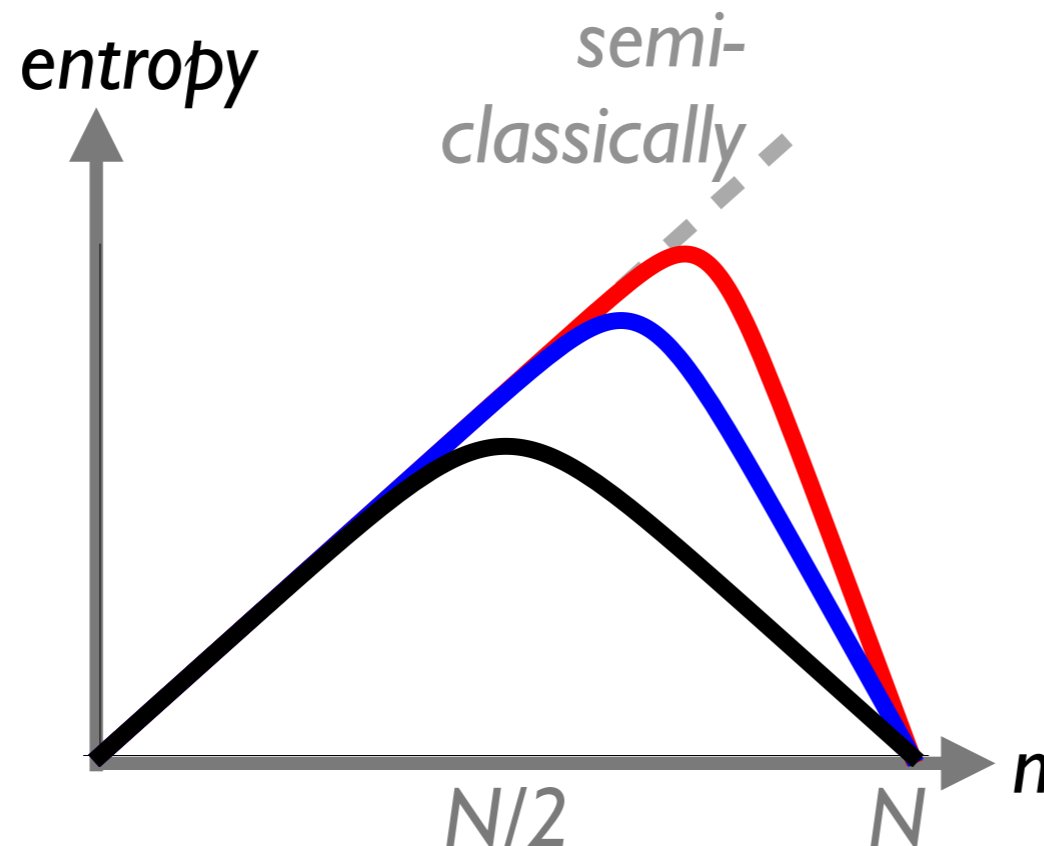


# The Page curve from less than the full quantum formalism

Markus P. Müller

Departments of Applied Mathematics and Philosophy, UWO  
Perimeter Institute for Theoretical Physics, Waterloo



- $r = 1$  : classical prob. theory
- $r = 2$  : quantum theory
- $r = 3$  : ???
- $r = 4$  : ???

## Motivation

The black-hole information paradox has motivated some people to consider **modifications of quantum theory**.

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The black-hole information paradox has motivated some people to consider **modifications of quantum theory**.

## The black hole final state

---

**Gary T. Horowitz**

*University of California at Santa Barbara  
Santa Barbara, CA 93106, U.S.A.  
E-mail: gary@physics.ucsb.edu*

**Juan Maldacena**

*Institute for Advanced Study  
Princeton, New Jersey 08540, U.S.A.  
E-mail: malda@ias.edu*

ABSTRACT: We propose that in quantum gravity one needs to impose a final state boundary condition at black hole singularities. This resolves the apparent contradiction between string theory and semiclassical arguments over whether black hole evaporation is unitary.

KEYWORDS: Black Holes in String Theory, AdS-CFT and dS-CFT Correspondence, Black Holes.

JHEP02(2004)008

# Motivation

The black-hole information paradox has motivated some people to consider **modifications of quantum theory**.

The bl

## Comment on “The black hole final state”

---

Gary T. H

**Daniel Gottesman**

*Universit  
Santa Ba  
E-mail: g*

*Perimeter Institute  
Waterloo ON N2V 1Z3, Canada  
E-mail: dgottesman@perimeterinstitute.ca*

Juan Mald

**John Preskill**

*Institute  
Princeton  
E-mail: m*

*Institute for Quantum Information, California Institute of Technology  
Pasadena, CA 91125, U.S.A.  
E-mail: preskill@theory.caltech.edu*

ABSTRACT  
condition &  
string theo

ABSTRACT: Horowitz and Maldacena have suggested that the unitarity of the black hole  $S$ -matrix can be reconciled with Hawking’s semiclassical arguments if a final-state boundary condition is imposed at the spacelike singularity inside the black hole. **We point out that, in this scenario, departures from unitarity can arise** due to interactions between the collapsing body and the infalling Hawking radiation inside the event horizon. **The amount of information lost** when a black hole evaporates depends on the extent to which these interactions are entangling.

KEYWORD  
Black Hole

# Motivation

Ad-hoc modifications of QT's formalism have proven to be problematic in general...

ANNALS OF PHYSICS **194**, 336–386 (1989)

## Testing Quantum Mechanics

STEVEN WEINBERG\*

*Theory Group, Department of Physics,  
University of Texas, Austin, Texas 78712*

Received March 6, 1989

This paper presents a general framework for introducing nonlinear corrections into ordinary quantum mechanics, that can serve as a guide to experiments that would be sensitive to such corrections. In the class of generalized theories described here, the equations that determine the time-dependence of the wave function are no longer linear, but are of Hamiltonian type. Also, wave functions that differ by a constant factor represent the same physical state and satisfy the same time-dependence equations. As a result, there is no difficulty in combining separated subsystems. Prescriptions are given for determining the states in which observables have definite values and for calculating the expectation values of observables for general states, but the calculation of probabilities requires detailed analysis



# Motivation

Ad-hoc modifications of QT's formalism have proven to be problematic in general...

ANN

Volume 143, number 1,2

PHYSICS LETTERS A

1 January 1990

## WEINBERG'S NON-LINEAR QUANTUM MECHANICS AND SUPRALUMINAL COMMUNICATIONS

N. GISIN

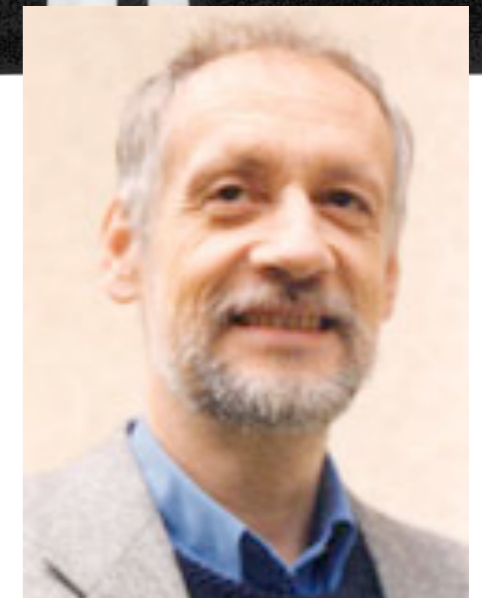
*Group on Applied Physics, University of Geneva, 1211 Geneva 4, Switzerland*

Received 16 October 1989; accepted for publication 3 November 1989  
Communicated by J.P. Vigiér

We show with an example that Weinberg's general framework for introducing non-linear corrections into quantum mechanics allows for arbitrarily fast communications.

Recently Weinberg has proposed a general framework for introducing non-linear corrections into ordinary quantum mechanics [1,2]. Although we fully support his emphasis on the importance of testing quantum mechanics, we would like in this Letter to draw attention to the difficulty of modifying quantum mechanics without introducing arbitrarily fast actions at a distance. Below we show how to construct, within Weinberg's framework, an arbitrarily fast telephone line. In ordinary quantum mechanics

to know what such an apparatus is... do you know what is inside your phone?) In order to simplify we consider only a single-bit message. The two directions  $z$  and  $u$  are in the  $xz$ -plane orthogonal to the incoming flow of particles, and are  $45^\circ$  from each other. The way the inhomogeneous magnetic field acts on the particles is well-known from experimental evidence. After the apparatus there are two counters. For each particle one of the counters will click. This click will be amplified until all readers of



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Ad-hoc modifications of QT's formalism have proven to be problematic in general...

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Volume 143, number 1,2

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1 January 1990

## WEINBERG'S NON-LINEAR QUANTUM MECHANICS

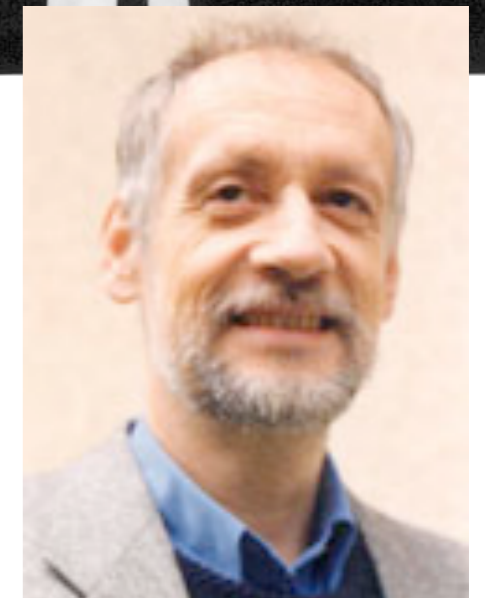
Work on the level of **physical principles**,  
not of the **formalism!**

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# Our approach

**MM**, J. Oppenheim, and O. Dahlsten, JHEP **09**, 116 (2012)

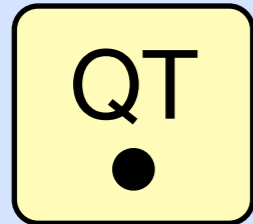


# Our approach

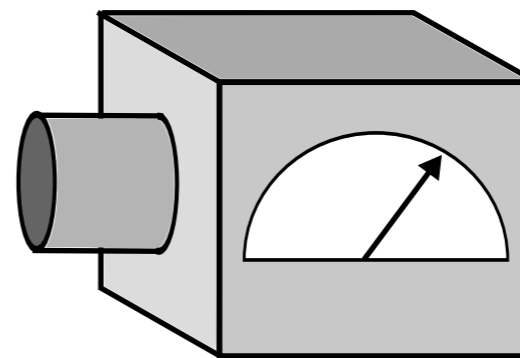
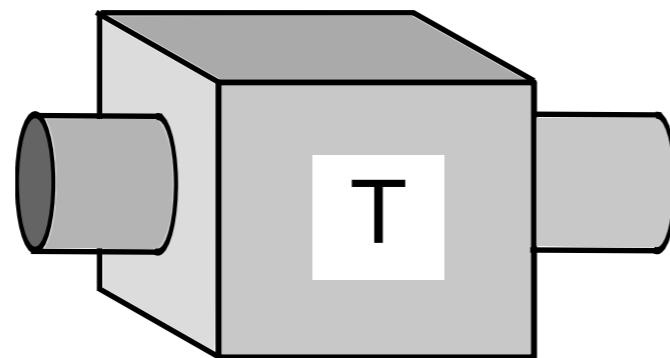
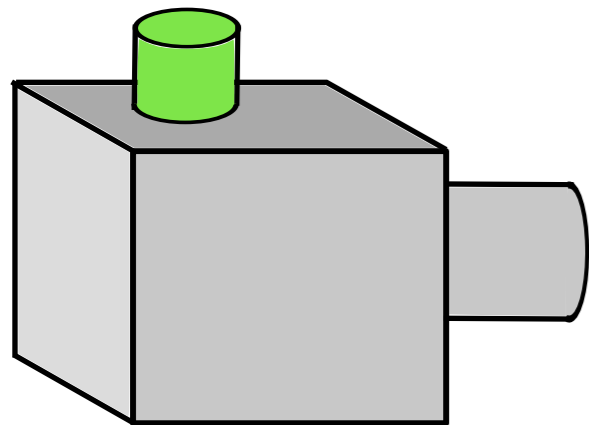
MM, J. Oppenheim, and O. Dahlsten, JHEP **09**, 116 (2012)

All probabilistic theories

●  
PR boxes



●  
CPT



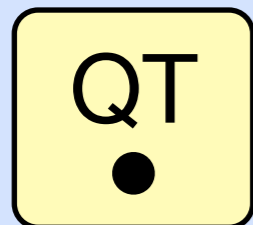
Preparation,  
transformation,  
measurement.

# Our approach

MM, J. Oppenheim, and O. Dahlsten, JHEP **09**, 116 (2012)

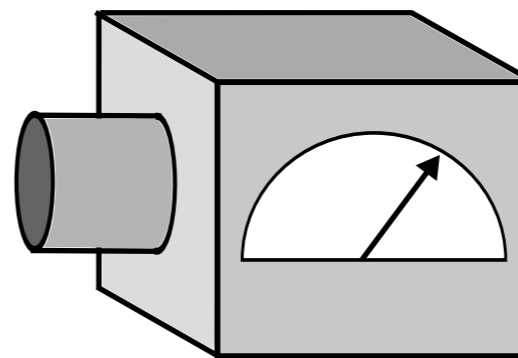
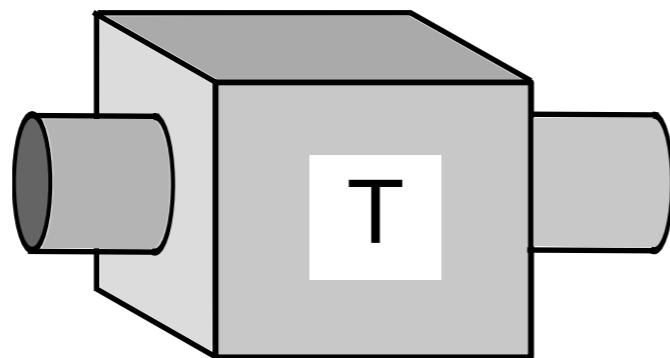
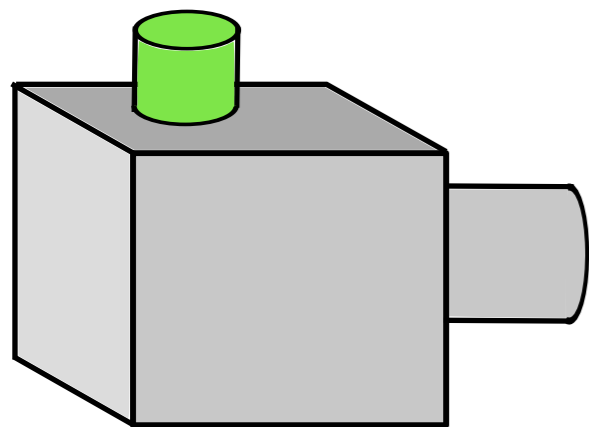
All probabilistic theories

●  
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●  
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Consider BH information  
without assuming the  
full quantum formalism.



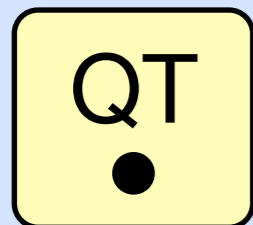
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MM, J. Oppenheim, and O. Dahlsten, JHEP **09**, 116 (2012)

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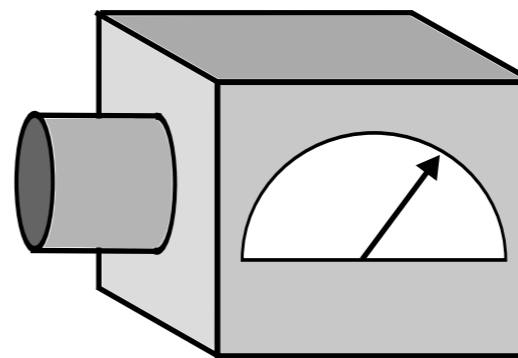
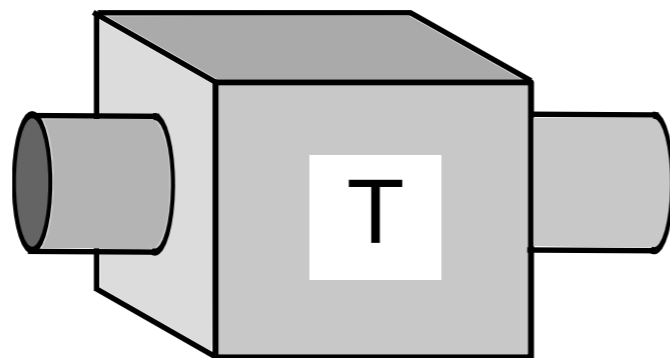
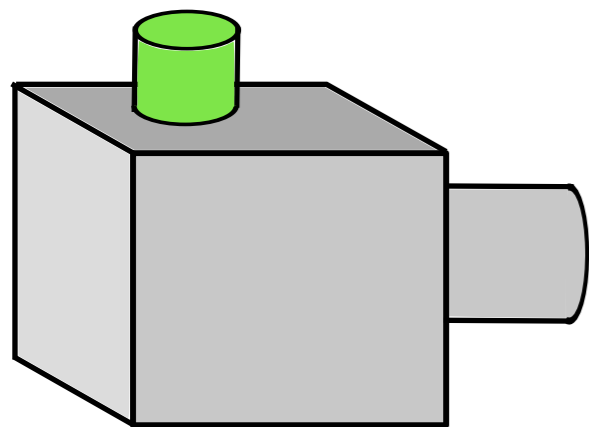
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CPT  
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Consider BH information  
without assuming the  
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Assume only  
a few **principles!**



Preparation,  
transformation,  
measurement.

# Our approach

MM, J. Oppenheim, and O. Dahlsten, JHEP **09**, 116 (2012)

All probabilistic theories

●  
PR boxes

QT



Theories that satisfy  
the principles

CPT



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MM, J. Oppenheim, and O. Dahlsten, JHEP **09**, 116 (2012)

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The **Principles** in a nutshell:

- Theories must allow **decoherence to classical**.
- Composition of systems satisfies "**tomographic locality**".
- Every two pure states are connected by **reversible time evolution**

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MM, J. Oppenheim, and O. Dahlsten, JHEP **09**, 116 (2012)

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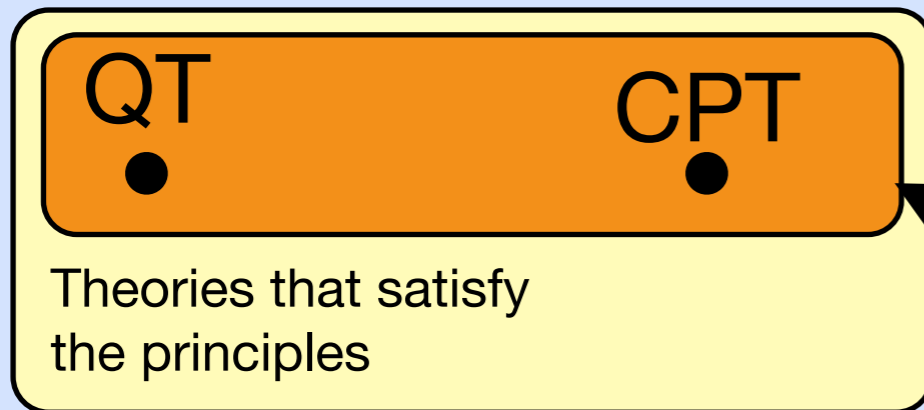
→ Assumes **less than full QT formalism**, respecting  
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# Our approach

MM, J. Oppenheim, and O. Dahlsten, JHEP **09**, 116 (2012)

All probabilistic theories

●  
PR boxes



Theories that satisfy the principles

Consider BH information without assuming the full quantum formalism.

Assume only a few **principles!**

Subset of theories for which results look simple (will only talk about those)

The **Principles** in a nutshell:

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→ Assumes less than full QT formalism, respecting "unitarity" (information preservation), causality, subsystem structure.

# The Page curve

D. N. Page, *Information in Black Hole Radiation*, Phys. Rev. Lett. **71**, 3743 (1993)

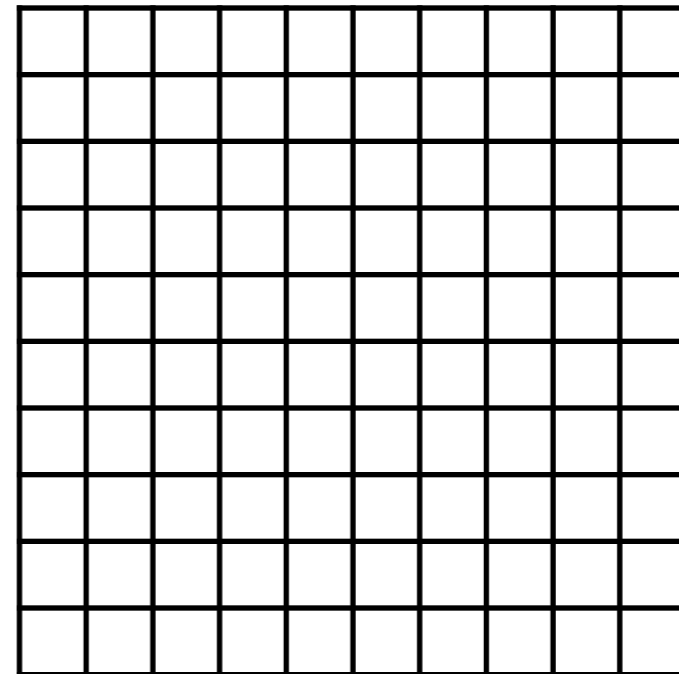


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MM, O. C. O. Dahlsten, and V. Vedral, *Commun. Math. Phys.* **316** (2), 441-487 (2012)

MM, J. Oppenheim, and O. C. O. Dahlsten, *JHEP* **2012** (9), 116 (2012)

$N$  qubits



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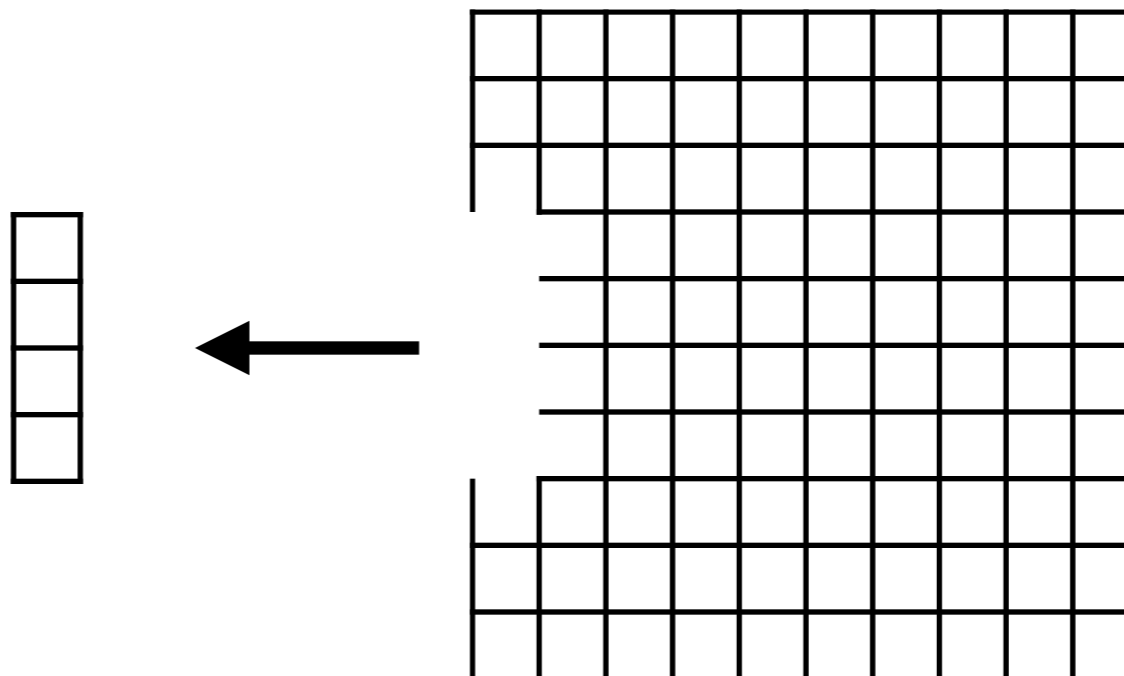
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$n$  qubits

$N-n$  qubits

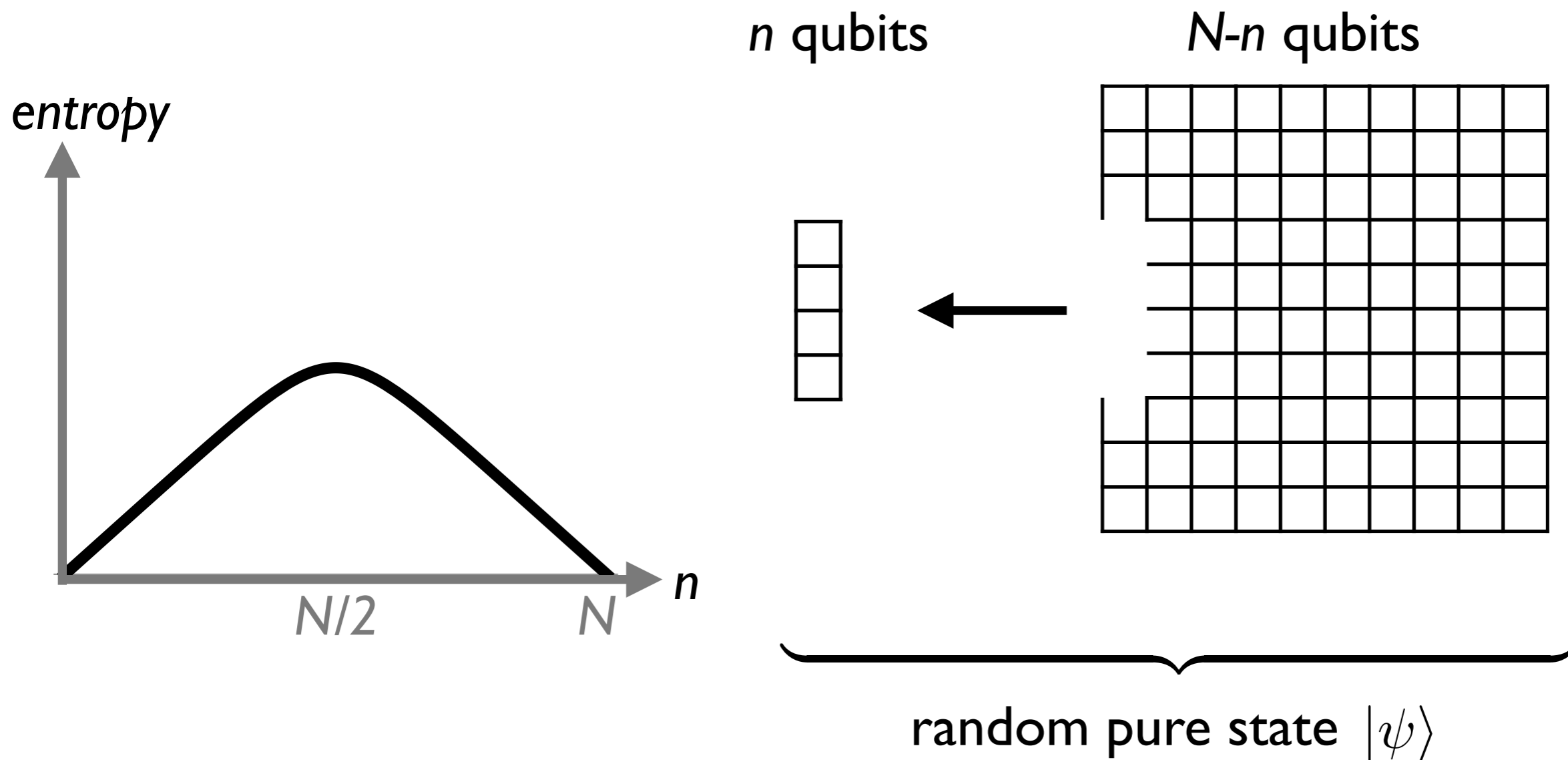


random pure state  $|\psi\rangle$

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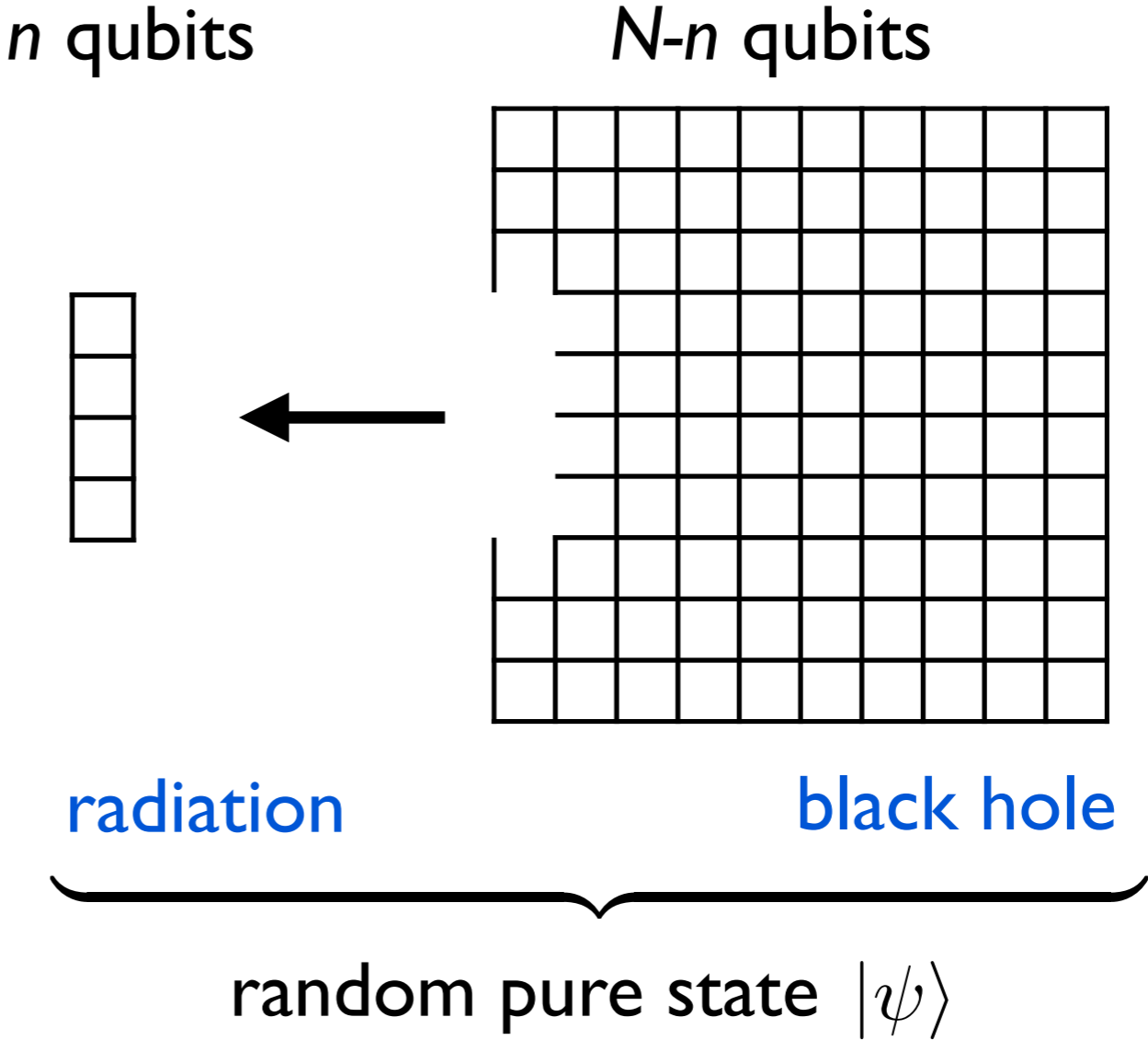
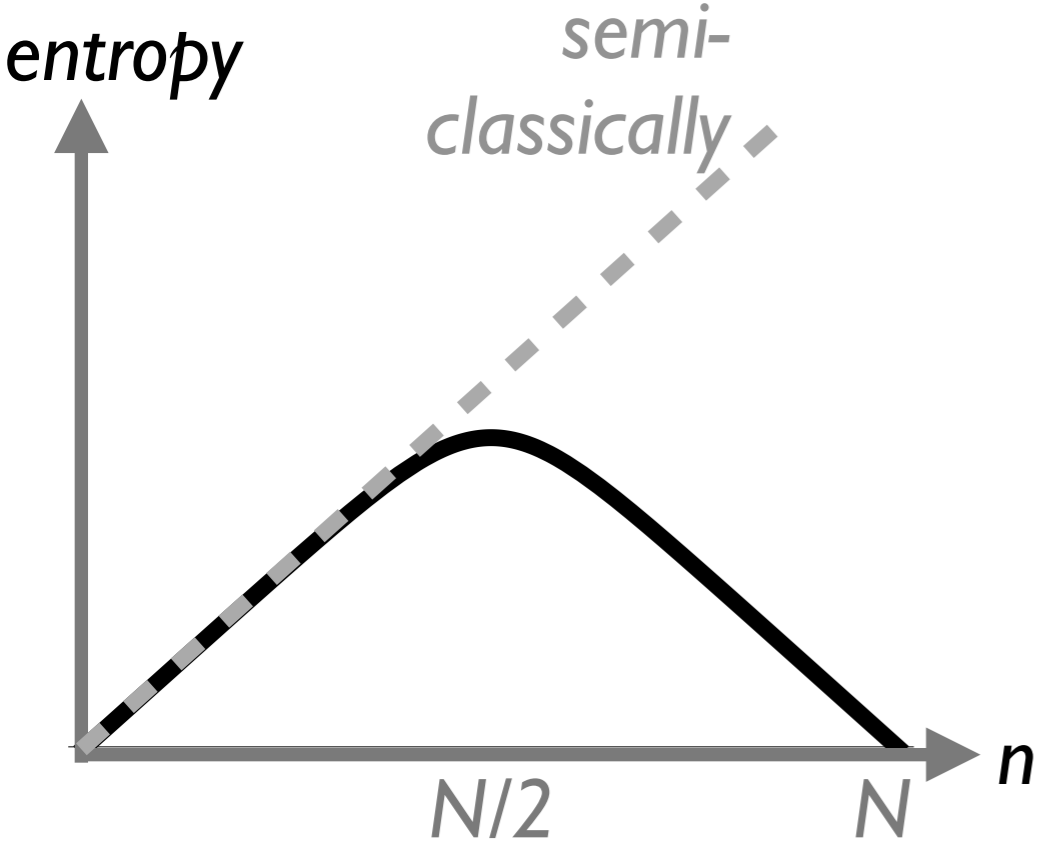
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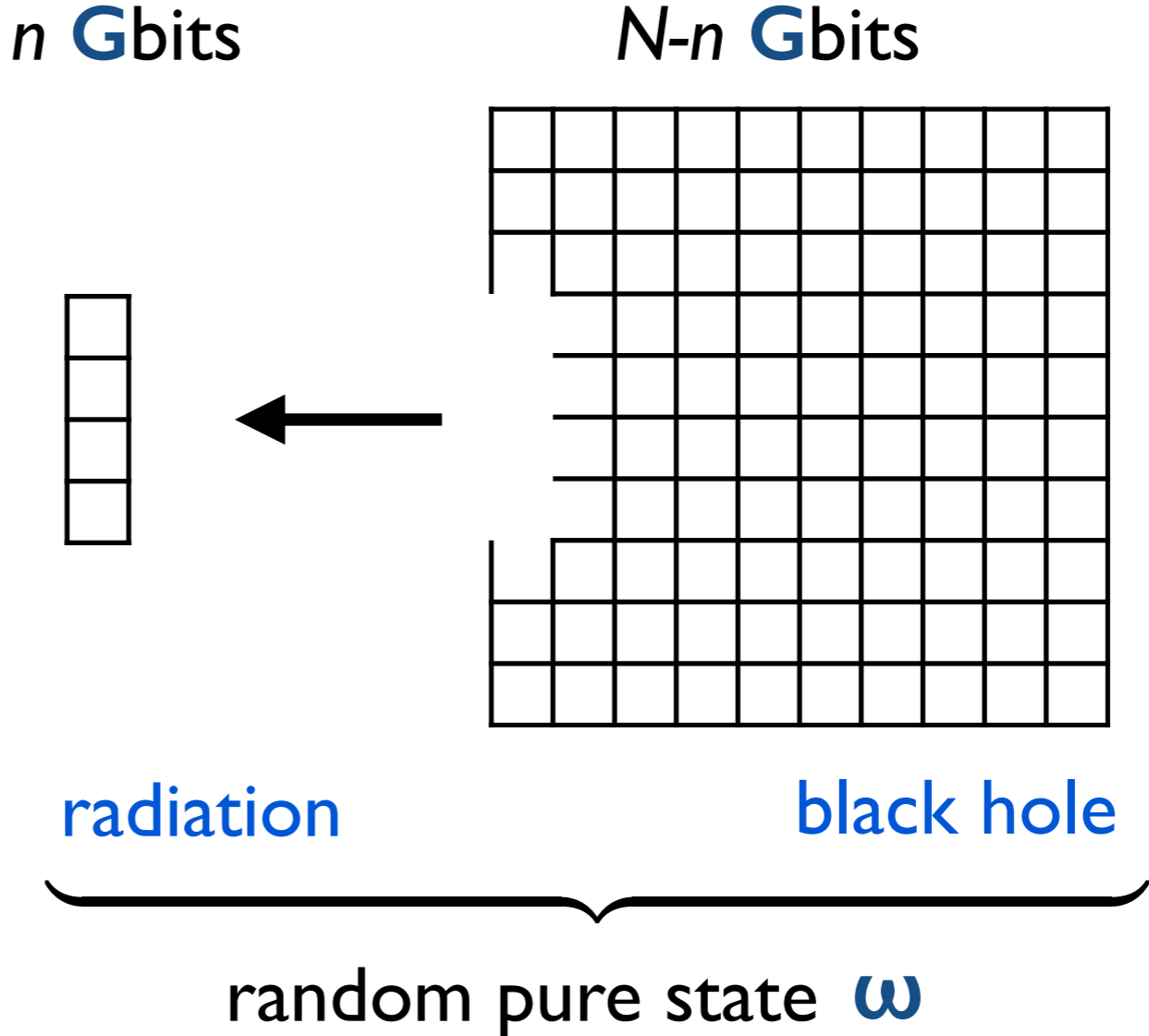
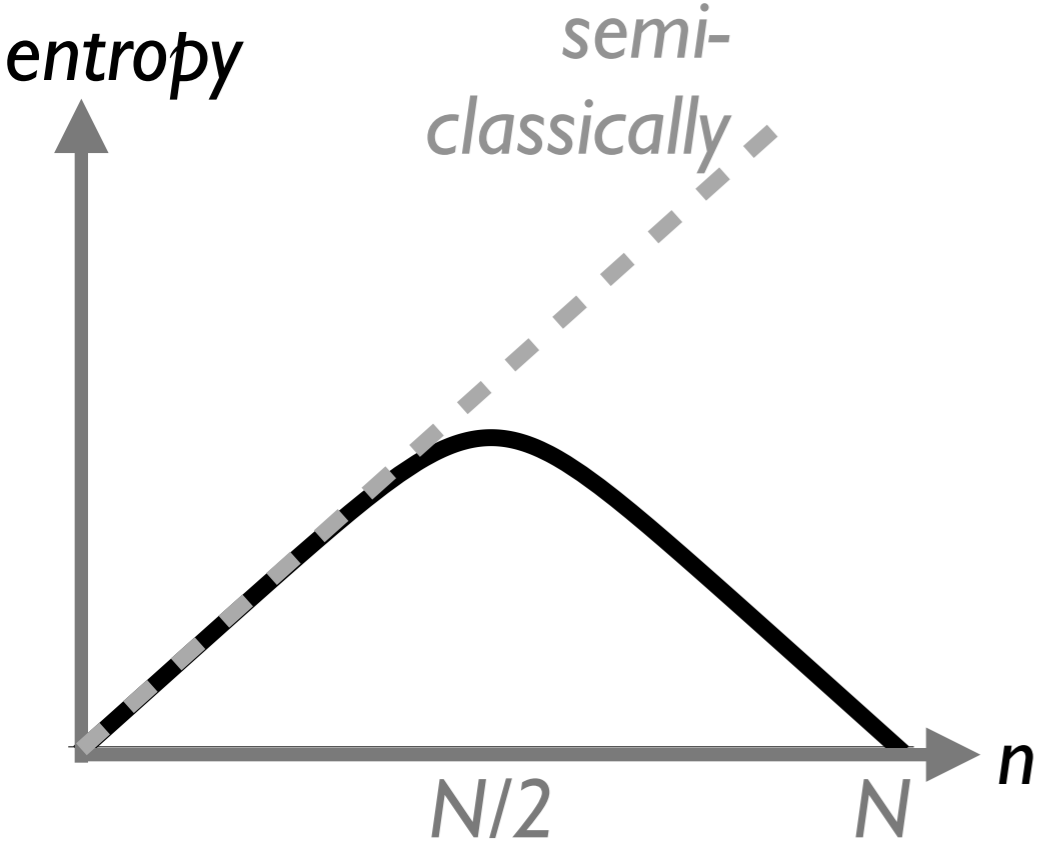
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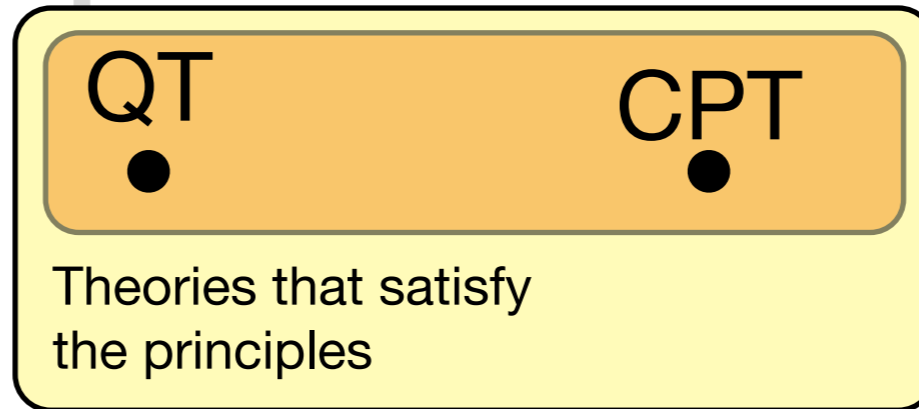
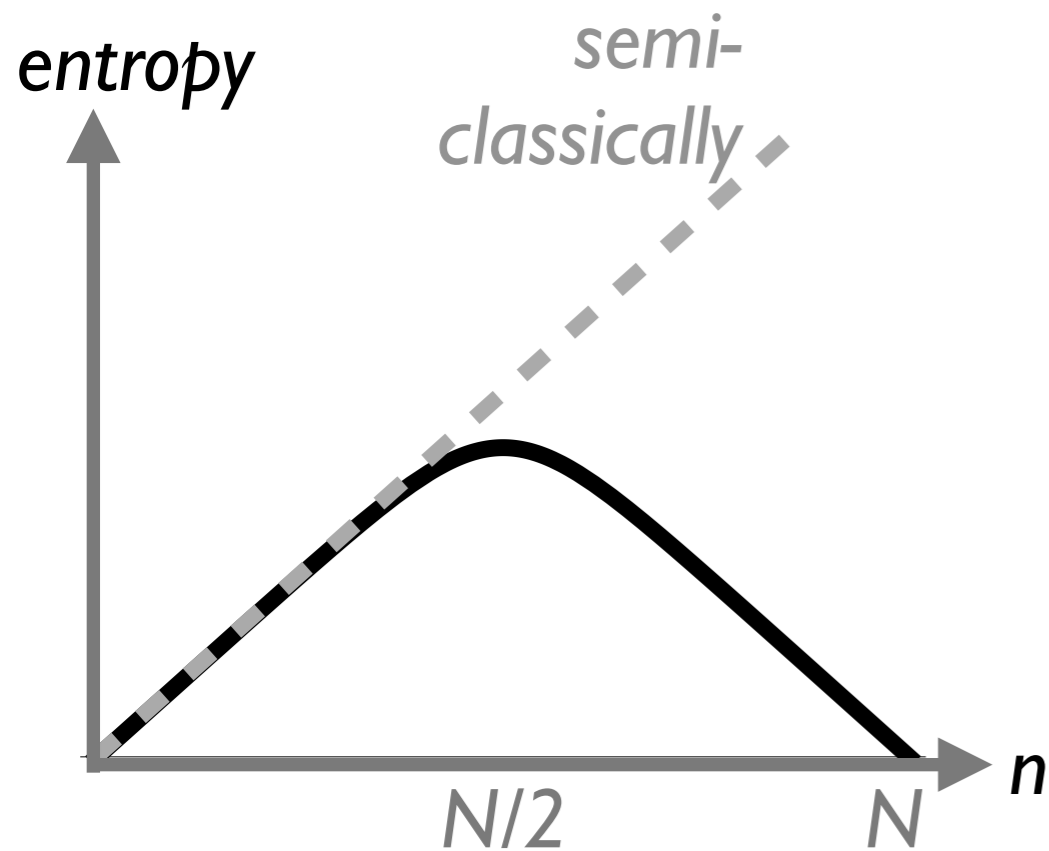


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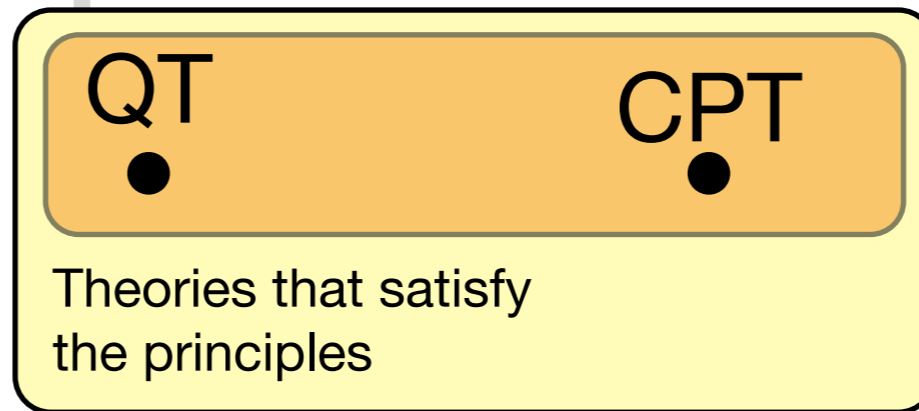
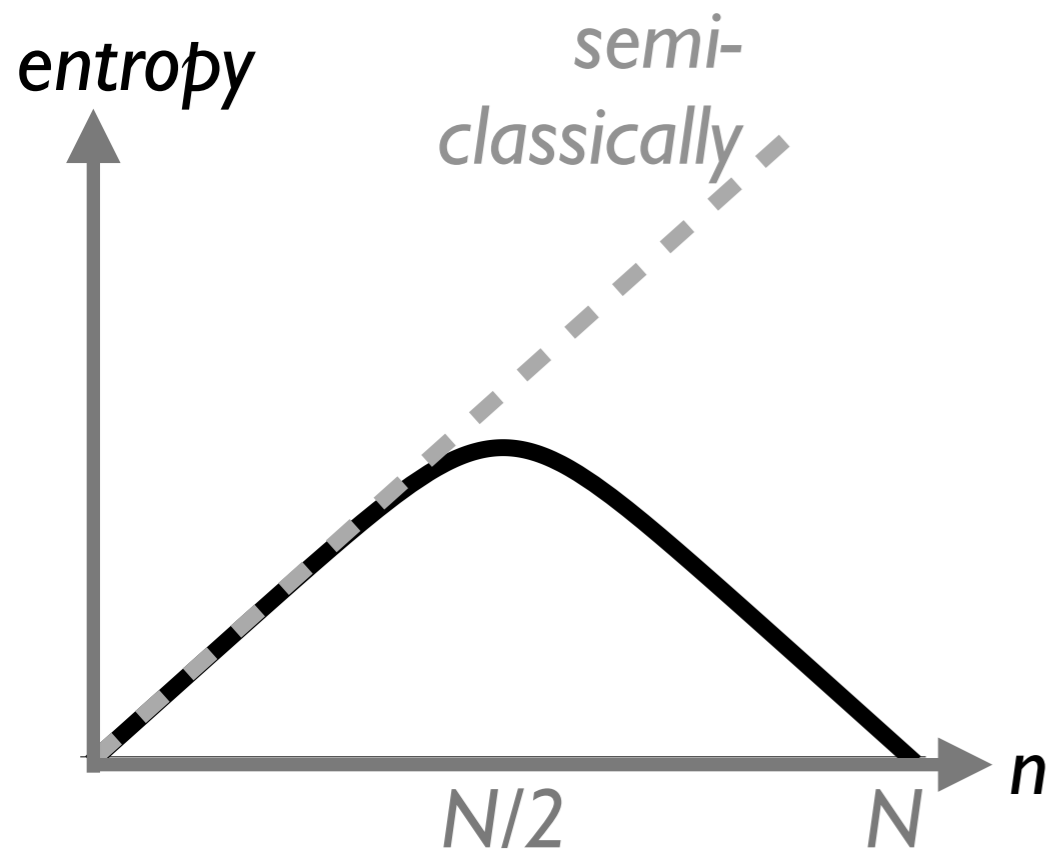


Result: "Interference parameter"  $r \in \mathbb{N}$

- $r = 1$  : CPT
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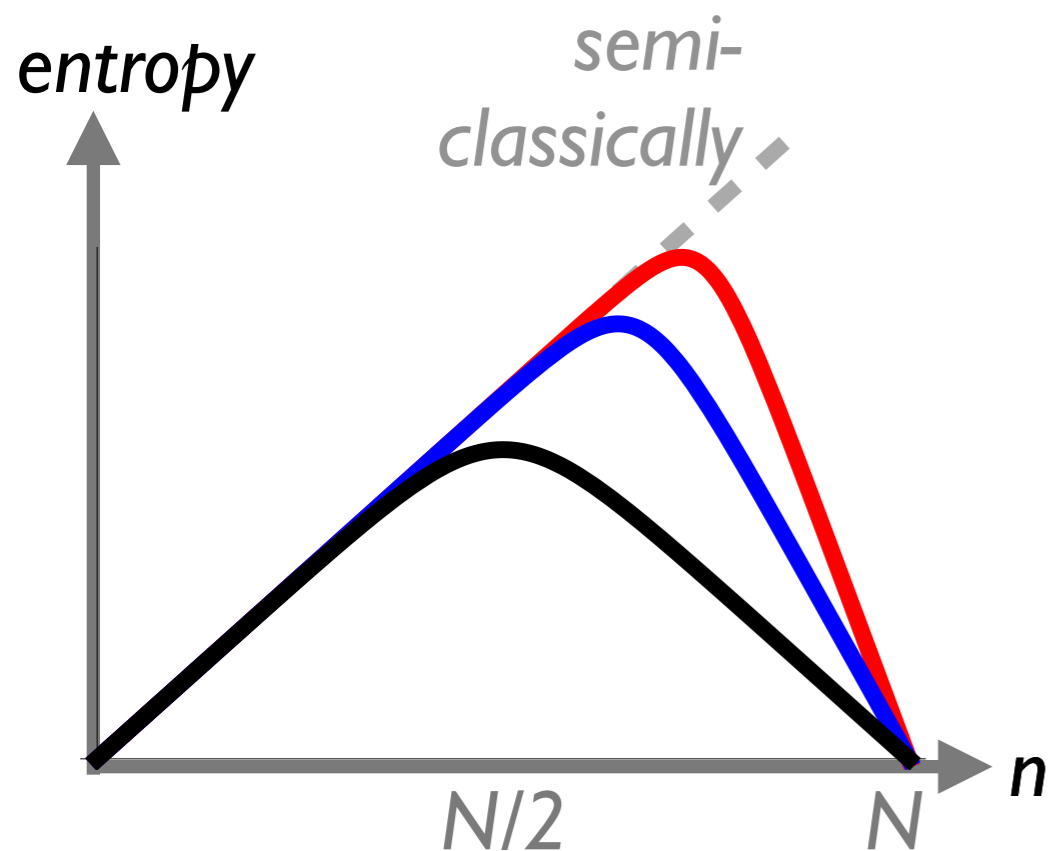
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**QT**

●

**CPT**

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Theories that satisfy the principles

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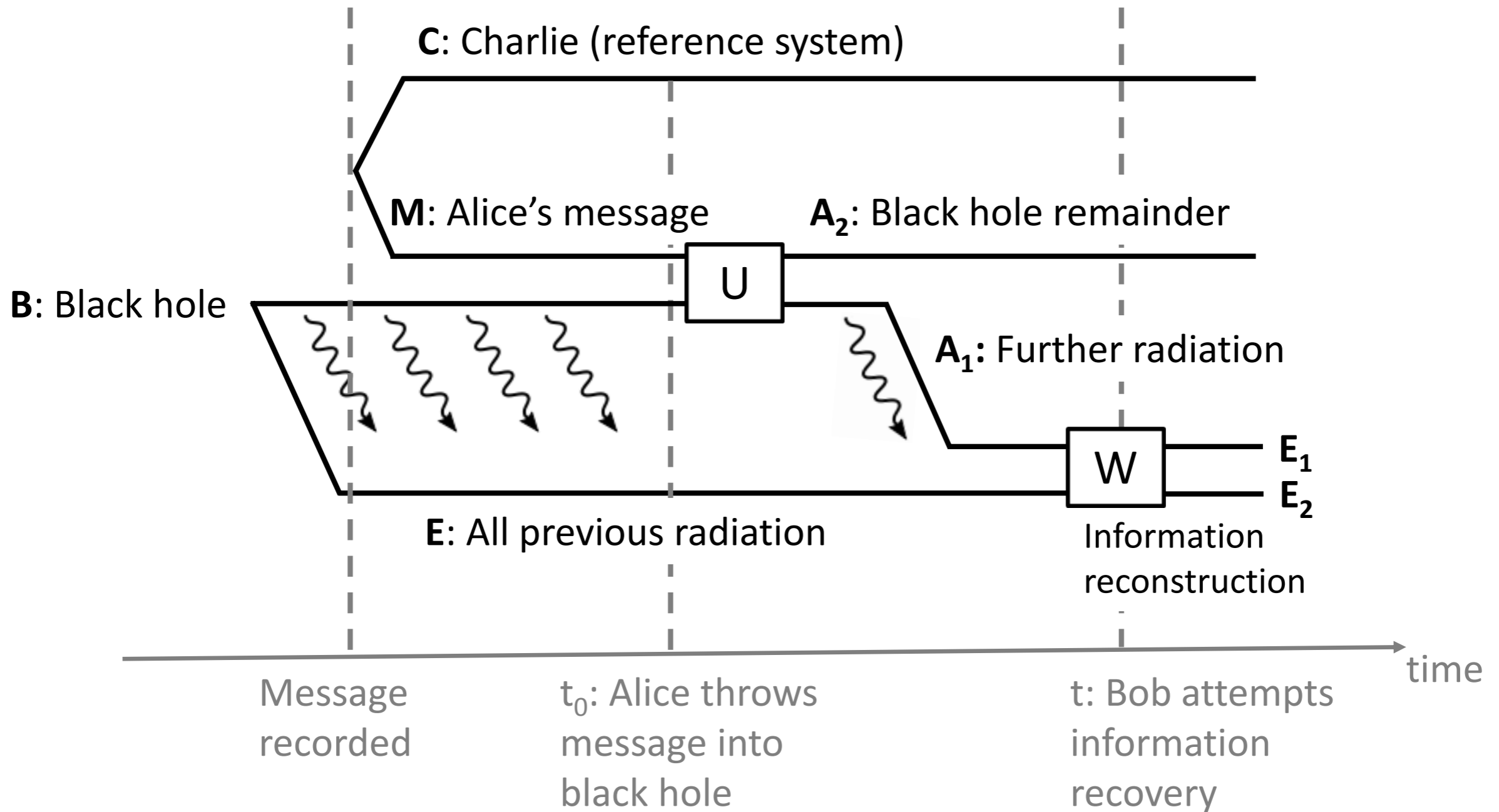
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Rényi-2 entropy of Hawking radiation:

$$H_2 = n - \log_2 \left( 1 + \frac{(2^{nr} - 1)(2^N - 1)}{2^{Nr} - 1} \right).$$

# More general scenario

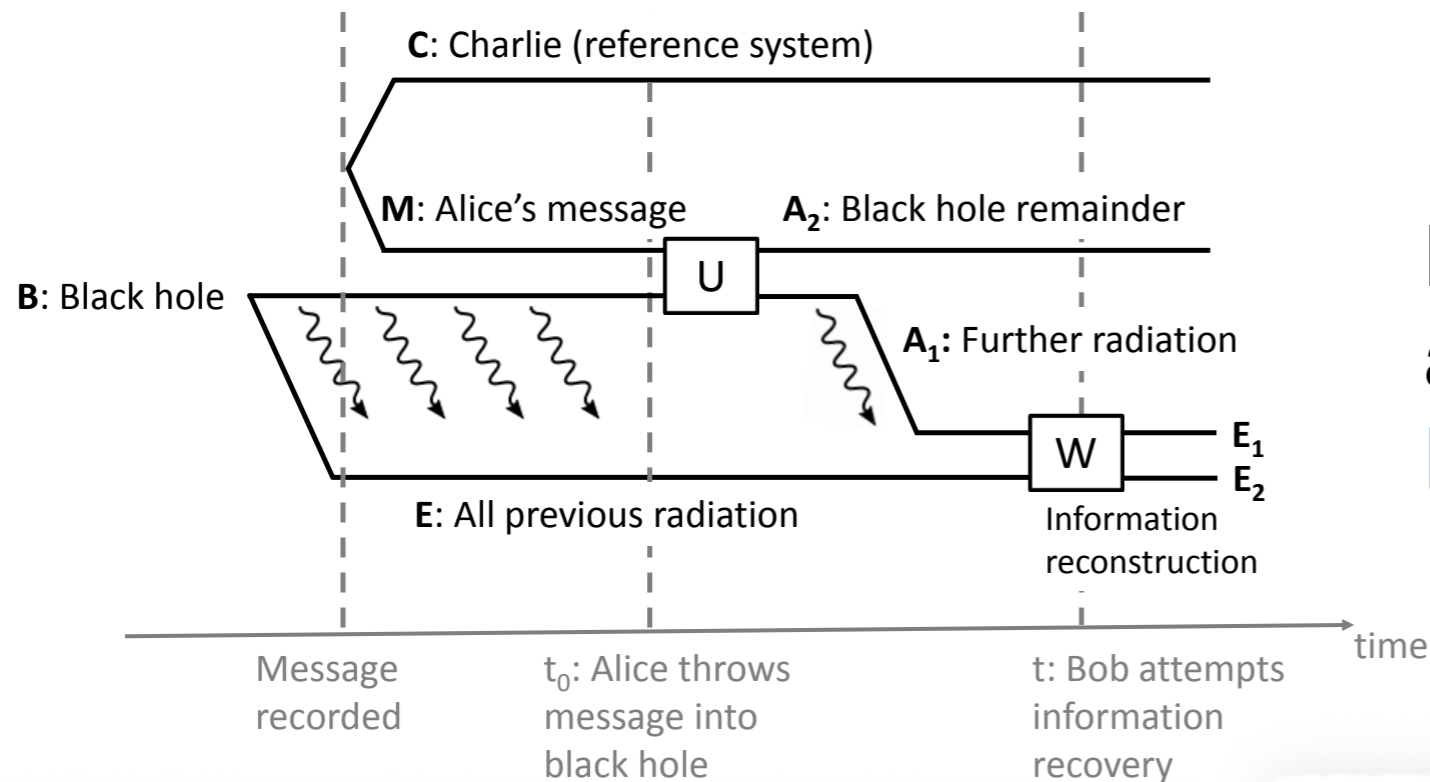
Generalizes P. Hayden and J. Preskill, *Black holes as mirrors*, JHEP **0709**:120 (2007)





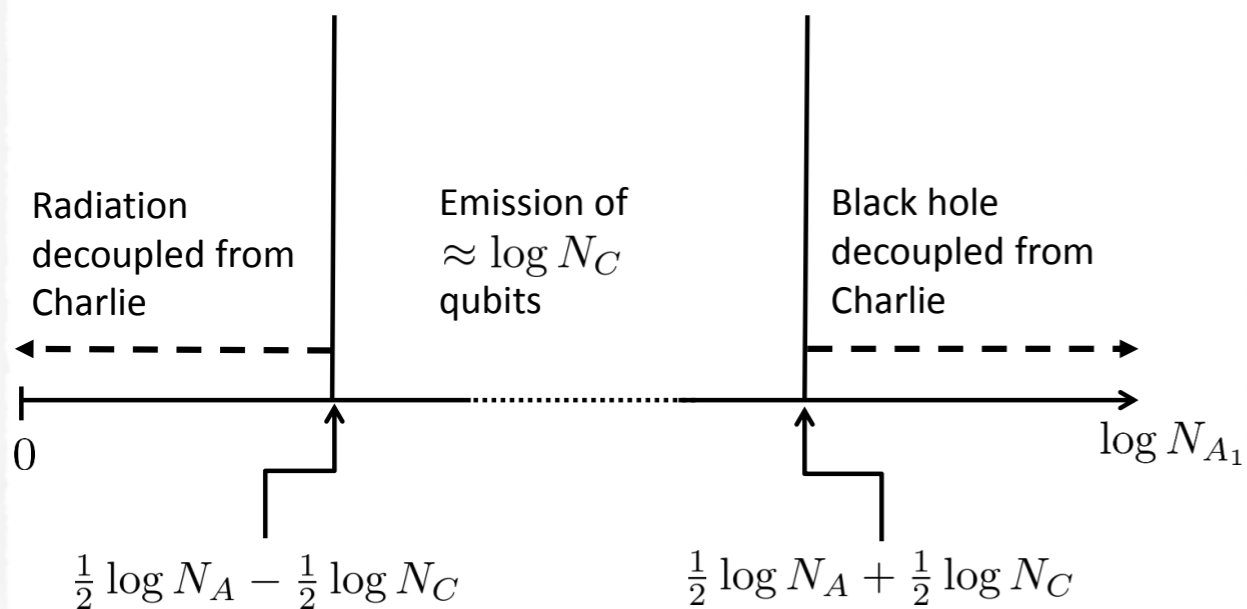
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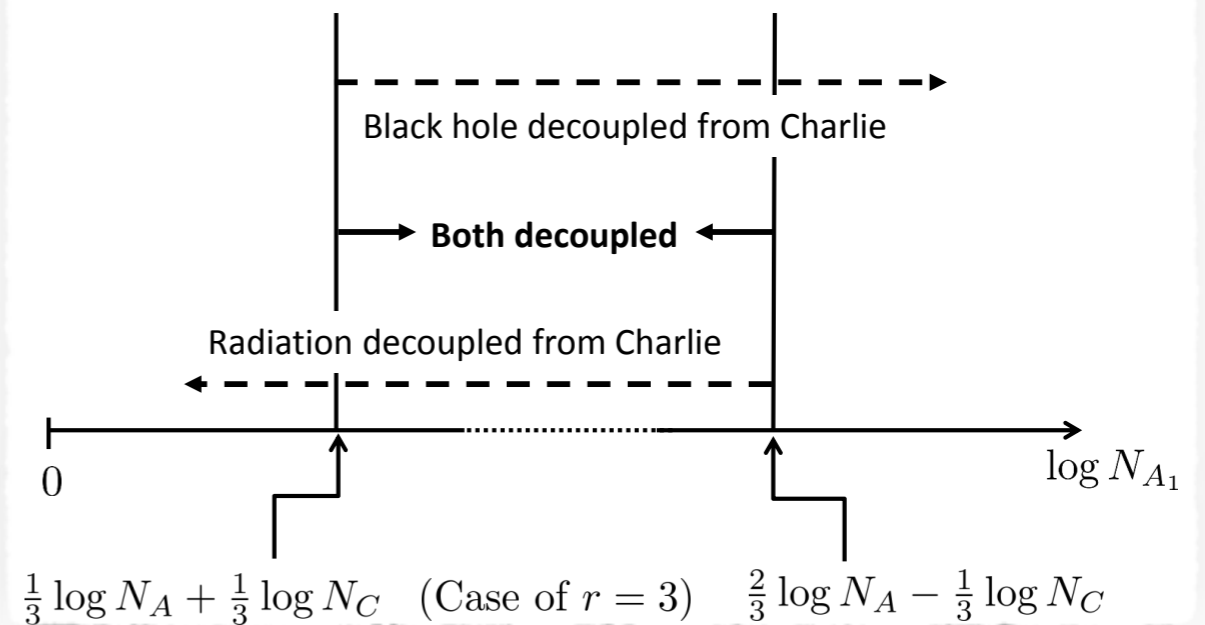


If Alice throws her diary into a very young BH, then QT's **No-Hiding Theorem** is violated:

Decoupling when  $K = N^2$  (Quantum case)



Both decoupled for  $K = N^r, r \geq 3$



## Why we think this is a good idea

- Probably **most conservative** way to generalize QT: just *drop* assumptions of QT, and don't add new ones.
- Satisfies information preservation ("**unitarity**"), **causality**, preserves **subsystem structure**, clear operational meaning.
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Shows what elbow room there is, within one's favorite physical properties / principles.

## Drawback

- Not clear how to combine with the rest of physics (Hamiltonians etc.)

## Some thoughts as a QG non-expert

**Q:** Do you *really* think QT is modified in some regime?

**A:**

## Some thoughts as a QG non-expert

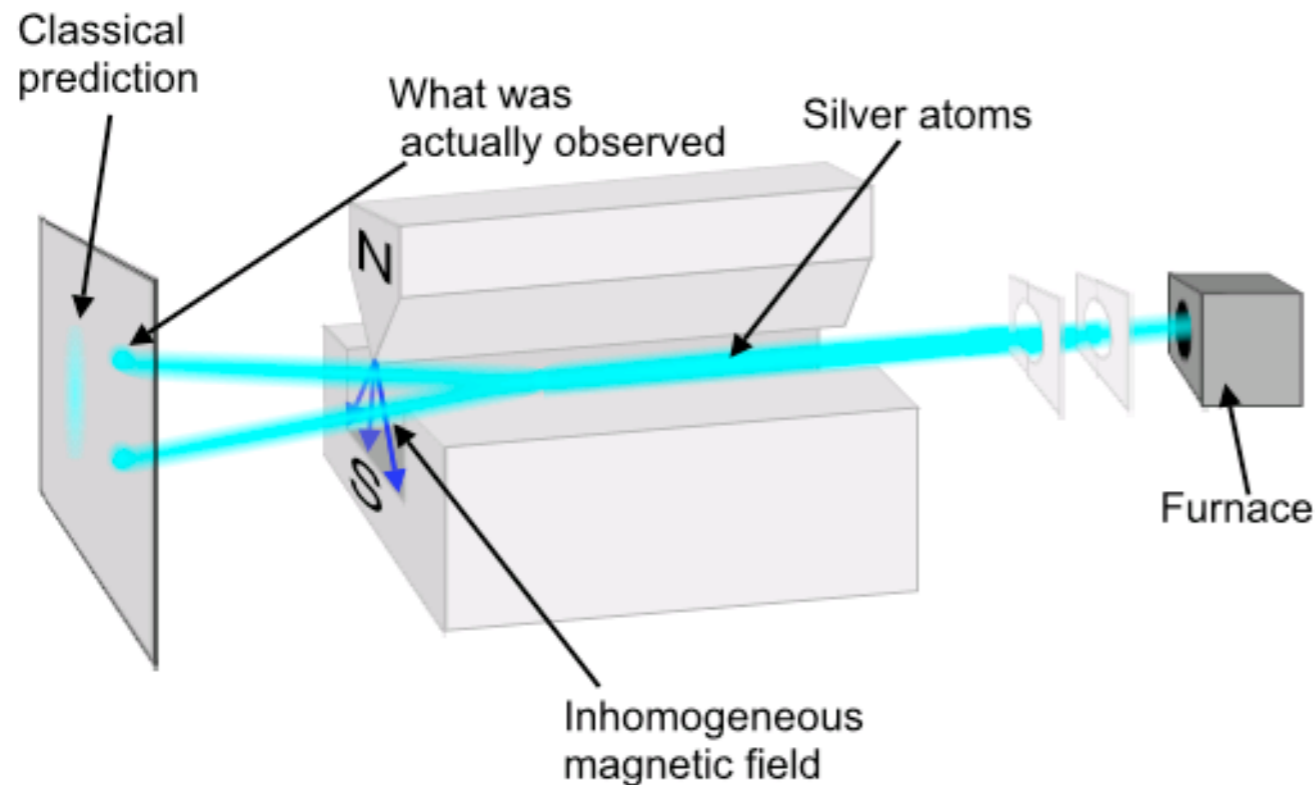
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QT seems suspiciously fine-tuned to fit into ordinary **spacetime**.

So if the latter is only an approximation, then maybe also the former.



quantum 2-level  
state space



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