Axiomatic reconstructions and generalizations of quantum theory

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Howard Barnum (University of New Mexico)
Cozmin Ududec (Perimeter Institute)



PERIMETER INSTITUTE
FOR THEORETICAL PHYSICS

Outline

1. General probabilistic theories

Quantum theory is just one possible probabilistic theory.

2. Geometry and probability

Deriving QT and 3D of space from axioms on their relation.

3. Third-order interference

Searching and testing for "QT's closest cousins".

1. General probabilistic theories

QT violates Bell inequalities, but not maximally:

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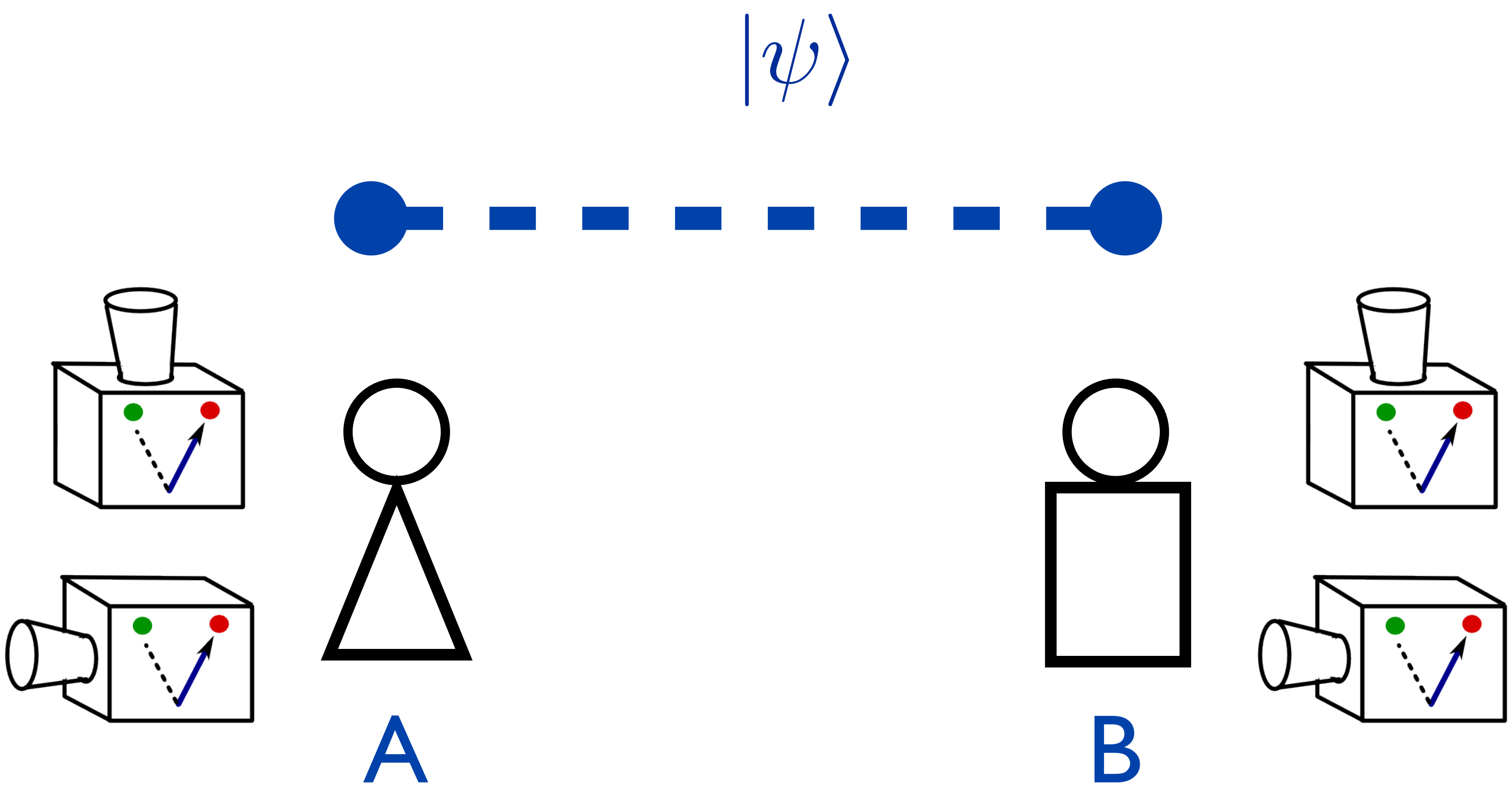

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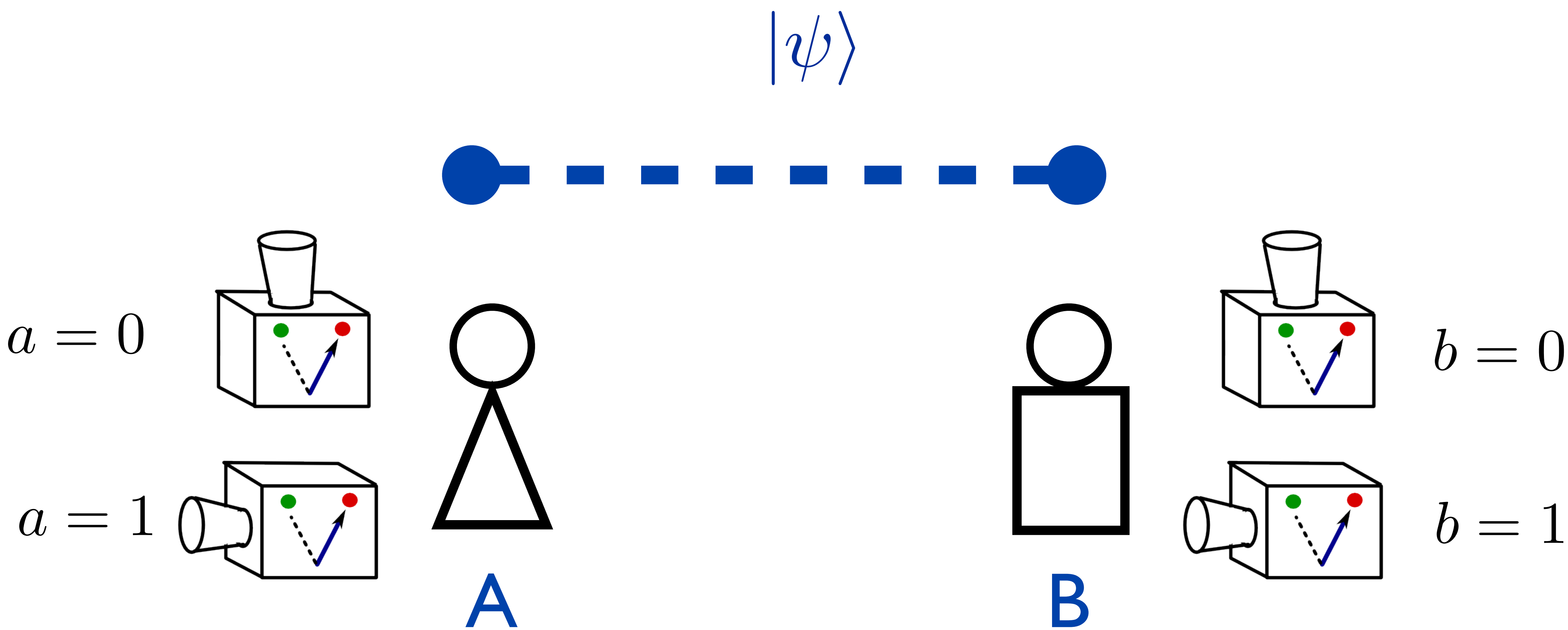

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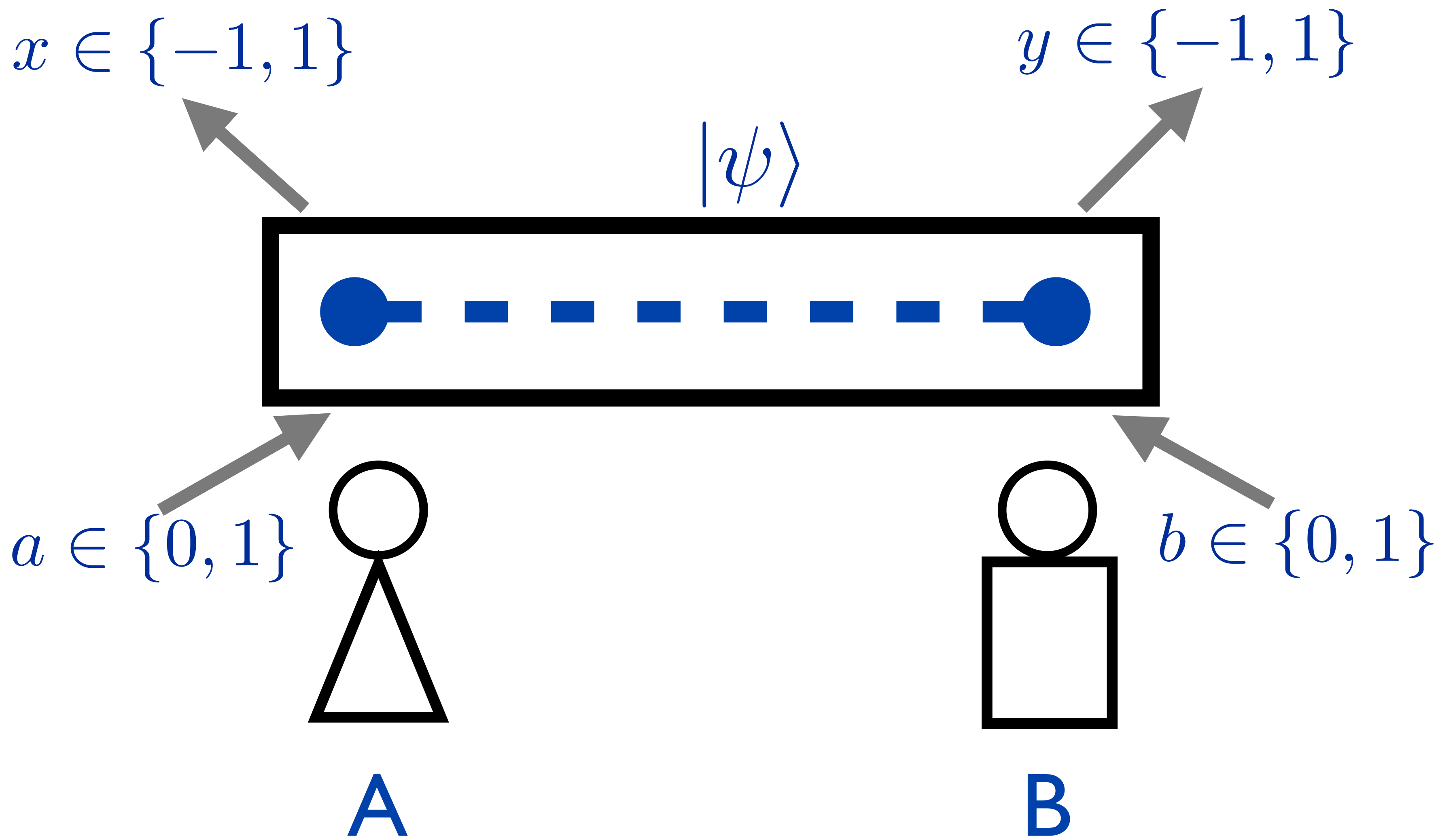

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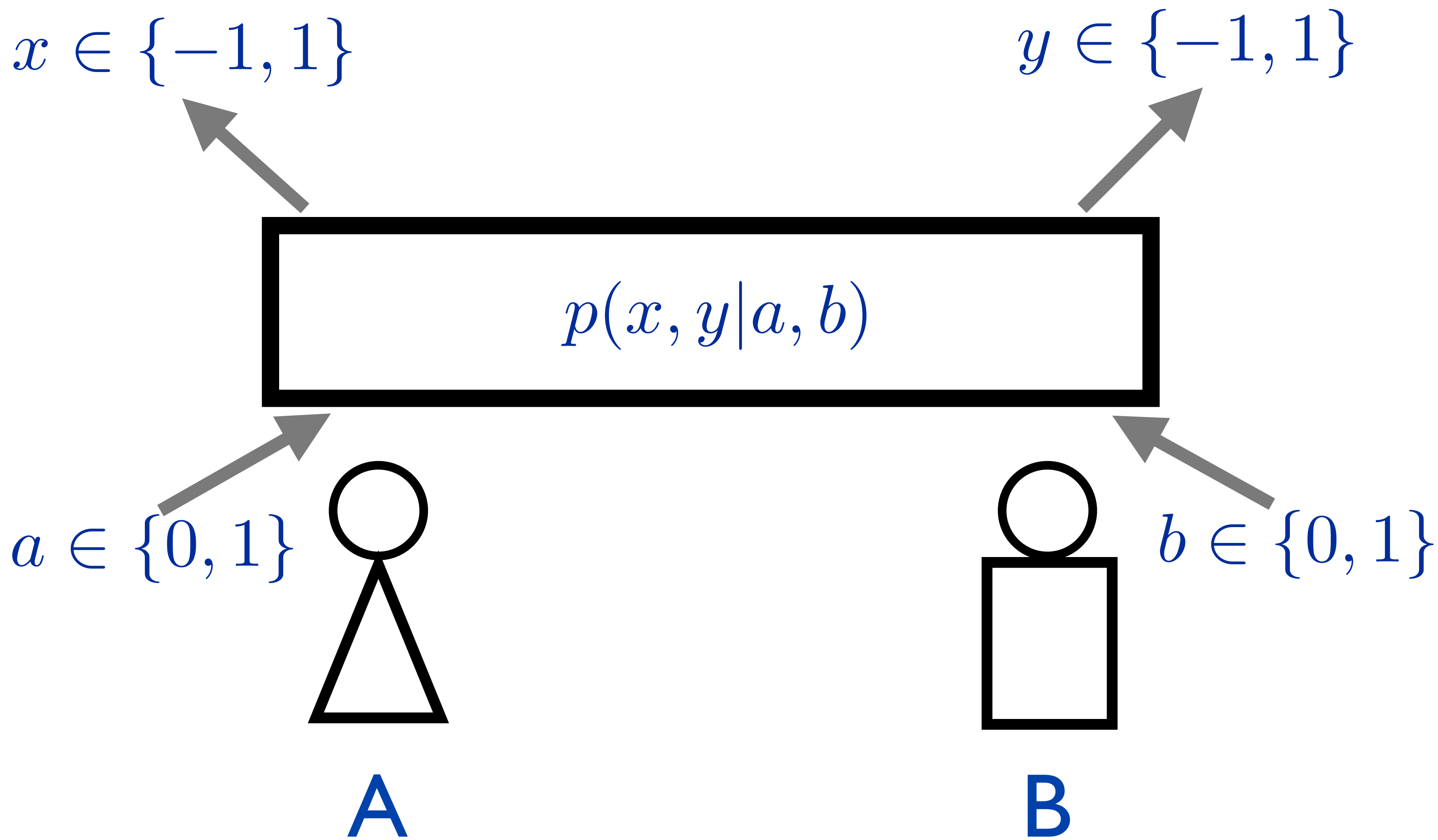

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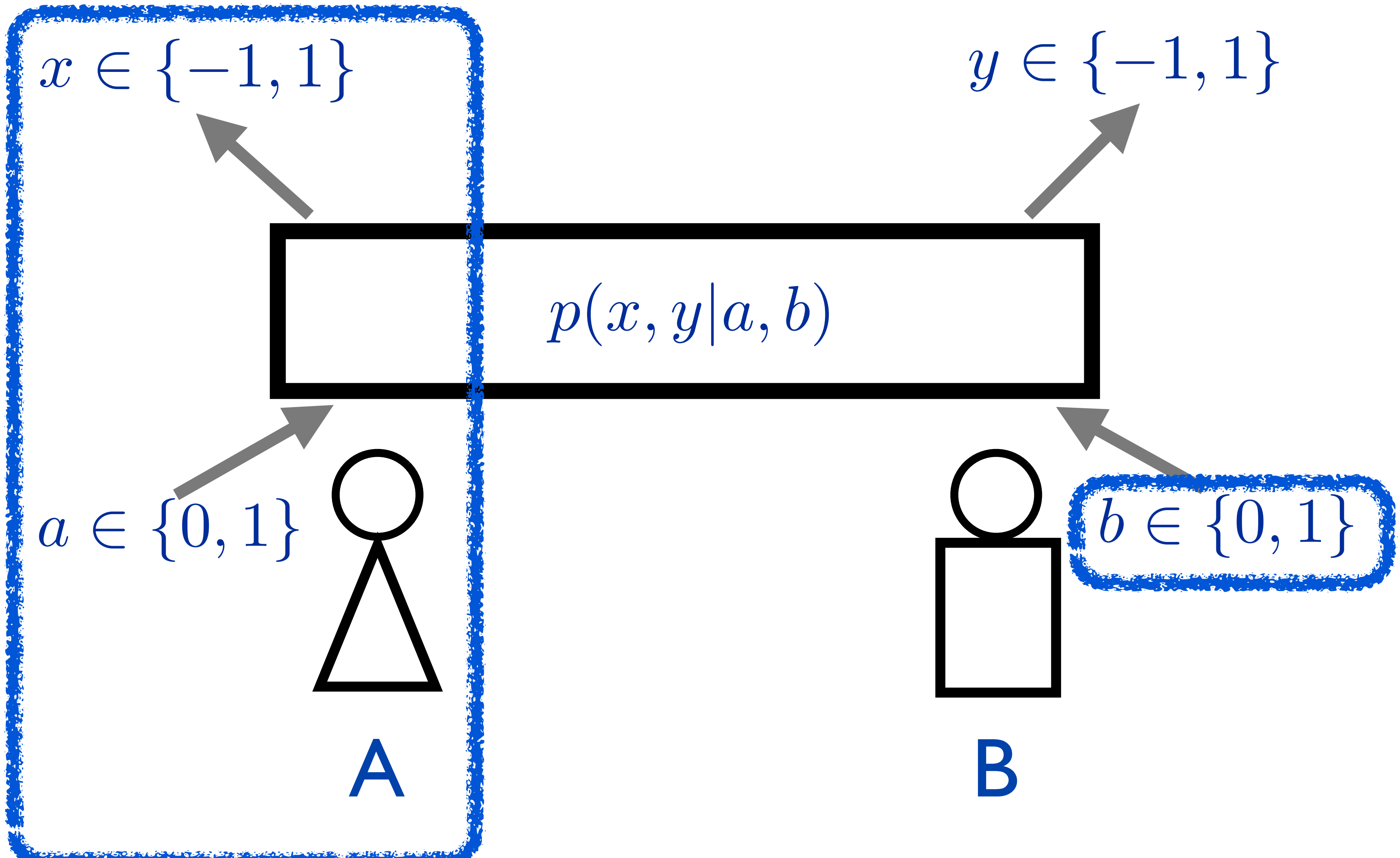

No-signalling: $p(x|a)$ does not depend on b (and vice versa)

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Classical probability distributions satisfy Bell inequality:

$$\text{CHSH} := |C_{00} + C_{01} + C_{10} - C_{11}| \leq 2 \quad \text{where} \quad C_{ab} := \mathbb{E}\left(x \cdot y | a, b\right).$$

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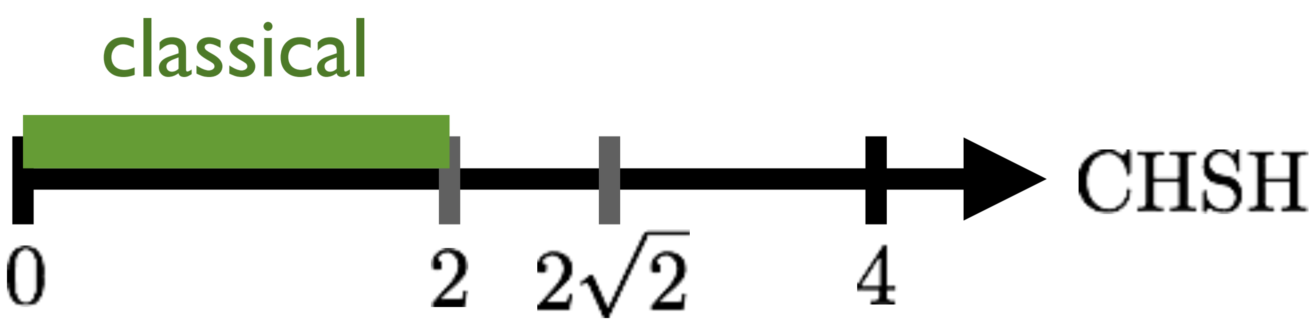

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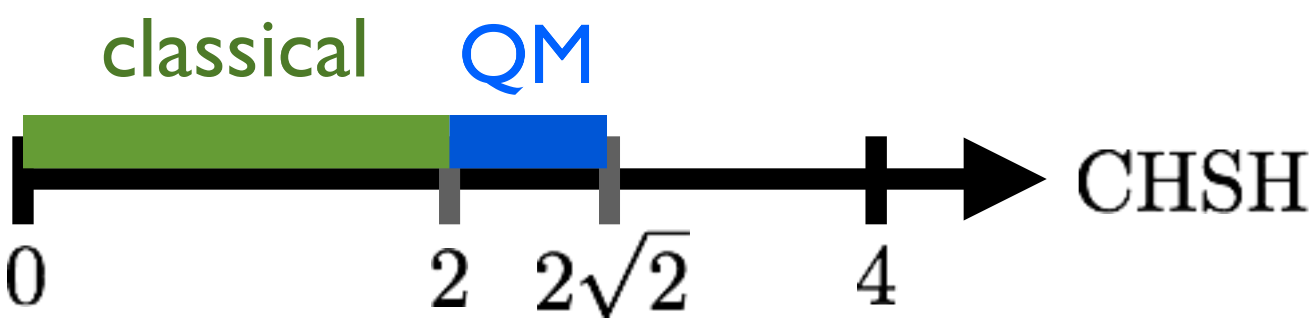

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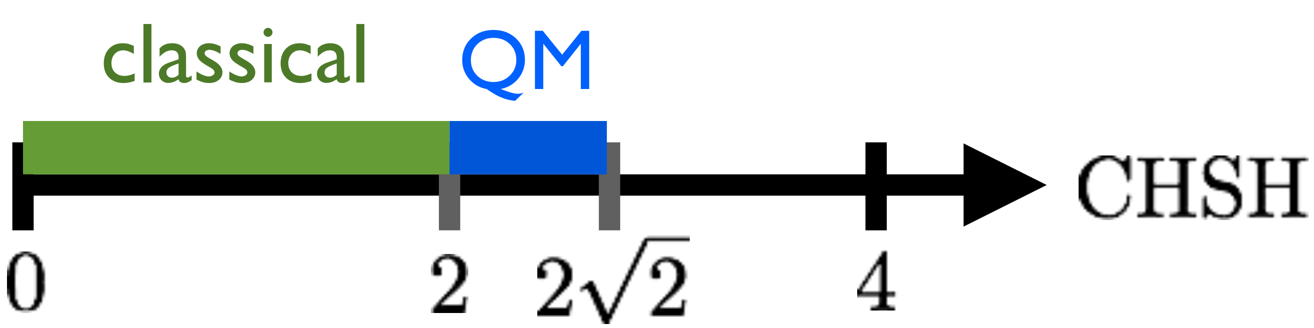

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$$p(+1,+1|a,b) = p(-1,-1|a,b) = \frac{1}{2}$$
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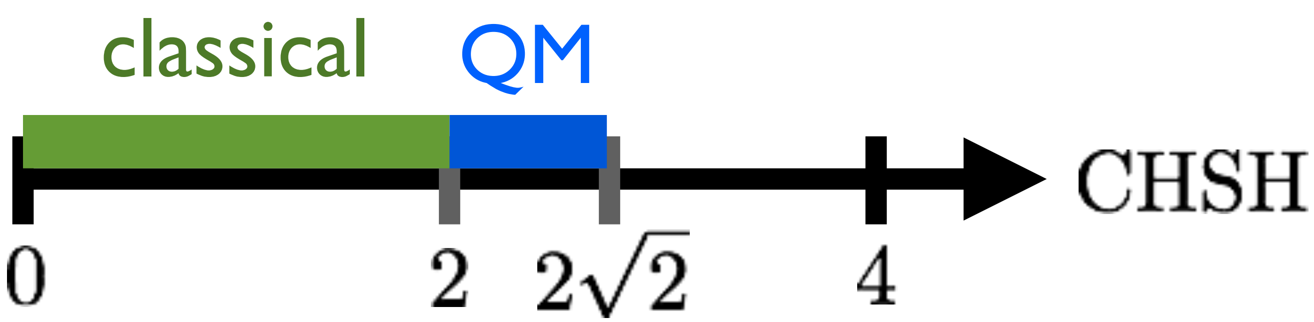

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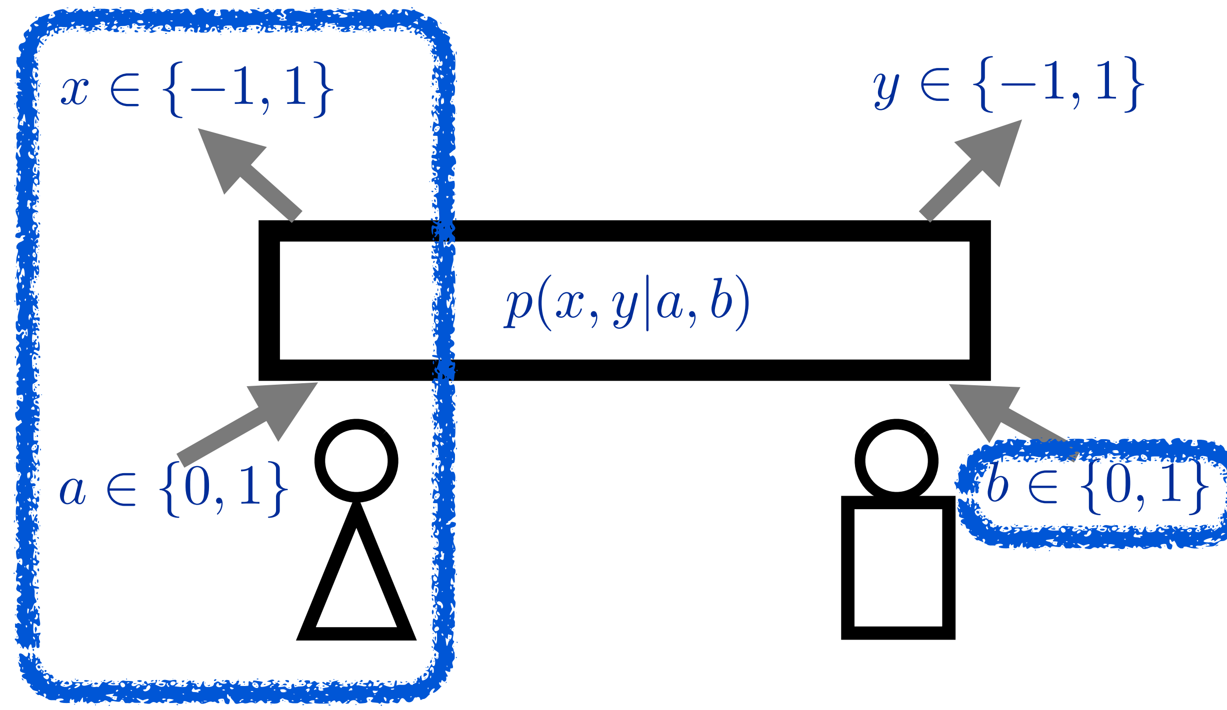

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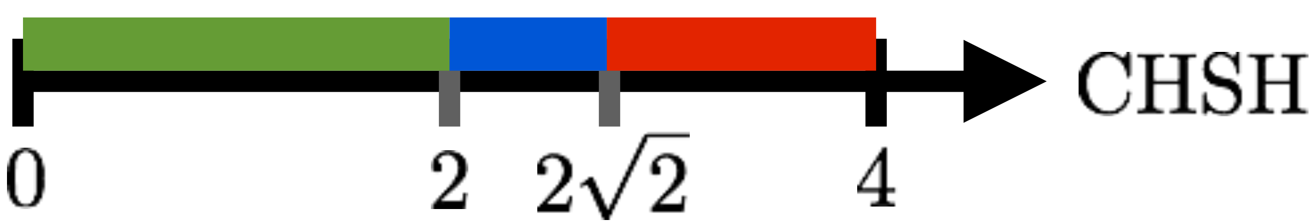

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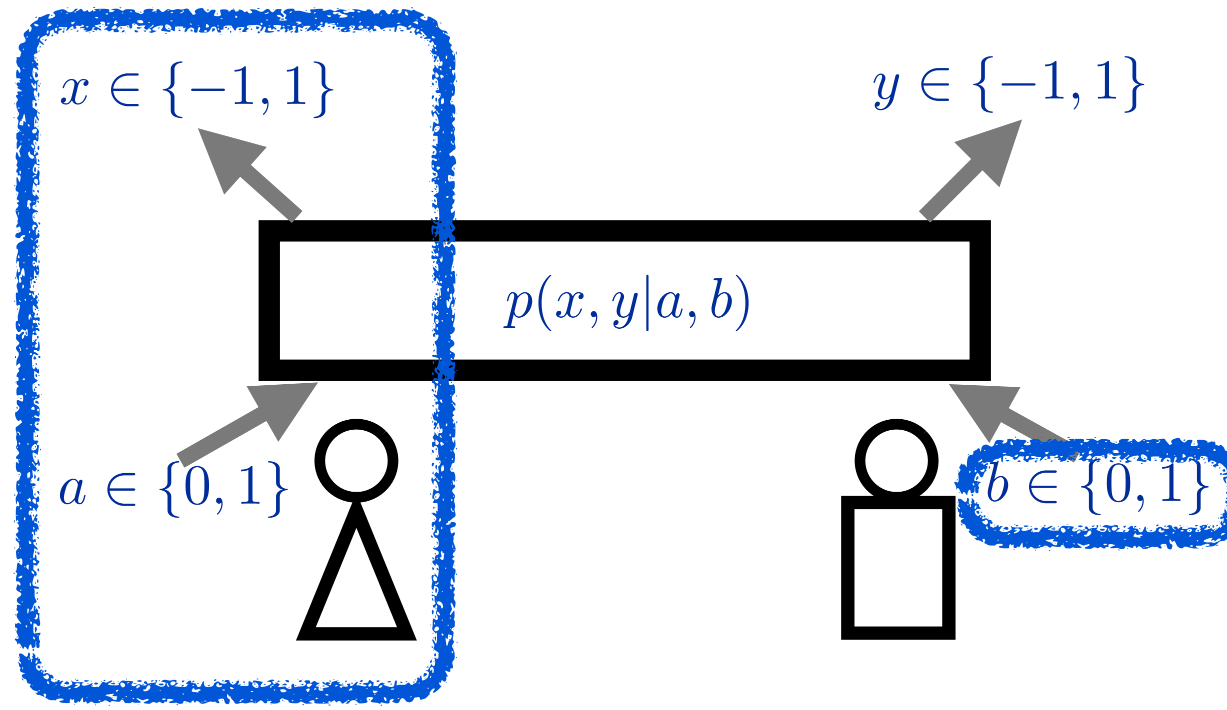

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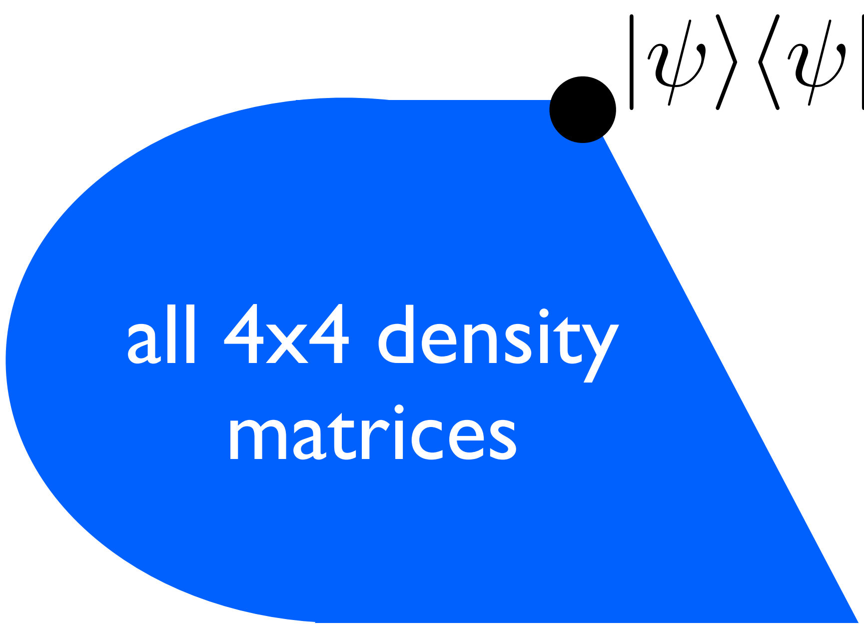

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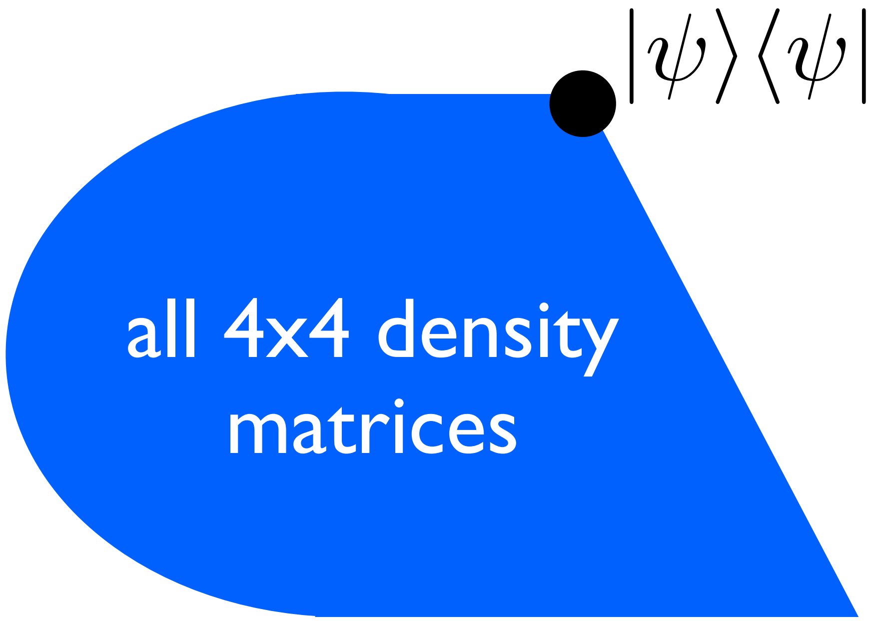

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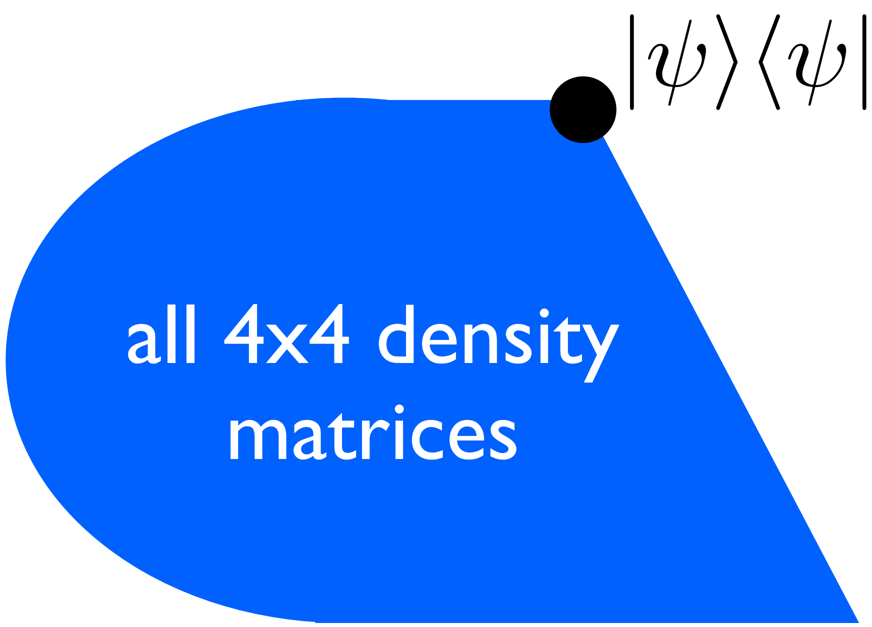

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For 2 parties, 2 measurements, 2 outcomes each:

$$\omega = \left(\begin{array}{c} p(+1,+1|0,0) \\ p(+1,-1|0,0) \\ p(-1,+1|0,0) \\ \vdots \\ p(+1,+1|1,1) \end{array} \right).$$

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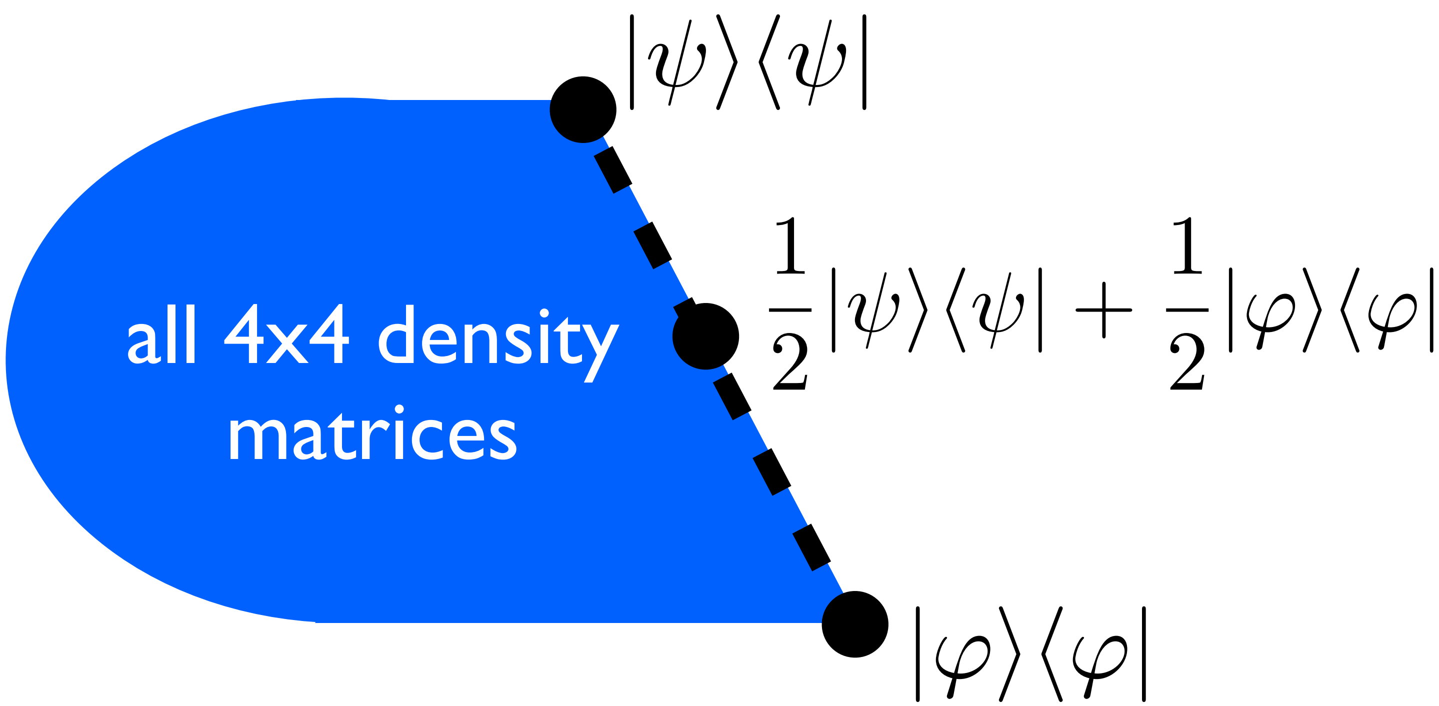

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State spaces are convex.

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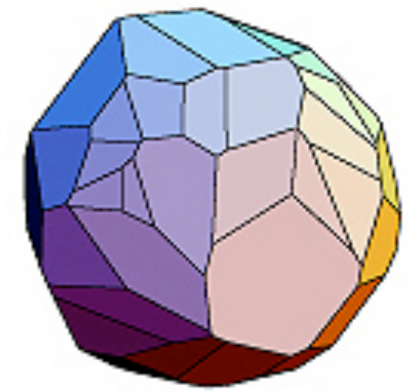

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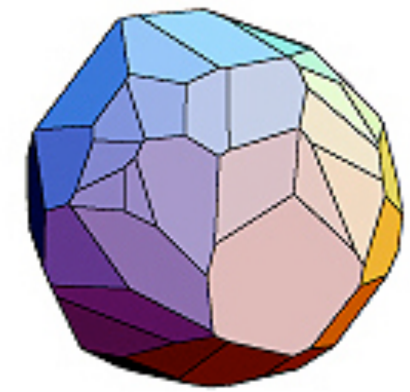

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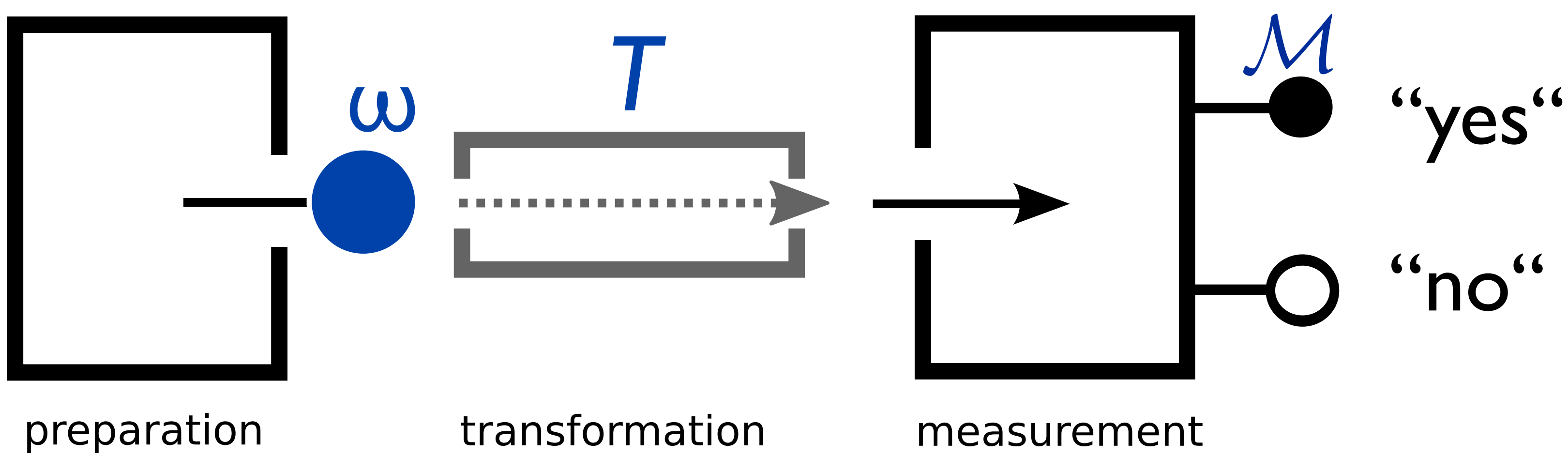
That is the set of all states ω that can be prepared.



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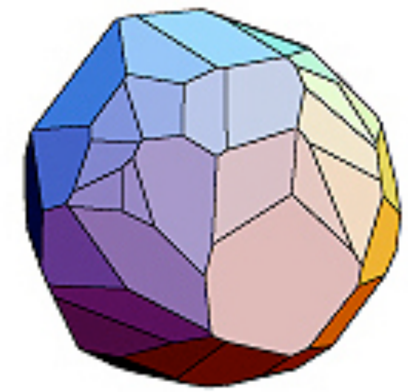

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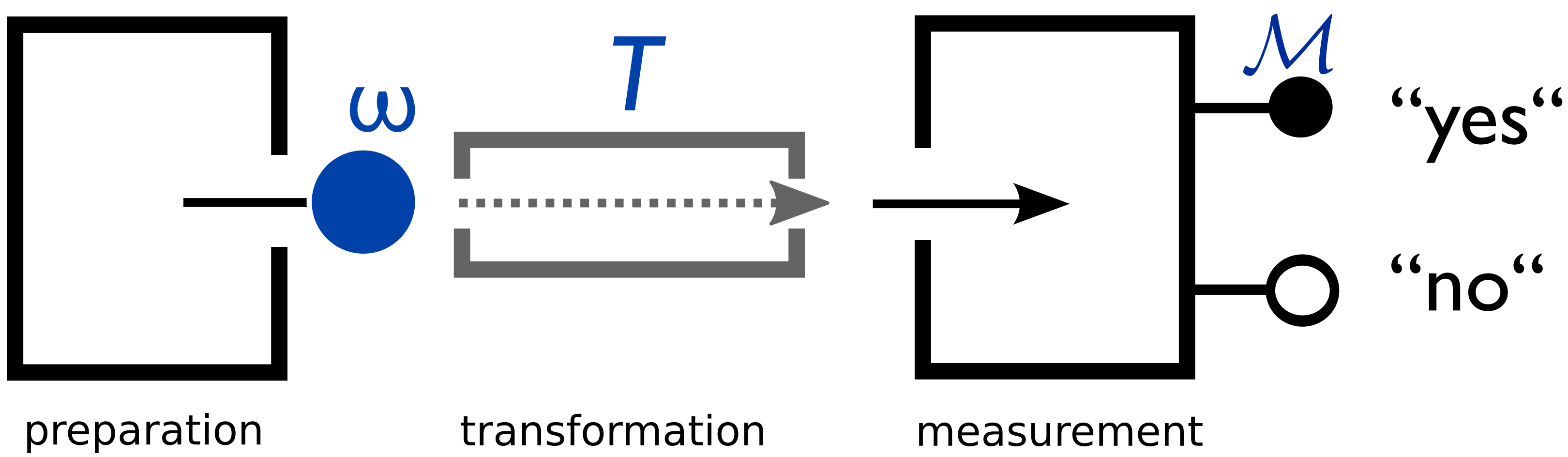

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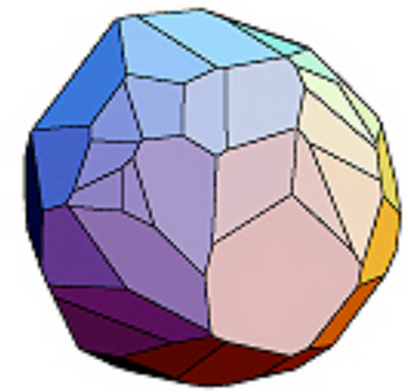

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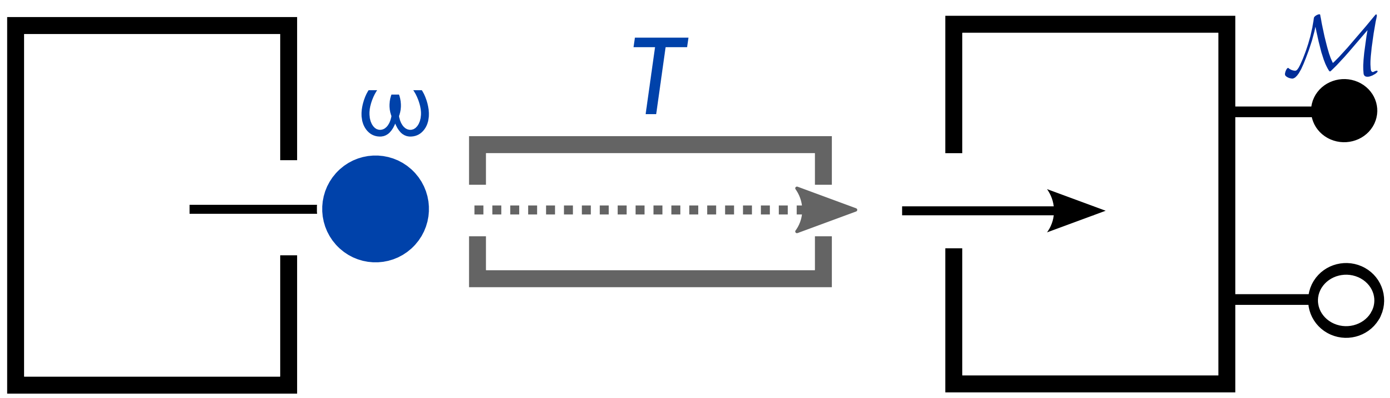

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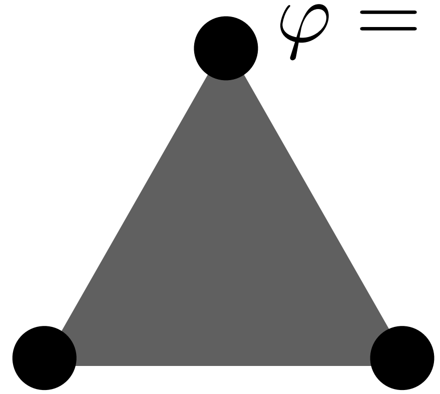
Probabilities of measurement outcomes $\mathcal{M}$ are linear functionals on state space.

1. General probabilistic theories

Classical N-level state spaces:

bit (N=2)



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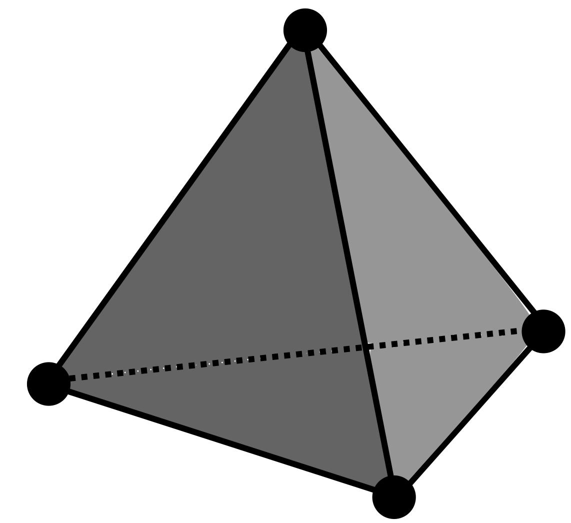

N=4

1. General probabilistic theories

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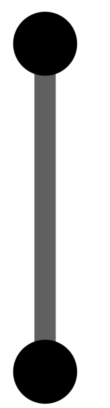

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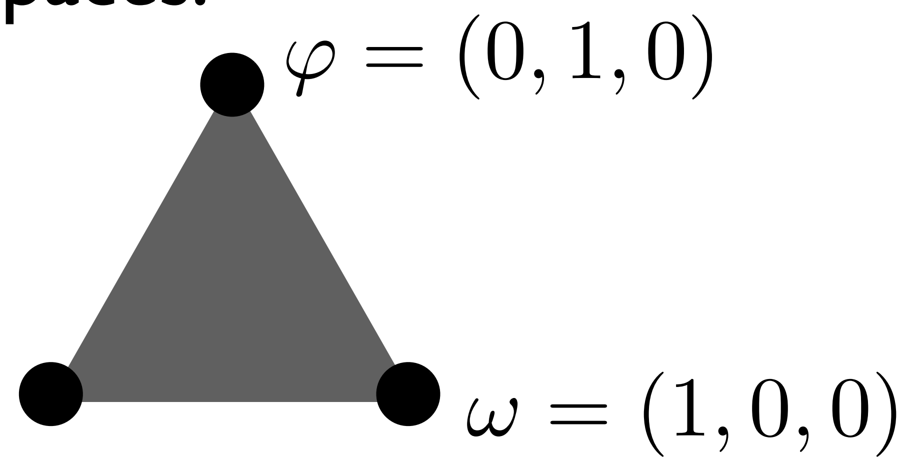

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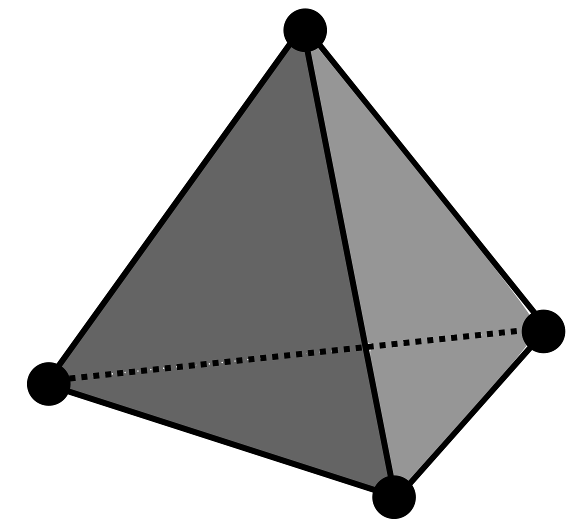

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Quantum N-level state spaces: NxN density matrices

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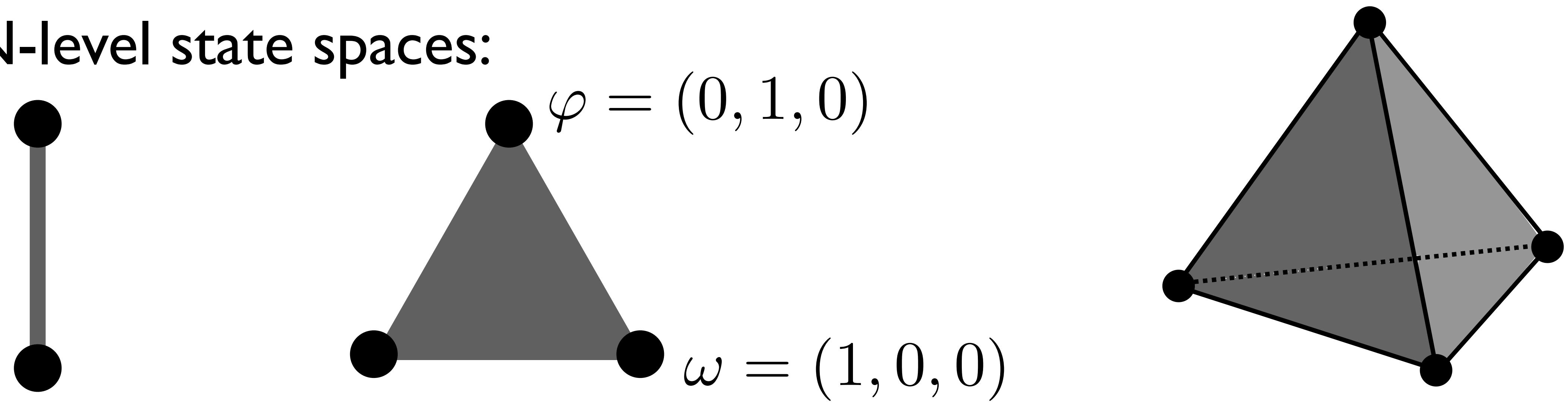

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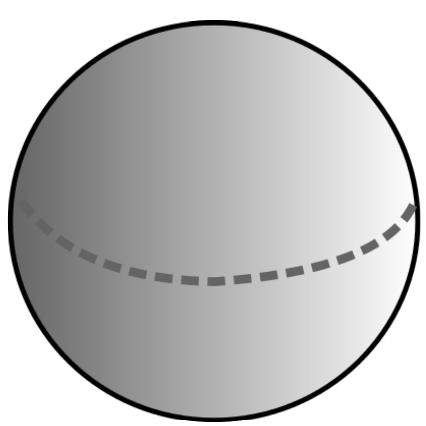

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle.$$

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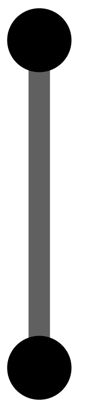

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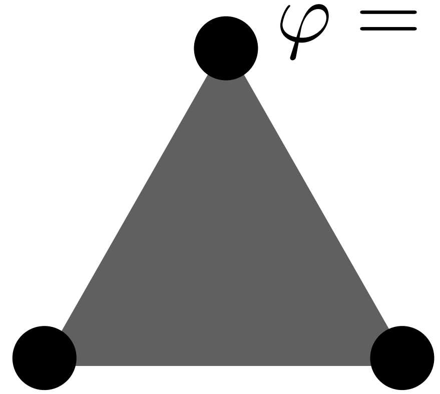

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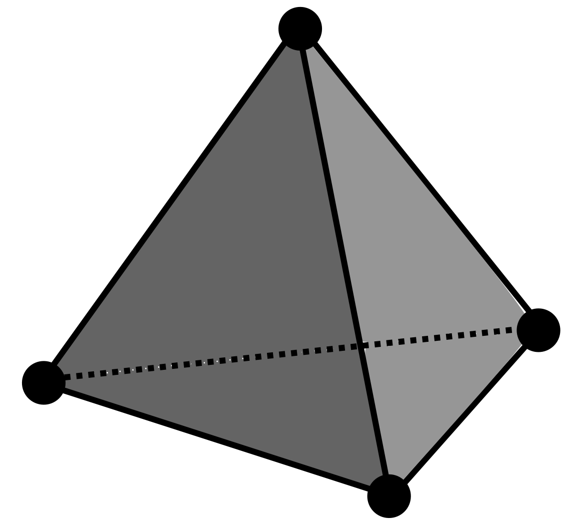

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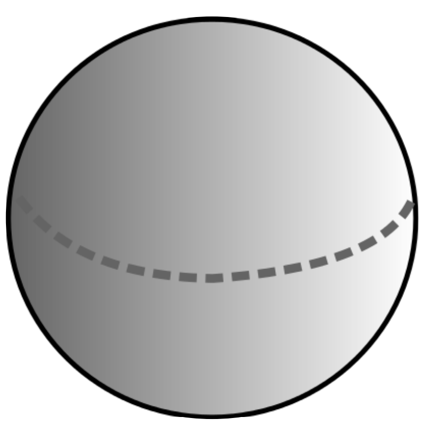

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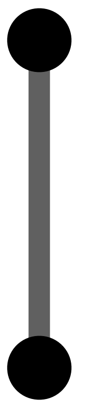

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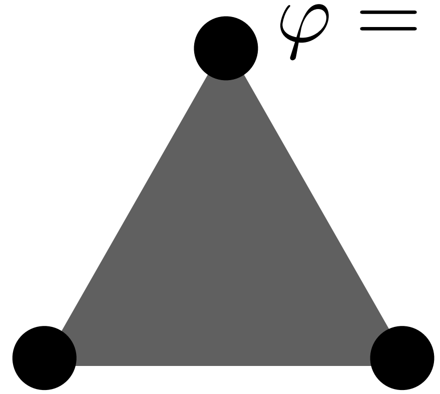

$\varphi = (0, 1, 0)$

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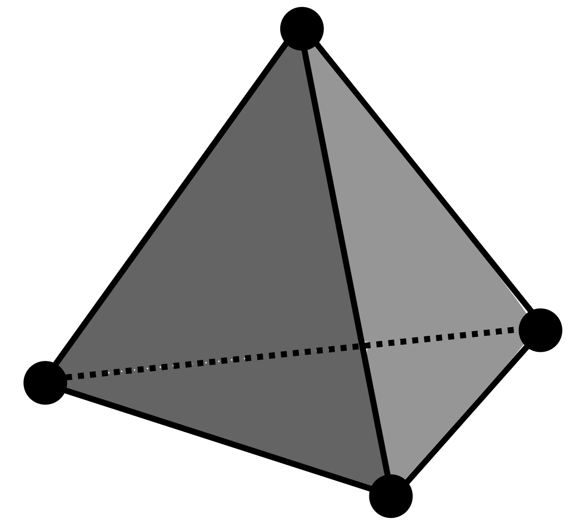

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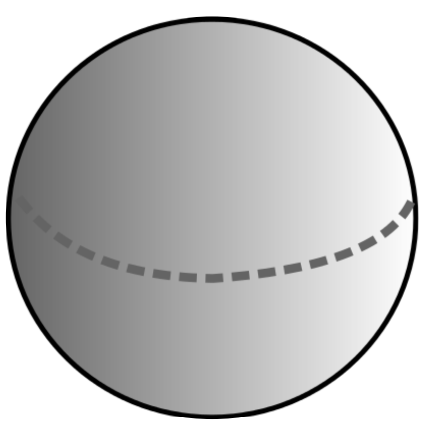

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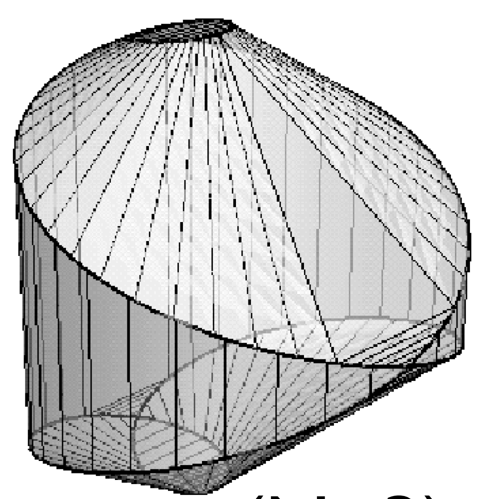

qutrit (N=3)

Complicated 8D convex set, mixed states in its boundary.

Bengtsson et al., arXiv:1112.2347

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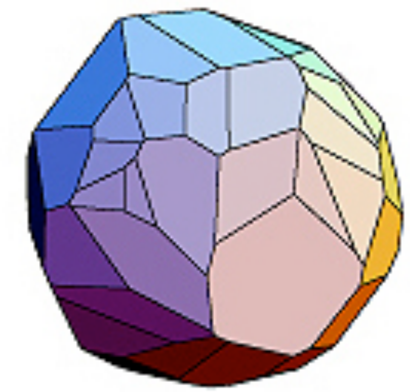

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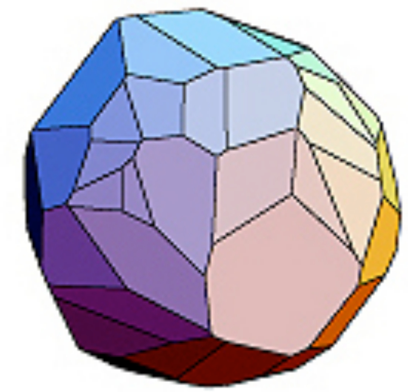
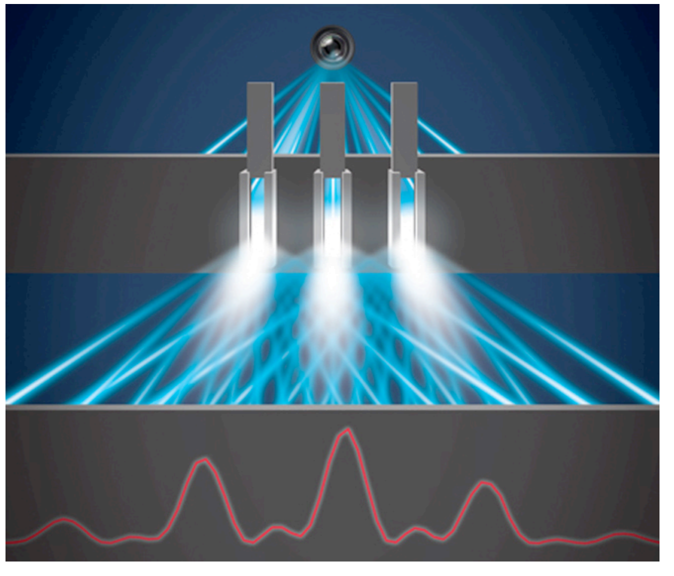


- More or less non-locality, complementarity, computational power than QT, no-cloning,
- many allow for teleportation, analogs of "unitaries" and the "Schrödinger equation",
- physical predictions different from QT.



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In what follows: sketch 2 ways with different goals.

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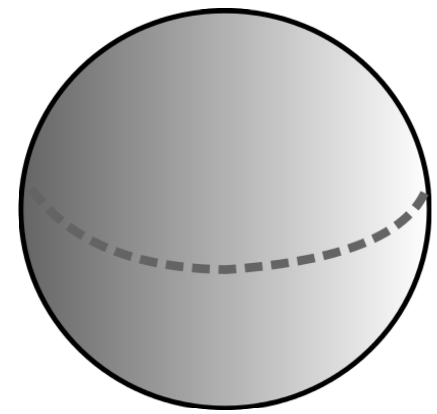
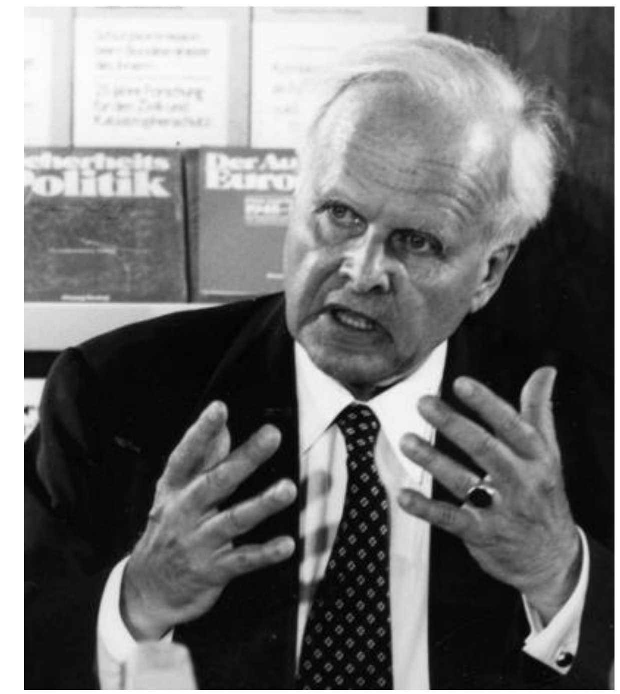
Space is 3D because the qubit is!





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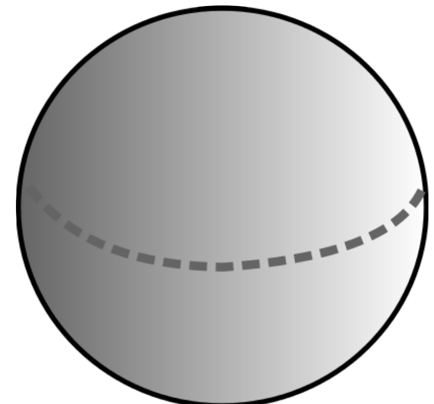
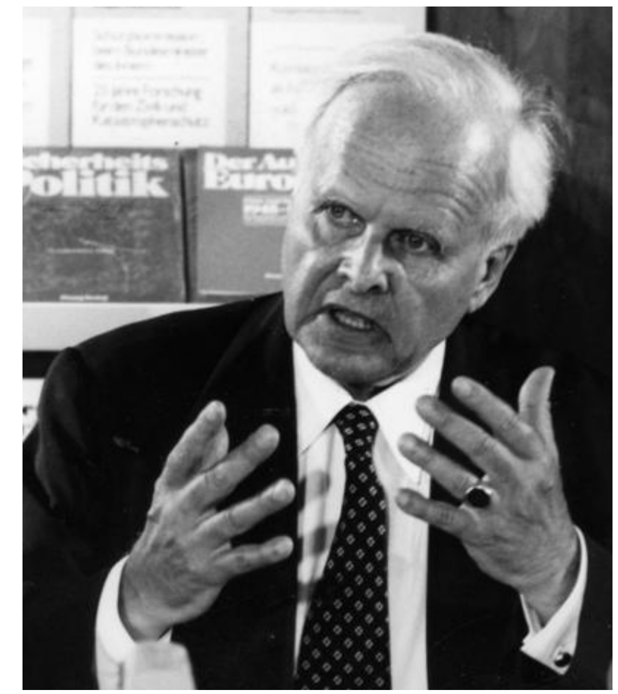
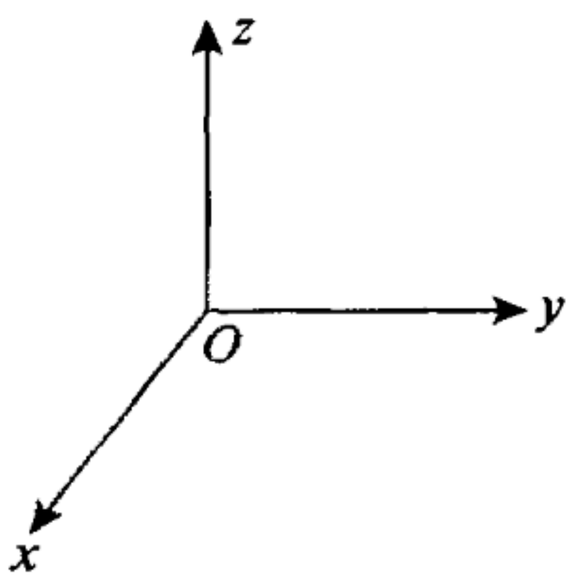
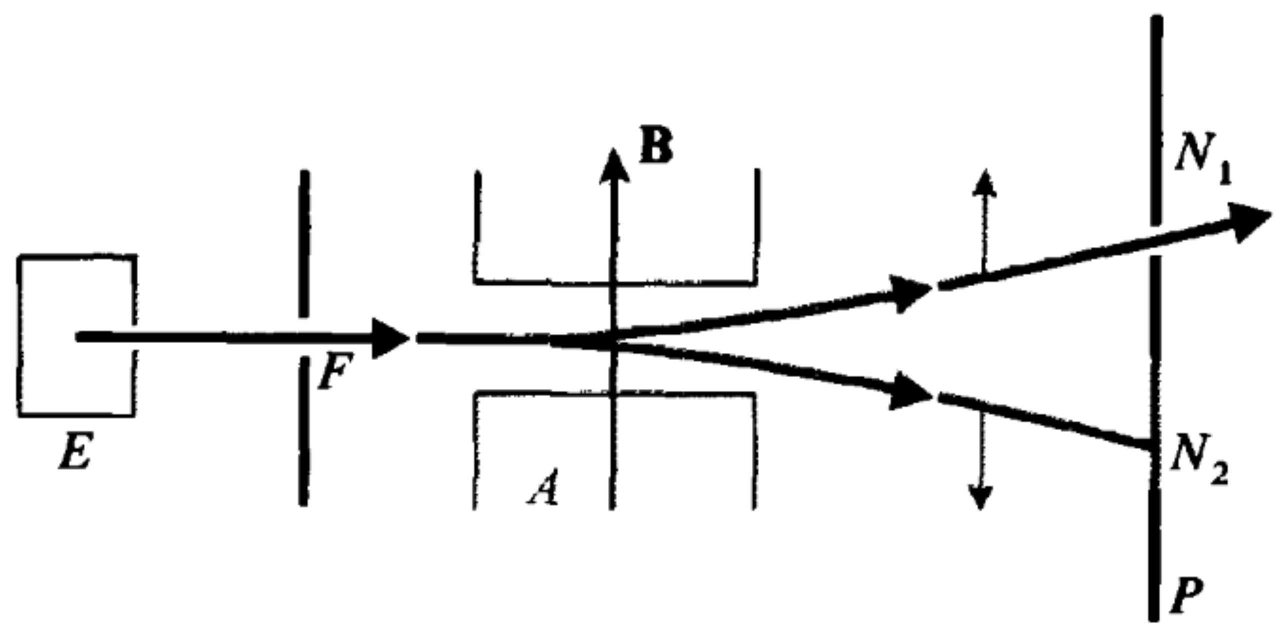




Spatial rotations of Stern-Gerlach device can prepare any pure qubit state.





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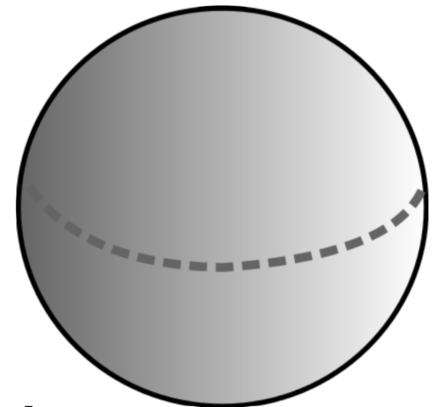
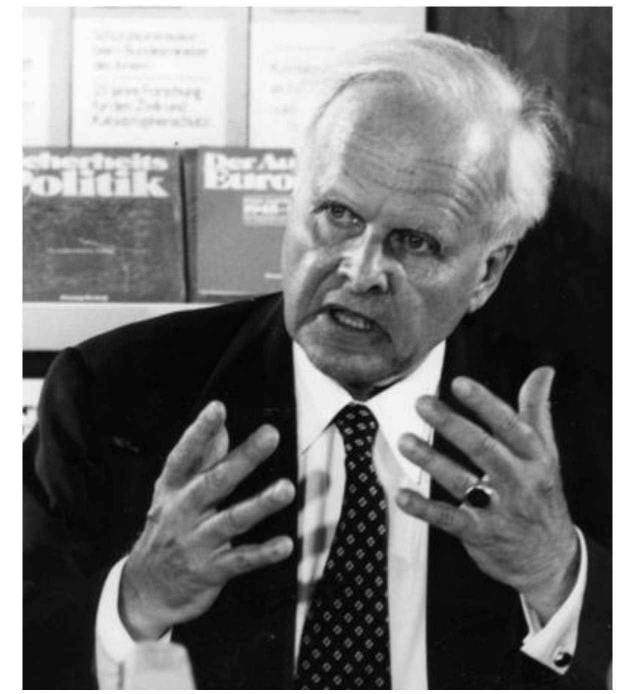
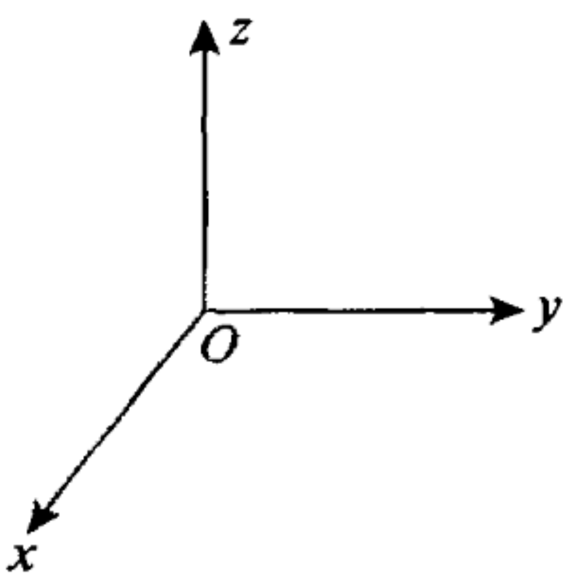
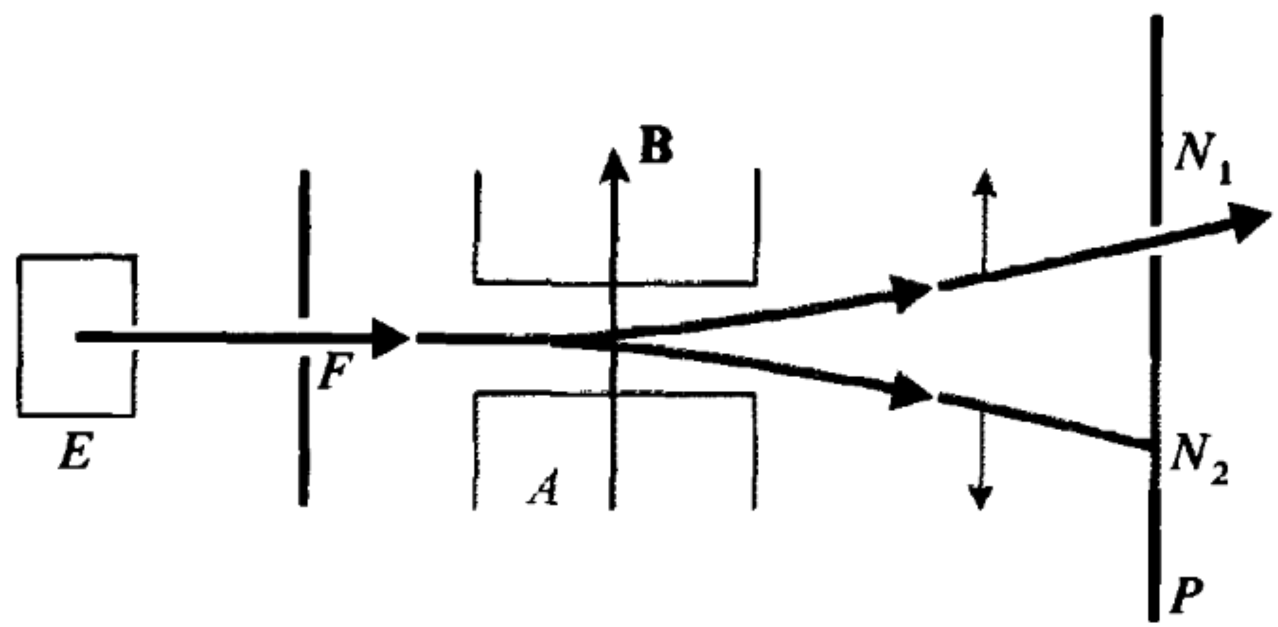
Space is 3D because the qubit is!





Spatial rotations of Stern-Gerlach device
can prepare any pure qubit state.



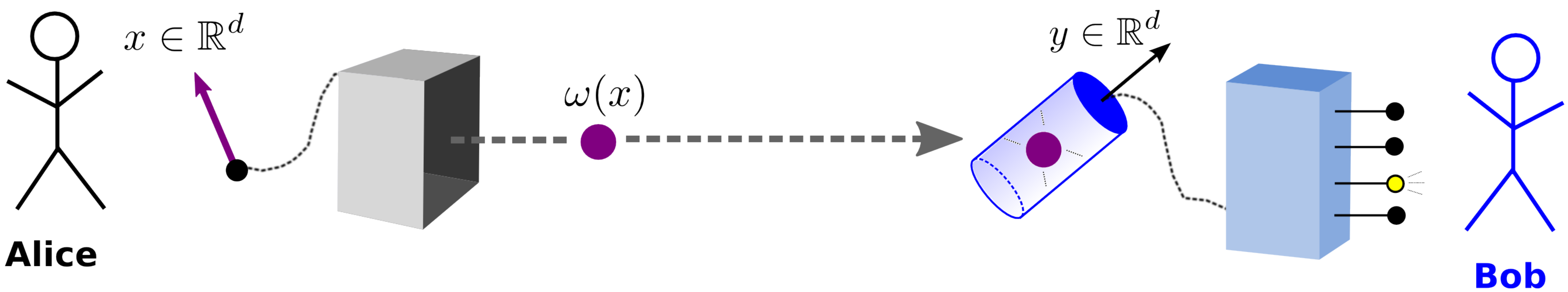


Could a similar relationship exist in d≠3 ?

2. Geometry and probability

2. Geometry and probability

An information-theoretic task in *d* spatial dimensions:



MM and Ll. Masanes, New J. Phys. **15**, 053040 (2013), arXiv:1206.0630

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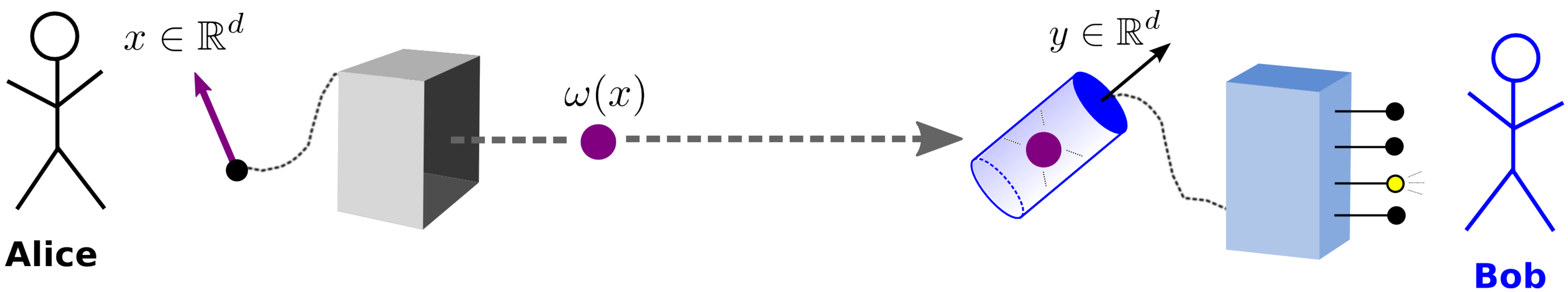

4 Postulates: There is a probabilistic system such that...

1. Alice can send any spatial direction $x \in \mathbb{R}^d, \ \ |x| = 1,$
2. but not more.

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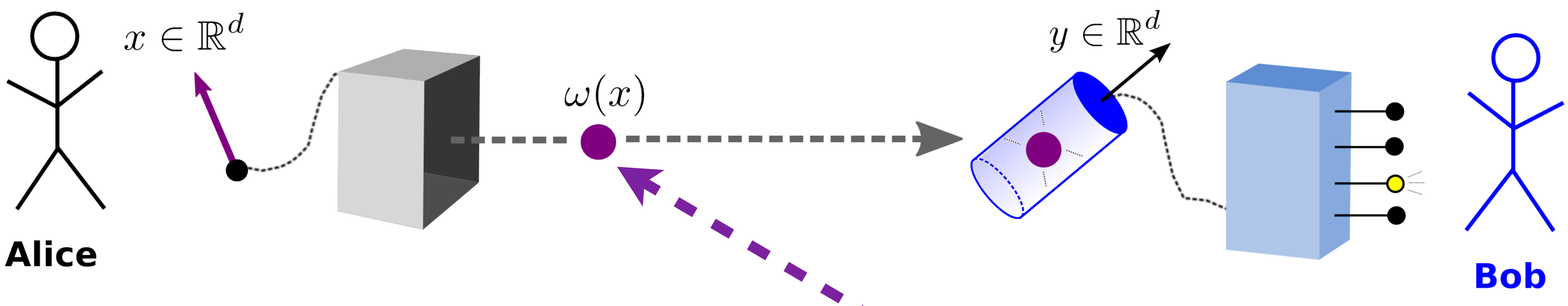
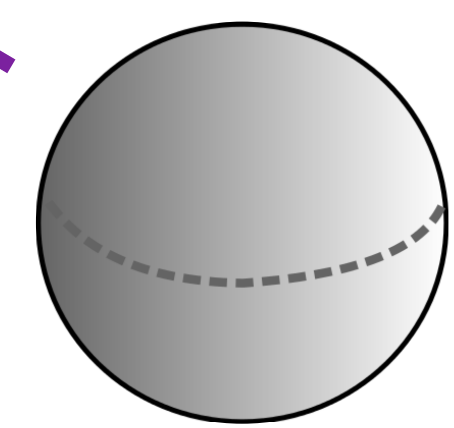


4 Postulates: There is a probabilistic system such that...

1. Alice can send any spatial direction, $x \in \mathbb{R}^d, \quad |x| = 1,$

2. but not more.

Consequence: state space is *d*-dim. unit ball.



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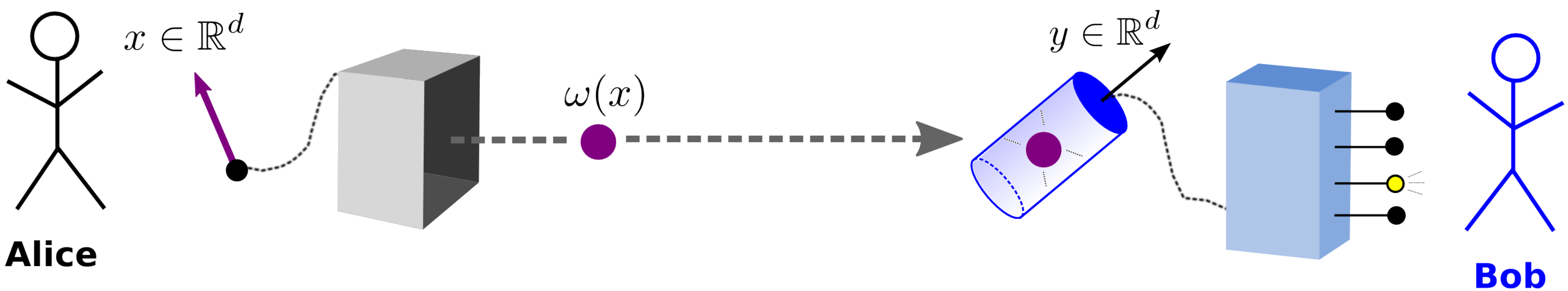

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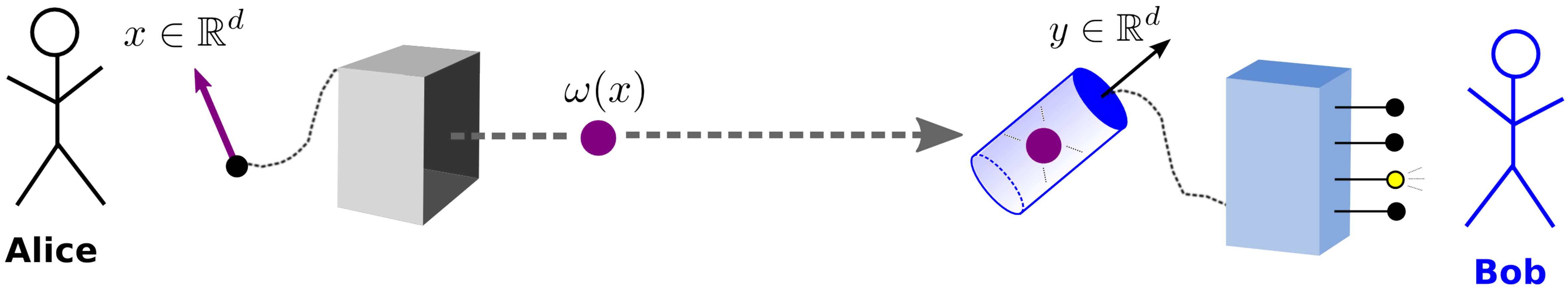

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3. (Coordinate) transformations on pairs of systems are uniquely determined by action on single systems.

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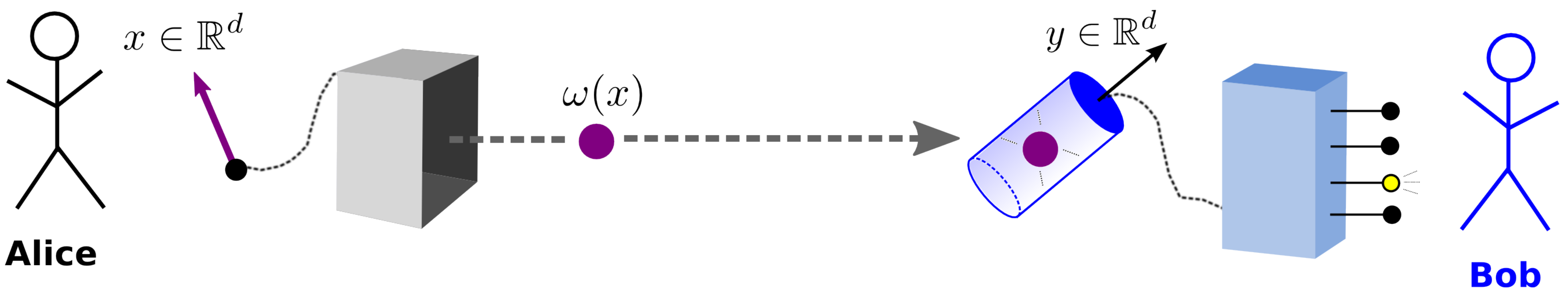

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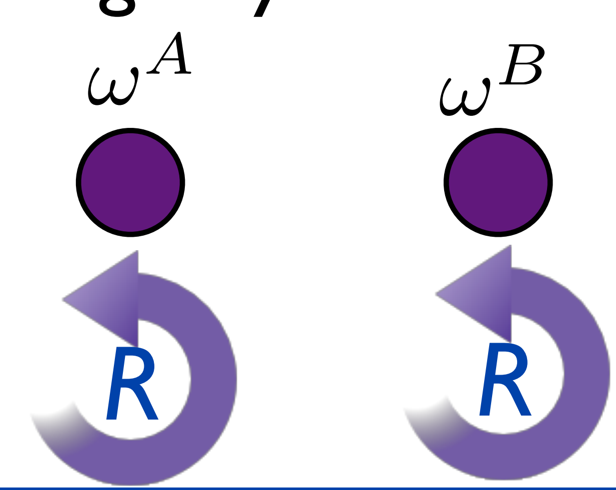

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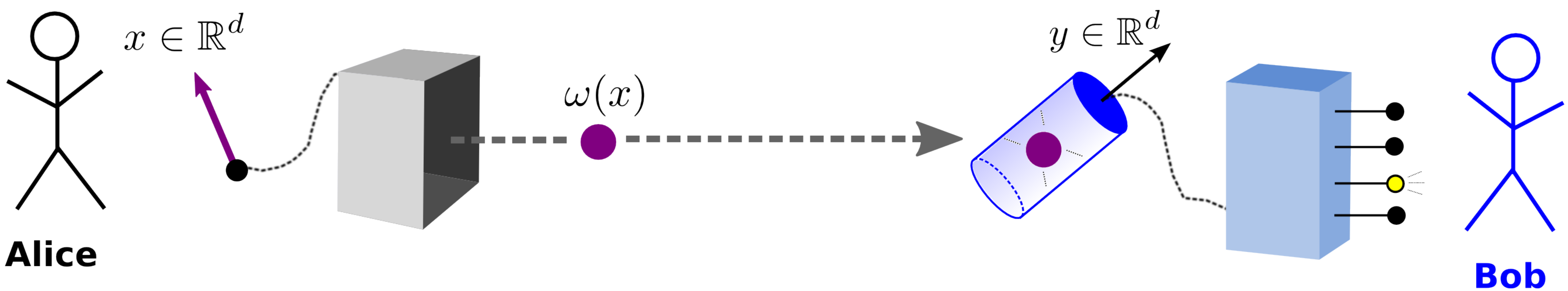

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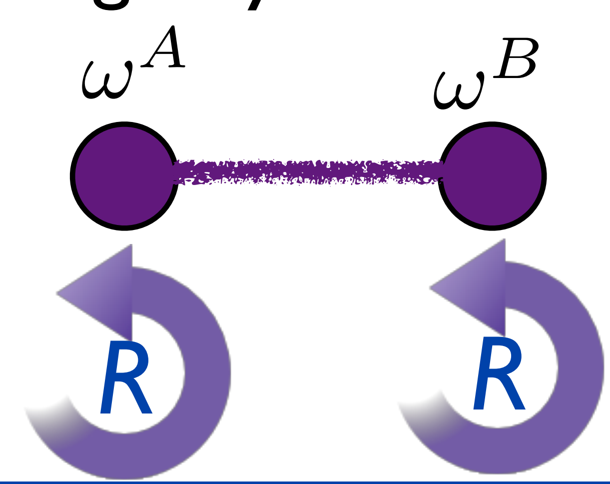

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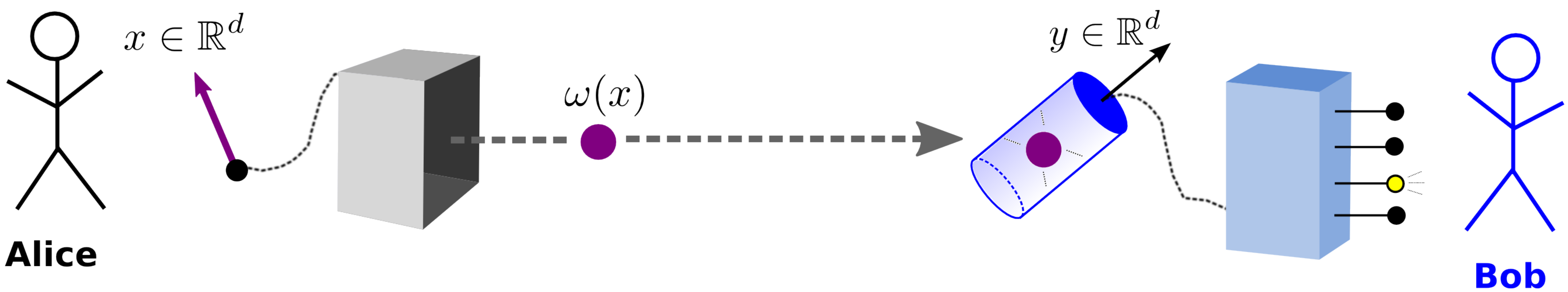

4 Postulates: There is a probabilistic system such that...

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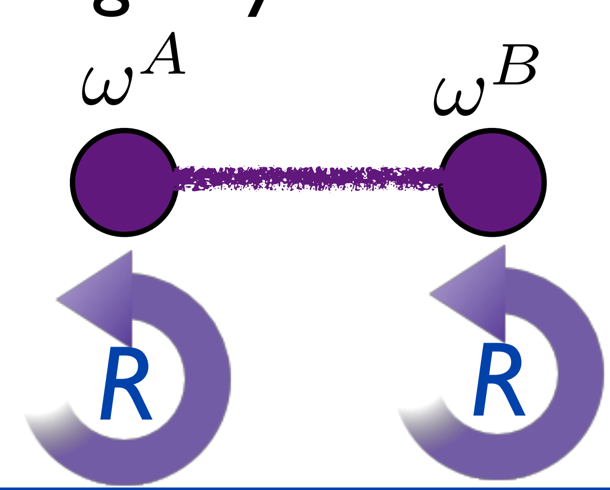
Consequence:

$$d_{AB} = (d+1)^2 - 1.$$



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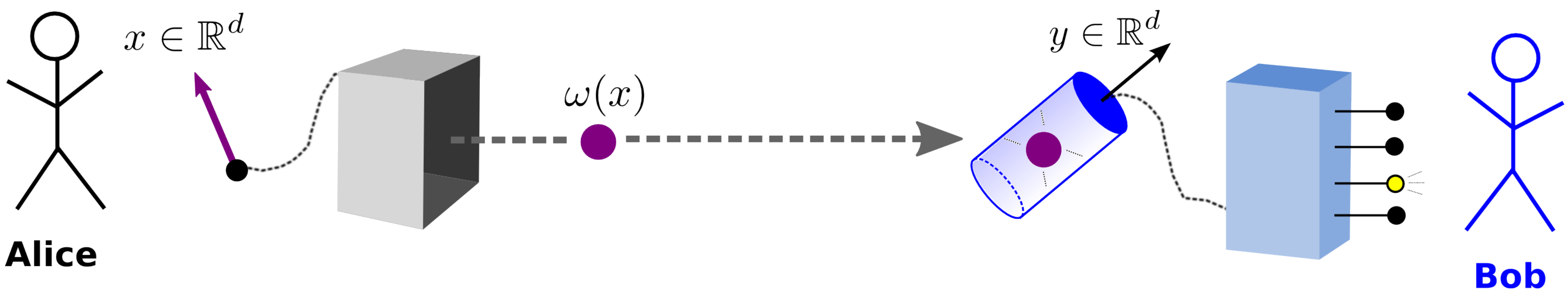

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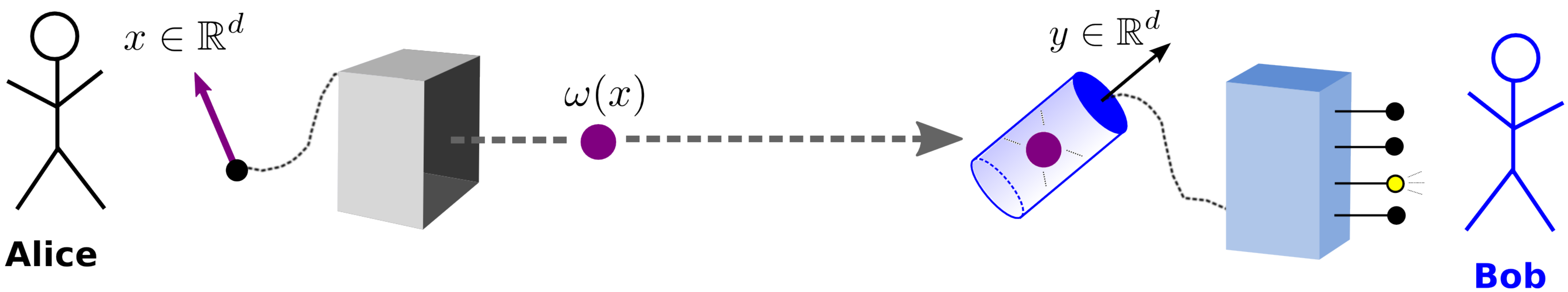

4 Postulates: There is a probabilistic system such that...

4. Pairs of systems can interact reversibly and continuously in time.

MM and Ll. Masanes, New J. Phys. **15**, 053040 (2013), arXiv:1206.0630

2. Geometry and probability

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4. Pairs of systems can interact reversibly and continuously in time.

$$\omega^{AB}(t) = G(t)\left(\omega^A(0)\omega^B(0)\right).$$

$\uparrow$

$$\neq \omega^A(t)\omega^B(t)$$

2. Geometry and probability

4. Pairs of systems can interact reversibly and continuously in time.

$$\omega^{AB}(t) = G(t)\left(\omega^A(0)\omega^B(0)\right).$$

We don't know the global state space...

... and don't know what this group is.

d-dim. ball state spaces

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Theorem (Ll. Masanes, MM, D. Pérez-García, R. Augusiak, arXiv:1111.4060)
This is only possible if d=3.

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Theorem (Ll. Masanes, MM, D. Pérez-García, R. Augusiak, arXiv:1111.4060)
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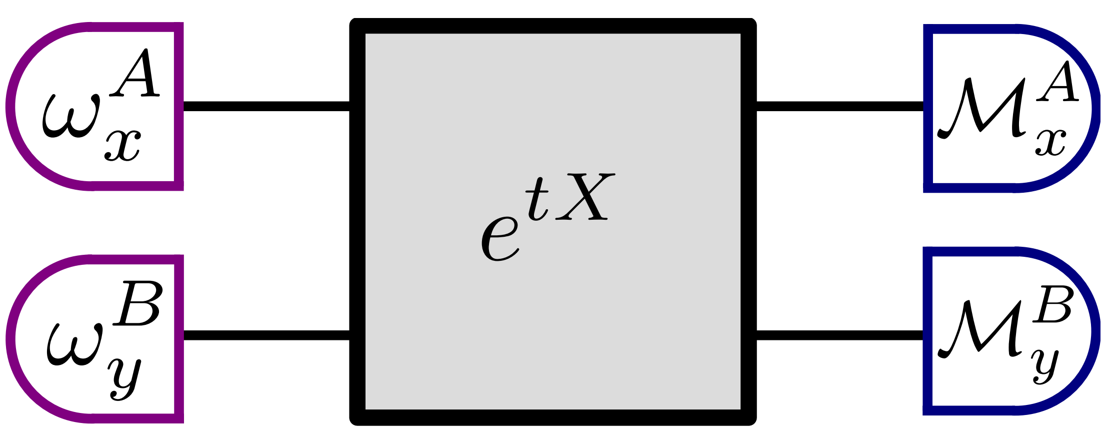
Proof idea:



If $d\neq 3$, probability negative unless $X=X^A+X^B$, i.e. no interaction.

2. Geometry and probability

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3-dim. ball state spaces

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3-dim. ball state spaces

If *d*=3 then $d_{AB} = (d+1)^2 - 1 = 15$

= # of real parameters in a 4x4 density matrix. In fact:

2. Geometry and probability

4. Pairs of systems can interact reversibly and continuously in time.

$$\omega^{AB}(t) = G(t)\left(\omega^A(0)\omega^B(0)\right).$$

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3-dim. ball state spaces

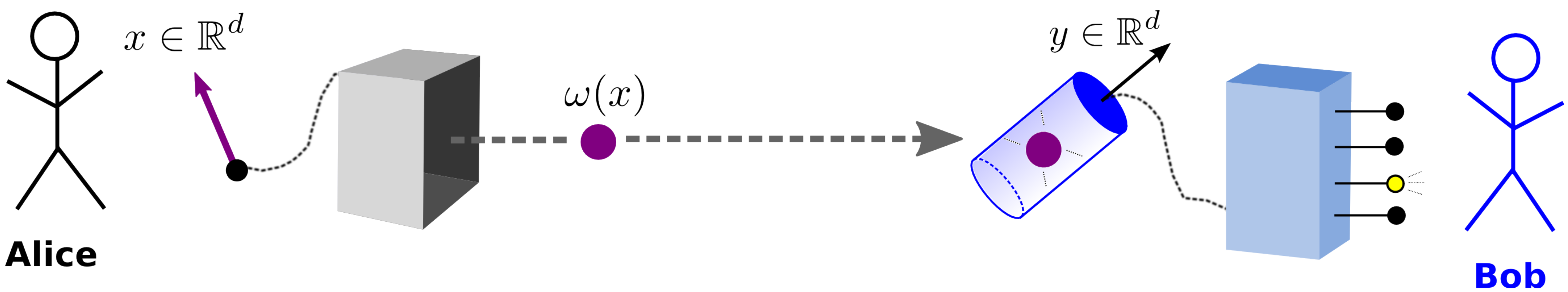
If *d=3* then $d_{AB} = (d+1)^2 - 1 = 15$

= # of real parameters in a 4x4 density matrix. In fact:

Theorem (G. de la Torre, Ll. Masanes, A. J. Short, MM, PRL **108** (2012)): Only solution for d=3 is two-qubit quantum theory, and interaction is of the form $\rho \mapsto U(t)\rho U(t)^\dagger$ with $U(t)$ unitary.

2. Geometry and probability

Result:

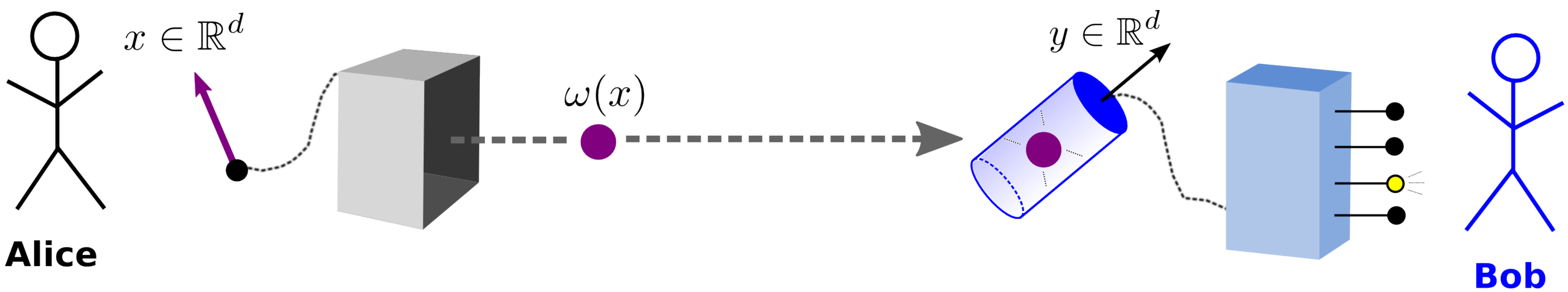


Information-theoretic task with 4 Postulates
uniquely determines spatial dimension d=3 and quantum theory.

MM and Ll. Masanes, New J. Phys. **15**, 053040 (2013), arXiv:1206.0630

2. Geometry and probability

Result:



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See also: B. Dakic and C. Brukner, arXiv:1307.3984

Additional solutions in case of tripartite interactions?

3. Third-order interference

3. Third-order interference

Science 23 July 2010:
Vol. 329 no. 5990 pp. 418-421
DOI: 10.1126/science.1190545

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REPORT

Ruling Out Multi-Order Interference in Quantum Mechanics

Urbasi Sinha[1,*], Christophe Couteau[1,2], Thomas Jennewein[1], Raymond Laflamme[1,3], Gregor Weihs[1,4,*]

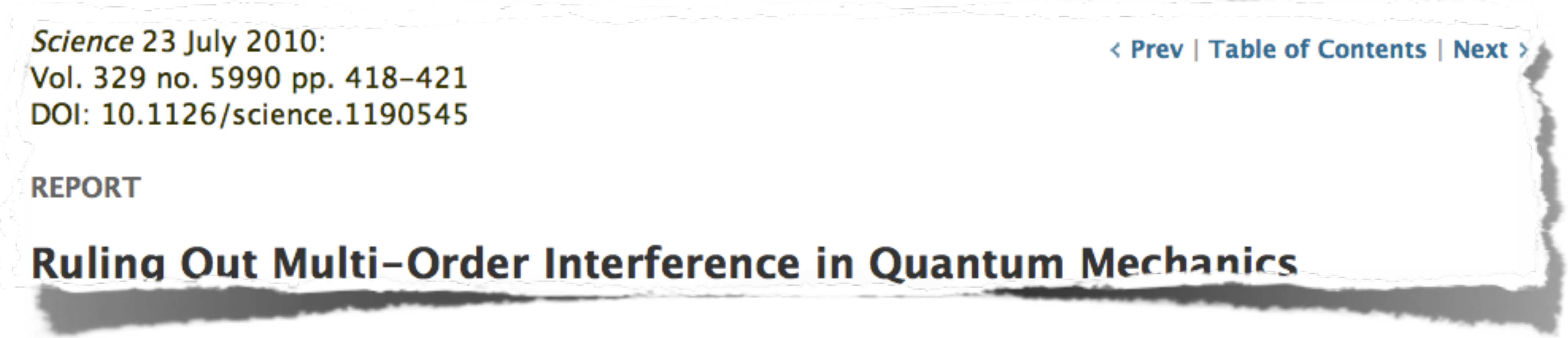
± Author Affiliations

*To whom correspondence should be addressed. E-mail: usinha@iqc.ca, gregor.weihs@uibk.ac.at

ABSTRACT

Quantum mechanics and gravitation are two pillars of modern physics. Despite their success in describing the physical world around us, they seem to be incompatible theories. There are suggestions that one of these theories must be generalized to achieve unification. For example, Born's rule—one of the axioms of quantum mechanics—could be violated. Born's rule predicts that quantum interference, as shown by a double-slit diffraction experiment, occurs from pairs of paths. A generalized version of quantum mechanics might allow multipath (i.e., higher-order) interference, thus leading to a deviation from the theory. We performed a three-slit experiment with photons and bounded the magnitude of three-path interference to less than 10^{-2} of the expected two-path interference, thus ruling out third- and higher-order interference and providing a bound on the

3. Third-order interference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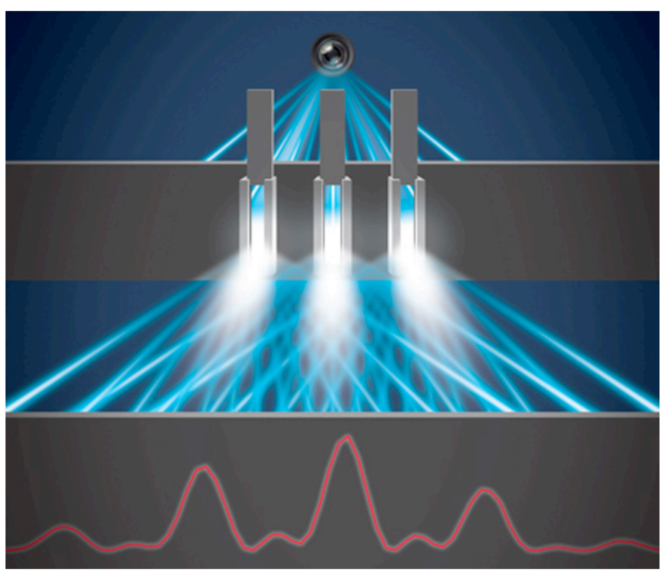

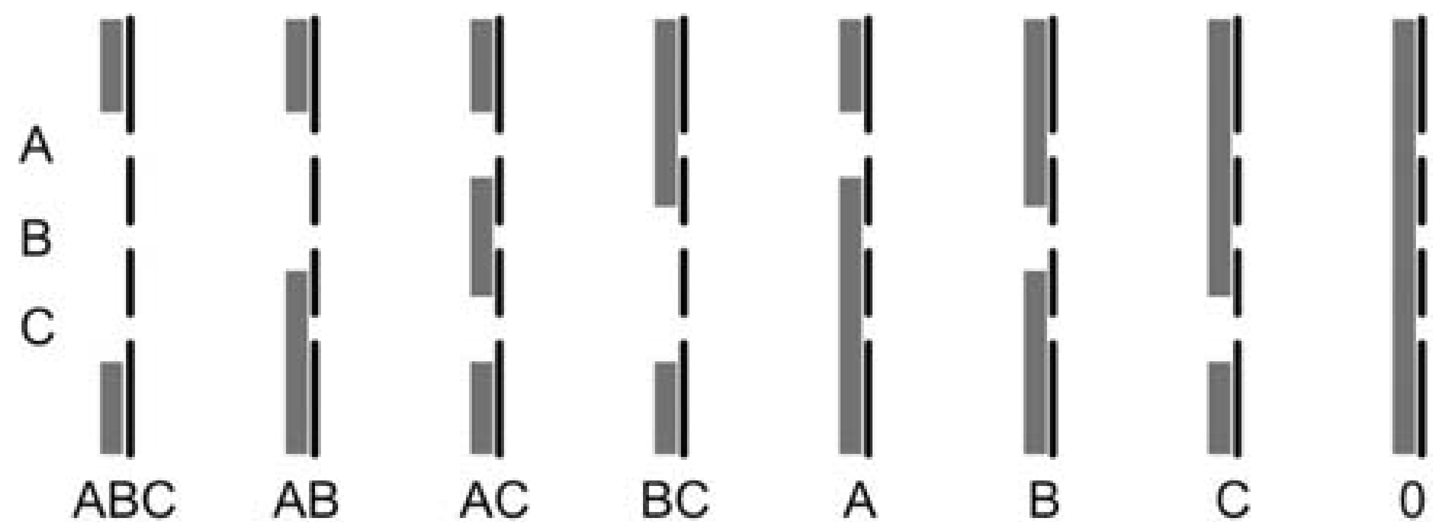

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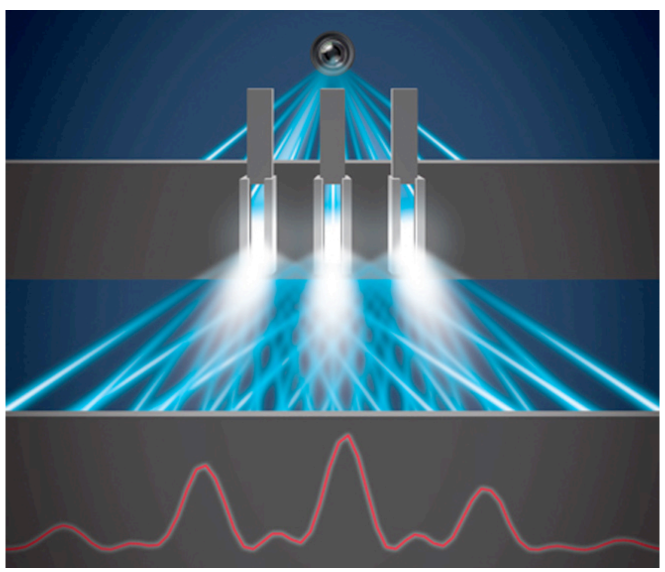
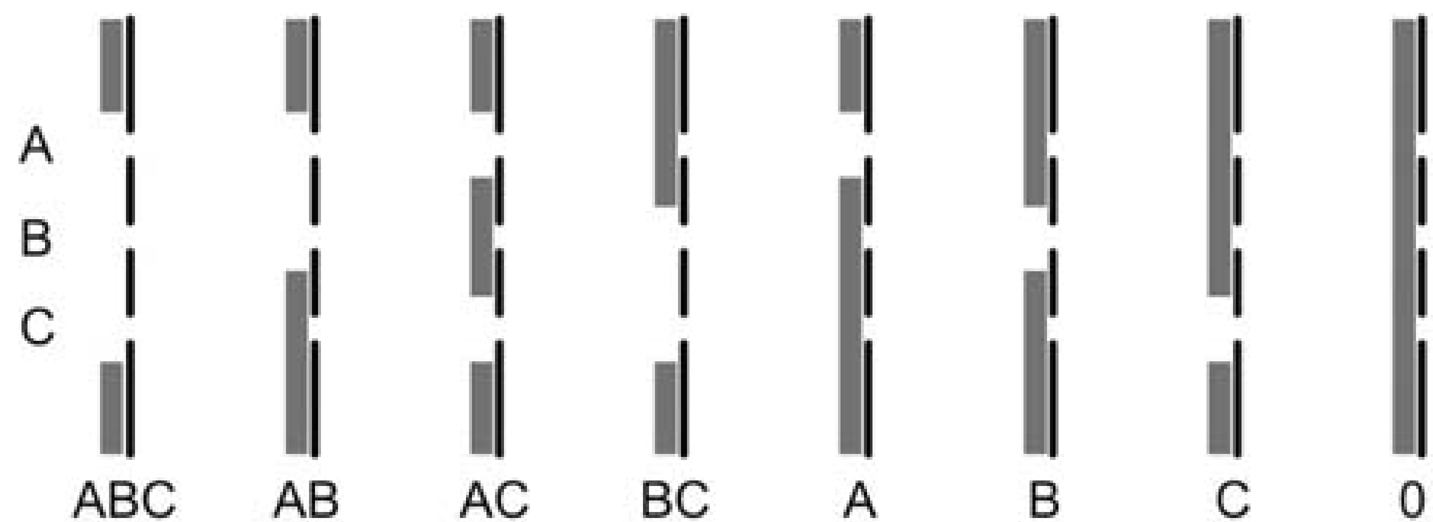
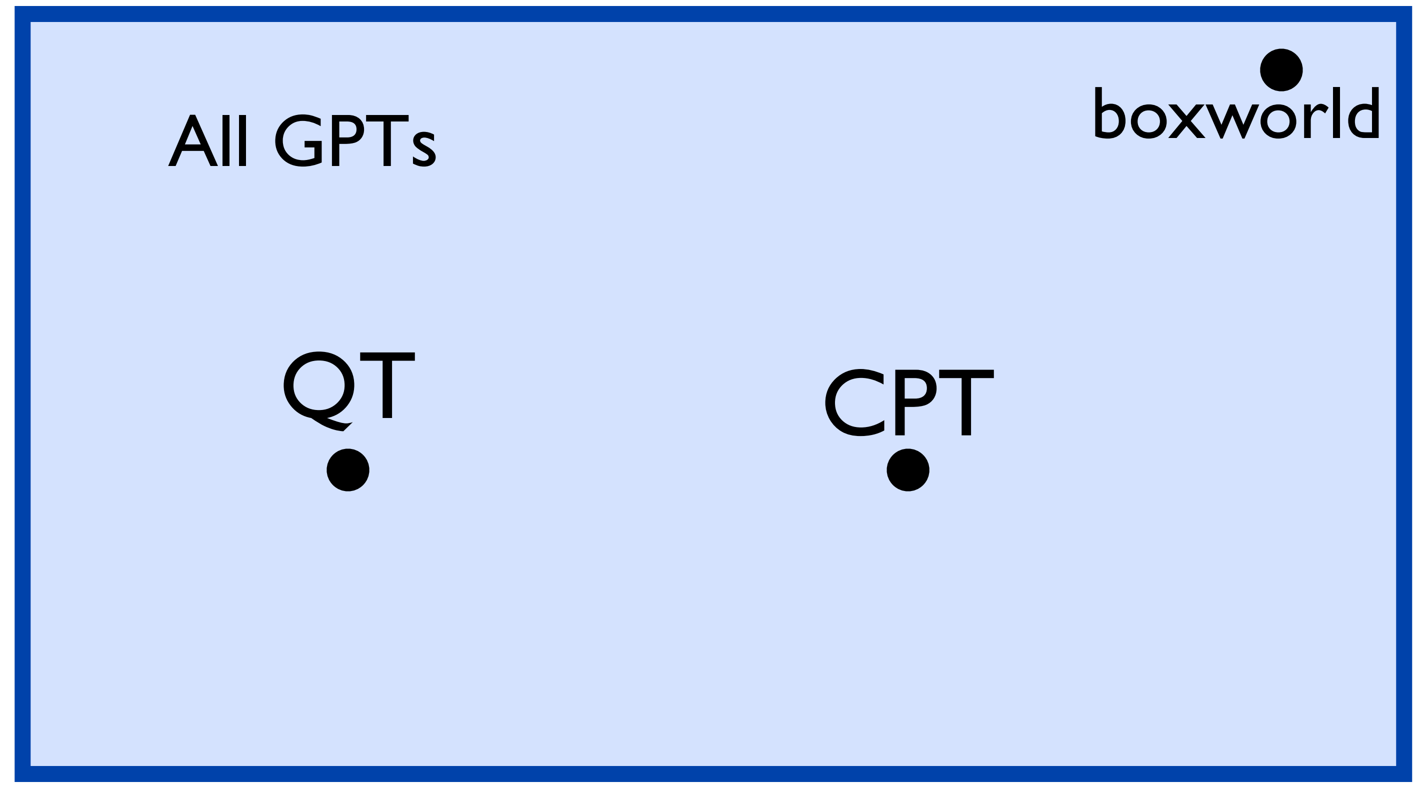




For 2 slits, QM predicts $P_{AB} \neq P_A + P_B$,

but for 3 slits $P_{ABC} = P_{AB} + P_{BC} + P_{AC} - P_A - P_B - P_C$.

3. Third-order interference



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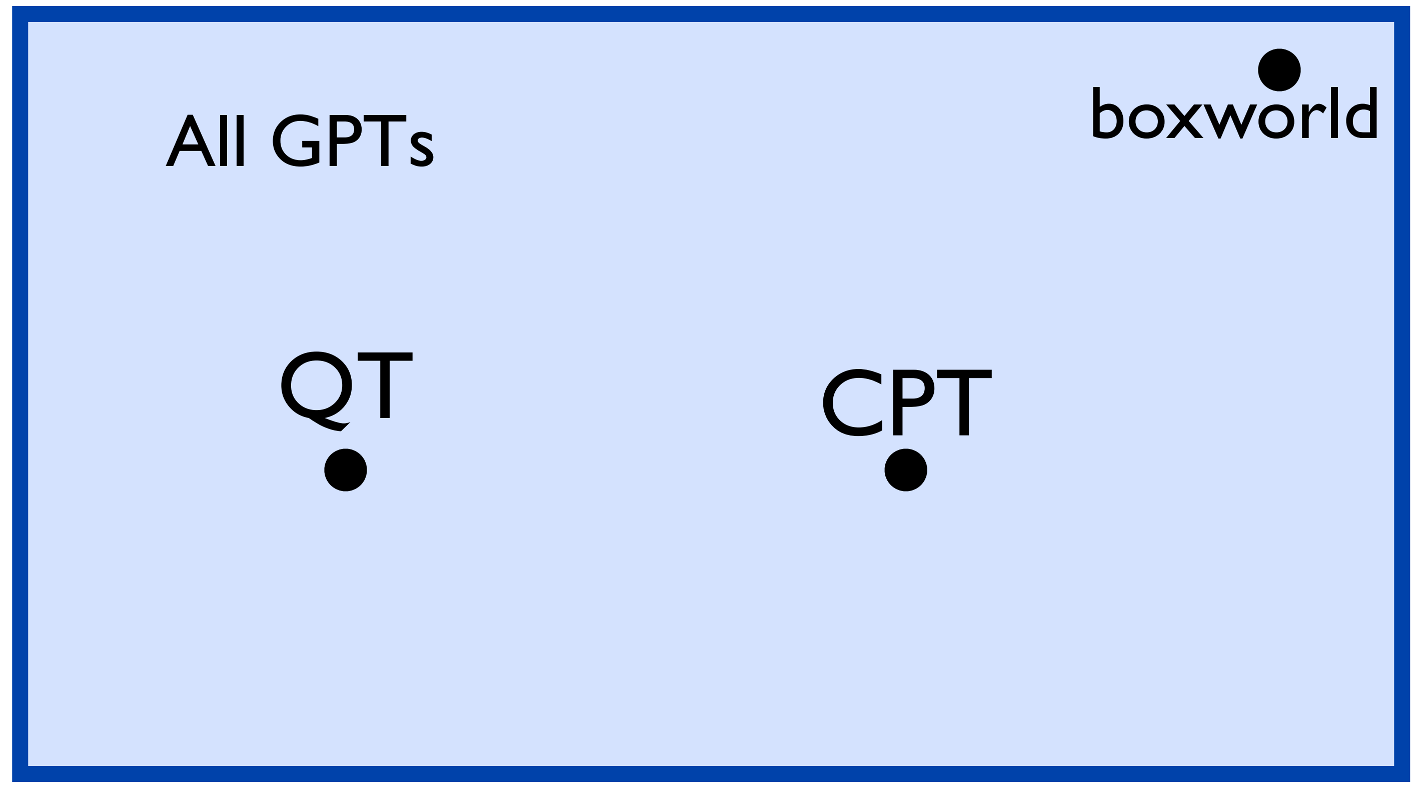


The axioms:

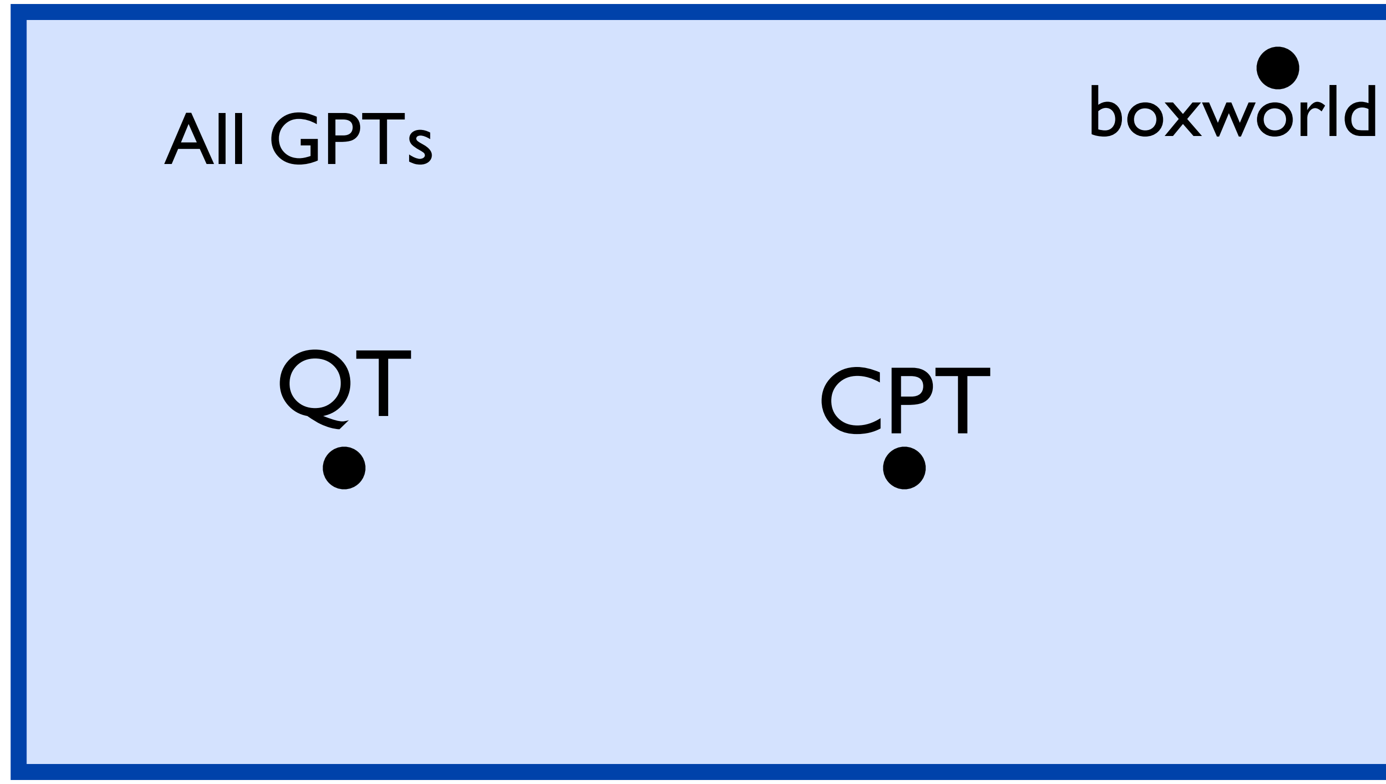
1. Every state belongs to a "classical subsystem",
2. lots of reversible dynamics,
3. no 3rd-order interference, and
4. energy is an observable.

H. Barnum, MM, and C. Ududec, in preparation (2013)

3. Third-order interference

Every state is a convex combination of pure, perfectly distinguishable states.

QT: $\rho = \sum_i \lambda_i |i\rangle\langle i|$.



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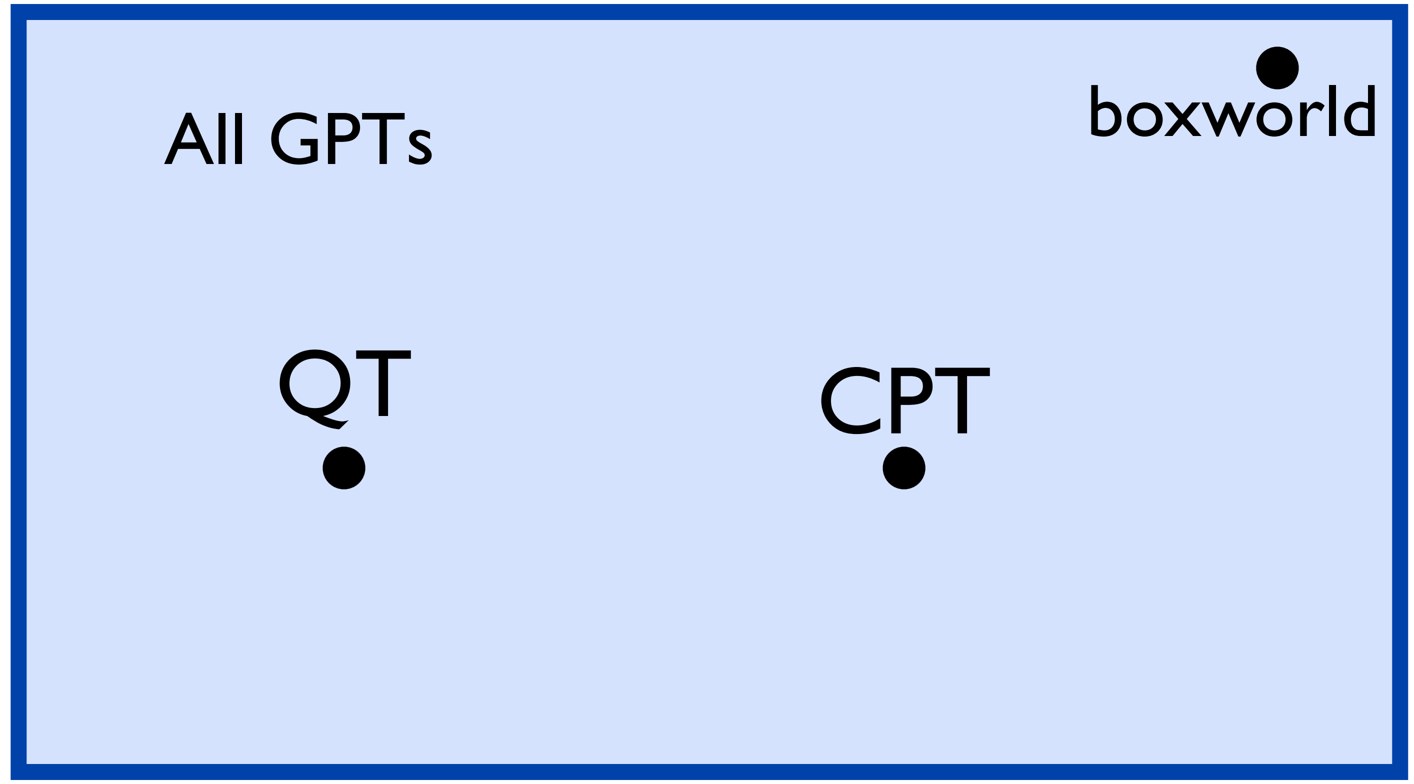
If $\omega_1, \ldots, \omega_k$ are pure and perfectly distinguishable, and so are $\varphi_1, \ldots, \varphi_k$, then there is a reversible transformation T with $T\omega_i = \varphi_i$.

QT: unitaries.



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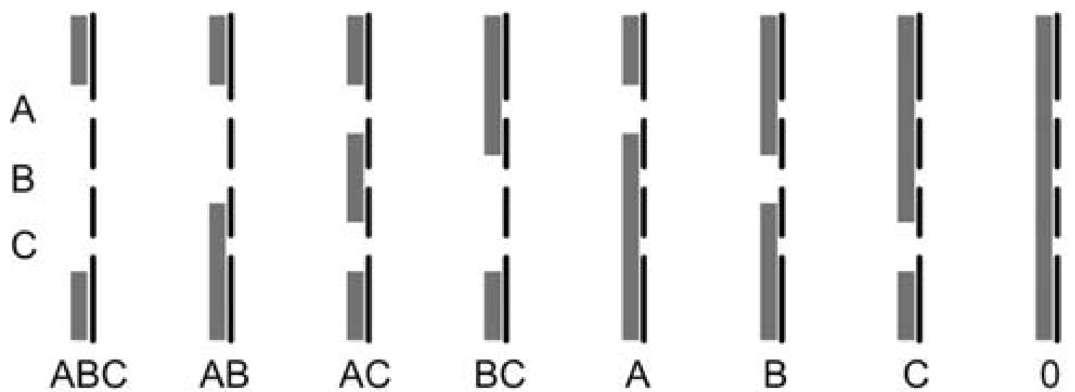
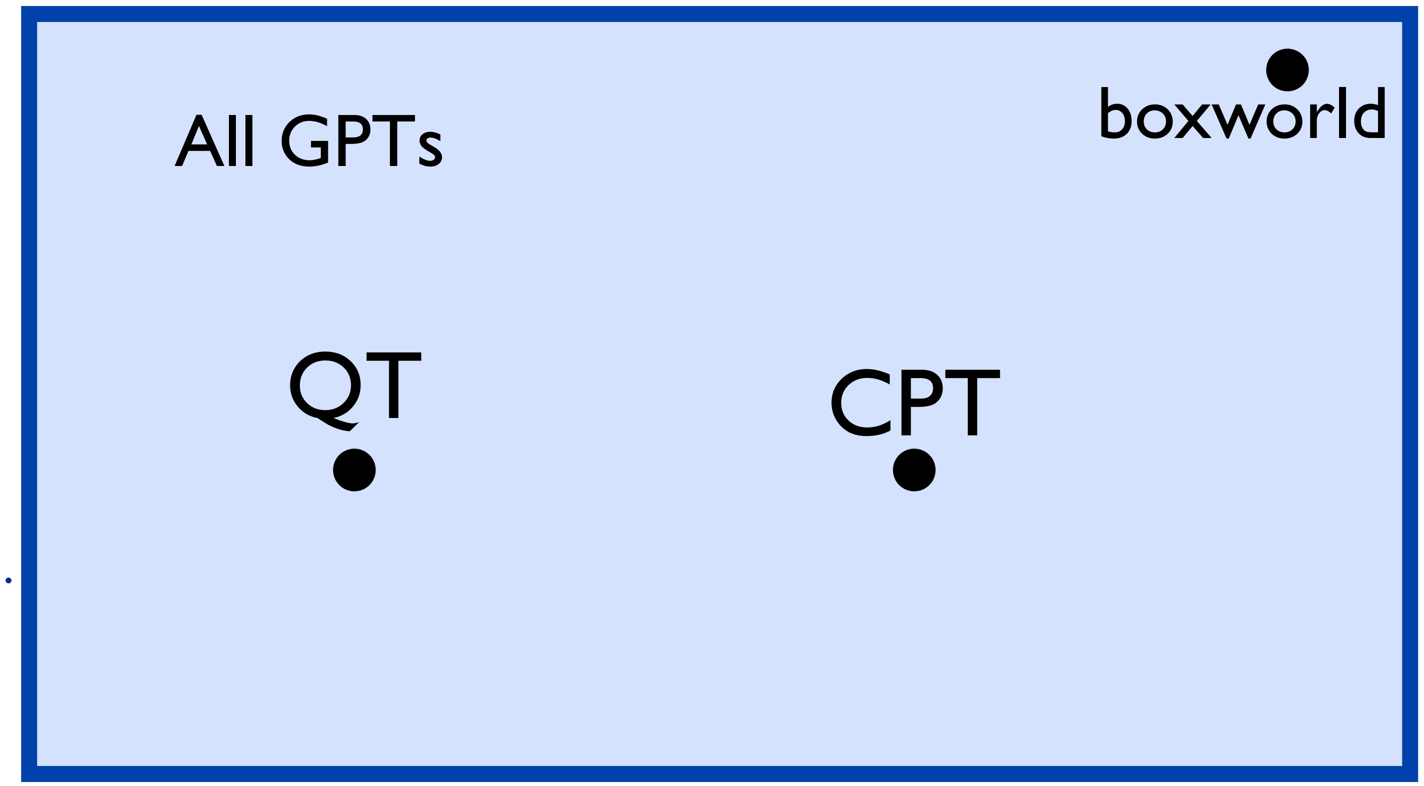


$$P_{ABC} = P_{AB} + P_{BC} + P_{AC} - P_A - P_B - P_C.$$



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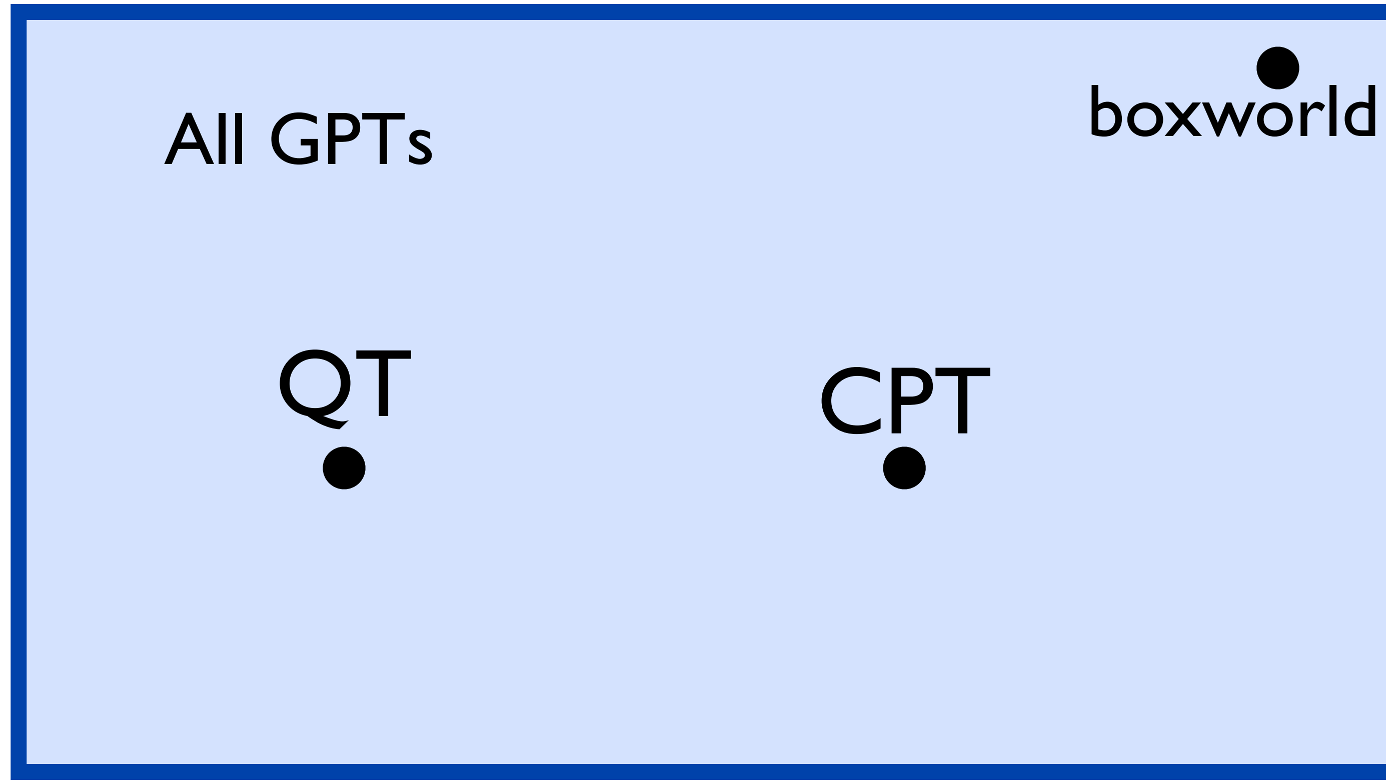
There is a meaningful way to associate observables to generators of time evolution.



QT: $\rho \mapsto -i[H,\rho]$

$\downarrow$

$\rho \mapsto \mathrm{tr}(H\rho).$

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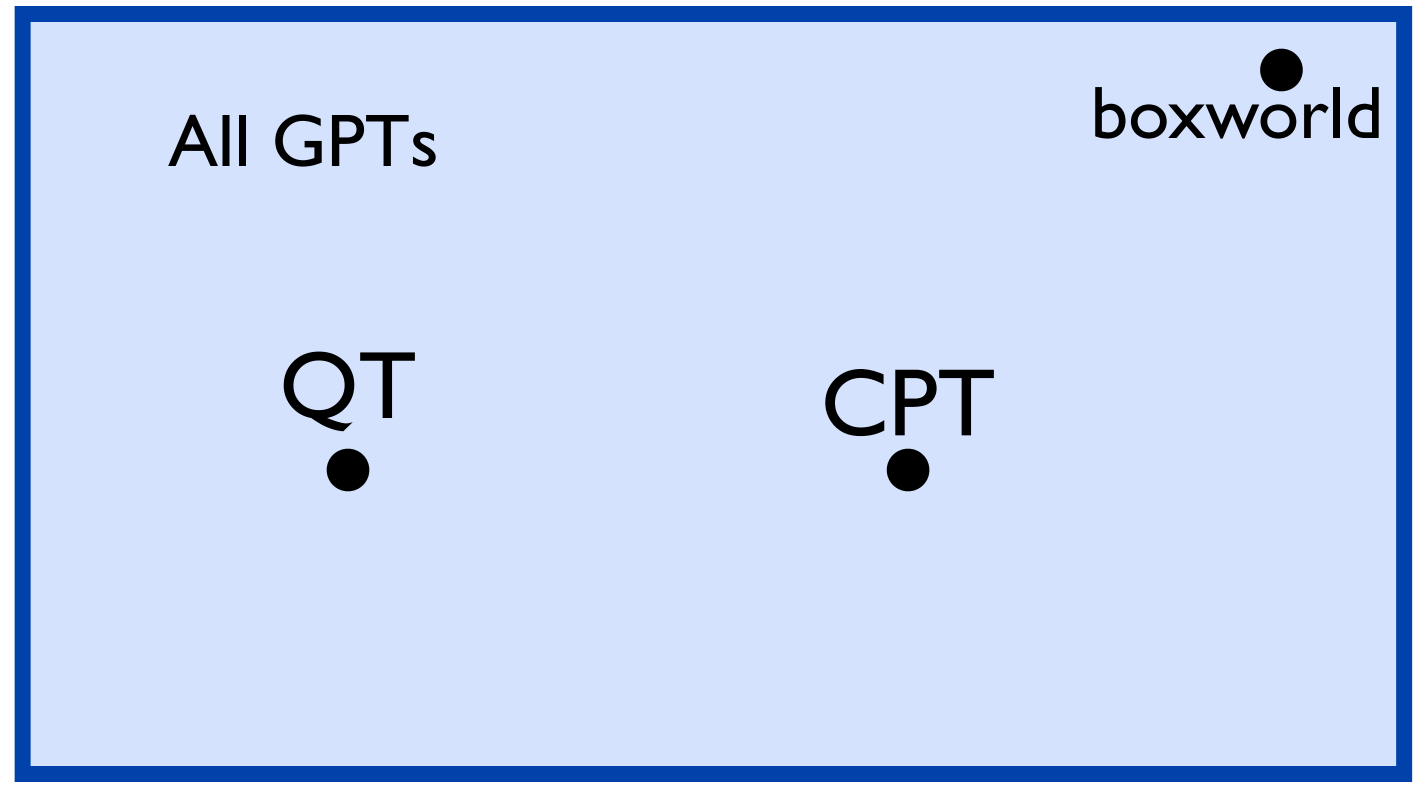

The axioms: *No mention of composite systems!*

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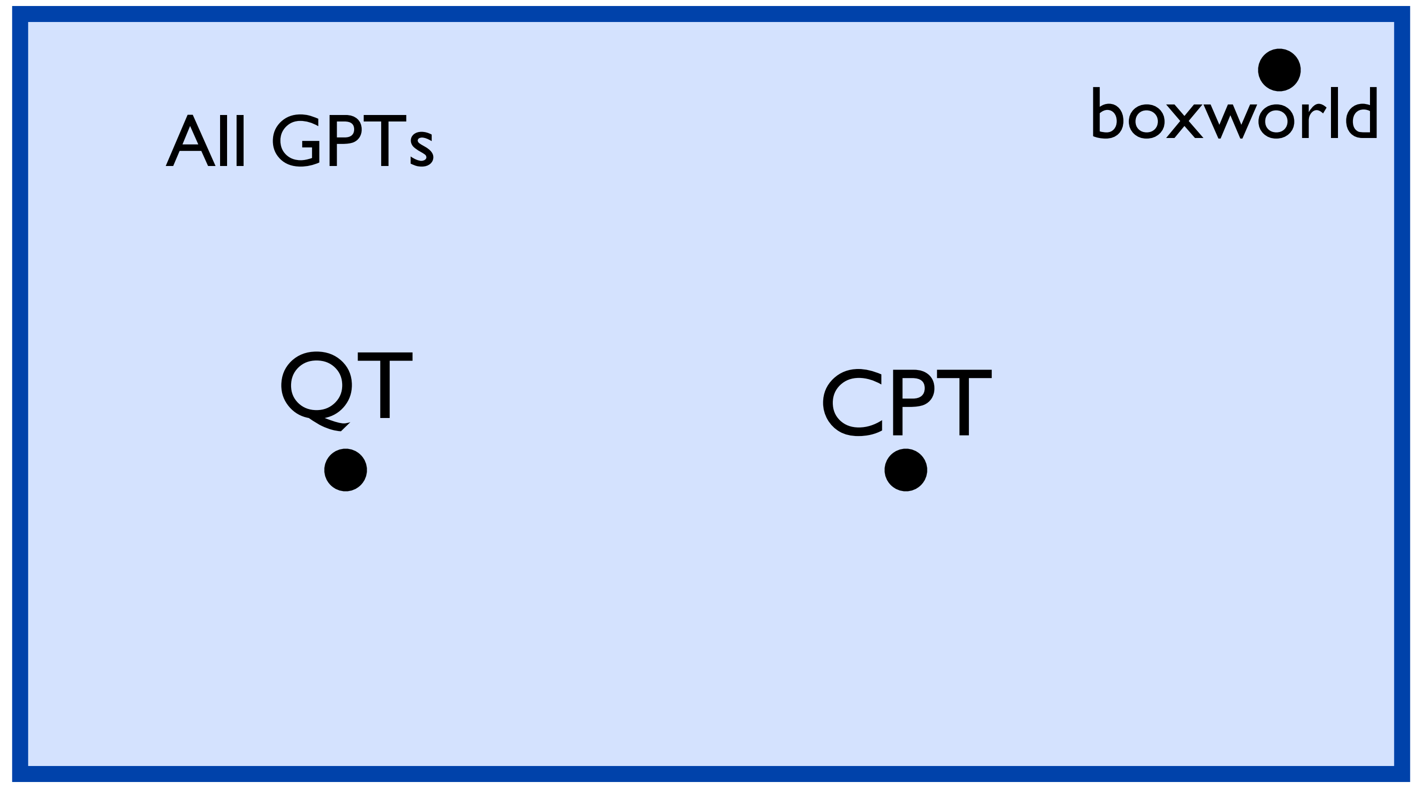

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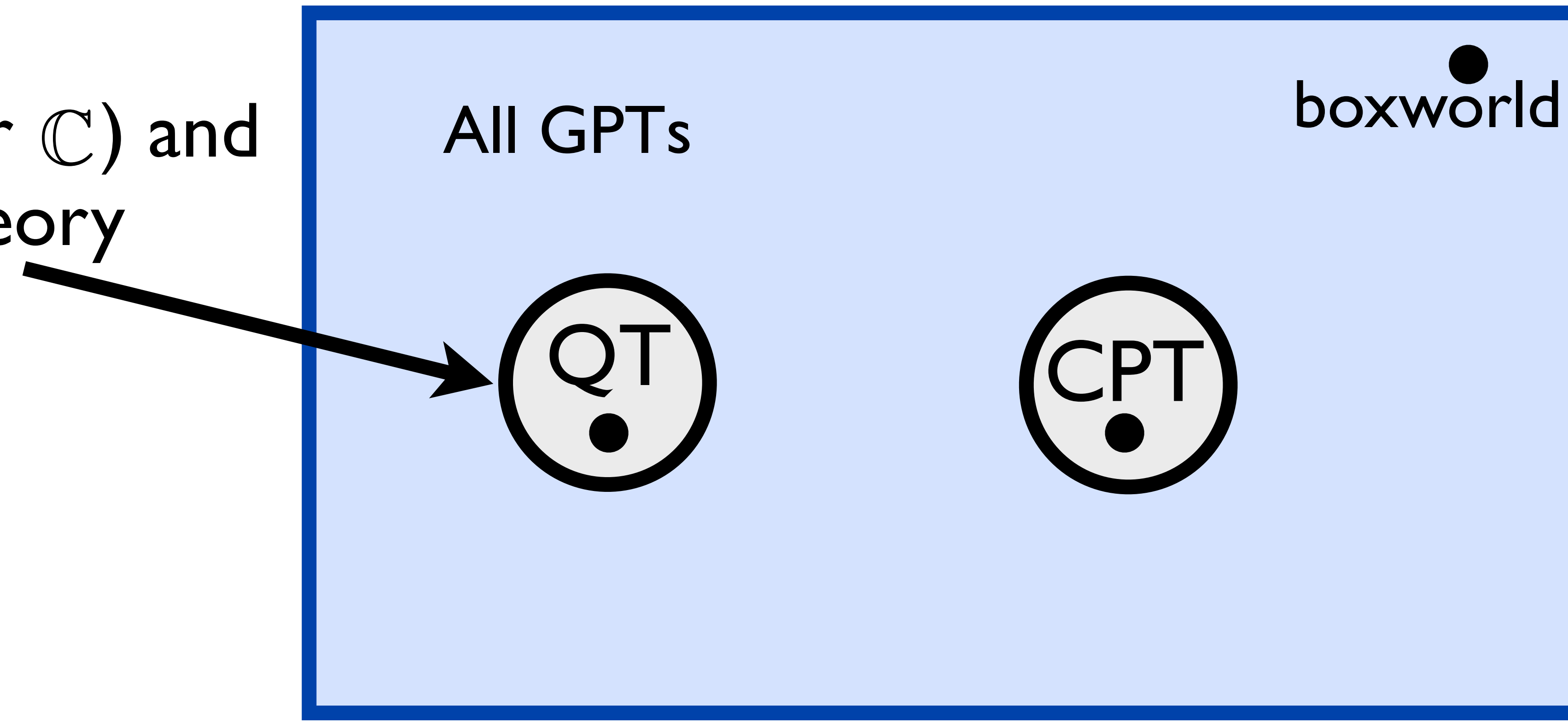
H. Barnum, MM, and C. Ududec, in preparation (2013)

3. Third-order interference

Quantum theory (over $\mathbb{C}$) and classical probability theory are the only solutions.



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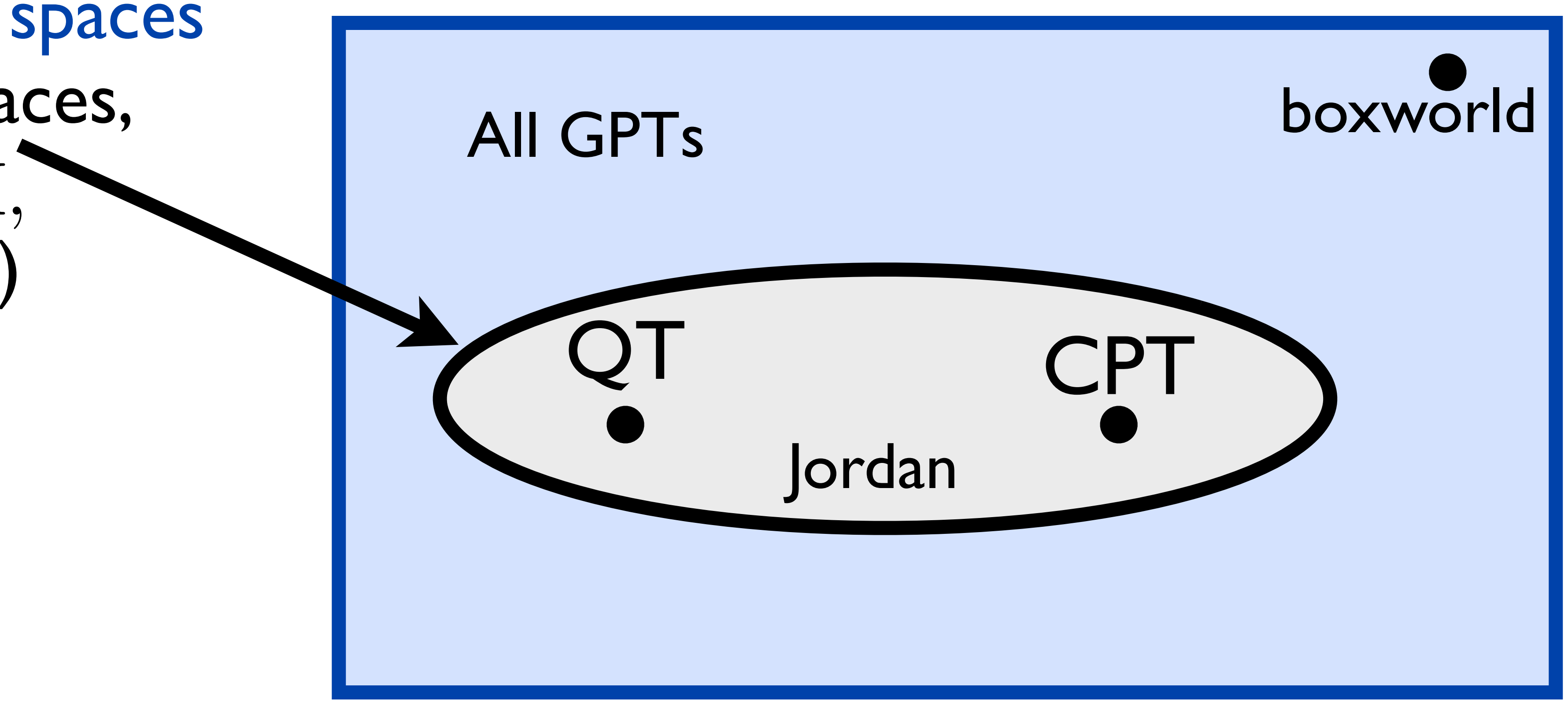
3. Third-order interference

Jordan algebra state spaces
(2-level ball state spaces,
QT over $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$,
3-level QT over $\mathbb{O}$.)



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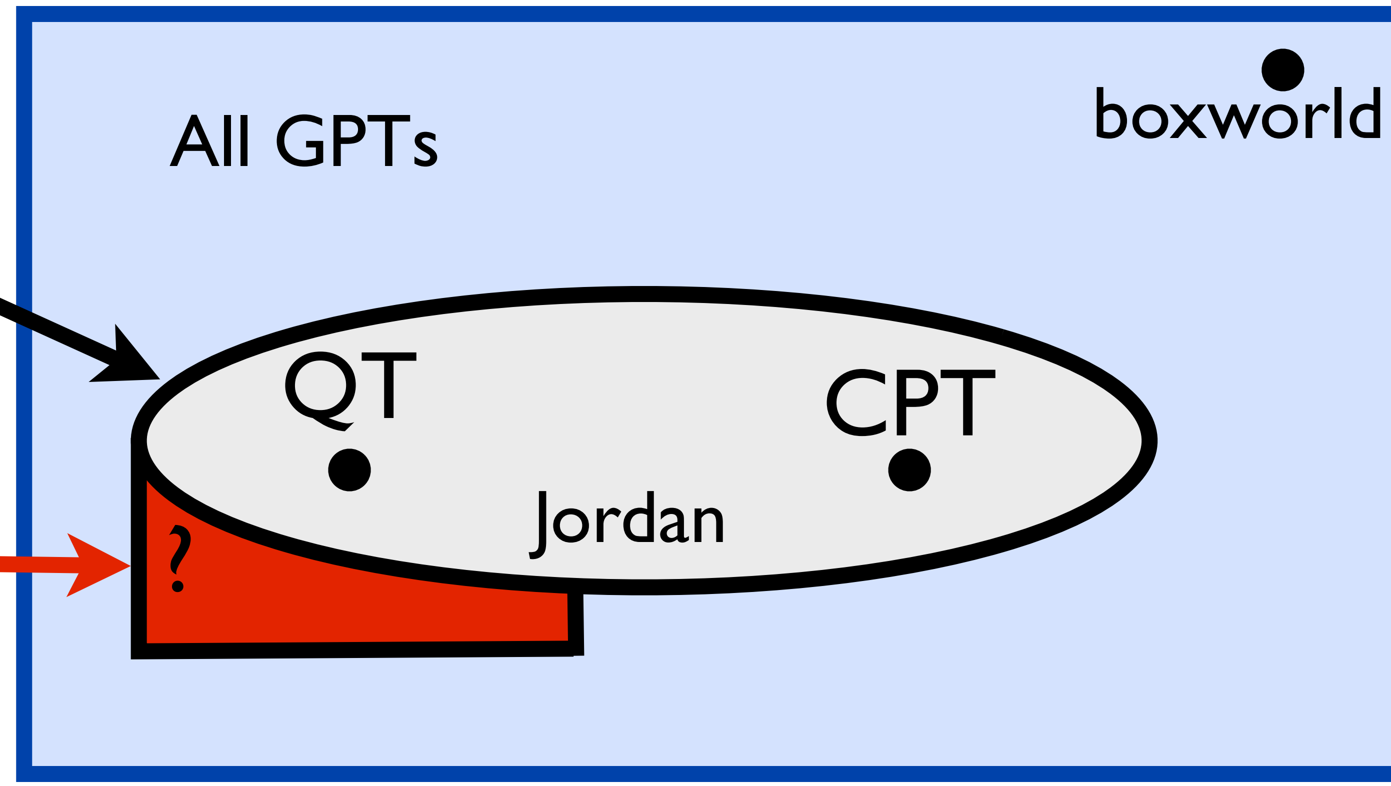

New solutions?
⇒ 3rd-order interference

The axioms:

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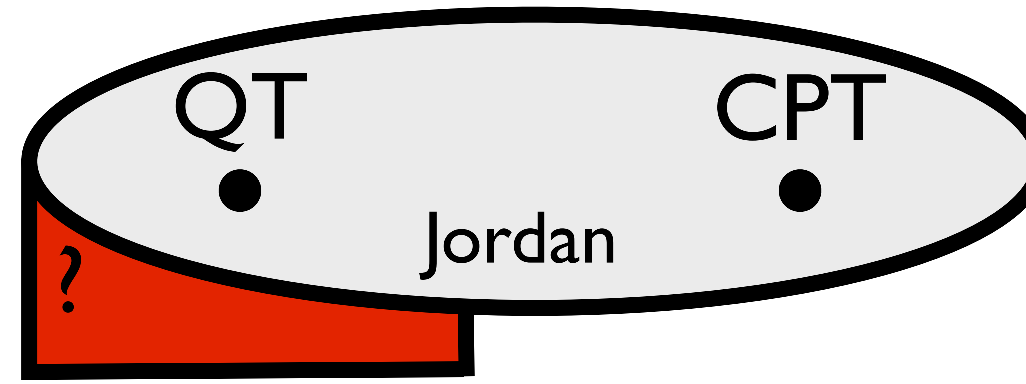
H. Barnum, MM, and C. Ududec, in preparation (2013)

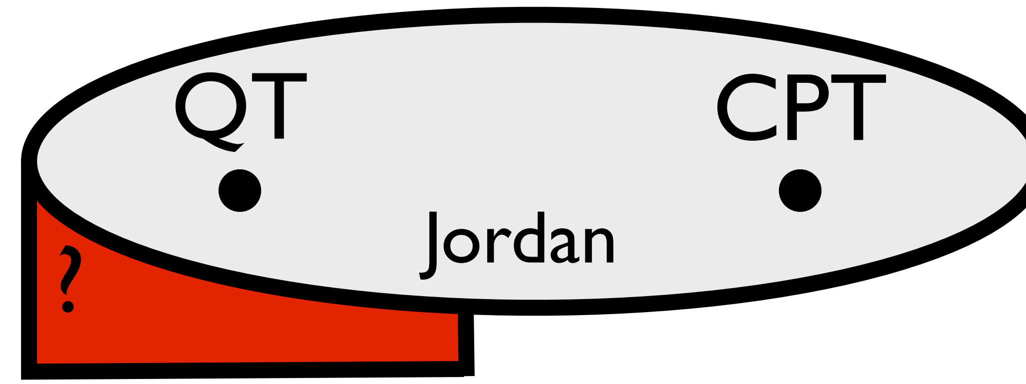
3. Third-order interference



We know a lot about these theories:

H. Barnum, MM, and C. Ududec, in preparation (2013)

3. Third-order interference

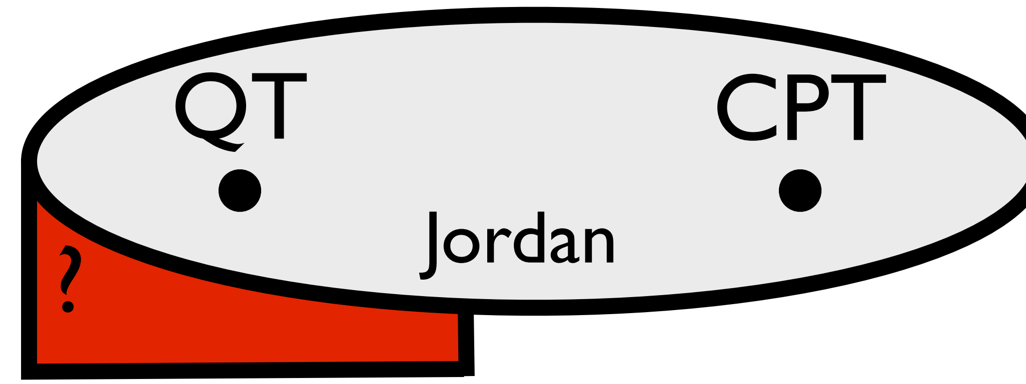


We know a lot about these theories:

- Unlike QT, they have 3rd-order interference,
- like QT, their elementary propositions are an orthomodular lattice,
- like QT, they satisfy Specker's Principle for contextuality,
- like QT, all bit subsystems are Euclidean ball state spaces,
- but two pure states can generate a 3-level subsystem (unlike QT),
- they violate the covering property of quantum logic,
- like QT, they should allow for powerful computation.

H. Barnum, MM, and C. Ududec, in preparation (2013)

3. Third-order interference



We know a lot about these theories:

- Unlike QT, they have 3rd-order interference,
- like QT, their elementary propositions are an orthomodular lattice,
- like QT, they satisfy Specker's Principle for contextuality,
- like QT, all bit subsystems are Euclidean ball state spaces,
- but two pure states can generate a 3-level subsystem (unlike QT),
- they violate the covering property of quantum logic,
- like QT, they should allow for powerful computation.

Do they exist? If yes: natural models, experimentally testable against QT.

H. Barnum, MM, and C. Ududec, in preparation (2013)

Conclusions

- Quantum theory is just one possible probabilistic theory.

- Information-theoretic task in *d* spatial dimensions.
  Result: "Nice" interplay between geometry & probability determines d=3 and QT uniquely.

  MM and Ll. Masanes, New J. Phys. **15**, 053040 (2013), arXiv:1206.0630

  See also:

  B. Dakic and C. Brukner, arXiv:1307.3984

- "No 3rd-order interference" as an axiom for QT, and the search for QT's "closest cousins".