entropy and free energy (block-diagonal states)

3. A no-broadcasting theorem for coherence and asymmetry

4. Conclusions

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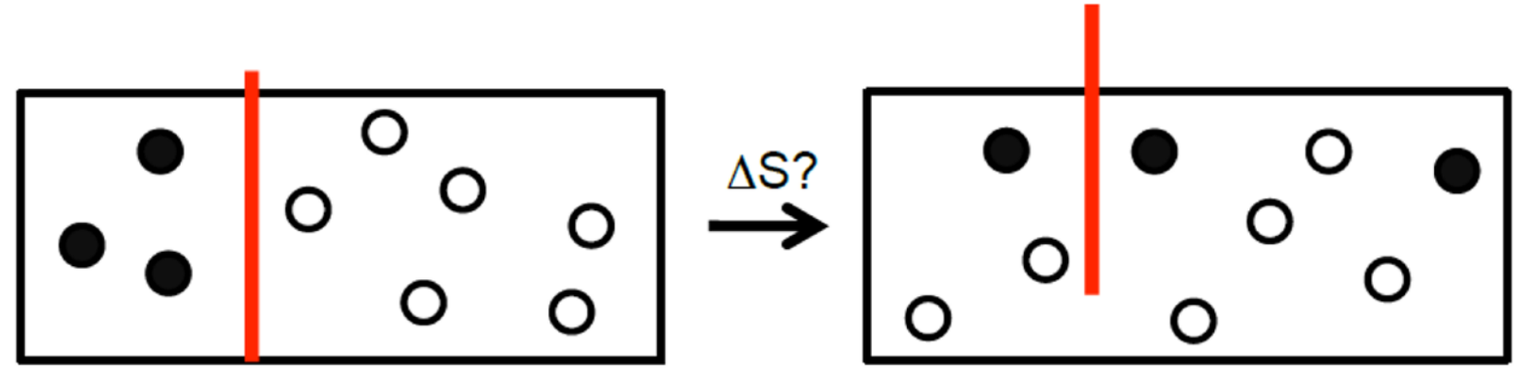
1. **Resource-theoretic approach to thermodynamics**

2. Single-shot interpretation of von Neumann entropy and free energy (block-diagonal states)

3. A no-broadcasting theorem for coherence and asymmetry

4. Conclusions

Standard view: thermodynamic limit



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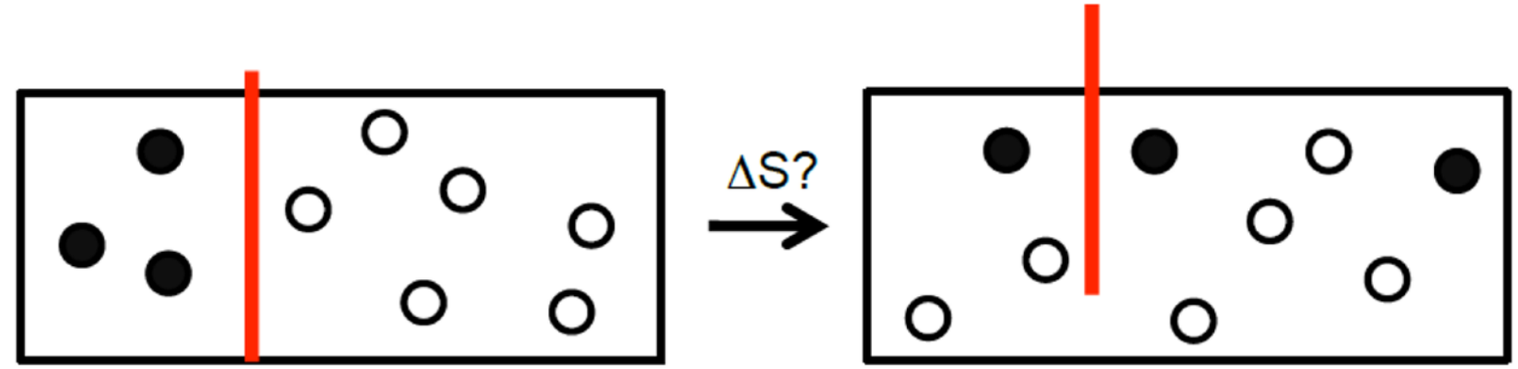

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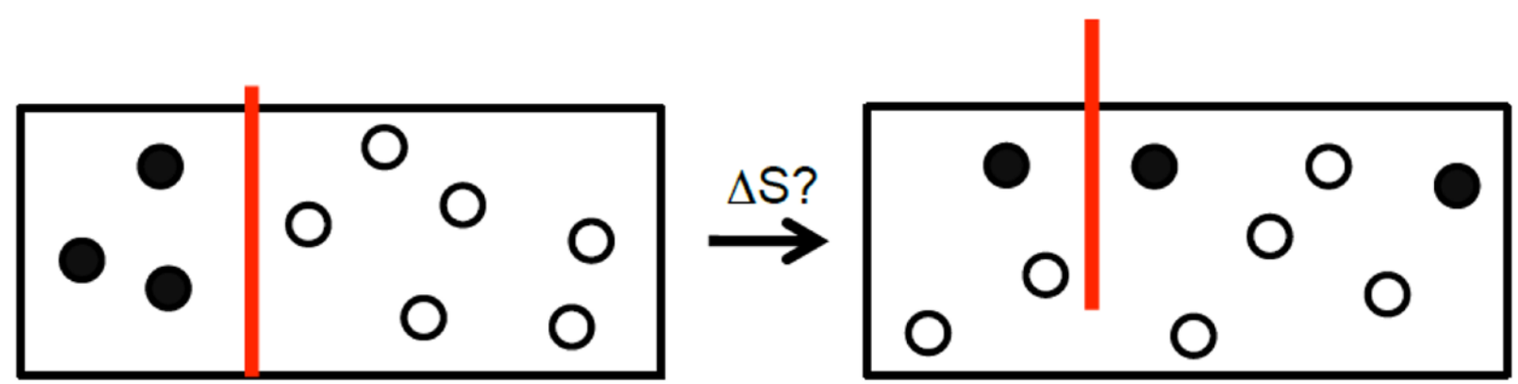
$$\Delta F \leq 0 \qquad \text{(2nd law)},$$

where $F = U - TS$.

If this is negative, then we can extract $|\Delta F|$ of work from the system.



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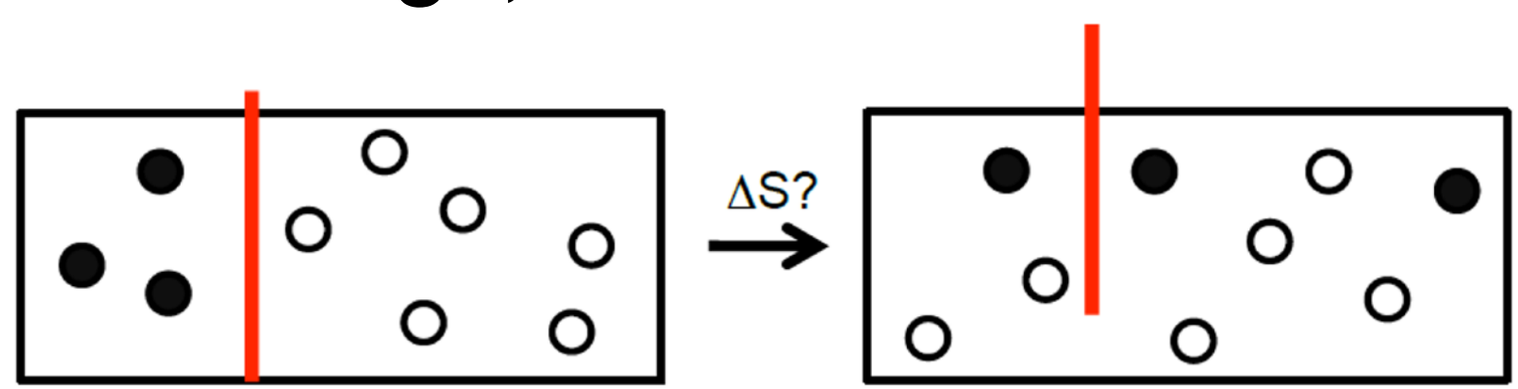
But this is a statement **on average**, since "work" is a random variable.



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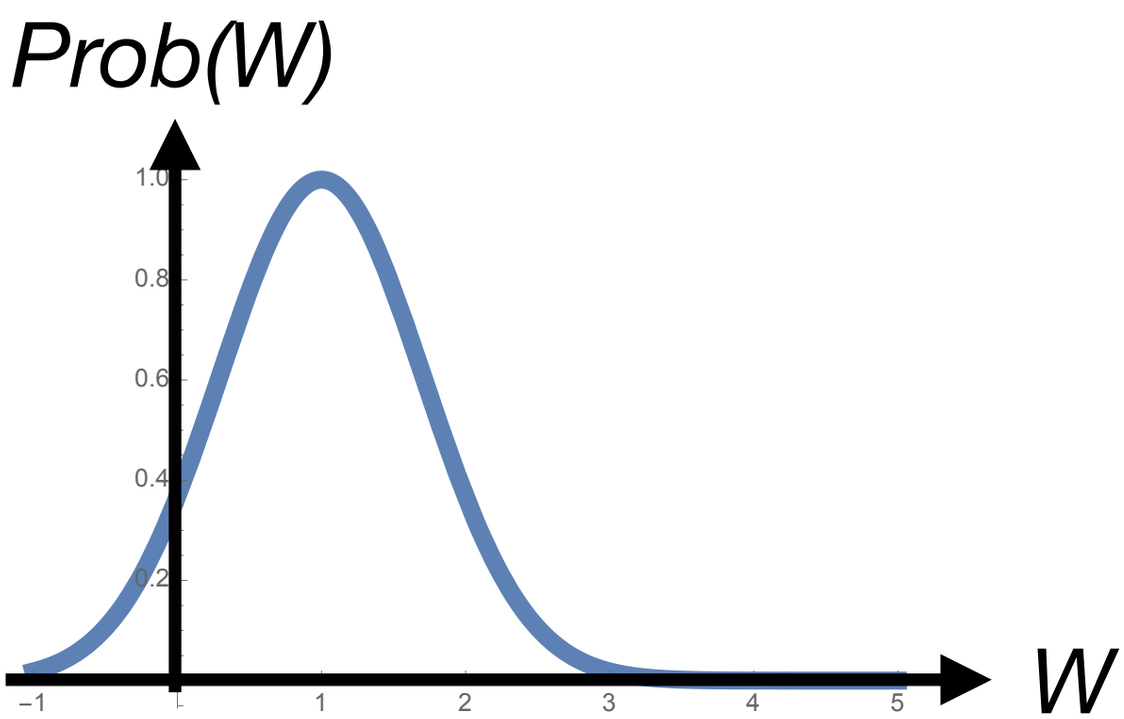
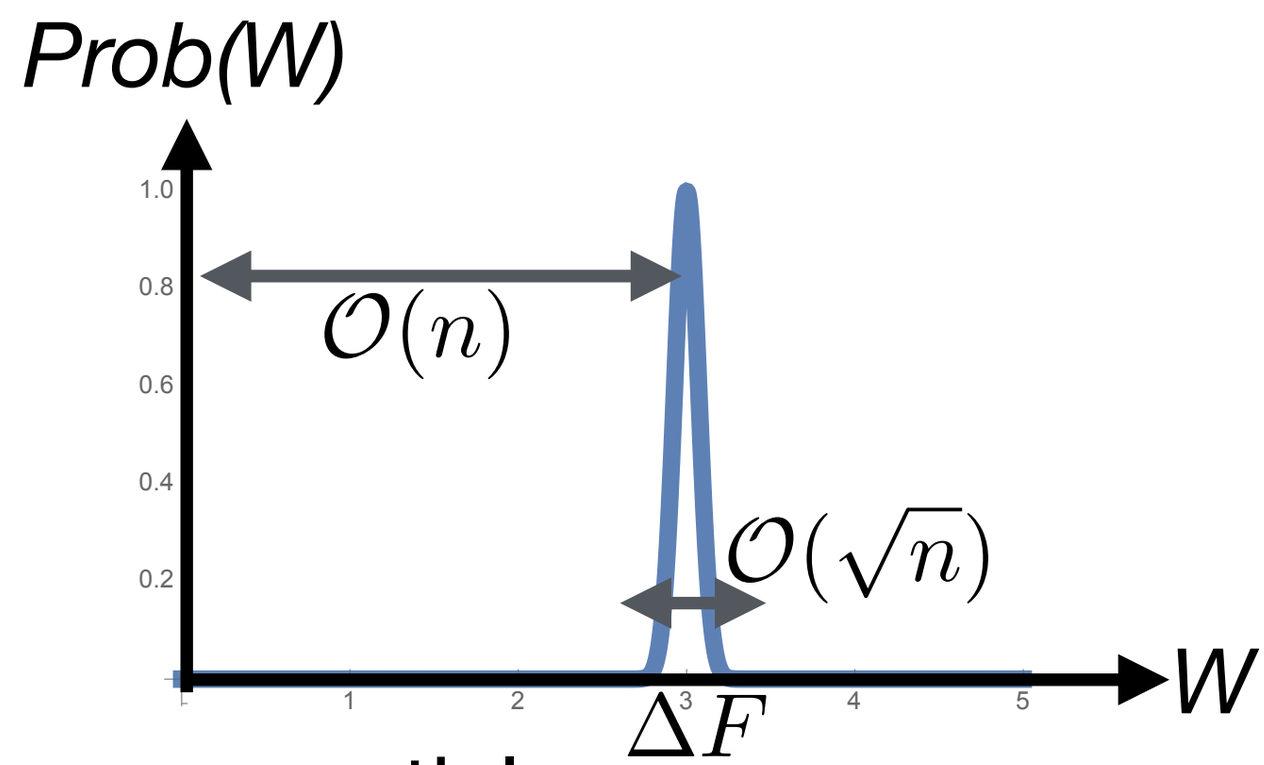


small # of particles



n particles

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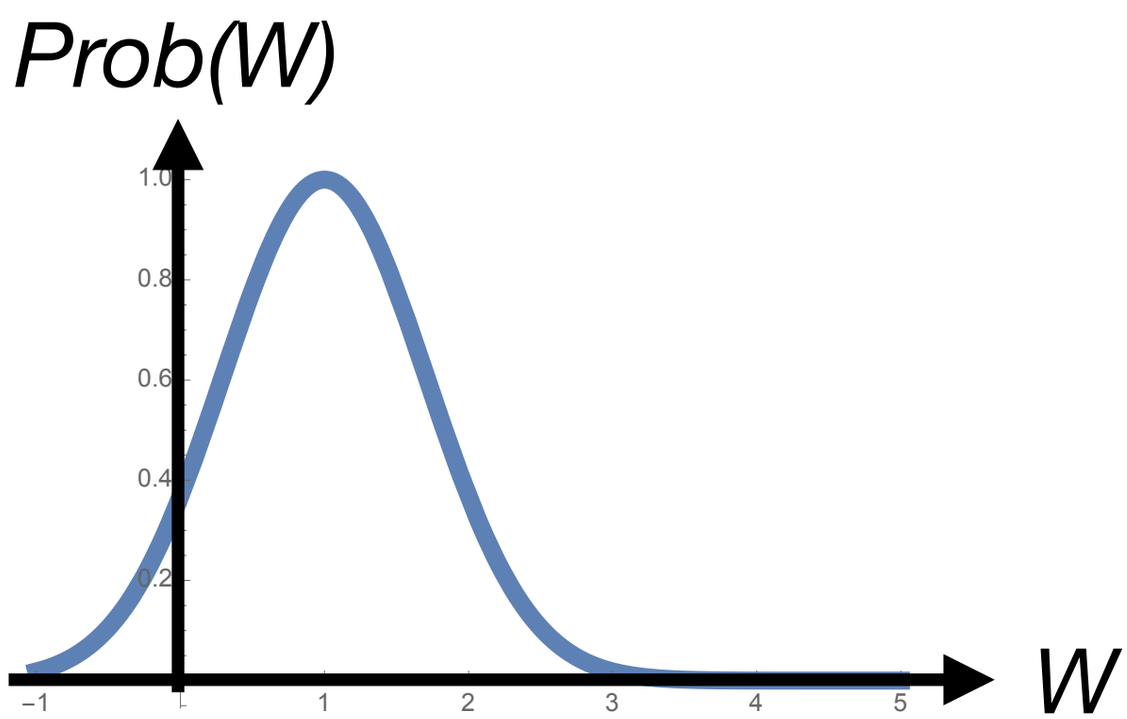

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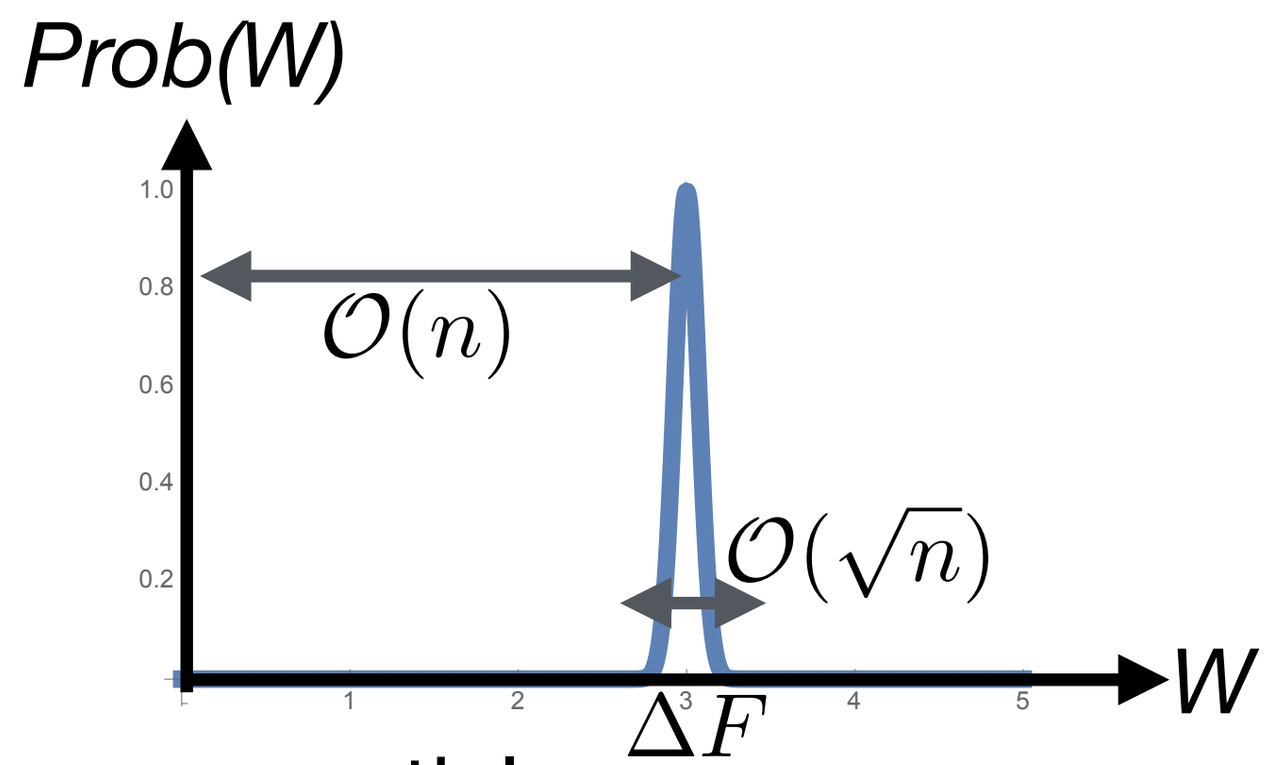

n particles

Extractable work "is" (optimally) ΔF :
only true in the thermodynamic limit $n \to \infty$
when fluctuations become irrelevant (law of large numbers).

Standard view: thermodynamic limit

But what do we do for “small” (quantum?) or strongly correlated systems? Work $\approx$ its fluctuations $\longrightarrow$ reliability?

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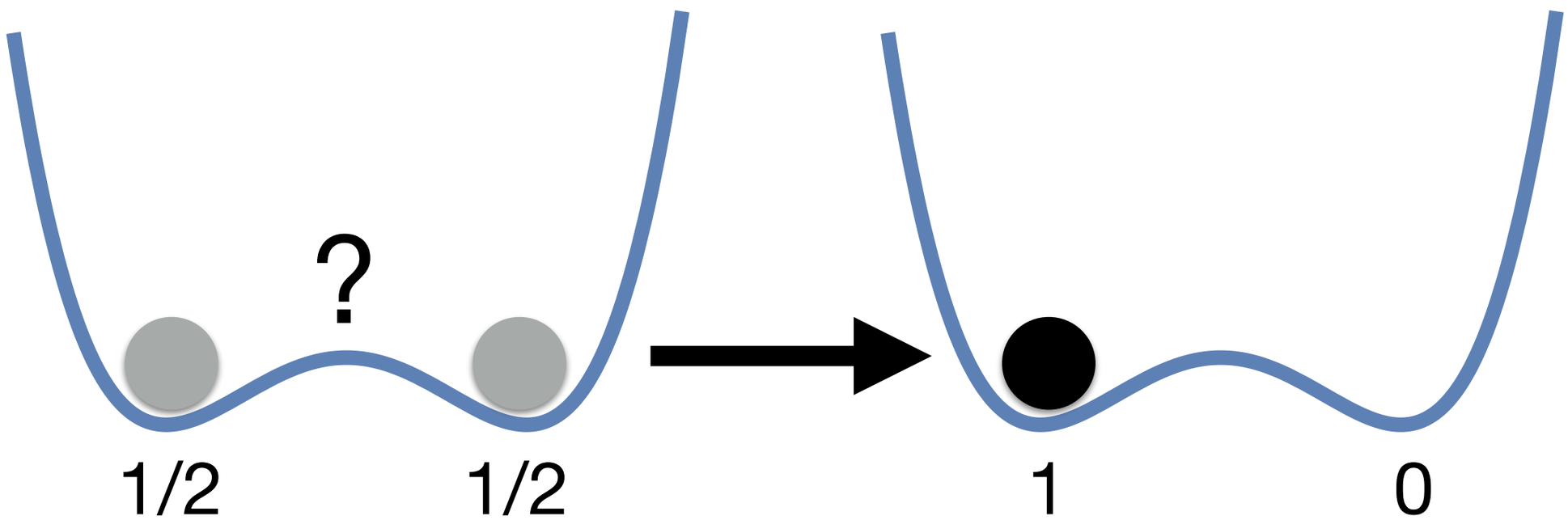
Landauer erasure:



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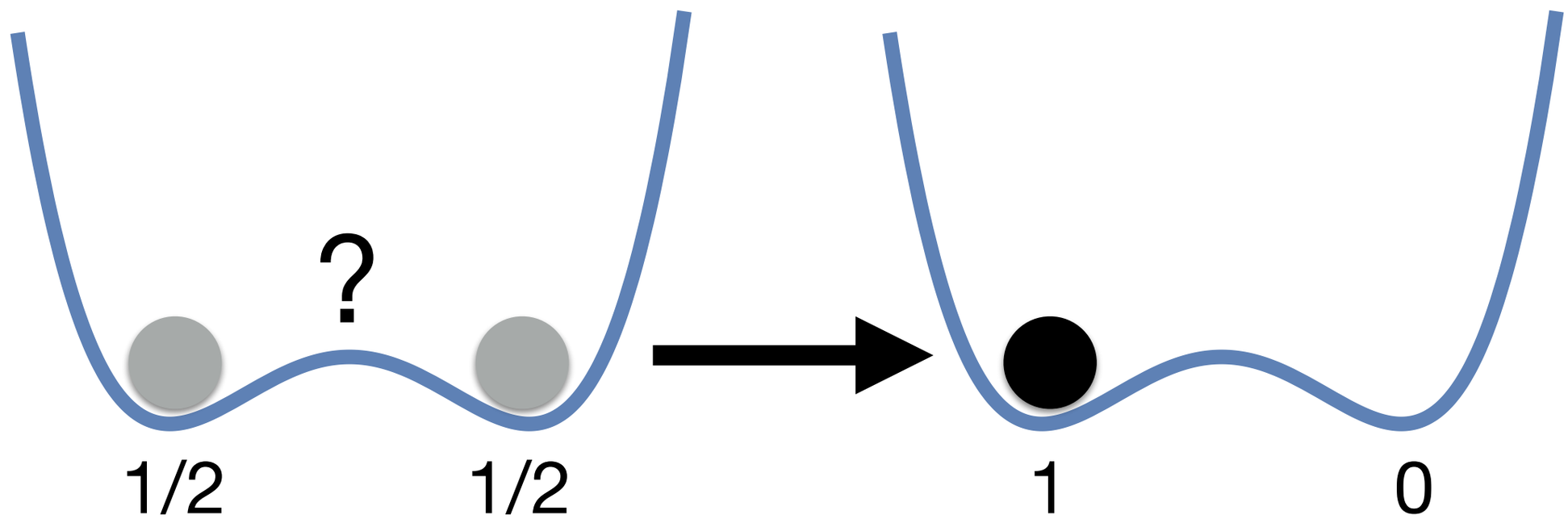

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But: Bennett’s puzzle: $\left(\frac{1}{2}, \frac{1}{2}, 0, \ldots, 0\right) \longrightarrow \left(1-\epsilon, \frac{\epsilon}{N}, \frac{\epsilon}{N}, \ldots, \frac{\epsilon}{N}\right)$

has $\Delta S > 0 \Rightarrow \Delta F < 0$ but should be impossible

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Landauer

e

Free energy *F* determines possibility of state transitions **only in the thermodynamic limit.** For “small” systems, resource theory formulation gives **additional constraints** (and solves Bennett’s puzzle). More soon.

In

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Thermodynamics as a resource theory

The rules of the game:

- It is "free" to bring in any system *B* in its thermal state $\gamma_B = \exp(-H_B/(k_B T))$,
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Question: Which transitions (work extraction etc.) are possible via thermal operations?

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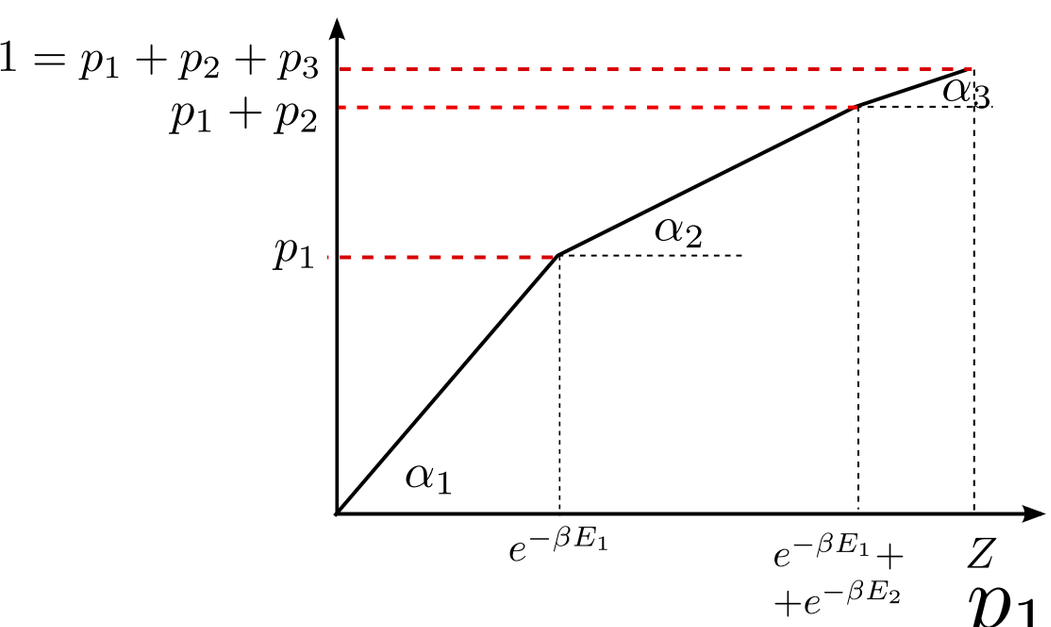

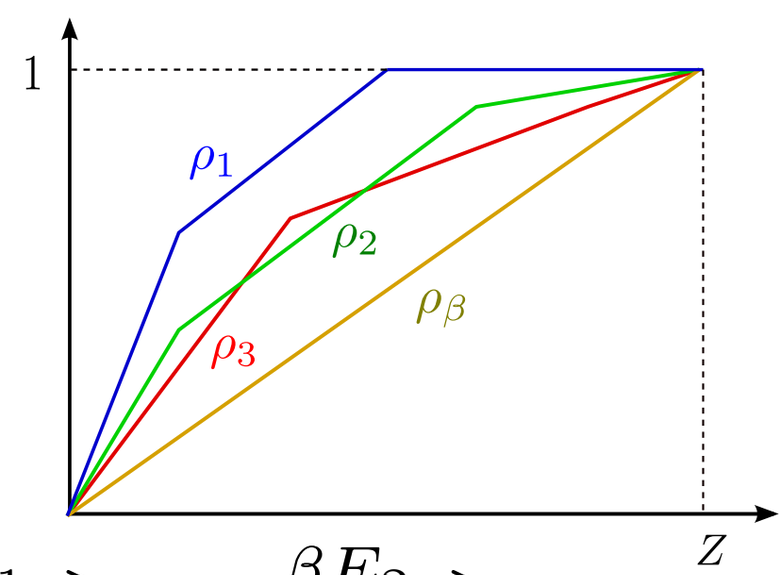

$$p_1 e^{\beta E_1} \geq p_2 e^{\beta E_2} \geq \ldots$$

Thermodynamics as a resource theory

The second laws of quantum thermodynamics

Fernando Brandão[a,1], Michał Horodecki[b], Nelly Ng[c], Jonathan Oppenheim[c,d,2], and Stephanie Wehner[c,e]

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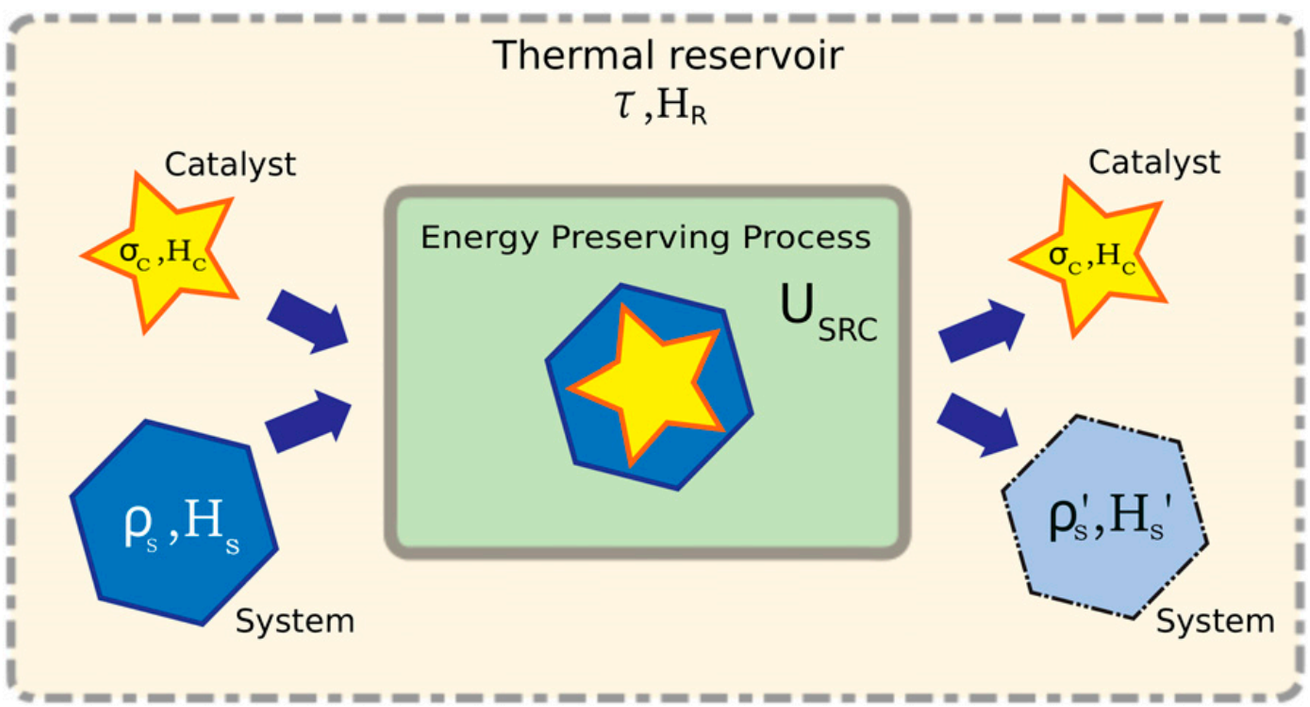

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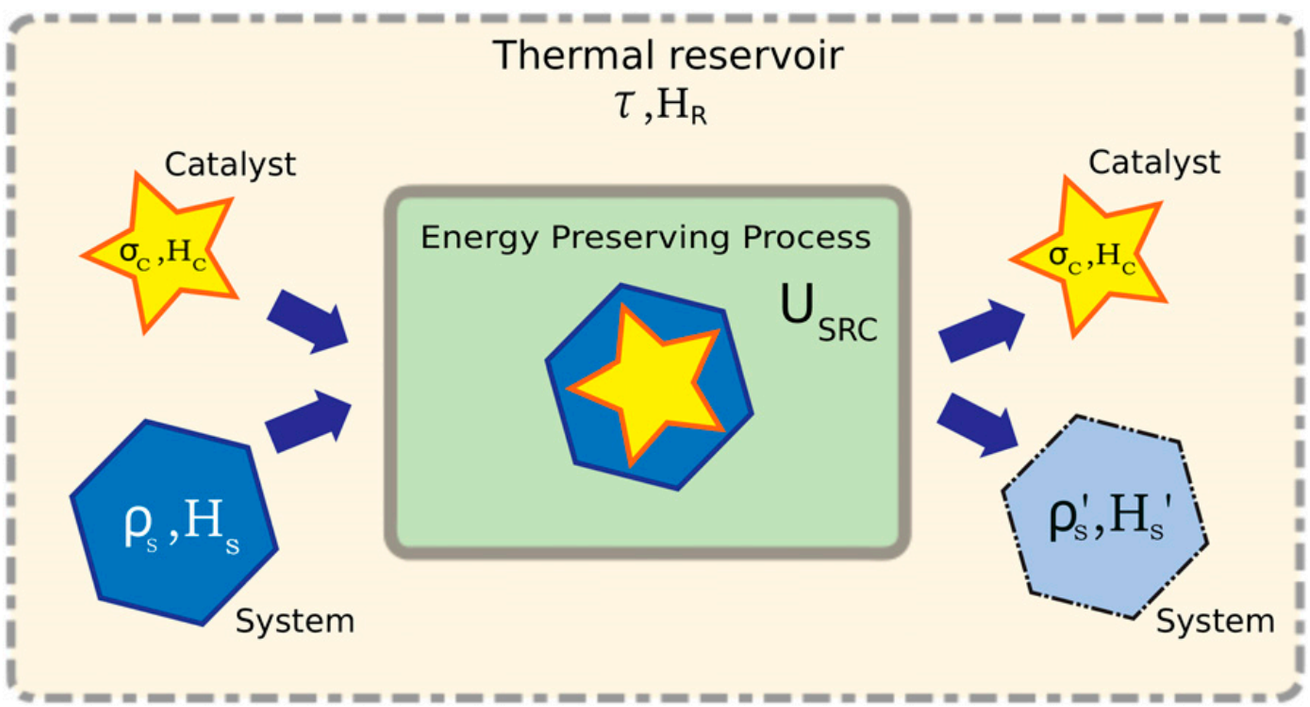

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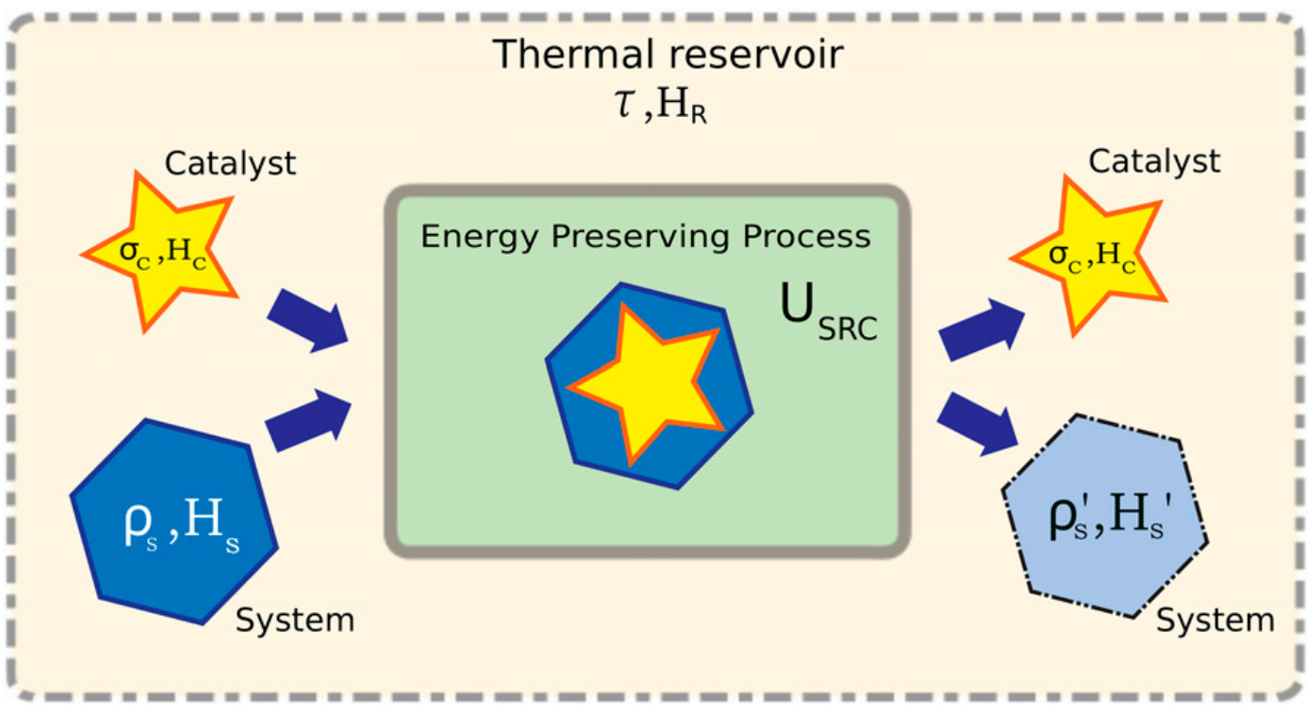

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thermal reservoir

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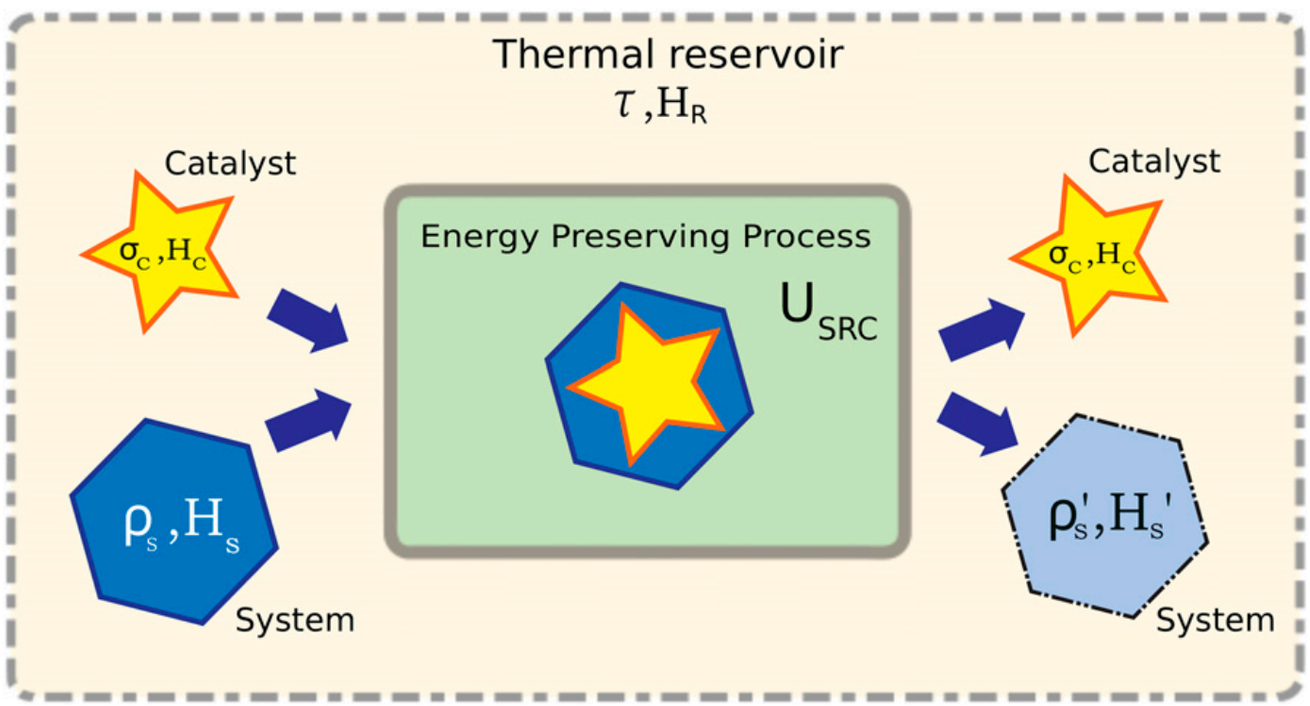

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thermal reservoir

$$[U_{SRC}, H_S + H_R + H_C] = 0$$

(energy strictly preserved)

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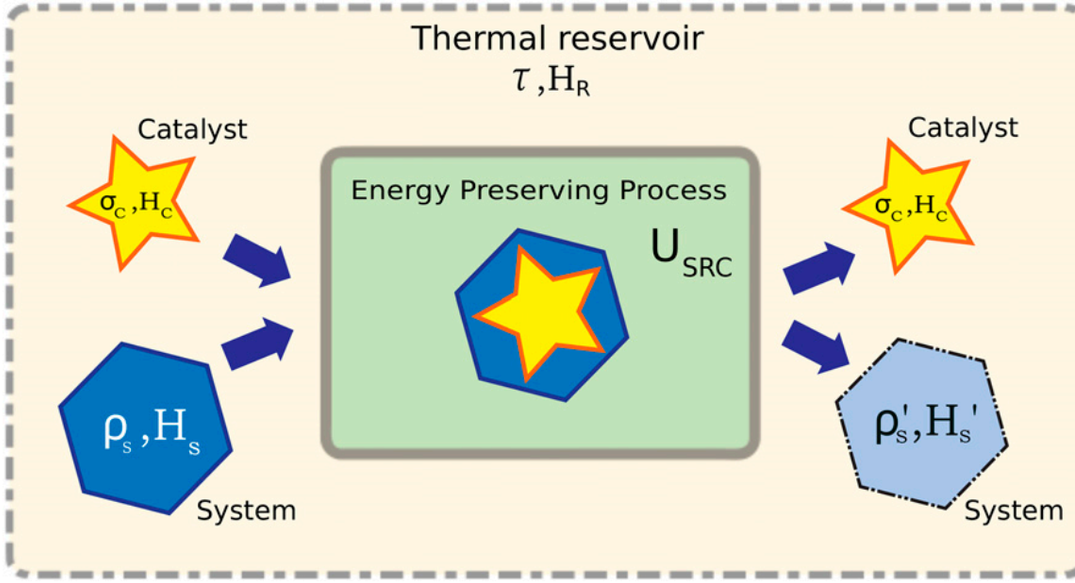

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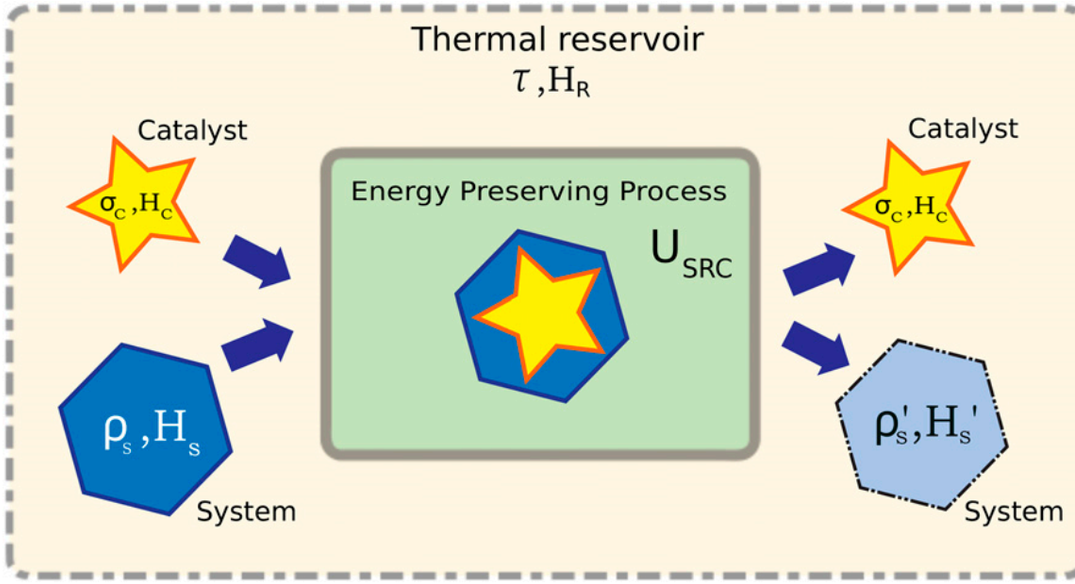
Theorem: $\rho \to \rho'$ is possible (for block-diagonal states) iff

$$F_\alpha(\rho) \geq F_\alpha(\rho') \;\; \forall \alpha$$

("free energies").



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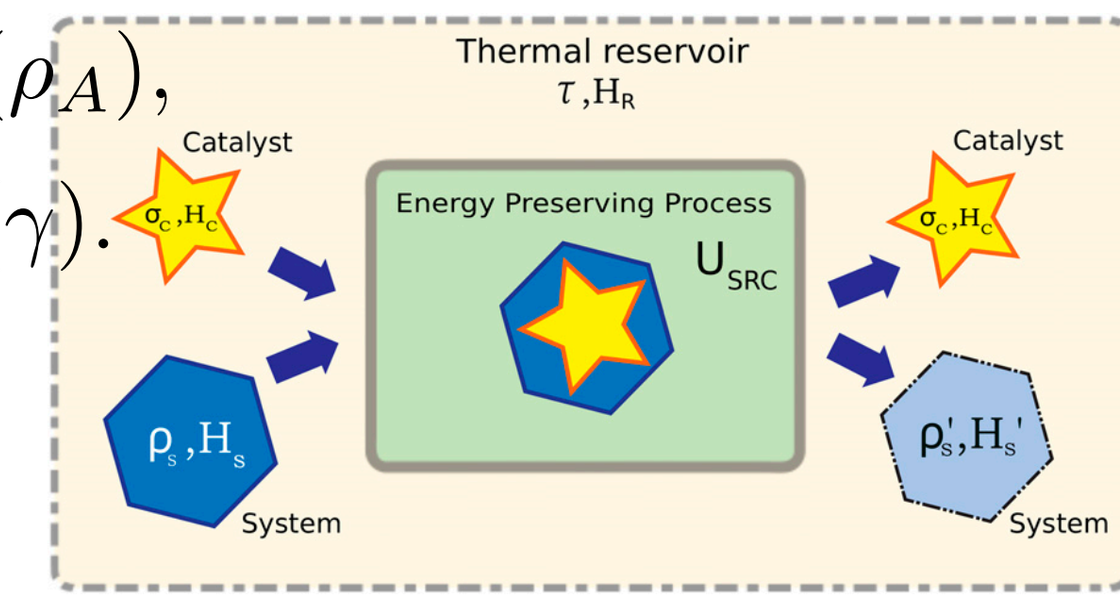
$$F(\rho_A) \equiv F_1(\rho) = \mathrm{tr}(\rho_A H_A) - k_B T S(\rho_A),$$

$$F_\alpha(\rho) = k_B T S_\alpha(\rho \| \gamma) + F_\alpha(\gamma).$$

Rényi divergence



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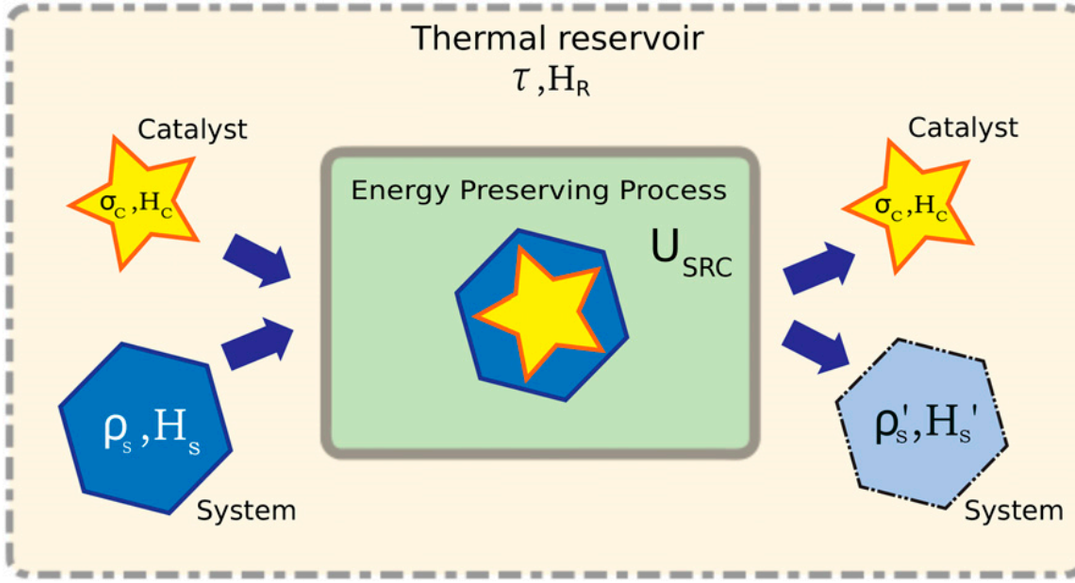

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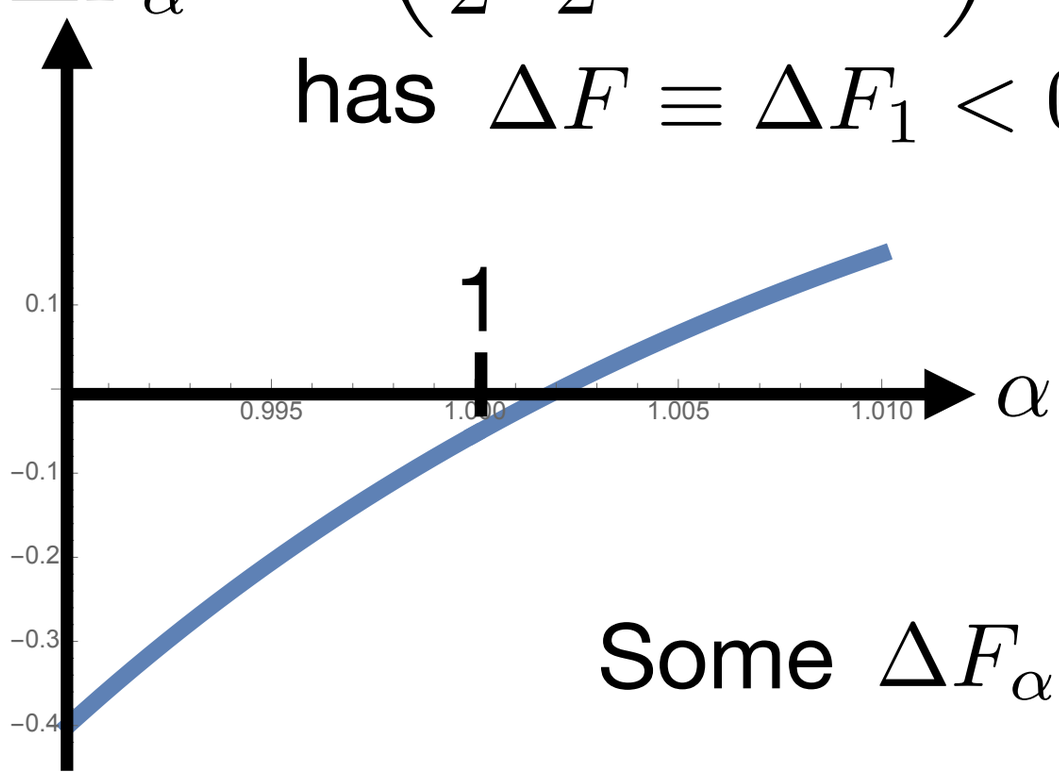

$$\epsilon = \frac{1}{100}, \quad N = 10^{30}.$$

Some $\Delta F_\alpha > 0$ hence **indeed impossible.**

How does the thermodynamic limit enter?

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Allowing small errors ε, we have

$$\frac{1}{n} F_\alpha^{(\varepsilon)}(\rho^{\otimes n}) \overset{n\to\infty}{\longrightarrow} F(\rho).$$

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$\Rightarrow$ For large numbers *n* of weakly interacting particles, it is only the free energy *F* that remains relevant.

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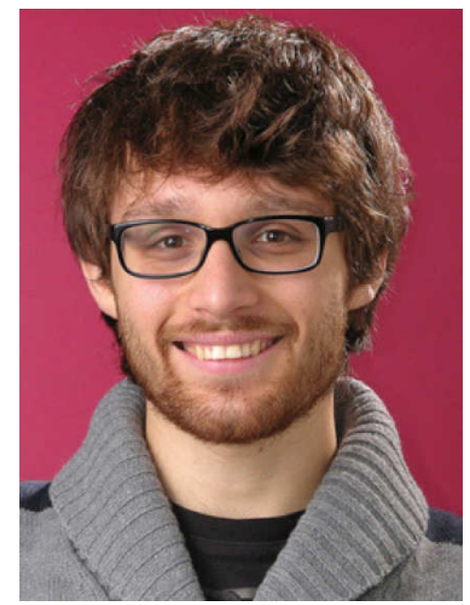
New one-shot interpretation of free energy

MM, arXiv:1707.03451, to appear in PRX

Building on earlier work with my students Jakob Scharlau and Michele Pastena, and with Matteo Lostaglio.



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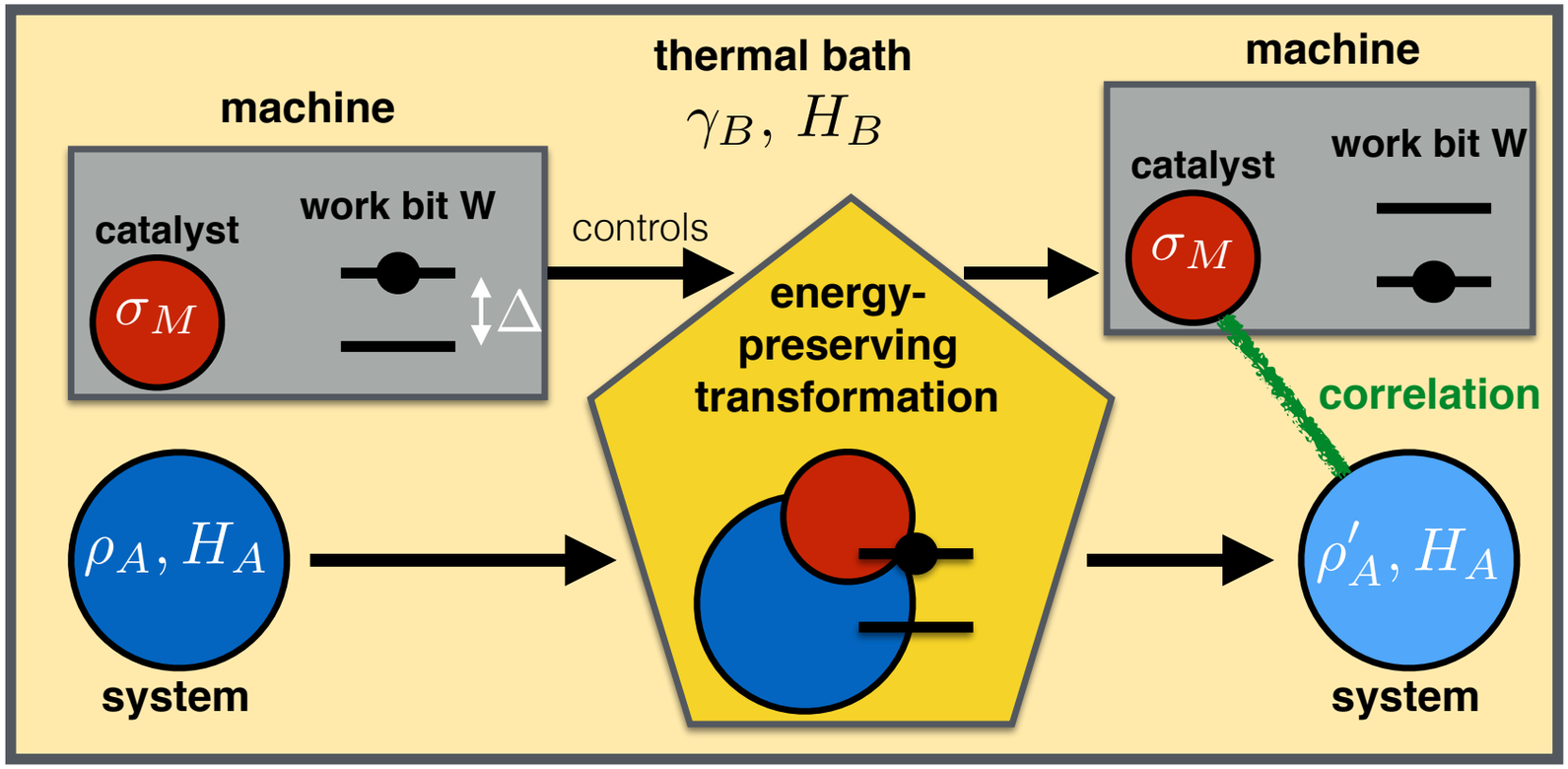

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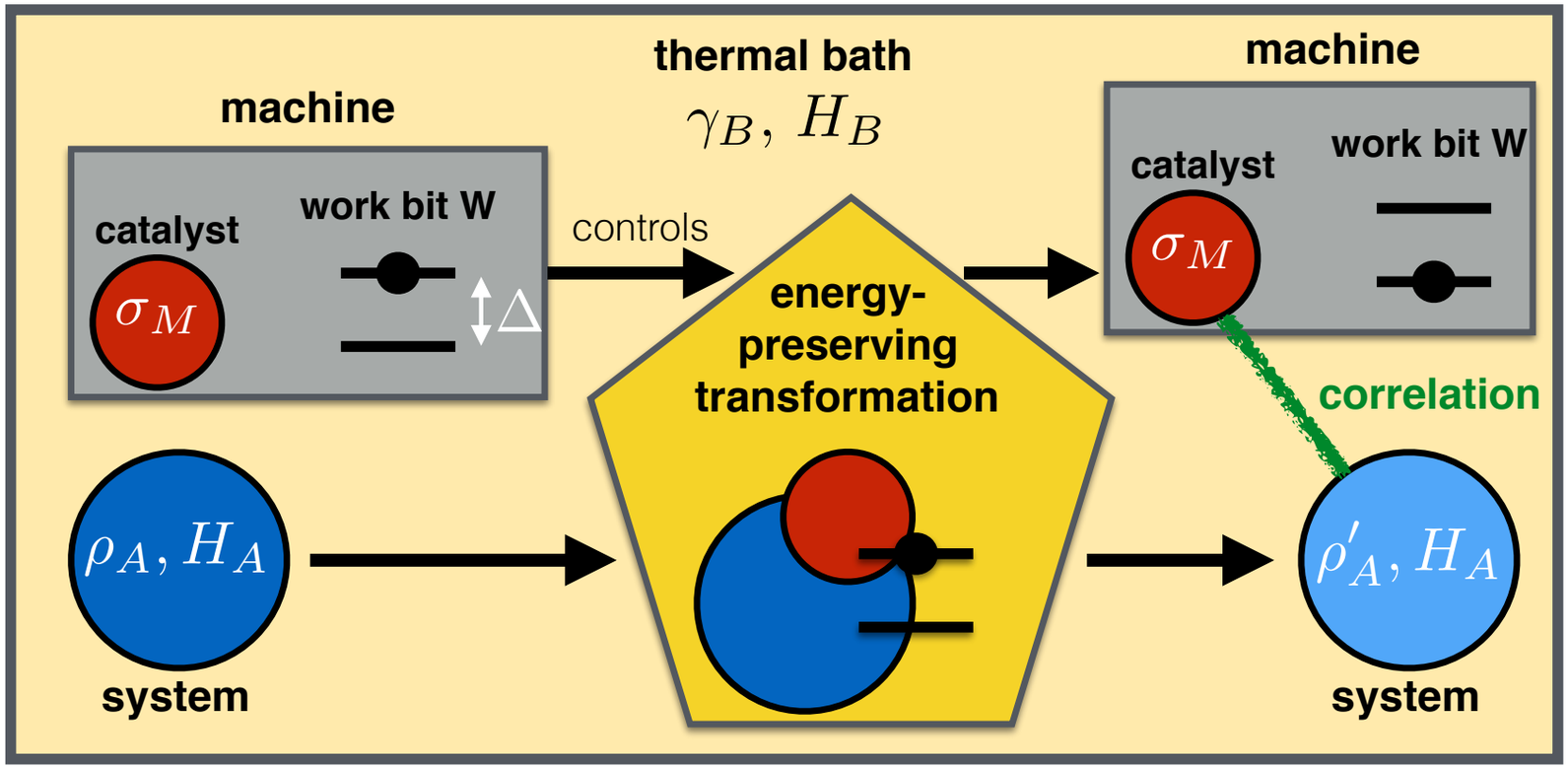

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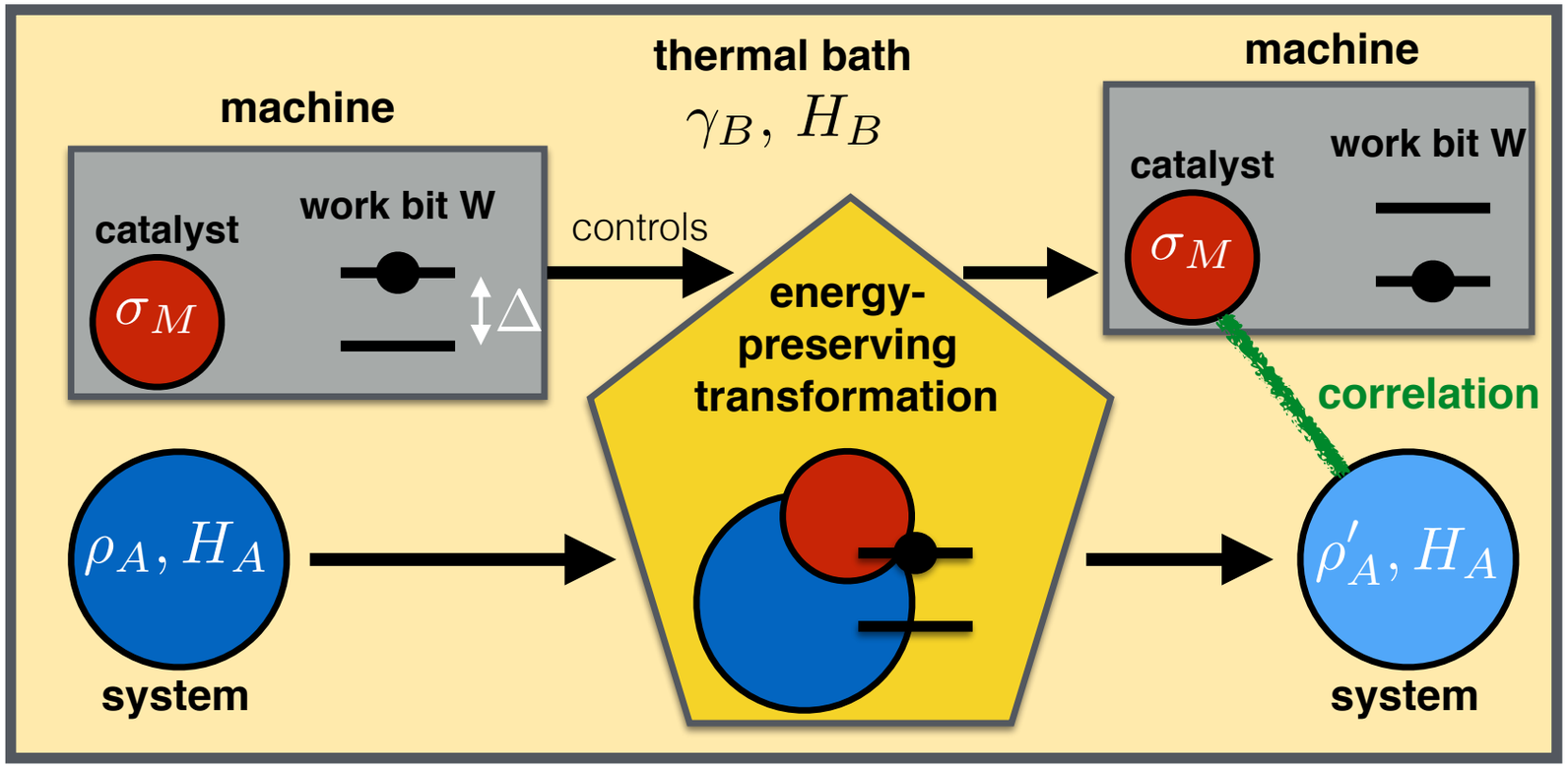

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thermal machine,
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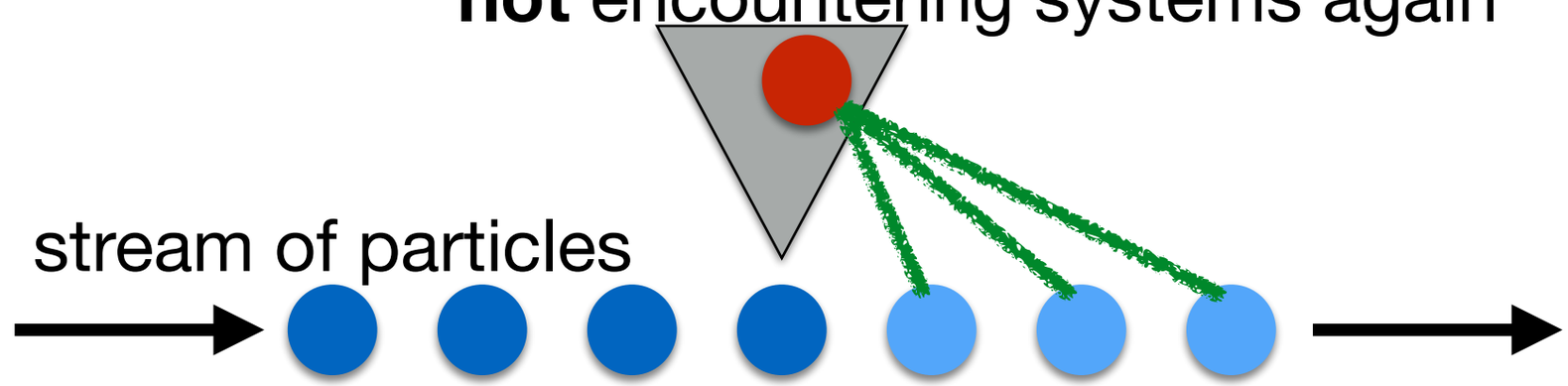

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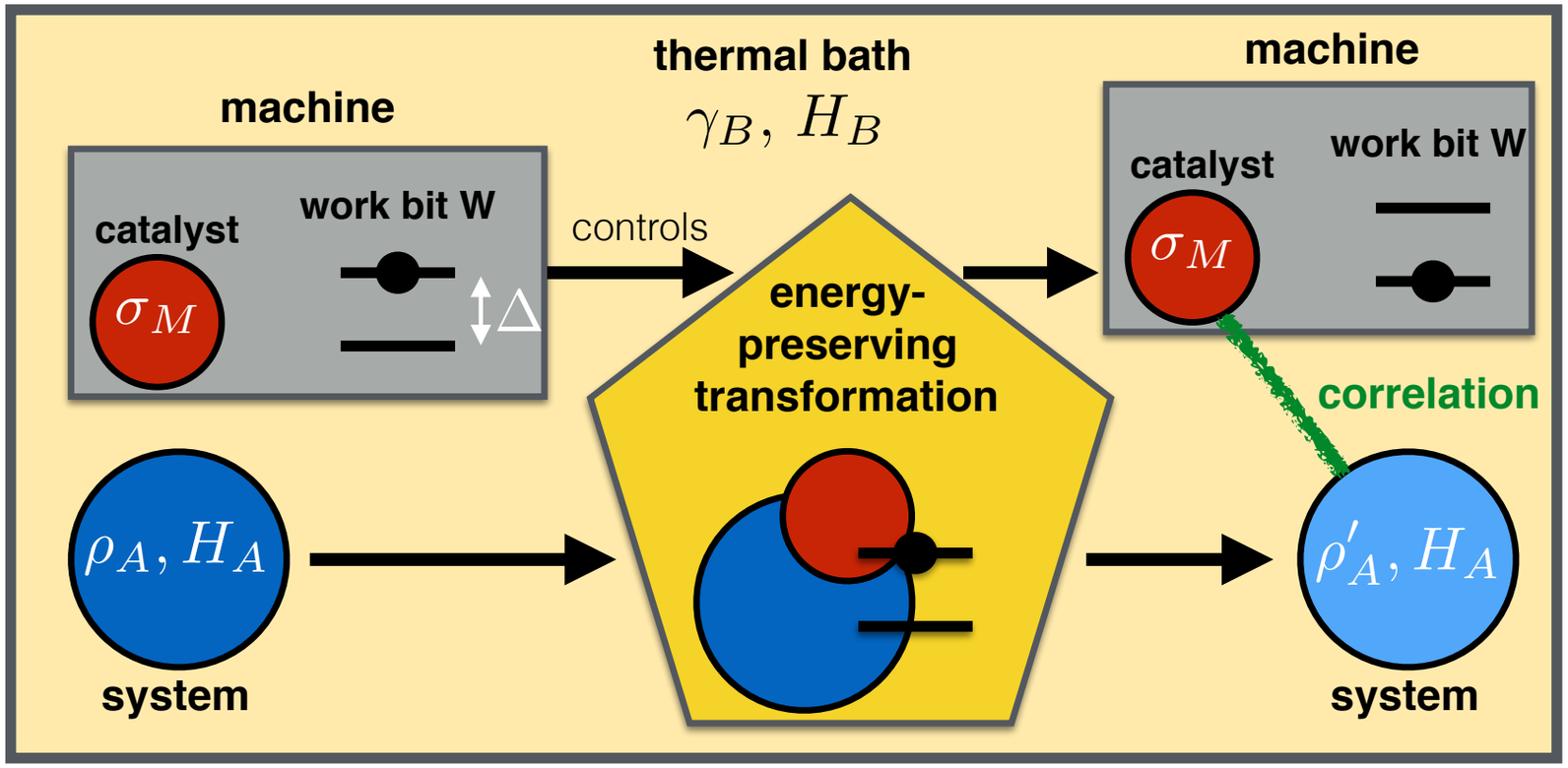

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Can this help? **Yes!**

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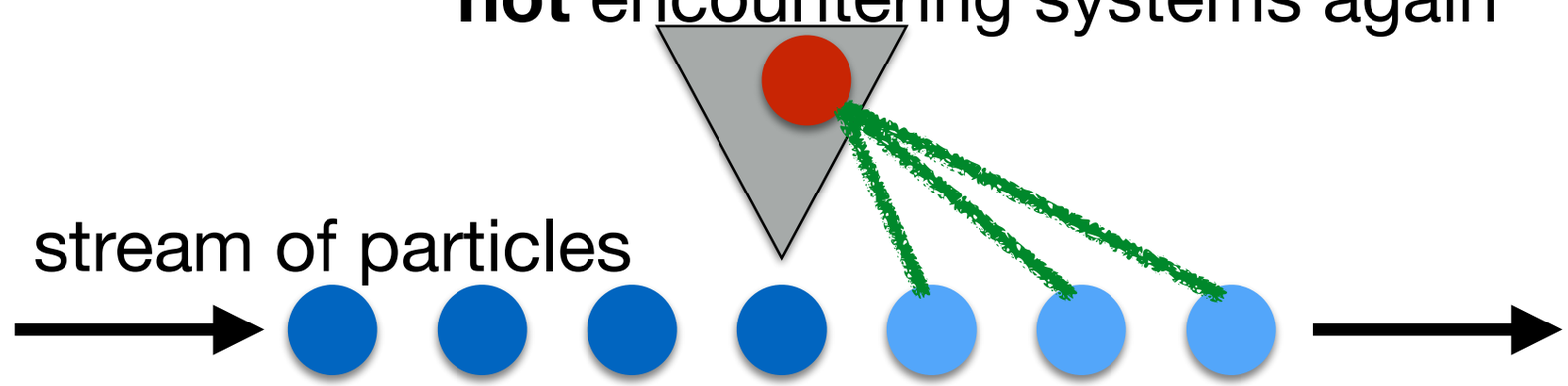

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Theorem. Let ρ_A, ρ'_A be block-diagonal states. Then, for every $\varepsilon > 0$, there is a thermal operation $\mathcal{T}_\varepsilon$, a state $\rho'_A(\varepsilon)$ with $\|\rho'_A - \rho'_A(\varepsilon)\| < \varepsilon$ and a finite-dimensional catalyst σ_C such that

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if and only if $F(\rho_A) \geq F(\rho'_A)$.

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"Single-shot" interpretation of $F(\rho) = \mathrm{tr}(\rho H) + k_B T\, \mathrm{tr}(\rho \log \rho)$.

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$F(\rho) = \mathrm{tr}(\rho H) + k_B T\, \mathrm{tr}(\rho \log \rho)$.

Notation: $\omega_{AC} = \rho'_A \sigma_C$ means that

$$\mathrm{Tr}_C \omega_{AC} = \rho'_A, \;\; \mathrm{Tr}_A \omega_{AC} = \sigma_C.$$

Main theorem

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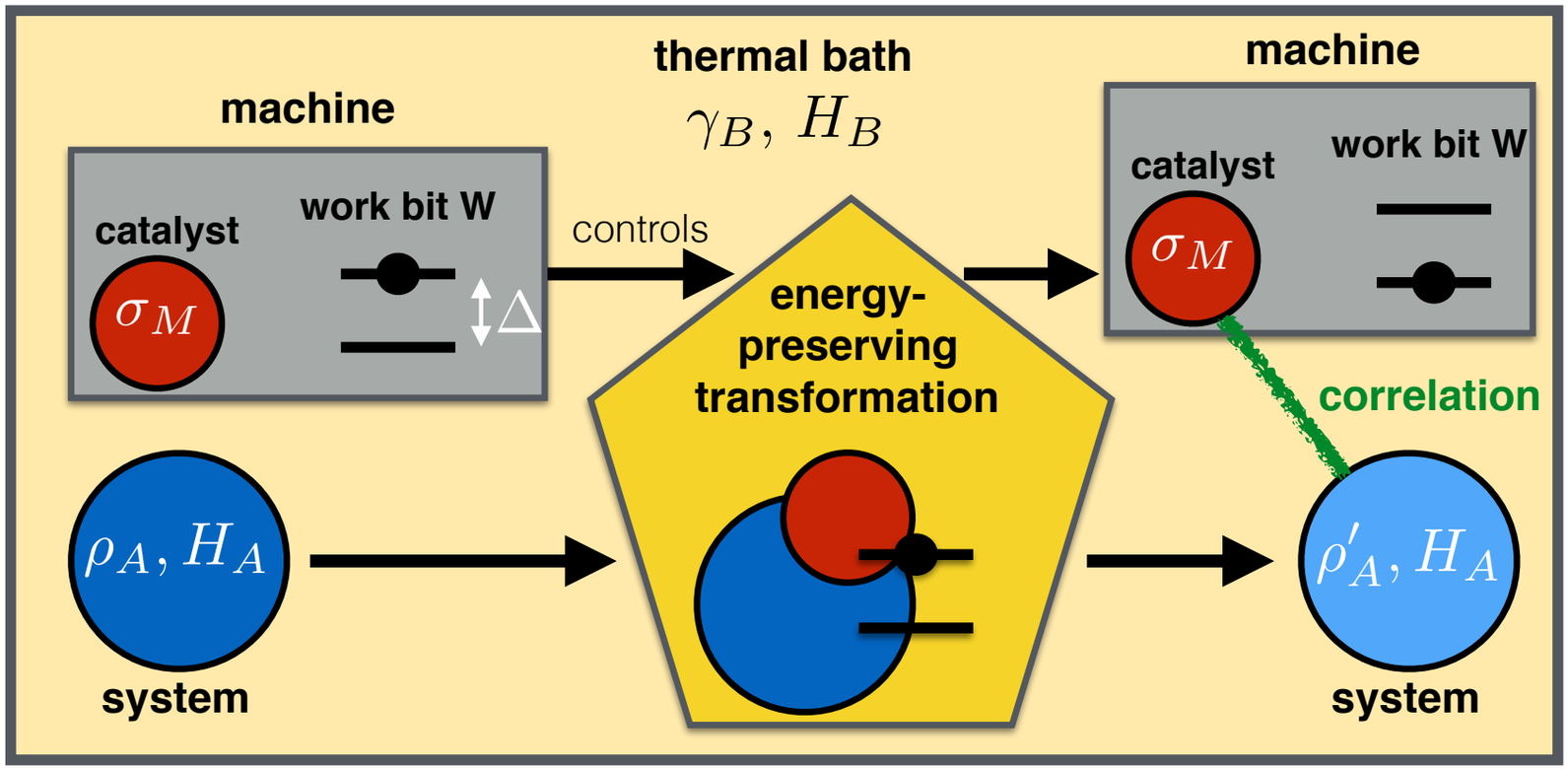

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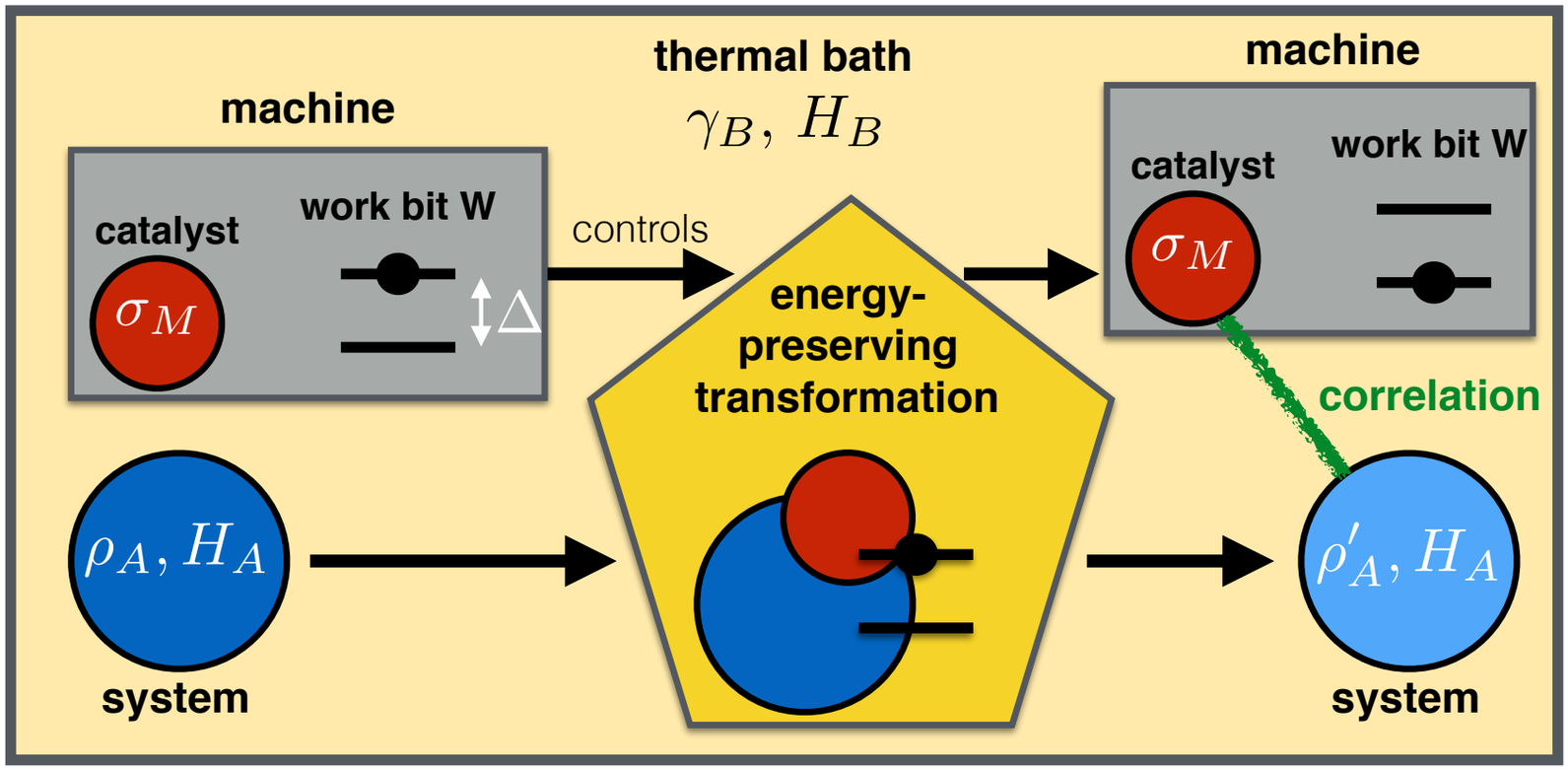

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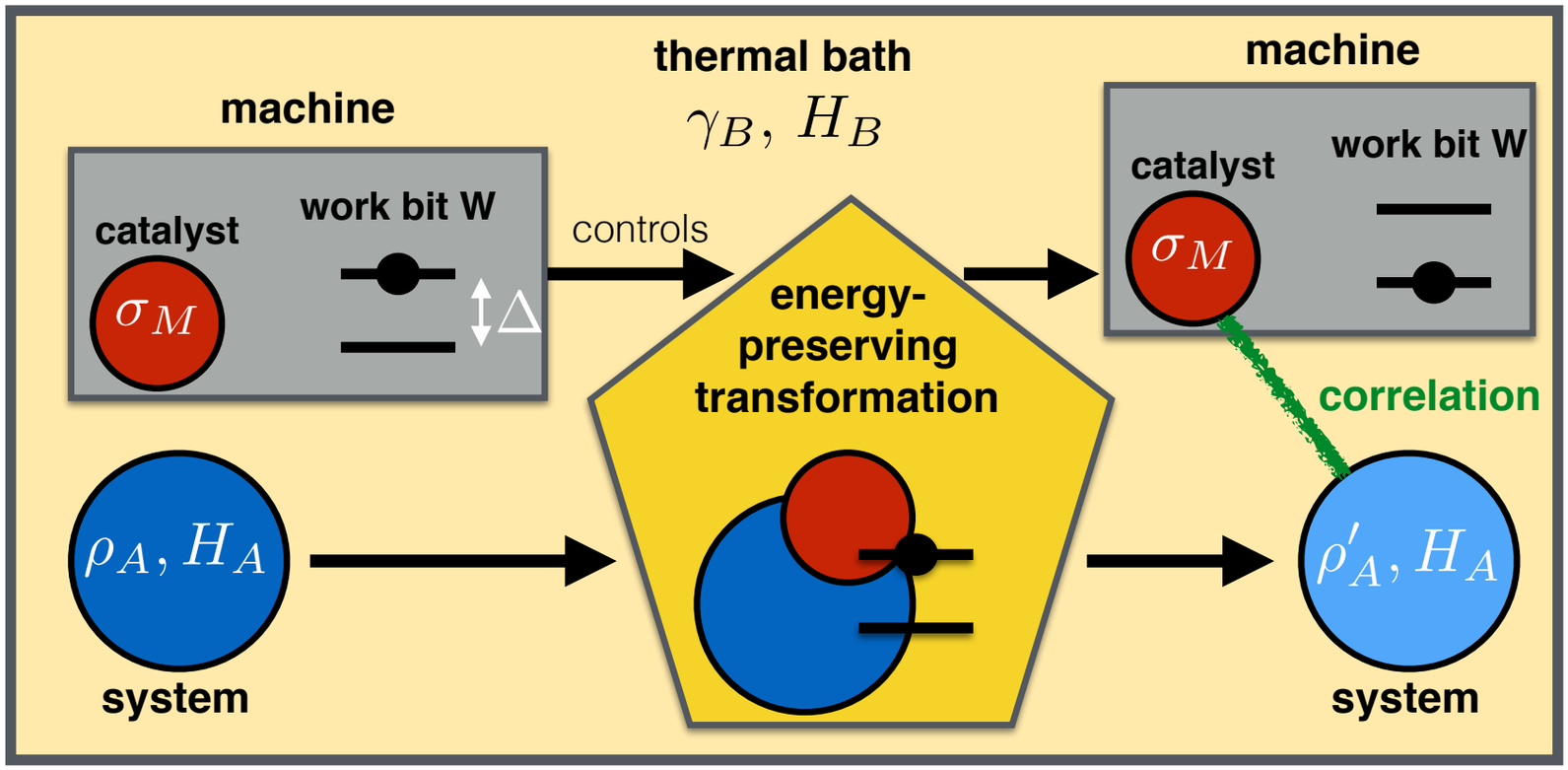

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- *Almost* fluct.-free **work extraction**: If $F(\rho_A) - F(\rho'_A) > \Delta > 0$, then transition possible while work bit (for arbitrary $\delta > 0$)

$$|g\rangle\langle g|_W \mapsto (1-\delta)|e\rangle\langle e|_W + \delta\,\mathbf{1}/d.$$

Outline

1. Resource-theoretic approach to thermodynamics

2. **Single-shot interpretation of von Neumann entropy and free energy (block-diagonal states)**

3. A no-broadcasting theorem for coherence and asymmetry

4. Conclusions

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1. Resource-theoretic approach to thermodynamics

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3. **A no-broadcasting theorem for coherence and asymmetry**

4. Conclusions

What if the states are **not block-diagonal?**

Theorem. Let ρ_A, ρ'_A be block-diagonal states. Then, for every $\varepsilon > 0$, there is a thermal operation $\mathcal{T}_\varepsilon$, a state $\rho'_A(\varepsilon)$ with $\|\rho'_A - \rho'_A(\varepsilon)\| < \varepsilon$ and a finite-dimensional catalyst σ_C such that

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Note that thermal operations are time-translation **covariant**:

$$\mathcal{T}_\varepsilon(\mathcal{U}_t(\rho_{AC})) = \mathcal{U}_t(\mathcal{T}_\varepsilon(\rho_{AC})), \text{ where } \mathcal{U}_t = e^{-itH} \bullet e^{itH}.$$

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Is it possible to have $\mathcal{T}(\rho_A \otimes \sigma_C) = \rho'_A \sigma_C,$

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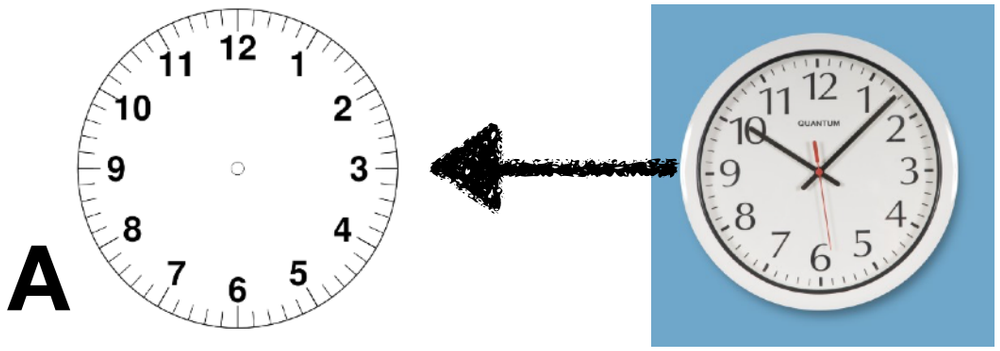
"Broadcasting coherence (=timing information) of C"



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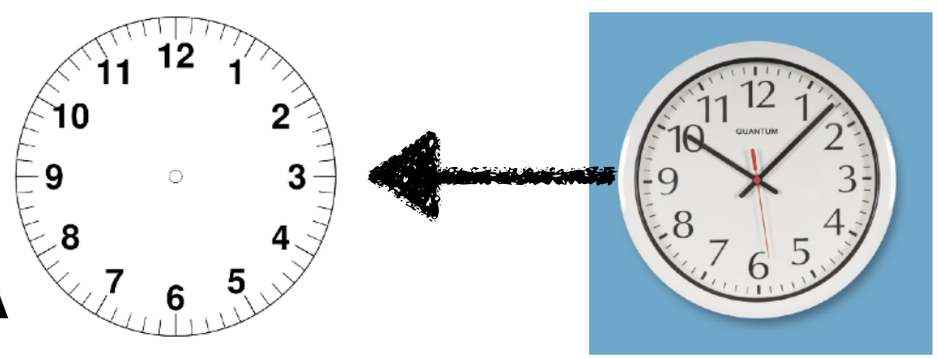
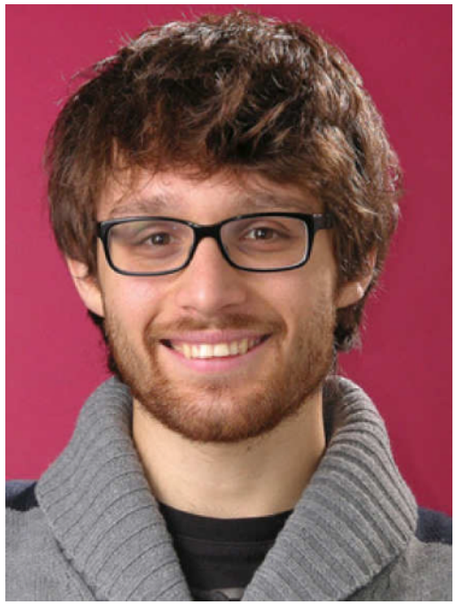


(finite-dimensional)

Is is **not obviously impossible**: cf. for the Fisher information,

$I(\rho_{AC}) \ll I(\rho_A) + I(\rho_C)$ in some cases.

No-broadcasting theorem for coherence and asymmetry



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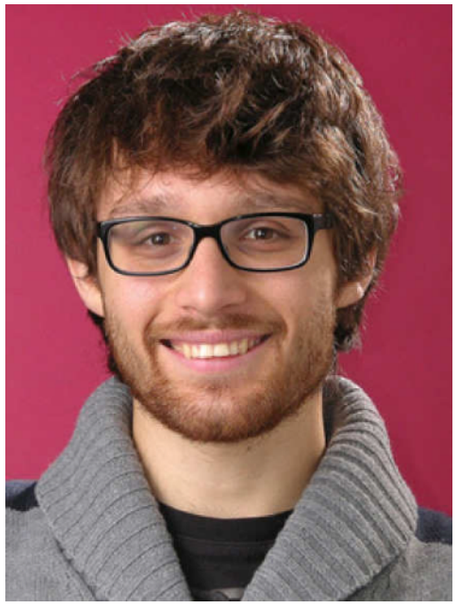

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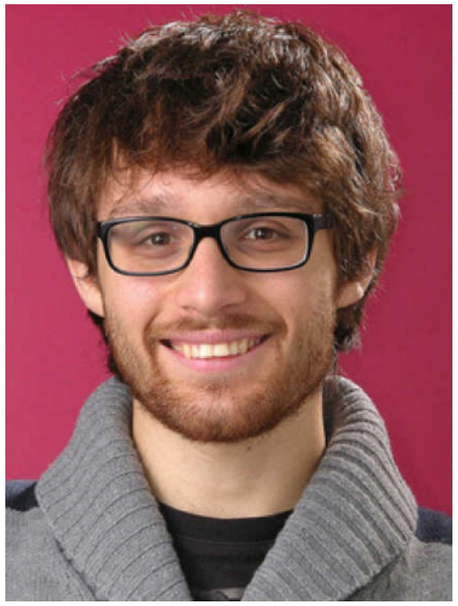

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Theorem. Let $\mathcal{G}$ be any connected Lie group. Then $\mathcal{G}$-asymmetry broadcasting is impossible.

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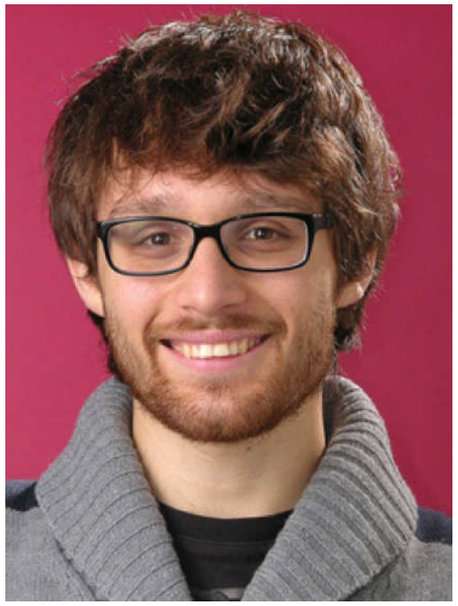

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Thus, result on free energy *cannot* hold in general for states with quantum coherence.

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Åberg has shown: a weak version of coherence broadcasting is possible with an infinite-dimensional “catalyst”.

PRL **113,** 150402 (2014) PHYSICAL REVIEW LETTERS week ending 10 OCTOBER 2014

Catalytic Coherence

Johan Åberg*
Institute for Physics, University of Freiburg, Hermann-Herder-Strasse 3, D-79104 Freiburg, Germany
(Received 10 December 2013; revised manuscript received 30 July 2014; published 7 October 2014)

Because of conservation of energy we cannot directly turn a quantum system with a definite energy into a superposition of different energies. However, if we have access to an additional resource in terms of a system with a high degree of coherence, as for standard models of laser light, we can overcome this

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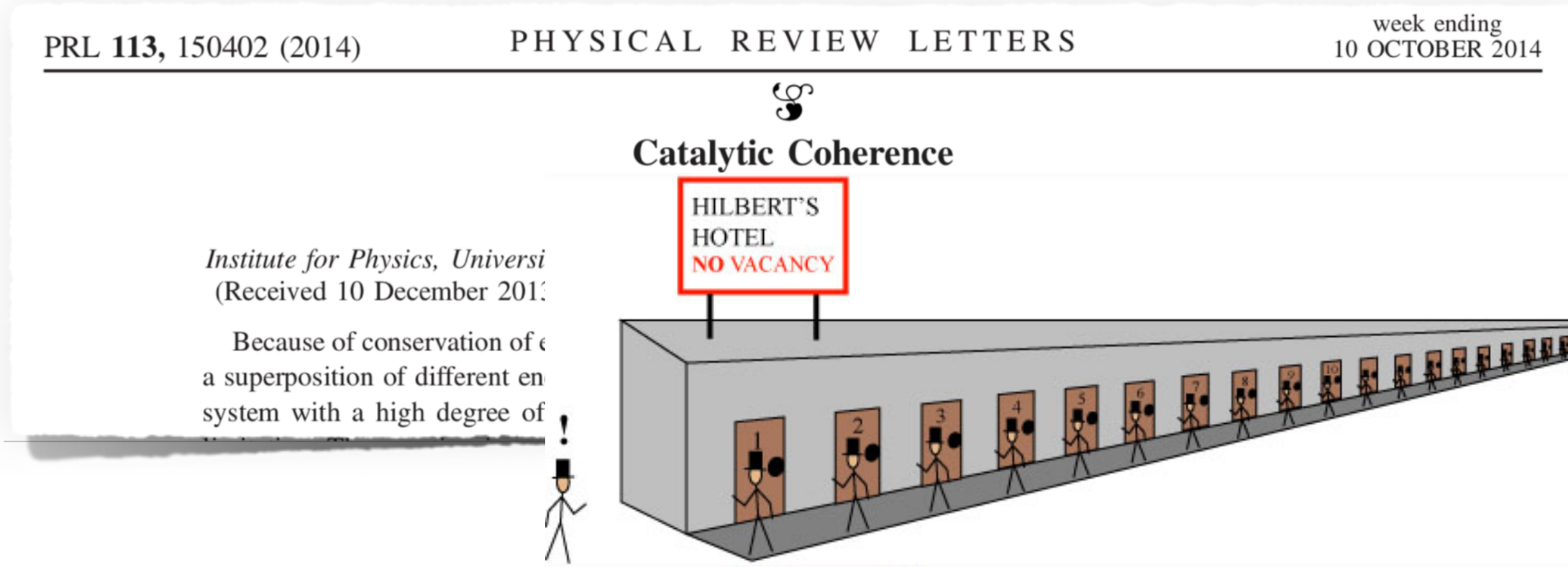
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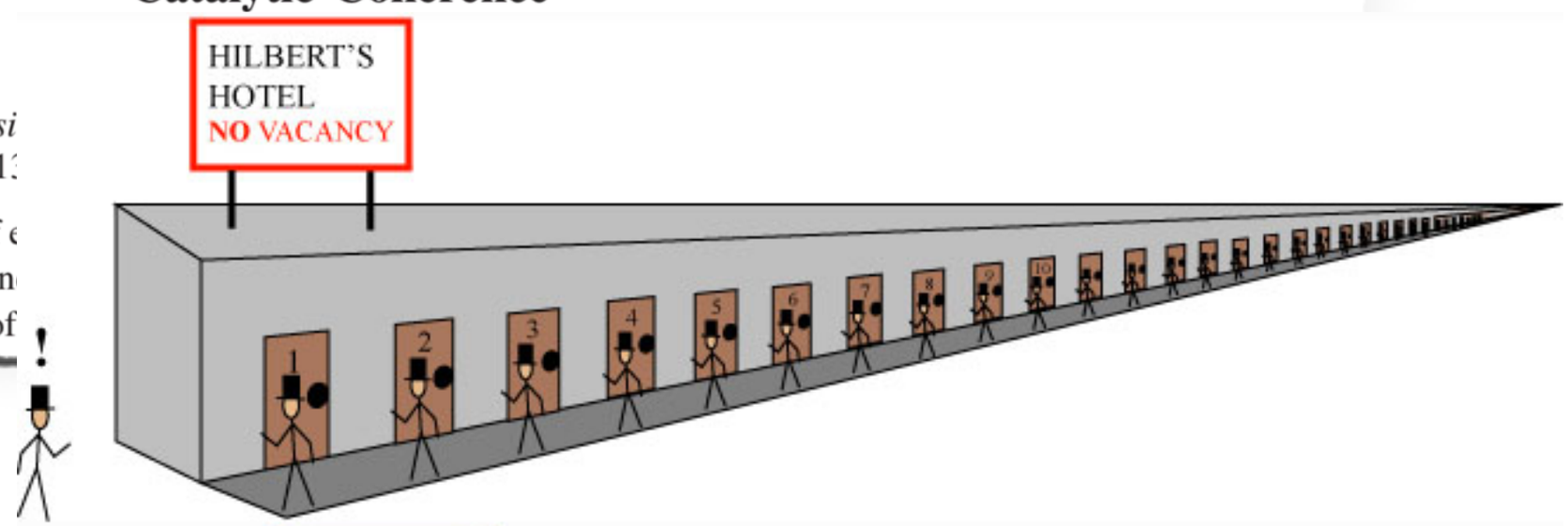

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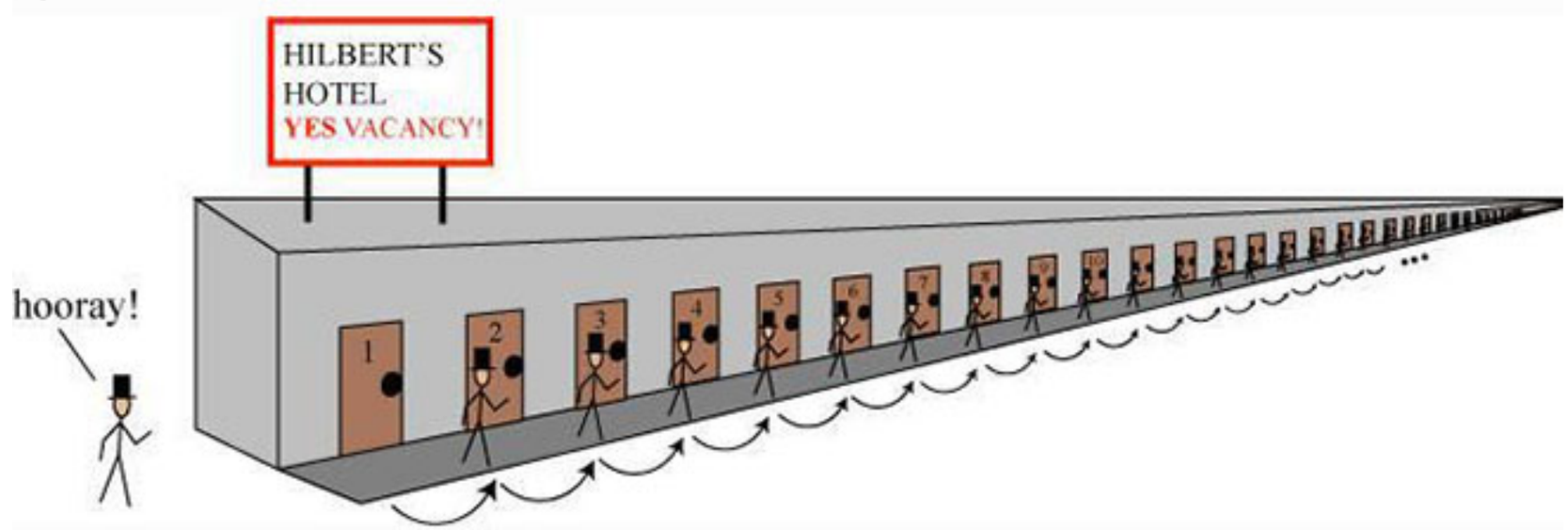

Conclusions

- Thermodynamics as a **resource theory**
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