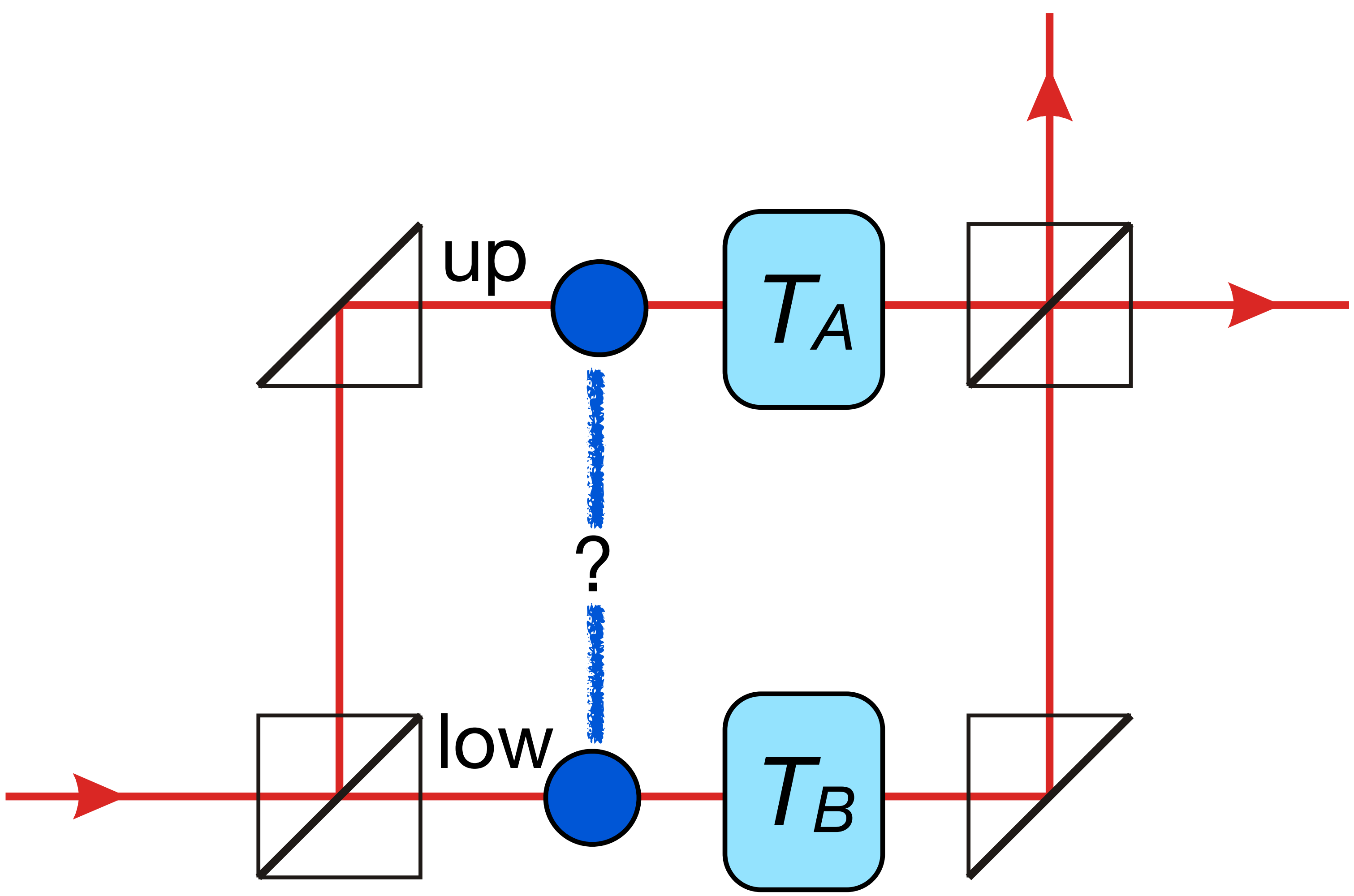
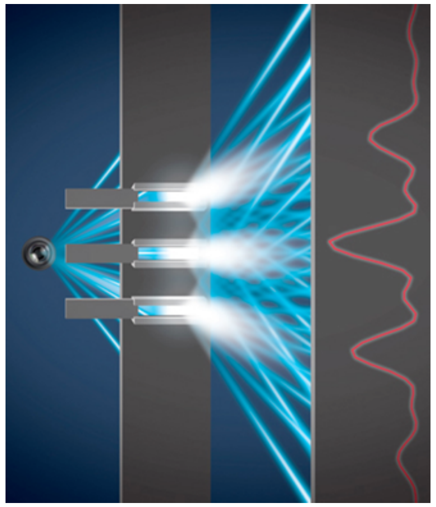
Interference, spacetime, and the structure of quantum information

Markus P. Müller
Departments of Applied Mathematics and Philosophy, UWO
Perimeter Institute for Theoretical Physics, Waterloo



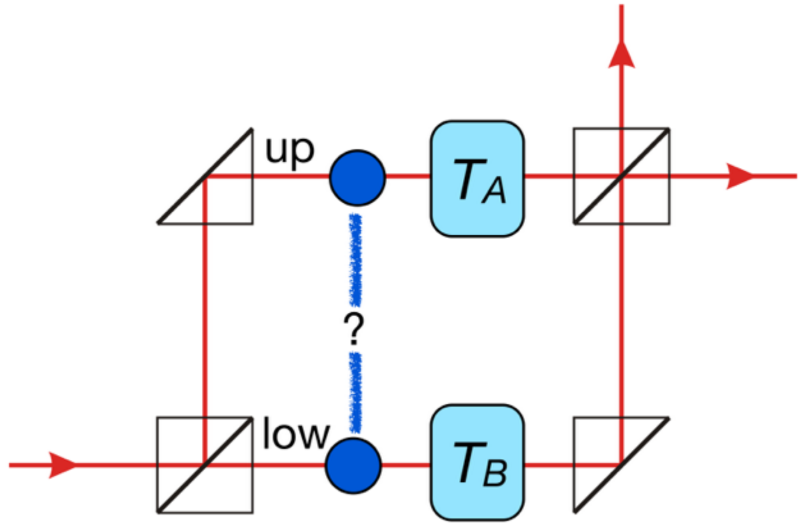
Outline

- Quantum theory from principles

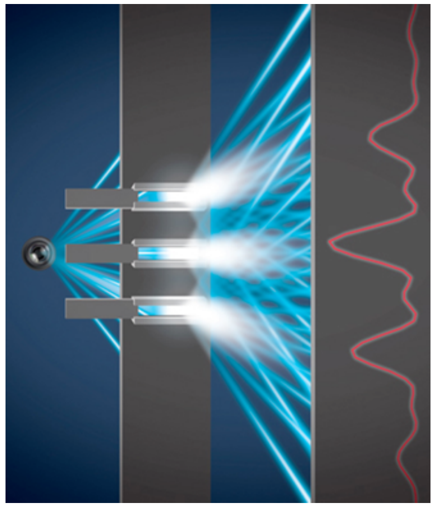


"Why does the qubit have 3 degrees of freedom?"

- Take 1: continuous-reversible interaction
- Take 2: relativity of simultaneity on interferometer



- **Quantum theory from principles**



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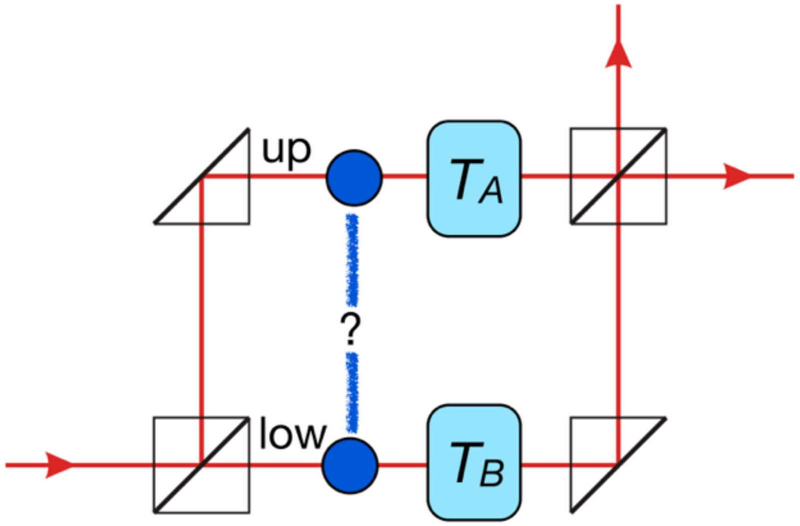


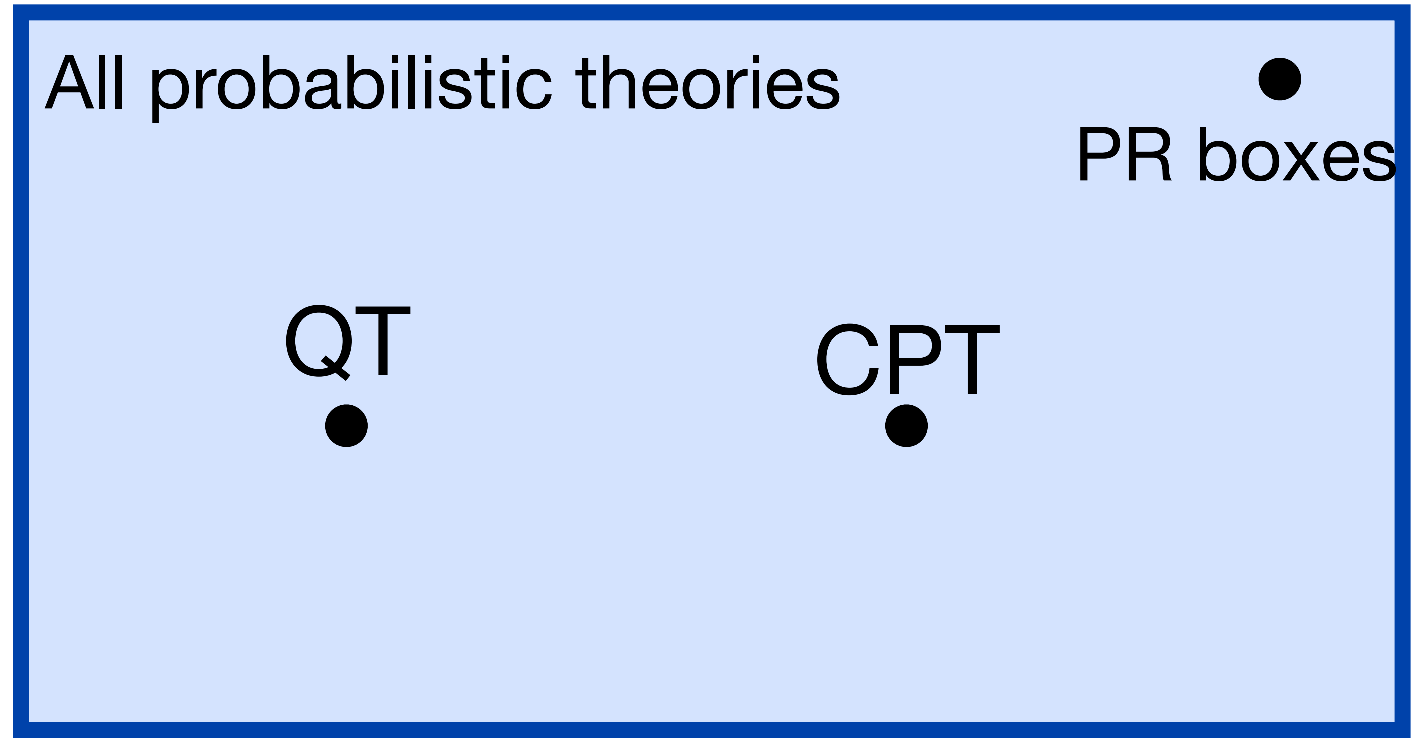
John A. Wheeler, NY Times, 2000:

„Quantum physics […] has explained the structure of atoms and molecules, […] the behavior of semiconductors […] and the comings and goings of particles from neutrinos to quarks.

The New York Times

Successful, yes, but mysterious, too.
Why does the quantum exist?“

1. Quantum theory from simple principles



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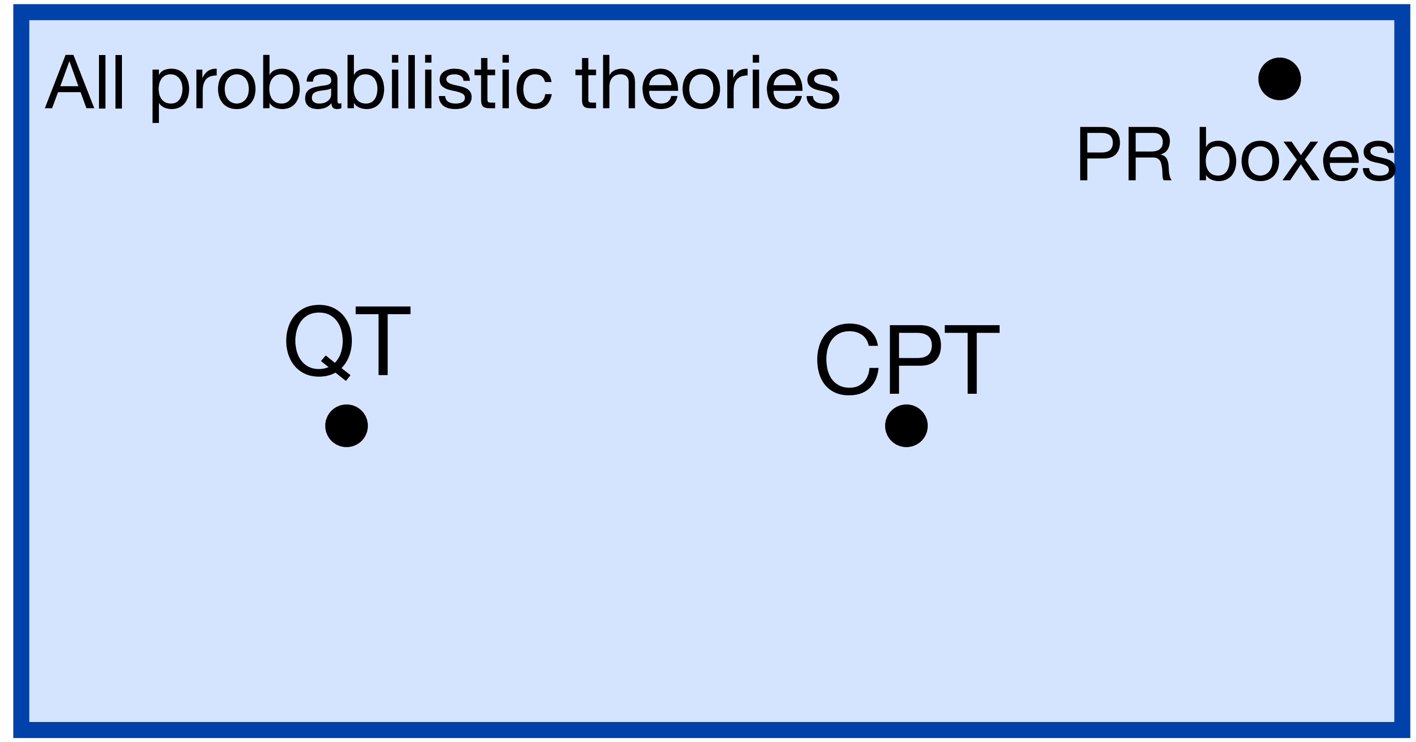
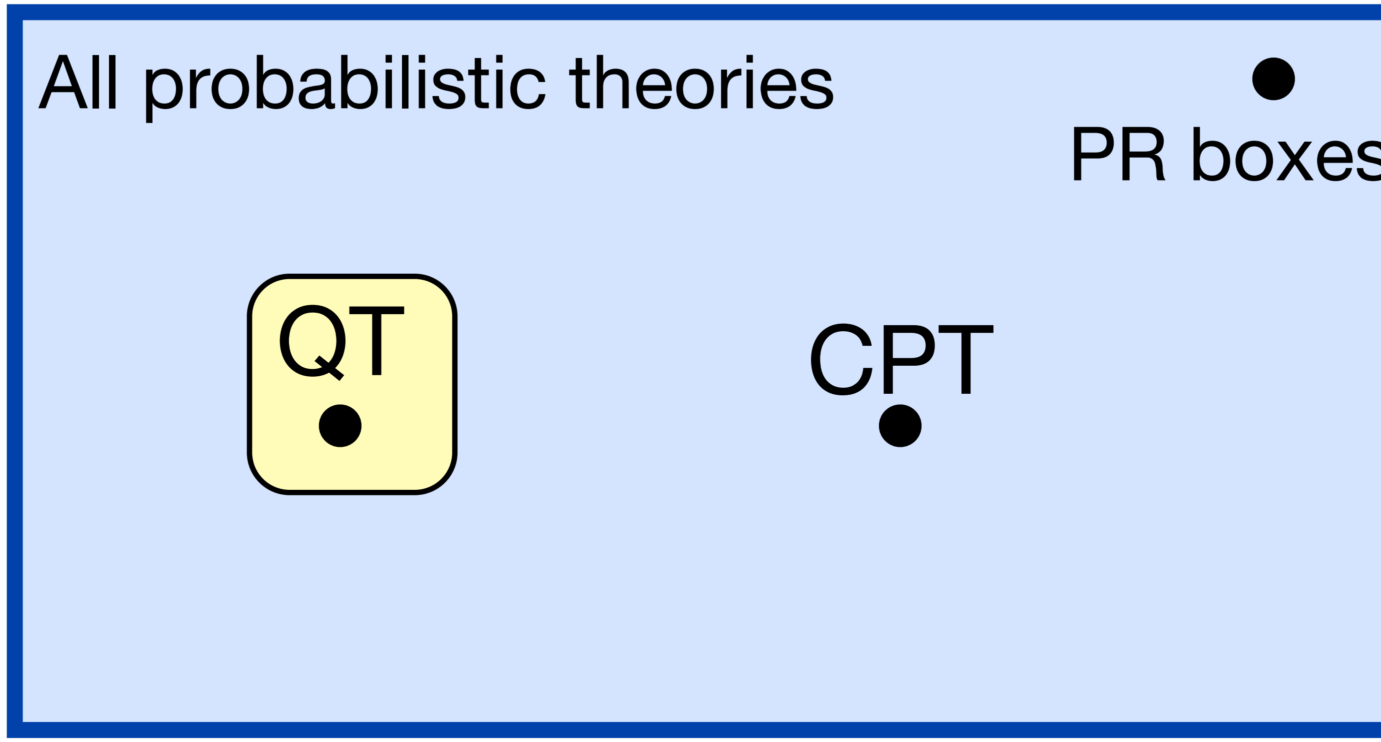


- Some more non-local than QT;
- share some features with QT: no-cloning, entanglement, …

Goal:
Simple principles that yield exactly QT.



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Lorentz transformations from
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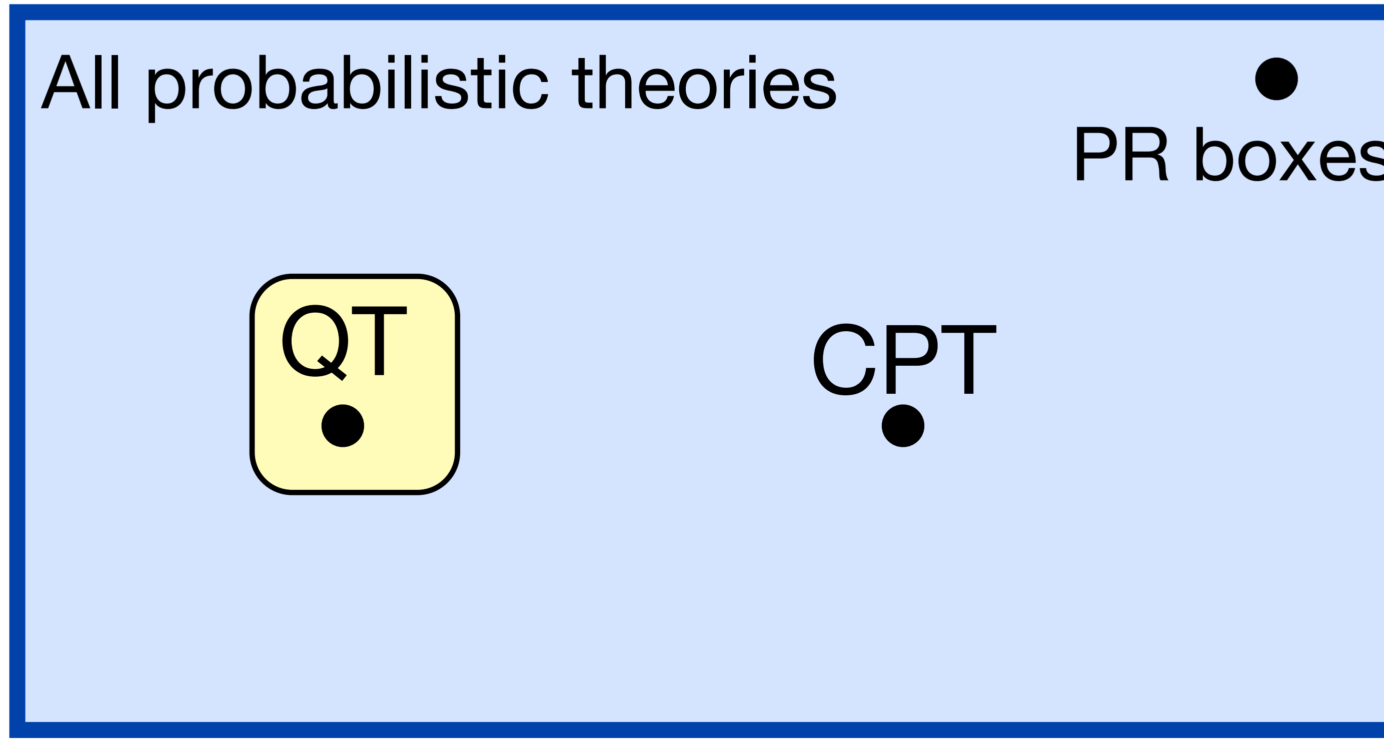

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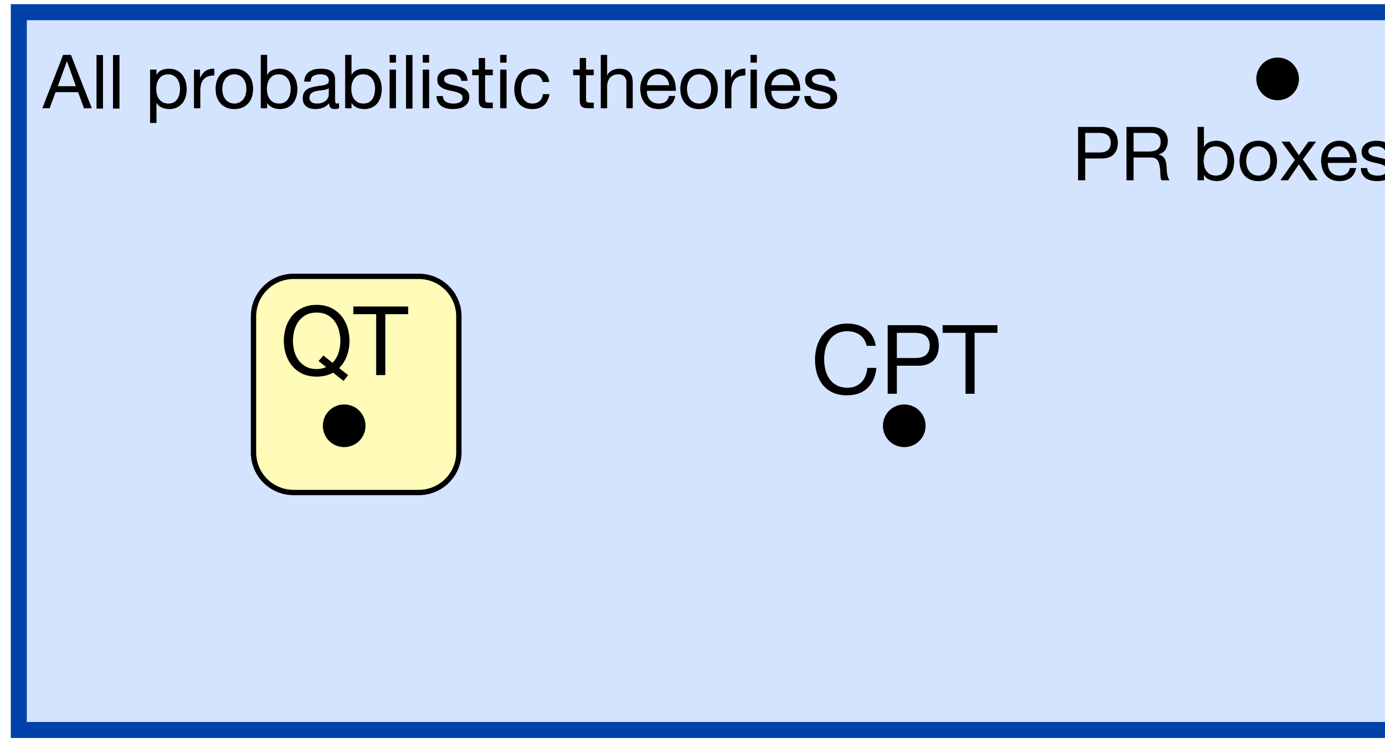

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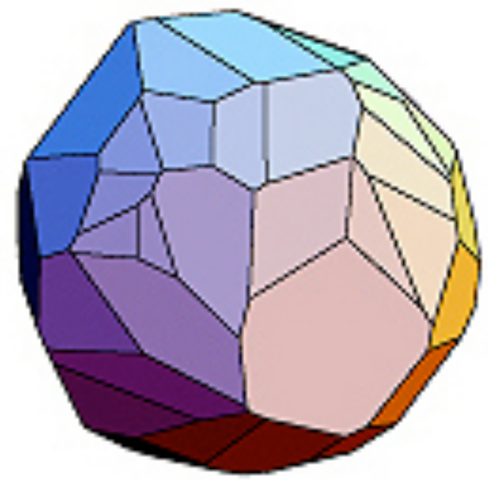
Now: I. Sketch how to describe those theories;
II. give a set of principles for QT.

How to describe a "general probabilistic theory"

Essentially by an arbitrary convex state space.
And here's why & how.



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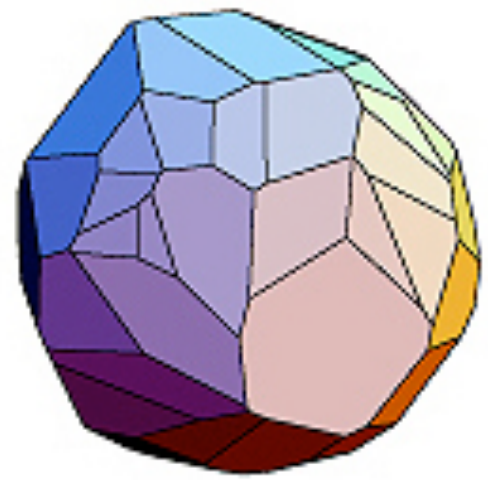
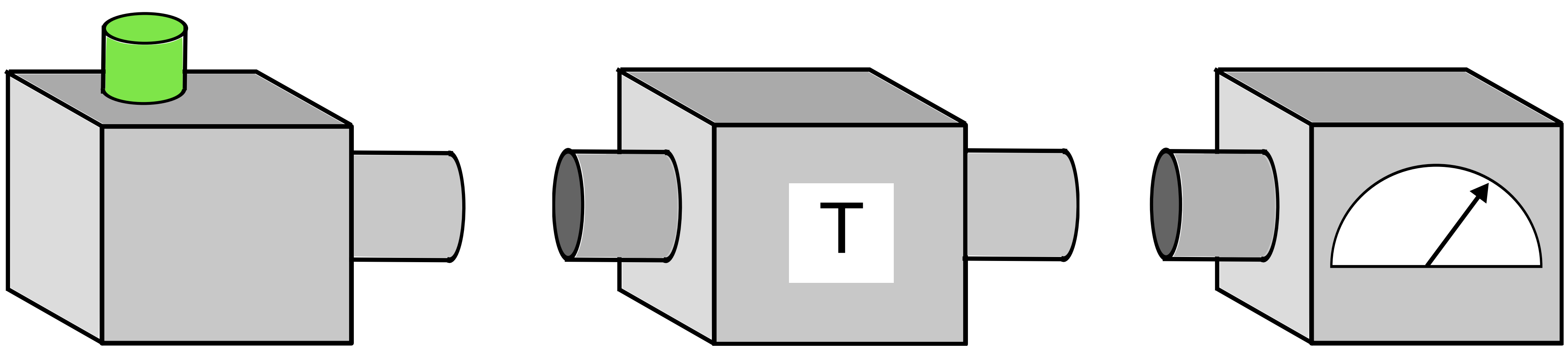


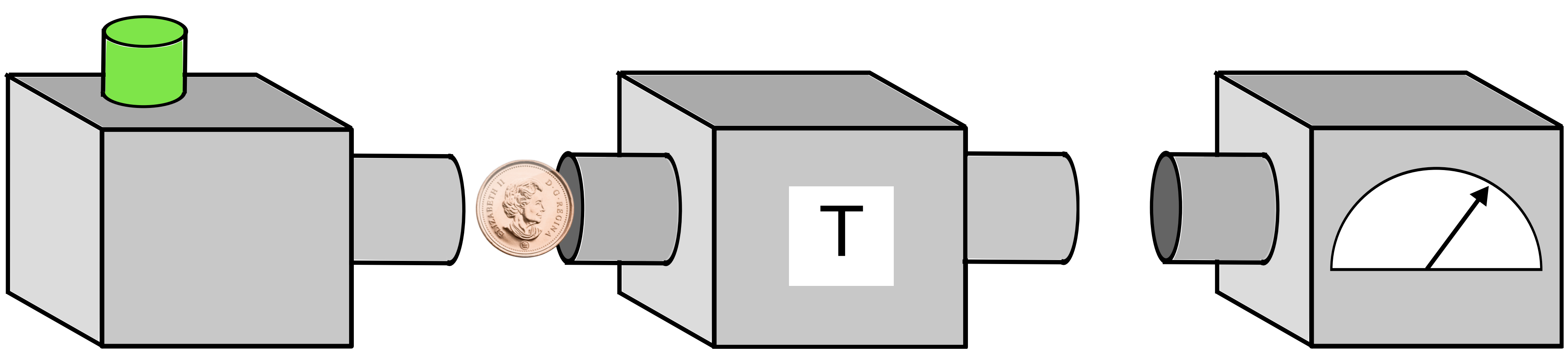


Preparation, transformation, measurement.

How to describe a "general probabilistic theory"

Example: classical coin toss.

- On every push of button, the preparation device performs a biased coin toss.



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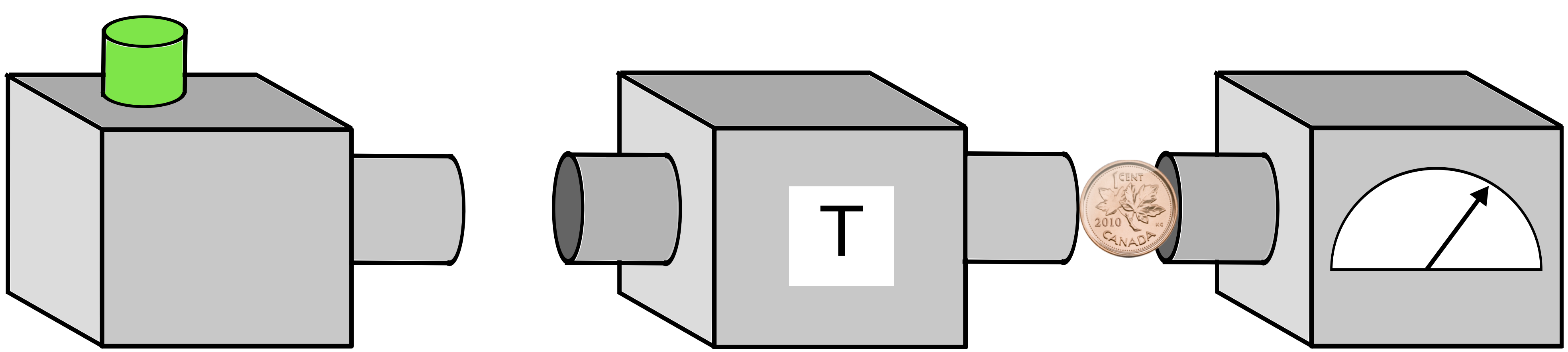

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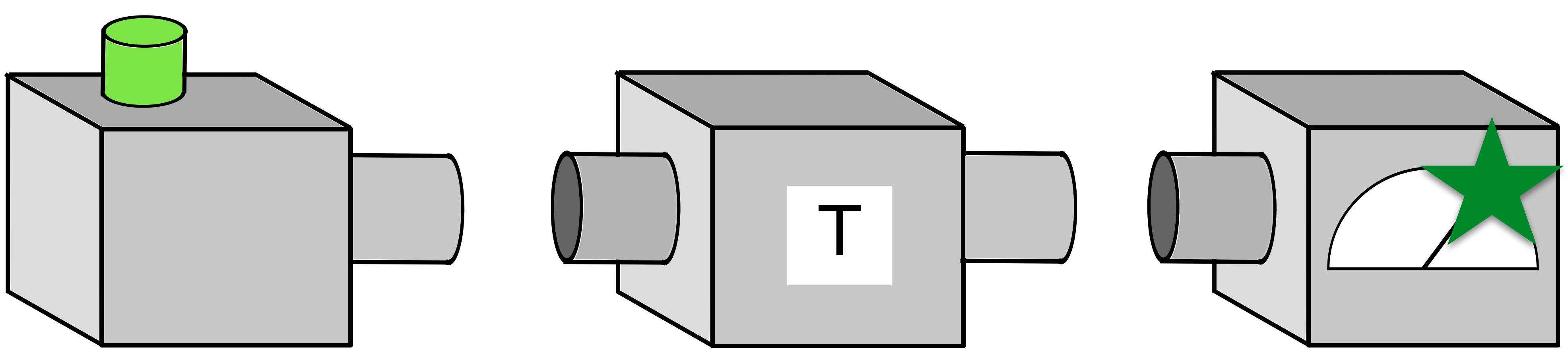
- The measurement outcome is "heads" or "tails".



Preparation, transformation, measurement.

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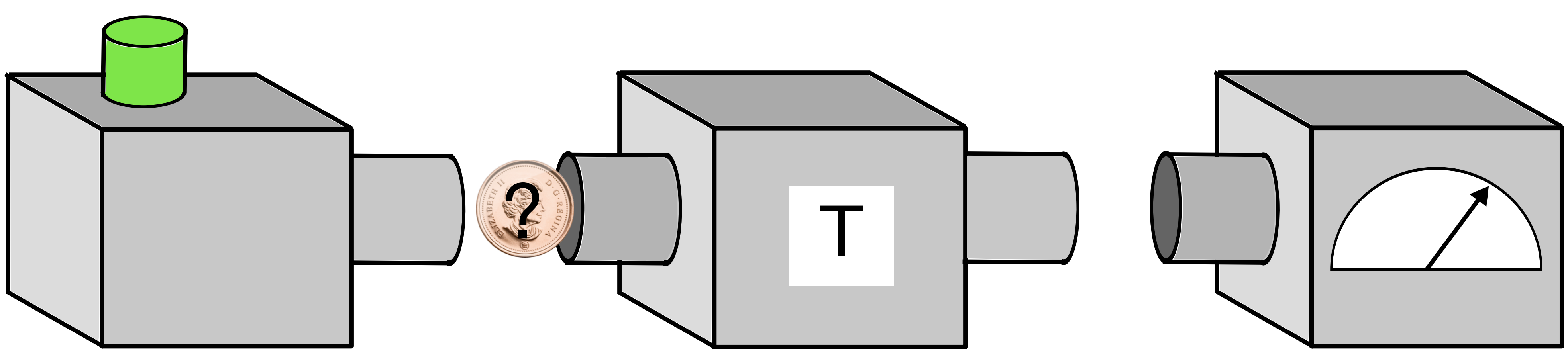
- The preparation device prepares a physical system in a state ω. Here

$$\omega = \begin{pmatrix} \text{Prob(heads)} \\ \text{Prob(tails)} \end{pmatrix} = \begin{pmatrix} p \\ 1-p \end{pmatrix}.$$



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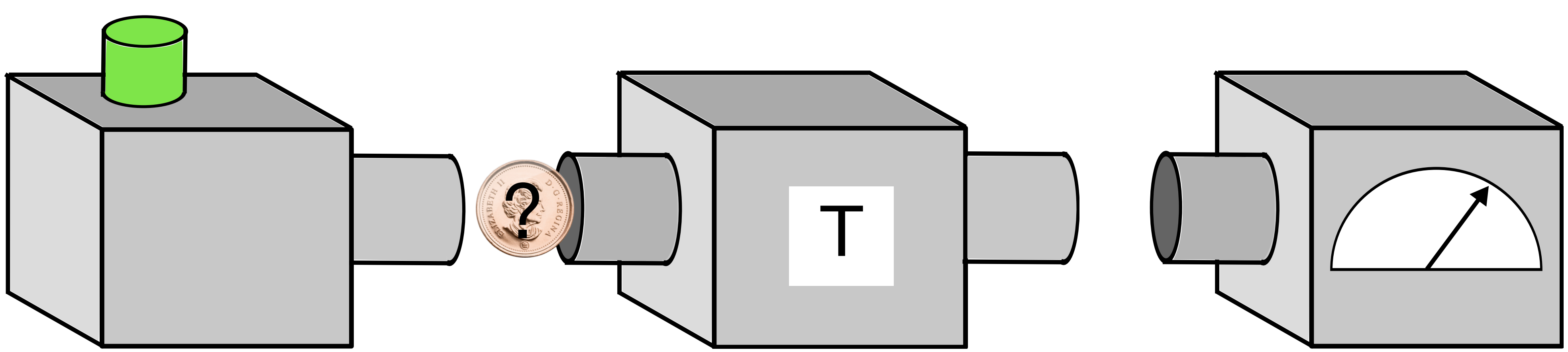
State space Ω: the set of all possible states



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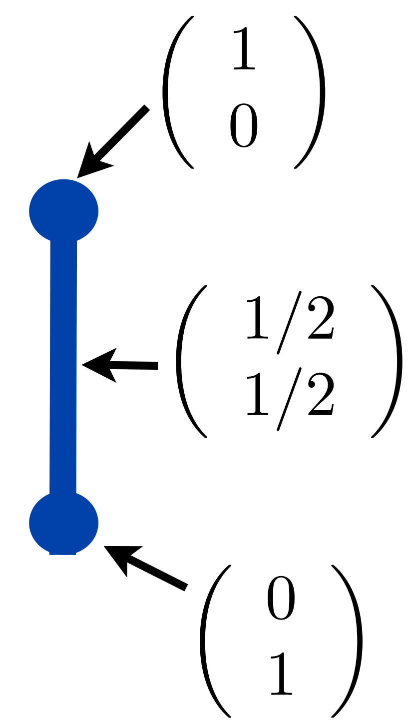

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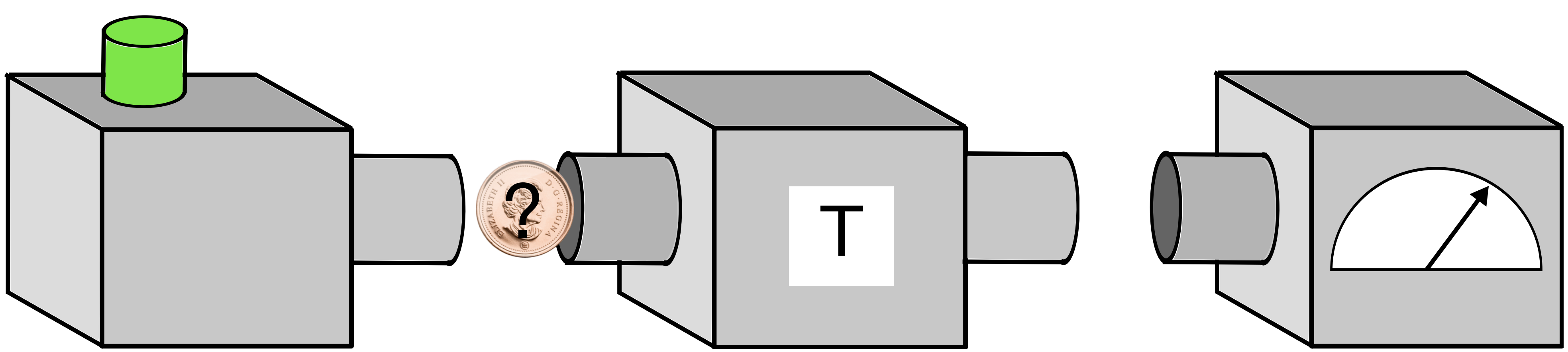

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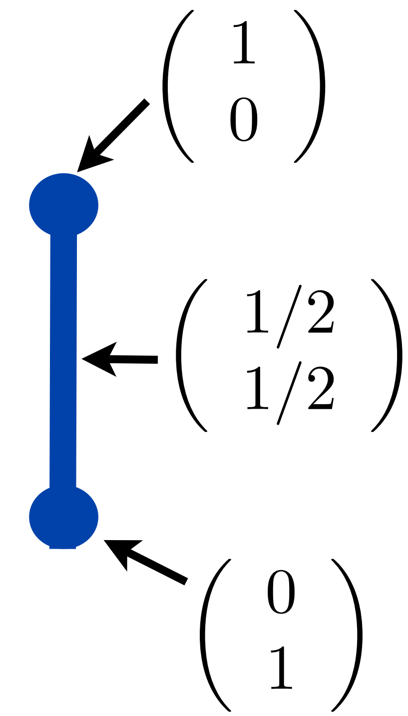

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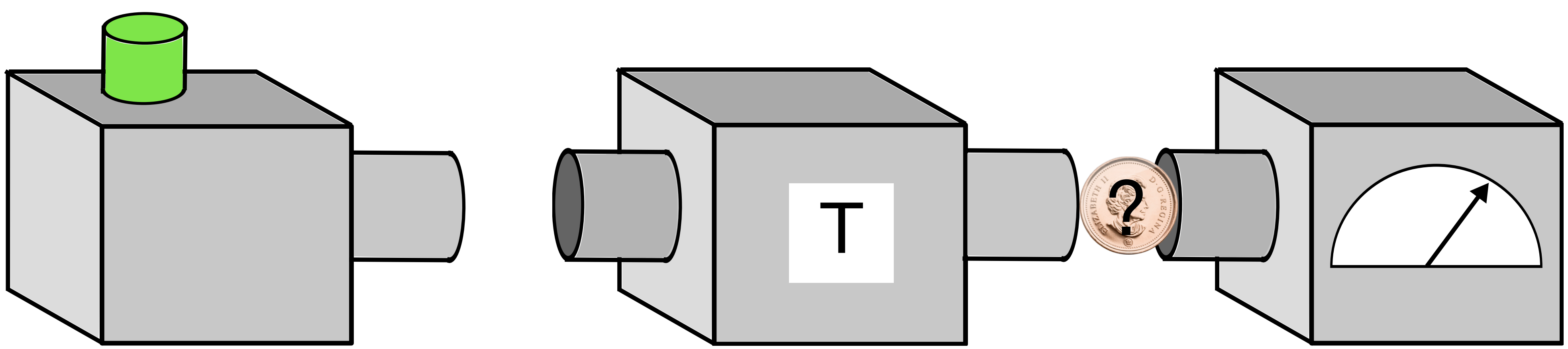

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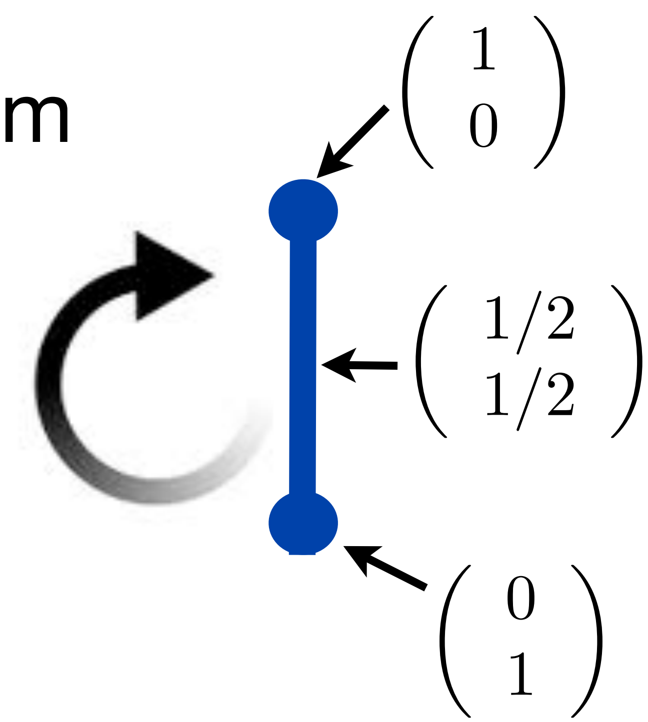

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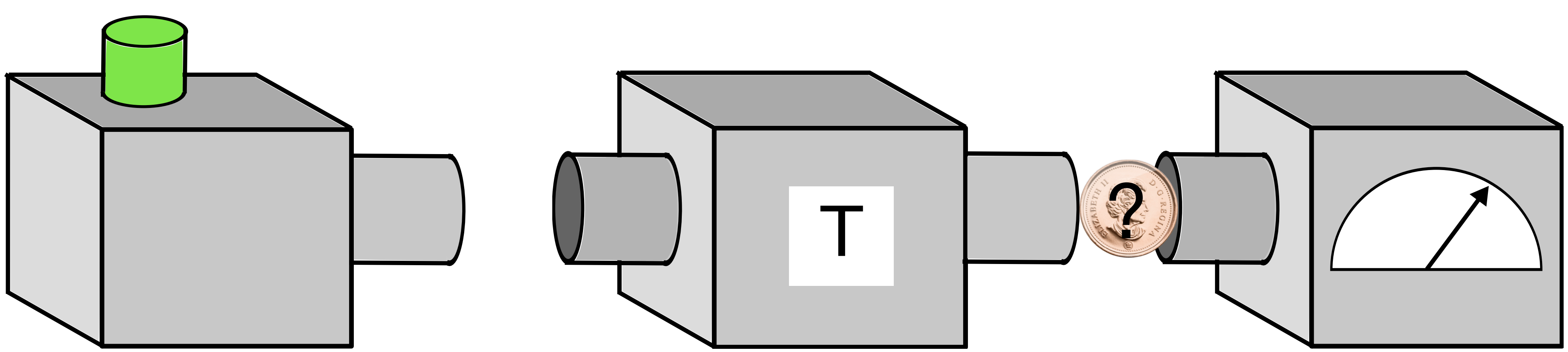
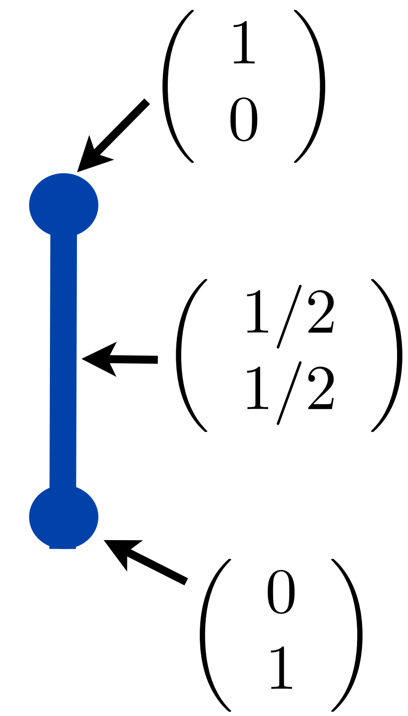
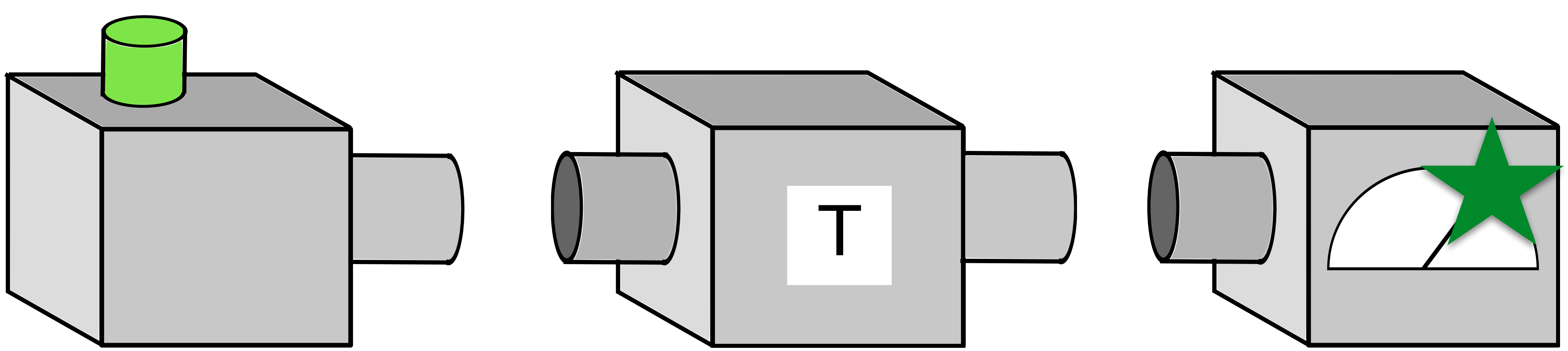
Maps states to states and is linear.



Example: classical coin toss.

- Every measurement outcome has a probability, depending linearly on the state:

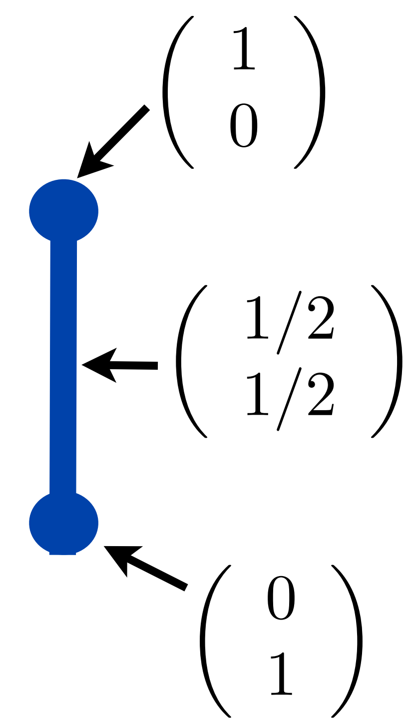


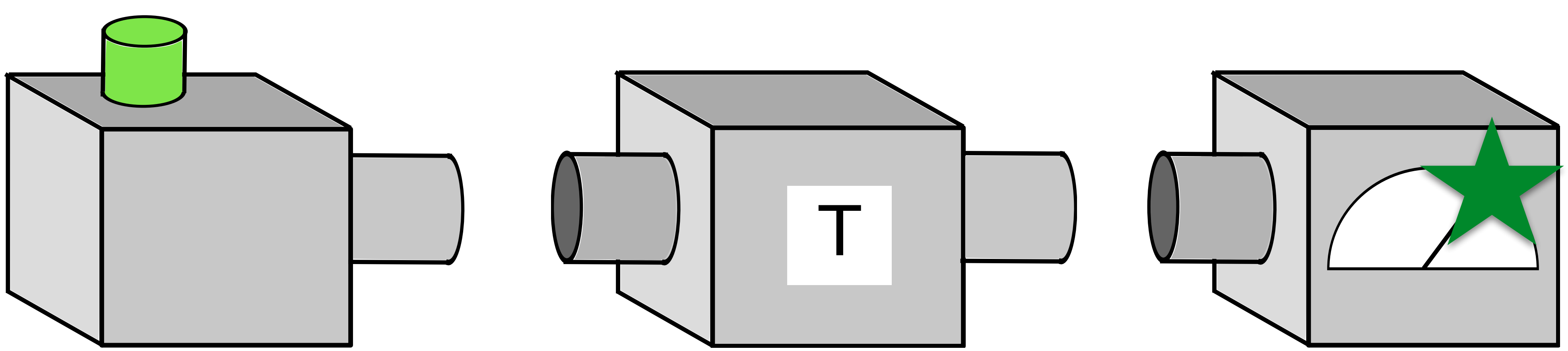


Example: classical coin toss.

- Every measurement outcome has a probability, depending linearly on the state:

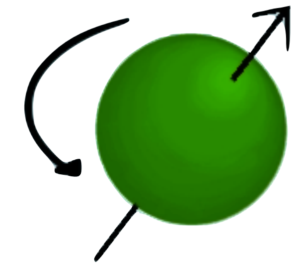
$$\mathrm{Prob}(\mathrm{heads}|\omega)=p=\begin{pmatrix}1\\0\end{pmatrix}\cdot\begin{pmatrix}p\\1-p\end{pmatrix}=e\cdot\omega.$$

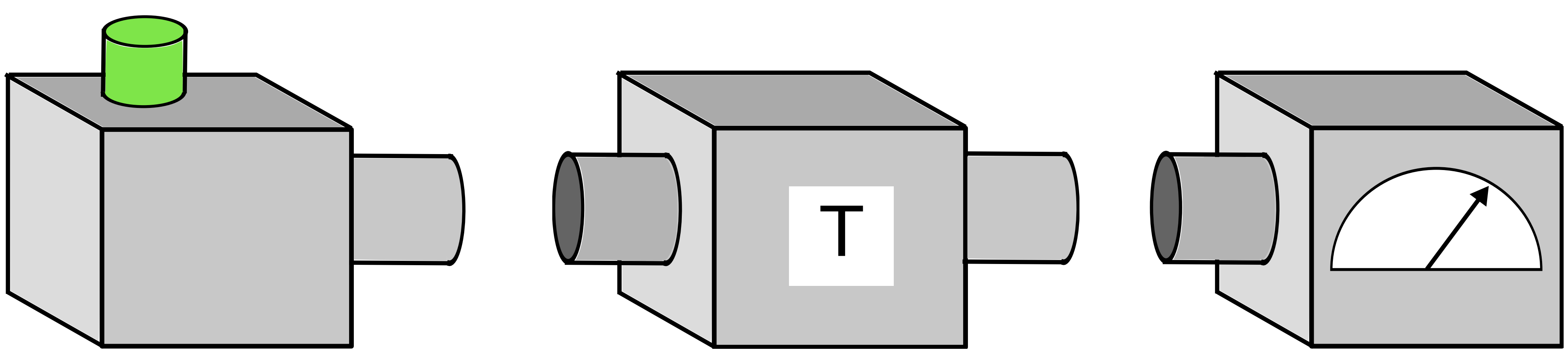




How to describe a "general probabilistic theory"

Example: quantum spin-1/2 particle.





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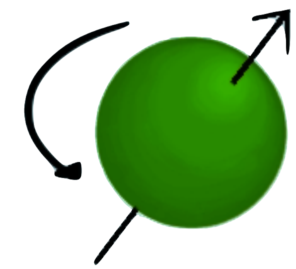
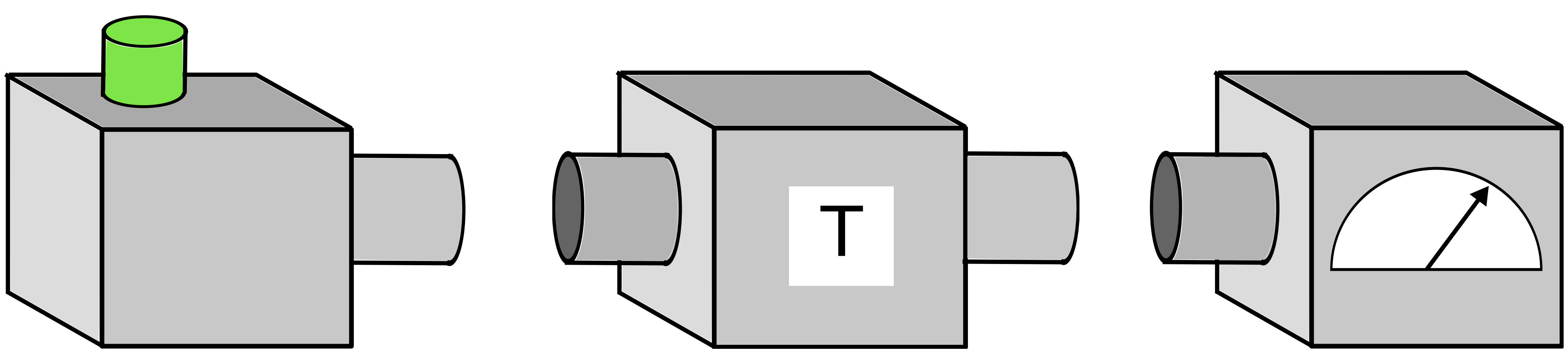


- The preparation device prepares a spin-1/2 particle in quantum state ω.

$$\alpha|\uparrow\rangle + \beta|\downarrow\rangle$$

More generally: ω is 2x2 density matrix.



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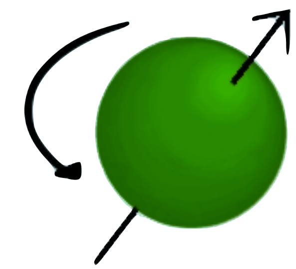

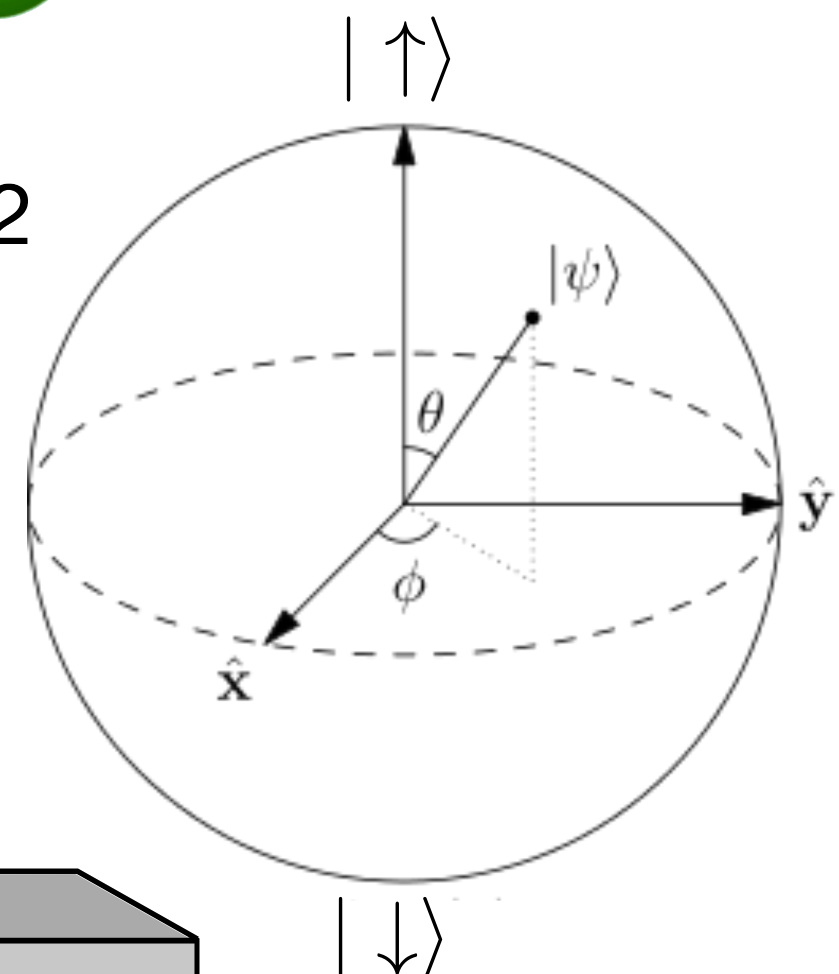

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$$\cos\frac{\theta}{2}|\uparrow\rangle + e^{i\phi}\sin\frac{\theta}{2}|\downarrow\rangle$$

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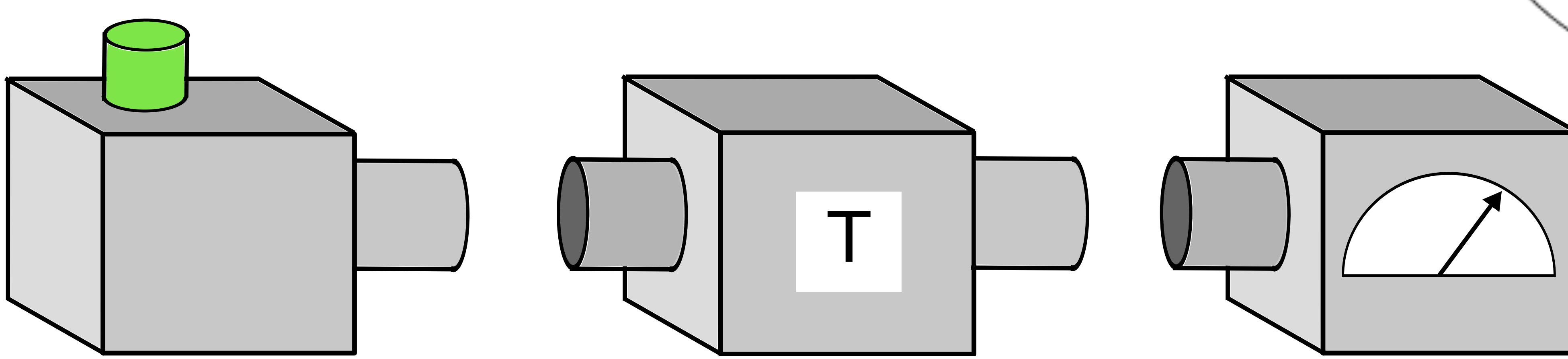

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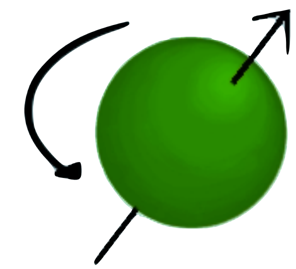
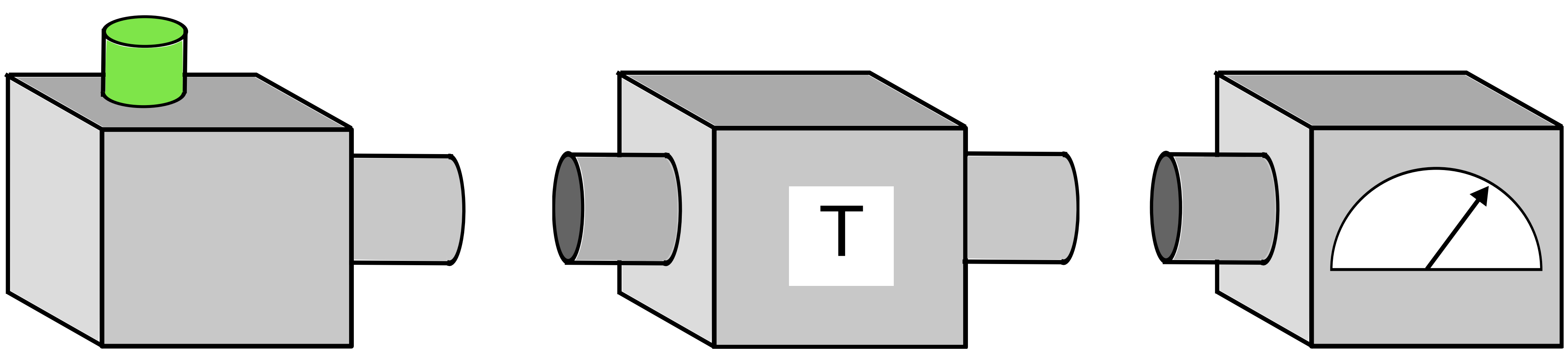


- Unitary transformation of the density matrix:

$$\omega \mapsto U\omega U^\dagger.$$



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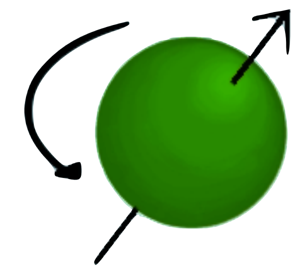

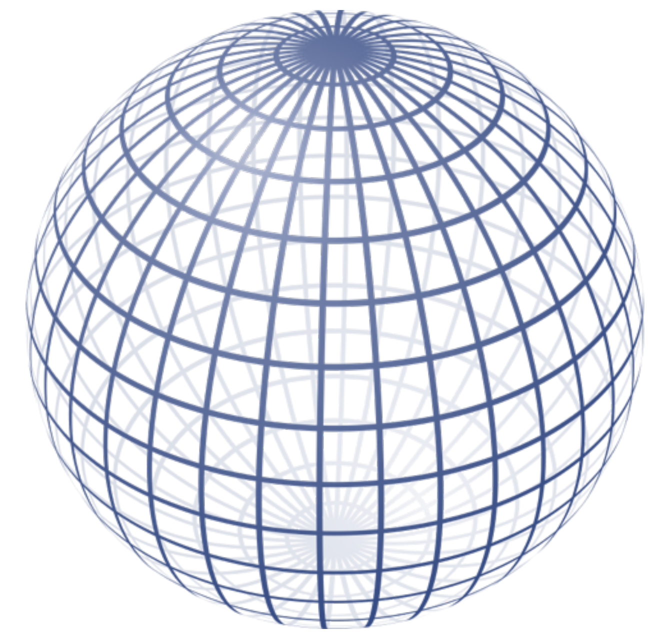

- Unitary transformation of the density matrix:

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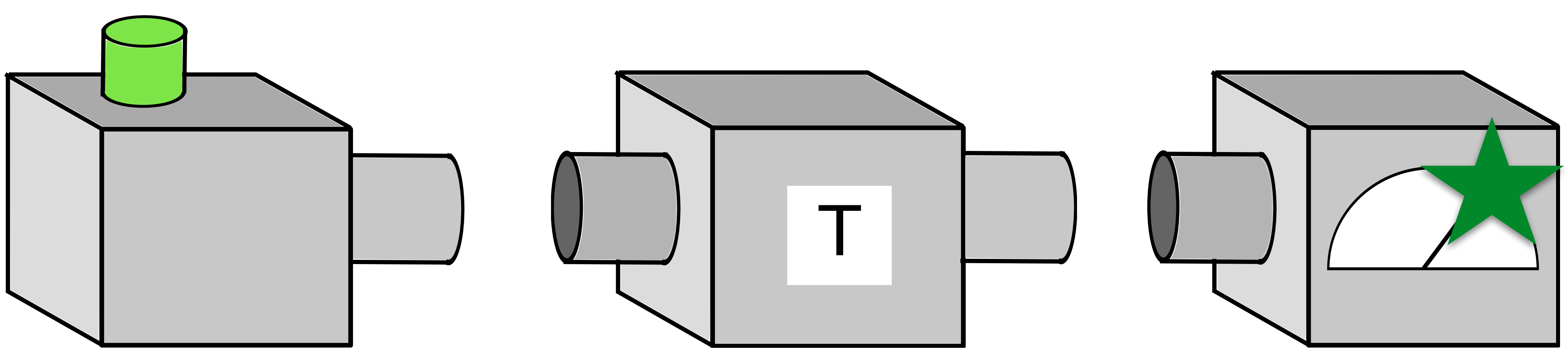
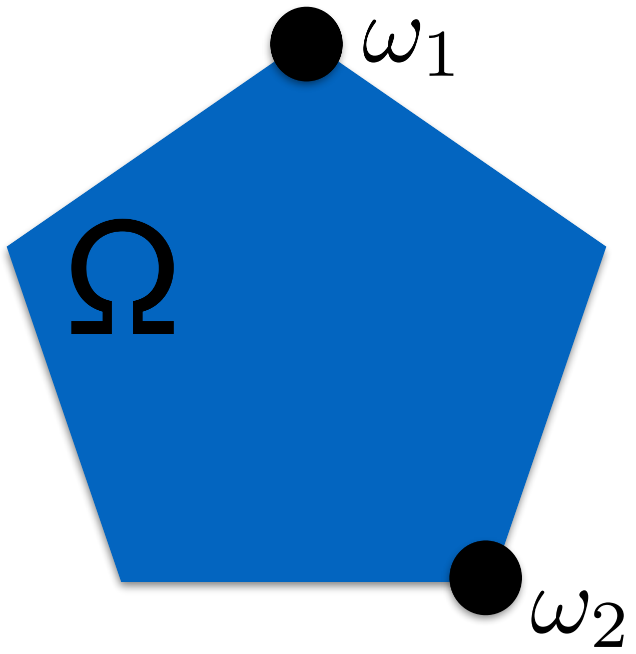
- Measurement in arbitrary spin direction *d*:

$$\mathrm{Prob}(\uparrow | \omega) = \mathrm{Tr}(P_d\, \omega)$$



How to describe a "general probabilistic theory"

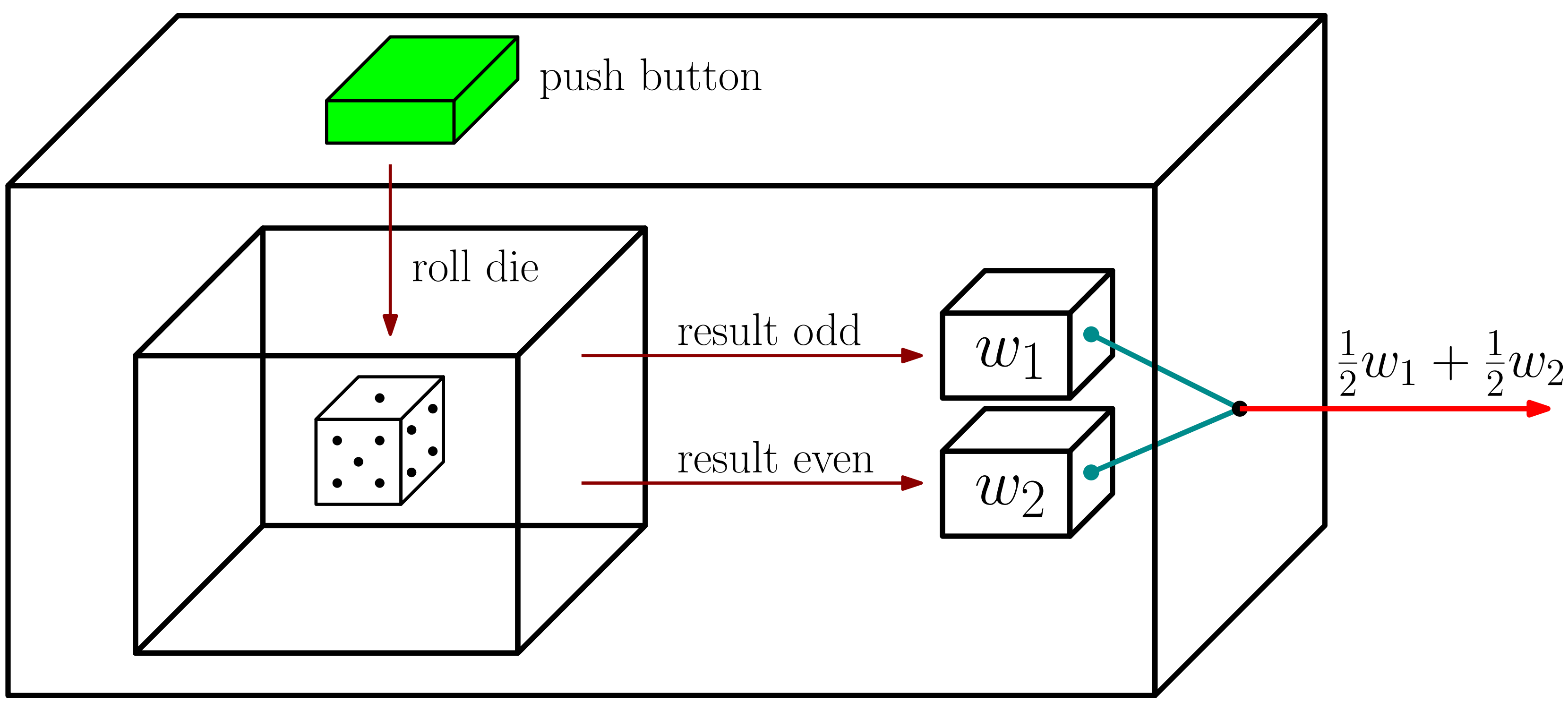
The set of all possible states of a given physical system is called the state space Ω.

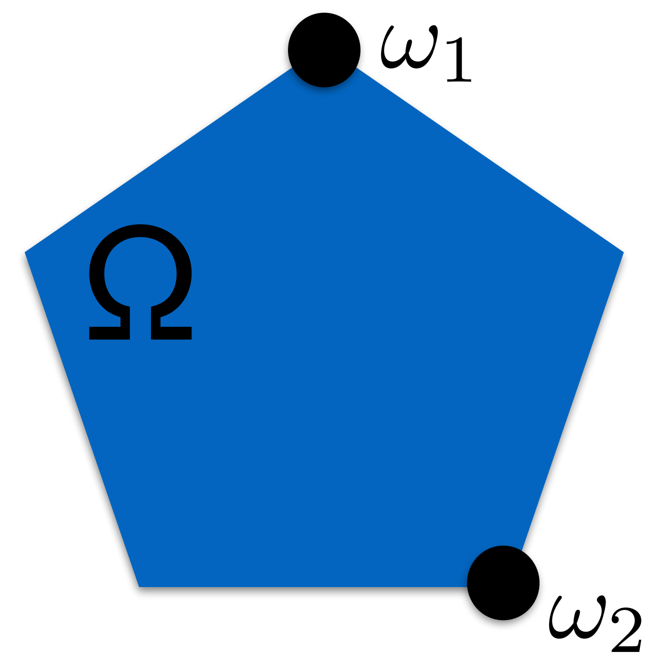


How to describe a "general probabilistic theory"

The set of all possible states of a given physical system is called the state space Ω.

Preparation of statistical mixtures: $\omega = \lambda\omega_1 + (1-\lambda)\omega_2$





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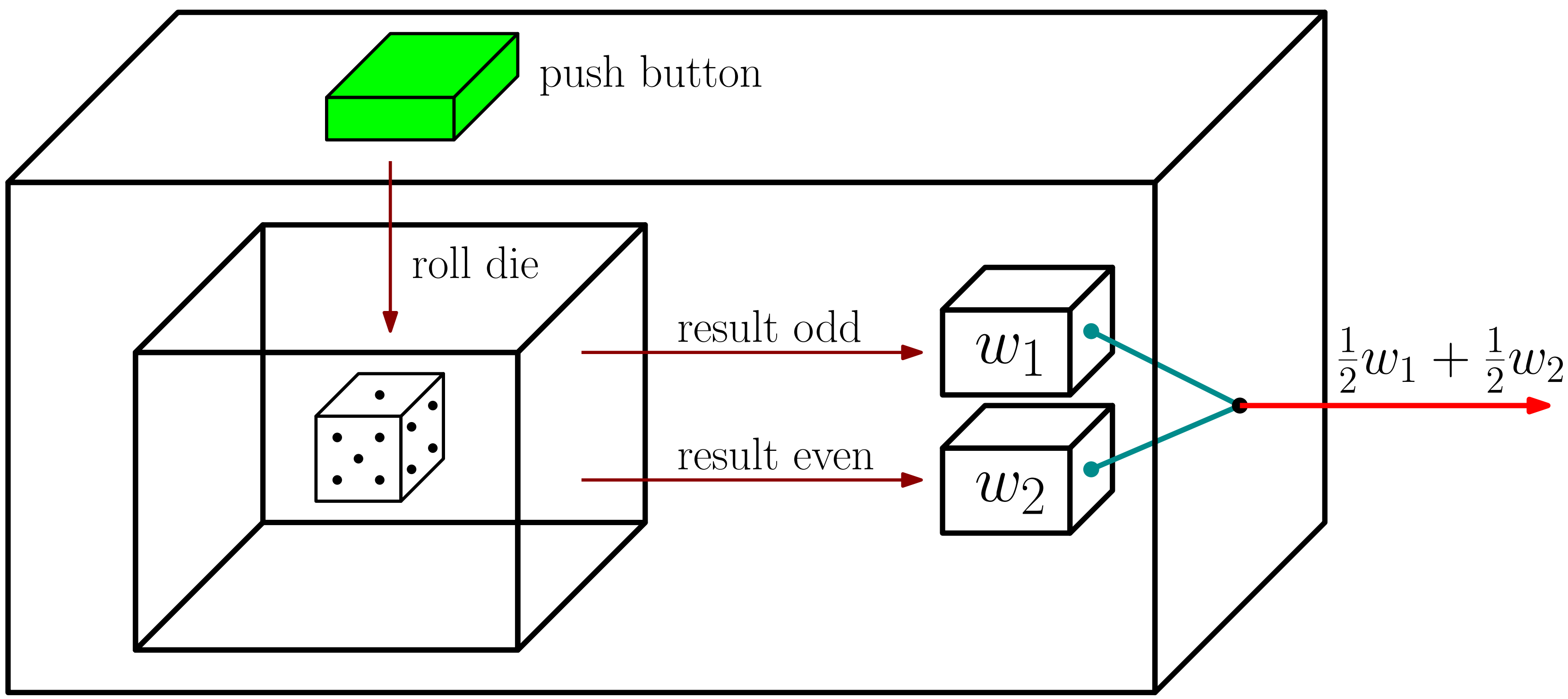

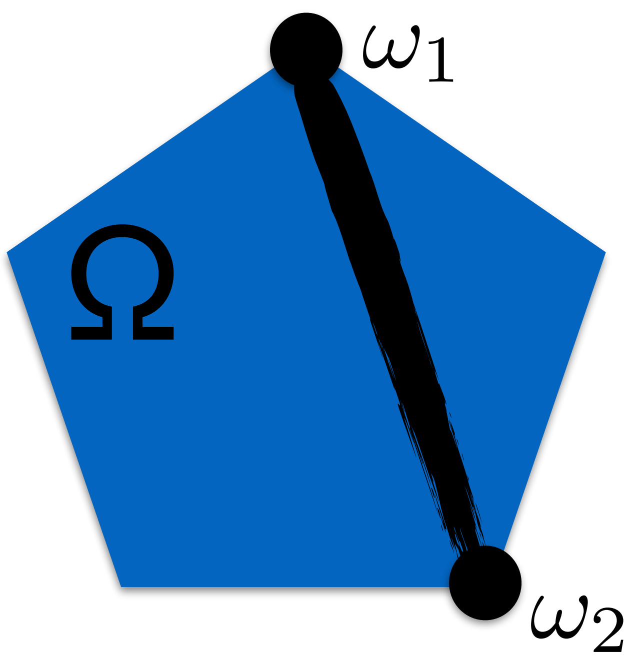

Thus Ω is a convex set.

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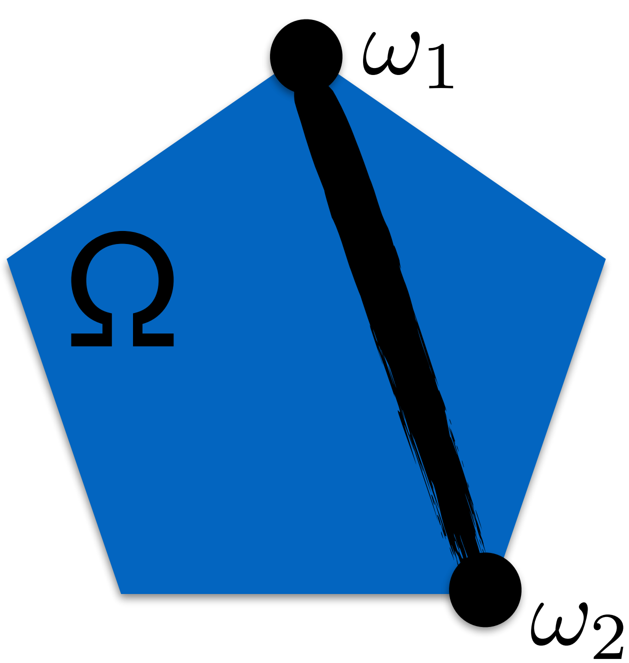
Preparation of statistical mixtures: $\omega = \lambda\omega_1 + (1-\lambda)\omega_2$

QT: Ω_N = set of N×N density matrices
CPT: Ω_N = set of prob. distributions $(p_1, \ldots, p_N)$.



Thus Ω is a convex set.

How to describe a "general probabilistic theory"

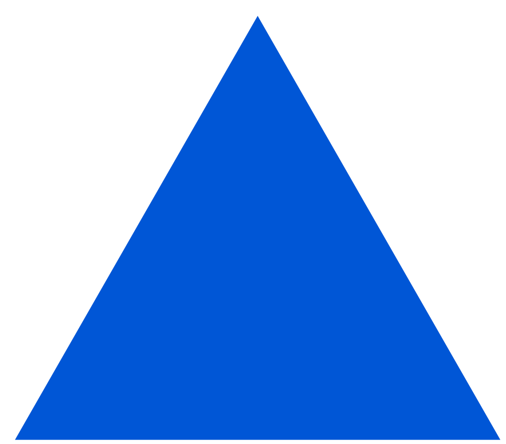
(Almost) everything can be inferred from shape of state space.

classical
bit

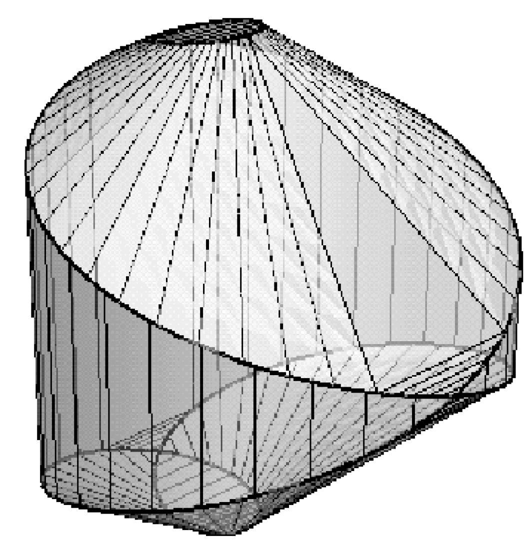
quantum
bit

"gbit"

Arbitrary convex
state space



Classical trit
(3-level-system)



Quantum trit:
8D "orbitope"

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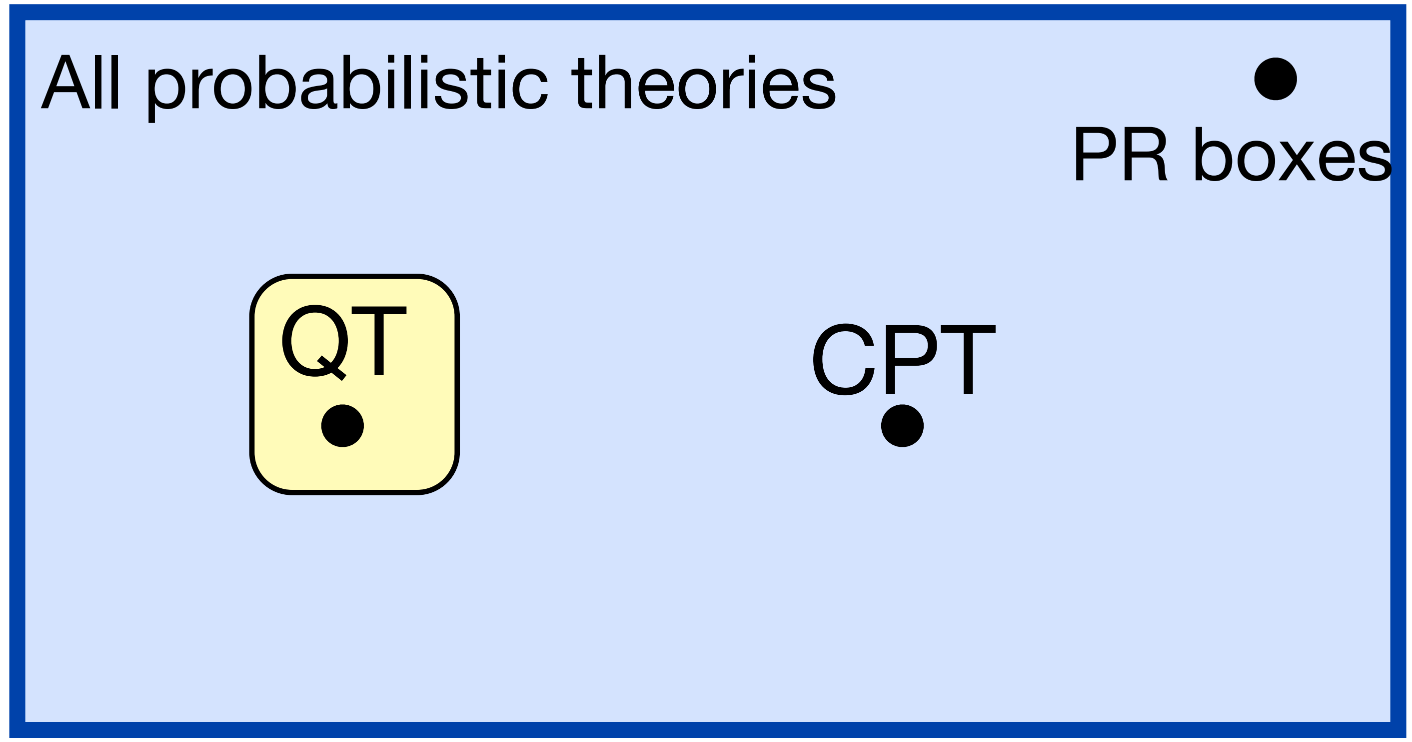

2. Quantum theory from simple principles

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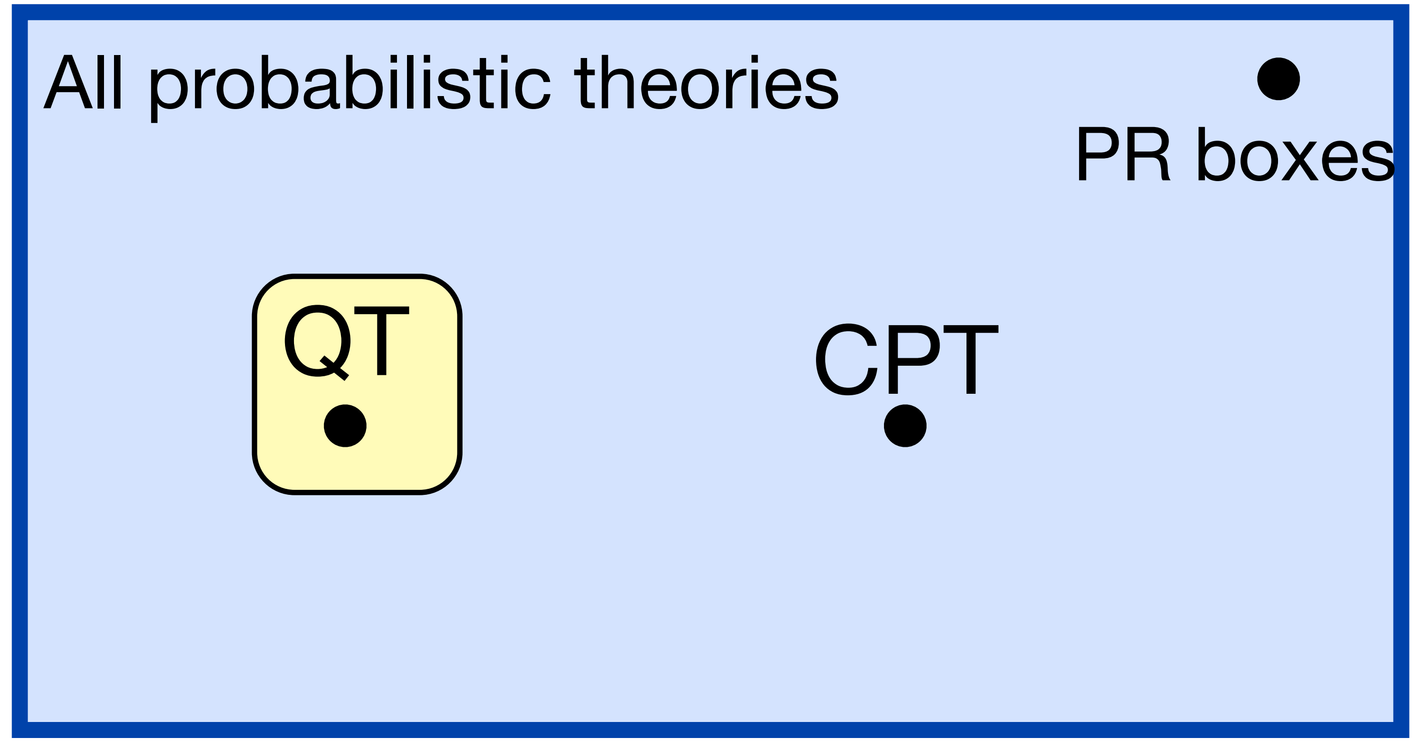
Simple principles that yield exactly QT.

Starting with Lucien Hardy 2001, lots of recent activity:



- L. Hardy, *Quantum theory from five reasonable axioms*, arXiv:quant-ph/0101012
- B. Dakic and C. Brukner, *Quantum Theory and Beyond: Is Entanglement Special?*, arXiv:0911.0695 (also "Deep Beauty"-book)
- Ll. Masanes and **MM**, *A derivation of quantum theory from physical requirements,* New J. Phys. **13**, 063001 (2011)
- G. Chiribella, G. M. D'Ariano, and P. Perinotto, *Informational derivation of quantum theory*, Phys. Rev. A **84**, 012311 (2011)
- L. Hardy, *Reformulating and reconstructing quantum theory*, arXiv:1104.2066
- ….

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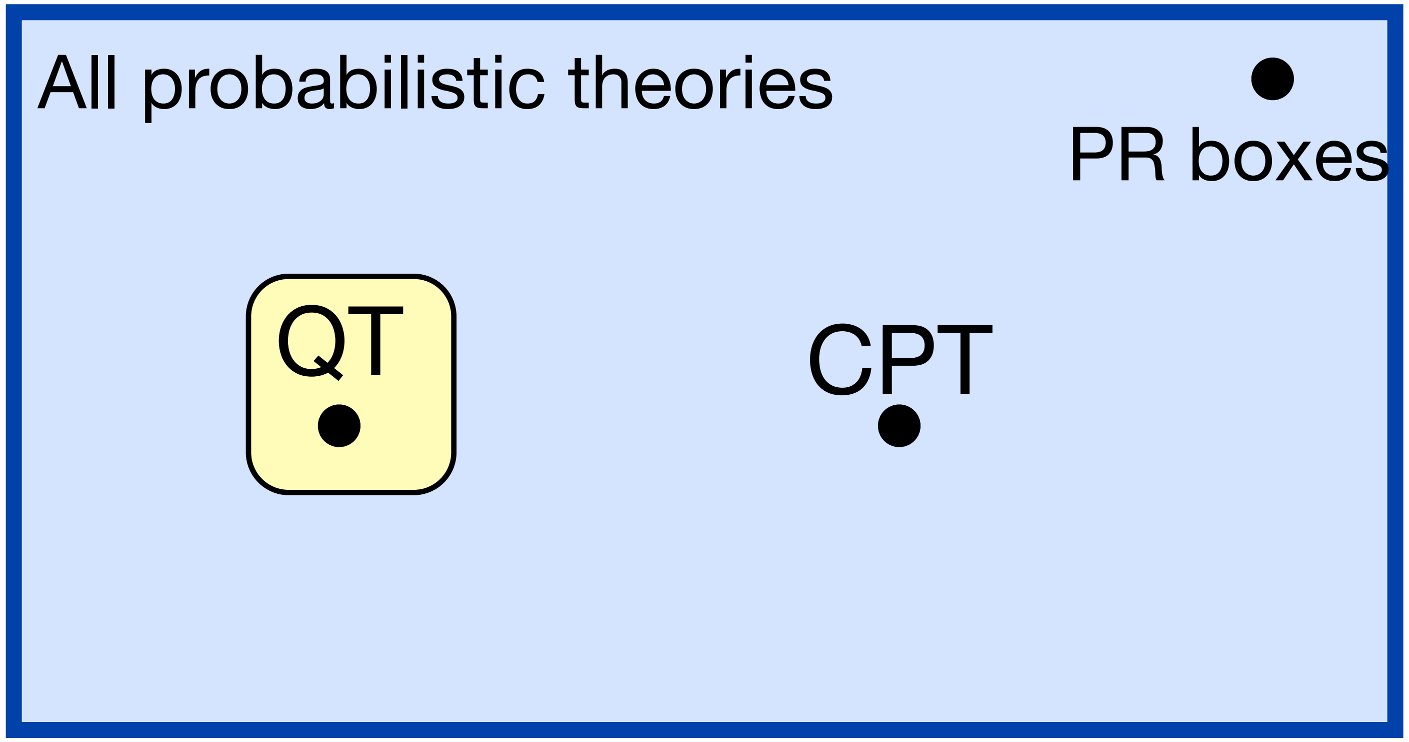

However, all these used assumptions on **composition of systems** in a crucial way. Disadvantages:

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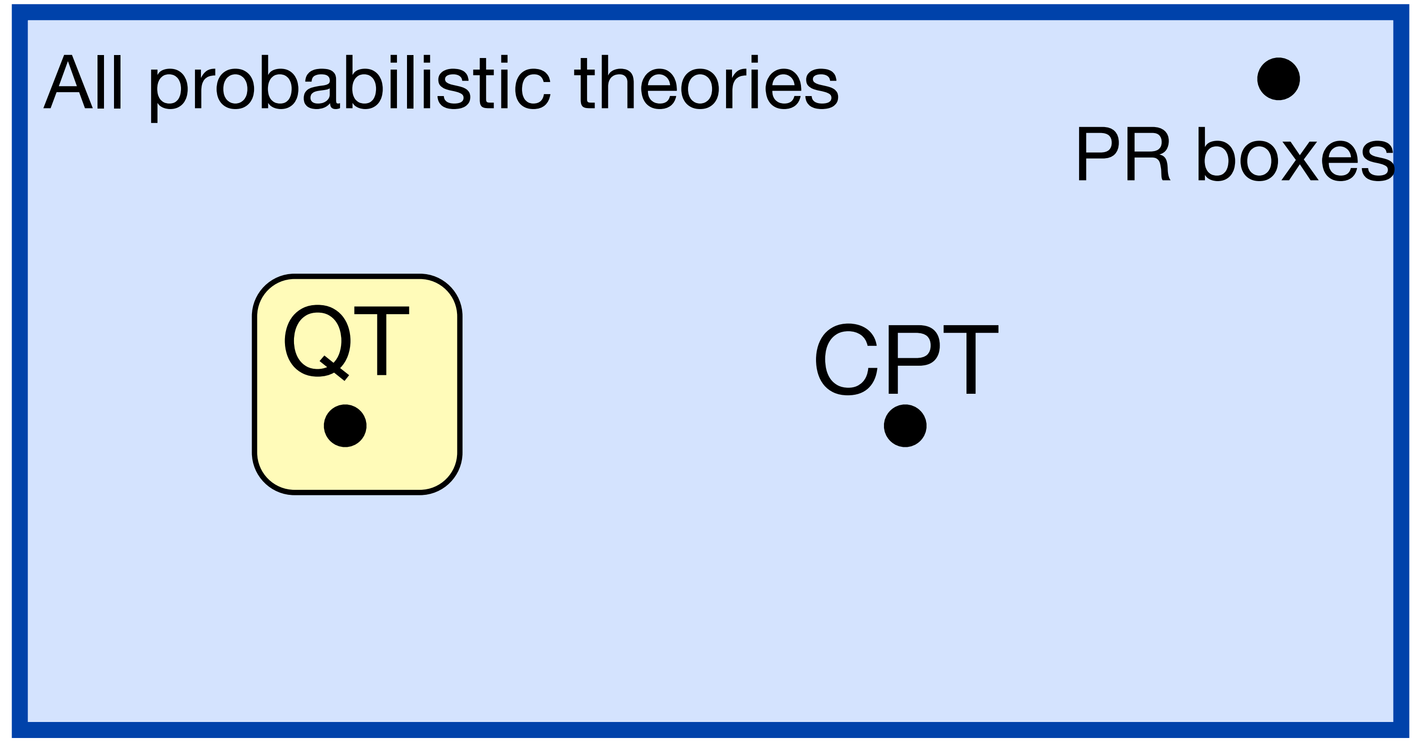

Starting with Lucien Hardy 2001, lots of recent activity.

However, all these used assumptions on **composition of systems** in a crucial way. Disadvantages:

- QT has already shown: we have **bad intuition** on composition!
- Very **hard to modify** postulates to get to "QT's closest cousins"

A **single-system** reconstruction of QT

H. Barnum, **MM**, and C. Ududec, *Higher-order interference and single-system postulates characterizing quantum theory*, New J. Phys. **16**, 123029 (2014).

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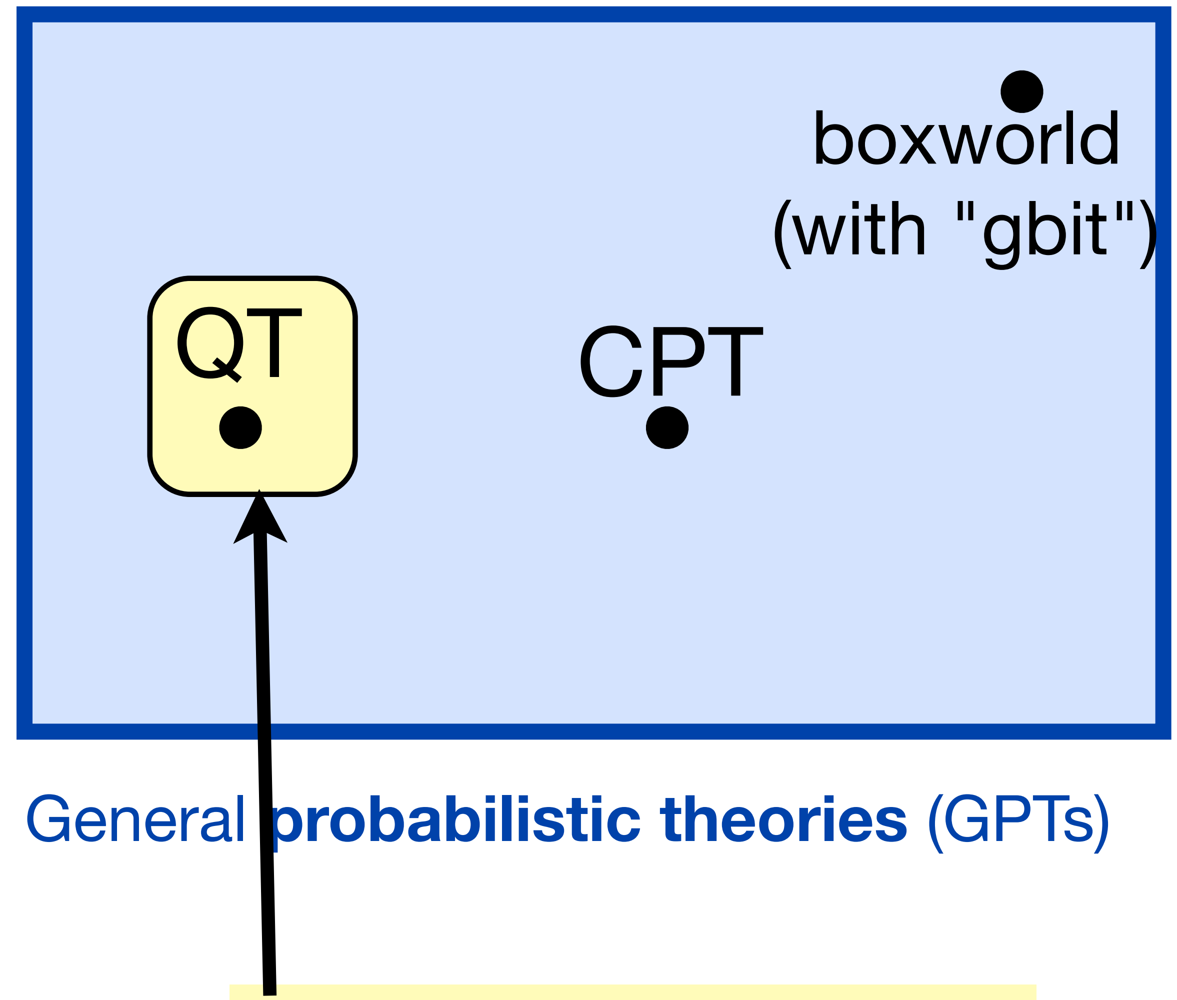


General **probabilistic theories** (GPTs)

Which principles
single out quantum theory?

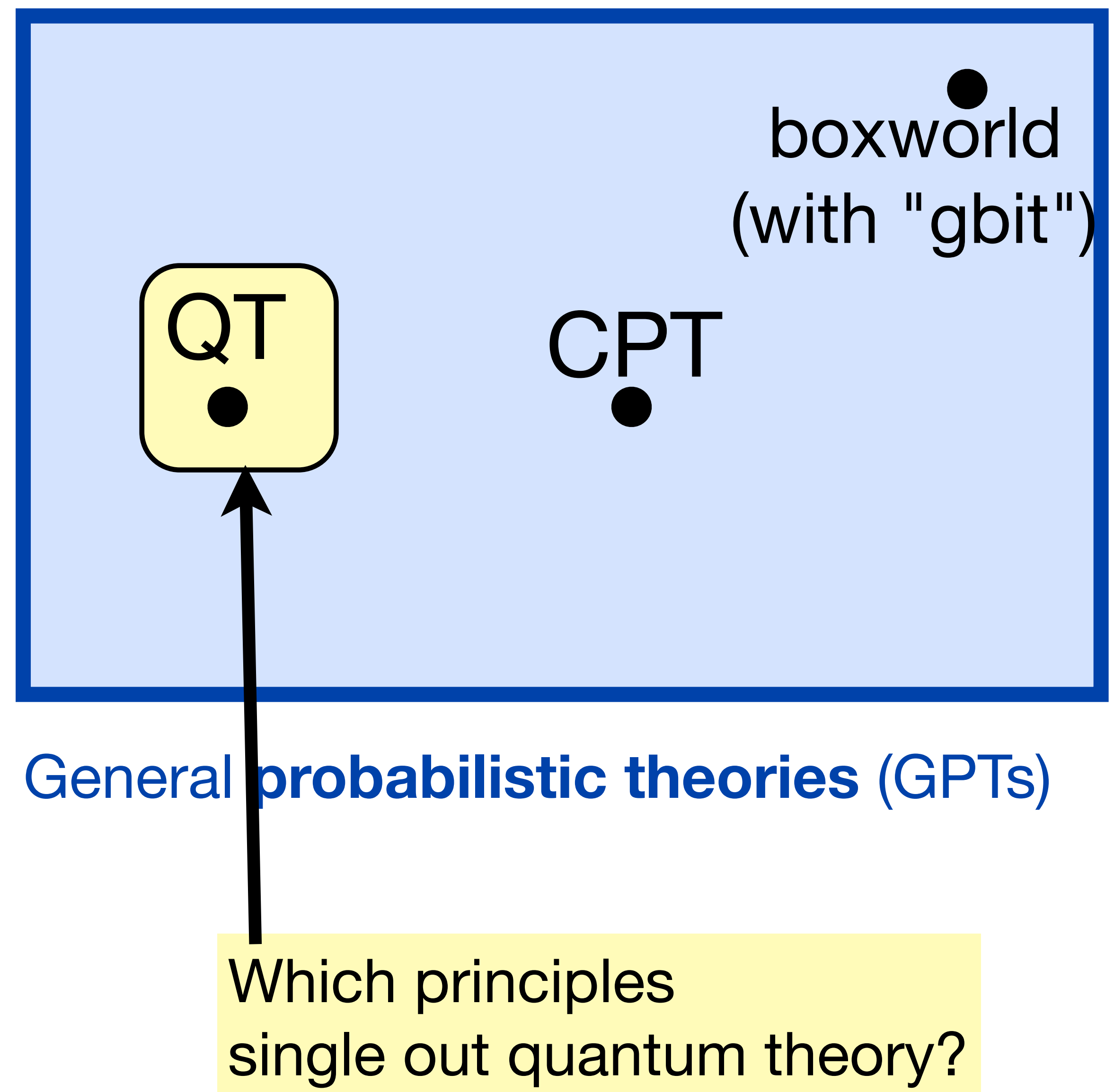
A **single-system** reconstruction of QT

H. Barnum, **MM**, and C. Ududec, *Higher-order interference and single-system postulates characterizing quantum theory*, New J. Phys. **16**, 123029 (2014).

Theorem: If a state space satisfies

1. Classical decomposability
2. Strong Symmetry
3. No Third-Order Interference
4. Energy Observability

then it is a **quantum state space**.



A **single-system** reconstruction of QT

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Theorem: If a state space satisfies

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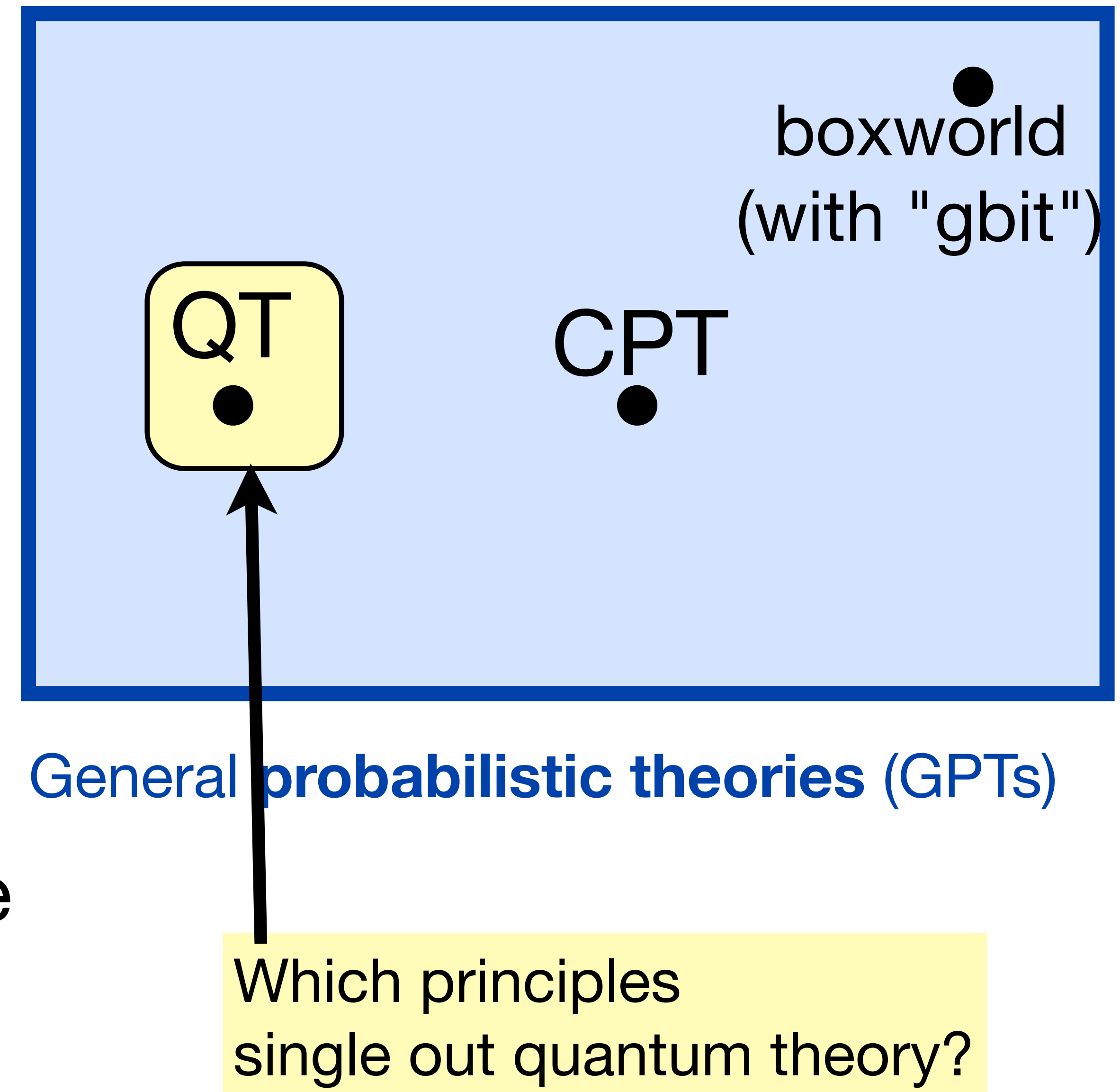
then it is a **quantum state space**, i.e. the states are the $N \times N$ complex density matrices, reversible transformations are $\rho \mapsto U\rho U^\dagger$ with *U* unitary or antiunitary, and the measurements are the POVMs.



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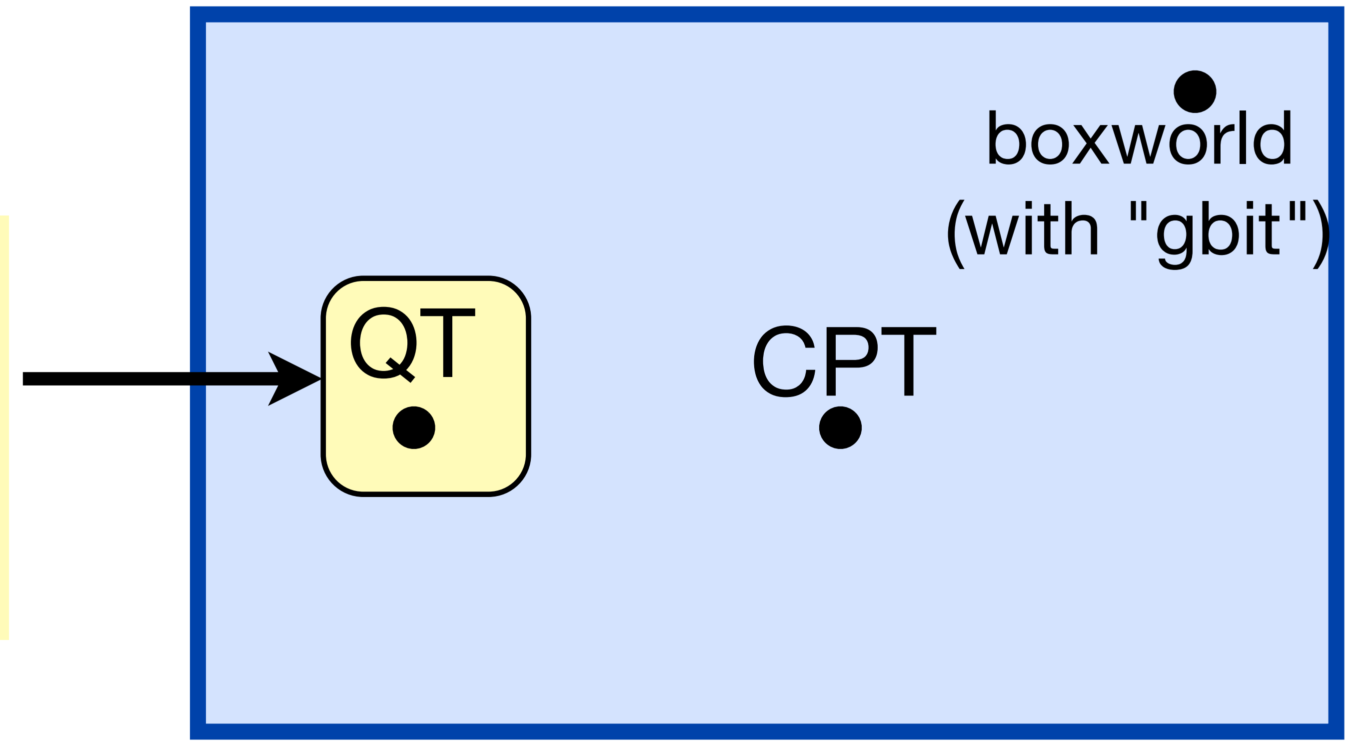

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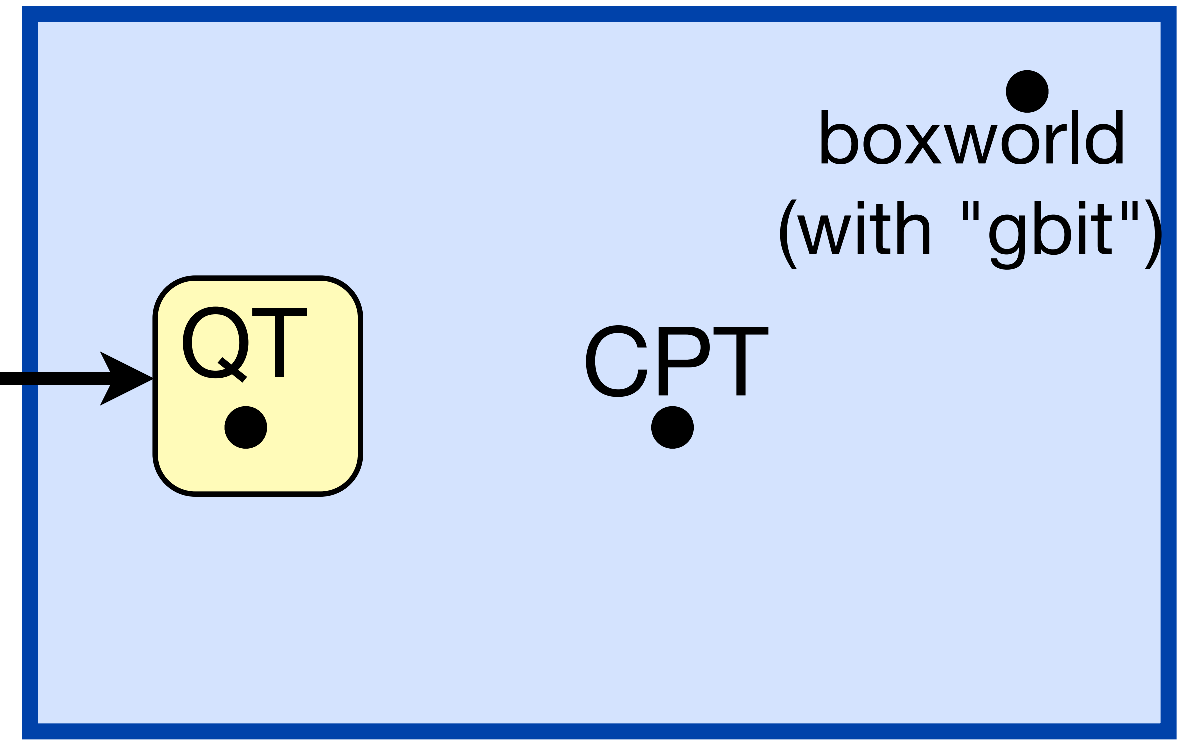

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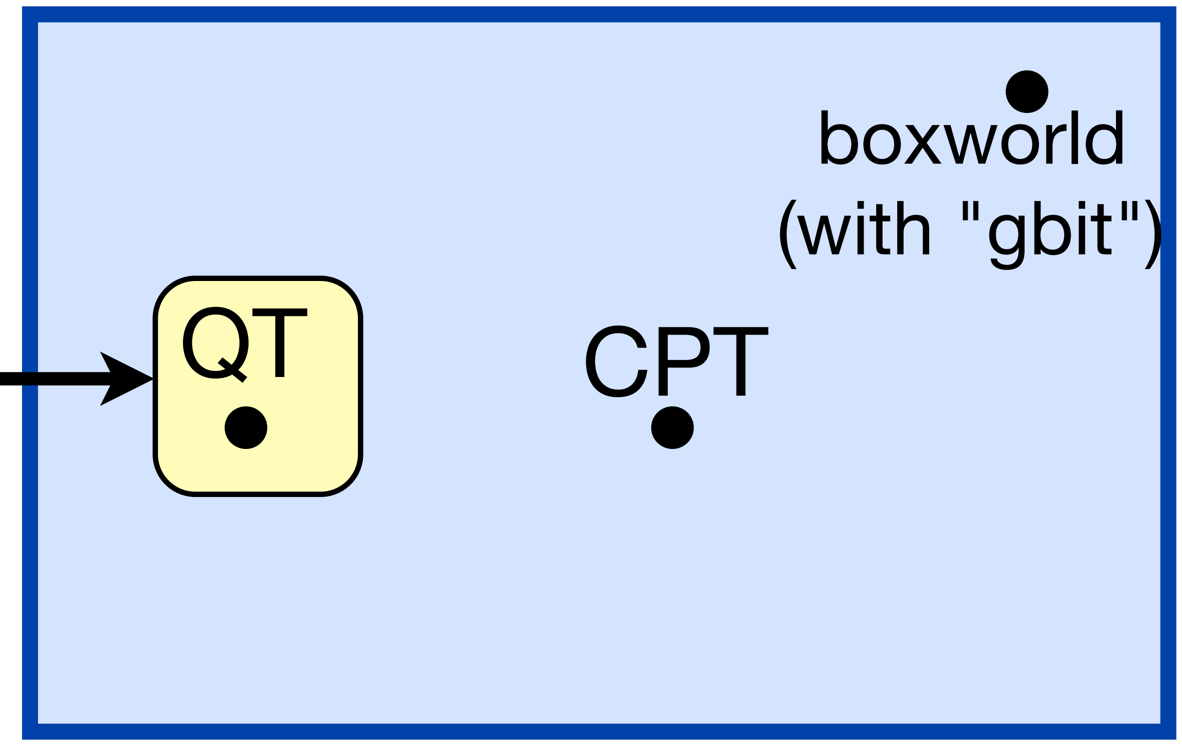

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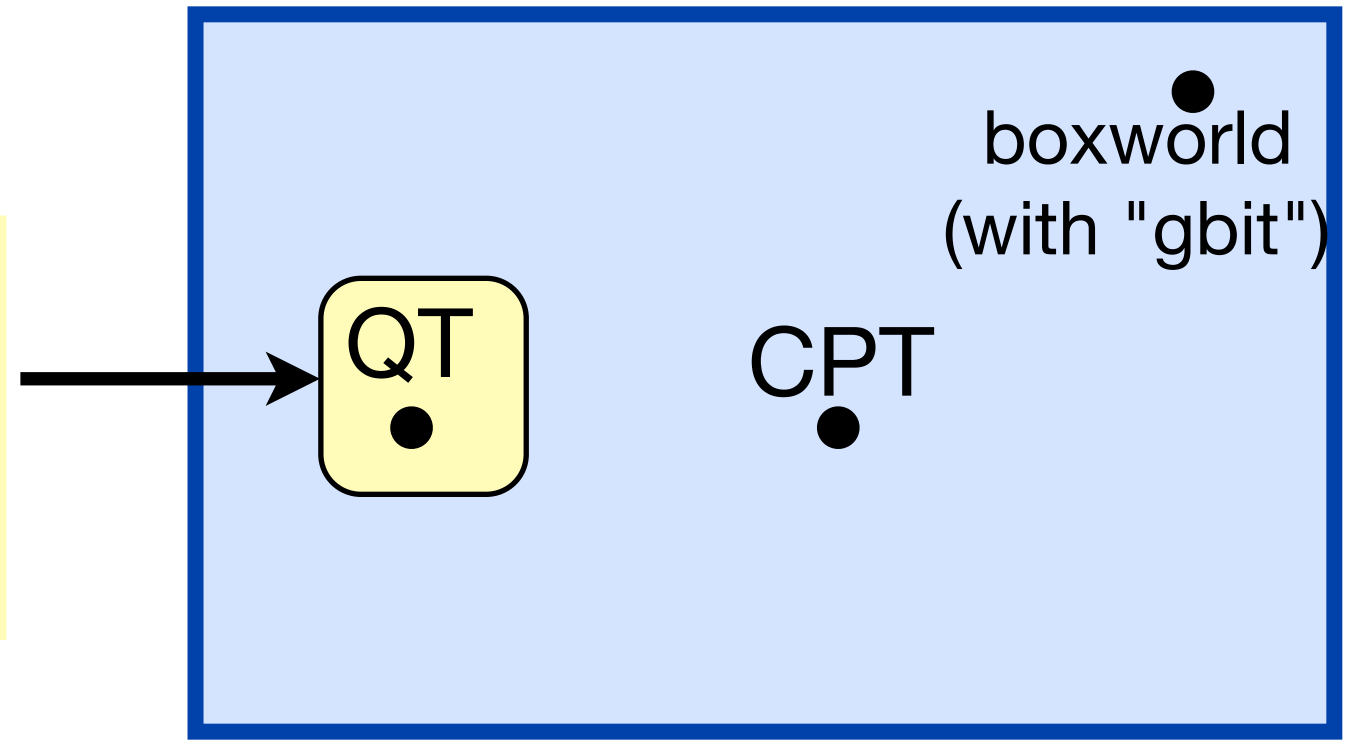

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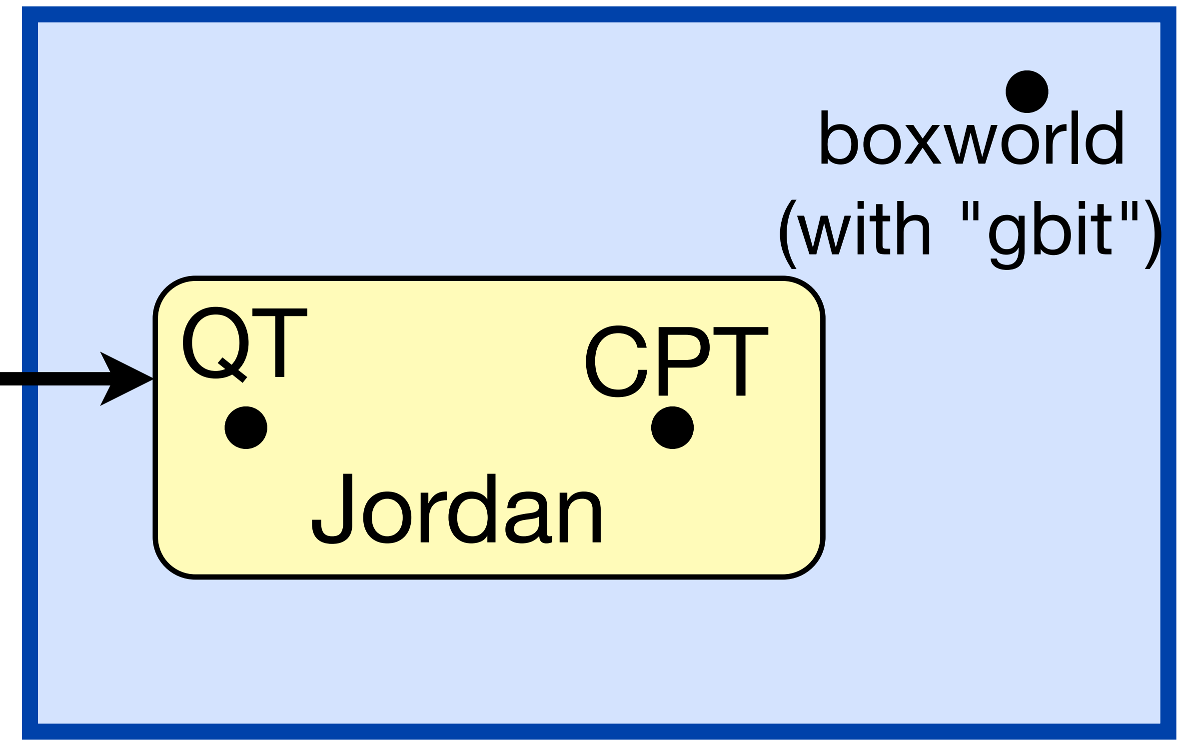

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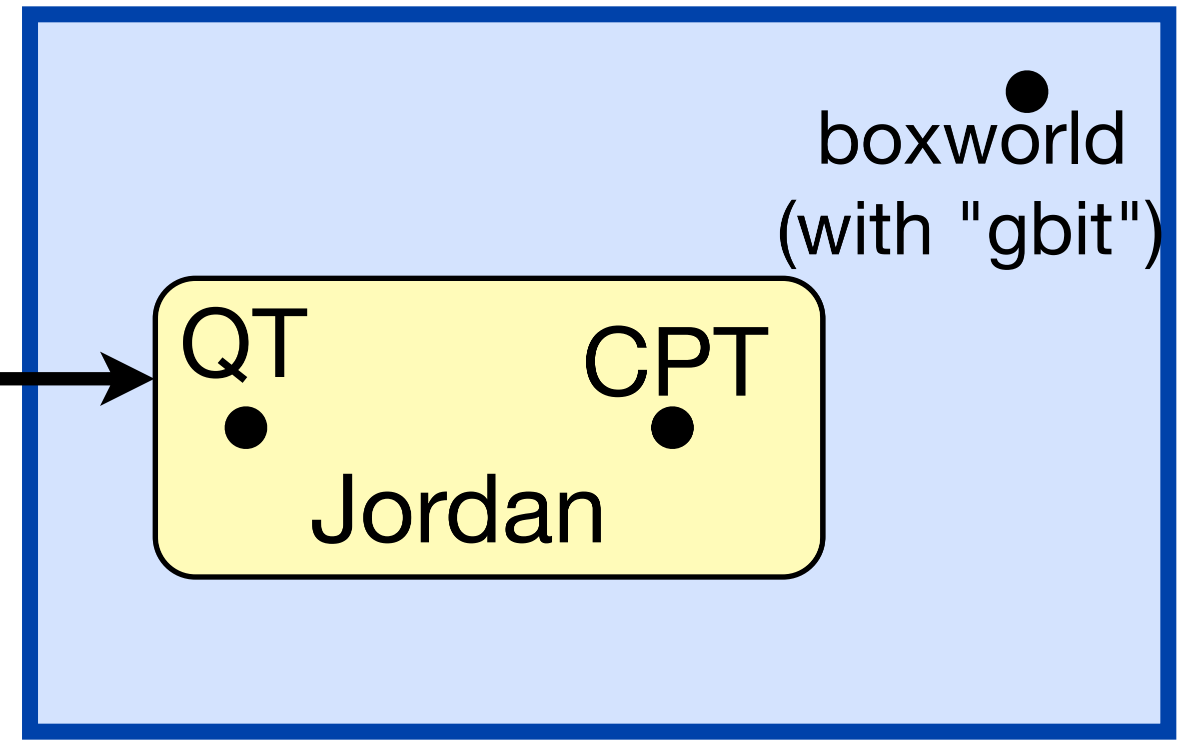

then it is one of the following:

- N-level quantum theory over $\mathbb{R}, \mathbb{C}$ or $\mathbb{H}$,
- 3-level quantum theory over the octonions,
- 2-level "Bloch balls" with any number of degrees of freedom (not necessarily 3 as in the qubit),
- N-level discrete classical probability distributions (CPT).

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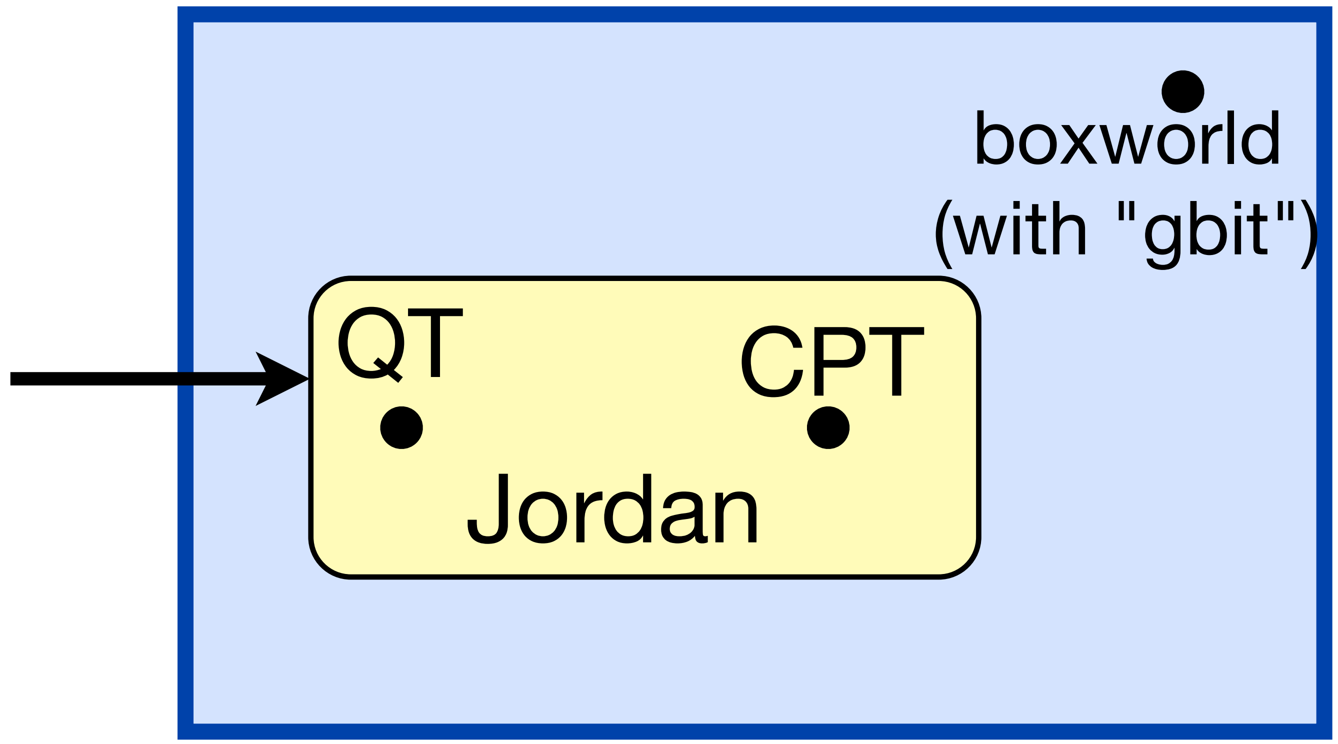

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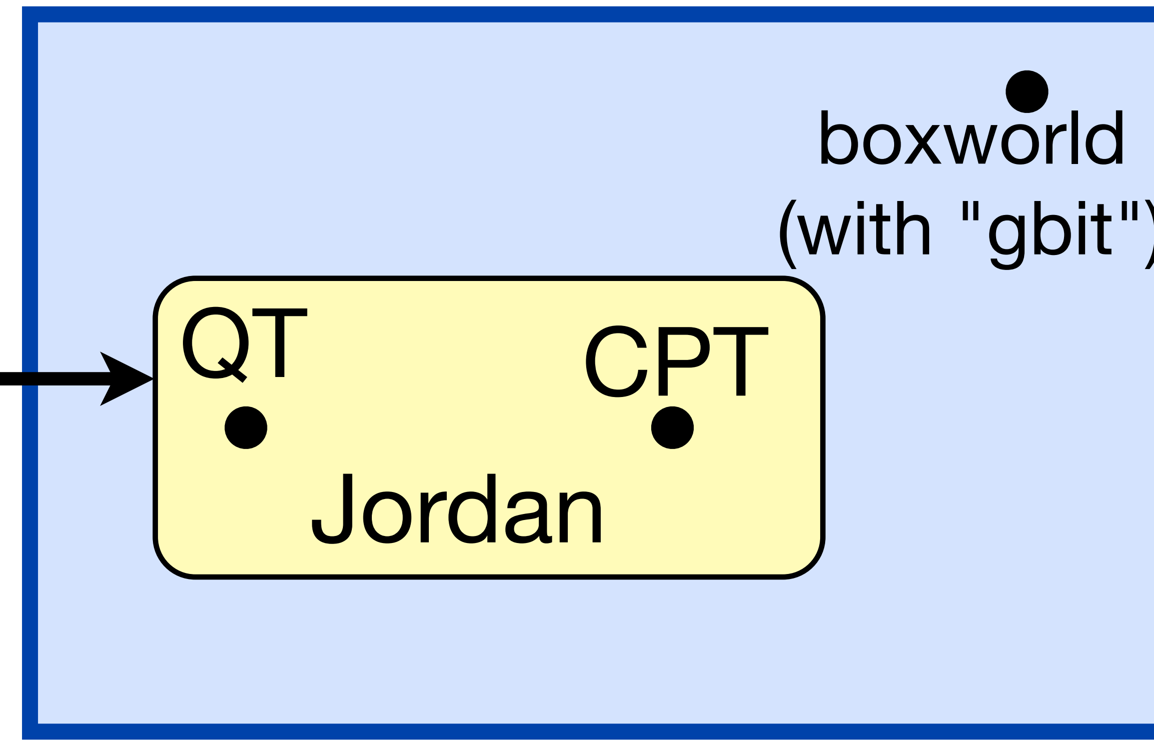

Most fascinating question will be:

What if we drop Postulate 3?

But for now, let's understand Postulates 1 and 2...

Classical decomposability

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Every state $\omega \in \Omega$ can be written as a convex combination of perfectly distinguishable pure states $\omega_1, \ldots, \omega_n$:

$$\omega = \sum_i \lambda_i \omega_i.$$

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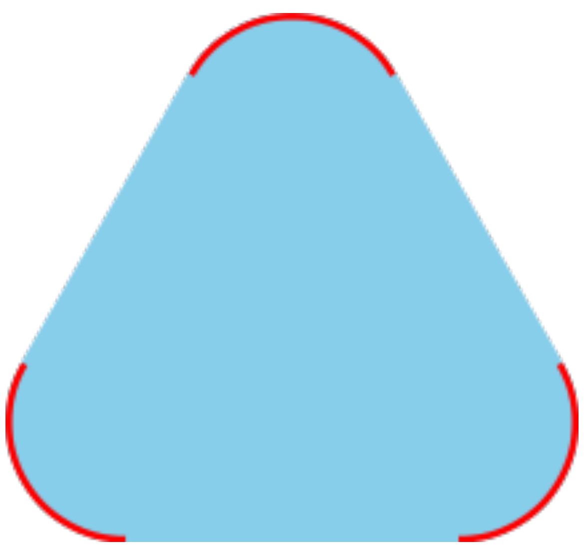

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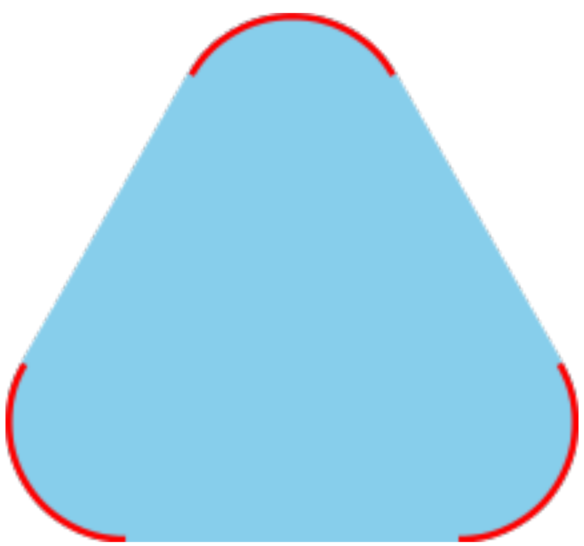

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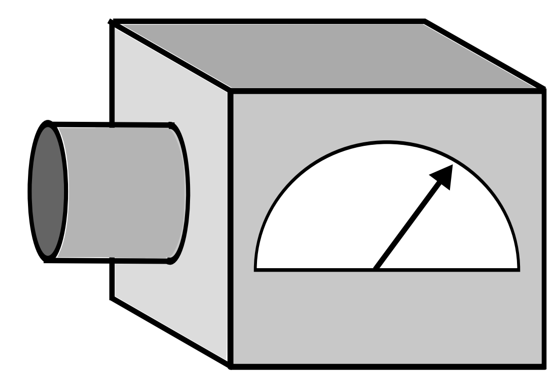

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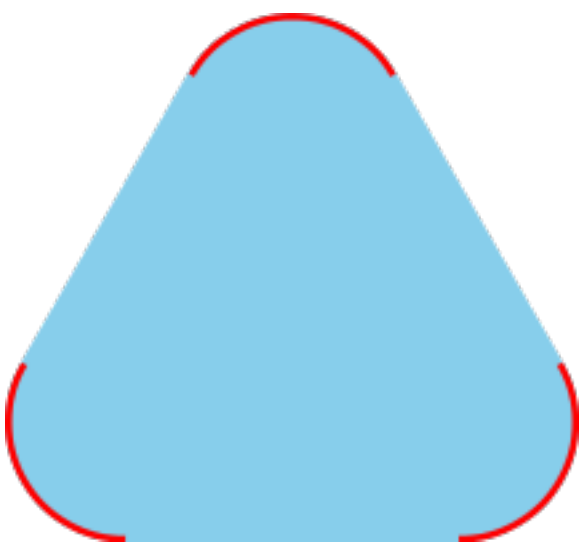

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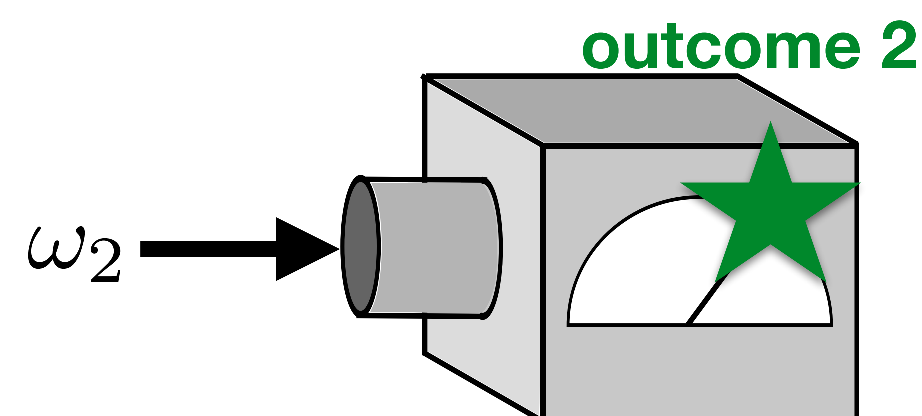

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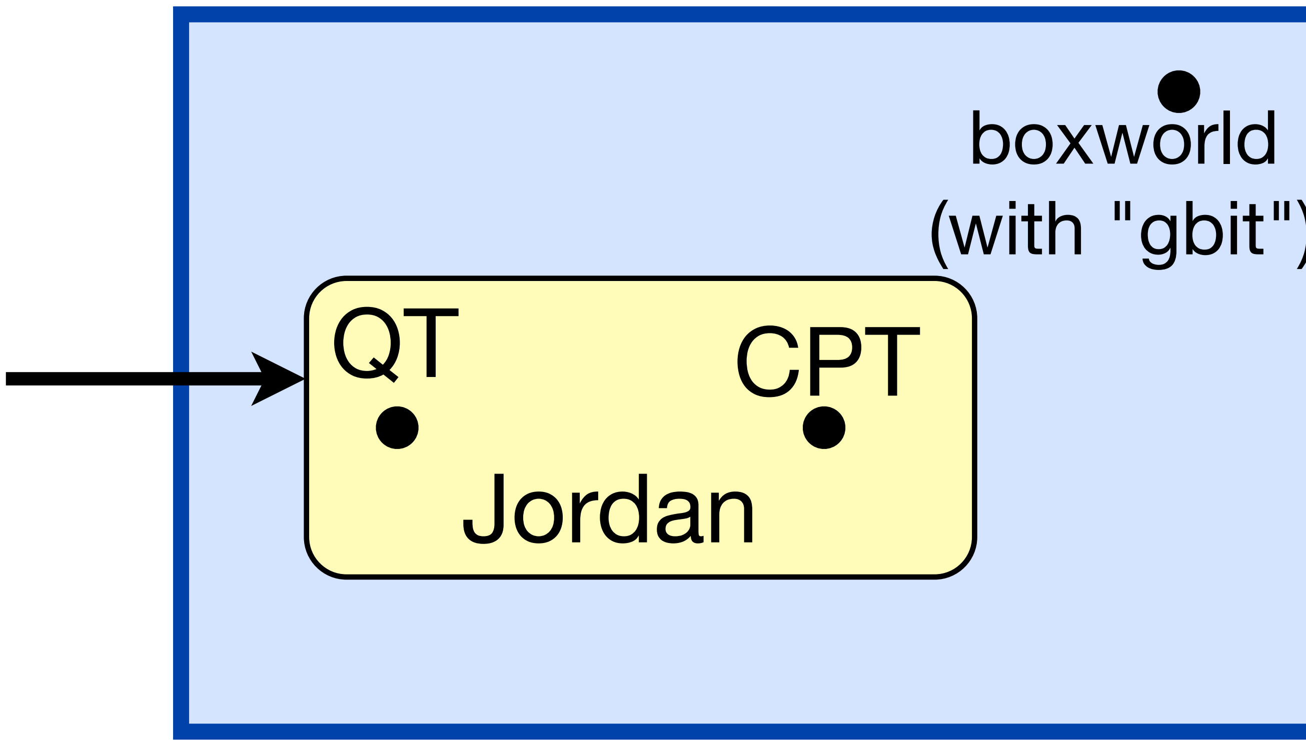

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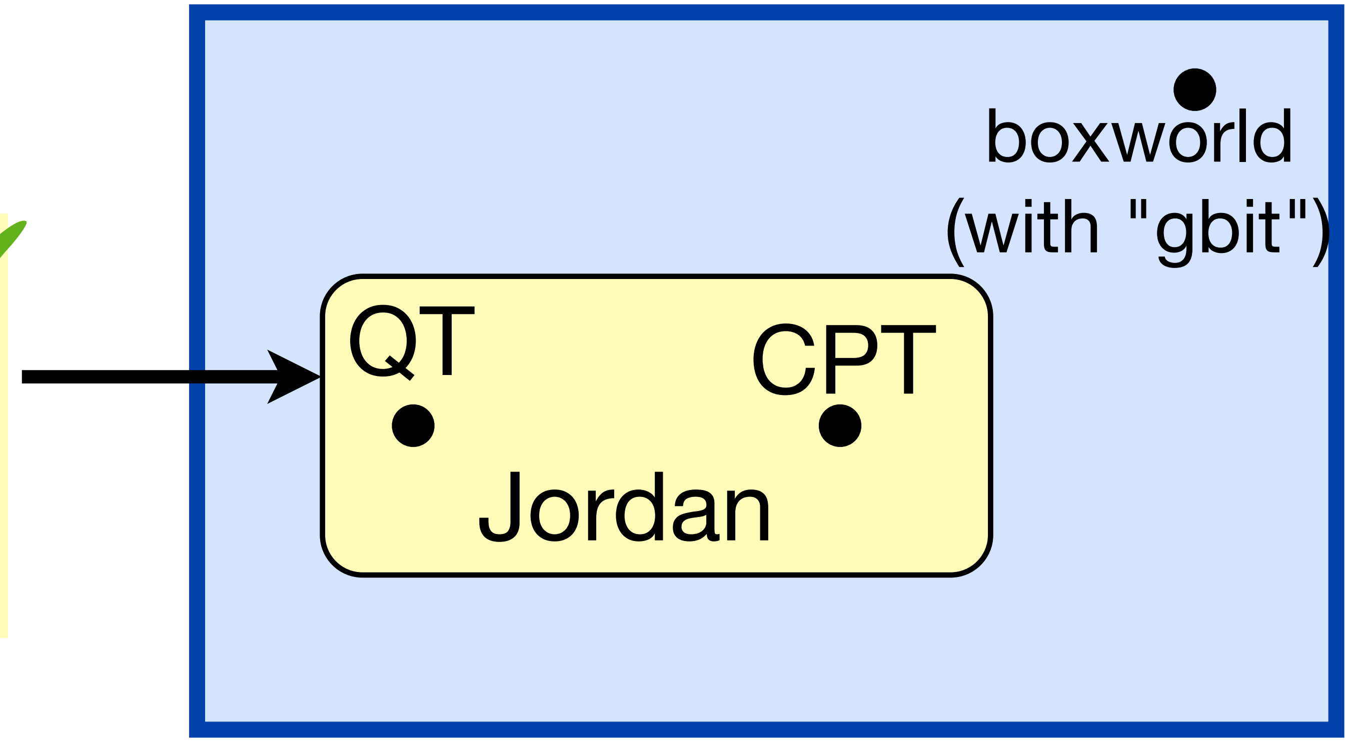

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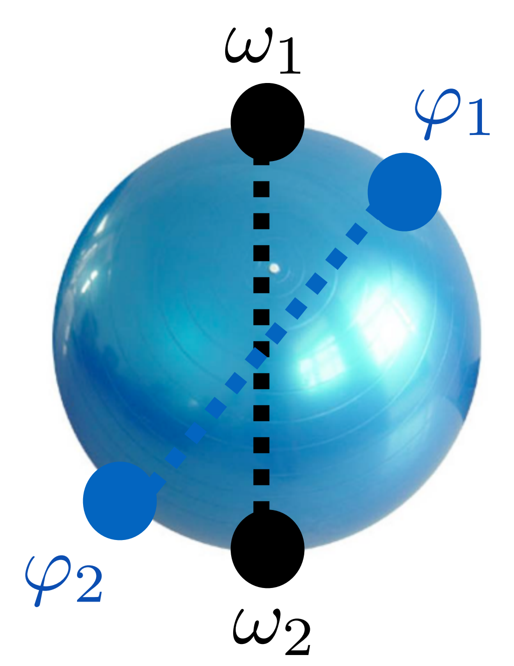
Strong symmetry for qubit easy to see in the Bloch ball representation:



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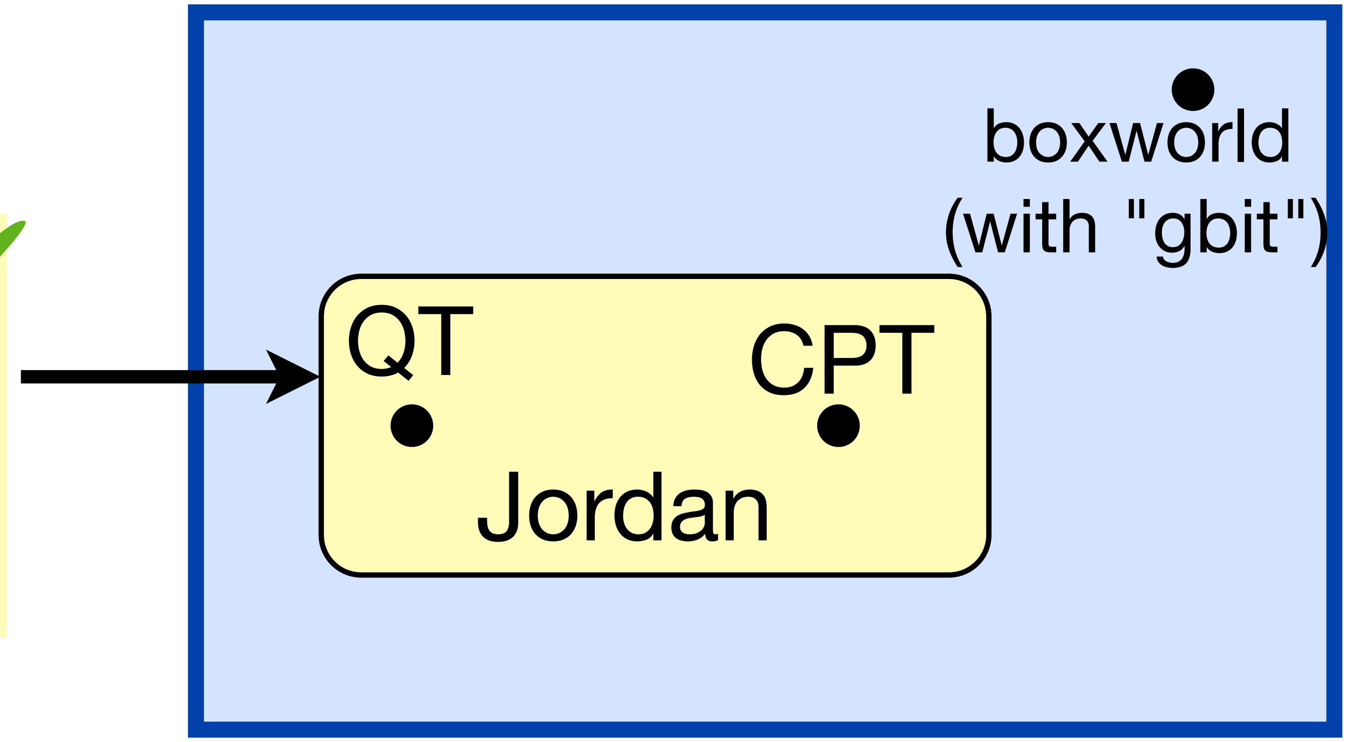

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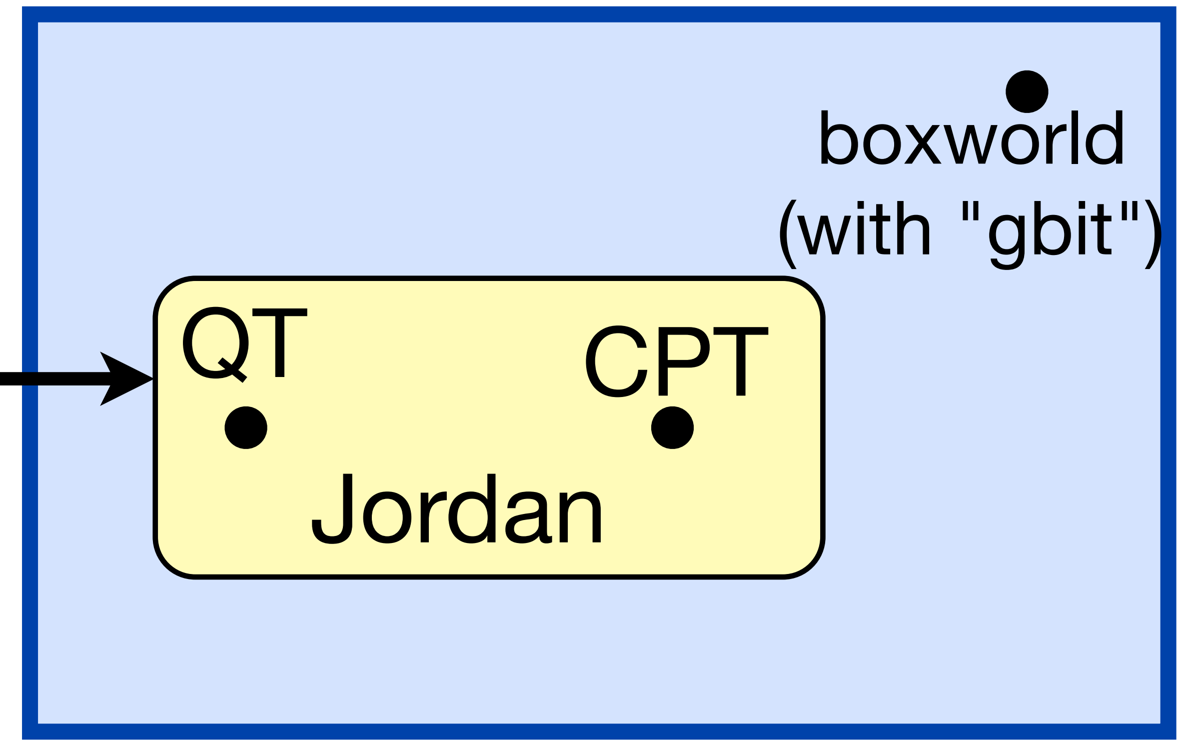

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$p_{i,j,\ldots} :=$ probability of event,
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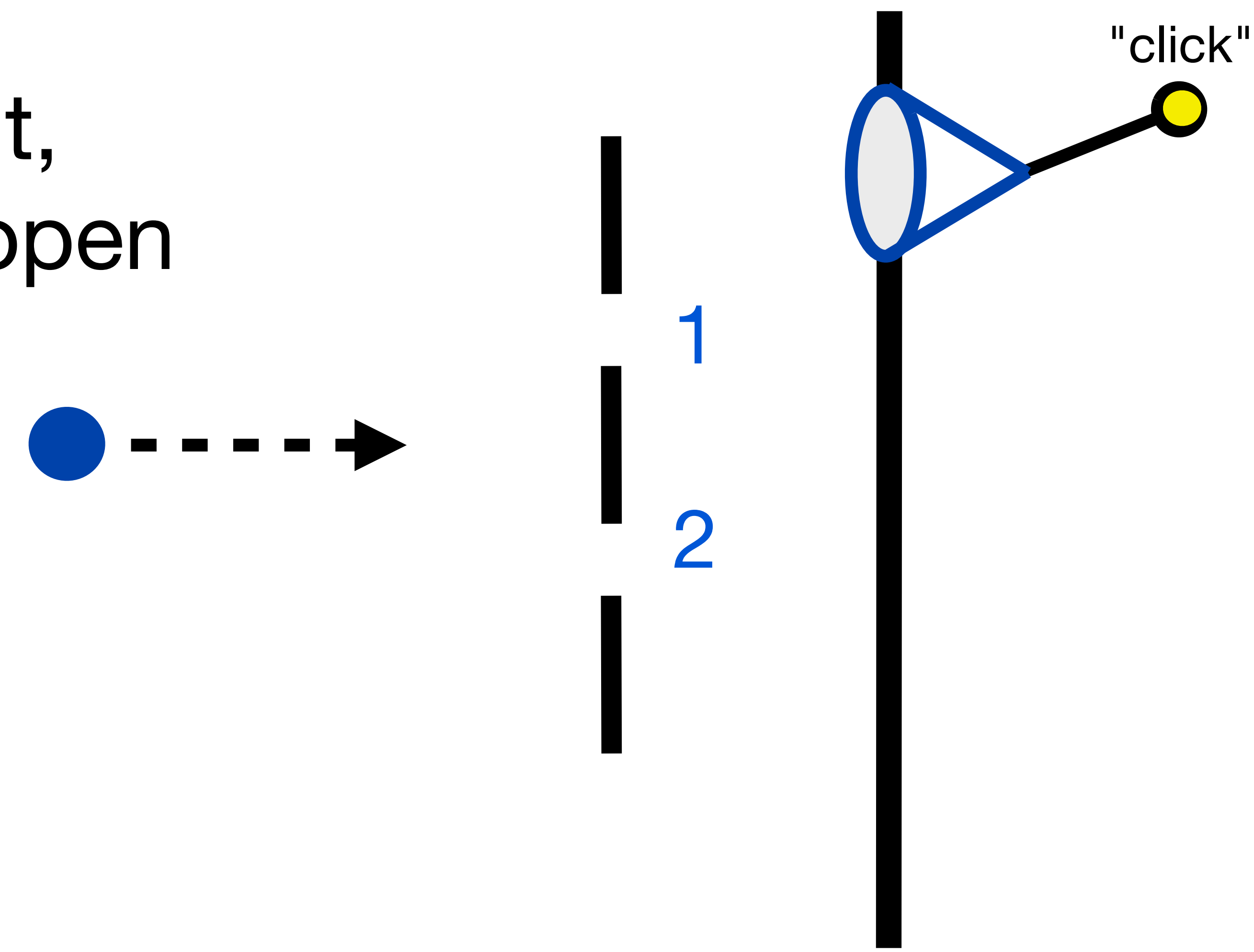

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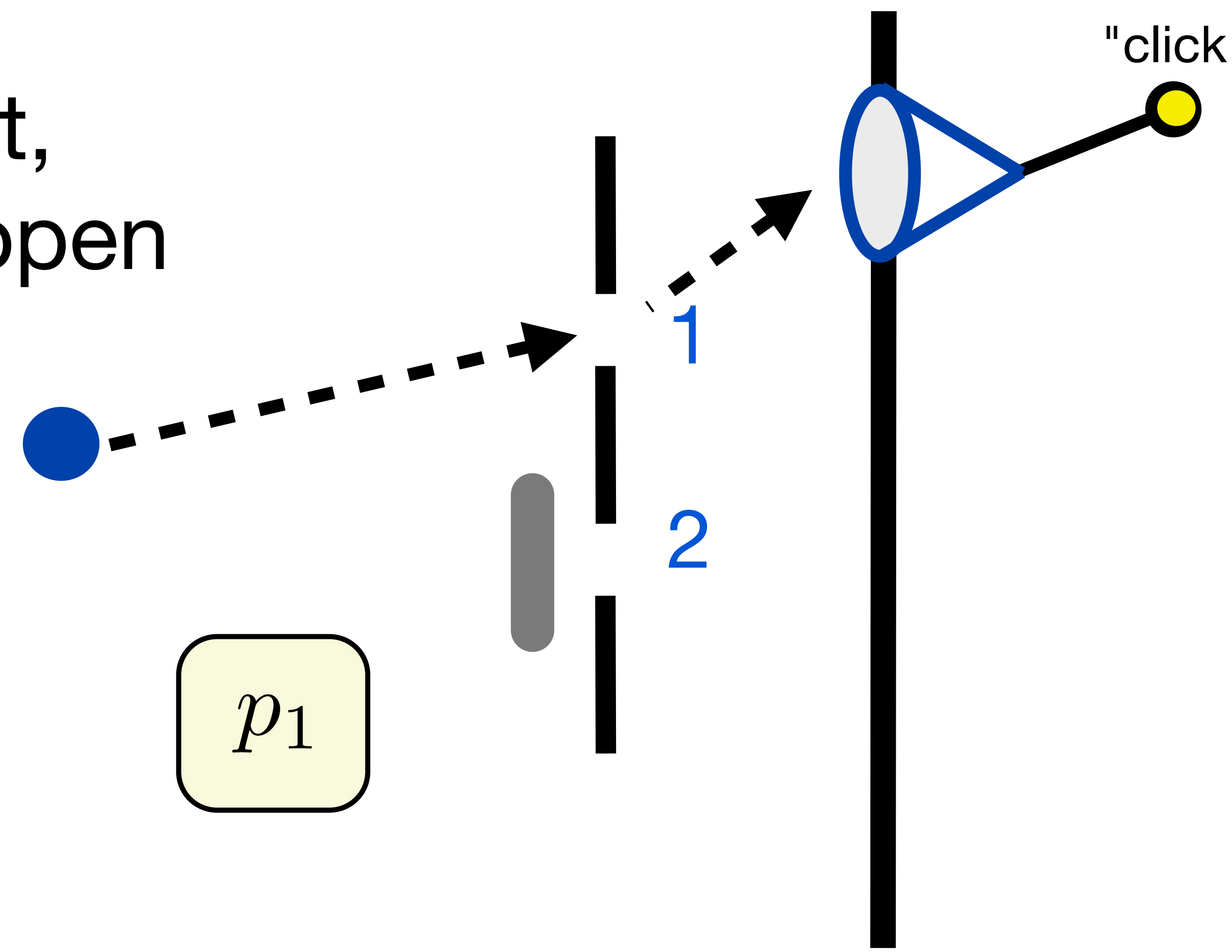

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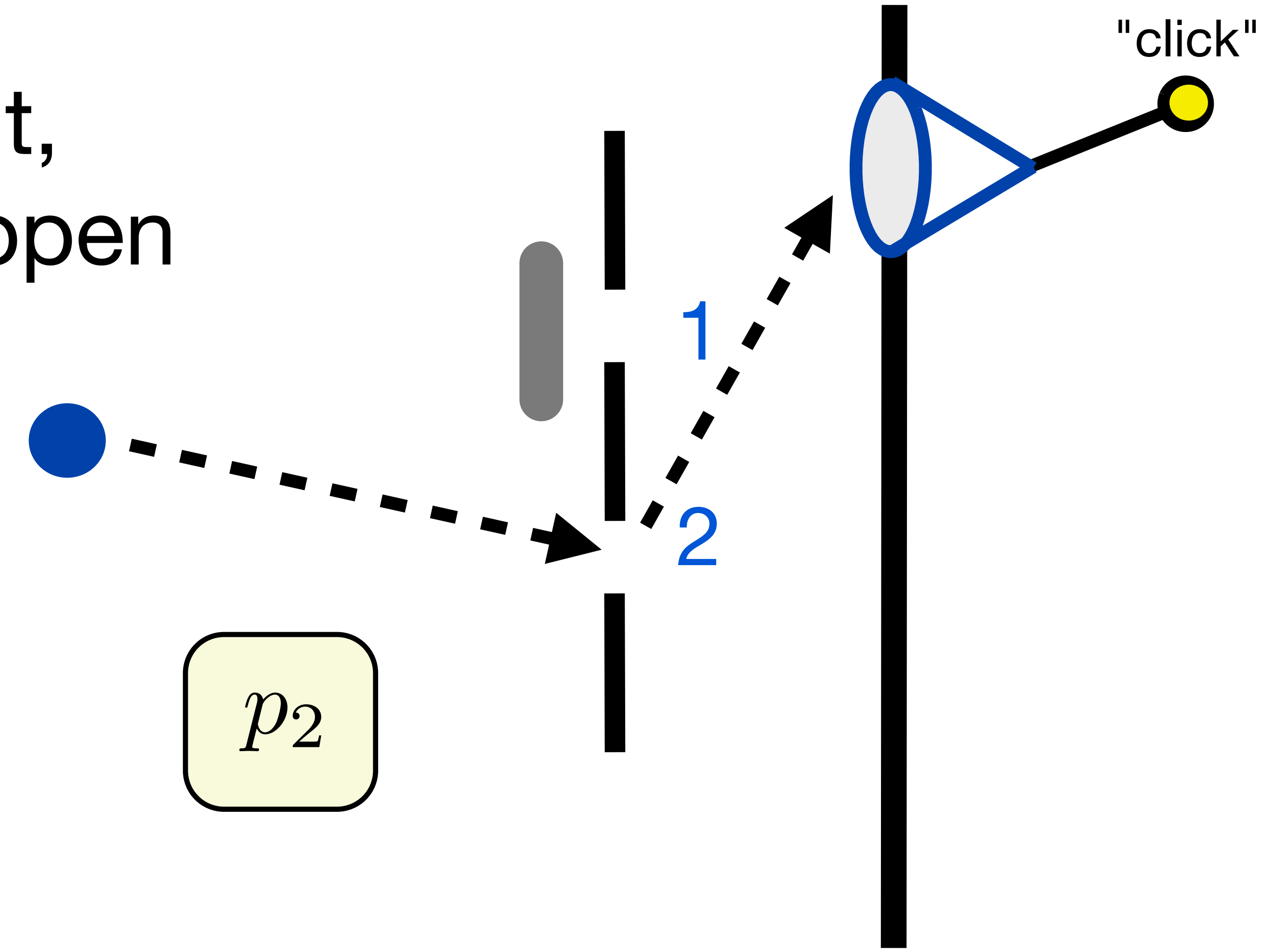

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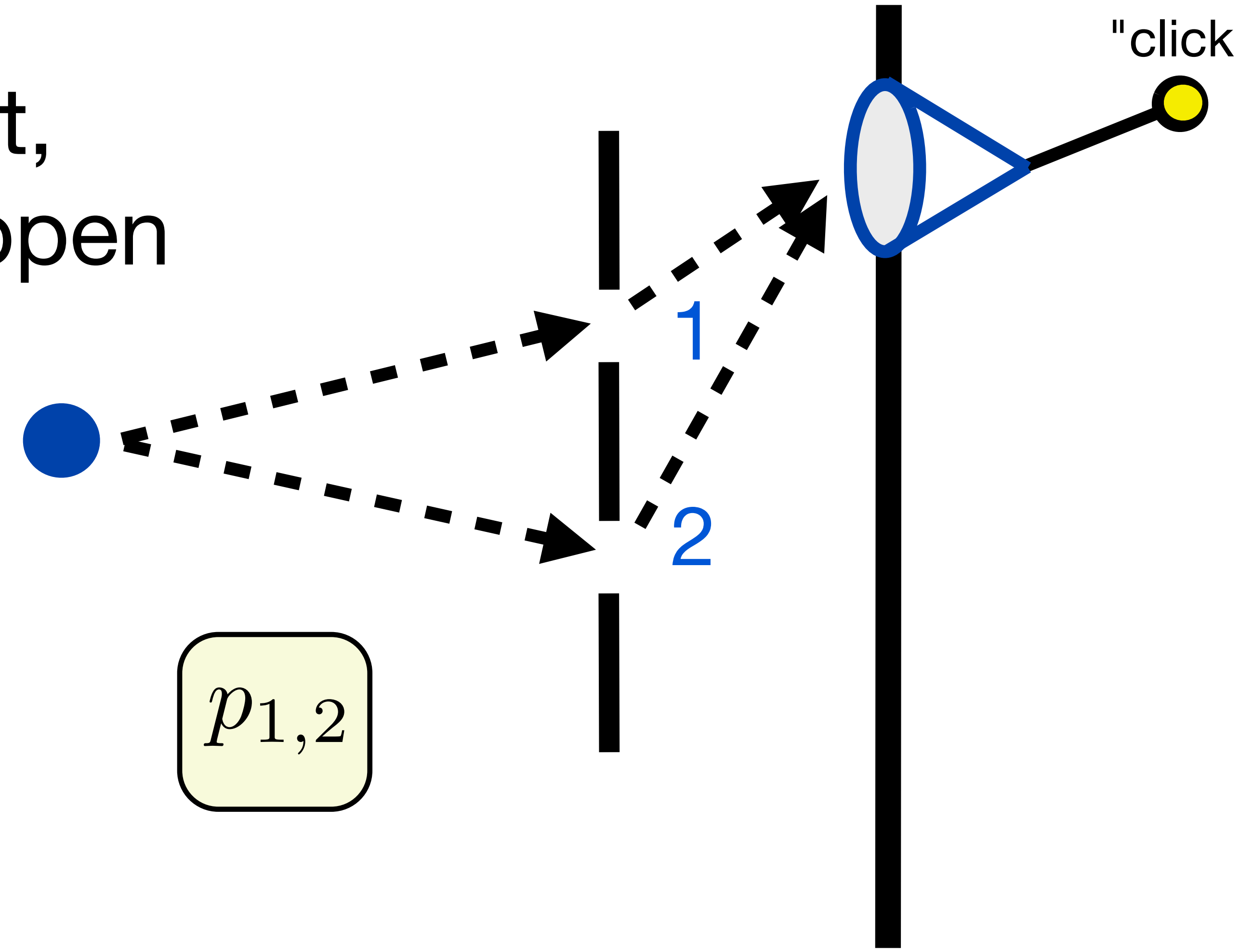

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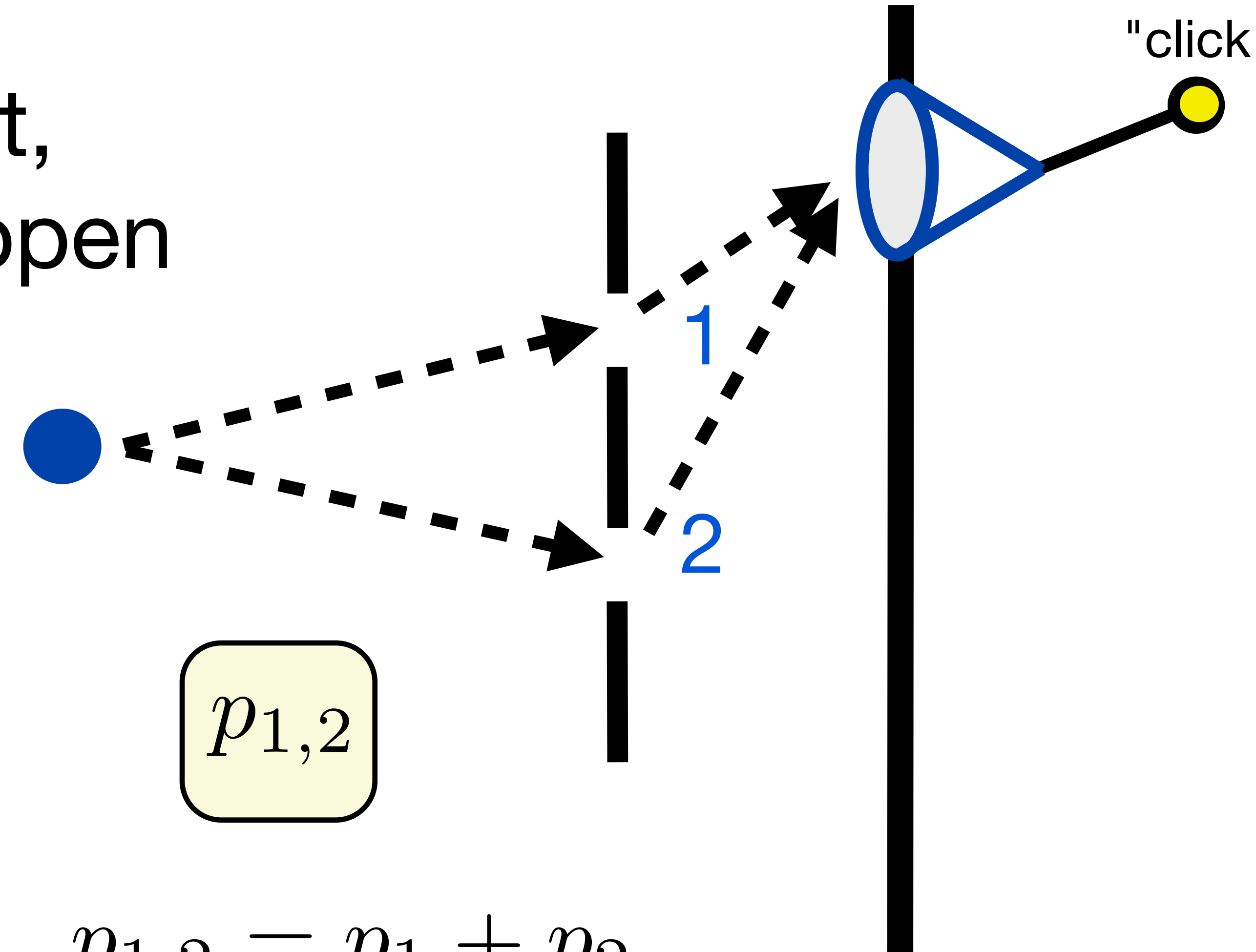

Classical probability theory: $p_{1,2} = p_1 + p_2$.

Quantum theory: $p_{1,2} \neq p_1 + p_2$.

Interference!

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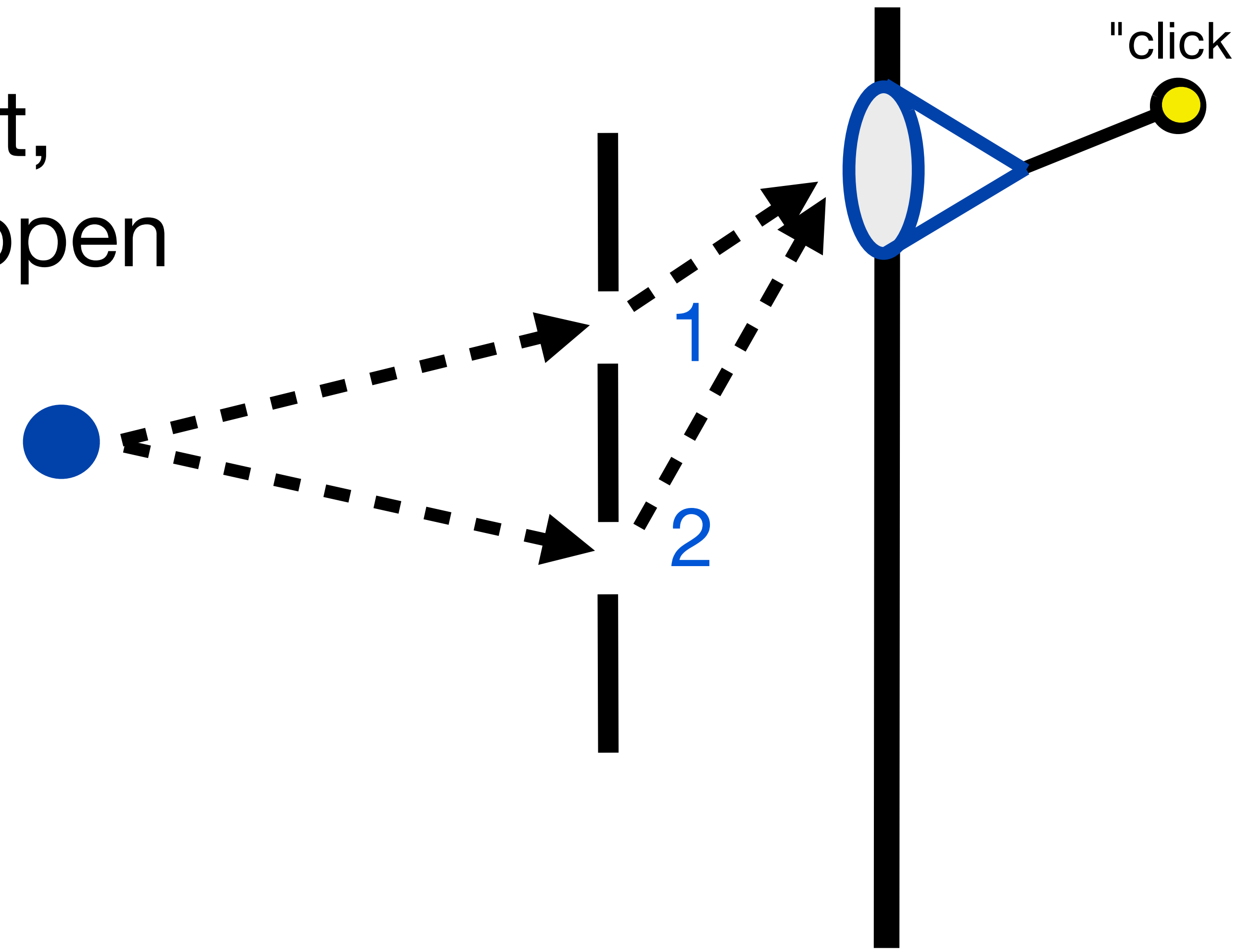

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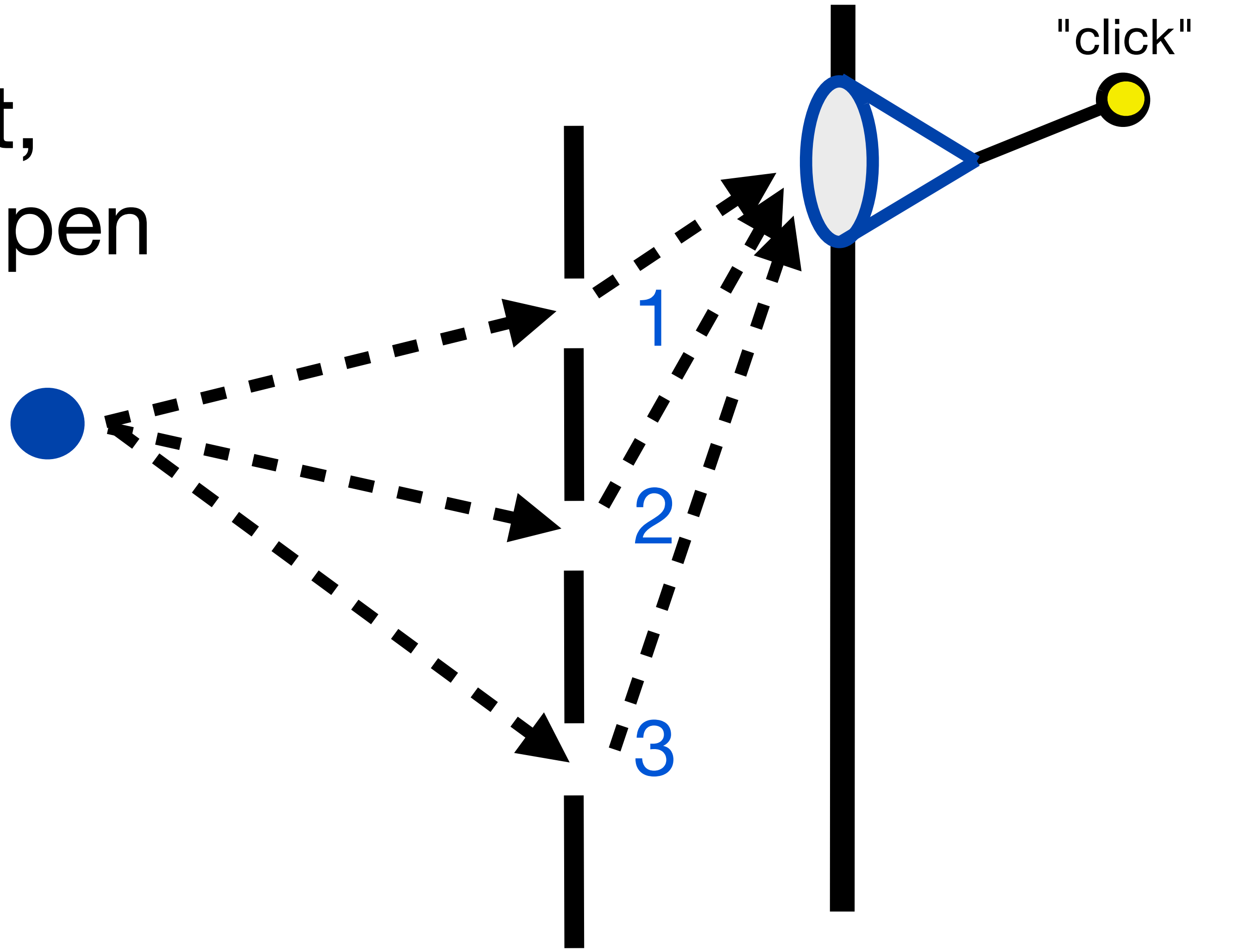

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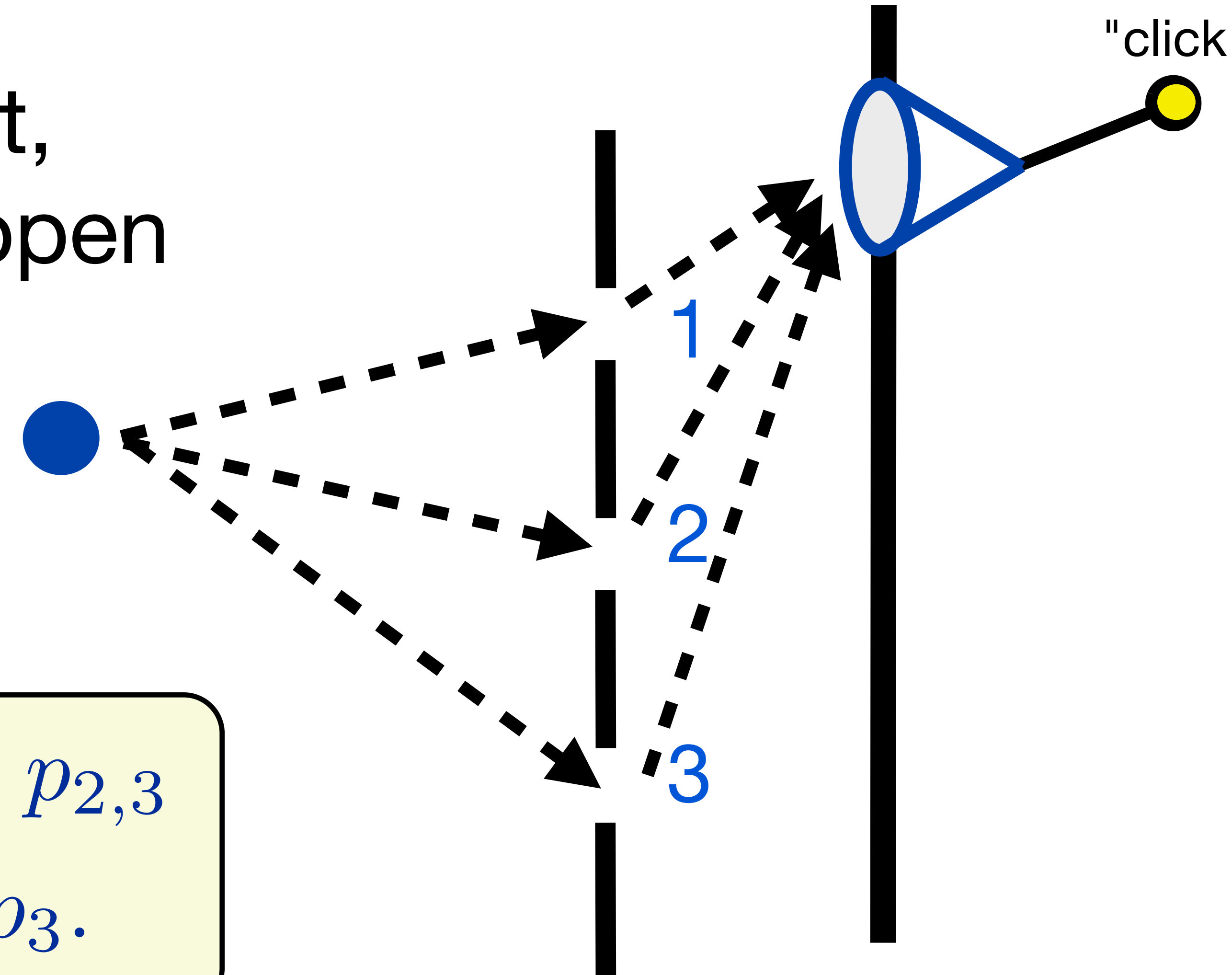

Surprisingly (?), quantum theory satisfies

$$p_{1,2,3} = p_{1,2} + p_{1,3} + p_{2,3} - p_1 - p_2 - p_3.$$

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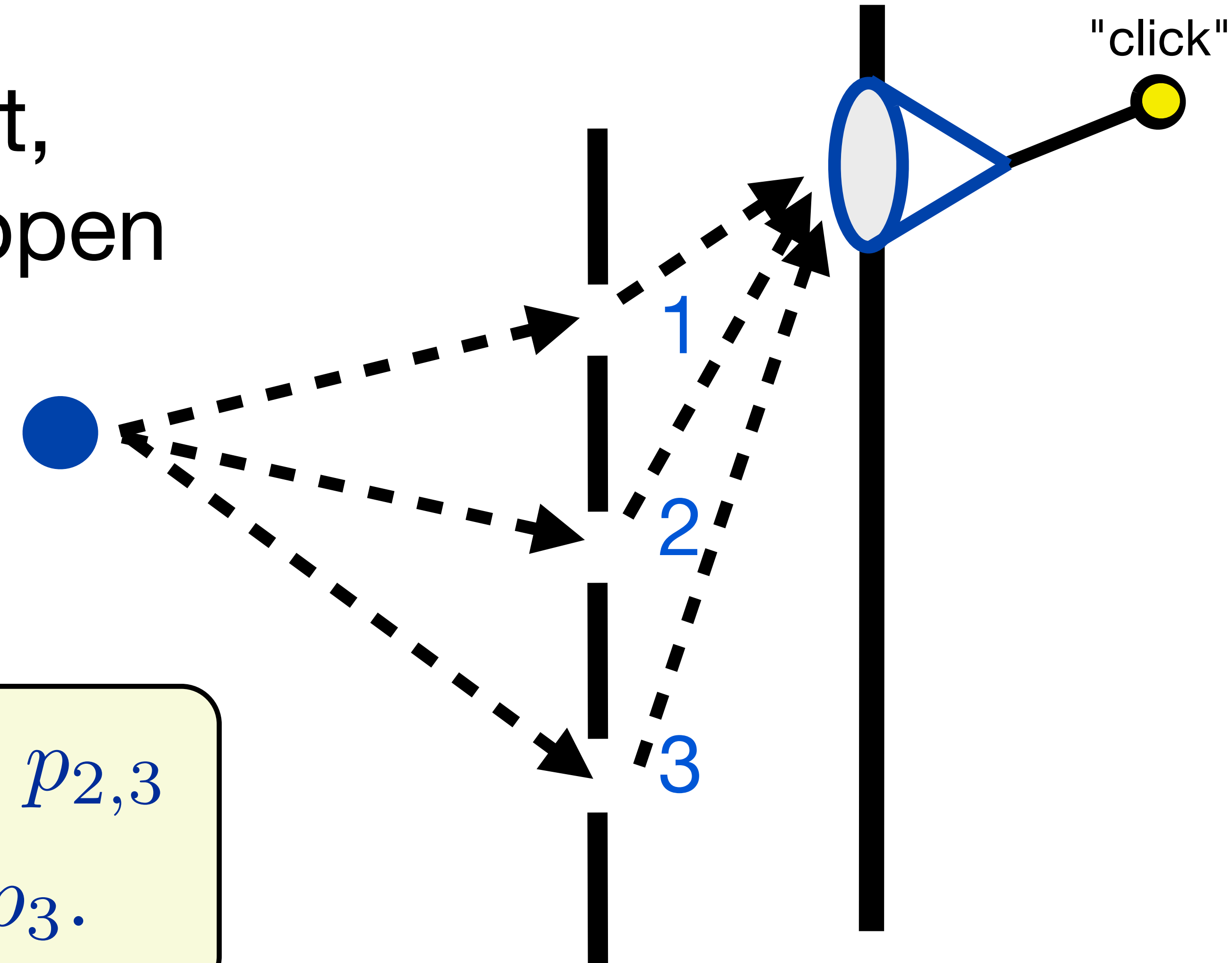

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No 3rd-order interference in QT!

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Sorkin:

$$I_2(A,B) \equiv |A \amalg B| - |A| - |B|$$

$$I_3(A,B,C) \equiv |A \amalg B \amalg C| - |A \amalg B| - |B \amalg C| - |A \amalg C| + |A| + |B| + |C$$

or in general,

$$\begin{aligned} I_n(A_1, A_2, \cdots, A_n) \equiv\ & |A_1 \amalg A_2 \amalg \cdots A_n| \\ & - \sum |(n-1)sets| + \sum |(n-2)sets| \cdots \\ & \pm \sum_{j=1}^{n} |A_j| \end{aligned}$$

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Classical probability theory: $I_2 = I_3 = I_4 = \ldots = 0.$

Quantum theory: $I_2 \neq 0, \ \ I_3 = I_4 = \ldots = 0.$

Science AAAS

Ruling Out Multi-Order Interference in Quantum Mechanics
Urbasi Sinha *et al.*
Science **329**, 418 (2010);
DOI: 10.1126/science.1190545
(U. Sinha, C. Couteau, T. Jennewein, R. Laflamme, G. Weihs)

$$\varepsilon = I_3 - \text{zerocount};$$

$$\kappa := \frac{\varepsilon}{\delta};$$

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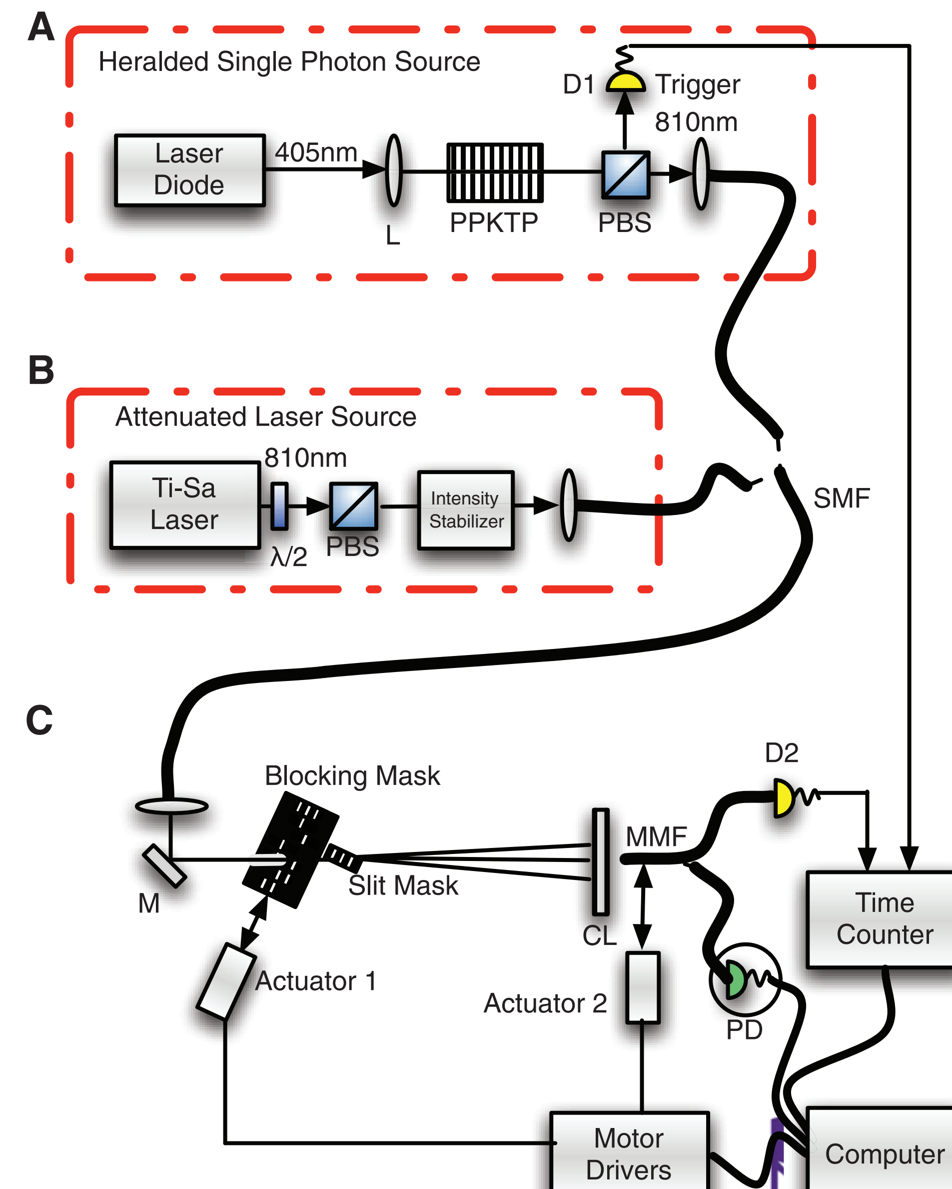

Experimental tests for higher-order interference

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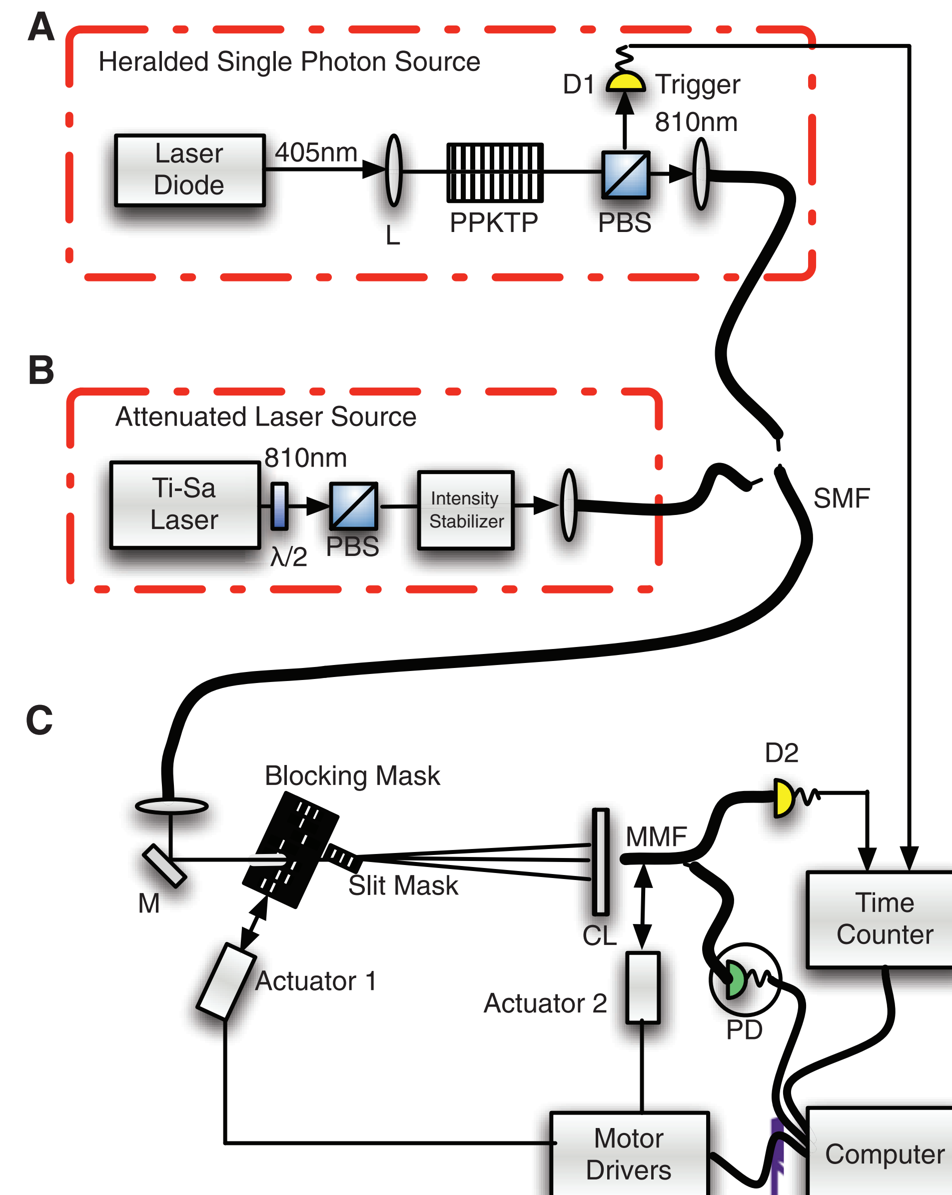
Result: $\kappa \leq 10^{-2}$.



Why does QT not have 3rd-order interference?

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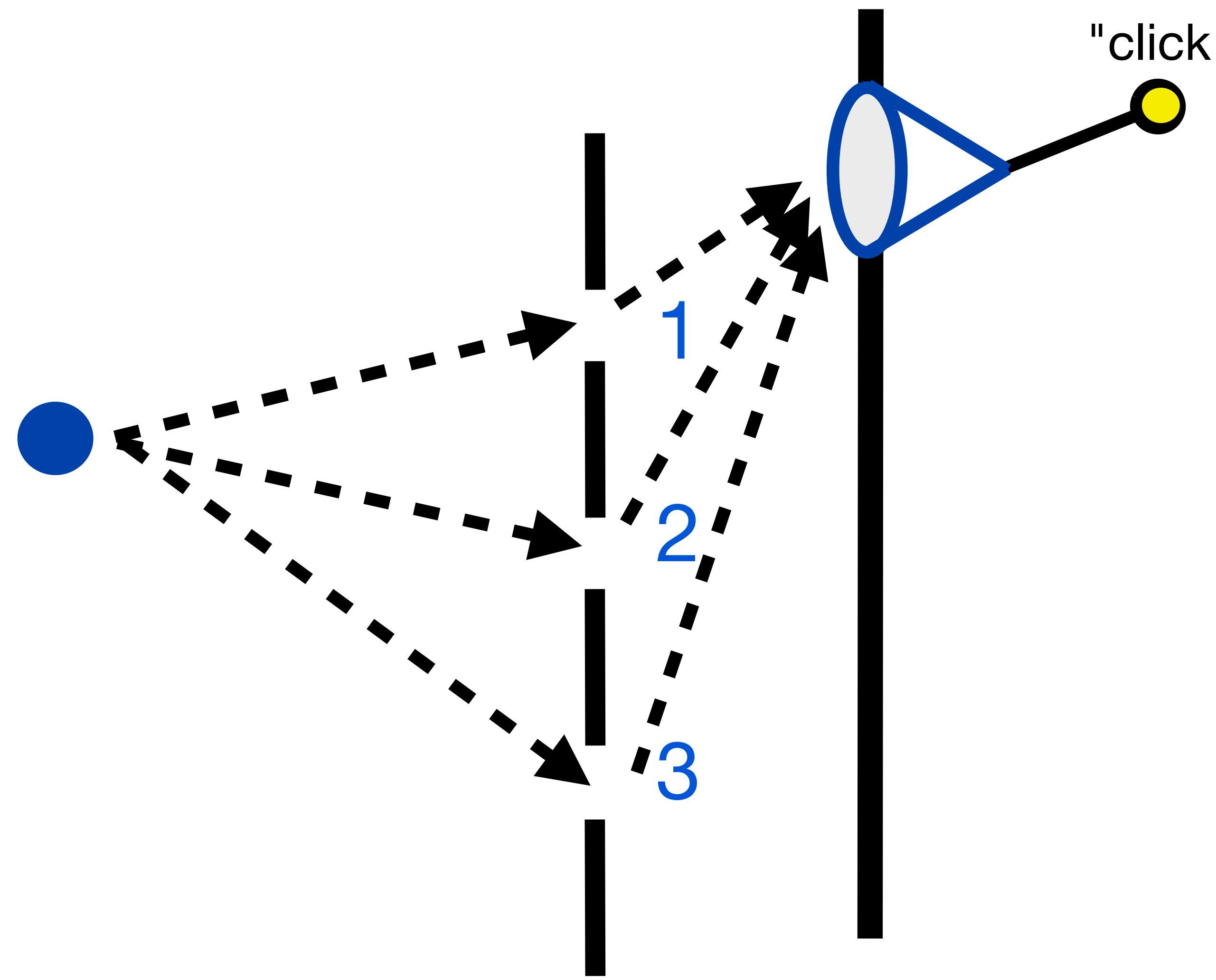

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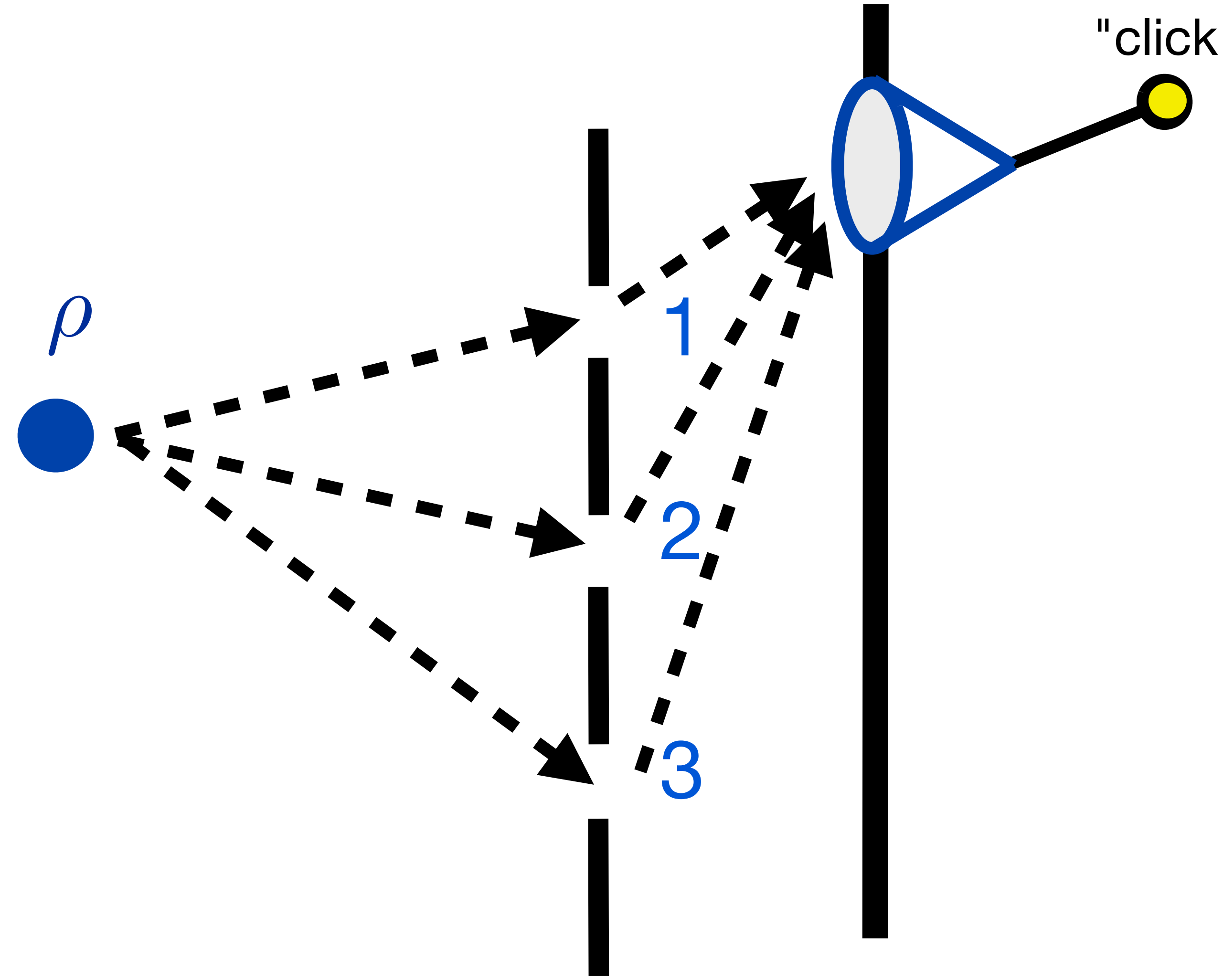

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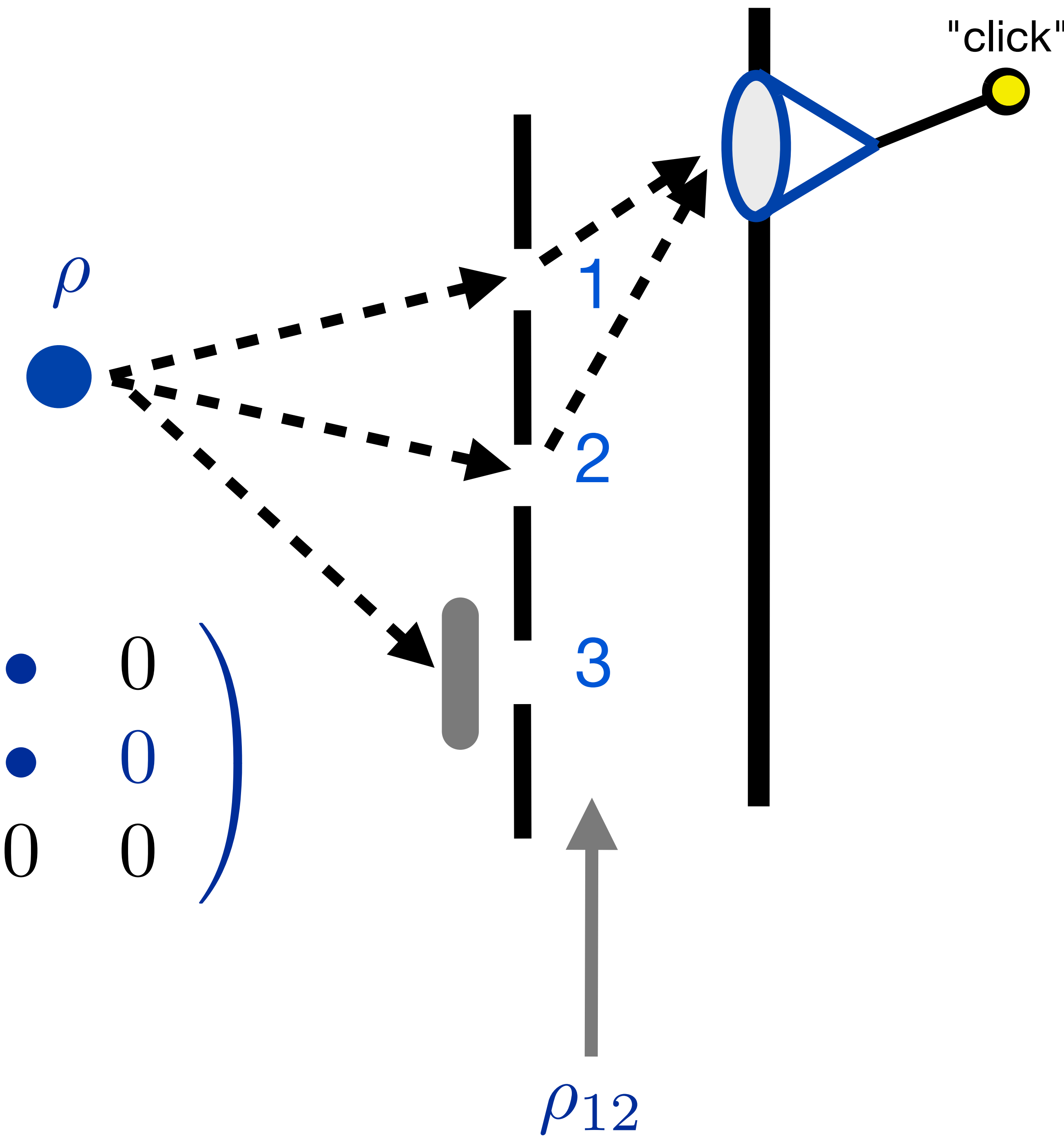

$$\rho \mapsto P_{12}\rho P_{12} =: \rho_{12} = \begin{pmatrix} \bullet & \bullet & 0 \\ \bullet & \bullet & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Why does QT not have 3rd-order interference?

$$\begin{pmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{pmatrix} = \begin{pmatrix} \bullet & \bullet & 0 \\ \bullet & \bullet & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} \bullet & 0 & \bullet \\ 0 & 0 & 0 \\ \bullet & 0 & \bullet \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & \bullet & \bullet \\ 0 & \bullet & \bullet \end{pmatrix}$$

$$- \begin{pmatrix} \bullet & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & \bullet & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \bullet \end{pmatrix}$$

$$\begin{aligned} p_{1,2,3} &= p_{1,2} + p_{1,3} + p_{2,3} \\ &\quad -p_1 - p_2 - p_3. \end{aligned}$$

Why does CPT not have 2nd-order interference?

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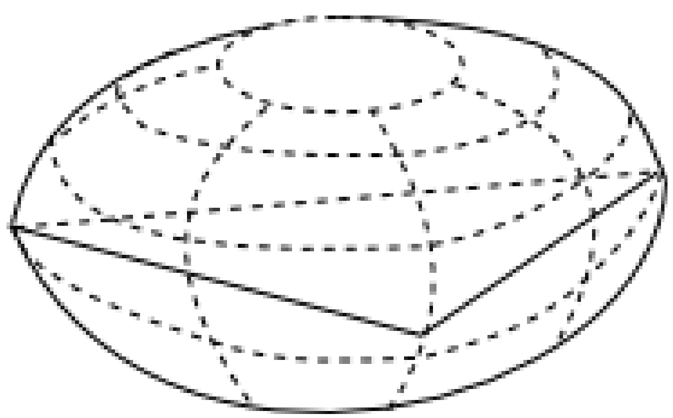
$$\begin{pmatrix} \bullet \\ \bullet \\ \bullet \end{pmatrix} = \begin{pmatrix} \bullet \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \bullet \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \bullet \end{pmatrix}$$

$$p_{1,2,3} = p_1 + p_2 + p_3.$$

Which natural GPTs have 3rd-order interference?

Some "artificial" GPTs exhibit order-3 interference:



C. Ududec, *Perspectives on the Formalism of Quantum Theory*, PhD thesis, University of Waterloo, 2012.

But what natural generalizations of QT could we test for in experiments?

Which natural GPTs have 3rd-order interference?

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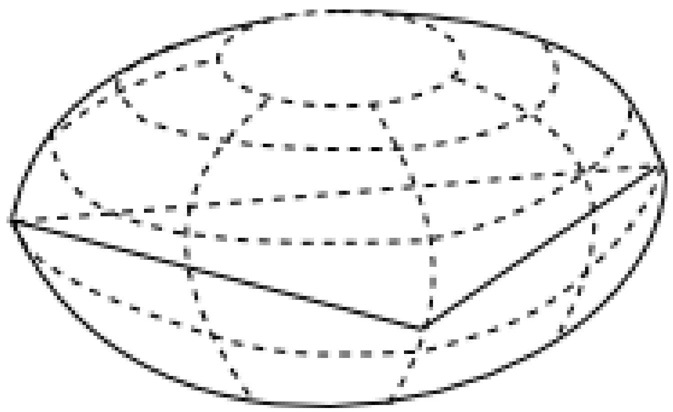

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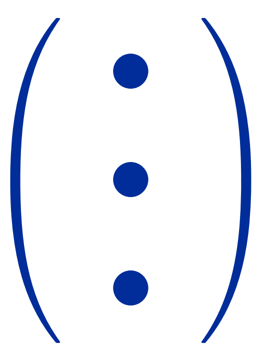
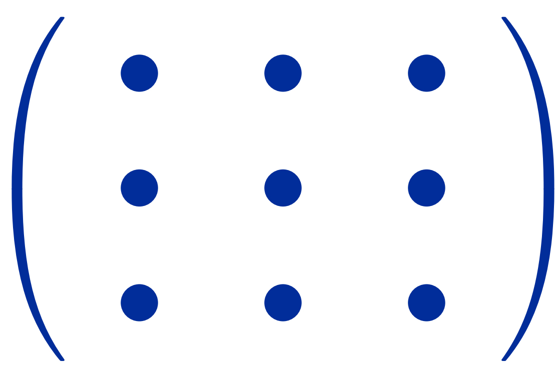
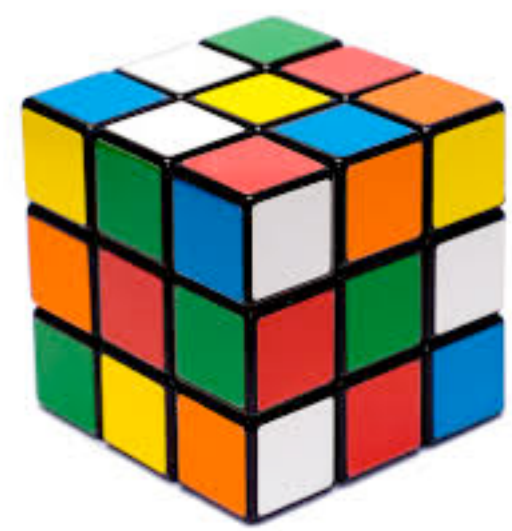


"1st-order" (trivial) interference



2nd-order interference



3rd-order interference?

A **single-system** reconstruction of QT

H. Barnum, **MM**, and C. Ududec, *Higher-order interference and single-system postulates characterizing quantum theory*, New J. Phys. **16**, 123029 (2014).

Theorem:

1. Classical decomposability ✓
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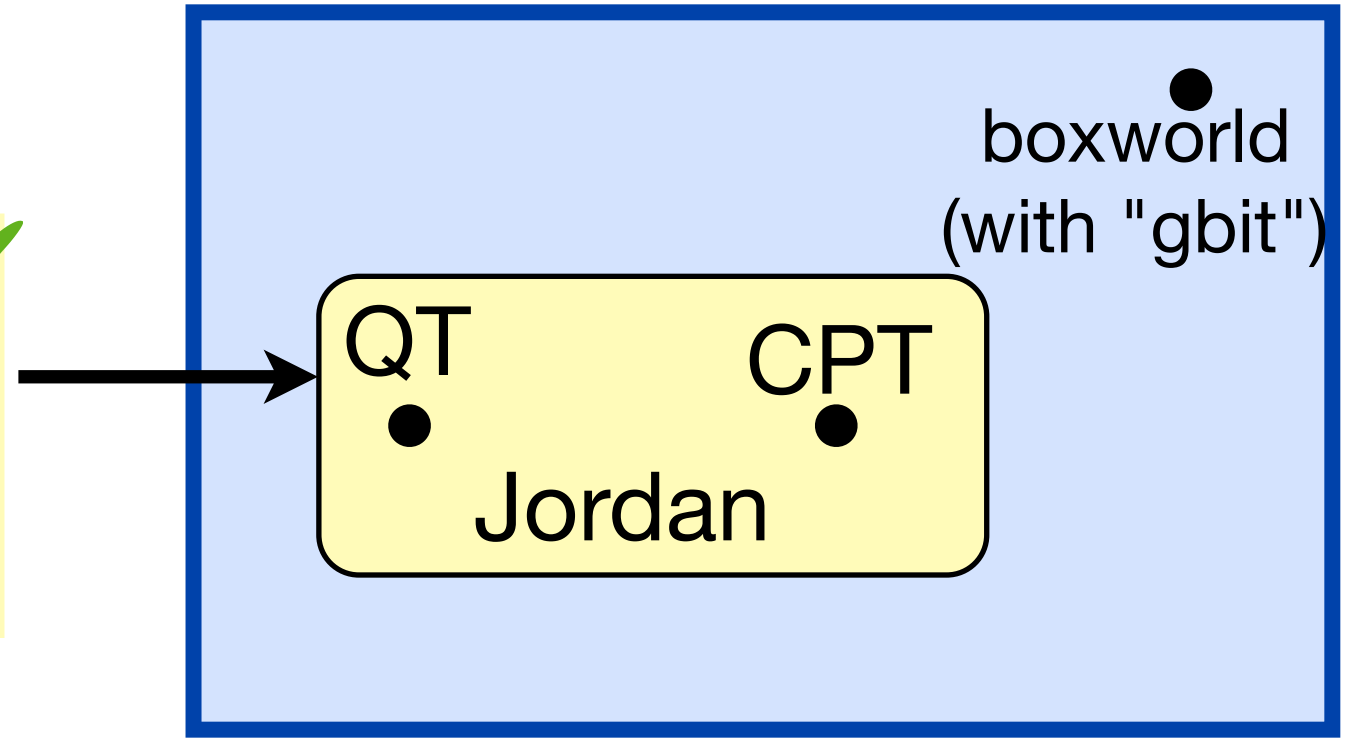

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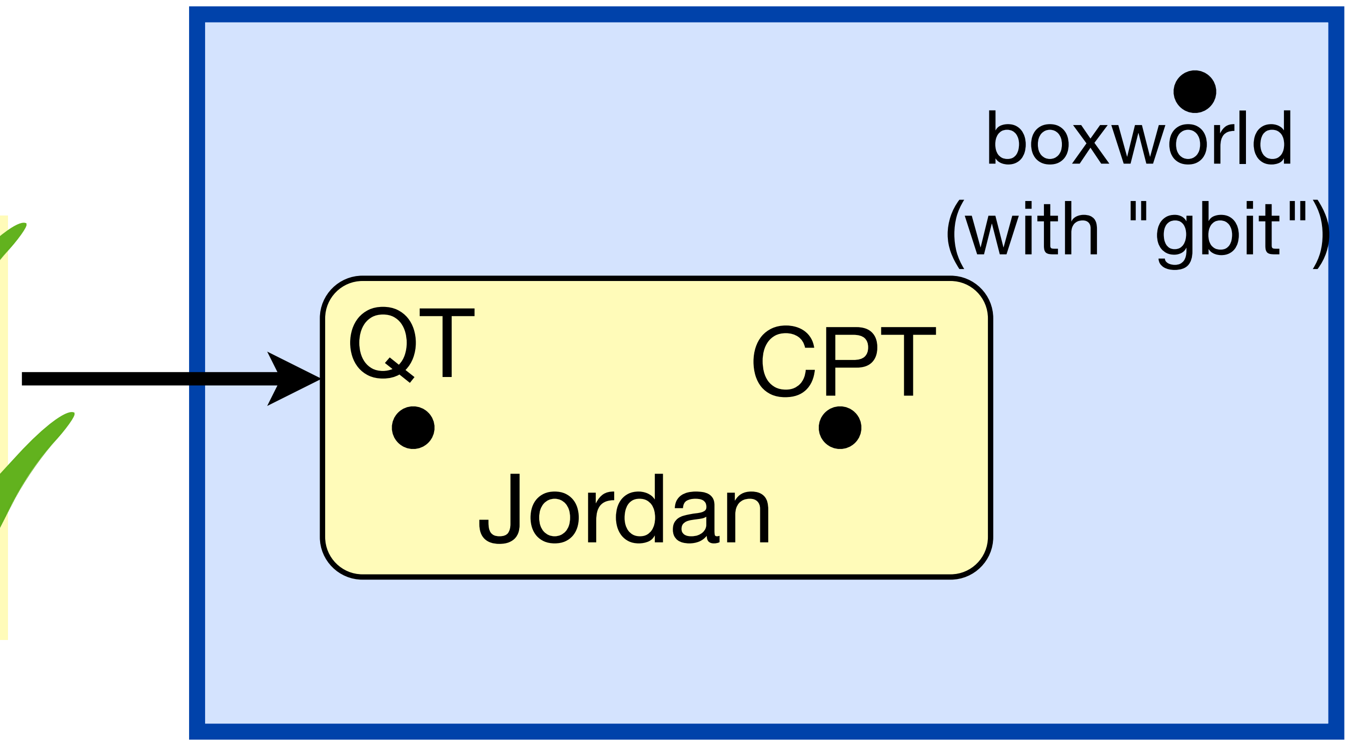

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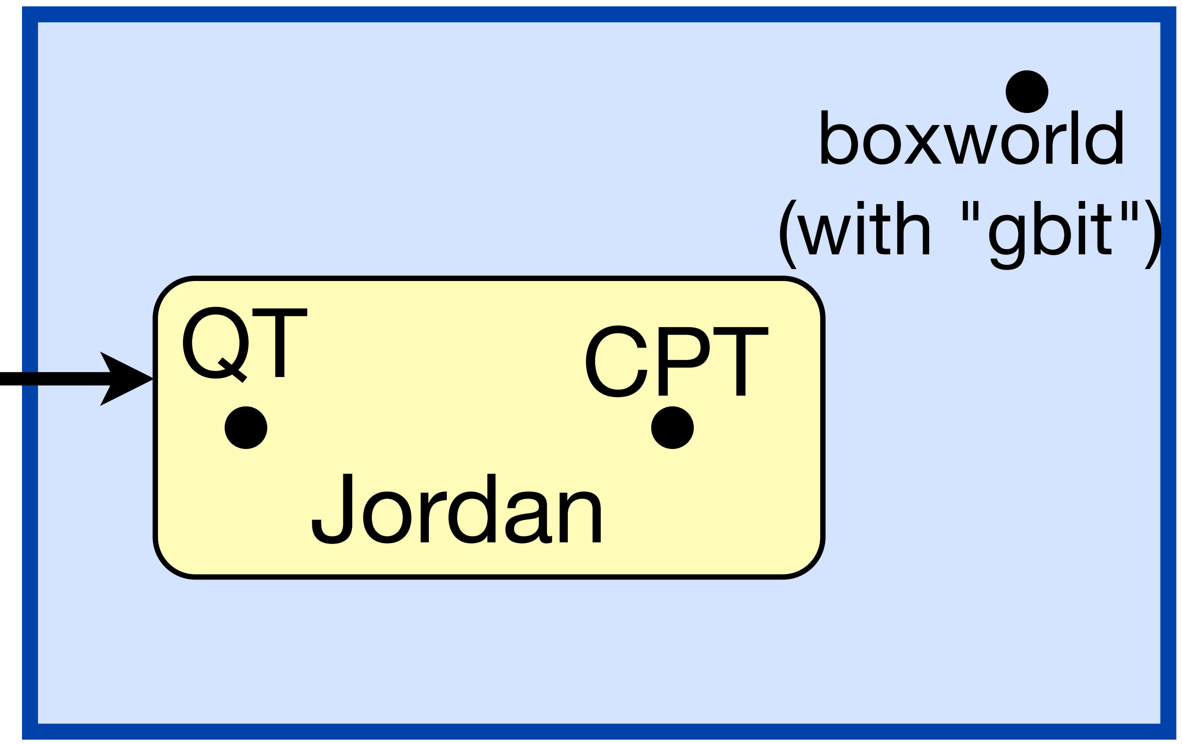

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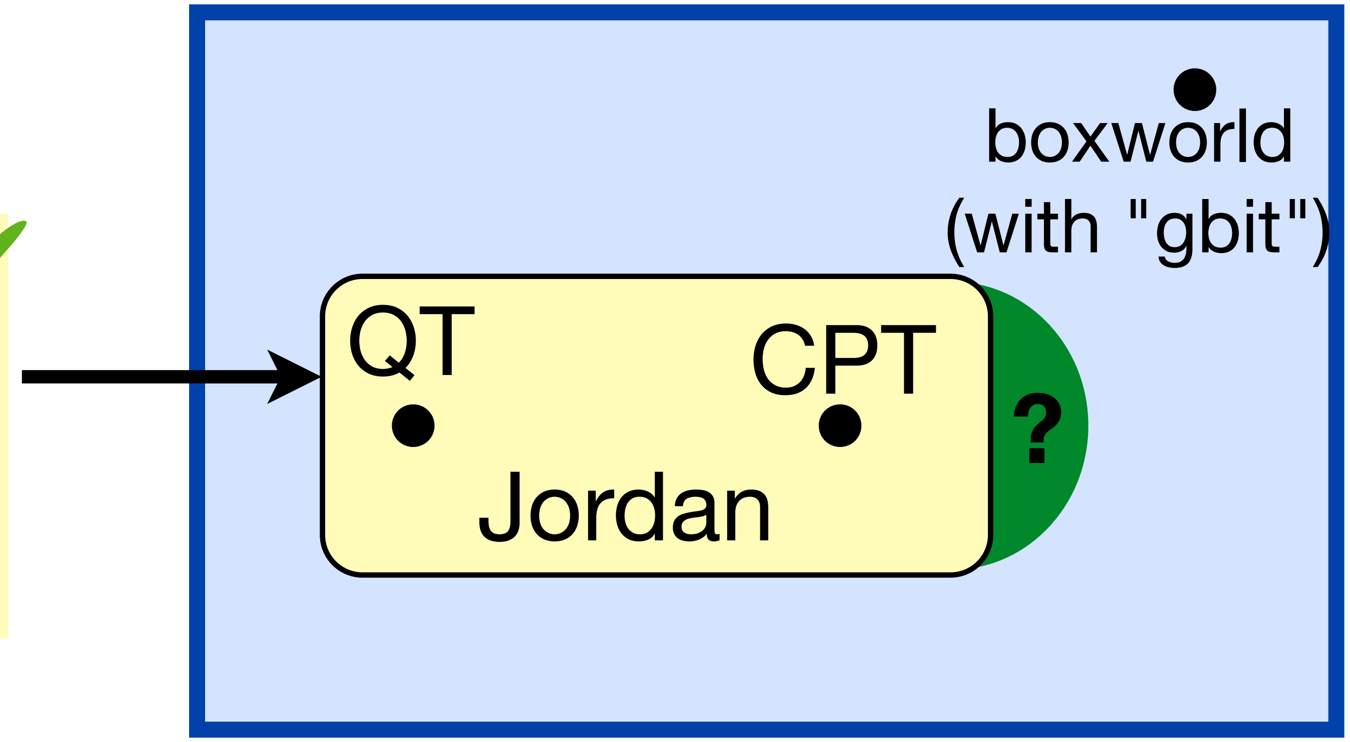

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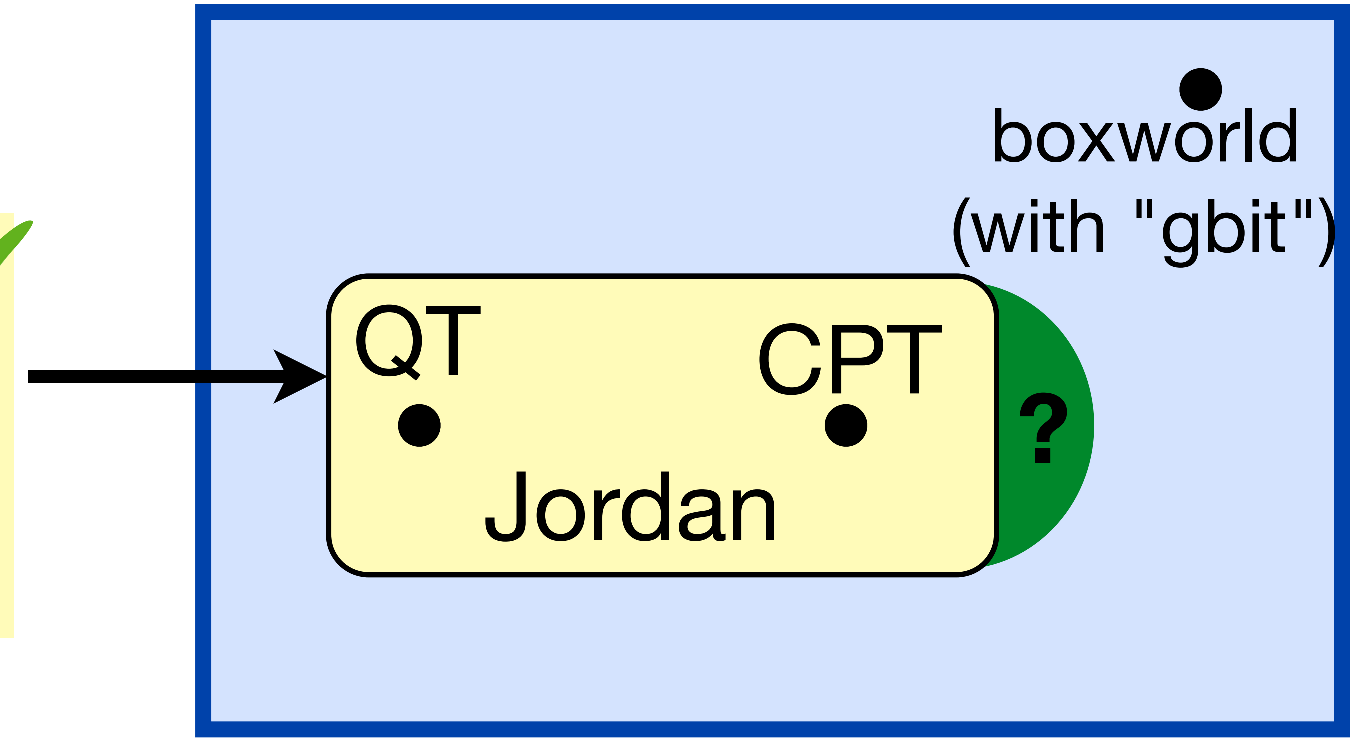

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If so, these are natural **models for higher-order interference!**

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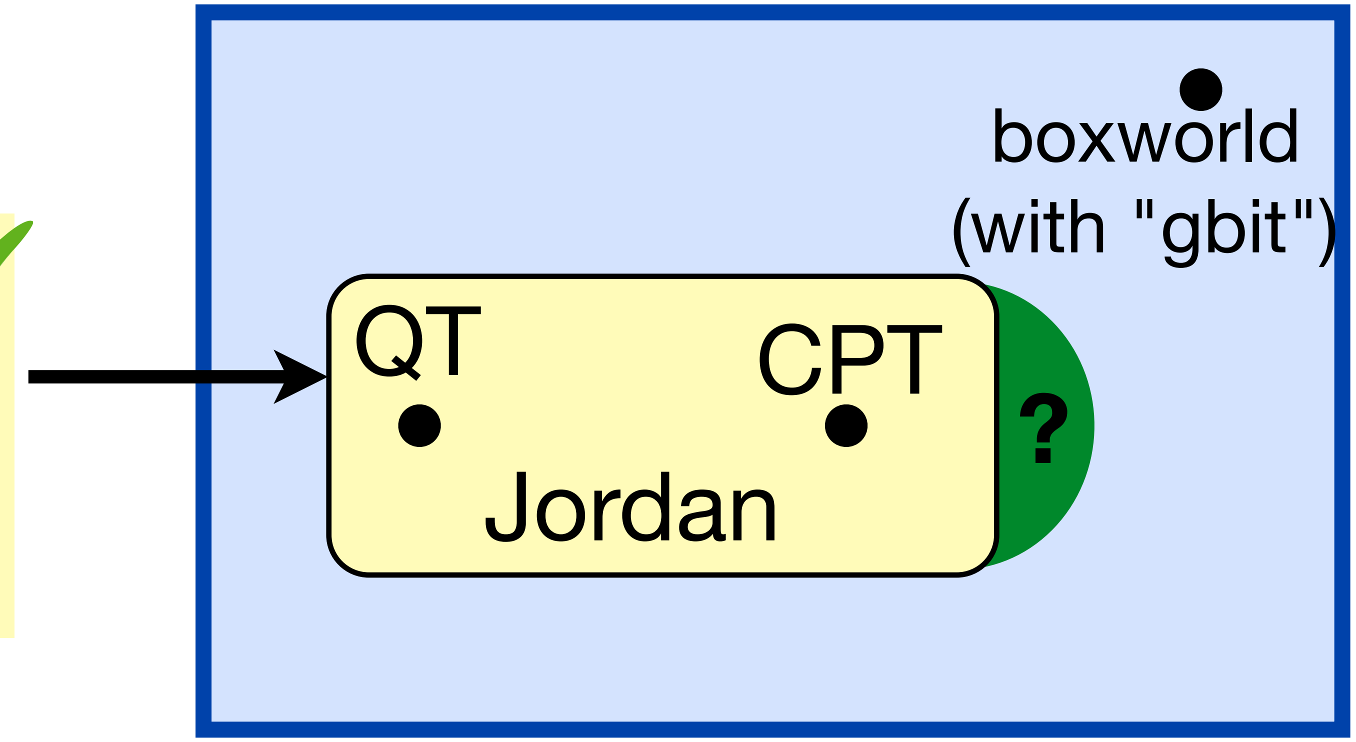

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OPEN QUESTION!

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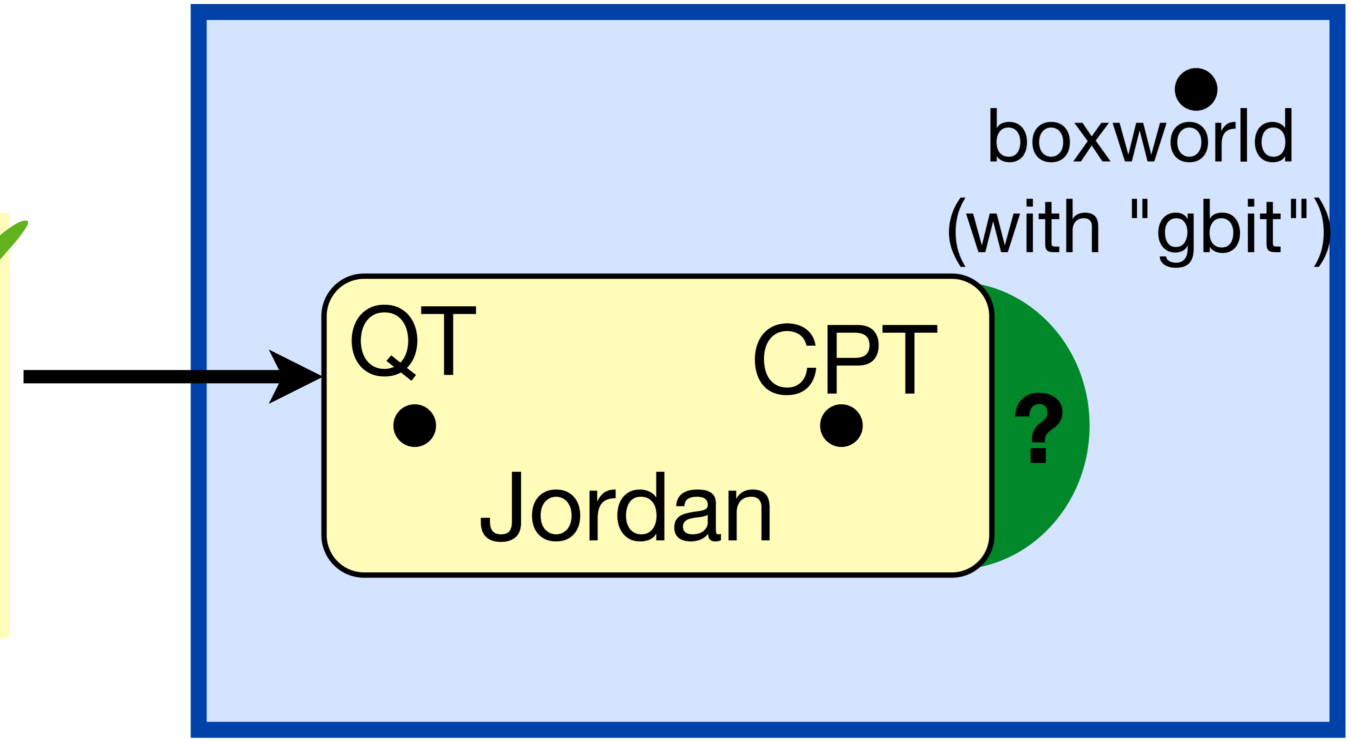

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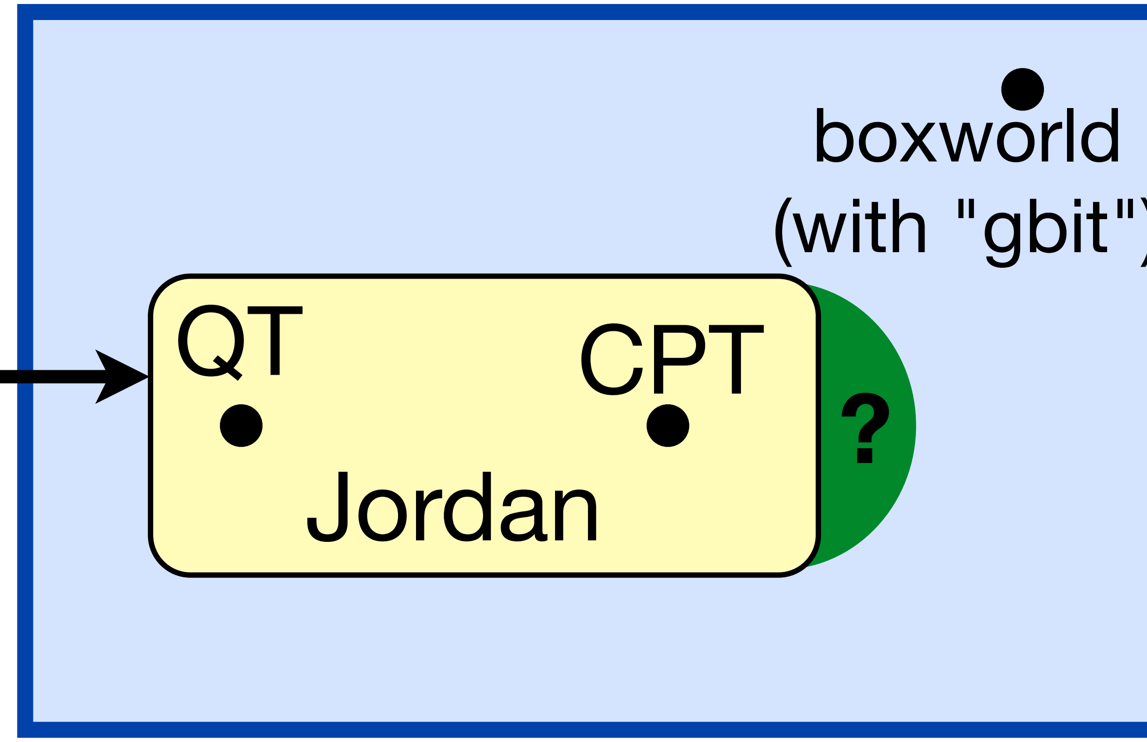

We know that 1+2 alone imply many things quantum:

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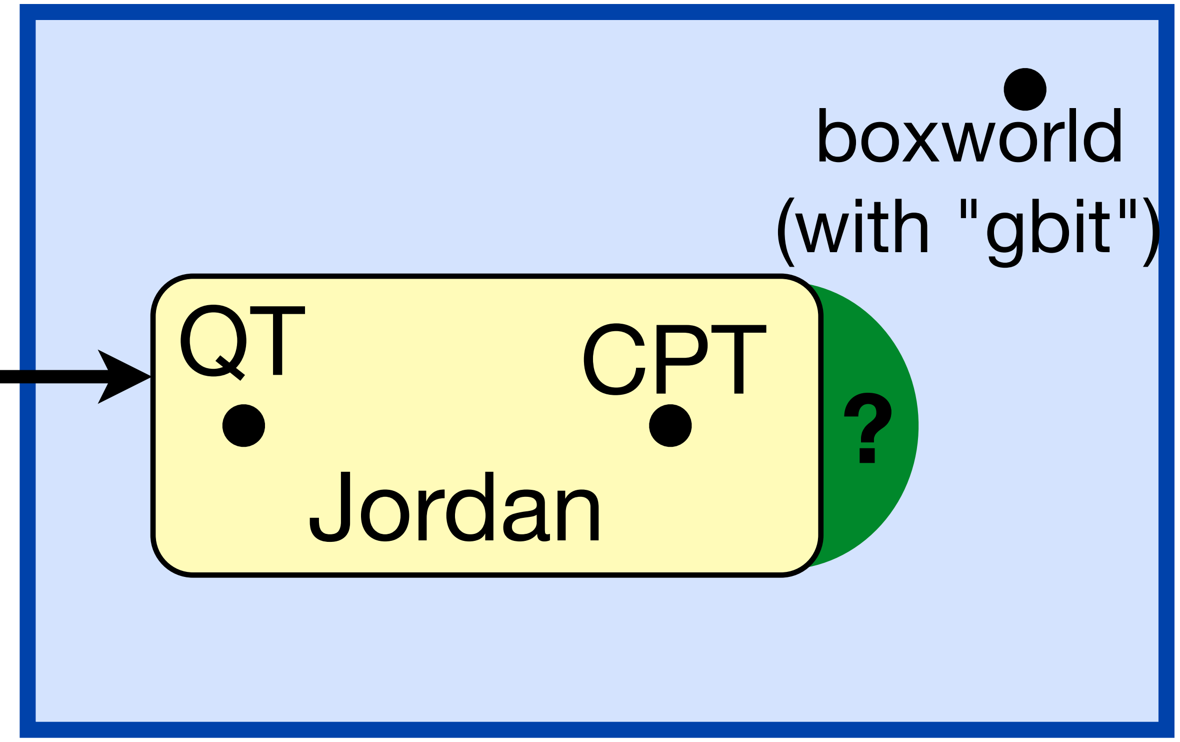

We know that 1+2 alone imply many things quantum:

- Analogues of **orthogonal projectors, eigenvalues, and eigenspaces**,
- their face lattice is an **orthomodular lattice** (→ quantum logic),
- they satisfy **Specker's Principle** (contextuality),
- all bit subsystems are **Bloch balls**,
- their state cones are **strongly self-dual**.

A **single-system** reconstruction of QT

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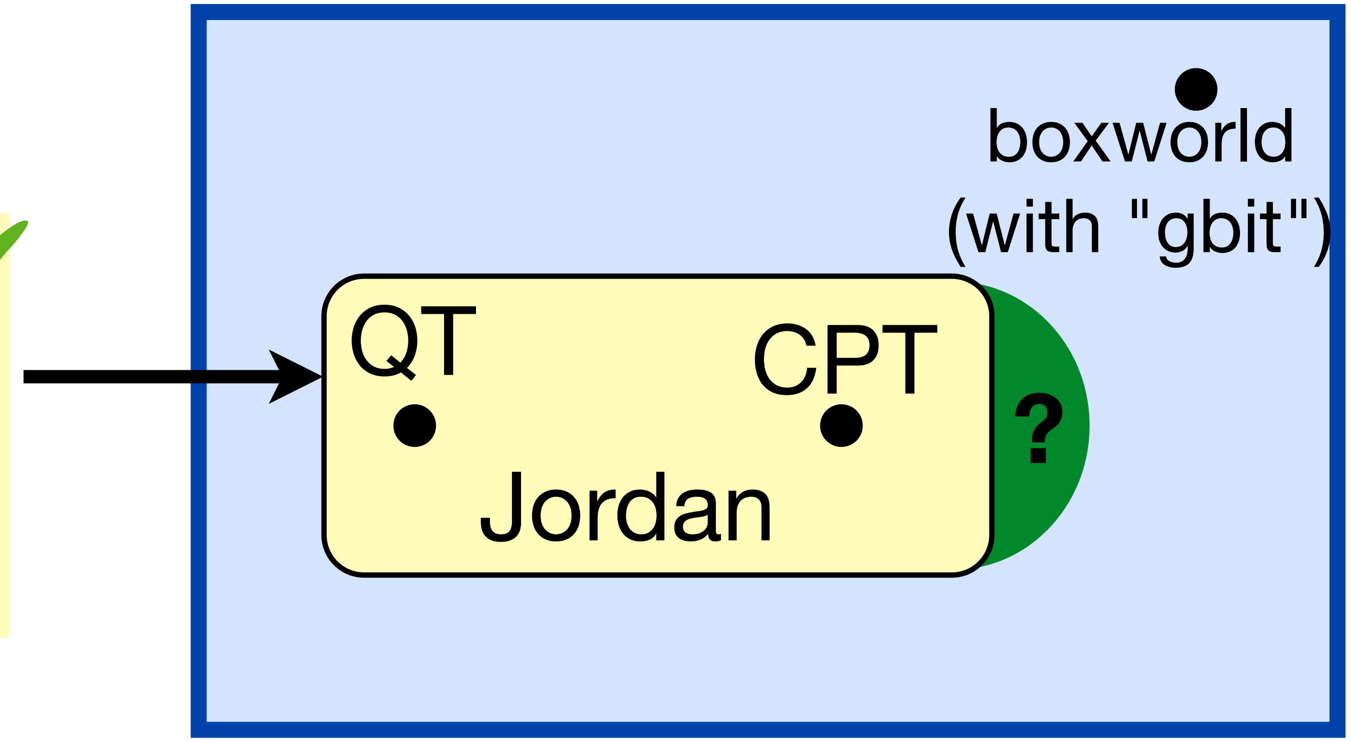

On the other hand, the new solutions violate some things quantum:

- They admit **higher-order interference**,
- the **covering law** of quantum logic is violated,
- the image of a pure state under a projection can be **mixed**,
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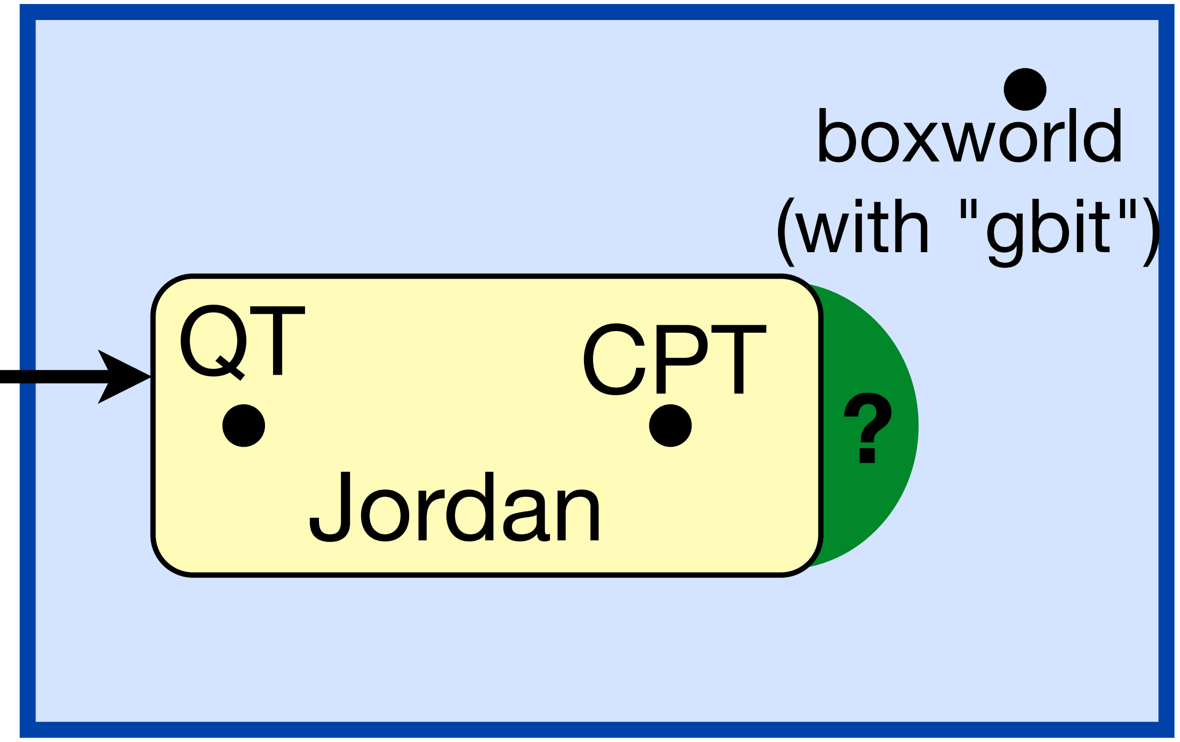

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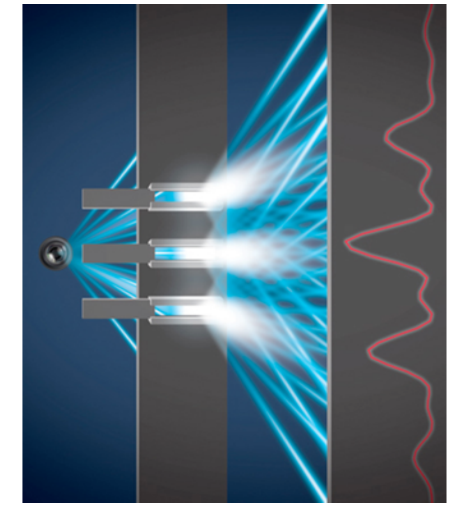
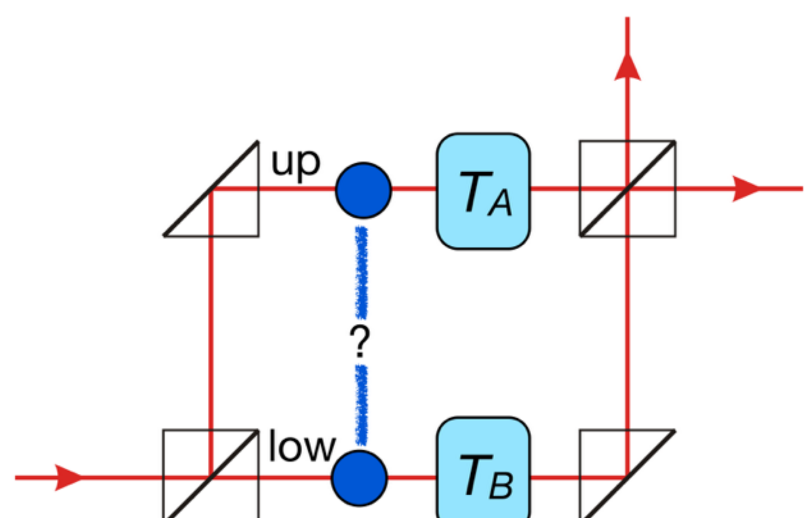
FIND AT LEAST ONE EXAMPLE!

- Quantum theory from principles



"Why does the qubit have 3 degrees of freedom?"

- Take 1: continuous-reversible interaction
- Take 2: relativity of simultaneity on interferometer



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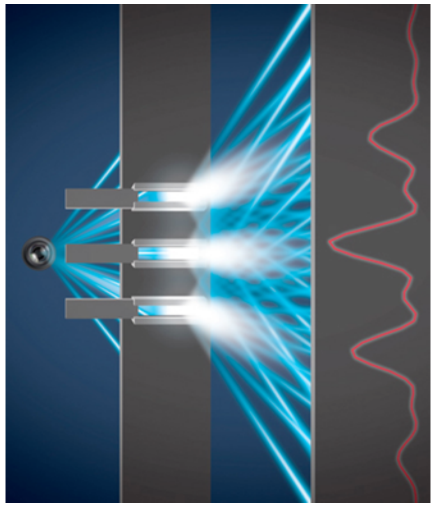

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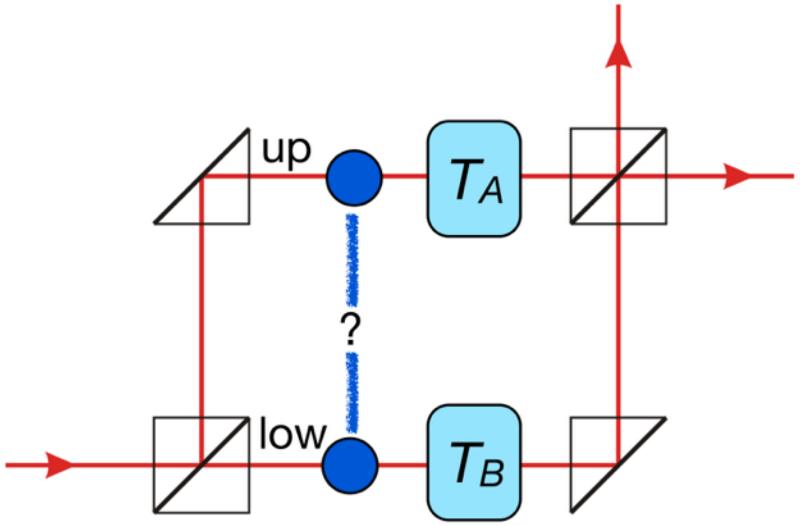
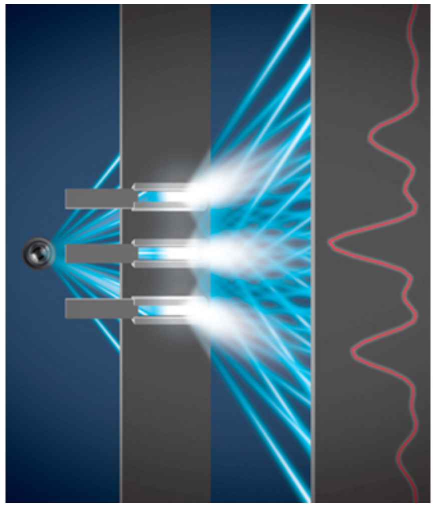


Outline



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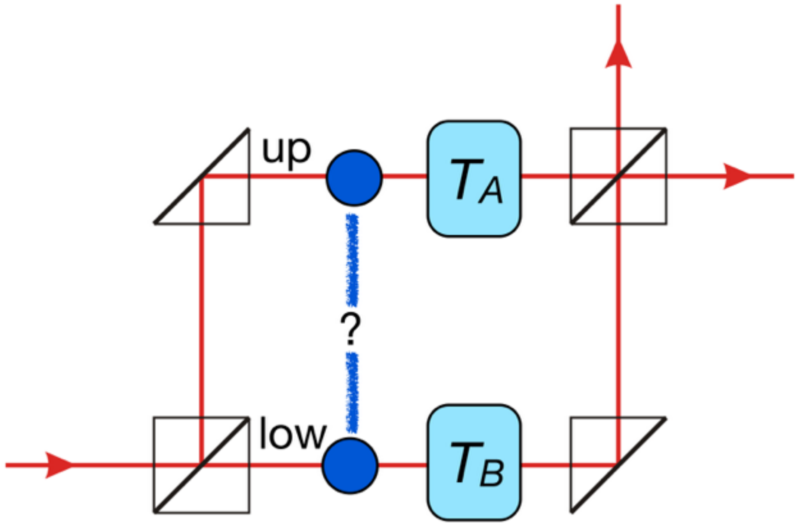

3D of the Bloch ball: continuous interaction

1. Classical decomposability
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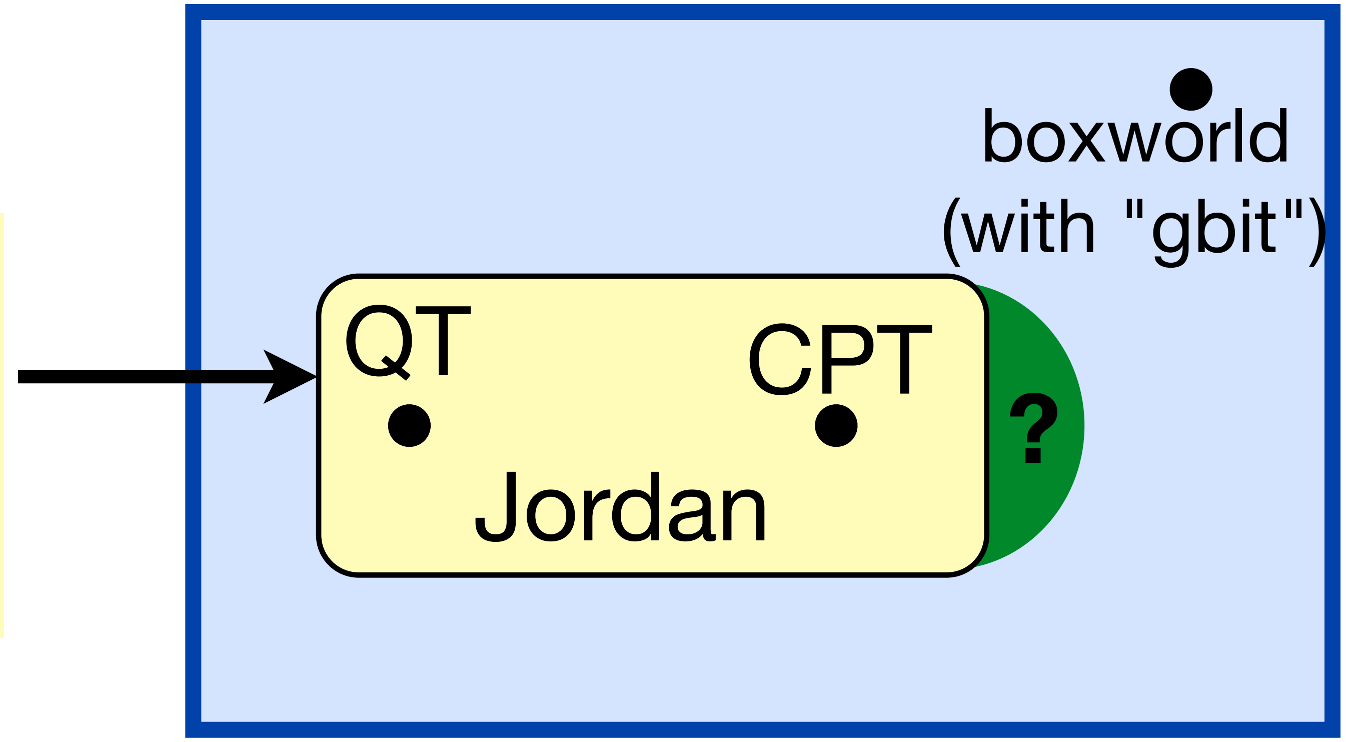

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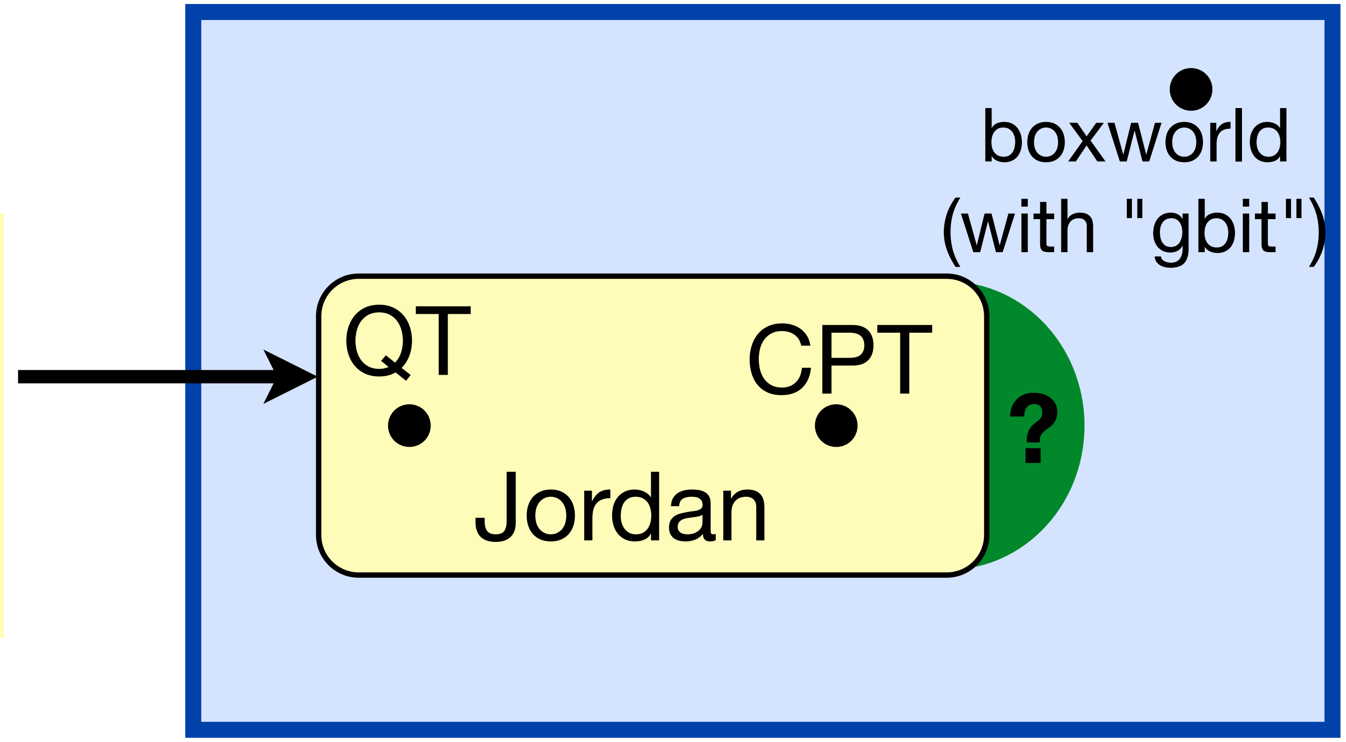
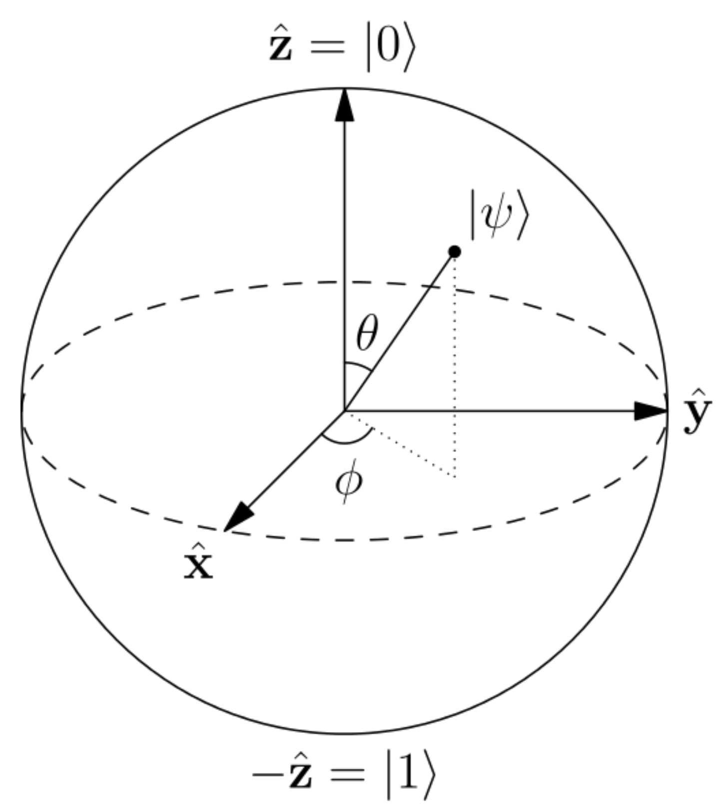


The quantum bit **Bloch ball** satisfies these postulates:

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$



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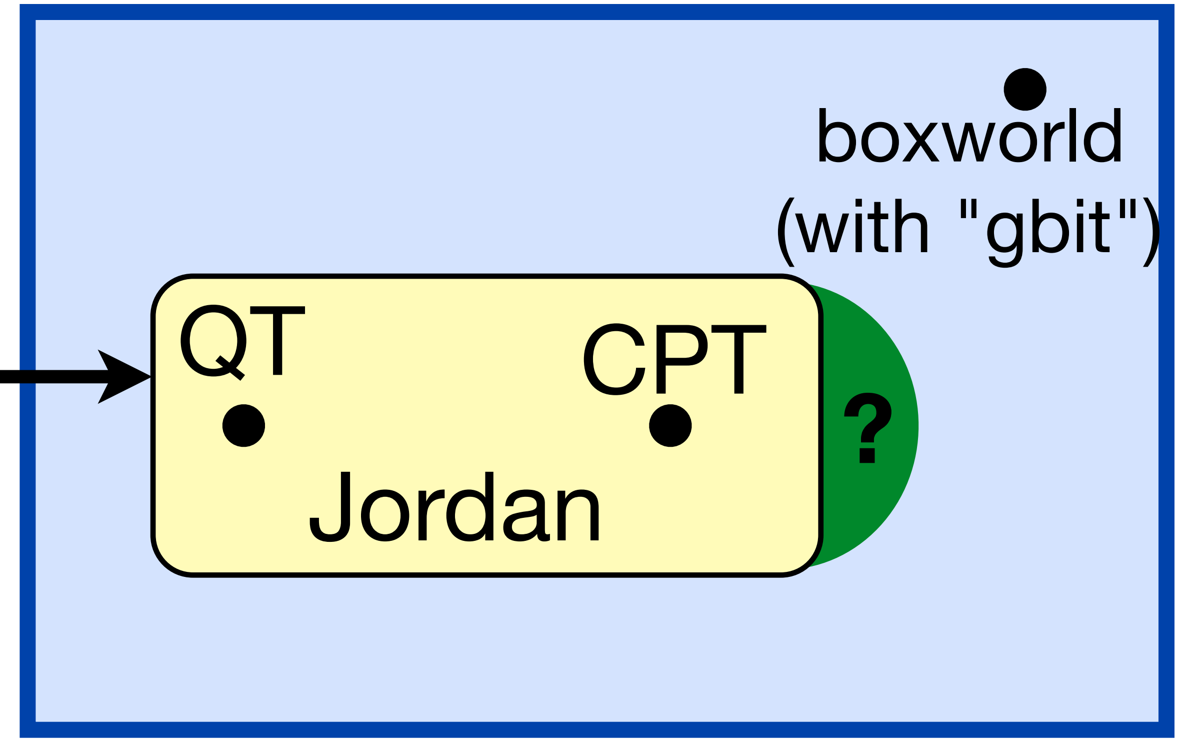
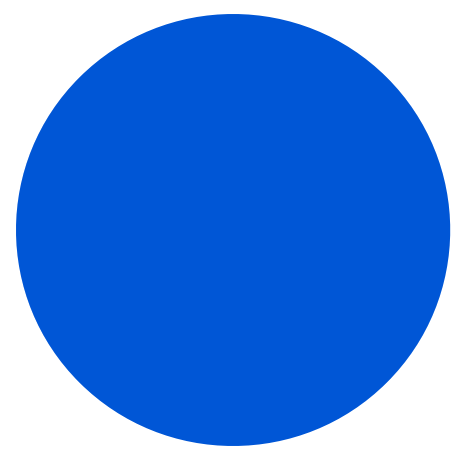


However,
Bloch ball bits of
arbitrary dimension
satisfy 1+2.

$d = 1$

classical
bit



$d = 2$

$d = 3$

quantum
bit

...

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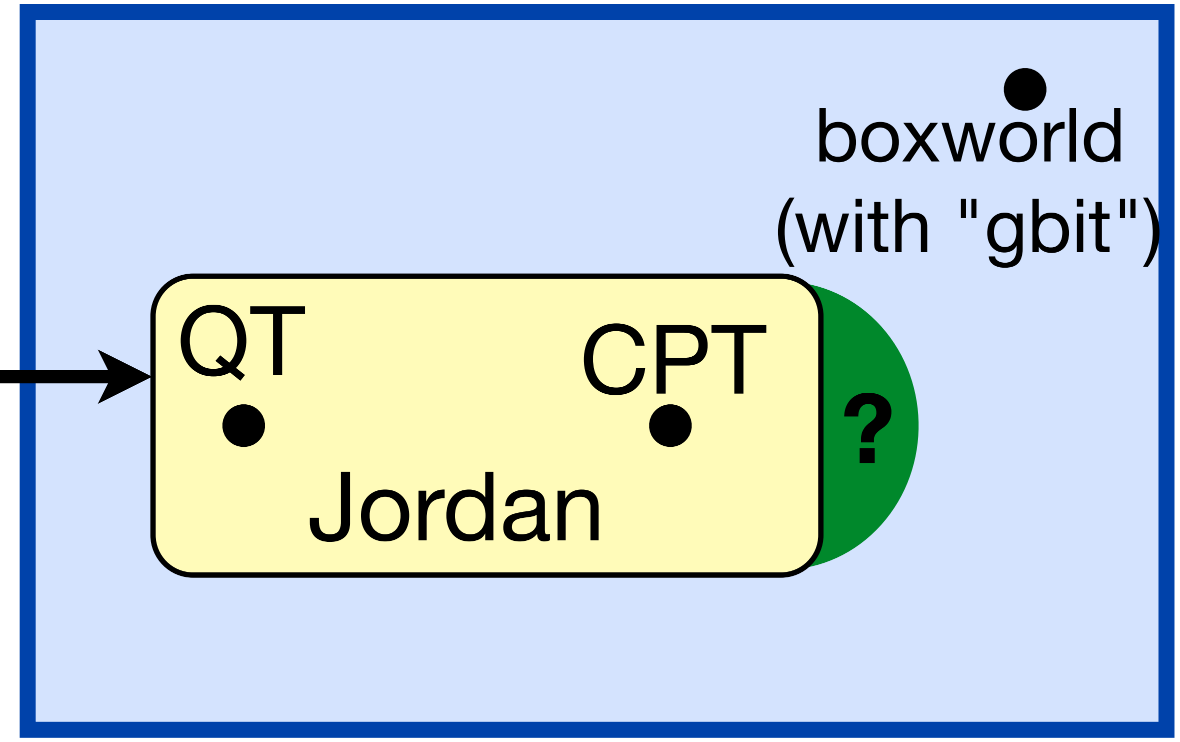

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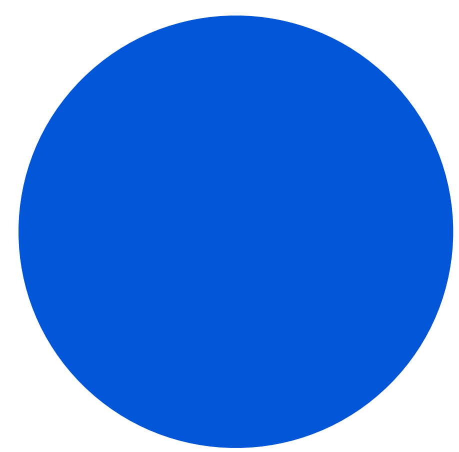

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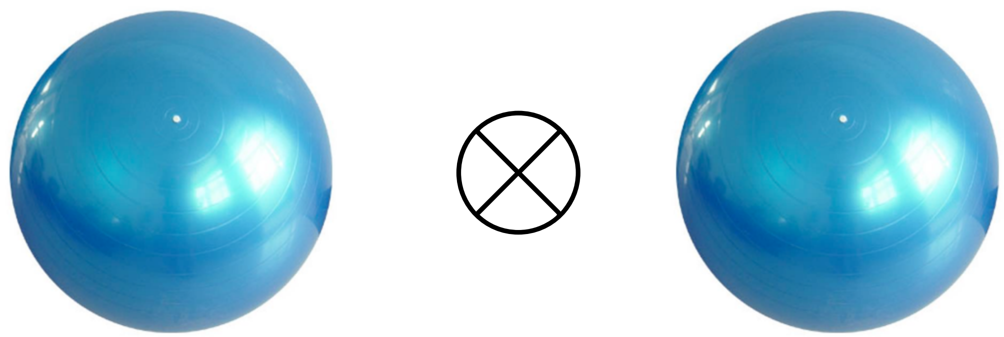
quantum bit

...

3D of the Bloch ball: continuous interaction

Suppose we want to combine two d-dim. Ball state spaces



A **B**

into a composite state space **AB**.

3D of the Bloch ball: continuous interaction

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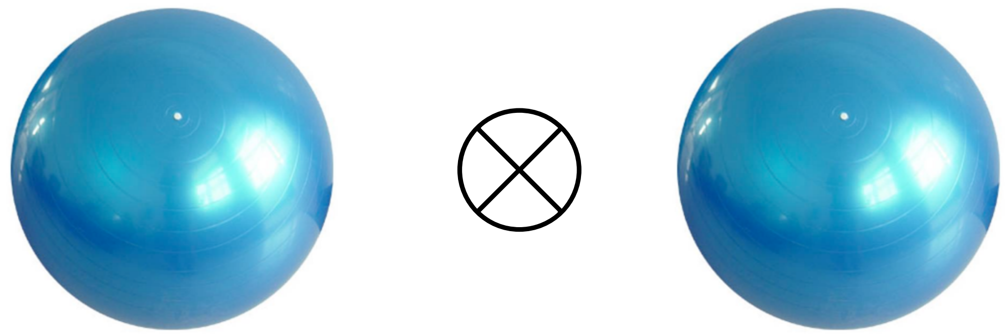

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into a composite state space **AB**, according to:

- **No-signalling**;
- **local tomography**: joint states are uniquely determined by the statistics and correlations of local measurements;
- AB contains all product states ("**independent preparations**"), product transformations, and product measurements.

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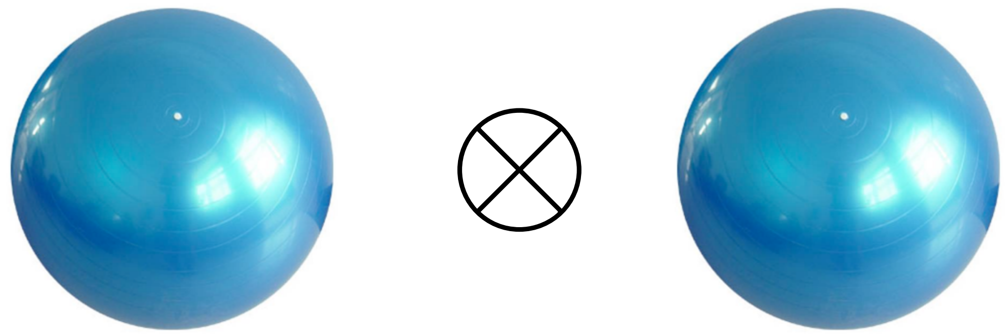

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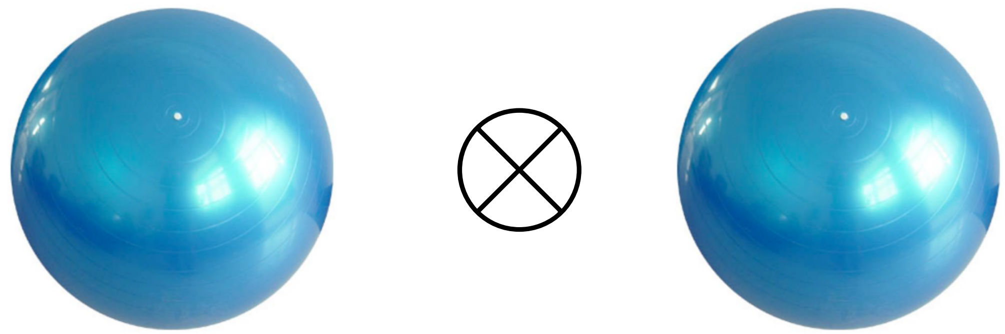
Then, for **any** $d \geq 2$, there are **infinitely many** possibilities!

3D of the Bloch ball: continuous interaction

Ll. Masanes, **MM**, D. Pérez-García, and R. Augusiak, *Entanglement and the three-dimensionality of the Bloch ball*, J. Math. Phys. **55**, 122203 (2014).



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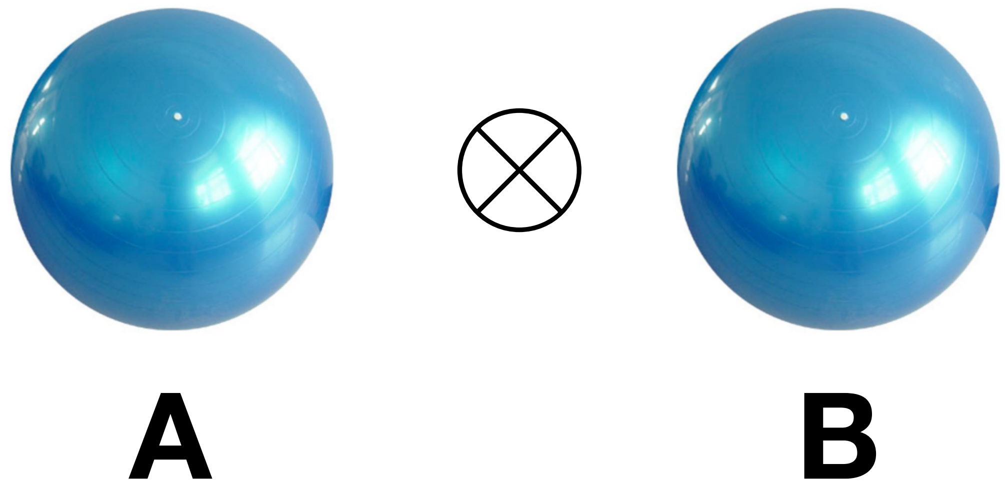

Theorem: Assume in addition:

There exists at least one continuous reversible transformation $T_{AB} \neq T_A \otimes T_B$ *("interaction").*

Then **only *d*=3** is possible, and only **one possible composite**, namely the quantum state space of **two qubits**.

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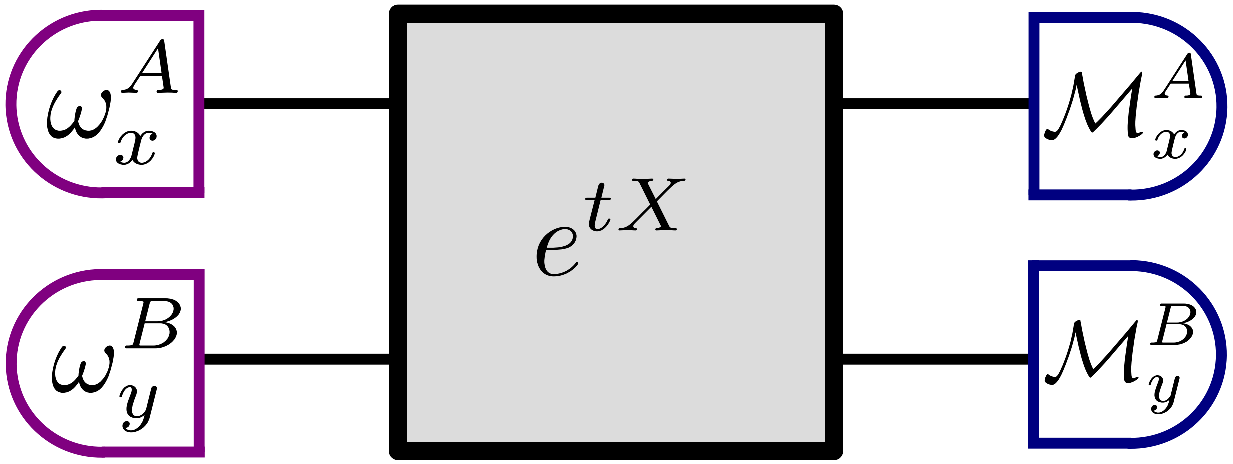

Product preparation; evolution for short time *t*; product measurement

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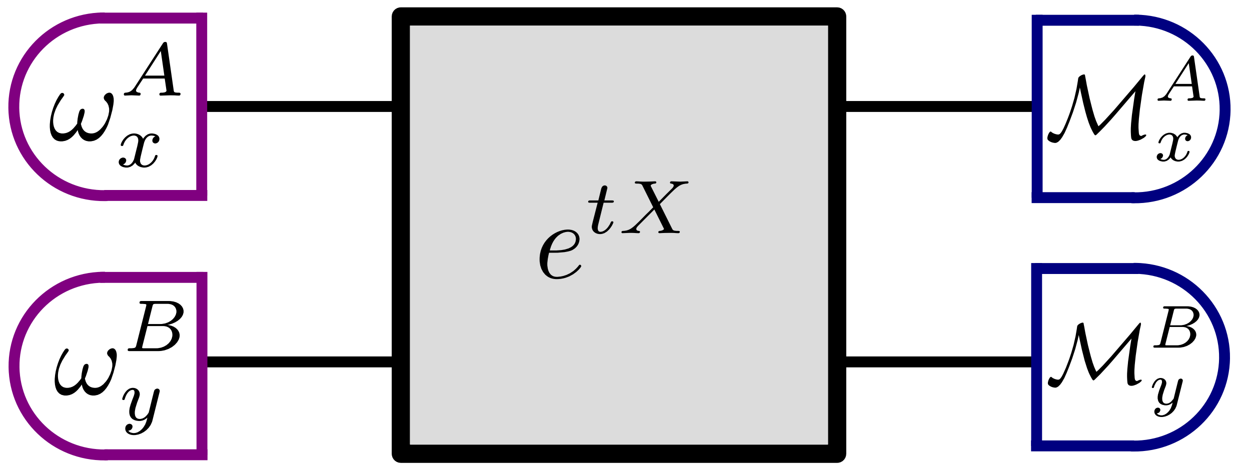

Product preparation; evolution for short time *t*; product measurement

$$\text{If } \mathcal{M}_x^A(\omega_x^A) = \mathcal{M}_y^B(\omega_y^B) = 1 \text{ then}$$

$$\frac{d}{dt}(\mathcal{M}_x^A \otimes \mathcal{M}_y^B)e^{tX}(\omega_x^A \otimes \omega_y^B) = 0.$$

(probabilities not larger than 1)

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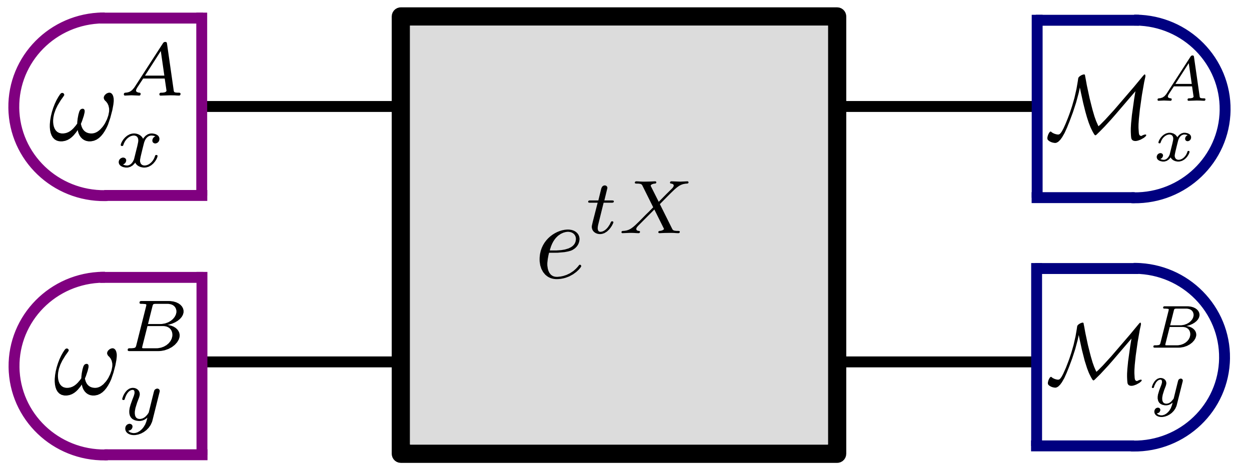

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$\Rightarrow$ Constraints on X.
If $d \neq 3$ then only $X = X^A + X^B$ possible
$\Rightarrow$ no interaction.

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For a while, we thought that there is an **additional 7-dimensional solution**, with Lie group G_2 acting locally...

... but in the end we showed that this is not the case, unfortunately.

$$W' = W - \int_{\mathcal{H}} dA\, (\hat{A}\otimes\hat{\mathbf{1}})W(\hat{A}\otimes\hat{\mathbf{1}})^{-1} - \int_{\mathcal{H}} dB\, (\hat{\mathbf{1}}\otimes\hat{B})W(\hat{\mathbf{1}}\otimes\hat{B})^{-1}$$

$$= \begin{bmatrix} 0 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \sum_i \mathbf{e}_i^{\mathrm{T}} \otimes Y_i \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \sum_i X_i \otimes \mathbf{e}_i^{\mathrm{T}} \\ \mathbf{0} & -\sum_i \mathbf{e}_i \otimes Y_i^{\mathrm{T}} & -\sum_i X_i^{\mathrm{T}} \otimes \mathbf{e}_i & \sum_j (U_j' \otimes S_j' + R_j' \otimes V_j') \end{bmatrix} \in \tilde{\mathfrak{g}}\,,$$

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$d=3$ is different for a **group-theoretic reason**. Namely:

There are $d=3$ independent measurements on a qubit because $SO(d-1)$ is commutative and non-trivial only for $d=3$.

3D of the Bloch ball: continuous interaction

Ll. Masanes, **MM**, D. Pérez-García, and R. Augusiak, *Entanglement and the three-dimensionality of the Bloch ball*, J. Math. Phys. **55**, 122203 (2014).

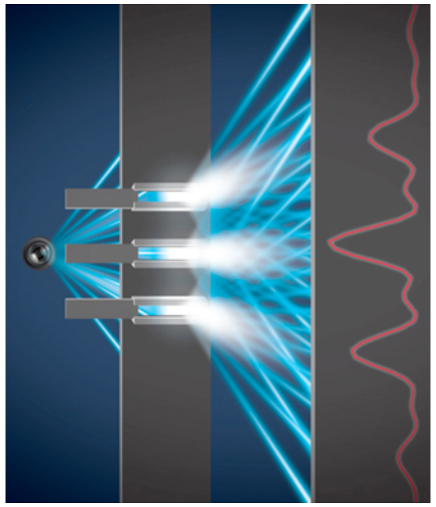
d=3 is different for a **group-theoretic reason**. Namely:

There are *d=3* independent measurements on a qubit because SO*(d-1)* is commutative and non-trivial only for *d=3*.

Surprisingly, this shows up in a completely different context: in **special relativity**!

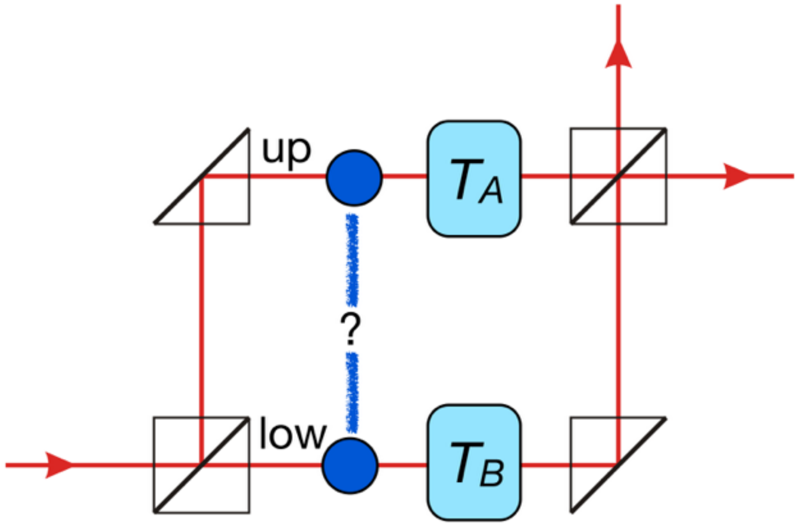
Outline

- Quantum theory from principles



"Why does the qubit have 3 degrees of freedom?"

- **Take 1: continuous-reversible interaction**
- Take 2: relativity of simultaneity on interferometer



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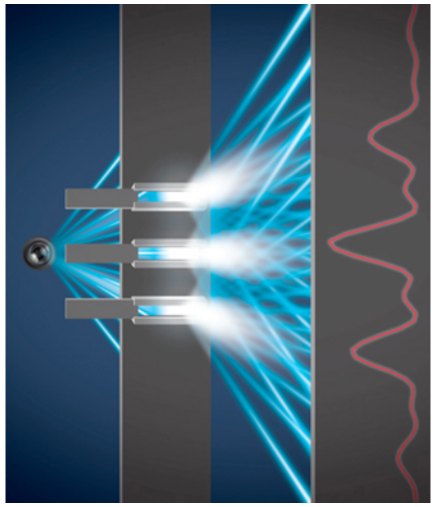

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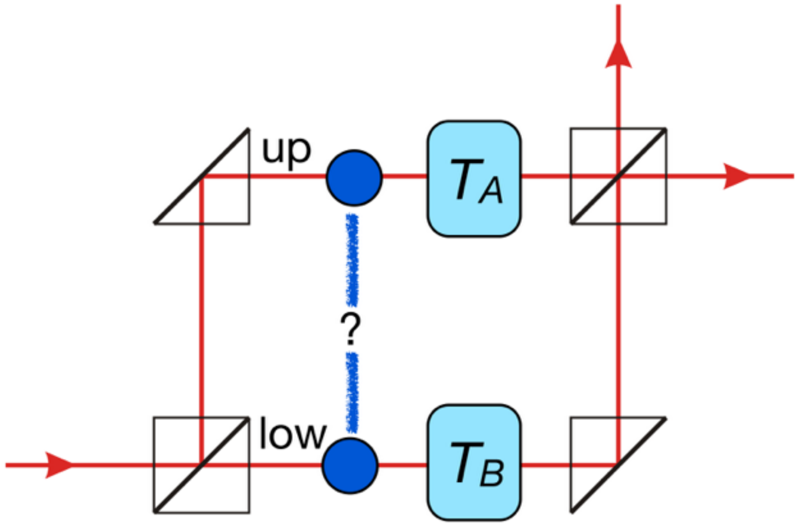
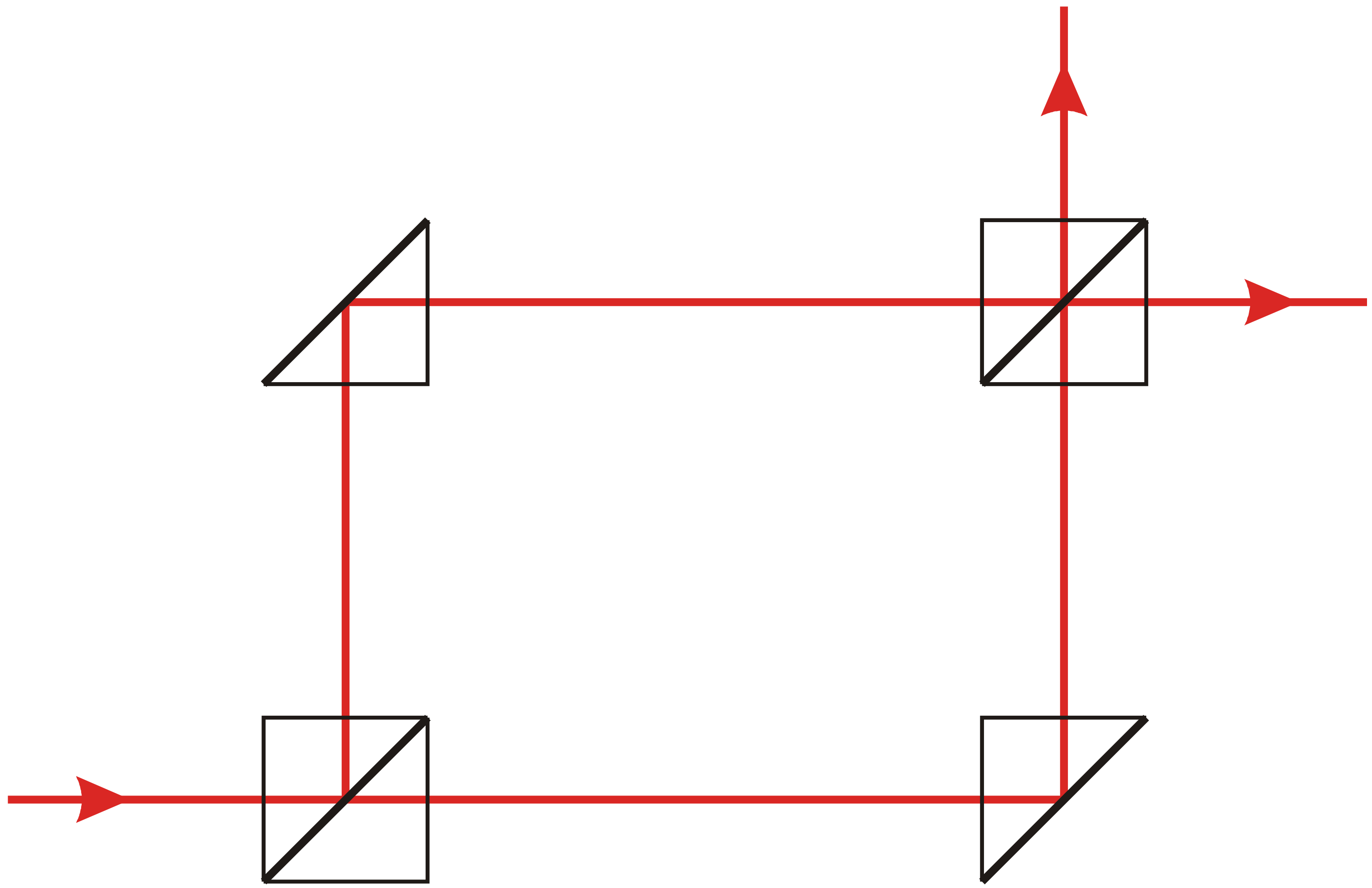


Relativistic constraints on the state space

A. Garner, **MM**, O. Dahlsten, arXiv:1412.7112



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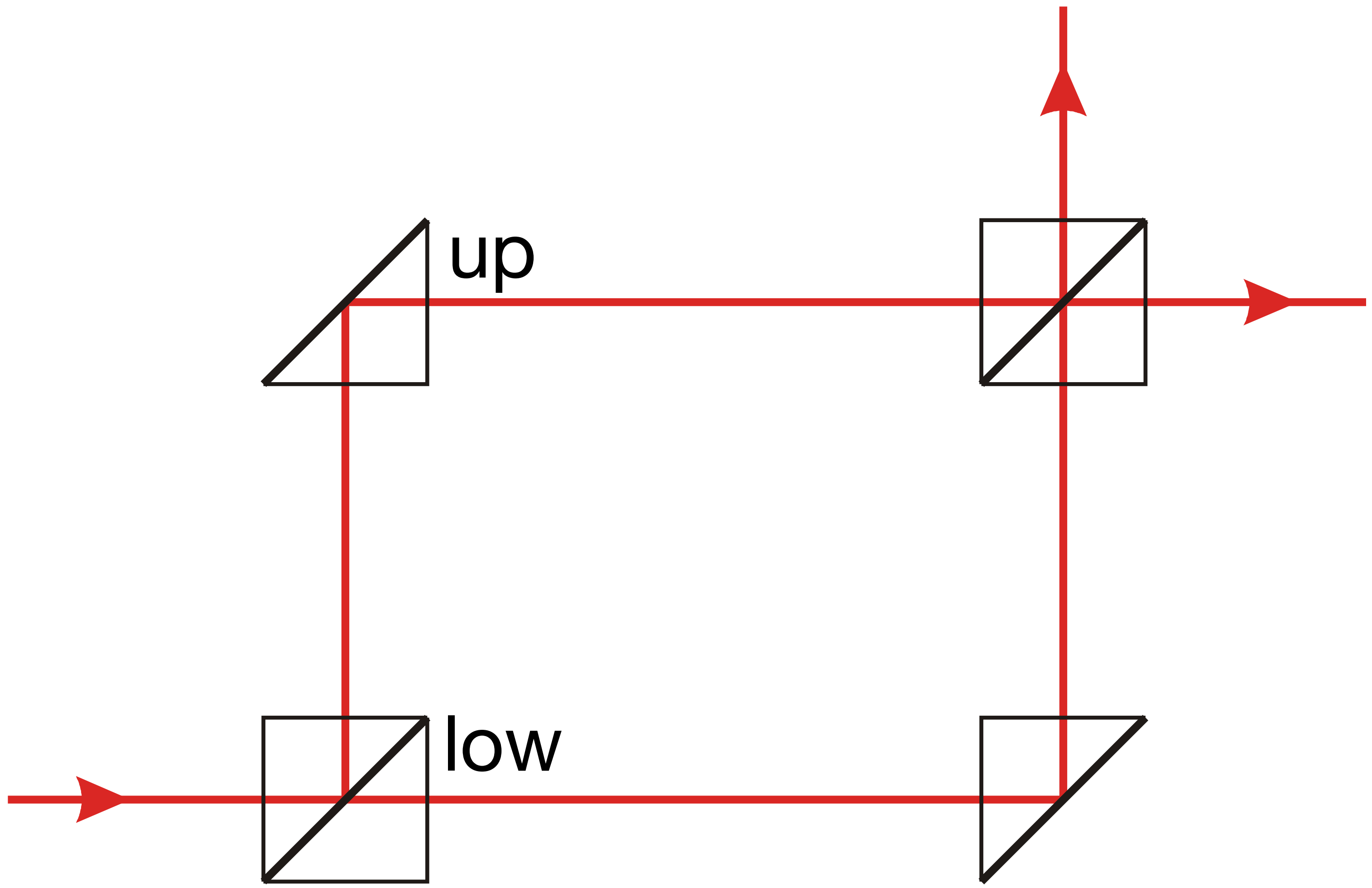

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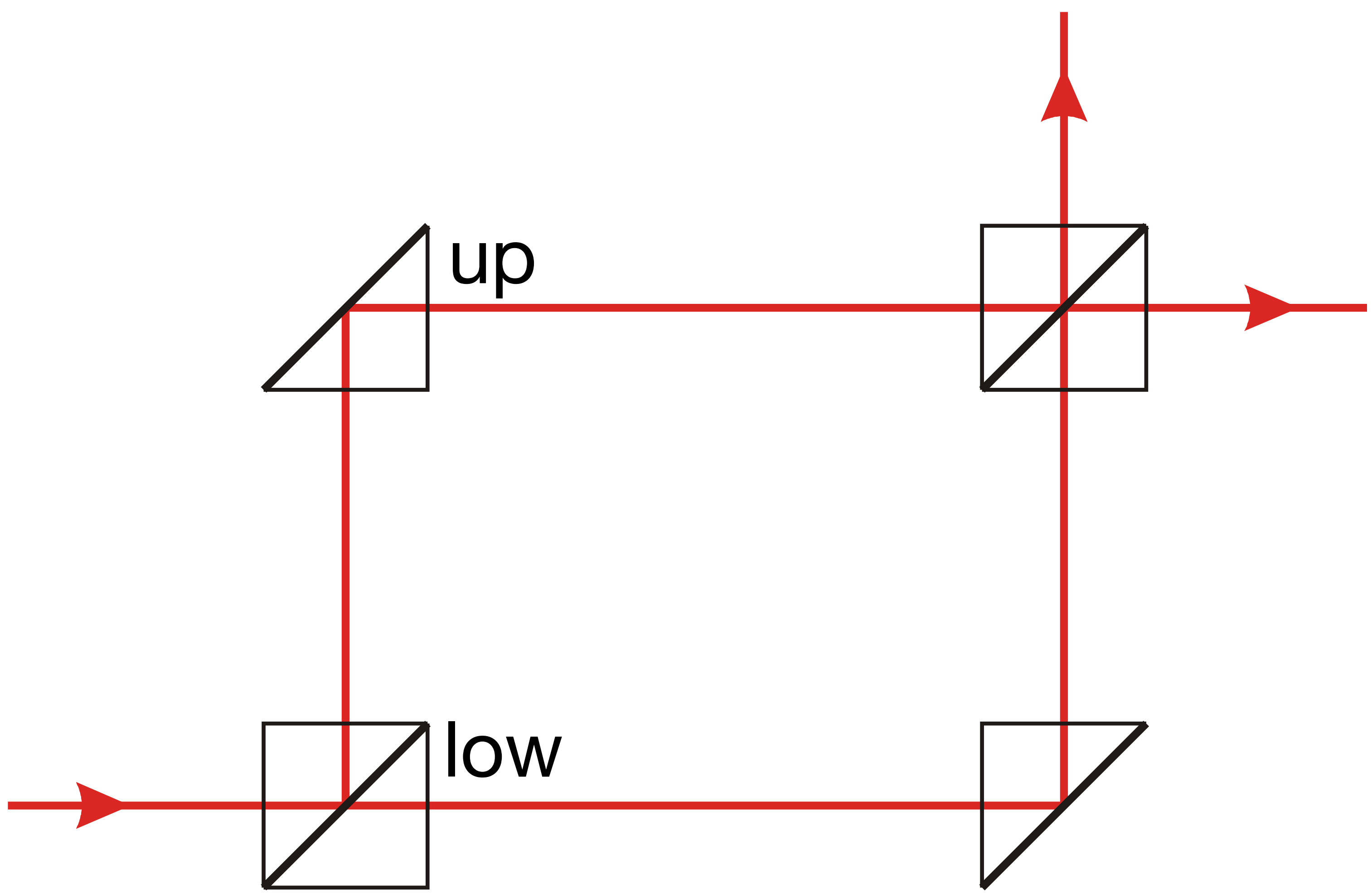

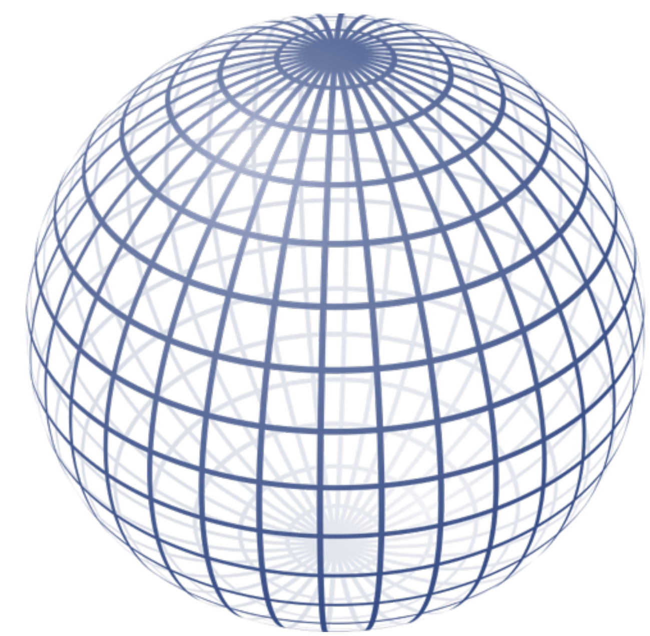

d-dim. "Bloch sphere"

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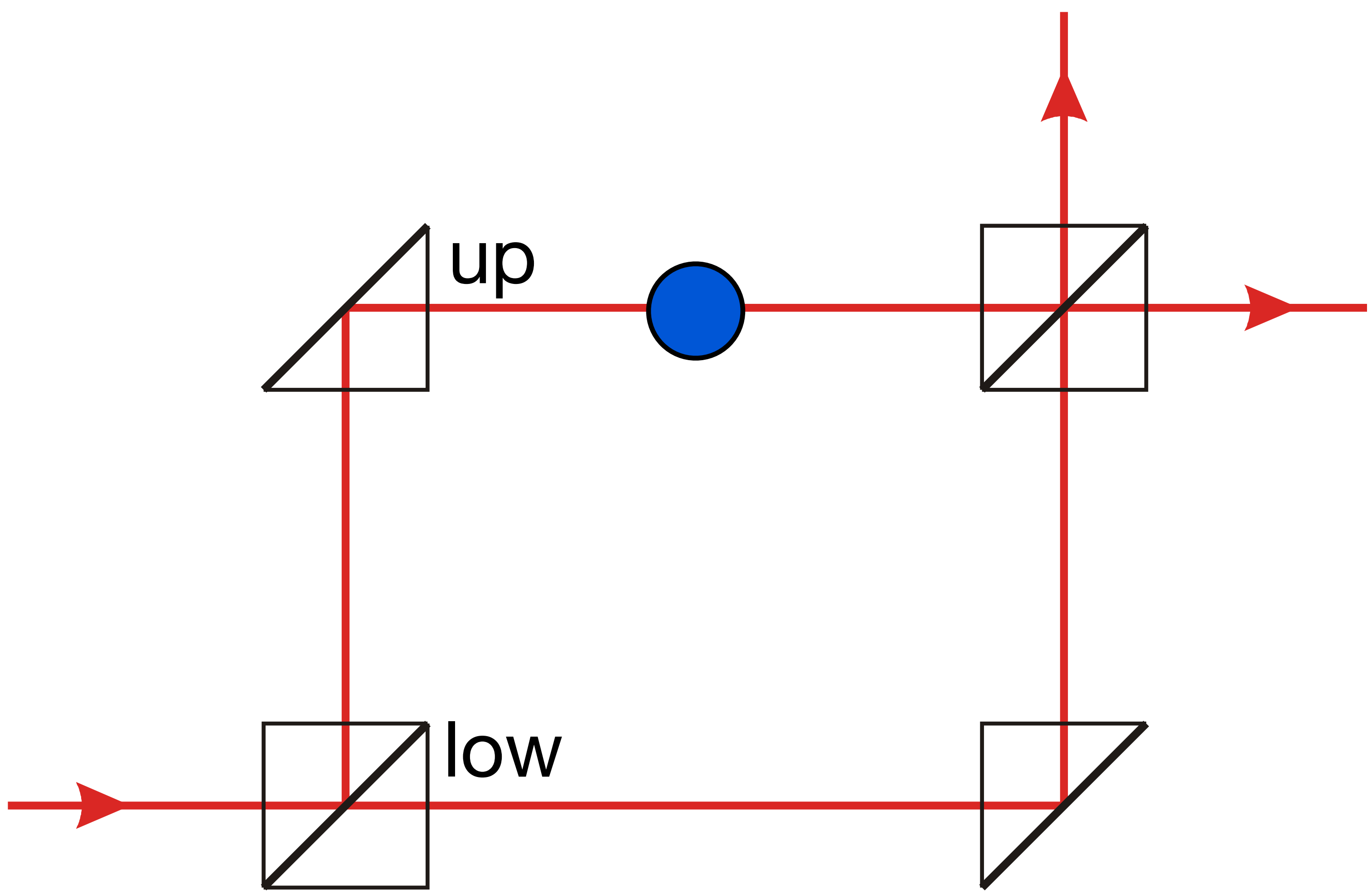

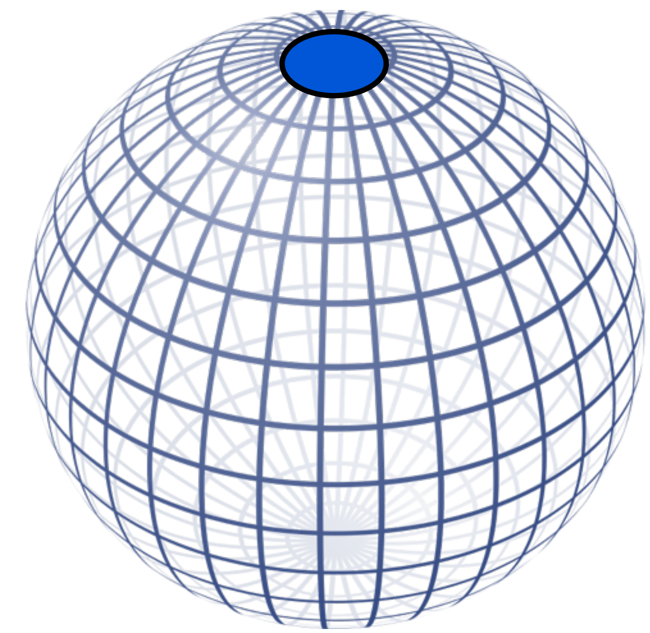

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North-pole state: particle definitely in upper branch.

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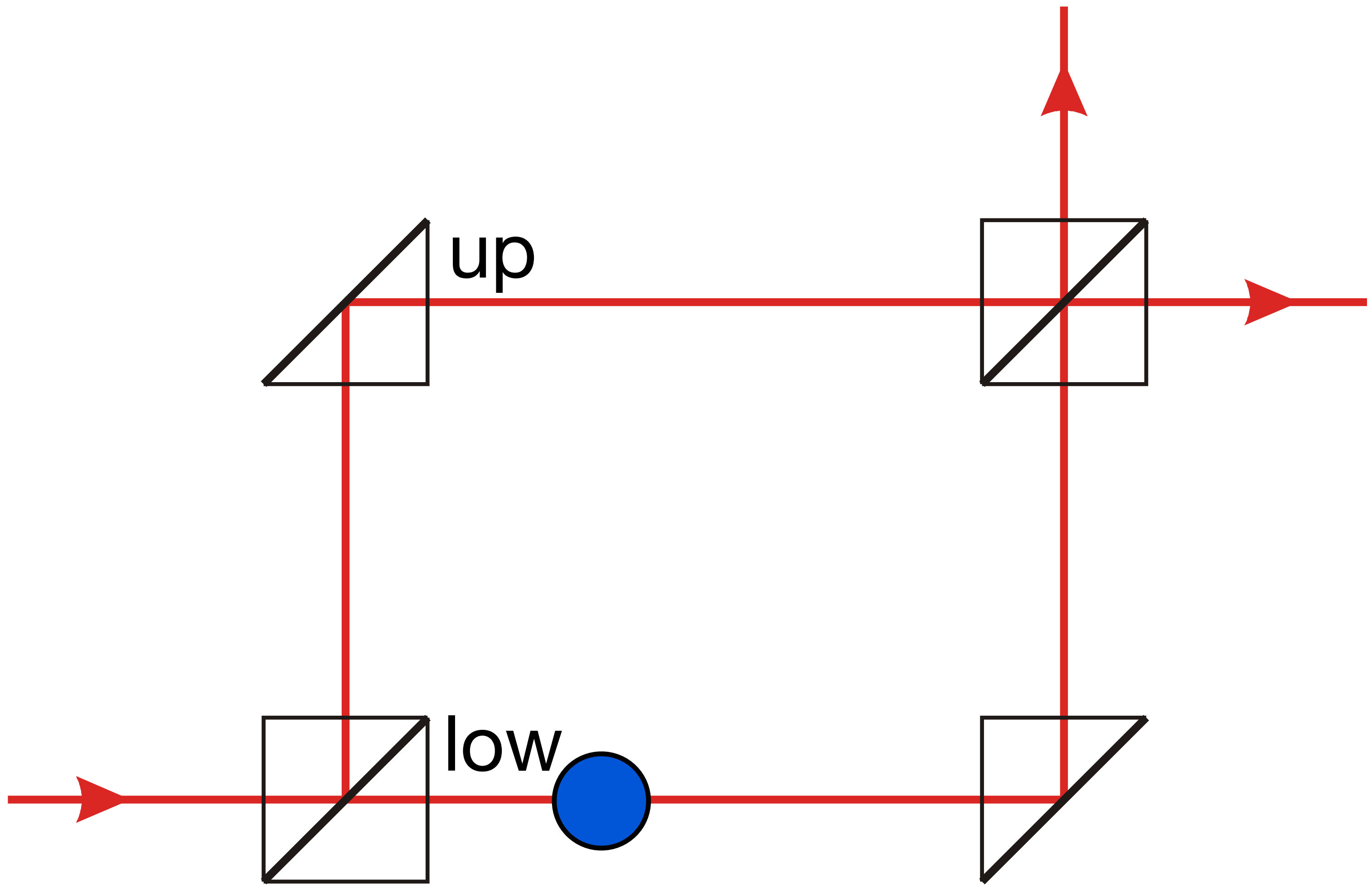

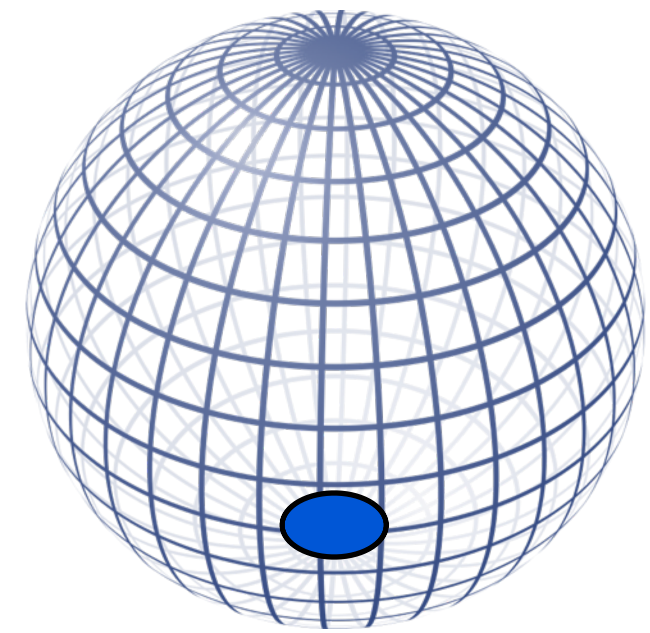

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South-pole state: particle definitely in lower branch.

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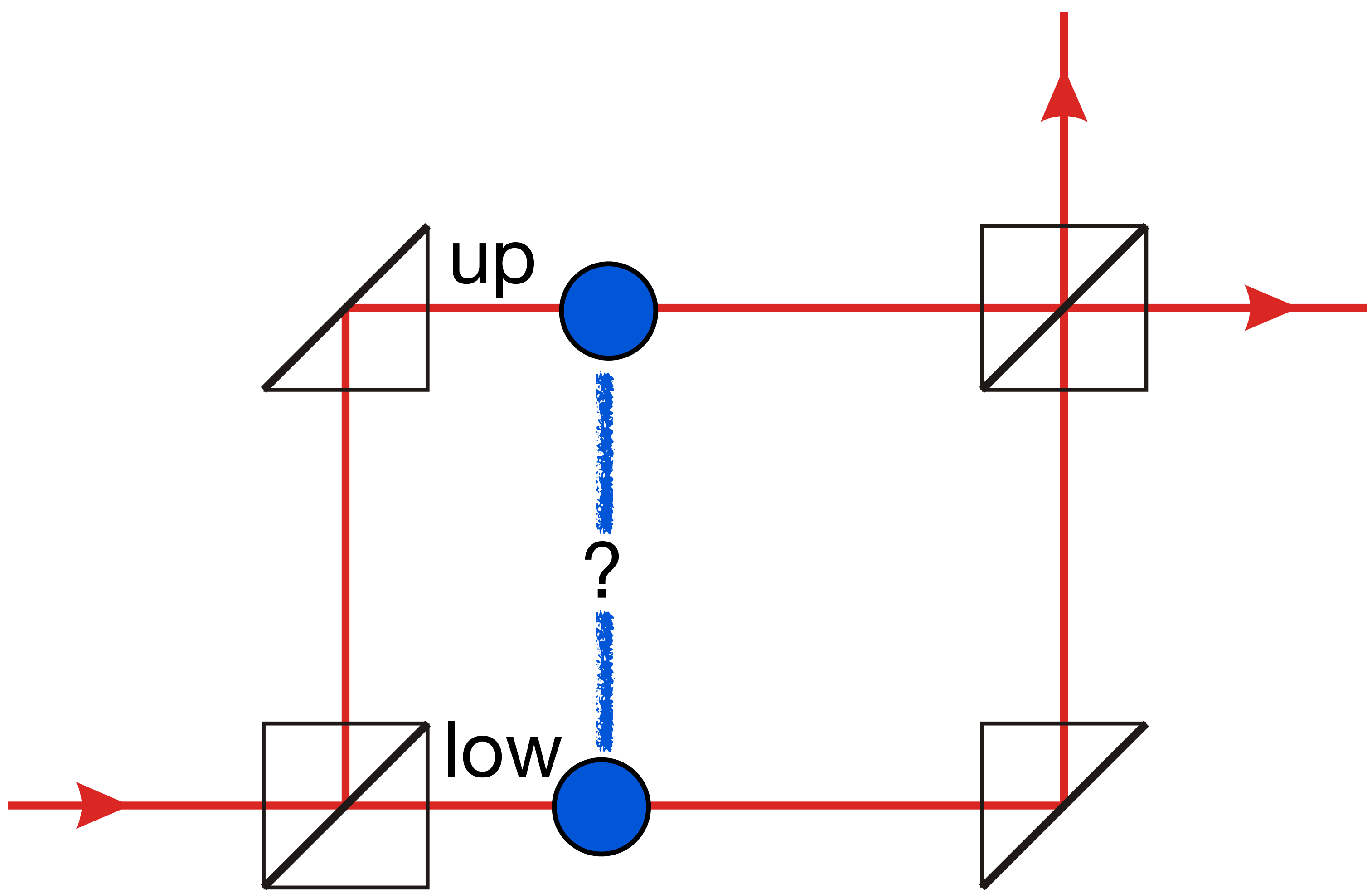

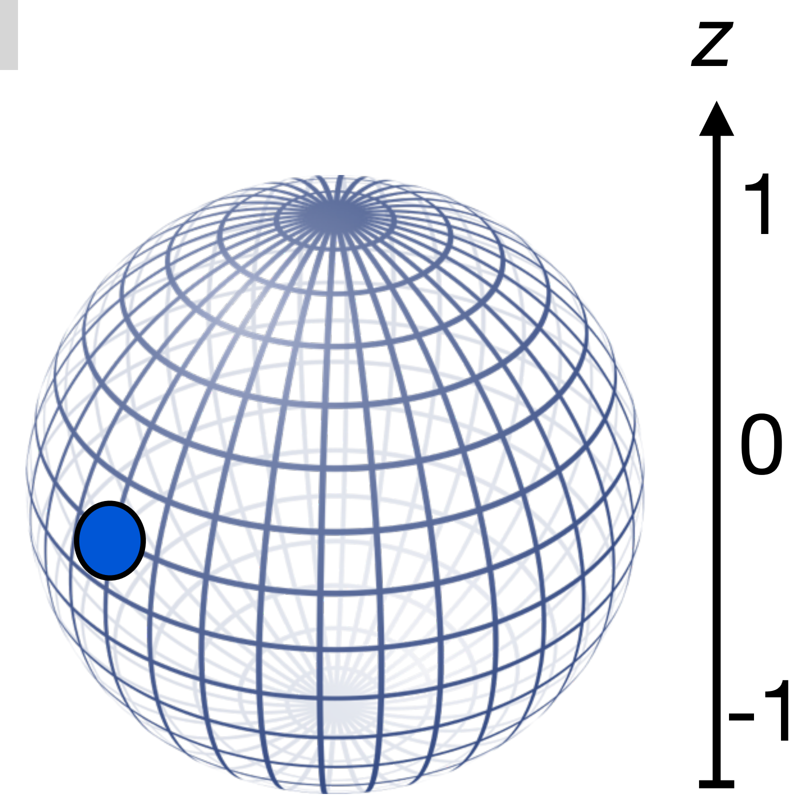

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State on equator *z=0*: probability 1/2 for each.

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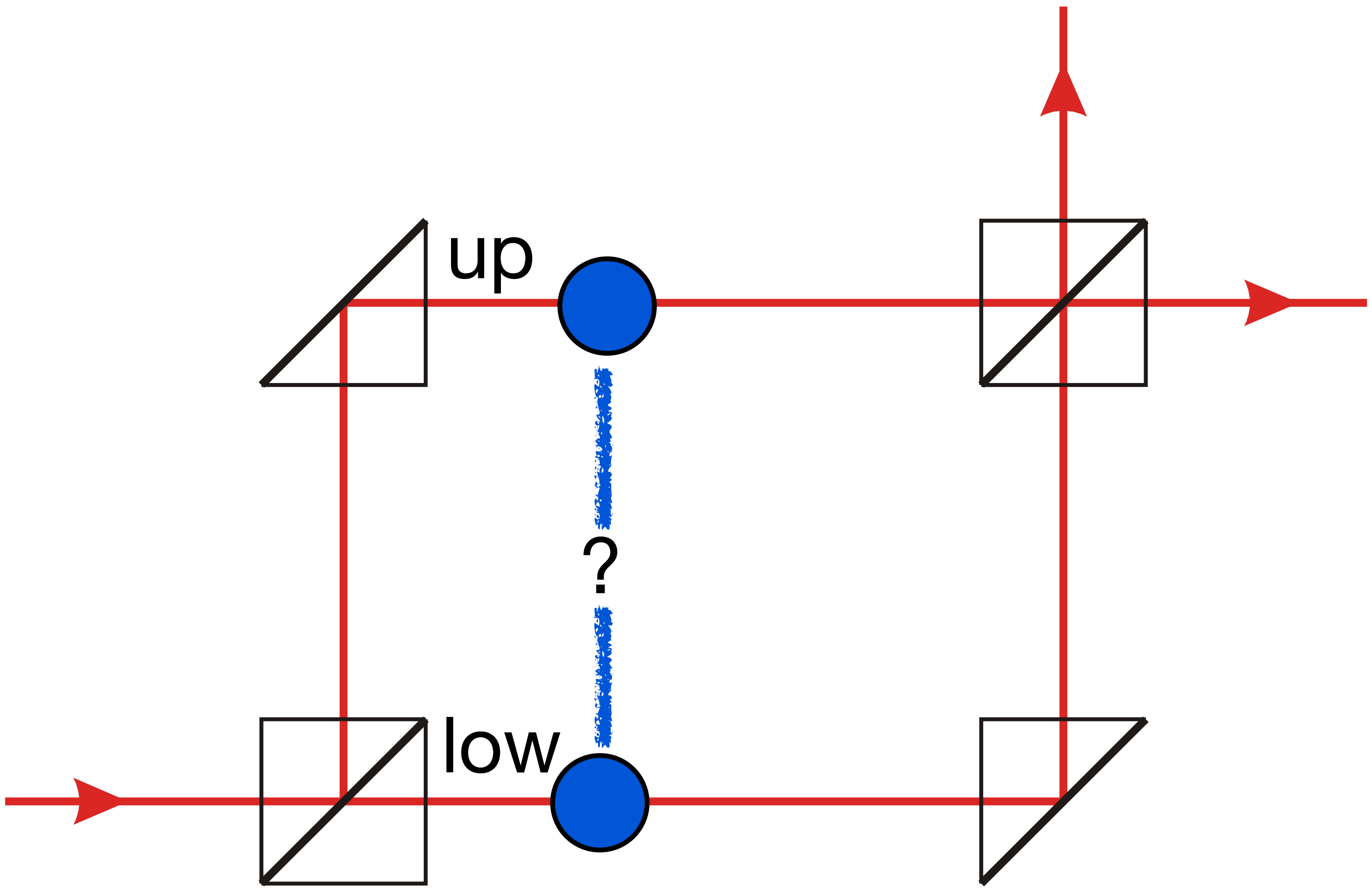

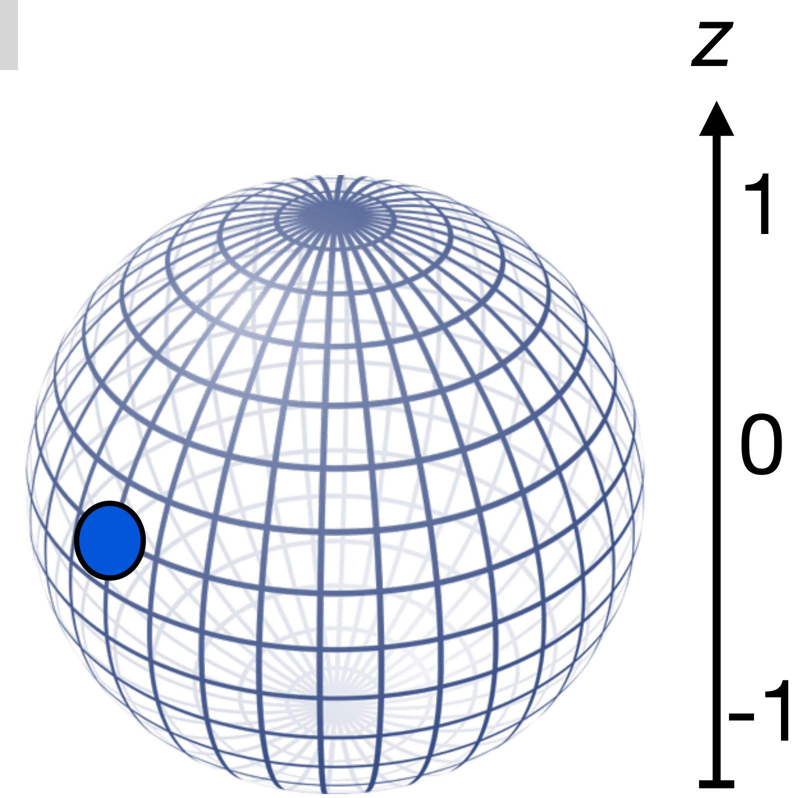

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$$p(\mathrm{up}) = \frac{1}{2}(z+1)$$

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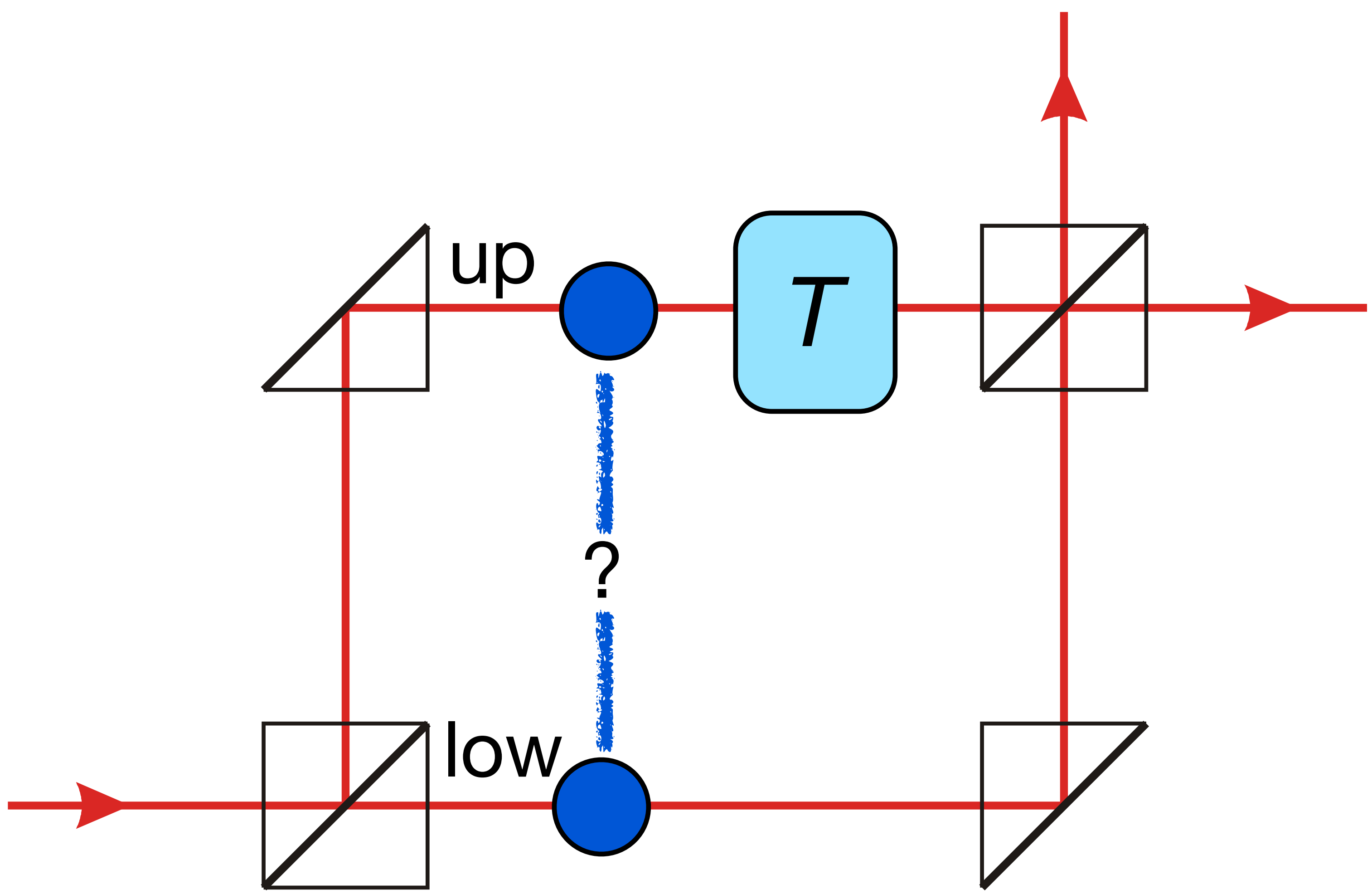

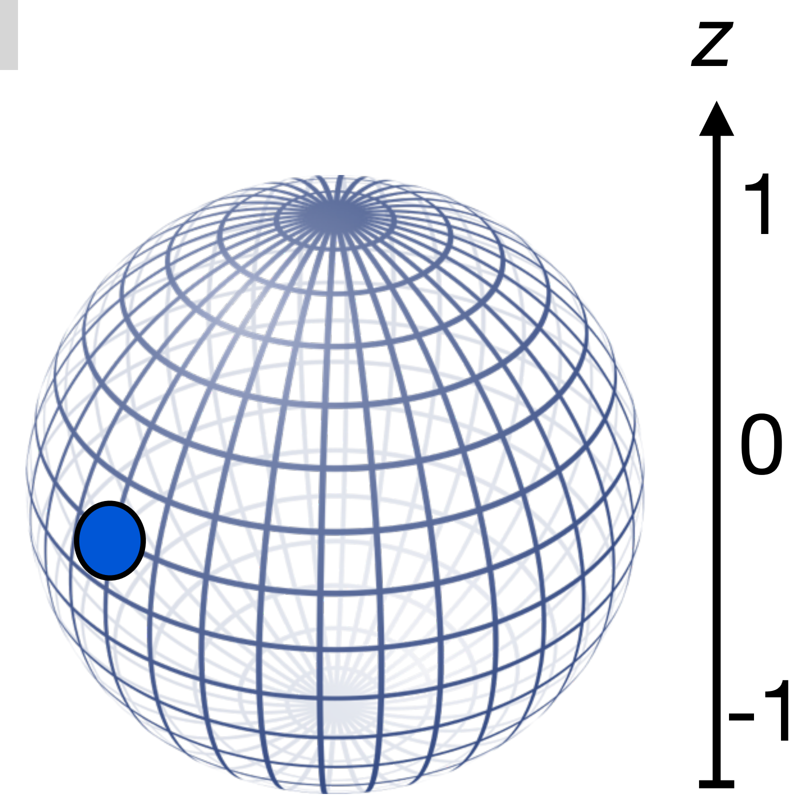

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What transformations *T* can we perform locally in one arm...
... without any information loss?

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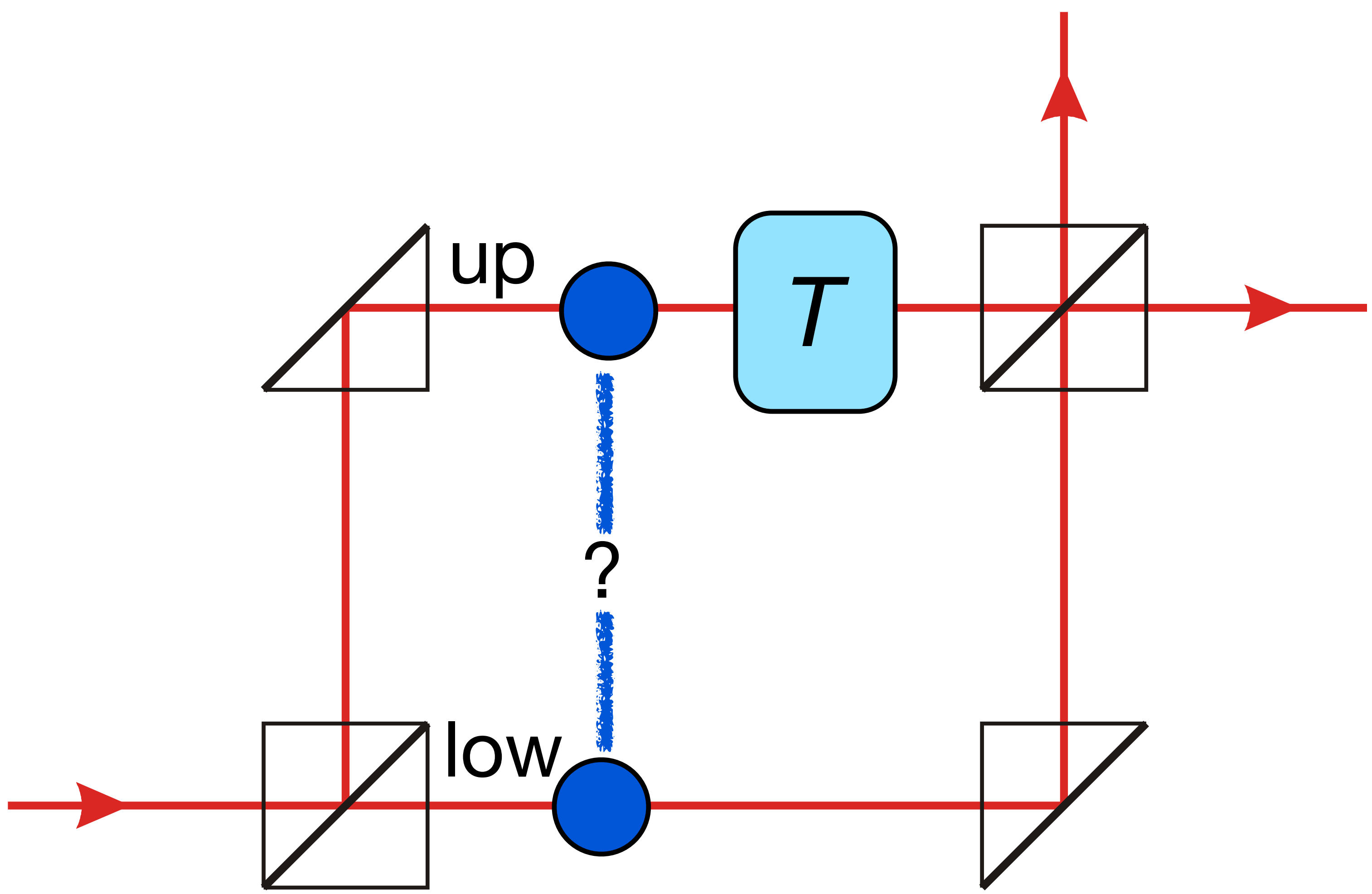

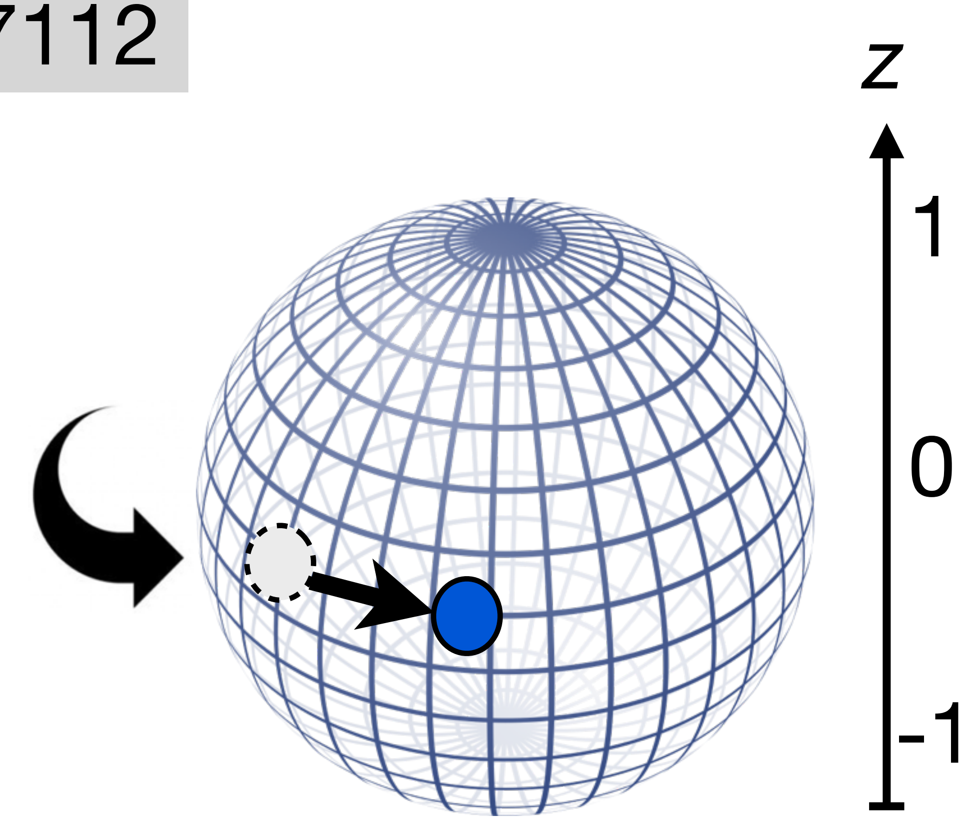

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T must be a rotation of the Bloch ball (reversible+linear)...
... and must preserve *p*(up), i.e. preserve the *z*-axis.

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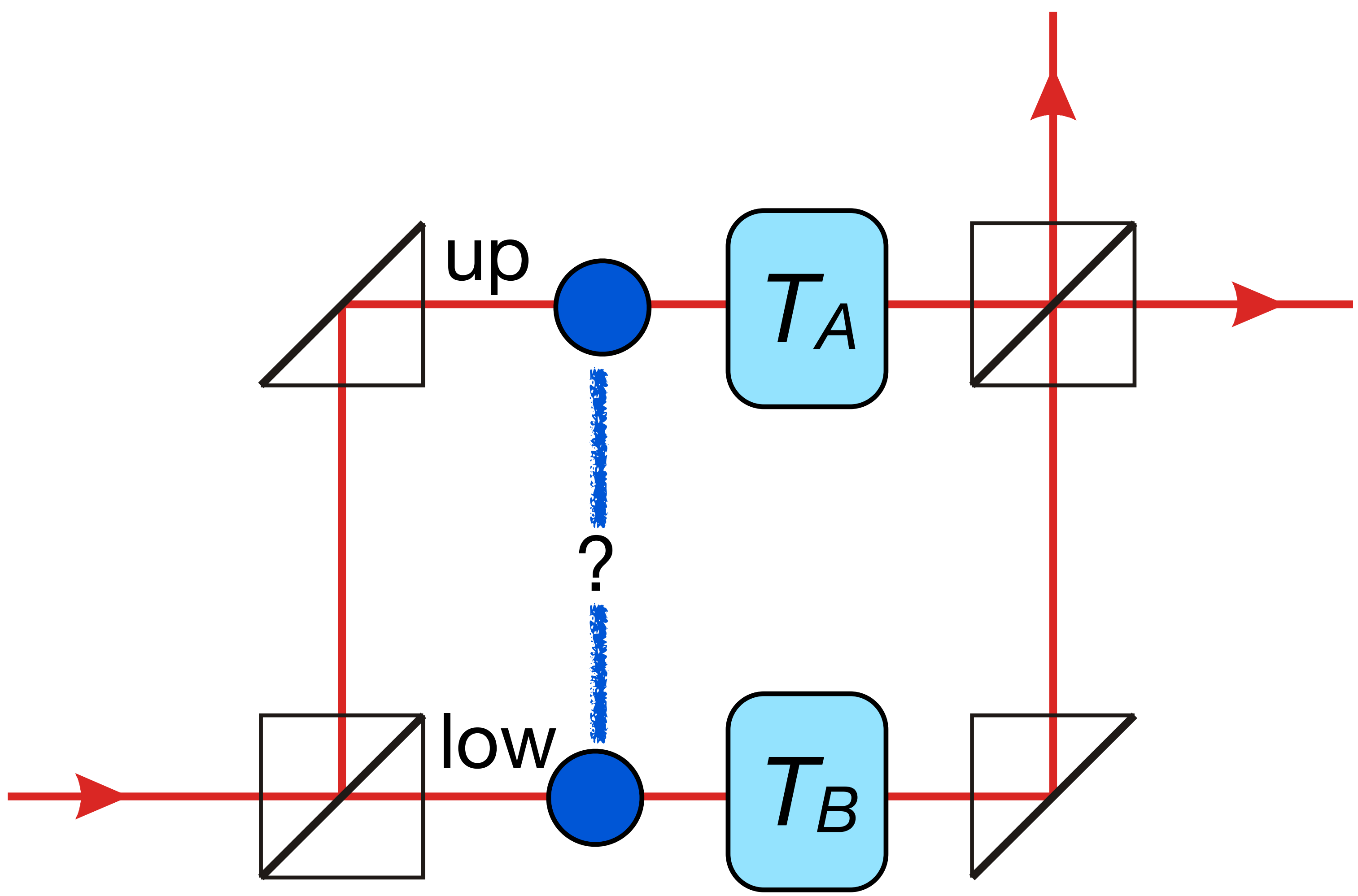

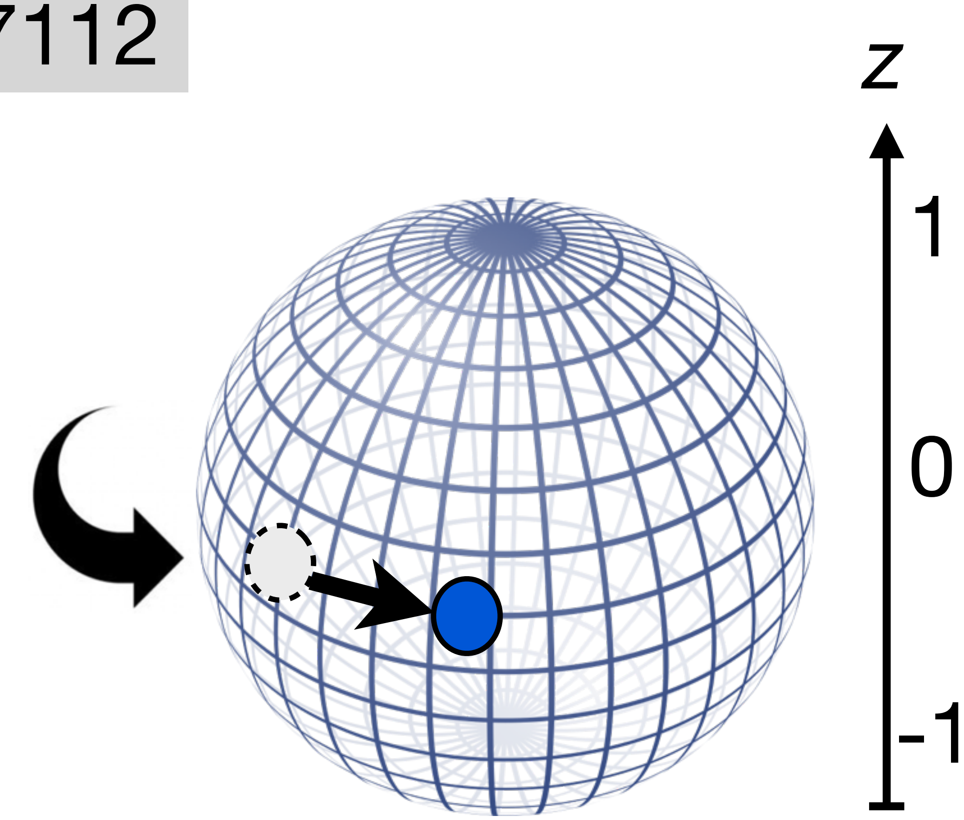

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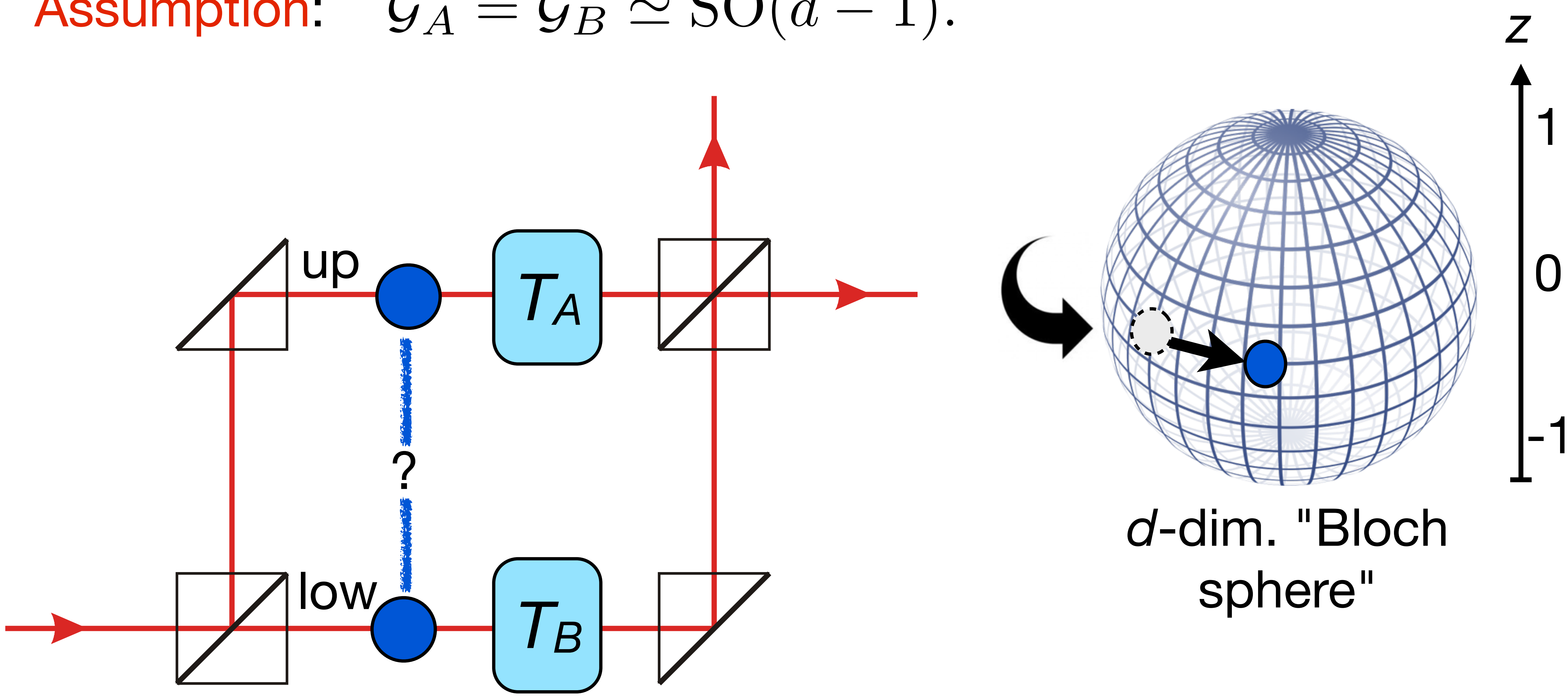
Assumption: $\mathcal{G}_A = \mathcal{G}_B \simeq \mathrm{SO}(d-1)$.



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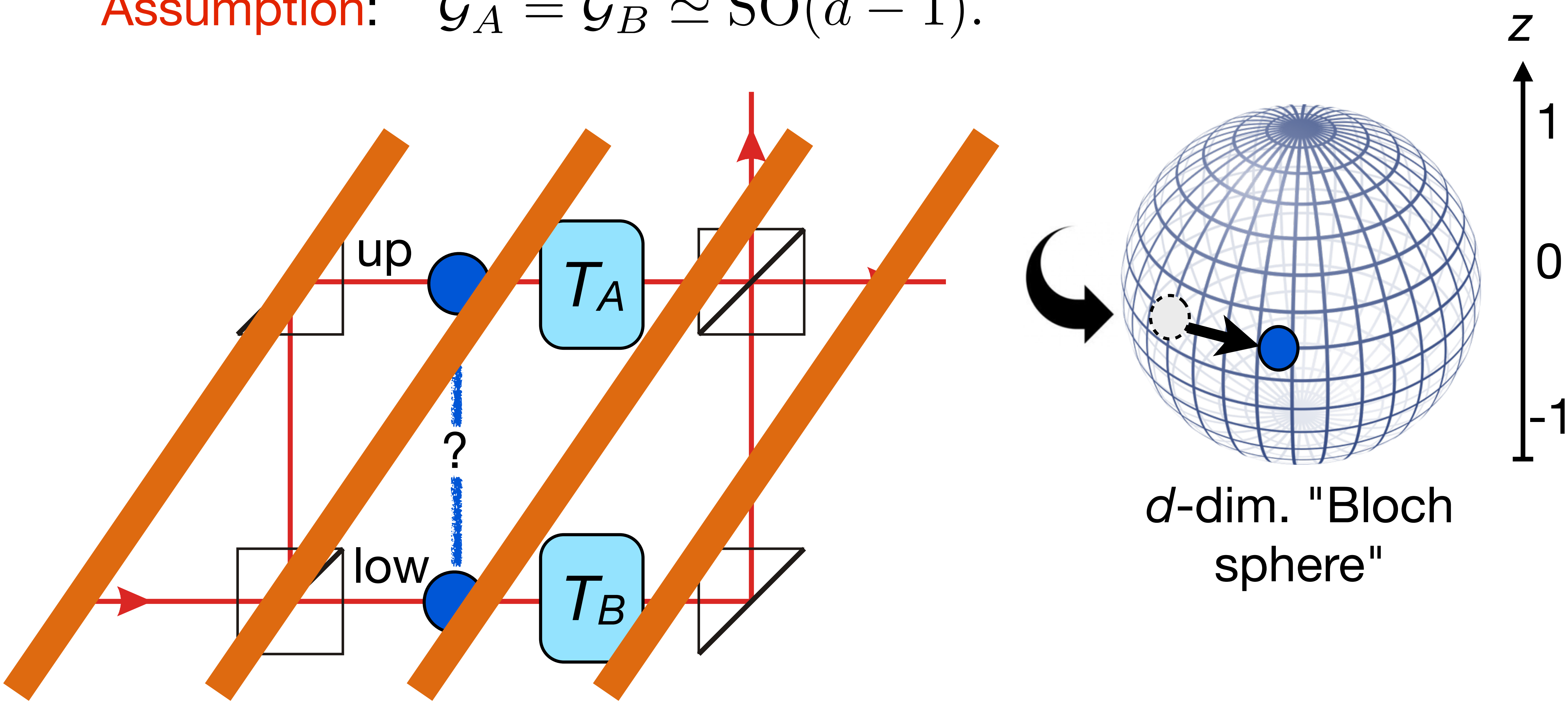

Relativity: there is one frame of reference in which T_A happens first, and then T_B...

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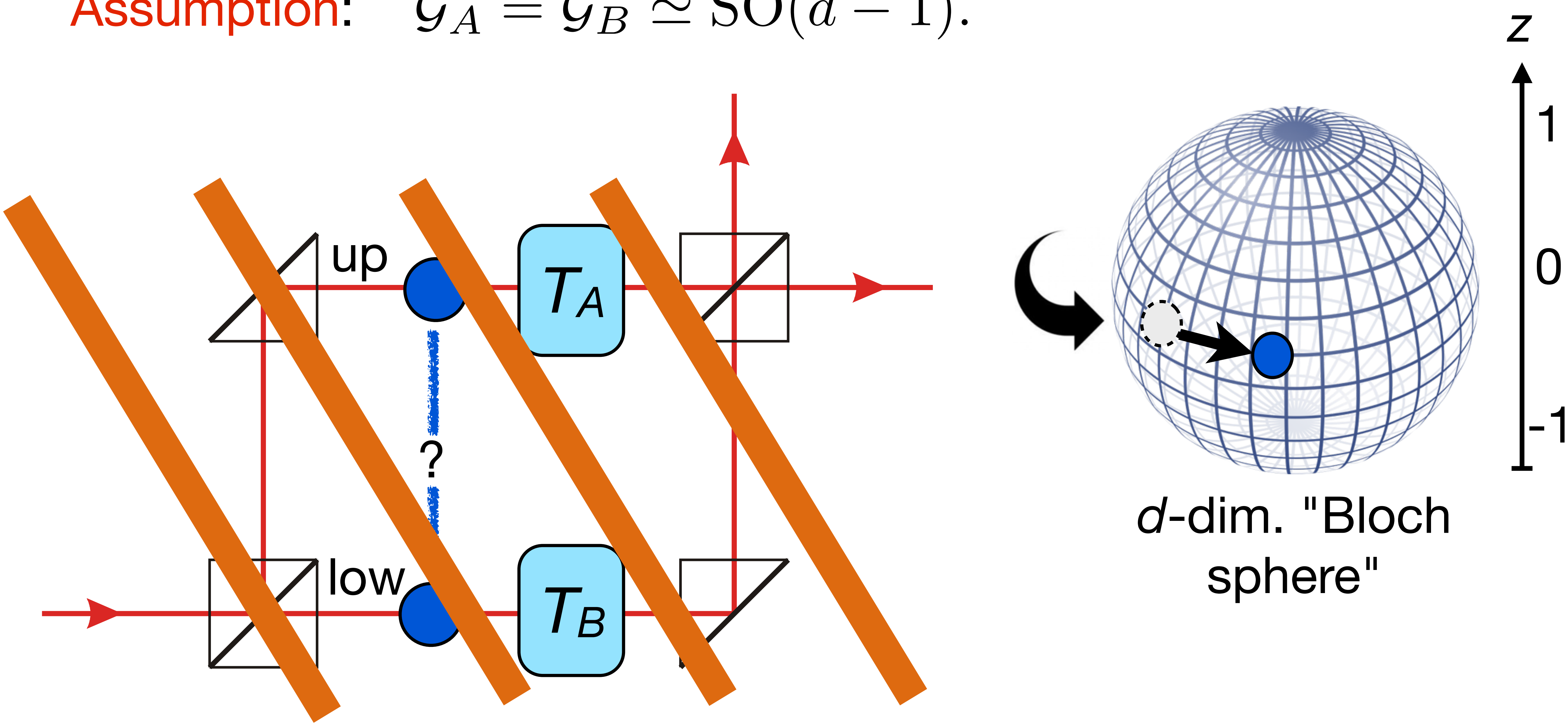

d-dim. "Bloch sphere"

Relativity: ... and another one in which it's the other way around.

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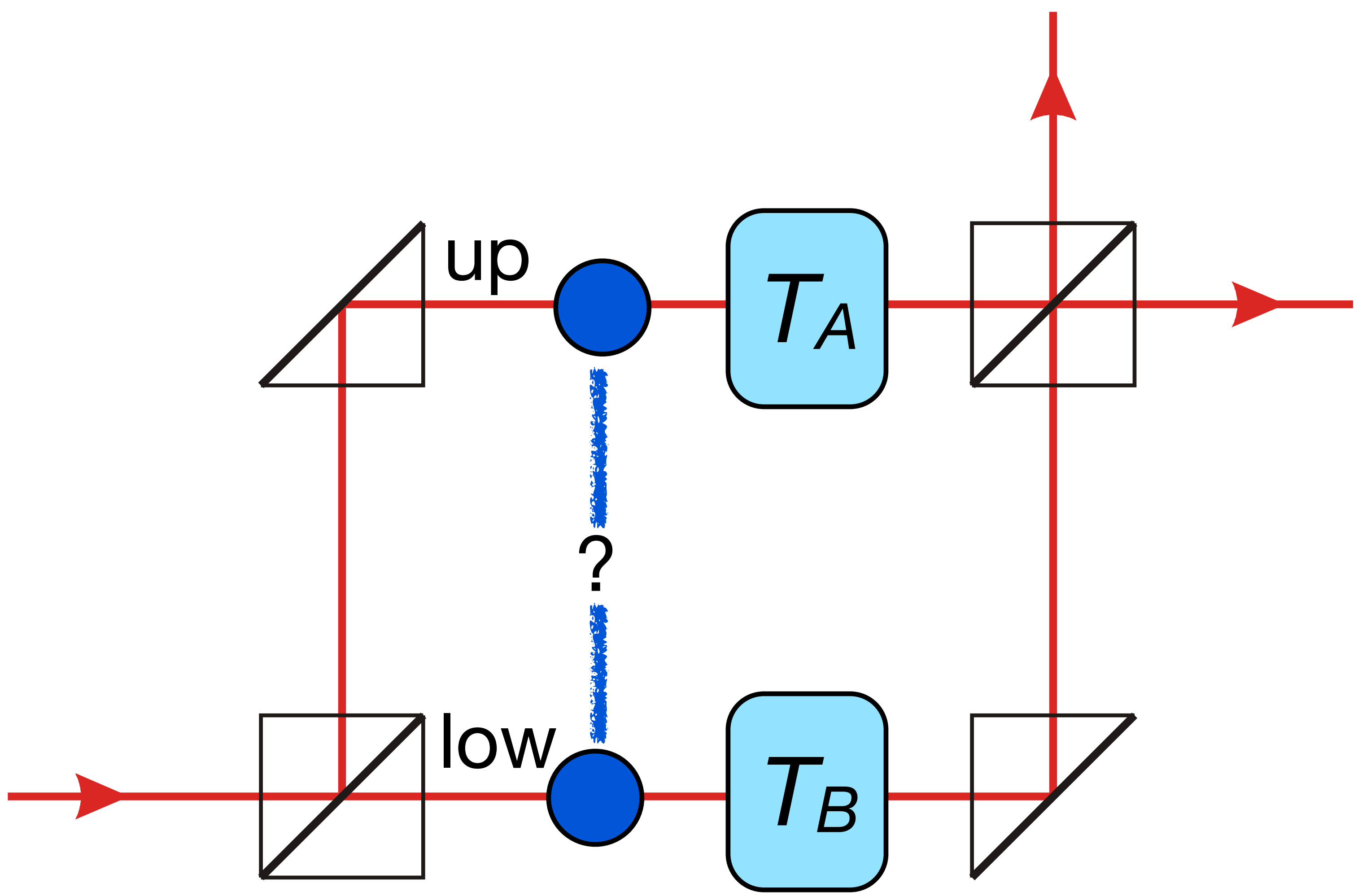
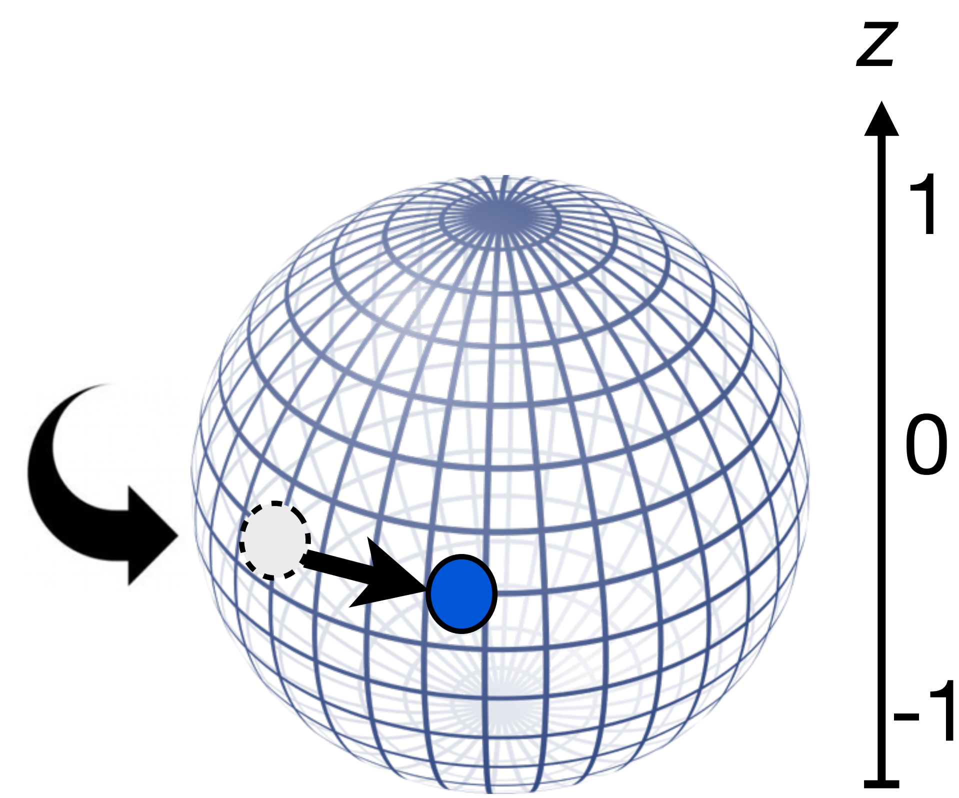




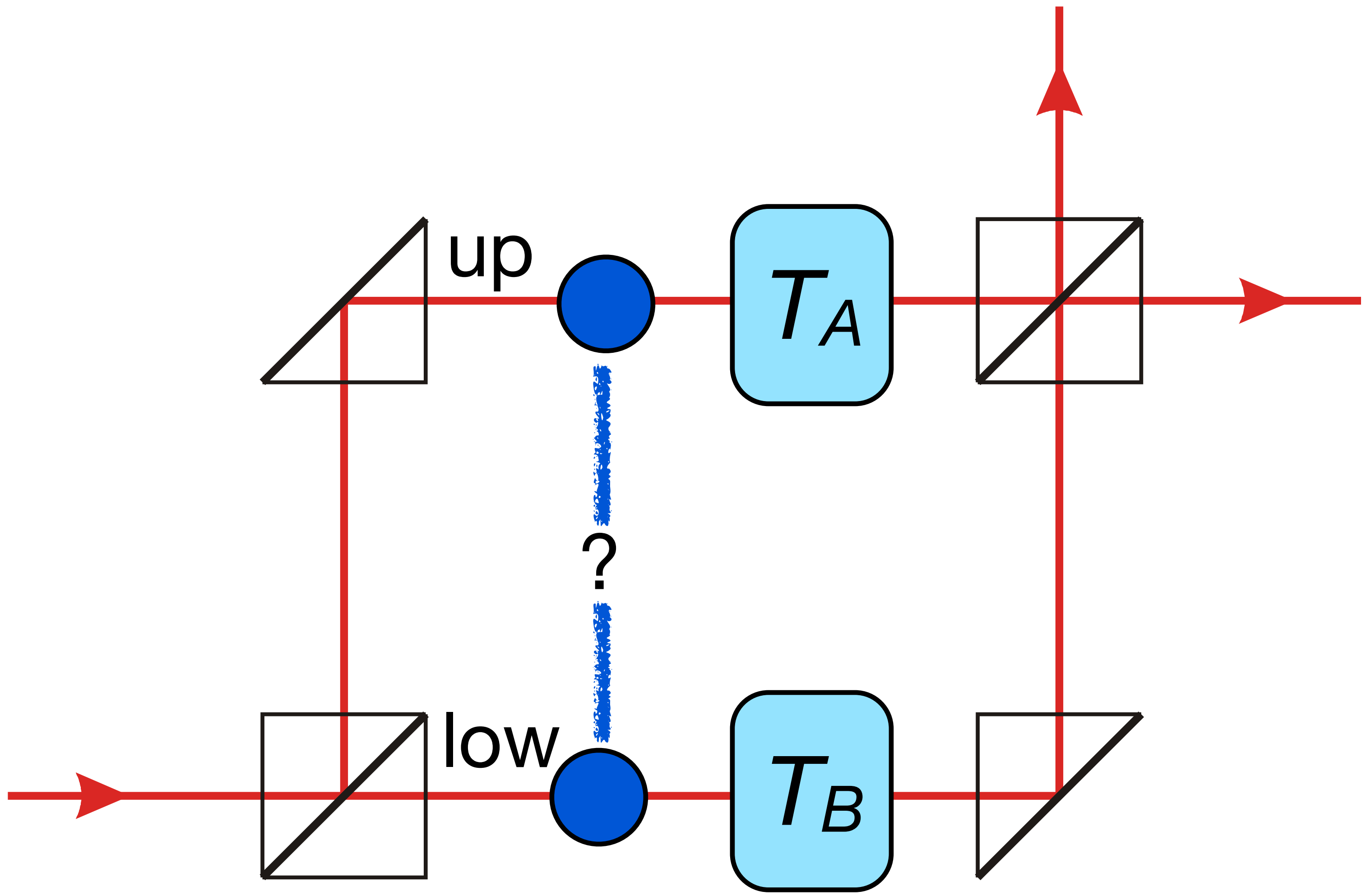
d-dim. "Bloch sphere"

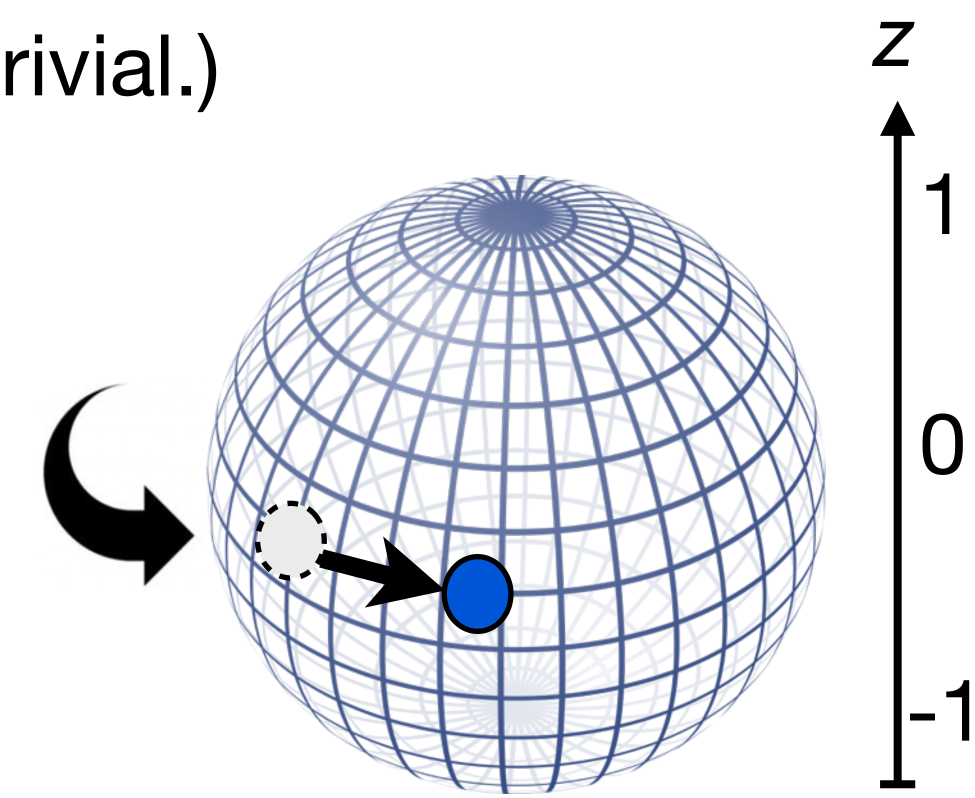
Detector click statistics is Lorentz-invariant

$$\Rightarrow T_A T_B = T_B T_A \text{ for all } T_A, T_B \in \mathrm{SO}(d-1).$$

Relativistic constraints on the state space

$\Rightarrow \quad d \leq 3$ (In fact, *d*=3, otherwise these transformations are all trivial.)





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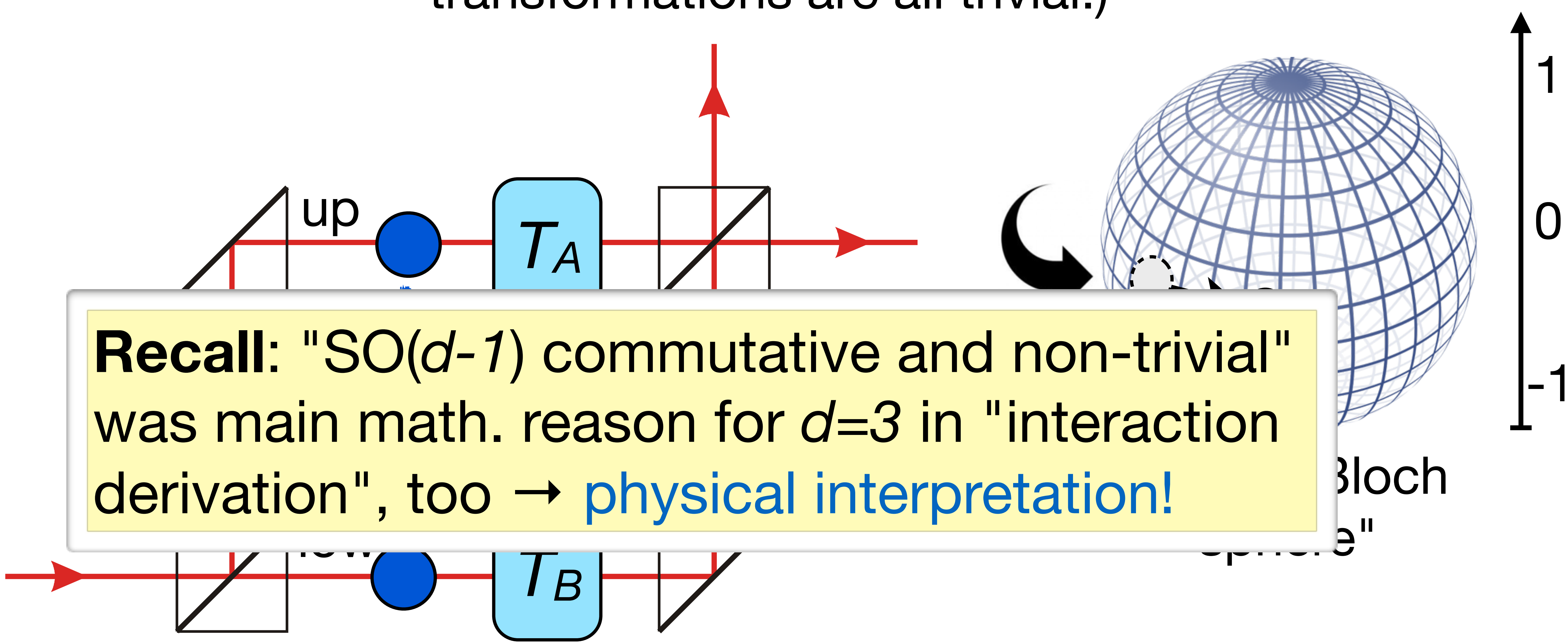

Recall: "SO(*d*-*1*) commutative and non-trivial" was main math. reason for *d*=*3* in "interaction derivation", too → physical interpretation!

Detector click statistics is Lorentz-invariant

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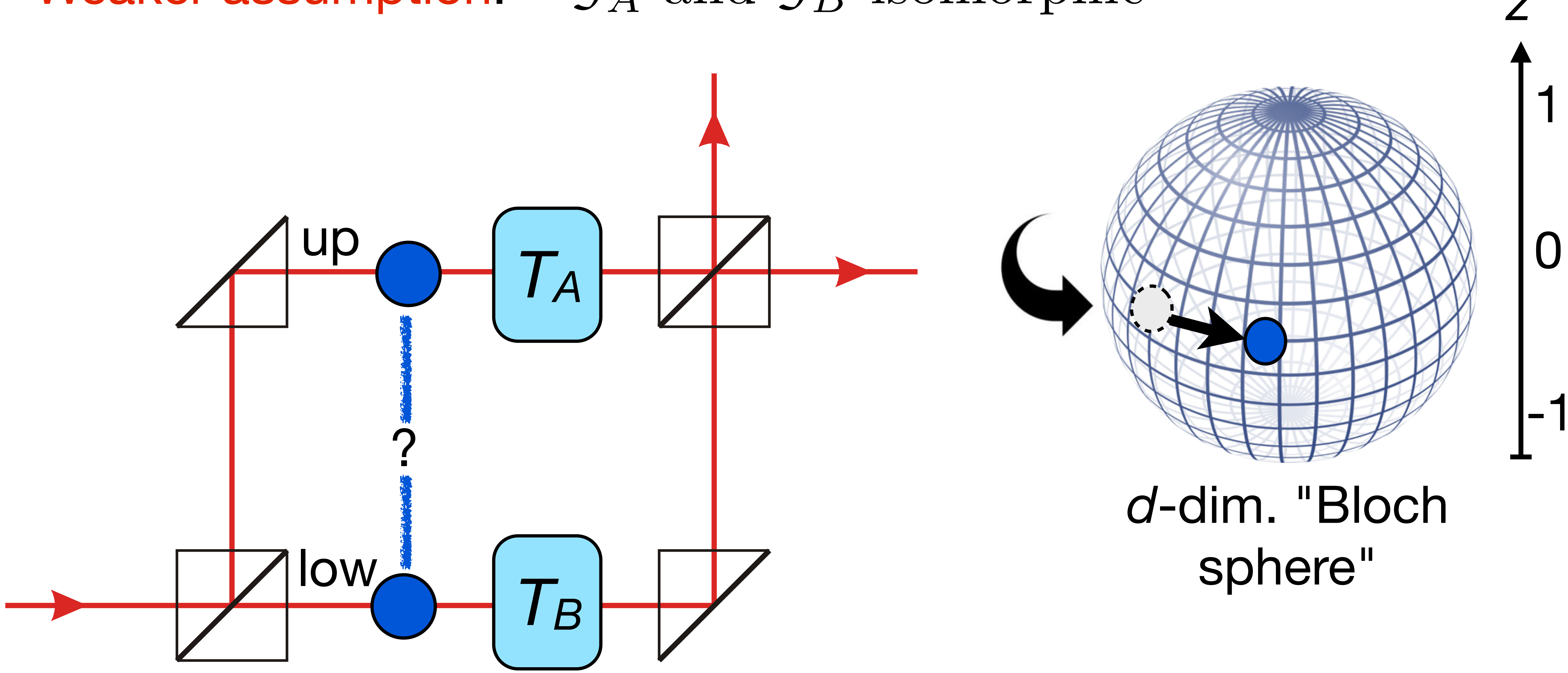
Western

Relativistic constraints on the state space

Weaker assumption: $\mathcal{G}_A$ and $\mathcal{G}_B$ isomorphic



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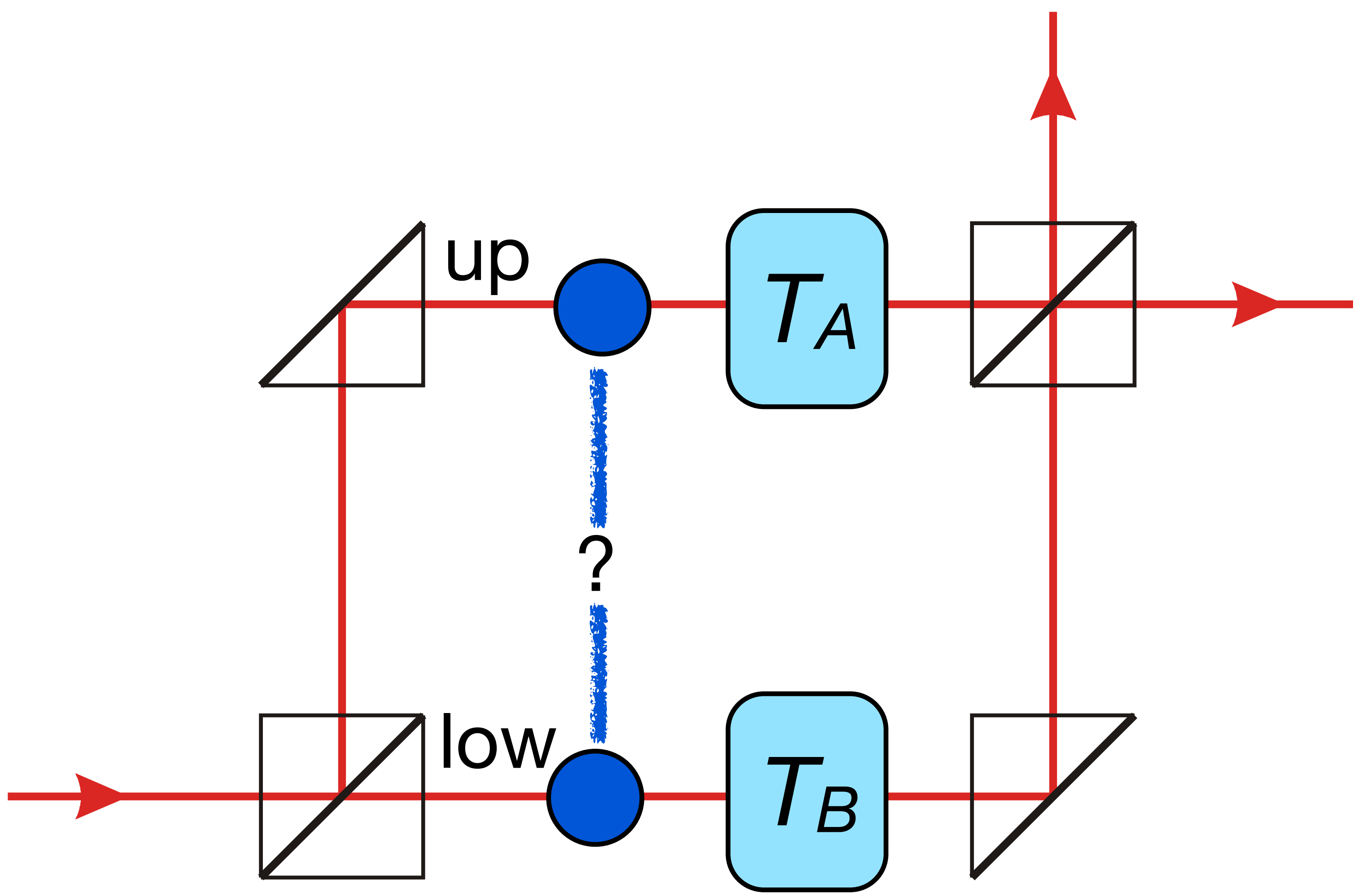

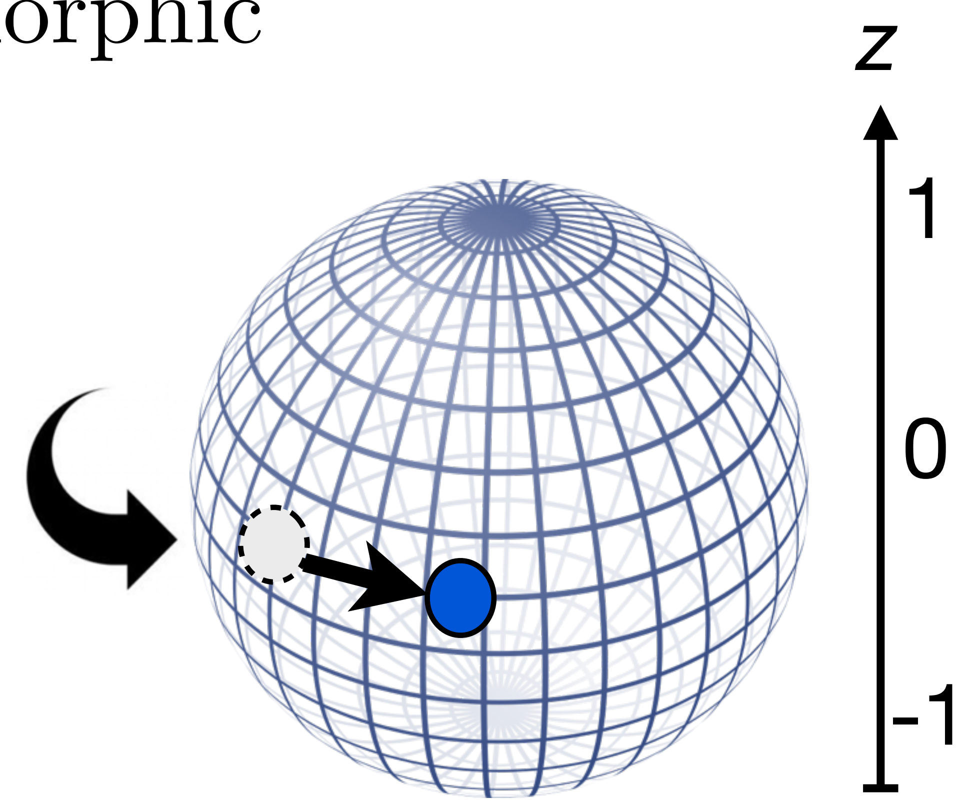

d-dim. "Bloch sphere"

$\Rightarrow \quad d \leq 5.$ Quaternionic QM survives.

Classification of possibilities

A. Garner, **MM**, O. Dahlsten, arXiv:1412.7112

Theorem 2. *Suppose that (i) $\mathcal{G}_A$ and $\mathcal{G}_B$ are isomorphic; (ii) they generate the full phase group; (iii) every pure state can be mapped to every other by a reversible transformation. Then relativity of simultaneity allows for the following possibilities and no more:*

- $d = 2$ *(the quantum bit over the real numbers), with* $\mathcal{G} = \mathrm{O}(2)$ *and* $\mathcal{G}_A = \mathcal{G}_B = \mathbb{Z}_2$*;*
- $d = 3$ *(the standard complex quantum bit), with* $\mathcal{G} = \mathrm{SO}(3)$ *and* $\mathcal{G}_A = \mathcal{G}_B = \mathrm{SO}(2) = \mathrm{U}(1)$*;*
- $d = 4$*, with* $\mathcal{G} \simeq \mathrm{U}(2)$ *and* $\mathcal{G}_A = \mathcal{G}_B = \mathrm{SO}(2) = \mathrm{U}(1)$*,*
- $d = 5$ *(the quaternionic quantum bit), with* $\mathcal{G} = \mathrm{SO}(5)$*,* $\mathcal{G}_A$ *the left- and* $\mathcal{G}_B$ *the right-isoclinic rotations in* $\mathrm{SO}(4)$ *(or vice versa), such that both are isomorphic to* $\mathrm{SU}(2)$ *and* $\mathcal{G}_A \cap \mathcal{G}_B = \{+\mathbb{1}, -\mathbb{1}\}$*.*

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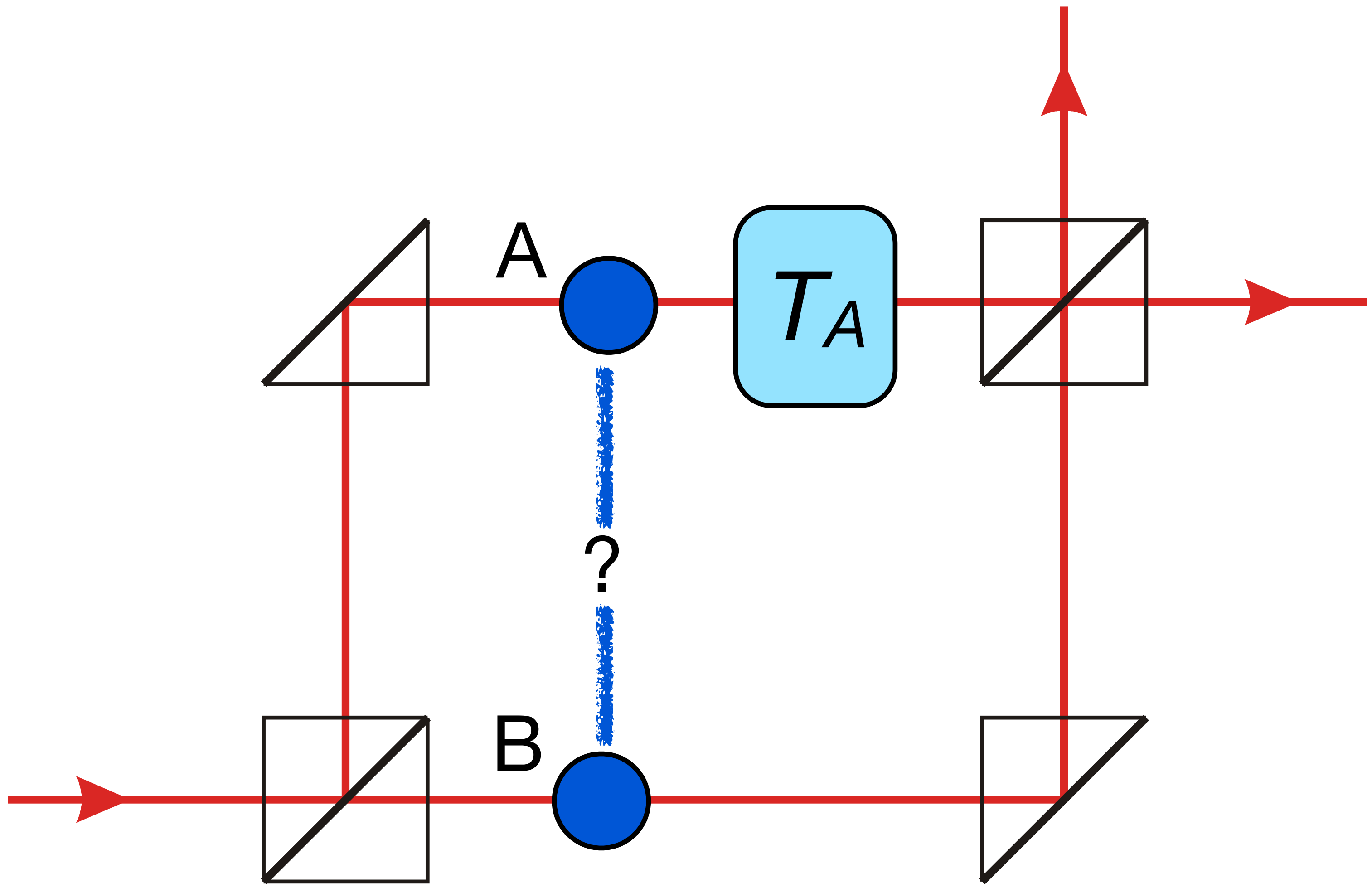

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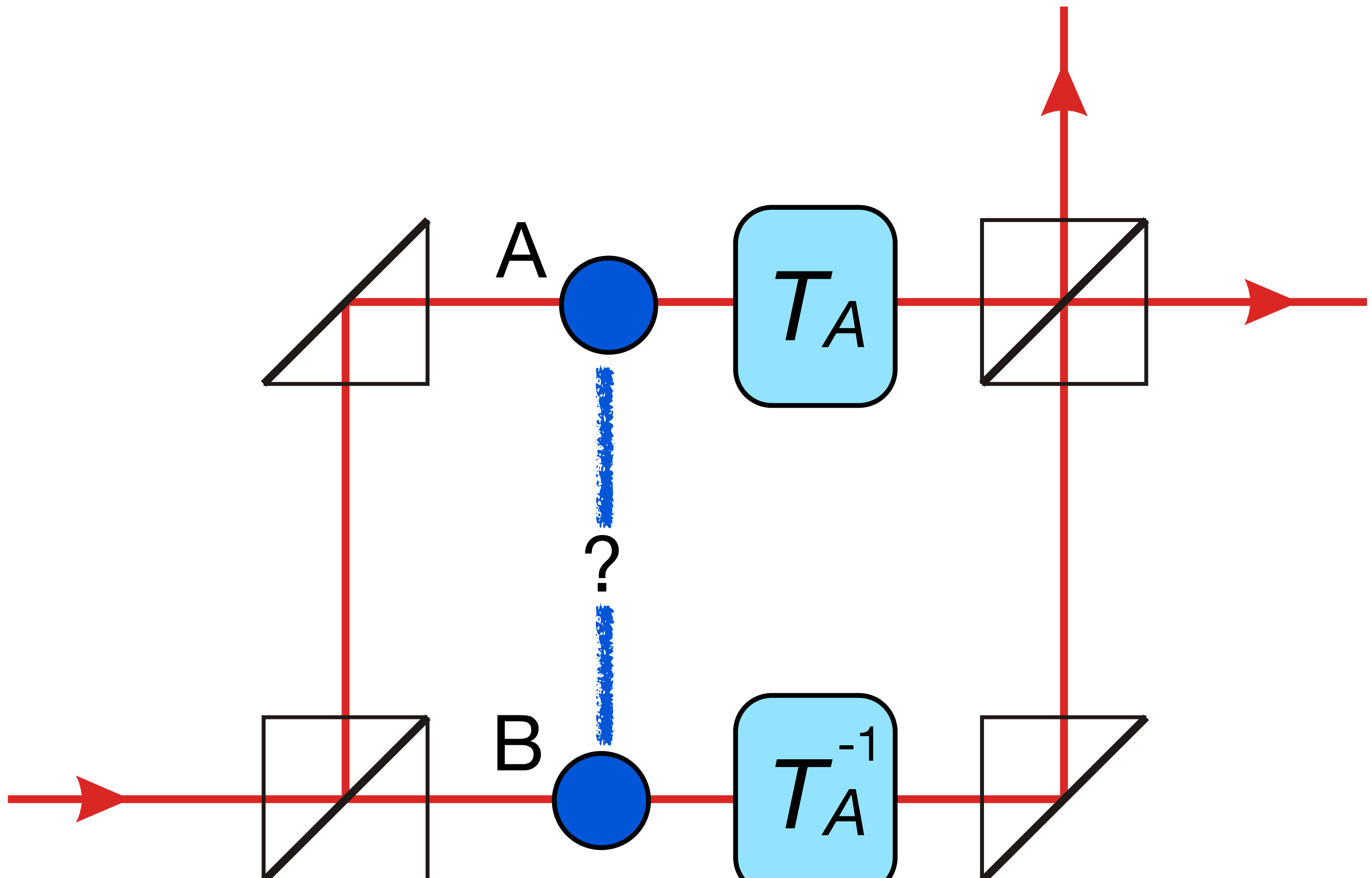

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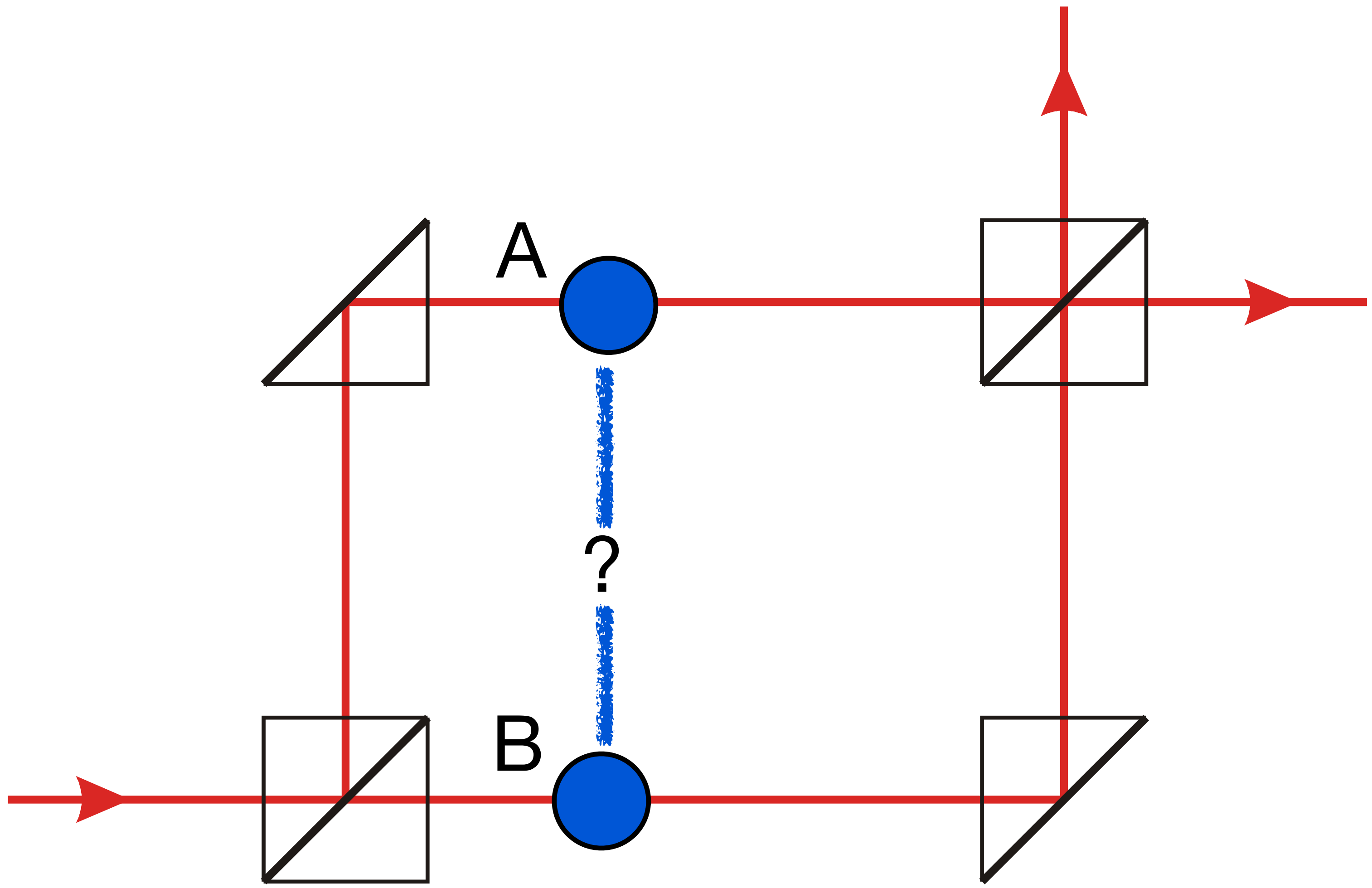

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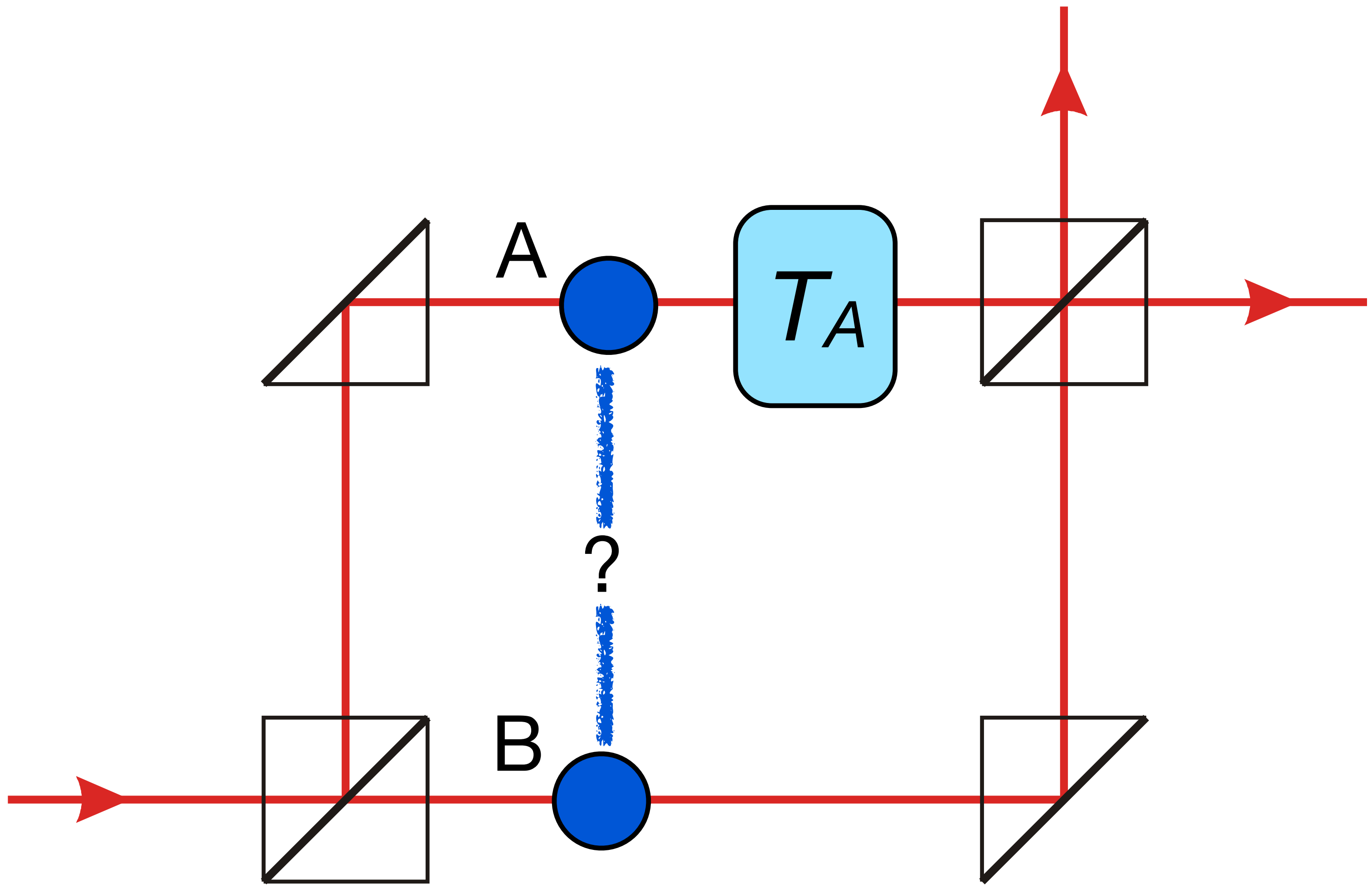

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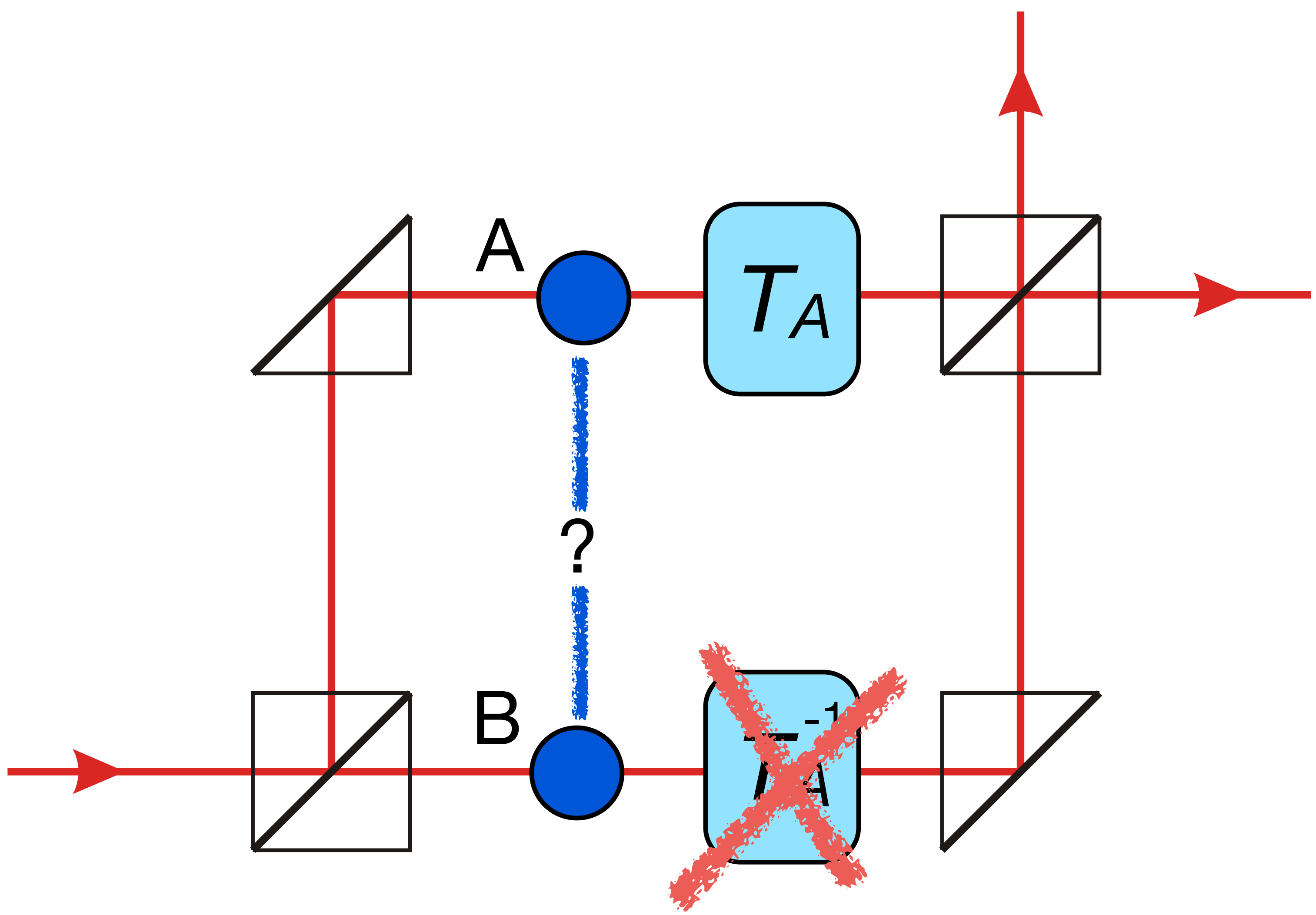

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Consequences for actual interference experiments:

PHYSICAL REVIEW LETTERS

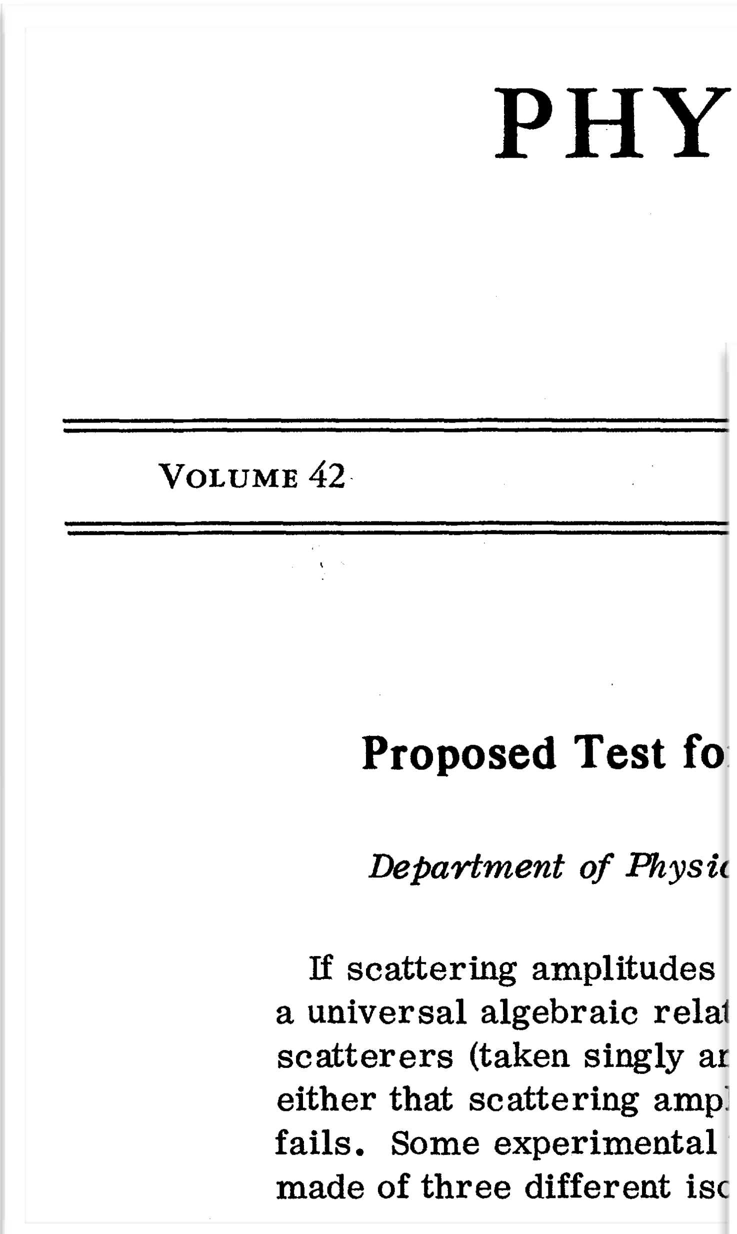
VOLUME 42 12 MARCH 1979 NUMBER 11

Proposed Test for Complex versus Quaternion Quantum Theory

Asher Peres
Department of Physics, Technion–Israel Institute of Technology, Haifa, Israel
(Received 7 December 1978)

If scattering amplitudes are ordinary complex numbers (not quaternions) then there is a universal algebraic relationship between the six coherent cross sections of any three scatterers (taken singly and pairwise). A violation of this relationship would indicate either that scattering amplitudes are quaternions, or that the superposition principle fails. Some experimental tests are proposed, involving neutron diffraction by crystals made of three different isotopes, neutron interferometry, and K_S-meson regeneration.

Consequences for actual interference experiments:


PHYSICAL REVIEW LETTERS

VOLUME 42

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Department of Physic

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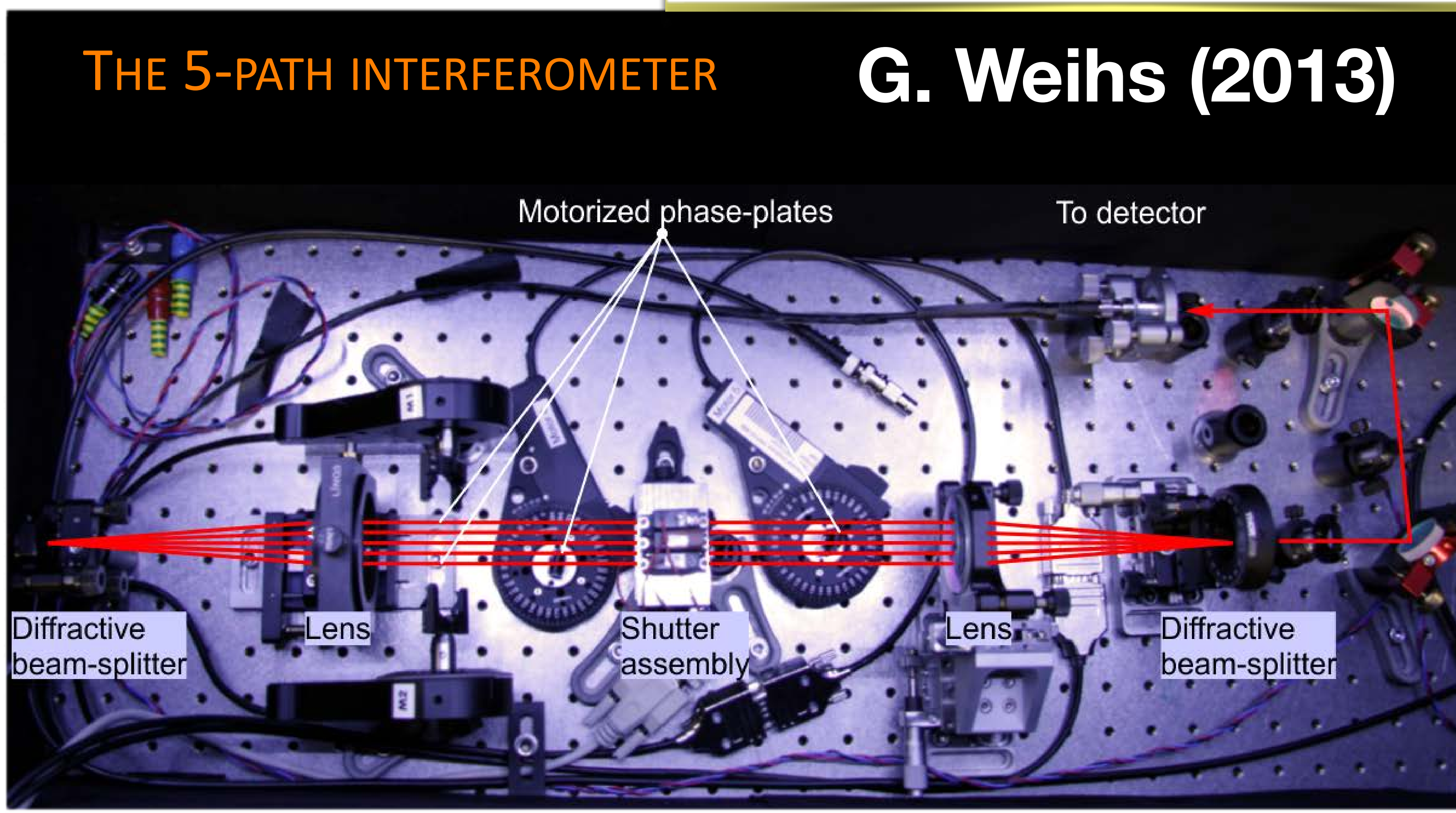
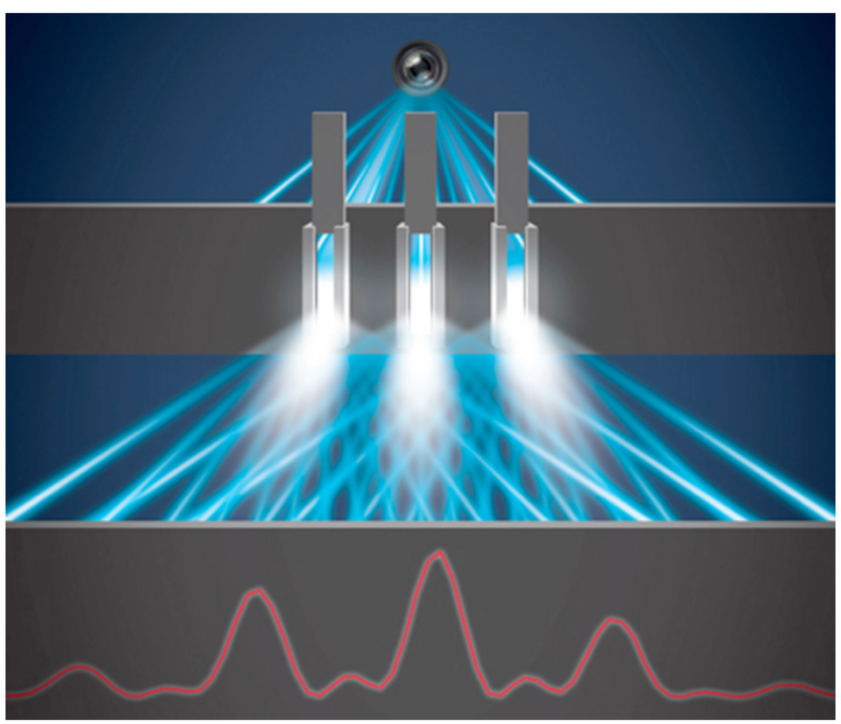
- Generalized Peres Test
  - Quaternion quantum mechanics?
  - Octonion quantum mechanics?

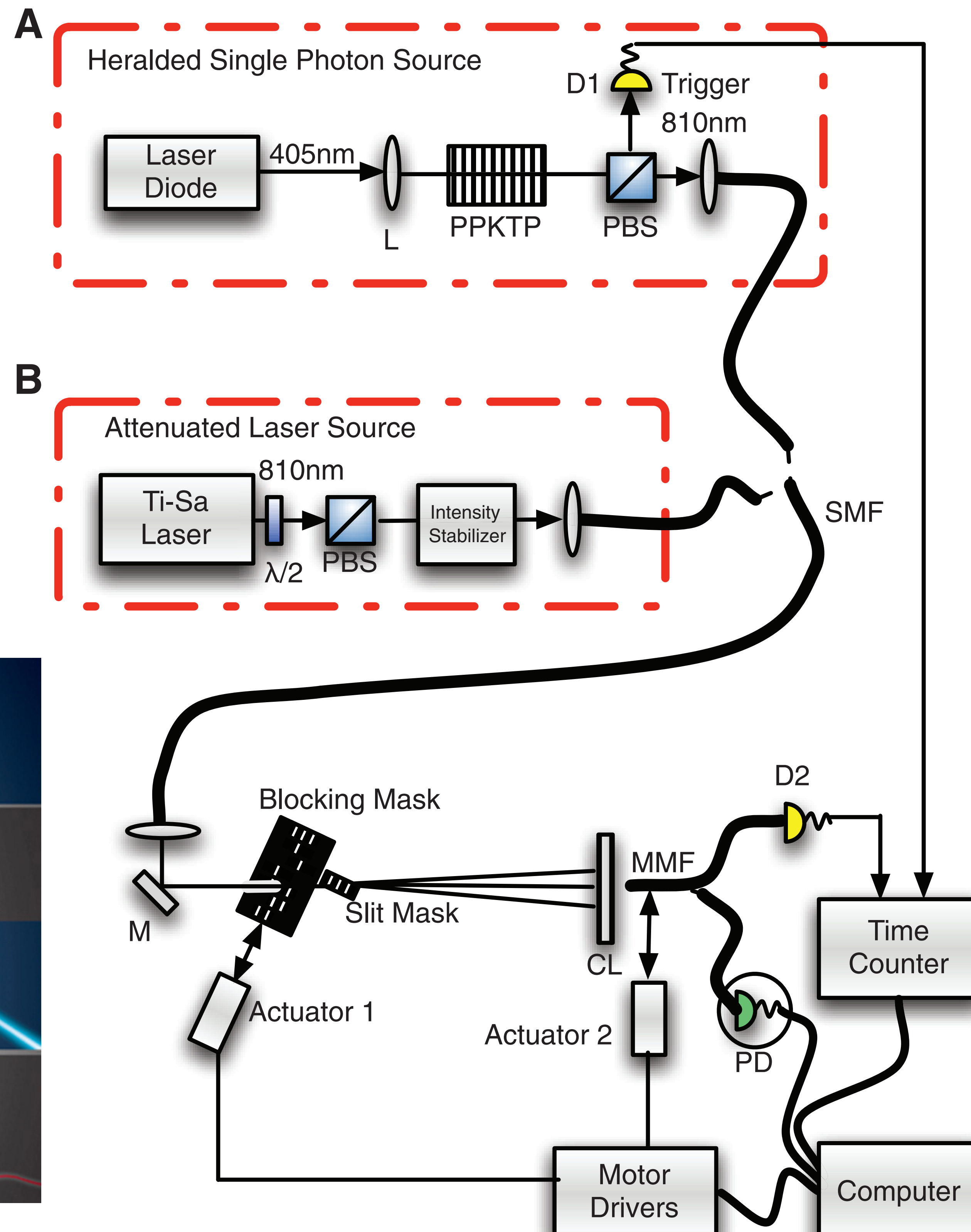


Relativistic constraints on the state space

Science
AAAS

U. Sinha, C. Couteau, T. Jennewein, R. Laflamme, G. Weihs, *Ruling Out Multi-Order Interference in Quantum Mechanics*, Science **329**, 418 (2010).





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- Structure of quantum theory is **closely related** to the structure of spacetime.
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- Is QT and the path integral the **only possible theory** describing detector click probabilities in relativistic spacetime?
- **Can we learn something about quantum gravity by studying this relationship?**
  Is the structure of QT modified in regimes where the structure of spacetime is modified?

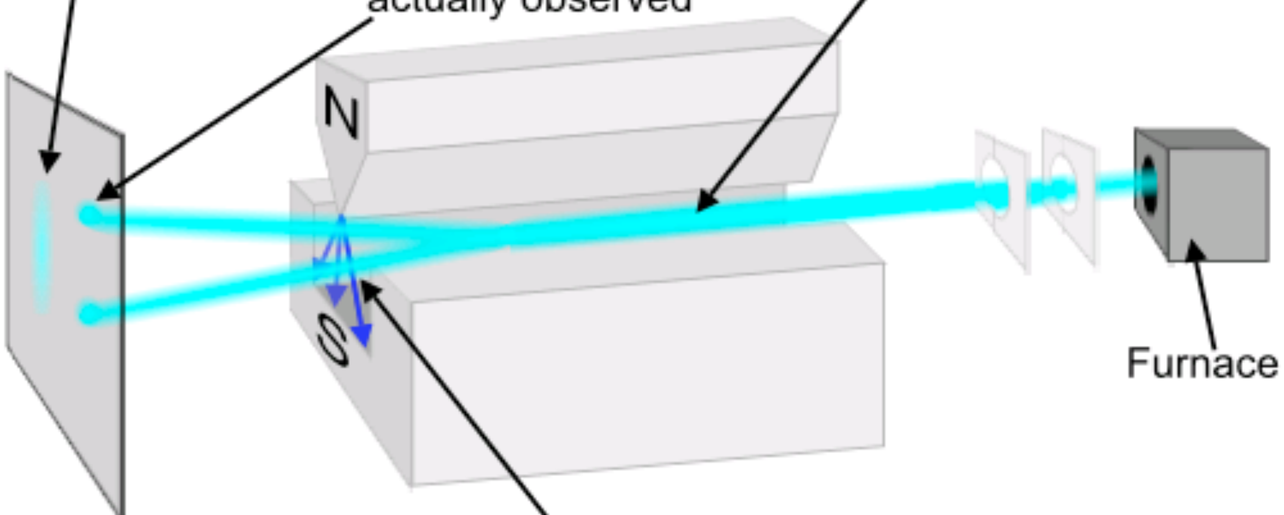
Further evidence

spatial rotations

↕ 1:1

quantum 2-level state space

transformations of the probabilistic state



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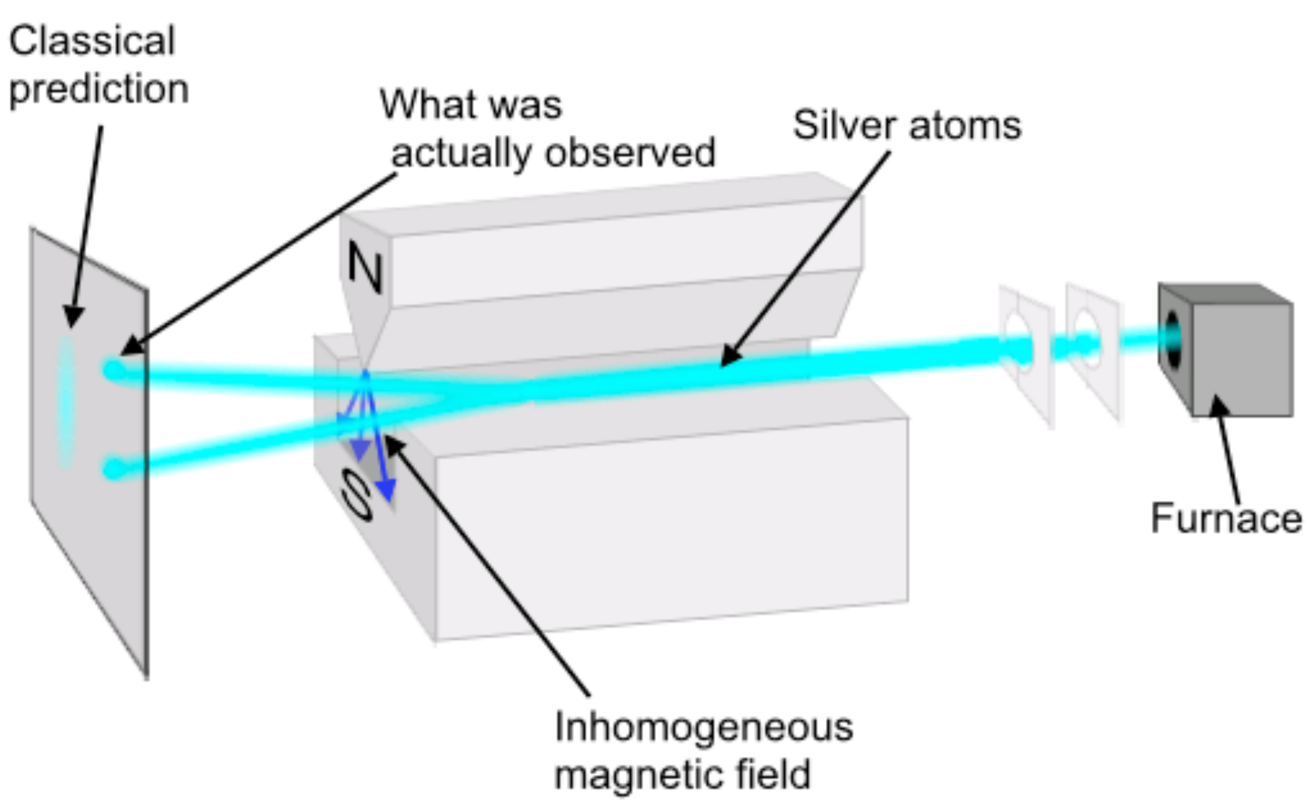

C. F. von Weizsäcker (>1954): "ur theory"

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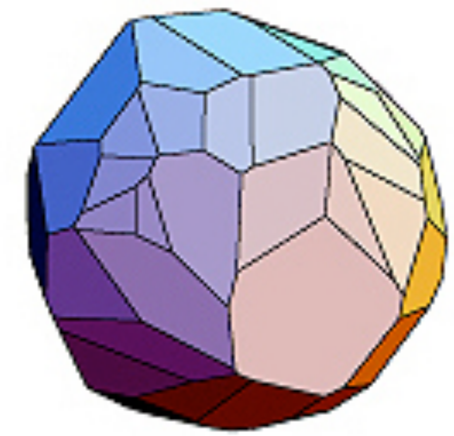

arbitrary state space

Standard perspective:

"That's all trivial, because the qubit *is* just a representation of SU*(2)*!"

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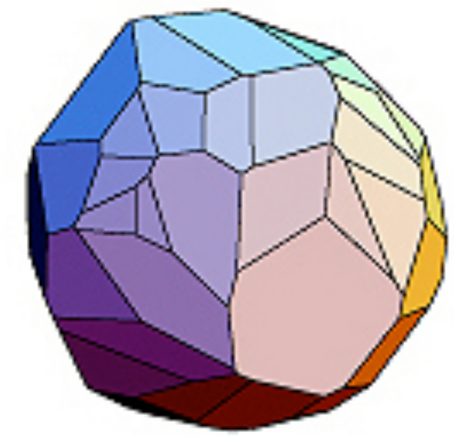

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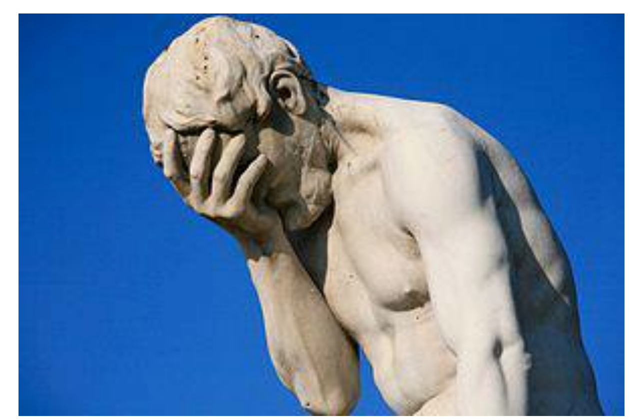


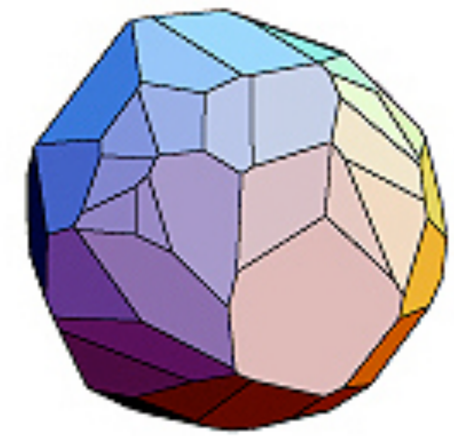
Further evidence

spatial rotations

↕ 1:1

transformations of the probabilistic state

quantum 2-level state space



arbitrary state space

Standard perspective:

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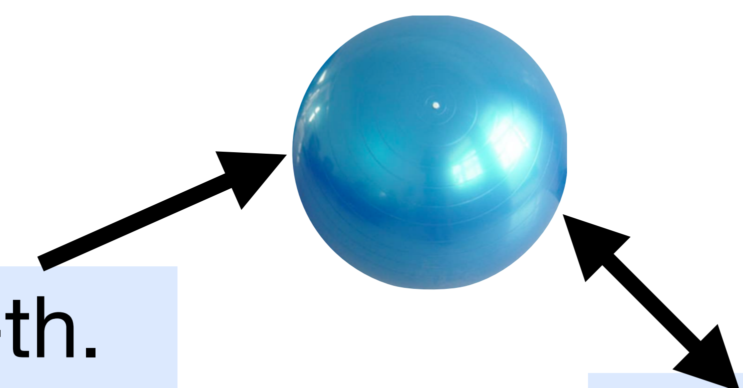

info-th. postulates

spatial rotations

Different view: it's highly remarkable!

4. Conclusions

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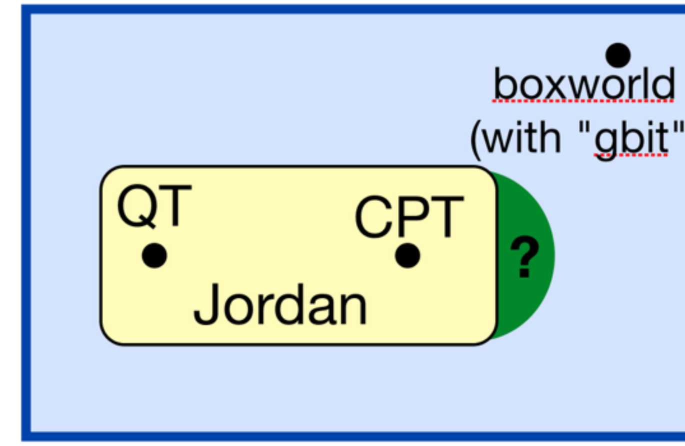
H. Barnum, **MM**, and C. Ududec, *Higher-order interference and single-system postulates characterizing quantum theory*, New J. Phys. **16**, 123029 (2014).

- QT can be derived from simple postulates.
- Open Problem: are there natural "higher-order interference" state spaces?



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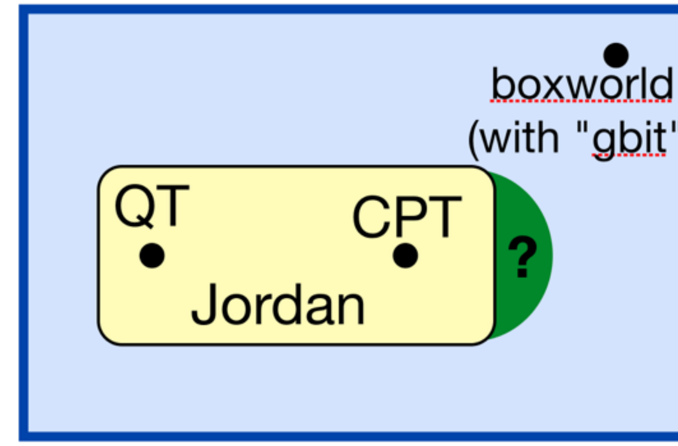


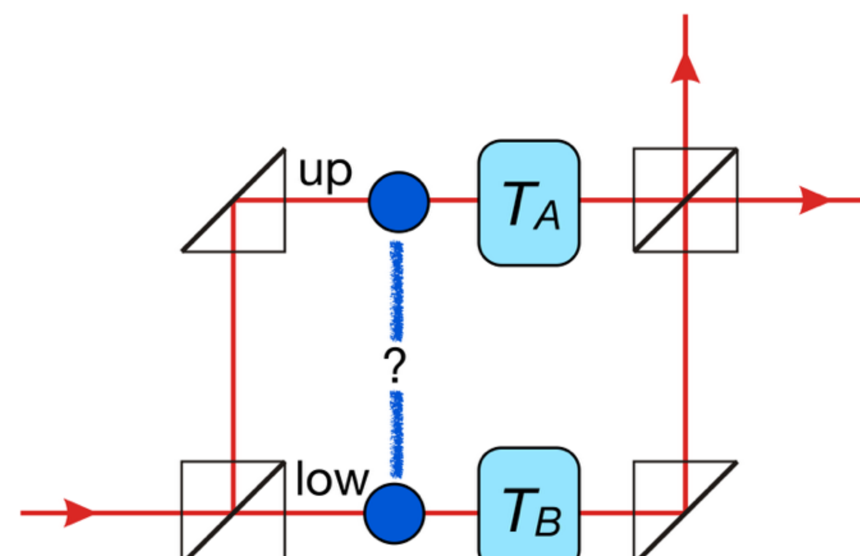
Ll. Masanes, **MM**, D. Pérez-García, and R. Augusiak, *Entanglement and the three-dimensionality of the Bloch ball*, J. Math. Phys. **55**, 122203 (2014).

- The Bloch ball is **3D** because otherwise bits could not interact.

A. Garner, **MM**, O. Dahlsten, arXiv:1412.7112

- The bloch ball is **3D** (or maybe 5D) due to relativity of simultaneity.

QT ⟷ spacetime



Western