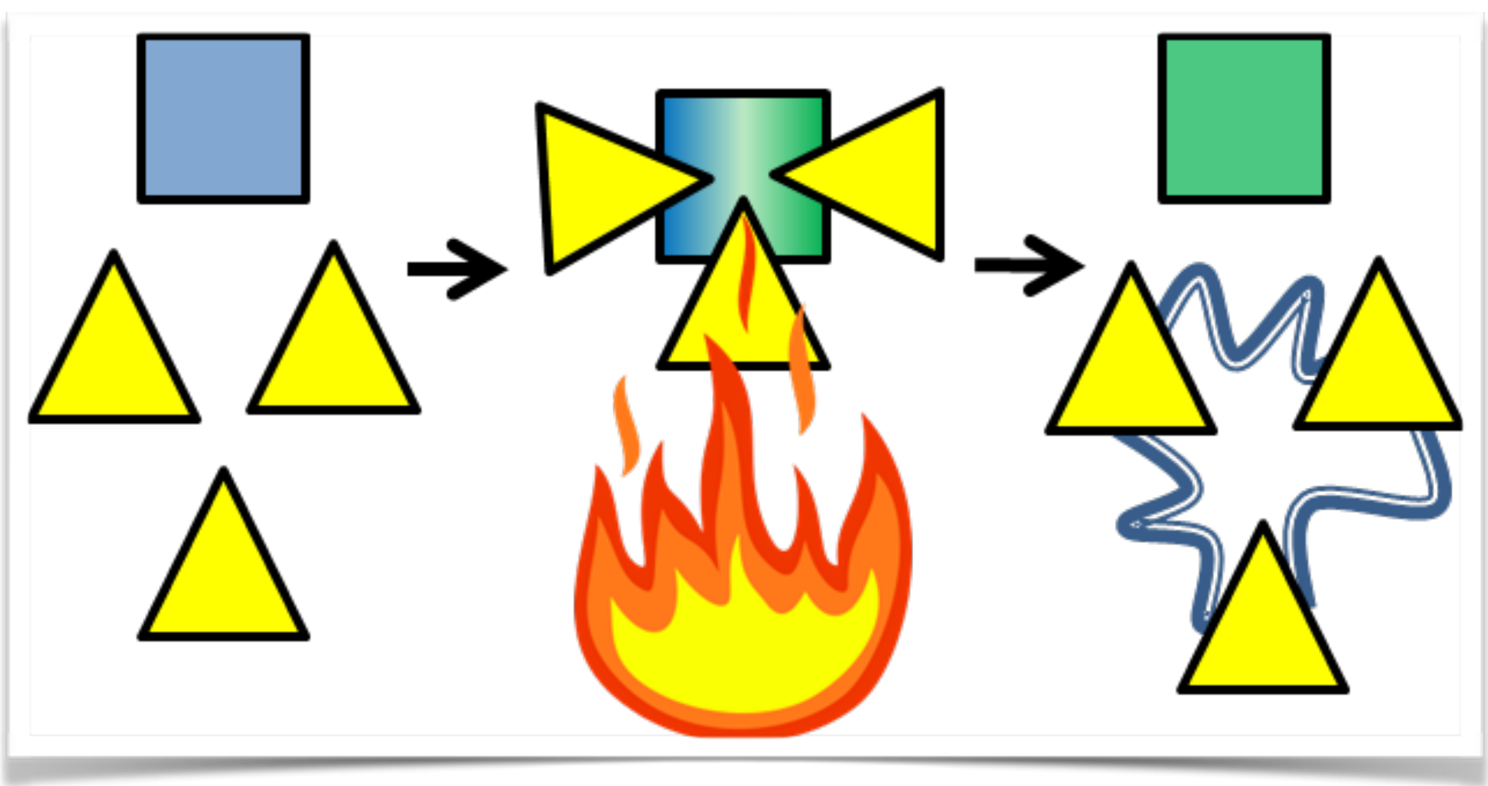
Stochastic independence as a resource in small-scale thermodynamics

Markus P. Müller
Departments of Applied Mathematics and Philosophy, UWO
Perimeter Institute for Theoretical Physics, Waterloo

Joint work with Matteo Lostaglio and Michele Pastena



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UNIVERSITÄT HEIDELBERG
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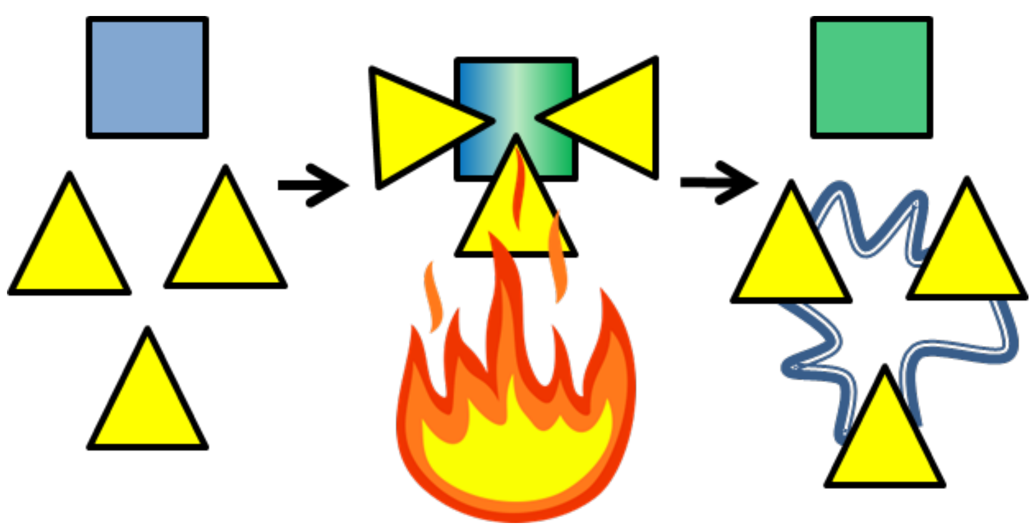
Outline

1. Background: the second law**s**

Severe constraints of small-scale thermodynamics

F. Brandao, M. Horodecki, N. Ng, J. Oppenheim, and S. Wehner, *The second laws of quantum thermodynamics*, Proc. Natl. Acad. Sci. USA **112** (2015)

2. Stochastic independence as a resource



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3. Conclusions

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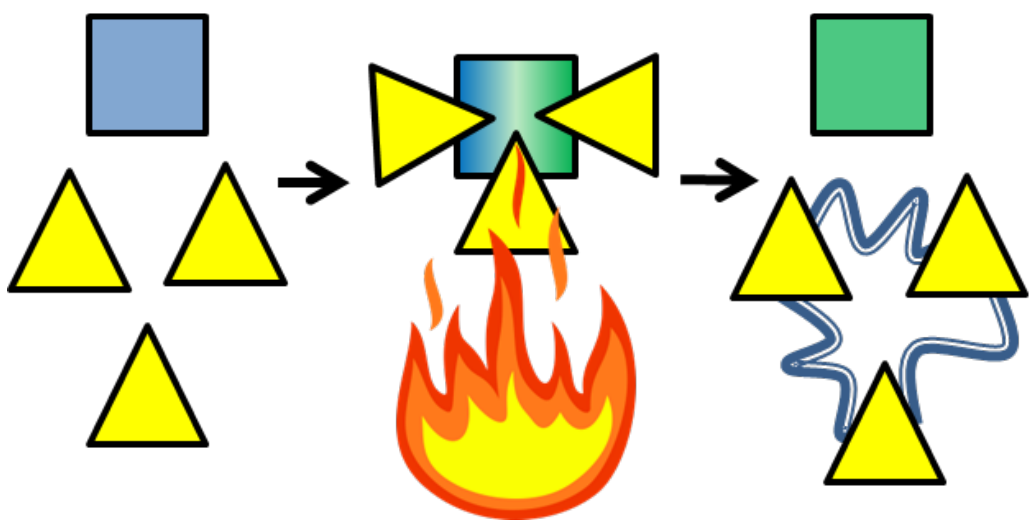

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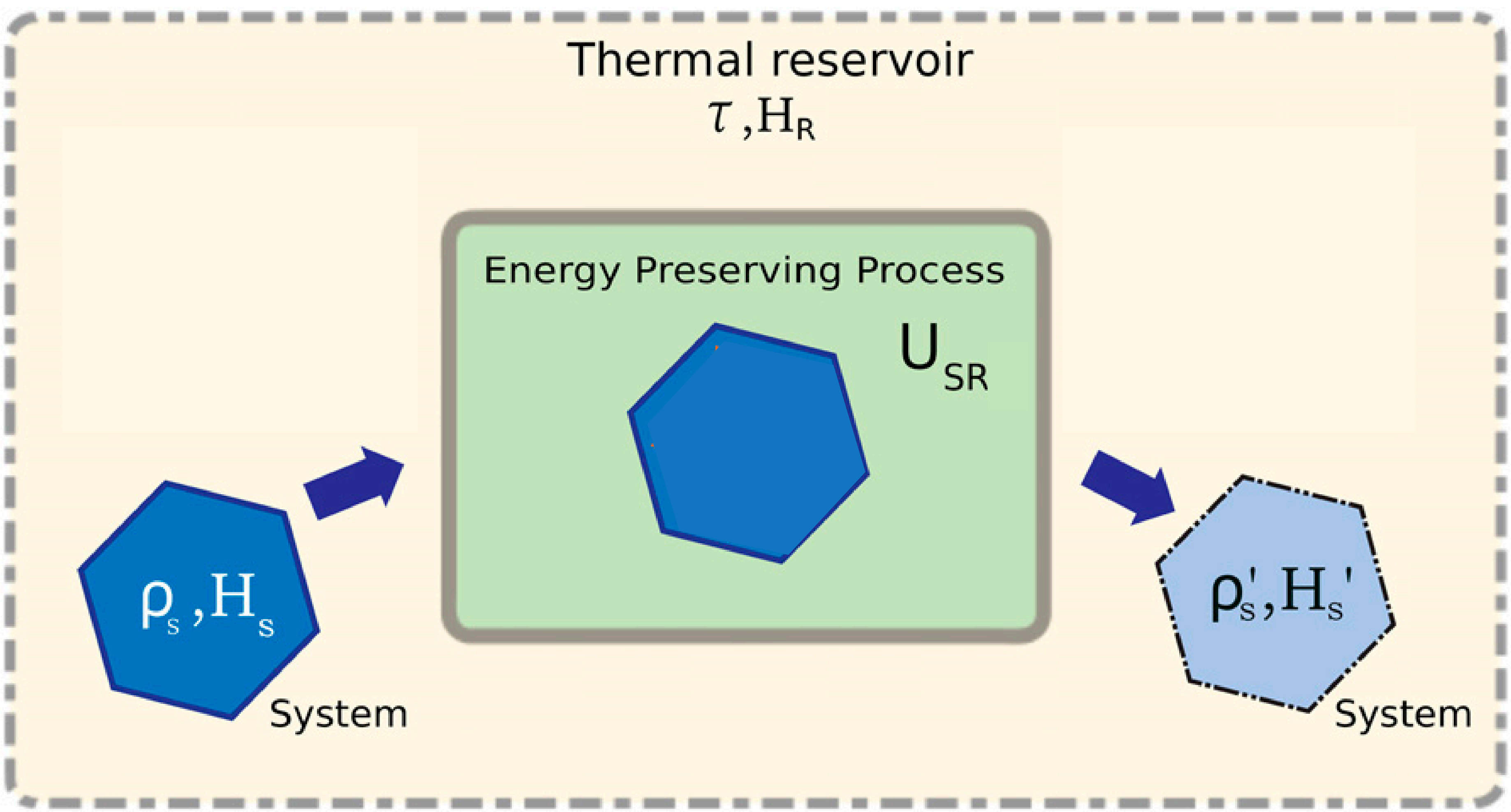

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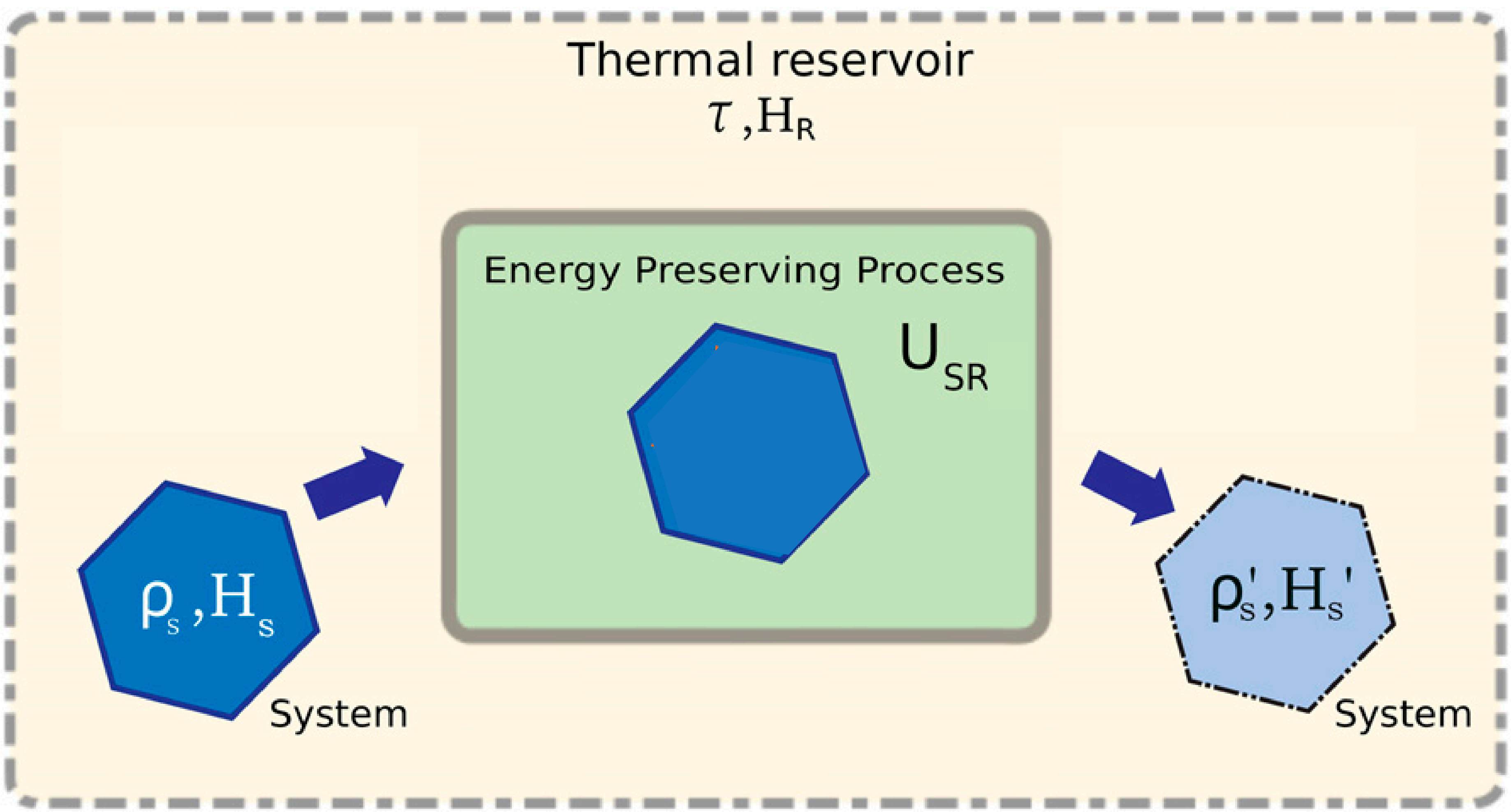

$$\rho'_S = \mathrm{Tr}_R\left[U_{SR}(\rho_S \otimes \tau_R)U_{SR}^\dagger\right]$$

$$[U_{SR}, H_S + H_R] = 0, \qquad \tau_R = \exp(-H_R/(k_B T))/Z.$$

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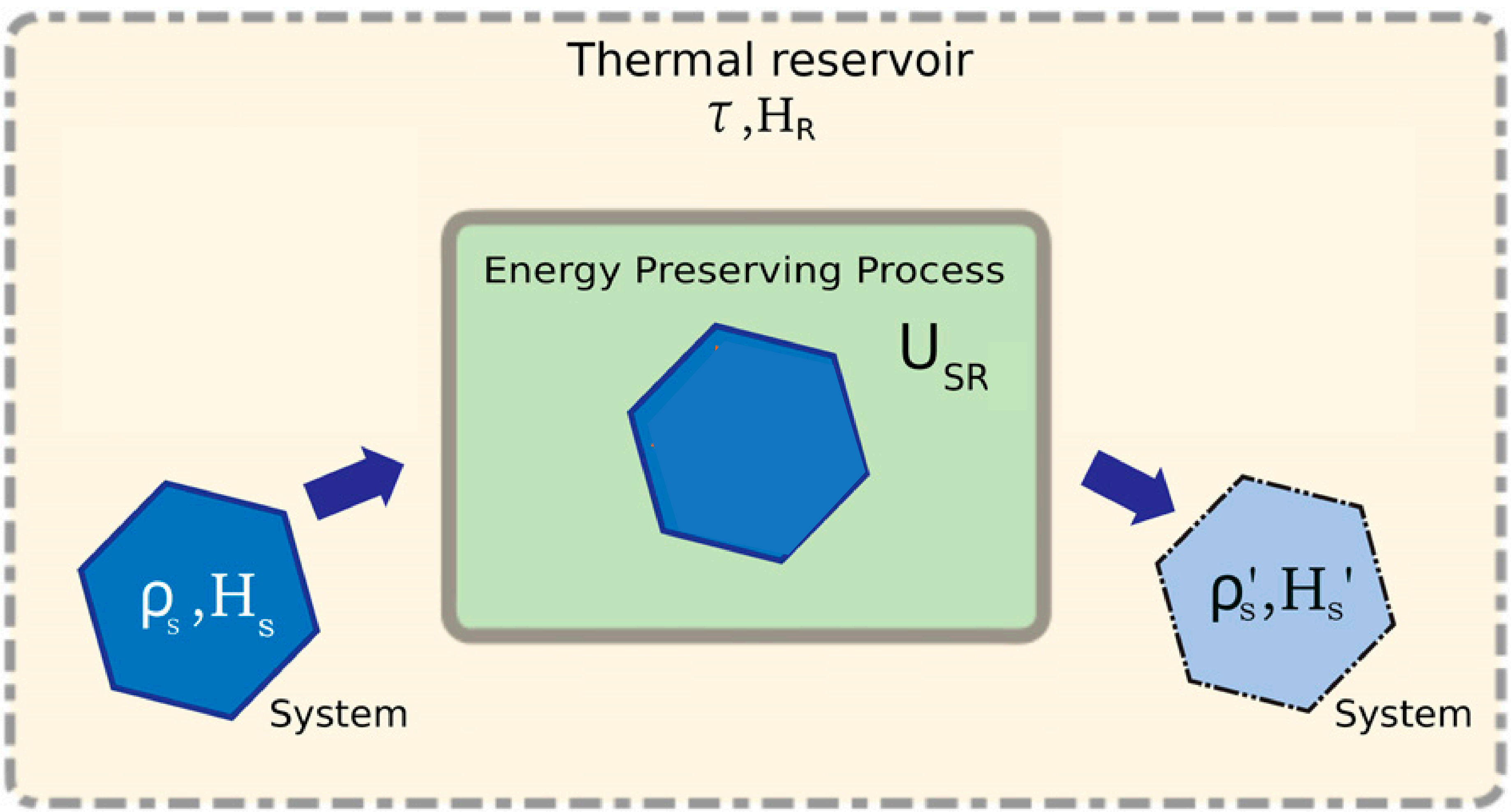

$$\rho_S \to \rho_S'$$

"thermal operation"

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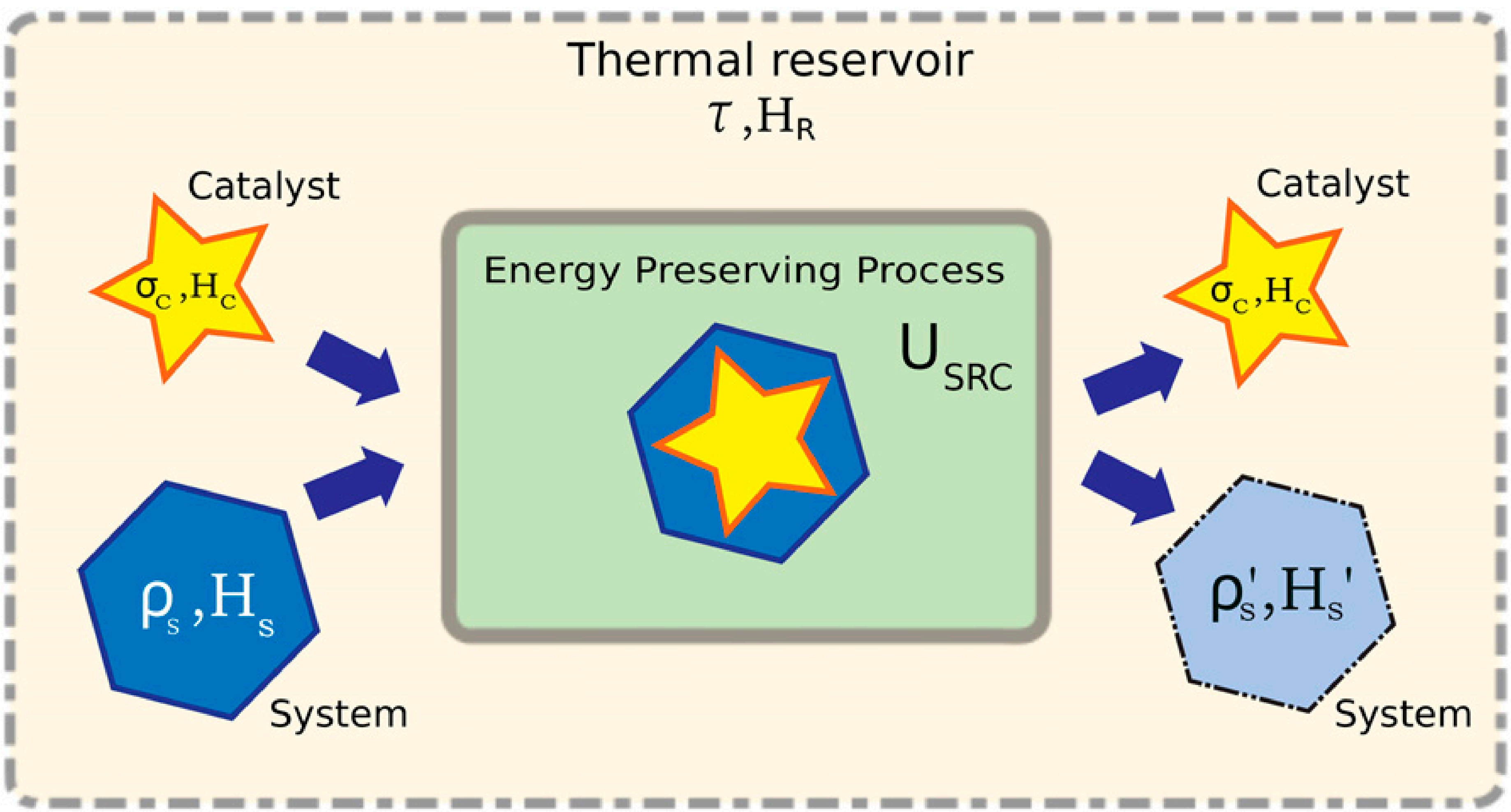

$$\rho_S \to \rho'_S$$

"catalytic thermal operation"

$$\rho'_S \otimes \sigma_C = \mathrm{Tr}_{RC}\left[U_{SRC}(\rho_S \otimes \tau_R \otimes \sigma_C)U^\dagger_{SRC}\right]$$

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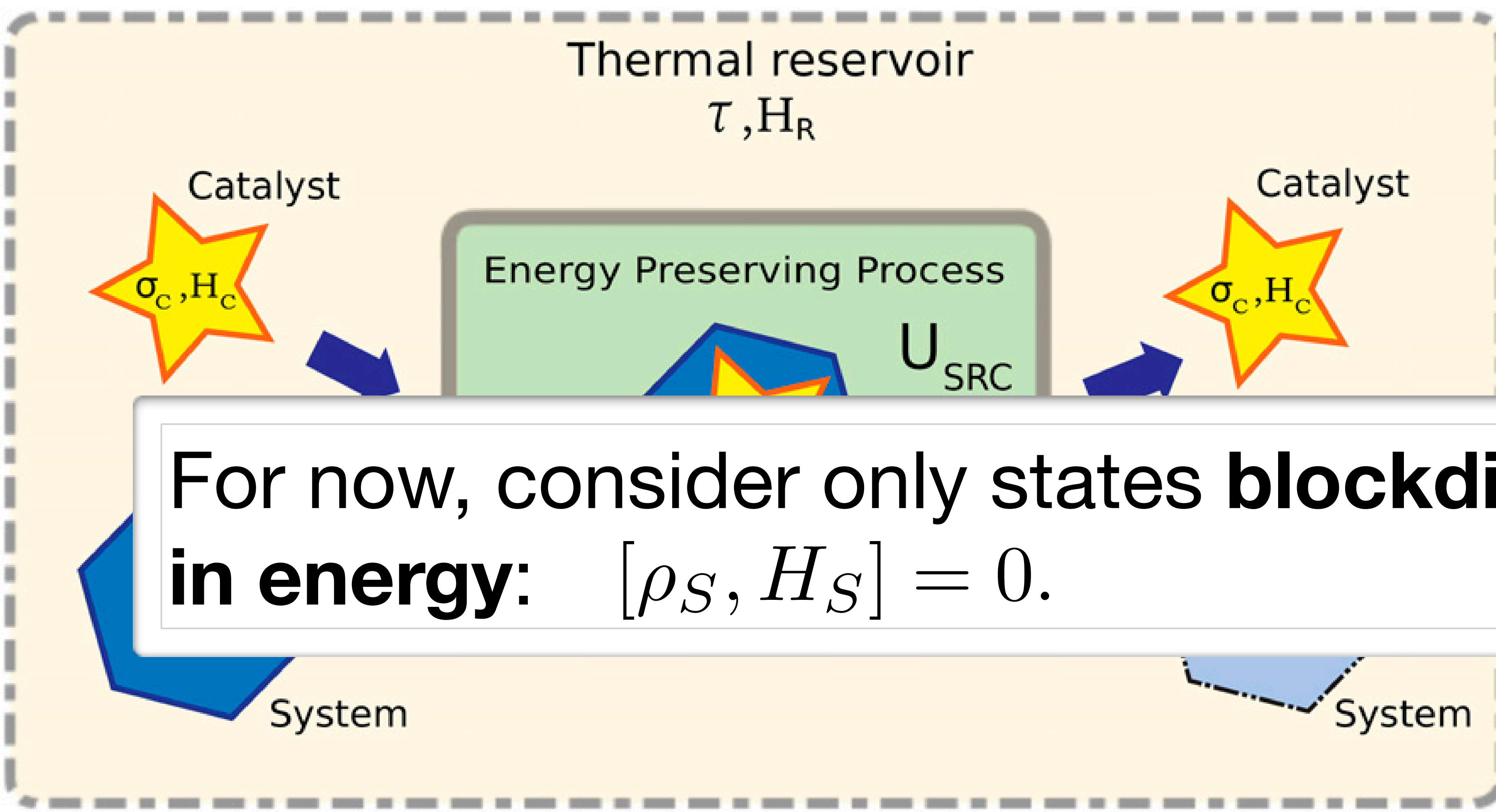

For now, consider only states **blockdiagonal in energy**: $[\rho_S, H_S]=0.$

$\rho_S \to \rho'_S$

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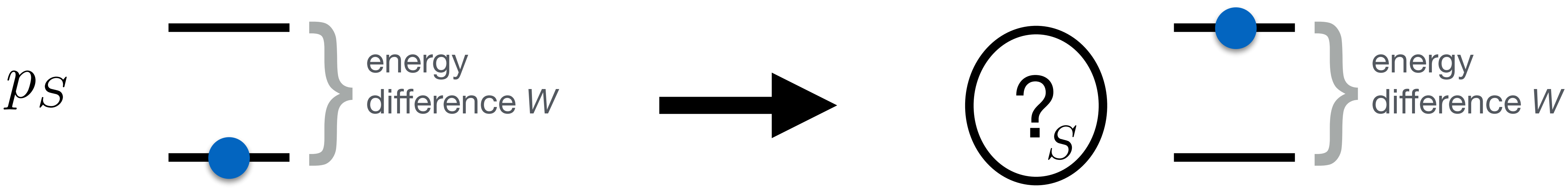
Work extraction and work cost

Work extraction: what is the largest possible W such that



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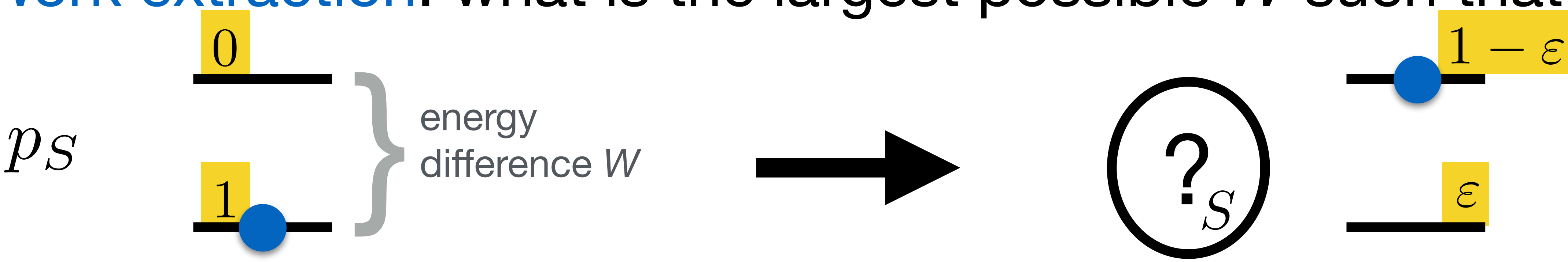

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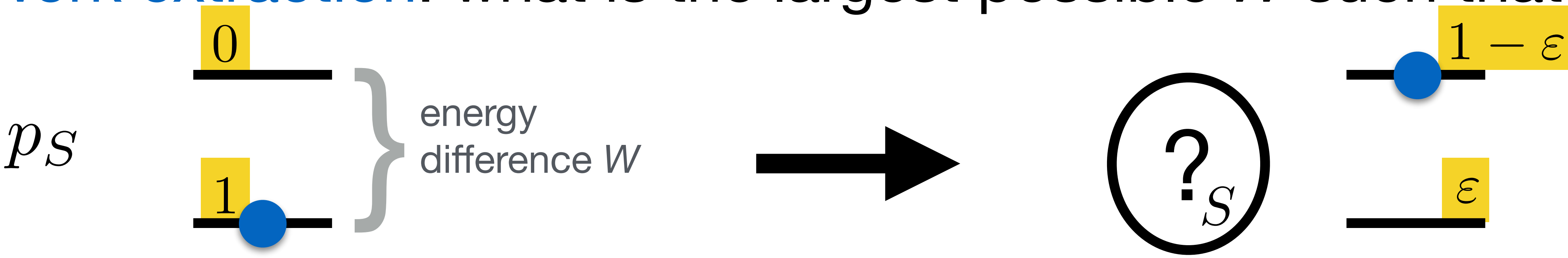

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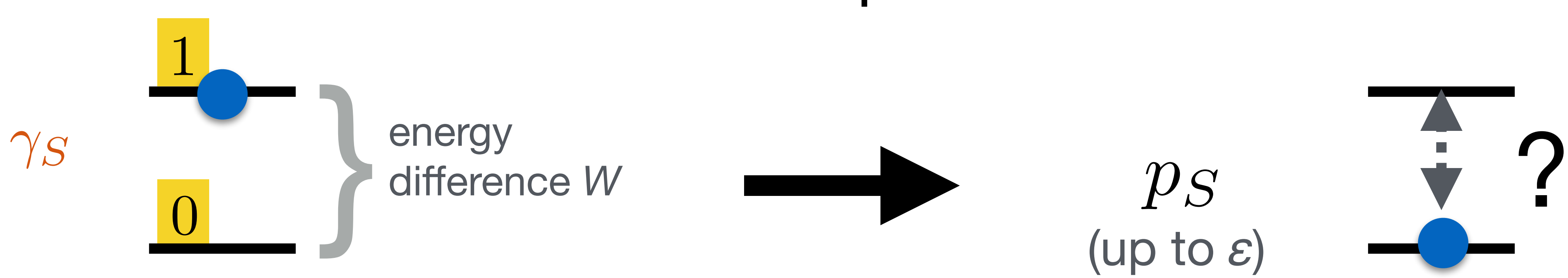

Work extraction and work cost

Theorem: The extractable work and work cost are

$$W_{\text{extr}} = k_B T \left(F_0^{\varepsilon}(p_S) - F(\gamma_S) \right),$$
$$W_{\text{cost}} = k_B T \left(F_{\infty}^{\varepsilon}(p_S) - F(\gamma_S) \right),$$

where F_α is the Rényi *α*-free energy:

$$F_\alpha(p_S) = k_B T \left(\frac{\operatorname{sgn}\alpha}{\alpha - 1} \log \sum_i p_i^\alpha \exp\left(\frac{-E_i(1-\alpha)}{k_B T} \right) \right) - k_B T \log Z,$$

and $F_1(p_S) = F(p_S) = \langle E \rangle - k_B T S(p_S)$

is the "standard" free energy.

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Fundamental thermodynamical irreversibility !

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But: in the thermodynamic limit,

n independent copies of p_S

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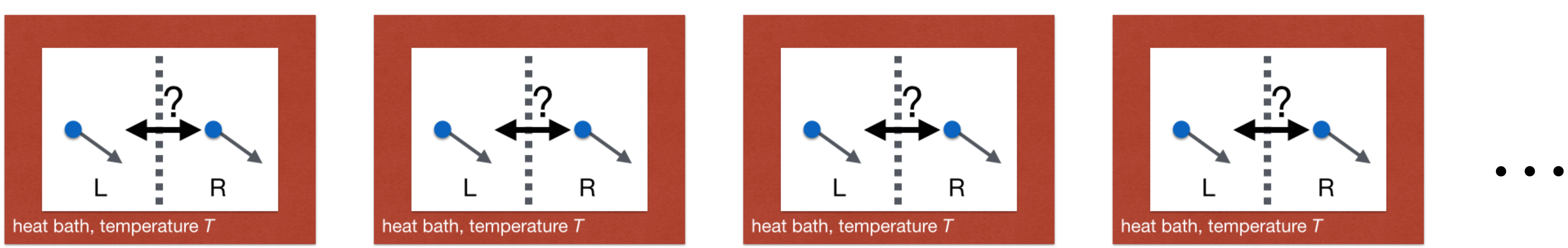

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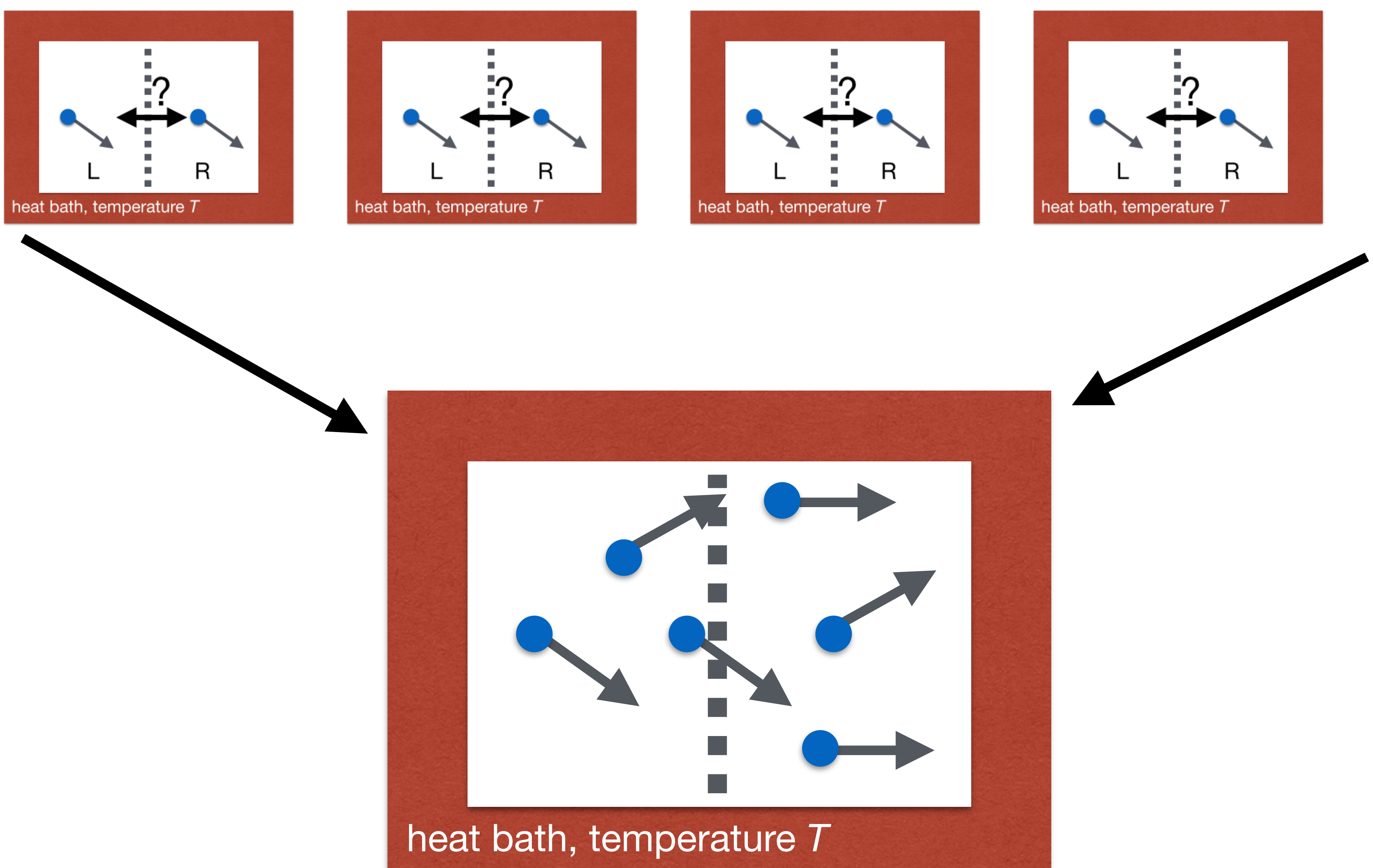

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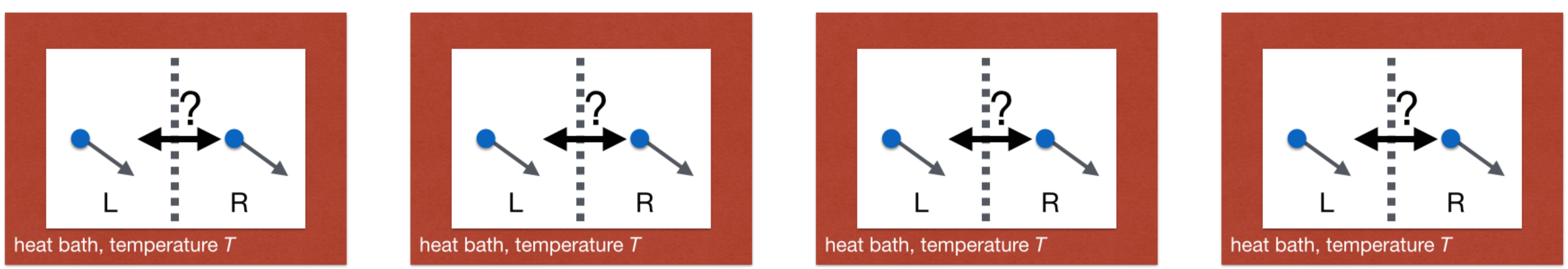

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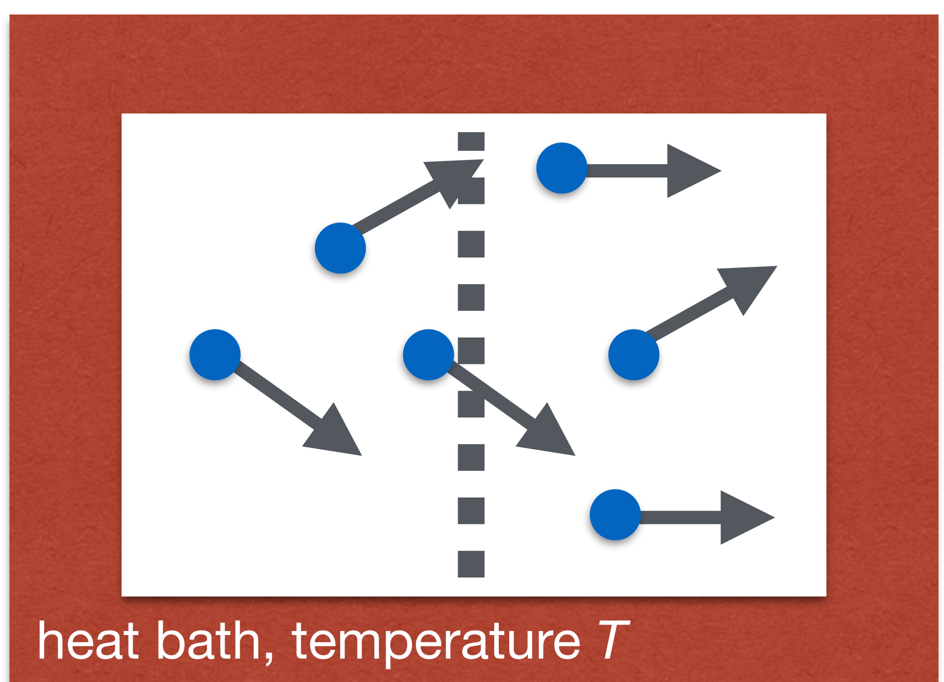

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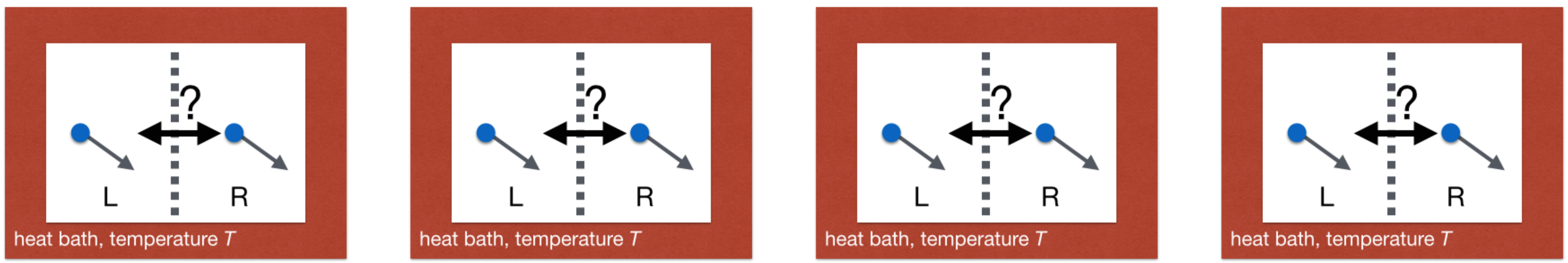

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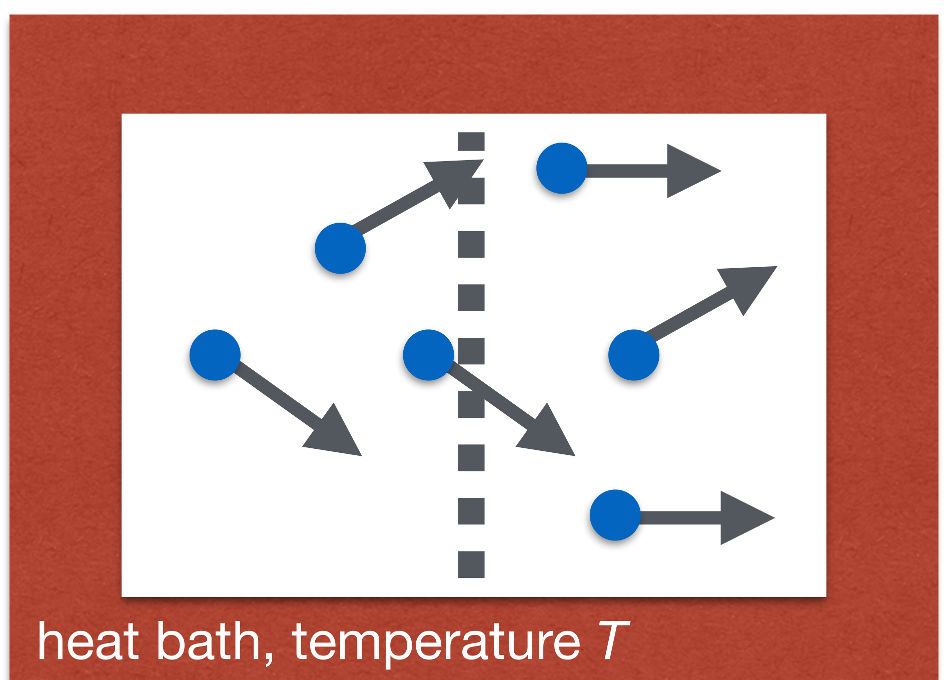

ideal gas;

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thermodynamical reversibility emerges !

The second laws

Theorem: A transition $p_S \to p'_S$ is possible by catalytic thermal operations if and only if

$$F_\alpha(p_S) \geq F_\alpha(p'_S) \text{ for all } \alpha \geq 0.$$

All *α*-free energies must go down!

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Consequence: some states are incomparable, i.e.

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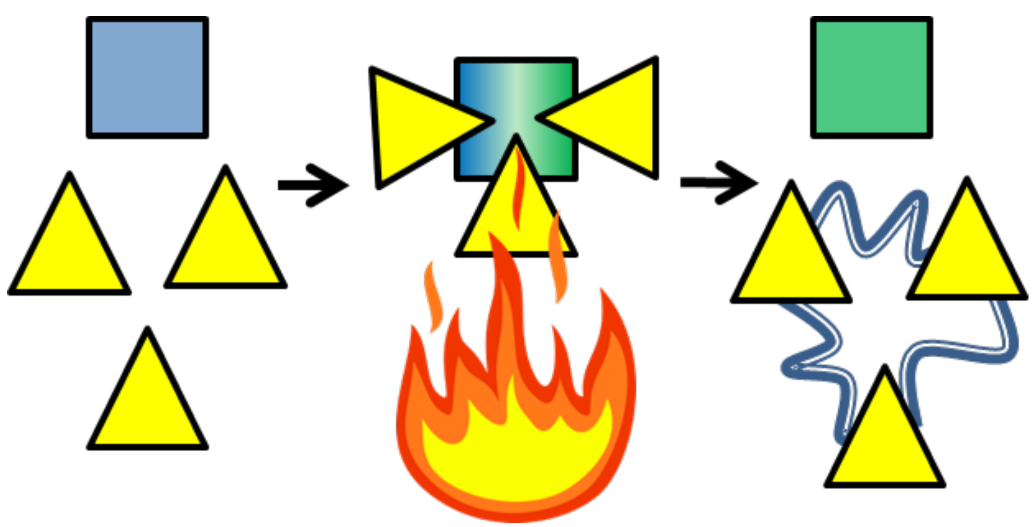

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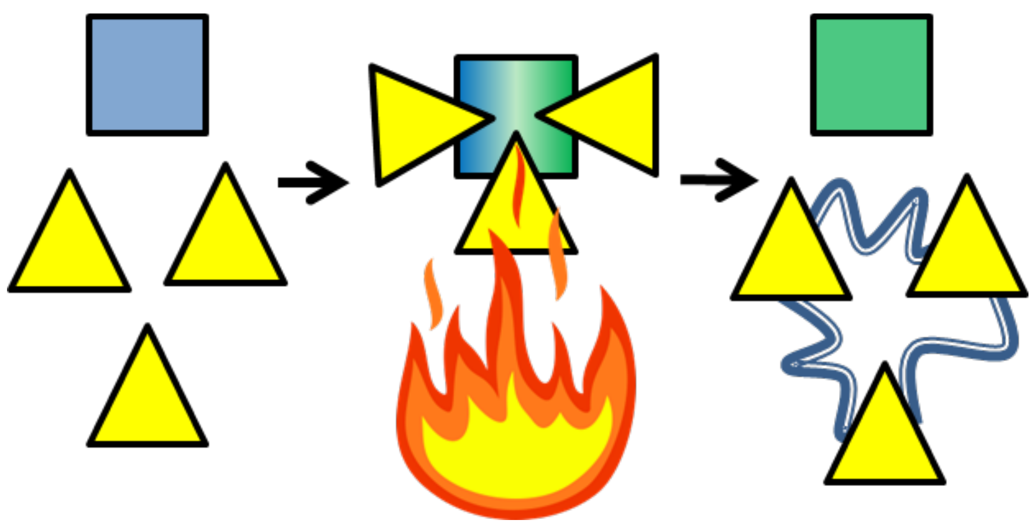

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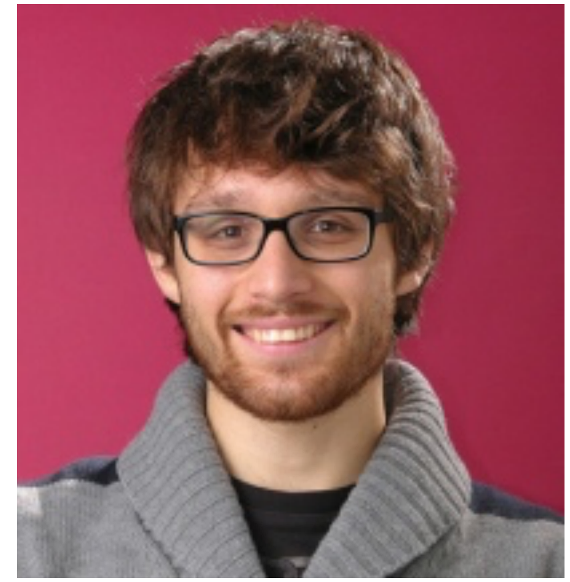

Standard free energy is superadditive:

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Equality iff $p_{AB} = p_A \otimes p_B$;
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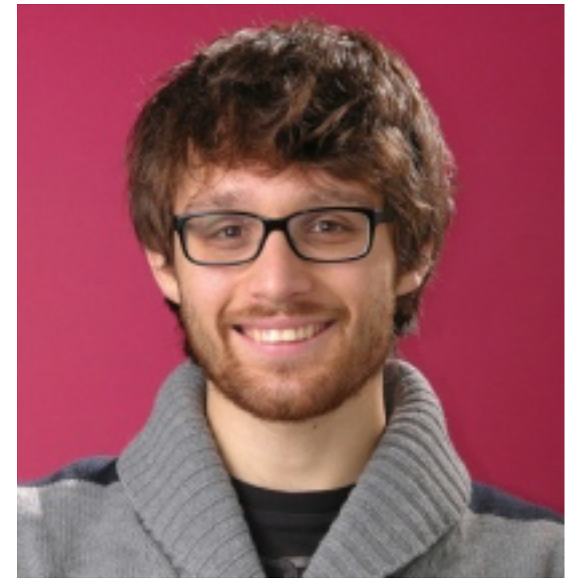

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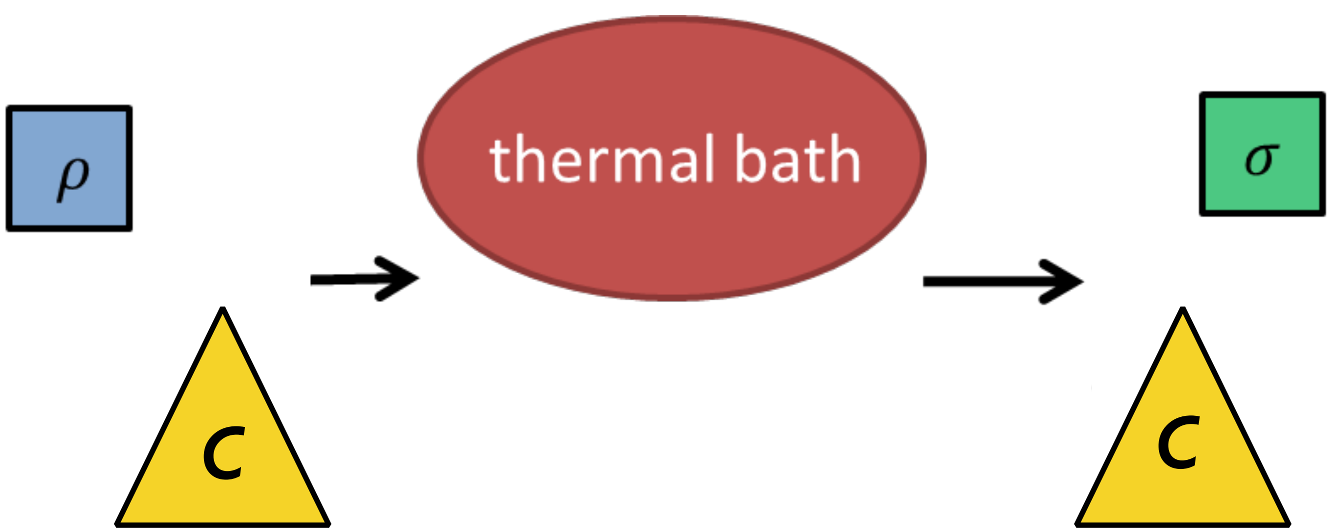

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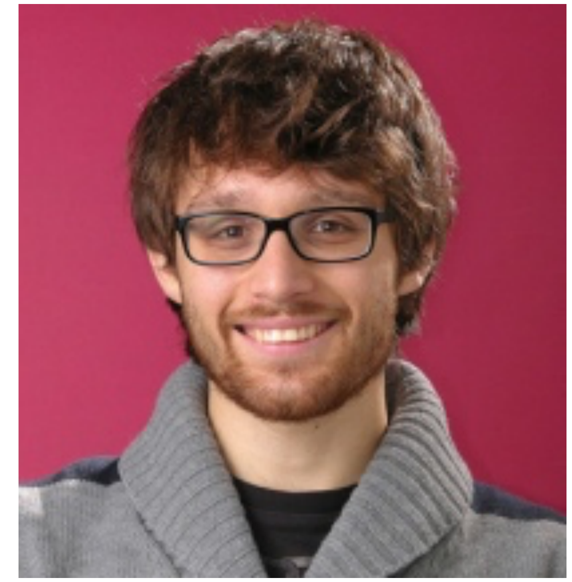

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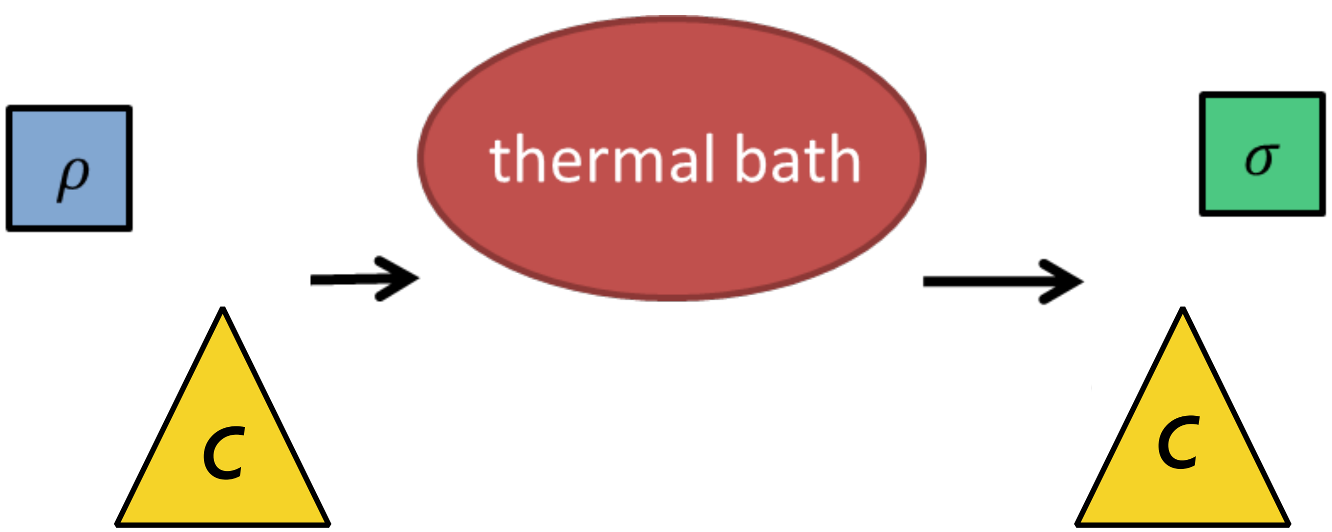

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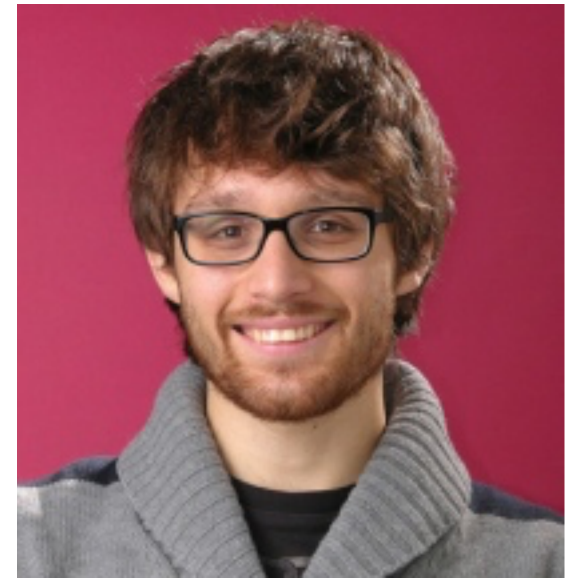

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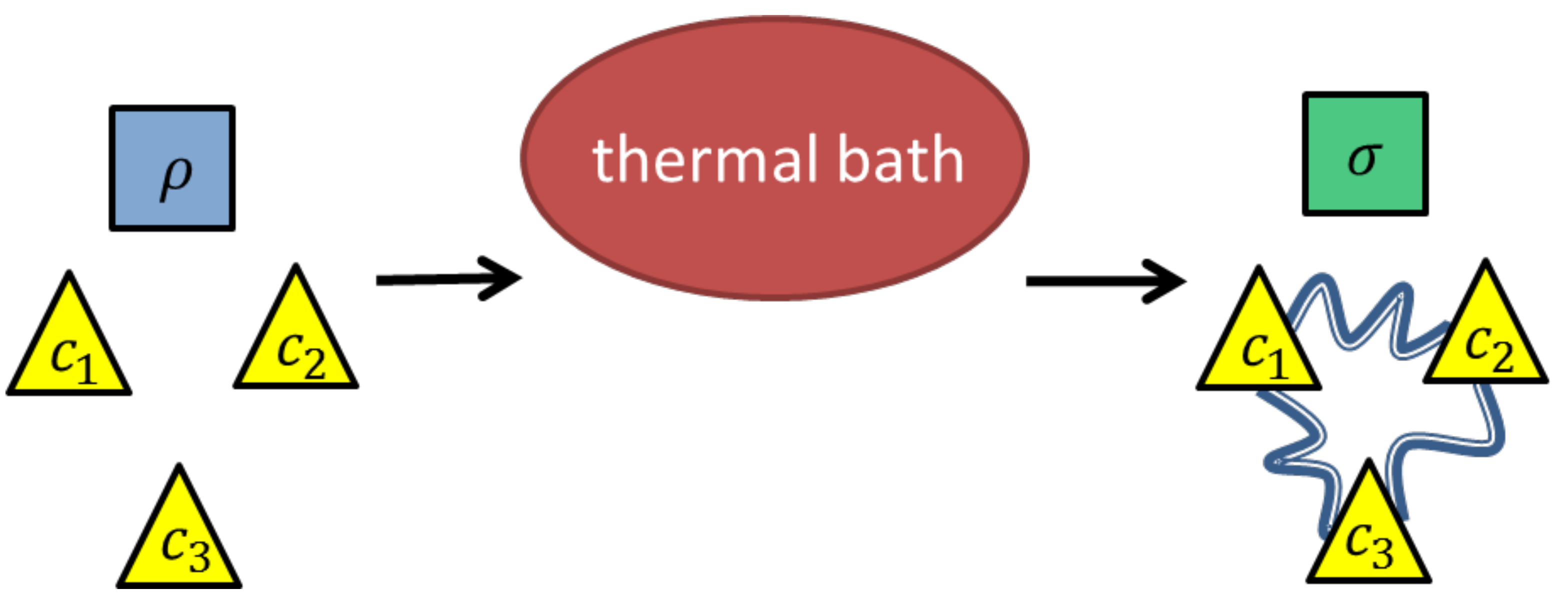

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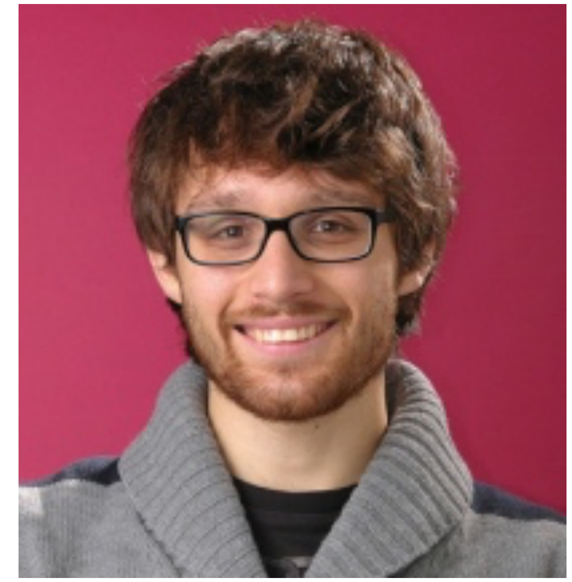

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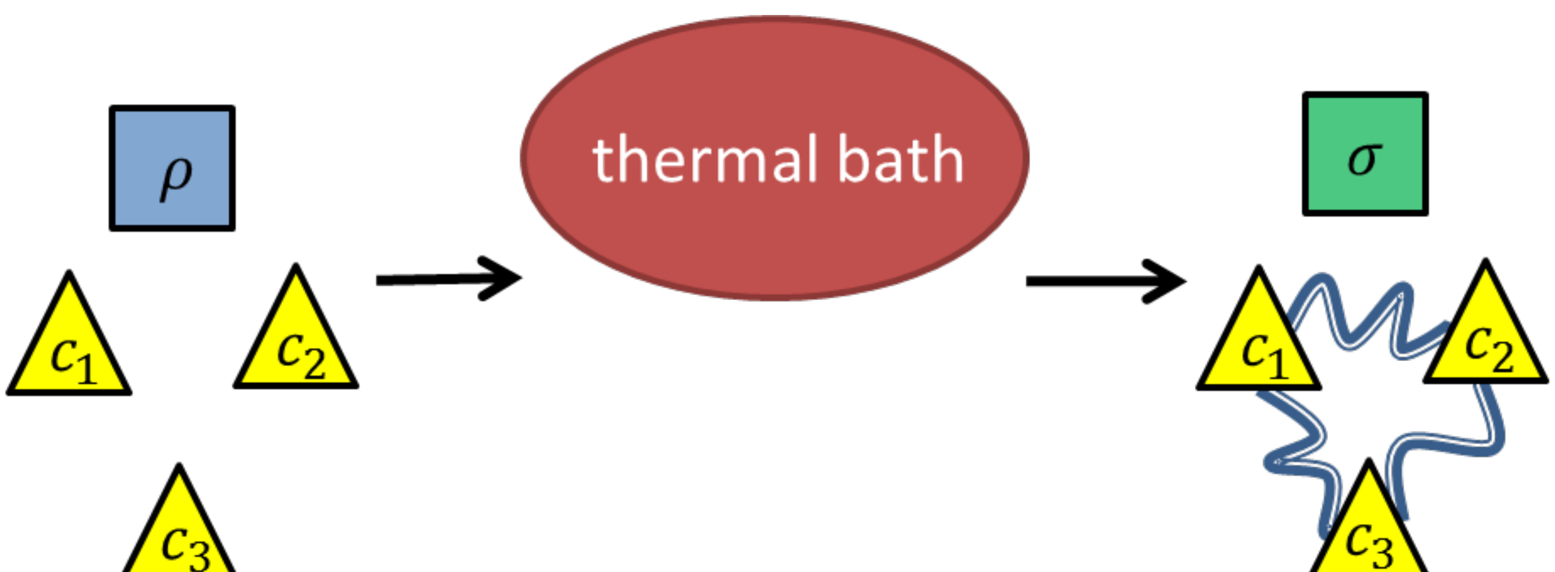

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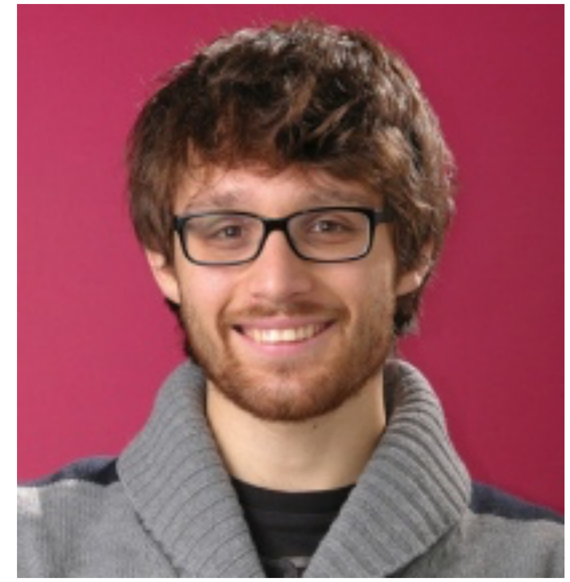

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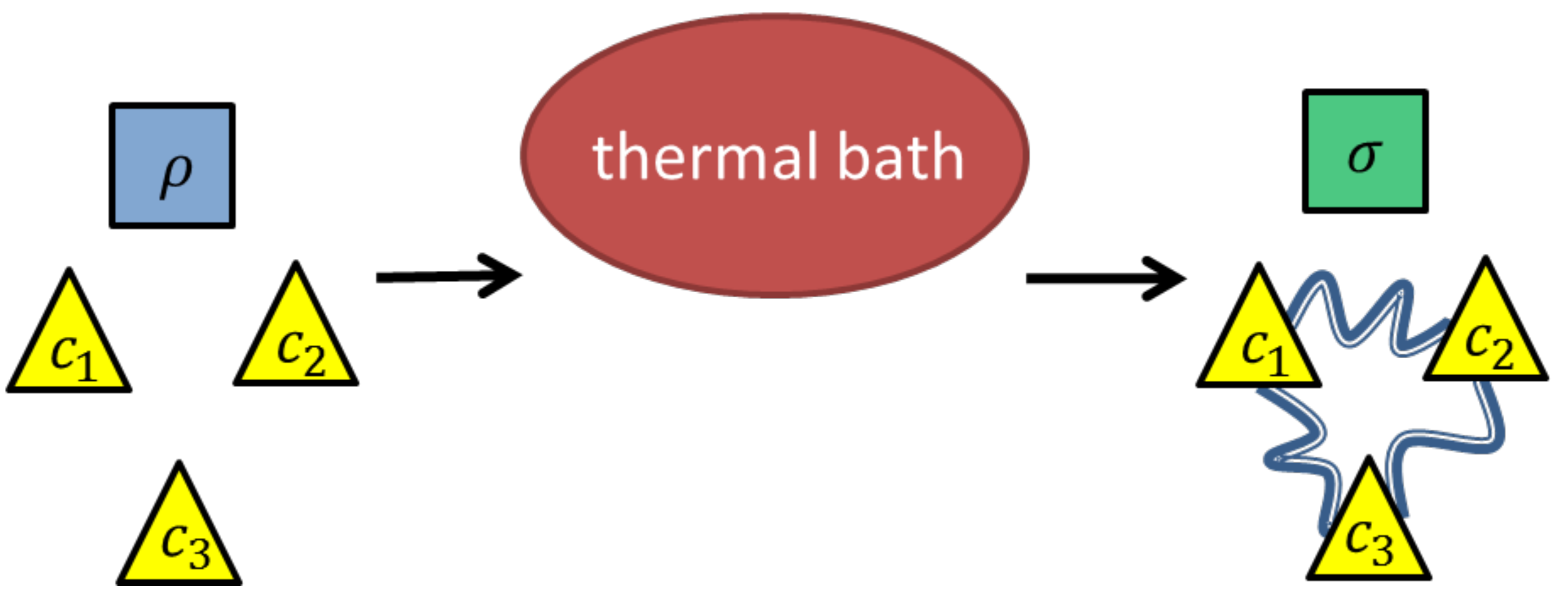

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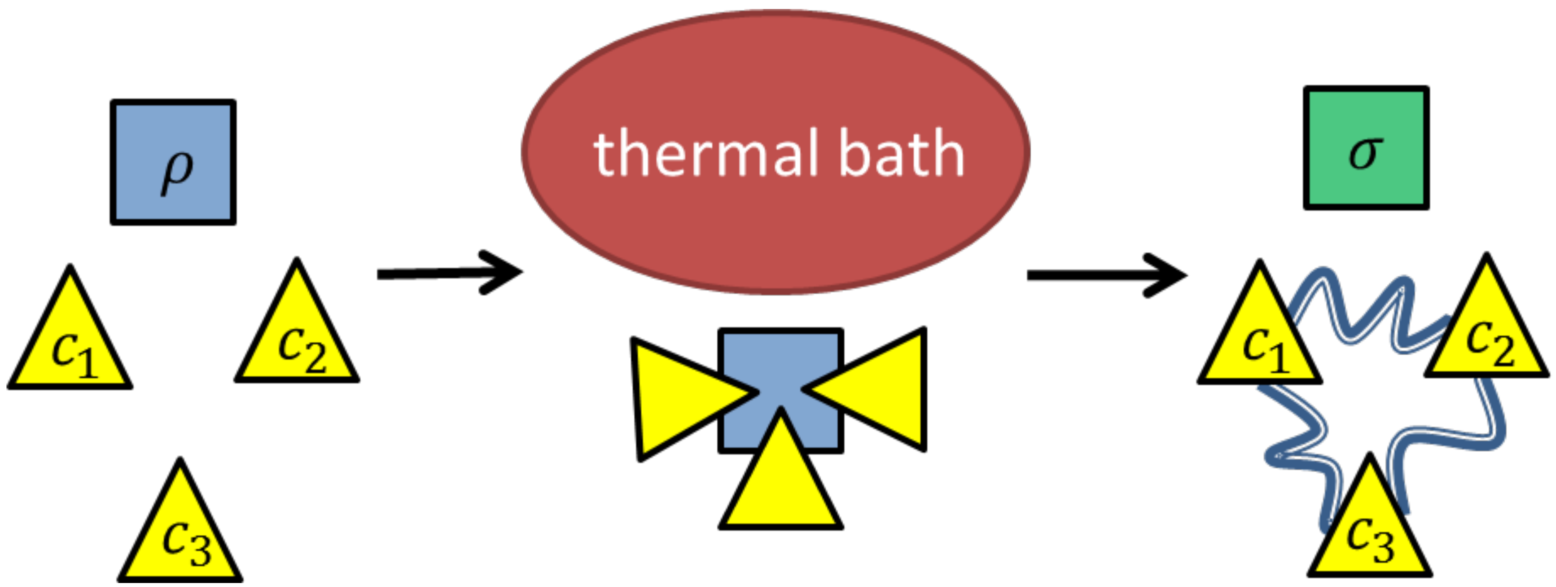

M. Lostaglio, **MM**, and M. Pastena, Phys. Rev. Lett. **115**, 150402 (2015); arXiv:1409.3258

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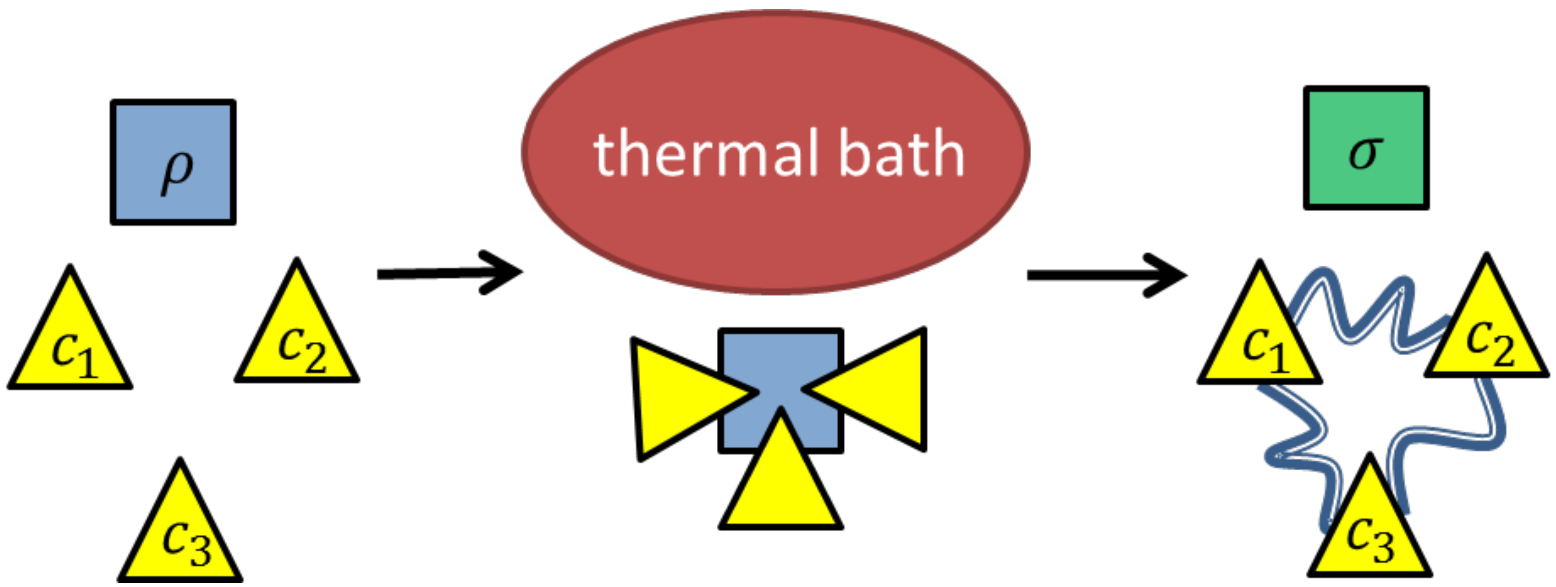

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⇒ Building up correlations can ease thermodynamic transitions.

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Theorem 1.—Consider a system with Hamiltonian $\mathcal{H}_S$ and states ρ and σ block diagonal in energy. The three following statements are equivalent.

1. There exists a thermodynamic process transforming ρ into a state σ_ϵ arbitrarily close to σ, by creating correlations among auxiliary systems, but without changing their local states:

$$\rho \otimes c_1 \otimes \cdots \otimes c_N \to \sigma_\epsilon \otimes c_{1,\ldots,N}. \tag{6}$$

One can always choose $N \leq 3$ and trivial Hamiltonians for the auxiliary systems.

2. There exists $c_1, \ldots, c_N$ and $c_{1,\ldots,N}$ such that anomalous α-entropy production ensures that all $\{F_\alpha\}$ constraints are satisfied in Eq. (6).

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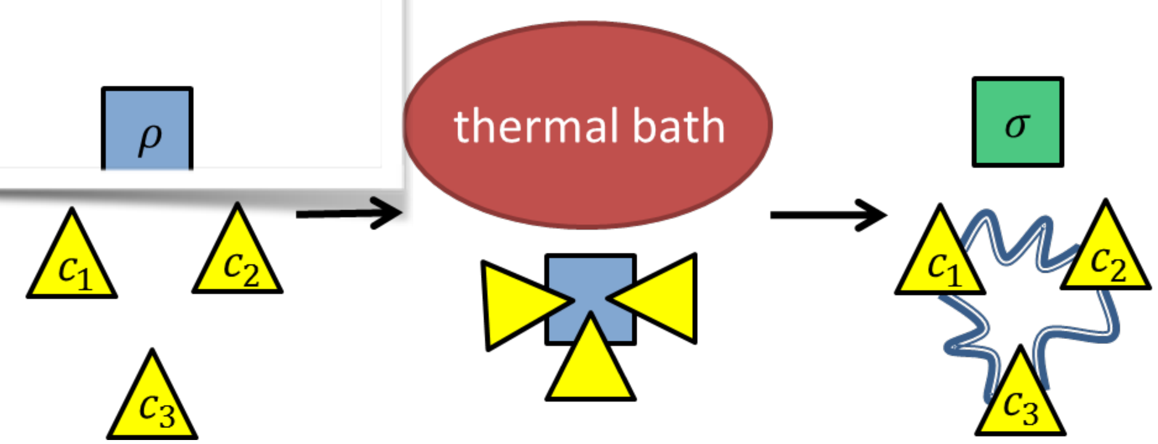

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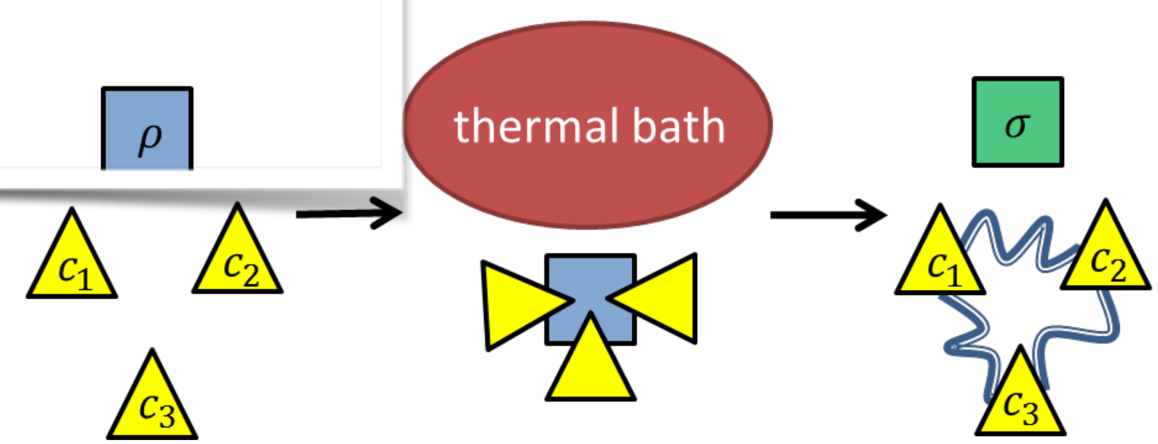

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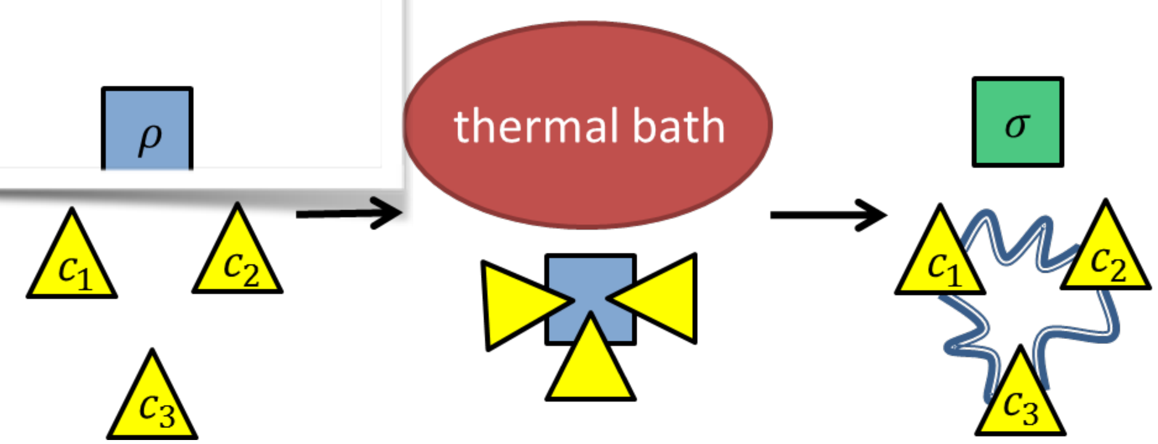
Total correlation

$$\sum_i H(c_i) - H(c_{1,\ldots,N})$$

can be made arbitrarily small.



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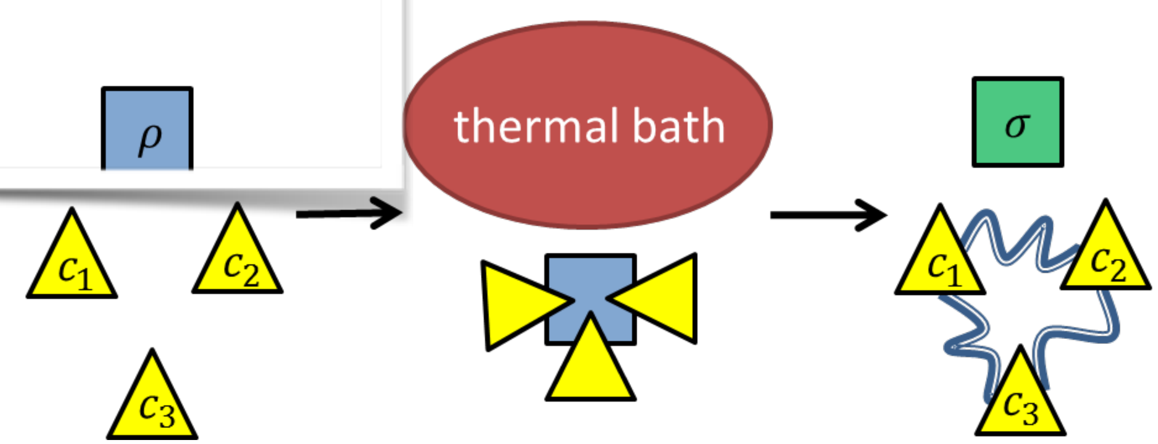
Does nature "really do that"?

Biology?

Natural interactions?



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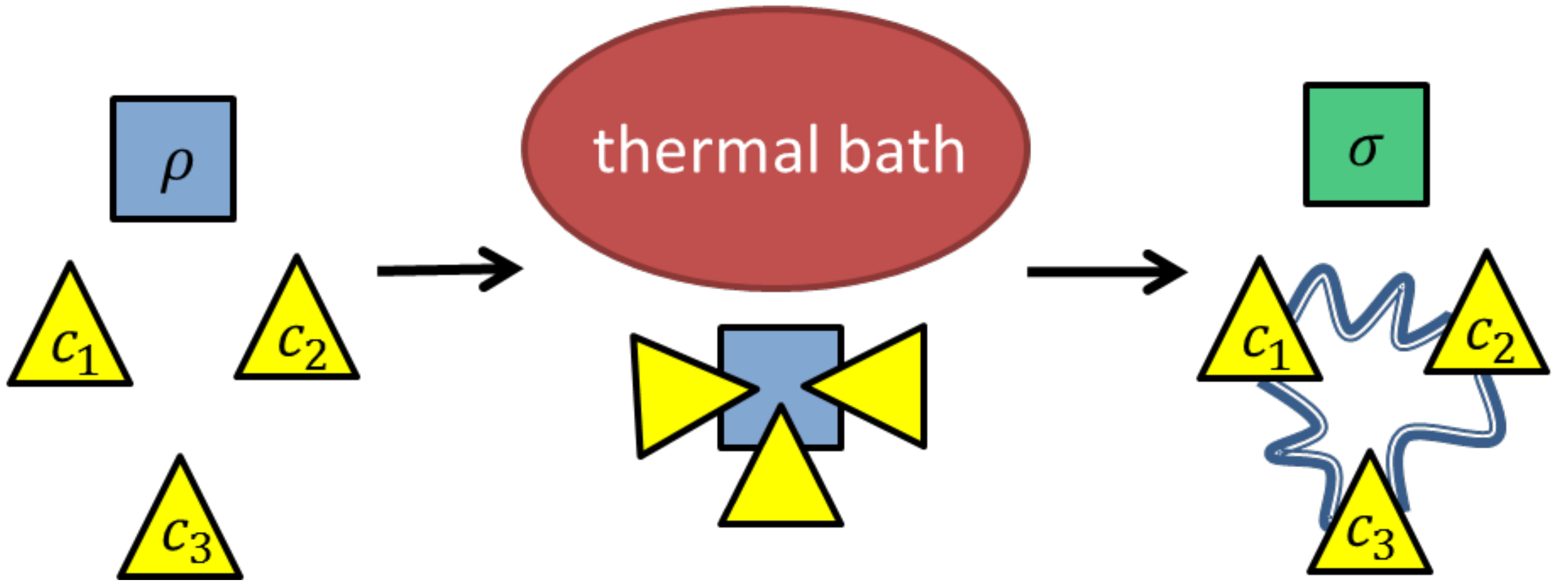

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Western

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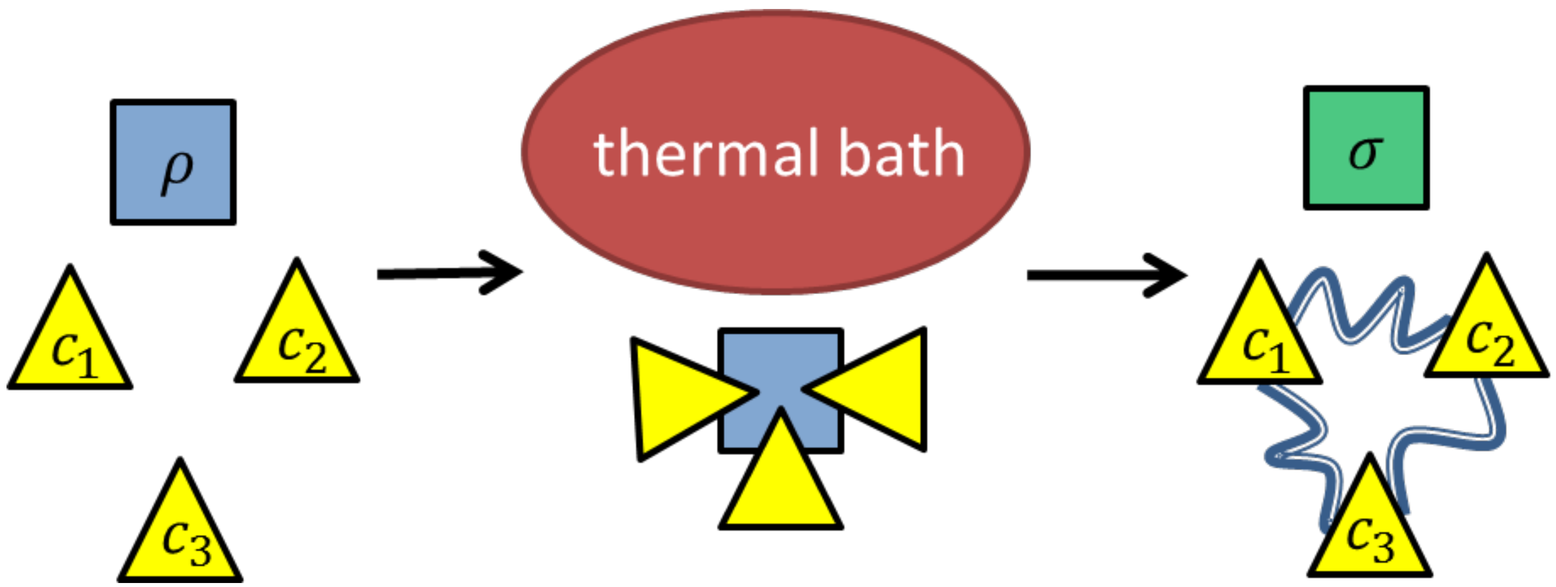

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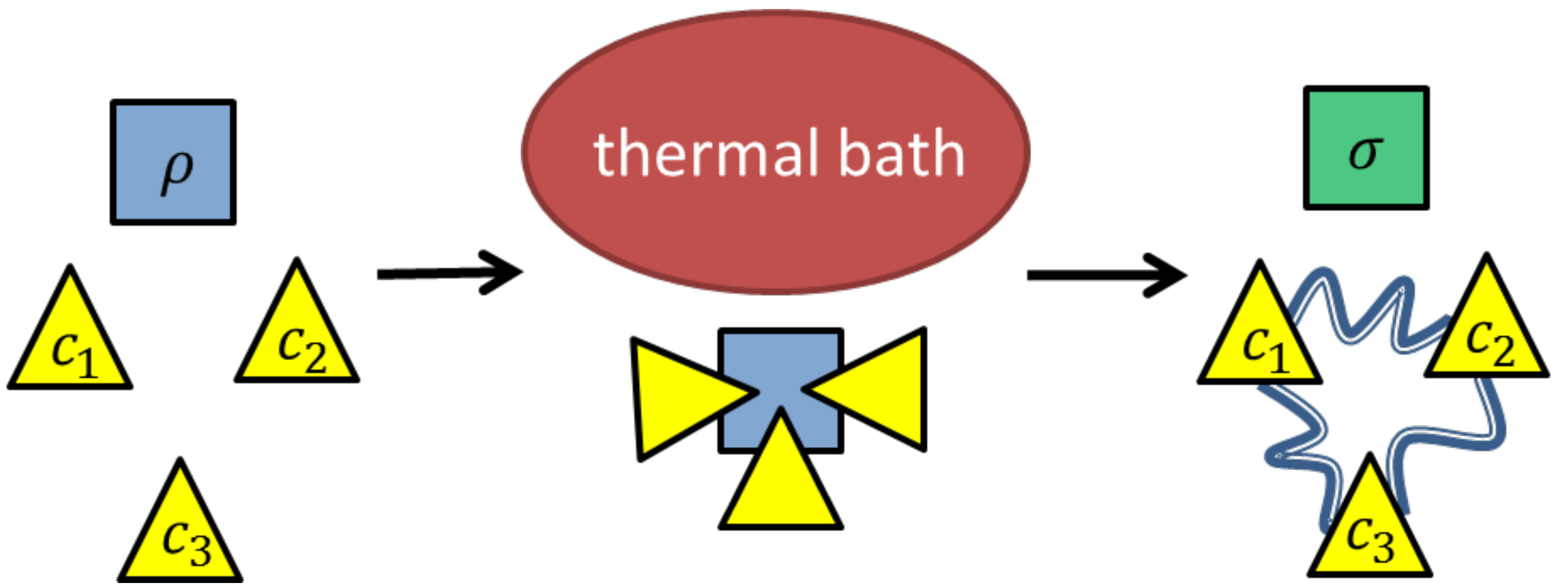

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$\Rightarrow$ Building up correlations can ease thermodynamic transitions.

In fact, it allows fluctuation-free work extraction of $F(\rho) - F(\tau_{\text{thermal}})$ (as in the thermodynamic limit, but on single copies deterministically.)

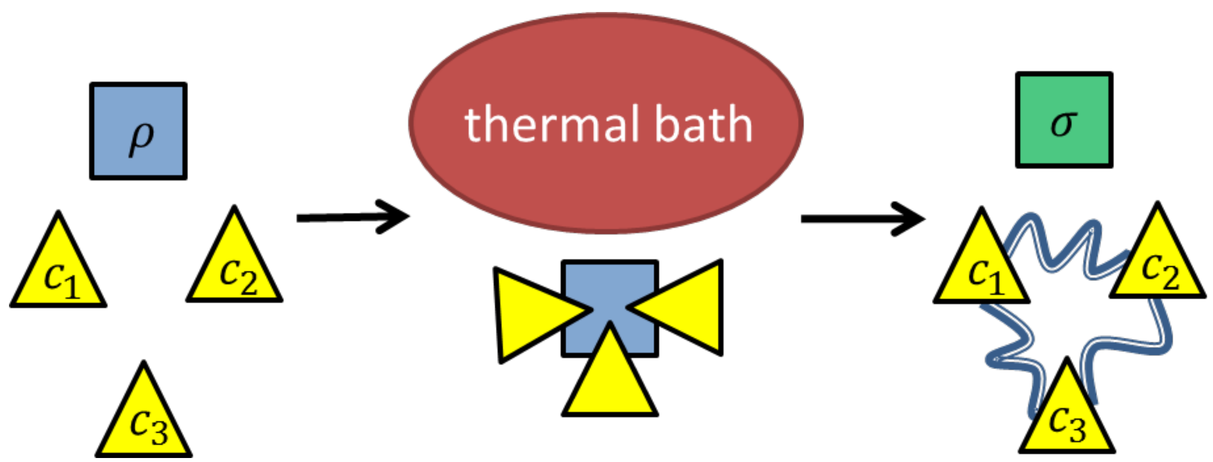
In contrast, no fluctuation-free work extraction at all is possible in the standard setting for full-rank quantum states ρ.

3. Conclusions

- Small-scale thermo: governed by ∞ many "second laws":

$$F_\alpha(\rho) \geq F_\alpha(\sigma)\,\forall\alpha \quad \Leftrightarrow \quad \rho \to \sigma.$$

- By building up correlations, these can be overcome.



$$\Leftrightarrow \quad F(\rho) \geq F(\sigma).$$

- Allows fluctuation-free extraction of work ΔF.
  "Fluctuations are dumped into the environment as correlations."

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Mathematical background: **MM** and M. Pastena, *A generalization of majorization that characterizes Shannon entropy*, arXiv:1507.06900.