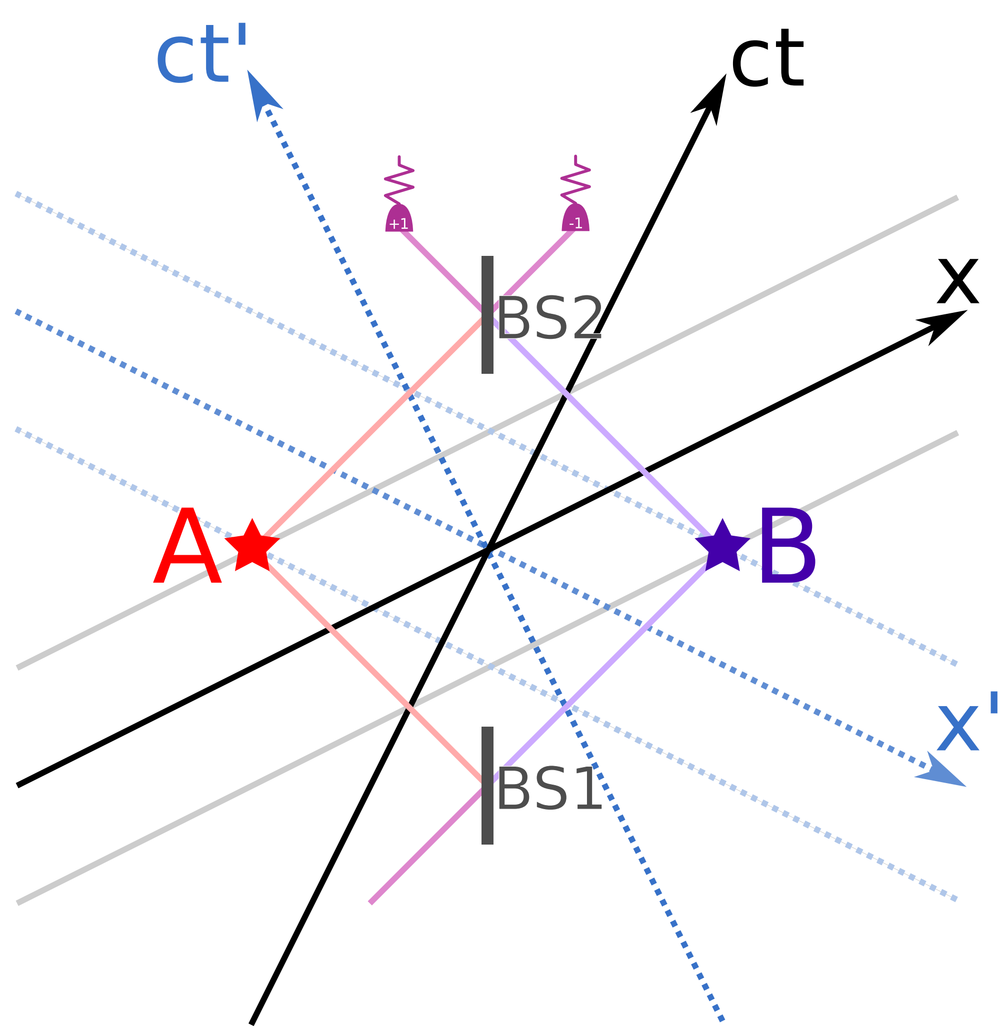
Interference and spacetime:
what GPTs can teach us about physics

Markus P. Müller

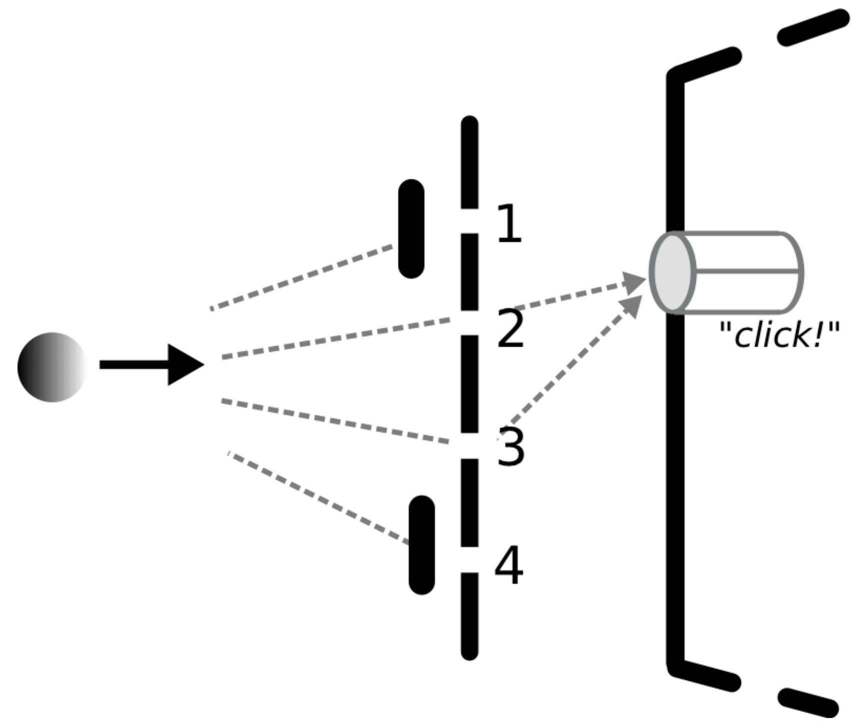
Institute for Theoretical Physics, Heidelberg University (Germany)



Outline

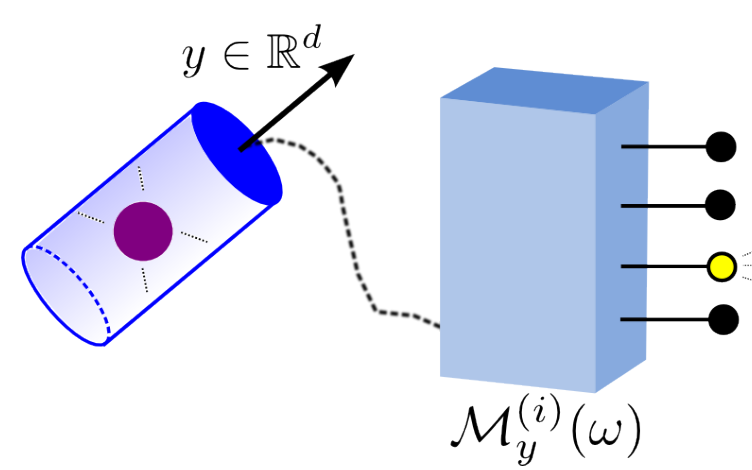
1. Interference

Sorkin's hierarchy
Density tensor theories?
An axiomatic approach



2. GPTs and spacetime

Relativistic constraints on interference experiments
Quantum theory and the dimensionality of space
Reverse-engineering physics



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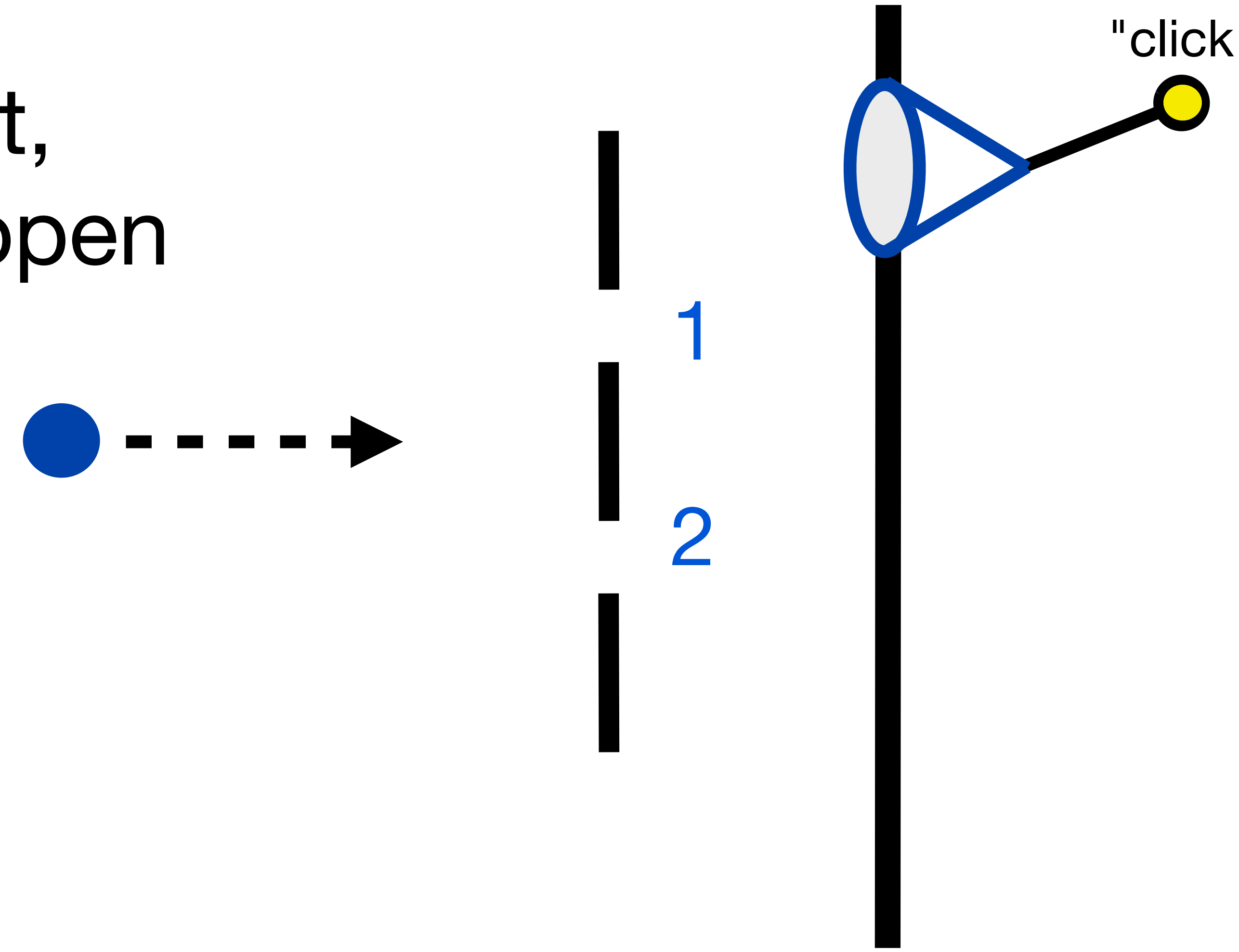
$p_{i,j,\ldots} :=$ probability of event,
if slits $i, j, \ldots$ are open



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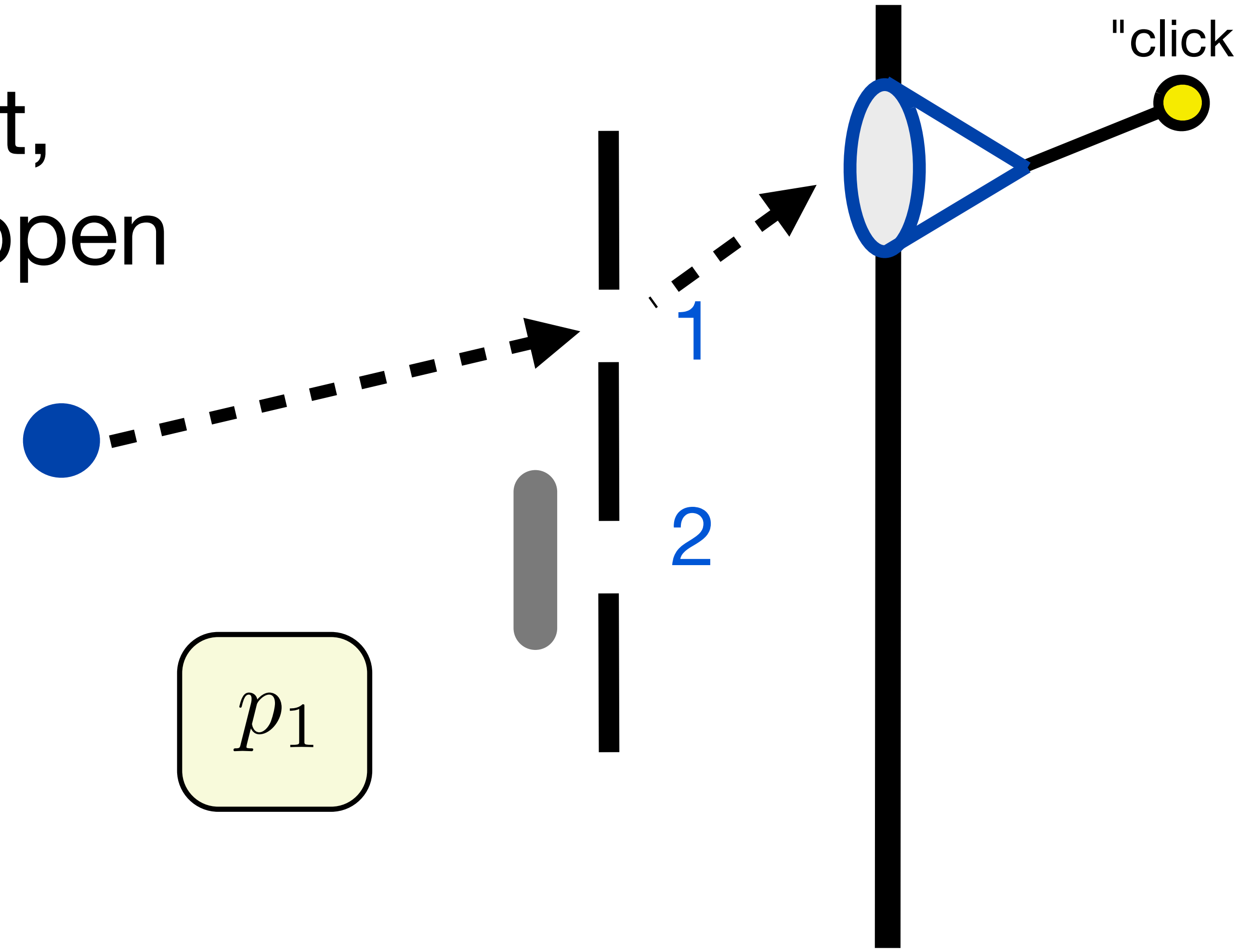

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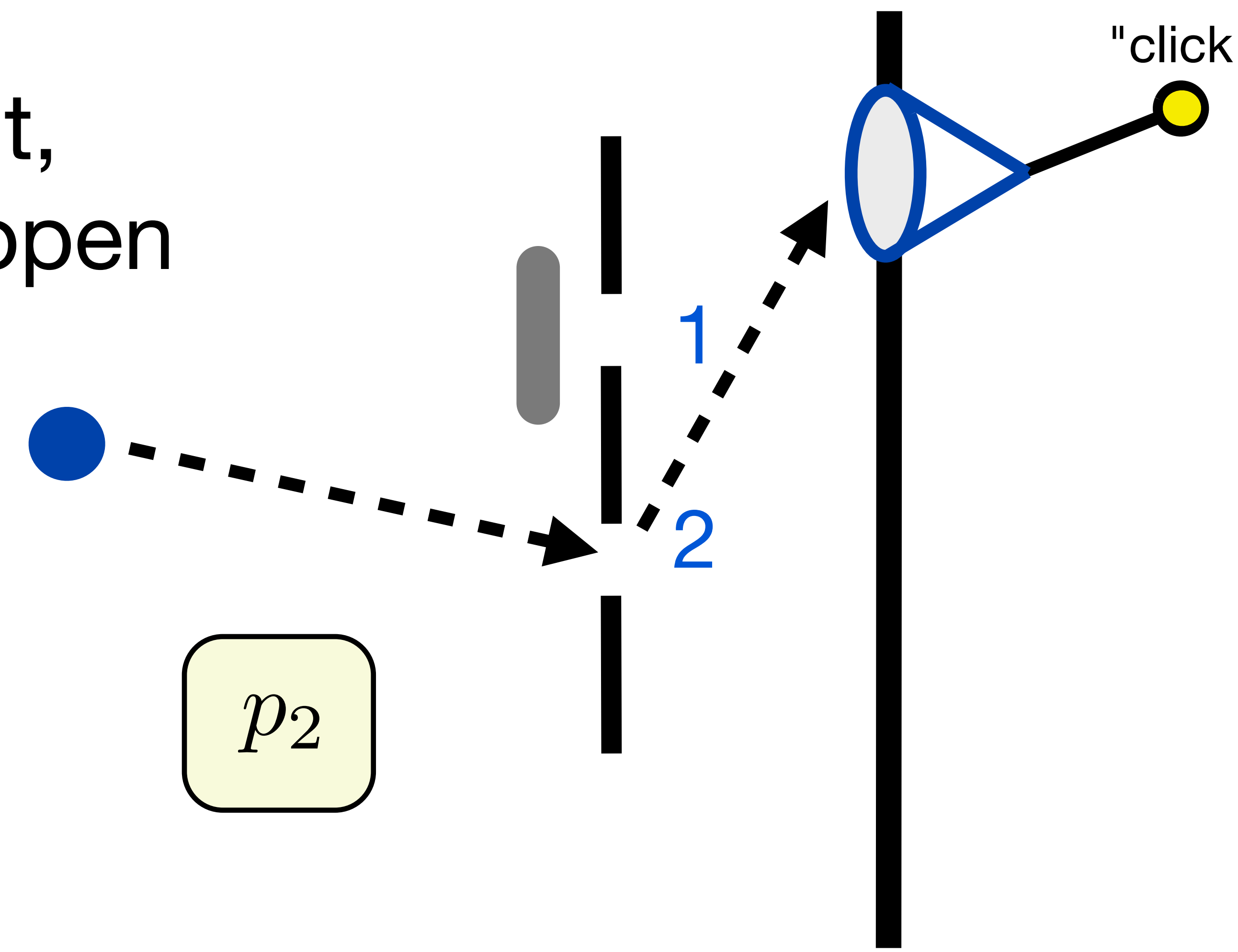

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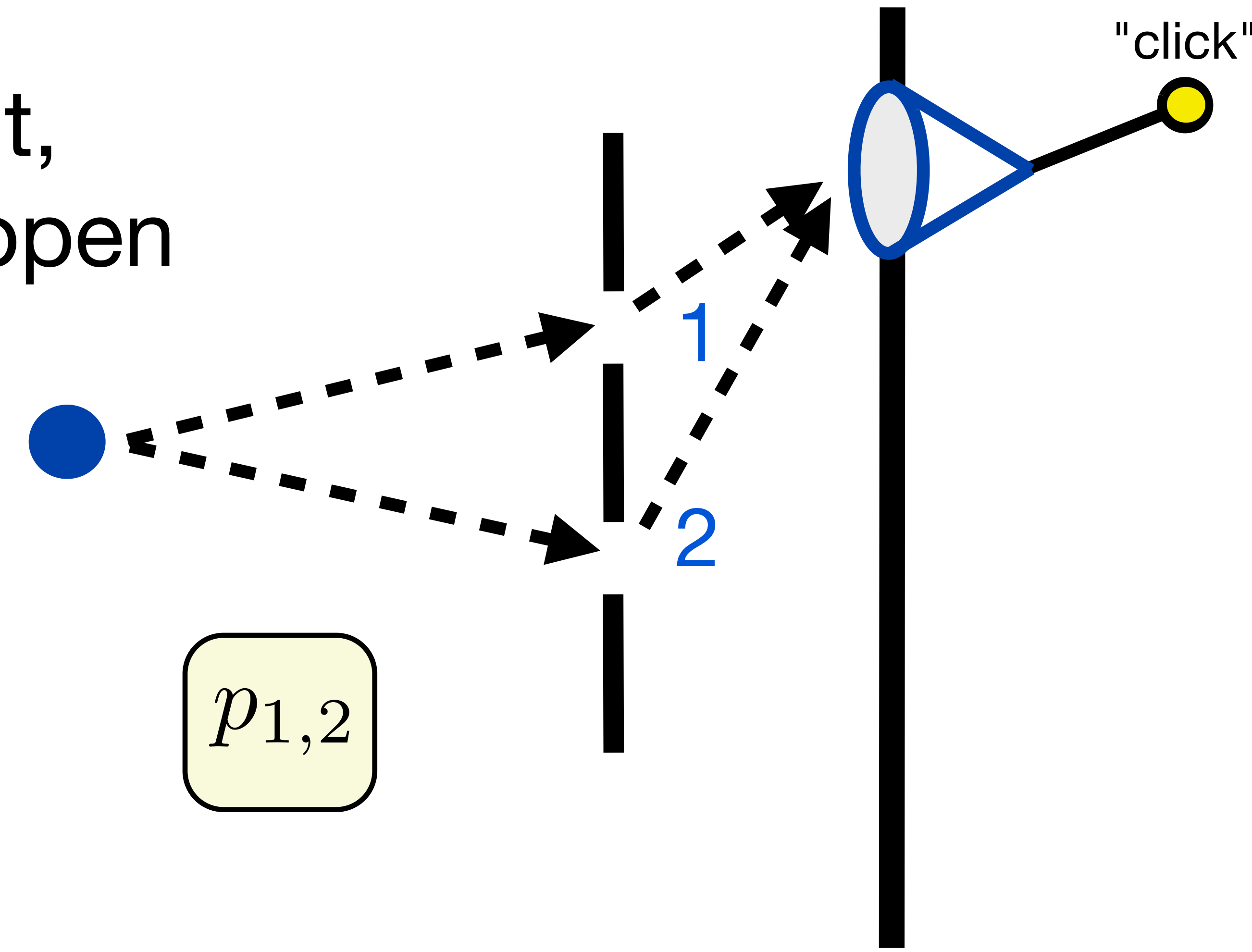

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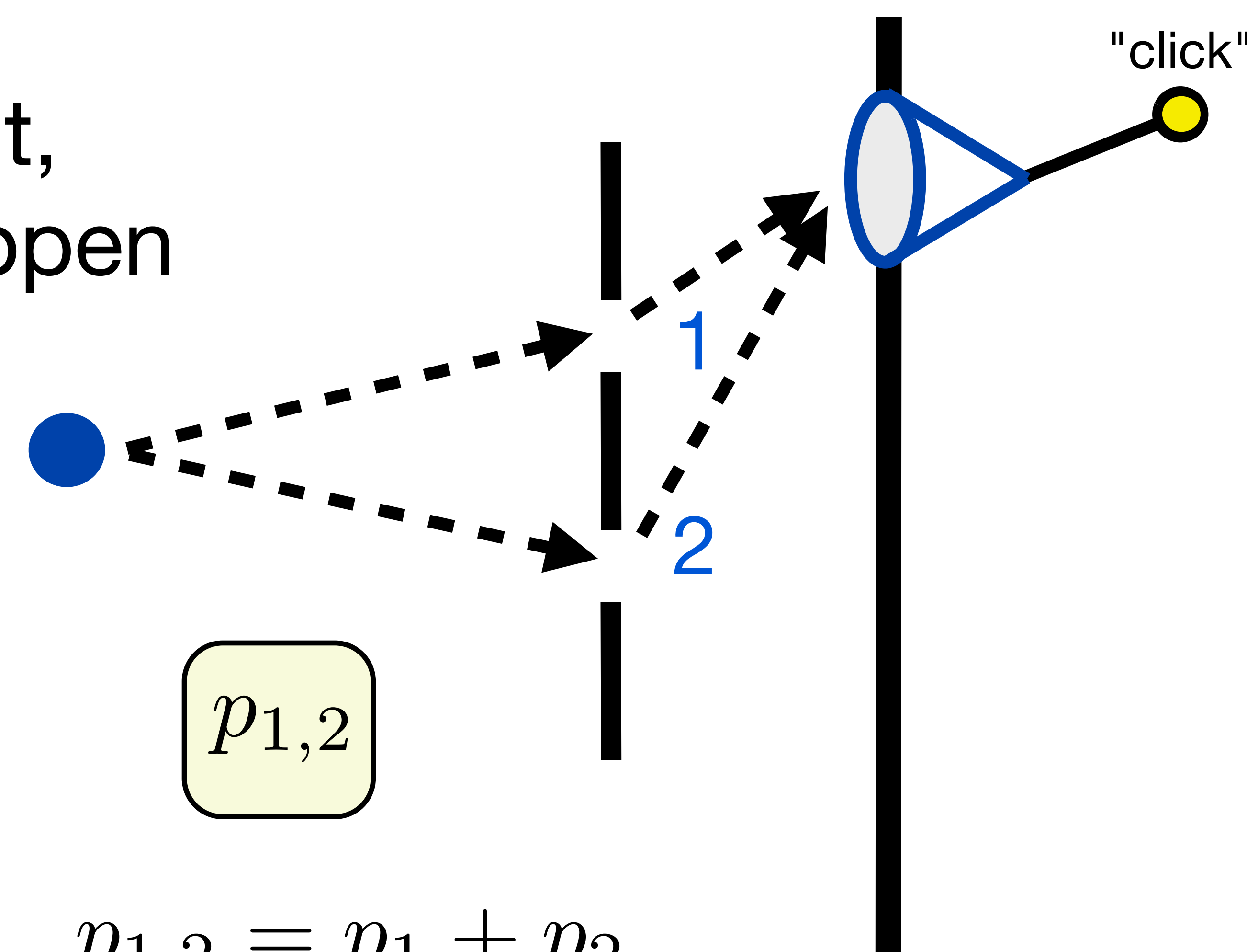

Classical probability theory: $p_{1,2} = p_1 + p_2.$

Quantum theory: $p_{1,2} \neq p_1 + p_2.$

Interference!

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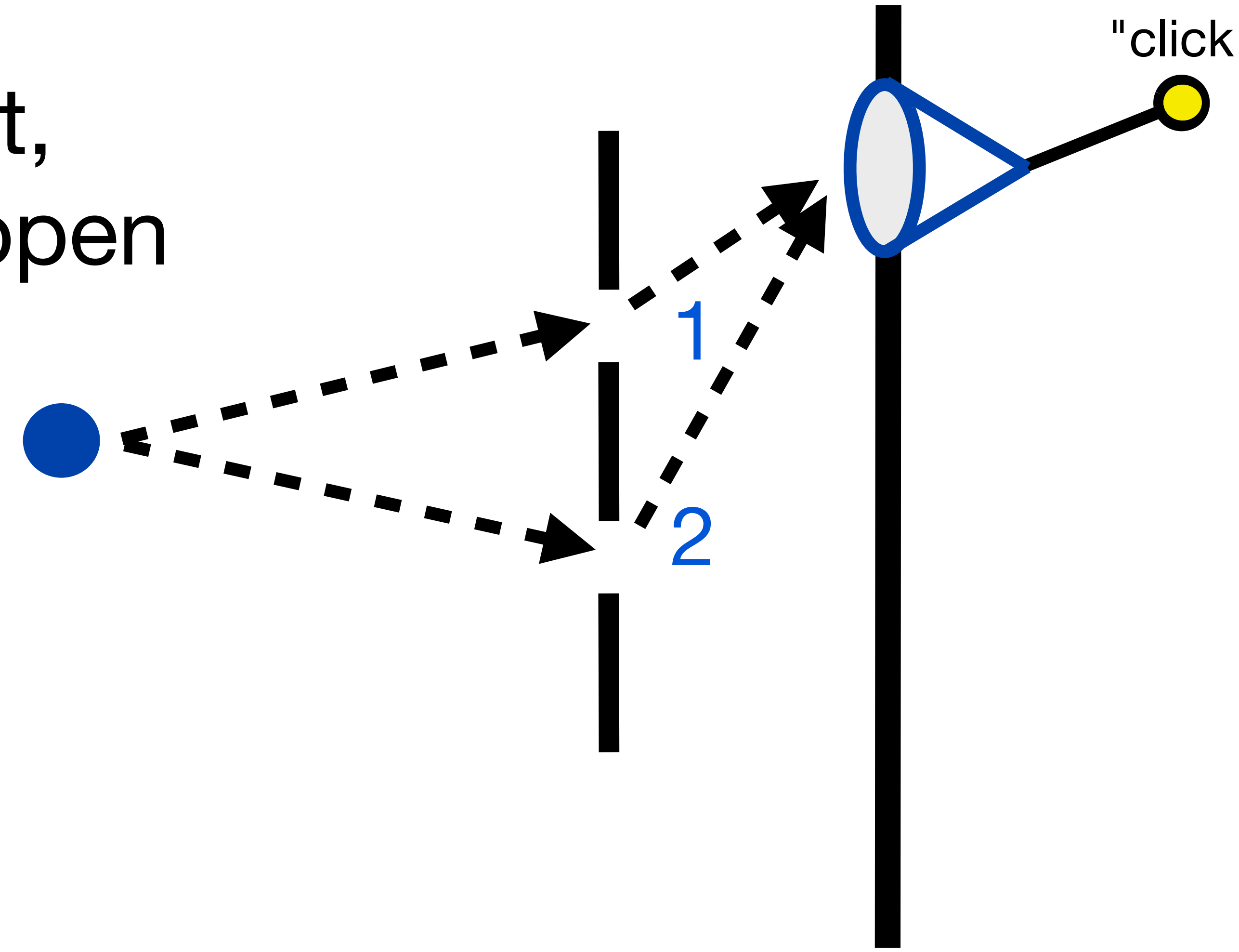

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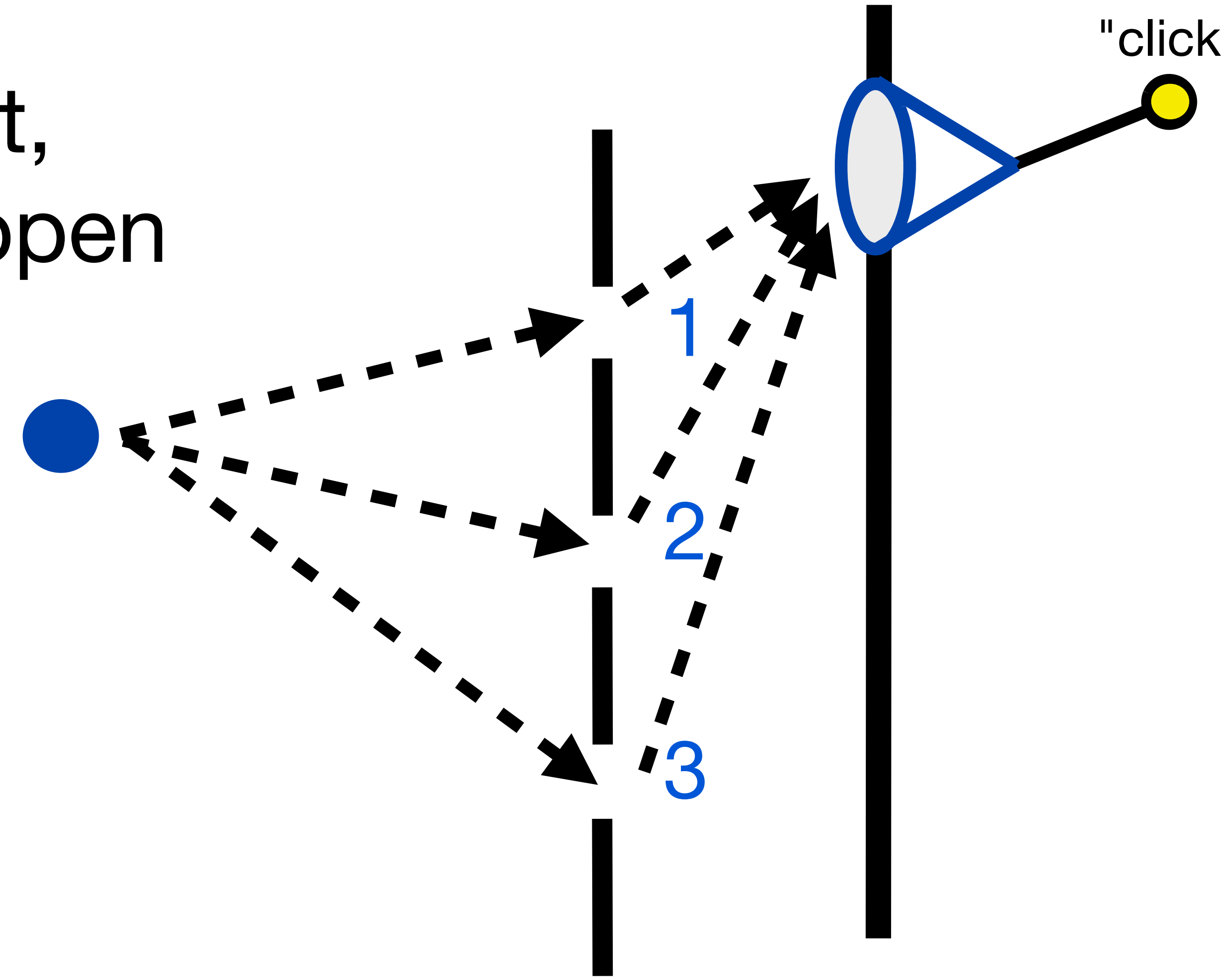

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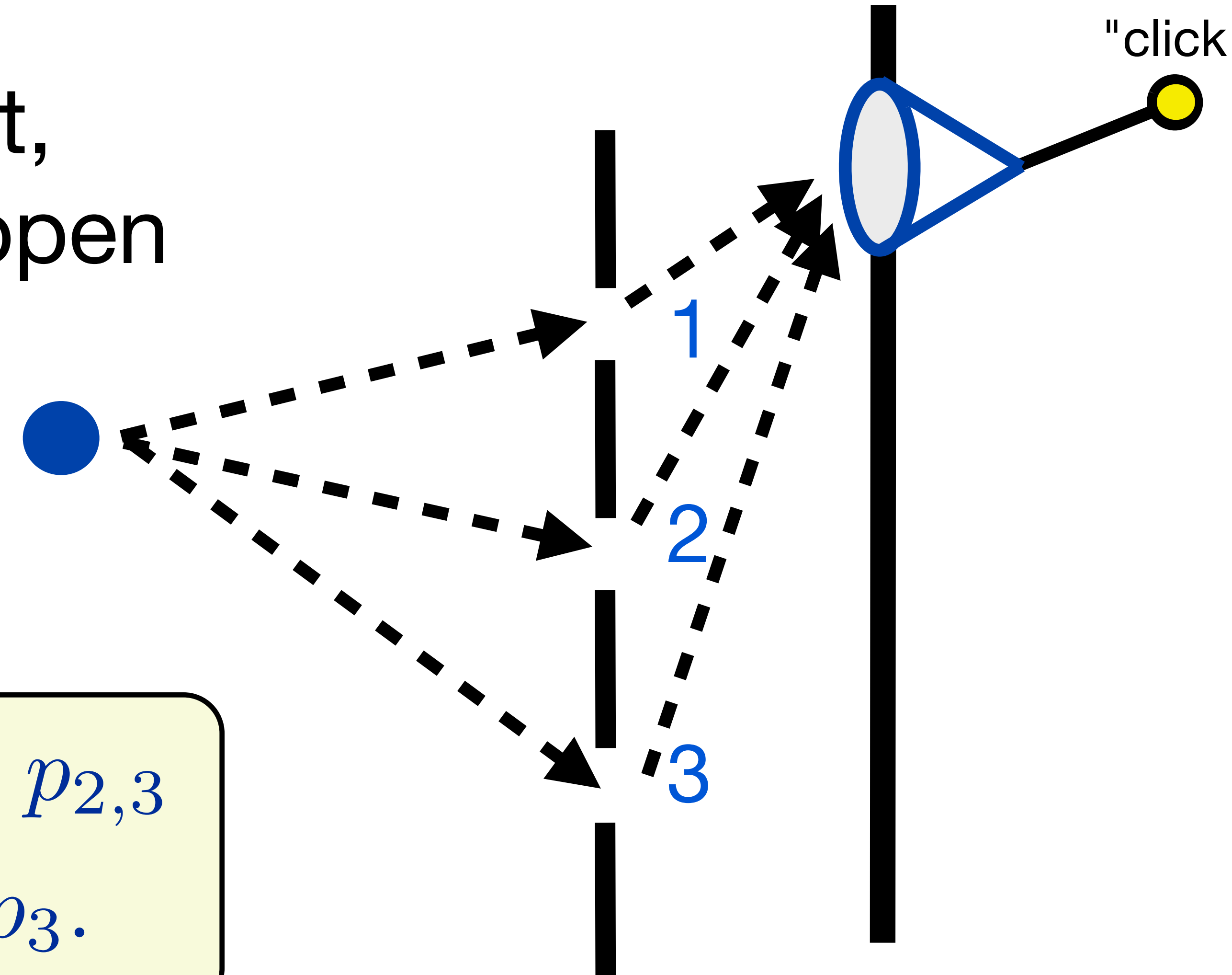

Surprisingly (?), quantum theory satisfies

$$p_{1,2,3} = p_{1,2} + p_{1,3} + p_{2,3} - p_1 - p_2 - p_3.$$

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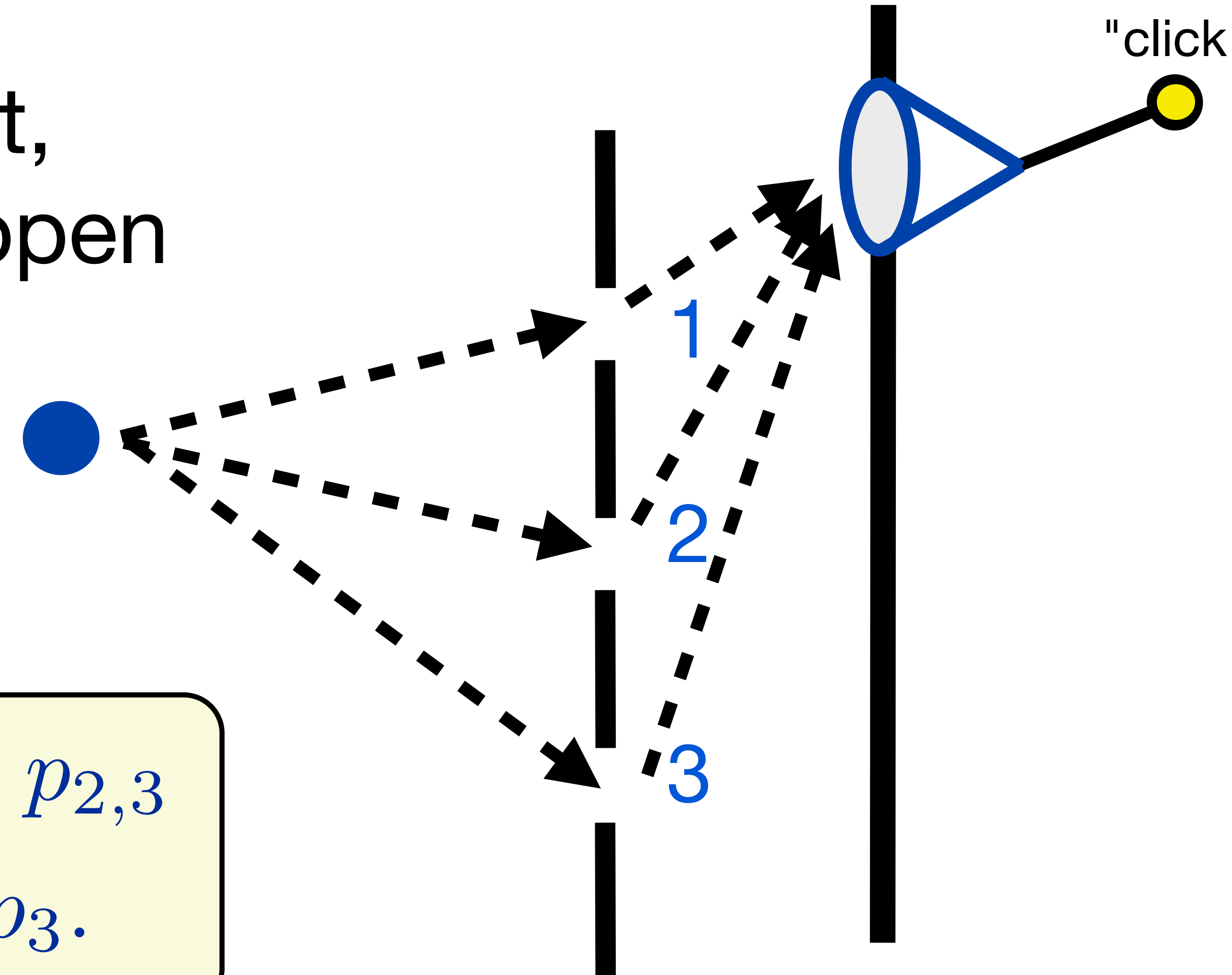

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$$\boxed{\begin{aligned} p_{1,2,3} \quad = \quad & p_{1,2} + p_{1,3} + p_{2,3} \\ & - p_1 - p_2 - p_3. \end{aligned}}$$

No 3rd-order interference in QT!

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Sorkin:

$$I_2(A,B) \equiv |A \amalg B| - |A| - |B|$$

$$I_3(A,B,C) \equiv |A \amalg B \amalg C| - |A \amalg B| - |B \amalg C| - |A \amalg C| + |A| + |B| + |C|$$

or in general,

$$\begin{aligned} I_n(A_1, A_2, \cdots, A_n) \equiv & \, |A_1 \amalg A_2 \amalg \cdots A_n| \\ & - \sum |(n-1) sets| + \sum |(n-2) sets| \cdots \\ & \pm \sum_{j=1}^{n} |A_j| \end{aligned}$$

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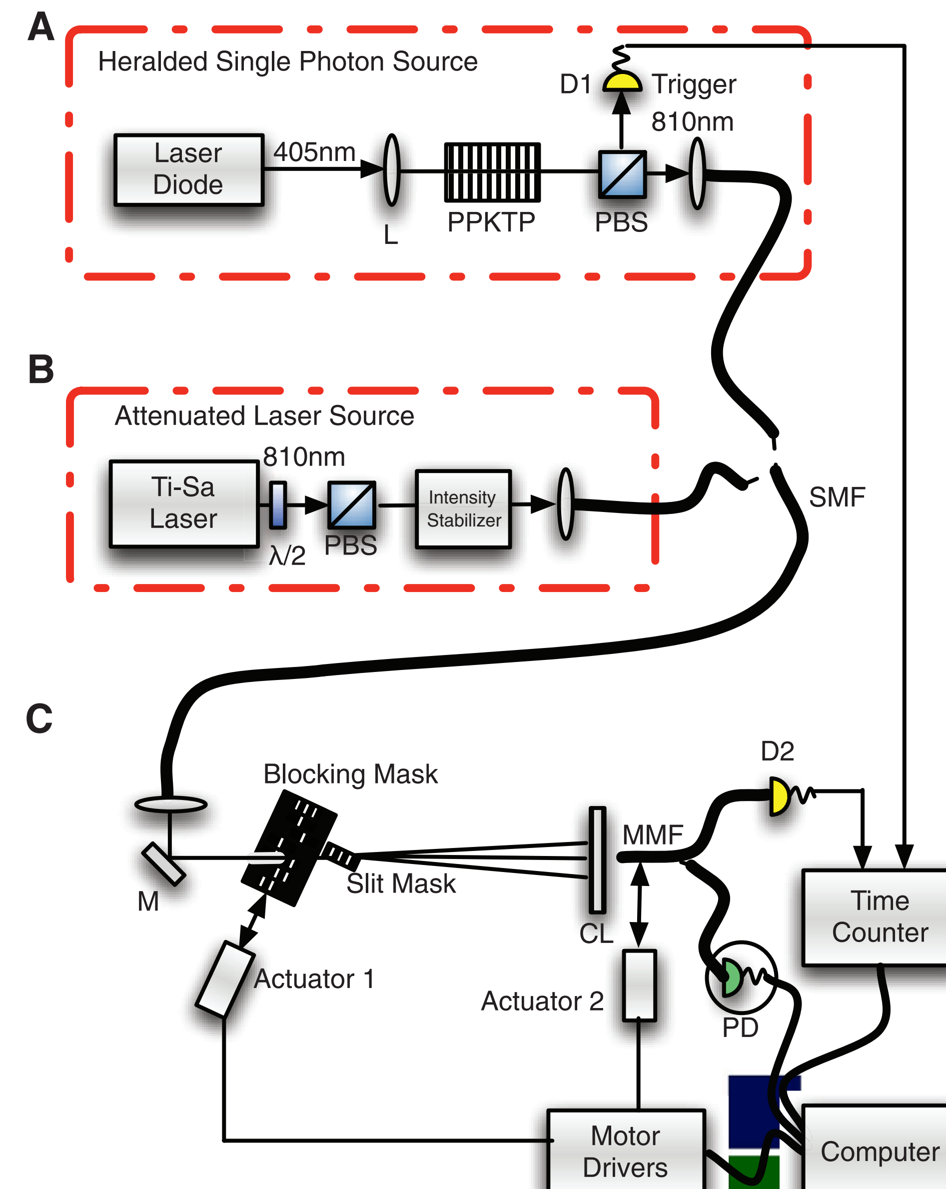
Classical probability theory: $I_2 = I_3 = I_4 = \ldots = 0.$

Quantum theory: $I_2 \neq 0, \;\; I_3 = I_4 = \ldots = 0.$

Experimental tests for higher-order interference

Ruling Out Multi-Order Interference in Quantum Mechanics
Urbasi Sinha *et al.*
Science **329**, 418 (2010);
DOI: 10.1126/science.1190545
(U. Sinha, C. Couteau, T. Jennewein, R. Laflamme, G. Weihs)

$$\varepsilon = I_3 - \text{zerocount};$$
$$\boxed{\kappa := \frac{\varepsilon}{\delta};}$$
$$\delta = |I_{12}| + |I_{13}| + |I_{23}|,$$
$$I_{12} = p_{12} - p_1 - p_2 \quad \text{etc.}$$



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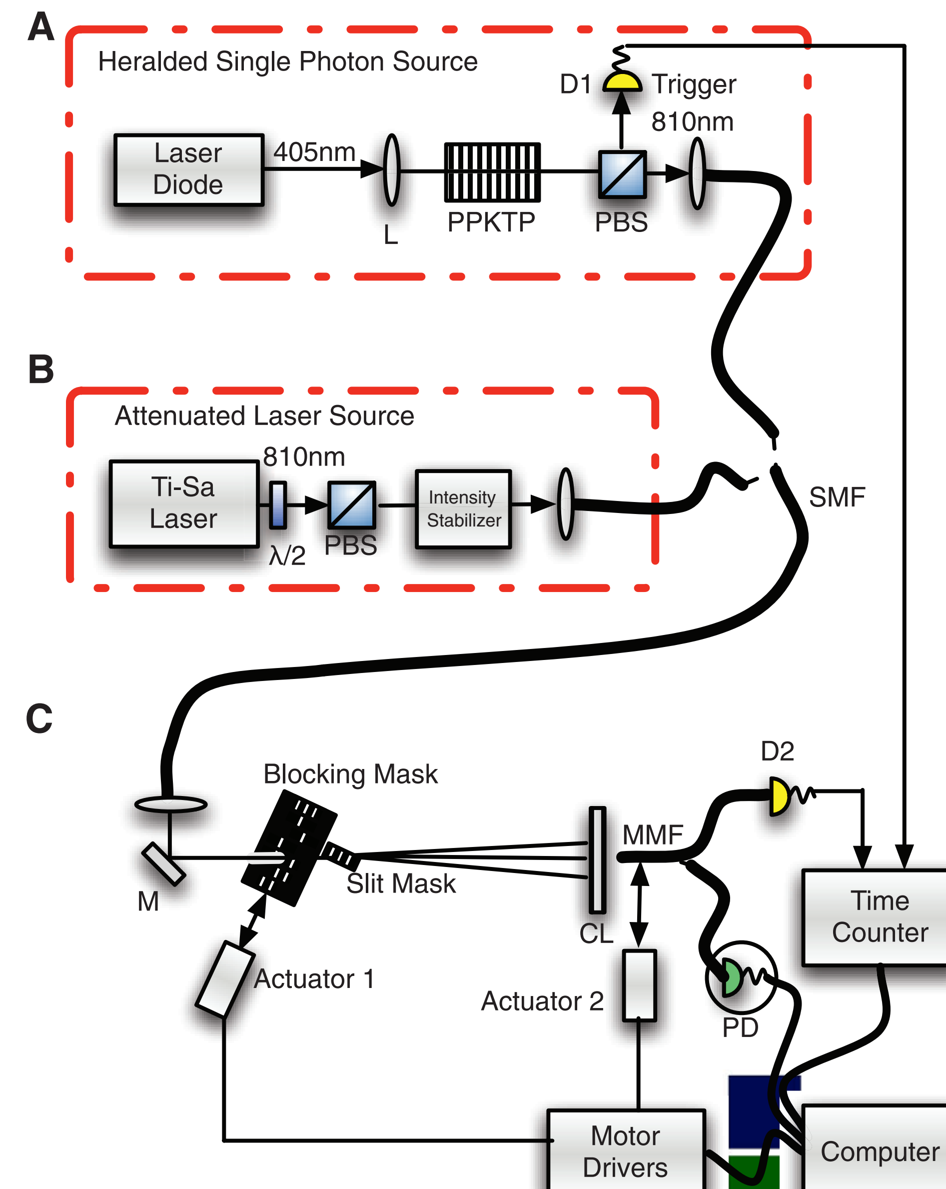
$$\begin{aligned} \varepsilon &= I_3 - \text{zerocount}; \\ \boxed{\kappa := \frac{\varepsilon}{\delta};} \\ \delta &= |I_{12}| + |I_{13}| + |I_{23}|, \\ & \quad I_{12} = p_{12} - p_1 - p_2 \quad \text{etc.} \end{aligned}$$



Result: $\boxed{\kappa \leq 10^{-2}.}$

Experimental tests for higher-order interference

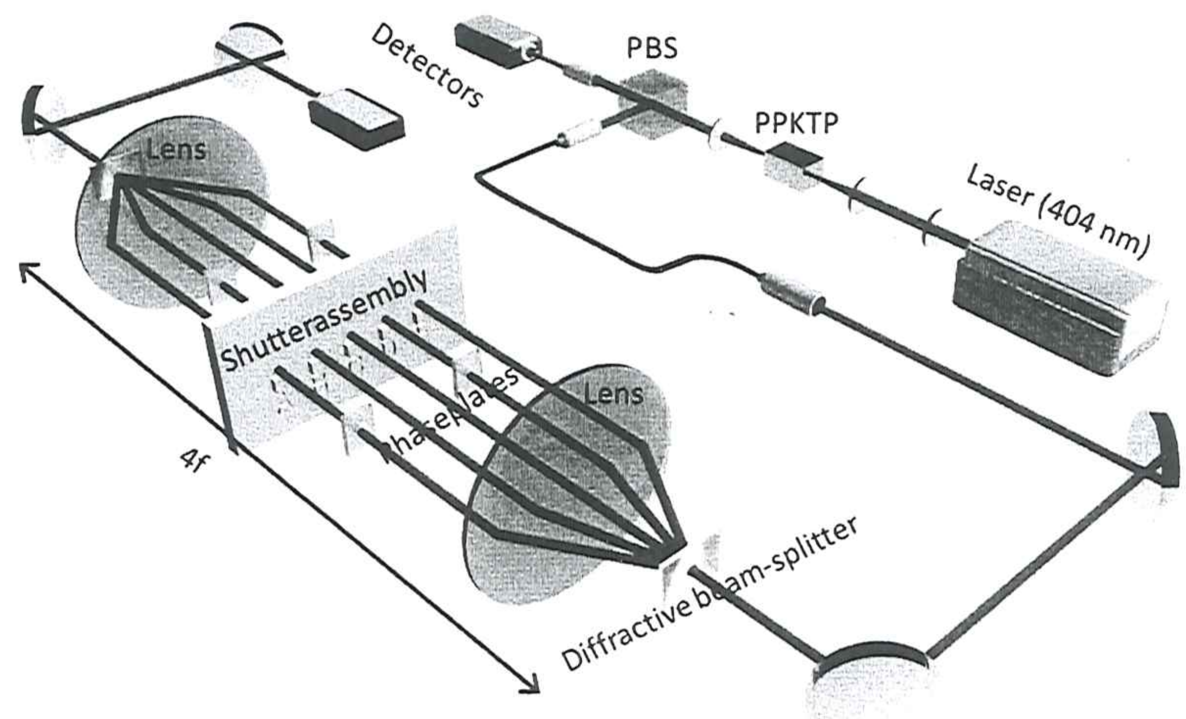
Presentation in Bad Honnef 2014:

Measuring higher-order interferences with a five-path interferometer

T. Kauten[1], T. Kaufmann[1], B. Pressl[1] and G. Weihs[1]

[1] *Institut für Experimentalphysik, Universität Innsbruck, Technikerstraße 25, 6020 Innsbruck, Austria*

E-mail: thomas.kauten@uibk.ac.at



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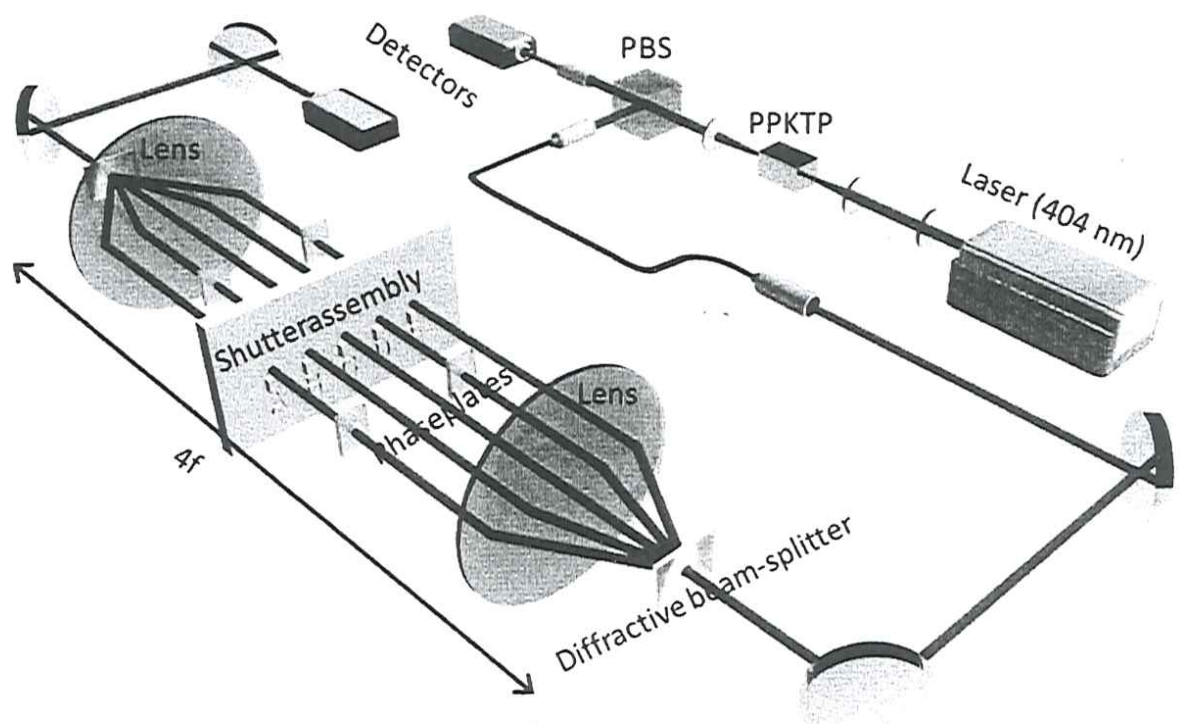

Result: $\boxed{\kappa \leq 10^{-4}.}$

Why does QT not have 3rd-order interference?

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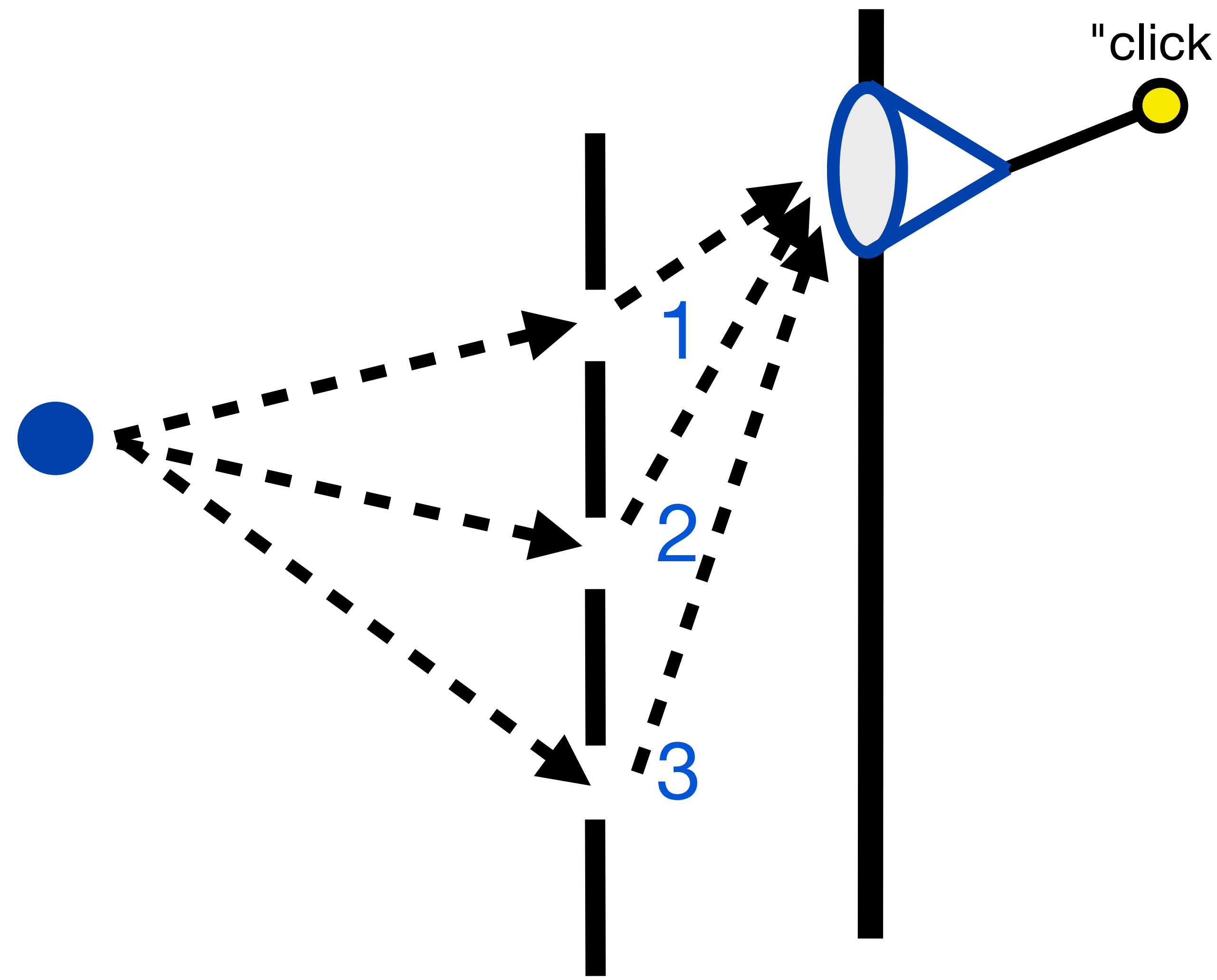
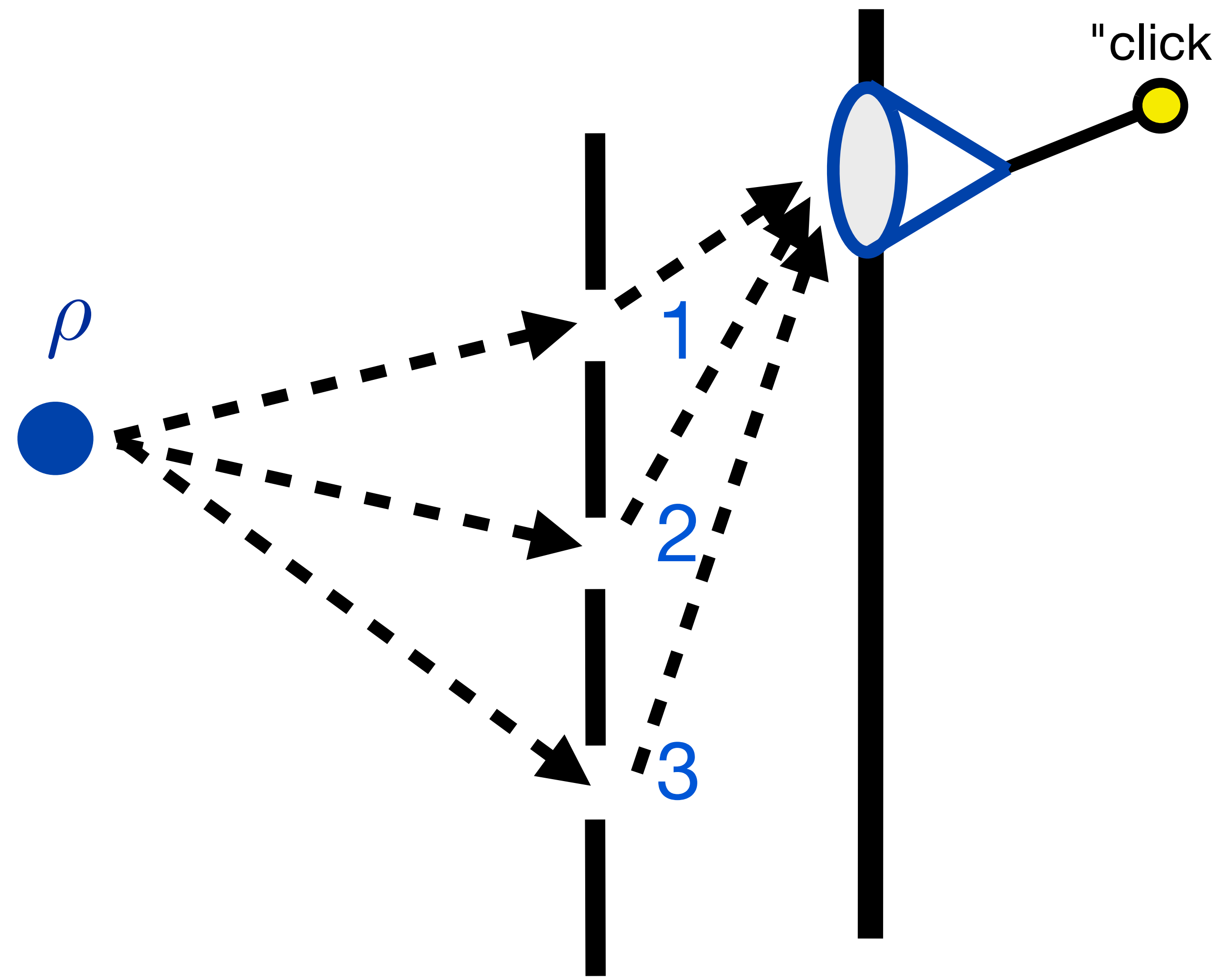


Why does QT not have 3rd-order interference?

$$\rho = \begin{pmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{pmatrix}$$



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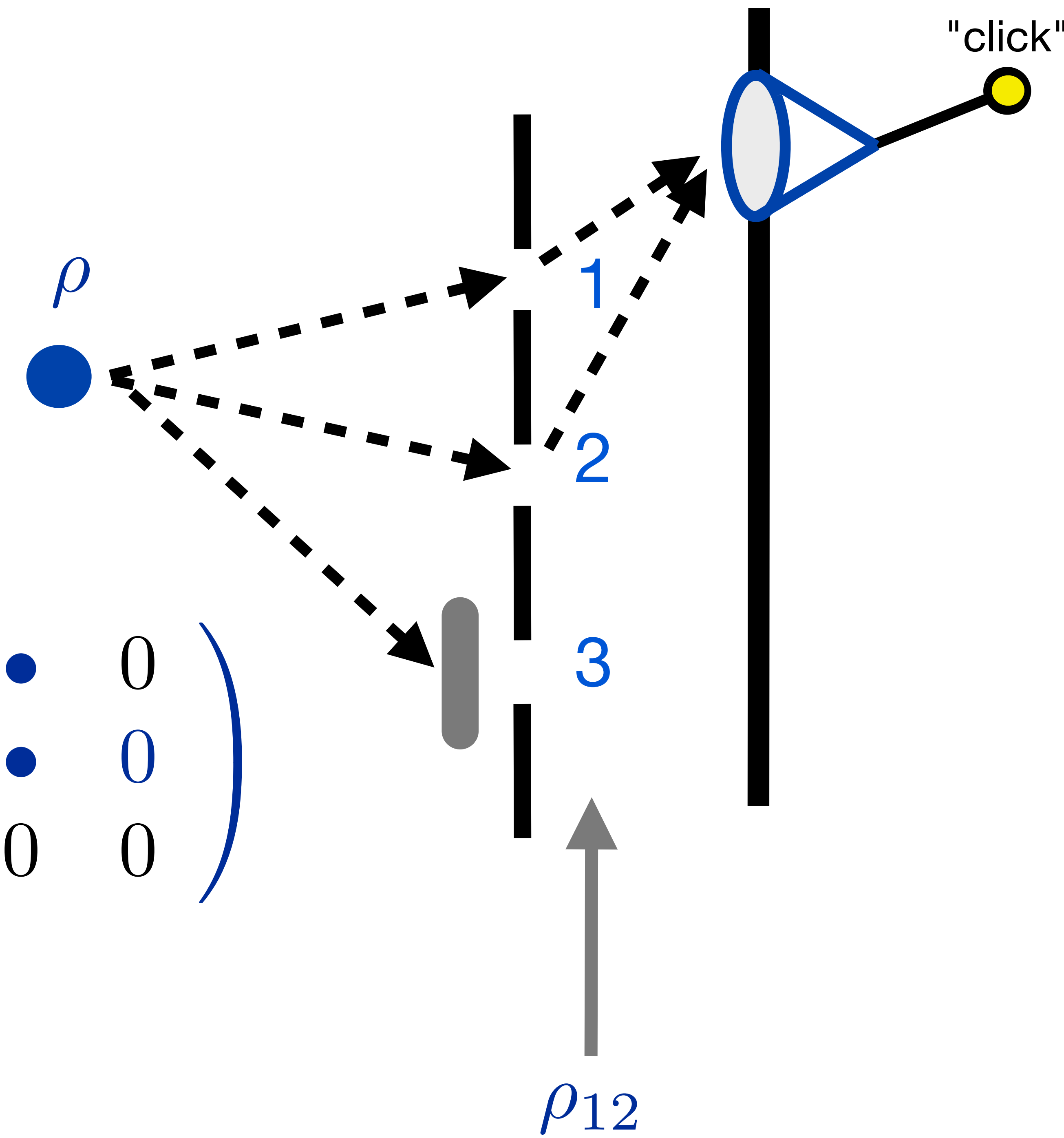

$$\rho \mapsto P_{12}\rho P_{12} =: \rho_{12} = \begin{pmatrix} \bullet & \bullet & 0 \\ \bullet & \bullet & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Why does QT not have 3rd-order interference?

$$\begin{pmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{pmatrix} = \begin{pmatrix} \bullet & \bullet & 0 \\ \bullet & \bullet & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} \bullet & 0 & \bullet \\ 0 & 0 & 0 \\ \bullet & 0 & \bullet \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & \bullet & \bullet \\ 0 & \bullet & \bullet \end{pmatrix}$$

$$- \begin{pmatrix} \bullet & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & \bullet & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \bullet \end{pmatrix}$$

$$\begin{aligned} p_{1,2,3} \quad &= \quad p_{1,2} + p_{1,3} + p_{2,3} \\ & \quad\quad -p_1 - p_2 - p_3. \end{aligned}$$

Why does CPT not have 2nd-order interference?

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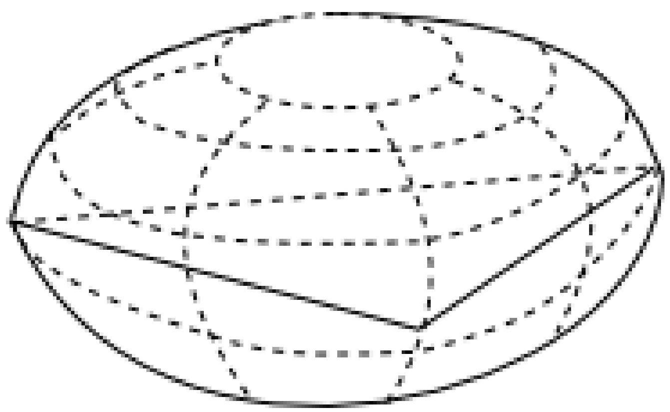
$$\begin{pmatrix} \bullet \\ \bullet \\ \bullet \end{pmatrix} = \begin{pmatrix} \bullet \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \bullet \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \bullet \end{pmatrix}$$

$$p_{1,2,3} = p_1 + p_2 + p_3.$$

Which natural GPTs have 3rd-order interference?

Some "artificial" GPTs exhibit order-3 interference:



C. Ududec, *Perspectives on the Formalism of Quantum Theory*, PhD thesis, University of Waterloo, 2012.

But what natural generalizations of QT could we test for in experiments?

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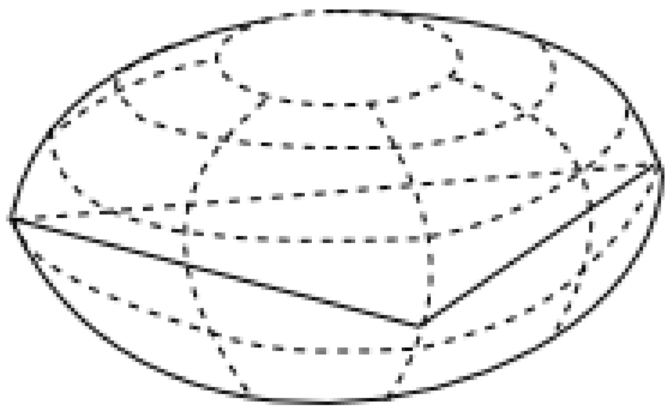

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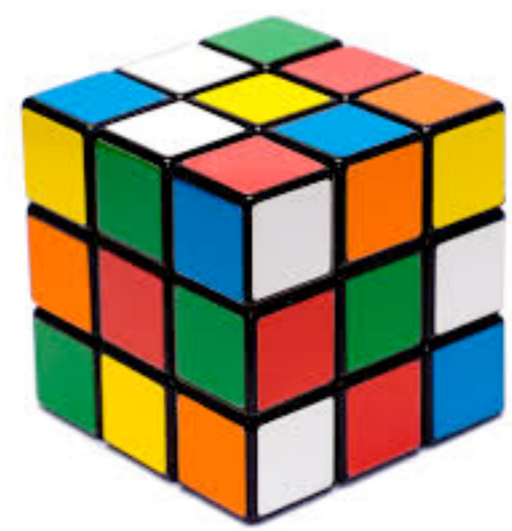
But what natural generalizations of QT could we test for in experiments?

$$\begin{pmatrix}\bullet\\ \bullet\\ \bullet\end{pmatrix}$$

"1st-order" (trivial) interference

$$\begin{pmatrix}\bullet & \bullet & \bullet\\ \bullet & \bullet & \bullet\\ \bullet & \bullet & \bullet\end{pmatrix}$$

2nd-order interference



3rd-order interference?

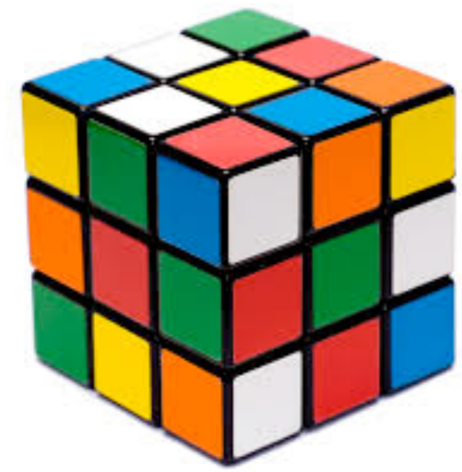
Density cubes? Density tensors?

$$\begin{pmatrix}\bullet\\\bullet\\\bullet\end{pmatrix}$$

CPT

$$\begin{pmatrix}\bullet&\bullet&\bullet\\\bullet&\bullet&\bullet\\\bullet&\bullet&\bullet\end{pmatrix}$$

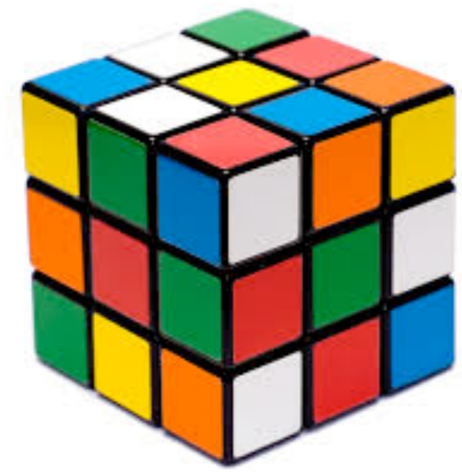
QT



???

Density cubes? Density tensors?

$$\begin{pmatrix} \bullet \\ \bullet \\ \bullet \end{pmatrix} \qquad \begin{pmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{pmatrix}$$



CPT

$K = N^1$

QT

$K = N^2$

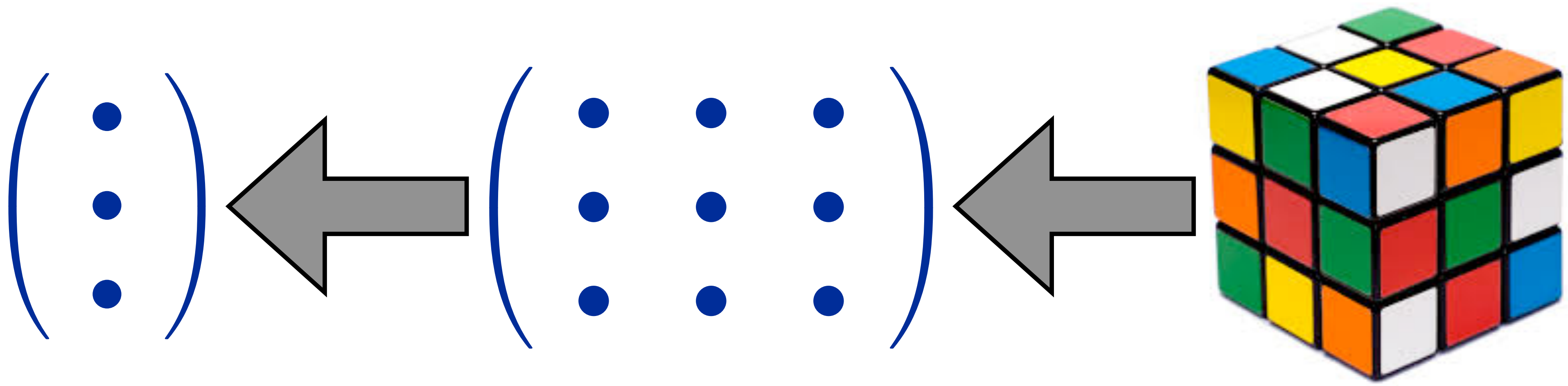
???

$K = N^3$

Would naturally fit into the Wootters-Hardy scheme $K = N^r \ldots$

L. Hardy, *Quantum Theory From Five Reasonable Axioms*, arXiv:quant-ph/0101012

Density cubes? Density tensors?



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Hyperdecoherence?

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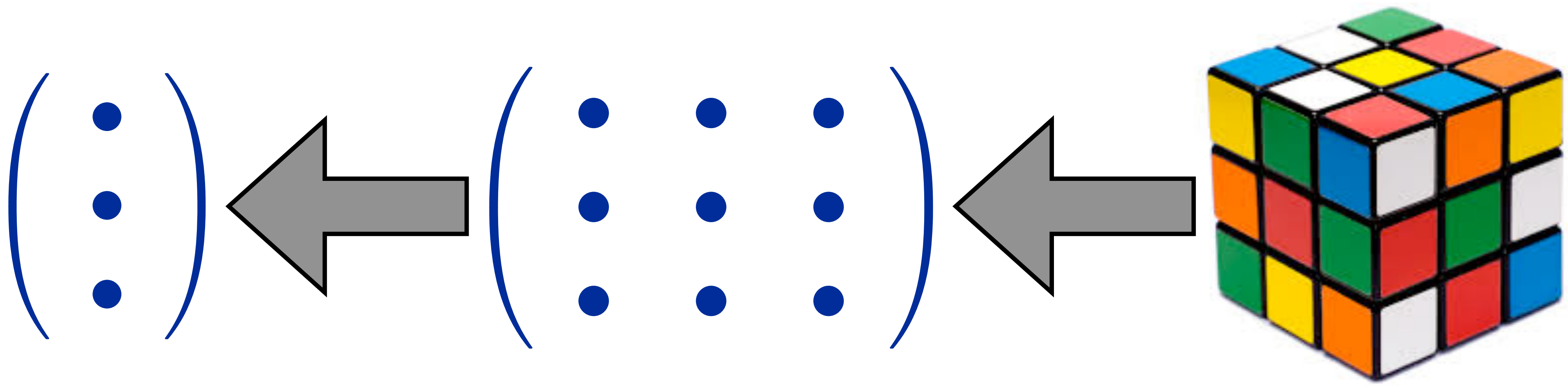

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Hyperdecoherence?

Despite lots of work, no concrete construction of state space so far...

L. Hardy, J. Barrett, M. Zyczkowski, J. Oppenheim, MM, ...

Density cubes? Density tensors?

... but there are a few hints:

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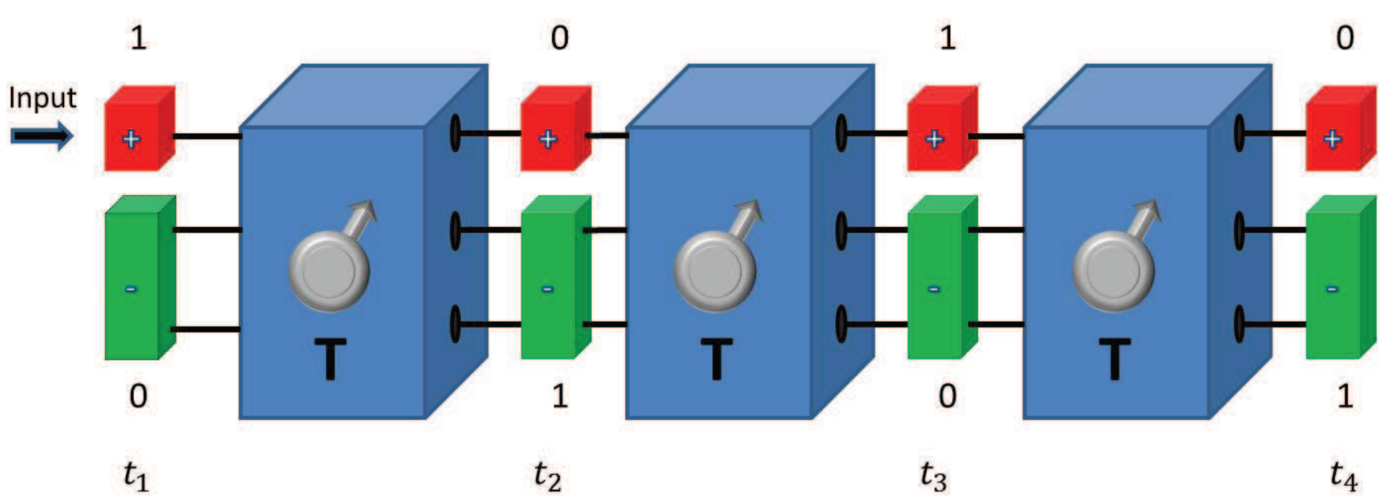
B. Dakic, T. Paterek, and C. Brukner, *Density cubes and higher-order interference theories*, New J. Phys. **16**, 023028 (2014).



Density cubes violate "Tsirelson bound of temporal correlations"

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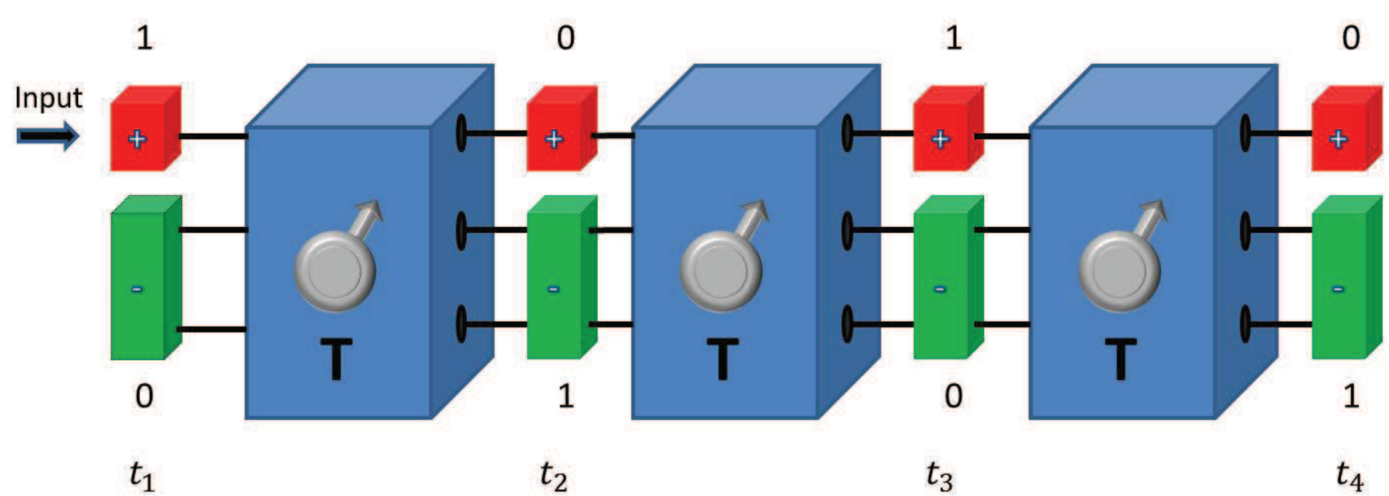


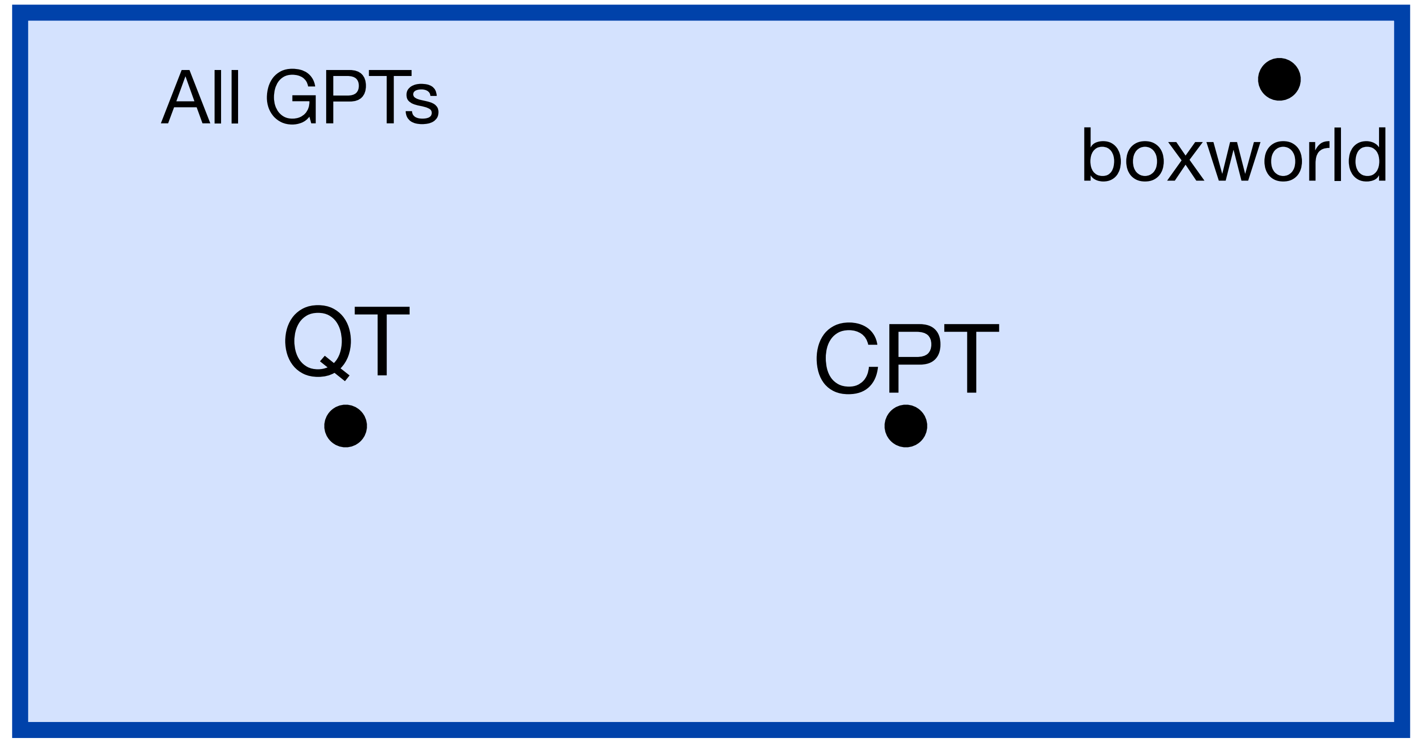
Density cubes violate "Tsirelson bound of temporal correlations"

G. Niestegge, *Three-slit experiments and quantum nonlocality,* Found. Phys. **43**(6), 805-812 (2013).

Absence of 3rd-order interference, together with a "conditional probability calculus", implies the Tsirelson bound.

An axiomatic approach (arXiv:1403.4147)

H. Barnum, MM, and C. Ududec, *Higher-order interference and single-system postulates characterizing quantum theory*.



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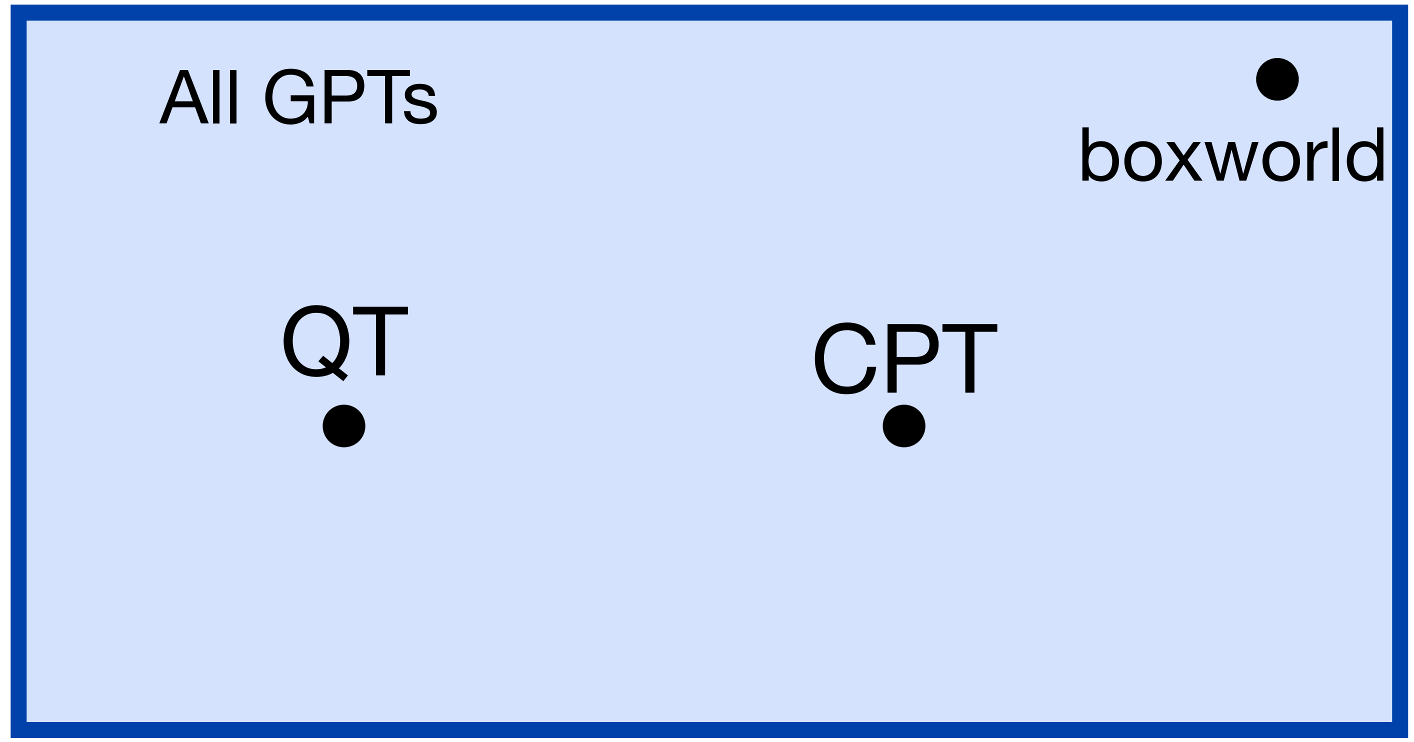
The following 4 postulates single out QT uniquely (for details, **see Howard's talk!**)



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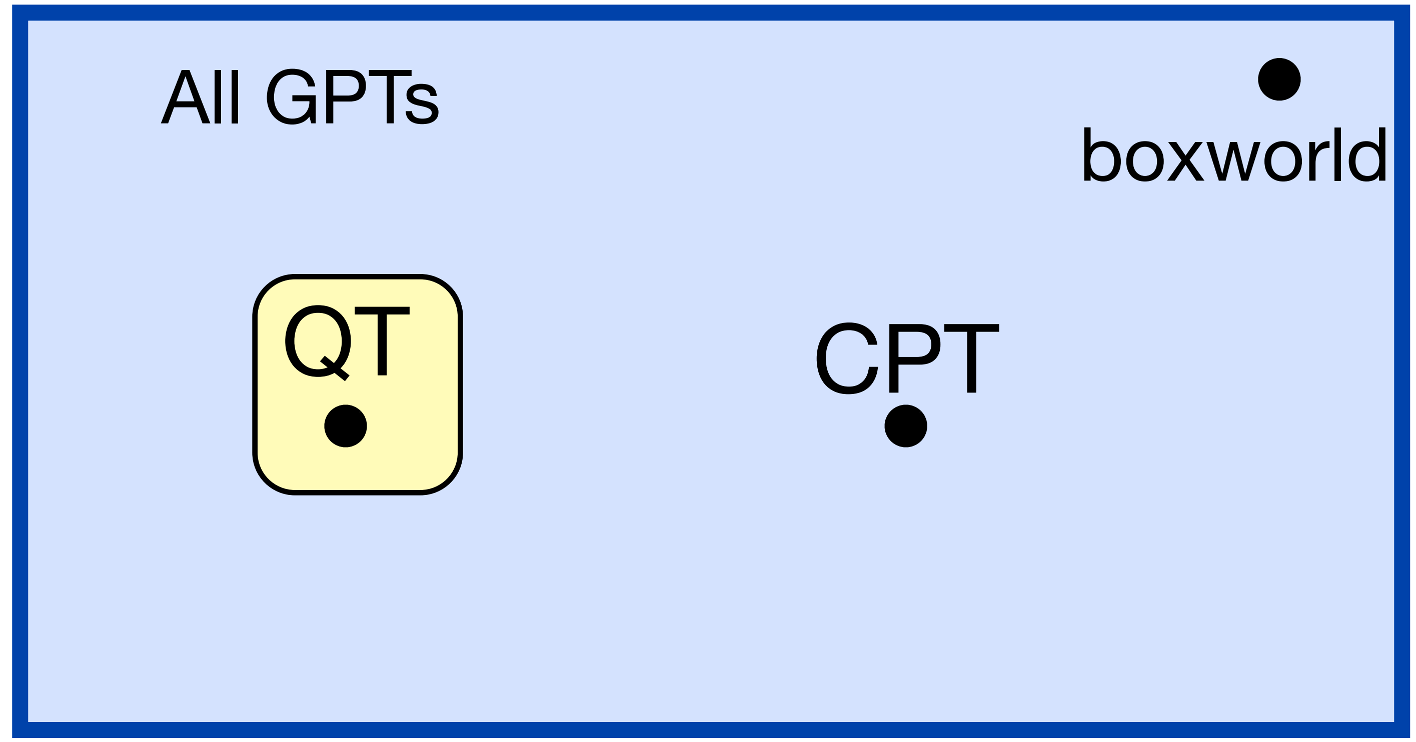

1. Every state belongs to a "classical subsystem",
2. lots of reversible dynamics,
3. **no 3rd-order interference**, and
4. energy is observable.

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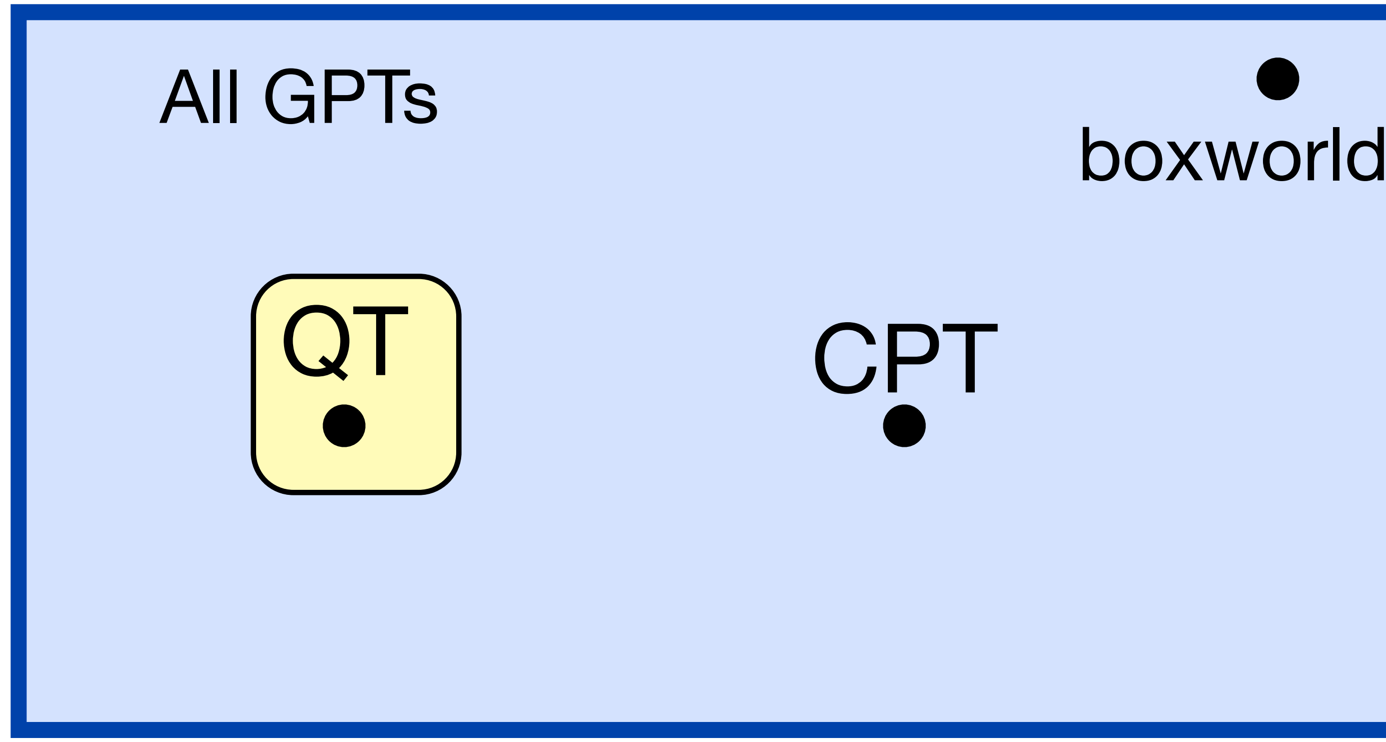
Let's drop #4...



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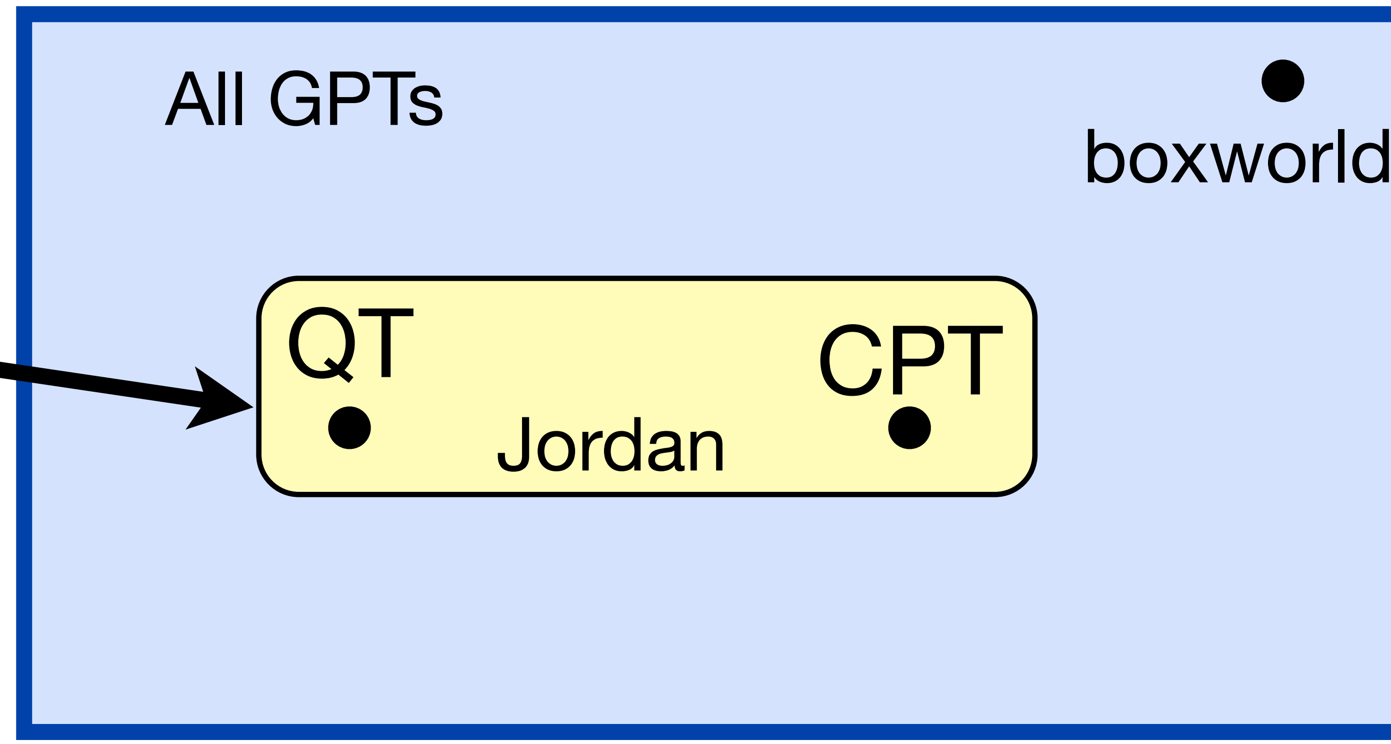
Jordan algebra state spaces
(2-level ball state spaces,
QT over $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$,
3-level QT over $\mathbb{O}$.)



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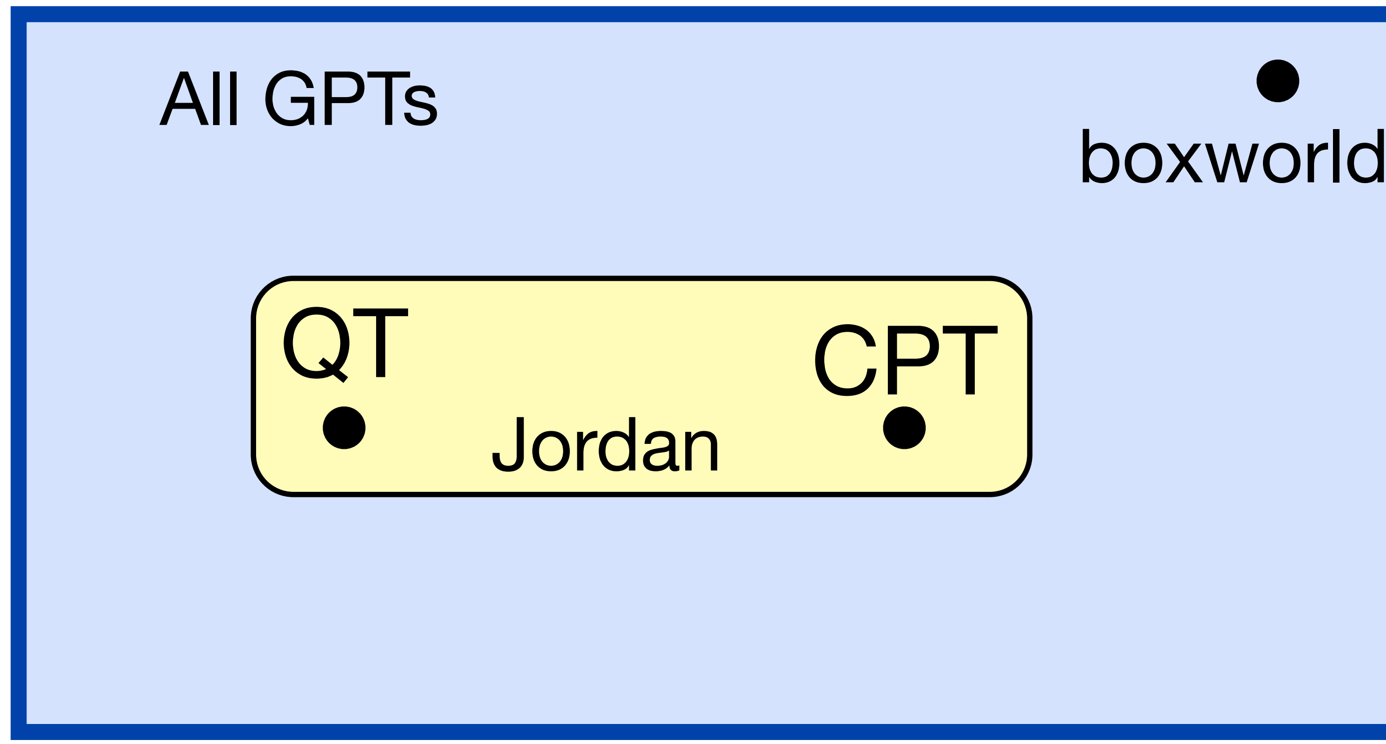
The point:
Let's drop #3 !

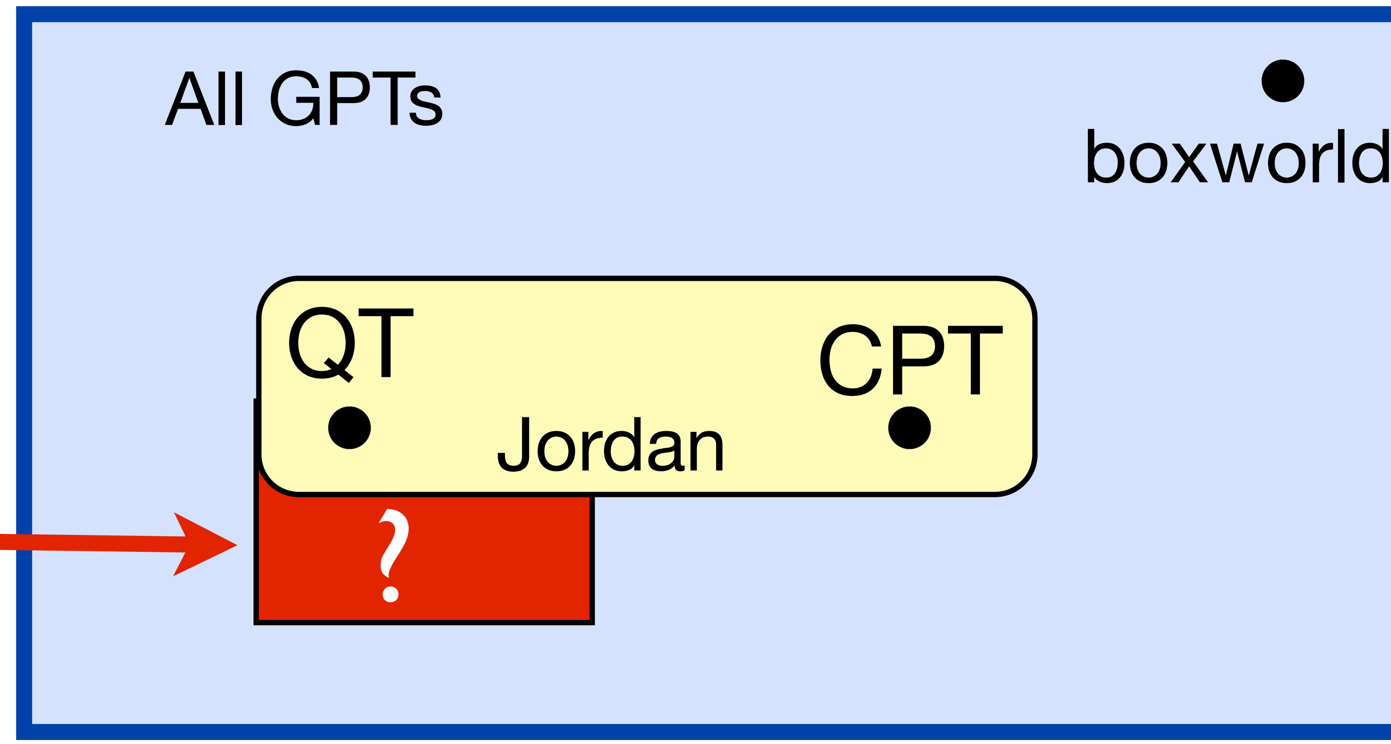


1. Every state belongs to a "classical subsystem",
2. lots of reversible dynamics,
3. **no 3rd-order interference**, and
4. ~~energy is observable.~~

An axiomatic approach (arXiv:1403.4147)

H. Barnum, MM, and C. Ududec, *Higher-order interference and single-system postulates characterizing quantum theory*.

New solutions?
⇒ 3rd-order interference



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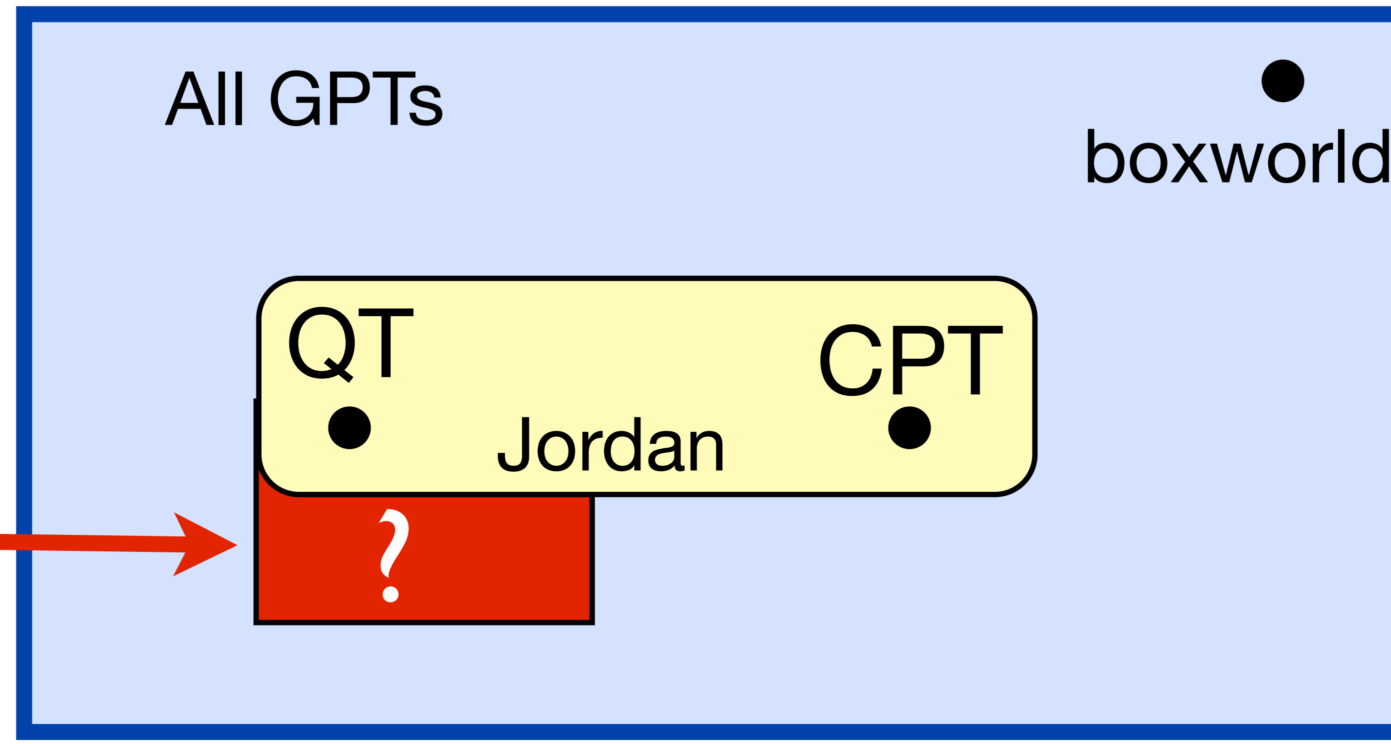

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Well-defined math problem: classify those state spaces!

HGSFP

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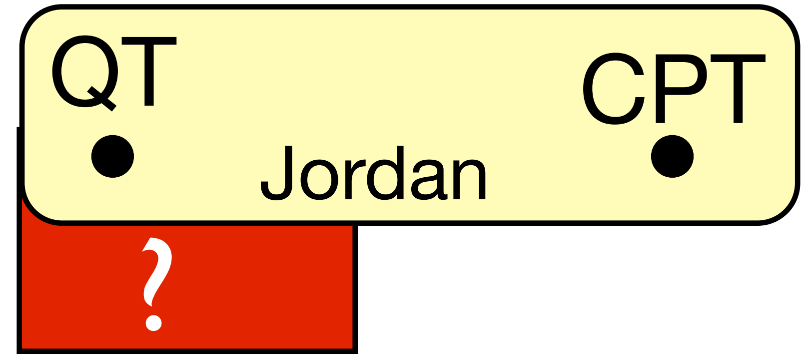
We know a lot about these theories:



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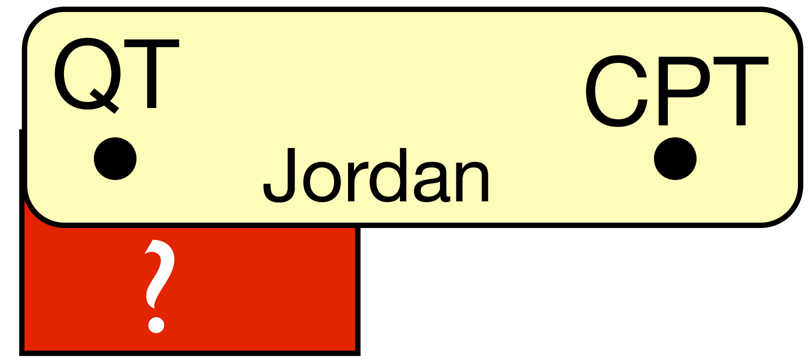

- Unlike QT, they have 3rd-order interference,
- like QT, their elementary propositions are an orthomodular lattice,
- like QT, they satisfy the **LO[1] principle** for contextuality,
- like QT, all bit subsystems are Euclidean ball state spaces,
- like QT, they allow for filters that are important in thermodynamics,
- but they violate the covering property of quantum logic,
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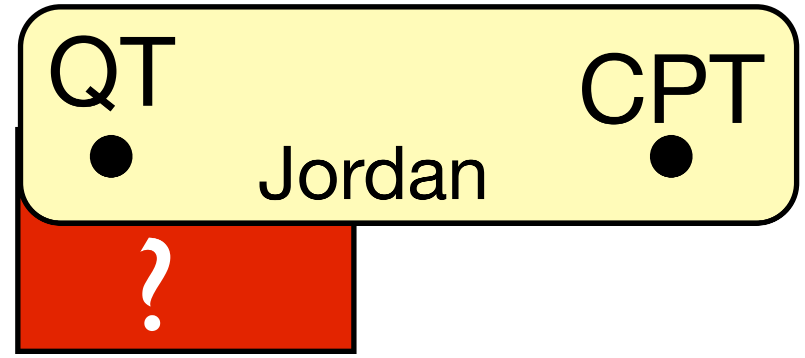

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Do they exist? If yes:
natural models, experimentally testable against QT.

A further interference experiment

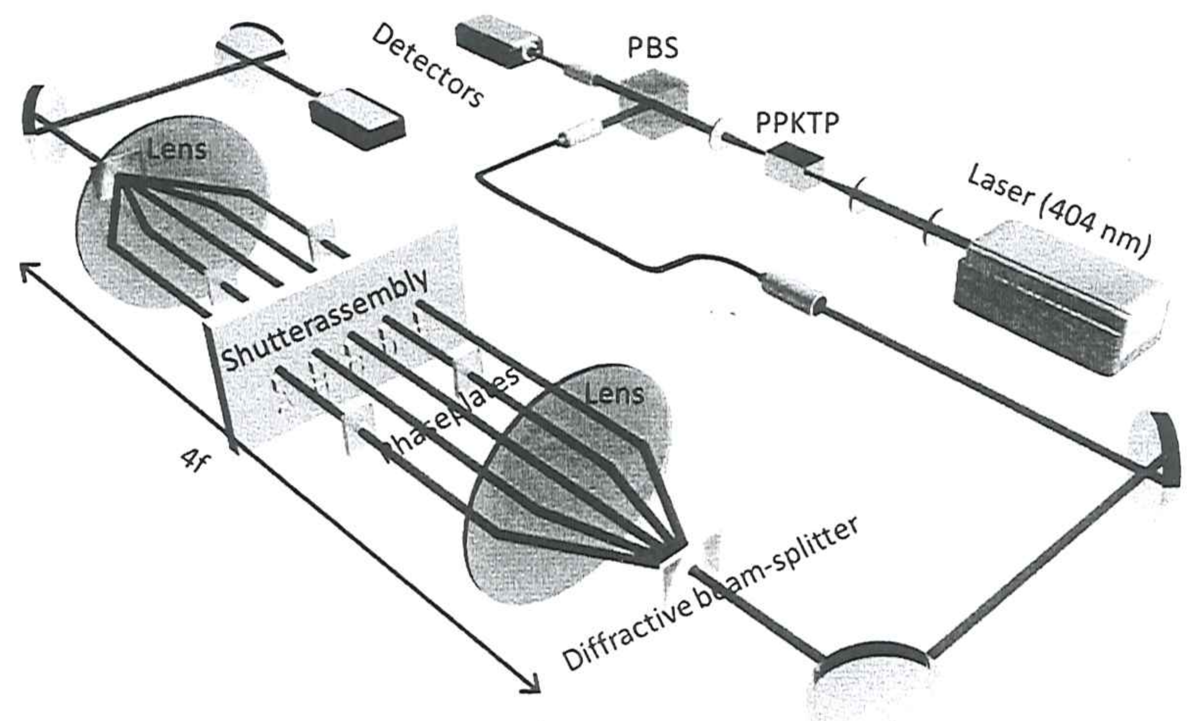
Recall this experiment:

Measuring higher-order interferences with a five-path interferometer

T. Kauten[1], T. Kaufmann[1], B. Pressl[1] and G. Weihs[1]

[1] *Institut für Experimentalphysik, Universität Innsbruck, Technikerstraße 25, 6020 Innsbruck, Austria*

E-mail: thomas.kauten@uibk.ac.at



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They are not only testing for higher-order interference, but also:

Proposed Test for Complex versus Quaternion Quantum Theory

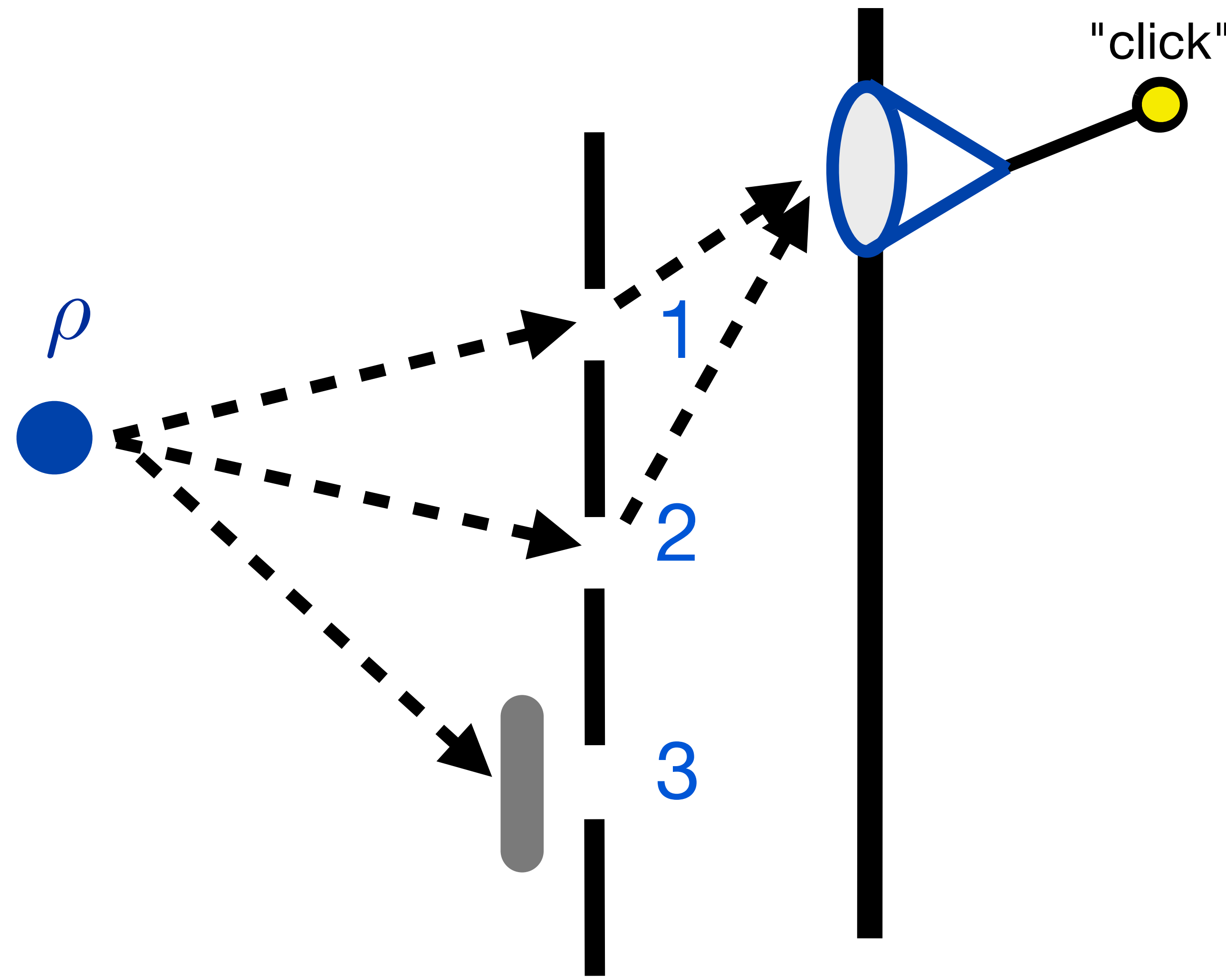
Asher Peres
Department of Physics, Technion—Israel Institute of Technology, Haifa, Israel
(Received 7 December 1978)

If scattering amplitudes are ordinary complex numbers (not quaternions) then there is a universal algebraic relationship between the six coherent cross sections of any three scatterers (taken singly and pairwise). A violation of this relationship would indicate either that scattering amplitudes are quaternions, or that the superposition principle fails. Some experimental tests are proposed, involving neutron diffraction by crystals made of three different isotopes, neutron interferometry, and K_S-meson regeneration.

A further interference experiment

A. Peres, *Proposed Test for Complex versus Quaternionic Quantum Theory*, Phys. Rev. Lett. **42**, 11 (1979).

By measuring combinations of single- and double-slit statistics, one can infer whether ρ is complex or quaternionic.



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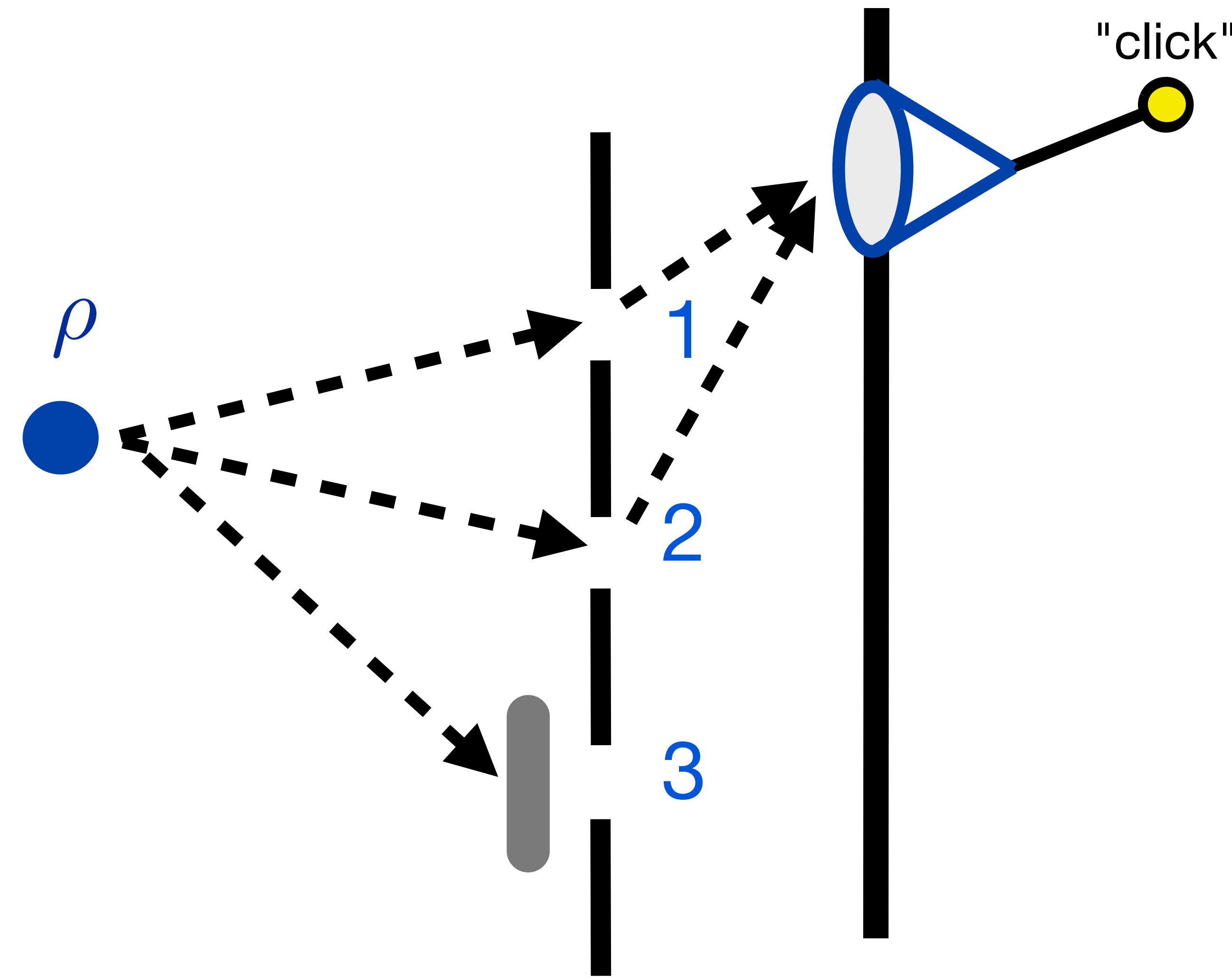

⇒ study general-probabilistic interference experiments

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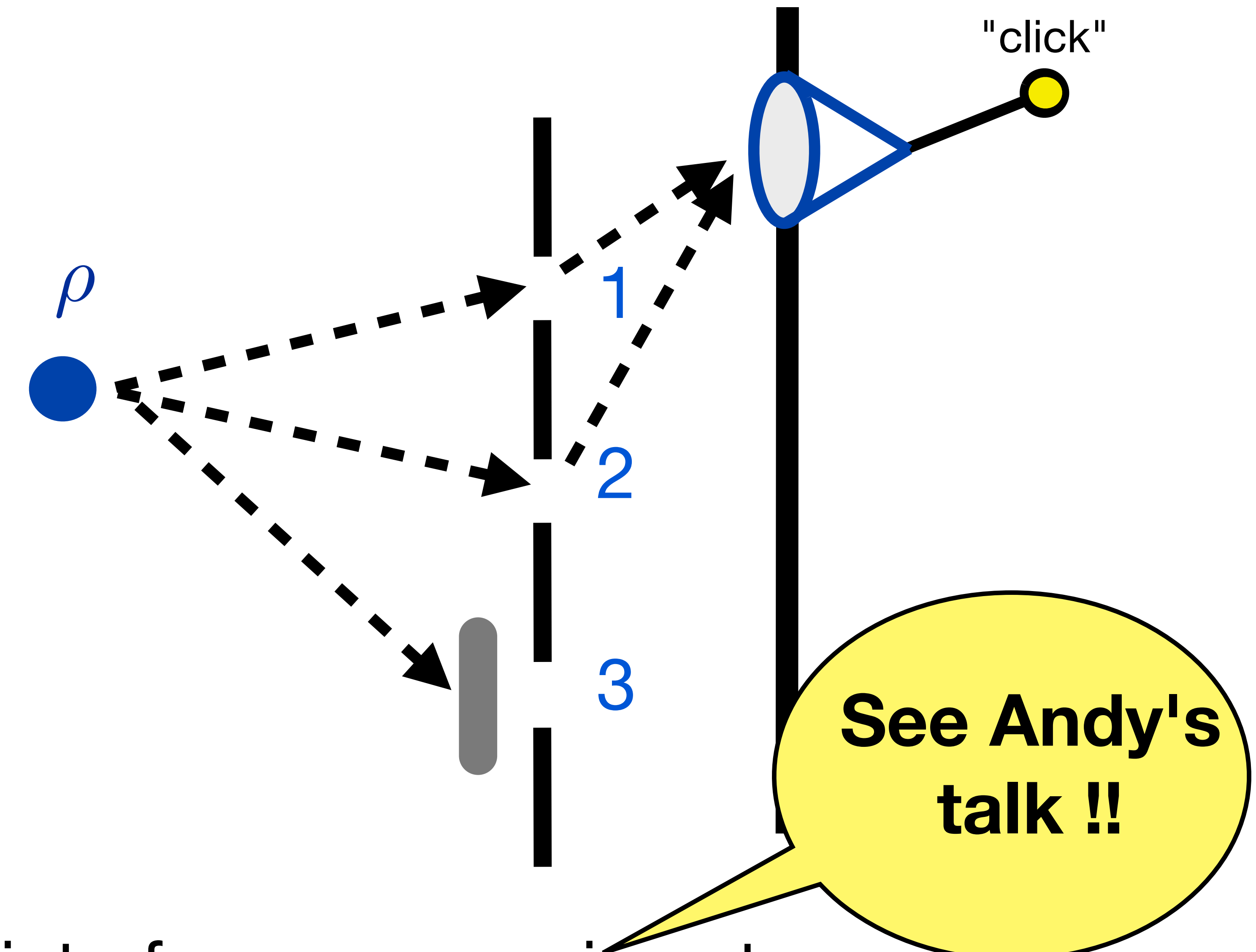

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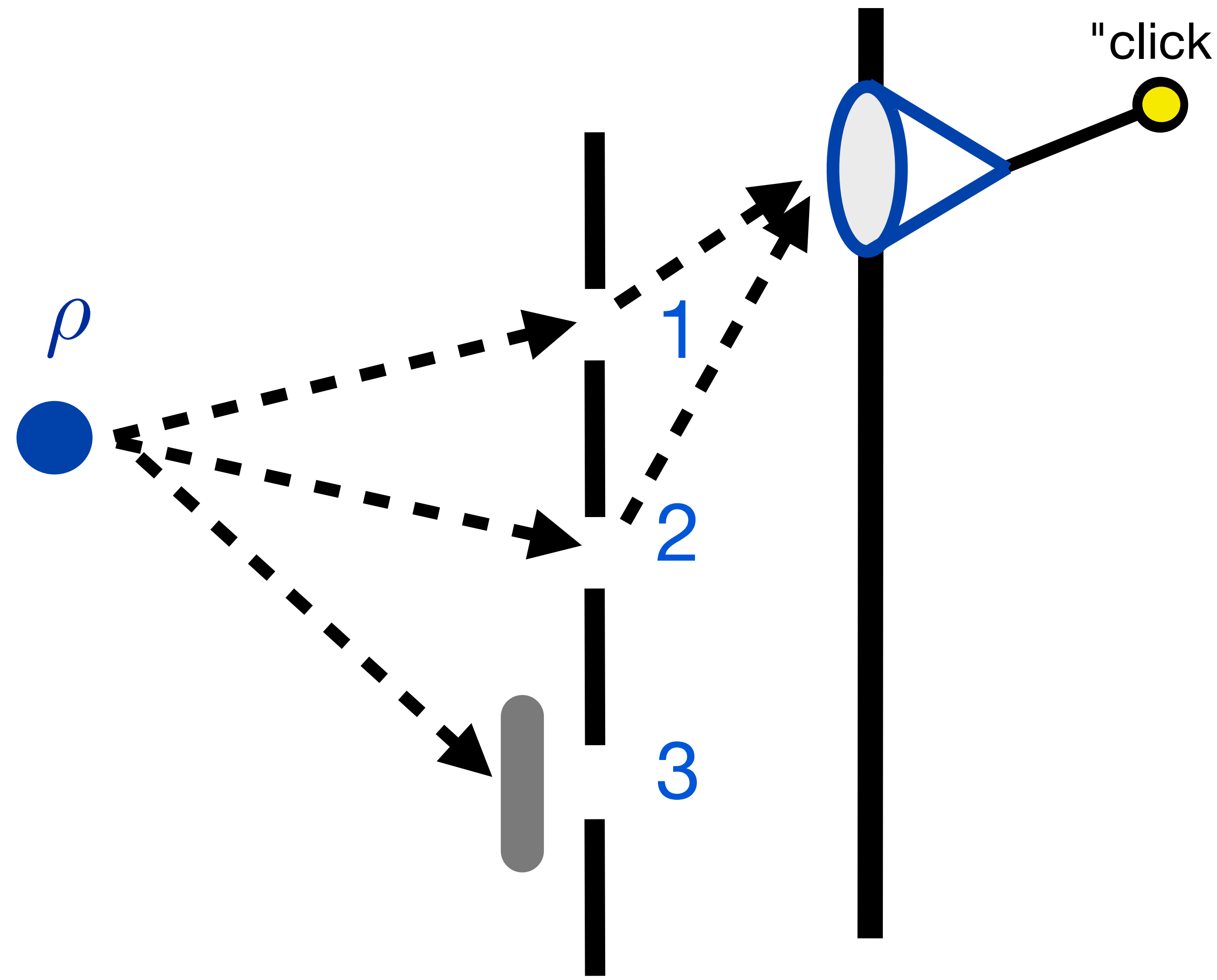


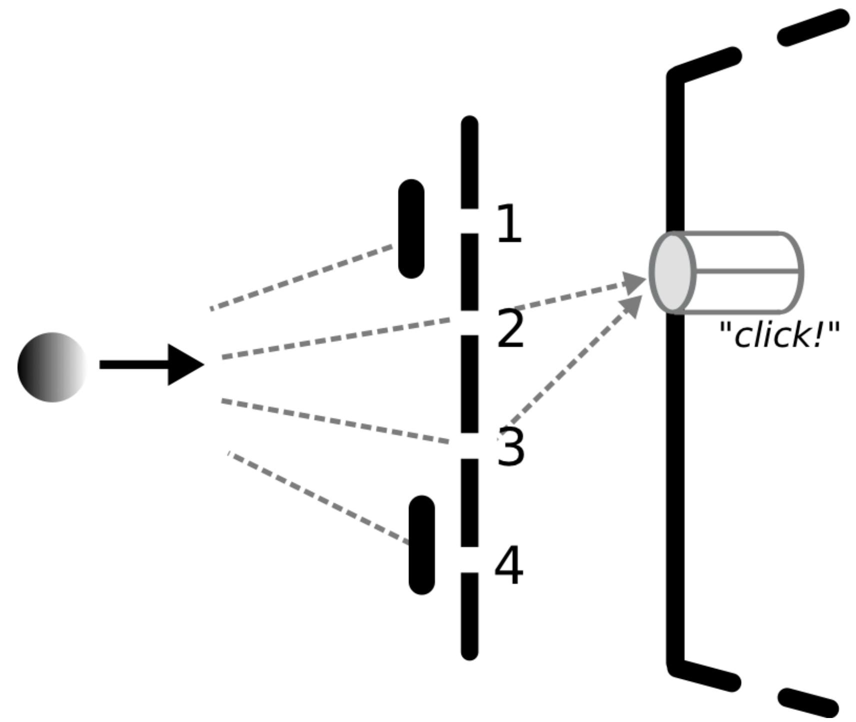
⇒ study general-probabilistic interference experiments

⇒ we will now see that relativity enforces complex QT!
(sometimes)

Outline

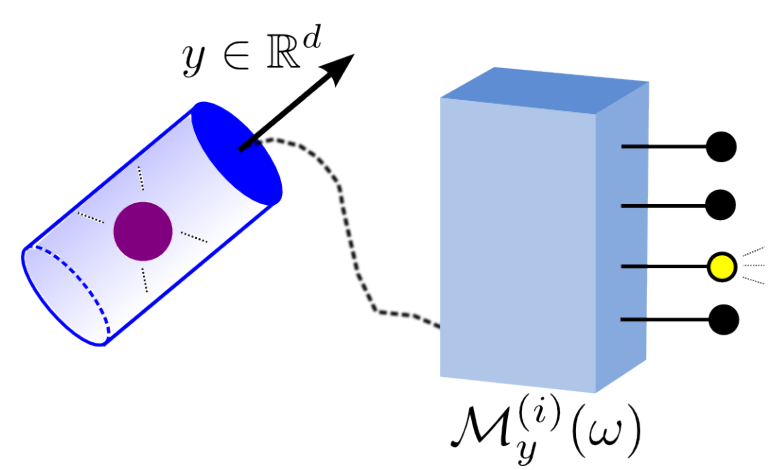
1. Interference

Sorkin's hierarchy
Density tensor theories?
An axiomatic approach



2. GPTs and spacetime

Relativistic constraints on interference experiments
Quantum theory and the dimensionality of space
Reverse-engineering physics



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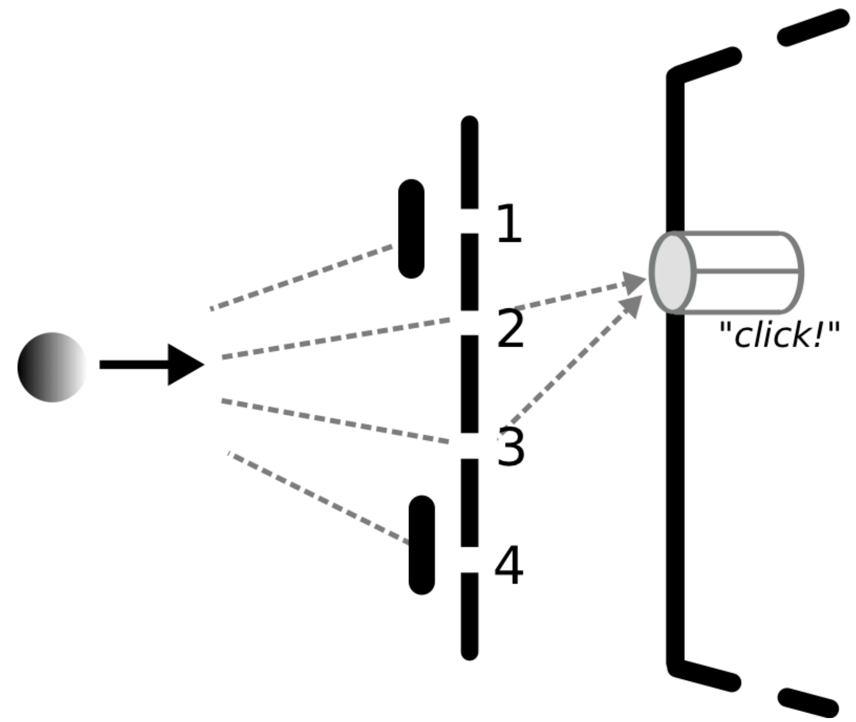

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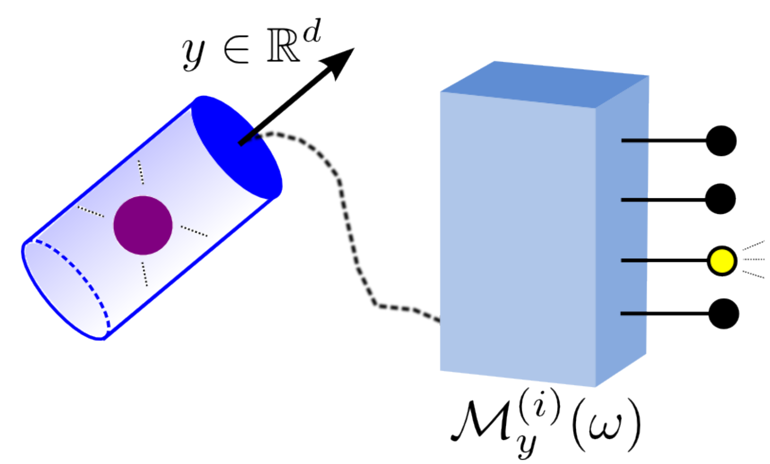

Relativistic constraints on interference experiments

Two-level state spaces of quantum theory over $\mathbb{R}, \quad \mathbb{C}, \quad \mathbb{H}, \quad \mathbb{O}$ are special cases of ball state spaces:

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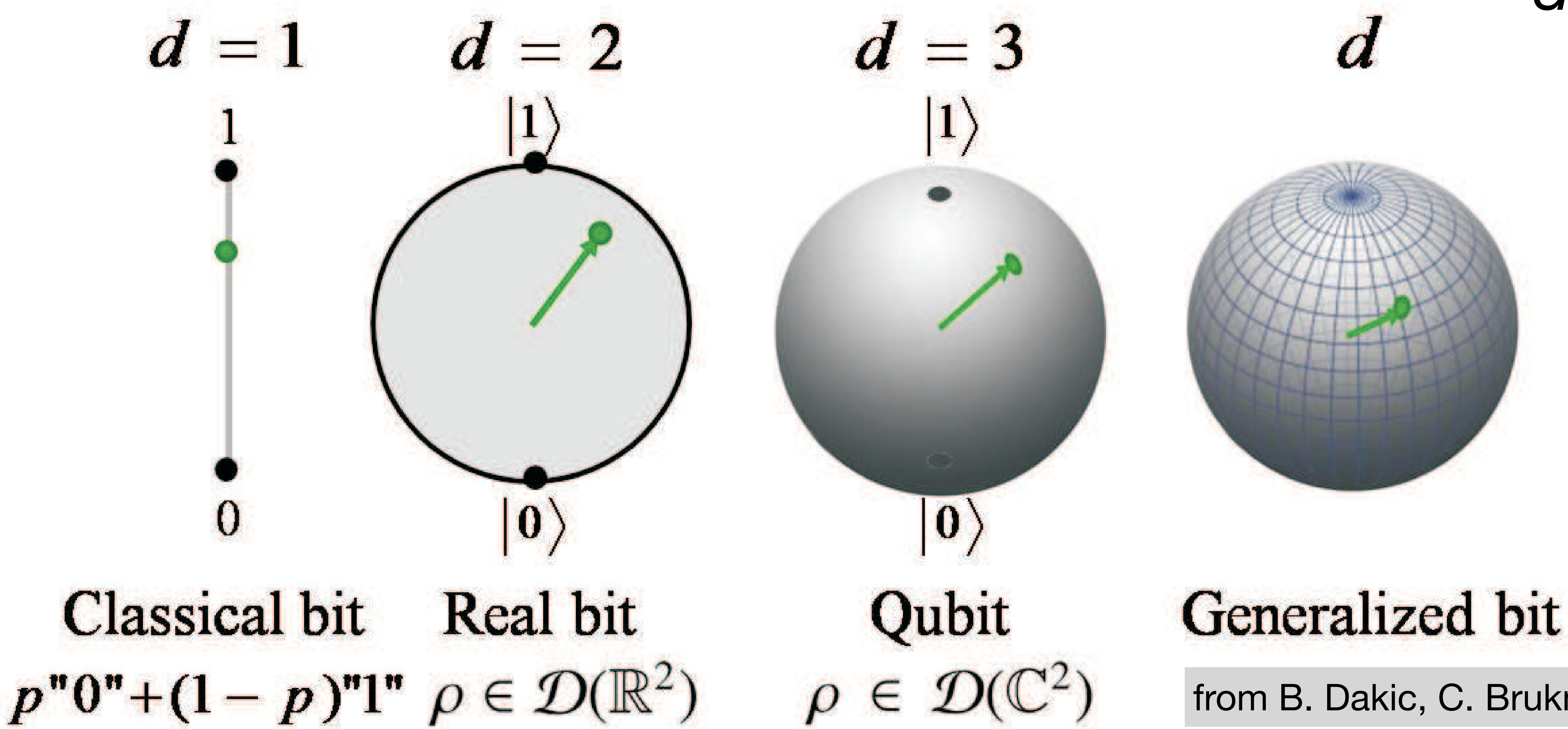
$\mathbb{H}$: *d*=5

$\mathbb{O}$: *d*=9



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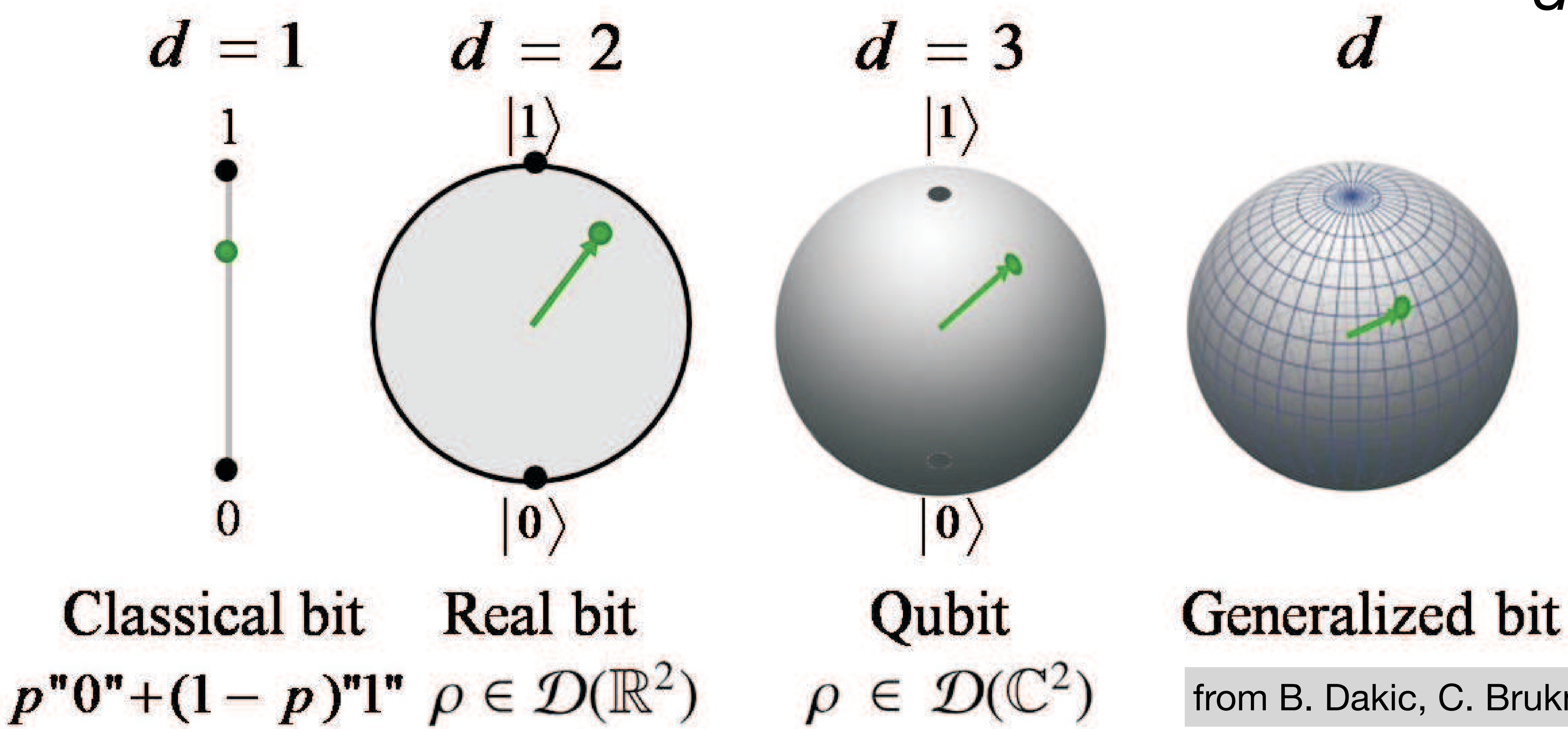

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All of them are 2-level state spaces ("hyperbits").

M. Pawlowski and A. Winter, *Hyperbits: The information quasiparticles*, Phys. Rev. A **85**, 022331 (2012).

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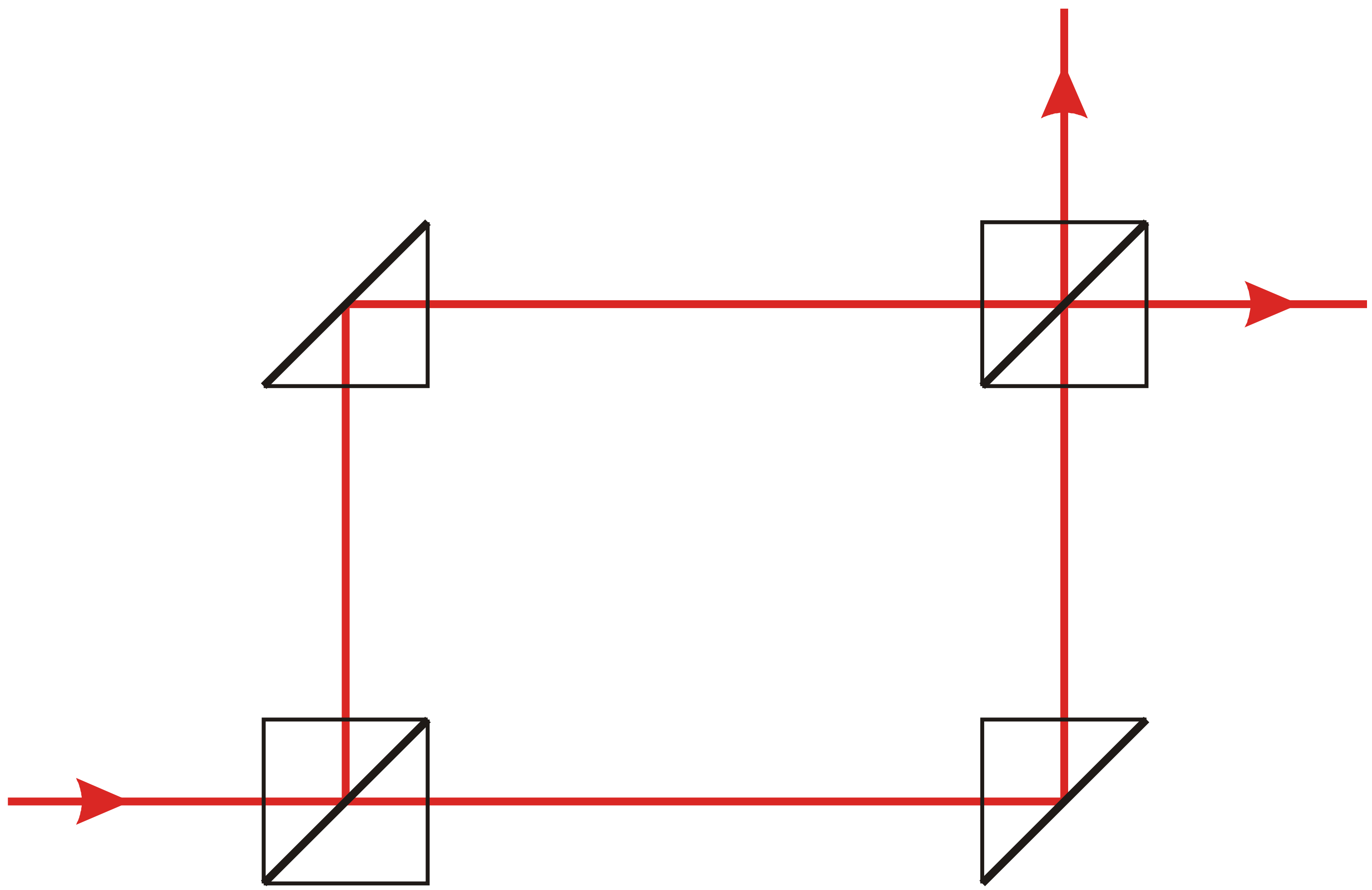
Joint work w/ Andy Garner & Oscar Dahlsten:



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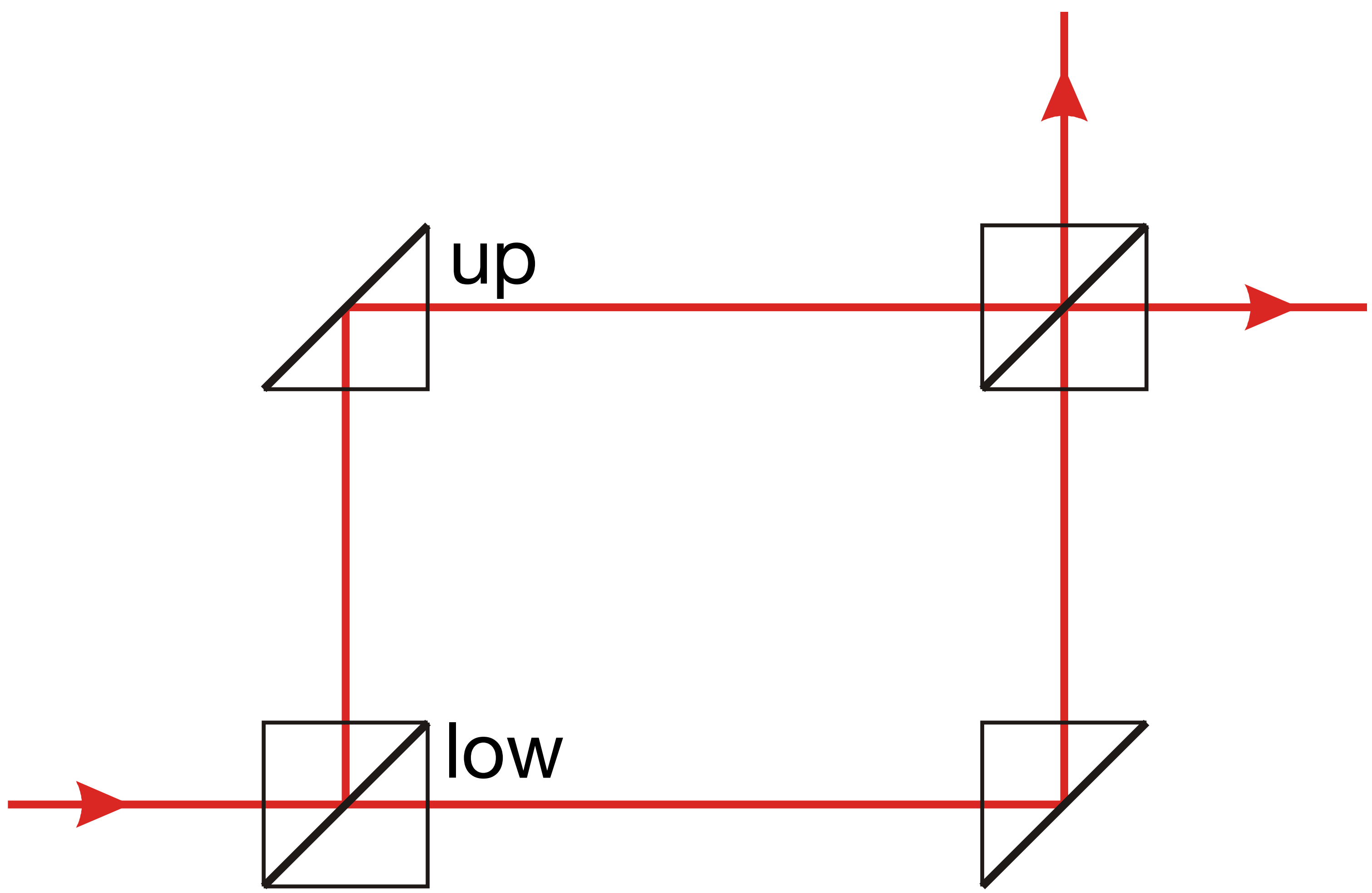

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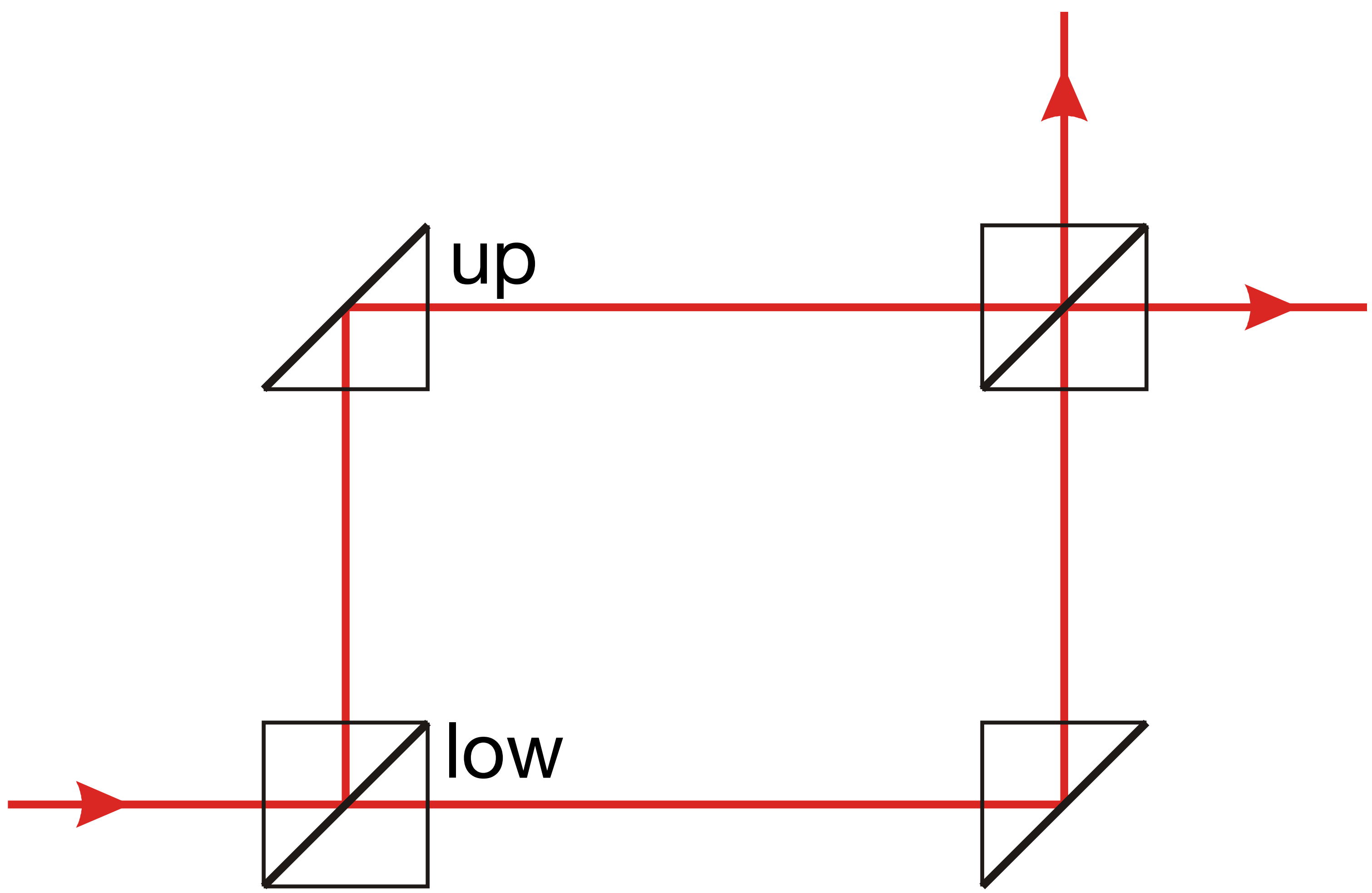

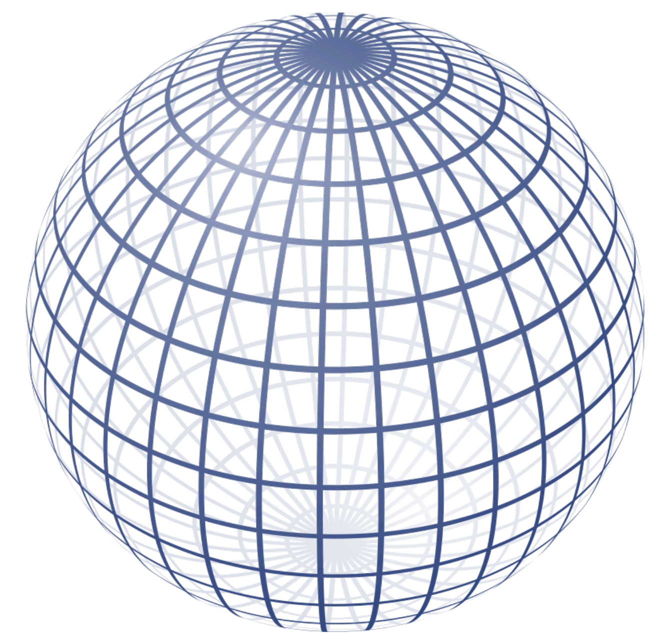

d-dim. "Bloch sphere"

Relativistic constraints on interference experiments

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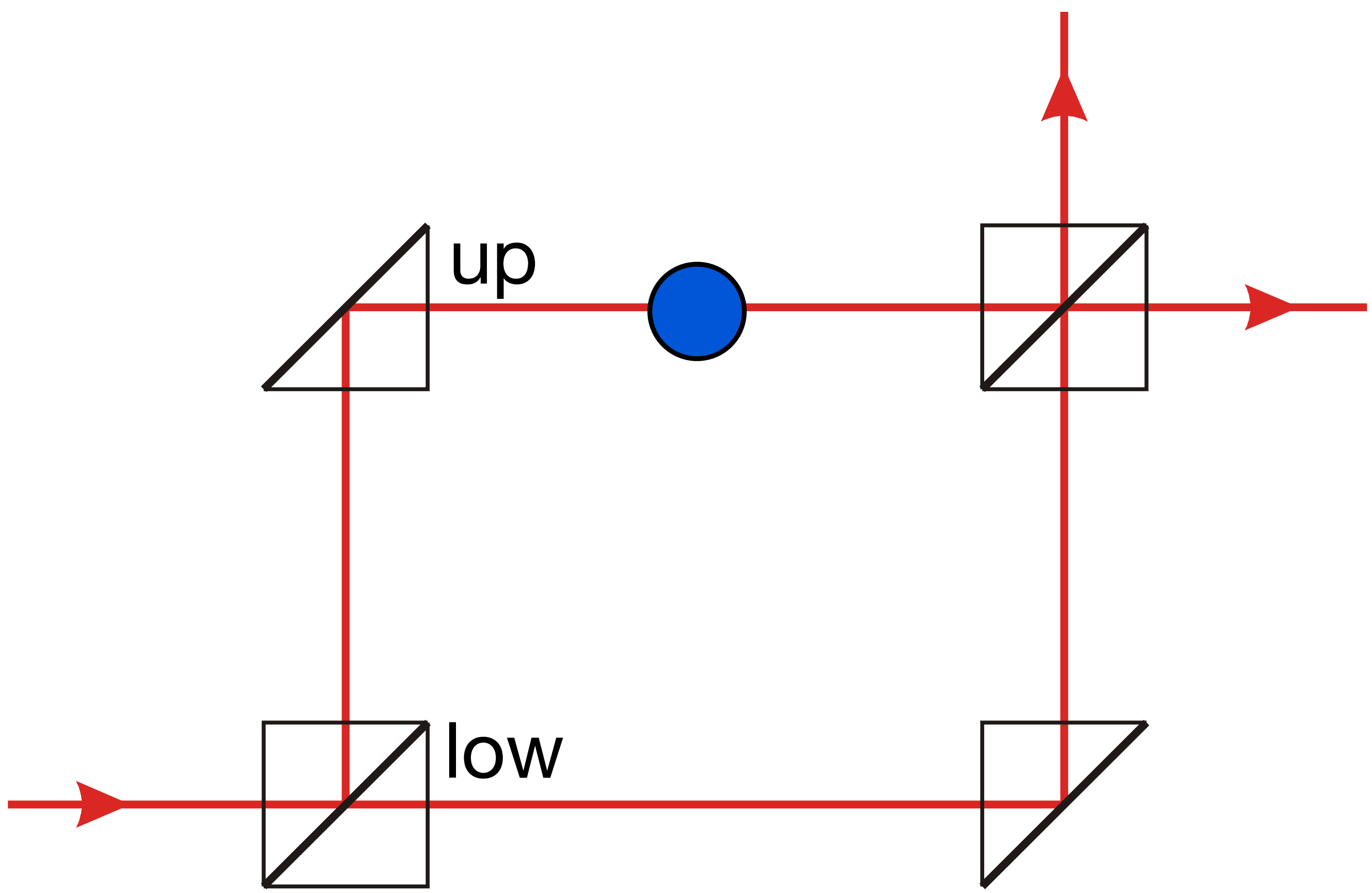

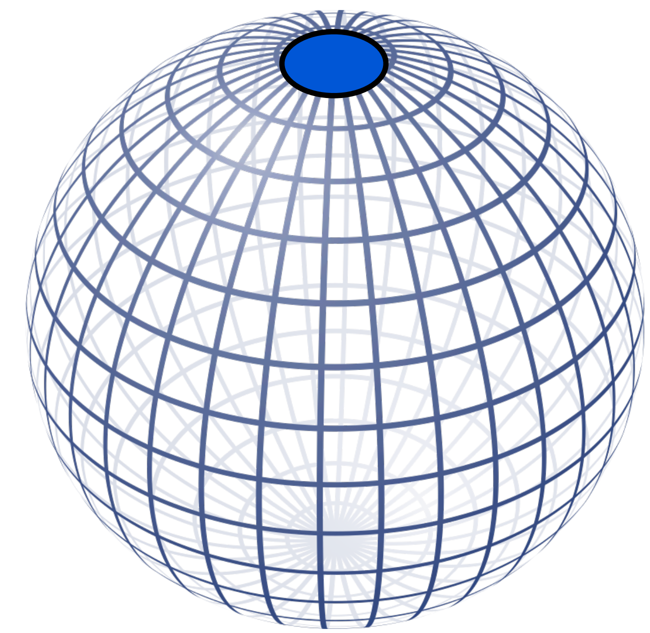

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North-pole state: particle definitely in upper branch.

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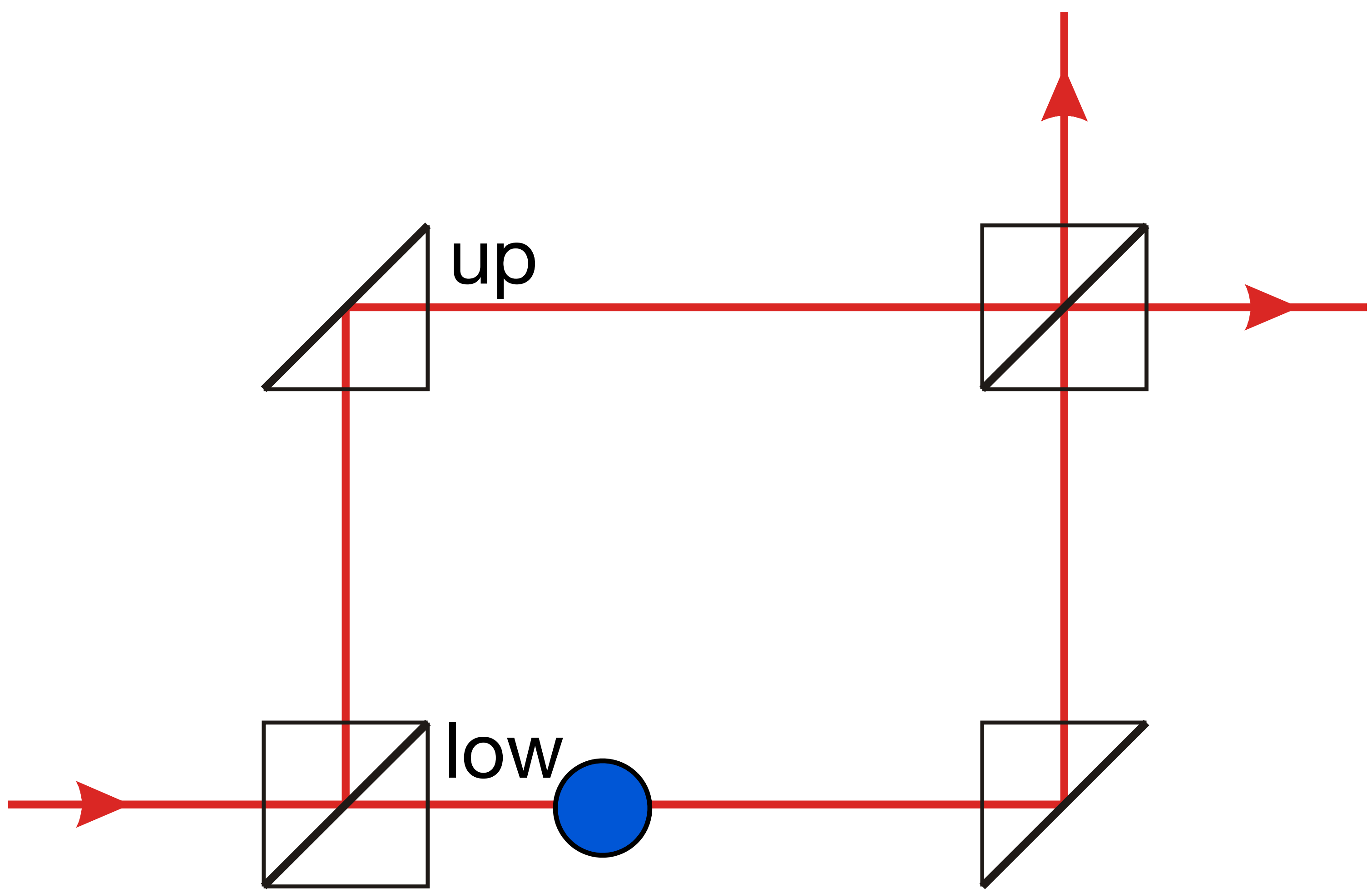

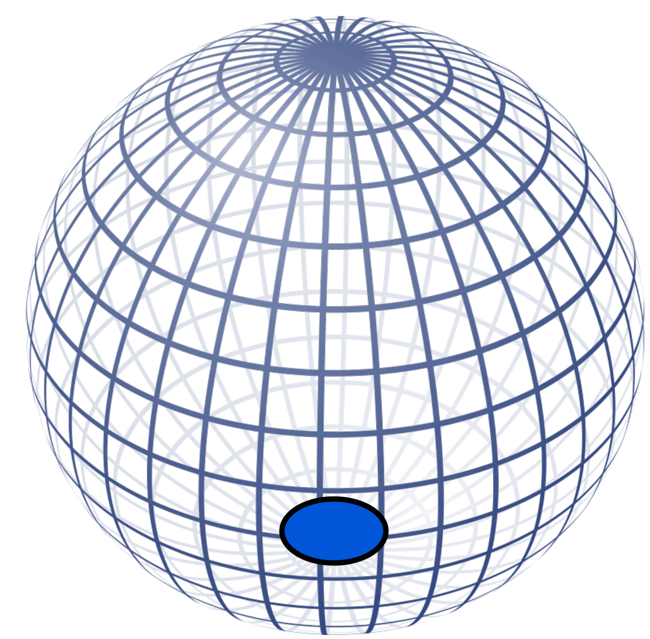

d-dim. "Bloch sphere"

South-pole state: particle definitely in lower branch.

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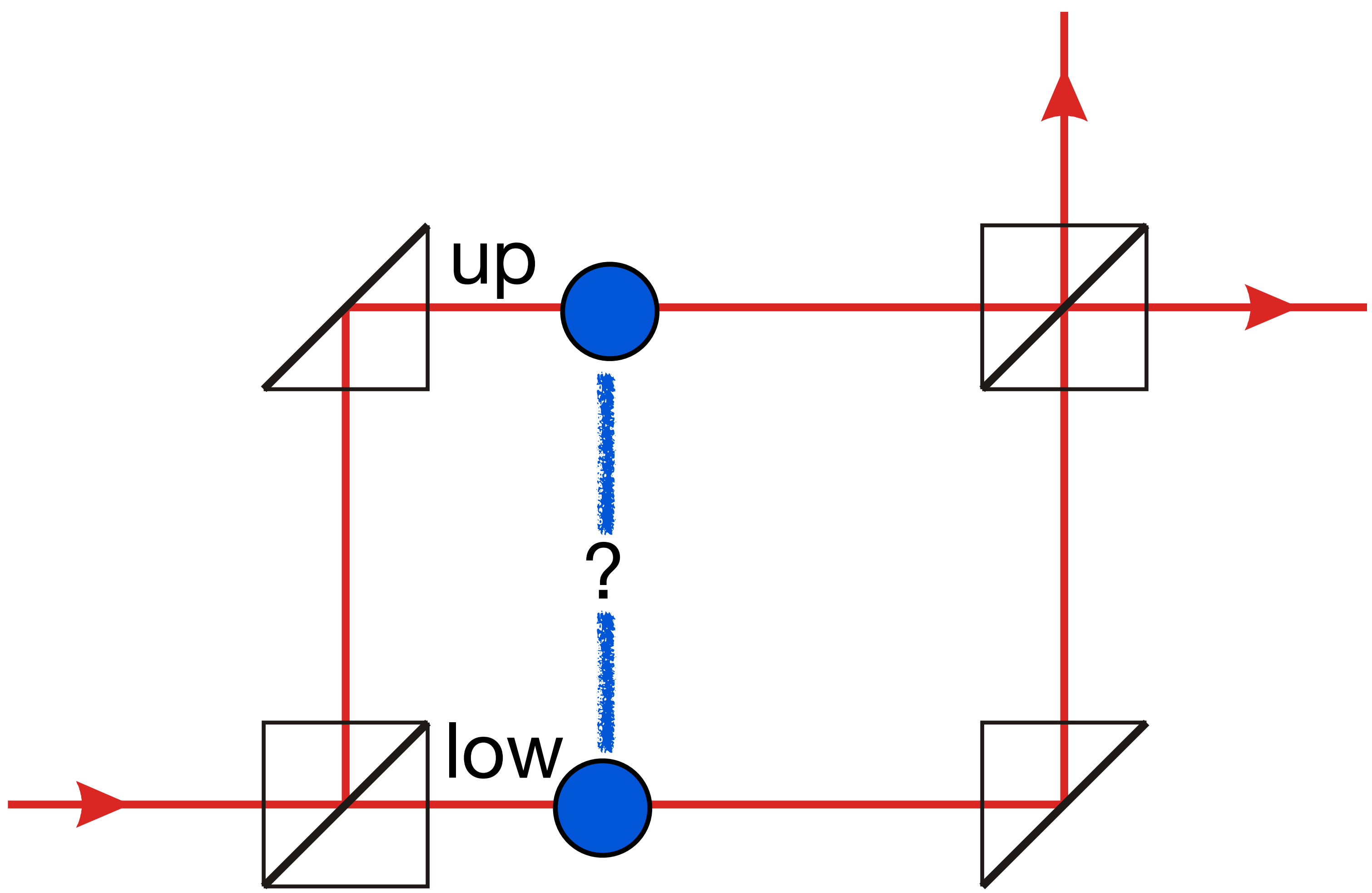

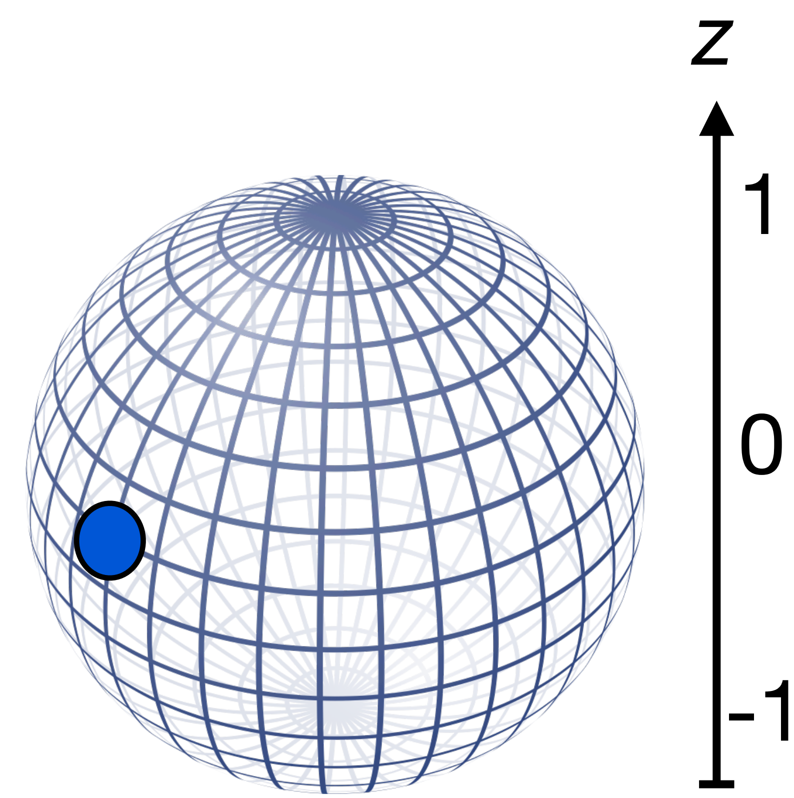

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State on equator *z=0*: probability 1/2 for each.

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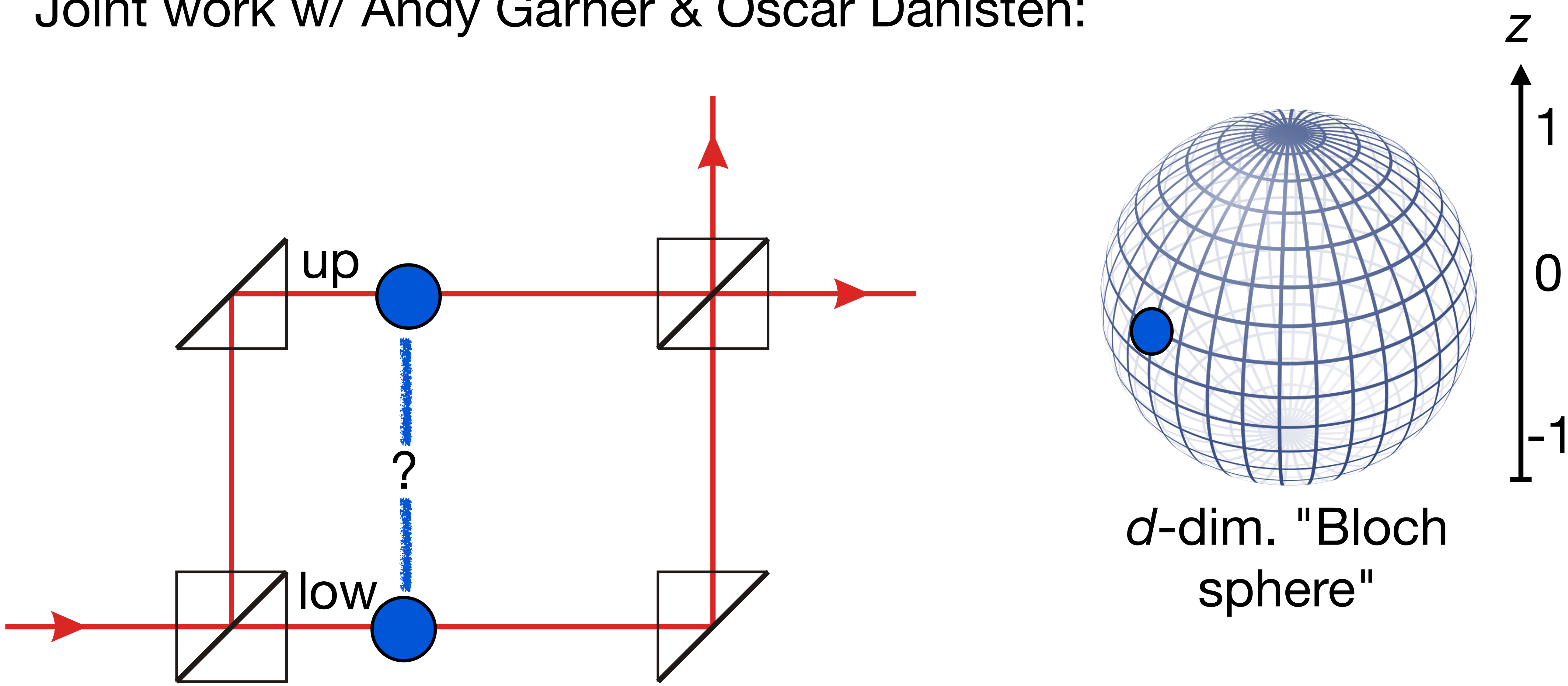

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$$p(\text{up}) = \frac{1}{2}(z+1)$$

Relativistic constraints on interference experiments

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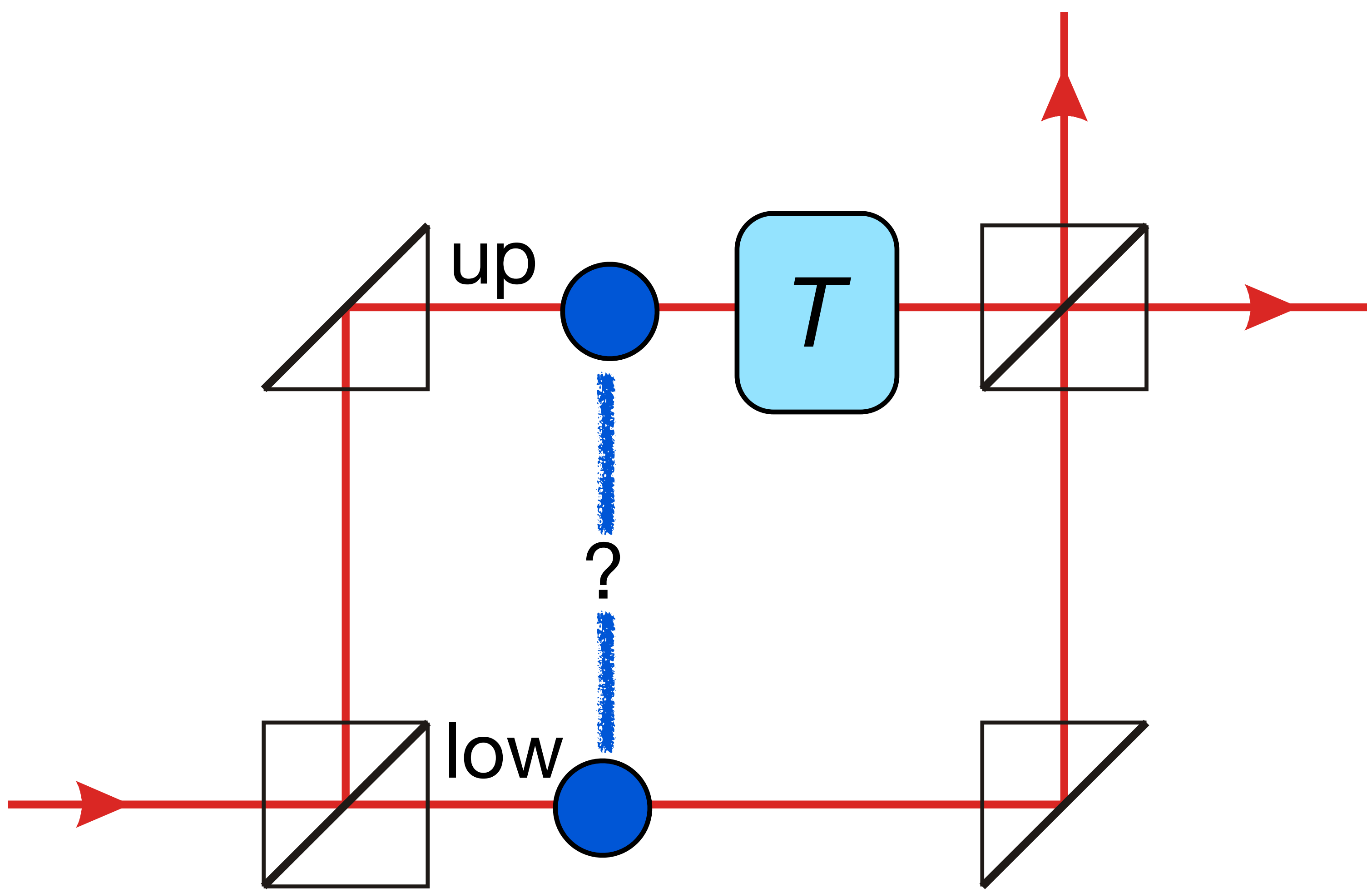

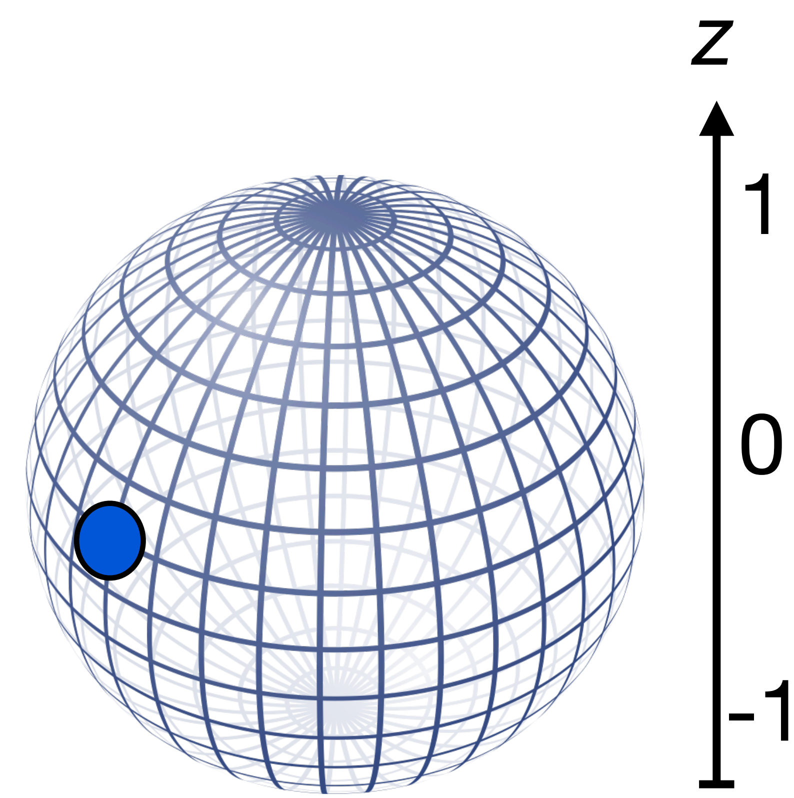

d-dim. "Bloch sphere"

What transformations *T* can we perform locally in one arm...
... without any information loss?

Relativistic constraints on interference experiments

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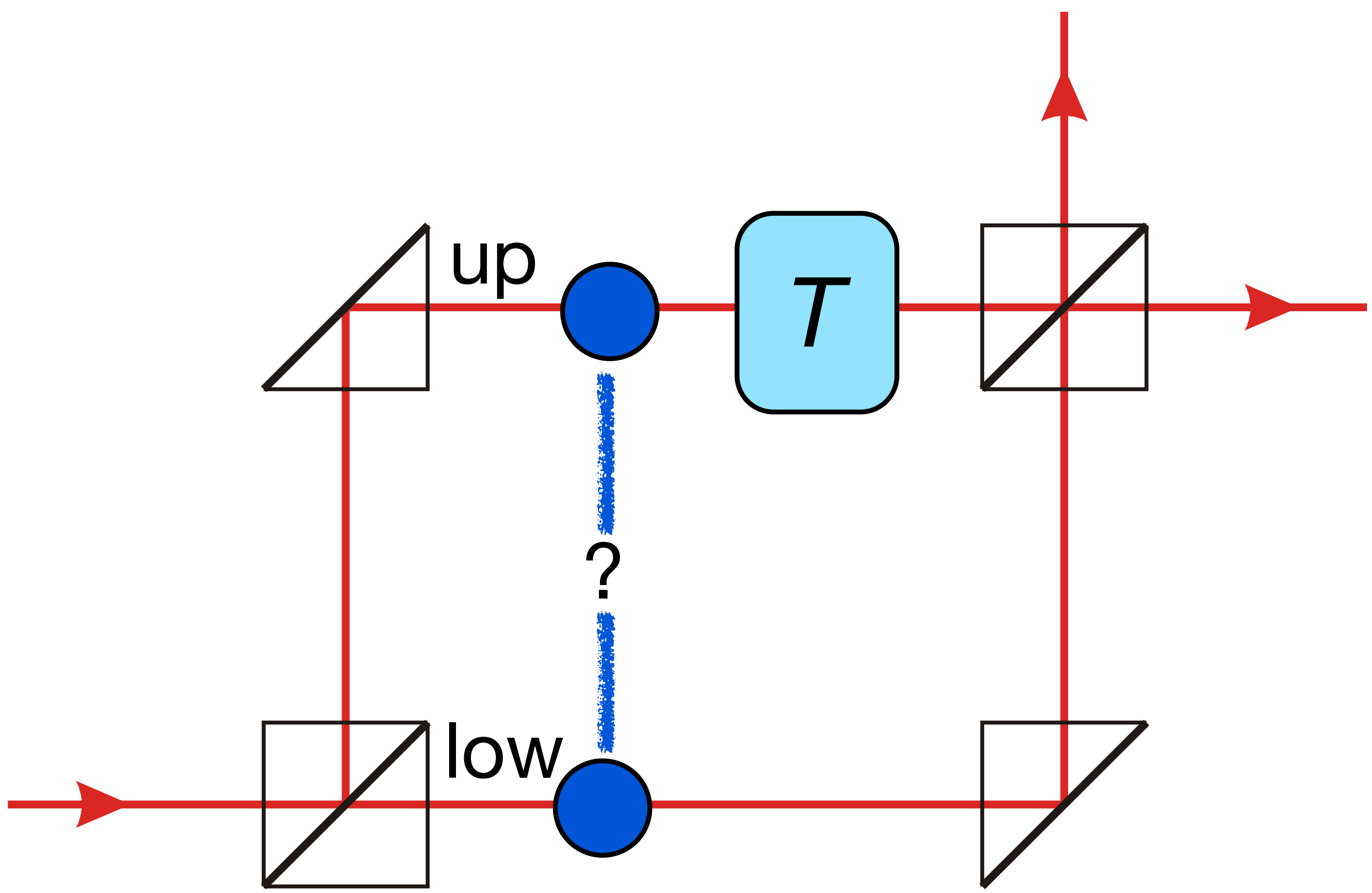
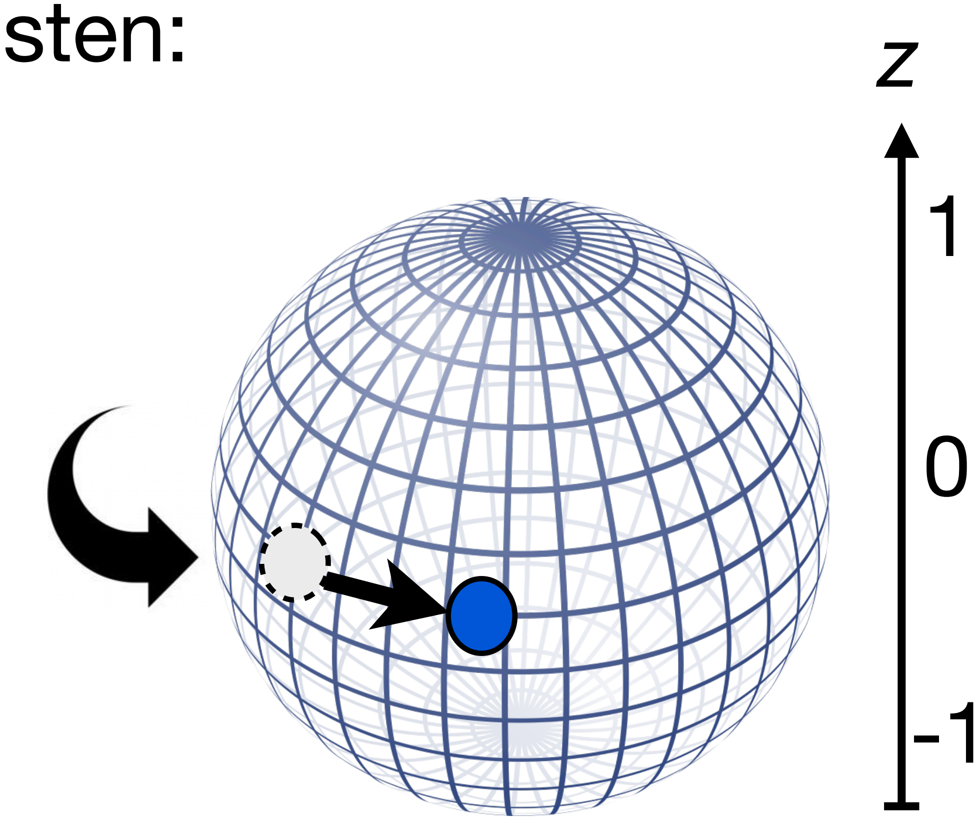


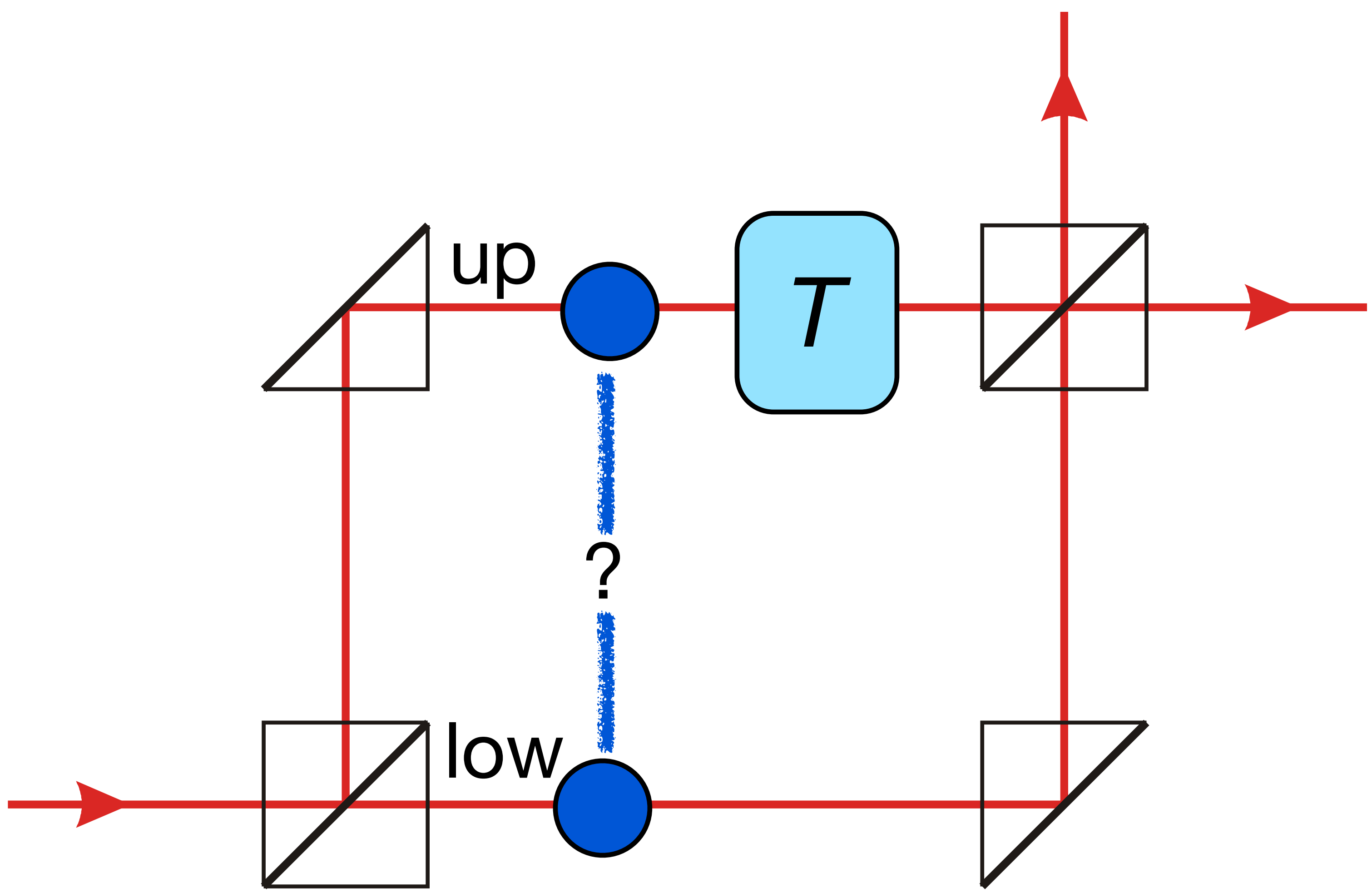


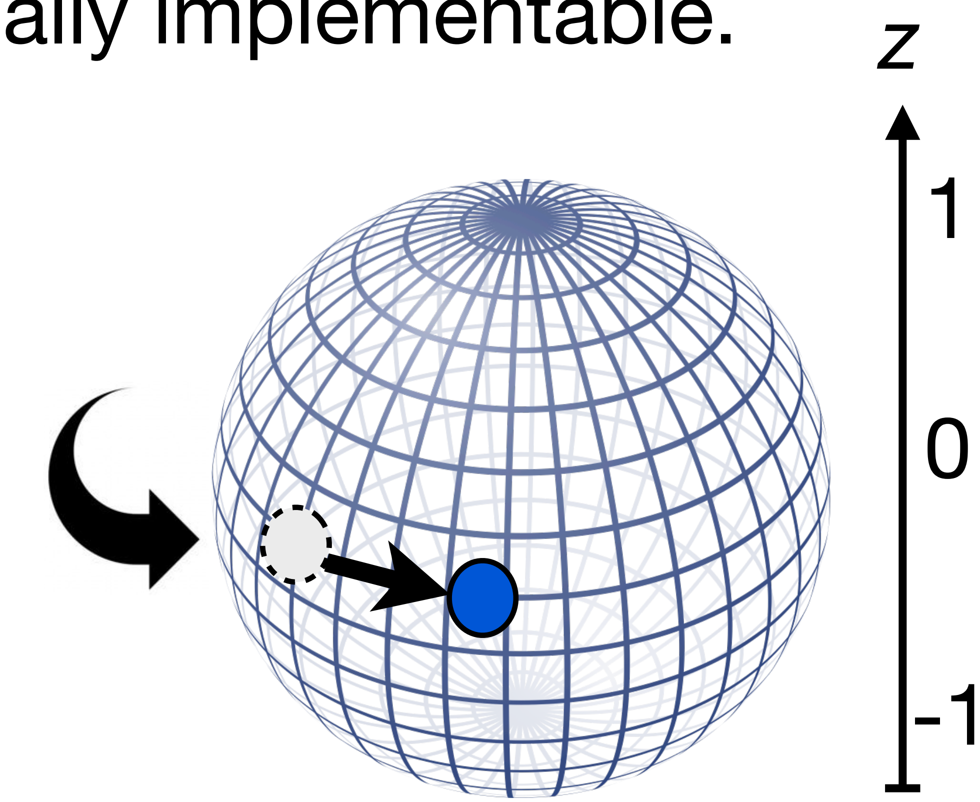
d-dim. "Bloch sphere"

T must be a rotation of the Bloch ball (reversible+linear)...
... and must preserve *p*(up), i.e. preserve the *z*-axis.

Relativistic constraints on interference experiments

Assumption: all maps of this kind are locally implementable.





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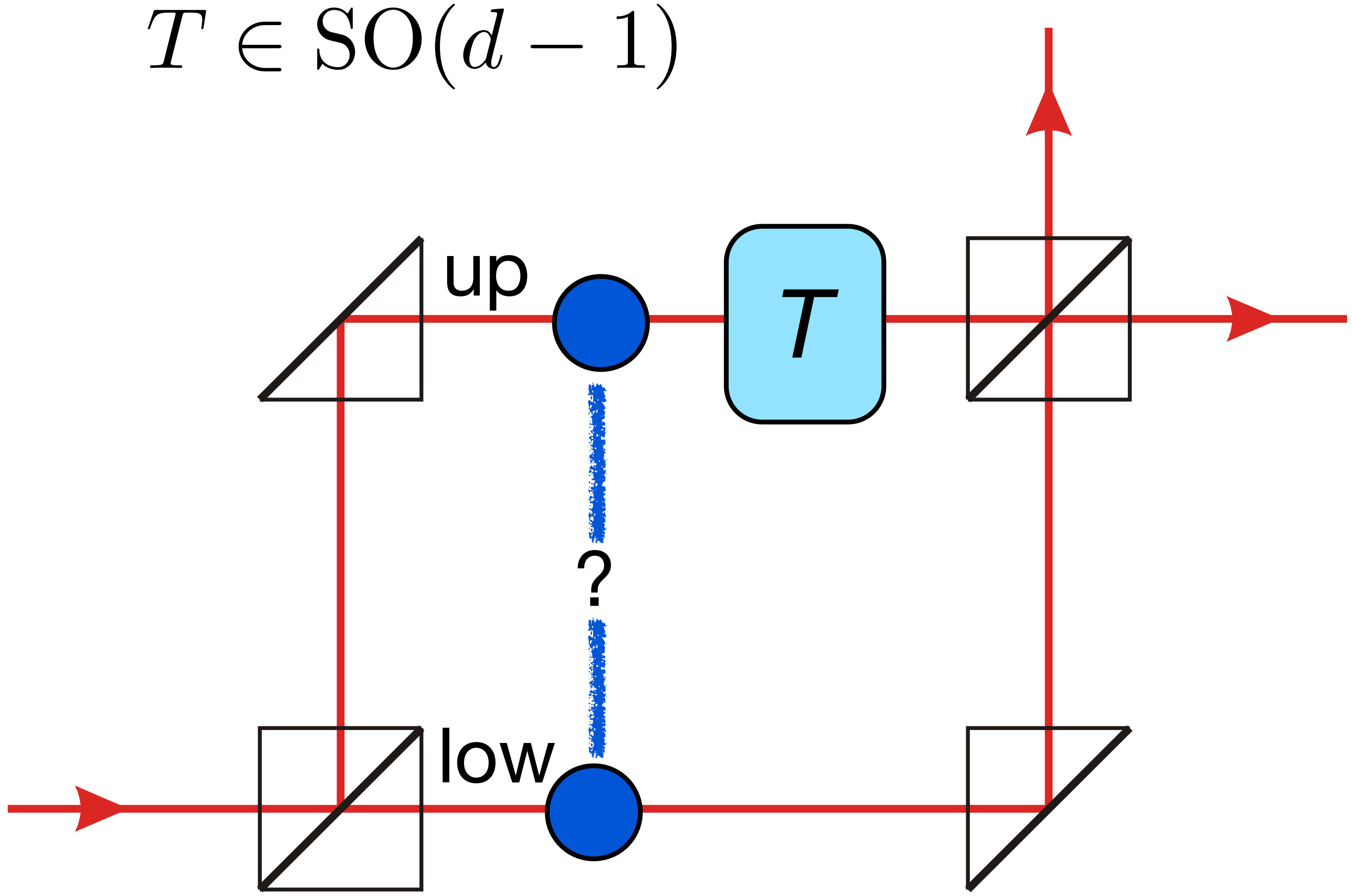

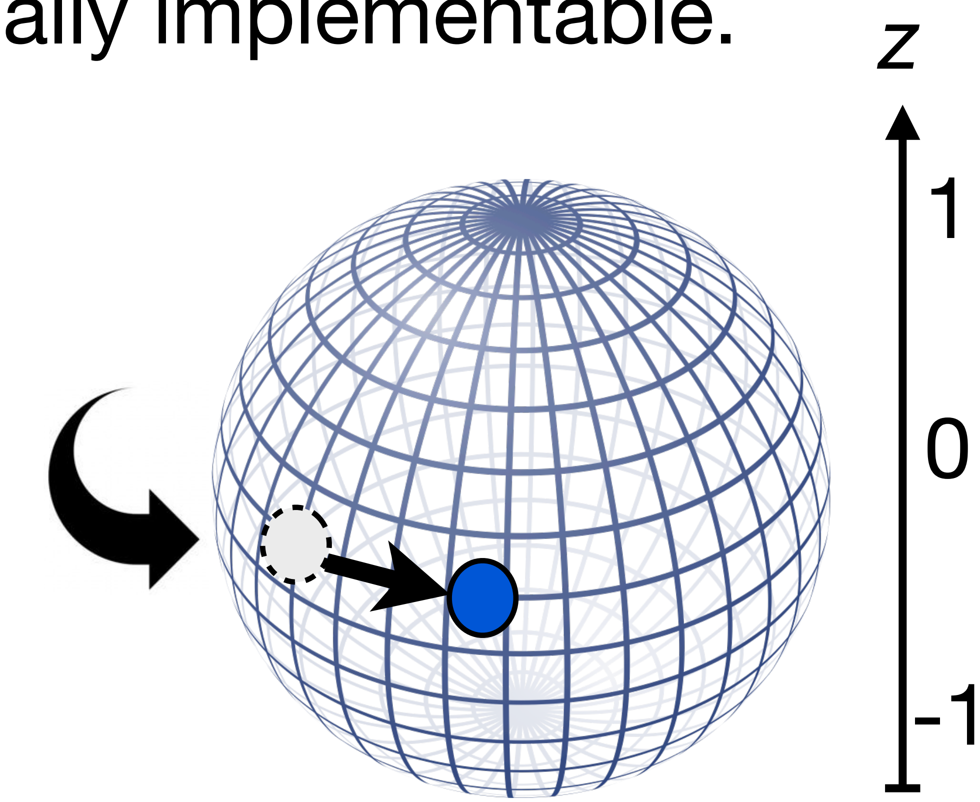

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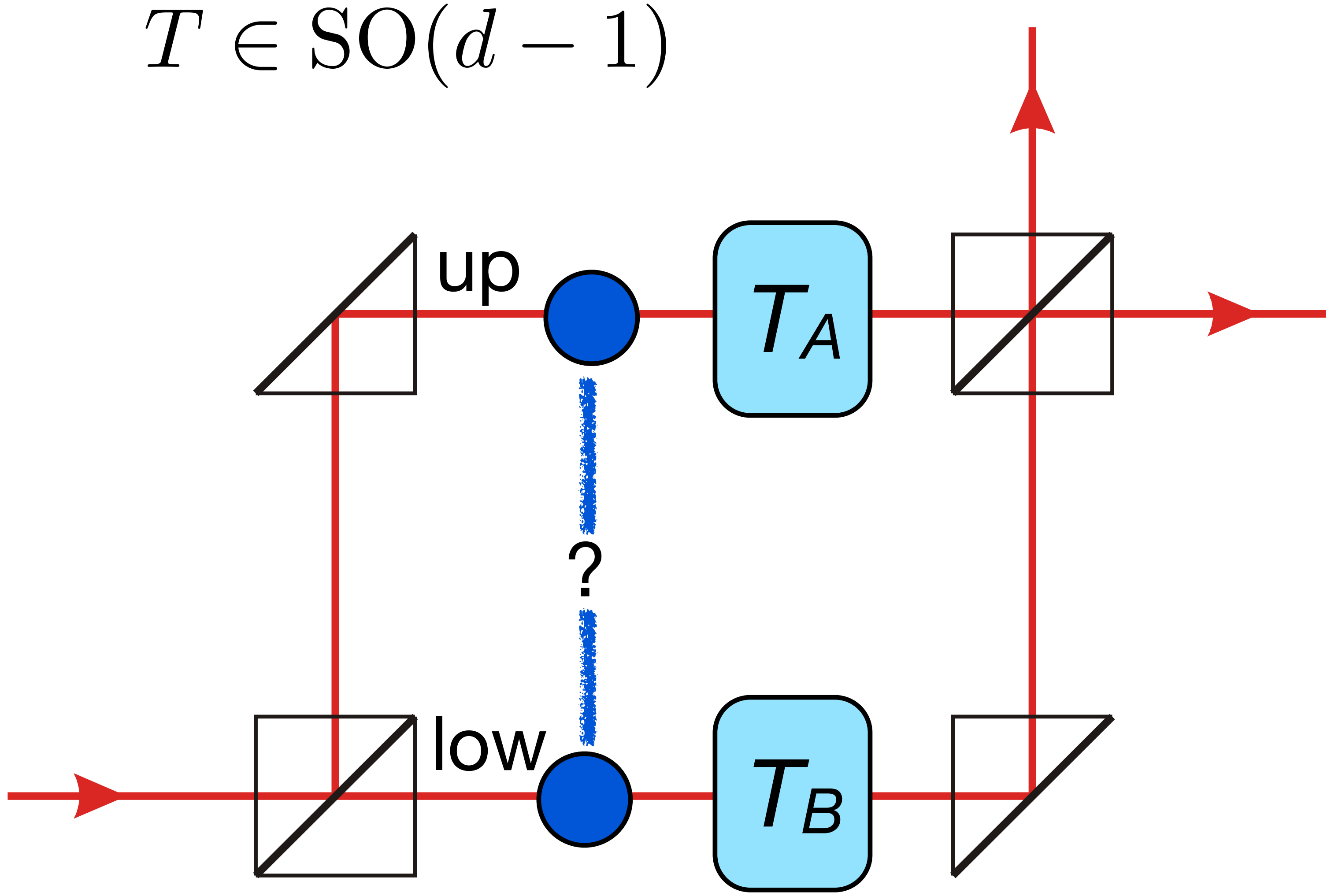

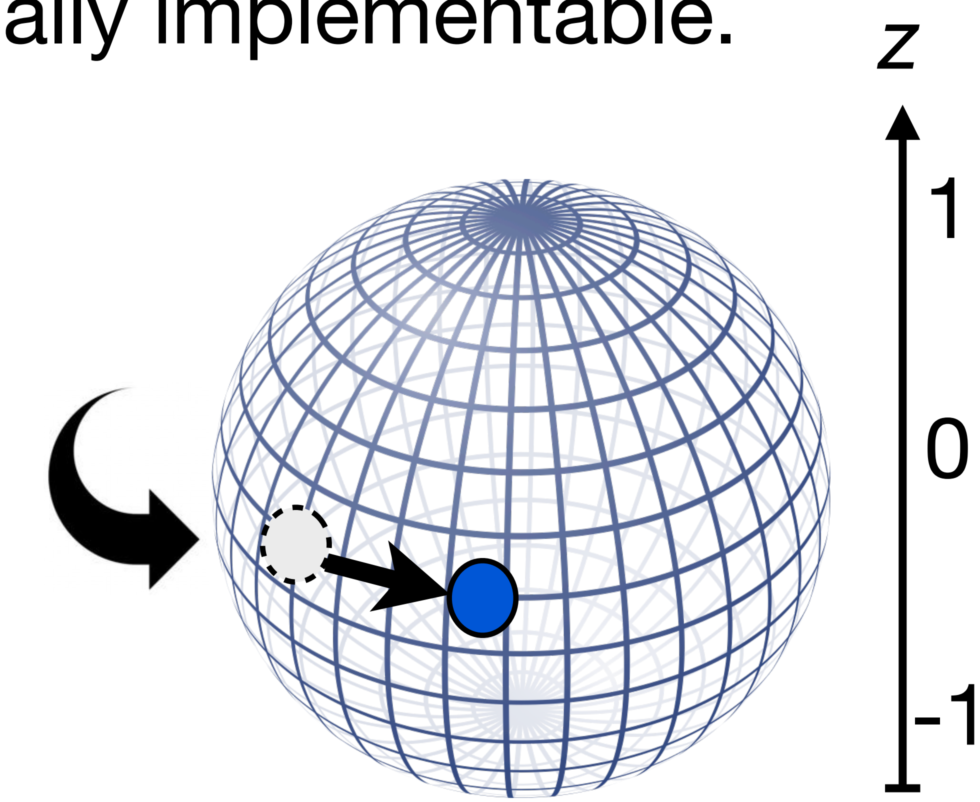

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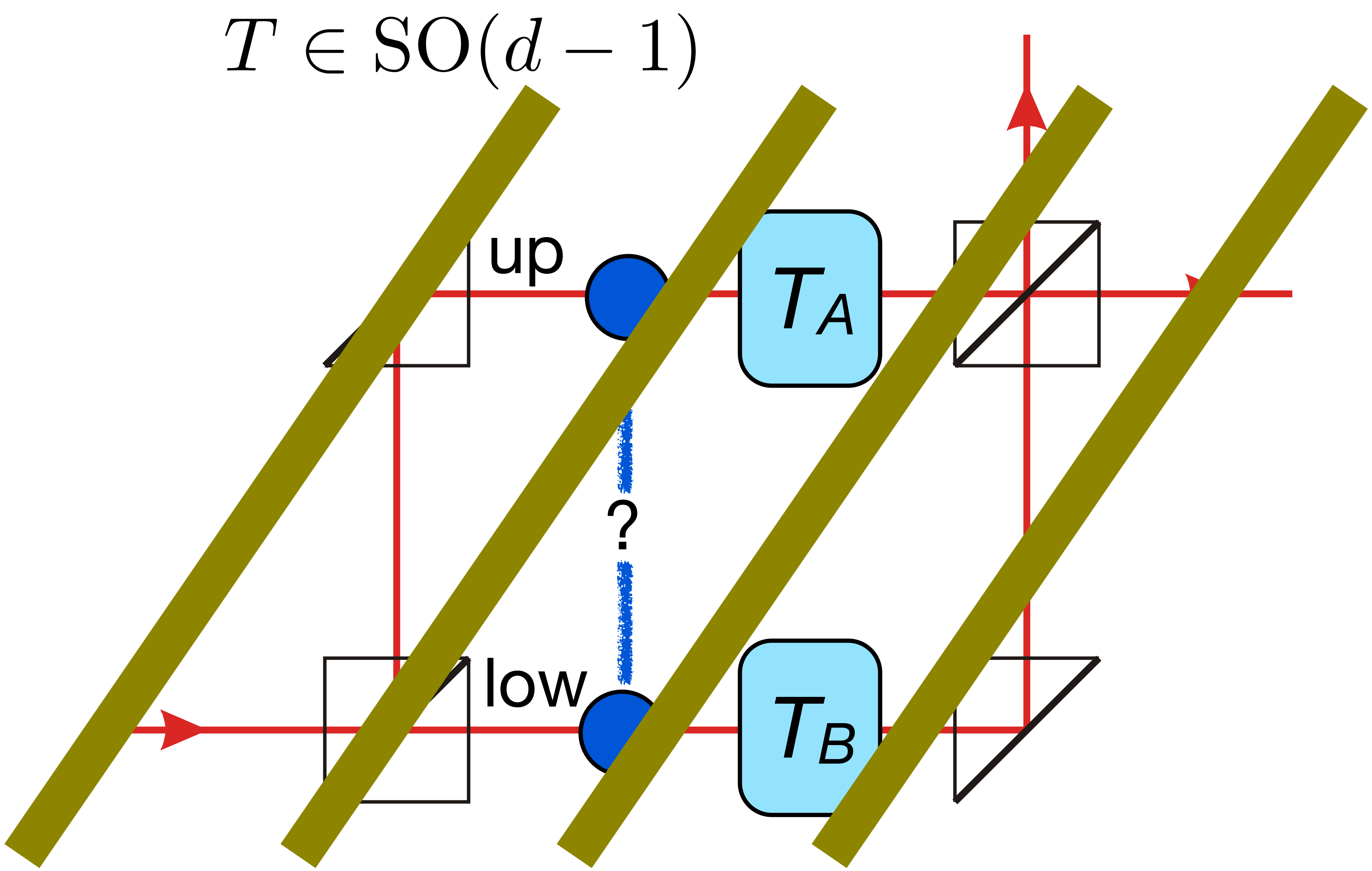

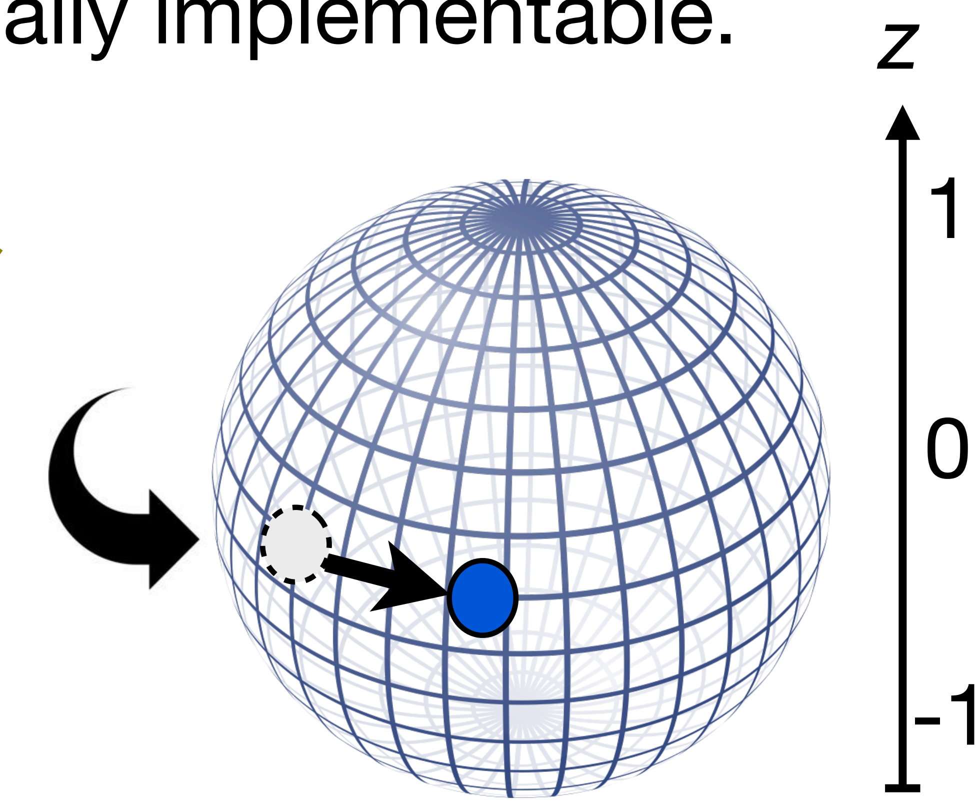

d-dim. "Bloch sphere"

Relativity: there is one frame of reference in which T_A happens first, and then T_B...

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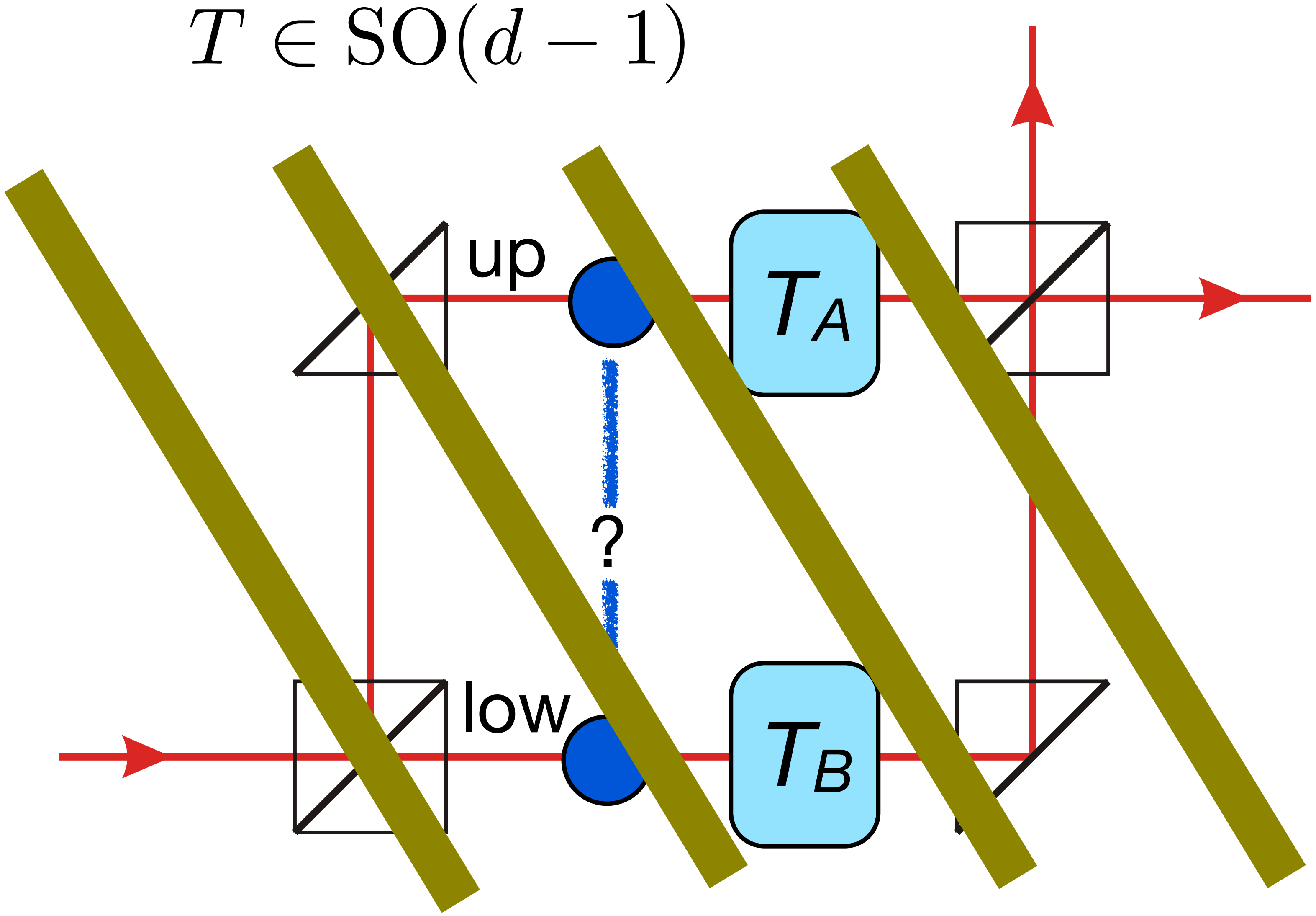

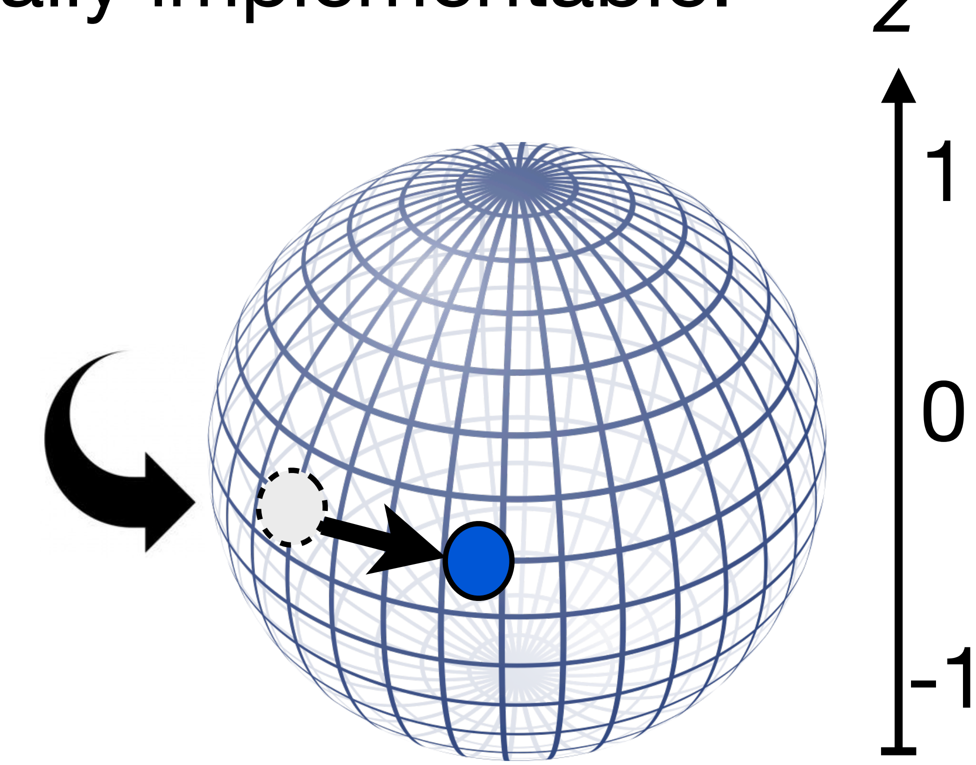

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Relativity: ... and another one in which it's the other way around!

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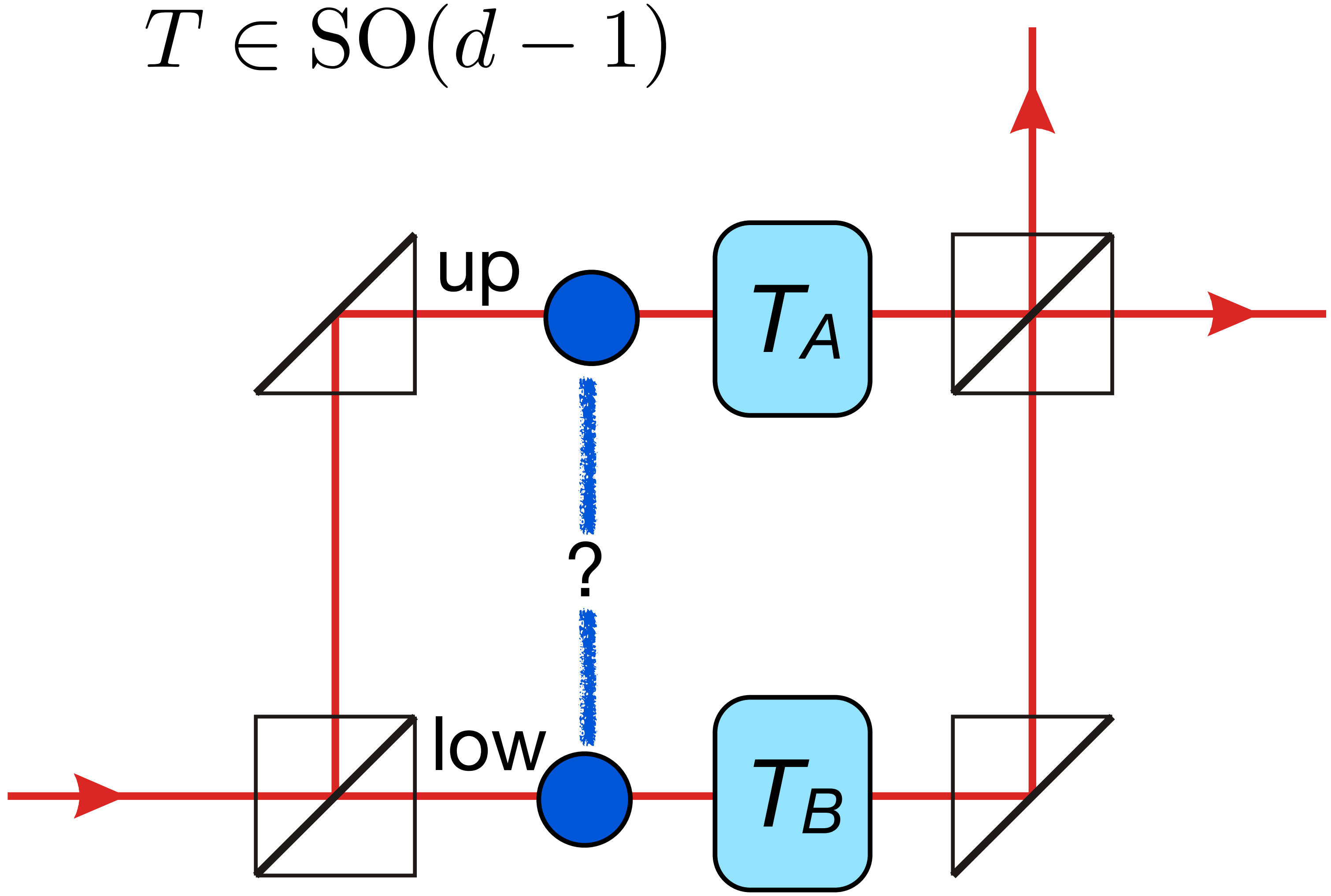
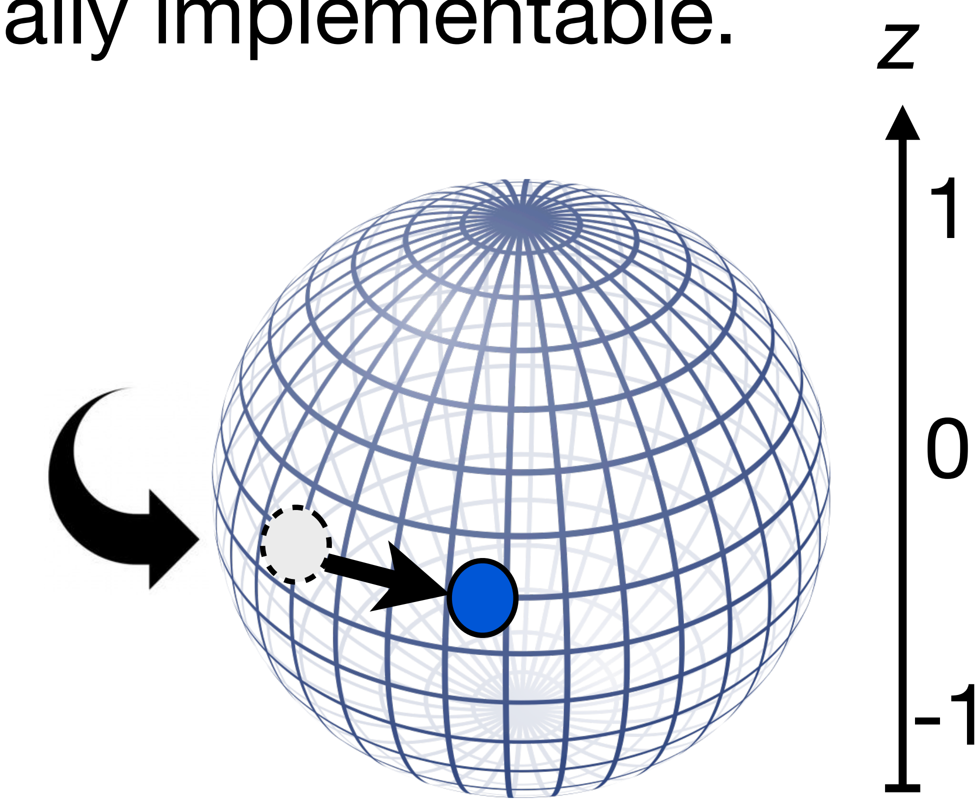




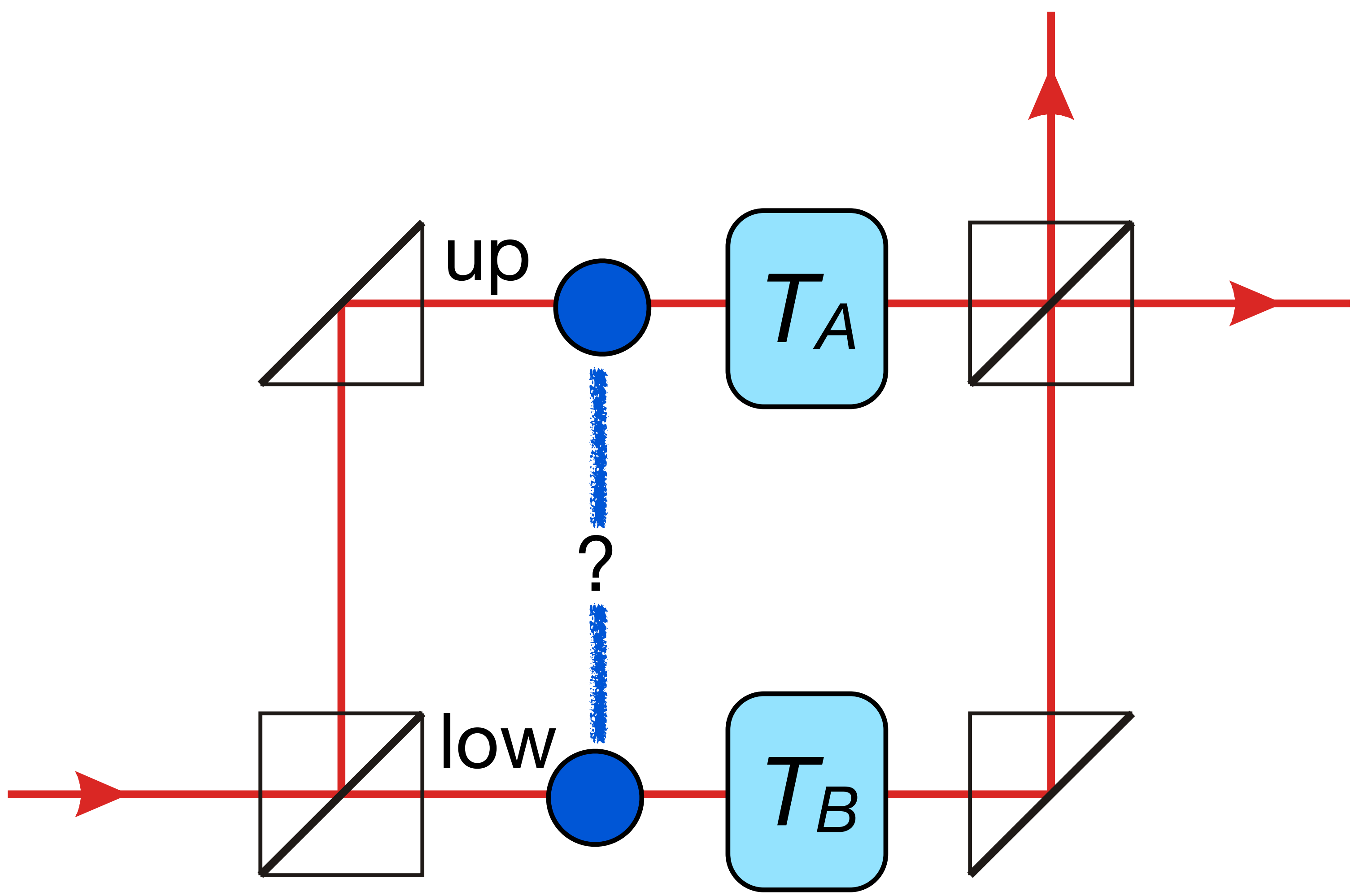
d-dim. "Bloch sphere"

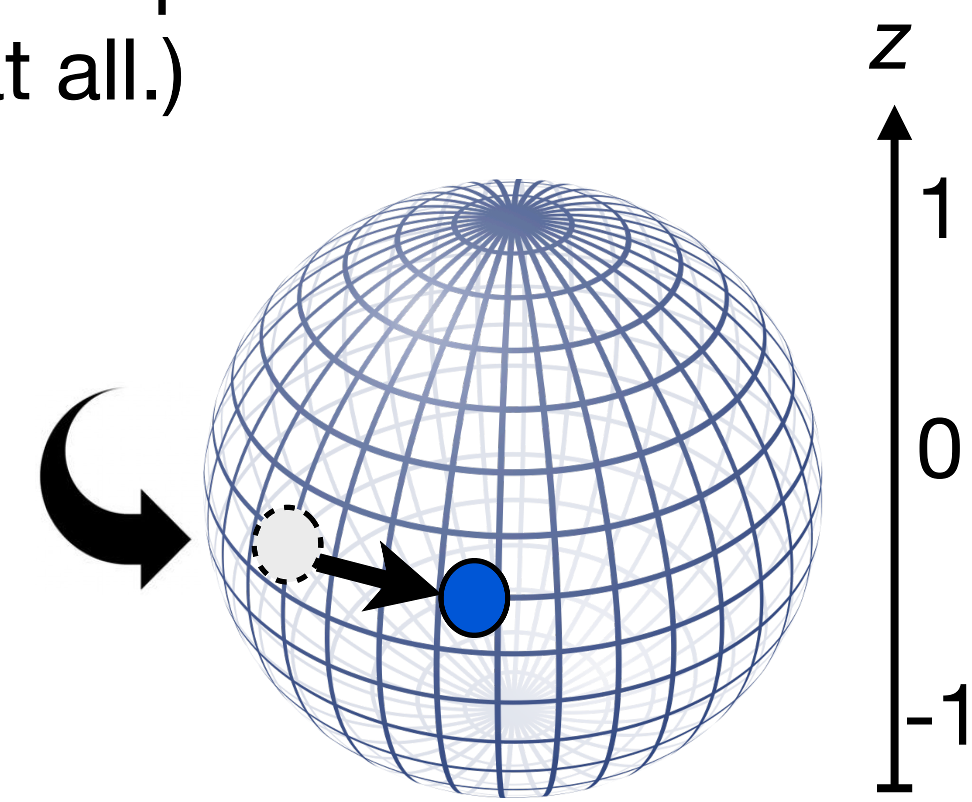
Detector click statistics is Lorentz-invariant

$\Rightarrow T_A T_B = T_B T_A$ for all $T_A, T_B \in \mathrm{SO}(d-1)$.

Relativistic constraints on interference experiments

$\Rightarrow \quad d \leq 3$ (In fact, *d*=*3*, otherwise no "phase transformations" exist at all.)





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Ll. Masanes and MM, *A derivation of quantum theory from physical requirements*, New J. Phys. **13** (2011)
Ll. Masanes, MM, D. Pérez-García, and R. Augusiak, *Entanglement and the three-dimensionality of the Bloch ball*, arXiv:1111.4060.

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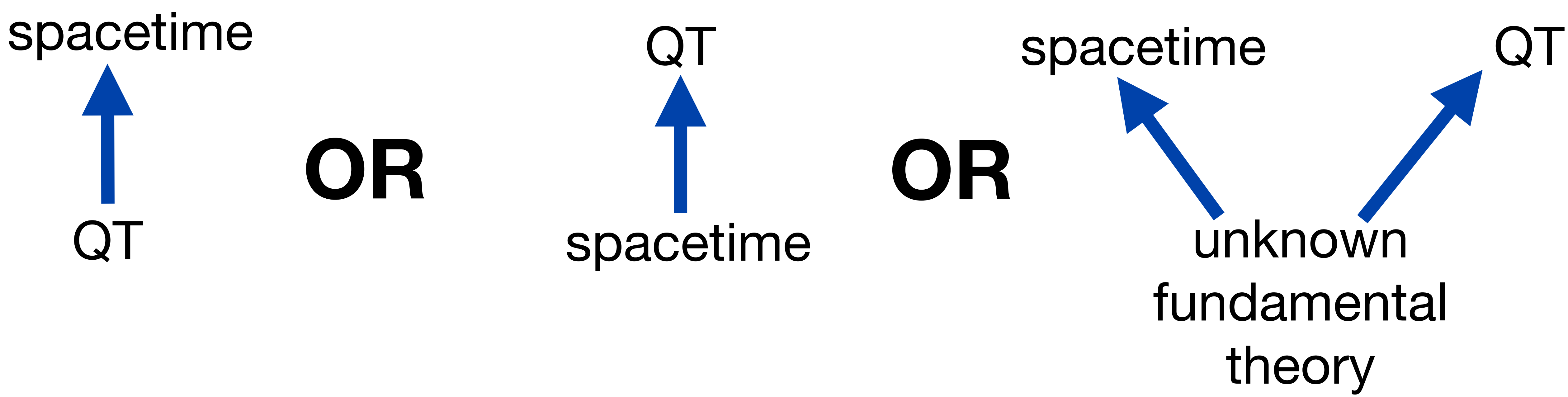
- Cf. talk by Daniel Rohrlich / PR-box idea: Continuous spacetime + probabilities are **hard** to combine → their structures are closely related!

Quantum theory and spacetime

Plausible hypothesis:



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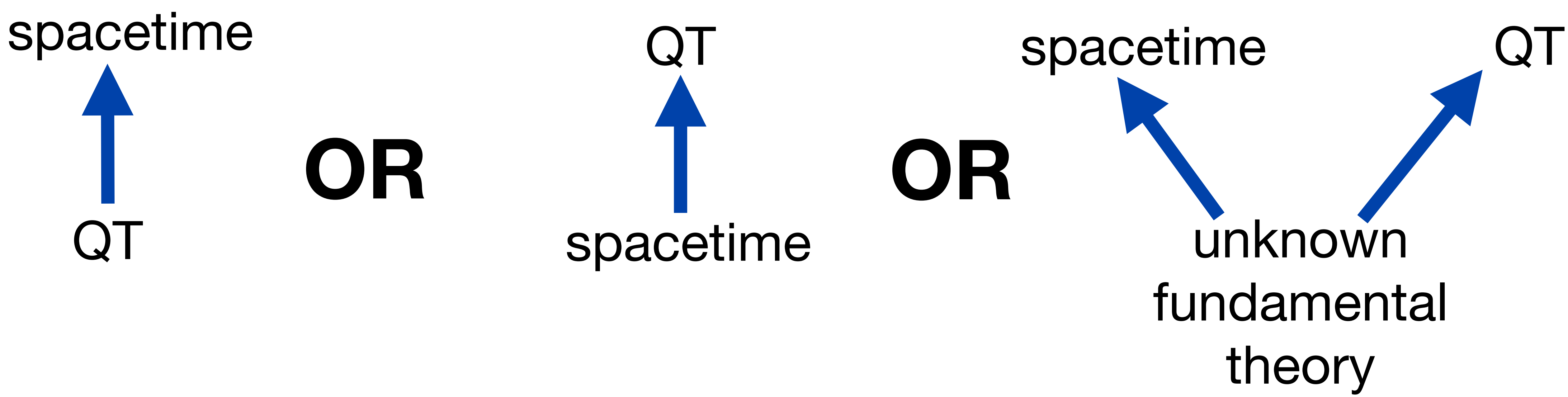

- Recall Borivoje Dakic's talk...

  "The classical limit of a physical theory and the dimensionality of space"

- ... and Mauro d'Ariano's talk.

  "Information-theoretic principles for QT and QFT" → relativistic covariance emergent.

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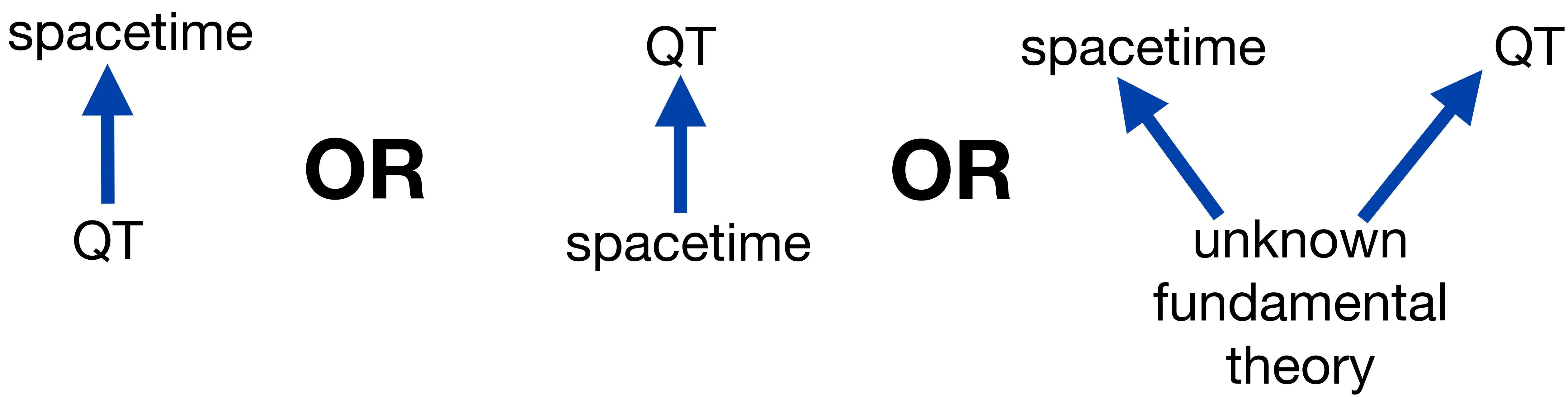

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To me, crucial hint is the spin-1/2 particle:

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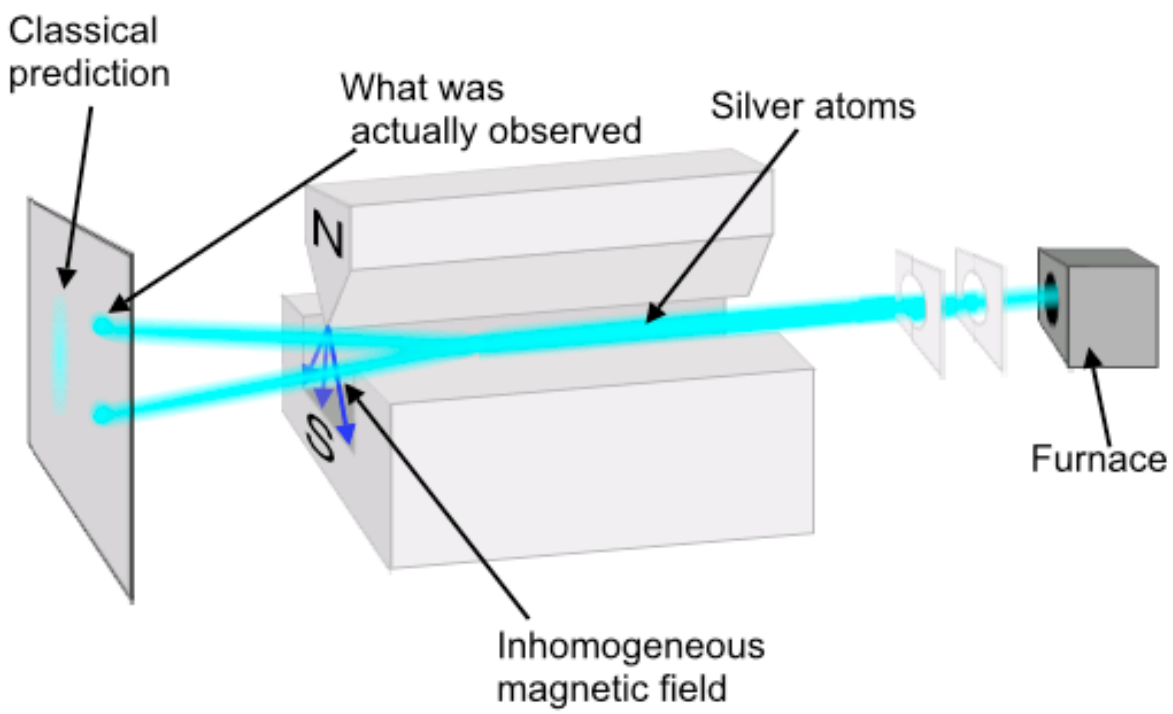
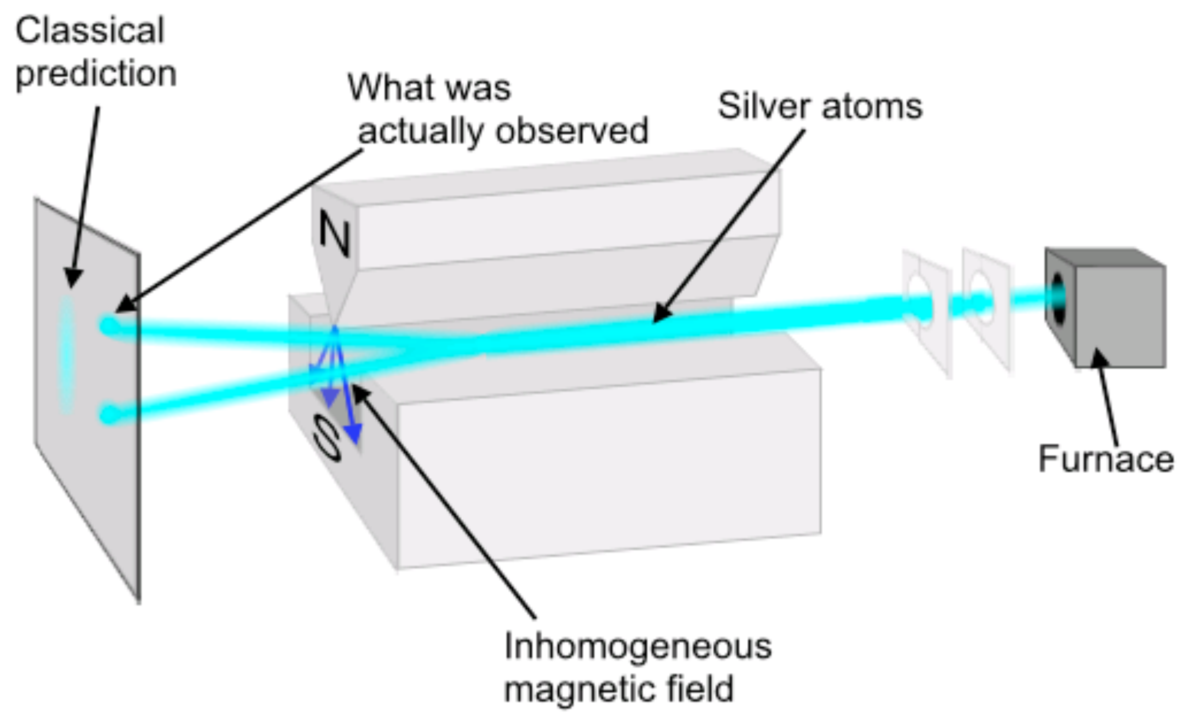


spatial rotations

$\updownarrow$ 1:1

transformations of the probabilistic state



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Most physicists are too used to it to wonder...

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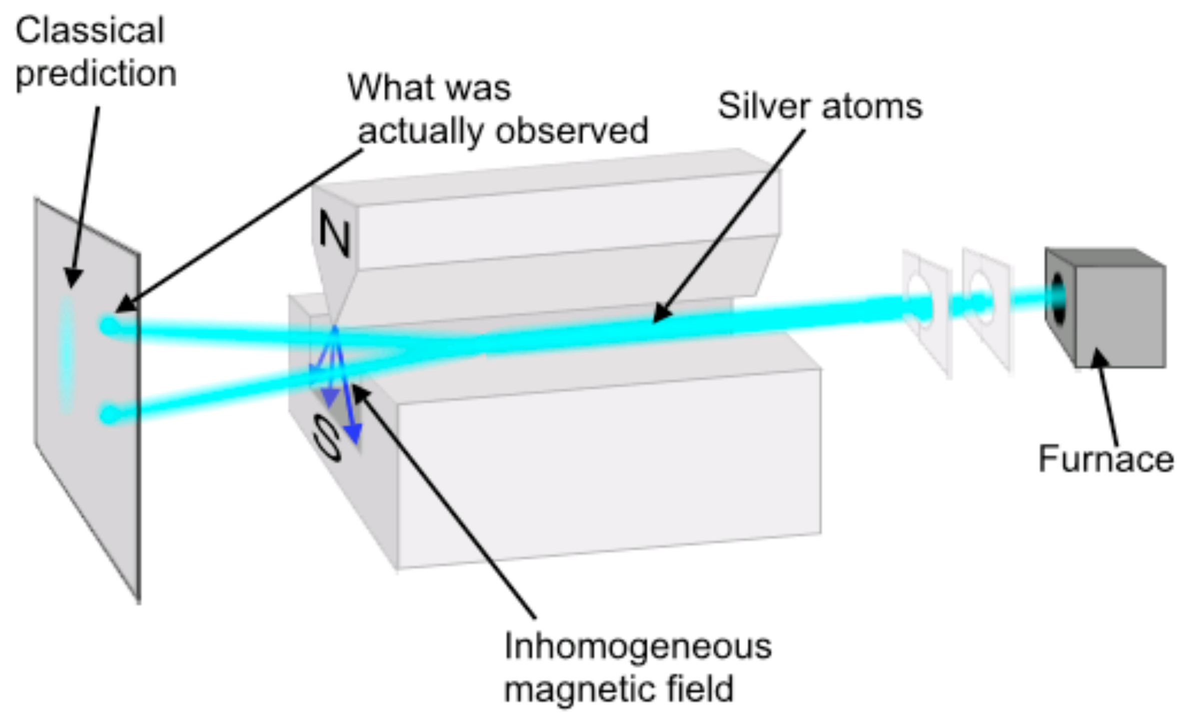

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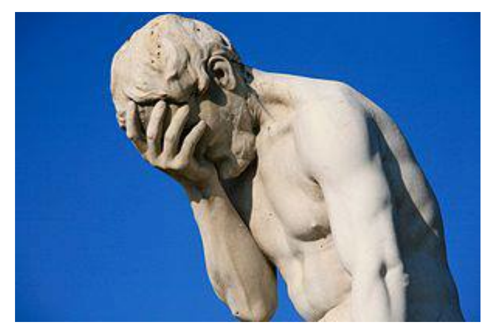

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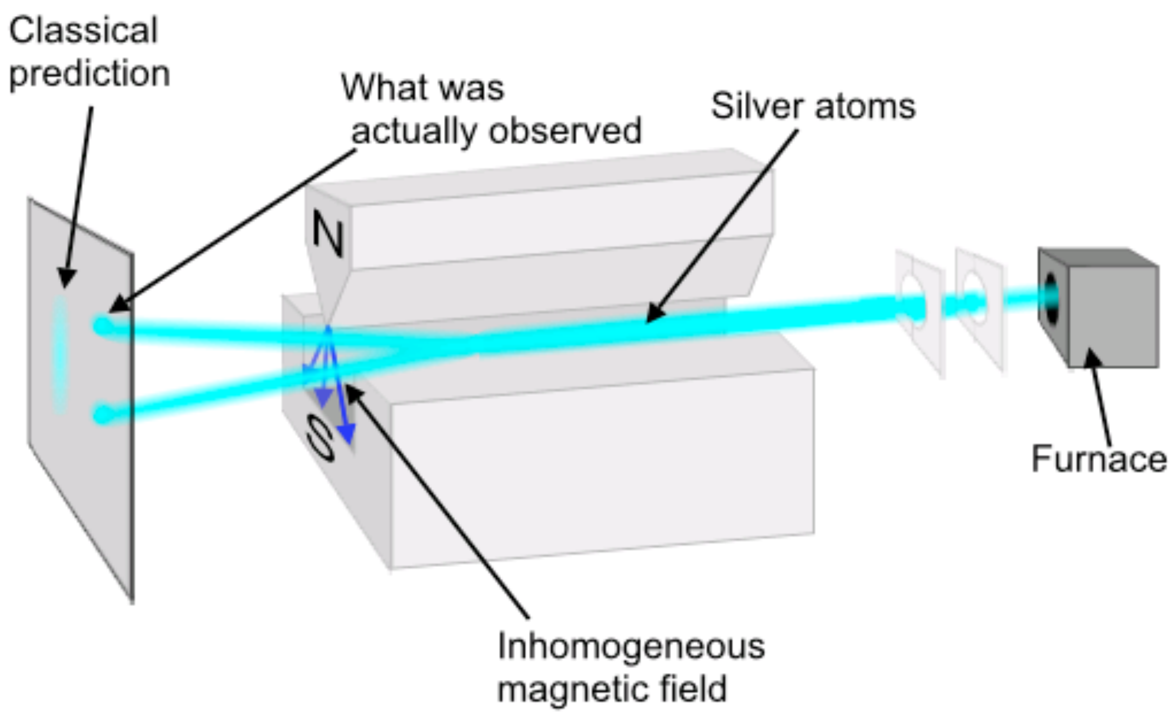

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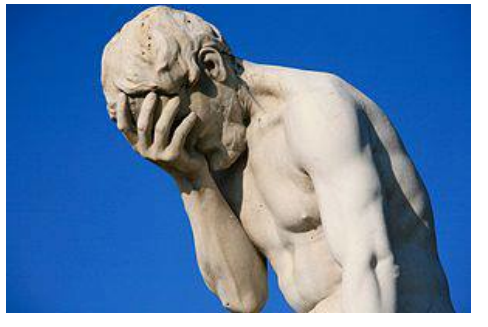

MM and Ll. Masanes, *Three-dimensionality of space and the quantum bit: an information-theoretic approach*, New J. Phys. **15**, 053040 (2013), arXiv:1206.0630.

Question: Could a similar 1:1 relation also hold in other spatial dimensions *$d \neq 3$* and other probabilistic theories?

Quantum theory and spacetime

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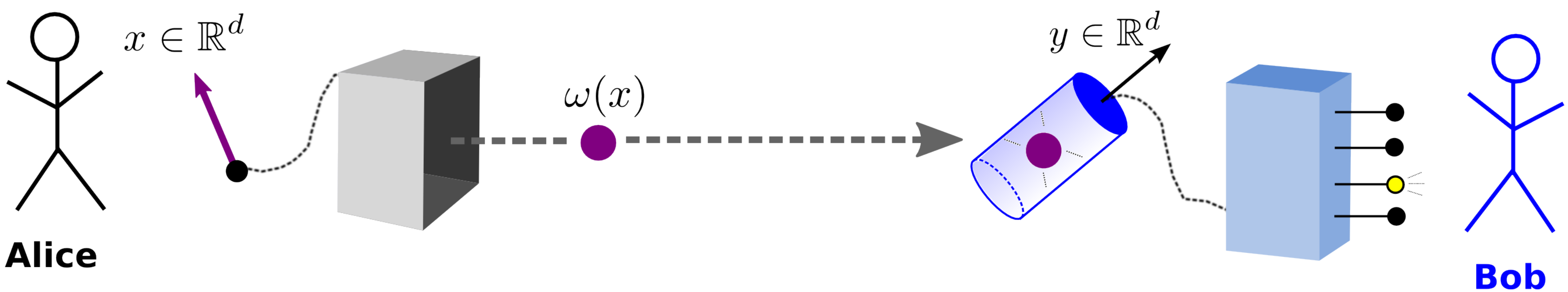
Formulate as information-theoretic task:



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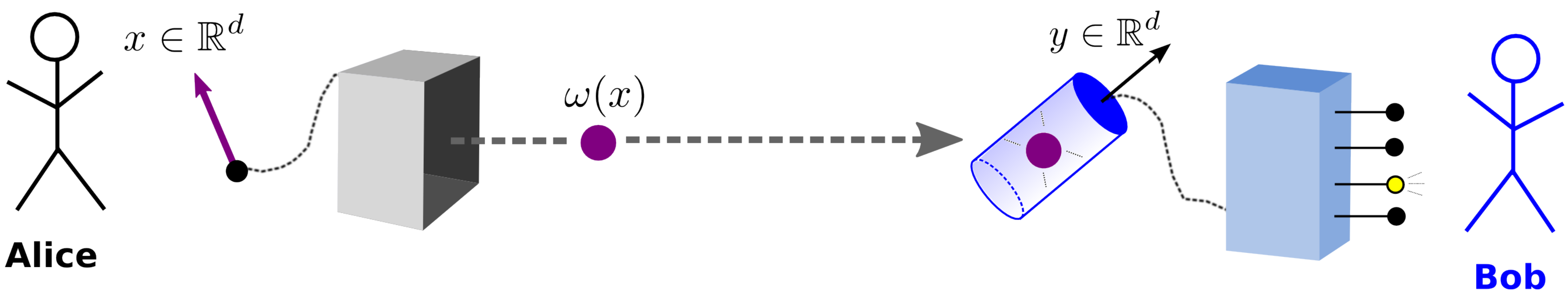

Suppose there is a probabilistic system such that...

1. Alice can encode any spatial direction into the state, but
2. any attempt to encode more results in information loss.
3. Coordinate transformations on pairs of these systems are uniquely determined by their action on single systems.
4. Pairs of these systems can interact reversibly and continuously in time.

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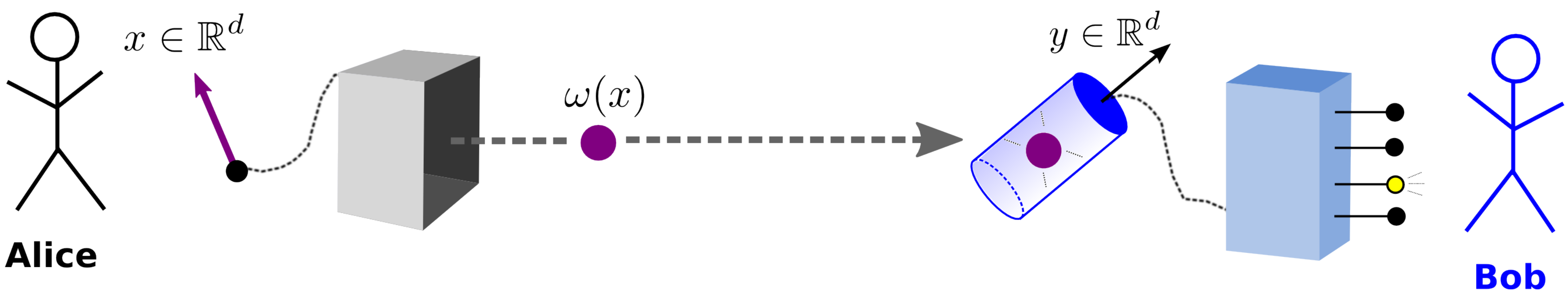
Theorem: Then the spatial dimension must be *d=3*, the systems are qubits, and pairs of these systems are quantum 4-level systems evolving unitarily in time.



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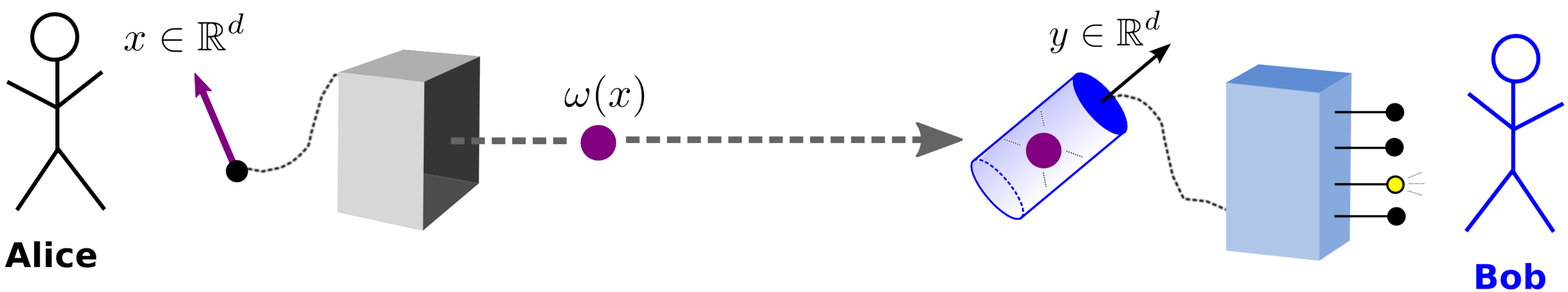
One more Theorem: If "spatial direction" $x \in \mathbb{R}^d, \quad |x| = 1$, is replaced by "spatial orientation" $X \in SO(d)$, then there is no solution (for topological reasons).



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Spatial coordinates from probabilities

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Appendix C

Spatial coordinates from probabilities

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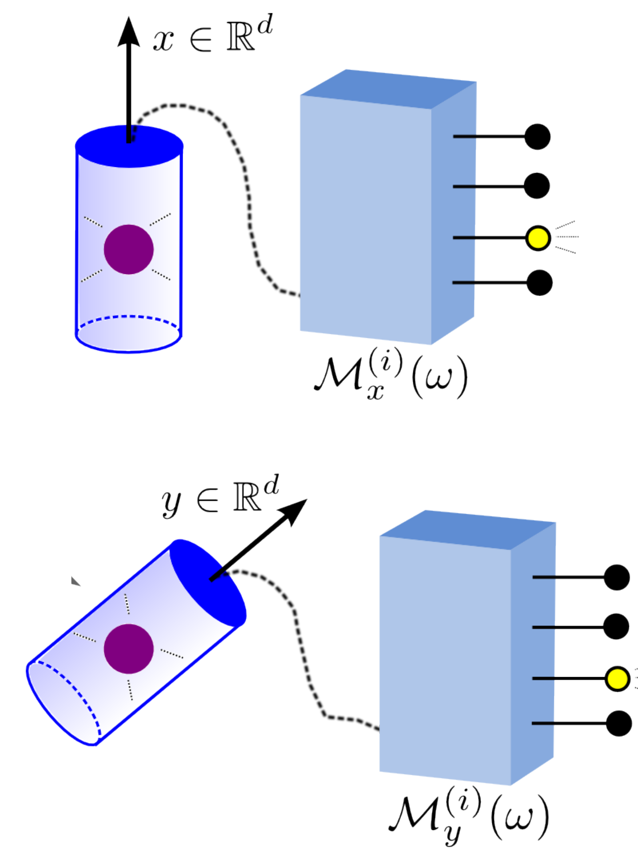
Appendix C

Physicist Alice wants to determine the angle between two measurement devices.



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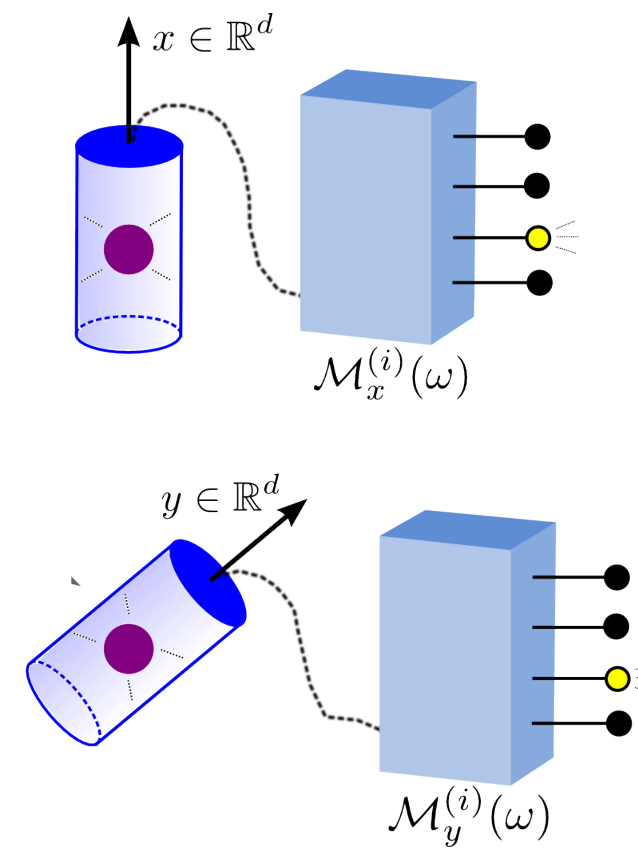

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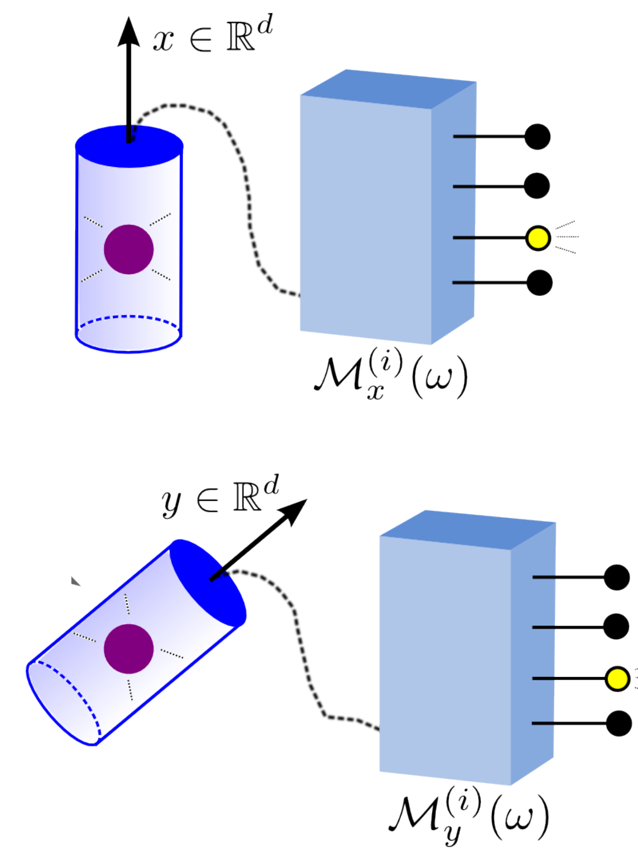

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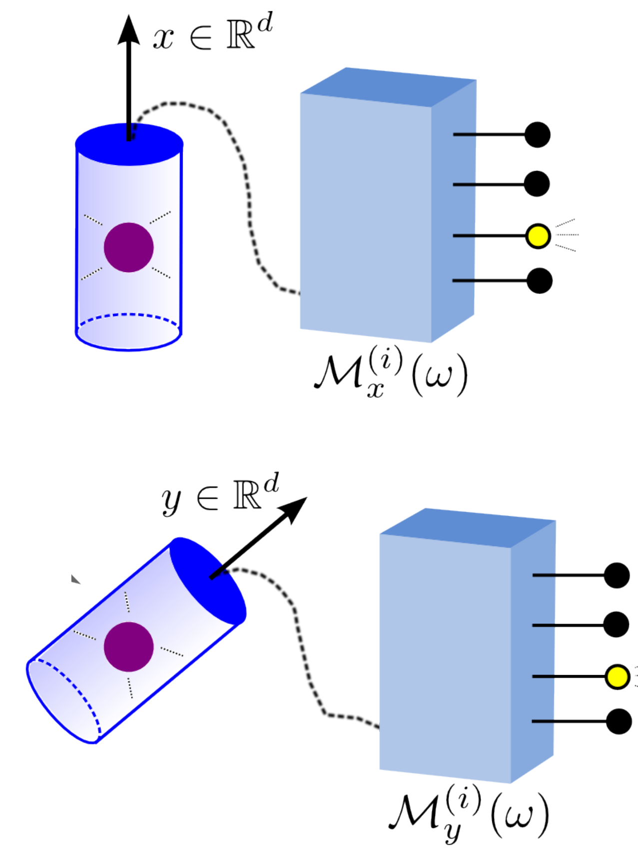

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$\Rightarrow$ Probabilities deliver Euclidean structure for free.

Reverse-engineering physics

Let's understand physics by analyzing how its parts (logically) work together.

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The Bloch ball is 3-dimensional because of...

- ... Minkowski structure on interferometers?
- ... locally-tomographic continuous interaction?
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There cannot be two independent reasons for a single fact.
Ultimately, all those points must be fundamentally related.

Conclusions

- The quest for higher-order interference theories is experimentally relevant, and still open...
- ... an axiomatic approach might help.

H. Barnum, MM, and C. Ududec, arXiv:1403.4147

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Thank you!