A resource-theoretic approach to thermodynamics

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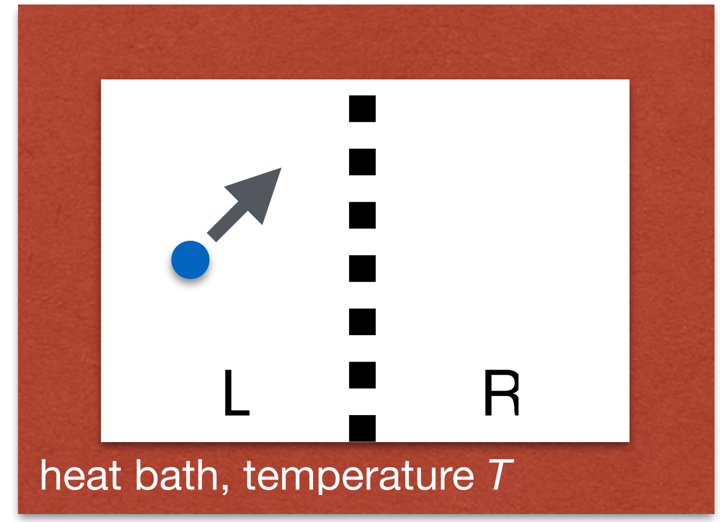
Drawing by Lidia del Rio, http://www.itp.phys.ethz.ch/itp/itp/people/delriol

Outline

1. Motivation

What is a resource theory?
Why and how thermo as a RT?

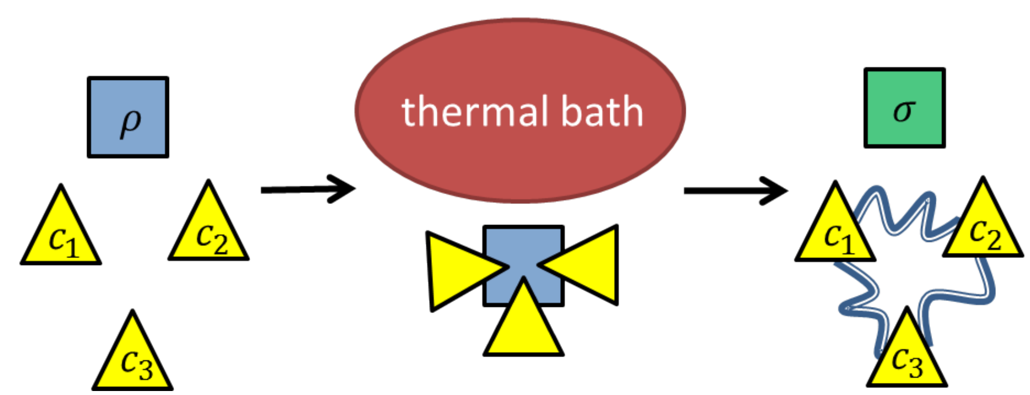


2. The resource theory of athermality

Definition, results, surprises.

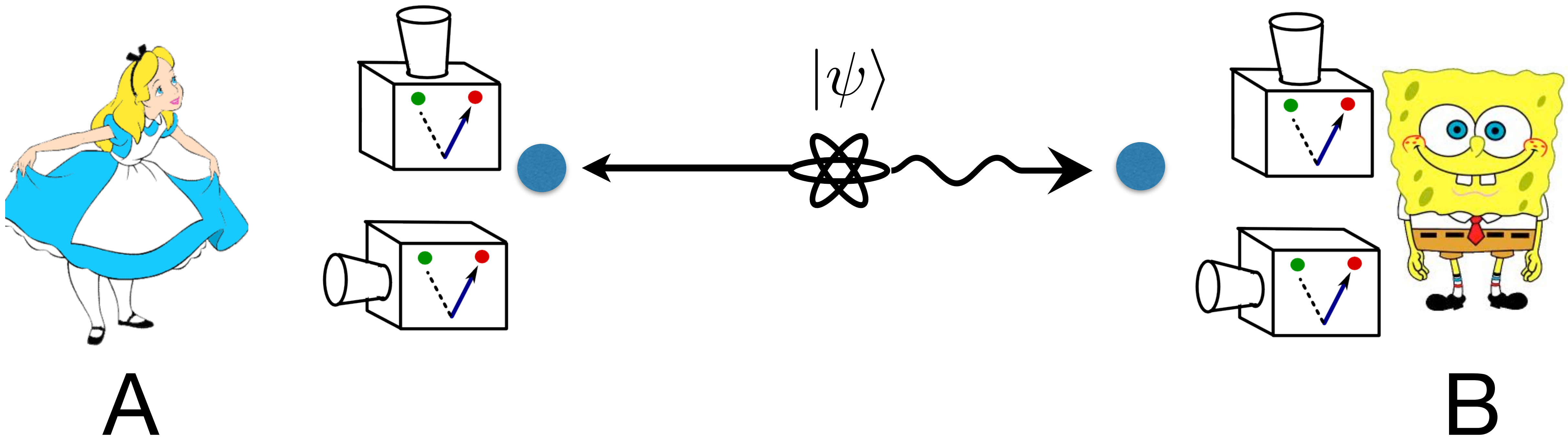
3. Extracting work from absence of correlations

How to do more by knowing less…



1. Motivation: what is a resource theory?

Origin: quantum information theory



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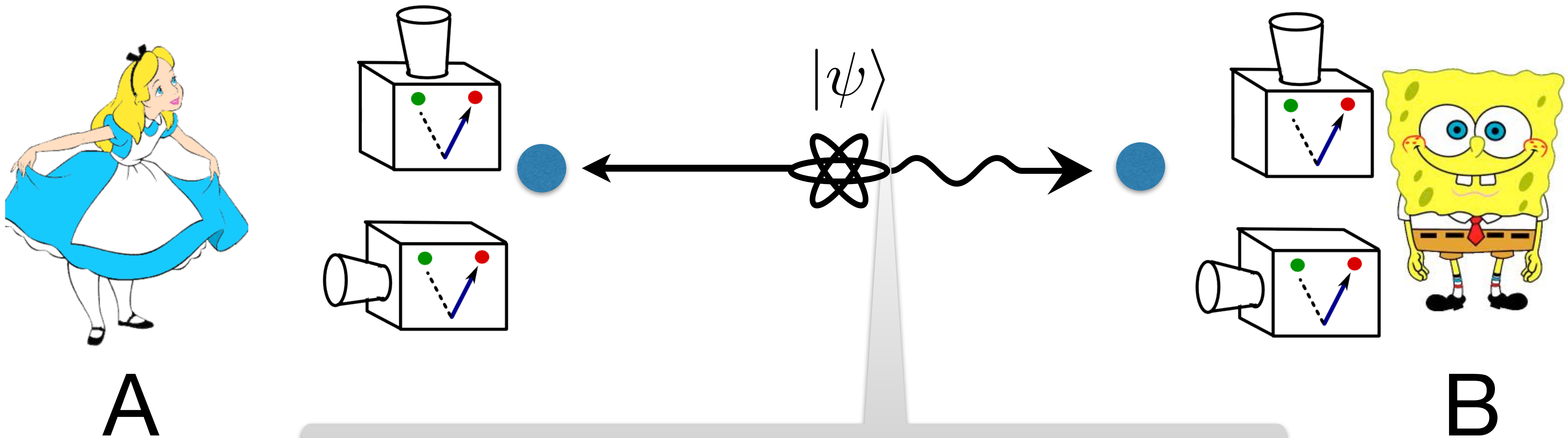

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|H_A\rangle|H_B\rangle - |V_A\rangle|V_B\rangle\right)$$

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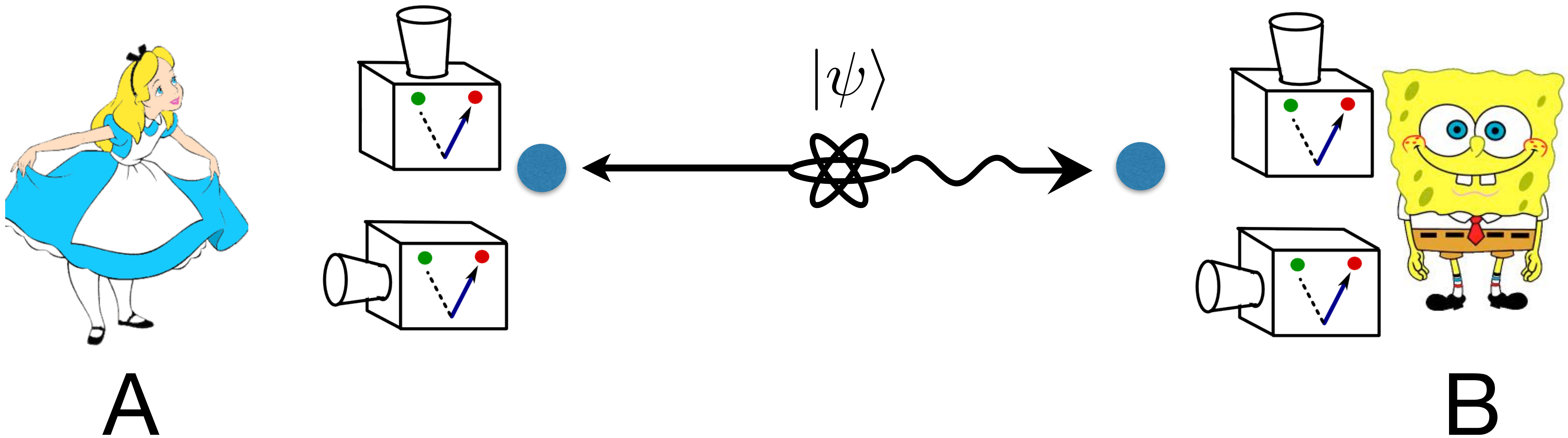

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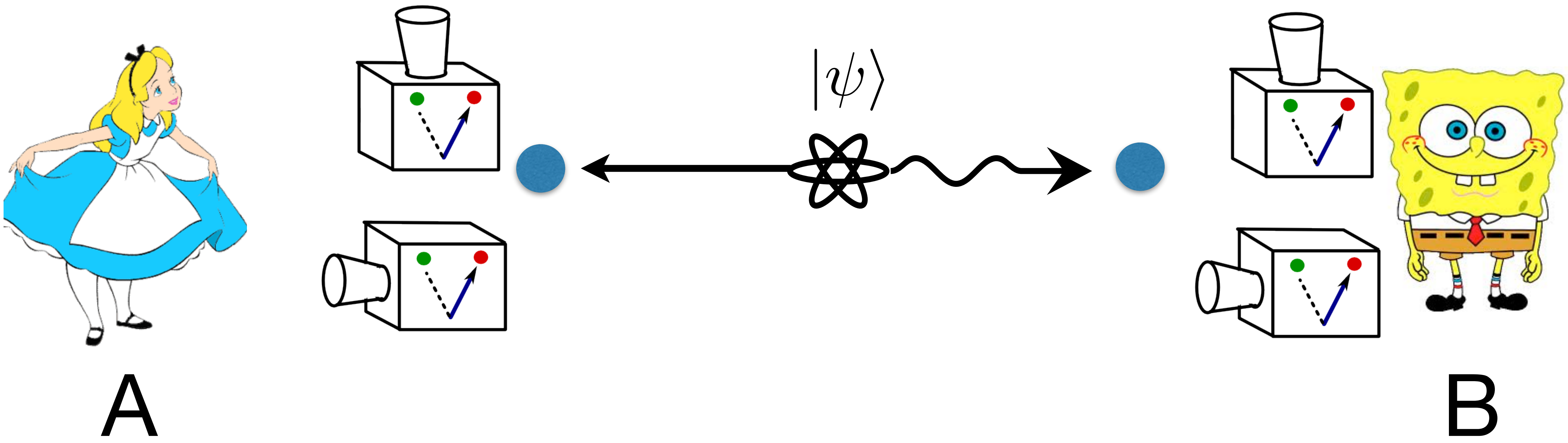
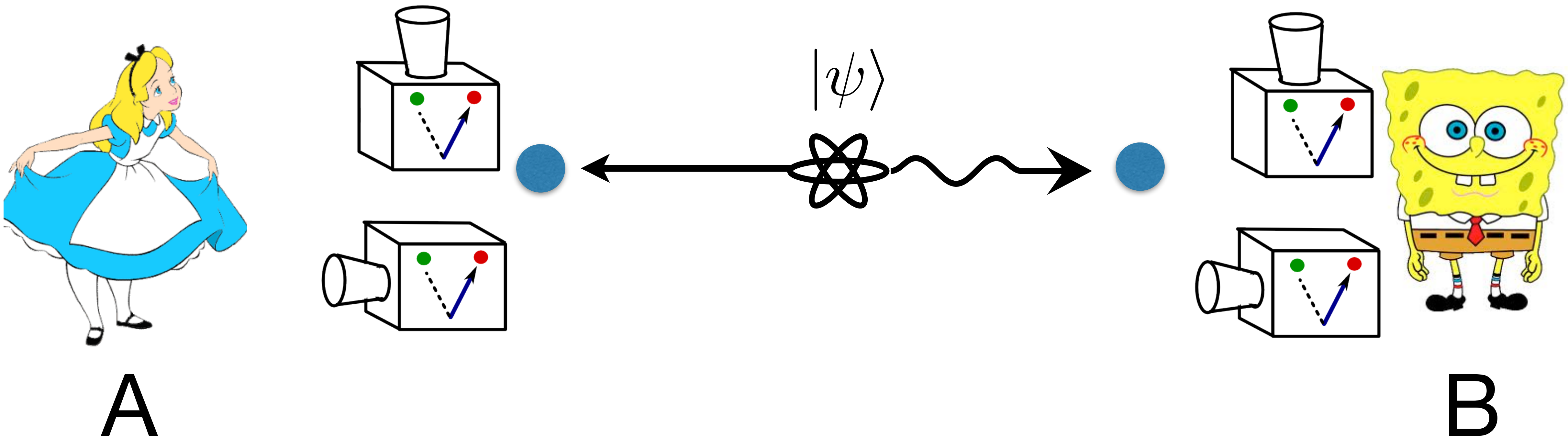


Alice and Bob are very far apart, so

- they can only act locally on the particle pair,
- can only produce new states of the form $|\varphi_A\rangle|\psi_B\rangle$,
- can talk on the telephone.

Origin: quantum information theory



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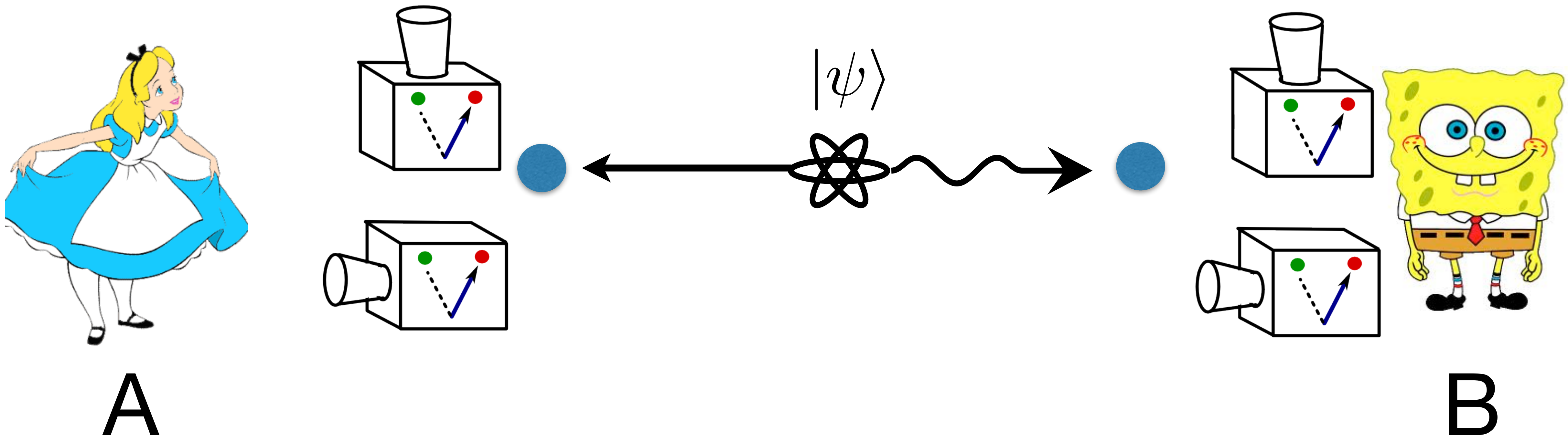

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Under these restrictions, **what can they do**?

For example, can they transform $|\varphi\rangle \longrightarrow |\psi\rangle$?

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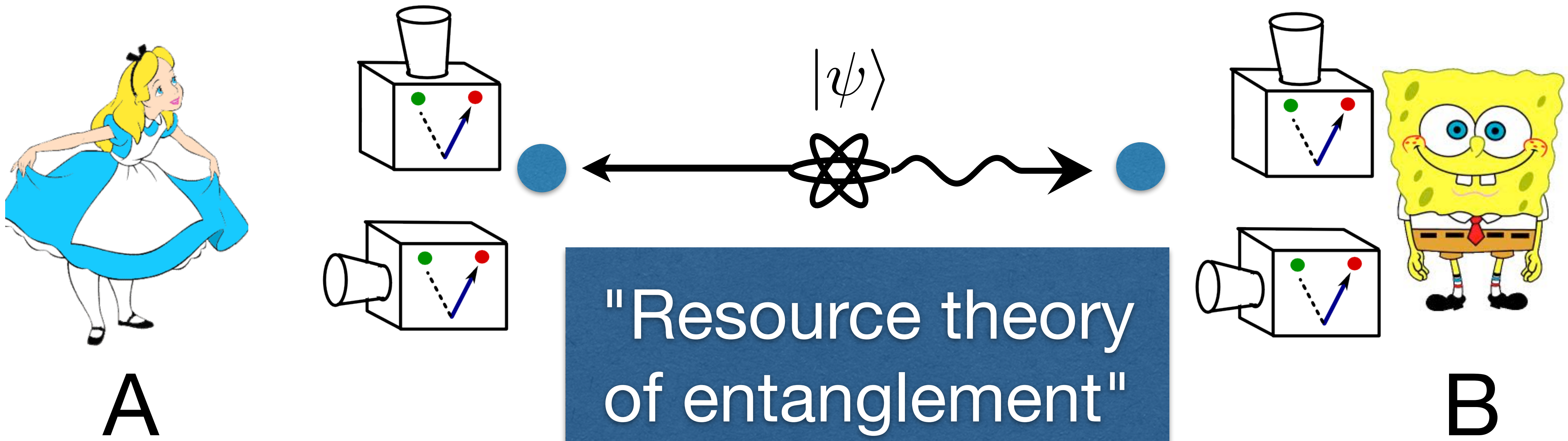

"Resource theory of entanglement"

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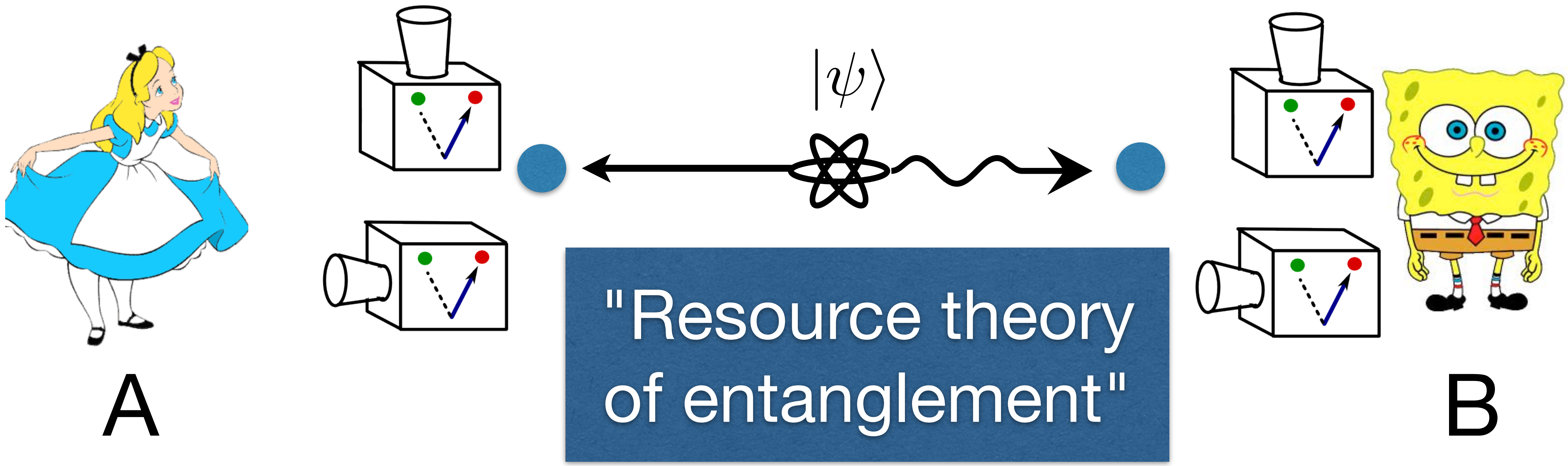

Free states / objects: $|\varphi_A\rangle|\psi_B\rangle$ (all other states are resources).
Allowed transformations: local operations + classical communication

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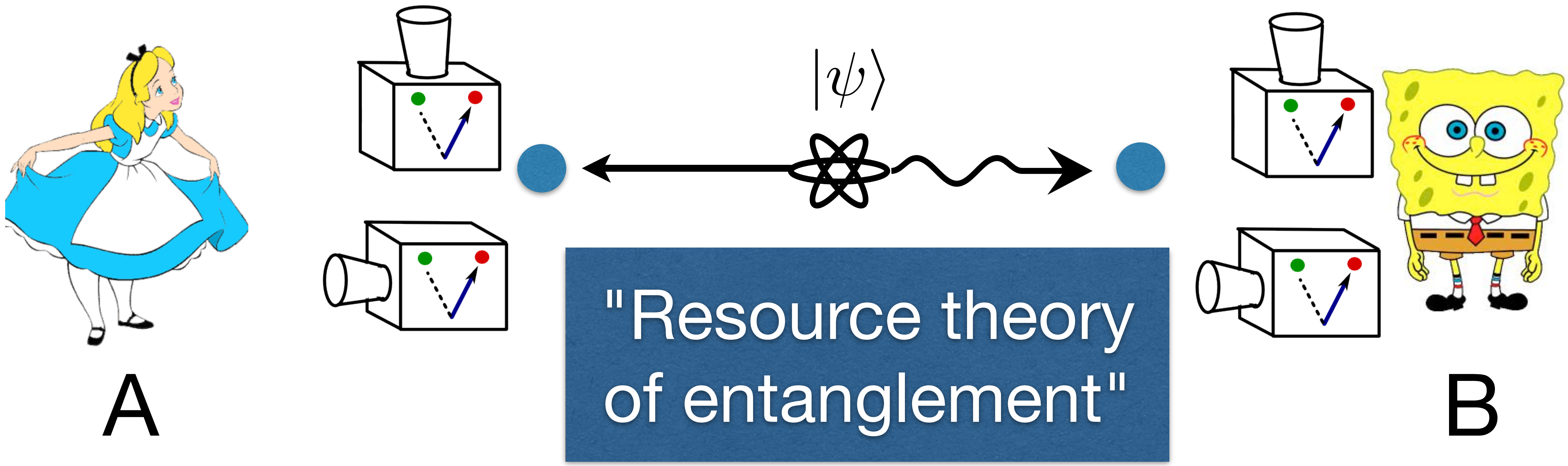

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Questions: can we transform $|\psi_1\rangle$ to $|\psi_2\rangle$?
How many copies of $|\psi_1\rangle$ do we need to obtain *n* copies of

Resource theory of cooking

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Free operations: cutting, mixing etc.

Free objects: water, electricity,...?

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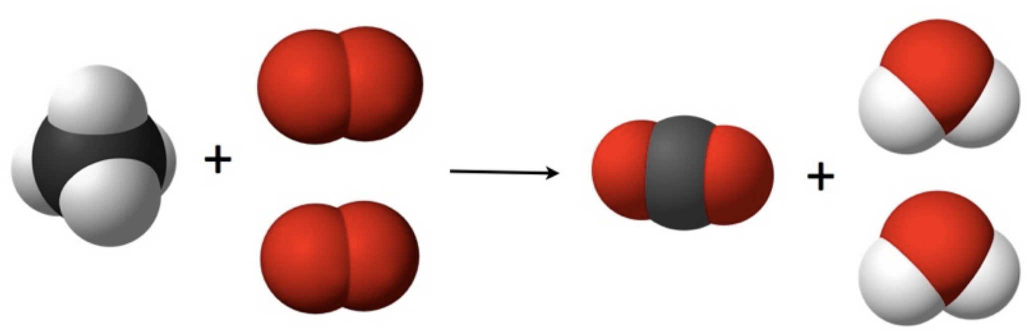
Resource theory of chemistry



$$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$$

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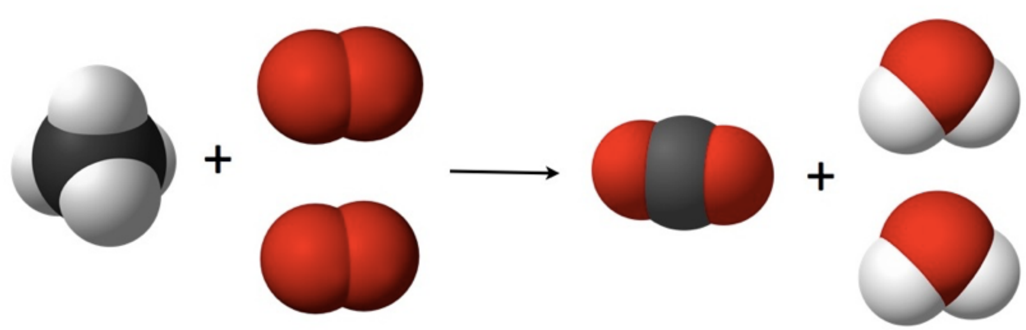
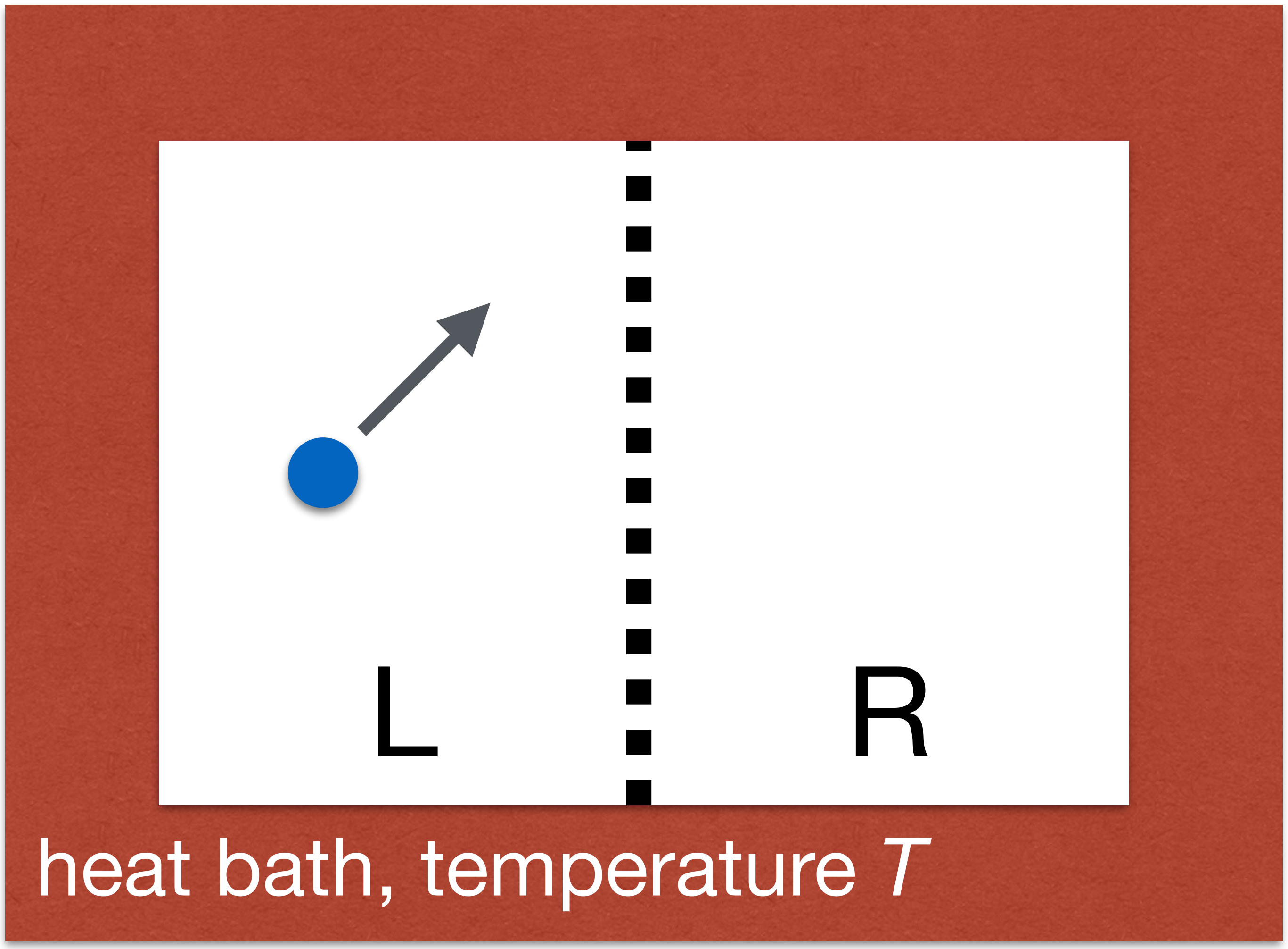


$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$

Category-theoretic formulation:
B. Coecke, T. Fritz, R. W. Spekkens, A mathematical theory of resources, arXiv:1409.5531

Thermodynamics as a resource theory - **how** and **why** ?

Thought experiment: single particle in a box



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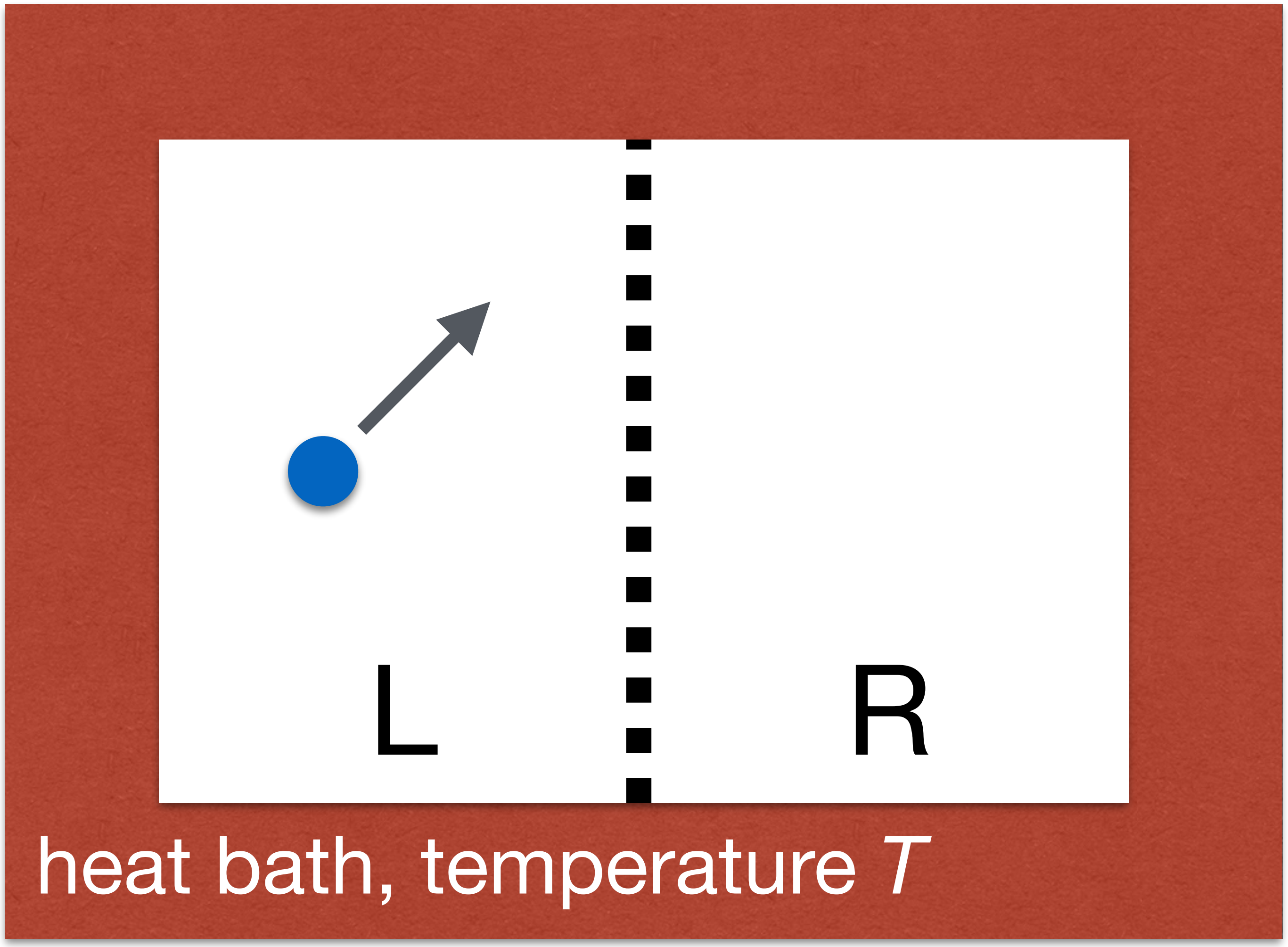

State of particle:

$$\left(\begin{array}{c} \mathrm{Prob}(L) \\ \mathrm{Prob}(R) \end{array} \right)$$

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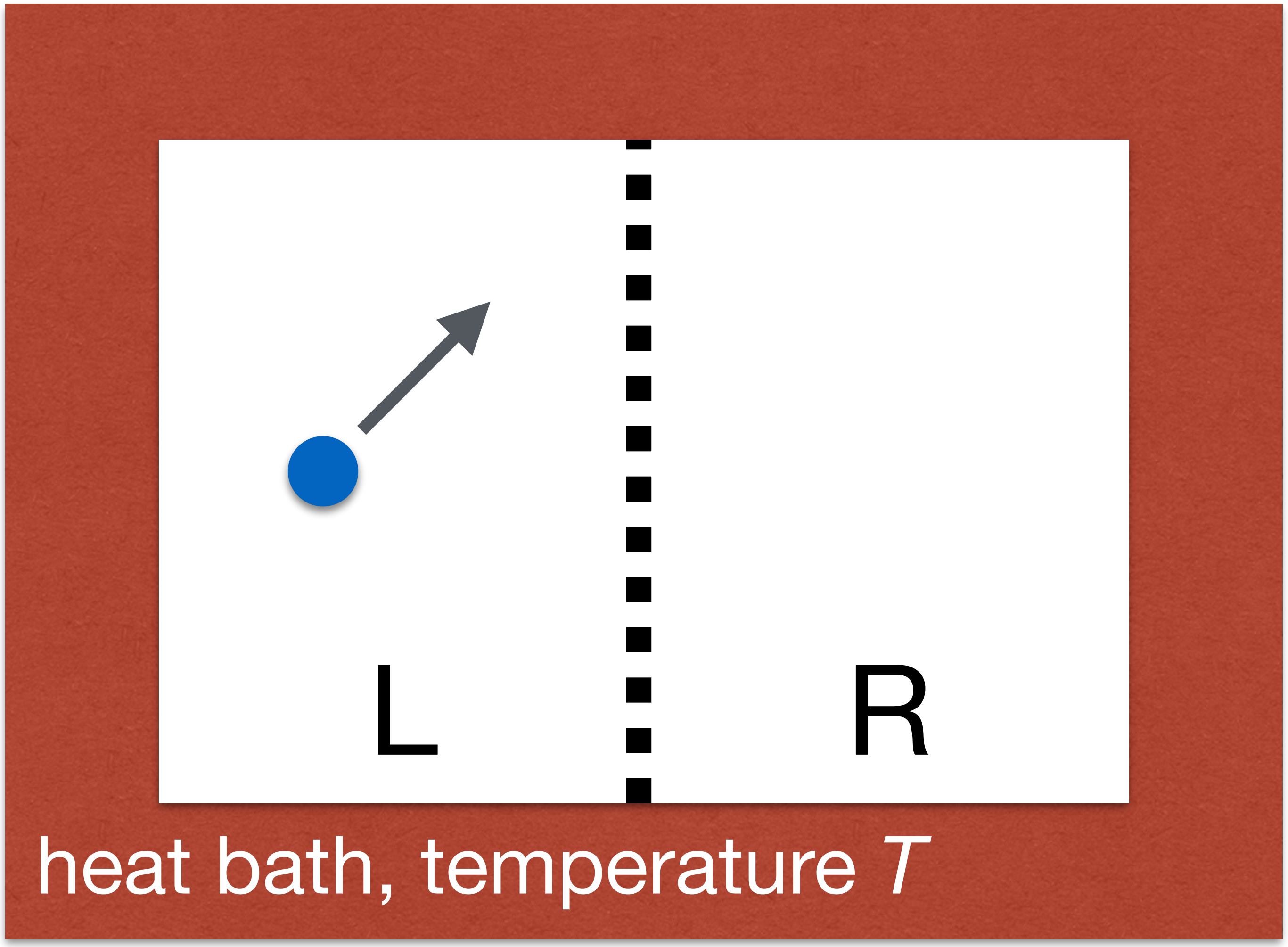

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Know for sure the particle is left
→ can extract energy by expansion

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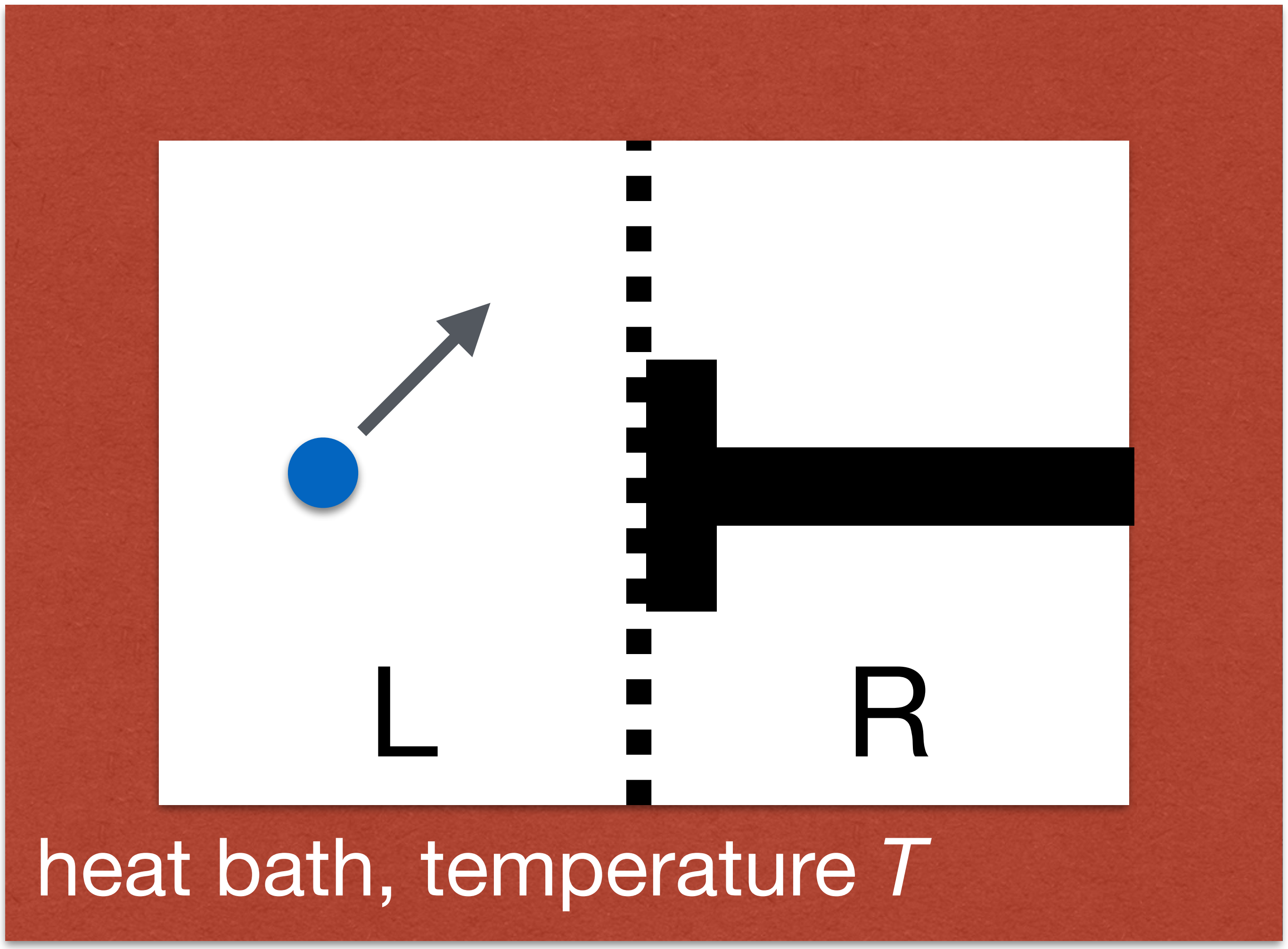

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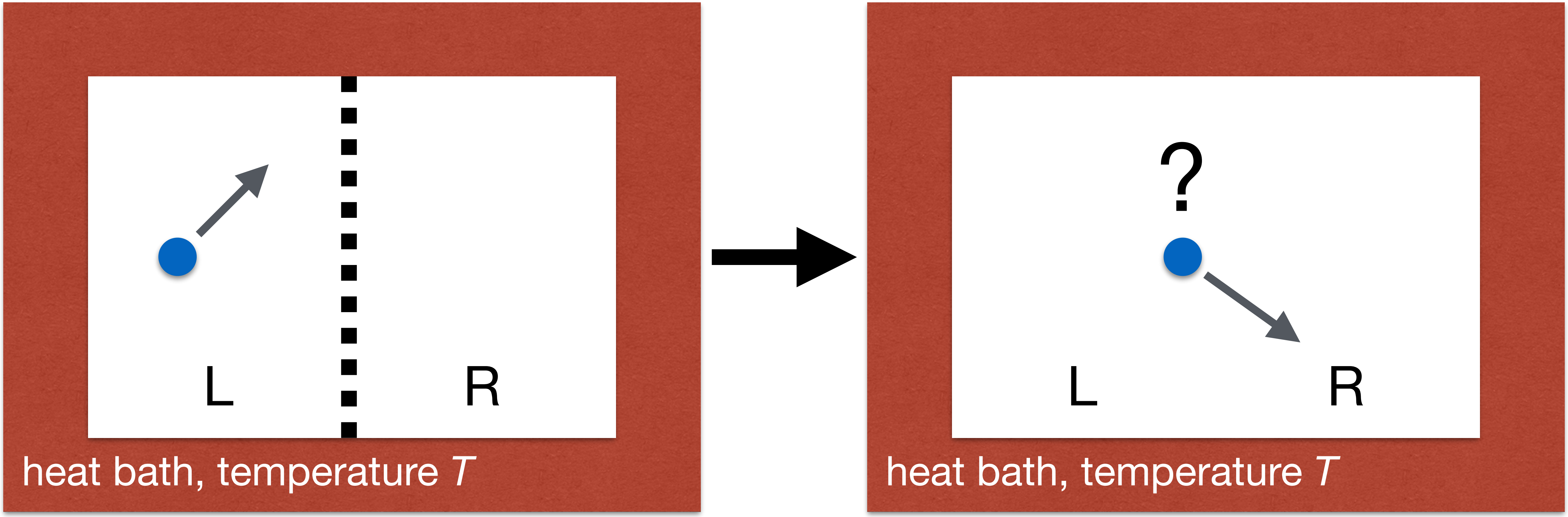

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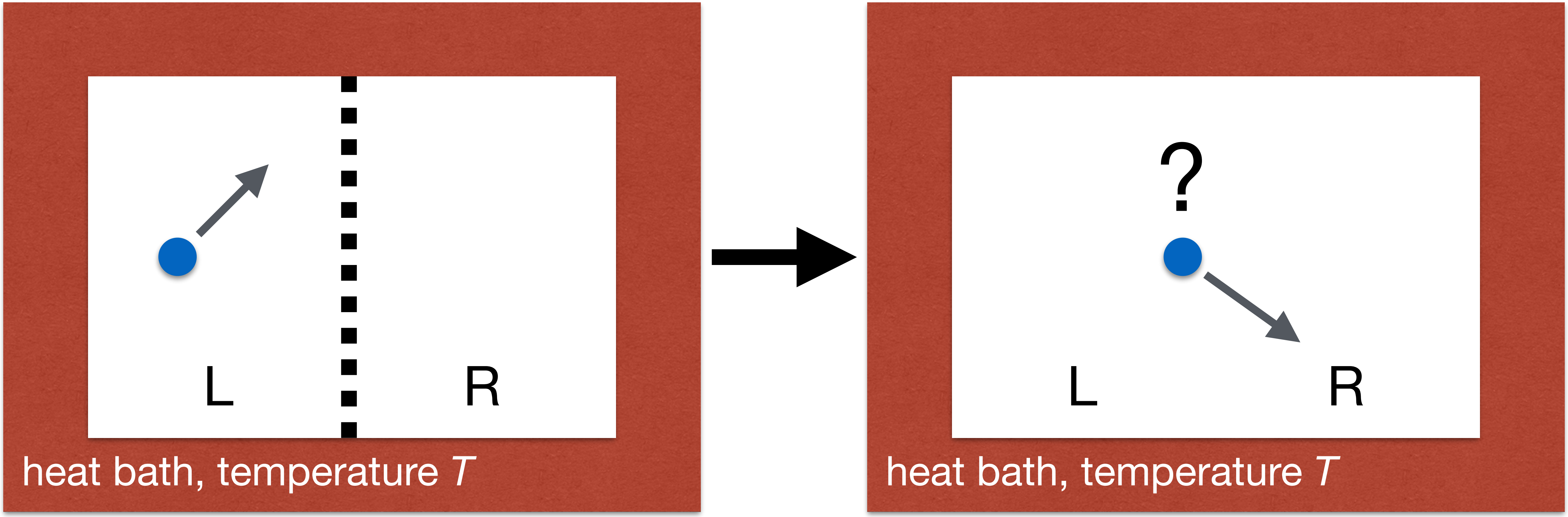

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix} + k_B T \ln 2$$

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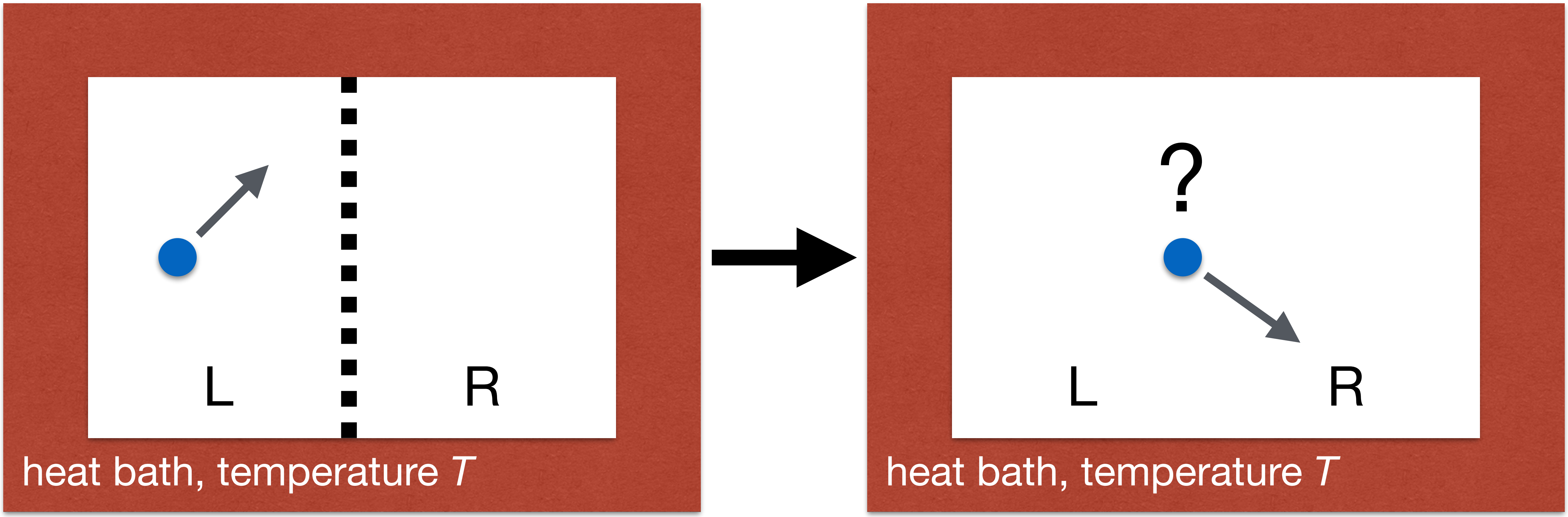

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Reverse process: **Landauer's Principle**.

Erasing 1 bit of information costs work.

Thermodynamics as a resource theory - **how** and **why** ?

This suggests that

- the objects of the theory are the observer's states of knowledge, i.e. probability distributions (on microstates),
- the free transformations are the microscopically reversible transformations (e.g. L↔R),
- the free states are $\begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$ or more generally $\begin{pmatrix} 1/N \\ \vdots \\ 1/N \end{pmatrix}$

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Something's missing: there are probabilities $\begin{pmatrix} p_1 \\ \vdots \\ p_N \end{pmatrix}$ and energies $\begin{pmatrix} E_1 \\ \vdots \\ E_N \end{pmatrix}$.

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  the thermal states (Gibbs states)

$$\gamma = \frac{1}{Z}\begin{pmatrix} e^{-E_1/(k_B T)} \\ \vdots \\ e^{-E_N/(k_B T)} \end{pmatrix}.$$

Why?

Thermodynamics as a resource theory - **how** and **why** ?

Why?

The inverse process is not dynamically impossible, but hard for us to implement.
→ Thermodynamics derives from limited knowledge about the physical system (thus, lack of control), together with energy conservation and microscopic reversibility.

Why?

R.T.: maximal generality; clear mathematical "rules of the game"

Reproduces and refines results of standard non-equilibrium thermodynamics.

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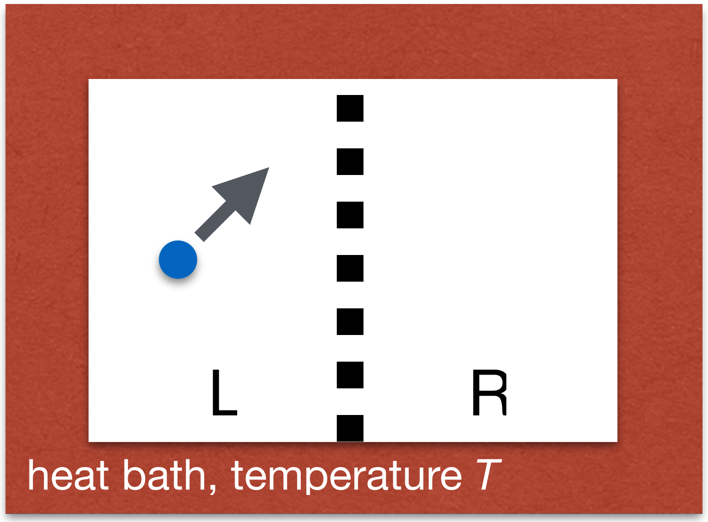

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Definition, results, surprises.

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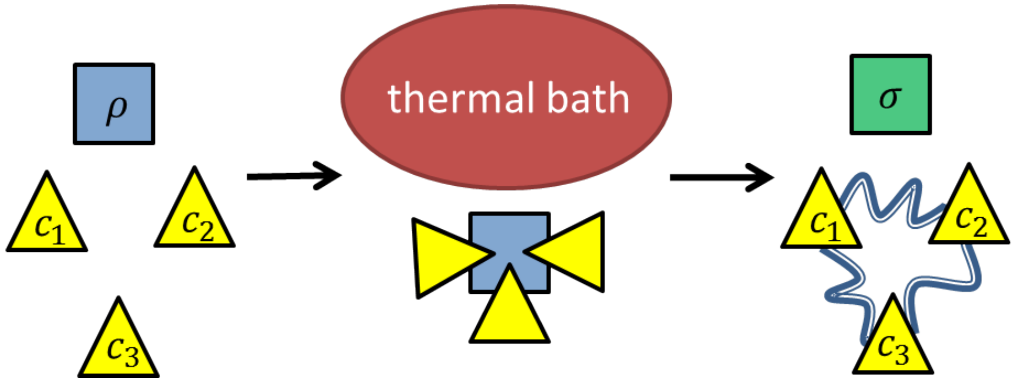

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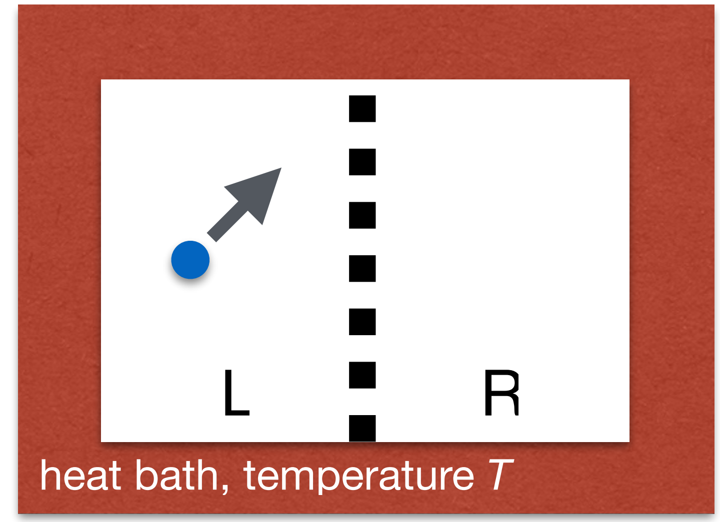

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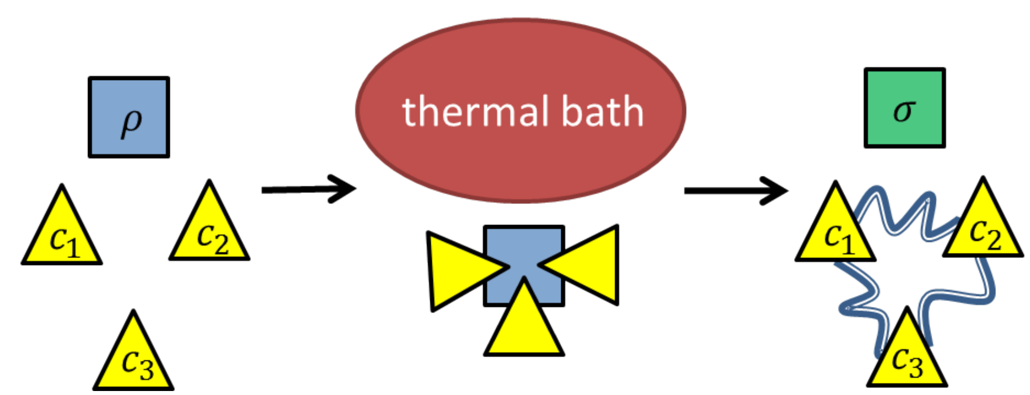

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The resulting transformations are called thermal operations:

2. Resource theory of athermality: precise definition

$\beta := 1/(k_B T)$ is now fixed.

Definition. A map Φ on a physical system S is a *thermal operation* if there is another (ancilla) system A with energy levels $(E_1, \ldots, E_N)$ and thus Gibbs state $\gamma_A = (e^{-\beta E_1}, \ldots, e^{-\beta E_n})/Z$, and an energy-preserving permutation π on SA such that

$$\Phi(p_S) = (\pi[p_S \otimes \gamma_A])_S \, .$$

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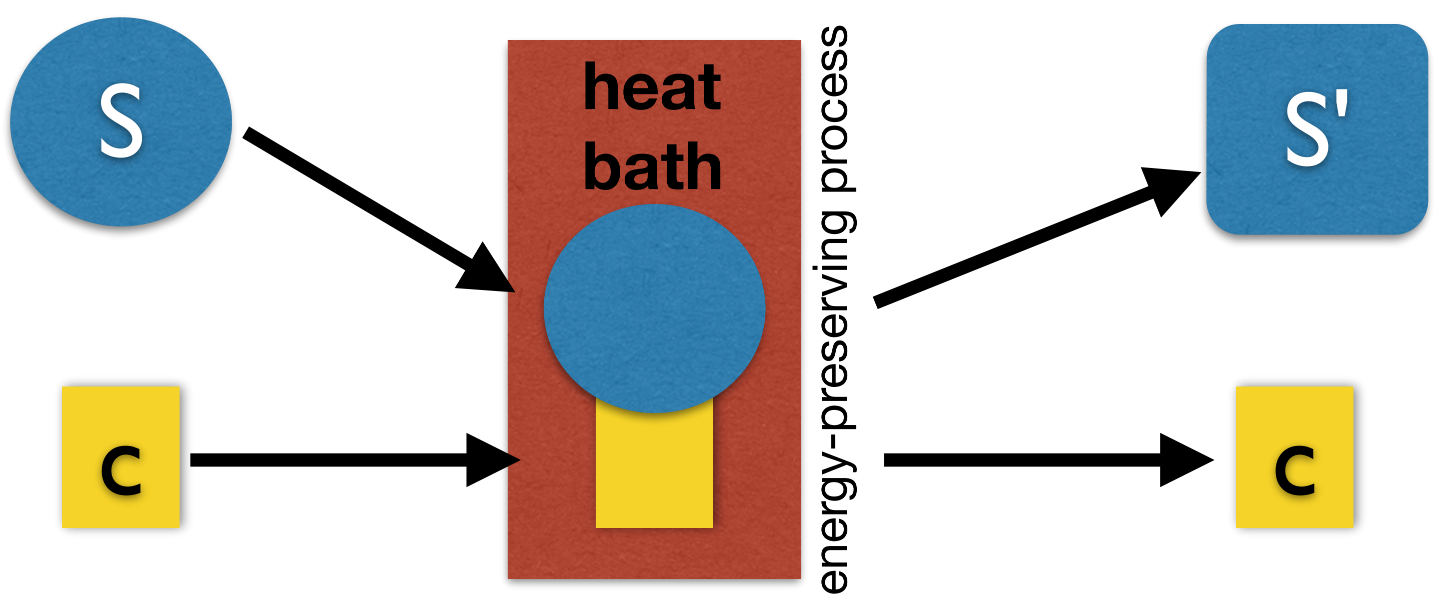
Slightly more general: catalytic thermal operation.



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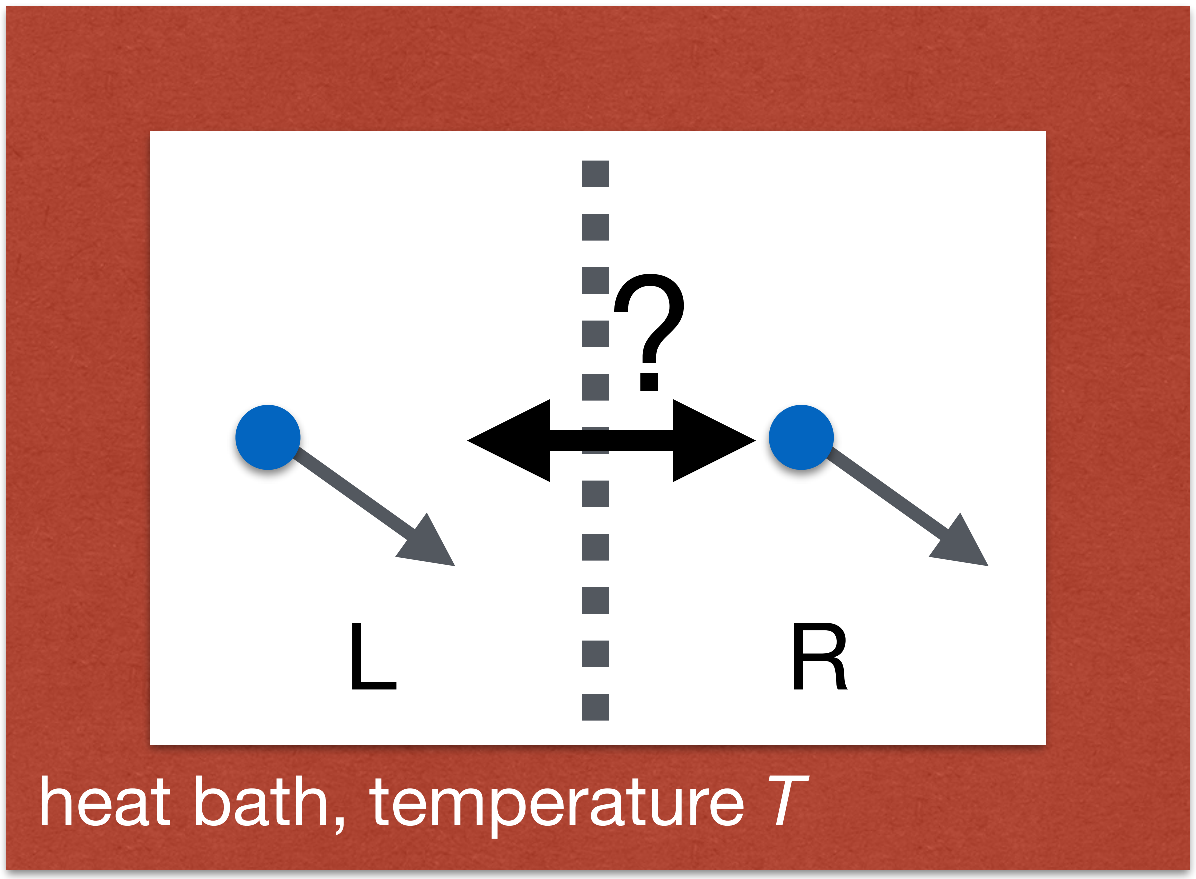
Completely general: $p_S = \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$ could mean



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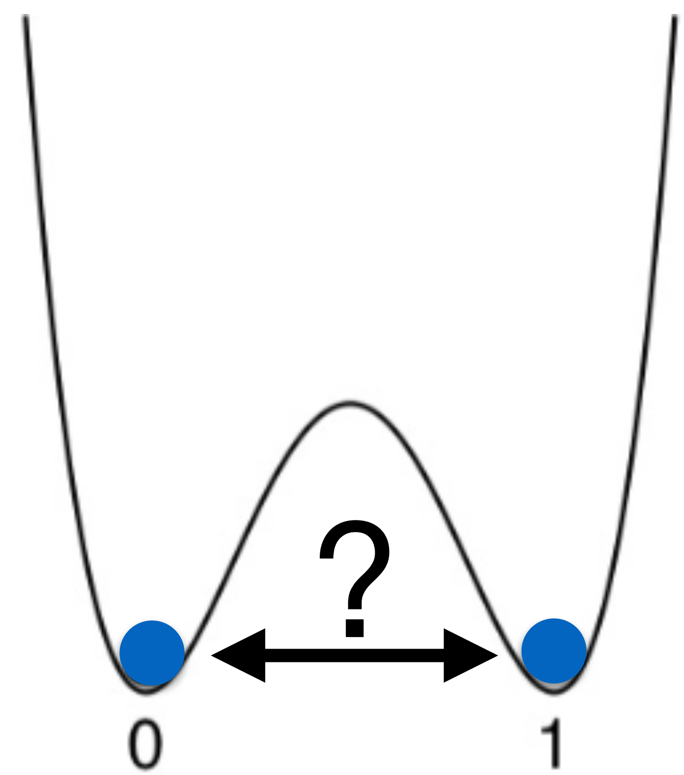

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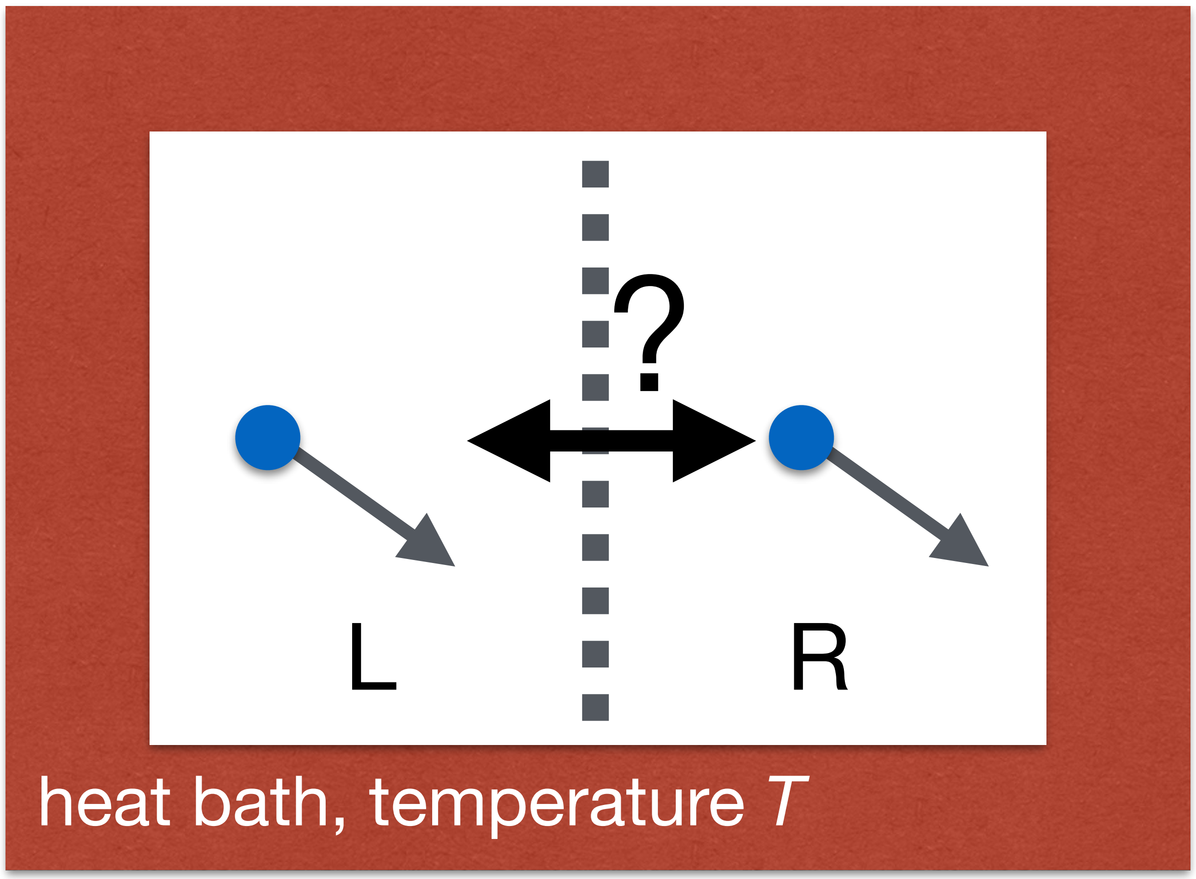

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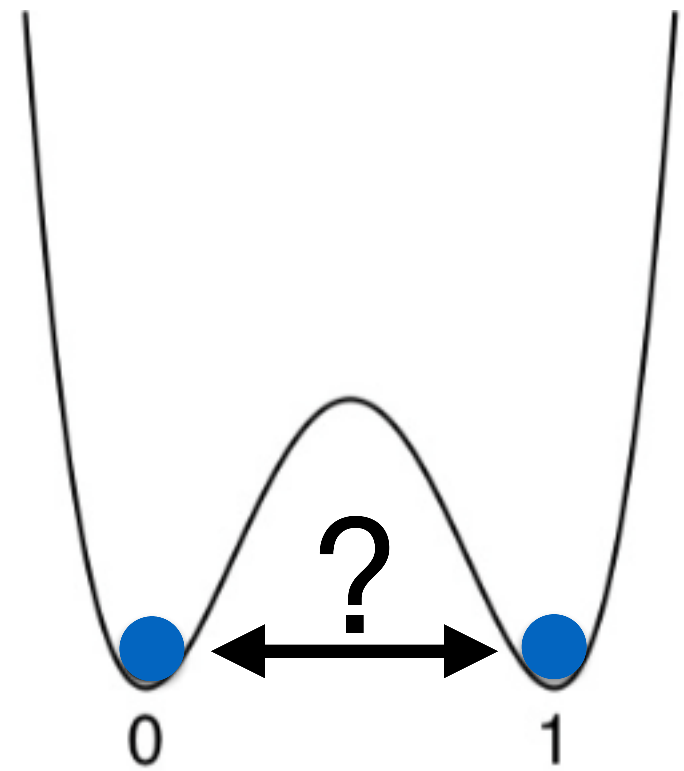

To be applicable, microstates should have well-defined energy…
… and those of identical energy should be permutable dynamically

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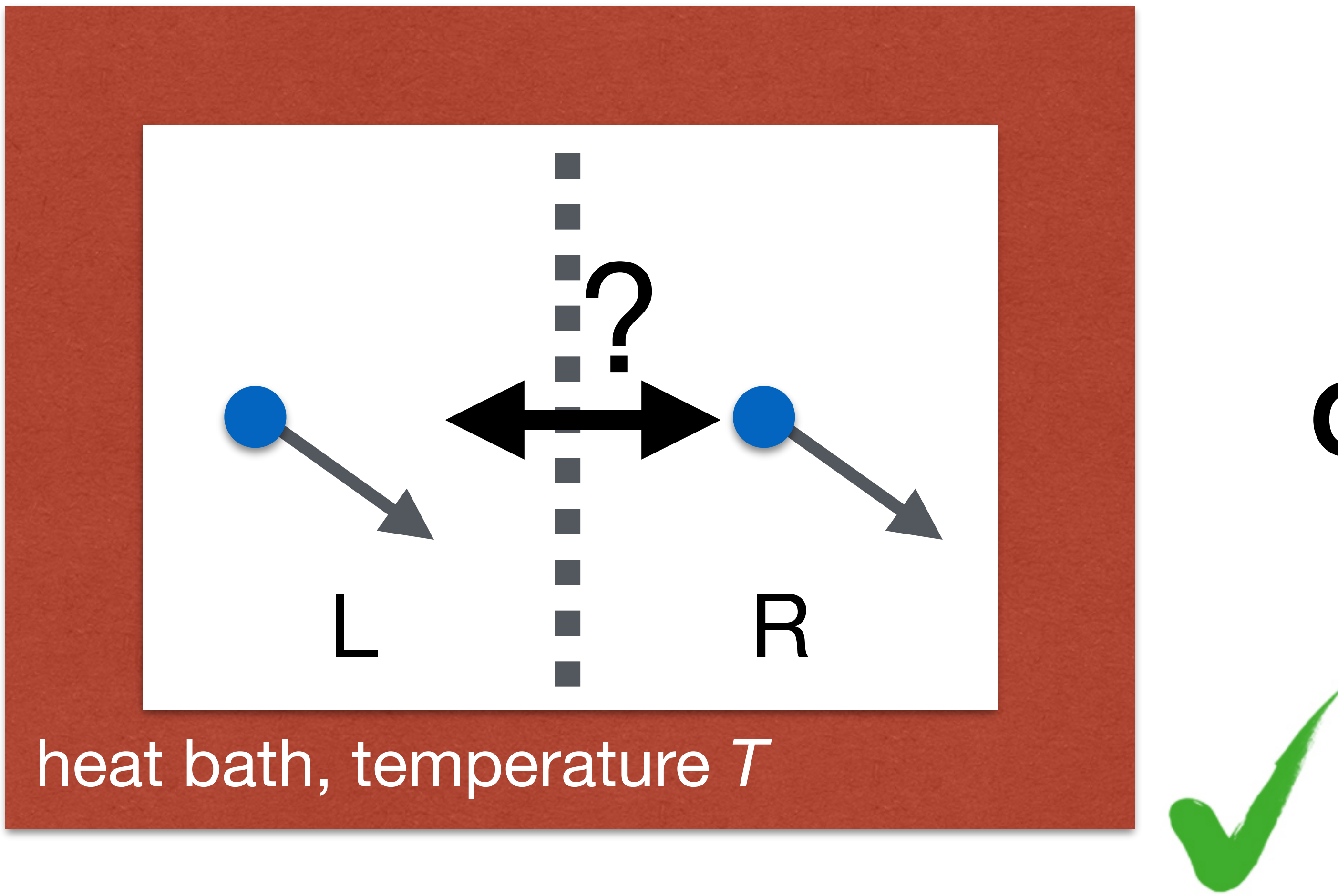

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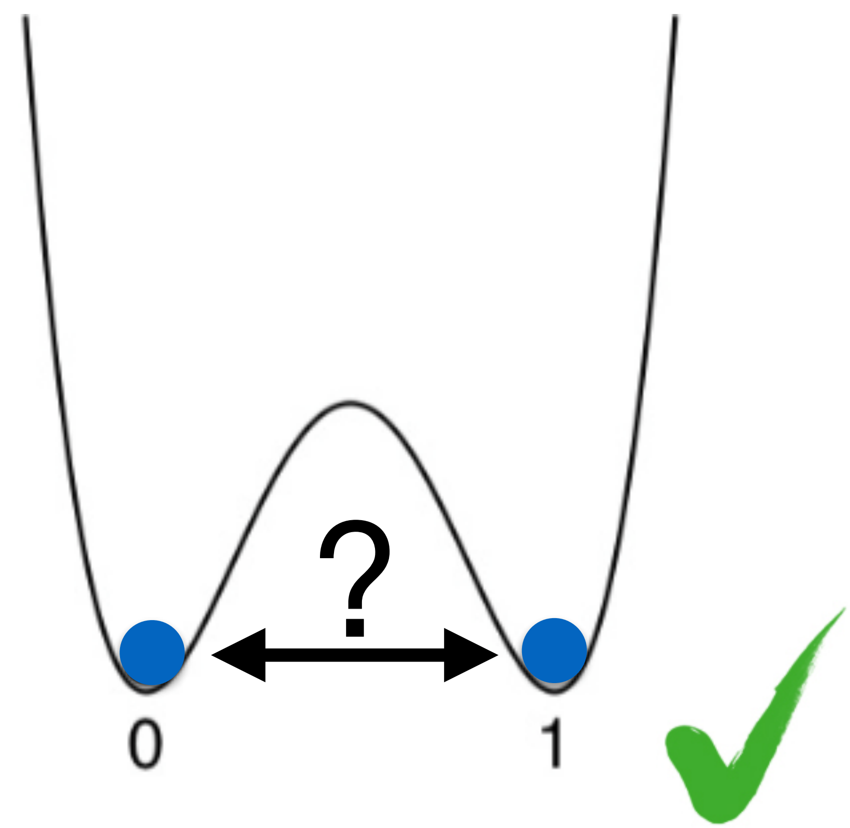

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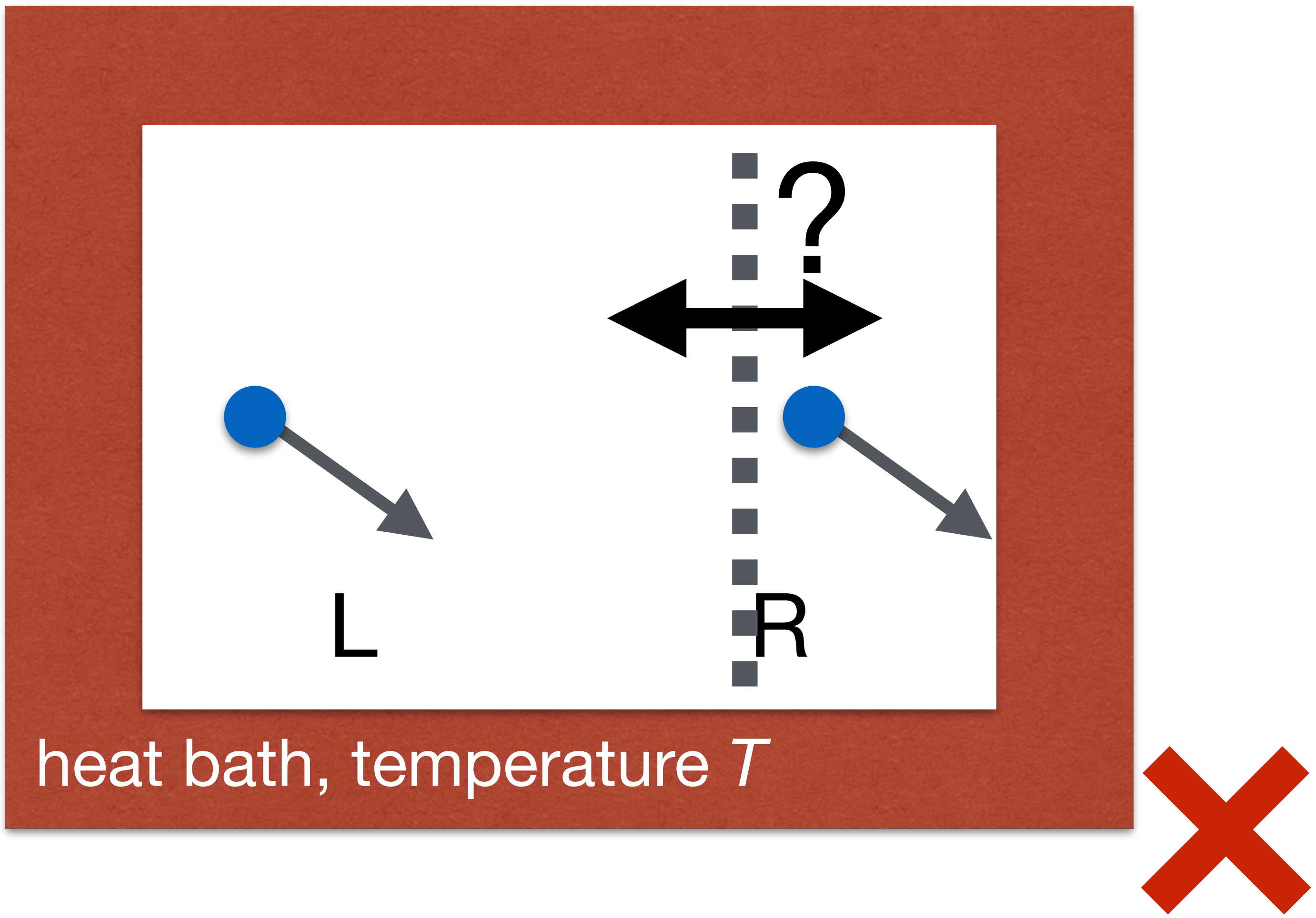

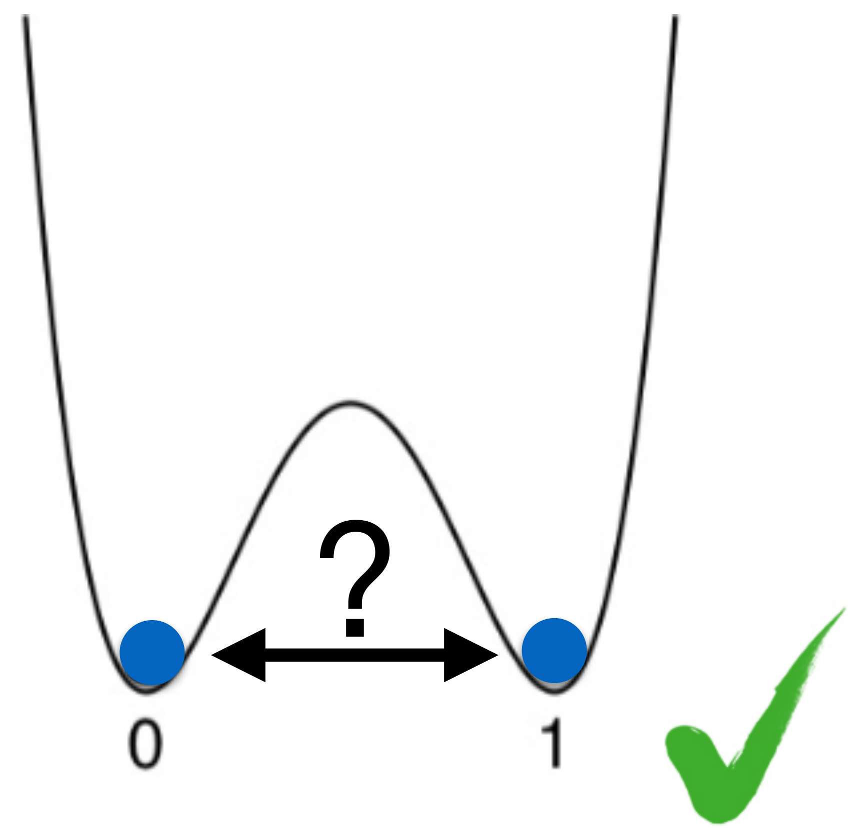

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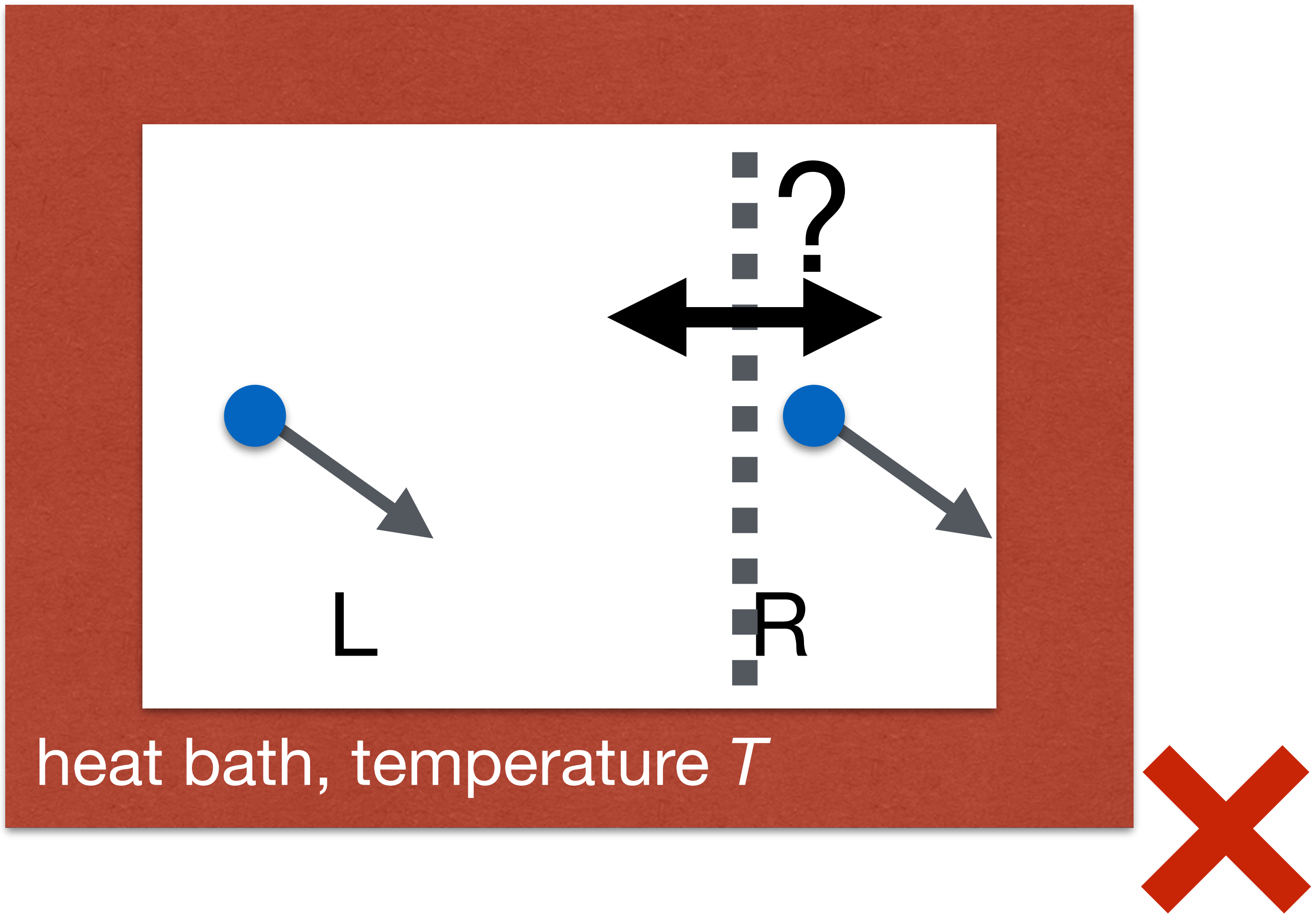

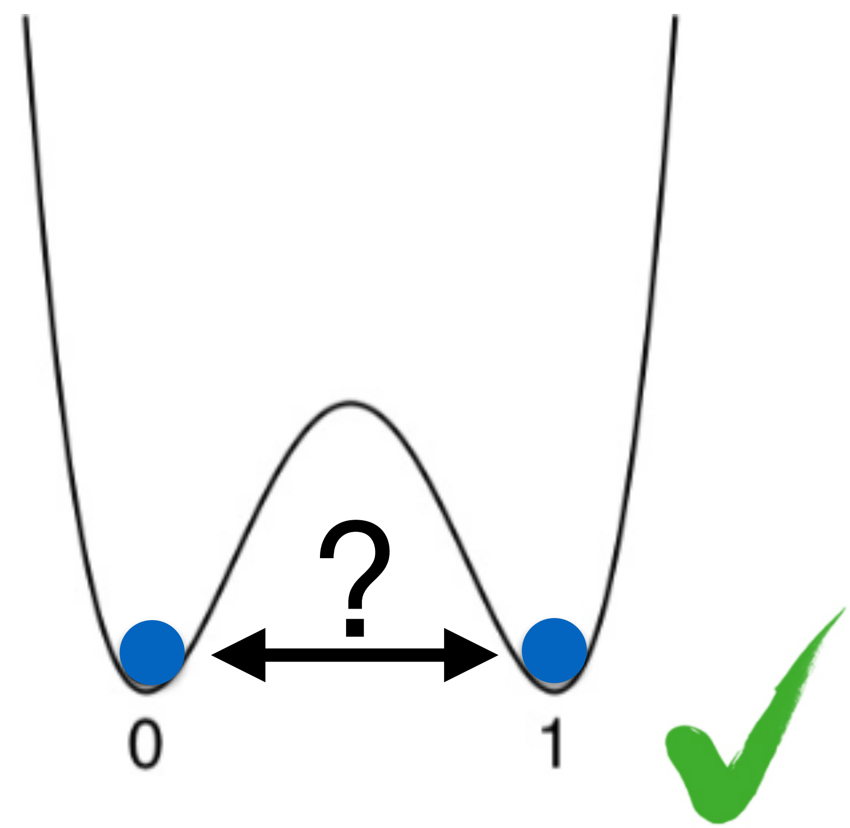

One can also allow quantum coherences, $\alpha|0\rangle + \beta|1\rangle$

→ (quantum) thermodynamics "at the nano scale"

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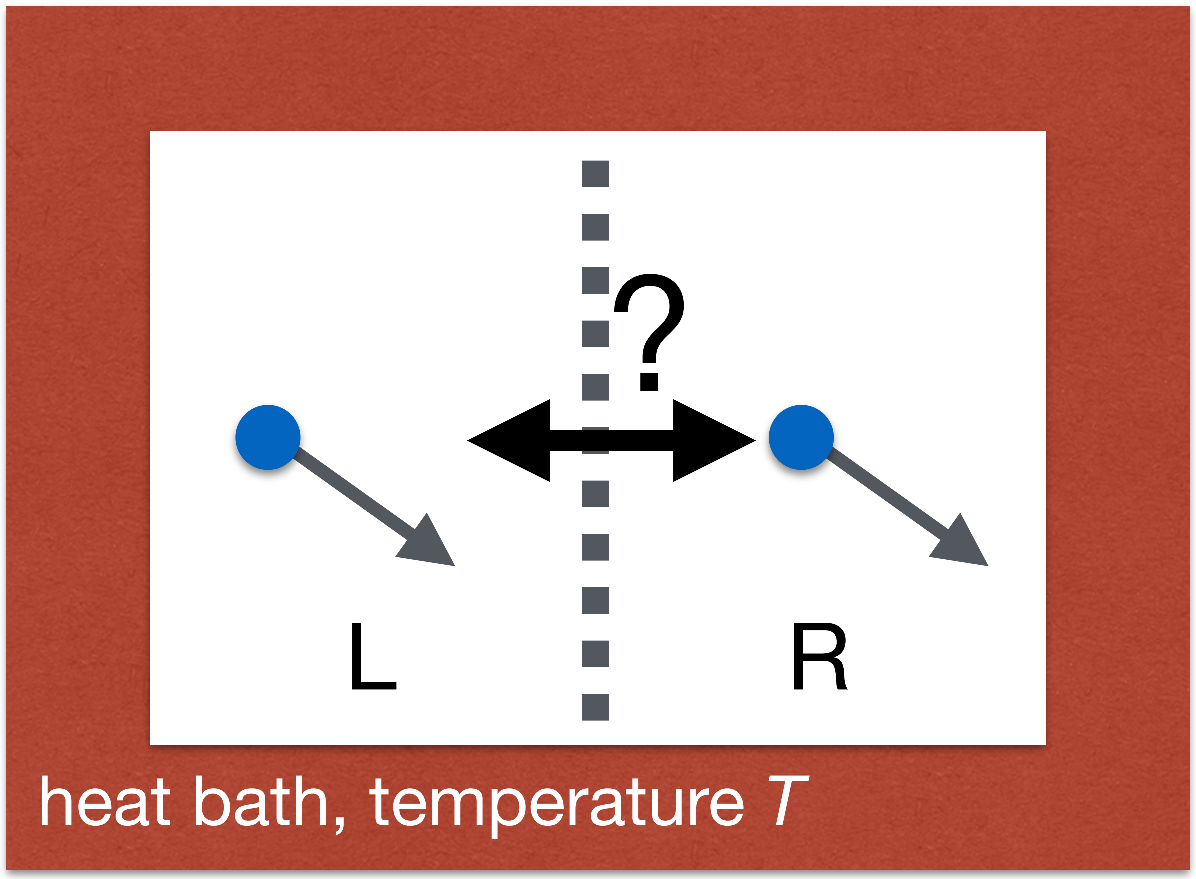

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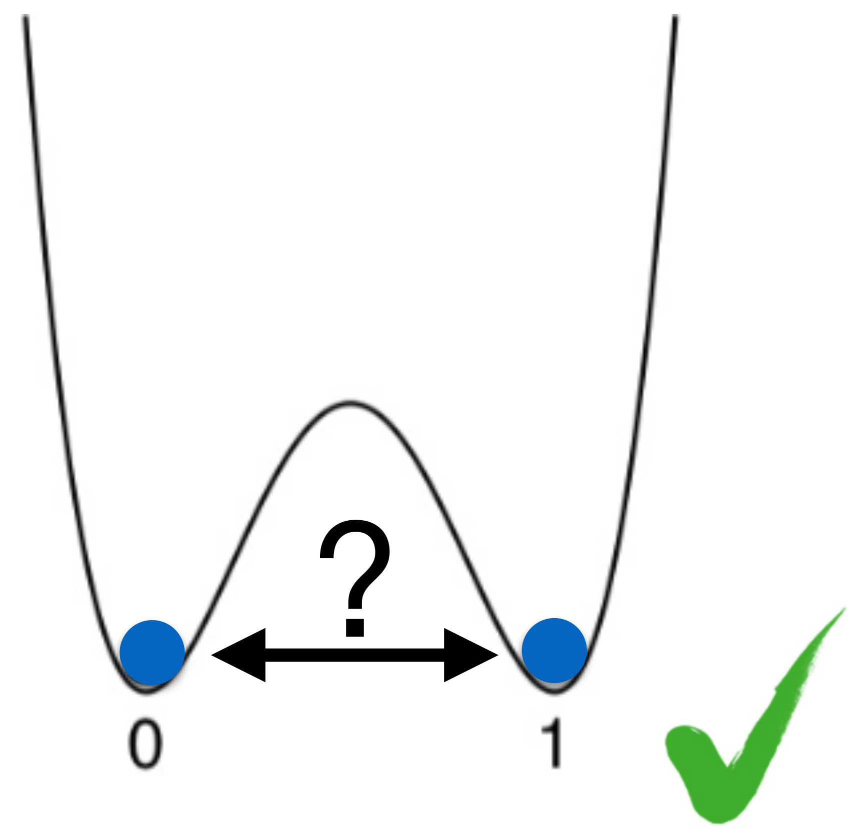
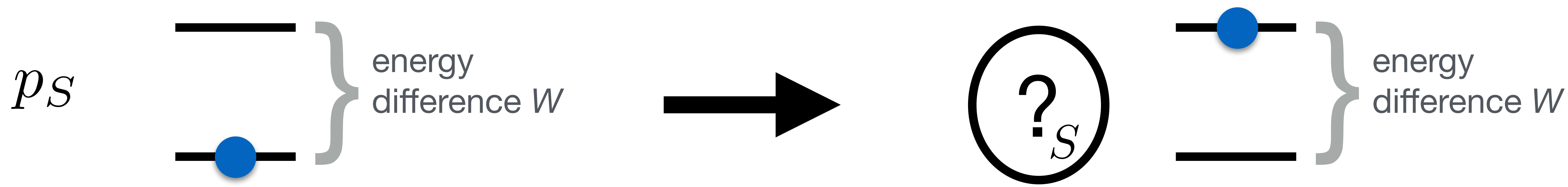


One can also allow quantum coherences, $\alpha|0\rangle + \beta|1\rangle$

→ (quantum) thermodynamics, not necessarily "at the nano scale"

Work extraction and work cost

Work extraction: what is the largest possible W such that



within the rules of the resource theory?

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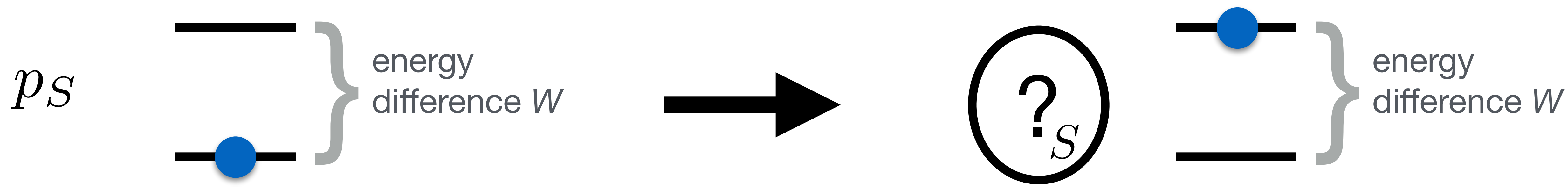

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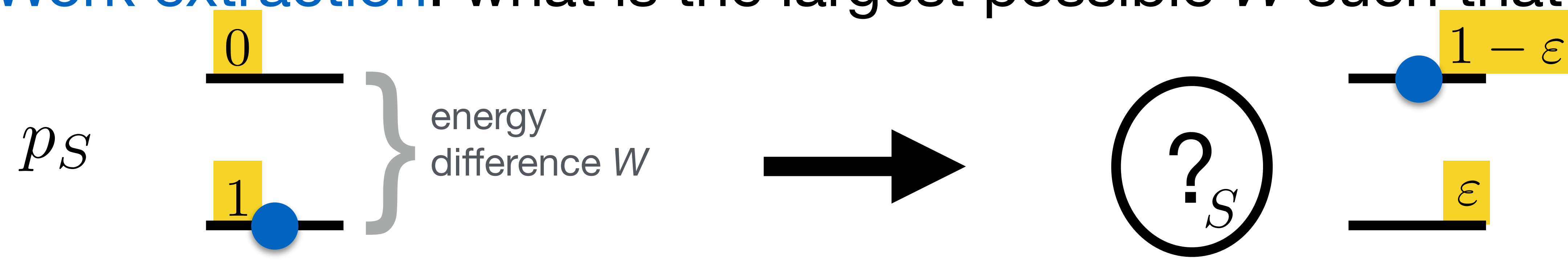

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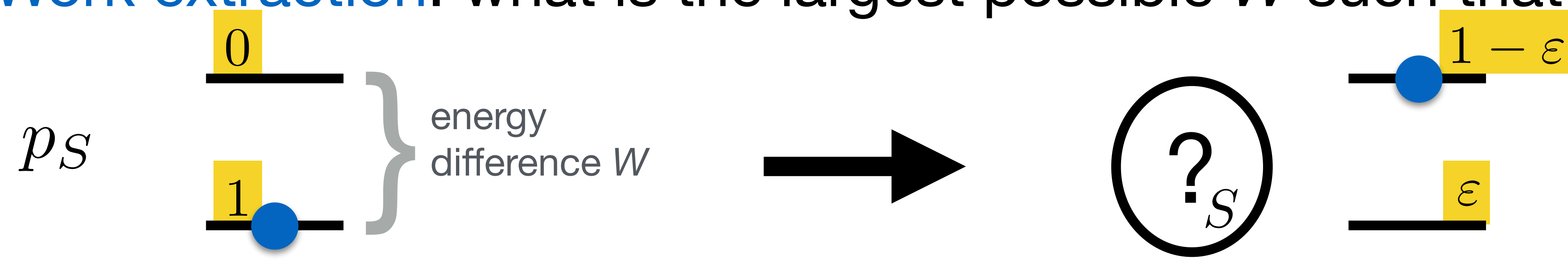

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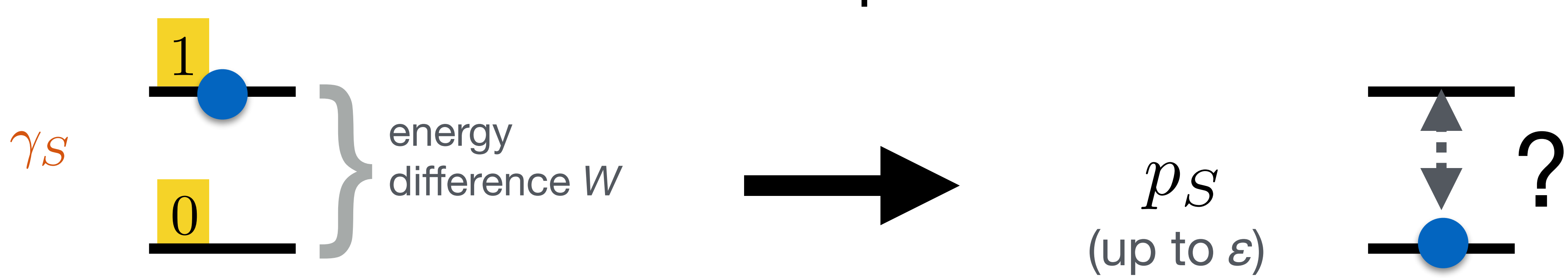

Work extraction and work cost

Theorem: The extractable work and work cost are

$$\begin{aligned} W_{\text{extr}} &= k_B T \left(F_0^{\varepsilon}(p_S) - F(\gamma_S) \right), \\ W_{\text{cost}} &= k_B T \left(F_\infty^{\varepsilon}(p_S) - F(\gamma_S) \right), \end{aligned}$$

where F_α is the Rényi α-free energy:

$$F_\alpha(p_S) = k_B T \left(\frac{\operatorname{sgn}\alpha}{\alpha - 1} \log \sum_i p_i^\alpha \exp\left(\frac{-E_i(1-\alpha)}{k_B T} \right) \right) - k_B T \log Z,$$

and $F_1(p_S) = F(p_S) = \langle E \rangle - k_B T S(p_S)$

is the "standard" free energy.

M. Horodecki and J. Oppenheim, *Fundamental limitations for quantum and nanoscale thermodynamics*, Nature Communications **4**, 2059 (2013).

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Landauer's Principle: if $p_S = (1,0)$ and two identical energies,

$$W_{\mathrm{extr}} = W_{\mathrm{cost}} = k_B T \ln 2.$$

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Fundamental thermodynamical irreversibility !

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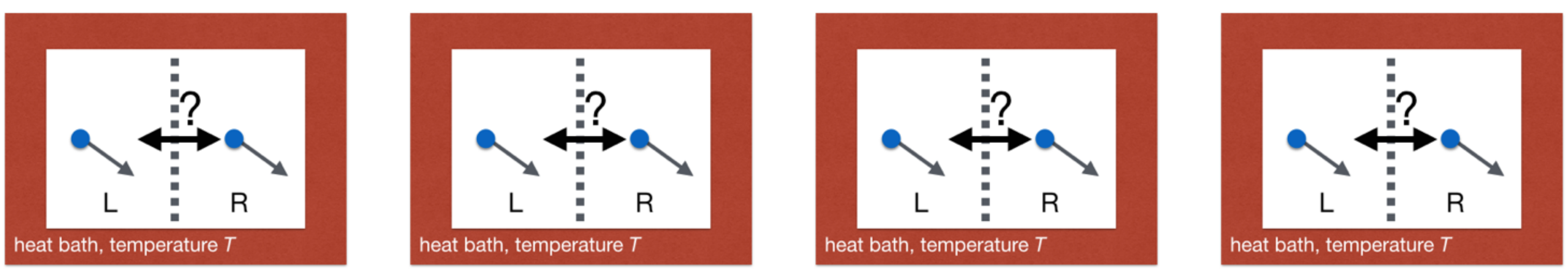

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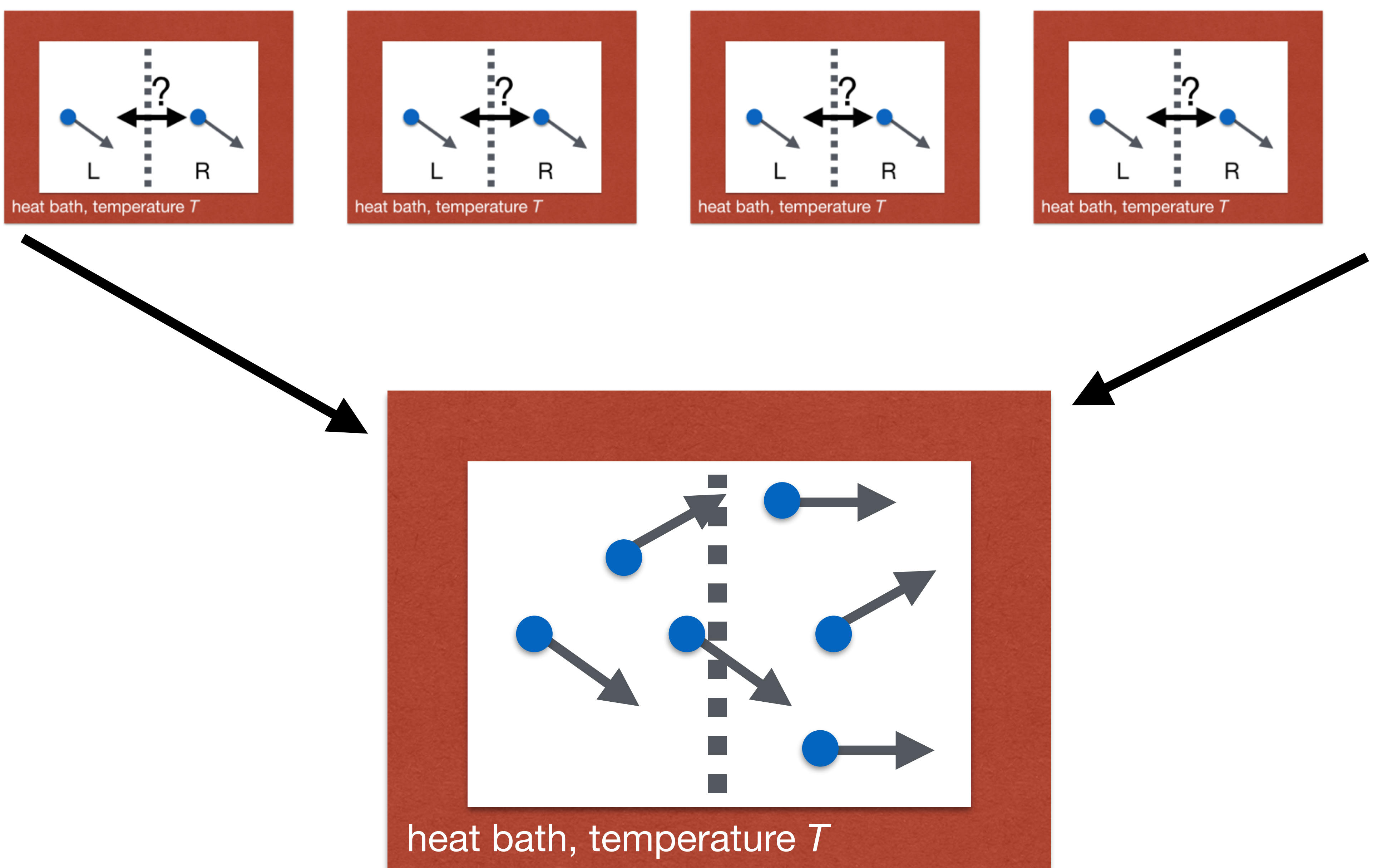

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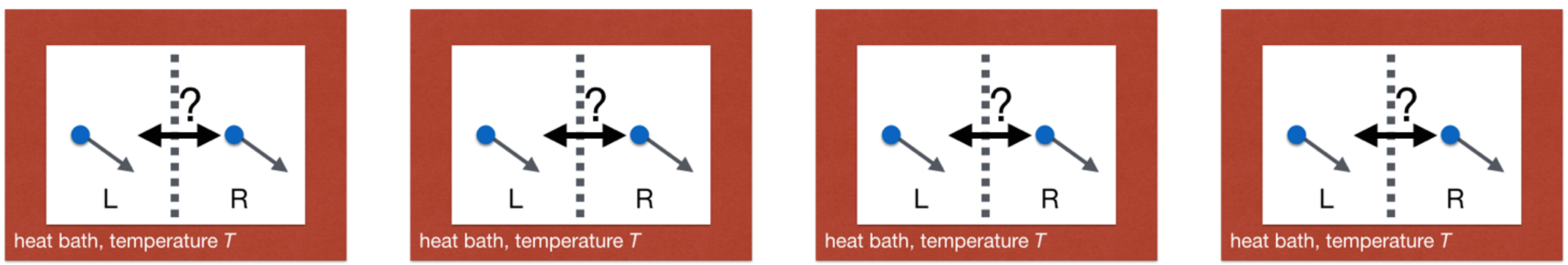

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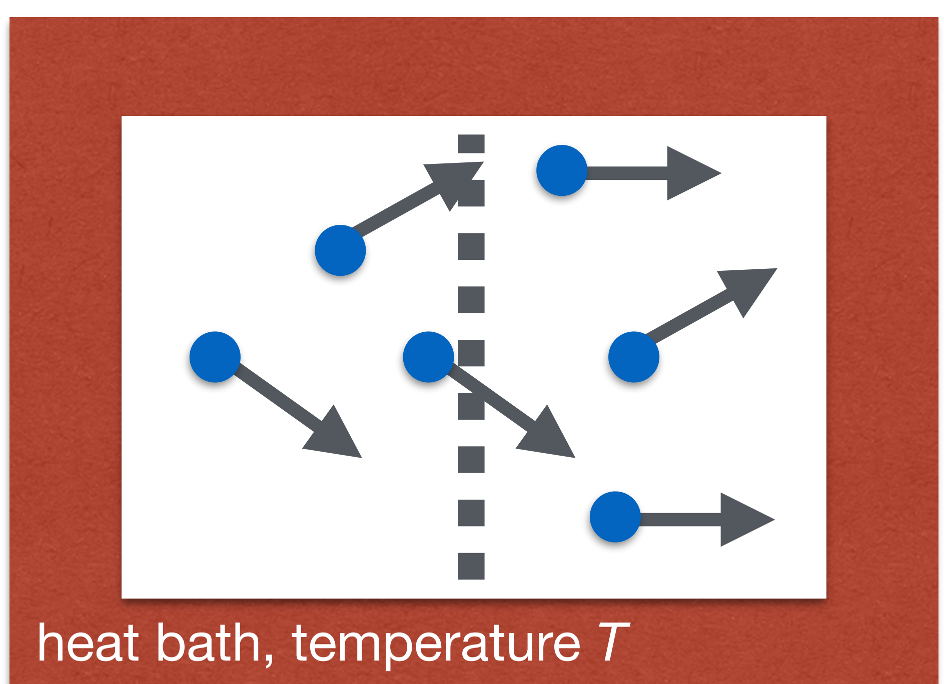

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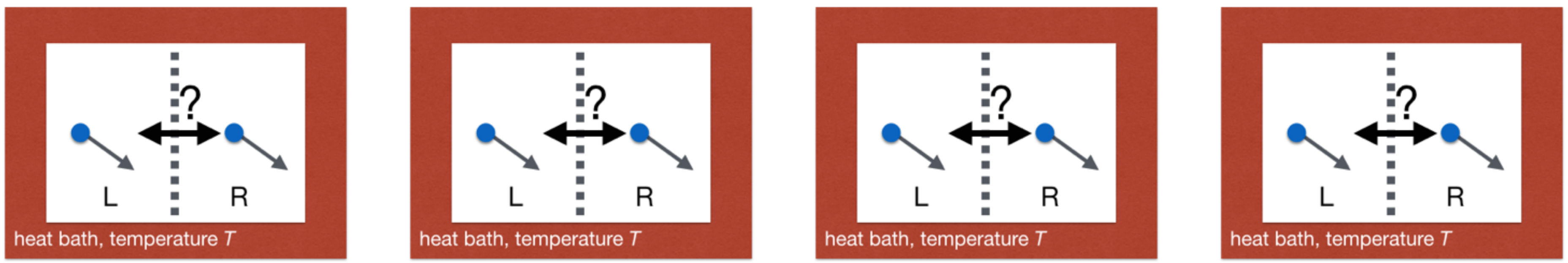

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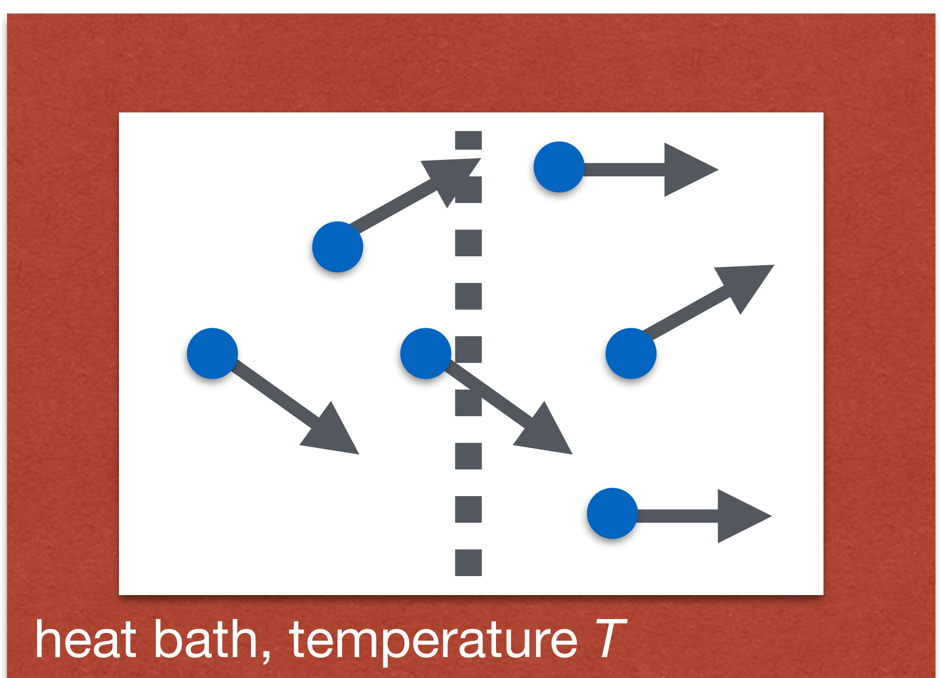

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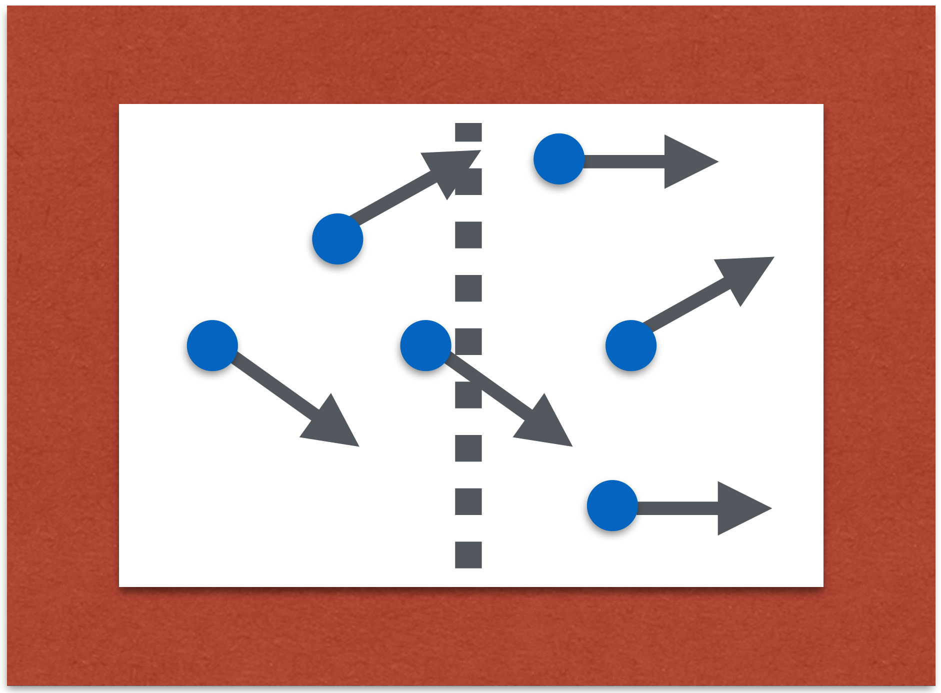

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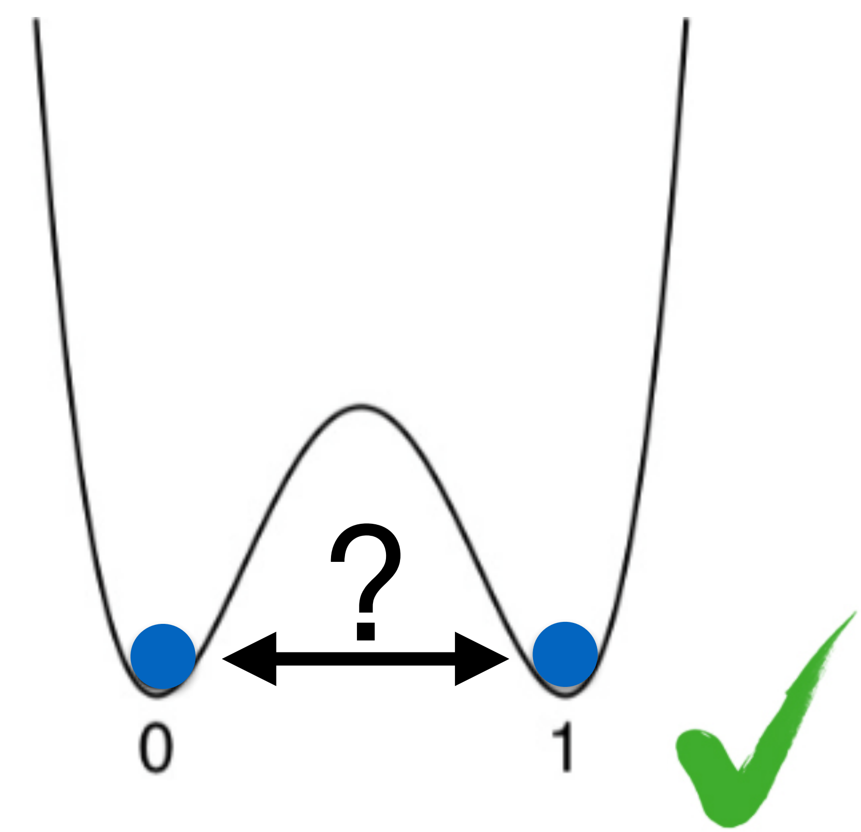
Remember: nowhere have we actually used that we have a gas.



The second law(s)

Theorem: A transition $p_S \to p'_S$ is possible if and only if

$$F_\alpha(p_S) \geq F_\alpha(p'_S) \text{ for all } \alpha \geq 0.$$

All α-free energies must go down!

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Outline

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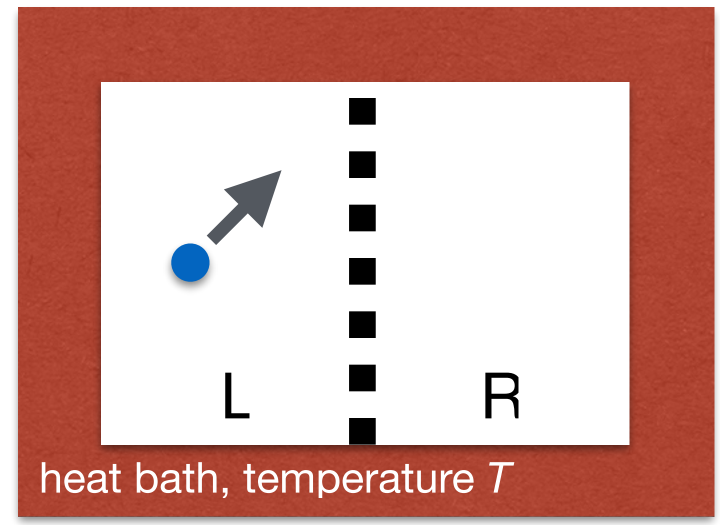
What is a resource theory?
Why and how thermo as a RT?



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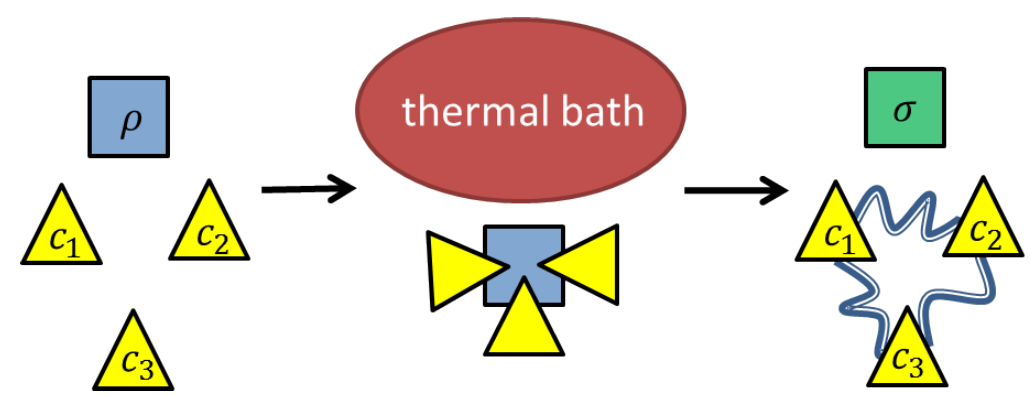
Definition, results, surprises.

3. Extracting work from absence of correlations

How to do more by knowing less…



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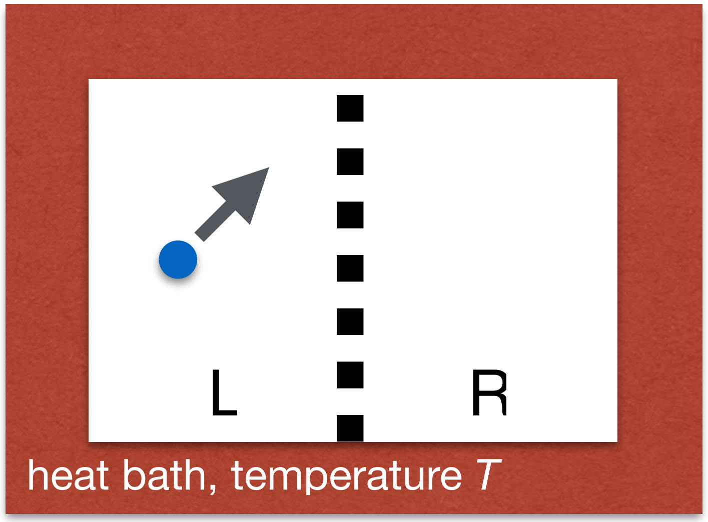

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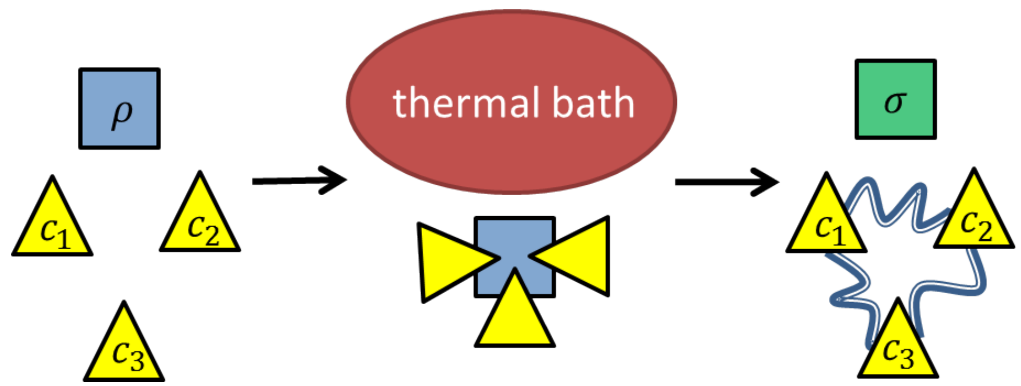

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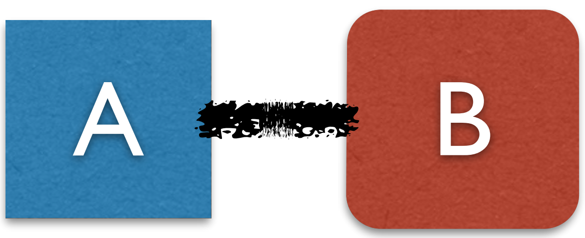

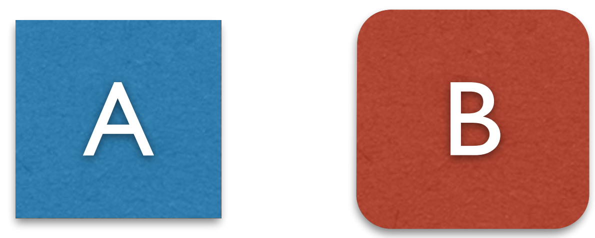

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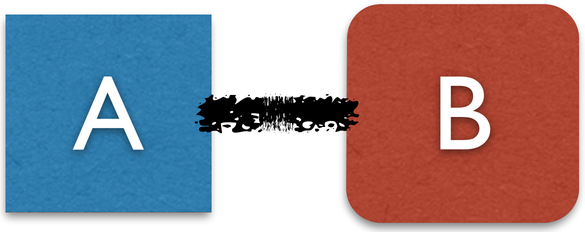

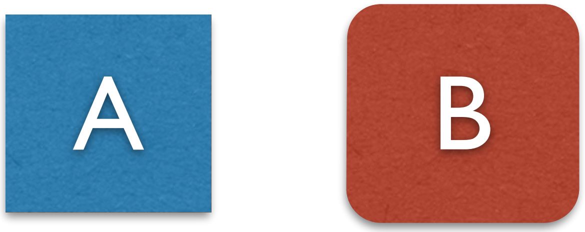

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One can "extract work from correlations".

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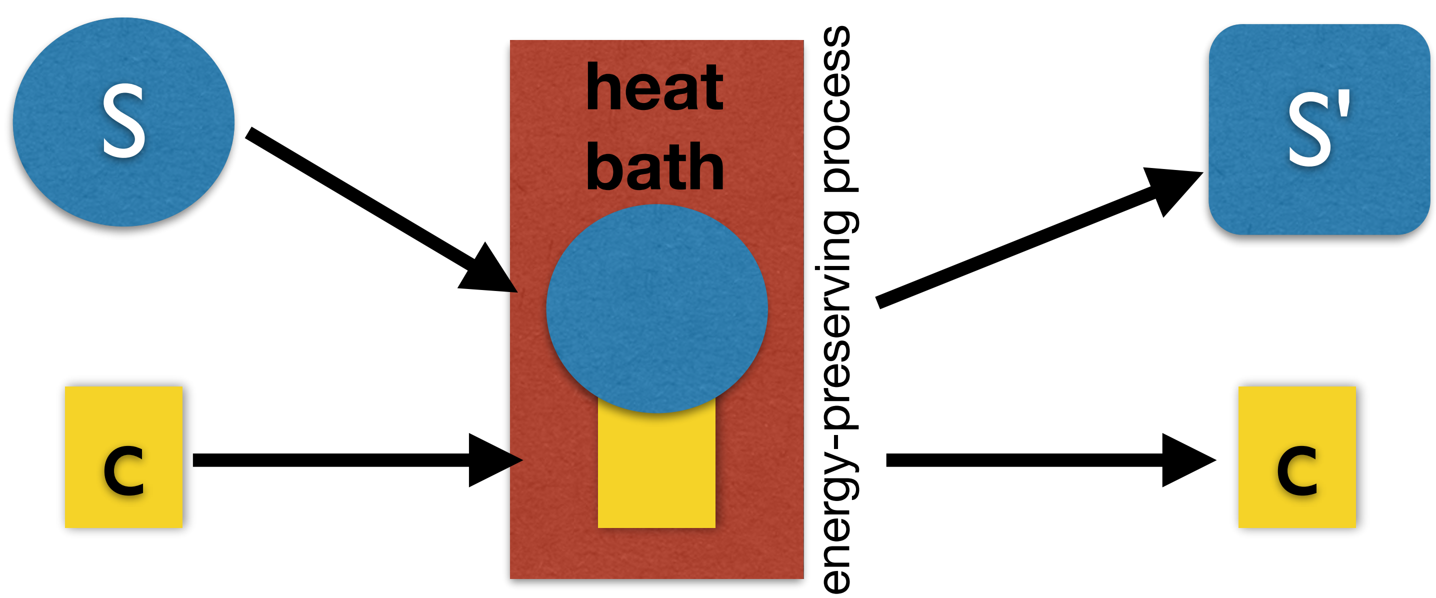
Resource theory ("single-shot thermodynamics"): recall as below.



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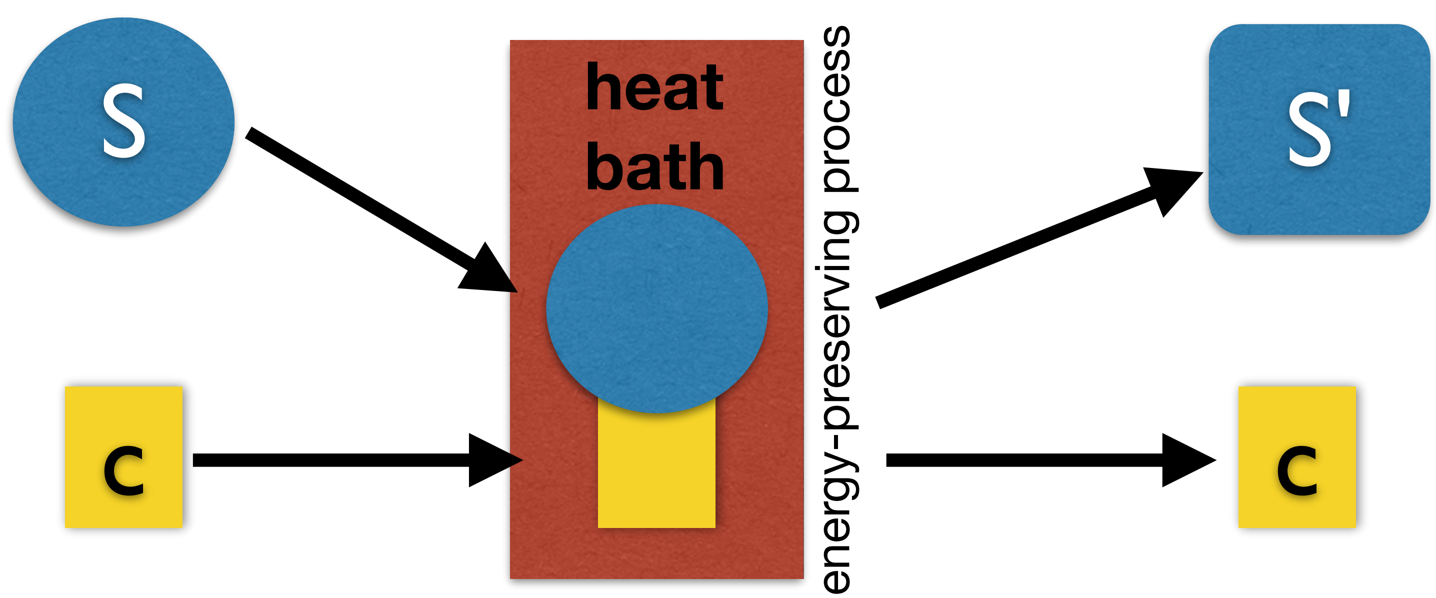

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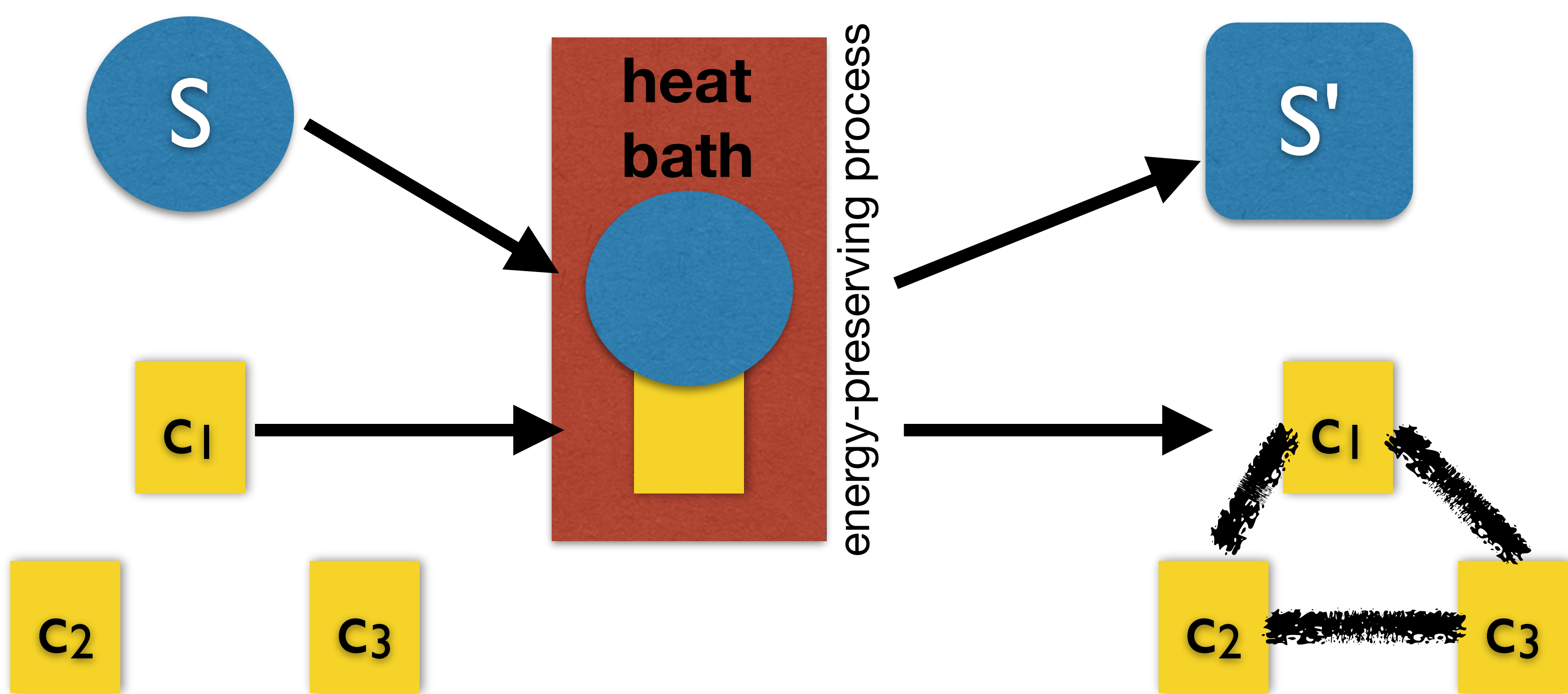

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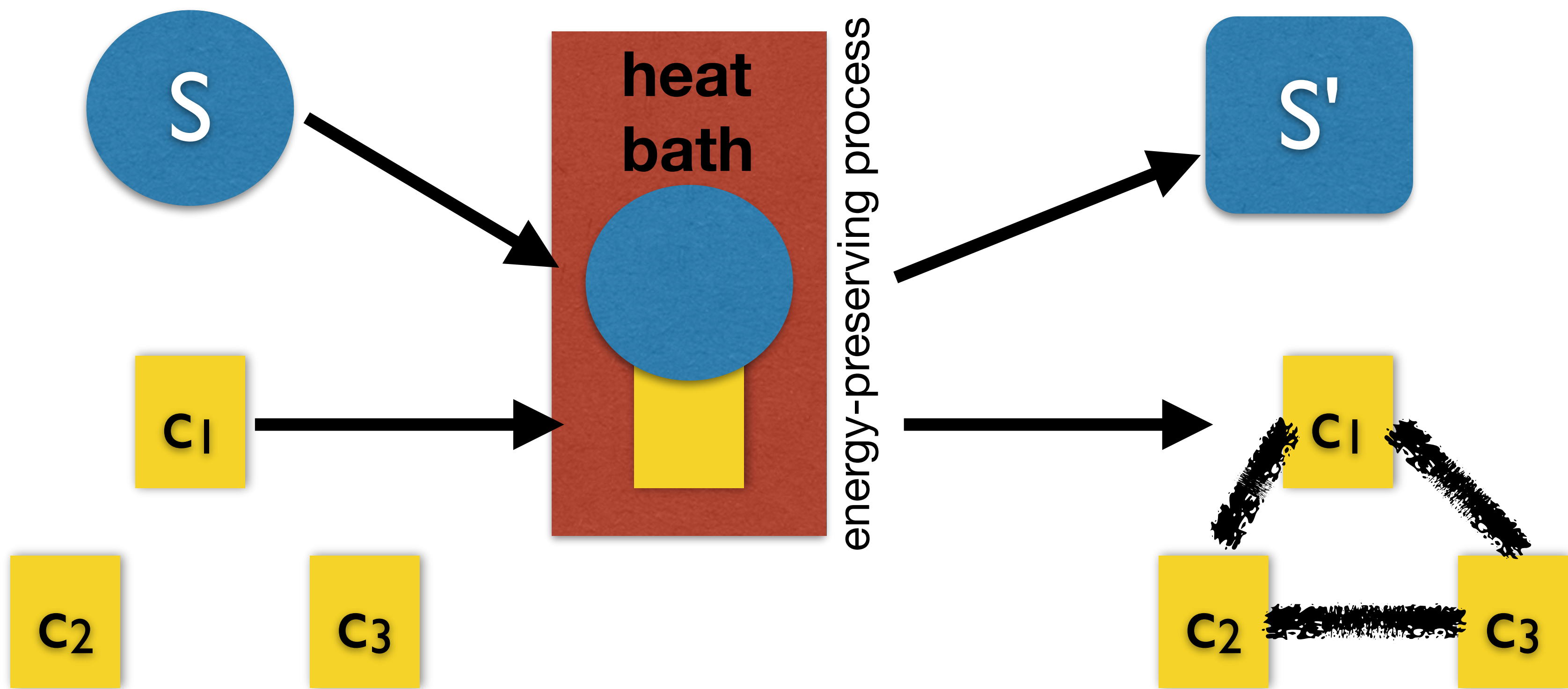

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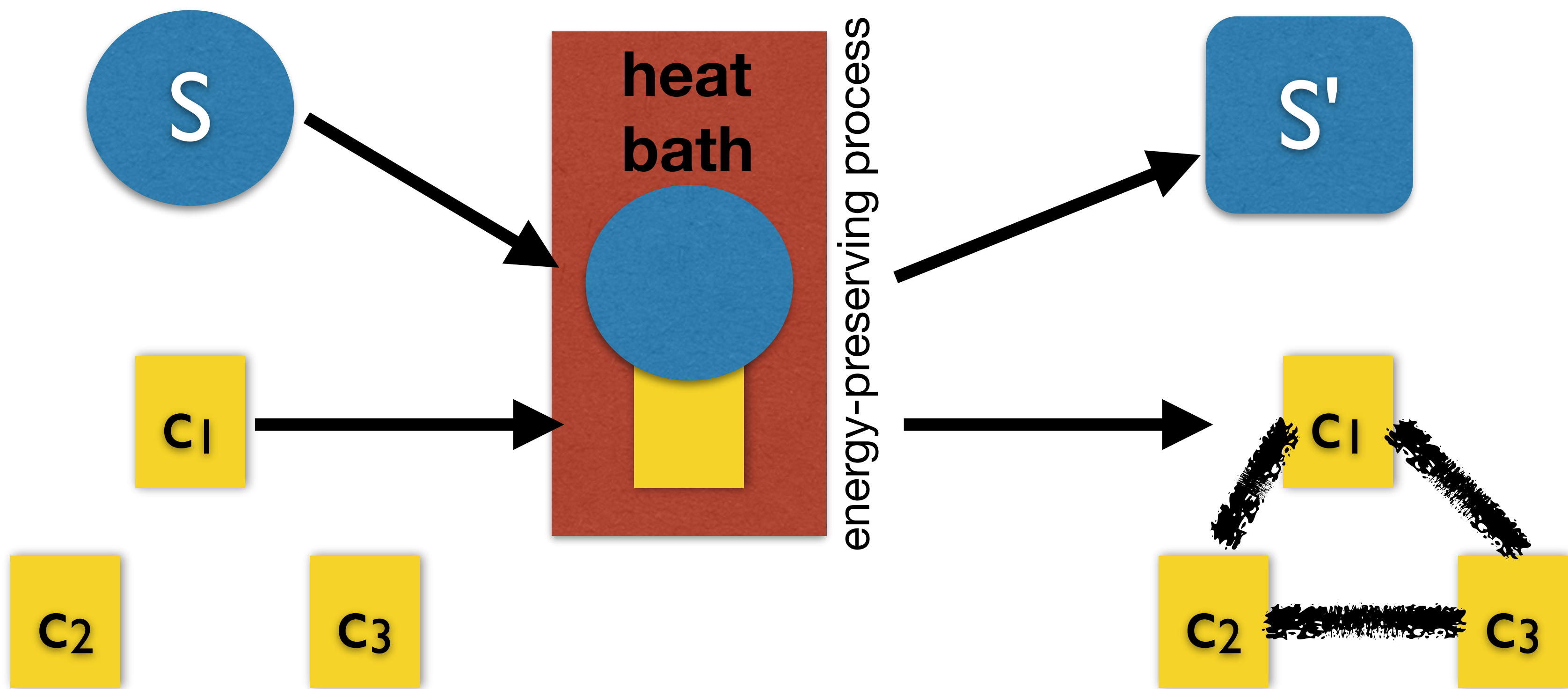

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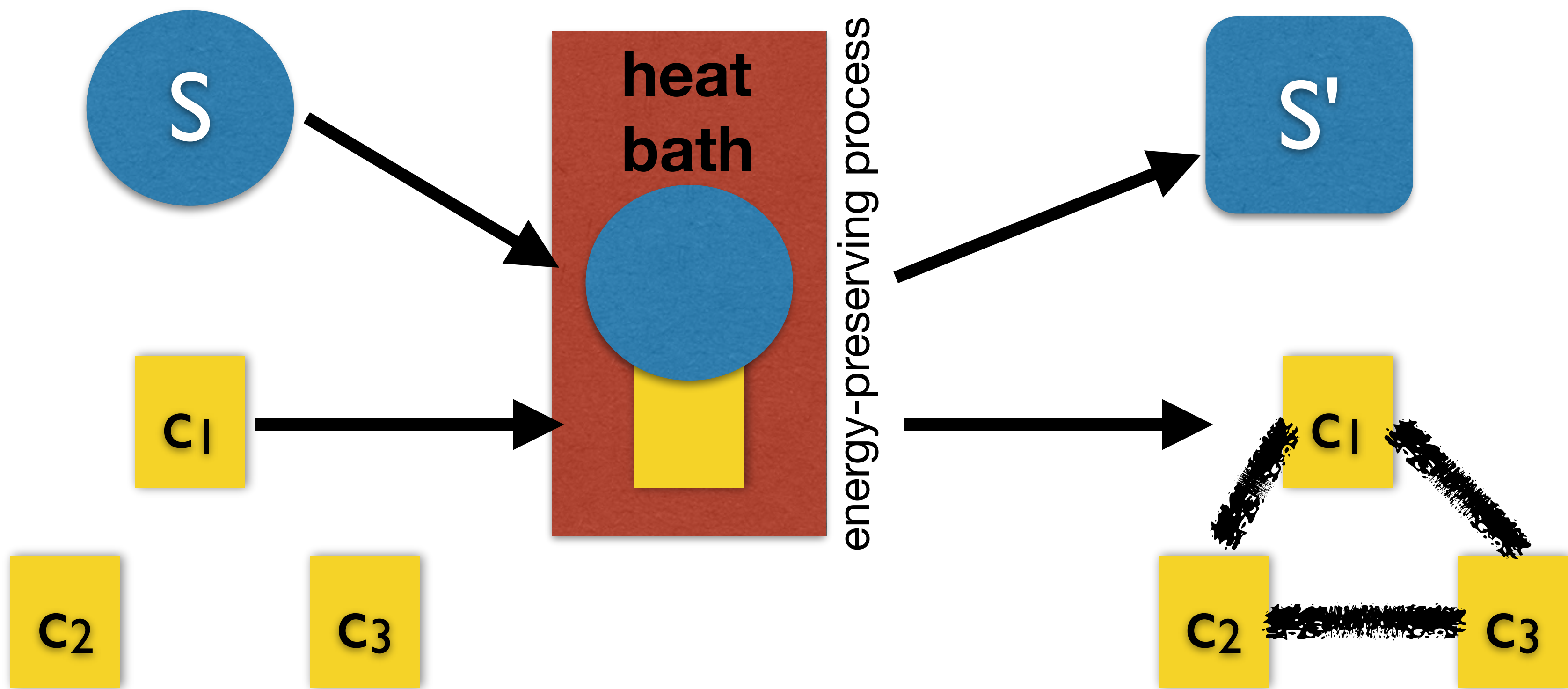

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It's actually easier - stochastic independence can be "burnt like a fuel"!

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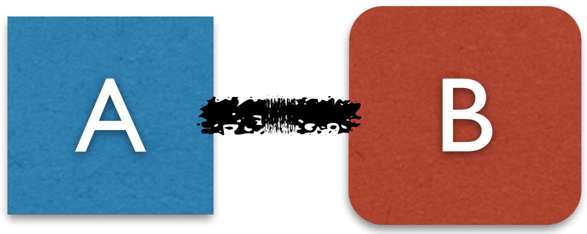

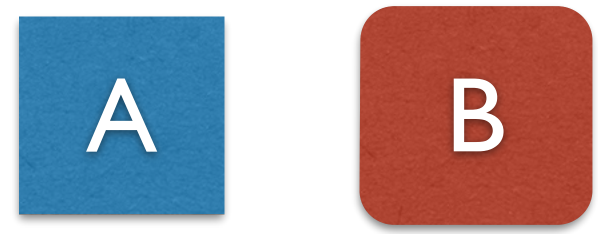

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Knowling less makes you "less α-confused" and allows you to do more.

Conclusions

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