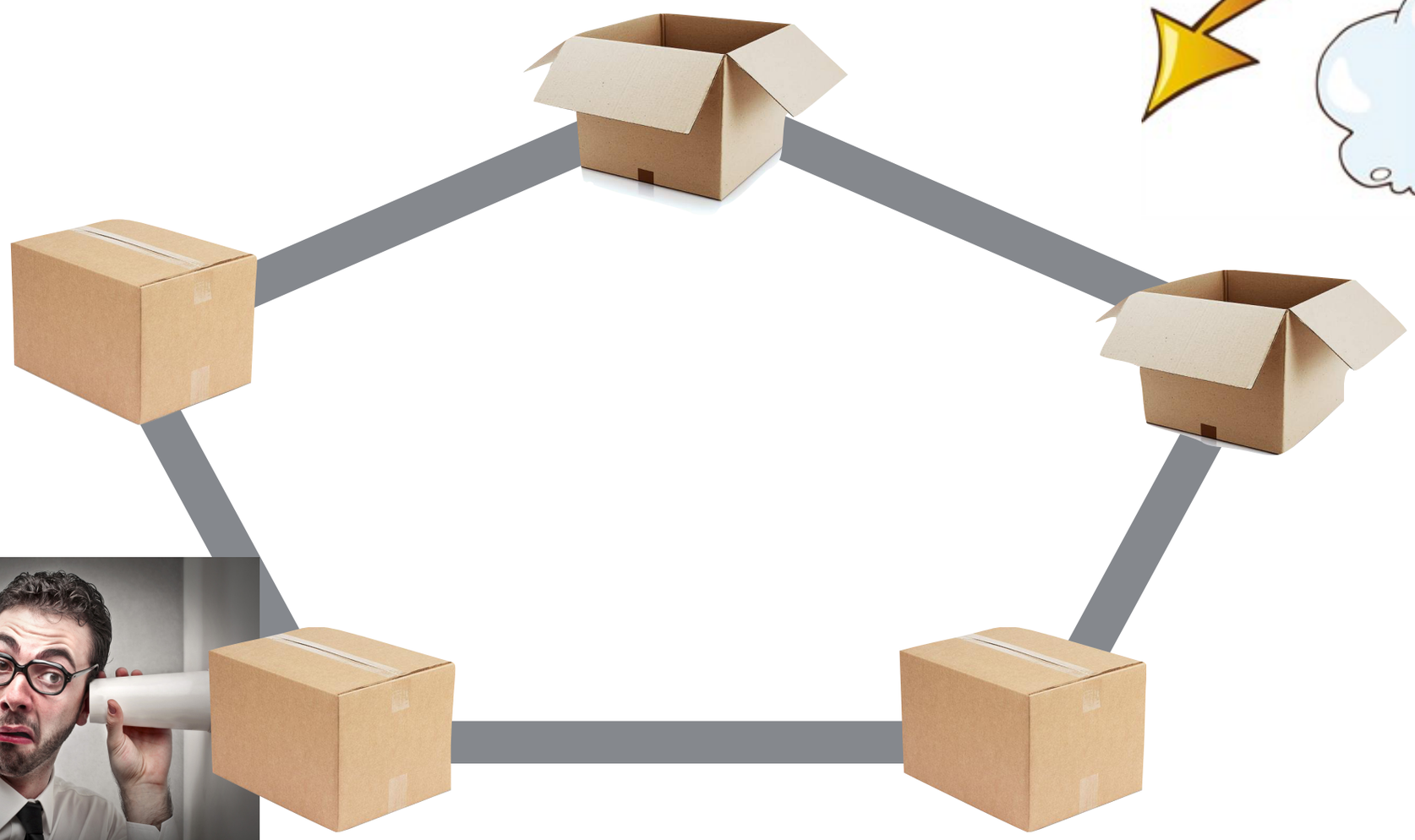
Consistent exclusivity and interference in probabilistic theories

Markus P. Müller
University of Western Ontario, and Perimeter

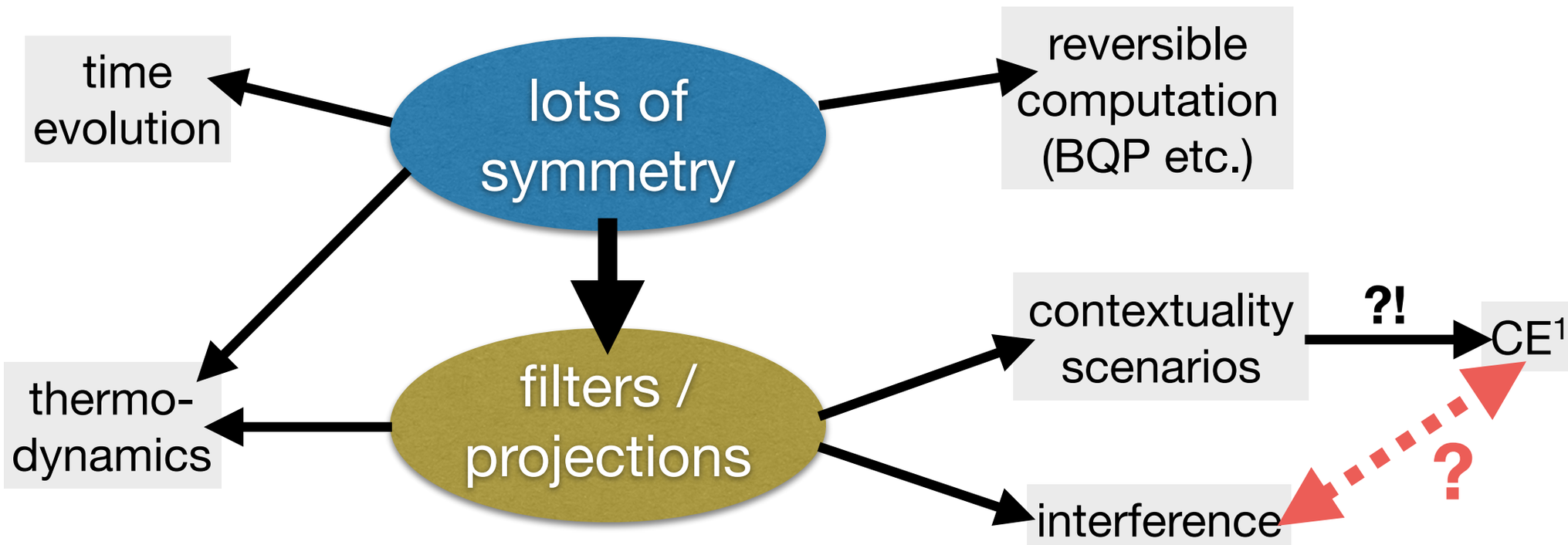


Outline

1. Questions on contextuality scenarios

Operational meaning of "compatibility"? Why CE[1]?

2. The general-probabilistic picture



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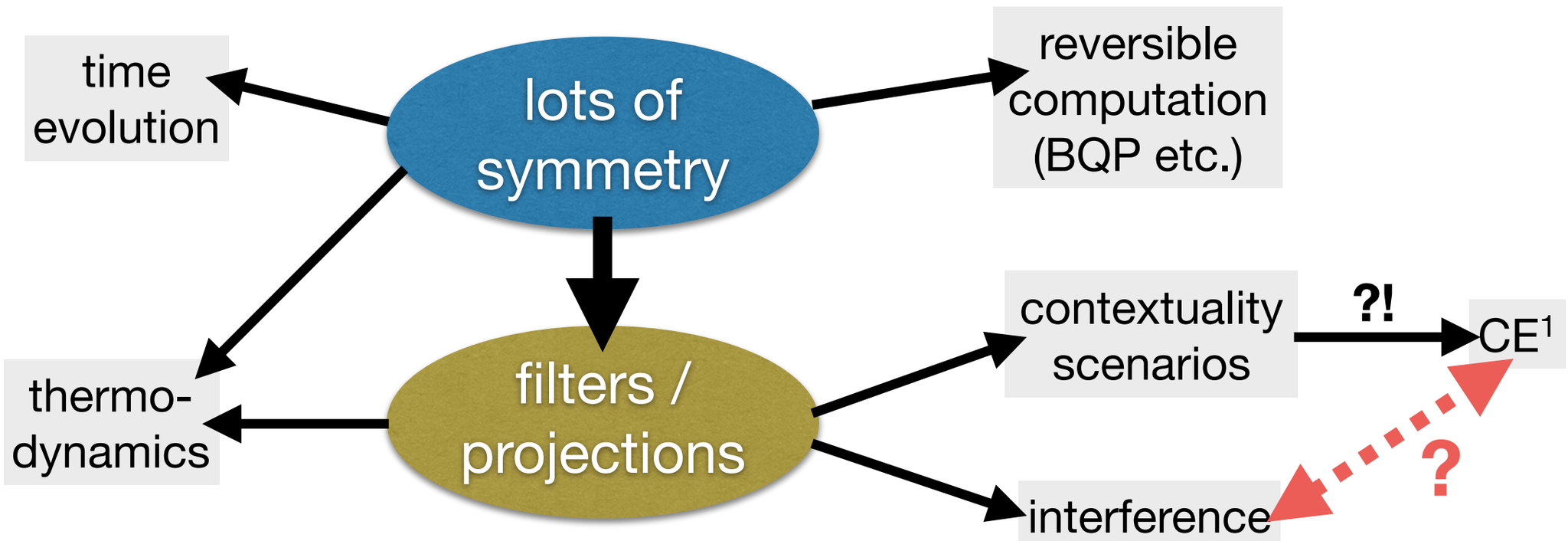

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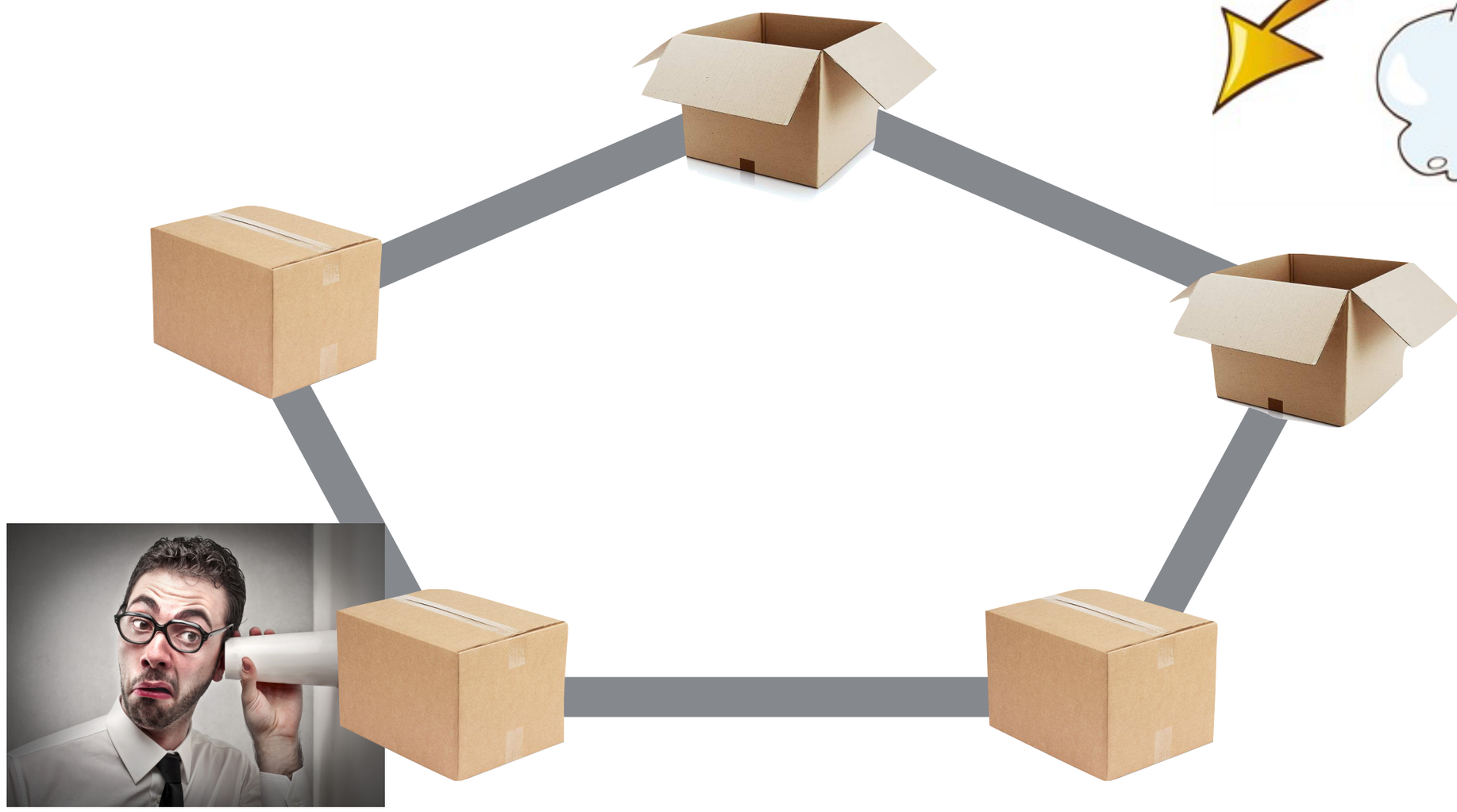
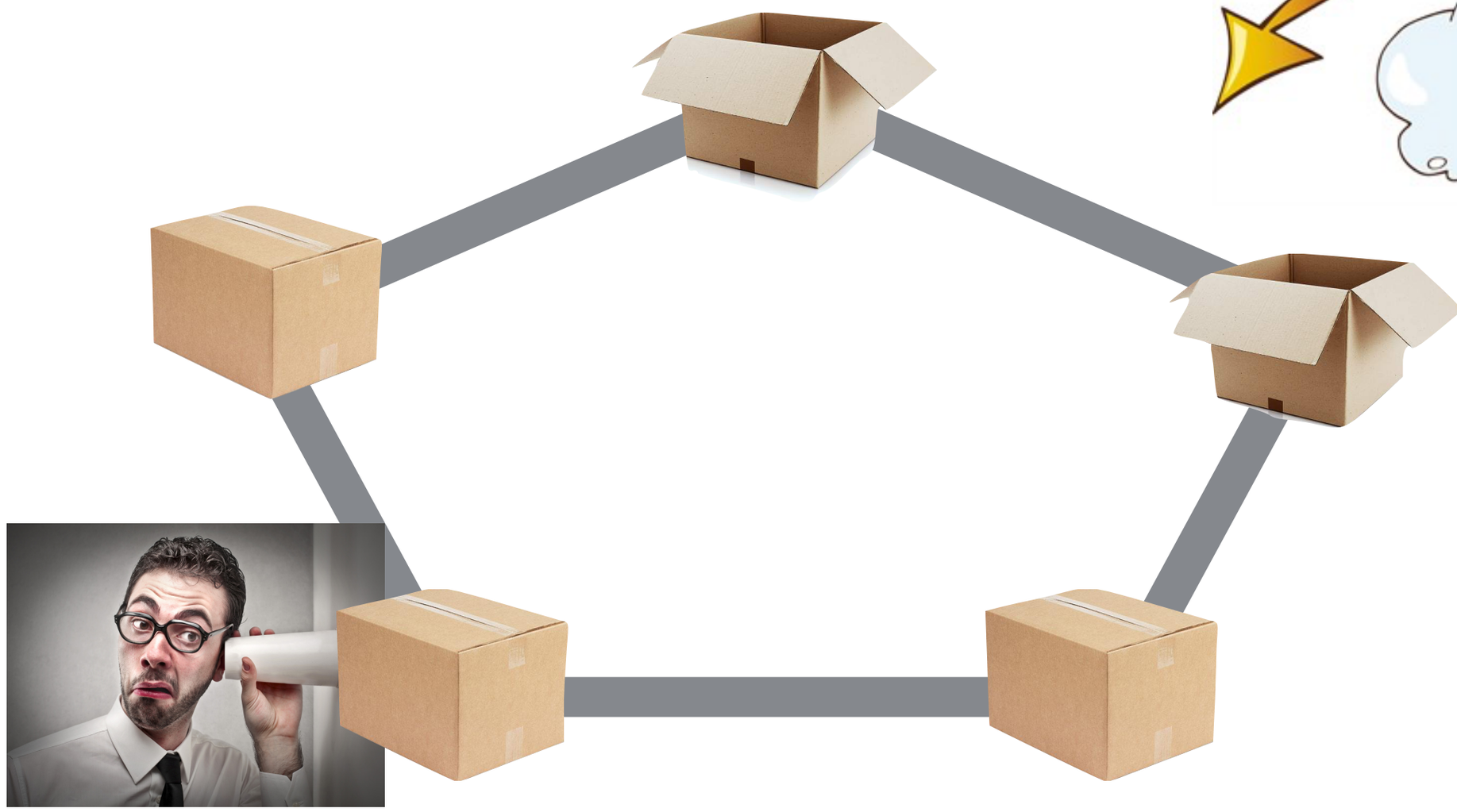
What does it mean that "*we can only open a pair of adjacent boxes at once*"?



1. Questions on contextuality scenarios

What does it mean that "*we can only open a pair of adjacent boxes at once*"?
What are "*compatible and exclusive questions*"?



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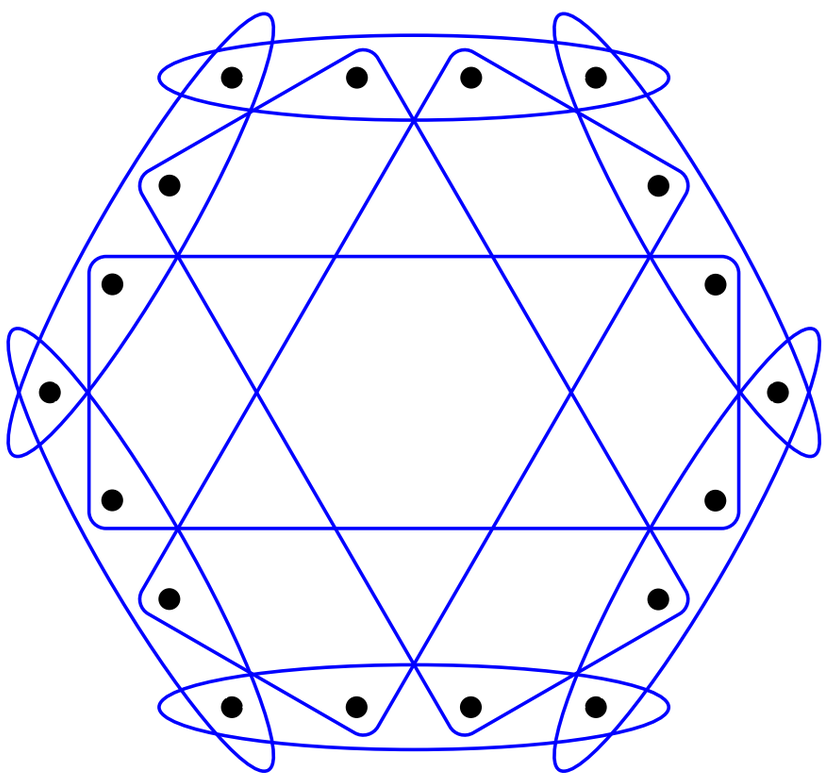
In the device-independent approach, these notions are taken as *primitives*, without attempt of *operational/formal definition*.

A. Acín, T. Fritz, A. Leverrier, and A. B. Sainz, *A Combinatorial Approach to Nonlocality and Contextuality*, Commun. Math. Phys. **334**(2), 533-628 (2015); arXiv:1212.4084.

2.2.1. Definition. *A* ***contextuality scenario*** *is a hypergraph H with set of vertices $V(H)$ and set of edges $E(H) \subseteq 2^{V(H)}$ such that $\bigcup_{e \in E(H)} e = V(H)$.*



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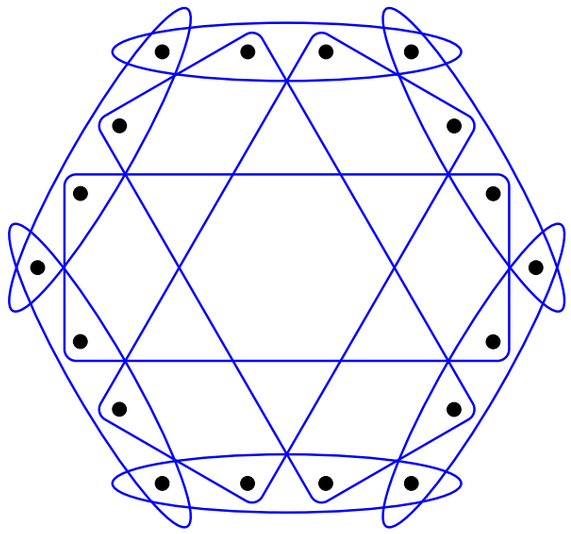
5.1.1. Definition. *Let H be a contextuality scenario. An assignment of probabilities $p : V(H) \to [0,1]$ is a* ***quantum model*** *if there exist a Hilbert space $\mathcal{H}$, a quantum state $\rho \in \mathcal{B}_{+,1}(\mathcal{H})$ and a projection operator $P_v \in \mathcal{B}(\mathcal{H})$ associated to every $v \in V$ which constitute projective measurements in the sense that*

$$\sum_{v \in e} P_v = \mathbb{1}_{\mathcal{H}} \quad \forall e \in E(H), \tag{5.1}$$

and reproduce the given probabilities,

$$p(v) = \mathrm{tr}\,(\rho P_v) \quad \forall v \in V(H). \tag{5.2}$$



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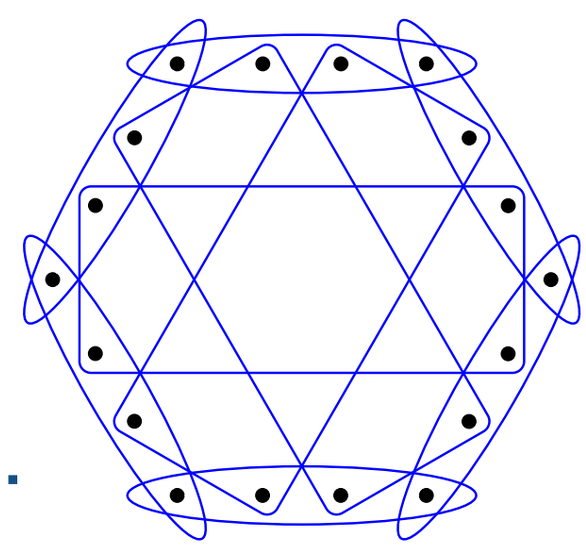


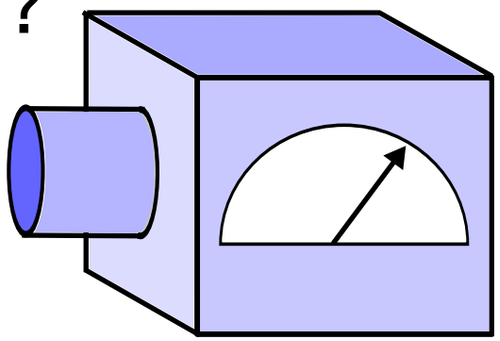
Furthermore,

CE[1] seems like a rather arbitrary property…

1. Questions on contextuality scenarios

Beyond QT, how do we know that the devices in our lab perform an analogue of a "projective measurement"?

What does that even *mean*?



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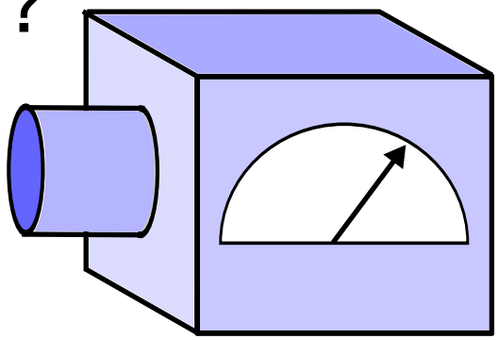

What does that even *mean*?

A. Cabello, S. Severini, and A. Winter, *(Non-)Contextuality of Physical Theories as an Axiom*, arXiv:1010.2163

GPT analog of projector was taken to be: *sum of extremal effects.*
But this is only one possible choice…

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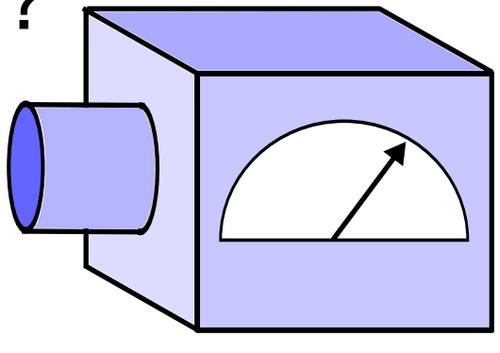

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"*Compatible* questions can be jointly asked *without mutual disturbance*" ⇒ need to talk about *post-measurement states*

⇒ need **GPTs**, in particular **projections / filters** in GPTs

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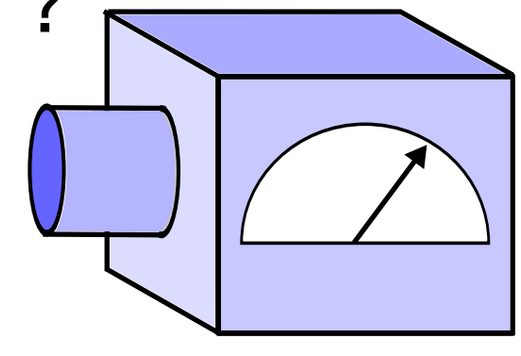

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⇒ need **GPTs**, in particular **projections / filters** in GPTs

replaced by *measurement disturbance.*

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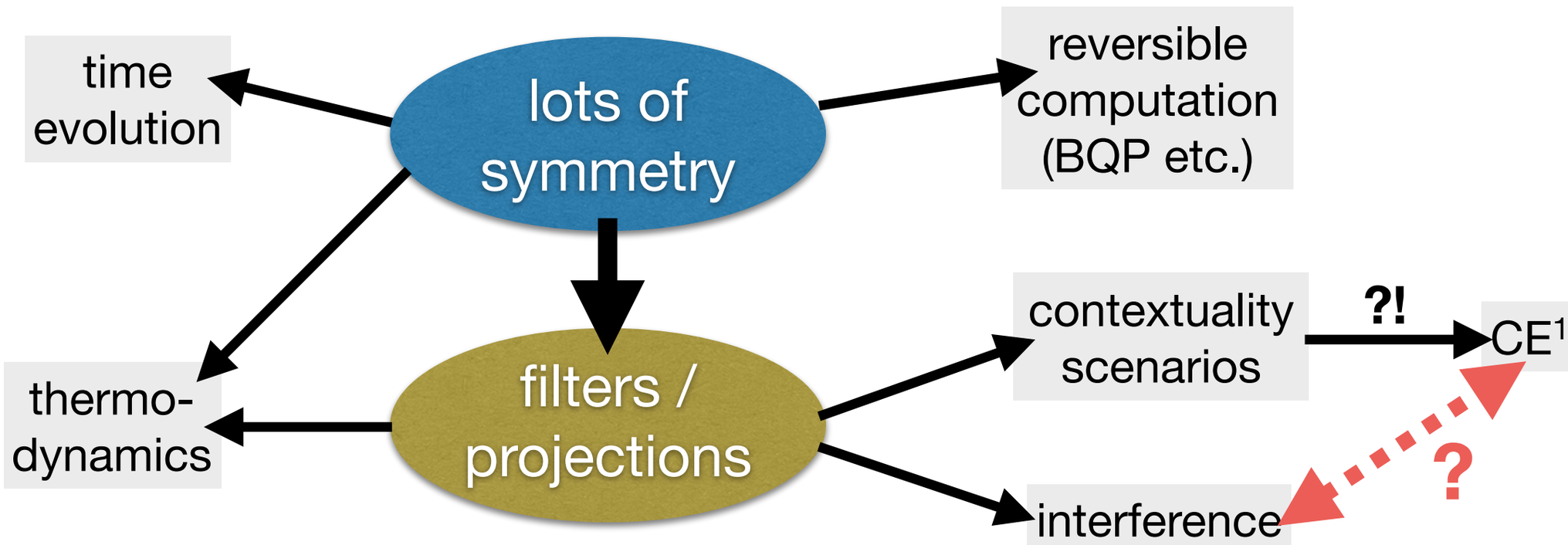

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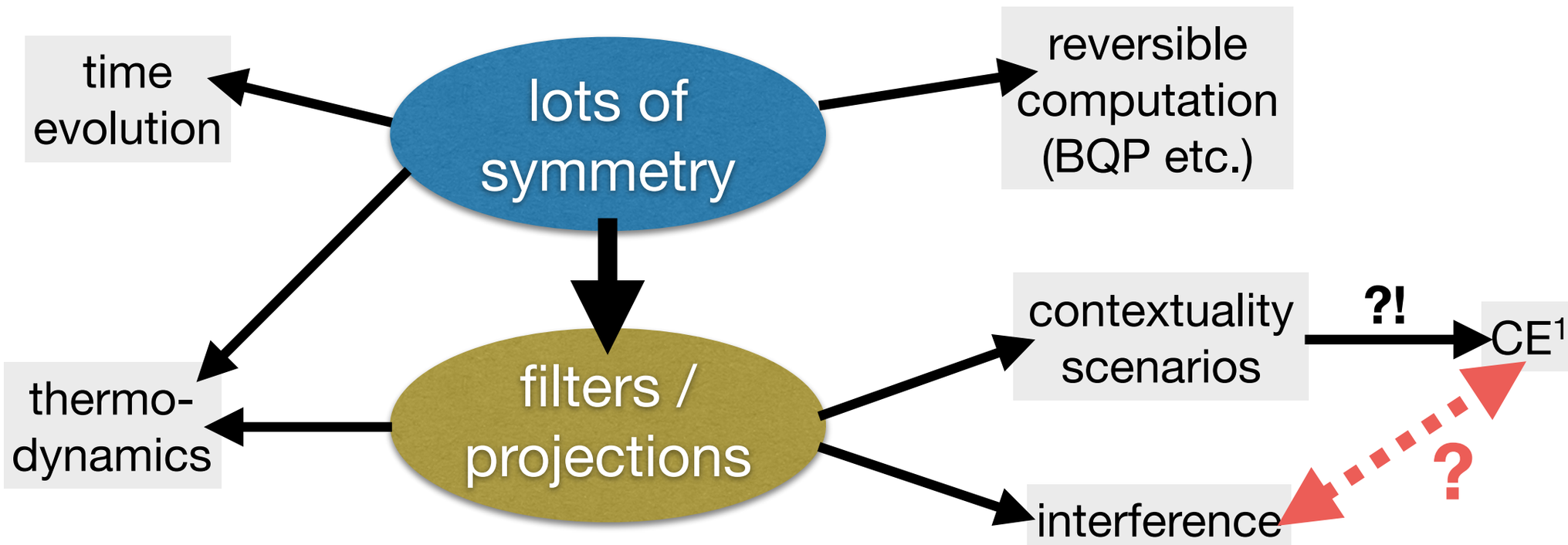

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Let's look at **interference** first…

Interference: Sorkin's hierarchy

R. D. Sorkin, *Quantum mechanics as quantum measure theory*, Mod. Phys. Lett. A **9**, 3119-3128 (1994).
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$p_{i,j,\ldots}$:= probability of event,
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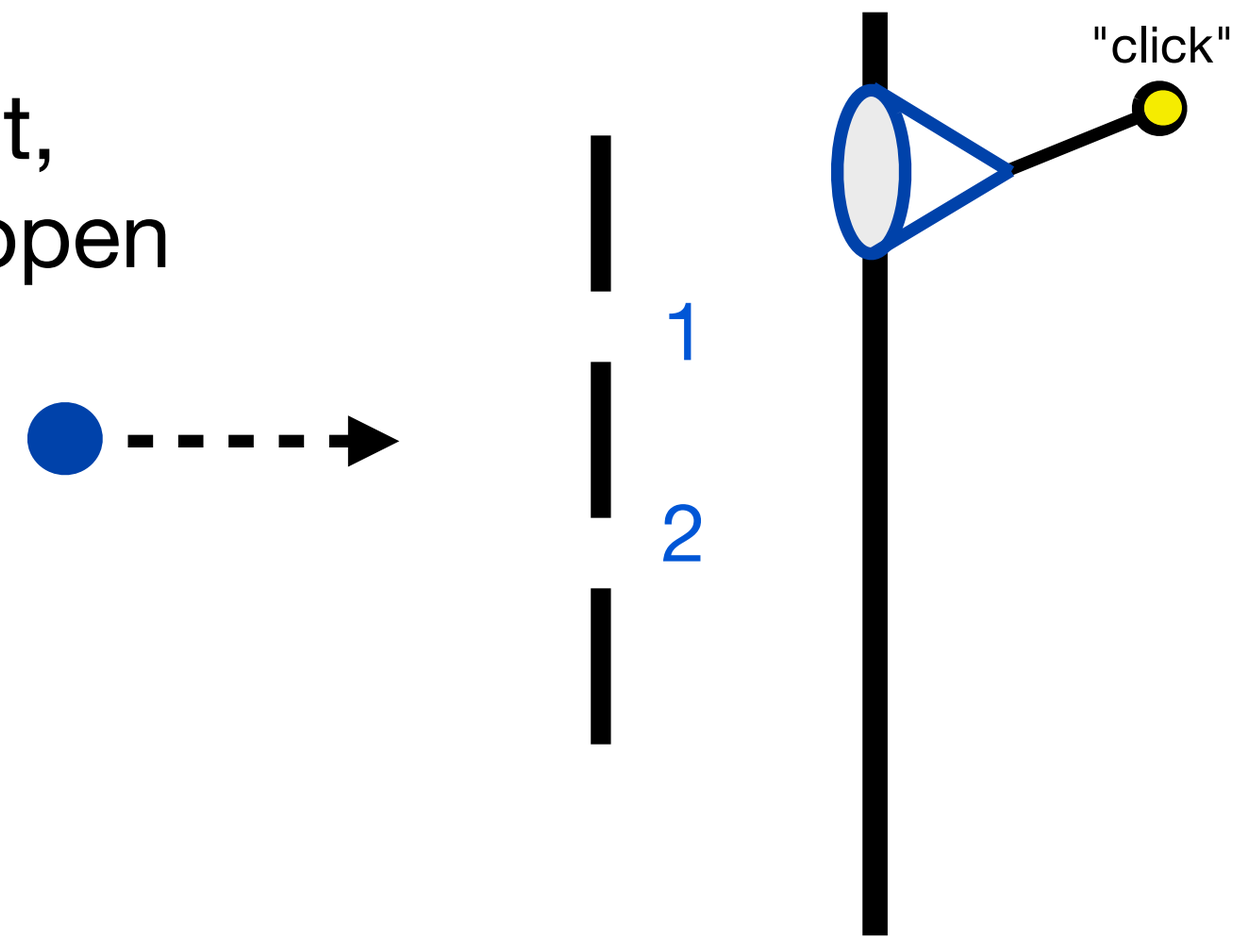

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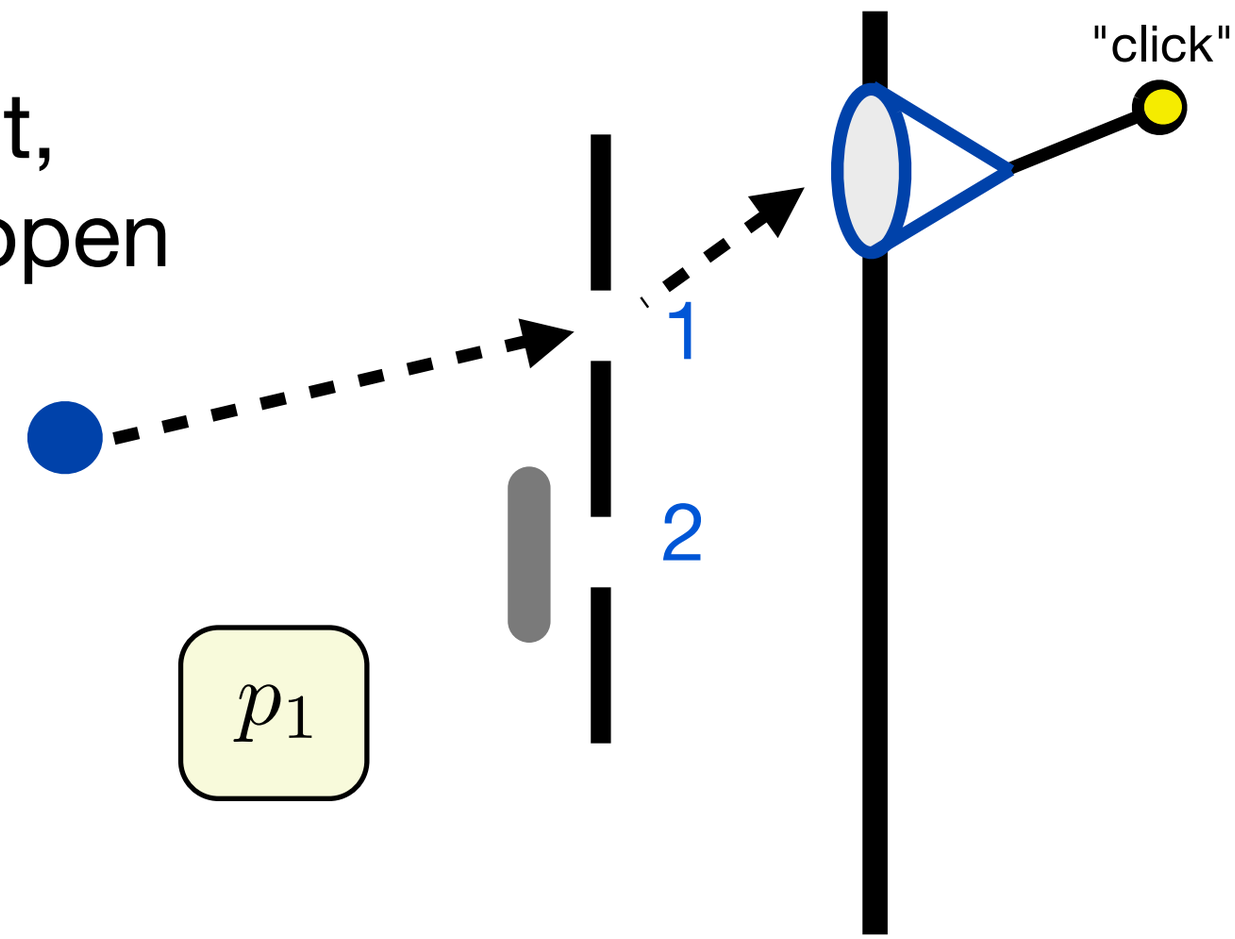

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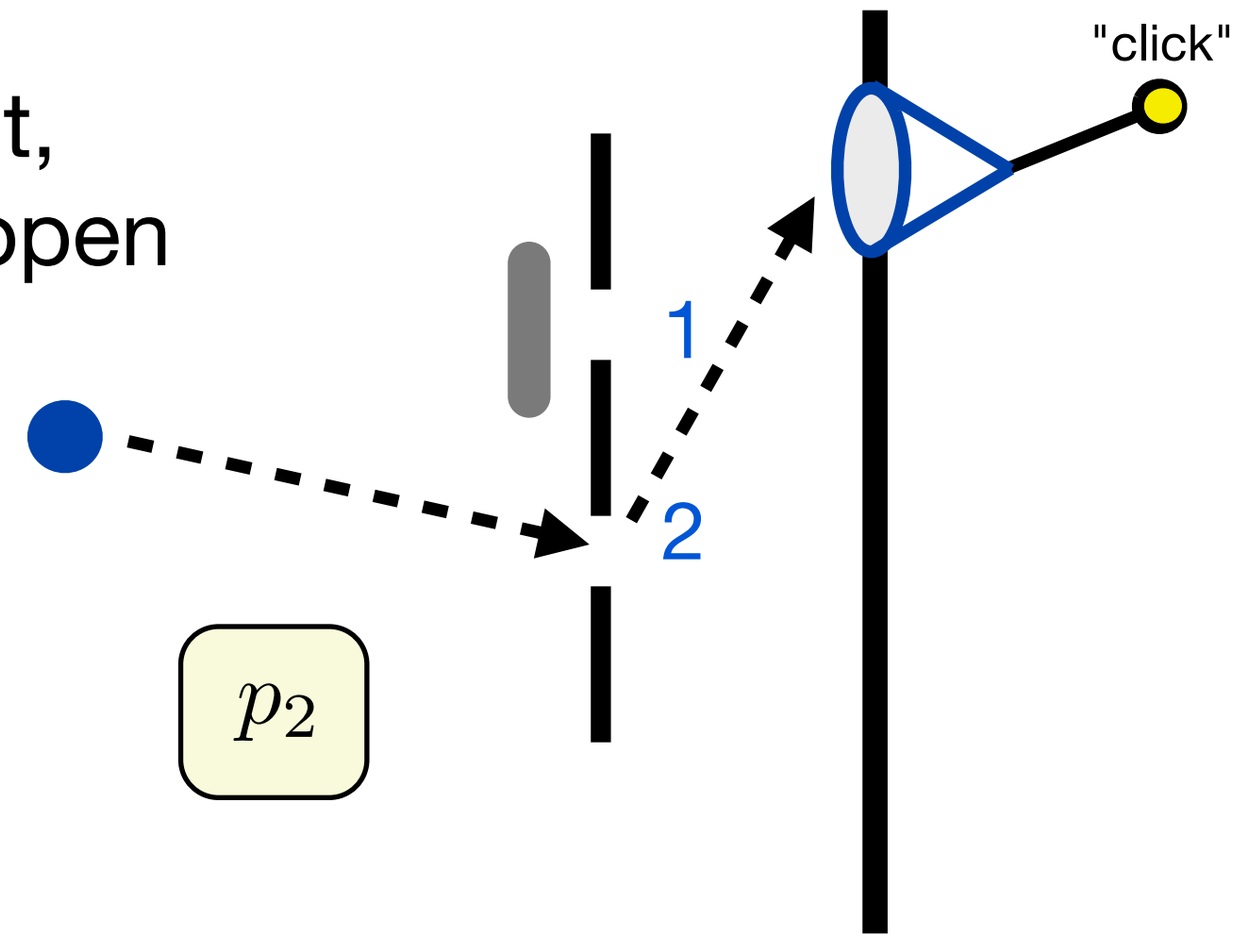

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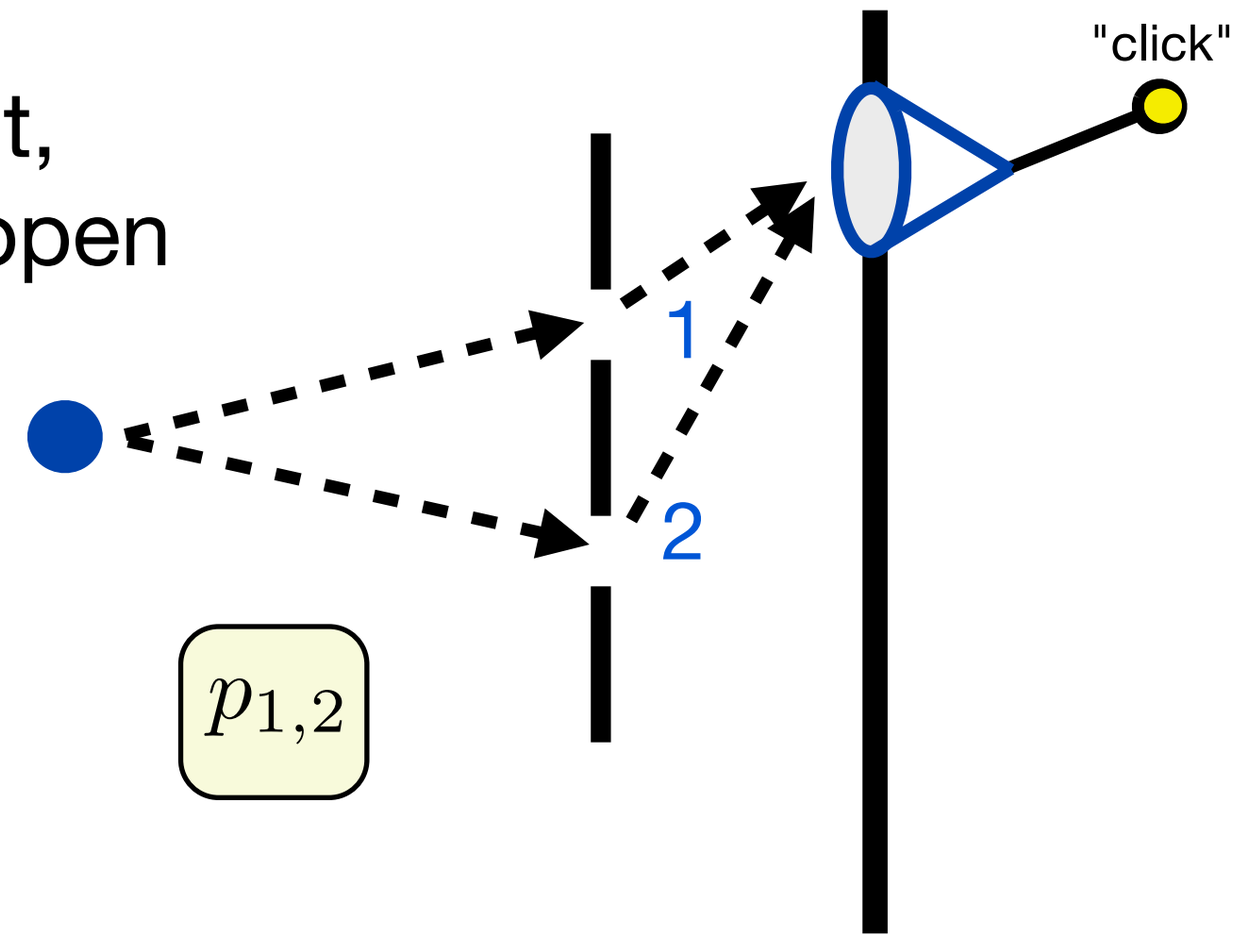

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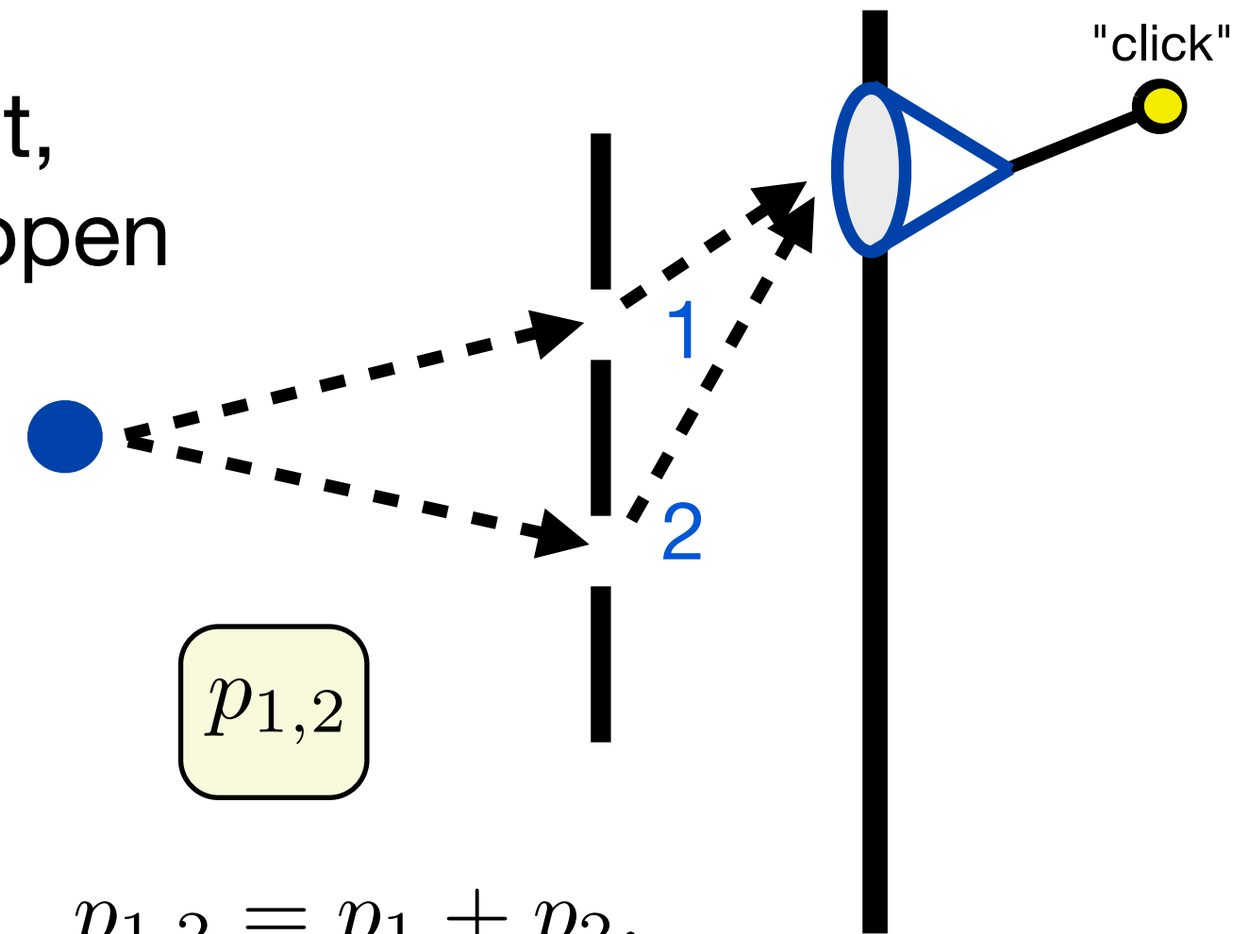

Classical probability theory: $p_{1,2} = p_1 + p_2.$

Quantum theory: $p_{1,2} \neq p_1 + p_2.$

Interference!

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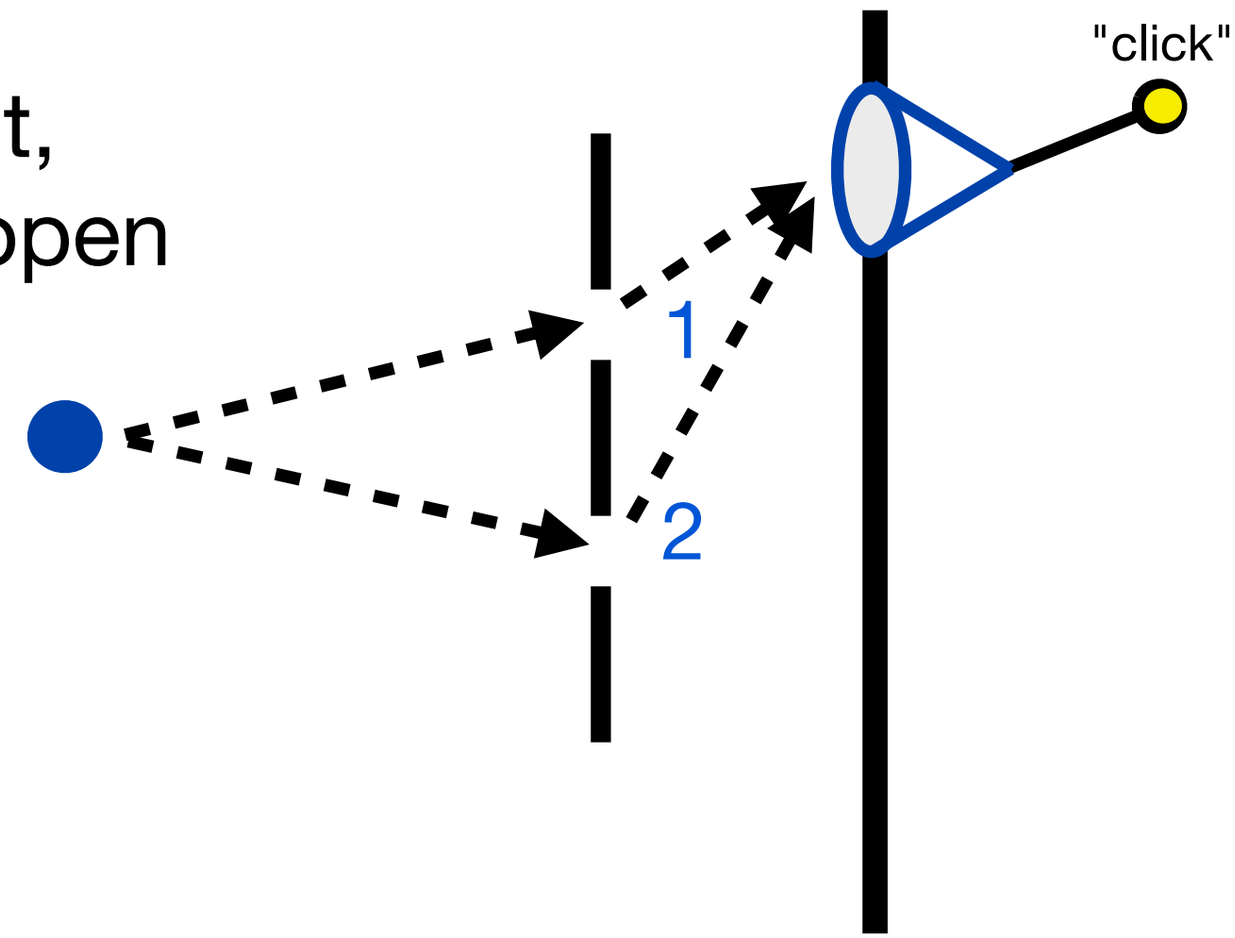

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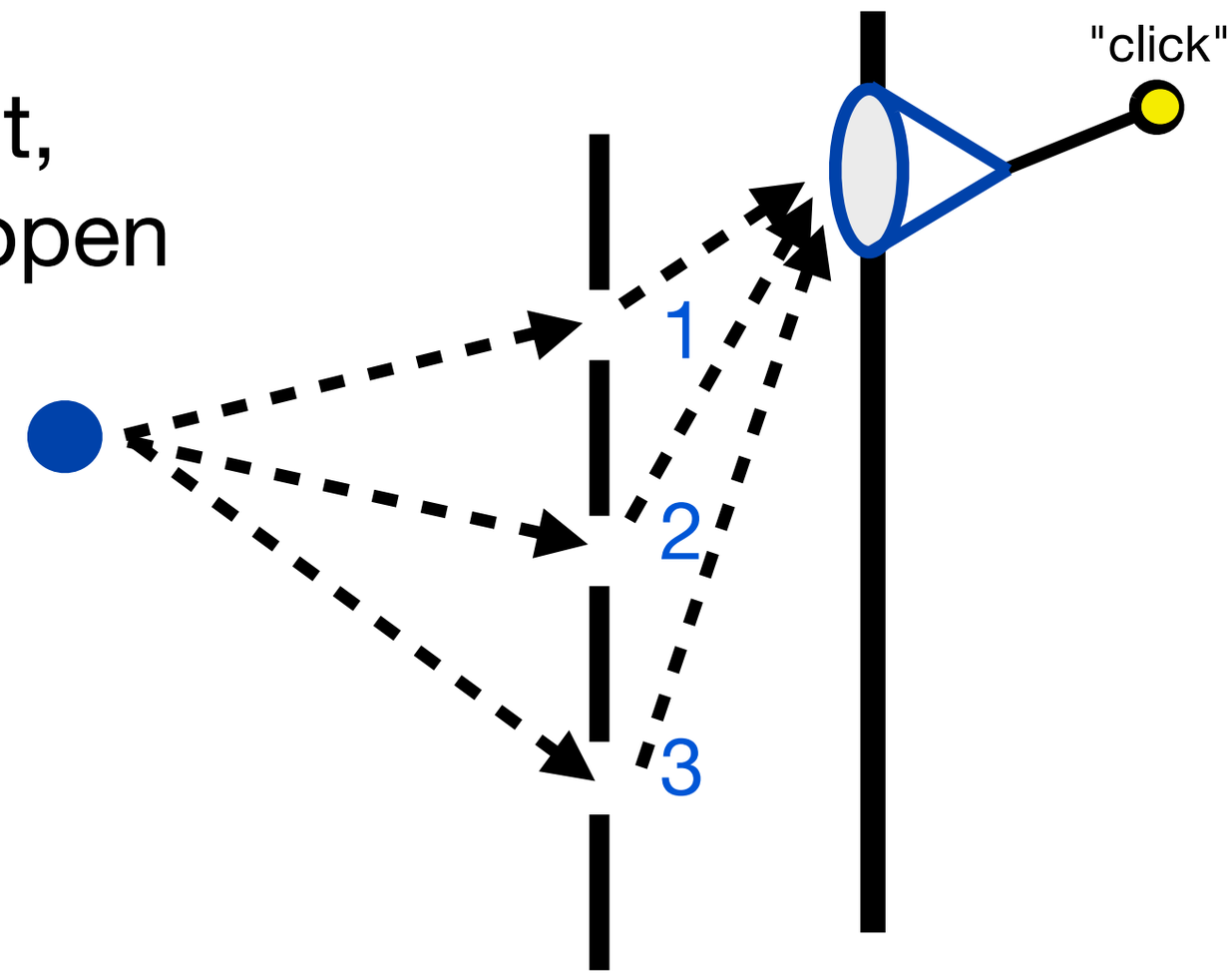

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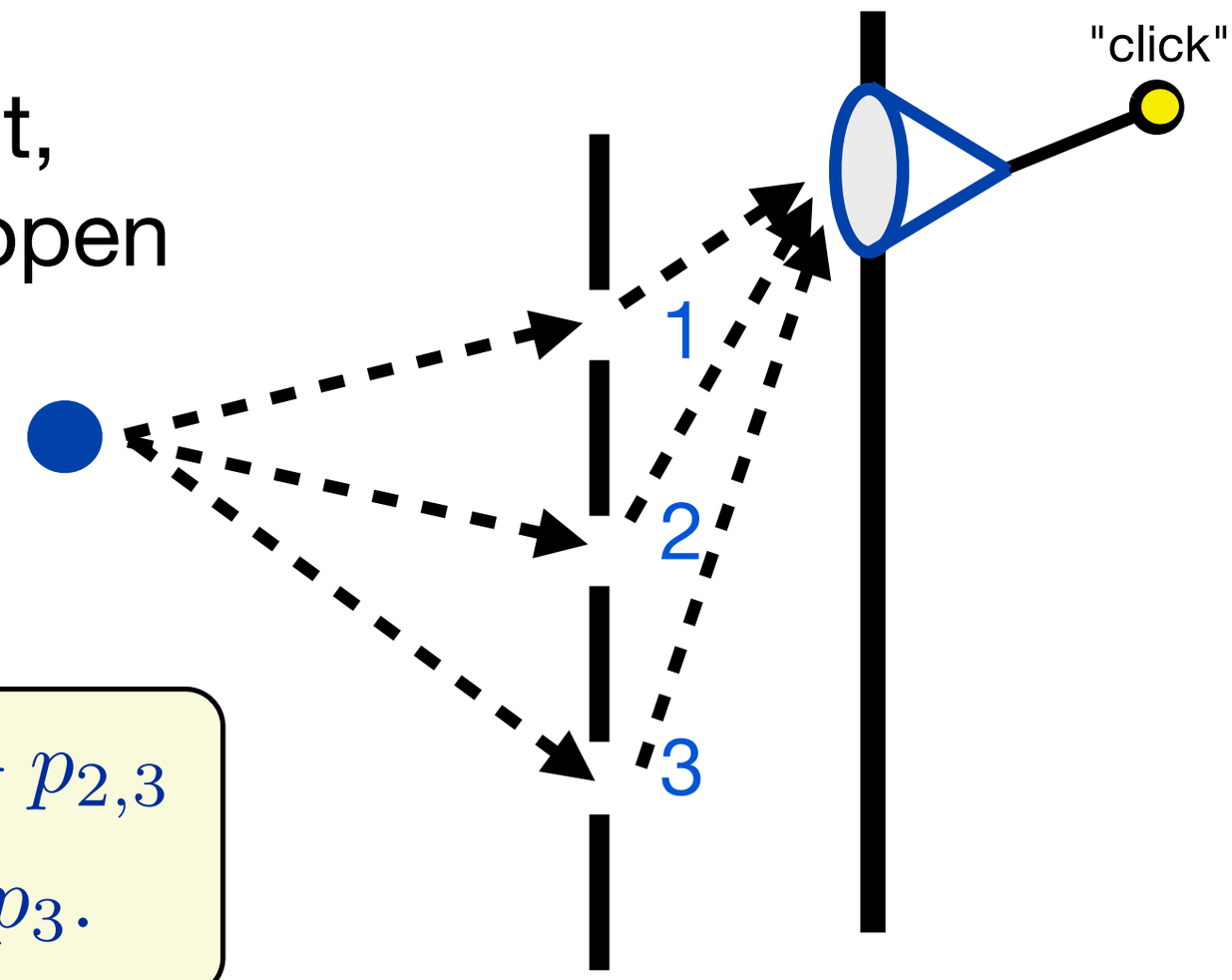

Surprisingly (?), quantum theory satisfies

$$p_{1,2,3} = p_{1,2} + p_{1,3} + p_{2,3} - p_1 - p_2 - p_3.$$

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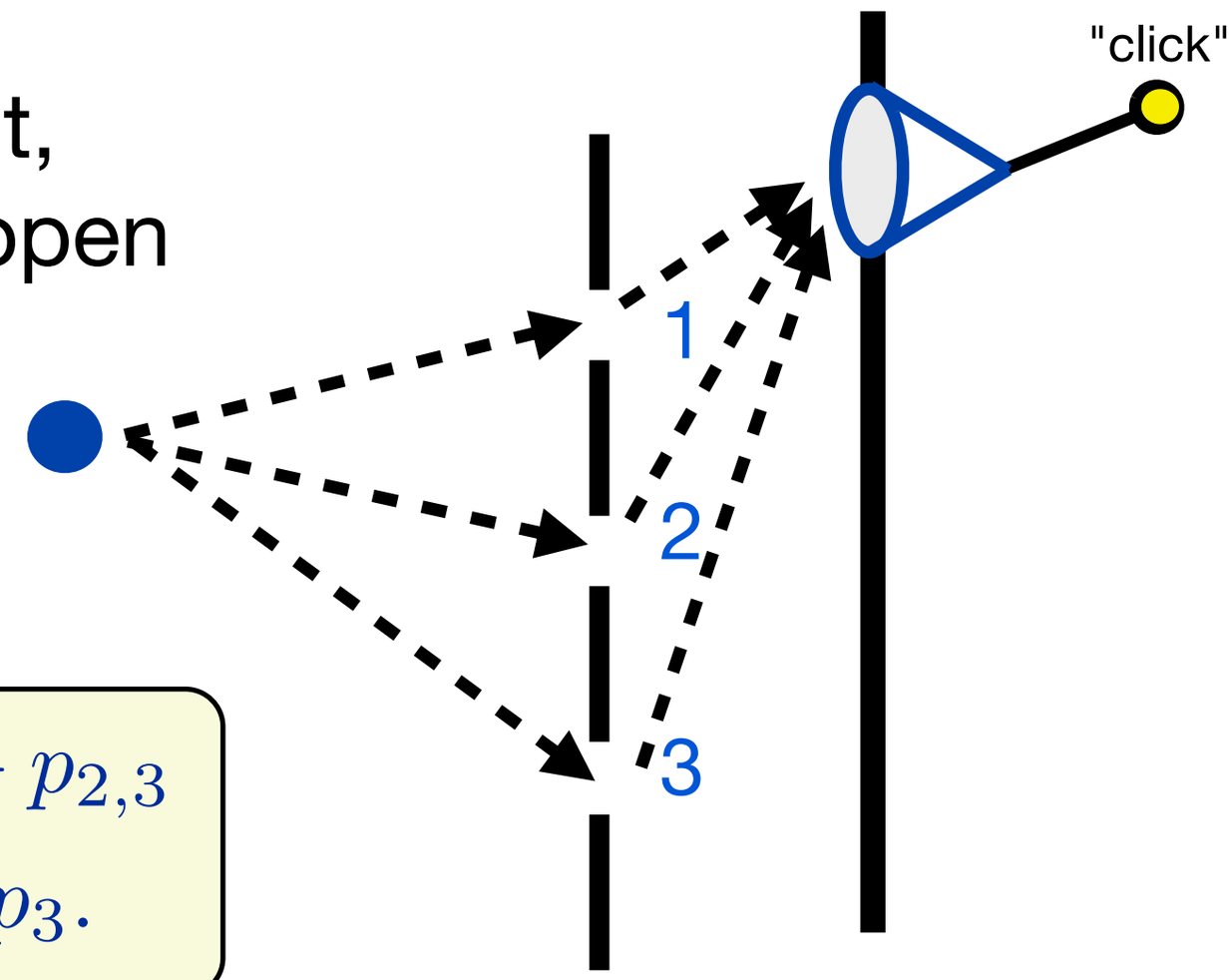

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No 3rd-order interference in QT!

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Sorkin:

$$I_2(A,B) \equiv |A \amalg B| - |A| - |B|$$

$$I_3(A,B,C) \equiv |A \amalg B \amalg C| - |A \amalg B| - |B \amalg C| - |A \amalg C| + |A| + |B| + |C$$

or in general,

$$I_n(A_1, A_2, \cdots, A_n) \equiv |A_1 \amalg A_2 \amalg \cdots A_n| - \sum |(n-1)sets| + \sum |(n-2)sets| \cdots \pm \sum_{j=1}^{n} |A_j|$$

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Classical probability theory: $I_2 = I_3 = I_4 = \ldots = 0.$

Quantum theory: $I_2 \neq 0, \;\; I_3 = I_4 = \ldots = 0.$

Experimental tests for higher-order interference

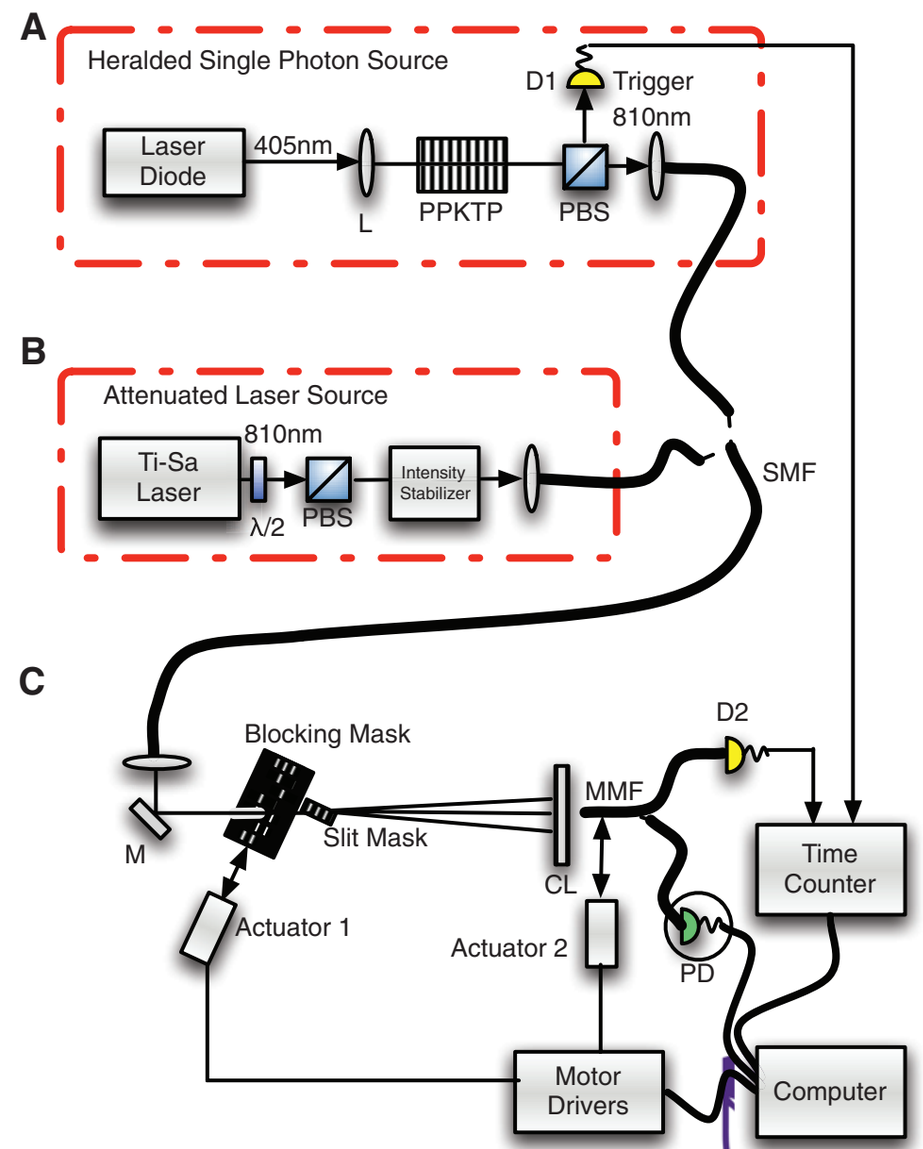
Ruling Out Multi-Order Interference in Quantum Mechanics
Urbasi Sinha *et al.*
Science **329**, 418 (2010);
DOI: 10.1126/science.1190545
(U. Sinha, C. Couteau, T. Jennewein, R. Laflamme, G. Weihs)

$$\varepsilon = I_3 - \text{zerocount};$$

$$\boxed{\kappa := \frac{\varepsilon}{\delta};}$$

$$\delta = |I_{12}| + |I_{13}| + |I_{23}|,$$

$$I_{12} = p_{12} - p_1 - p_2 \text{ etc.}$$



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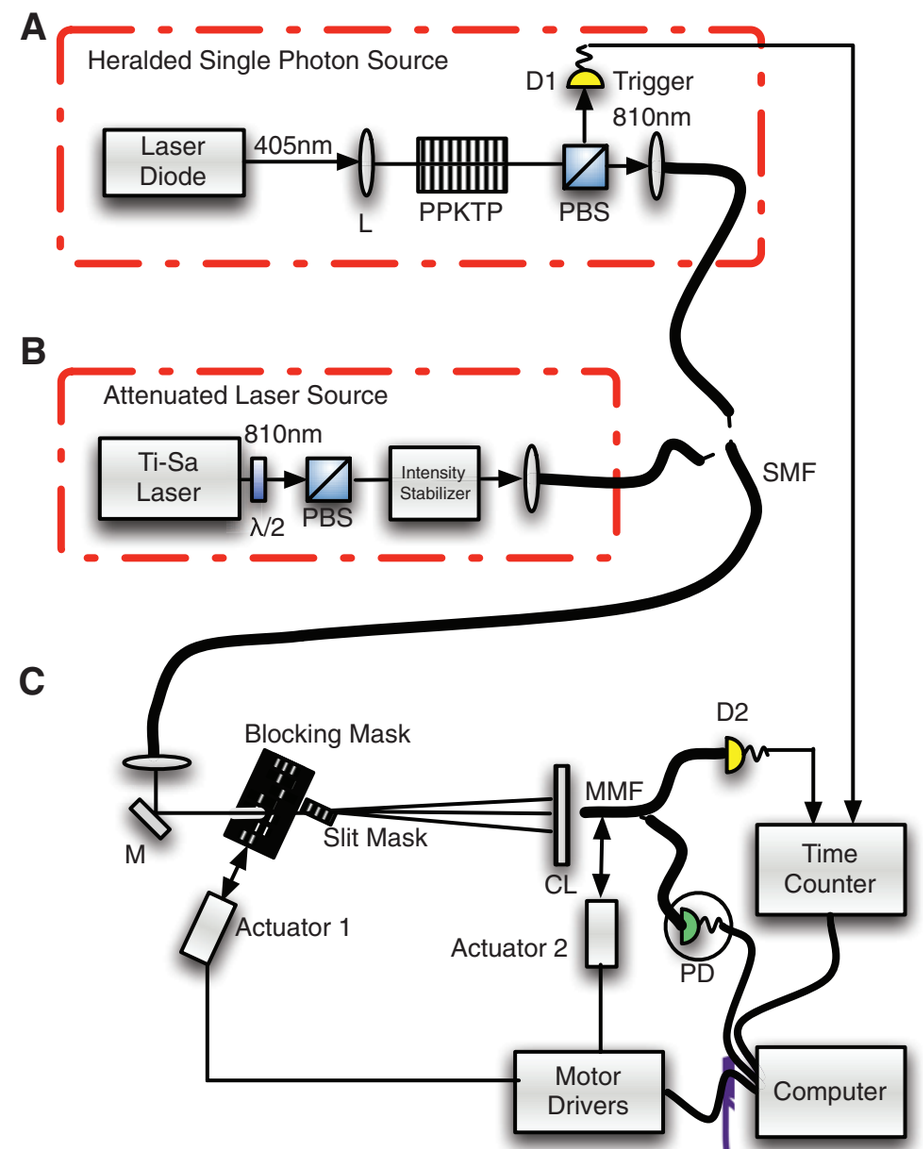
Result: $\boxed{\kappa \leq 10^{-2}.}$



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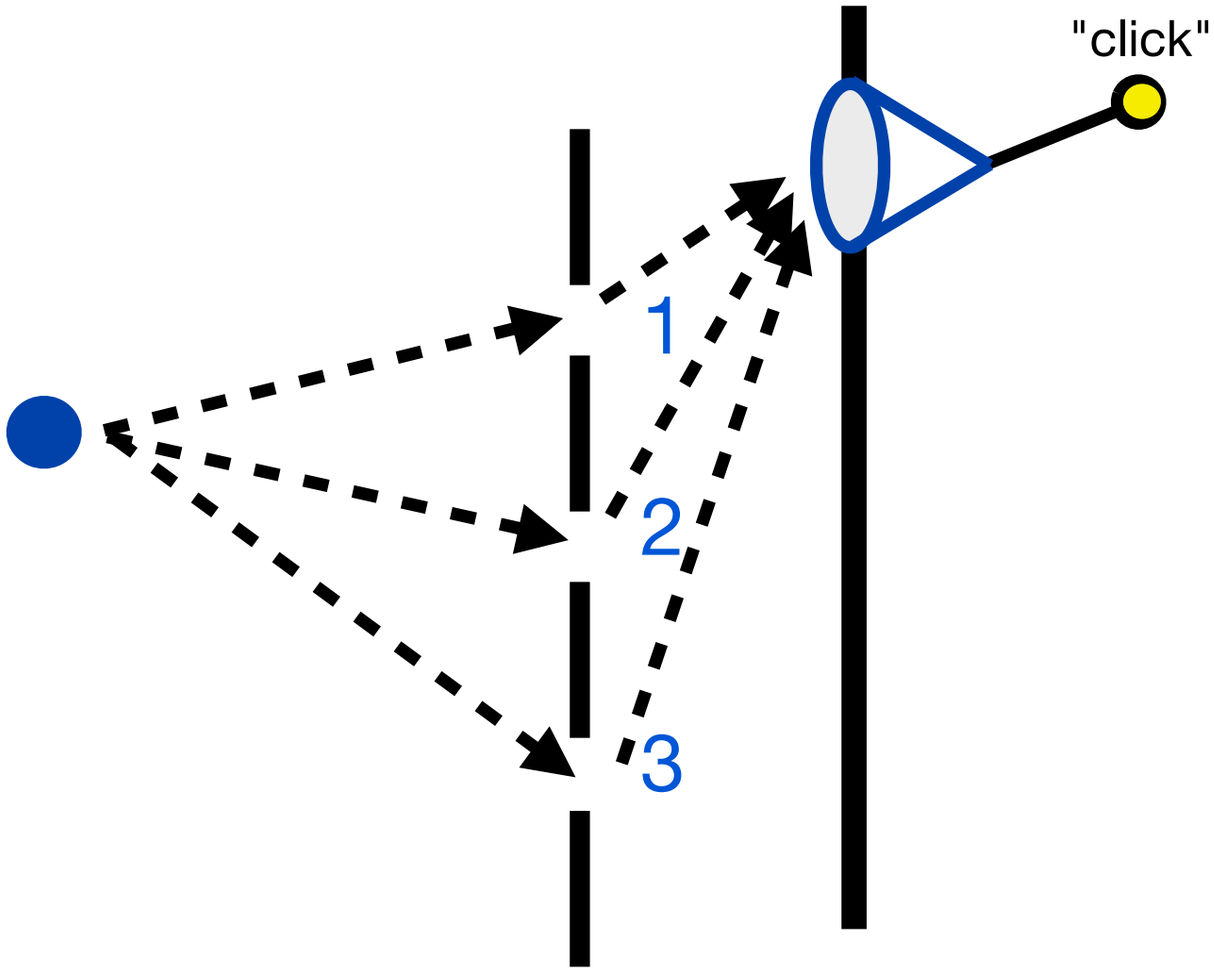

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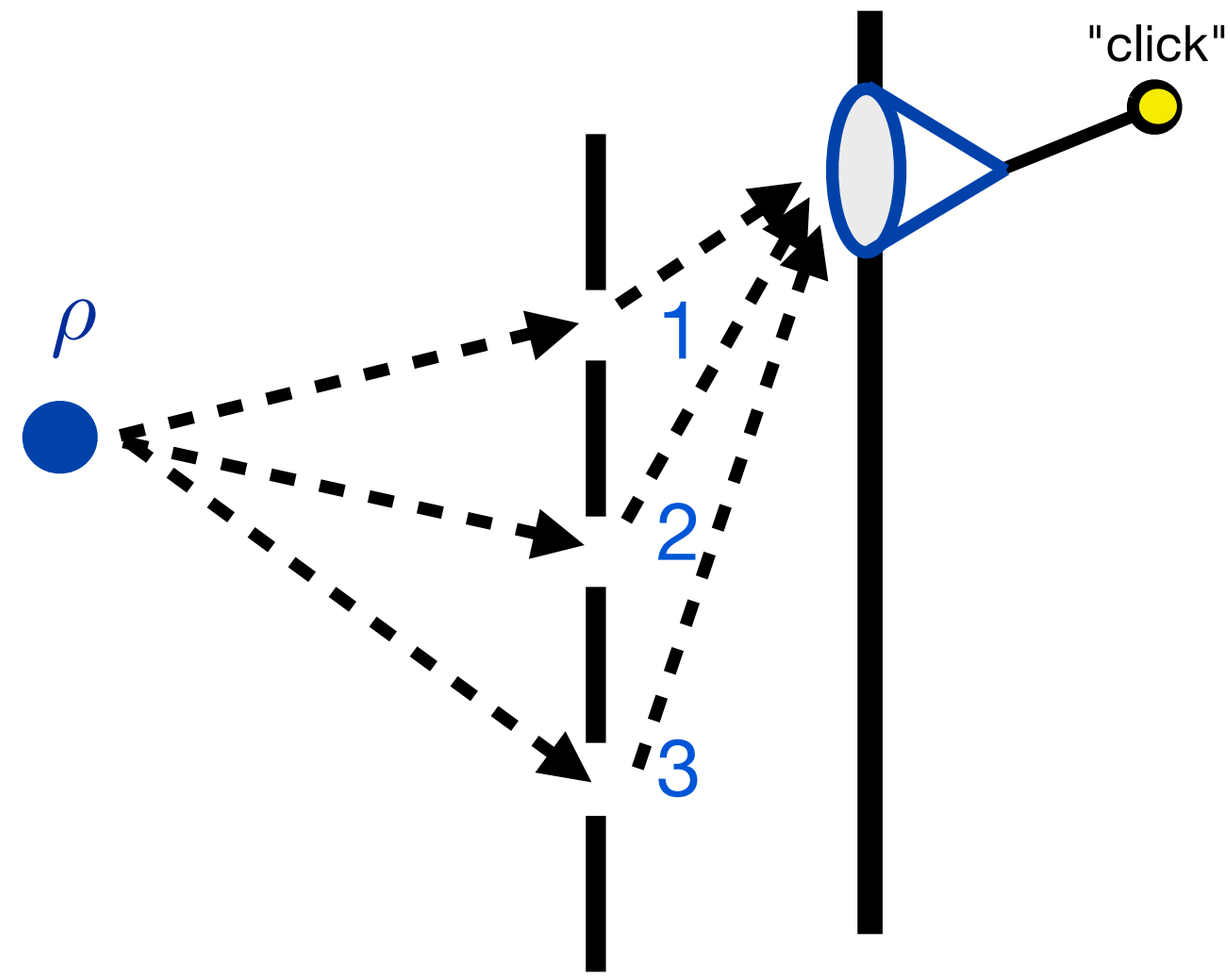

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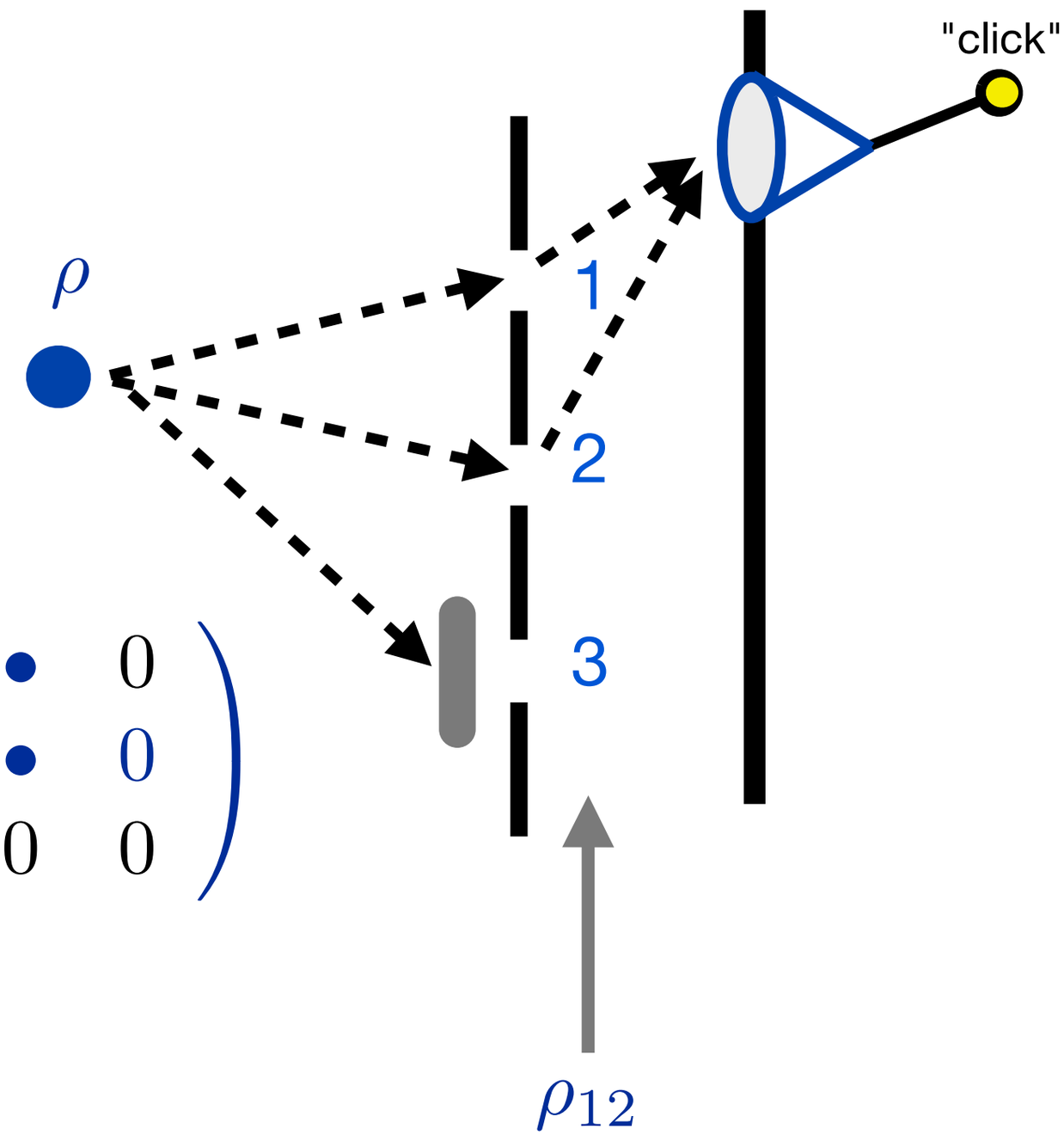
$$\rho \mapsto P_{12}\rho P_{12} =: \rho_{12} = \begin{pmatrix} \bullet & \bullet & 0 \\ \bullet & \bullet & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



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$$\begin{pmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{pmatrix} = \begin{pmatrix} \bullet & \bullet & 0 \\ \bullet & \bullet & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} \bullet & 0 & \bullet \\ 0 & 0 & 0 \\ \bullet & 0 & \bullet \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & \bullet & \bullet \\ 0 & \bullet & \bullet \end{pmatrix}$$

$$- \begin{pmatrix} \bullet & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & \bullet & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \bullet \end{pmatrix}$$

$$\begin{aligned} p_{1,2,3} \quad &= \quad p_{1,2} + p_{1,3} + p_{2,3} \\ &\quad -p_1 - p_2 - p_3 . \end{aligned}$$

Why does CPT not have 2nd-order interference?

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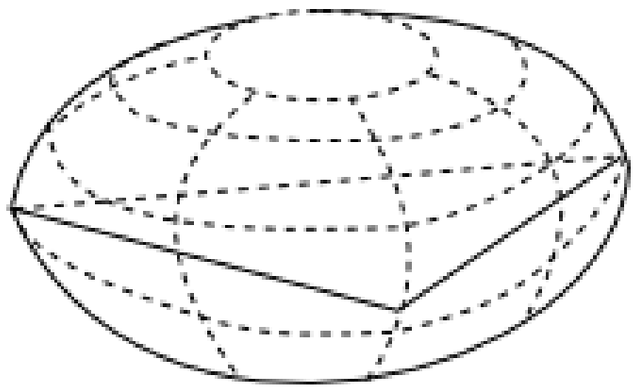
$$\begin{pmatrix} \bullet \\ \bullet \\ \bullet \end{pmatrix} = \begin{pmatrix} \bullet \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \bullet \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \bullet \end{pmatrix}$$

$$p_{1,2,3} = p_1 + p_2 + p_3.$$

Which natural GPTs have 3rd-order interference?

Some "artificial" GPTs exhibit order-3 interference:



C. Ududec, *Perspectives on the Formalism of Quantum Theory*, PhD thesis, University of Waterloo, 2012.

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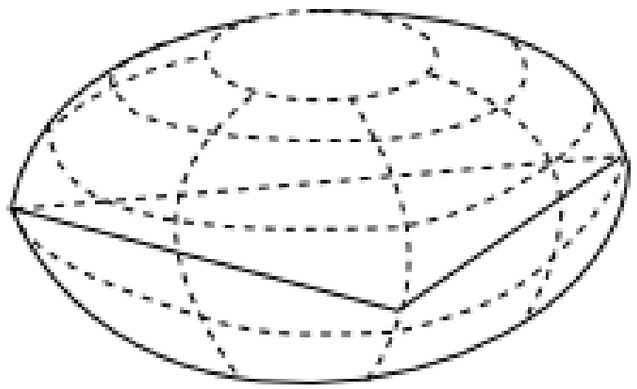

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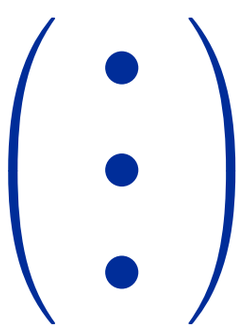
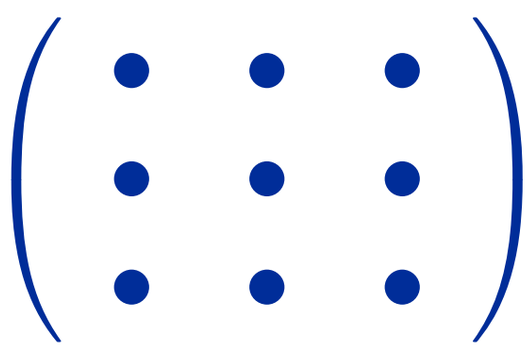
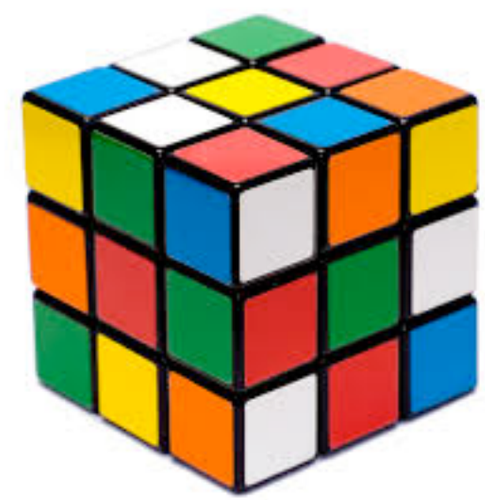


"1st-order" (trivial) interference

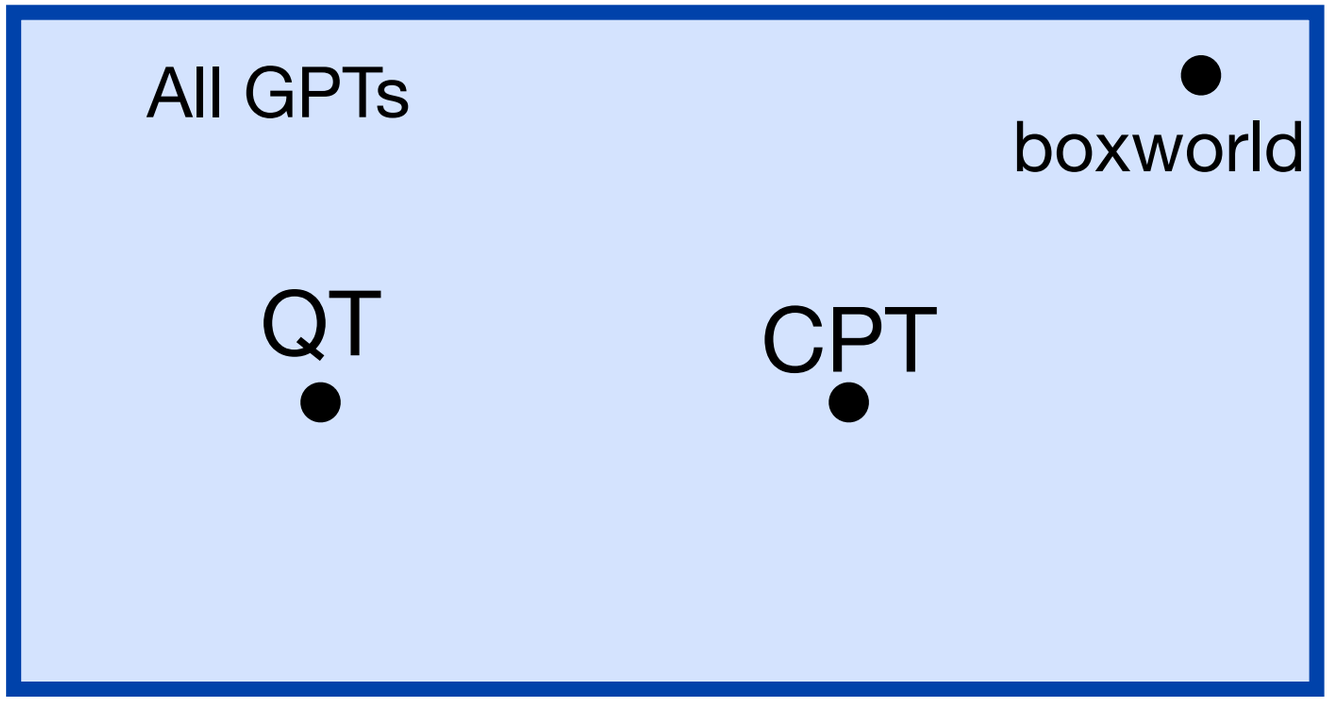


2nd-order interference



3rd-order interference?

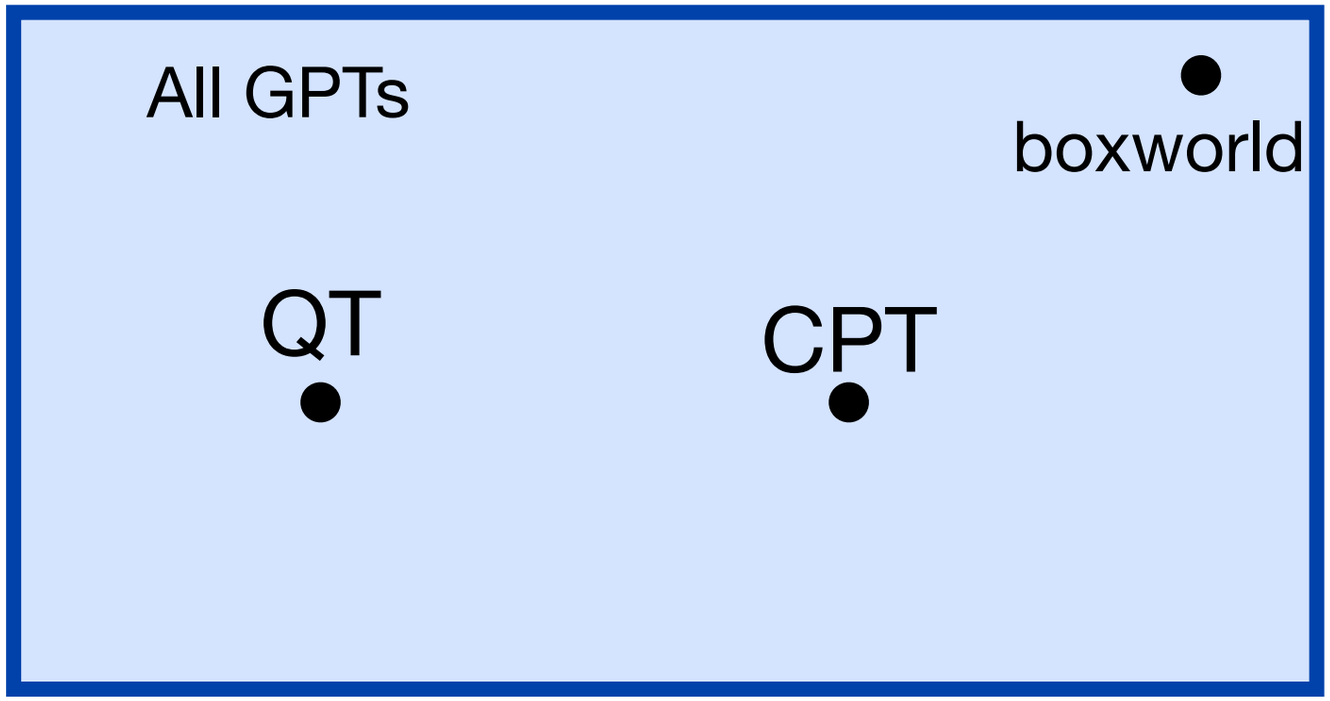
An axiomatic approach



An axiomatic approach

H. Barnum, MM, and C. Ududec, *Higher-order interference and single-system postulates characterizing quantum theory*, New J. Phys. **16**, 123029 (2014).

The following 4 postulates single out QT uniquely:



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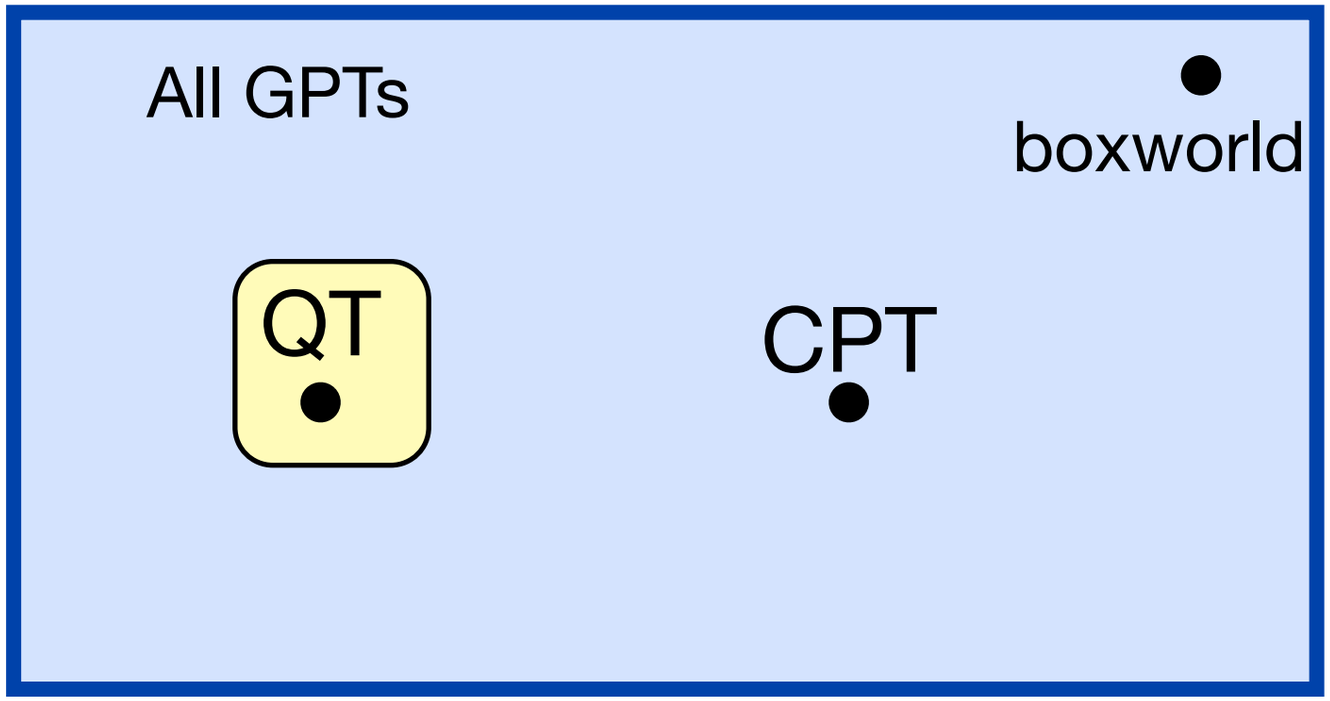
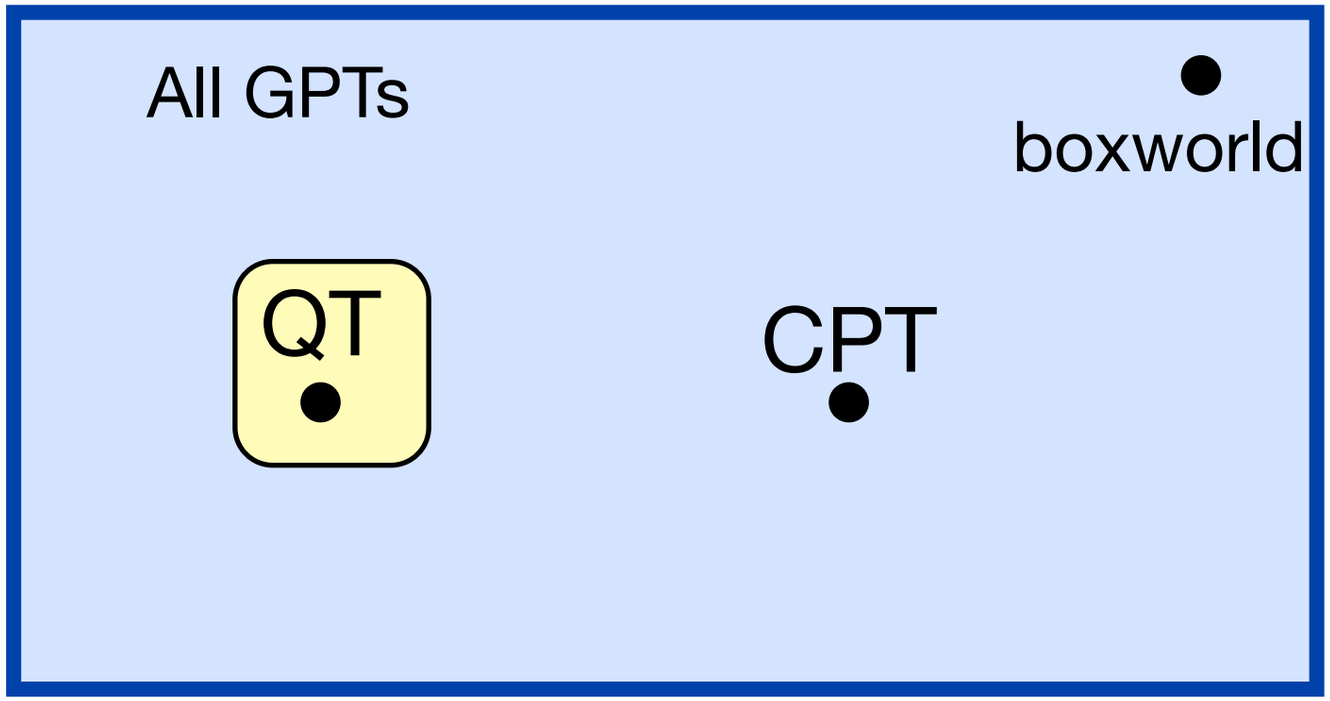


The following 4 postulates single out QT uniquely:

1. Every state has a "spectral decomposition",
2. lots of symmetry,
3. **no 3rd-order interference**, and
4. energy is observable.

An axiomatic approach

Let's drop #4...



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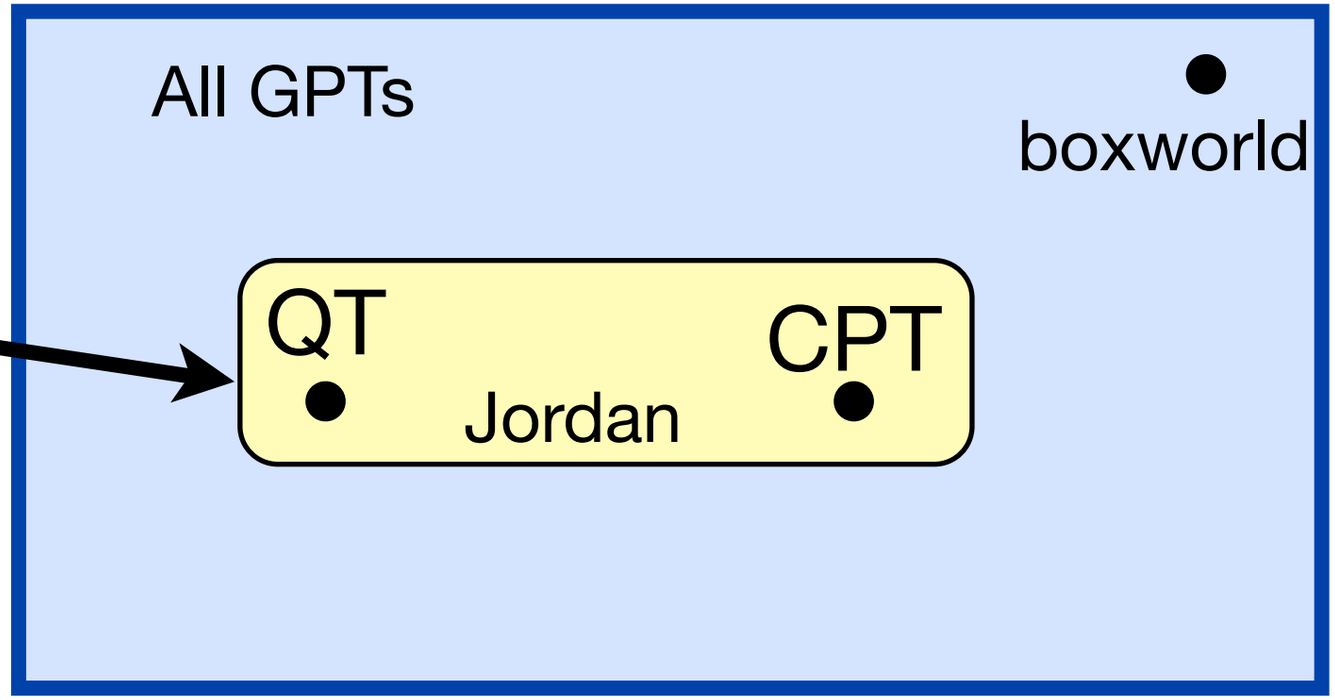
An axiomatic approach

Jordan algebra state spaces
(Bloch ball state spaces,
QT over $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$,
3-level QT over $\mathbb{O}$.)



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An axiomatic approach

The point:
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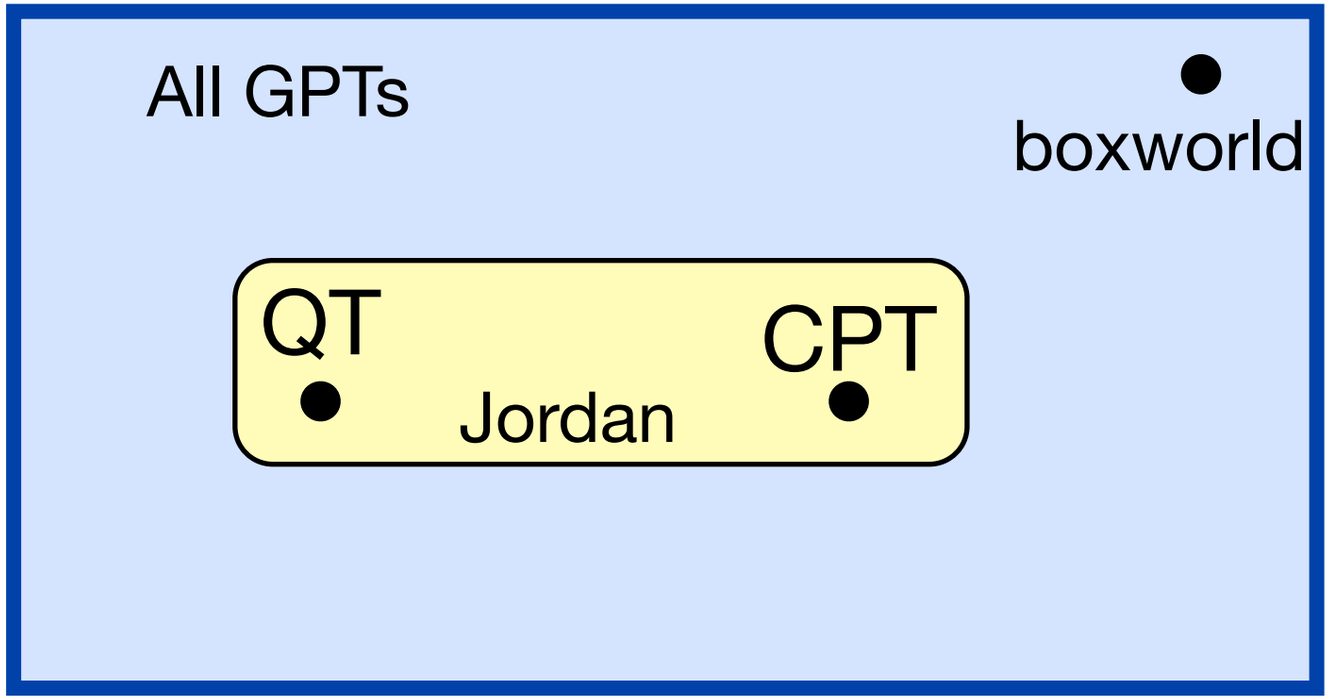

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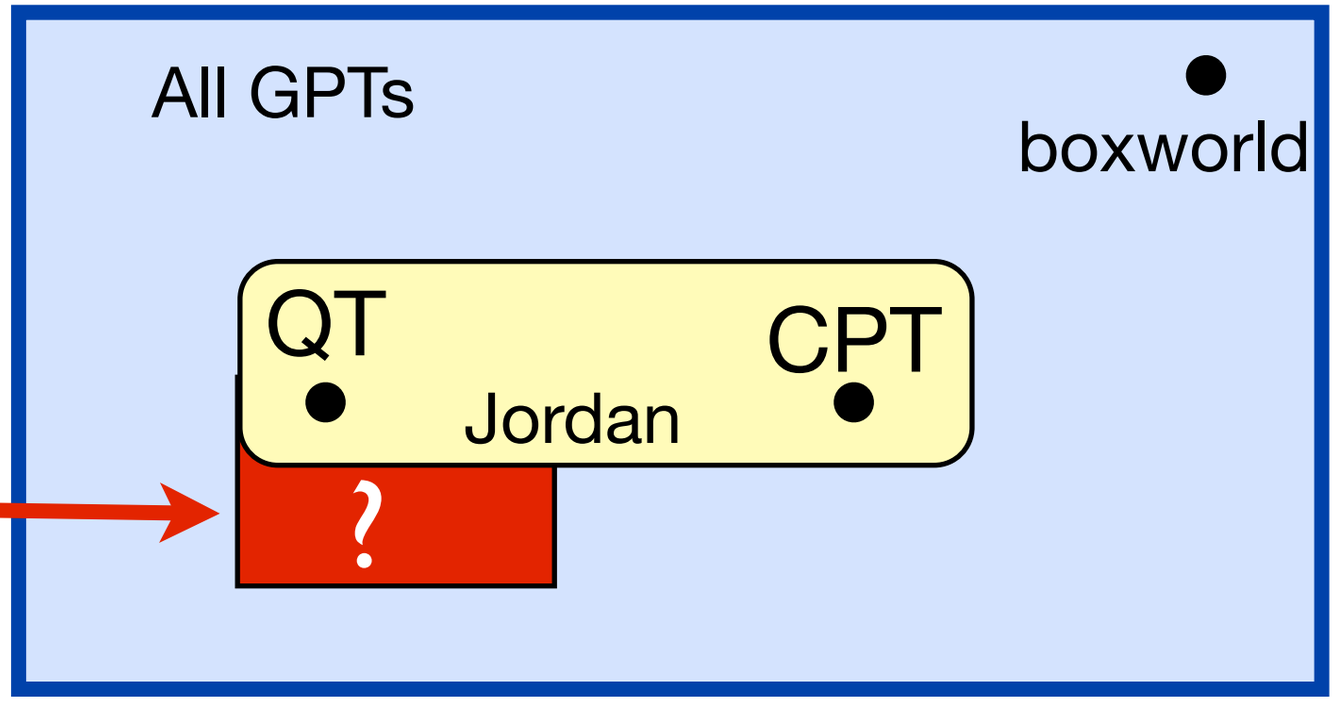

New solutions?
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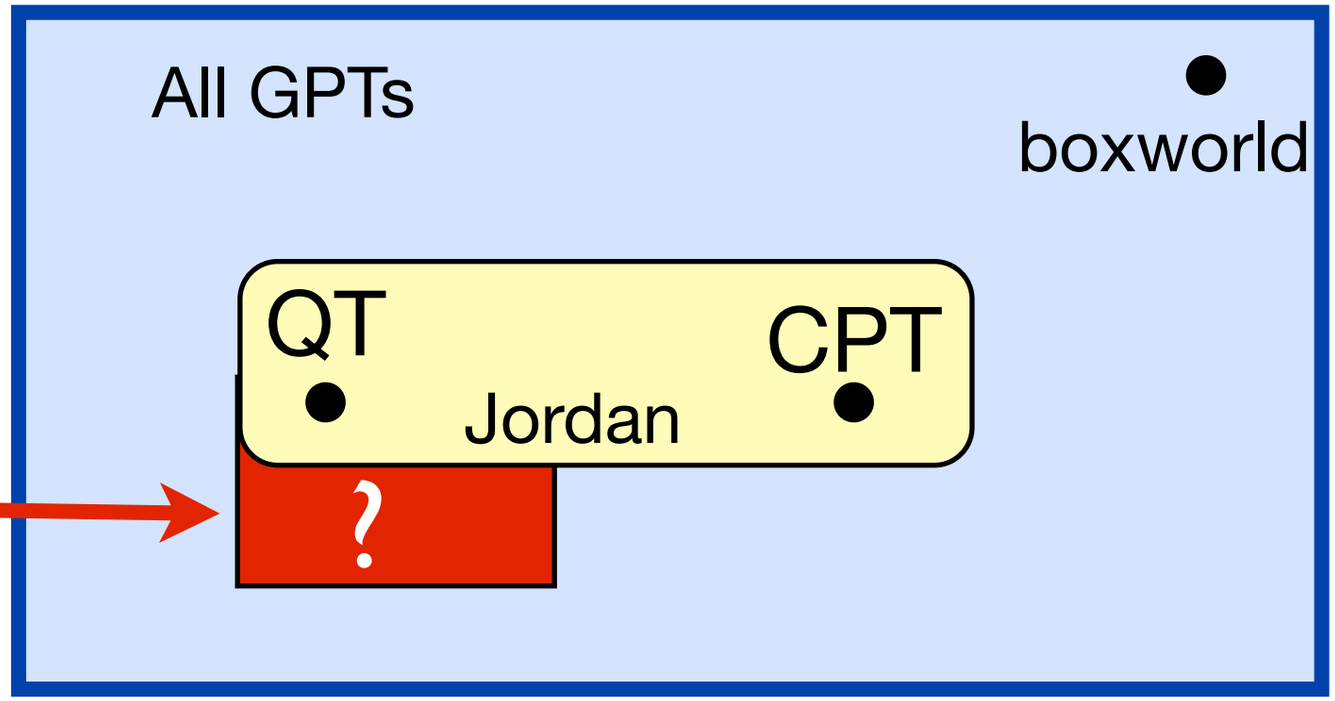

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Well-defined math problem: classify those state spaces!

The two axioms in more detail

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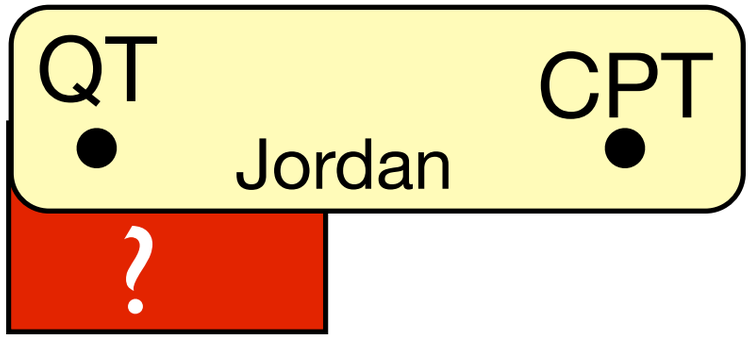

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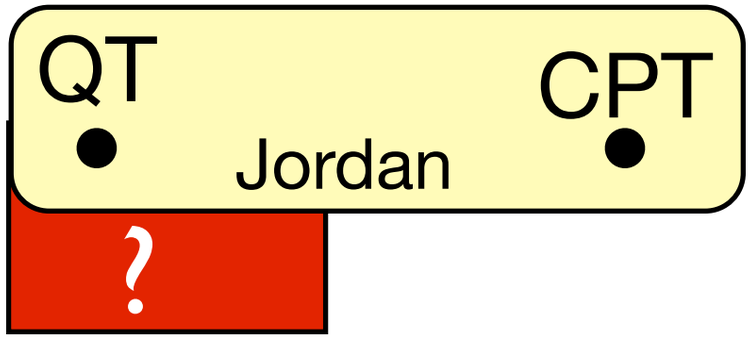

Postulate 1. Every state ω can be written as a convex combination

$$\omega = \sum_{i=1}^{n} \lambda_i \omega_i,$$

where $\omega_1, \ldots, \omega_n$ are pure and perfectly distinguishable (an "n-frame").

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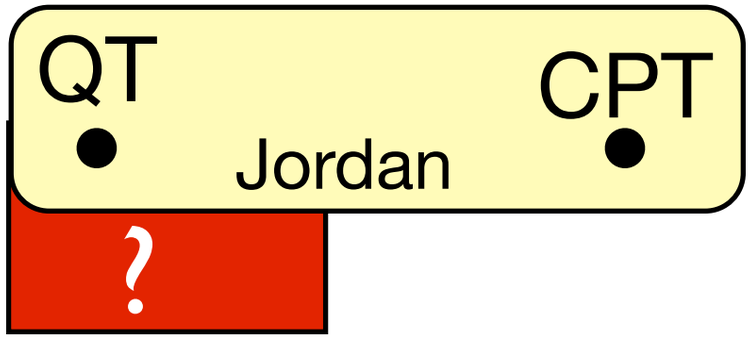

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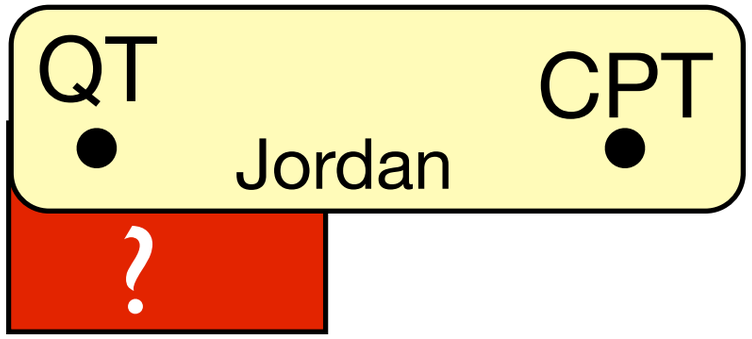

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QT: *n*-frame = orthonormal (sub-)basis

Postulate 1 = spectral decomposition of a density matrix

Postulate 2 = unitaries can map any basis to any other.

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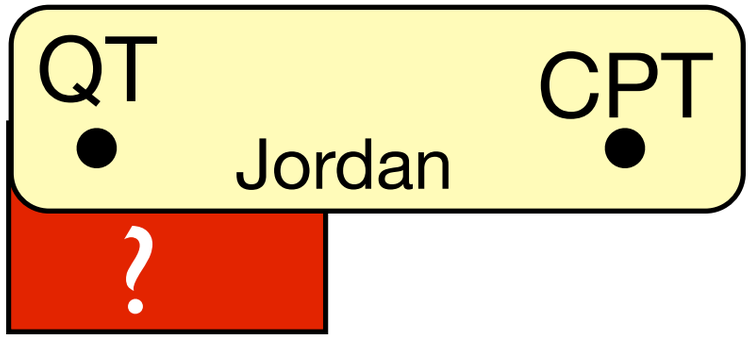

Theorem: All these theories satisfy consistent exclusivity, CE[1].

This is because there exist **orthogonal projections** in analogy to the quantum ones:

Remember:

$$\rho = \begin{pmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{pmatrix}$$

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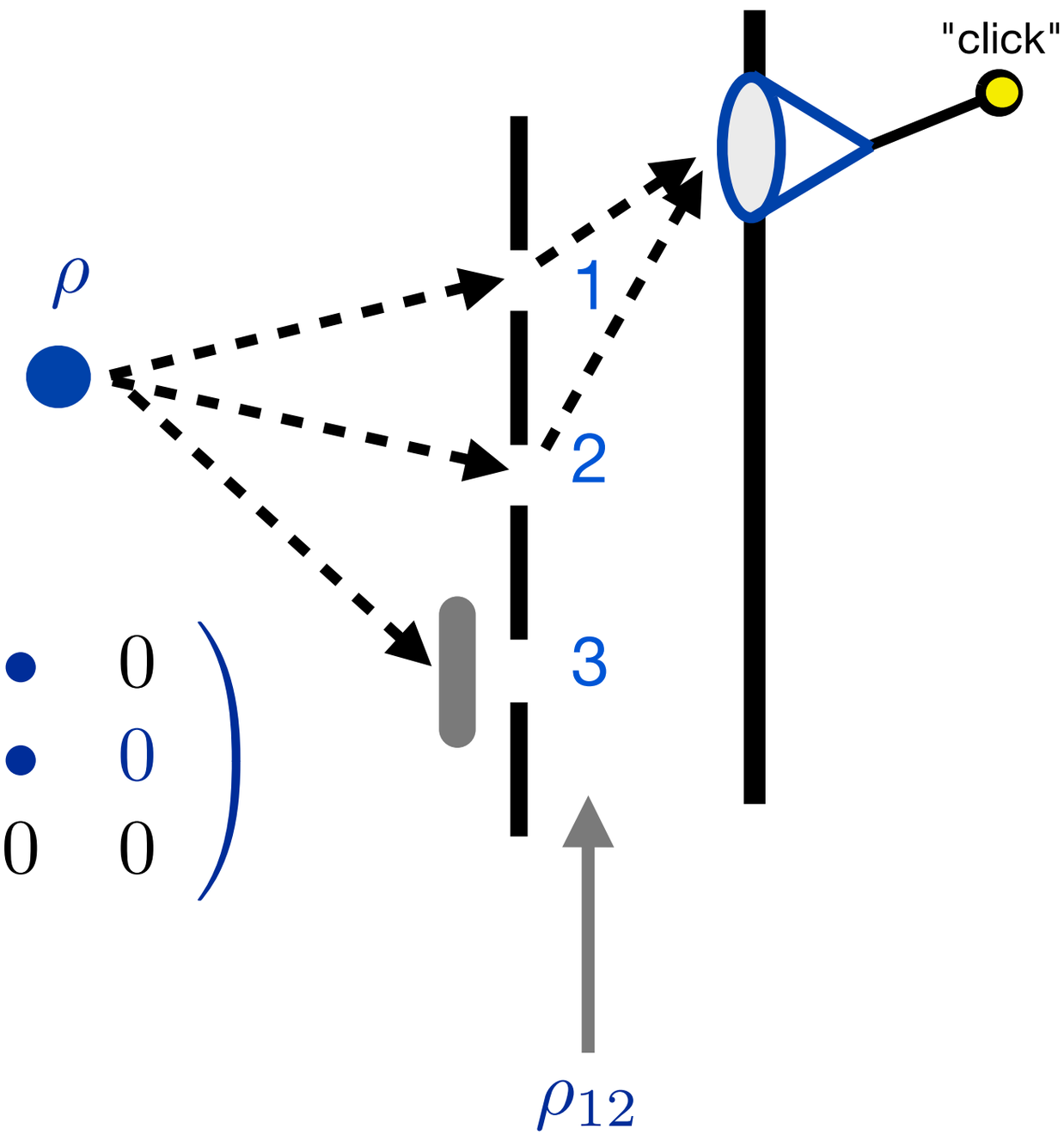

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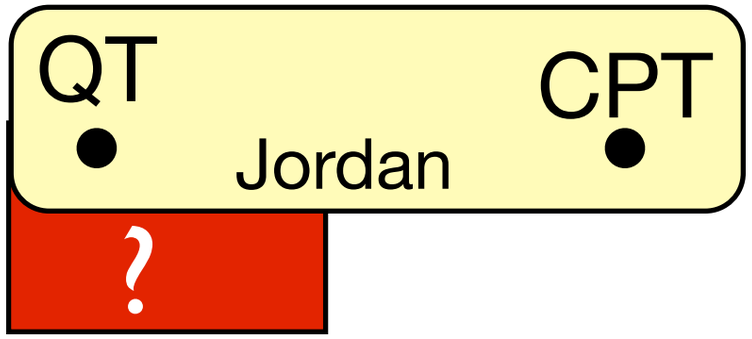

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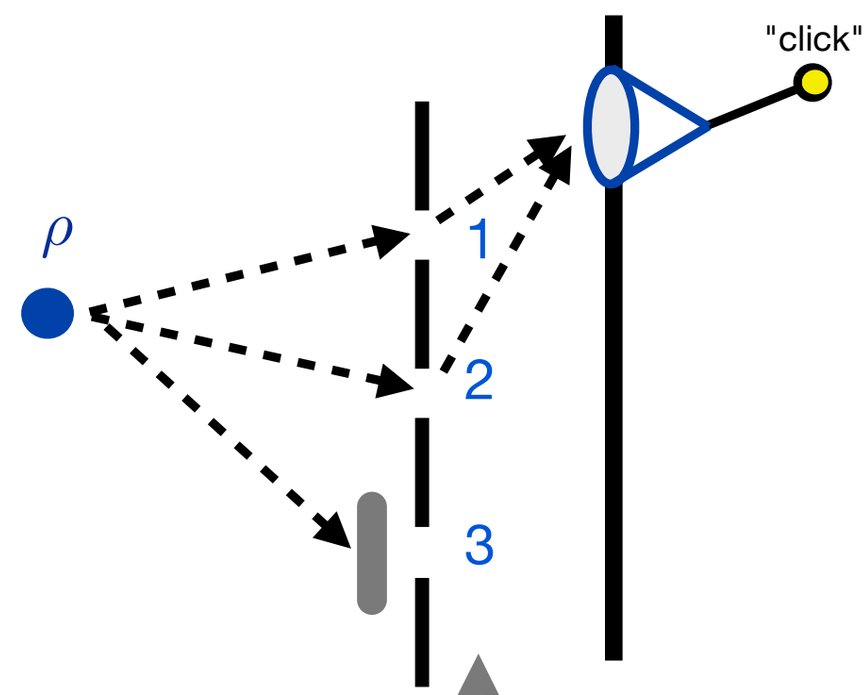

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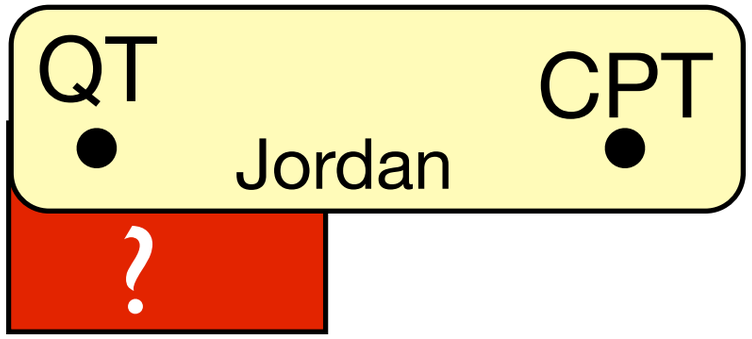

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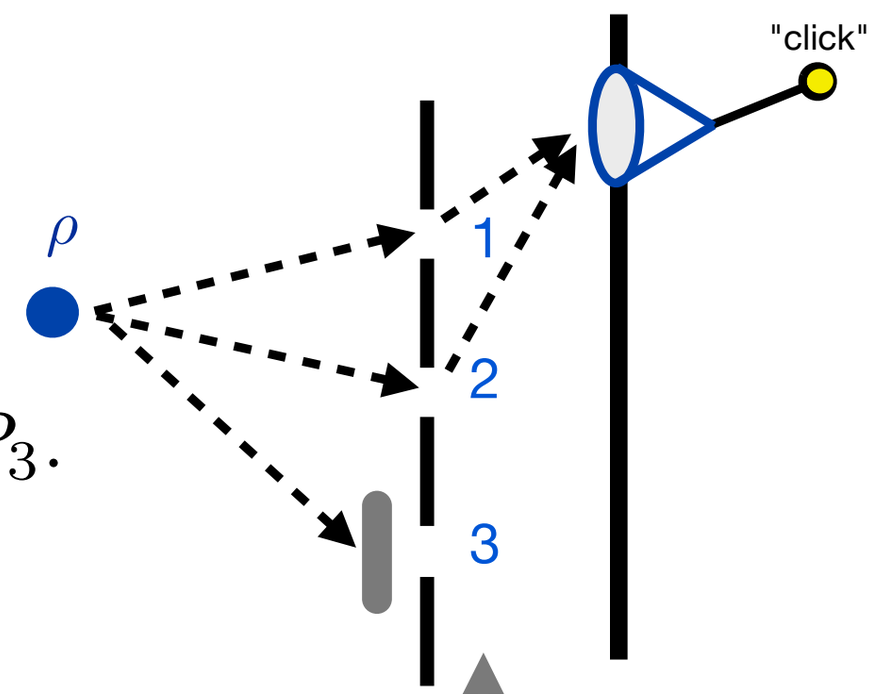

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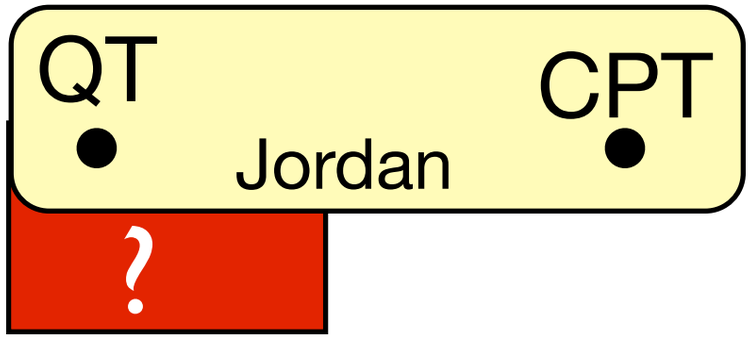

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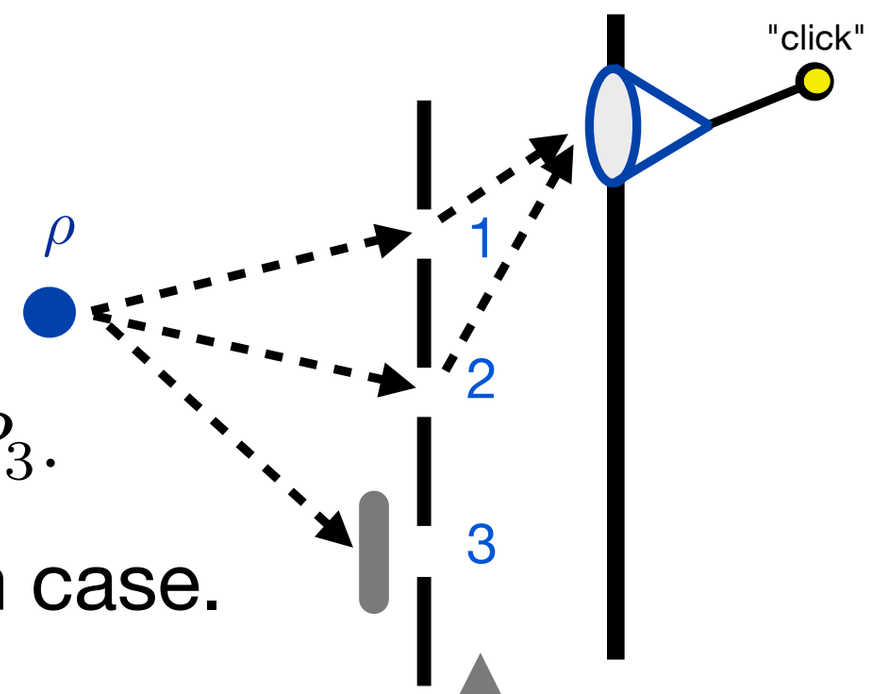
Proof of CE[1] identical to quantum case.



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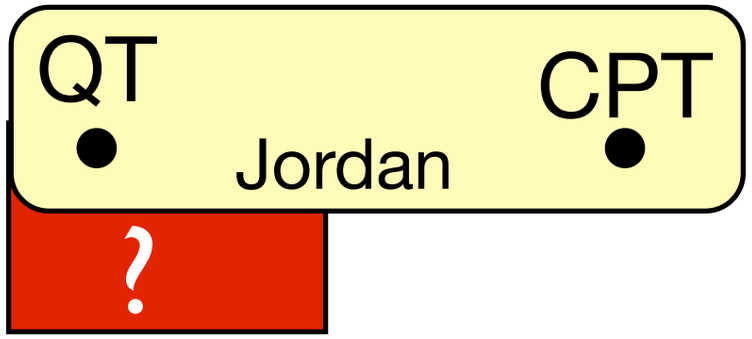

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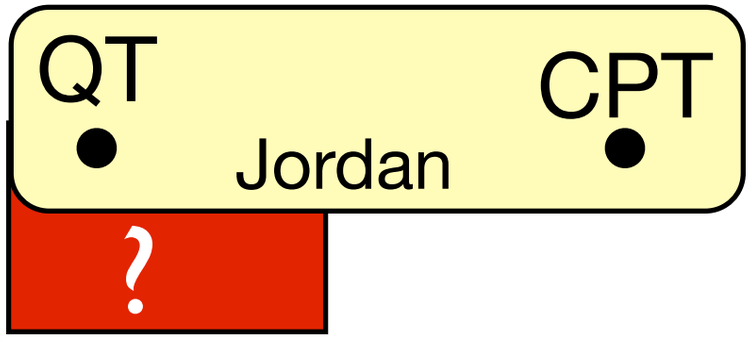

- Like QT, they allow for **consistent thermodynamics** & (rel.) entropy
- Unlike QT, they have higher-order interference.
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- Like in QT, "**decoherence** to classical" is a possible process.
- Like in QT, the projectors form an **orthomodular lattice**.
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- Like in QT, all sub-bits are **Bloch balls** (of some dimension).
- Unlike in QT, there are pure states ω, ρ that do not lie in a common 2-level subspace.
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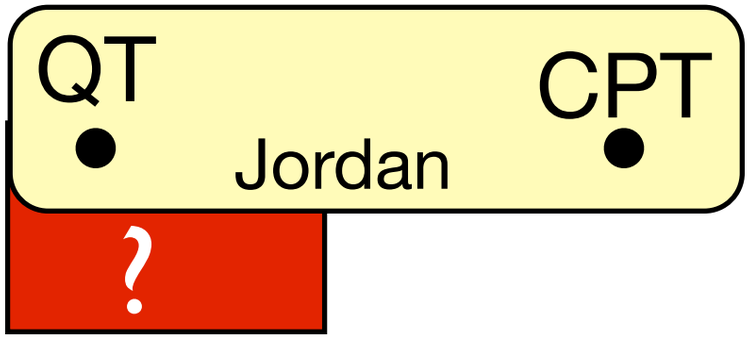

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... if they exist!

Some speculation…

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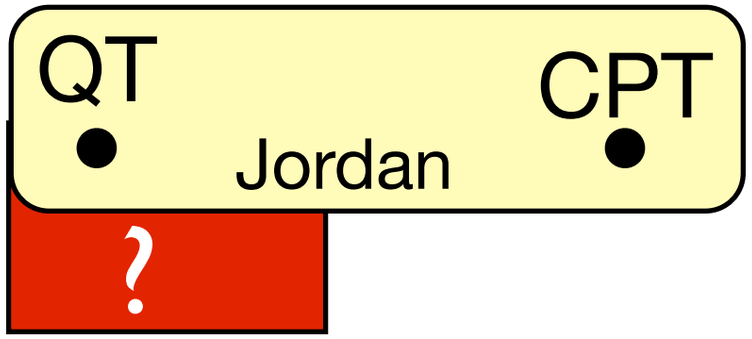

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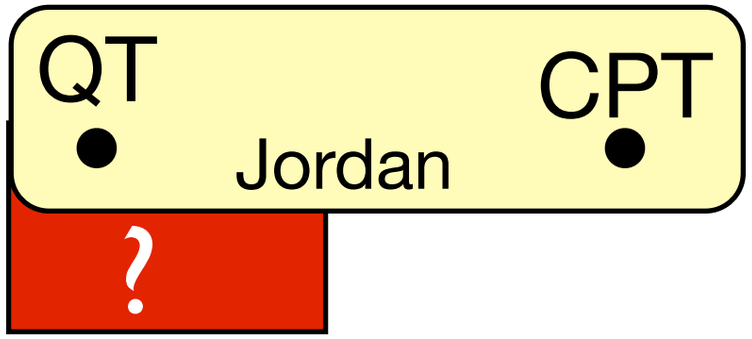

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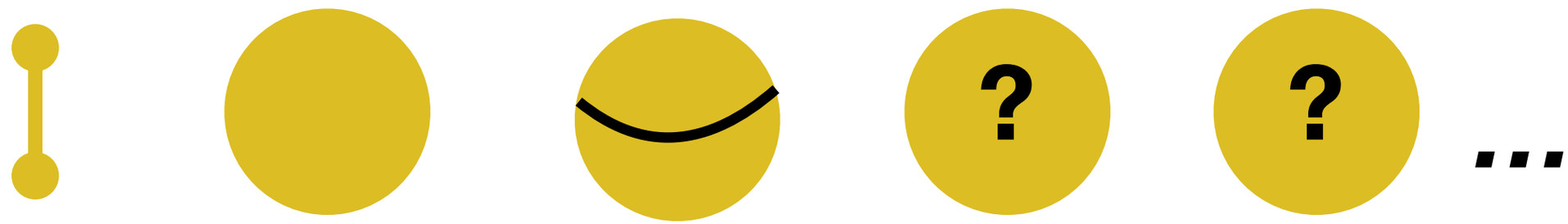

d=1 *d=2* *d=3* *d=4* *d=5* *…*

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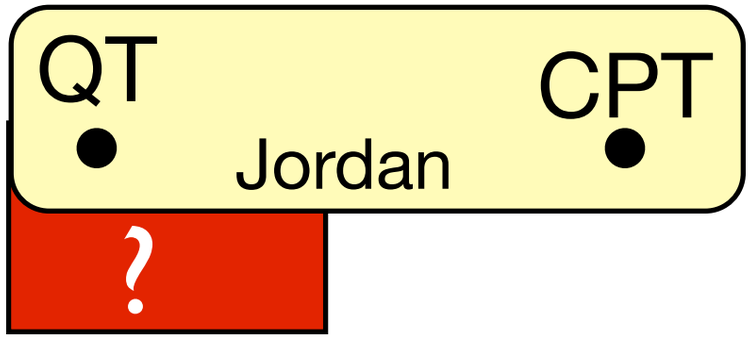

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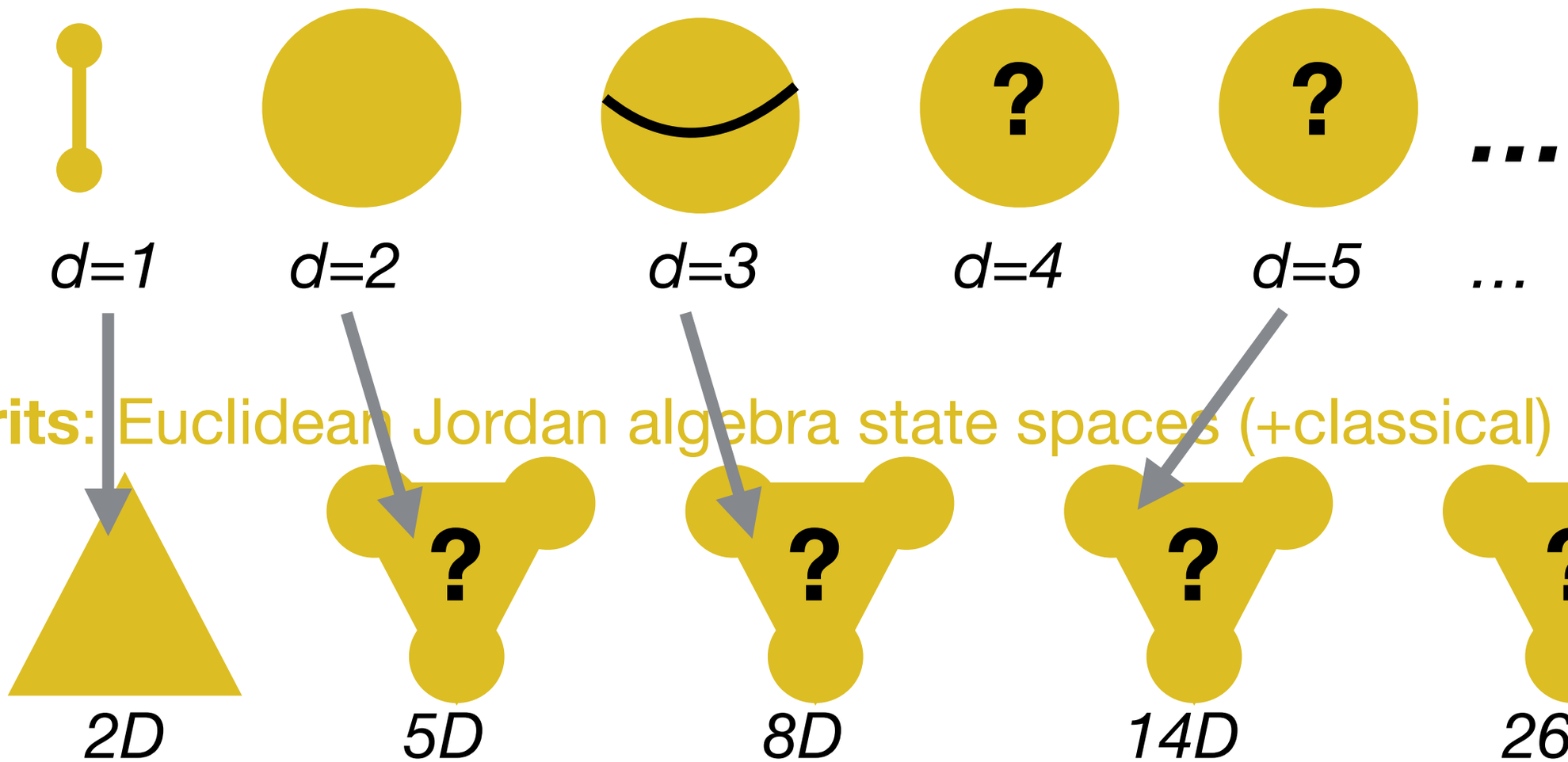

no others known!

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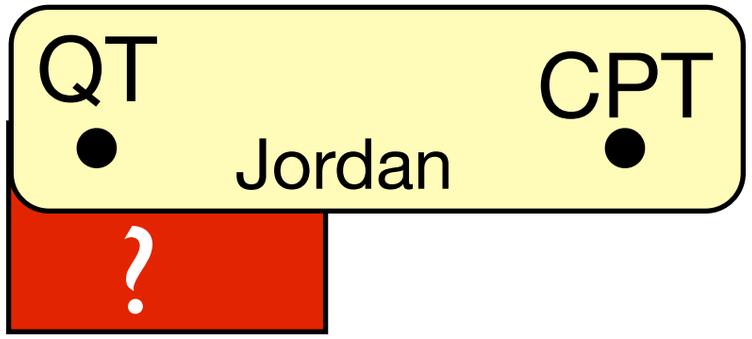

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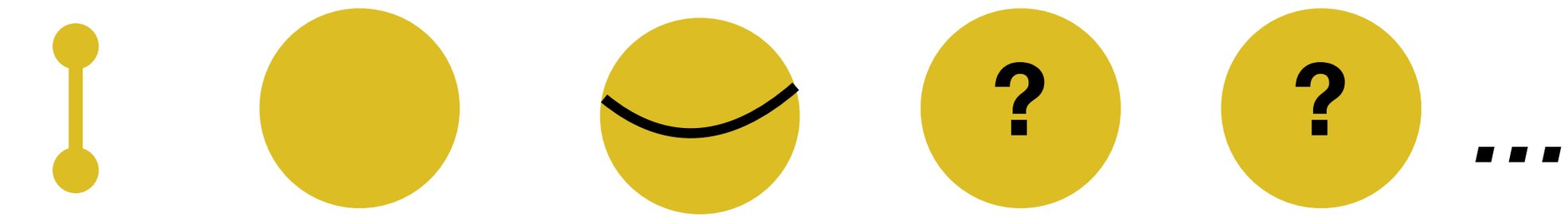

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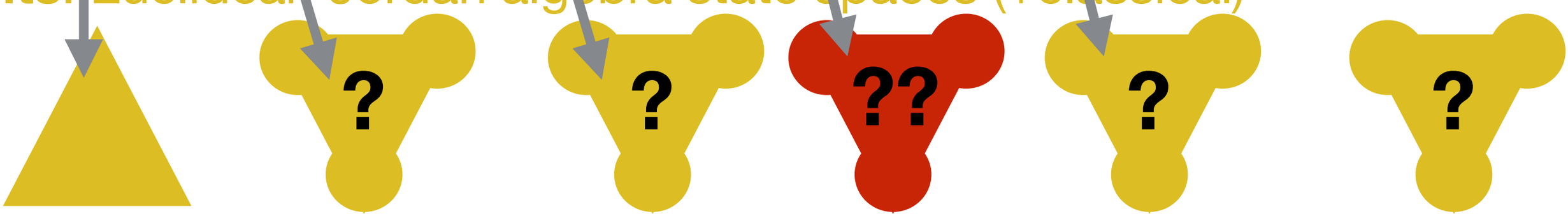
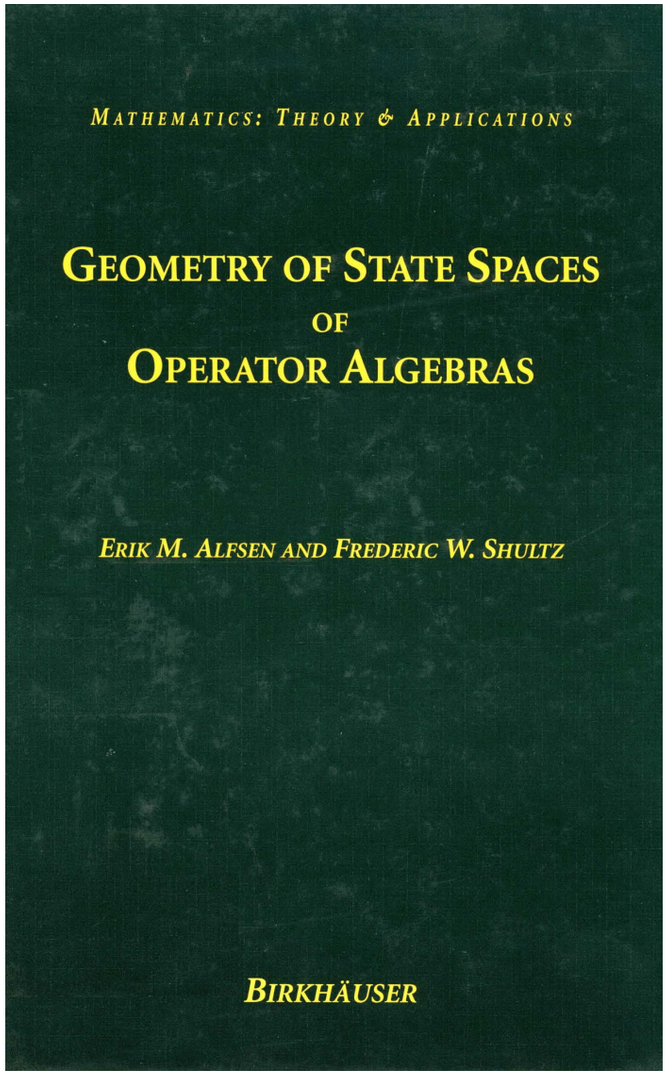
Trits: Euclidean Jordan algebra state spaces (+classical)



2D (classical) *5D* (real QM) *8D* (QM over $\mathbb{C}$) ***12D++*** ("quatrit"?) *14D* (QM over $\mathbb{H}$) *26D* (QM over $\mathbb{O}$

Orthogonal projections

Orthogonality of projections expresses **operationally intuitive properties** of compatibility and non-disturbance:



$$PQ = QP = QP^2 = PQP = \dots$$

Composition of filters / compressions / slits

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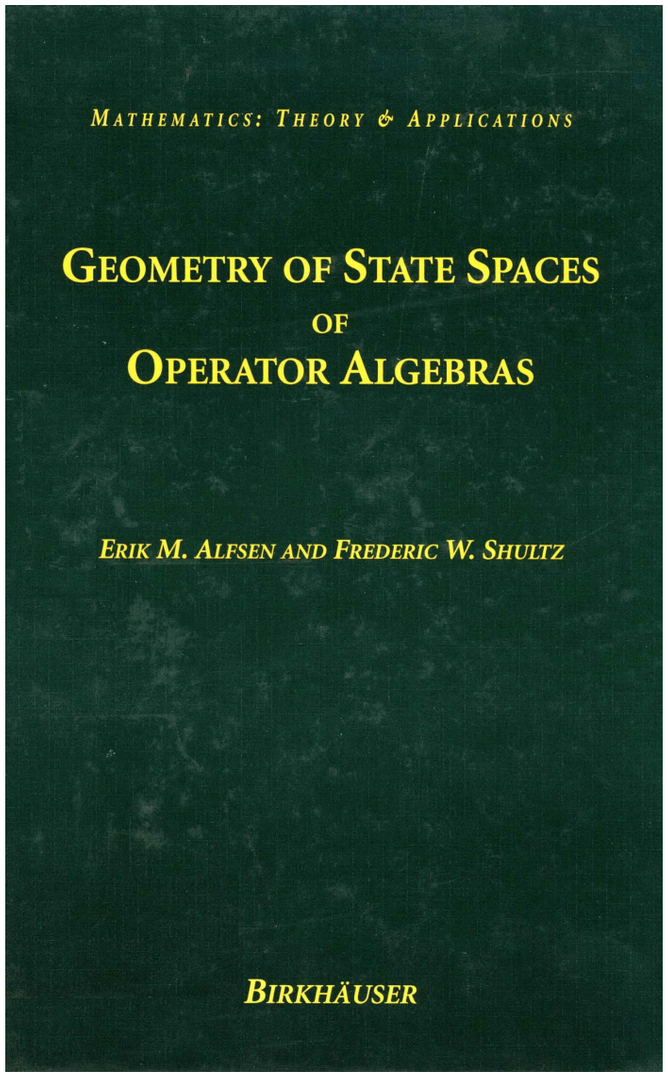

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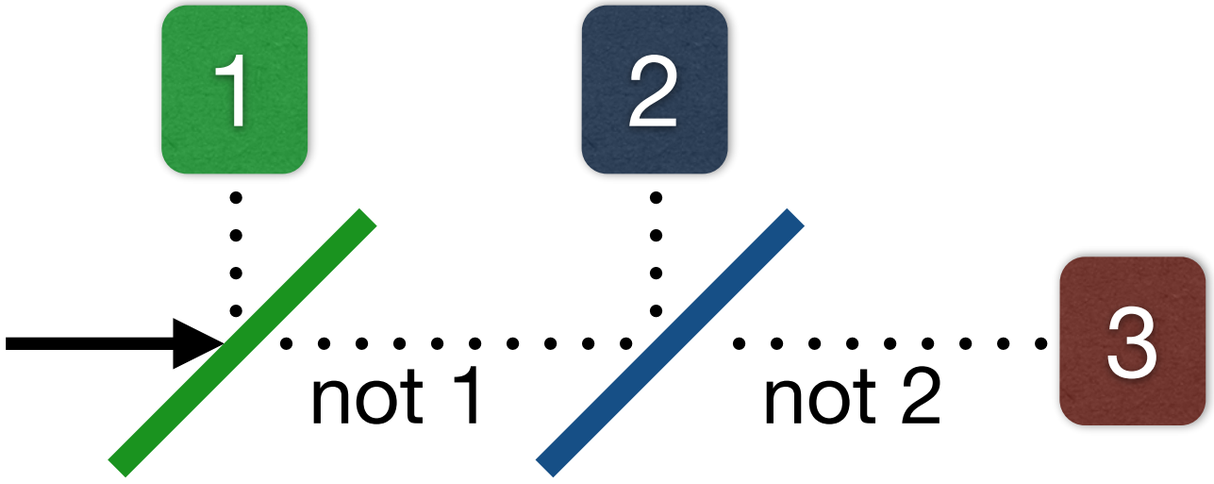
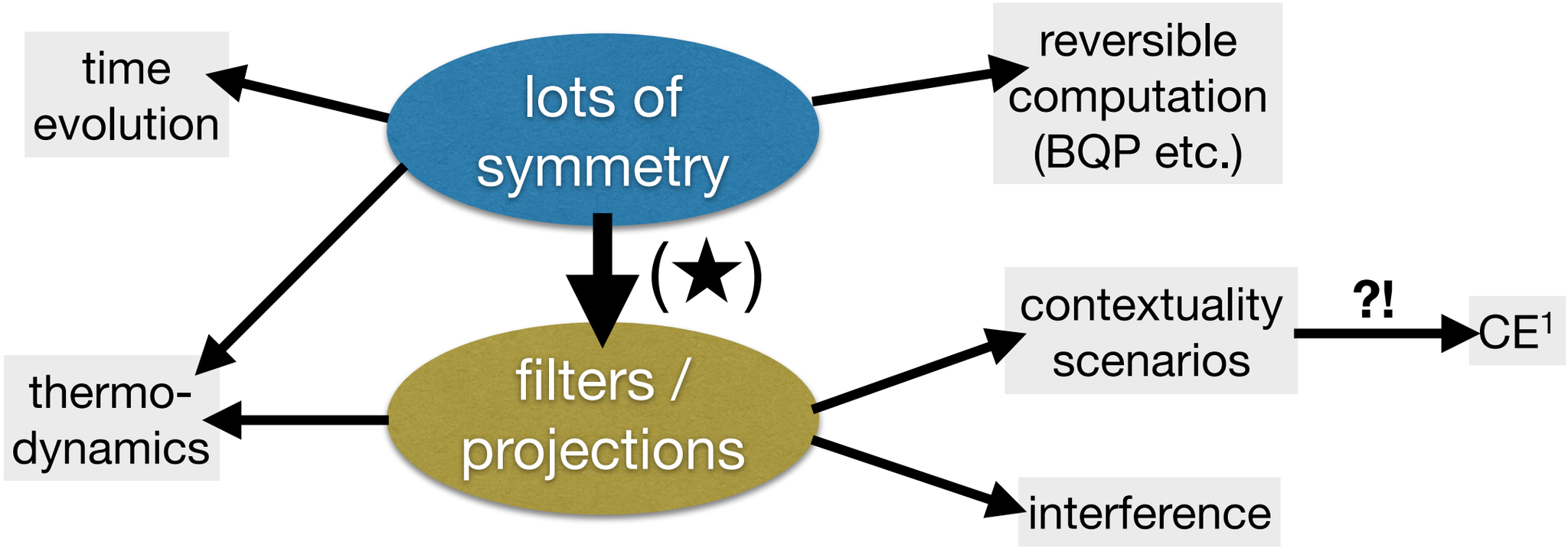
Composition of filters / compressions / slits

Consistent exclusivity:
build filters from pairwise filters



Orthogonal projections



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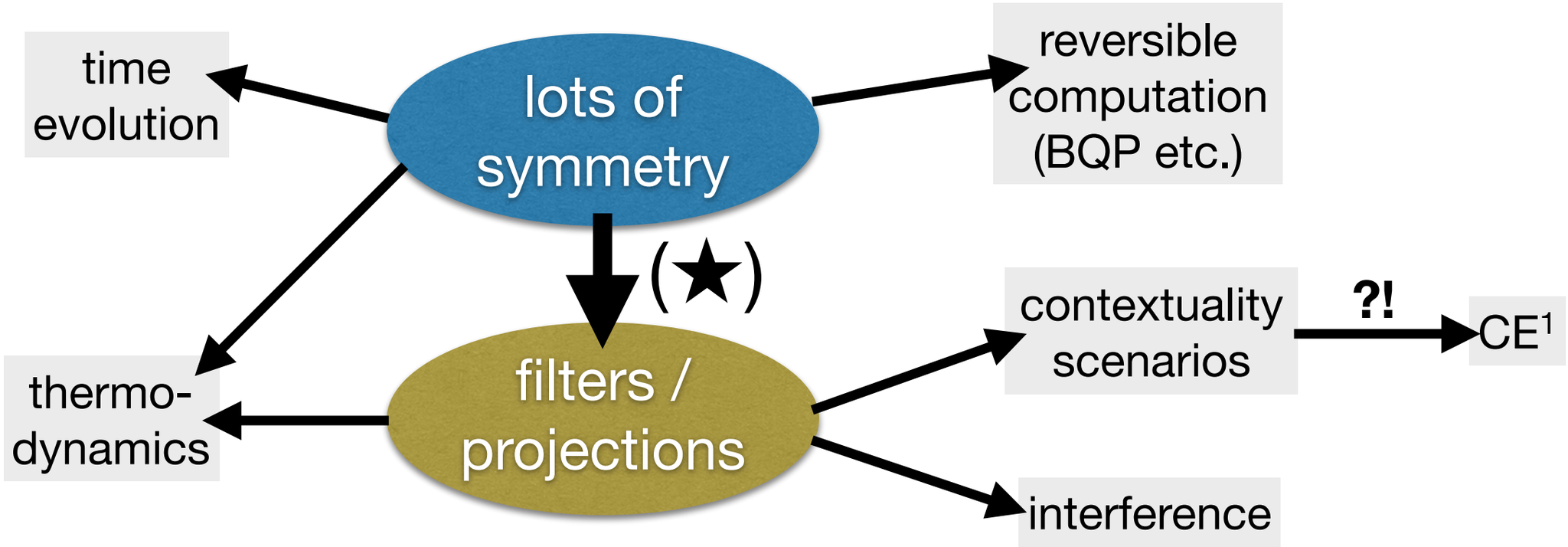

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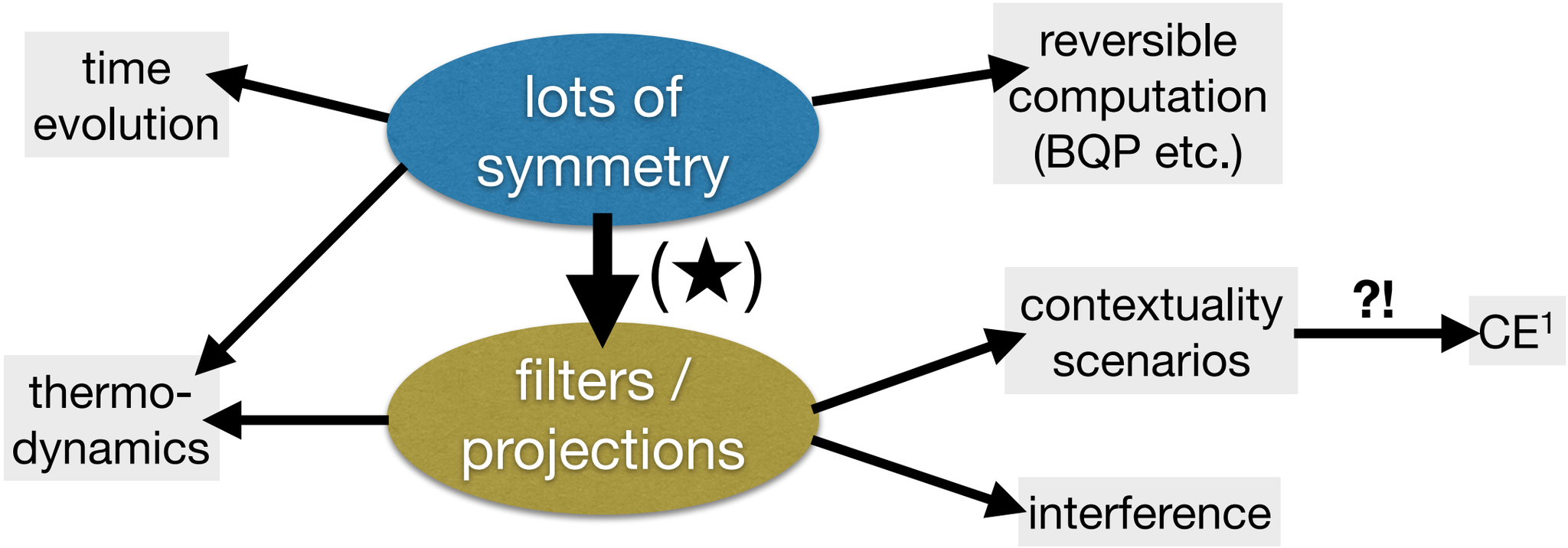

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In both approaches:

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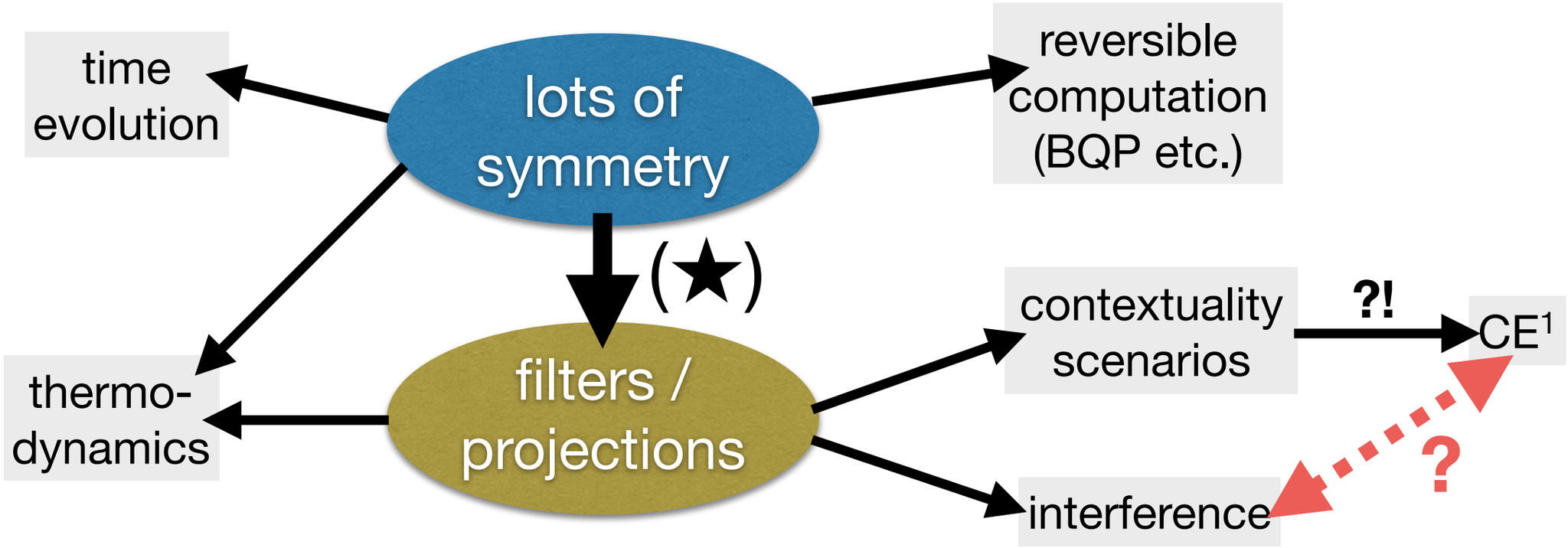

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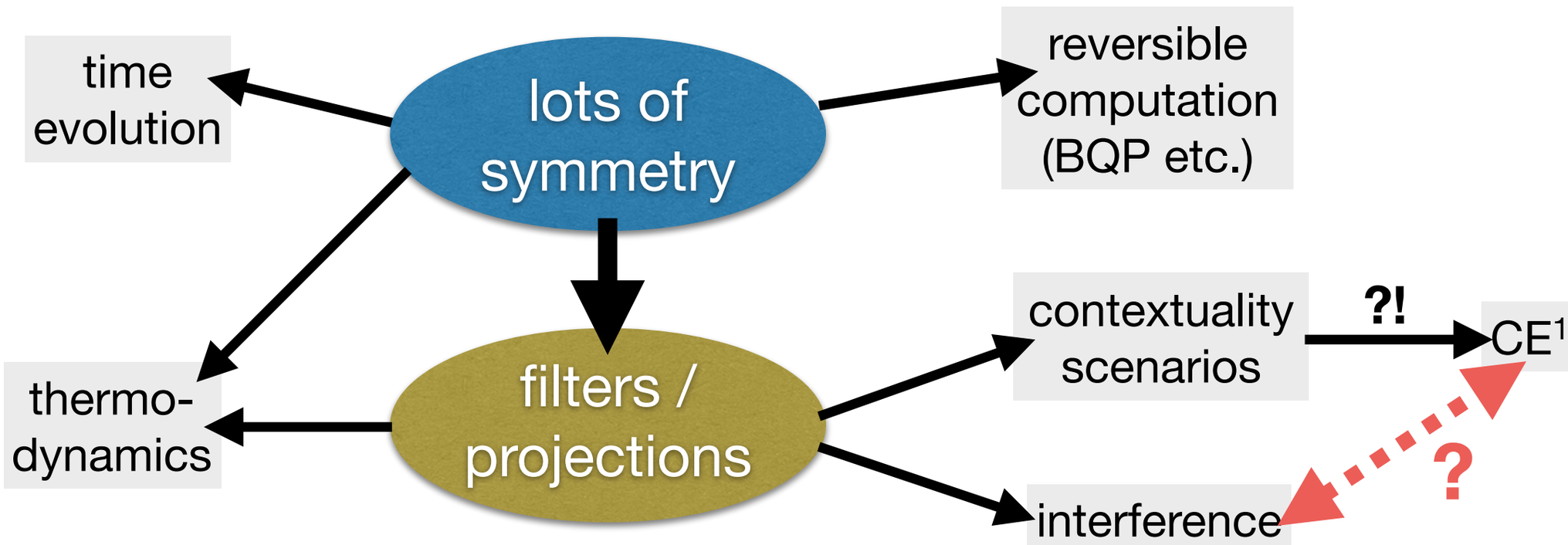
1. Questions on contextuality scenarios

Operational meaning of "compatibility"? Why CE[1]?

2. The general-probabilistic picture



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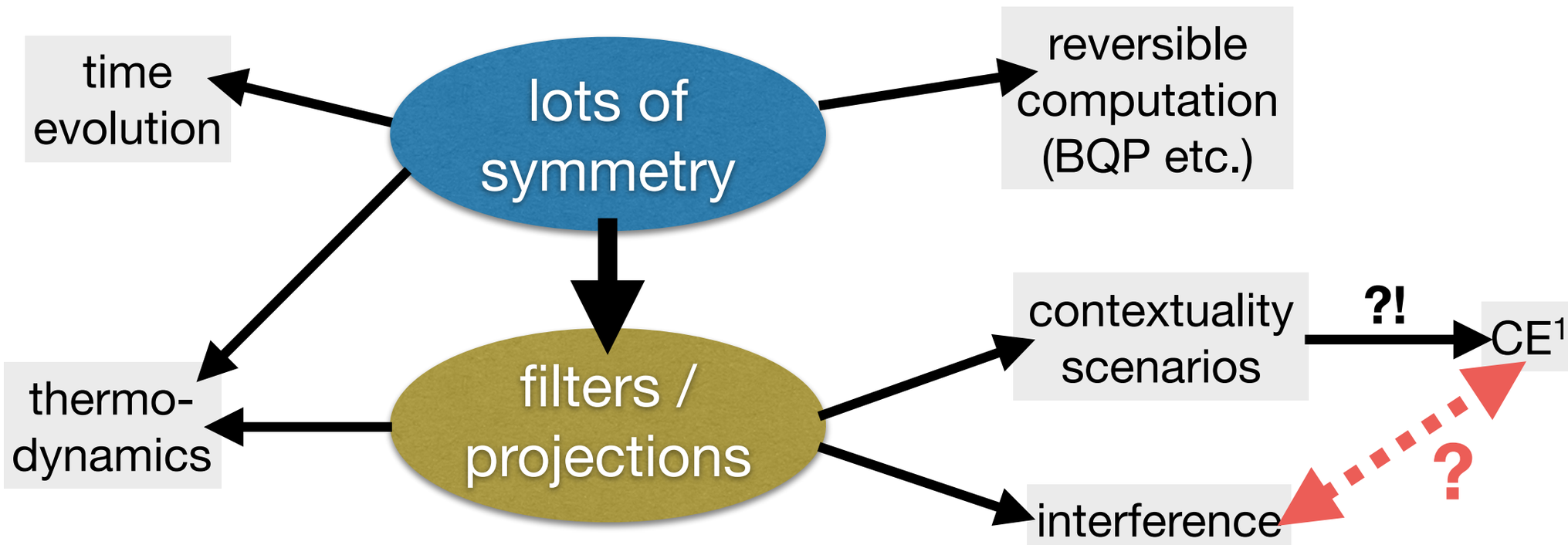

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J. Henson, *Bounding quantum contextuality with lack of third-order interference*, arXiv:1406.3281

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Basically, no 3rd order interference ⇒ consistent exclusivity.

Theorem 2. *Consider a probability function P on a scenario $\mathcal{S}$. If P admits a joint quantum measure then it obeys Consistent Exclusivity.*

$$\mu(A)+\mu(B)+\mu(C)$$
$$-\mu(A\cup B)-\mu(B\cup C)-\mu(C\cup A)$$
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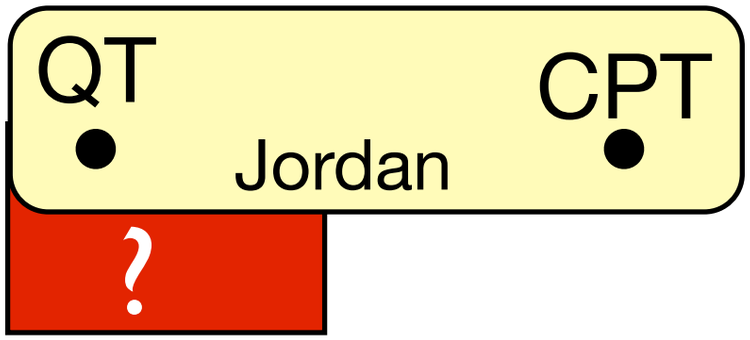
Q: What about the converse implication?
A: Seems very unlikely, given the above.
In particular, these theories are counterexamples (if they exist).



1. Every state has a "spectral decomposition",
2. lots of symmetry.

3. Interference ↔ CE[1]?

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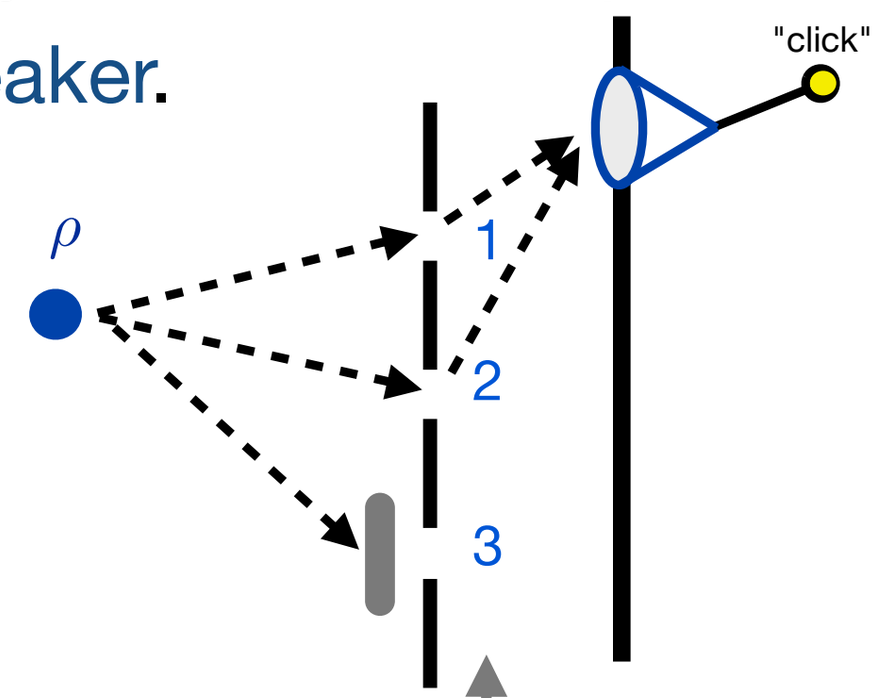
CE[1]: intuitive properties of composition of slit transformations

No 3rd order int.: decomposability of interference pattern into pairs

⇒ intuitively, CE[1] seems much weaker.



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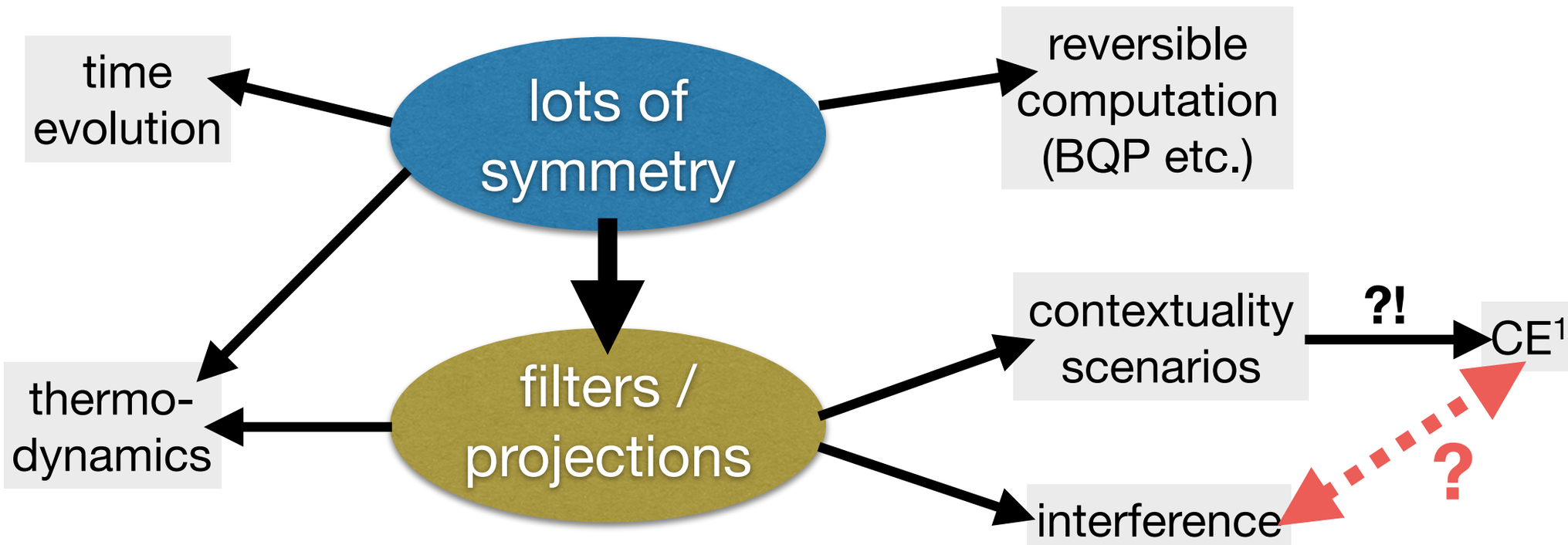

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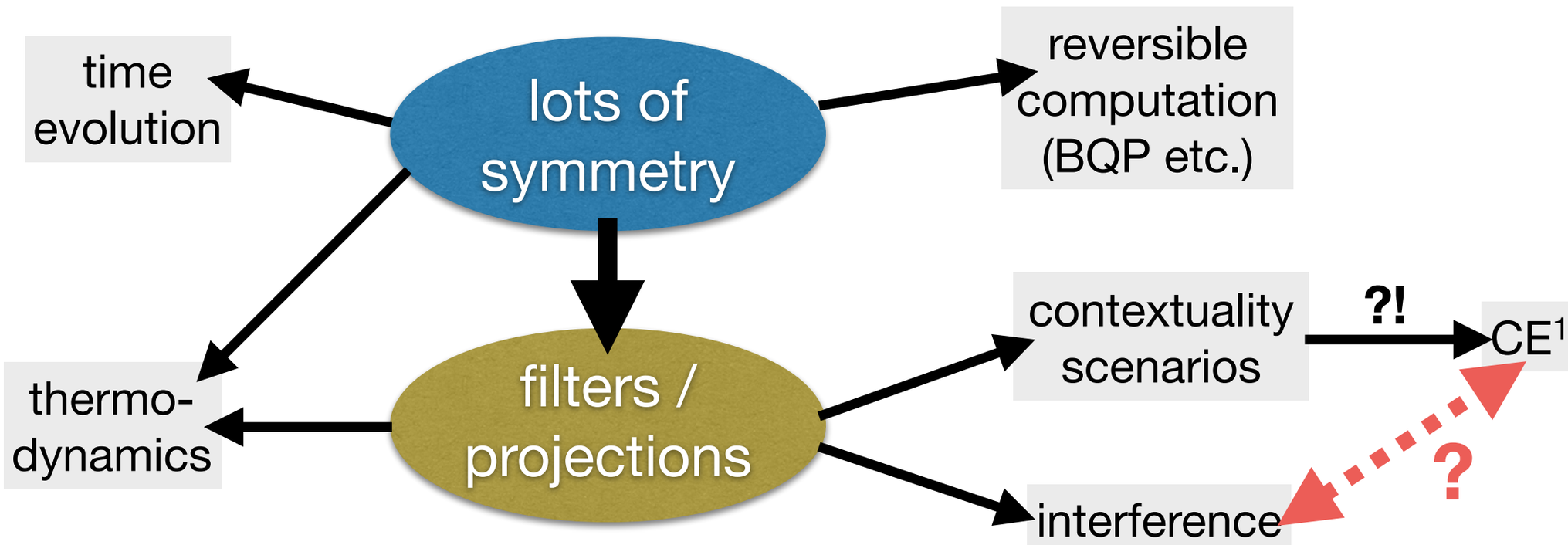

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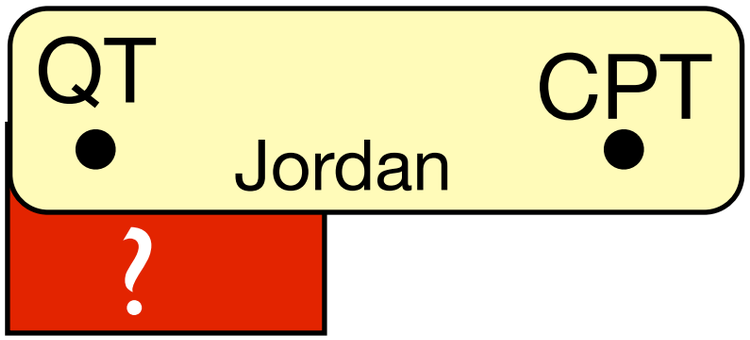
4. Conclusions

- Contextuality should be studied in the context of GPTs
  → post-measurement states

- Consistent exclusivity becomes much less mysterious.
  In particular
  lots of symmetry ⇒ projections ⇒ CE[1]

- Relation to 3rd-order interference goes probably only in one direction. →What are those?



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H. Barnum, MM, and C. Ududec, *Higher-order interference and single-system postulates characterizing quantum theory*, New J. Phys. **16**, 123029 (2014), arXiv:1403.4147

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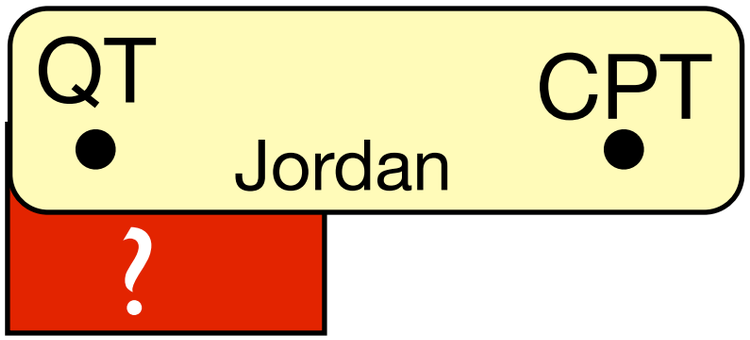

Thank you!