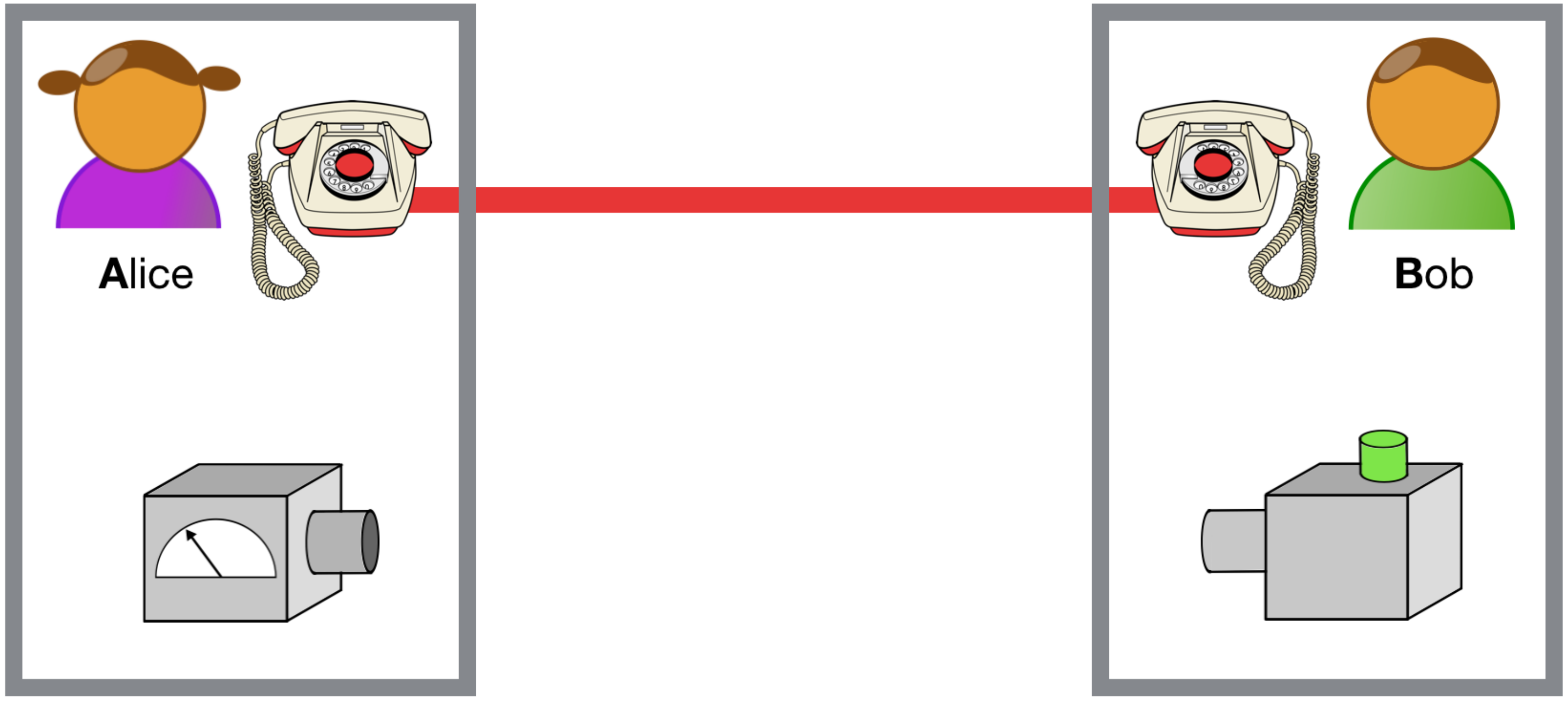
An operational approach to spacetime symmetries: Lorentz transformations from quantum communication

Markus P. Müller
Departments of Applied Mathematics and Philosophy, UWO
Perimeter Institute for Theoretical Physics, Waterloo

joint work with Philipp Höhn



Context

New paradigm in the last few years:
understand spacetime structure via quantum information.

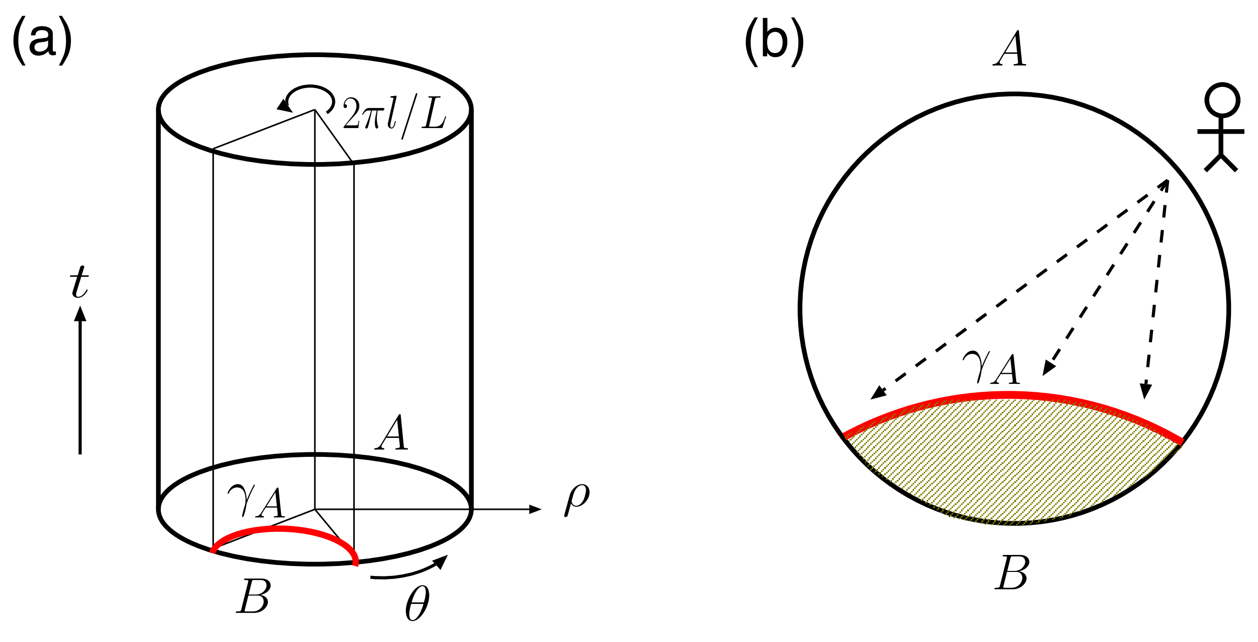


FIG. 1: (a) AdS_3 space and CFT_2 living on its boundary and (b) a geodesics γ_A as a holographic screen.

S. Ryu and T. Takayanagi, PRL **96**, 181602 (2006)

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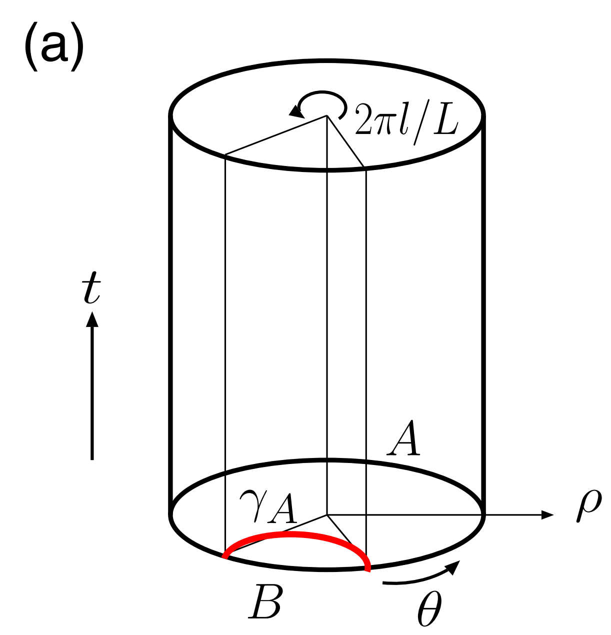

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Our question:
Can we understand the **symmetry group of spacetime** from a quantum information perspective?

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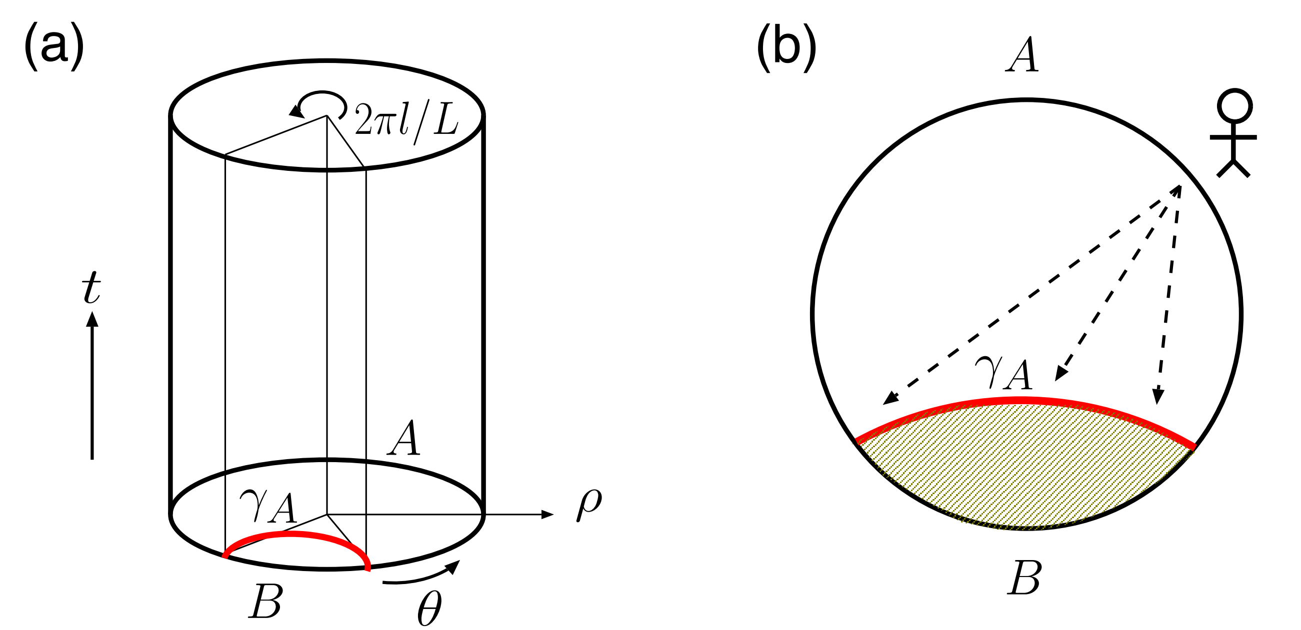

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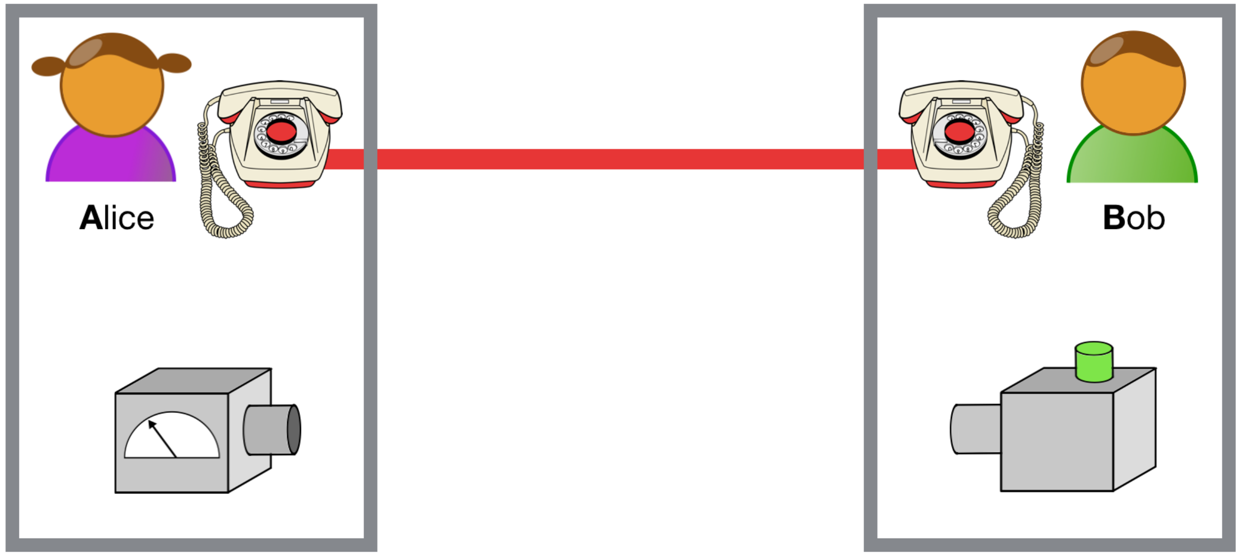
Can we understand the **symmetry group of spacetime** from a quantum information perspective?

Yes, under certain conditions+assumptions.

Outline

- **General setup:**
  two observers and $\mathcal{G}_{\min}$



- **Communicating quantum states:**
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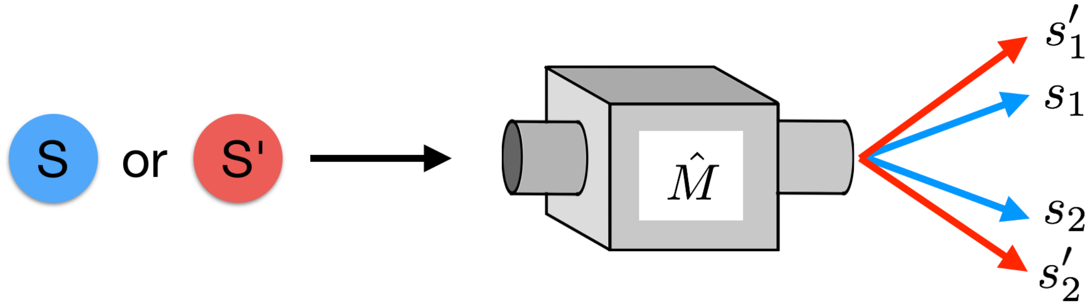

- **Spacetime interpretation:**
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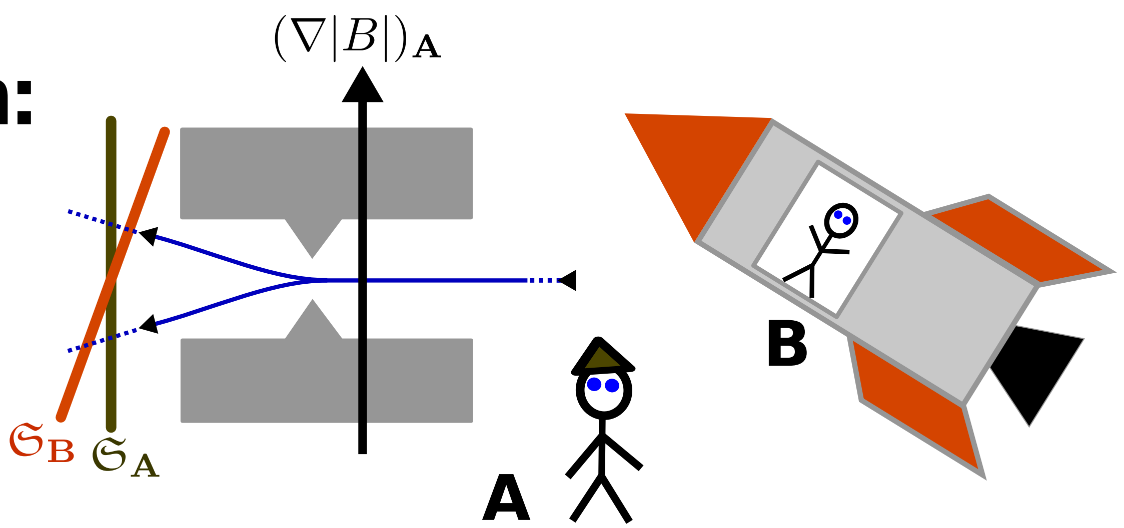

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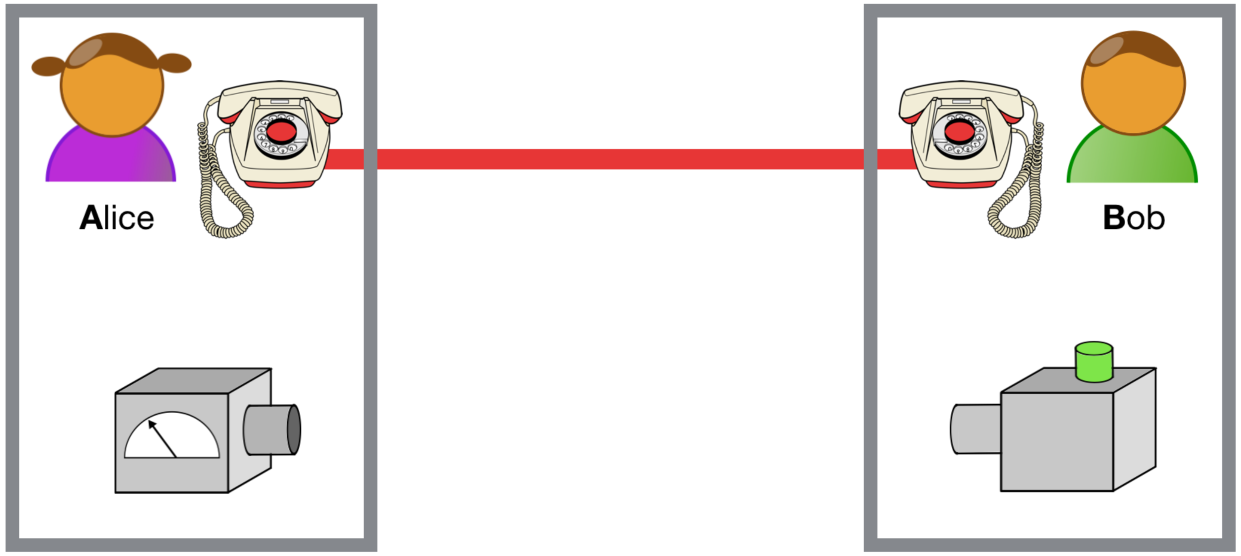

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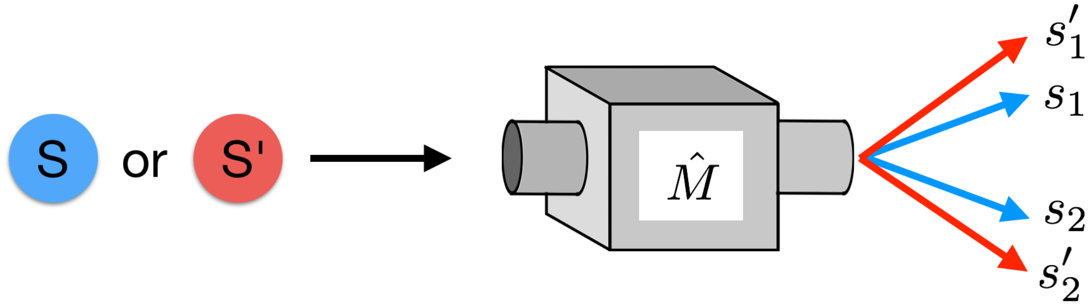

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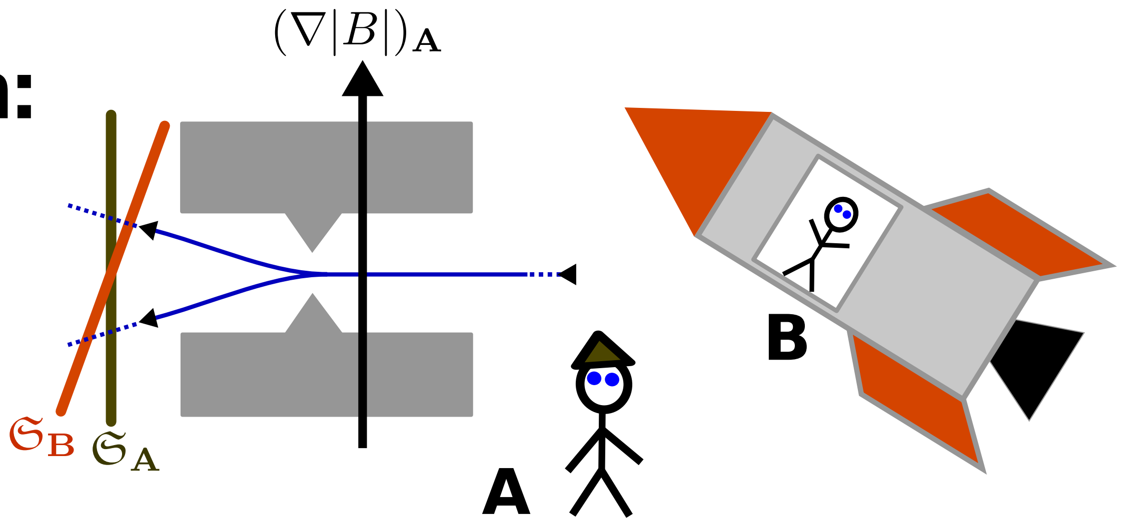

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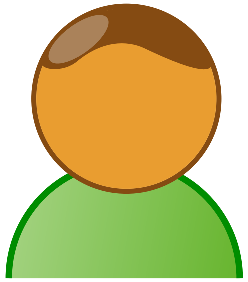
2. General setup

Two observers

Alice



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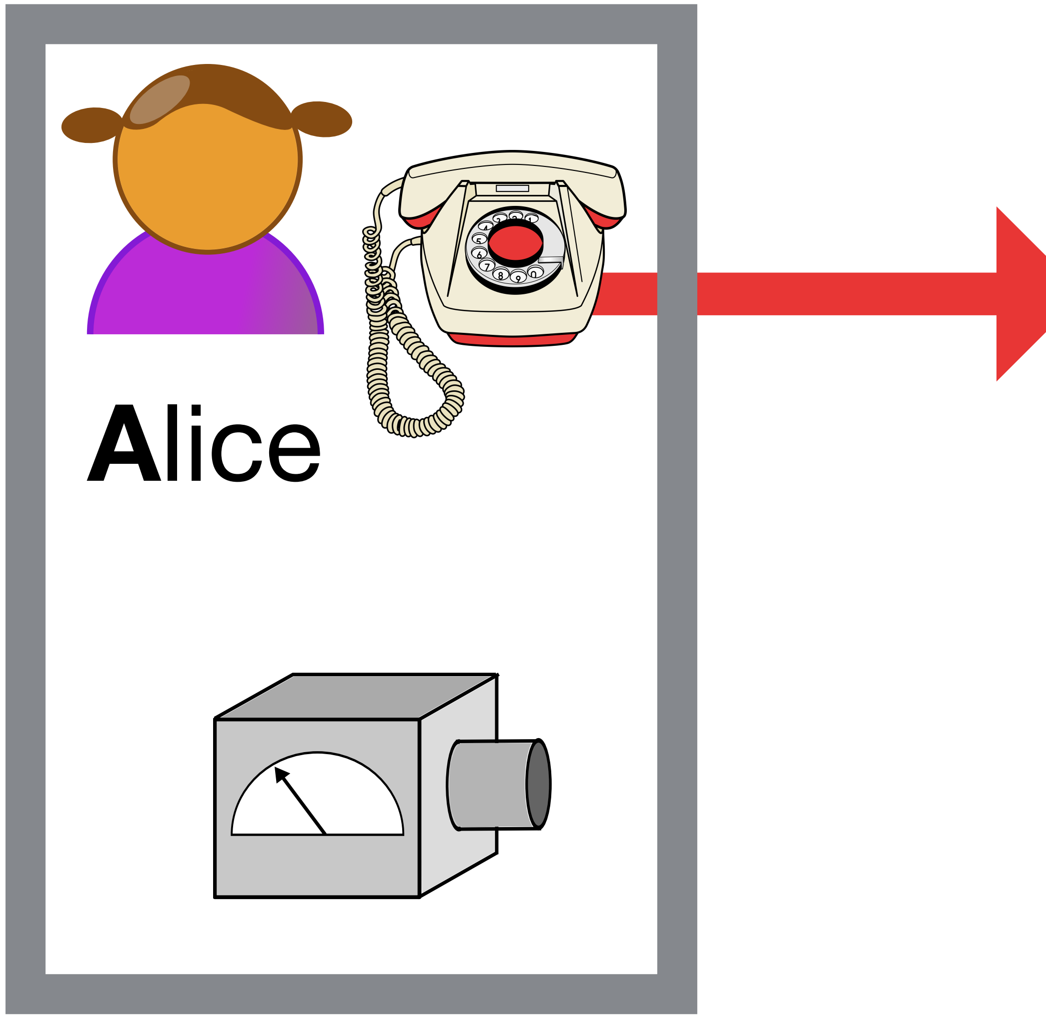

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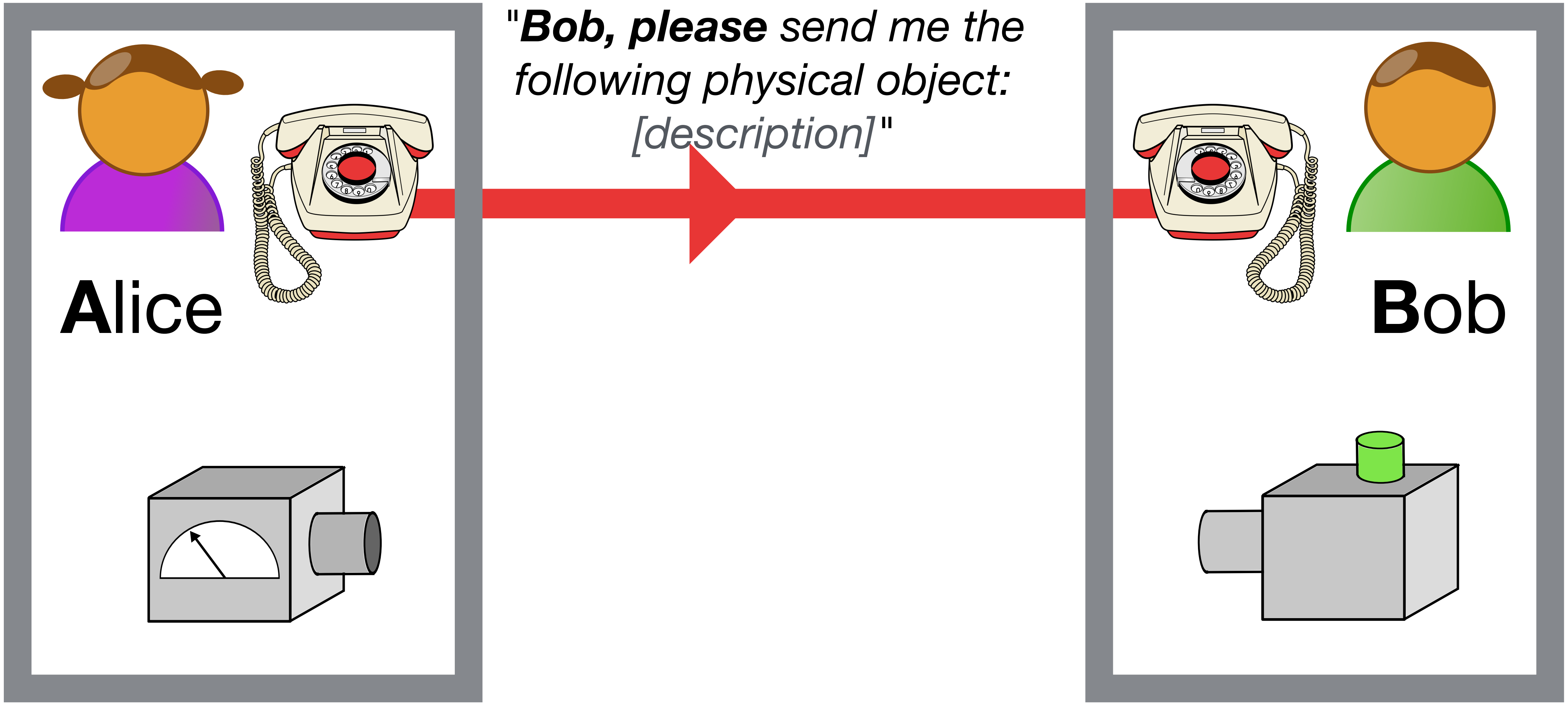

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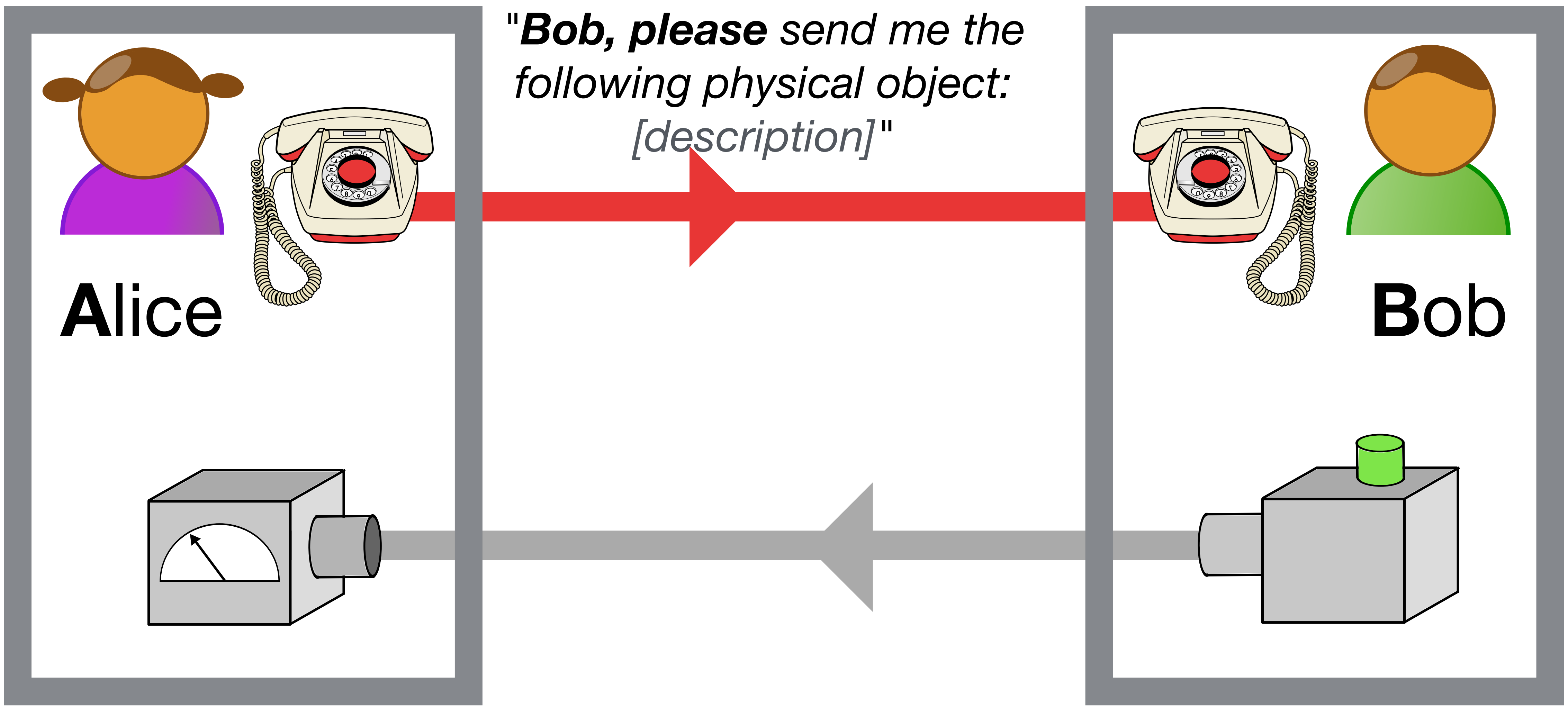

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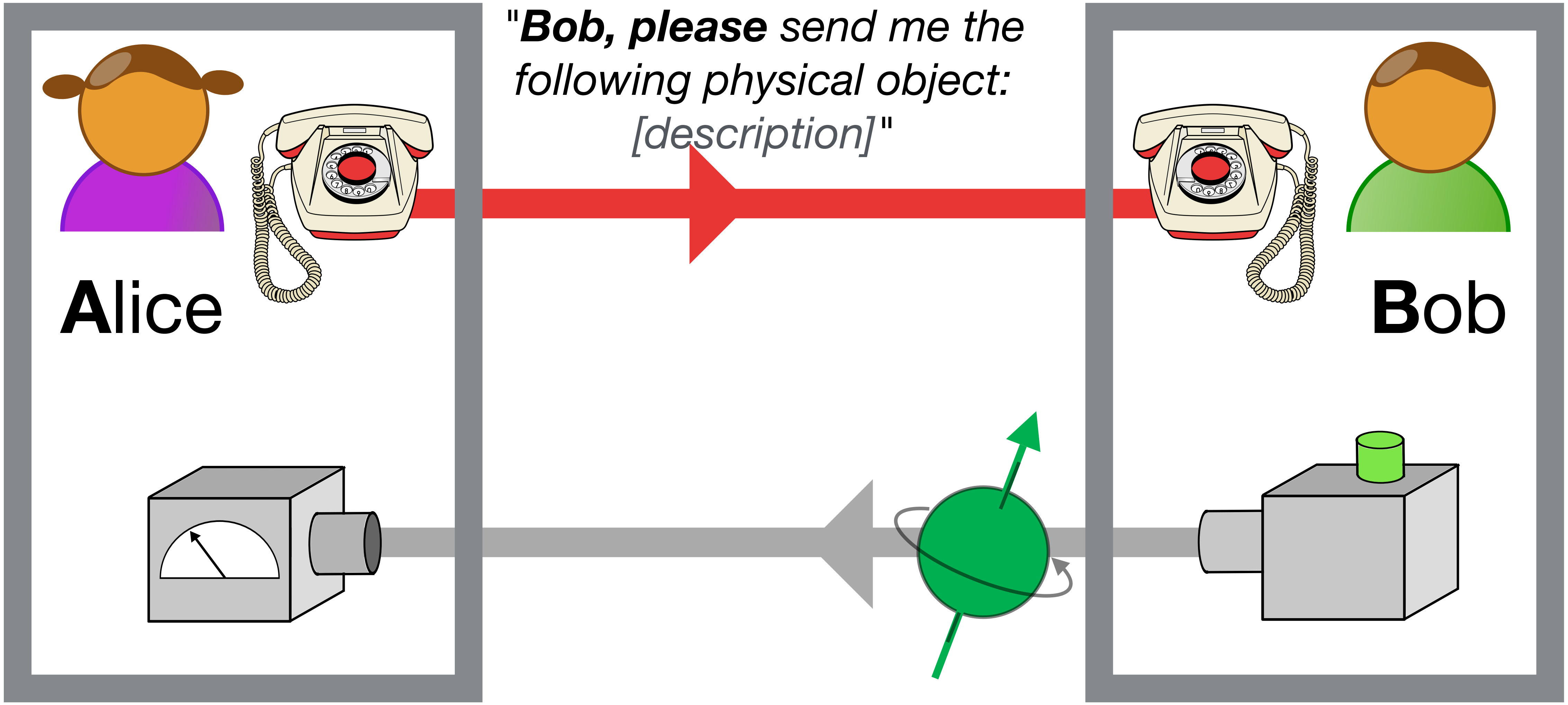

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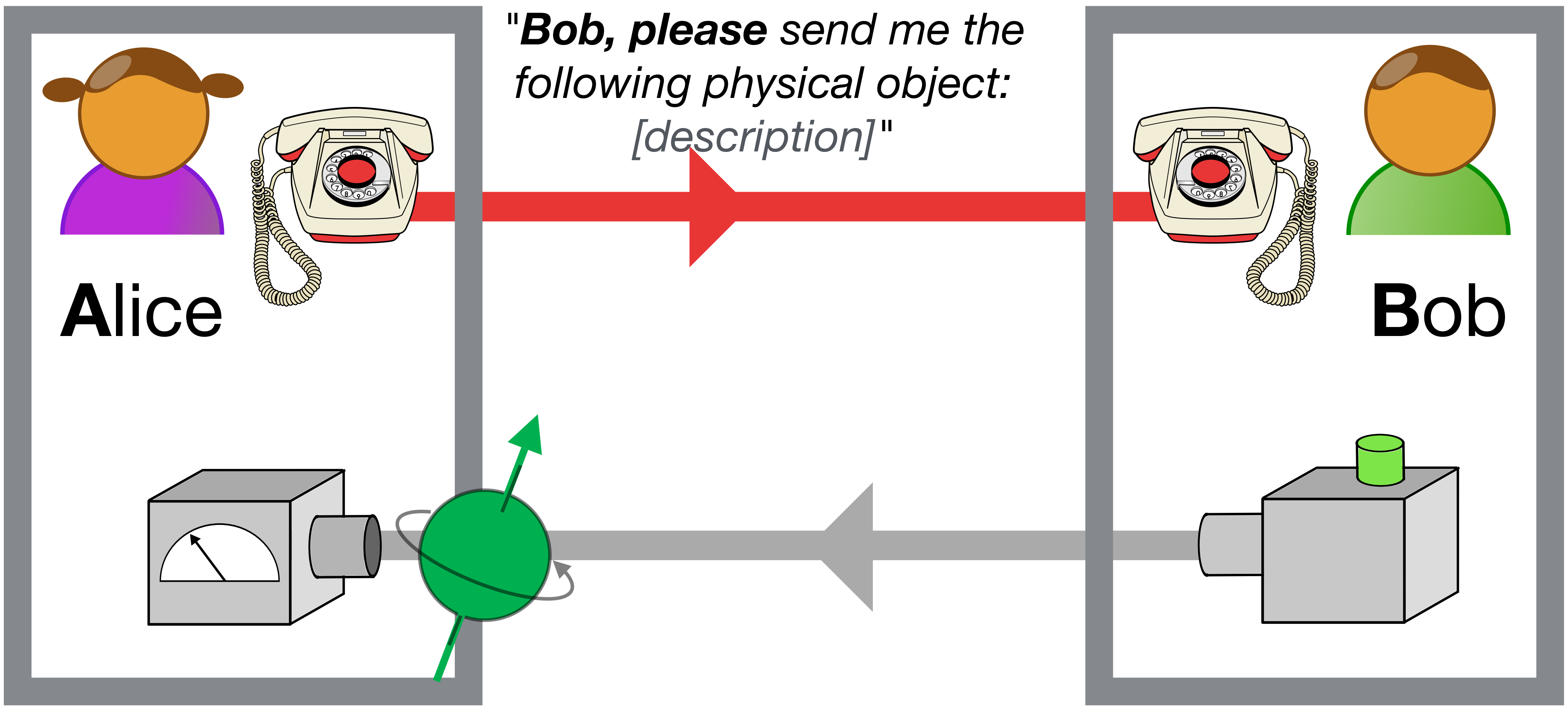

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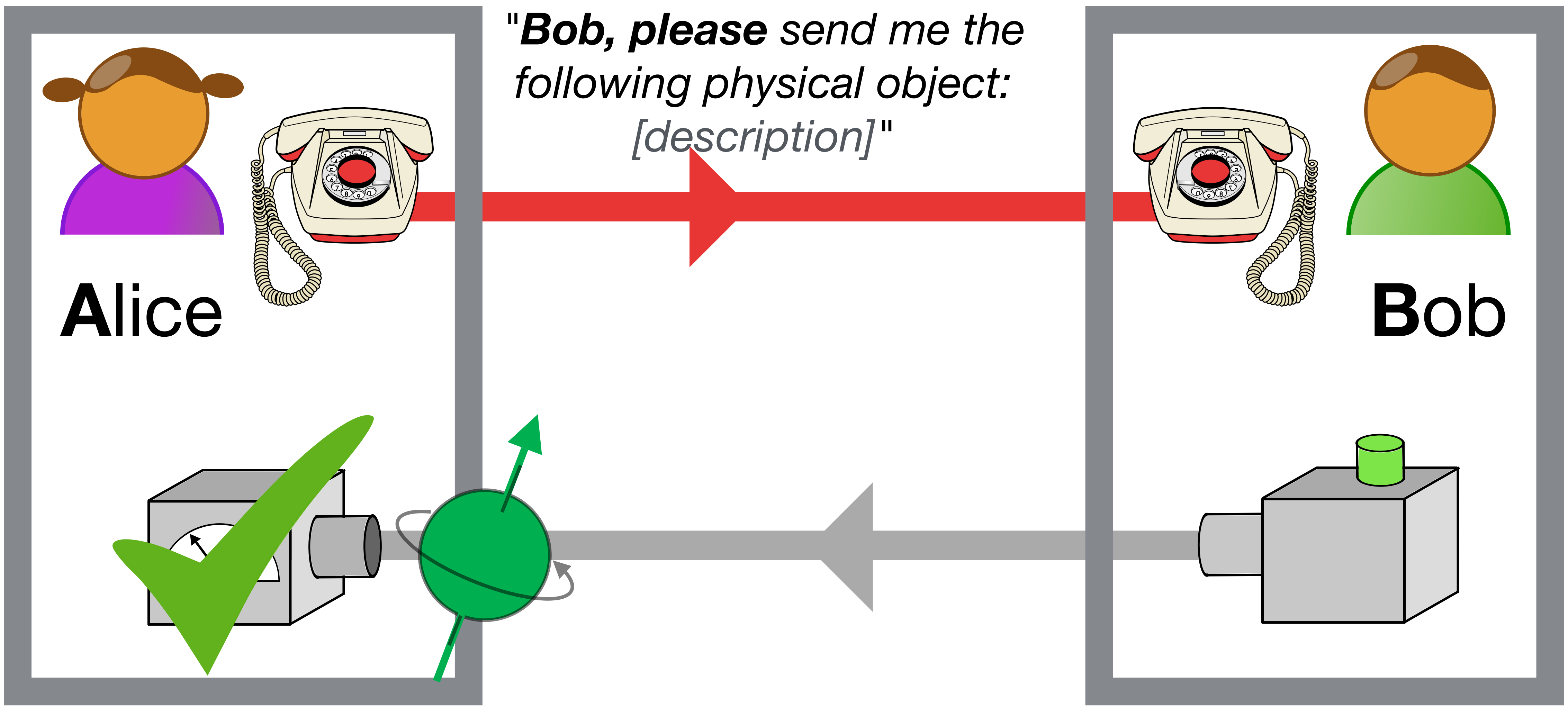

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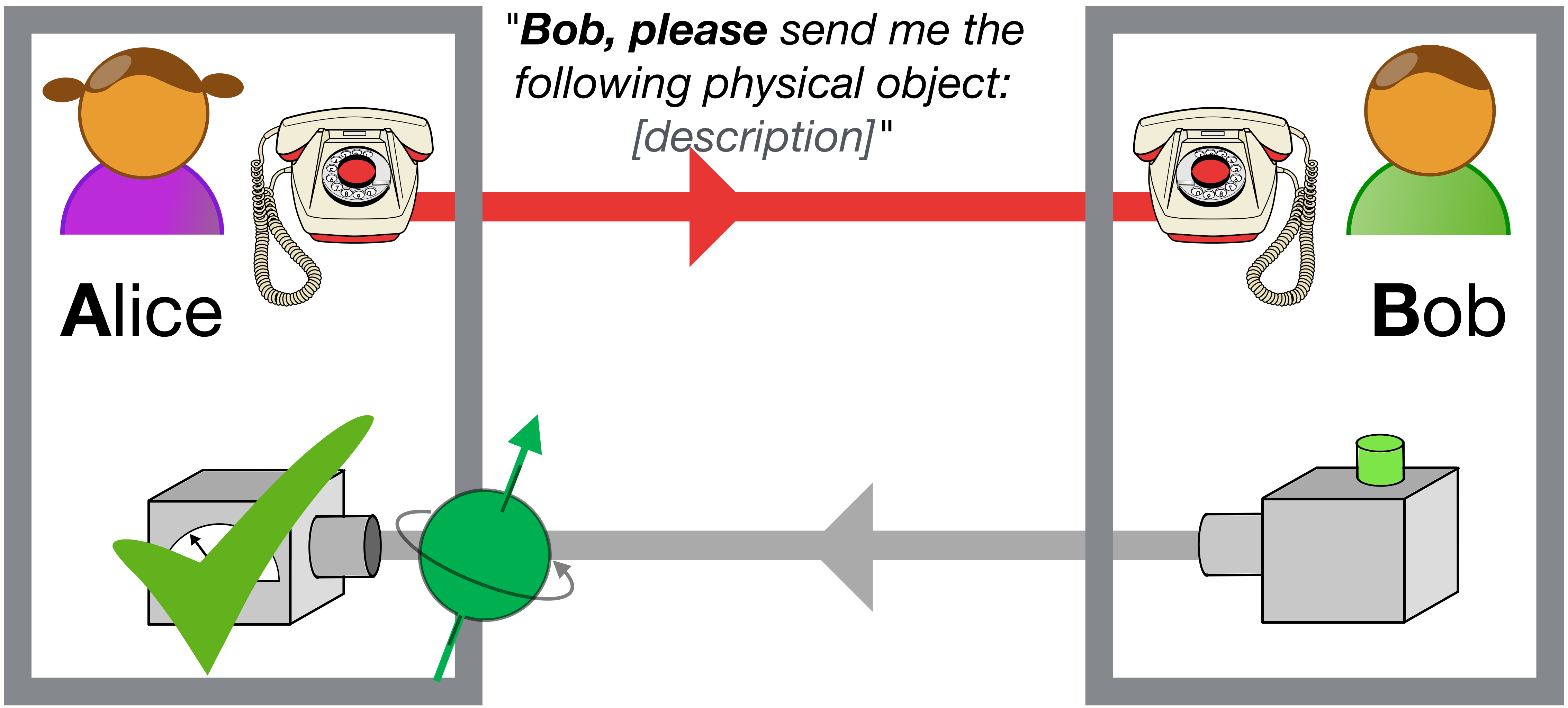

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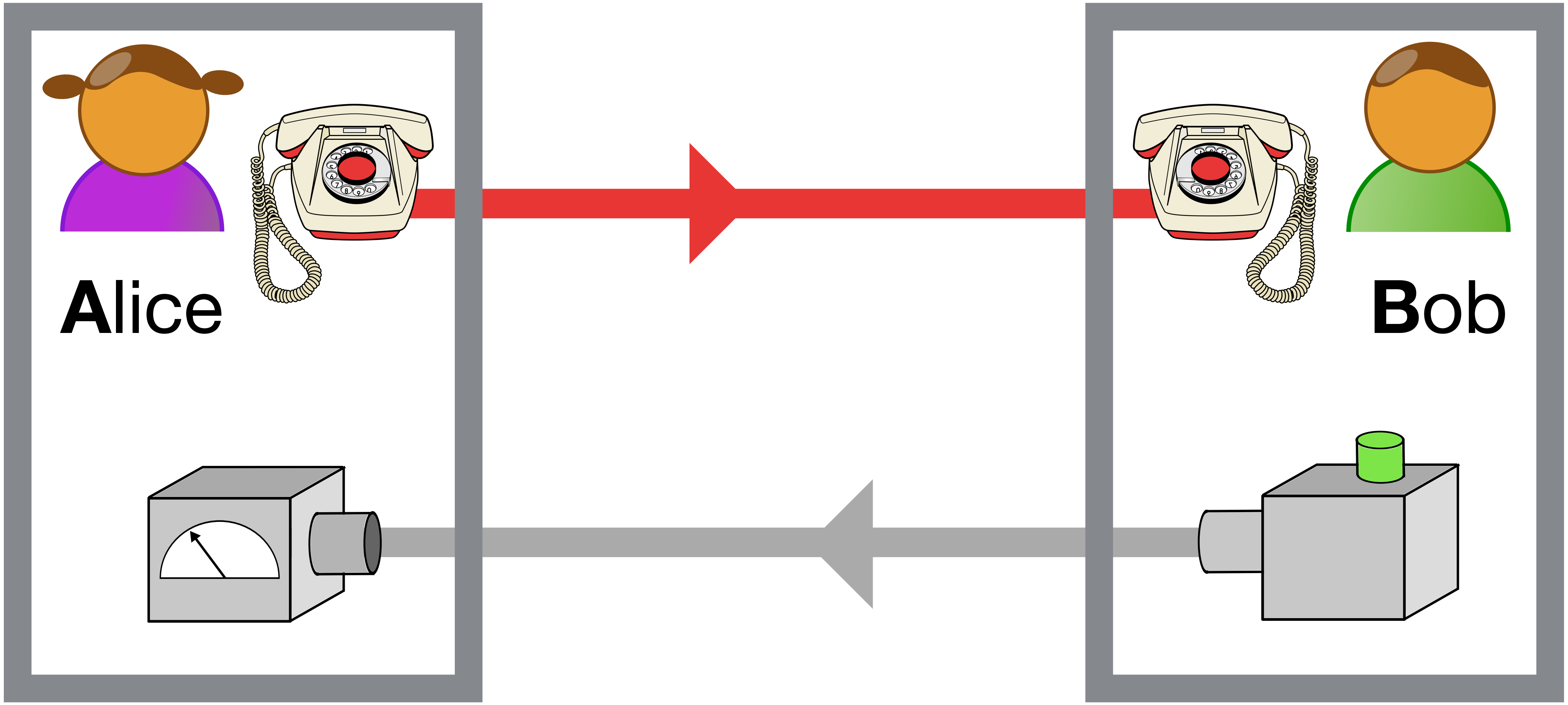

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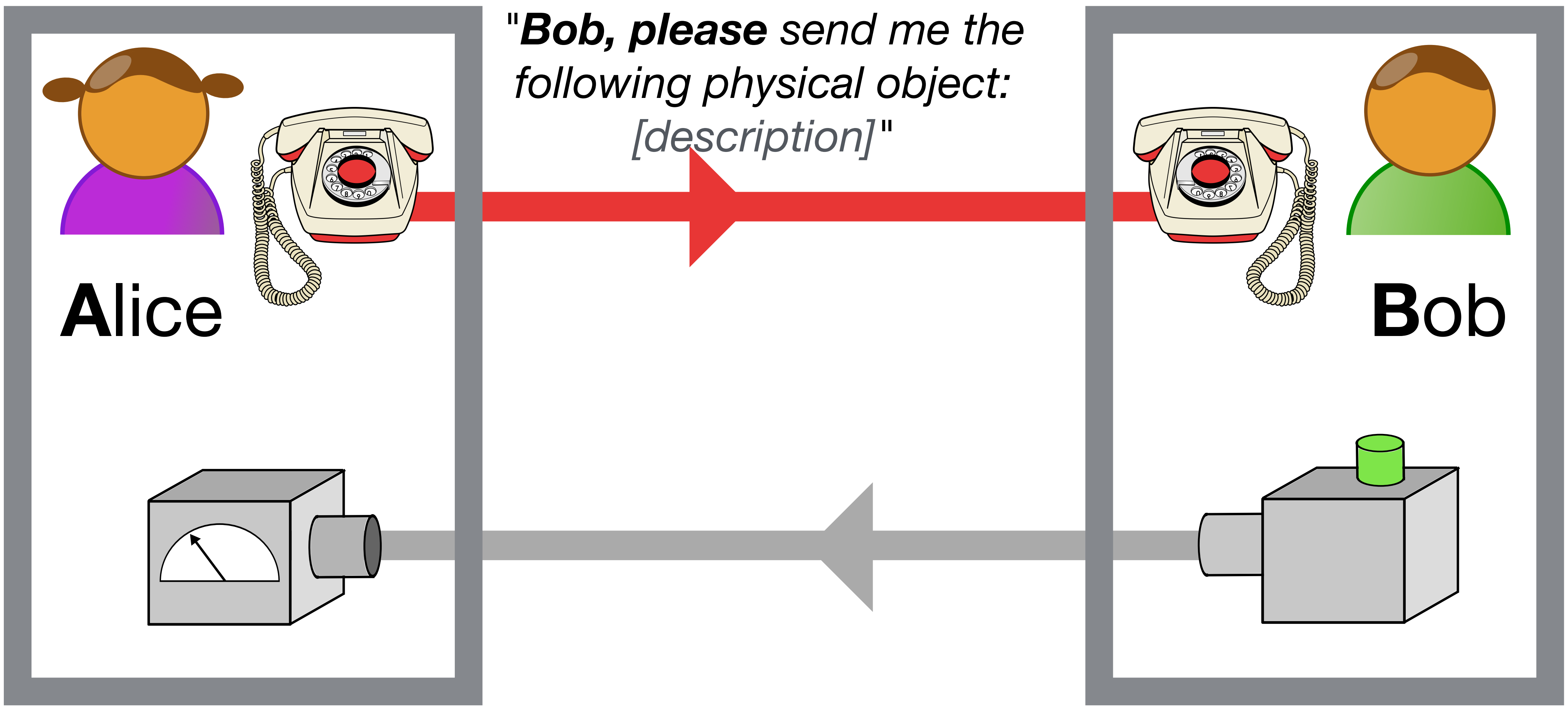

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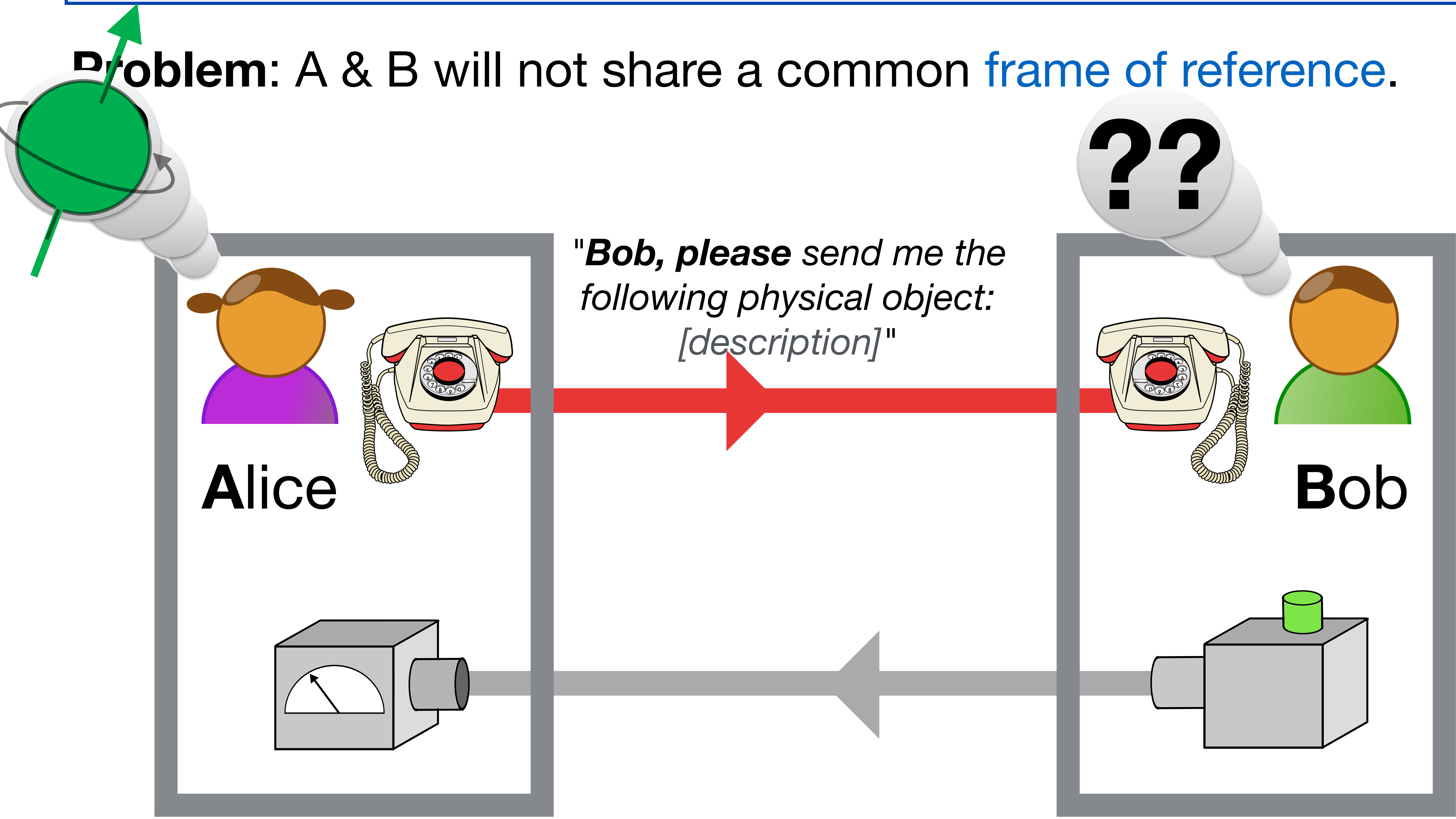

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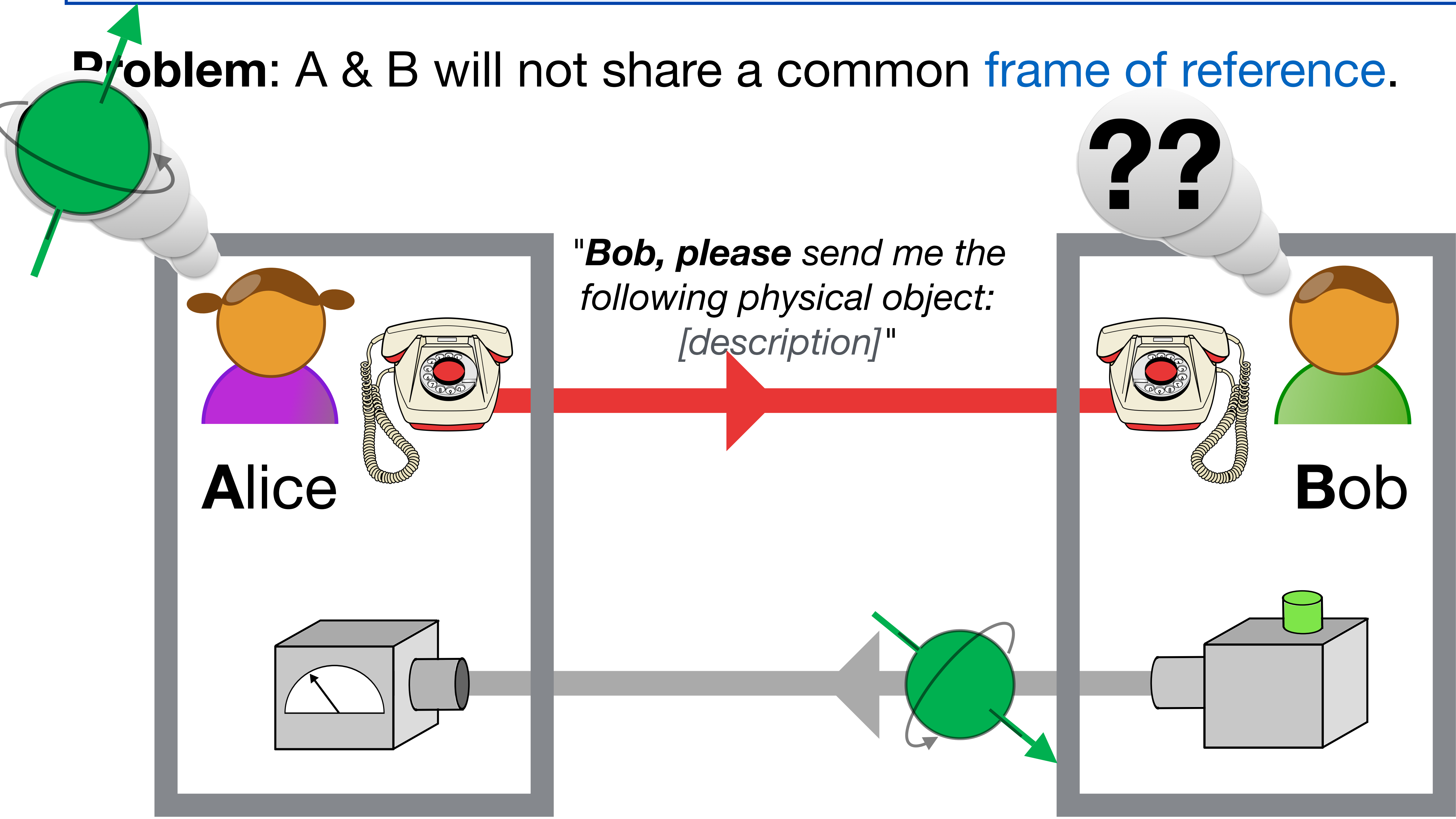

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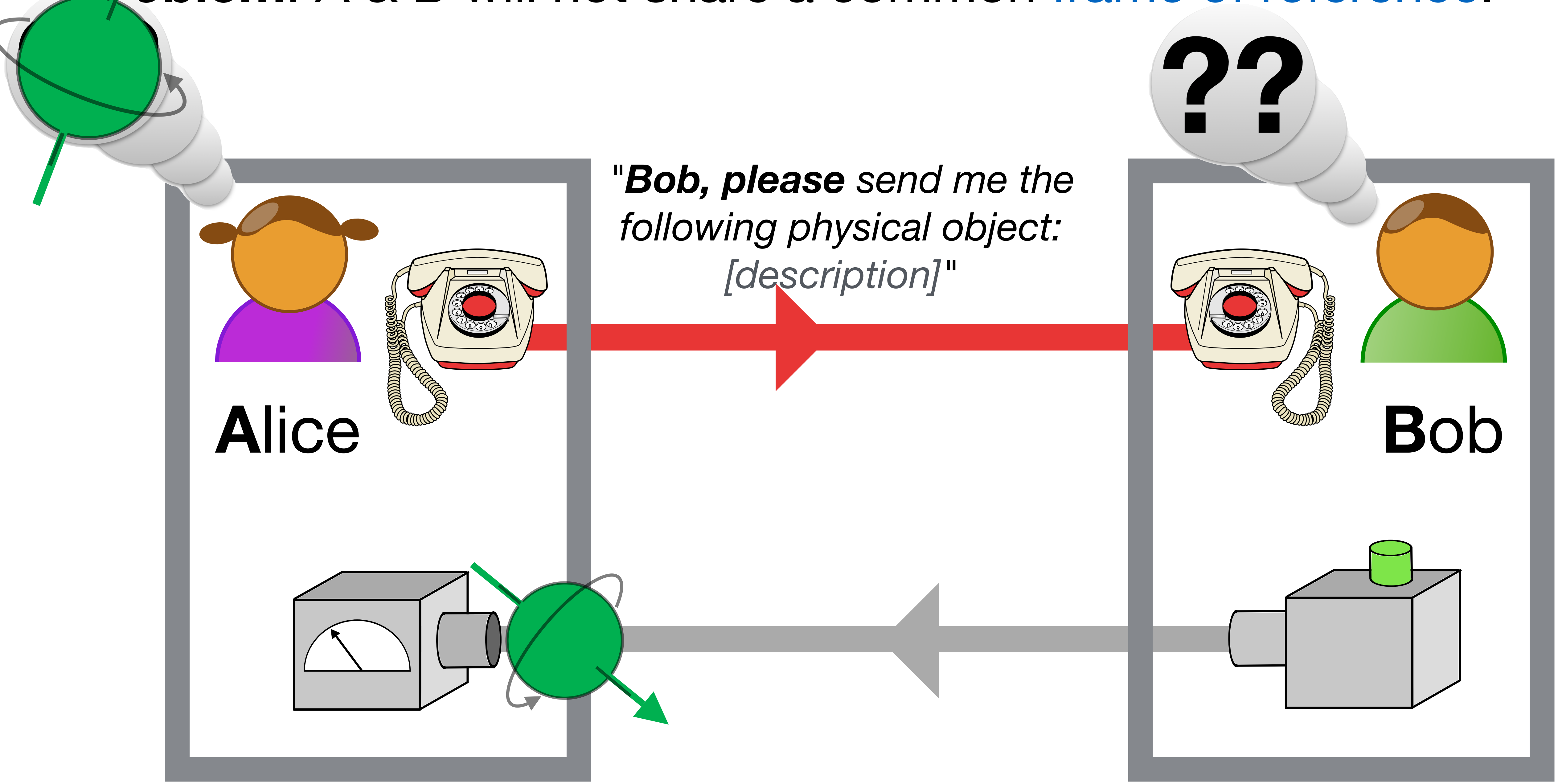

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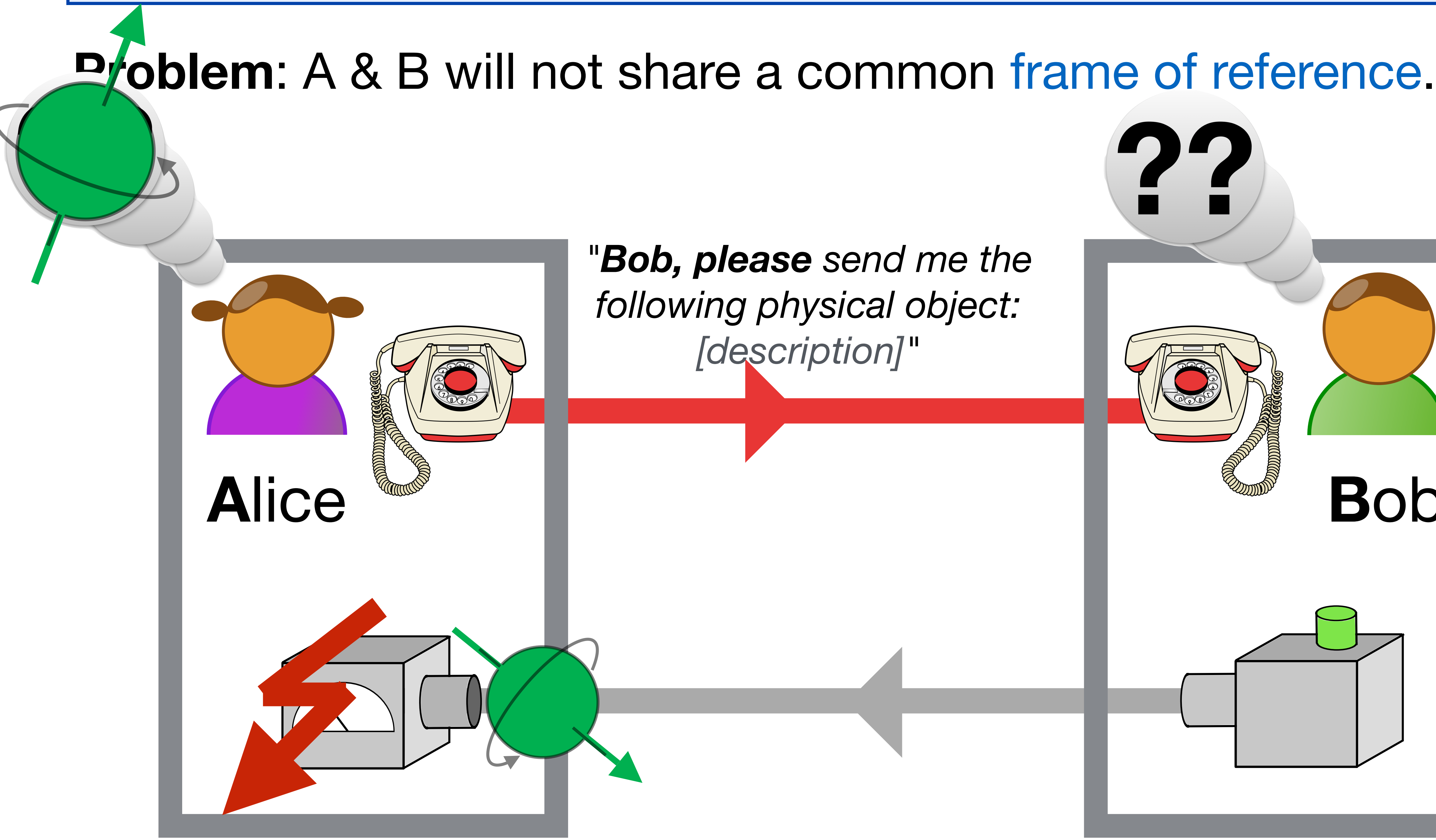

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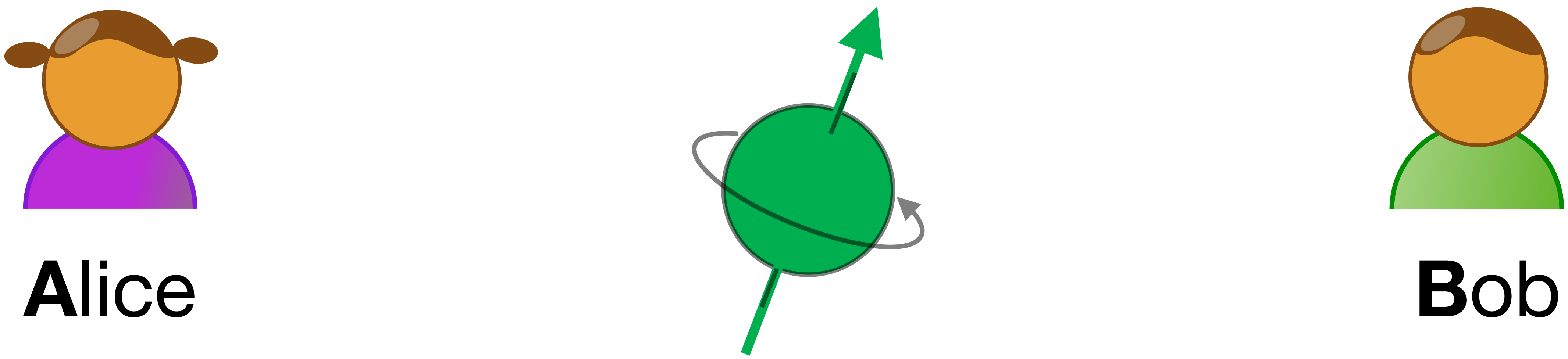

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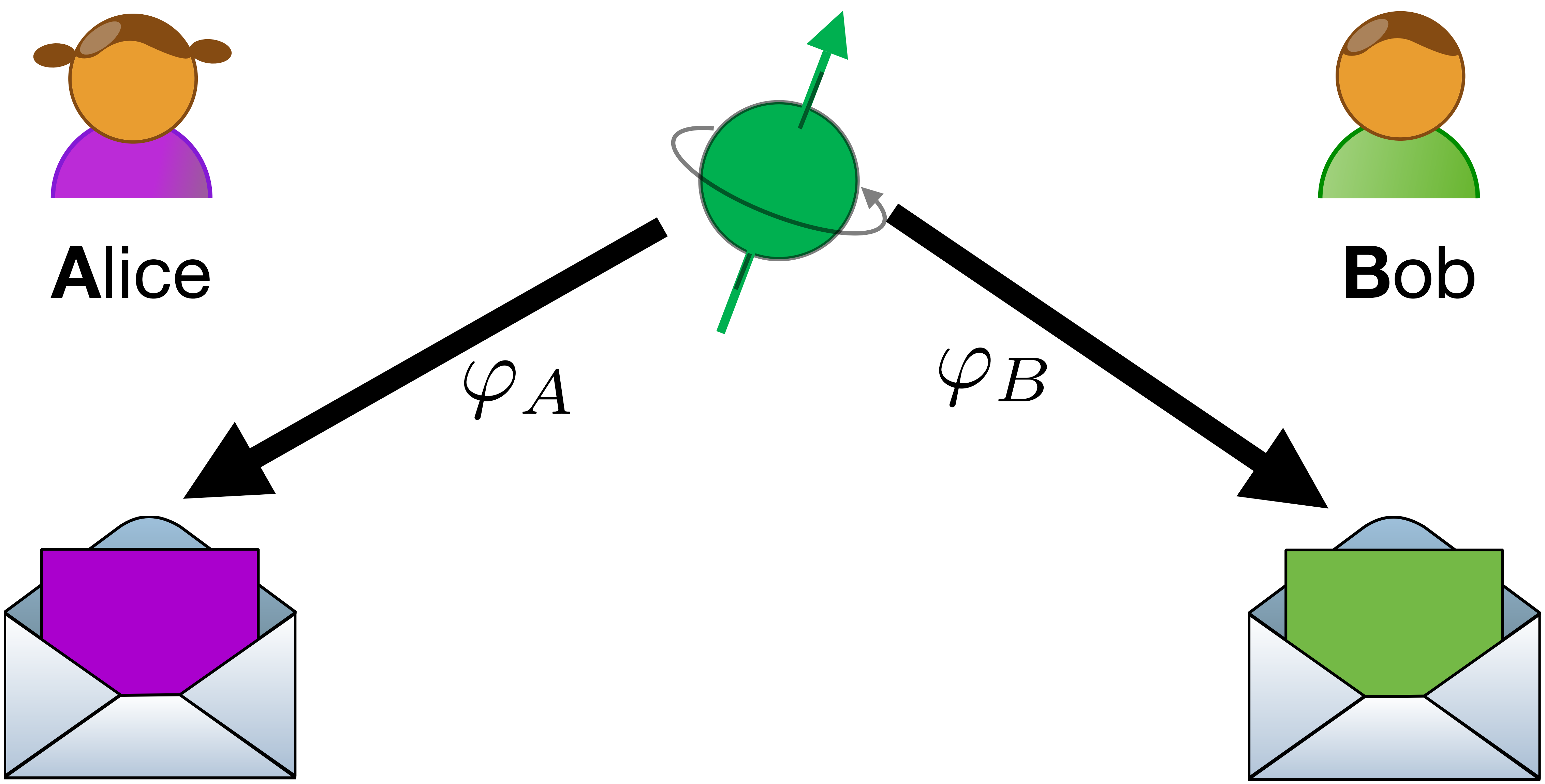

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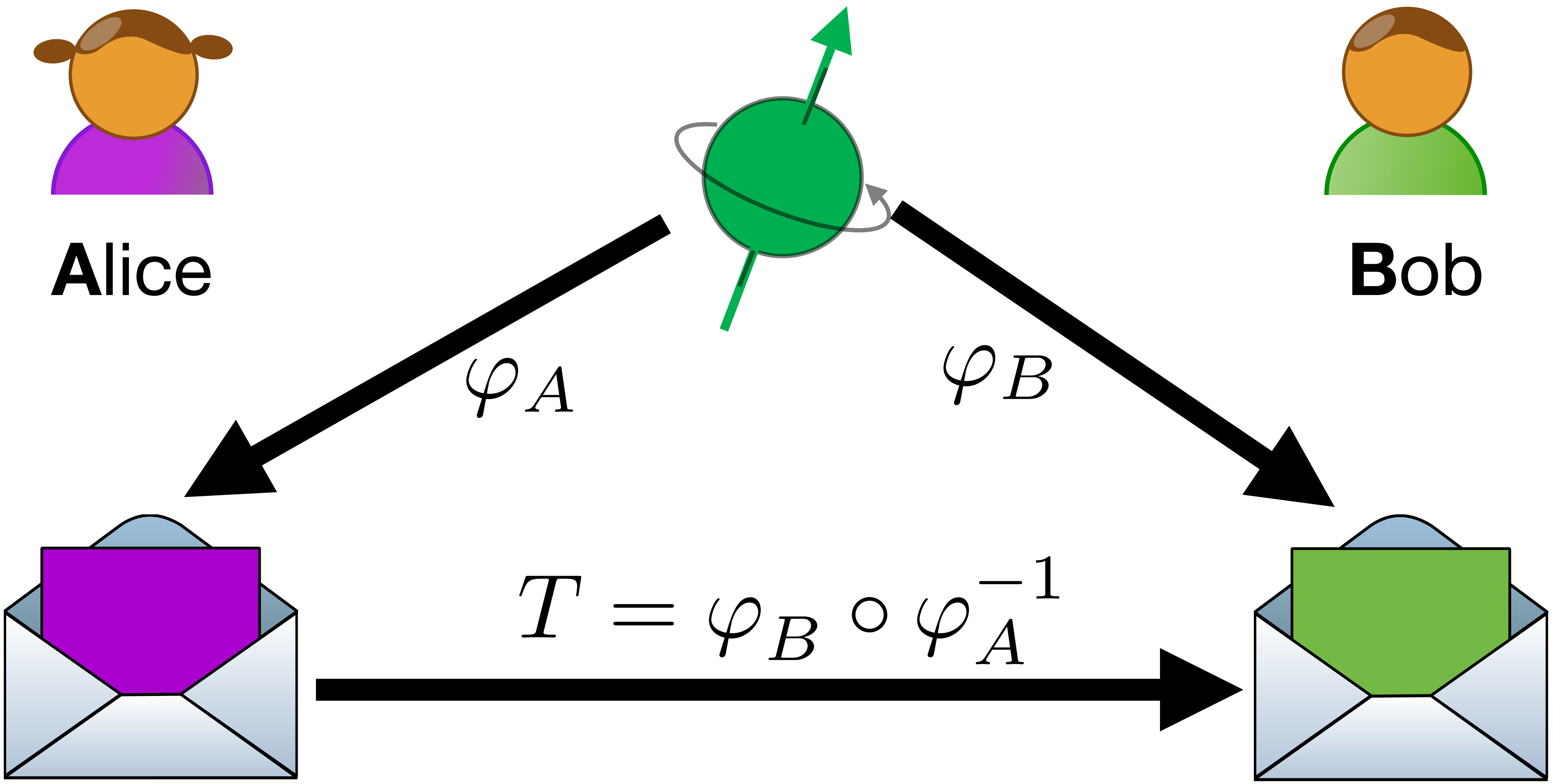

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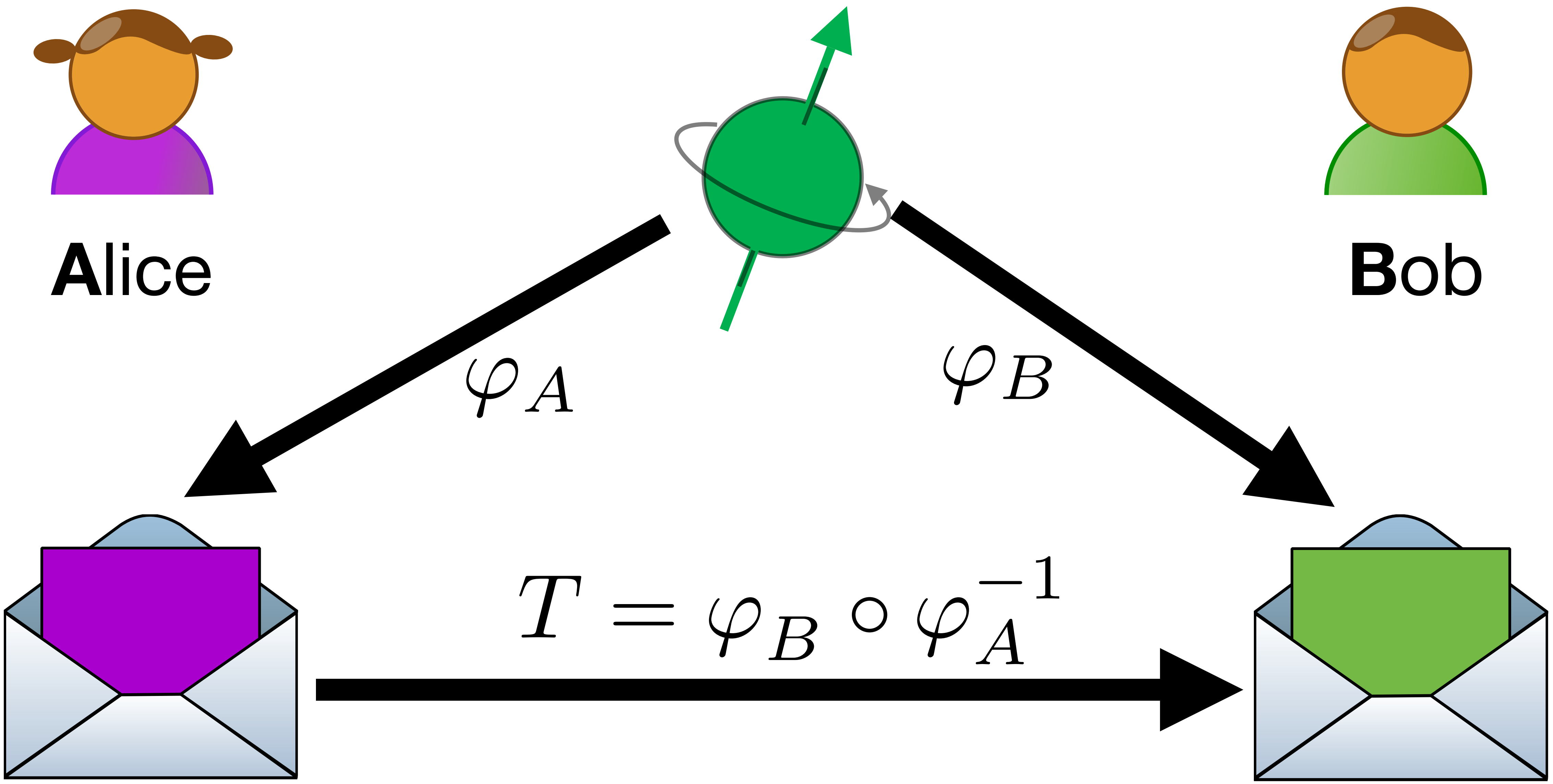
Can use this as a "correcting transformation".



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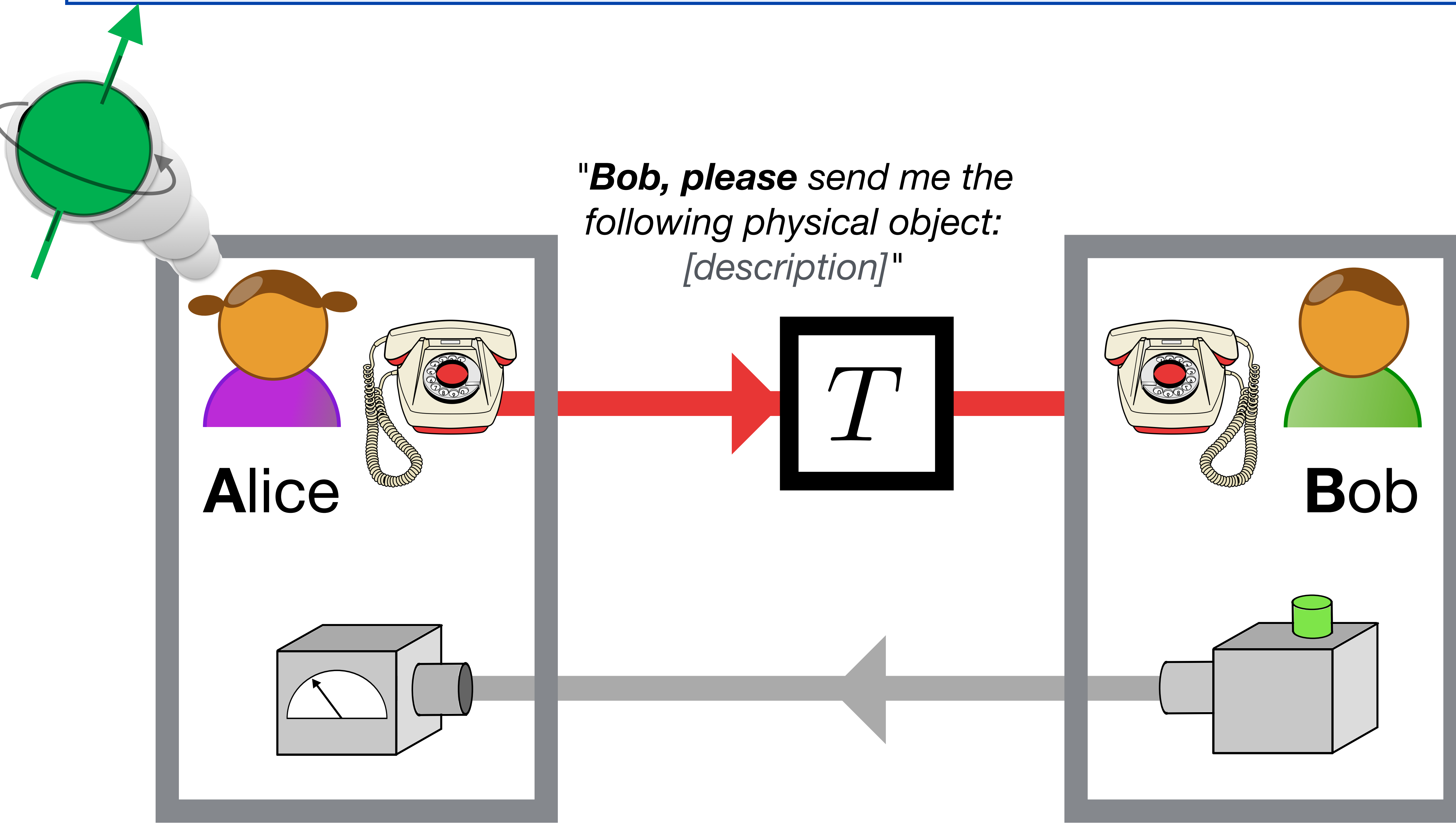

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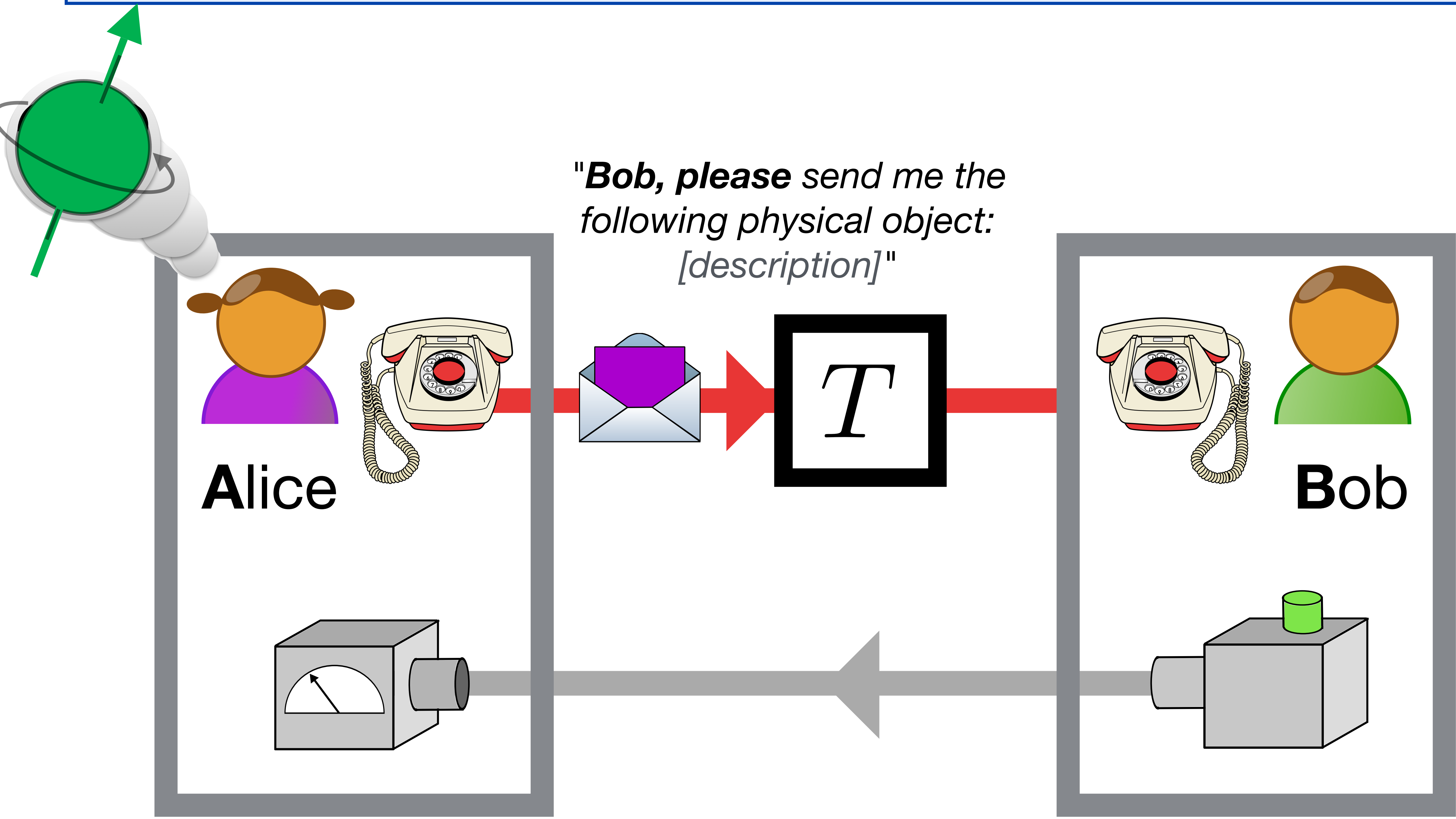

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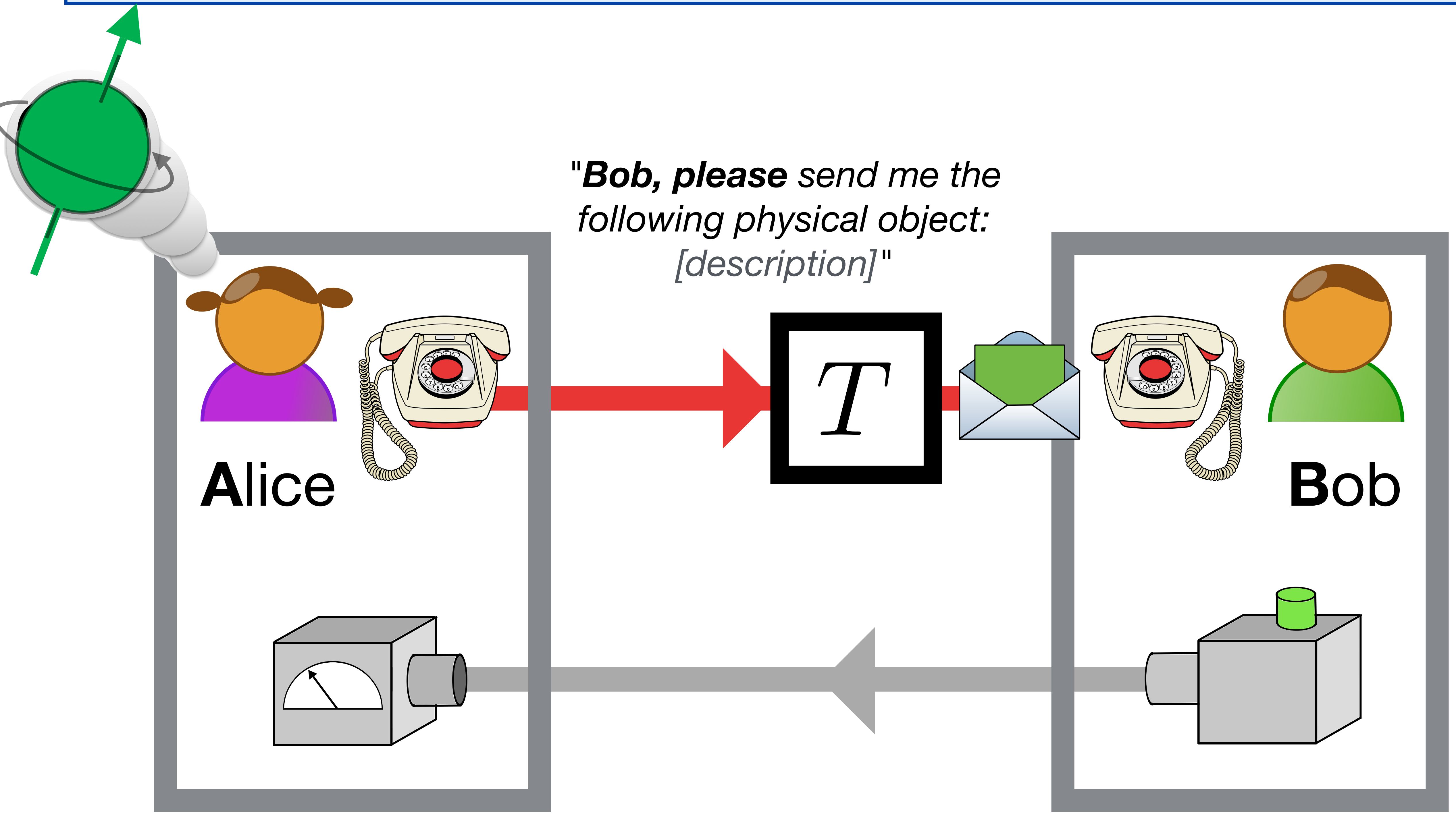

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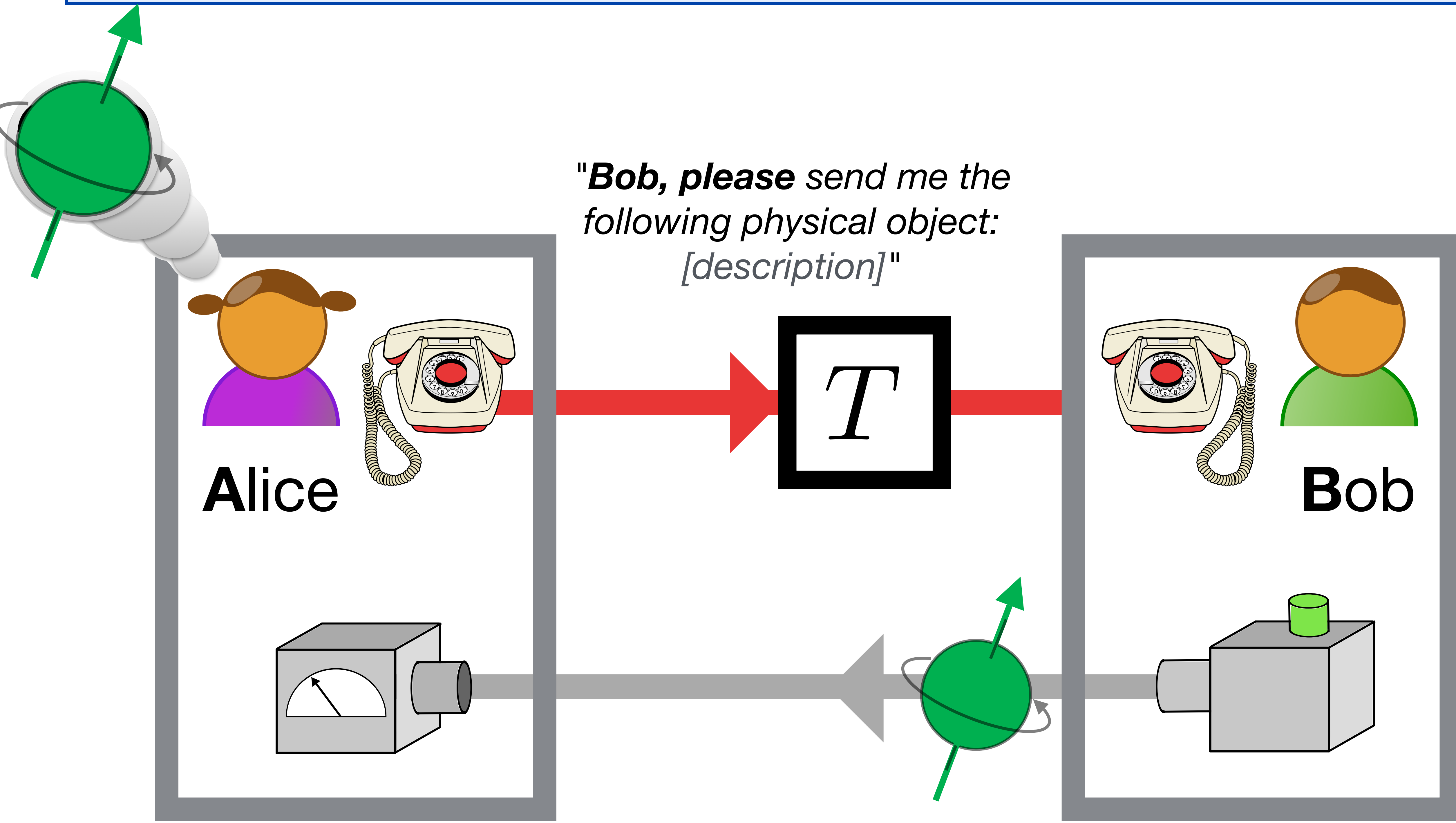

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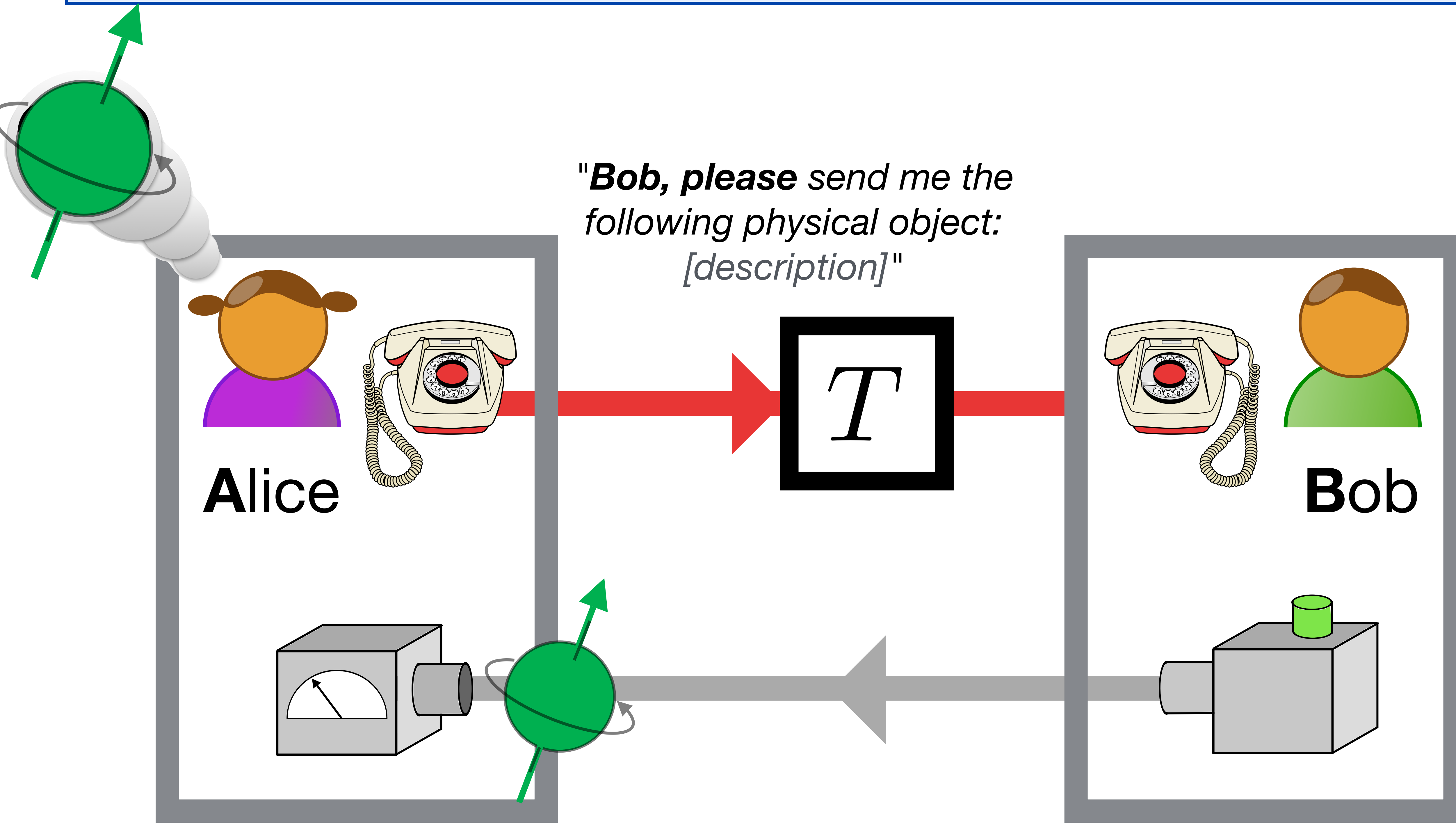

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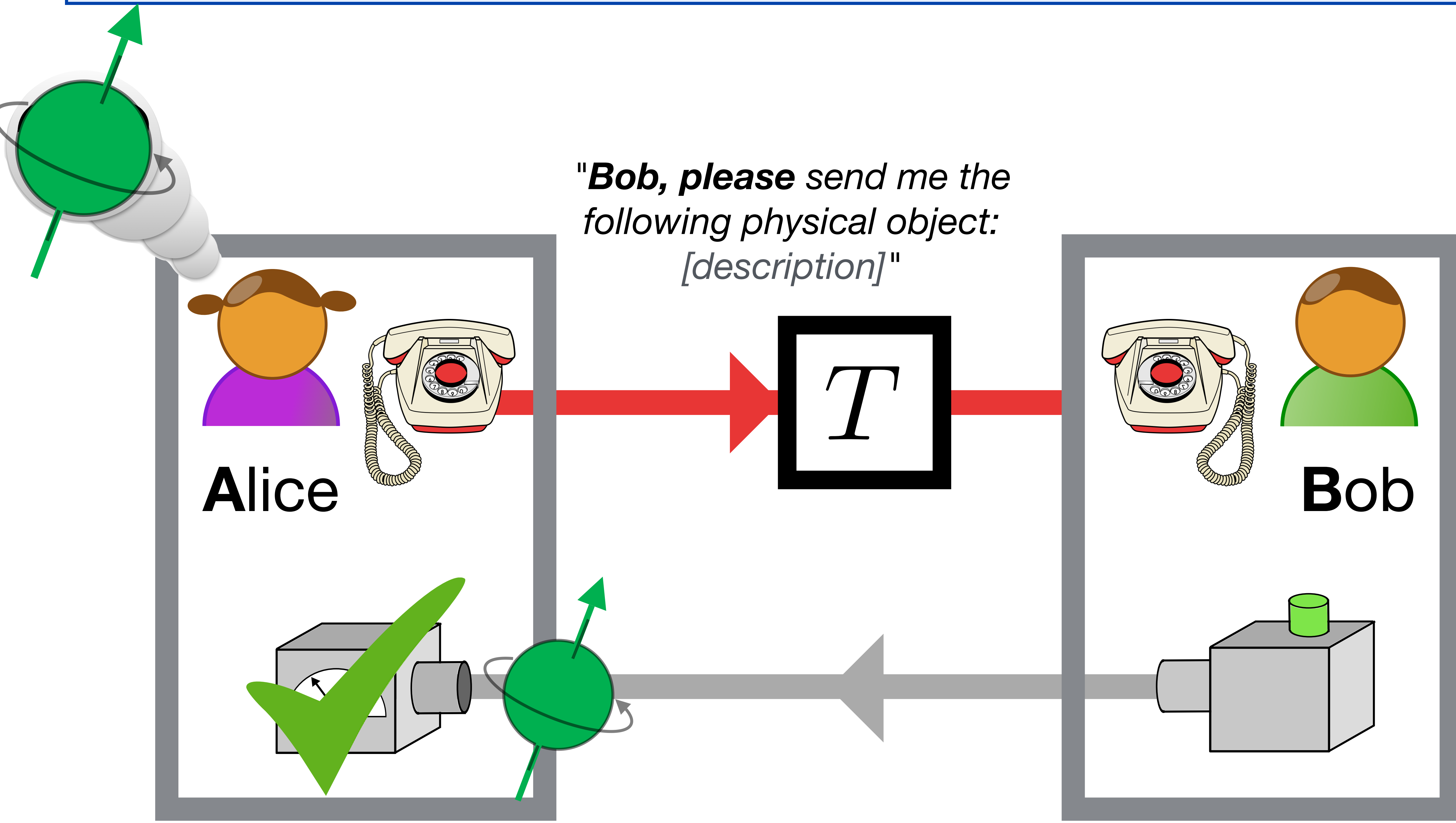

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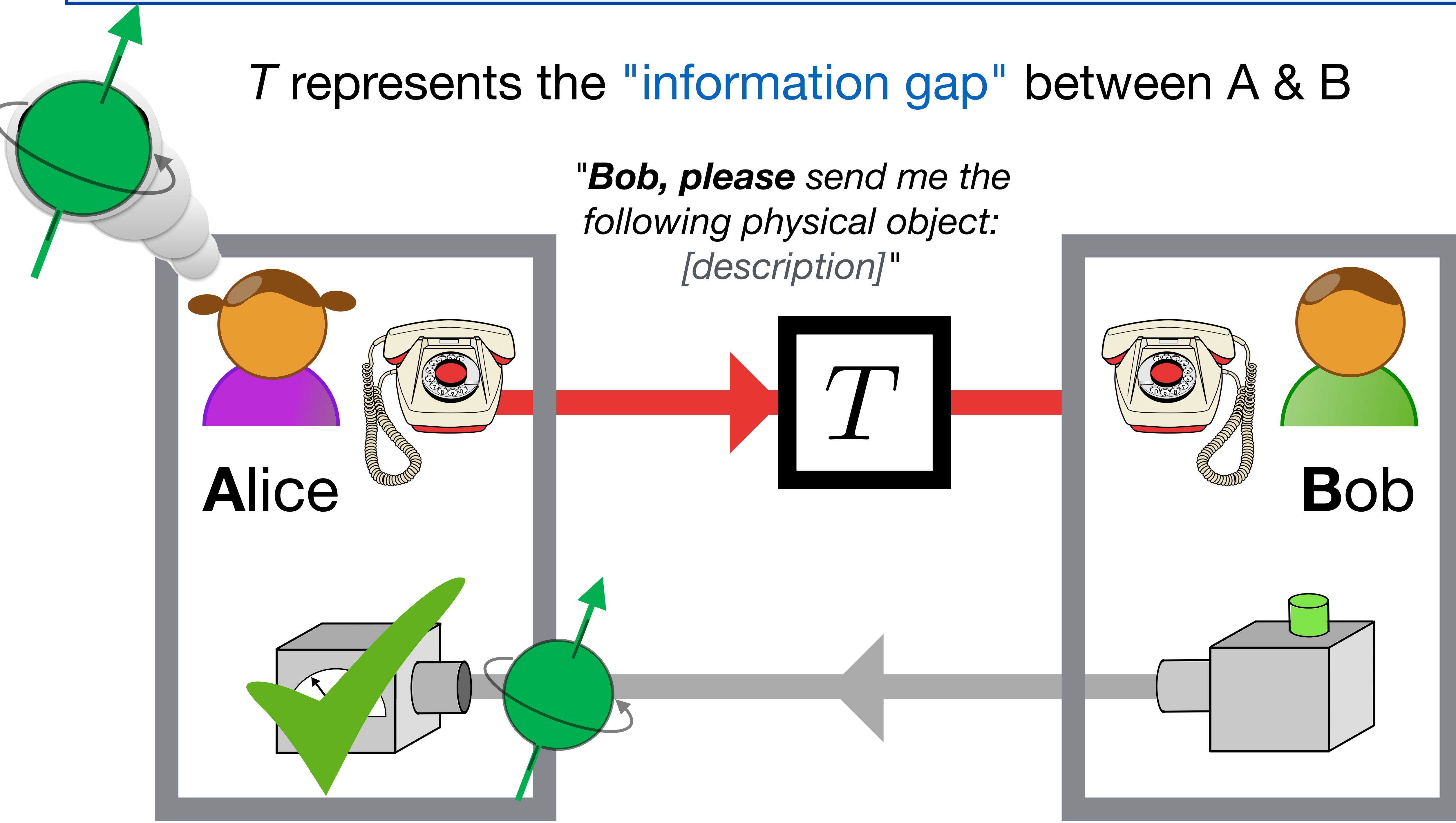
T represents the "information gap" between A & B



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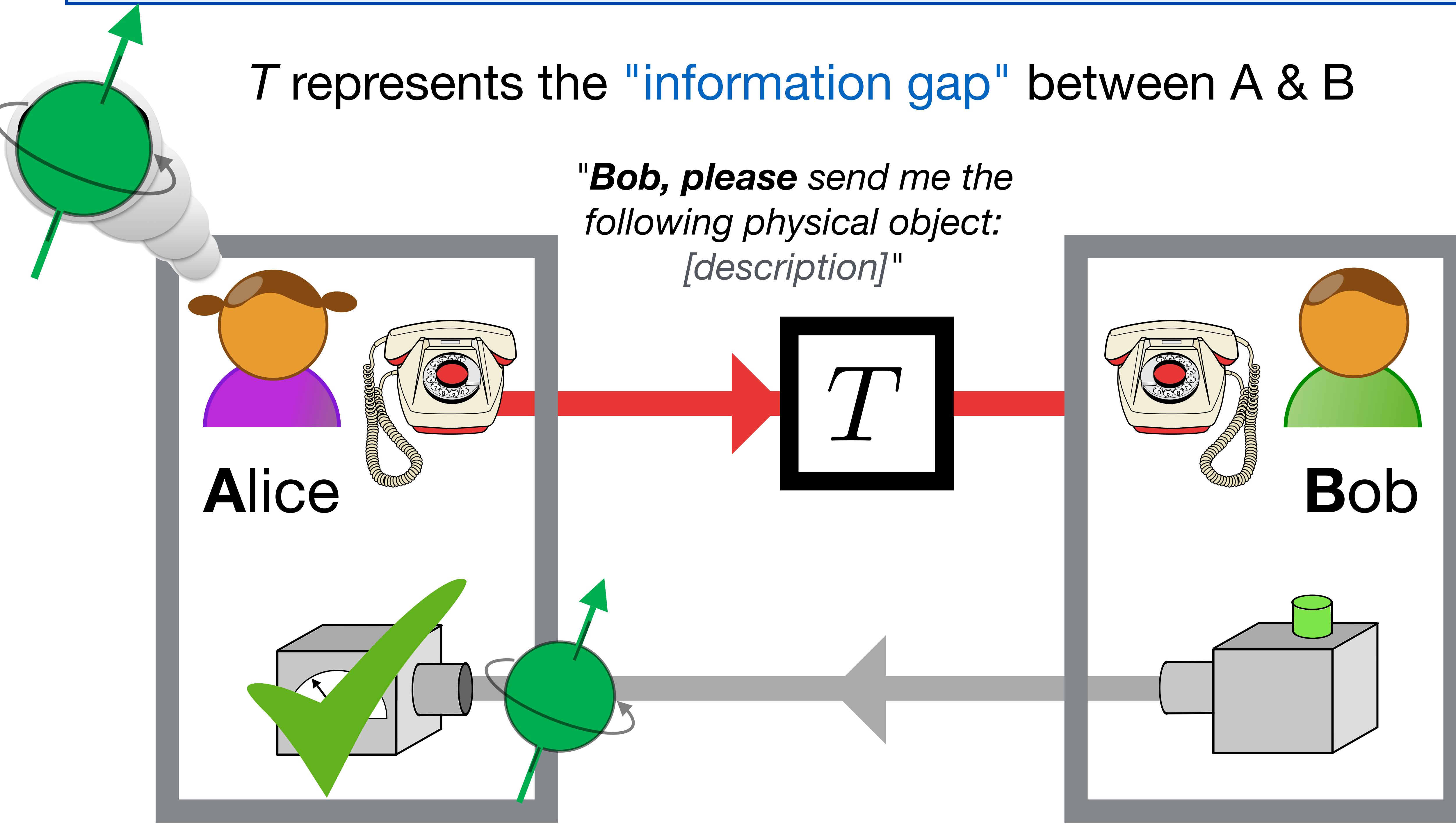

Collaboration: make this gap "as small as possible".

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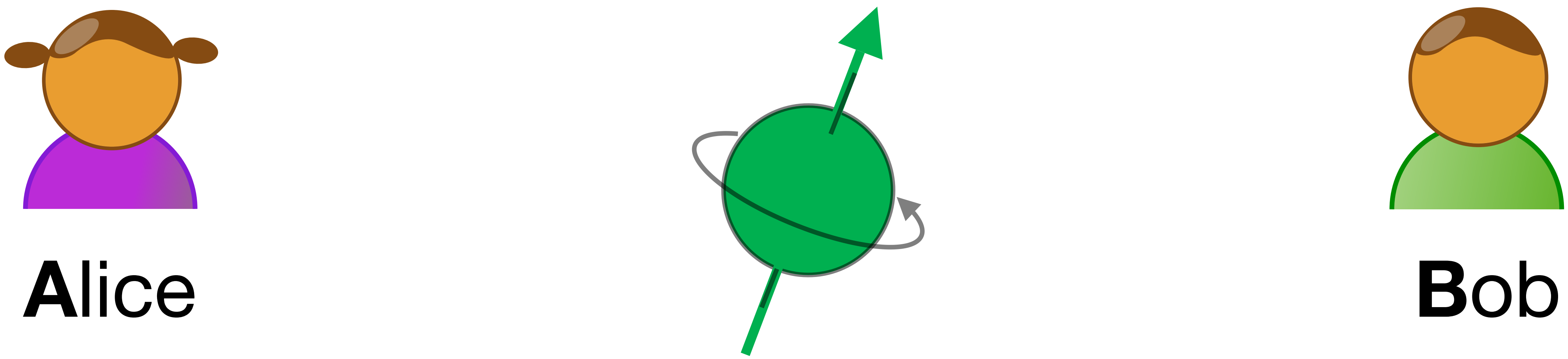
Example: Sending a spinning billard ball in classical mechanics.



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The minimal group $\mathcal{G}_{\min}$

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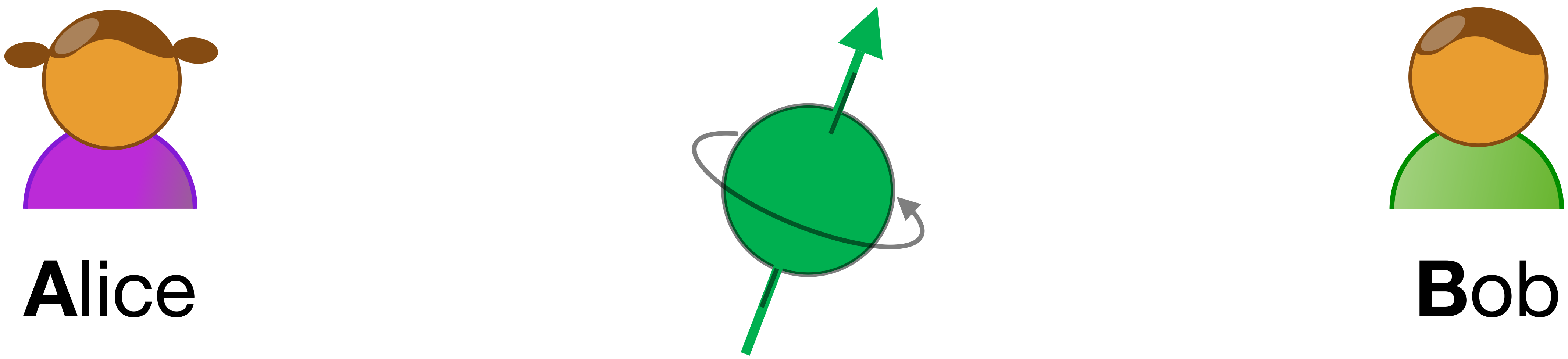

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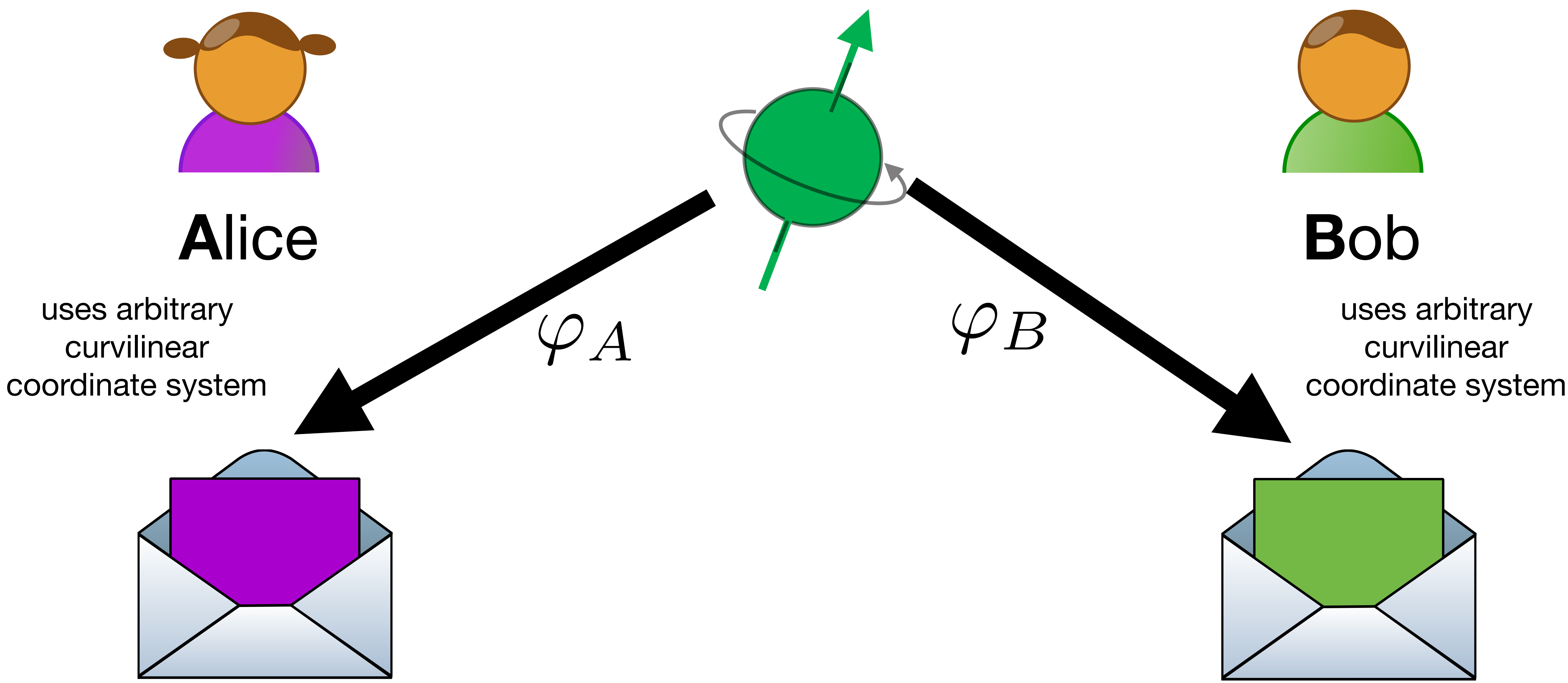

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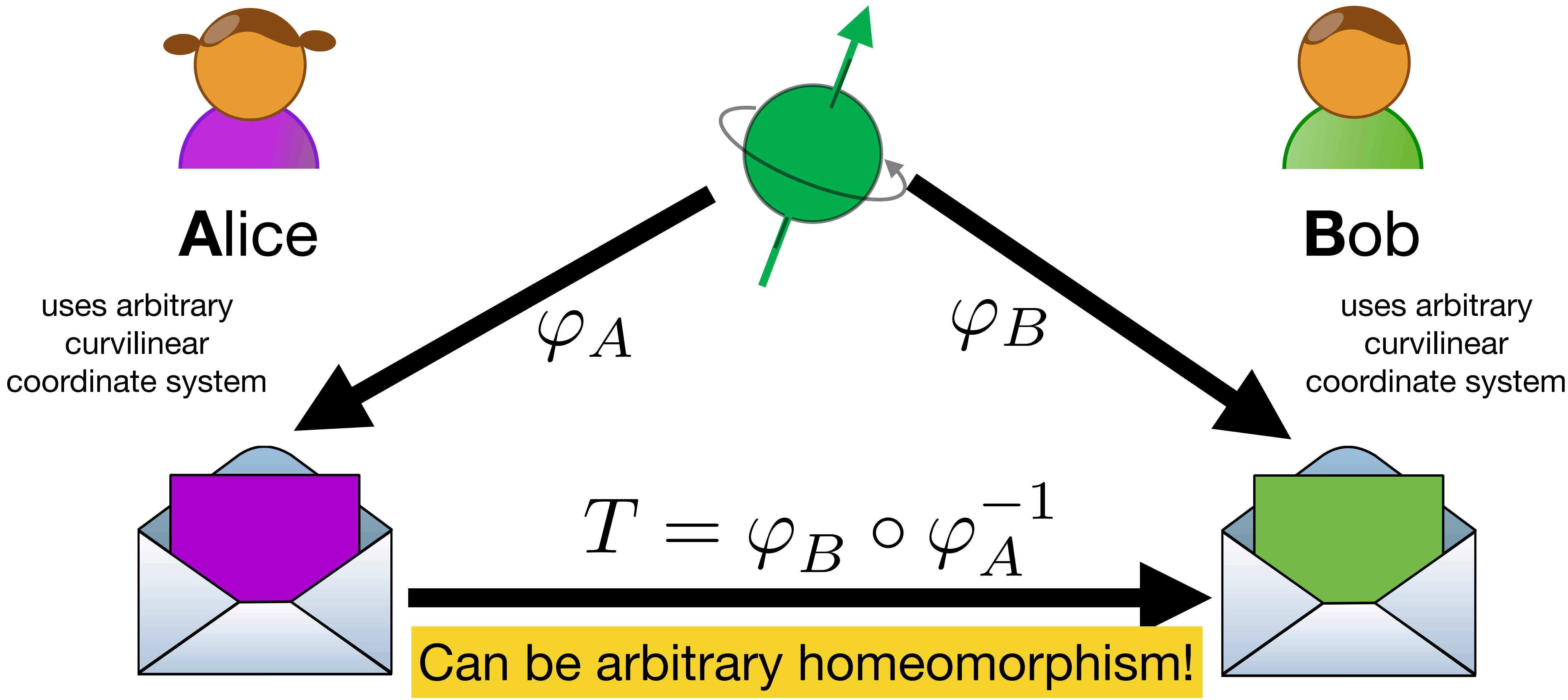

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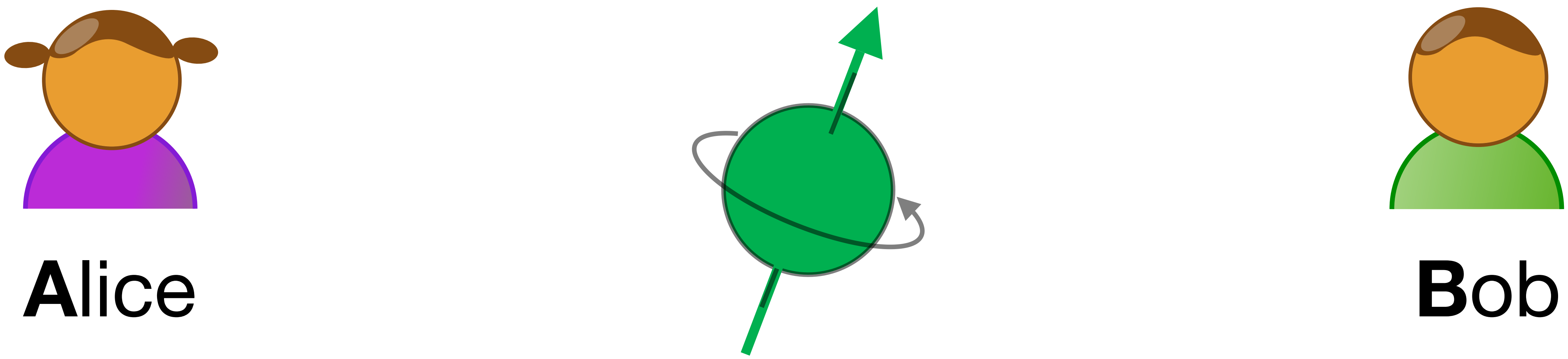

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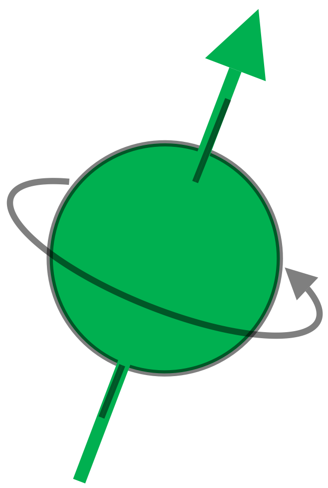
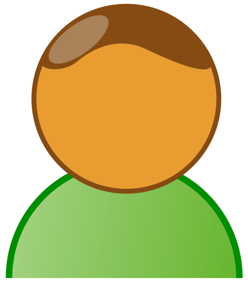
uses **inertial frame** coordinate system





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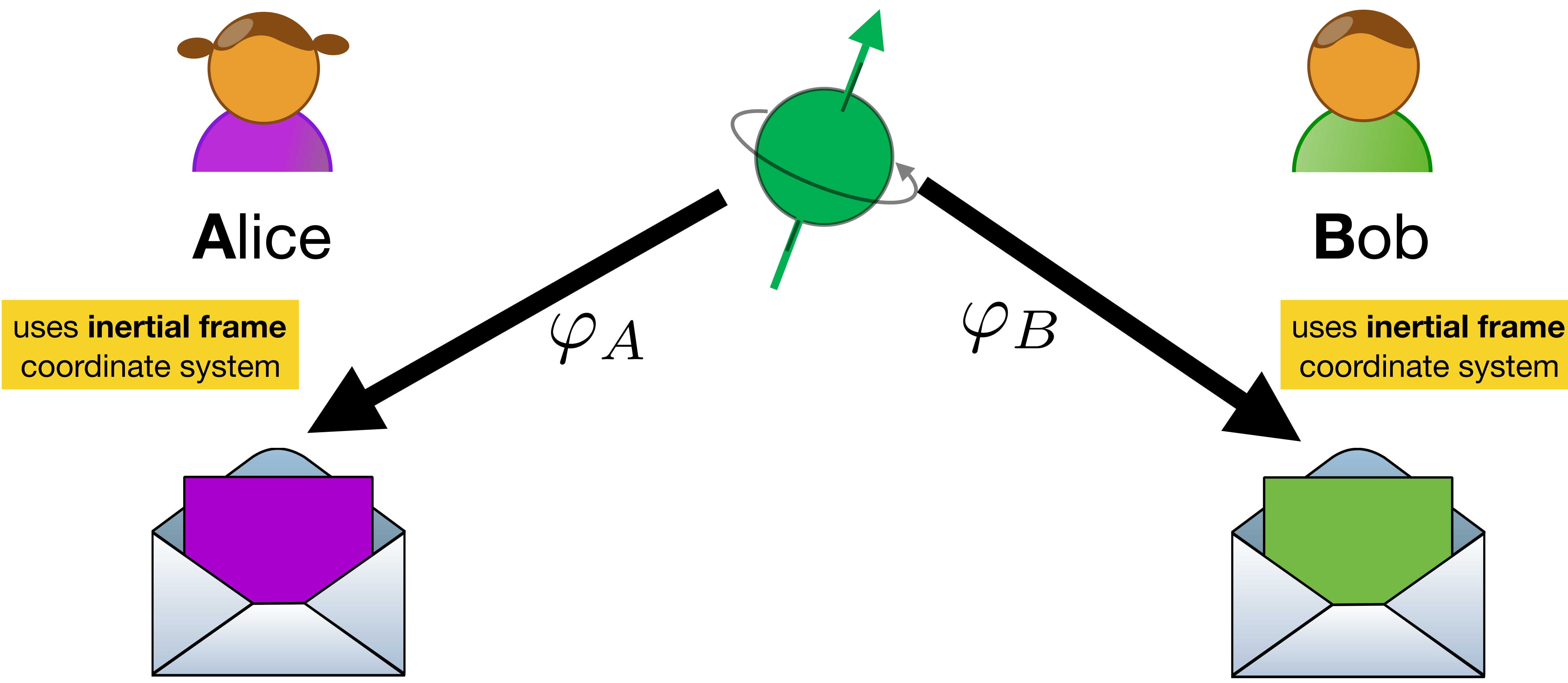

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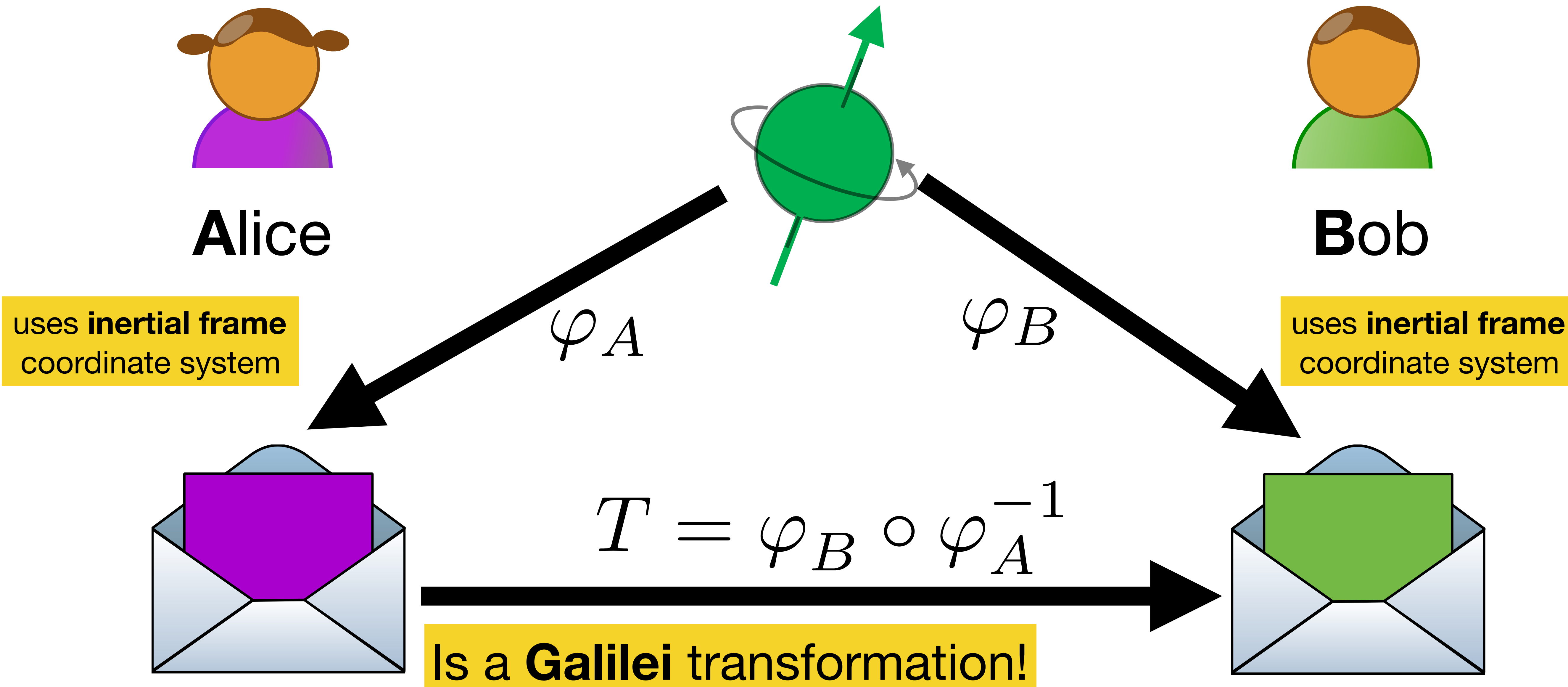

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otherwise it would be unnecessarily large.
Hence the set of possible transformations

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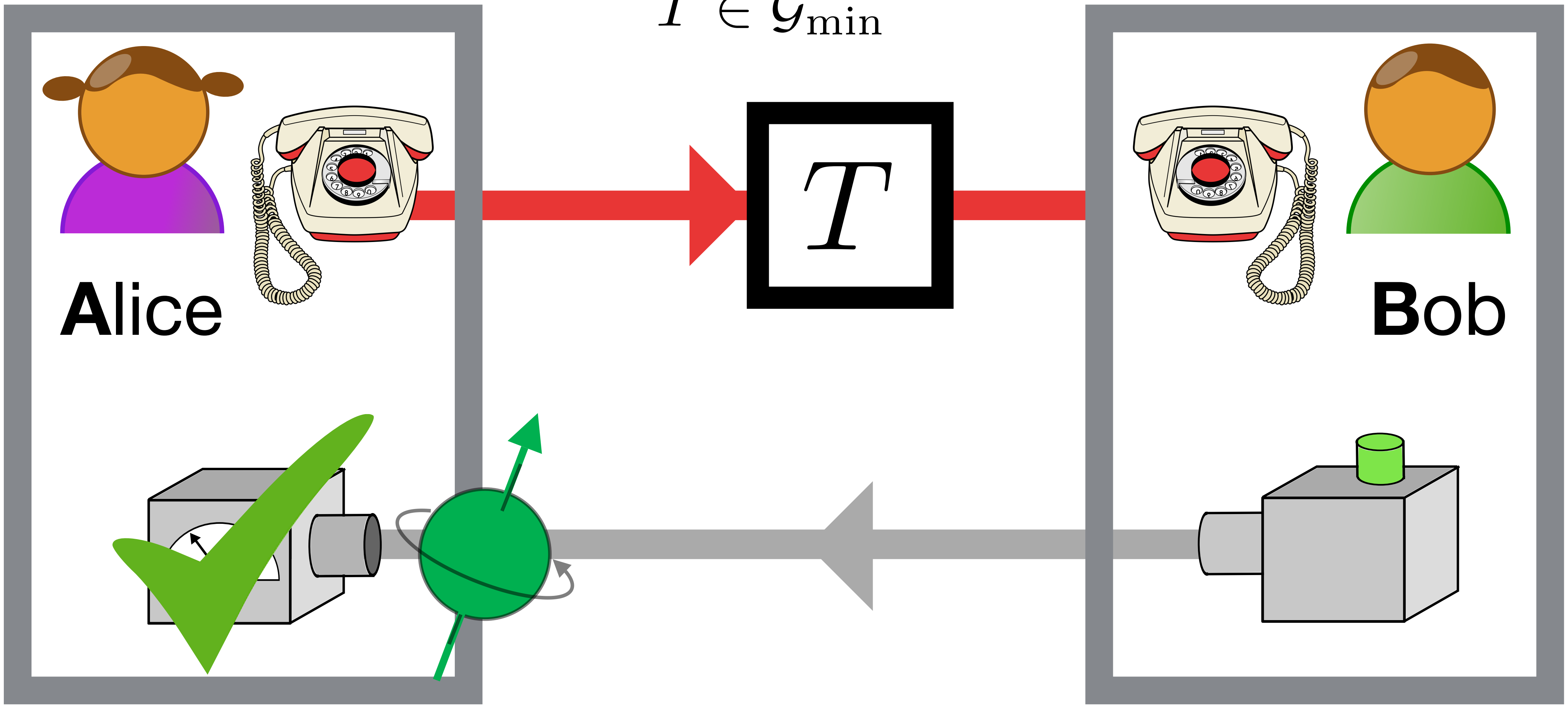
Theorem: Up to isomorphism, there is always a unique smallest group $\mathcal{G}_{\min}$ that A & B can agree upon.

P. Höhn and **M. P. Müller**, *An operational approach to spacetime symmetries: L.T. from Q.C.*, New J. Phys. **18**, 063026 (2016).

The minimal group $\mathcal{G}_{\min}$

Summary: Given any physical background assumptions, and choice of objects to send, there is a unique smallest group $\mathcal{G}_{\min}$ that relates A and B.

$T \in \mathcal{G}_{\min}$



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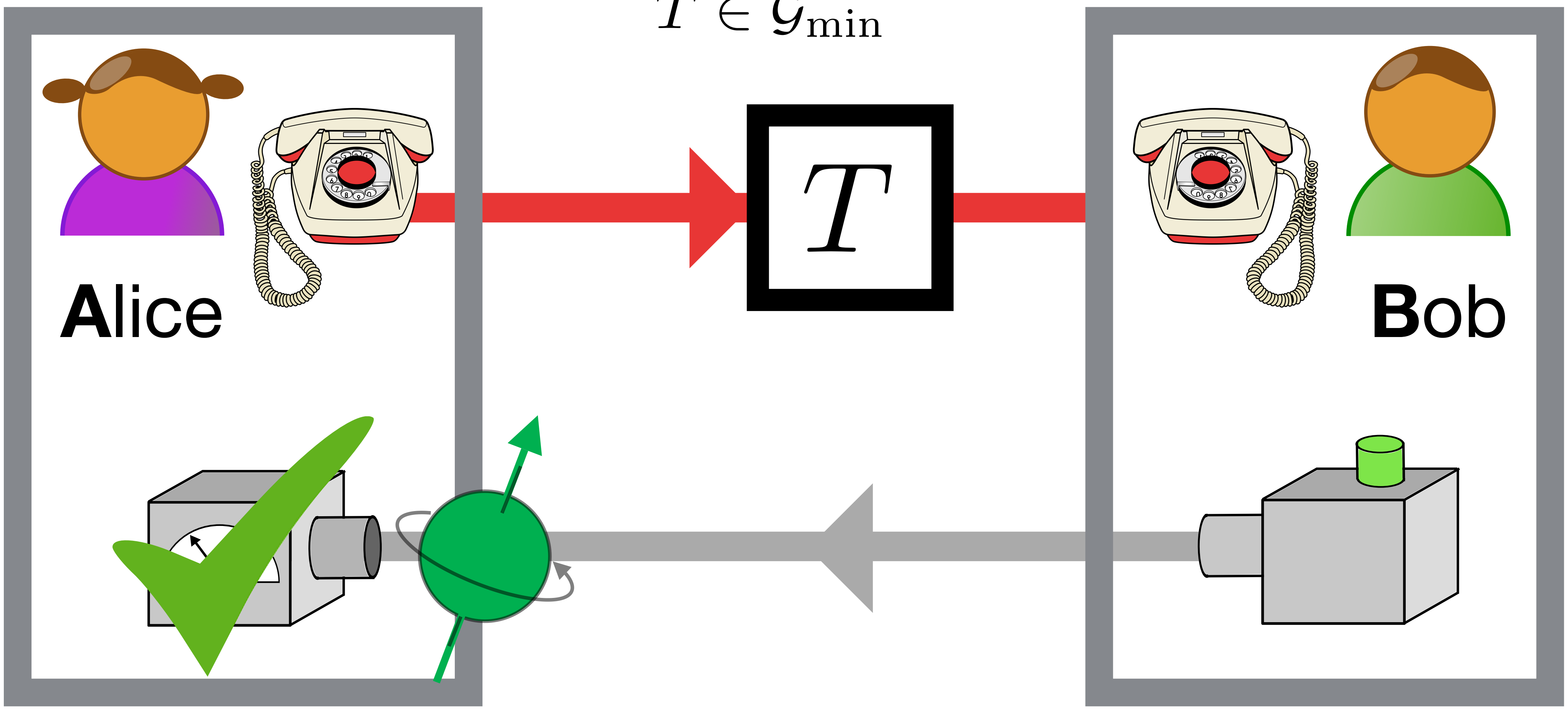

Example: Spinning/moving billard balls in class. mech.: Galilei group.

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The minimal group $\mathcal{G}_{\min}$

Apriori, every physical object has its own group $\mathcal{G}_{\min}$.

However, often different objects "hang together":

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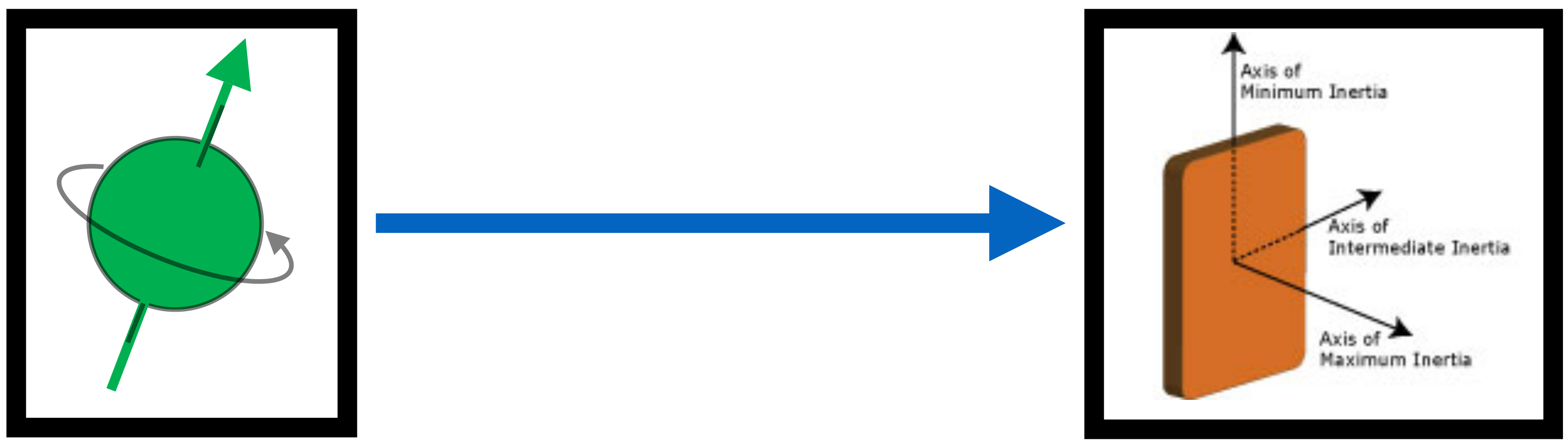

object 1

$T \in \mathcal{G}_{\min}$

object 2

$T \times T \times T$

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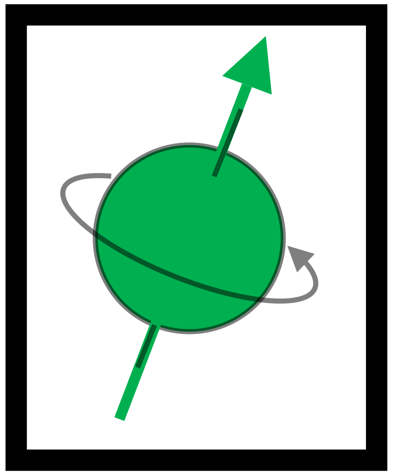

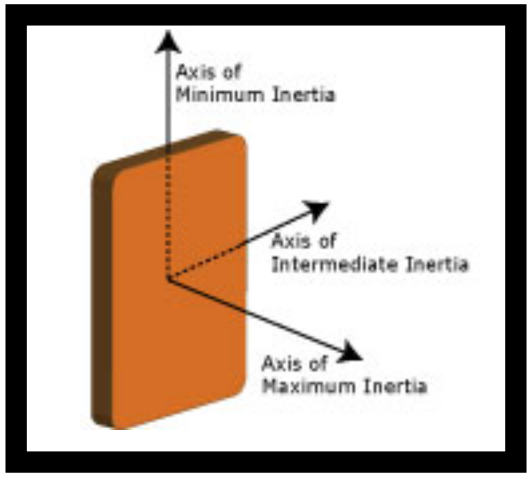

object 1 $T \in \mathcal{G}_{\min}$

object 2 $T \times T \times T$

Then A & B need to negotiate common description only for one of the objects. If this is true for many (all?) objects, then we get an operational definition of "reference frames".

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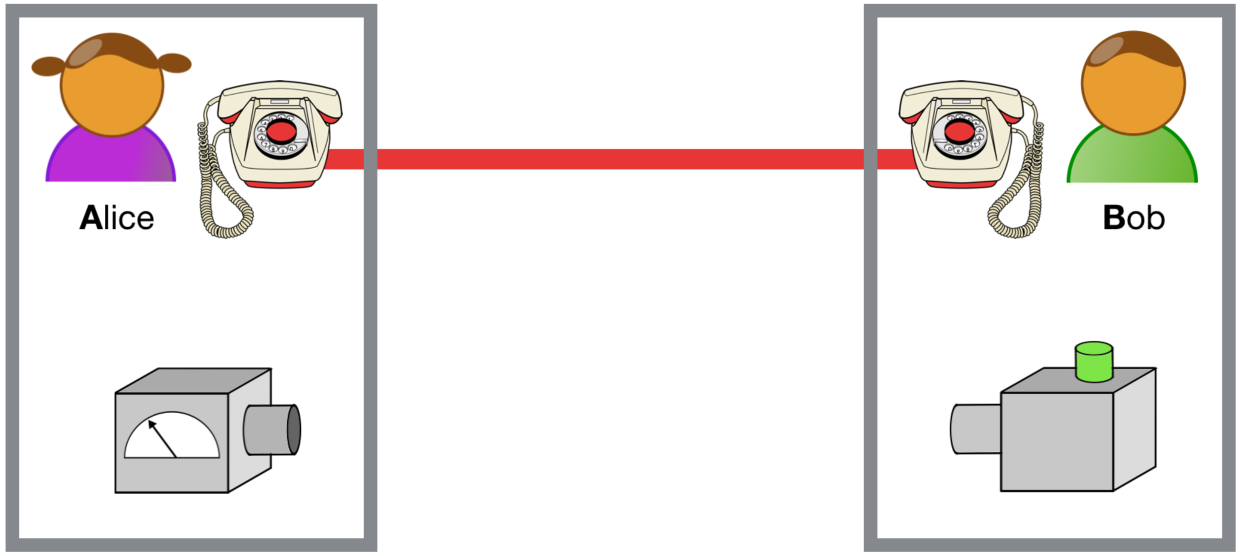
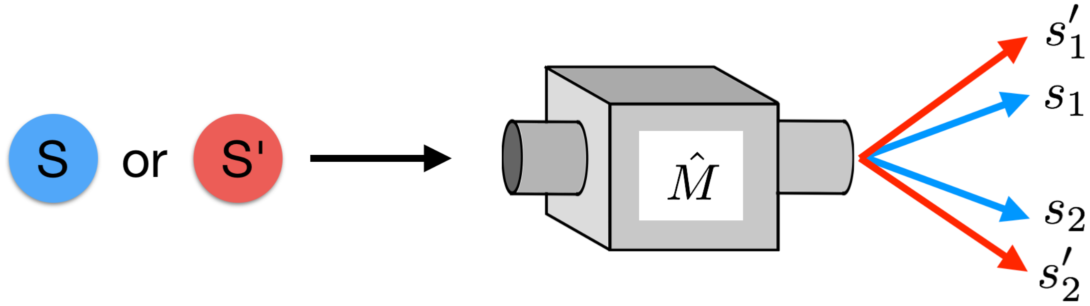
Outline

- **General setup:**
  two observers and $\mathcal{G}_{\min}$



- **Communicating quantum states:**
  emergence of the Lorentz group



- **Spacetime interpretation:**
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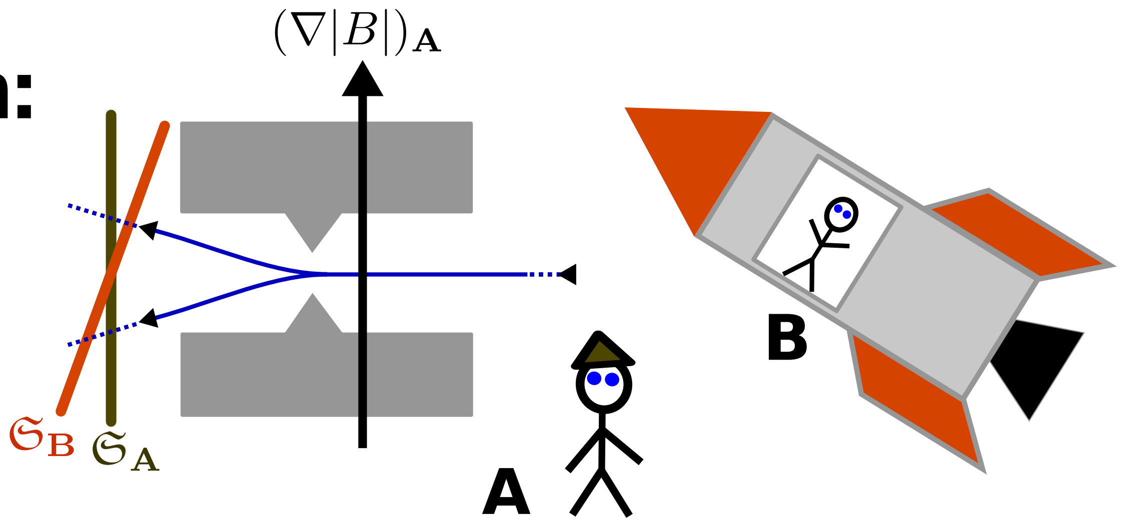

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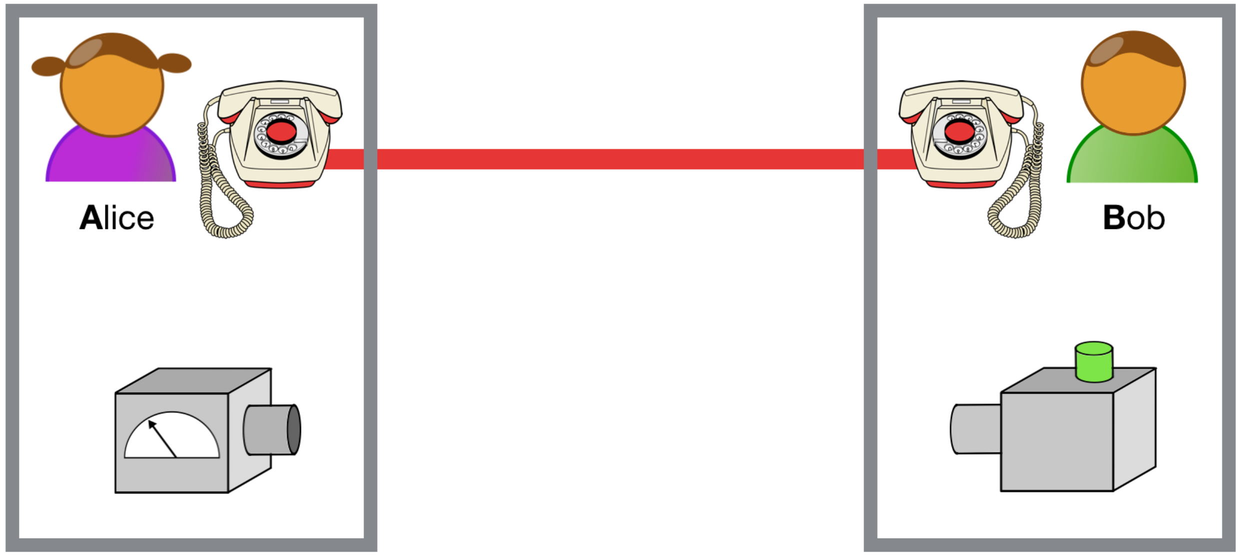

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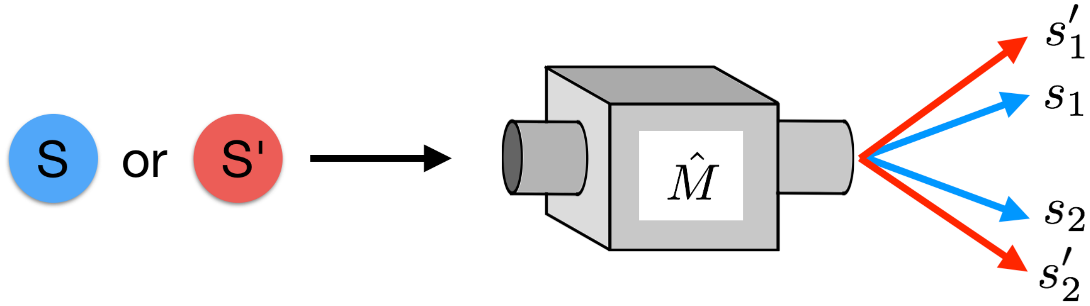

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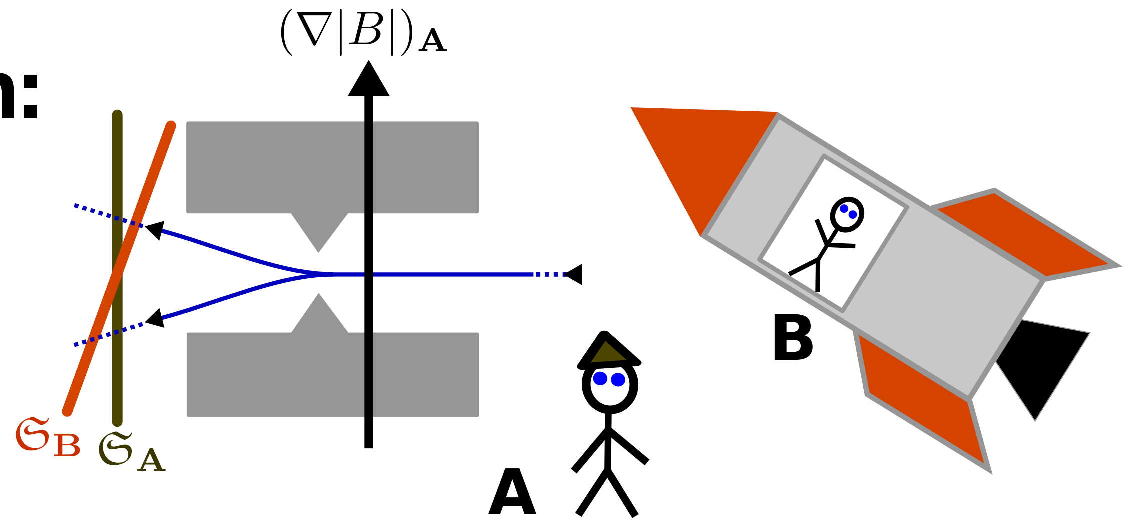

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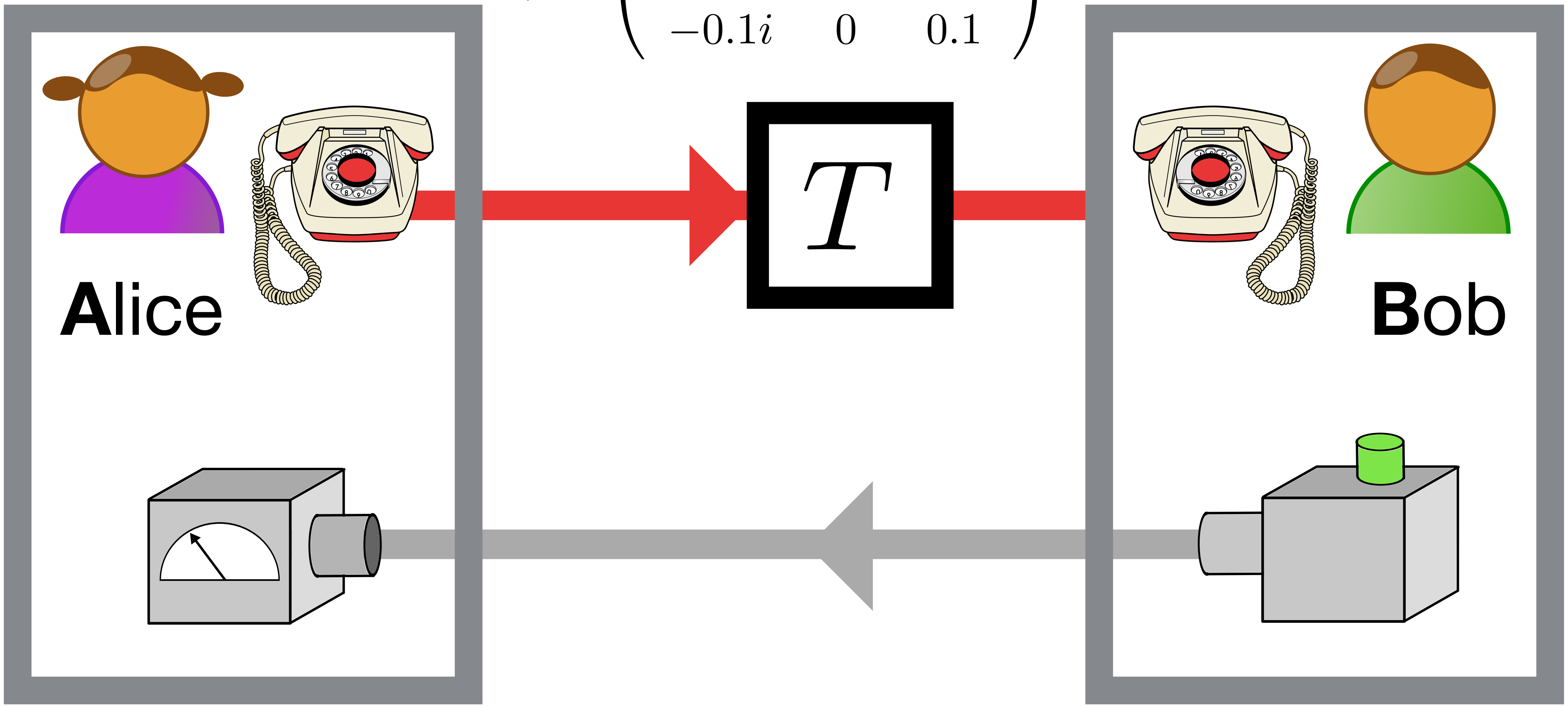
Communicating quantum states

! In what follows, we are **not** assuming any specific background space(time).

Communicating quantum states

***Bob, please** send me the following quantum state:*

$$\rho = \begin{pmatrix} 0.6 & 0 & 0.1i \\ 0 & 0.3 & 0 \\ -0.1i & 0 & 0.1 \end{pmatrix}$$



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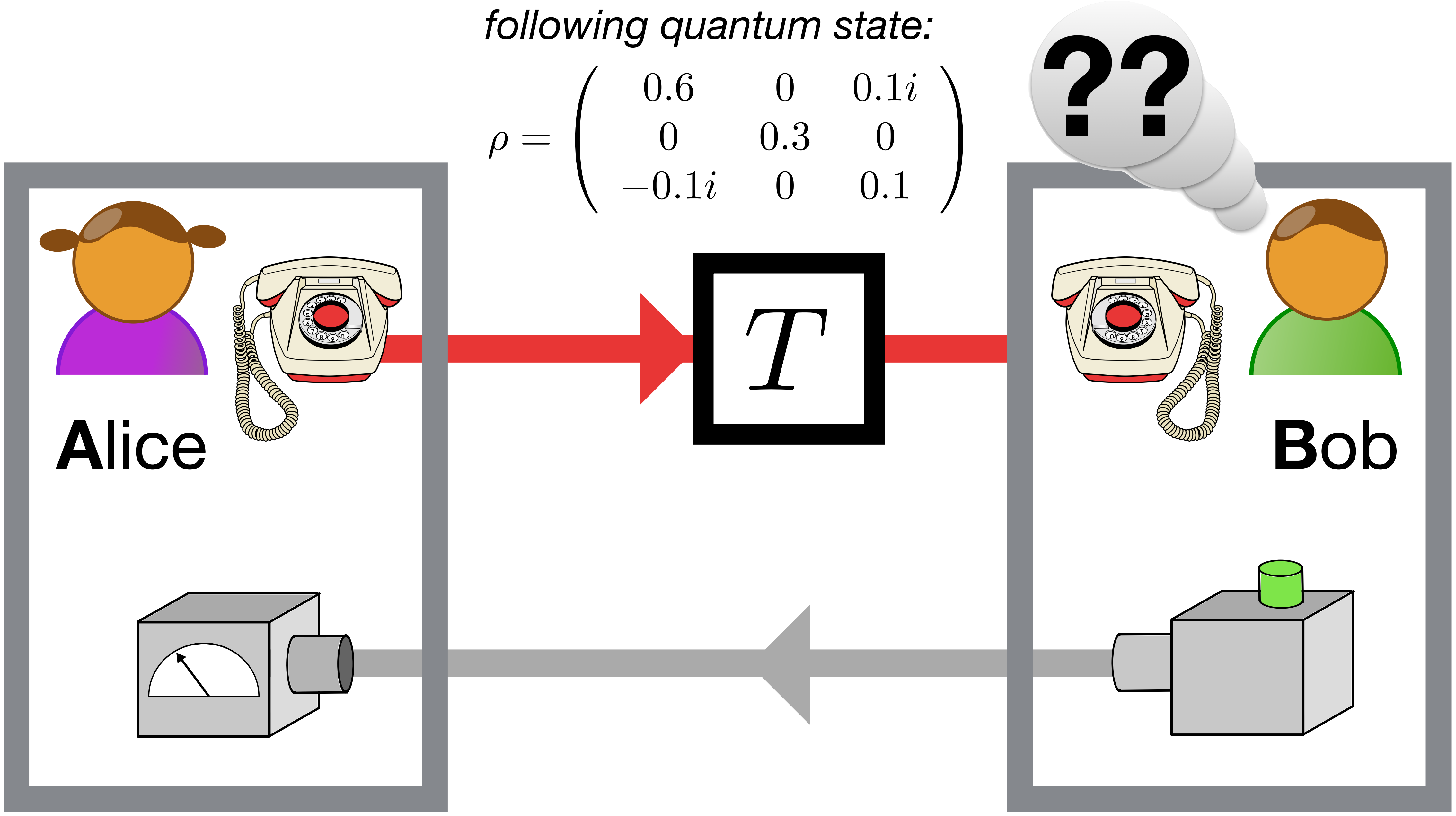

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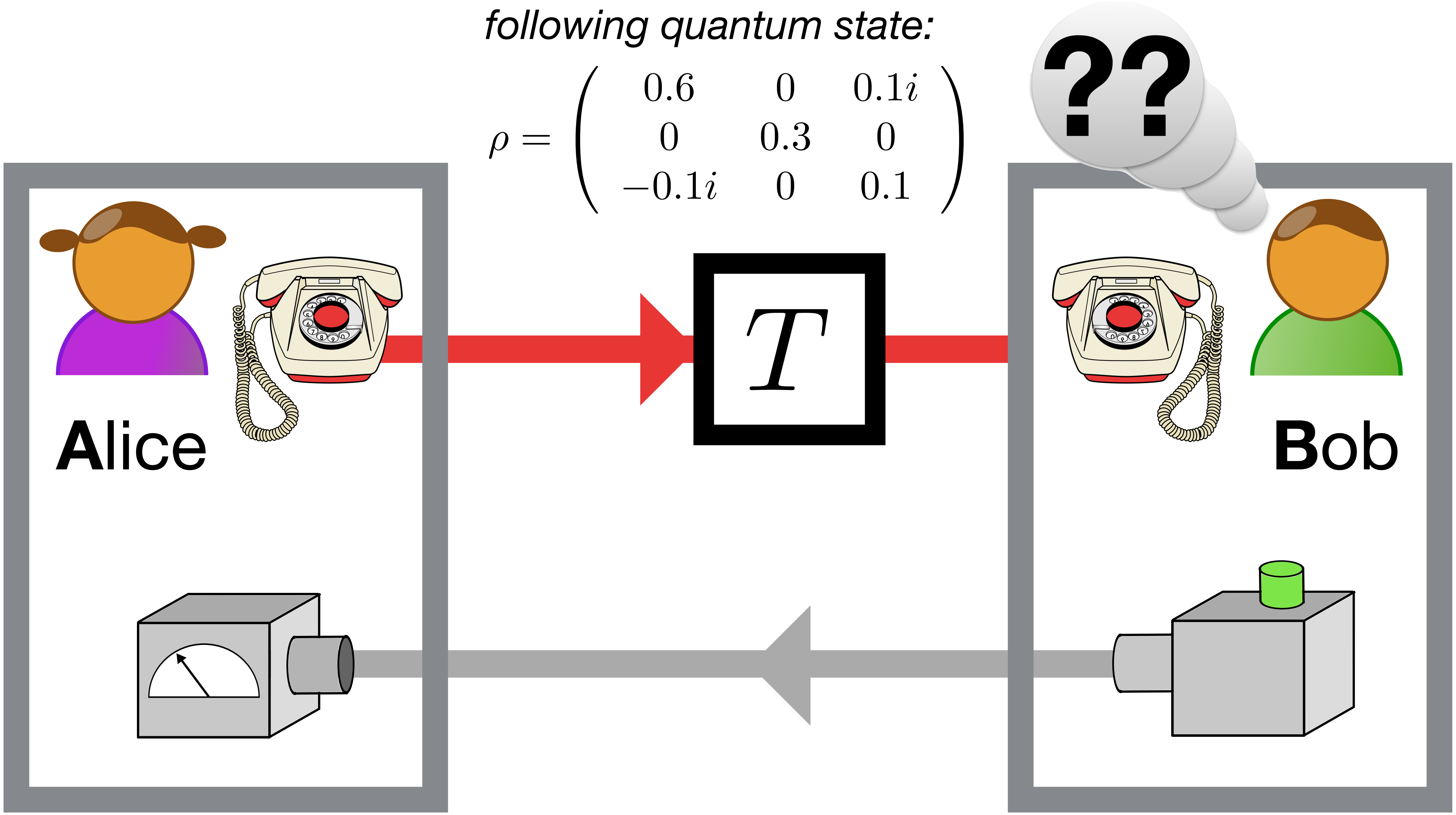

Problem: A & B have not agreed on a Hilbert space basis.

P. Höhn and **M. P. Müller**, *An operational approach to spacetime symmetries: L.T. from Q.C.*, New J. Phys. **18**, 063026 (2016).

Communicating quantum states

Situation looks like the following:

- A & B agree to choose encodings of quantum states ω as usual into **density matrices** $\rho_A = \varphi_A(\omega), \rho_B = \varphi_B(\omega)$ (convex-linear).
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For every Hilbert space $\mathcal{H}$, there is a unitary $U \in \mathrm{SU}(N)$ with $N = \dim \mathcal{H}$ such that

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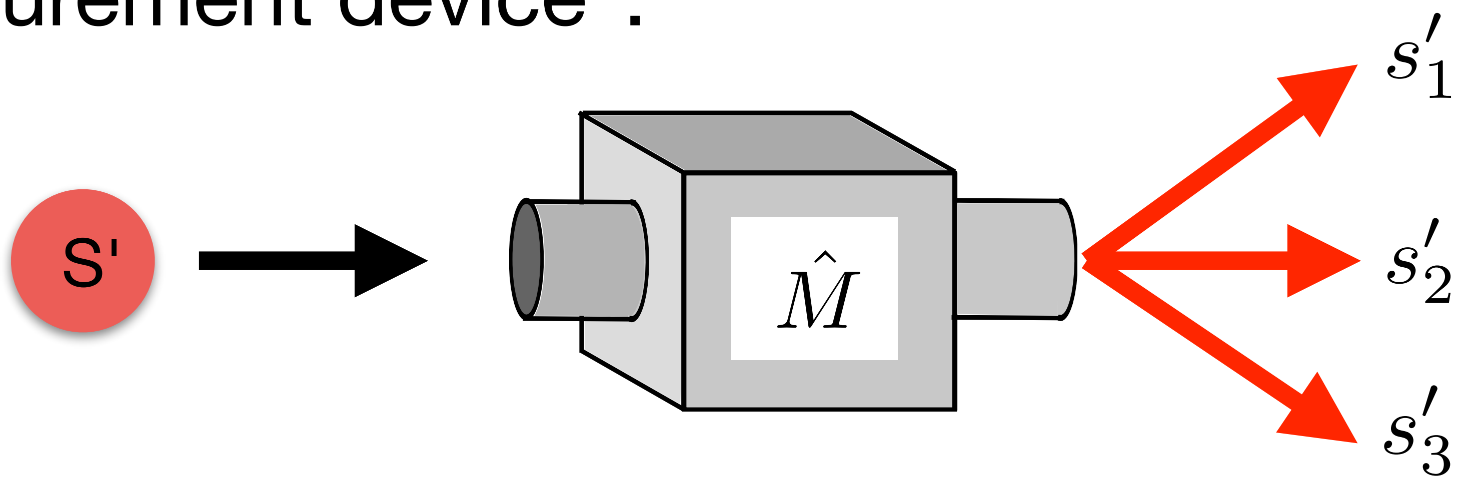
But this would mean: for every Hilbert space $\mathcal{H}$, A & B have to establish a separate transformation $T \in \mathcal{G}_{\min}(\mathcal{H})$.
Highly impractical! Can they do better? **Yes!**

P. Höhn and **M. P. Müller**, *An operational approach to spacetime symmetries: L.T. from Q.C.*, New J. Phys. **18**, 063026 (2016).

How do different Hilbert spaces "hang together"?

"universal measurement device":



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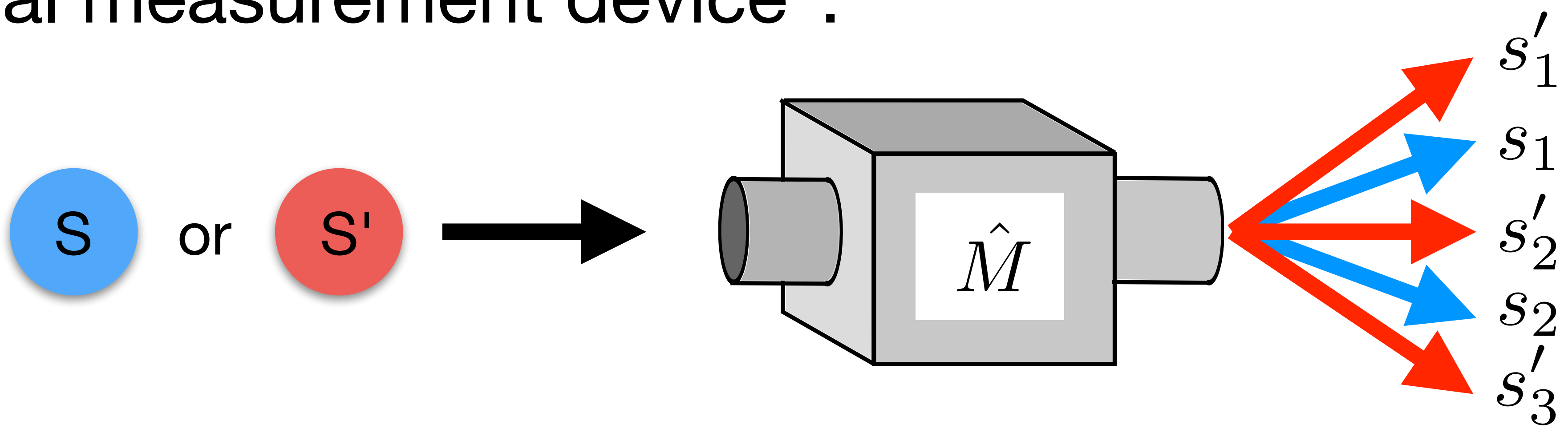

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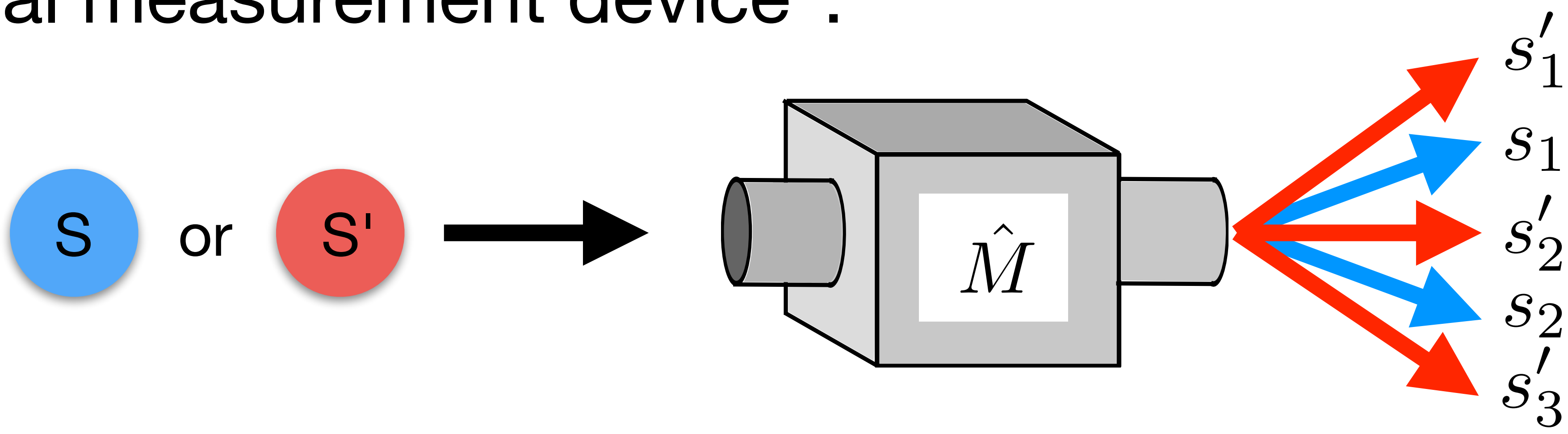

Measures "the same observable" on different quantum systems S, S' (with maybe $\dim S \neq \dim S'$).

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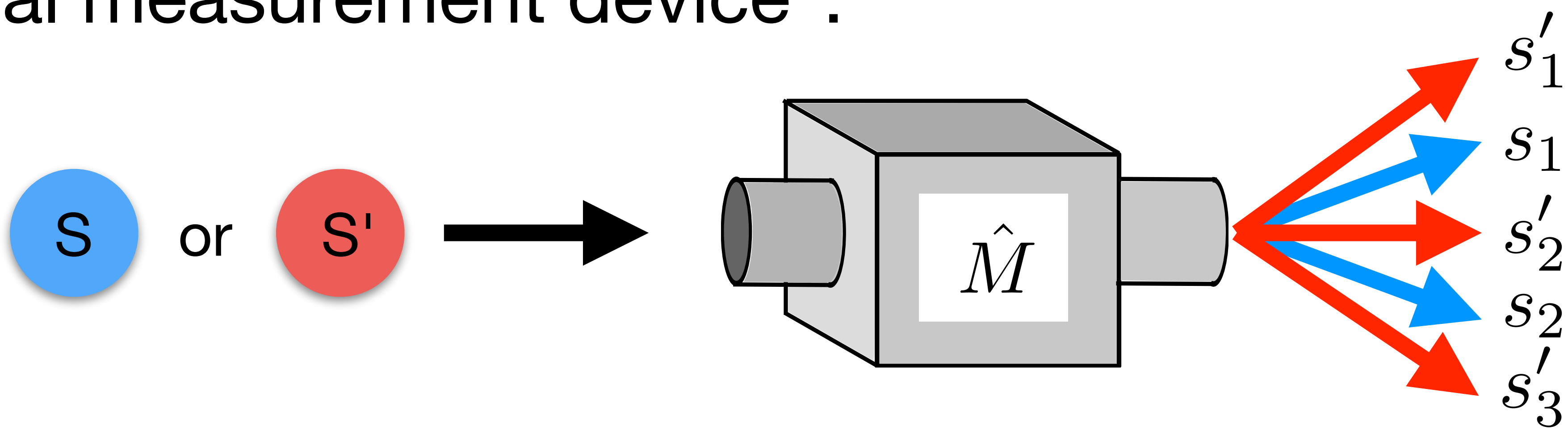

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Example: Stern-Gerlach device, $\hat{M}$=spin in z-direction, S=electron spin, S'=Z-Boson spin

$$\hat{M}(S) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \hat{M}(S') = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

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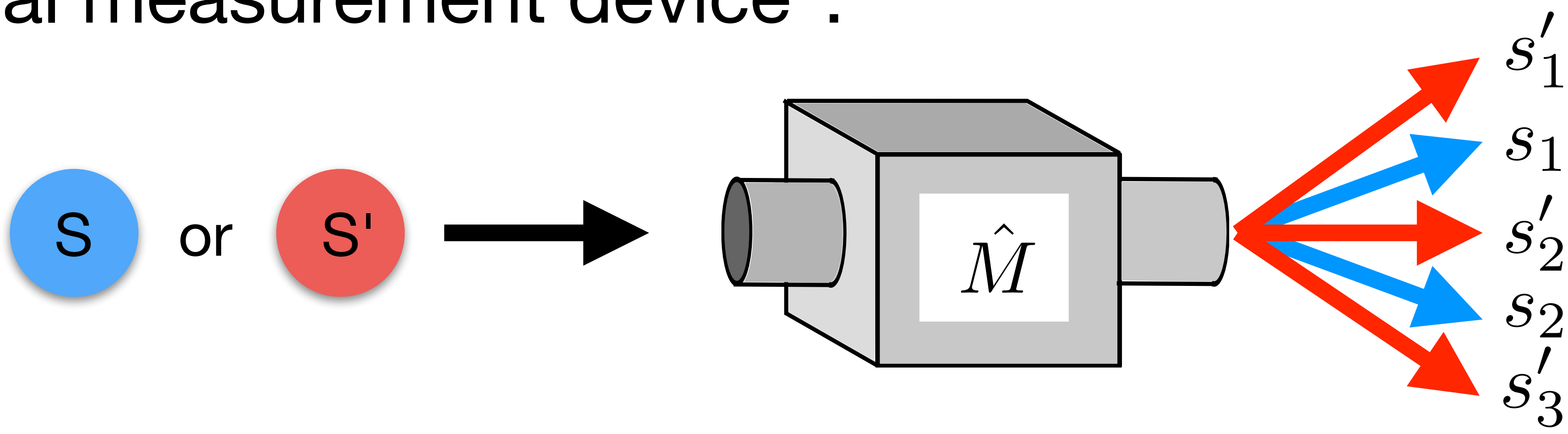

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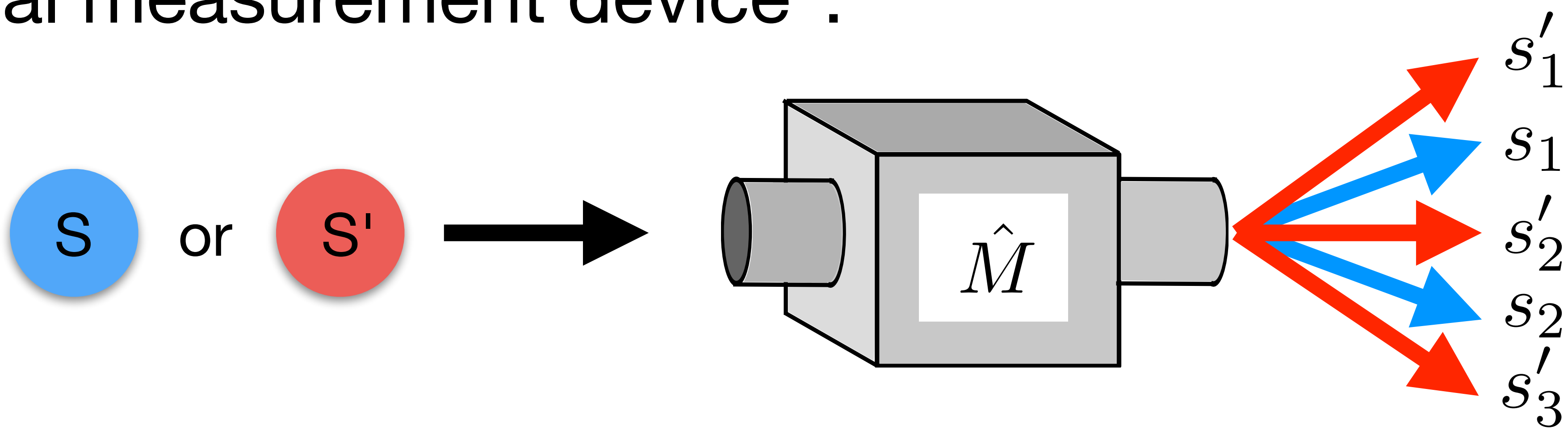

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This will **relate S and S'**. But how to define this abstractly?

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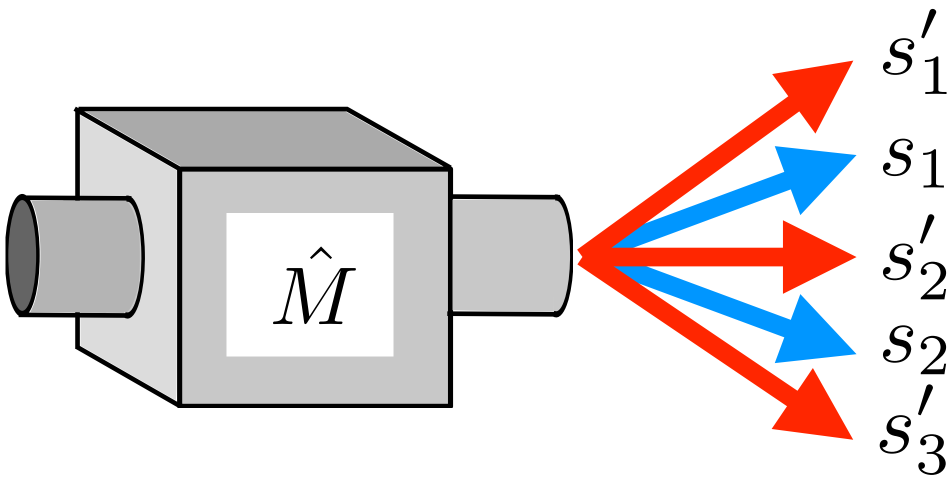

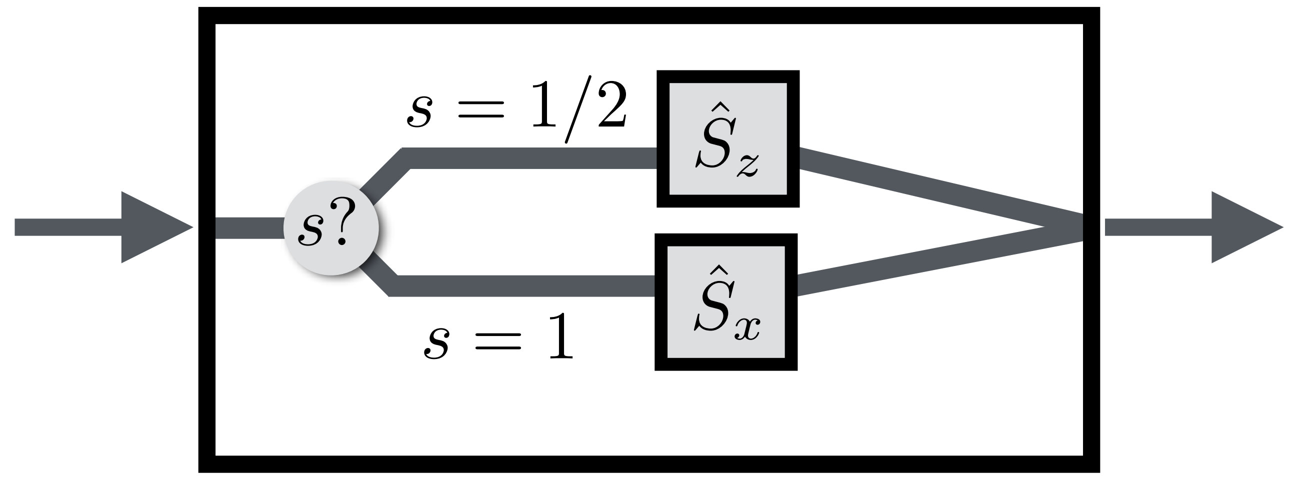

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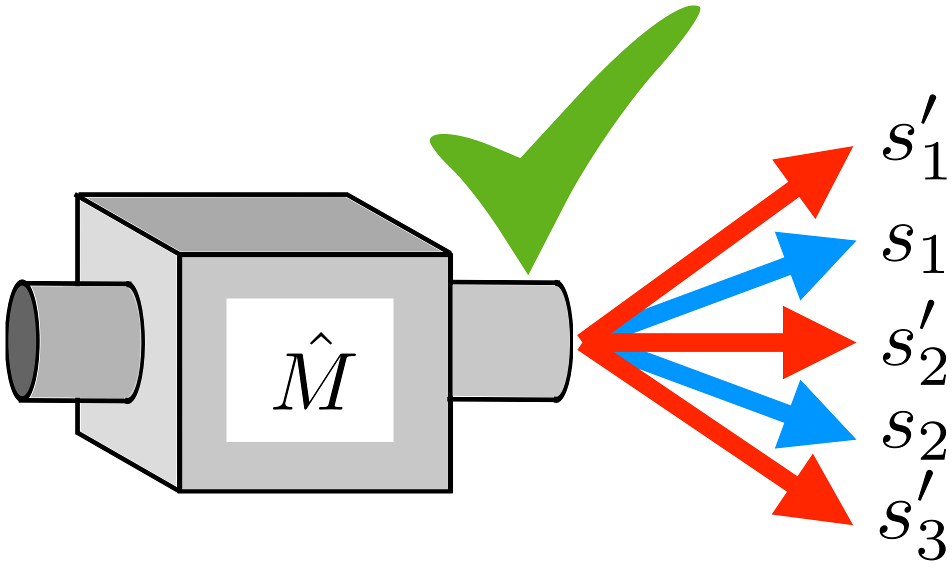

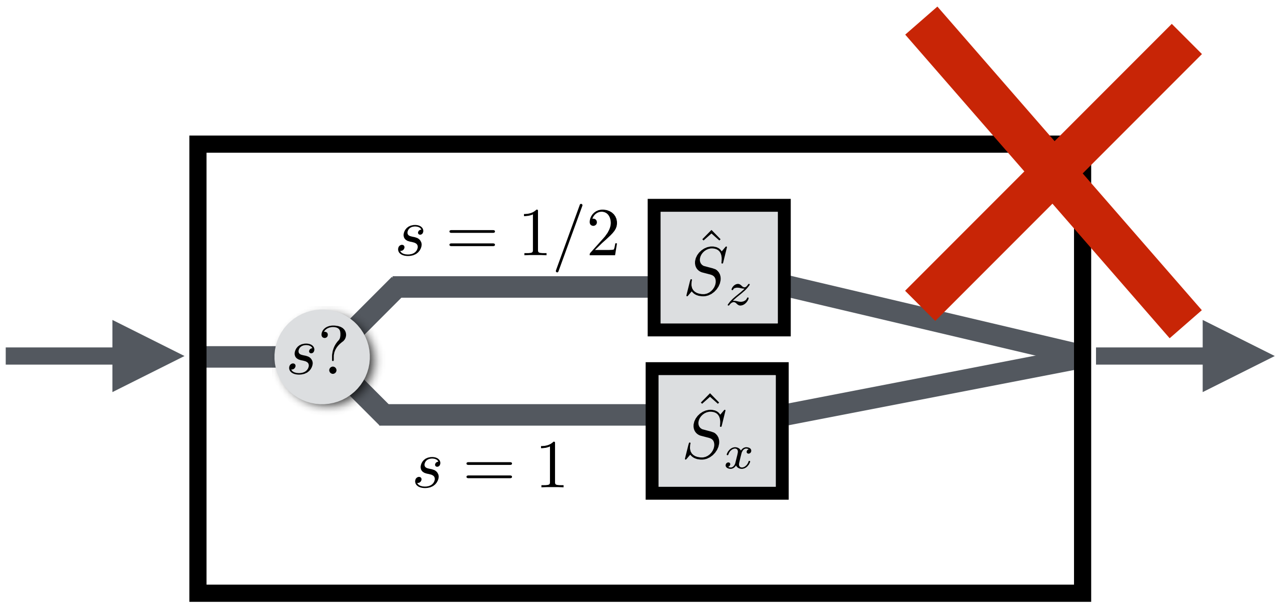

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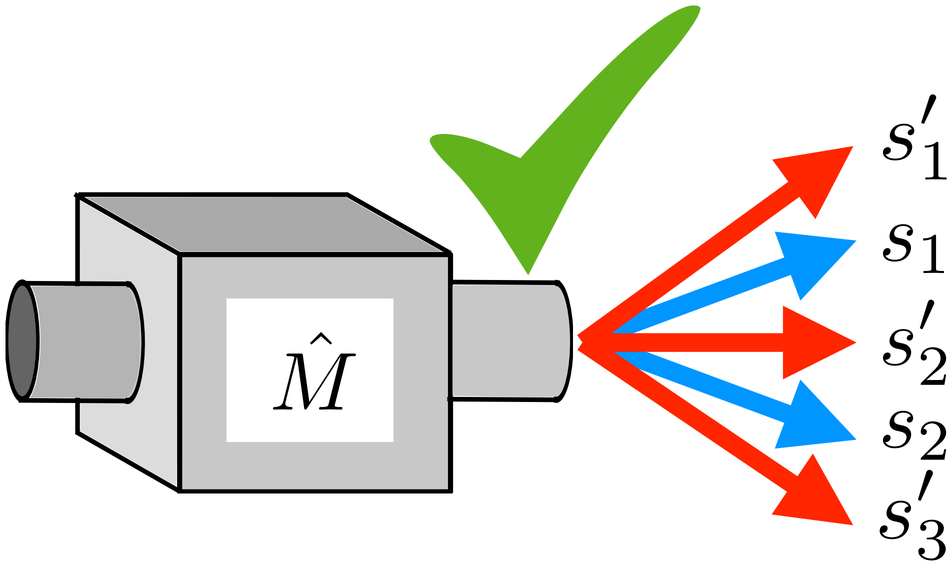

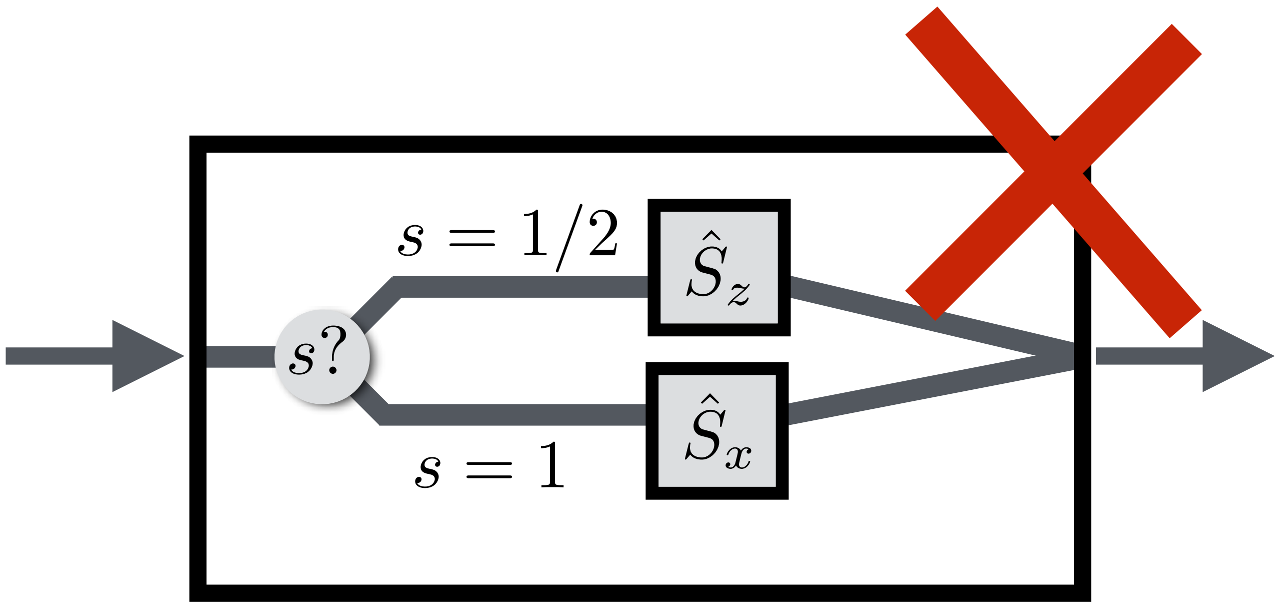

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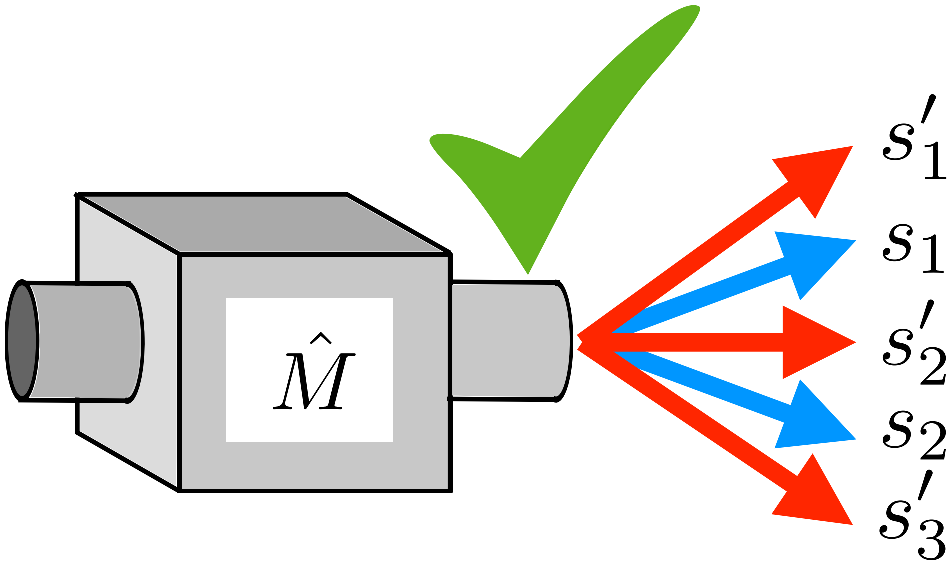

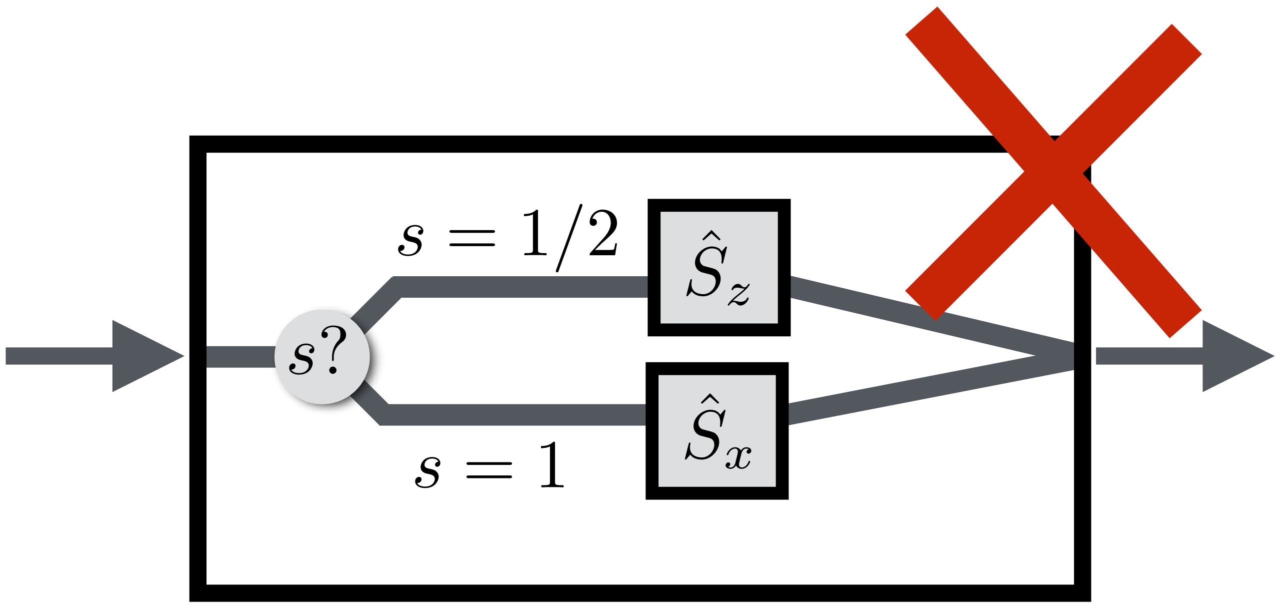

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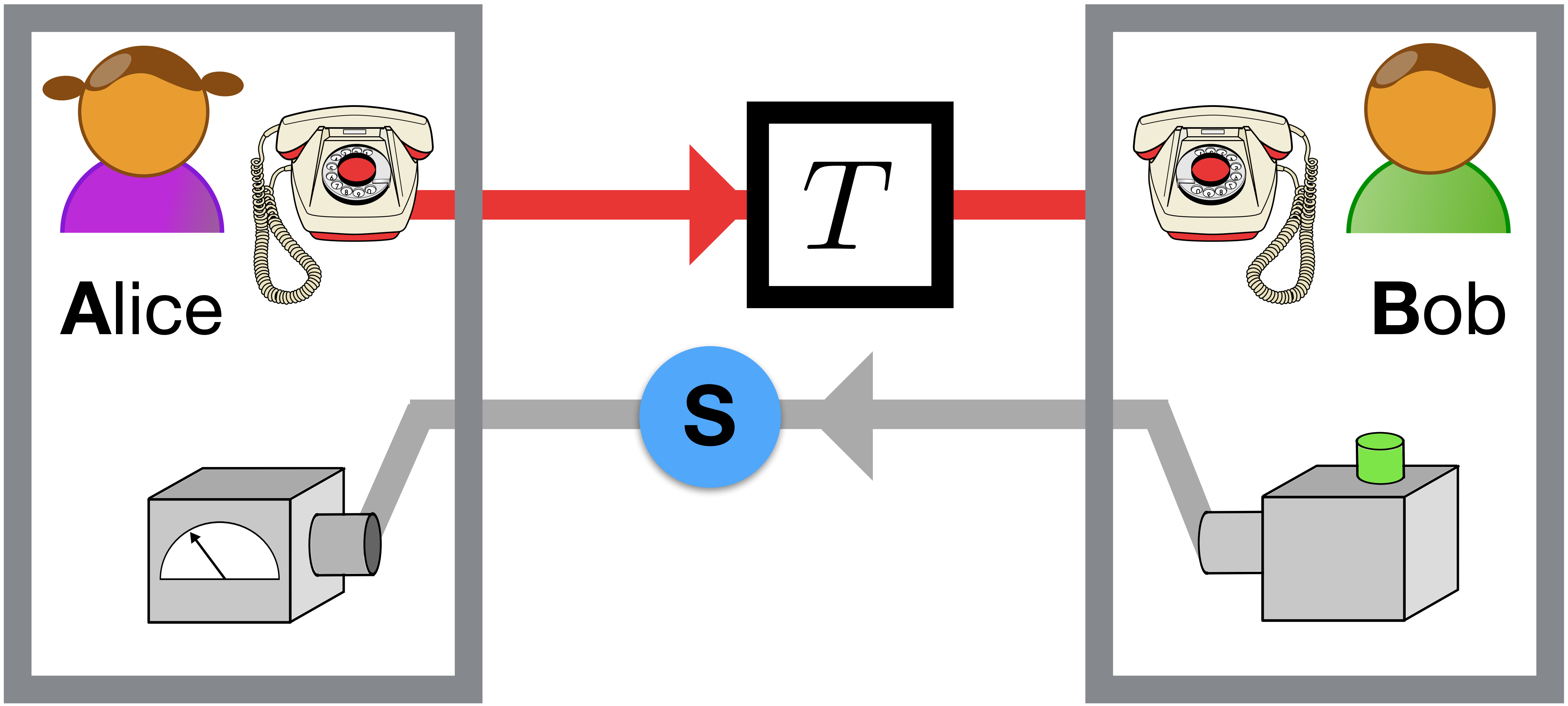

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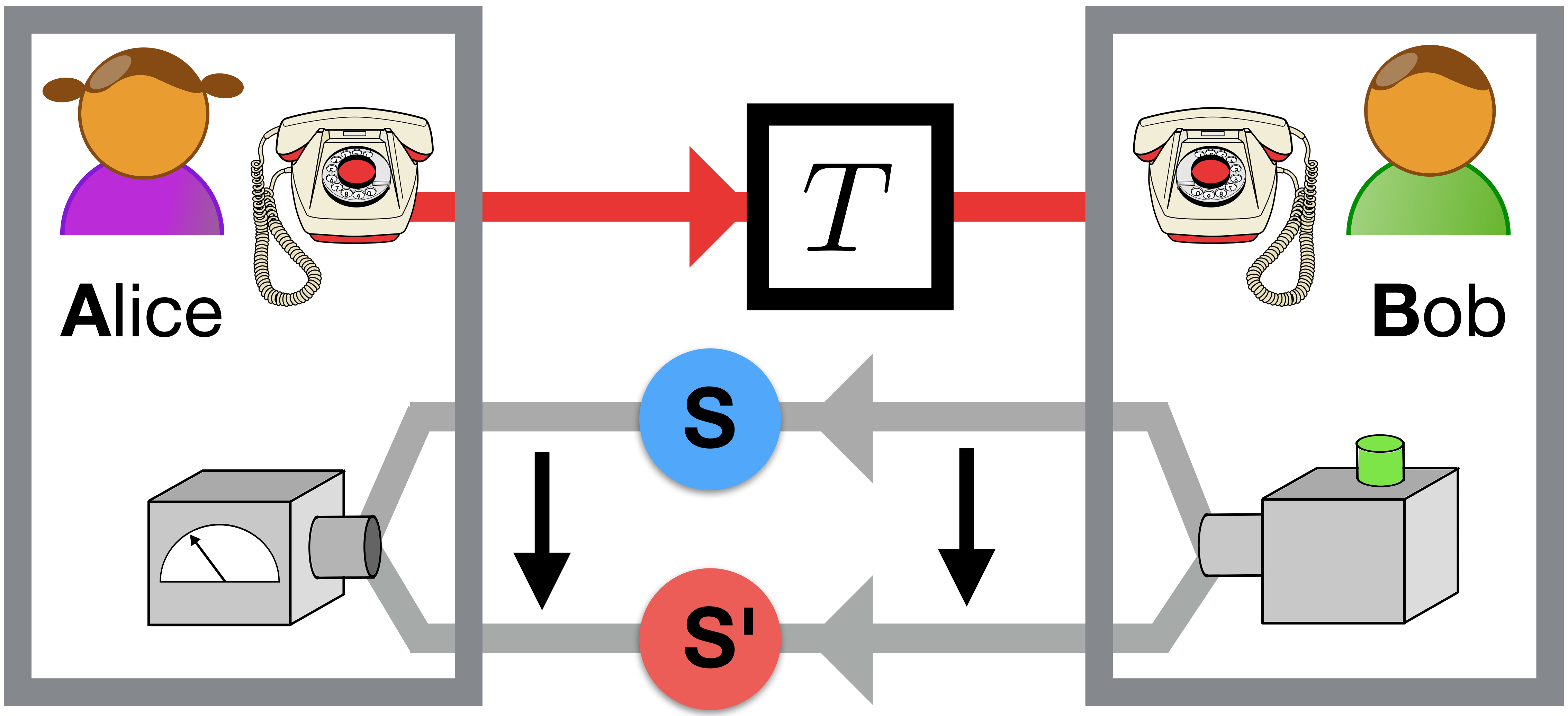

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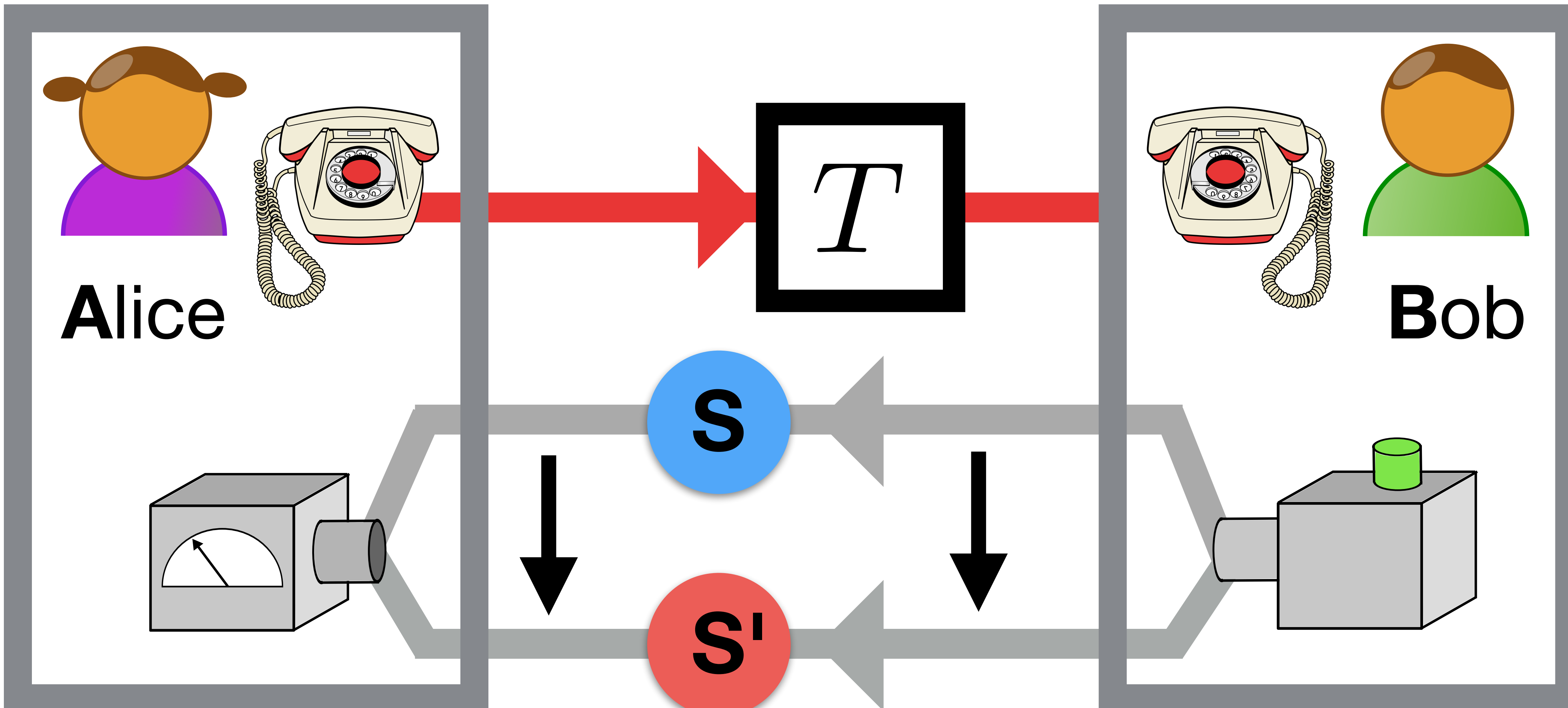

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What if different Hilbert spaces "hang together"?

Consequence: if all quantum systems hang together, then $\mathcal{G}_{\min} =$ symmetry group of **smallest-dim.** quantum system.

Recall consistency conditions for "universal devices"

- $\hat{M}(S) \leftrightarrow \hat{M}(S')$ continuous,
- $\hat{M}_1(S) \leq \hat{M}_2(S) \quad \Rightarrow \quad \hat{M}_1(S') \leq \hat{M}_2(S')$,
- $\hat{M}(S) = 0 \quad \Rightarrow \quad \hat{M}(S') = 0.$

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Further assumption: ***N*=2** (maybe redundant)

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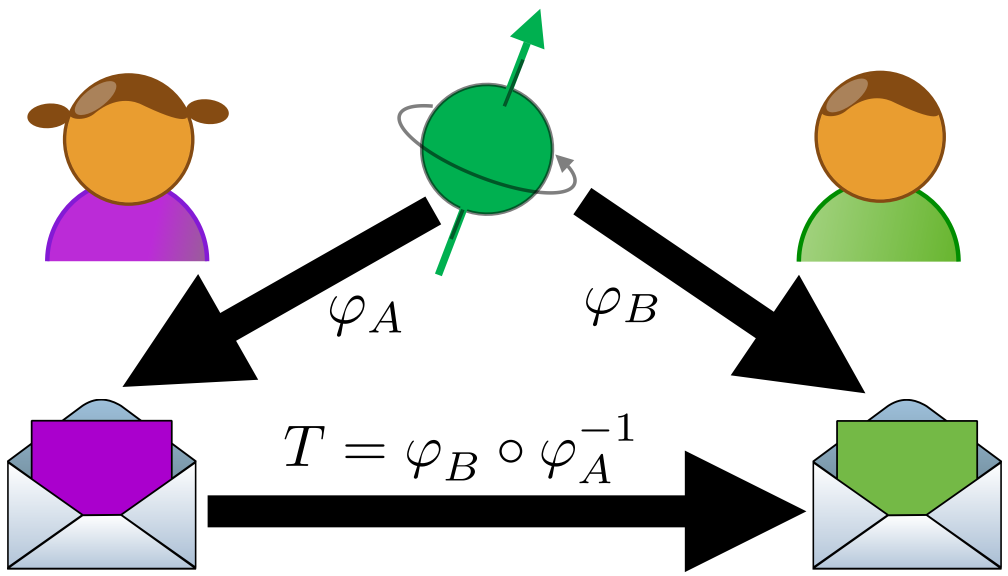

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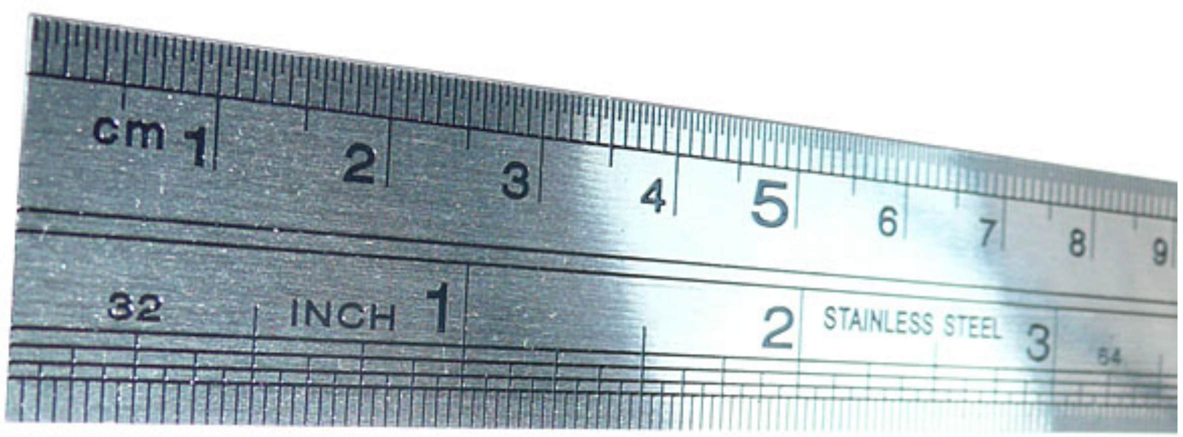
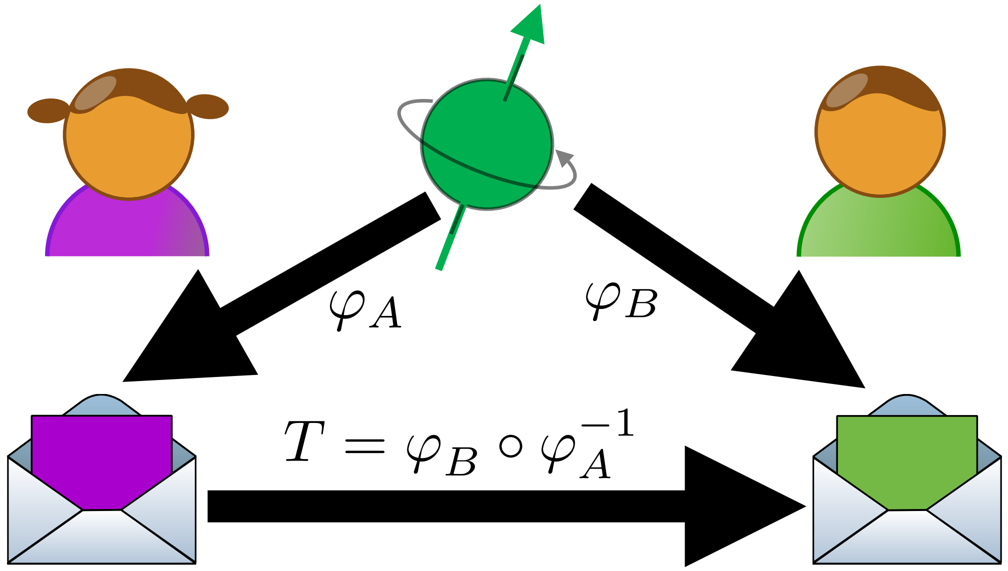
Interpretation of $\mathbb{R}^+$-factor:
Conversion factor for different choices of units (for eigenvalues)





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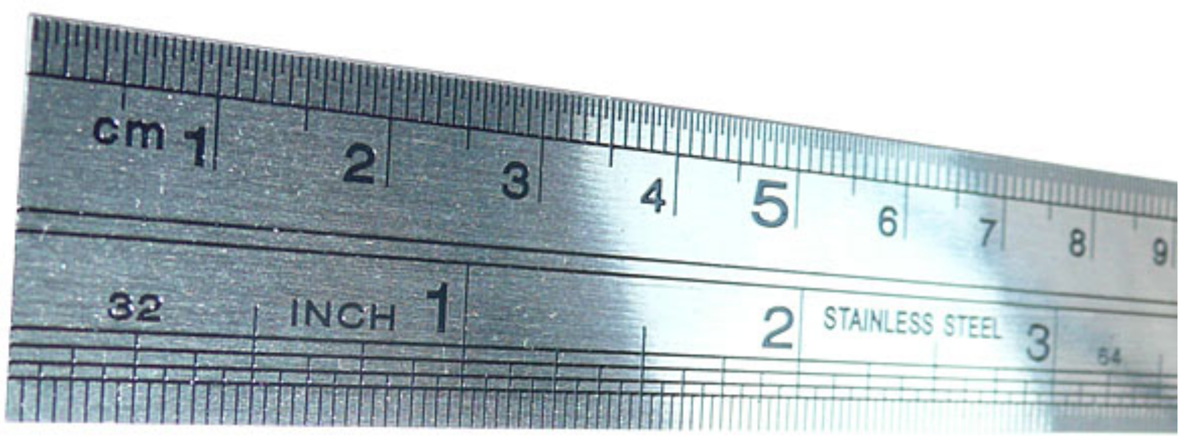

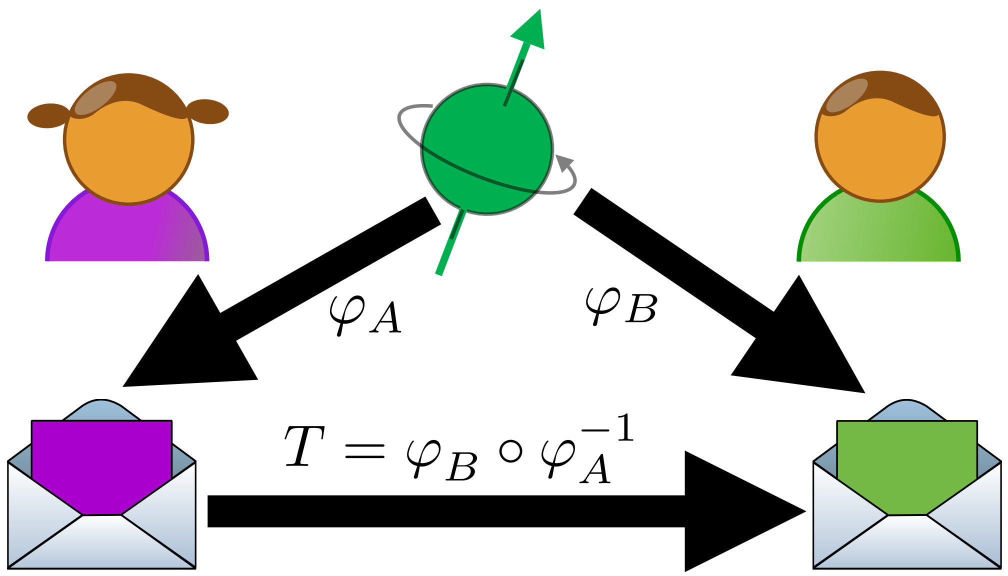

Translates between observers' descriptions of local quantum physics.

⟶ **spacetime interpretation?**

P. Höhn and **M. P. Müller**, *An operational approach to spacetime symmetries: L.T. from Q.C.*, New J. Phys. **18**, 063026 (2016).

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Theorem 4.12. *In the scenario above, the minimal group is the orthochronous Lorentz group, together with a scaling factor,* $\mathcal{G}_{\min} = \mathbb{R}^+ \times \mathrm{O}^+(3,1)$. *The subgroup of implementable transformations is* $\mathbb{R}^+ \times \mathrm{SO}^+(3,1)$, *the group of proper orthochronous Lorentz transformations, times a scaling factor.*

Furthermore, if $\mathbf{S}$ *is the 'root qubit' of assumptions 4.11, and* $\mathbf{S}'$ *any other quantum system such that all observables of* $\mathbf{S}$ *are universally measurable on* $\mathbf{S}$ *and* $\mathbf{S}'$, *then* $\mathbf{S}'$ *carries a projective representation of* $\mathrm{SO}^+(3,1)$; *the group elements act as isometries between different Hilbert spaces. All other quantum systems* $\mathbf{S}'$ *carry a projective representation of the subgroup of* $\mathrm{SO}^+(3,1)$ *which preserves the observables that are universally measurable on* $\mathbf{S}$ *and* $\mathbf{S}'$.

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We get projective rep's of SO*(3)* ⟶ different spin.

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But wait a minute - this is not unitary?!

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This is fine – the map $|\psi\rangle \mapsto X|\psi\rangle$ is an **isometry** between the two Hilbert spaces $(\mathbb{C}^N, \langle\cdot,\cdot\rangle)$ and $(\mathbb{C}^N, (\cdot,\cdot))$.

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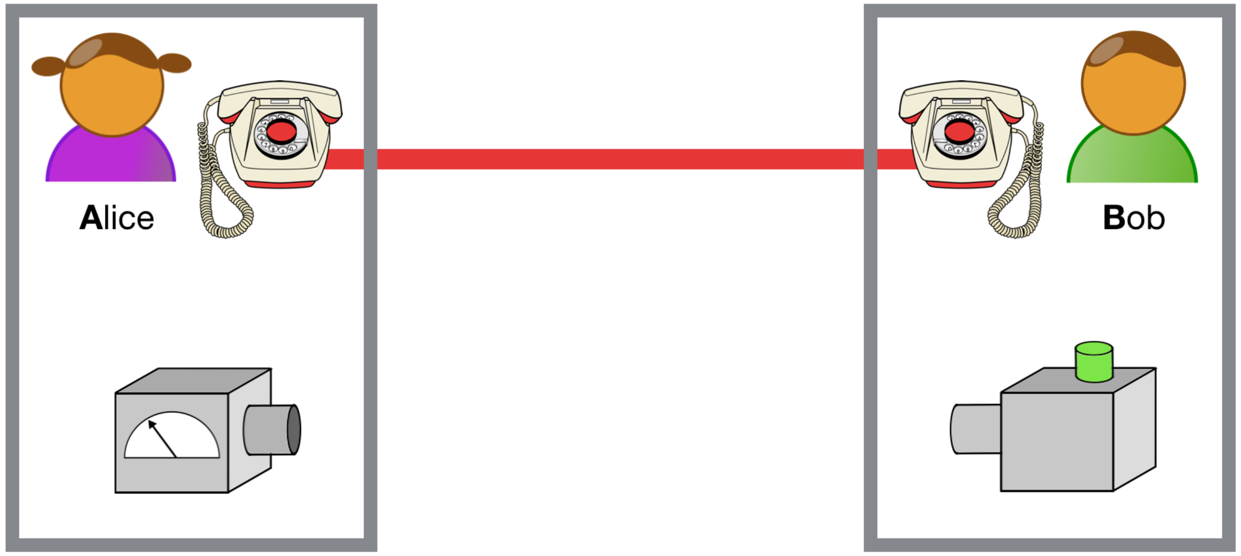
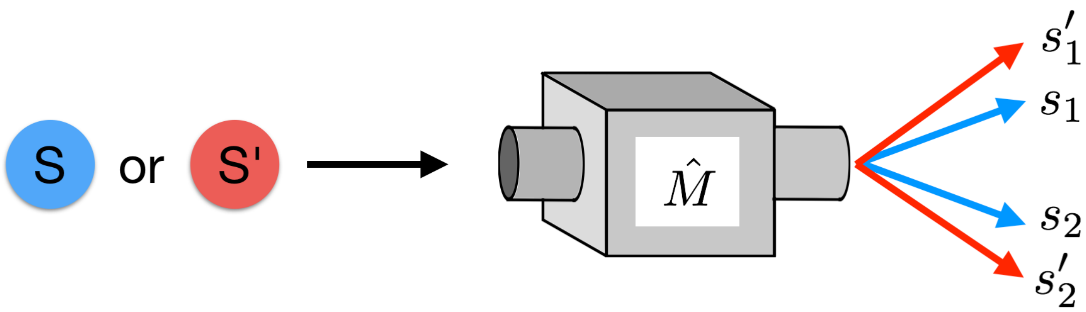
Outline

- **General setup:**
  two observers and $\mathcal{G}_{\min}$



- **Communicating quantum states:**
  emergence of the Lorentz group



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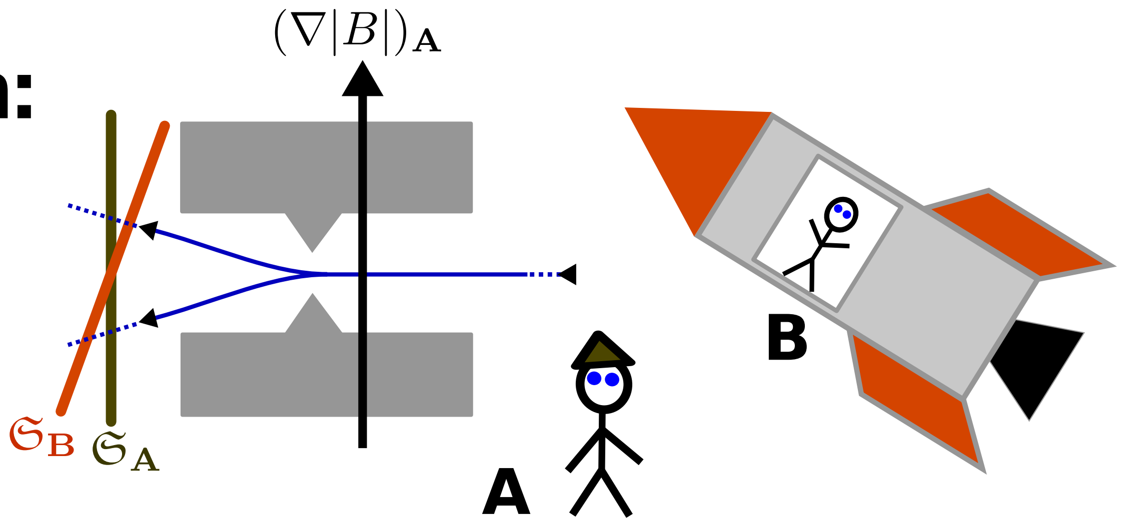

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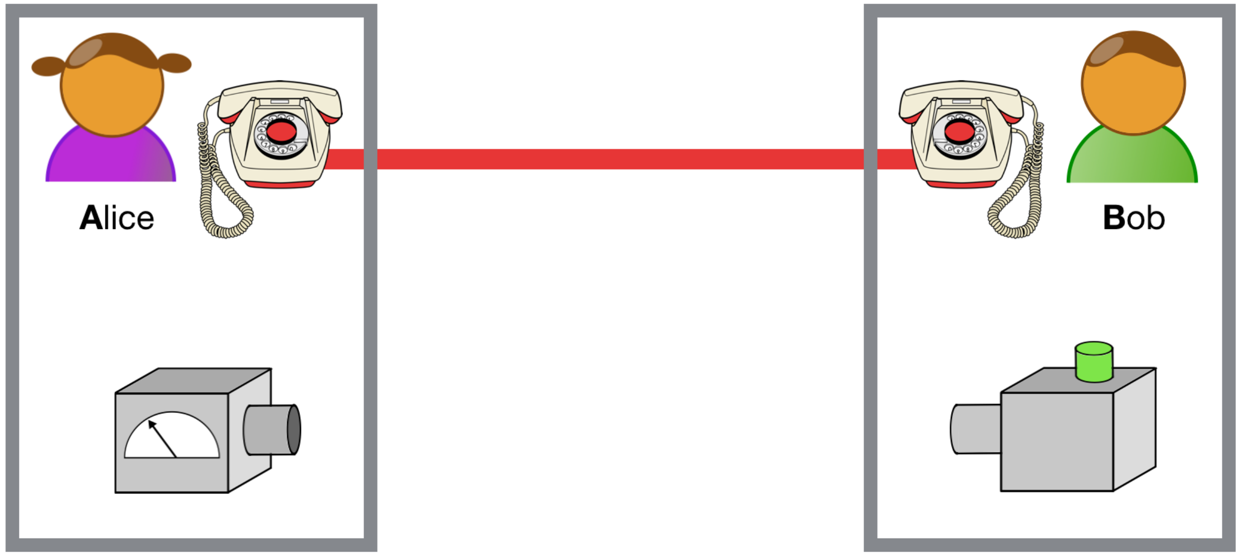

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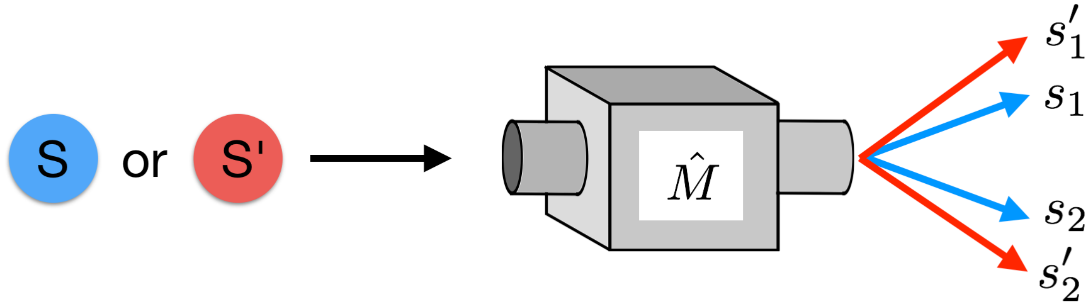

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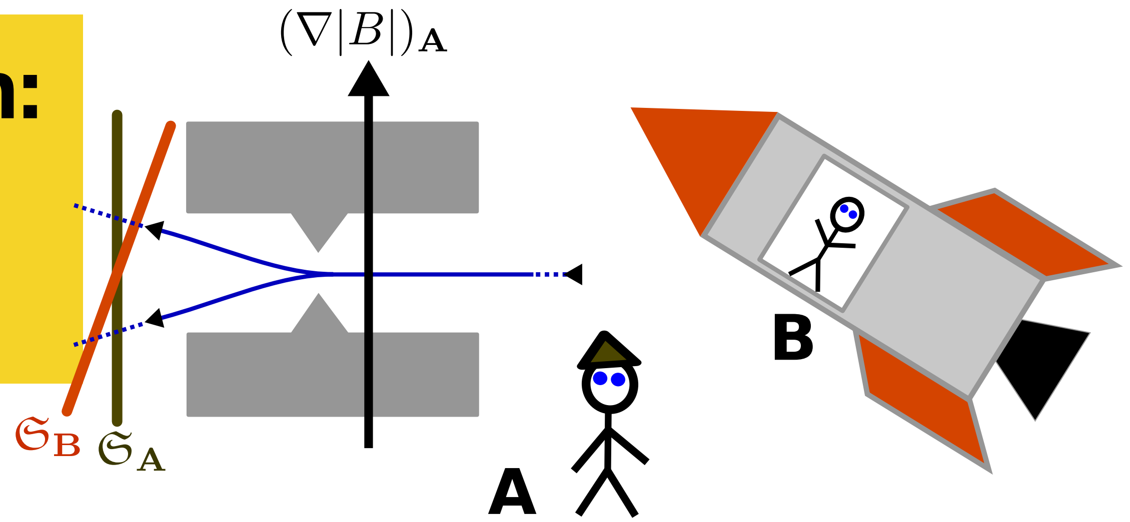

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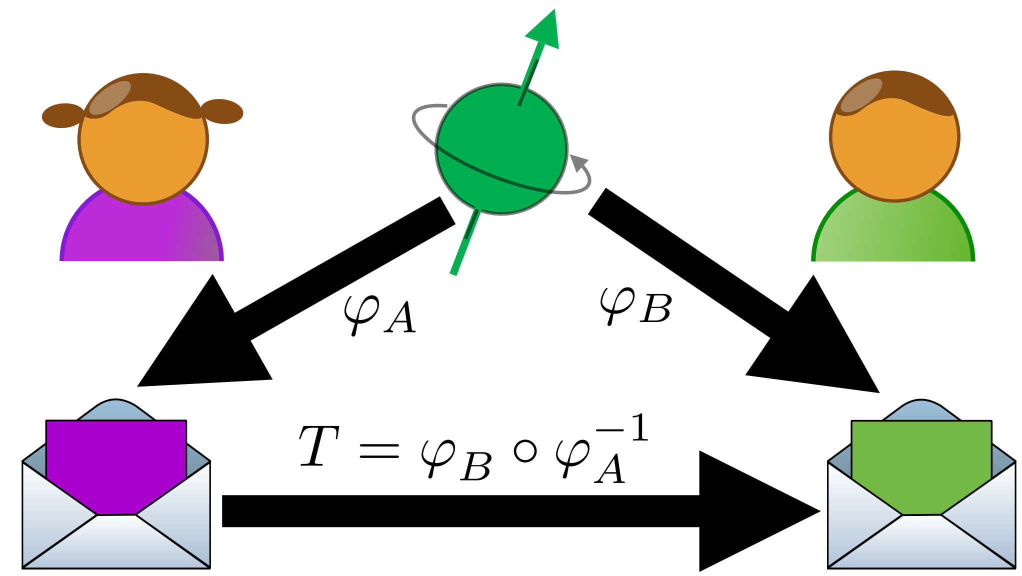

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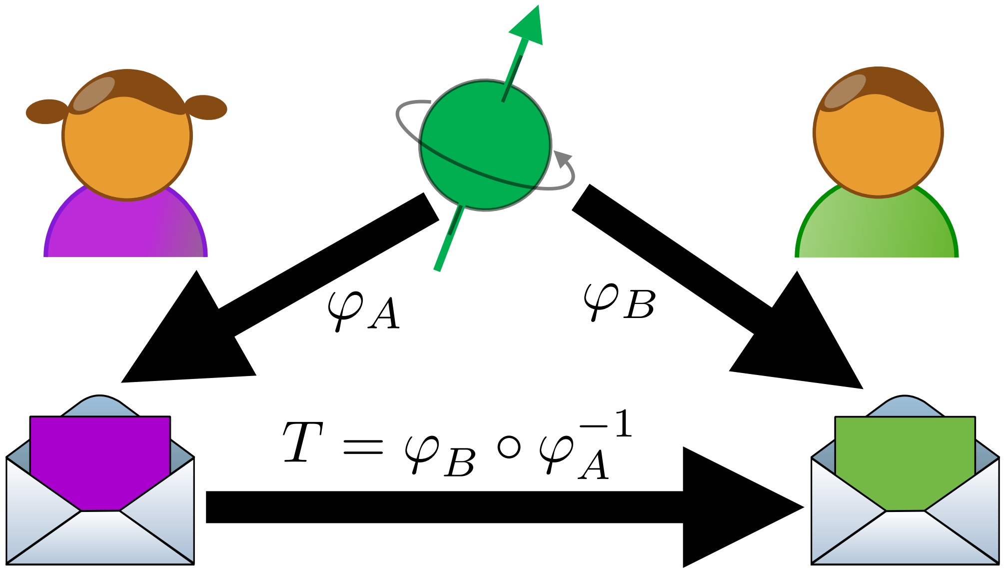

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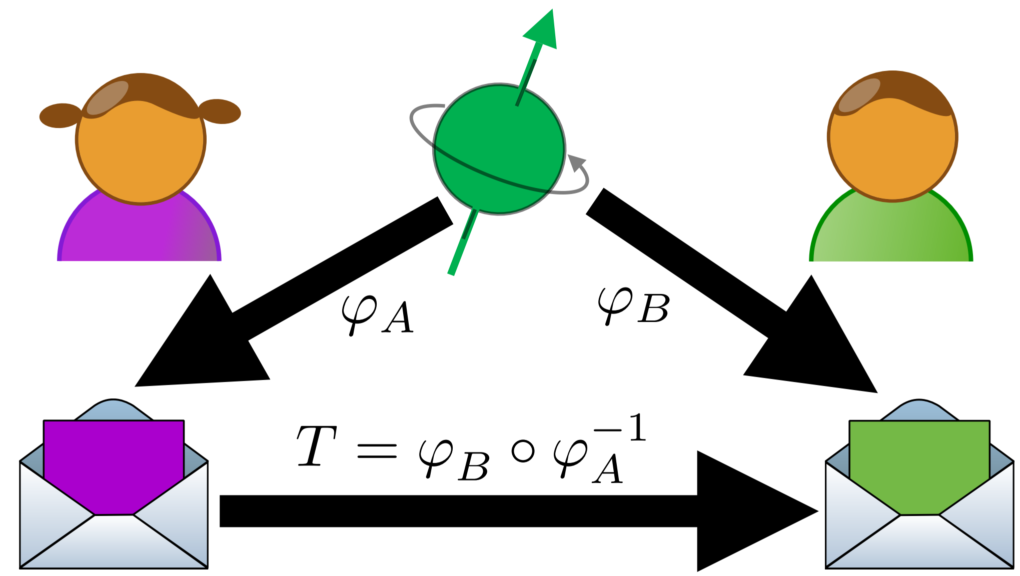

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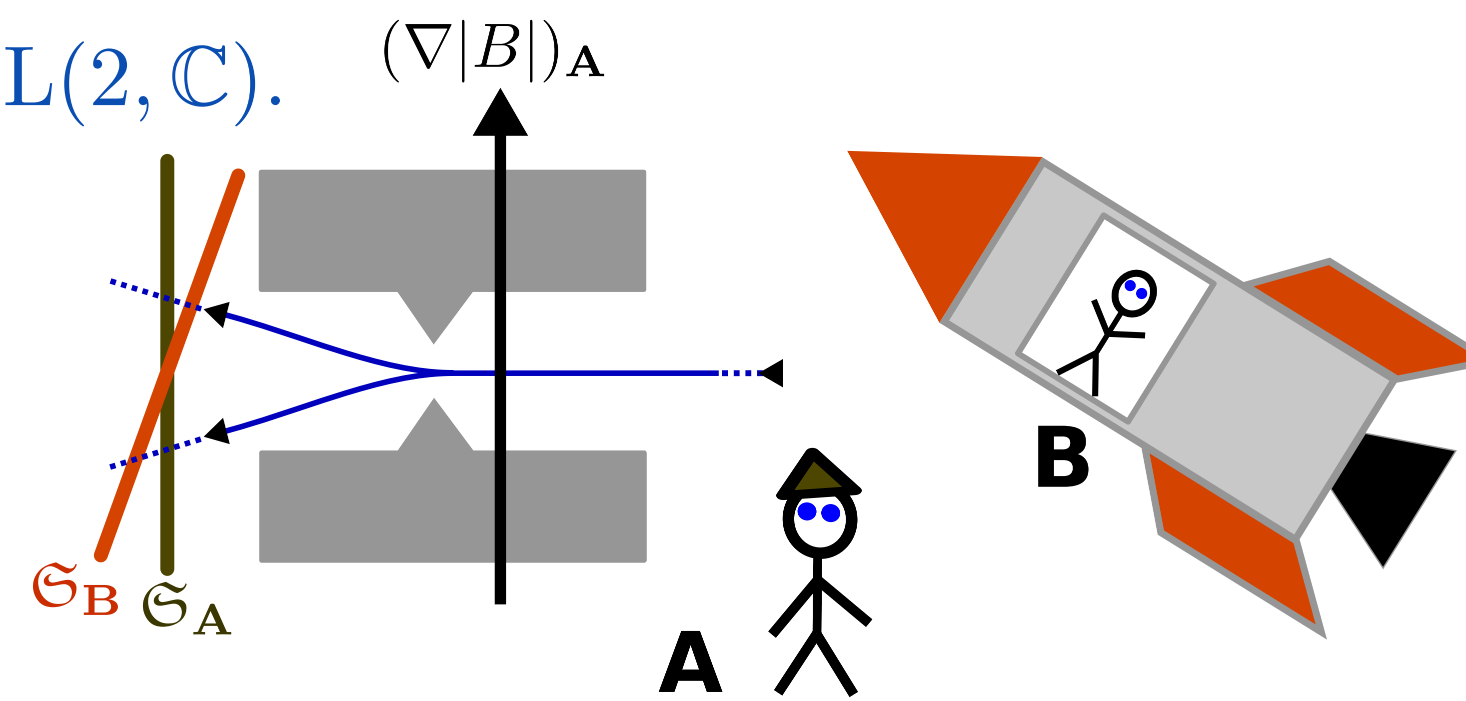

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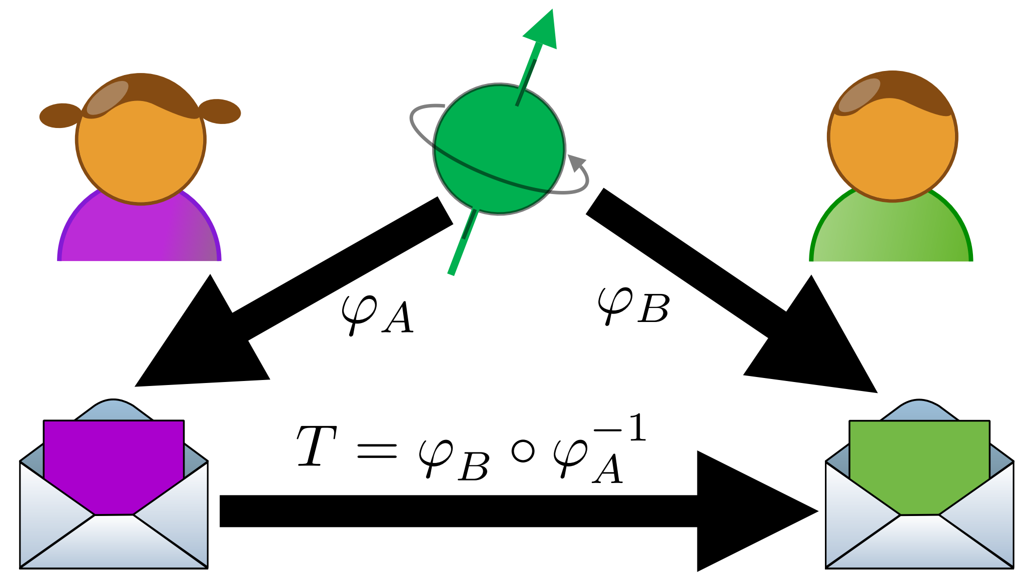

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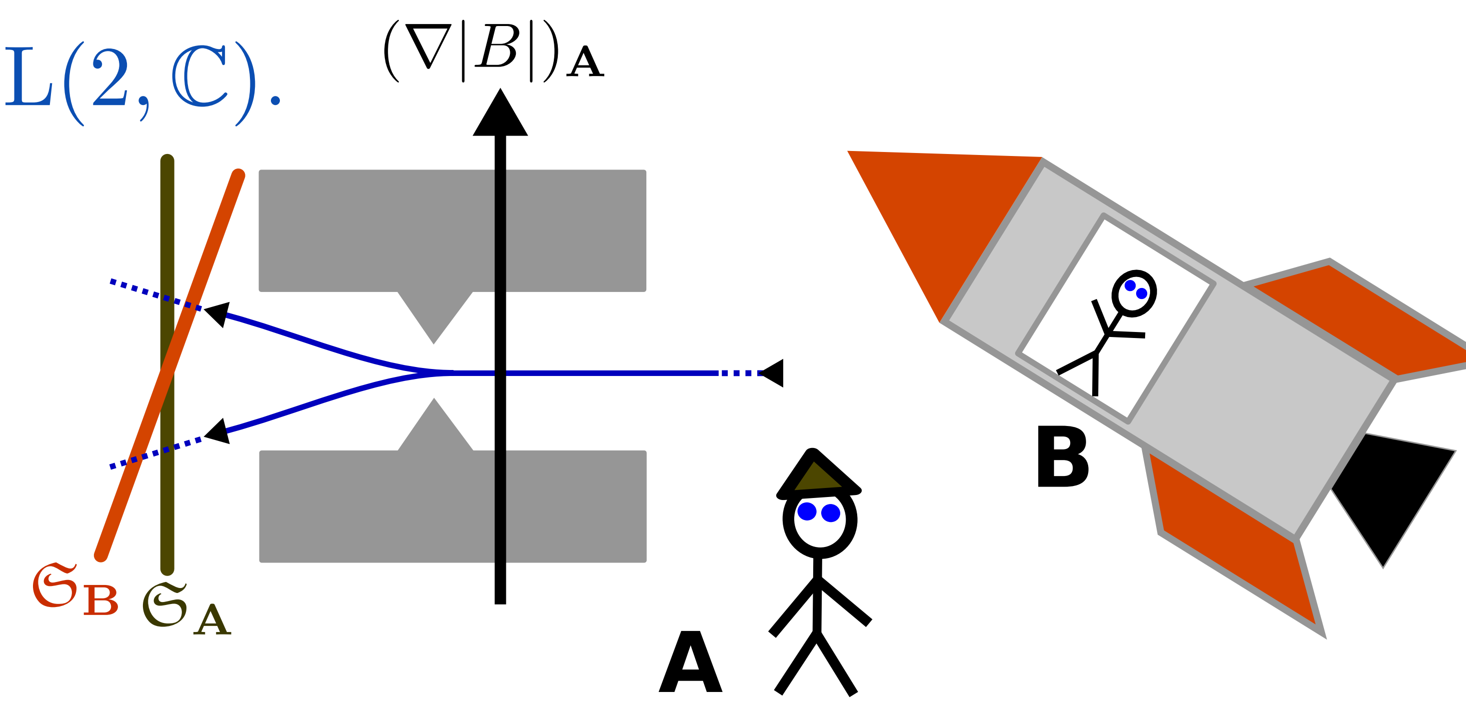

Boosted observers see different "deflection eigenvalues" in Stern-Gerlach device.

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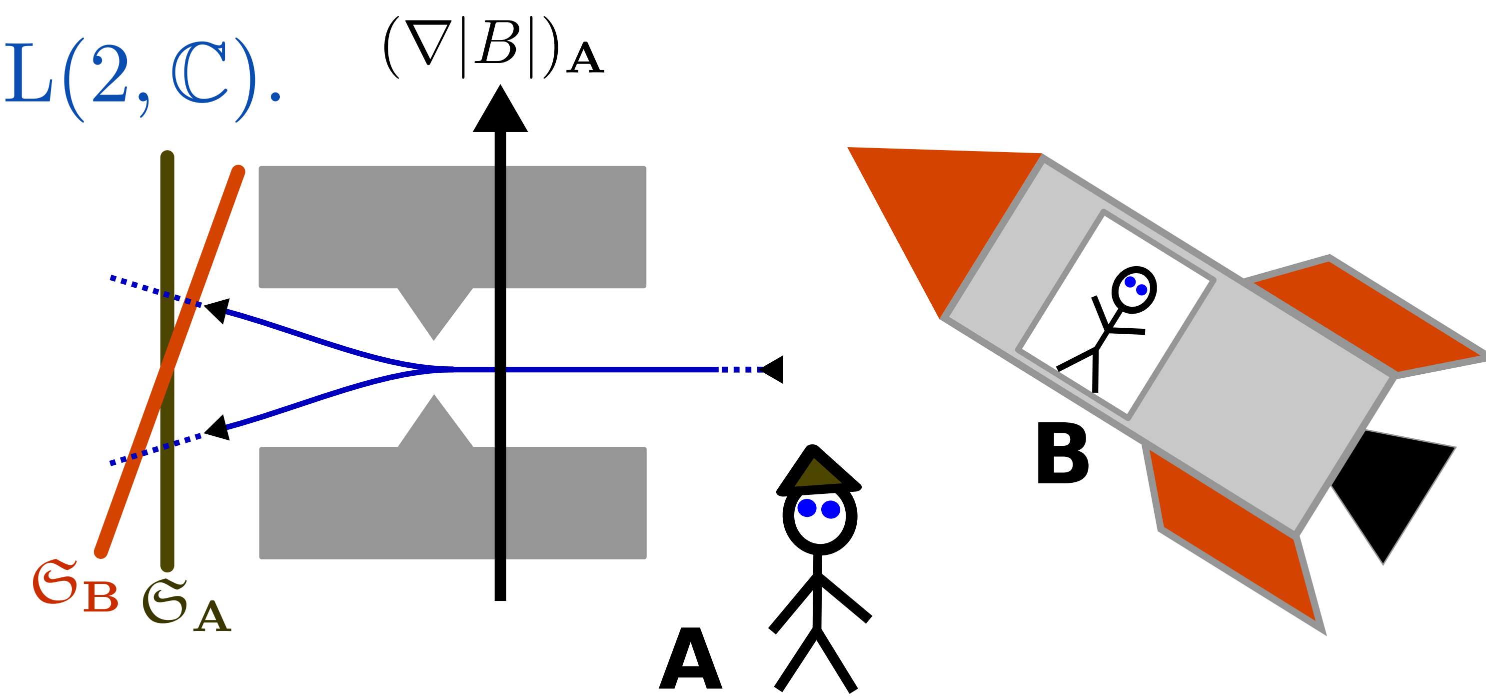

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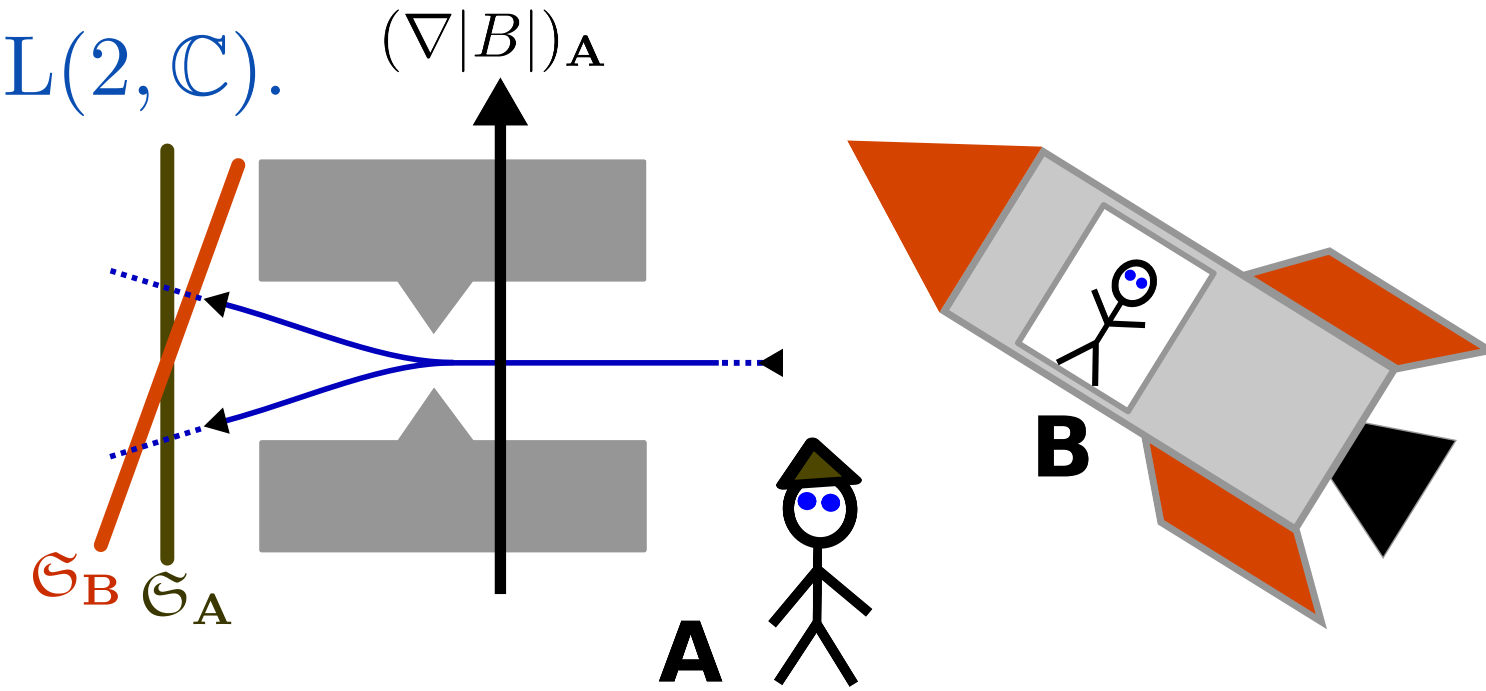

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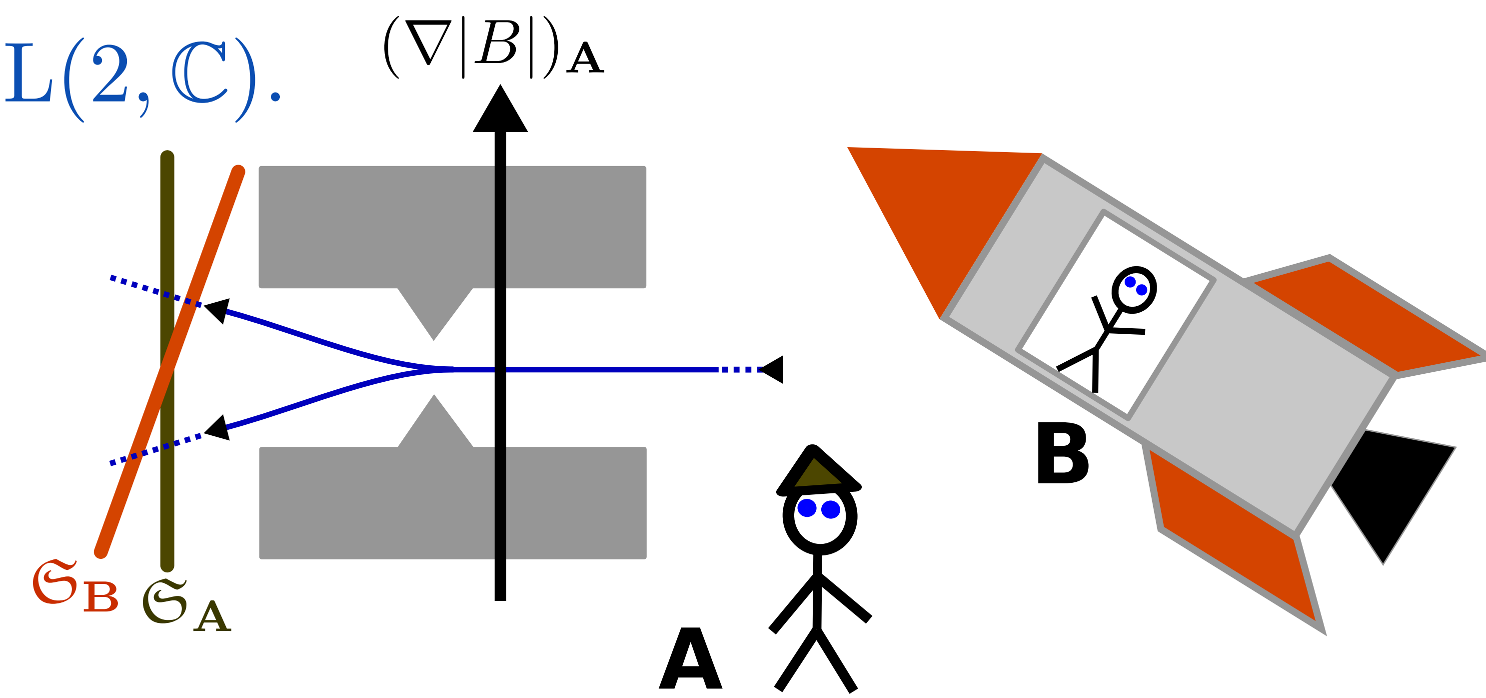

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Work in progress w/ Sylvain Carrozza: relate this to Wigner representation.

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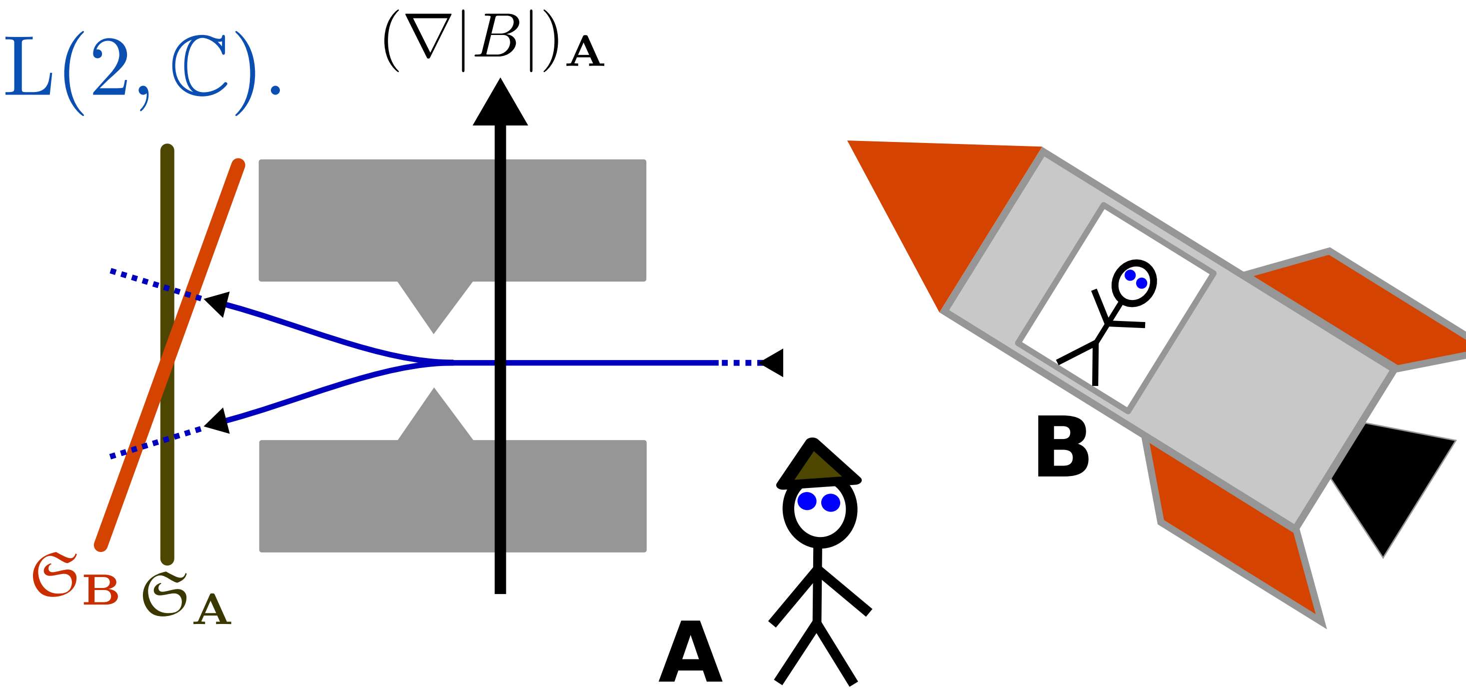

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Have **not at all** "derived relativity" (no manifold etc.!), and spacetime interpretation is not necessary -- but quite suggestive.

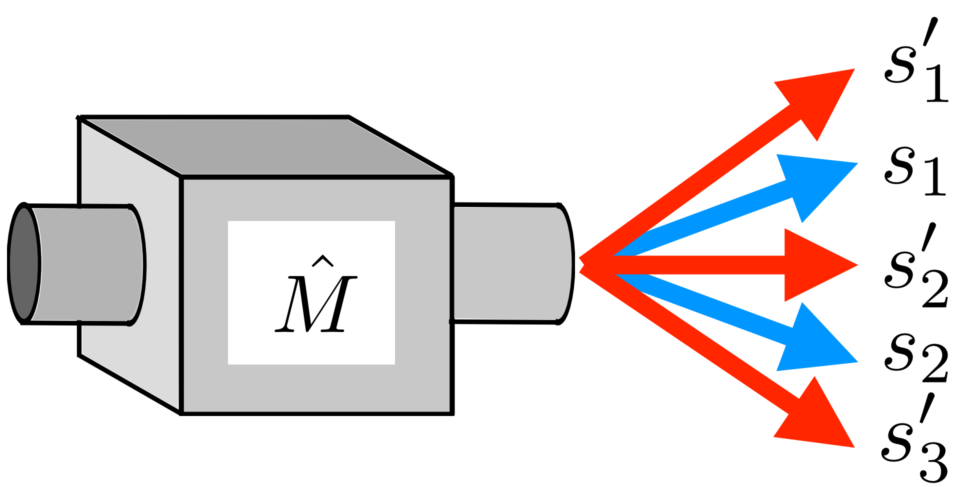
Summary

Usual line of reasoning:

Relativistic (3+1)-spacetime

$\Downarrow$

- symmetry group SO(3,1)
- rep's of SO(3) on quantum systems; spin
- existence of Stern-Gerlach measurement devices



Our arguments:

- operational "symmetry" group SO(3,1)
- rep's of SO(3) on quantum systems ("spin")

$\Uparrow$

Existence of "enough" universal quantum measurement devices; auxiliary assumptions (e.g. N=2)

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