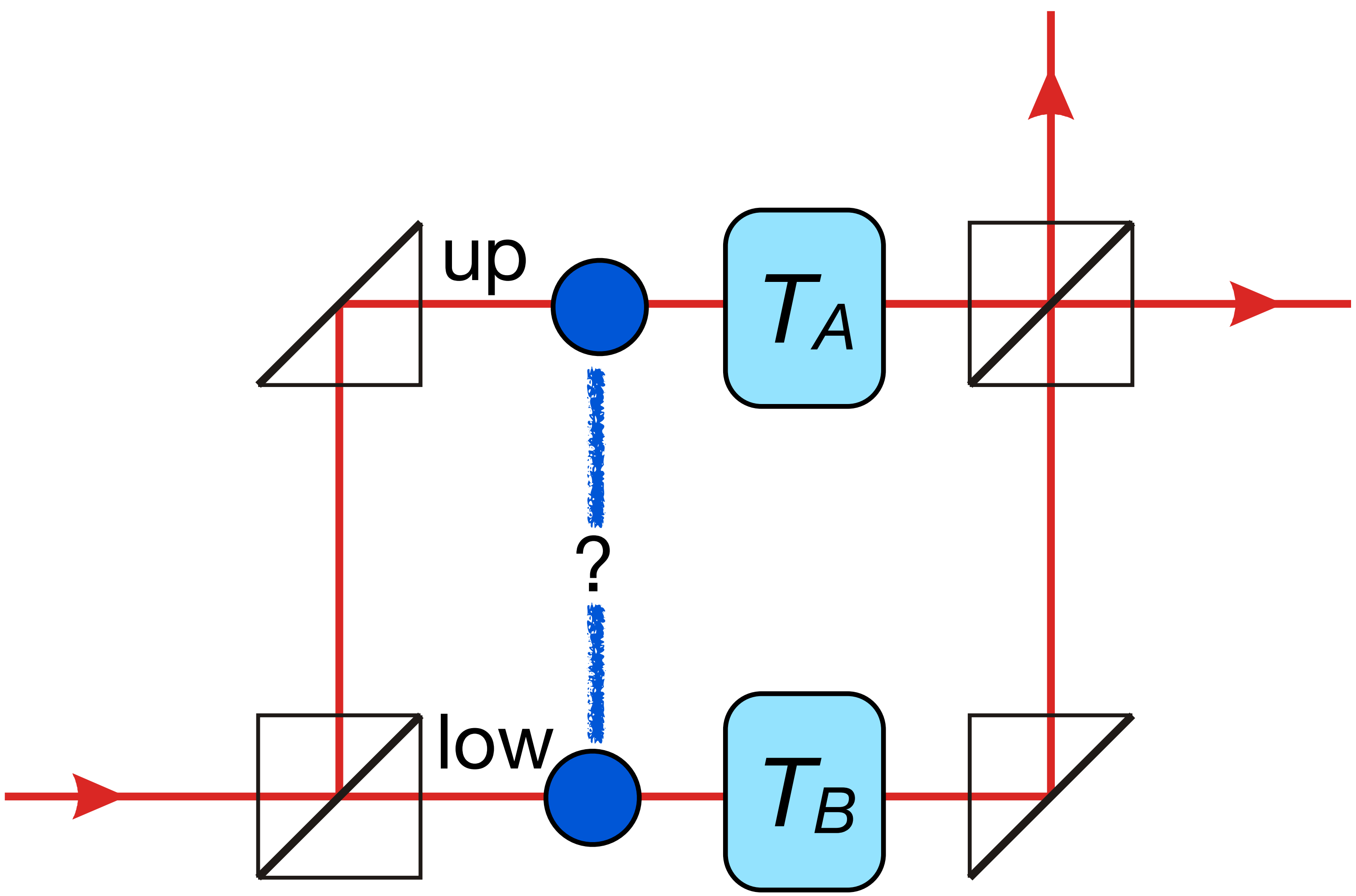
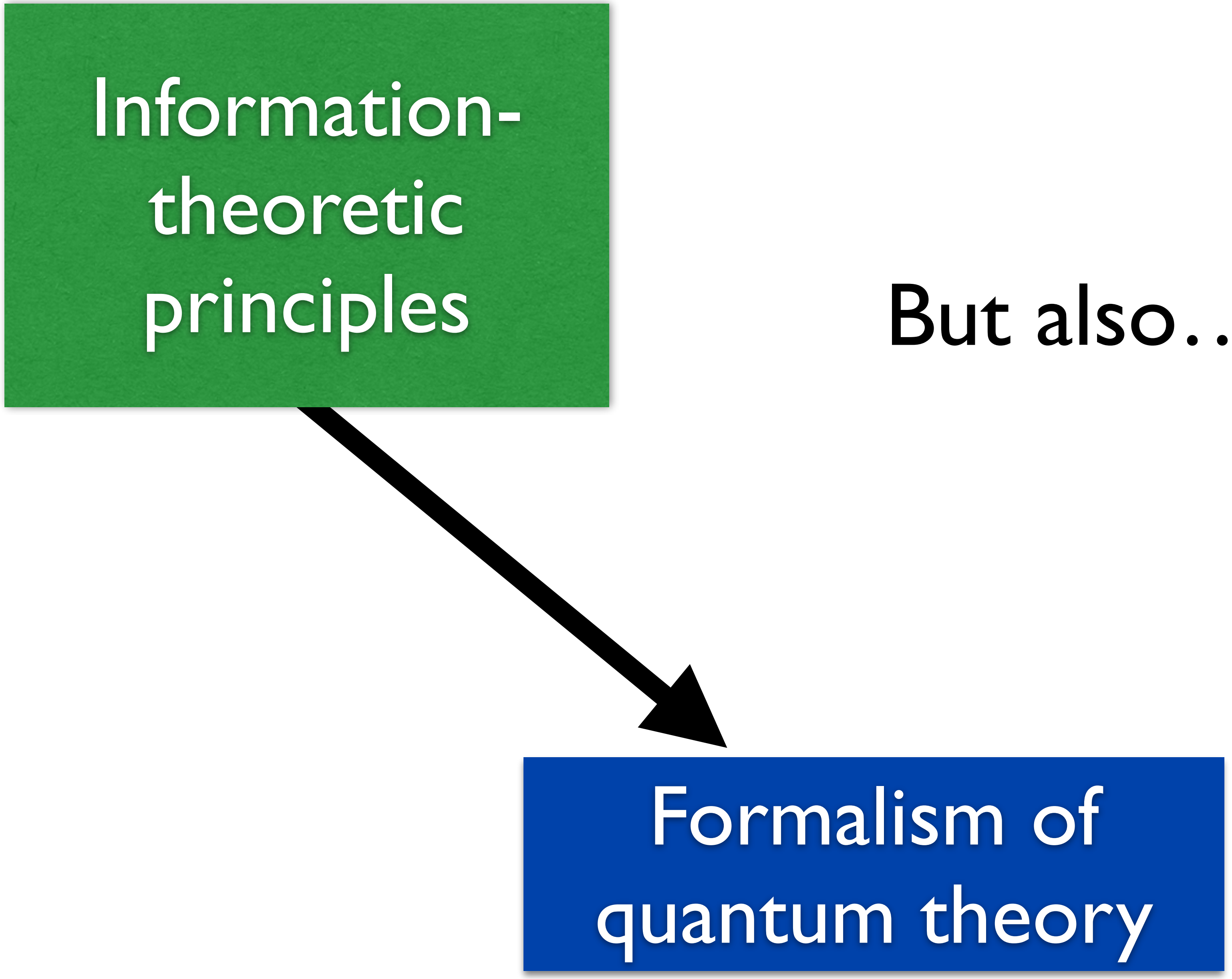
Quantum theory and spacetime: progress from principles

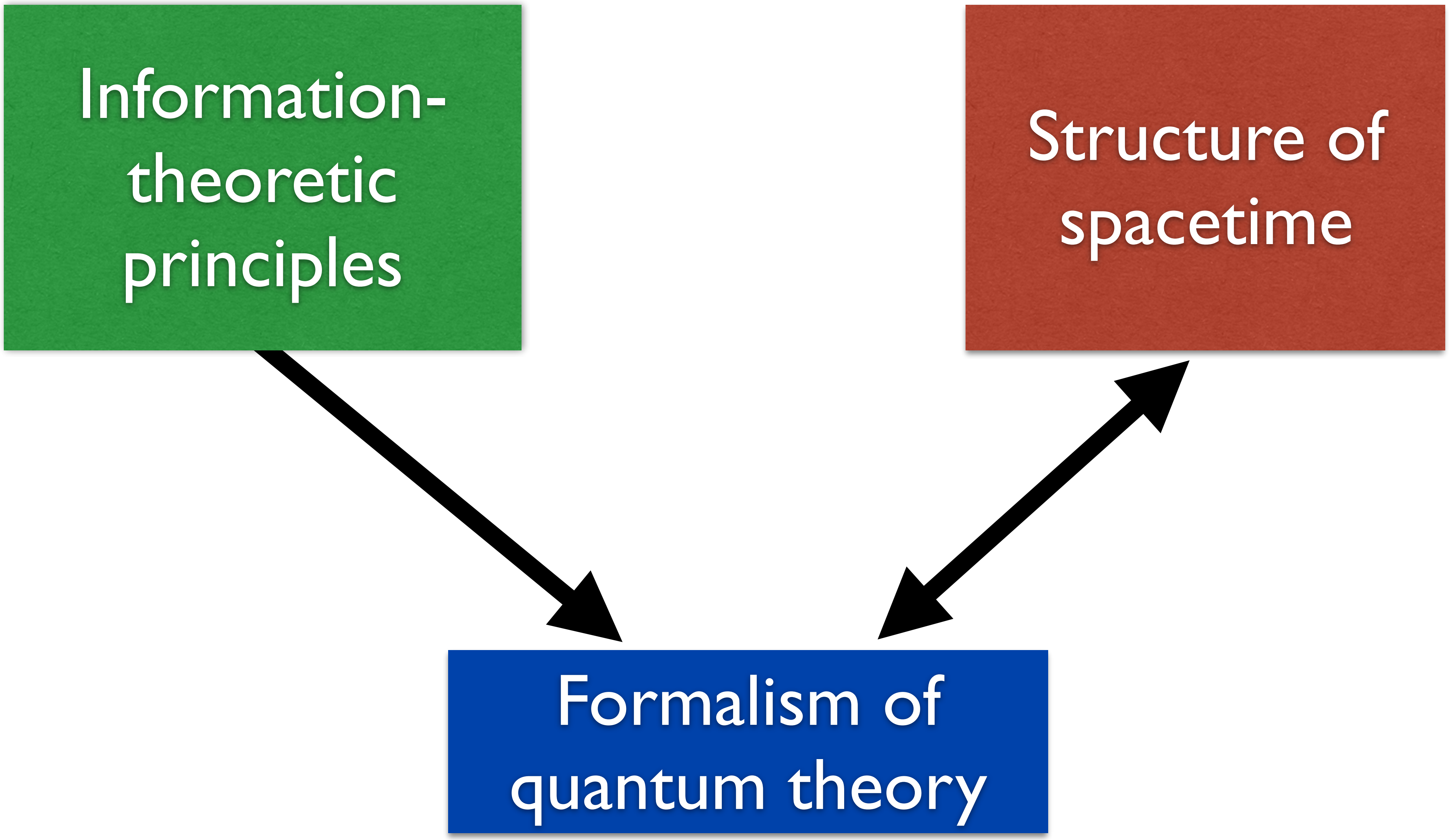
Markus P. Müller
Institute for Theoretical Physics, Heidelberg University (Germany)



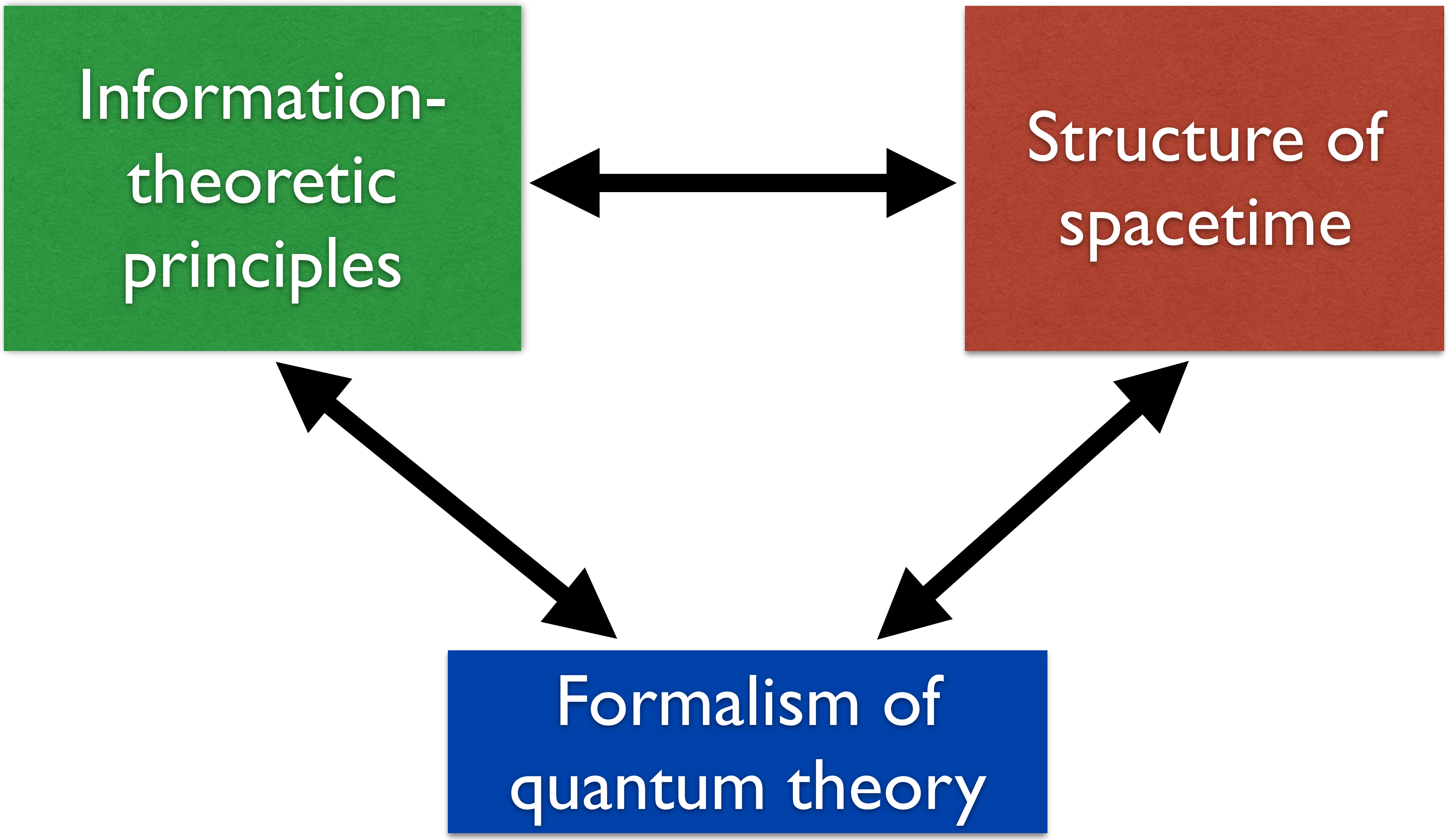

Information-theoretic principles
Formalism of quantum theory

But also…

Main message

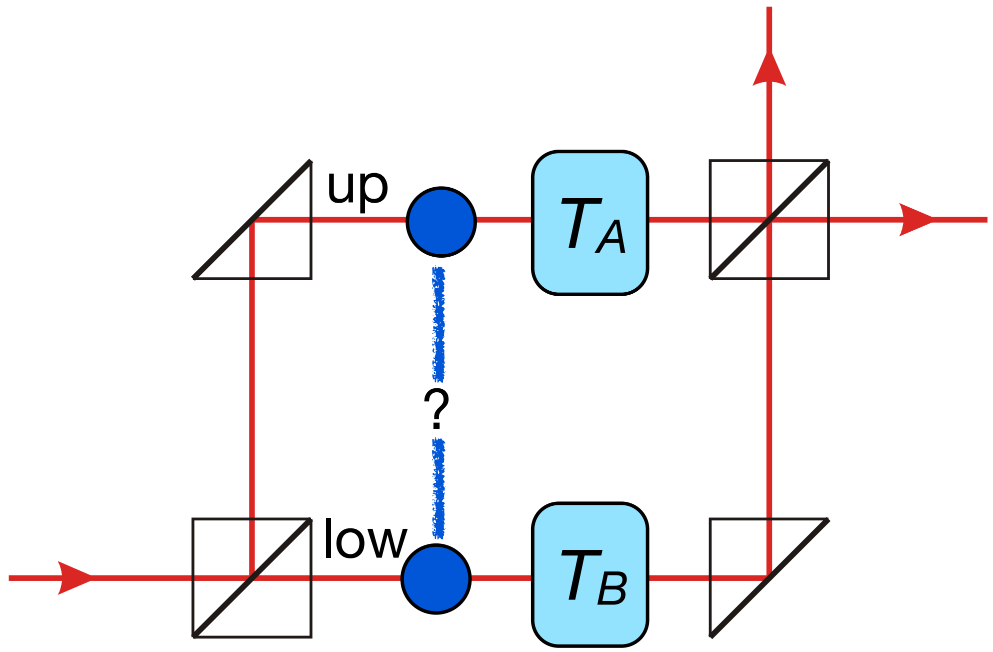


Main message

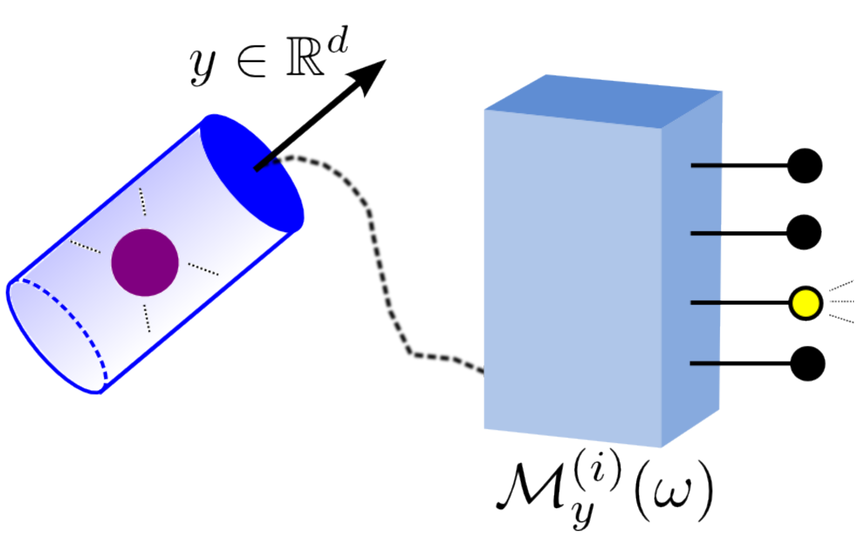


Insights into the “architecture“ of physics

1. Relativity and interference experiments

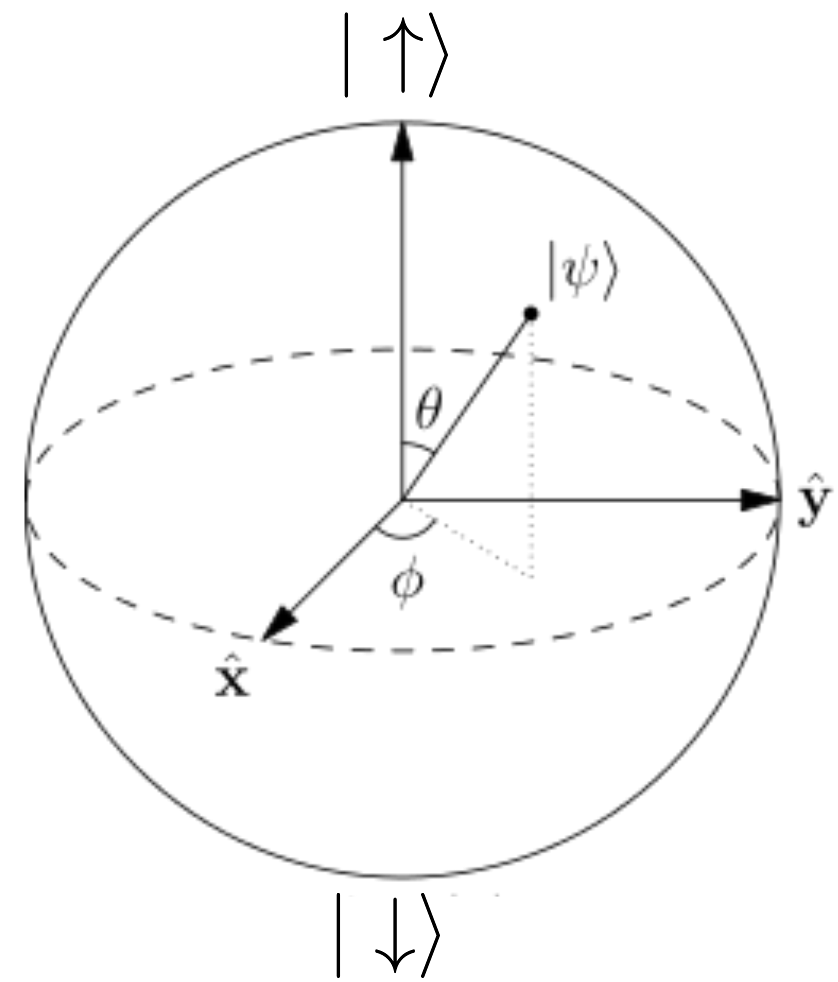


2. Quantum theory and the dimensionality of space



The state space of a quantum bit is a 3D ball - the Bloch ball.

$$\cos\frac{\theta}{2}|\uparrow\rangle + e^{i\phi}\sin\frac{\theta}{2}|\downarrow\rangle$$



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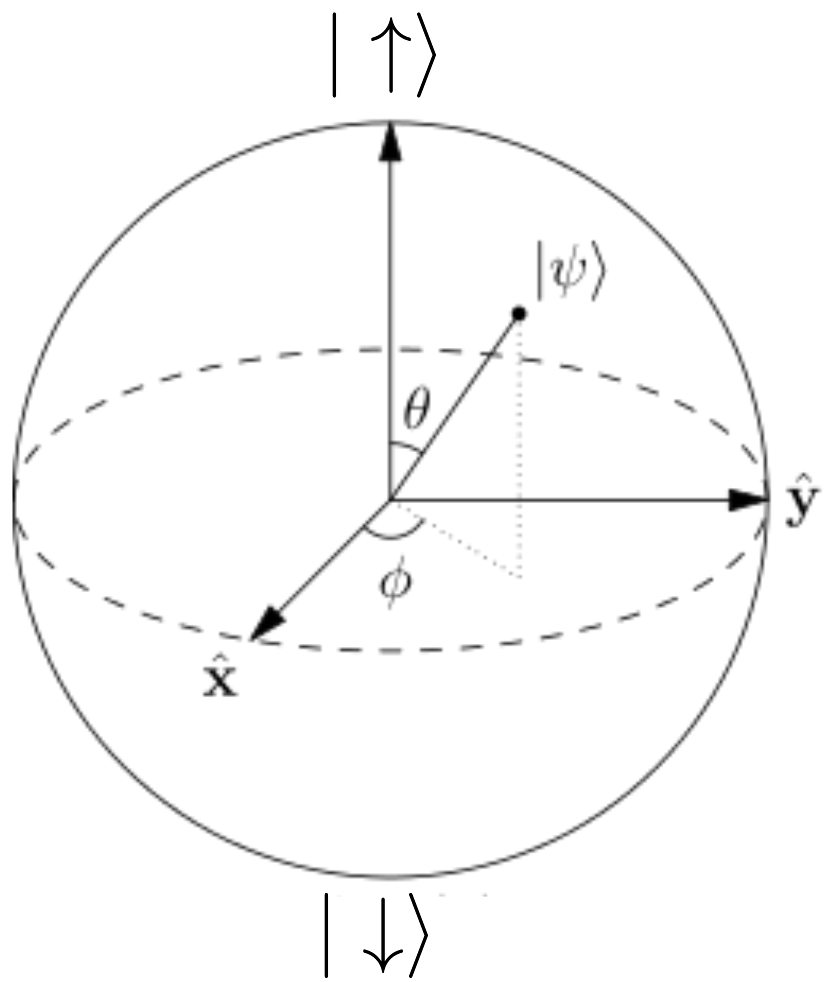


In most reconstructions of QT, it is

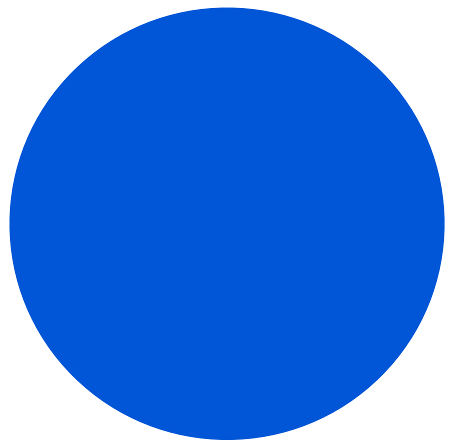
- first shown that a bit is a *d*-ball,
- then shown that *d*=3 (difficult!).

1. Relativity and interference experiments

Two-level state spaces ("bits") are naturally ball state spaces:

$d = 1$

classical bit



$d = 2$

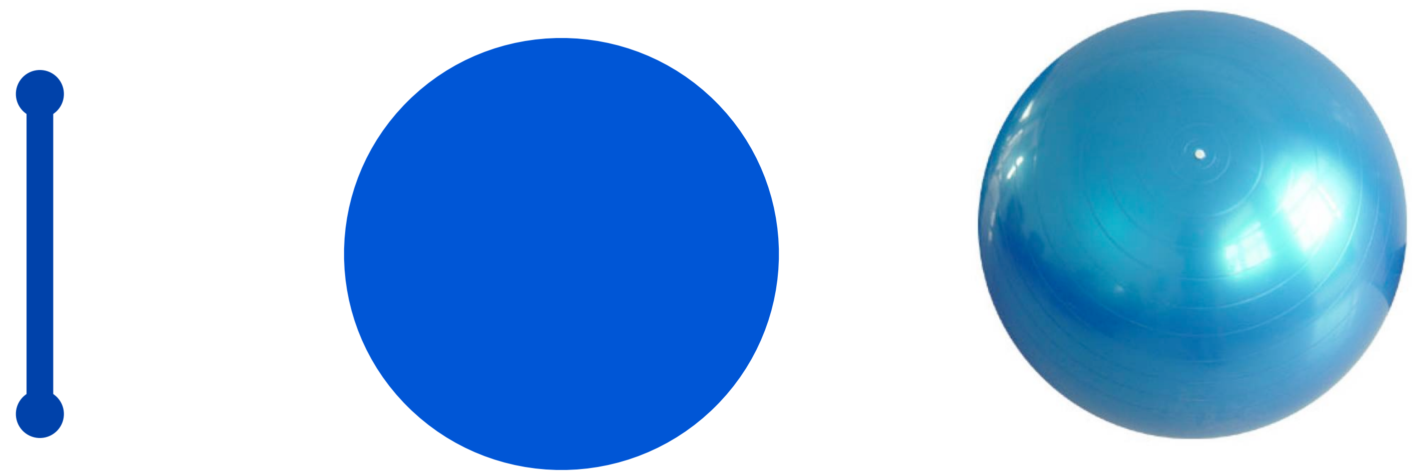
$d = 3$

quantum bit

...

1. Relativity and interference experiments

Two-level state spaces ("bits") are naturally ball state spaces:



$d=1$ classical bit

$d=2$

$d=3$ quantum bit

...

$d=2,5,9$ are bits in quantum theory over $\mathbb{R}$, $\mathbb{H}$, $\mathbb{O}$.

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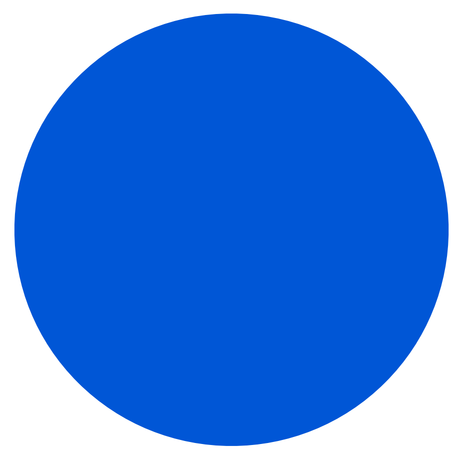


$d = 2$

$d = 3$

quantum bit

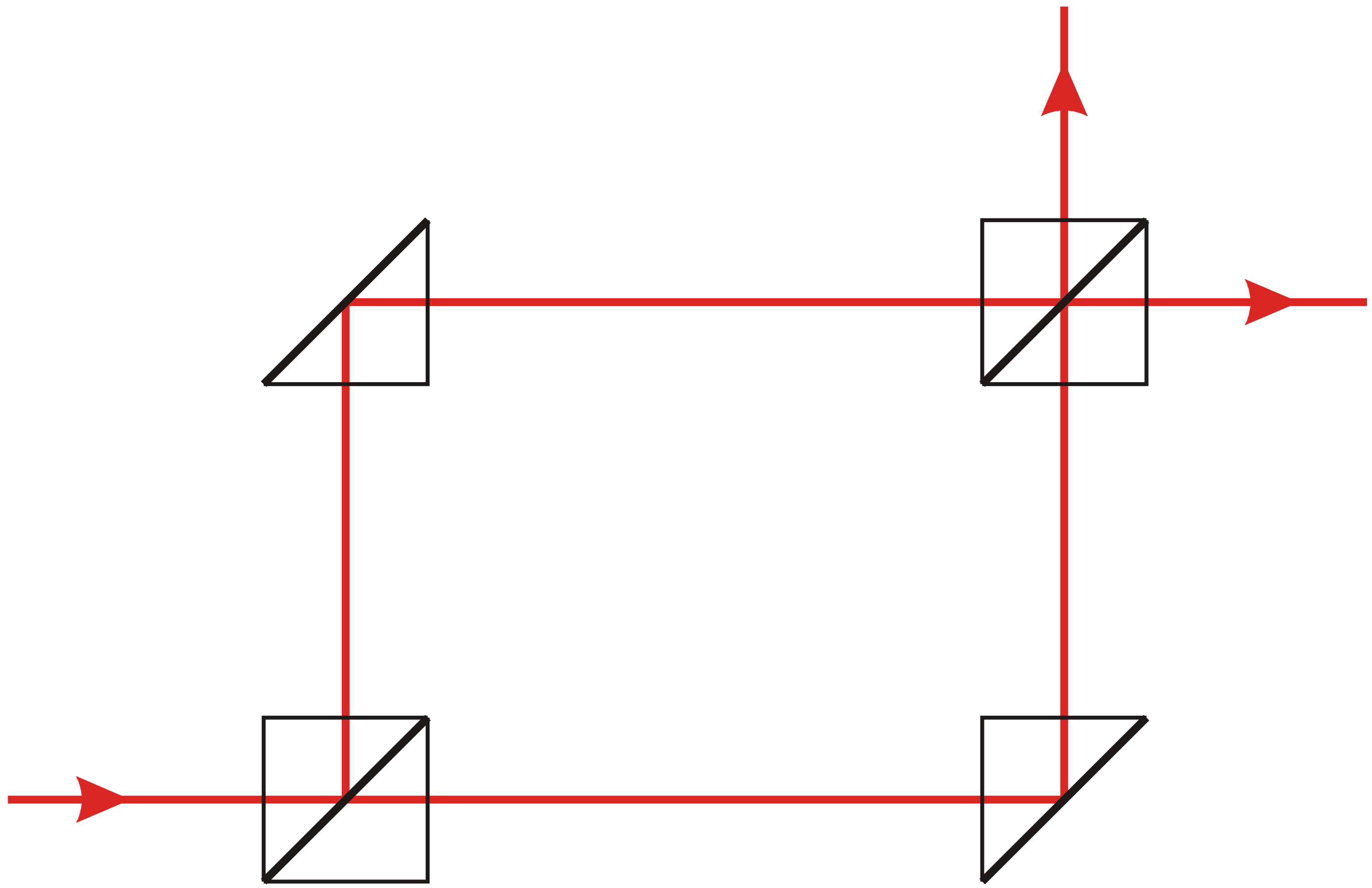
...

$d = 2, 5, 9$ are bits in quantum theory over $\mathbb{R}, \mathbb{H}, \mathbb{O}$.

We will now show that relativity of simultaneity rules out all $d \geqq 4$!

Relativistic constraints on interference experiments

Joint work w/ Andy Garner & Oscar Dahlsten:



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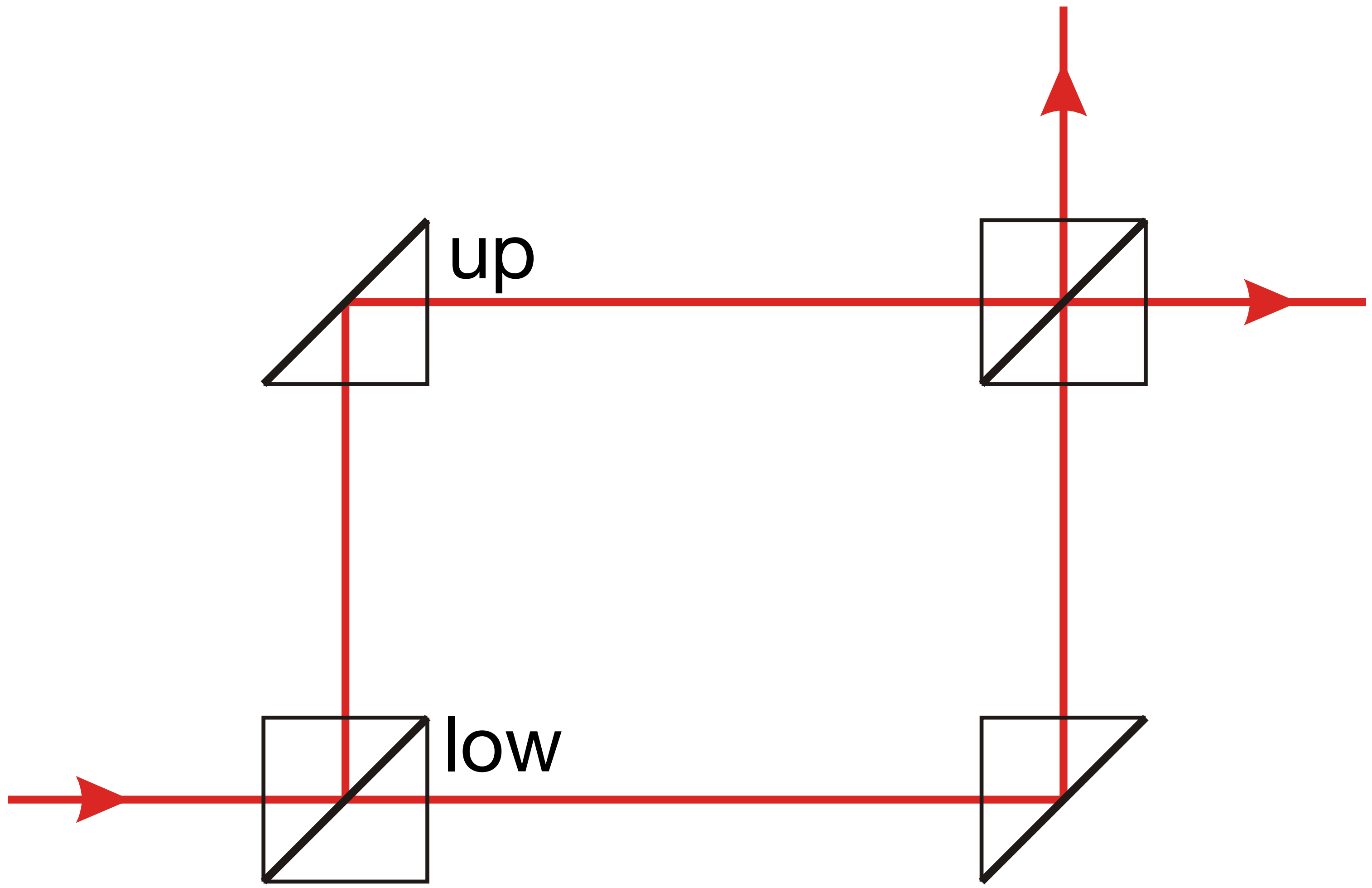

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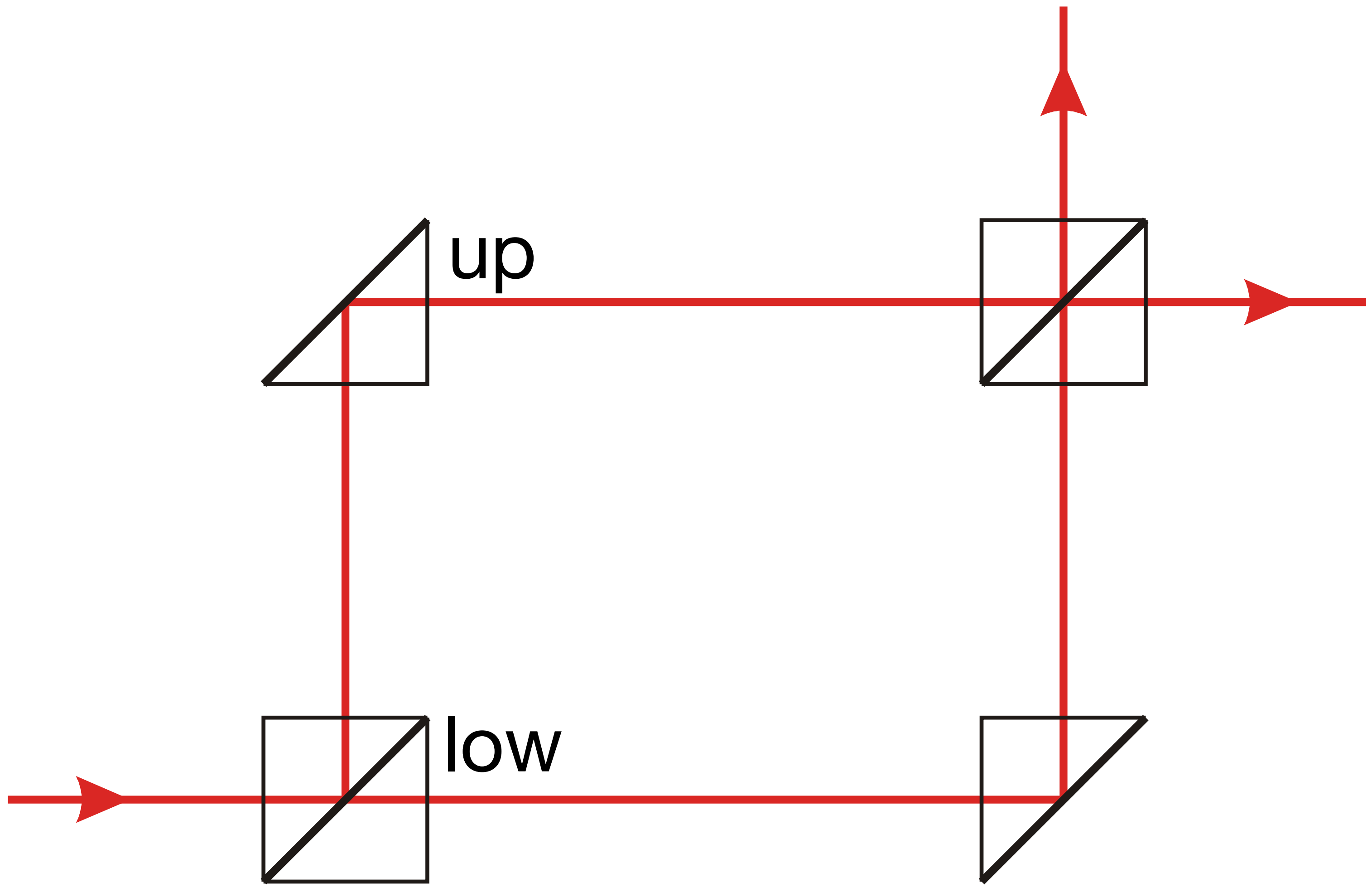

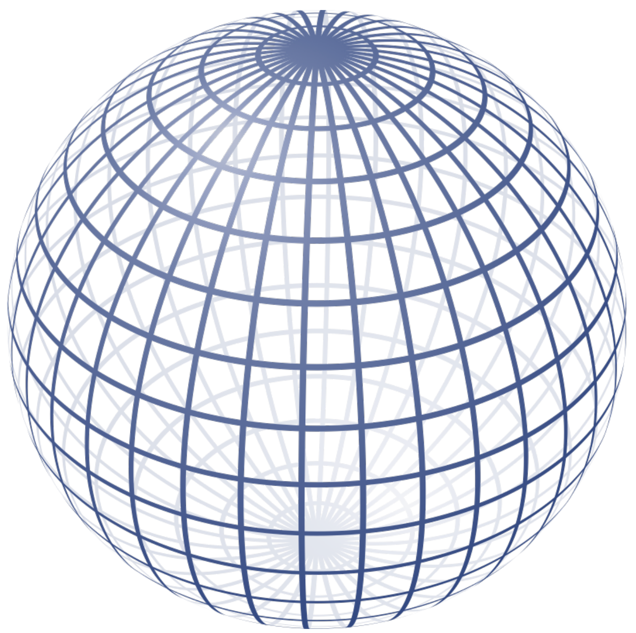

d-dim. "Bloch sphere"

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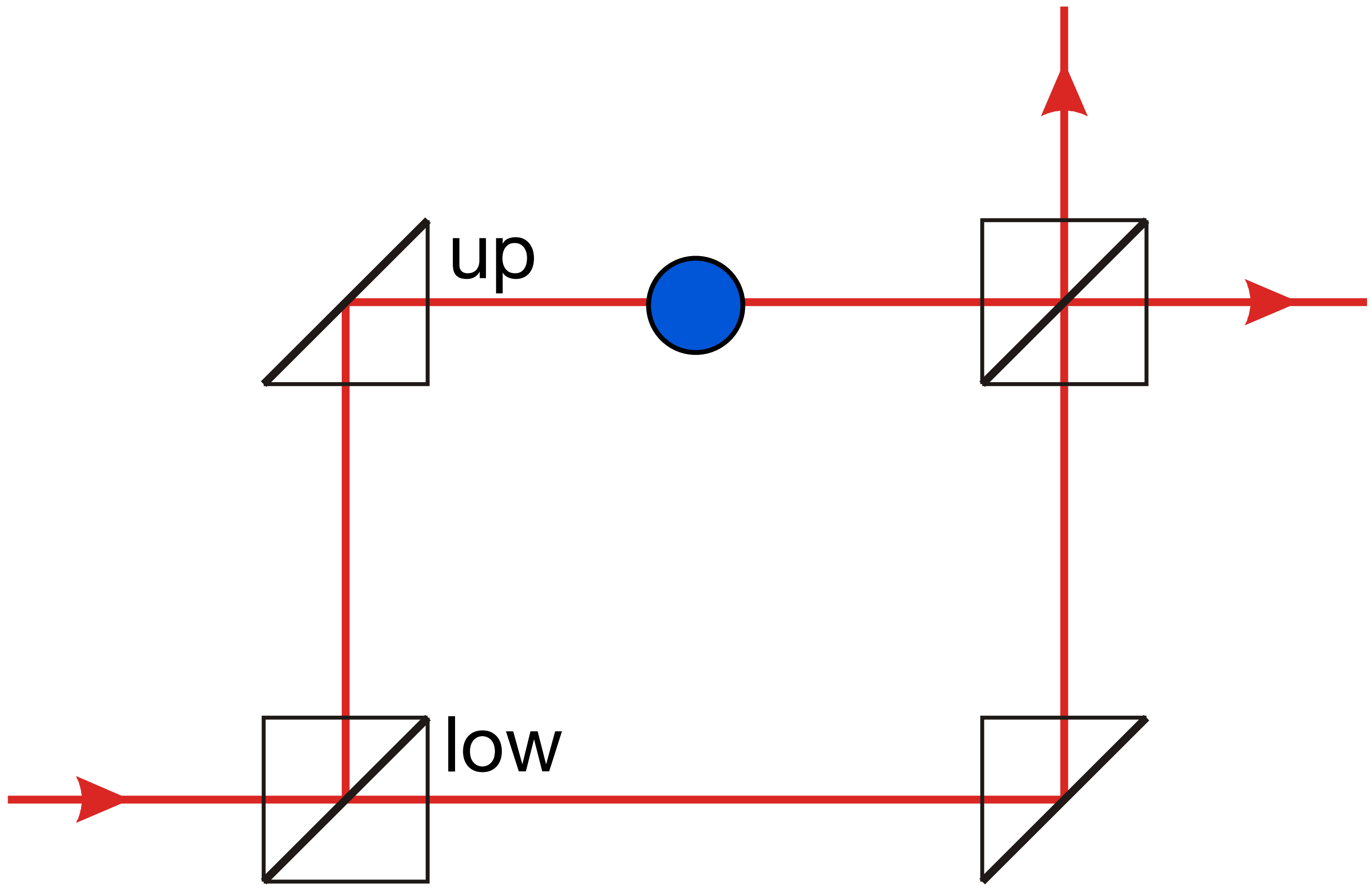

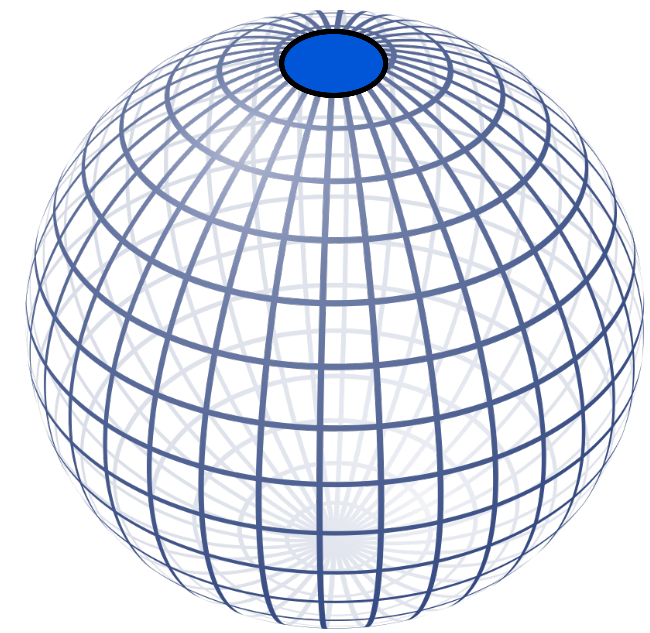

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North-pole state: particle definitely in upper branch.

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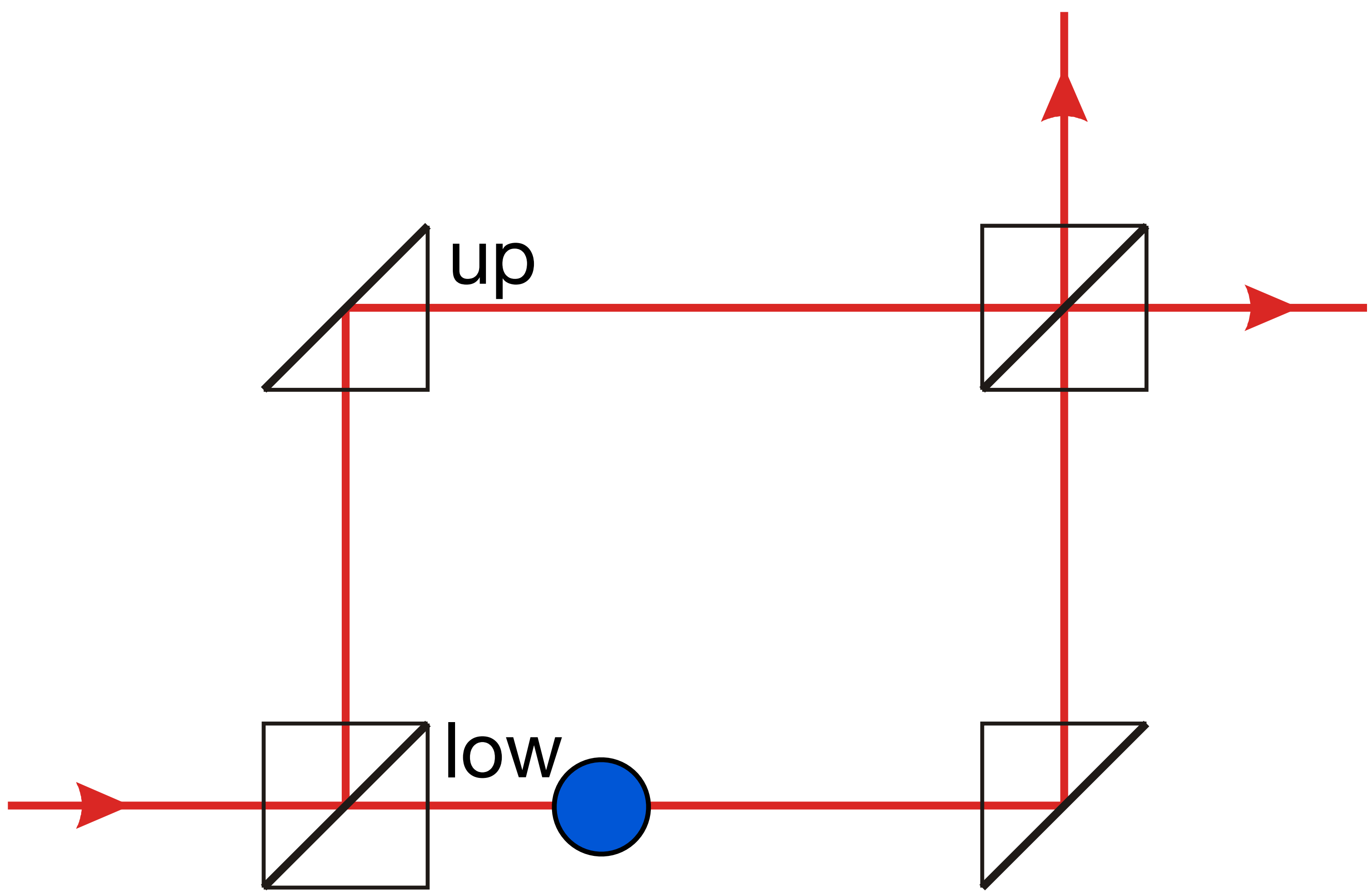

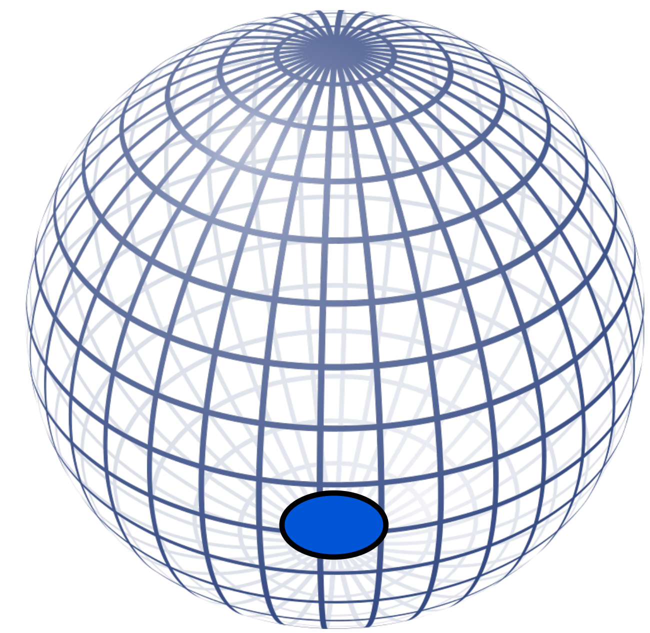

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South-pole state: particle definitely in lower branch.

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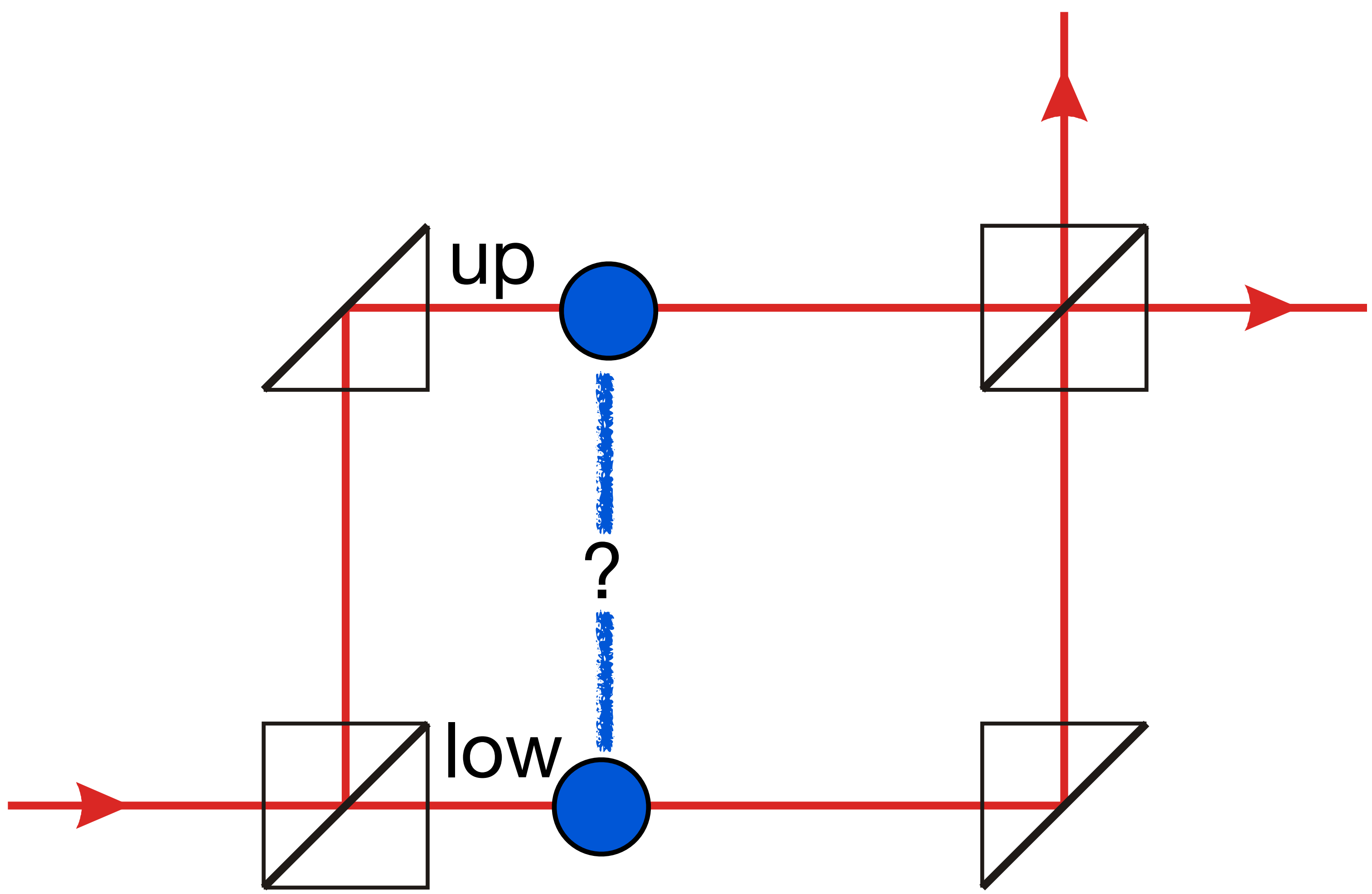

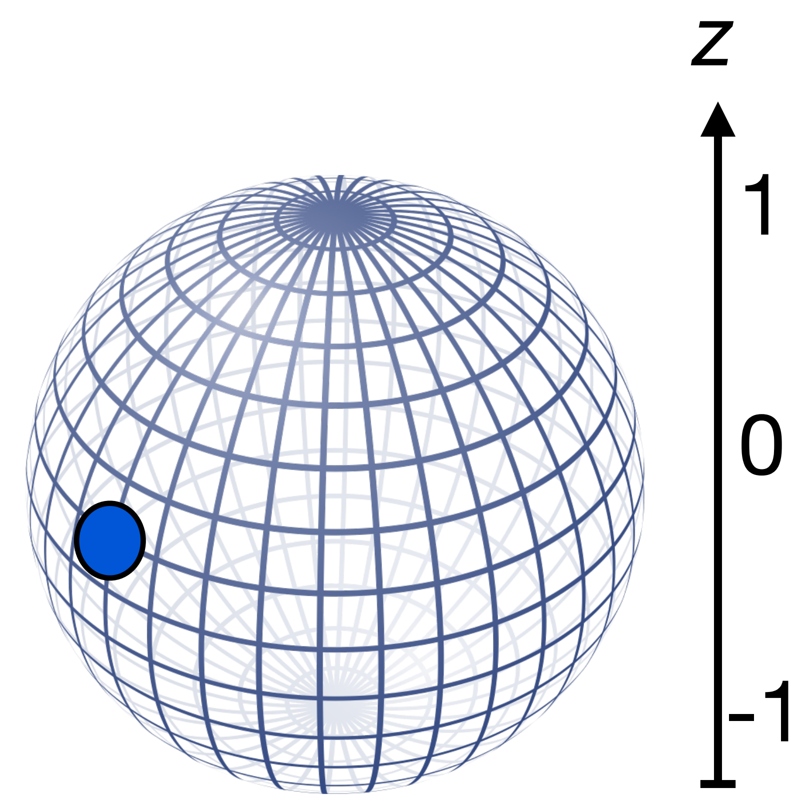

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State on equator *z=0*: probability 1/2 for each.

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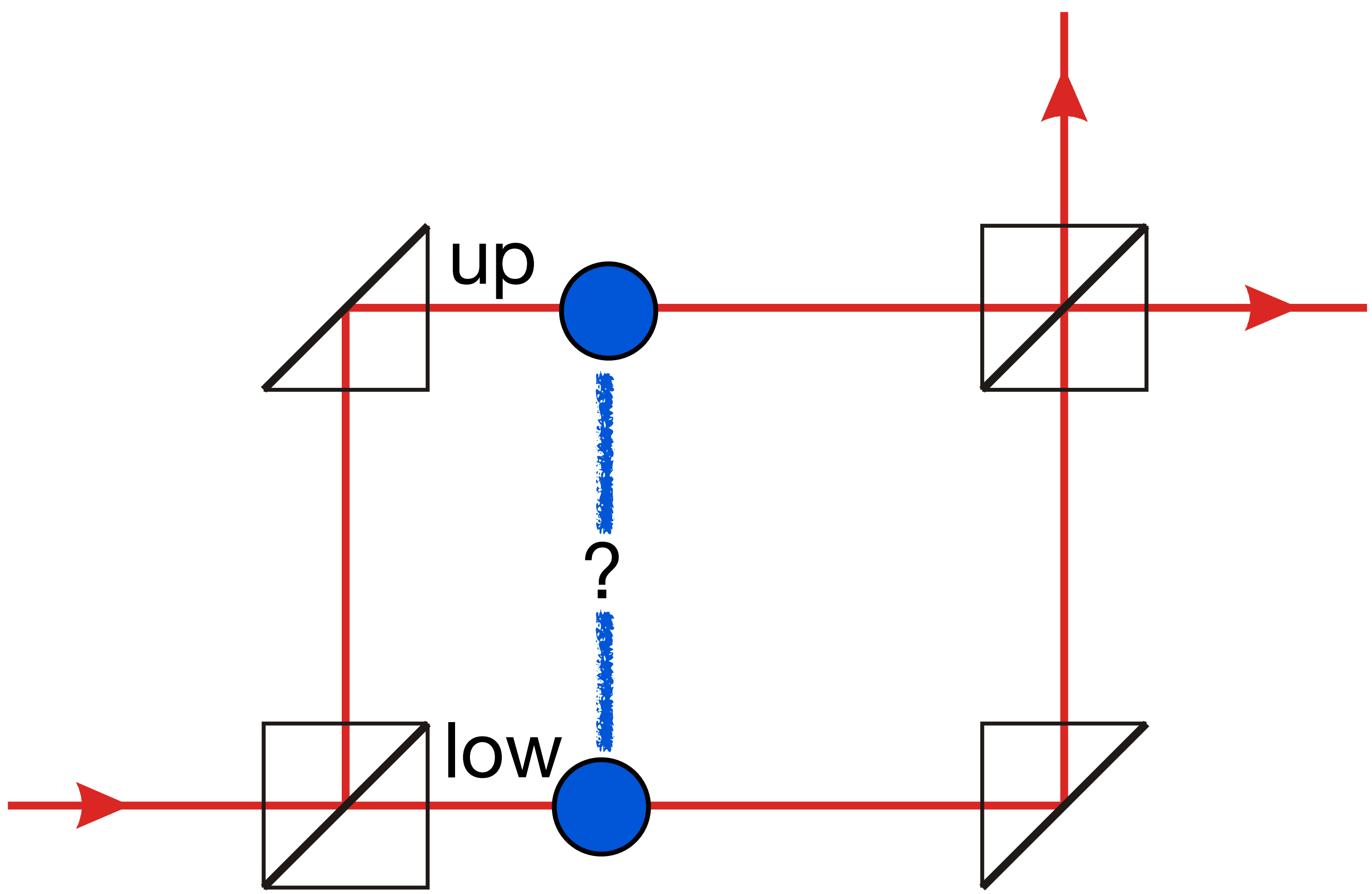

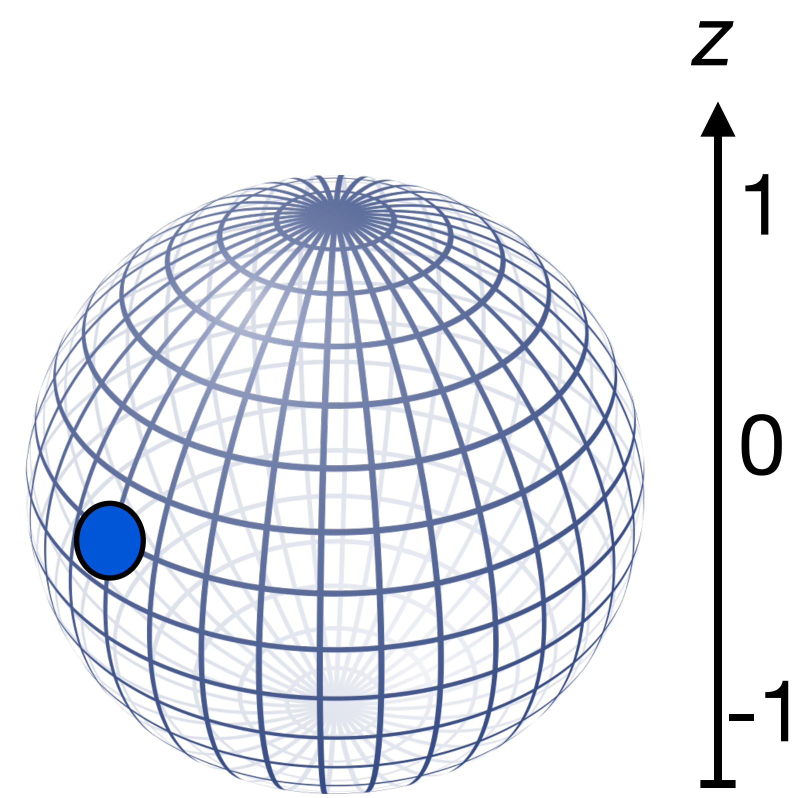

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State on equator *z=0*: probability 1/2 for each.

$$p(\mathrm{up}) = \frac{1}{2}(z+1)$$

Relativistic constraints on interference experiments

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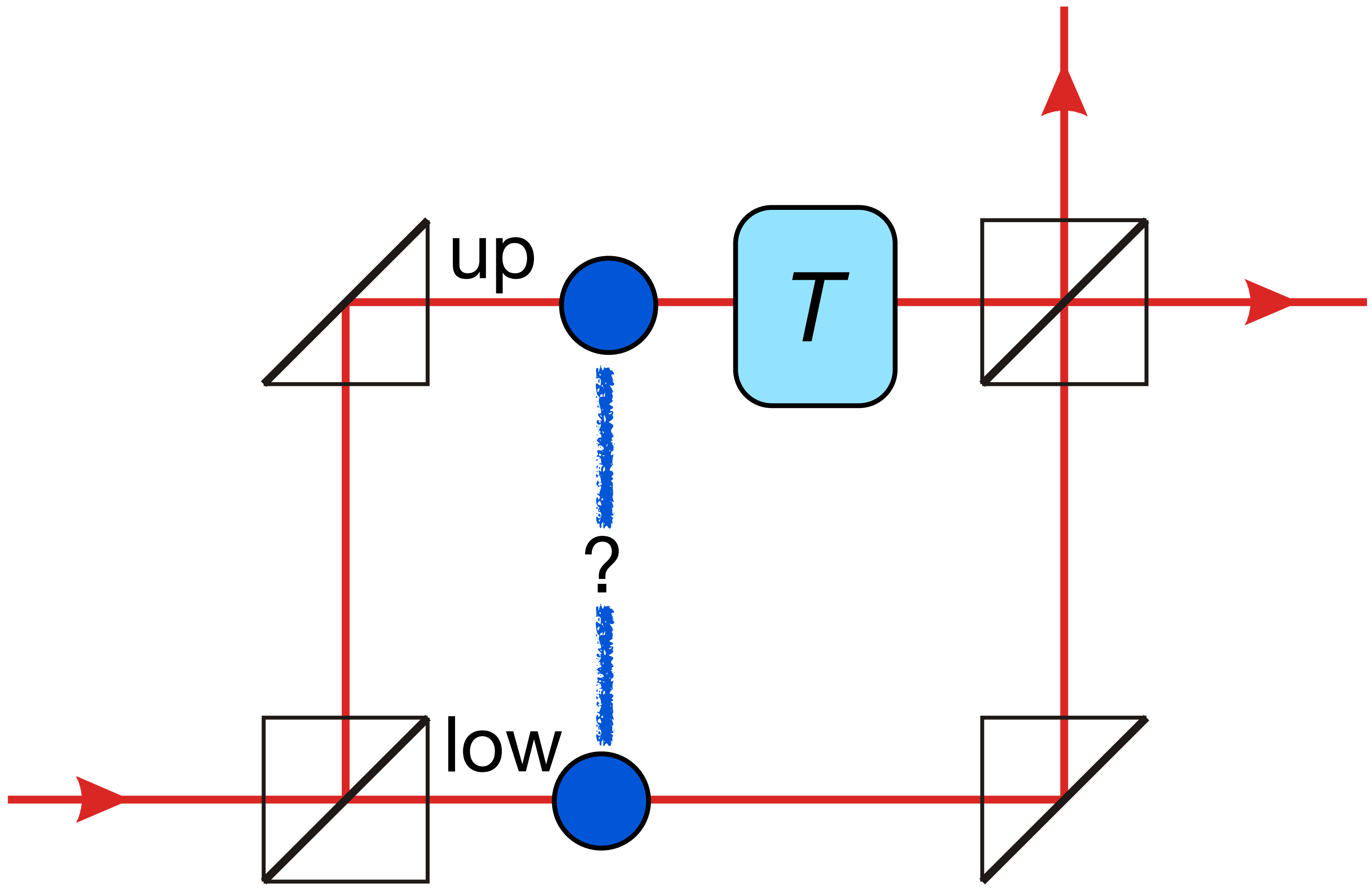

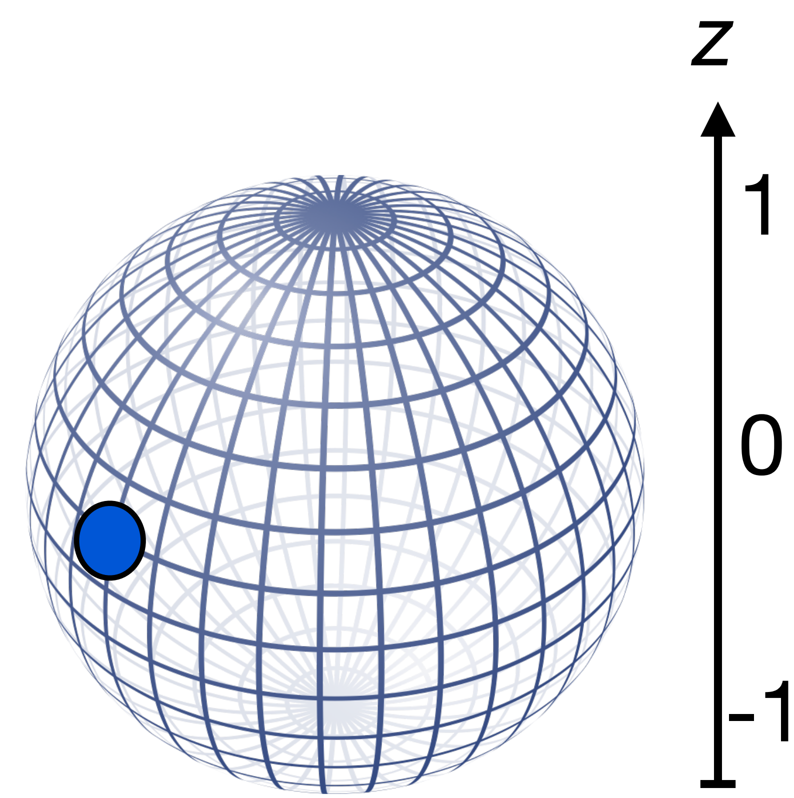

d-dim. "Bloch sphere"

What transformations T can we perform locally in one arm...
... without any information loss?

Relativistic constraints on interference experiments

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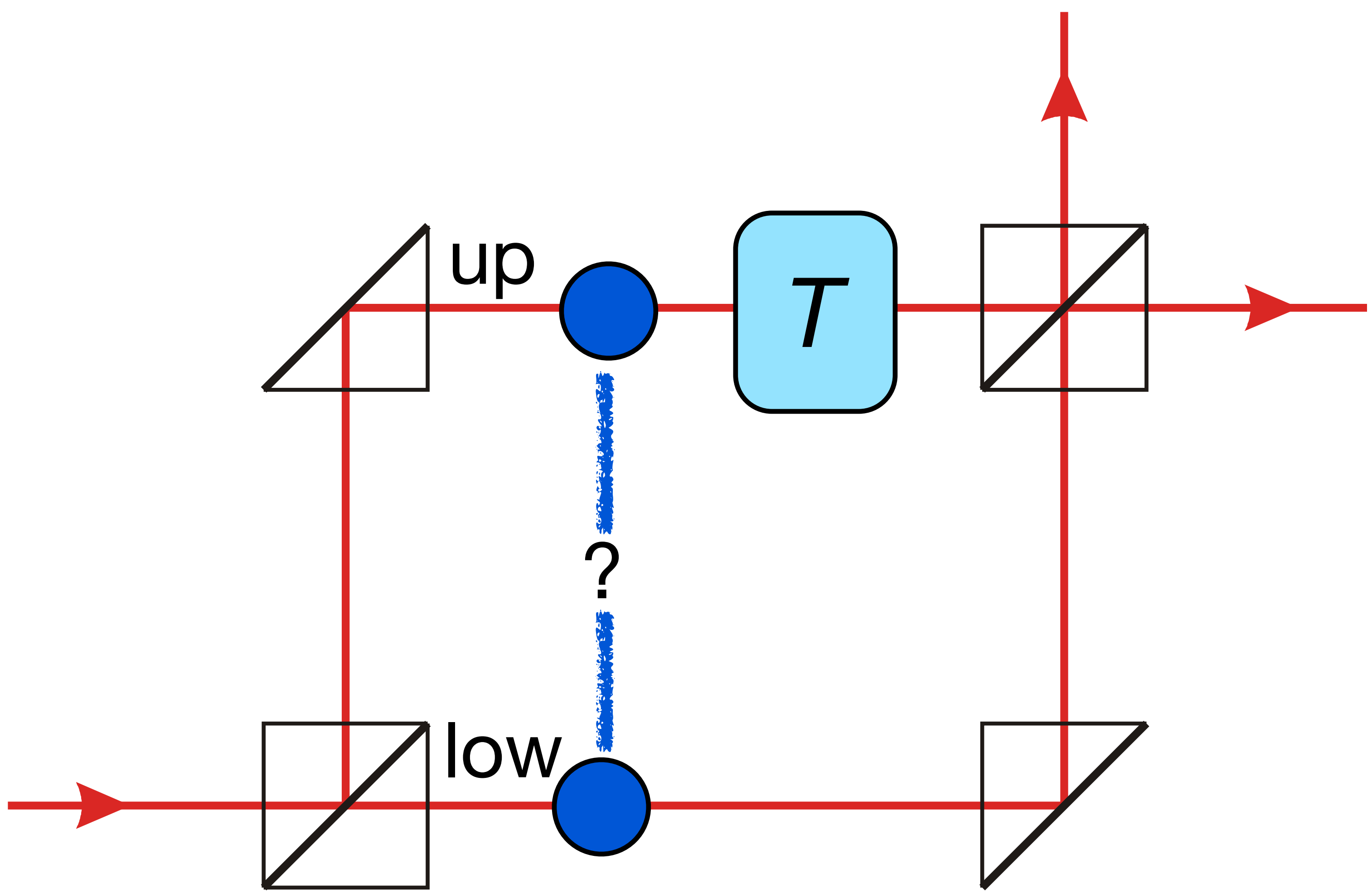
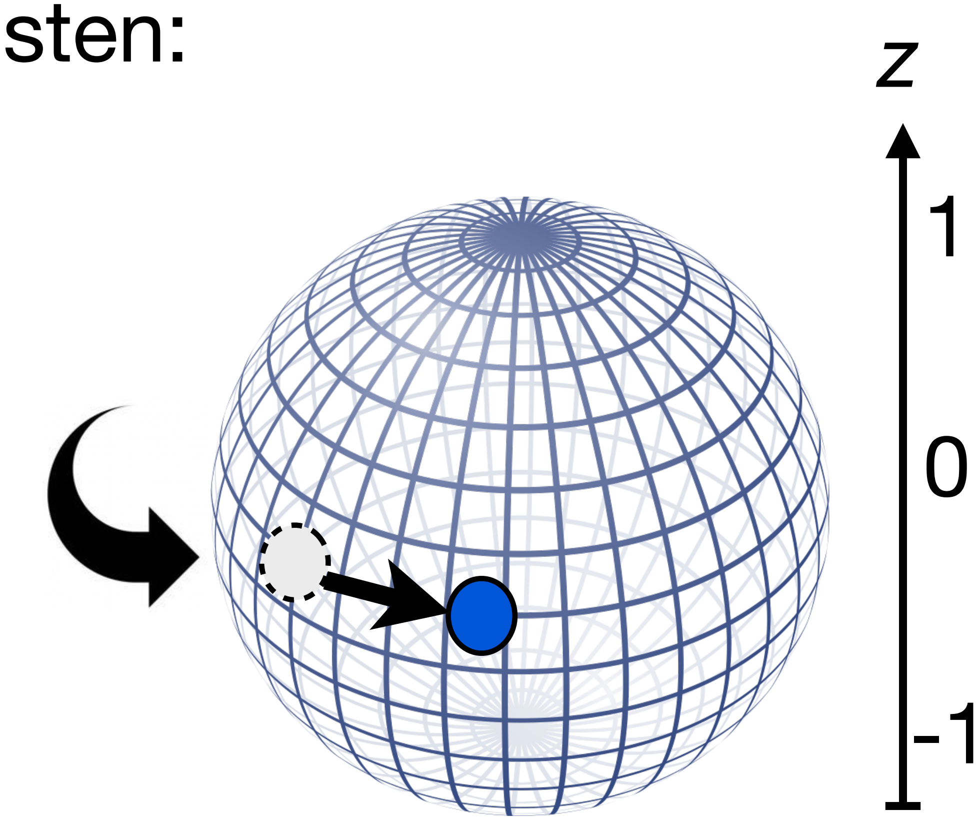
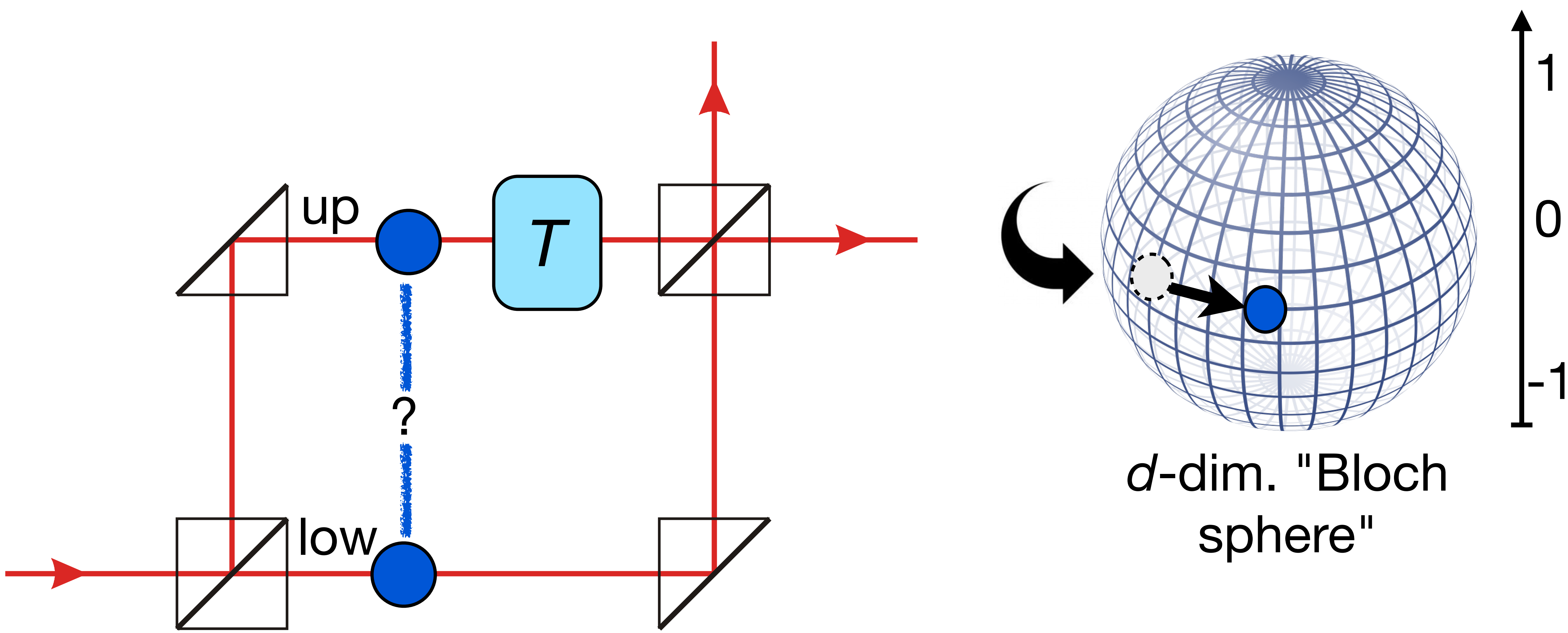




d-dim. "Bloch sphere"

T must be a rotation of the Bloch ball (reversible+linear)...
... and must preserve *p*(up), i.e. preserve the *z*-axis.

Assumption: all maps of this kind are locally implementable.



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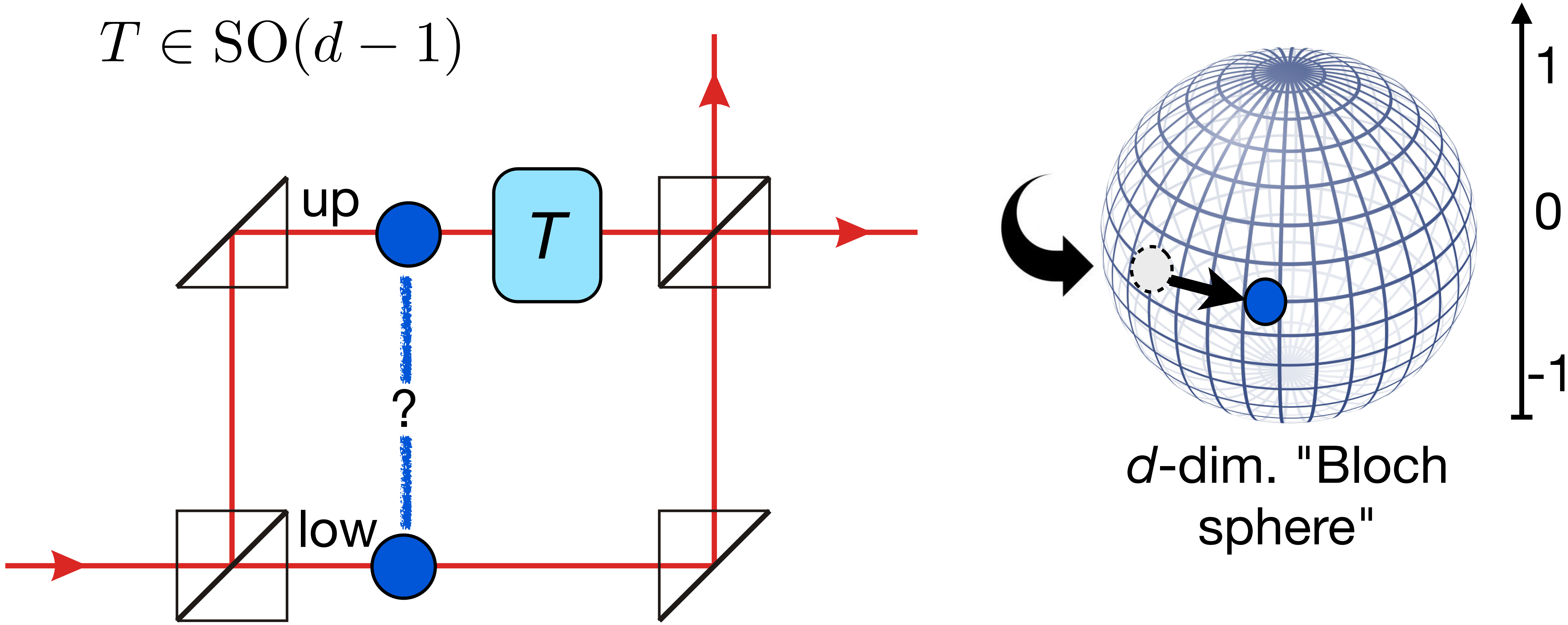

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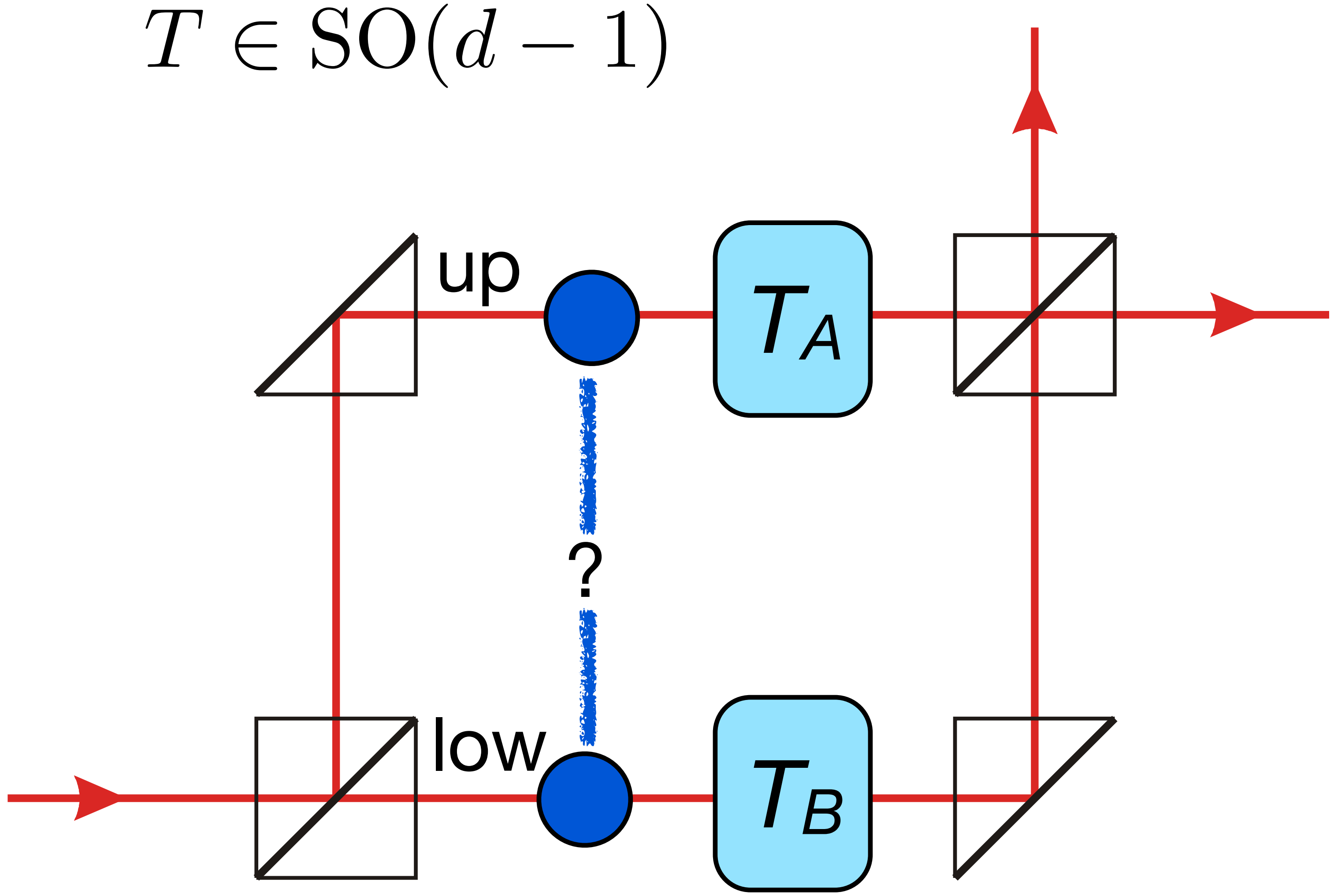
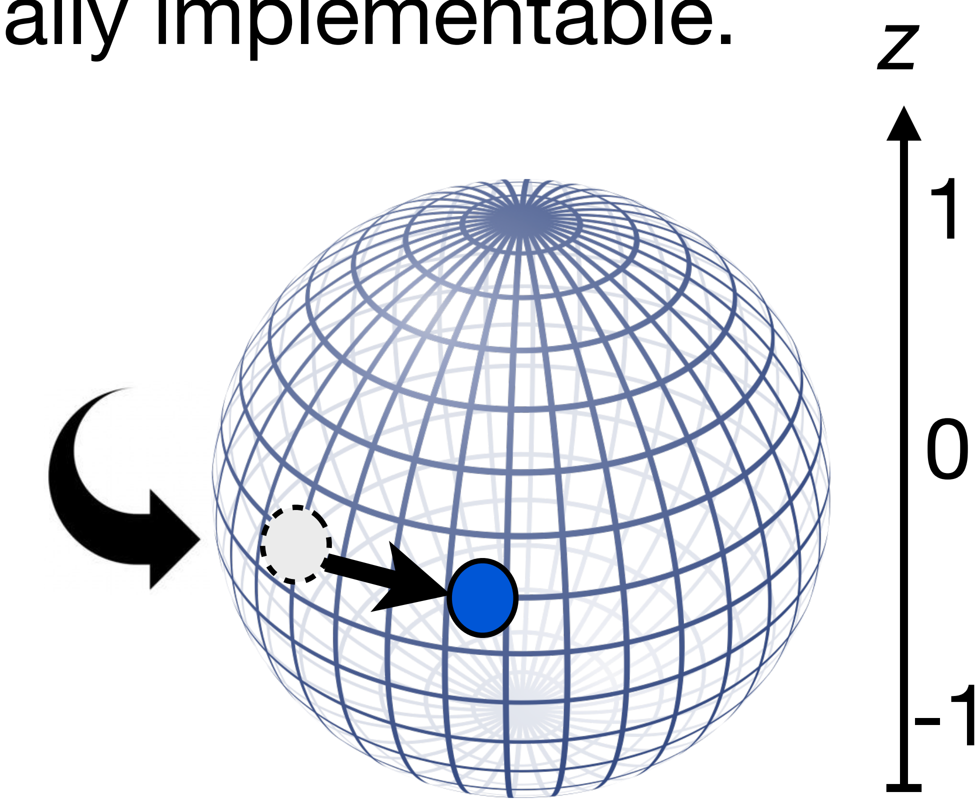
$T \in \mathrm{SO}(d-1)$





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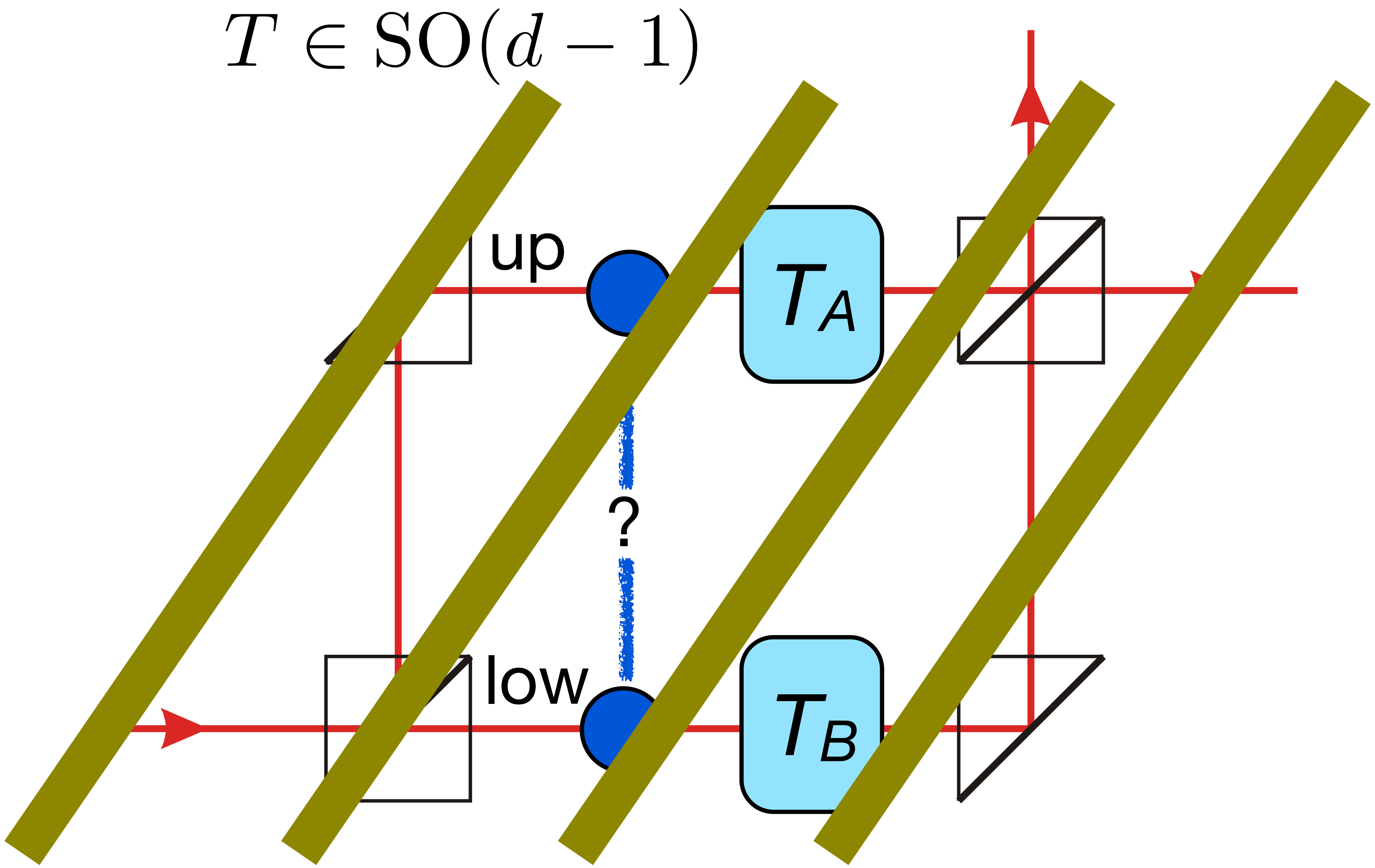

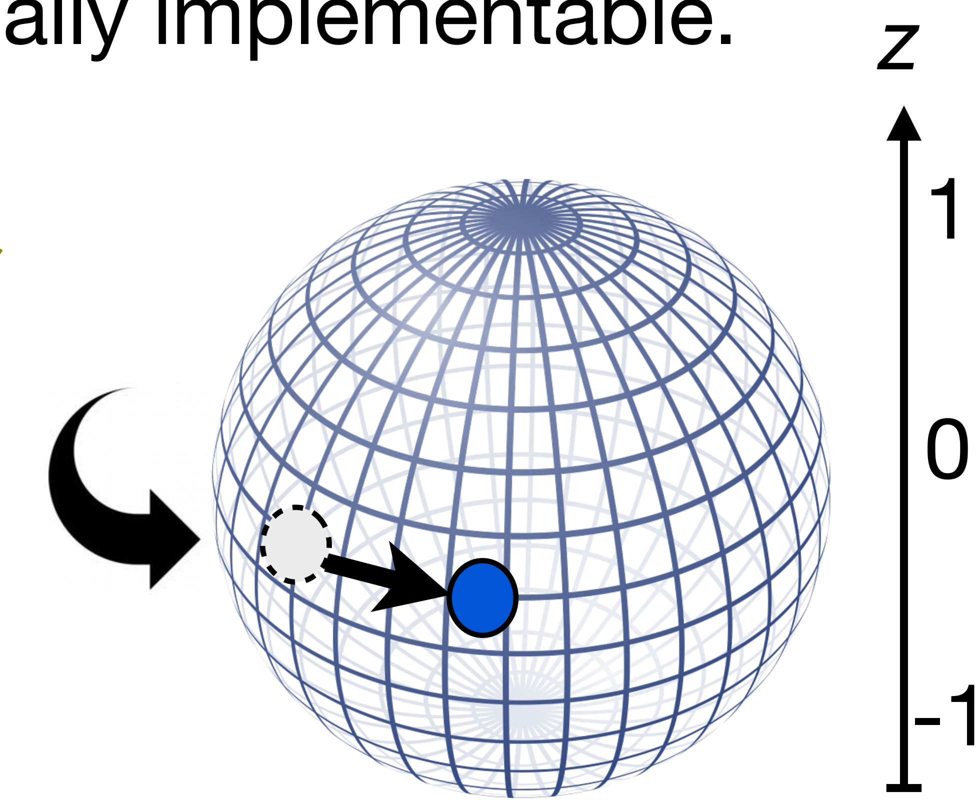

d-dim. "Bloch sphere"

Relativity: there is one frame of reference in which T_A happens first, and then T_B...

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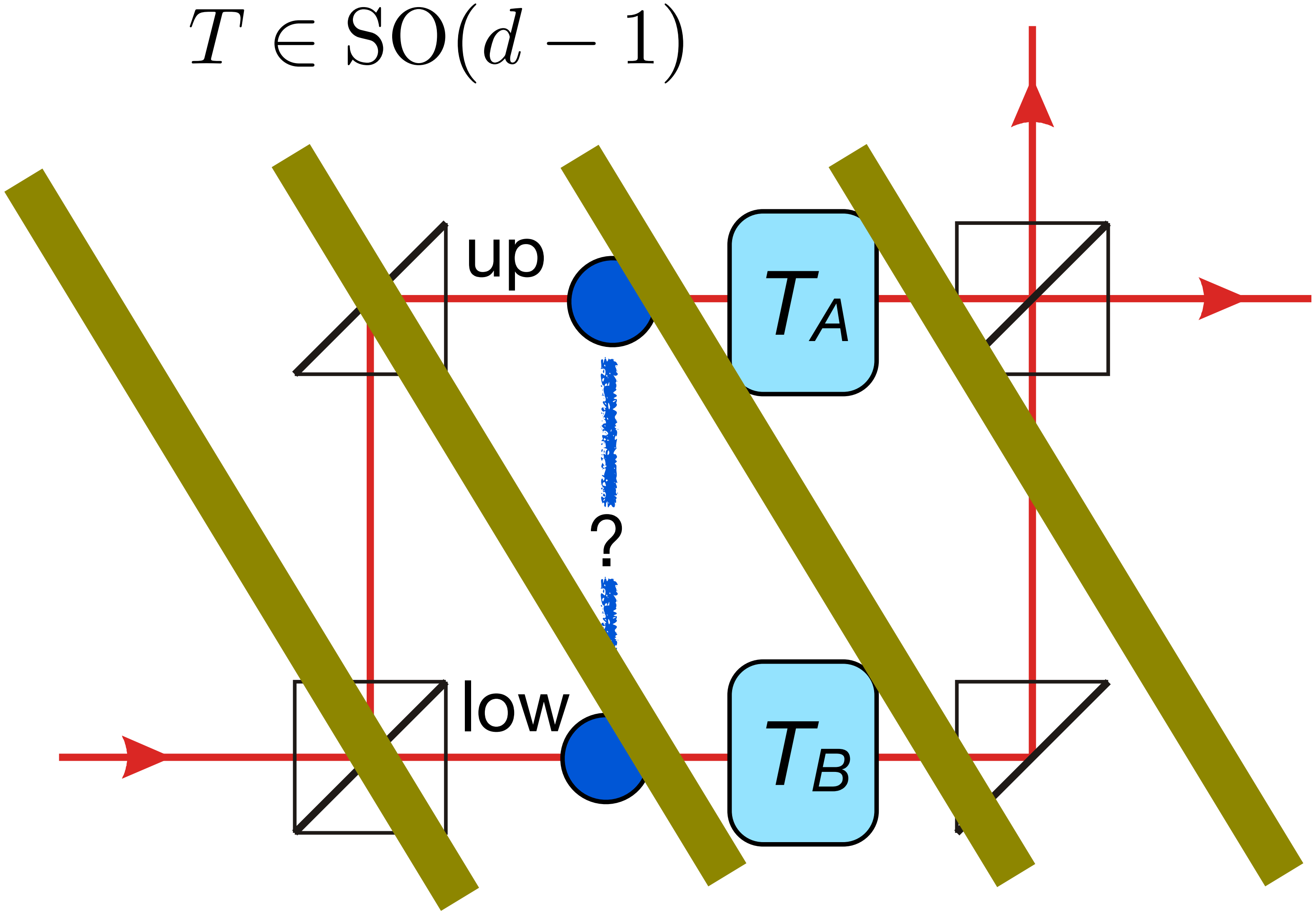

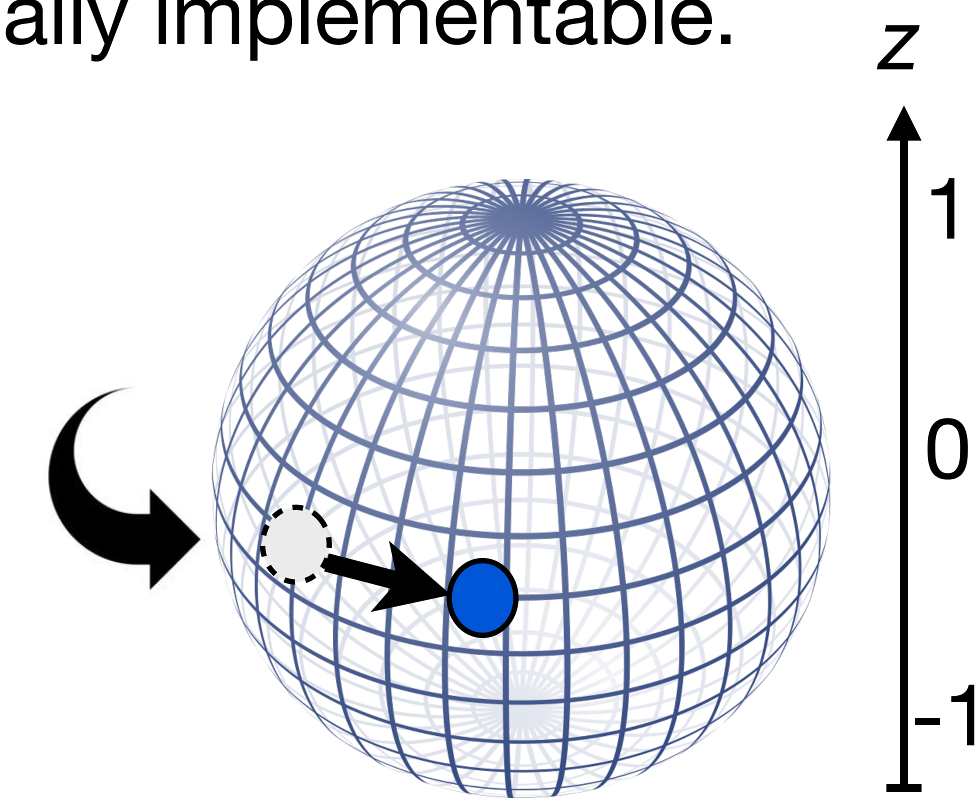

d-dim. "Bloch sphere"

Relativity: ... and another one in which it's the other way around!

Relativistic constraints on interference experiments

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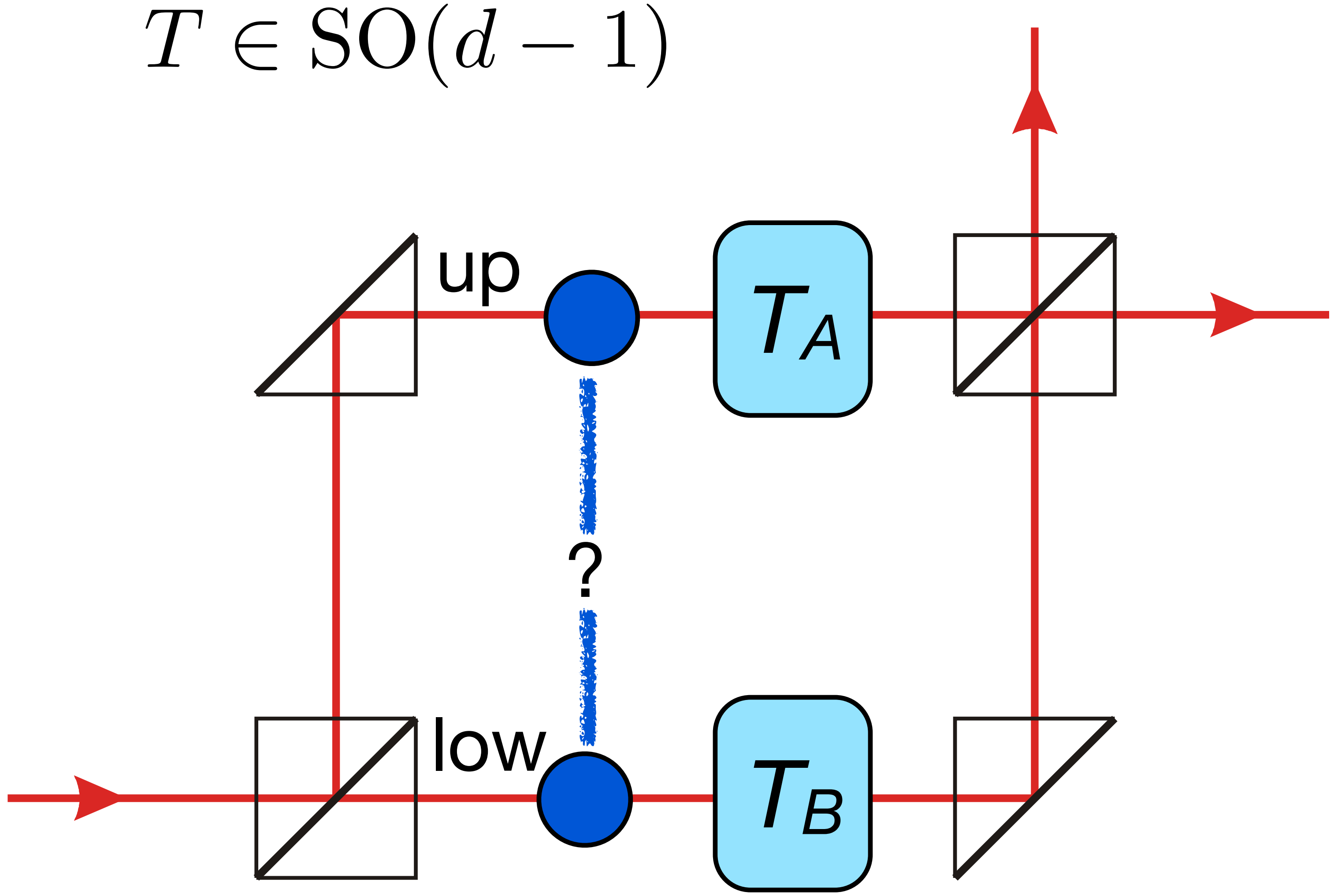
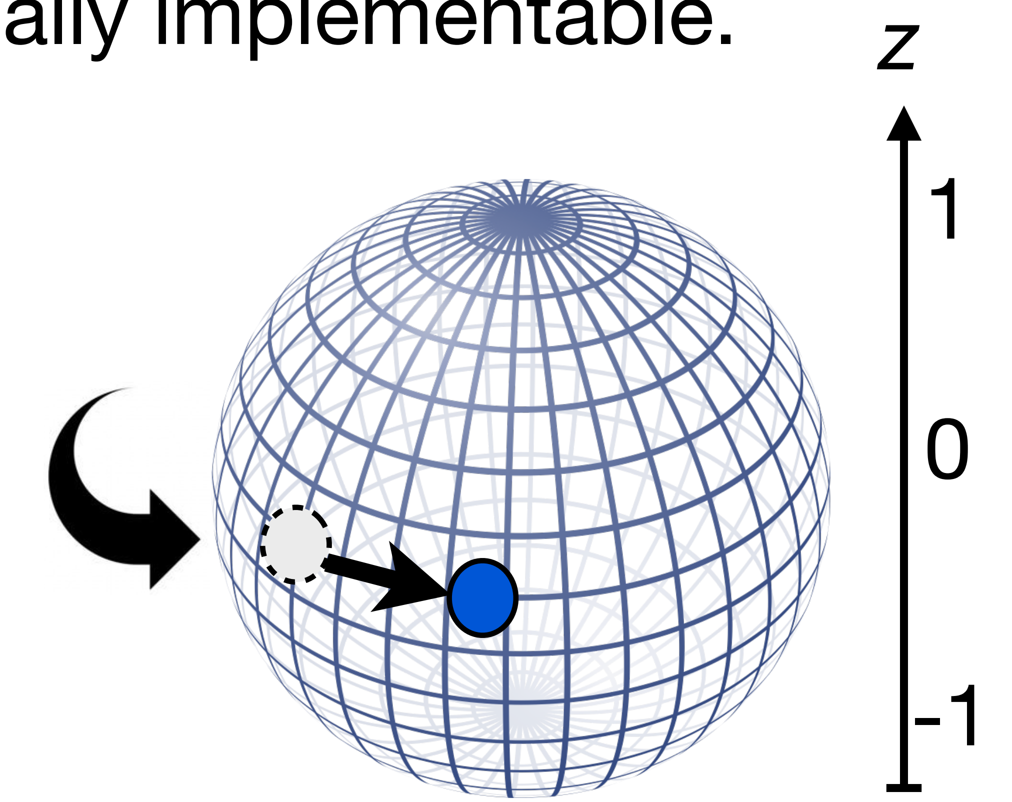


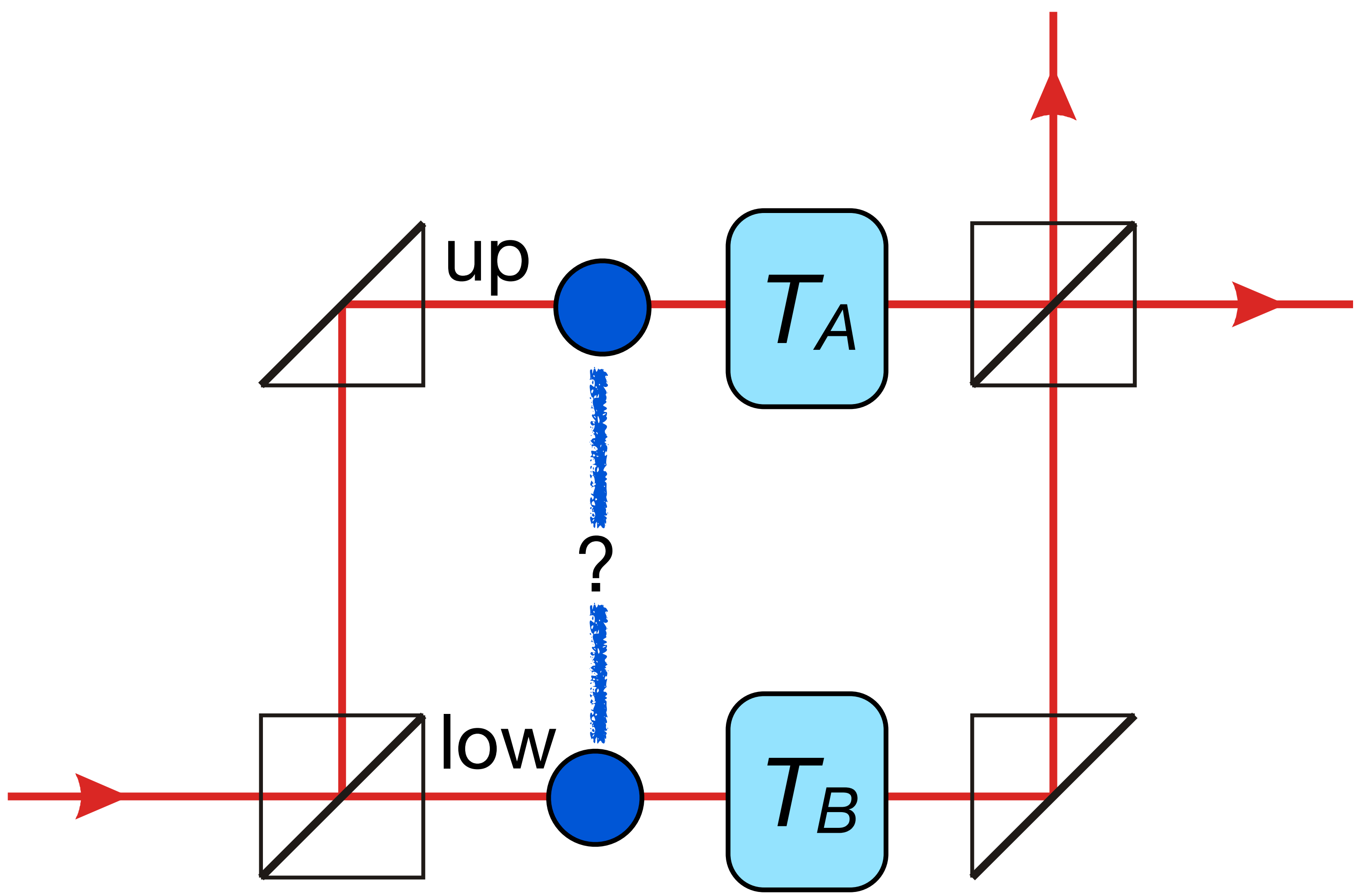


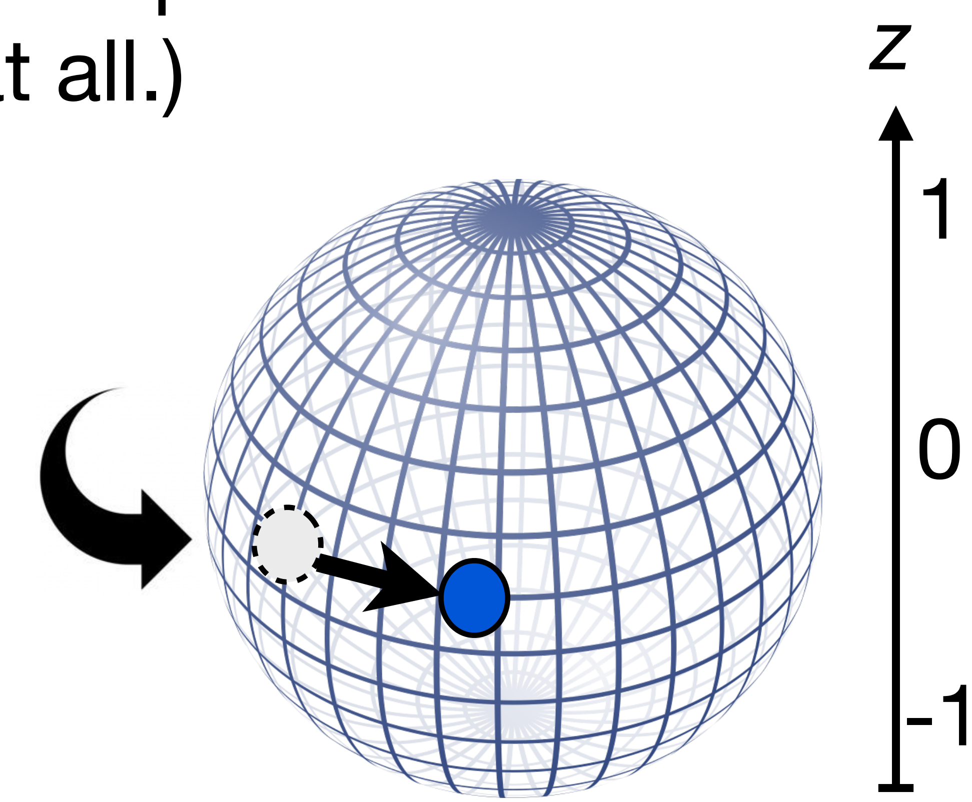
d-dim. "Bloch sphere"

Detector click statistics is Lorentz-invariant

$$\Rightarrow T_A T_B = T_B T_A \text{ for all } T_A, T_B \in \mathrm{SO}(d-1).$$

$\Rightarrow \quad d \leq 3$ (In fact, *d*=*3*, otherwise no "phase transformations" exist at all.)





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Remarks:

Relativistic constraints on interference experiments

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Remarks:

- In several axiomatic reconstructions of QT, the fact that "*SO(d-1) must be Abelian*" was a crucial intermediate proof step → physical interpretation!

Ll. Masanes and MM, *A derivation of quantum theory from physical requirements*, New J. Phys. **13** (2011)
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- Cf. original Popescu-Rohrlich box idea:
  Spacetime + probabilities are **hard** to combine
  → their structures constrain each other!

$\Rightarrow \quad d \leq 3$ (In fact, *d*=3, otherwise no "phase transformations" exist at all.)

Remarks:

- Work in progress: consequences for actual interference experiments.

Proposed Test for Complex versus Quaternion Quantum Theory

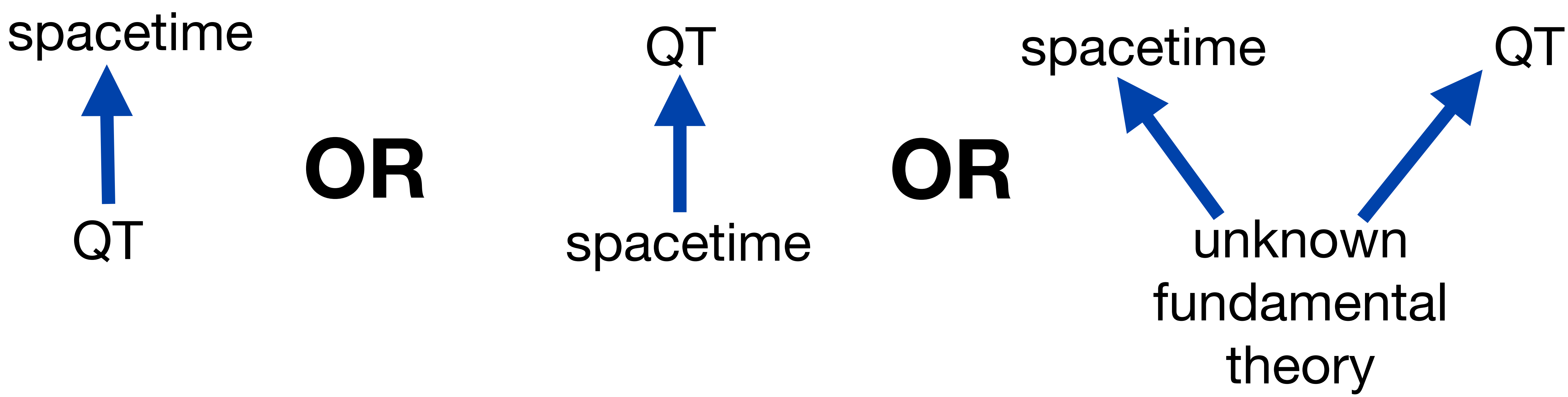
Asher Peres
Department of Physics, Technion—Israel Institute of Technology, Haifa, Israel
(Received 7 December 1978)

If scattering amplitudes are ordinary complex numbers (not quaternions) then there is a universal algebraic relationship between the six coherent cross sections of any three scatterers (taken singly and pairwise). A violation of this relationship would indicate either that scattering amplitudes are quaternions, or that the superposition principle fails. Some experimental tests are proposed, involving neutron diffraction by crystals made of three different isotopes, neutron interferometry, and K_S-meson regeneration.

Quantum theory and spacetime

Quantum theory and spacetime

Plausible scenarios:



Quantum theory and spacetime

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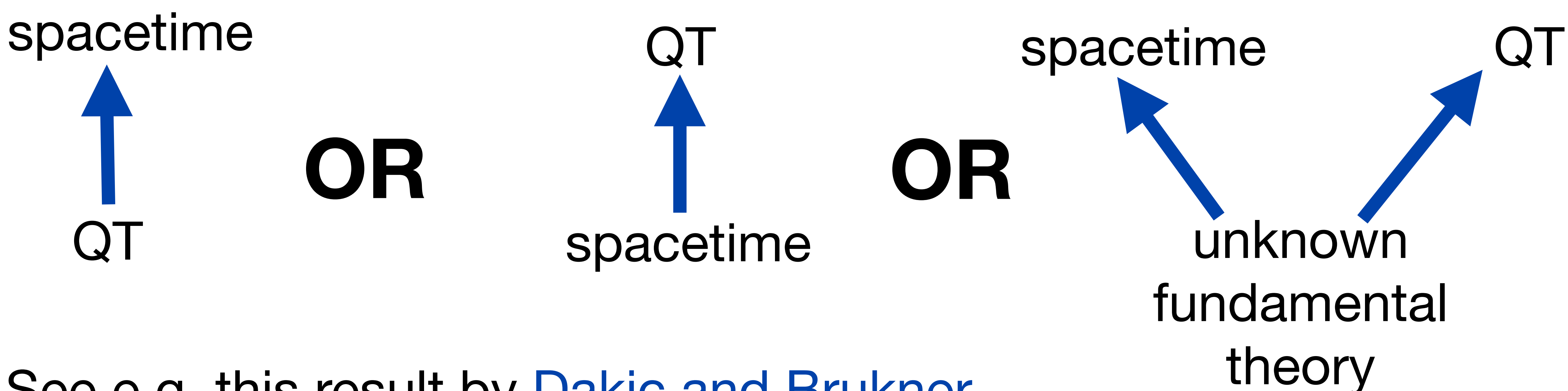

- See e.g. this result by Dakic and Brukner…

B. Dakic and C. Brukner, *The classical limit of a physical theory and the dimensionality of space*, arXiv:1307.3984

- ... or Mauro d'Ariano's approach.

Relativistic covariance emergent from underlying QCA.

Quantum theory and spacetime

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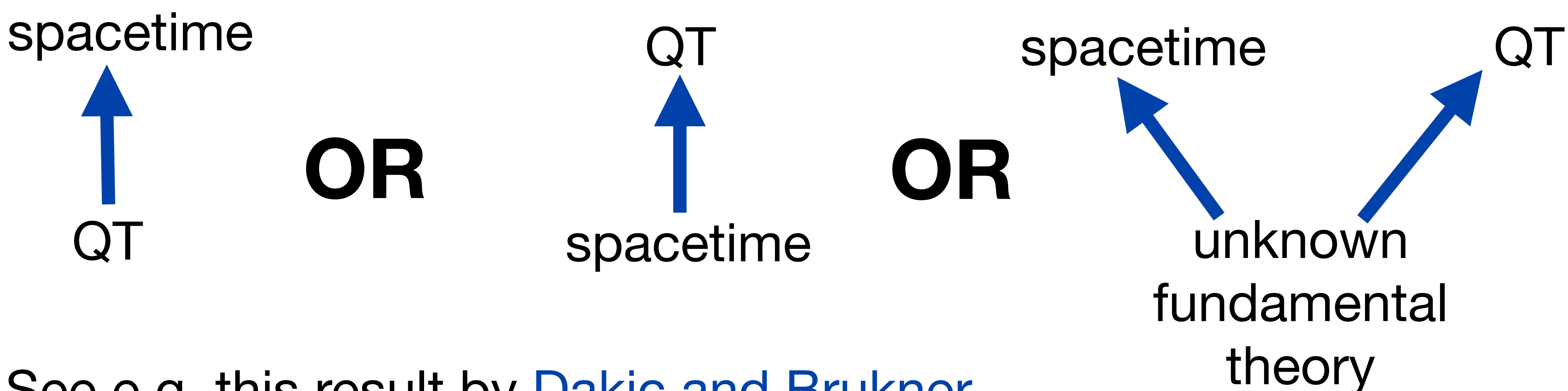

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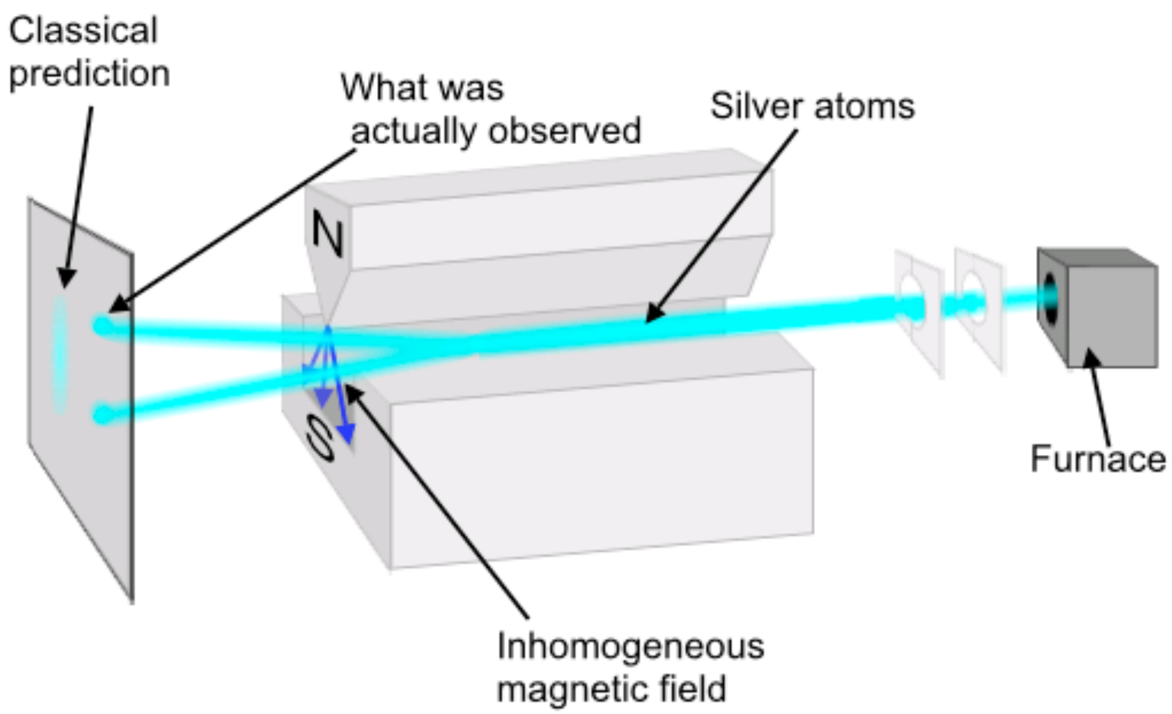
Relativistic covariance emergent from underlying QCA.

To me, crucial hint is the spin-1/2 particle:

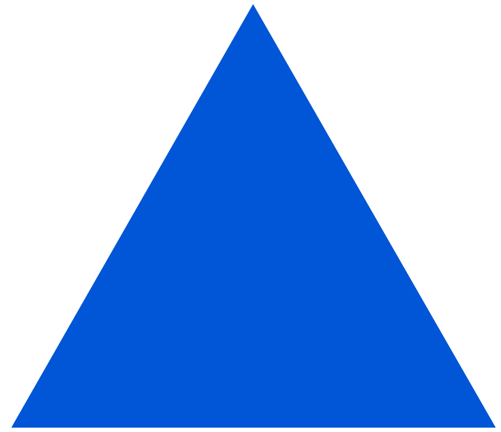


spatial rotations

↕ 1:1

transformations of the probabilistic state

quantum 2-level
state space



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Quantum theory and spacetime

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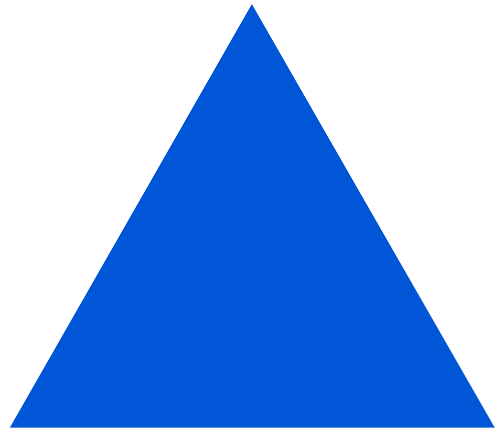

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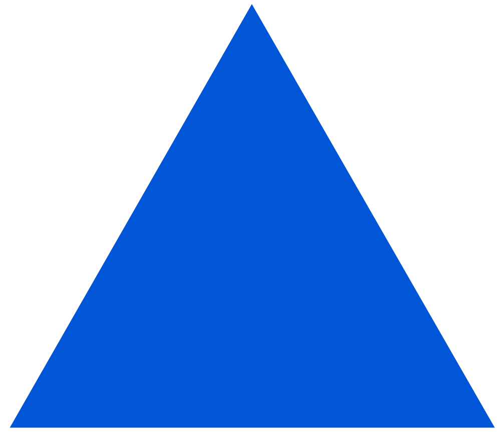
C. F. von Weizsäcker's suggestion (>1954):
Somehow, the Euclidean 3D structure of space follows from the qubit.

Quantum theory and spacetime

quantum 2-level state space



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Next part of talk:
Making some of this rigorous, via QIT tools.

2. Quantum theory and the dimensionality of space

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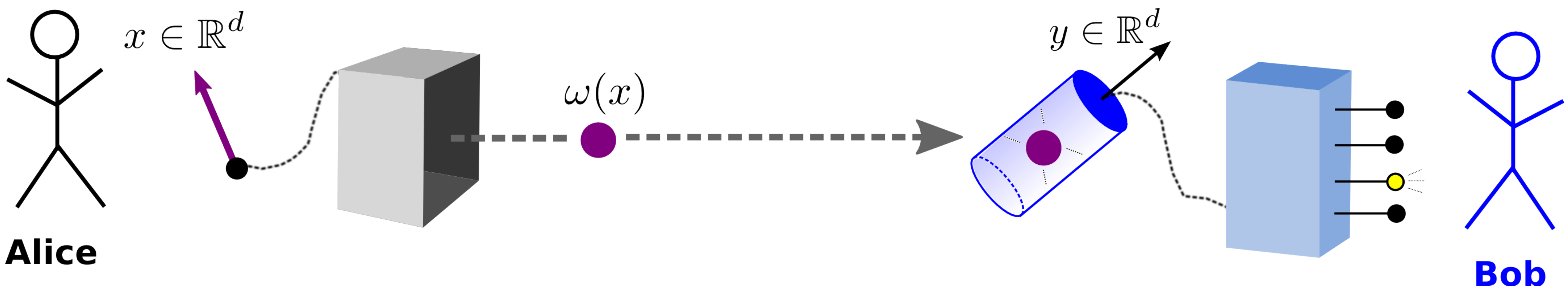
MM and Ll. Masanes, *Three-dimensionality of space and the quantum bit: an information-theoretic approach*, New J. Phys. **15**, 053040 (2013), arXiv:1206.0630.

Formulate as information-theoretic task:



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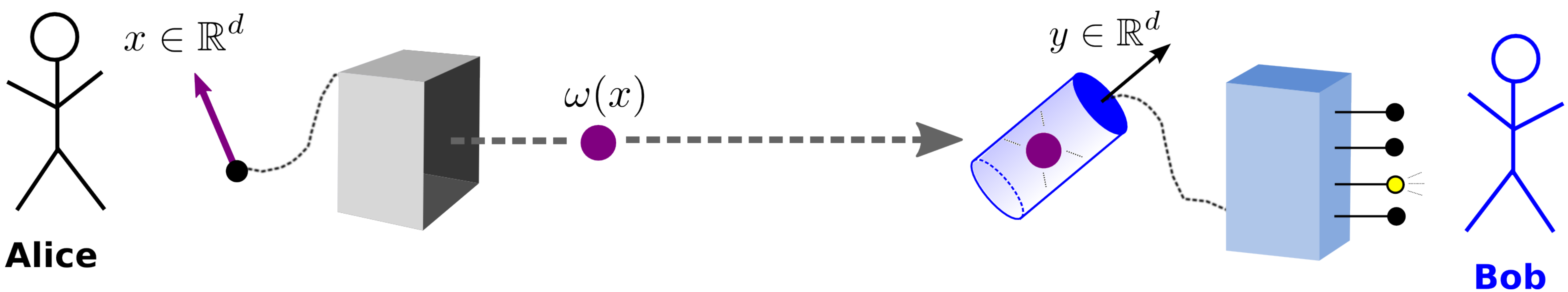


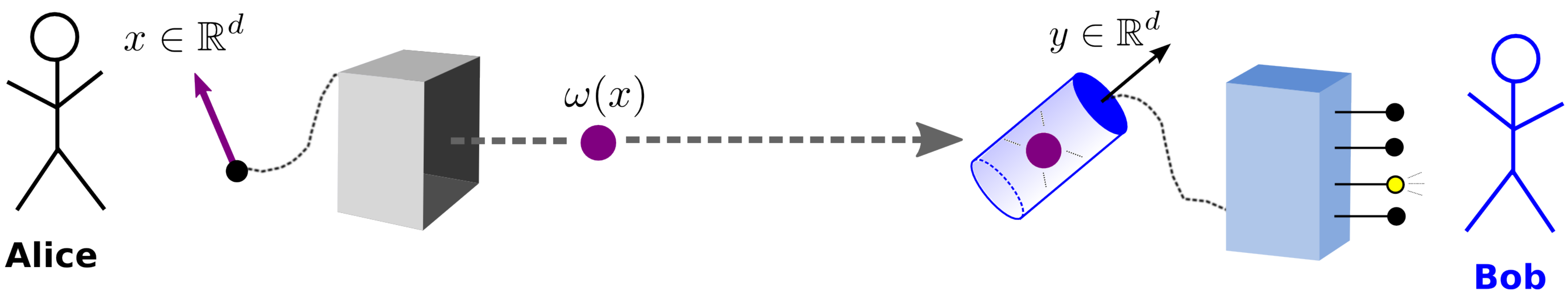
Suppose there is a probabilistic system such that...

1. Alice can encode any spatial direction into the state, but
2. any attempt to encode more results in information loss.
3. Coordinate transformations on pairs of these systems are uniquely determined by their action on single systems.
4. Pairs of these systems can interact reversibly and continuously in time.

2. Quantum theory and the dimensionality of space

MM and Ll. Masanes, *Three-dimensionality of space and the quantum bit: an information-theoretic approach*, New J. Phys. **15**, 053040 (2013), arXiv:1206.0630.

Theorem: Then the spatial dimension must be *d=3*, the systems are qubits, and pairs of these systems are quantum 4-level systems evolving unitarily in time.



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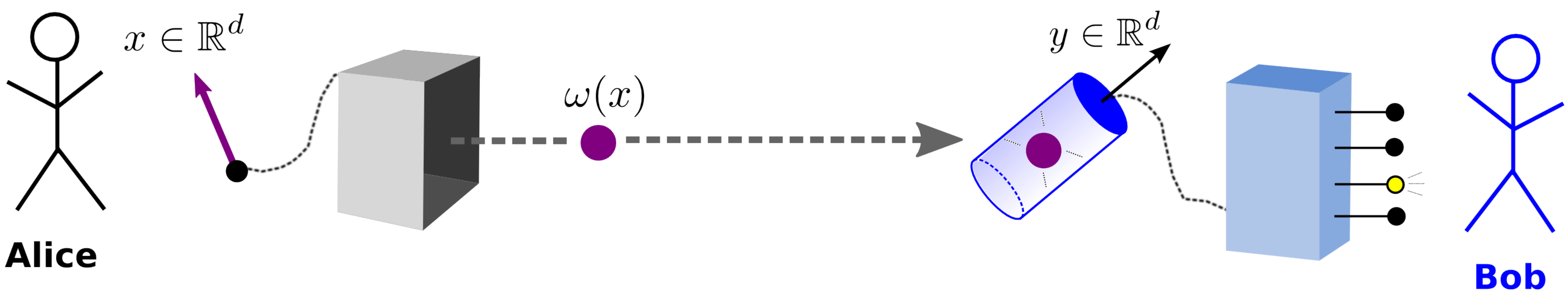
One more Theorem: If "spatial direction" $x \in \mathbb{R}^d, \quad |x| = 1$, is replaced by "spatial orientation" $X \in SO(d)$, then there is no solution (for topological reasons).



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3. Coordinate transformations on pairs of these systems are uniquely determined by their action on single systems.
4. Pairs of these systems can interact reversibly and continuously in time.

Spatial geometry from probabilities

MM and Ll. Masanes, *Three-dimensionality of space and the quantum bit: an information-theoretic approach*, New J. Phys. **15**, 053040 (2013), arXiv:1206.0630.

Appendix C

Spatial geometry from probabilities

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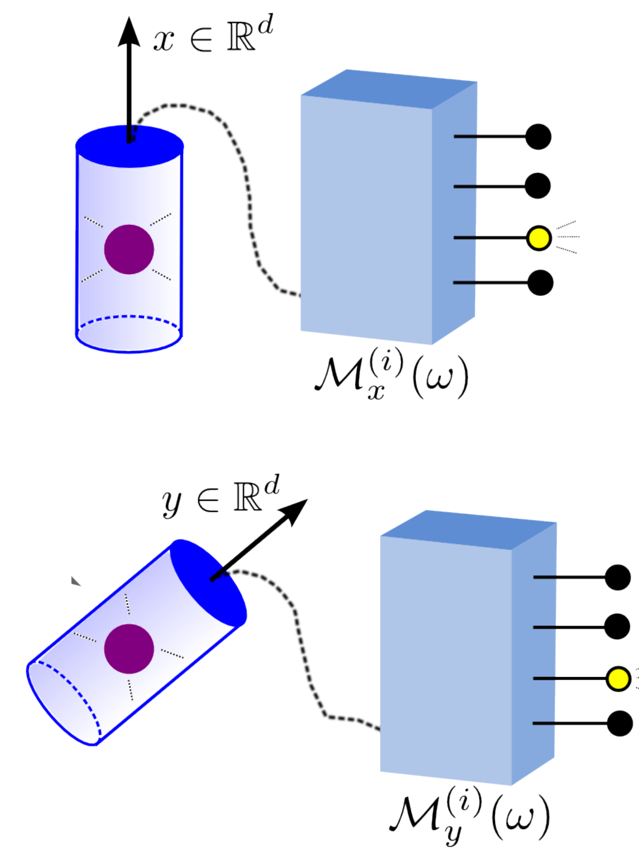
Appendix C

Physicist Alice wants to determine the angle between two measurement devices.



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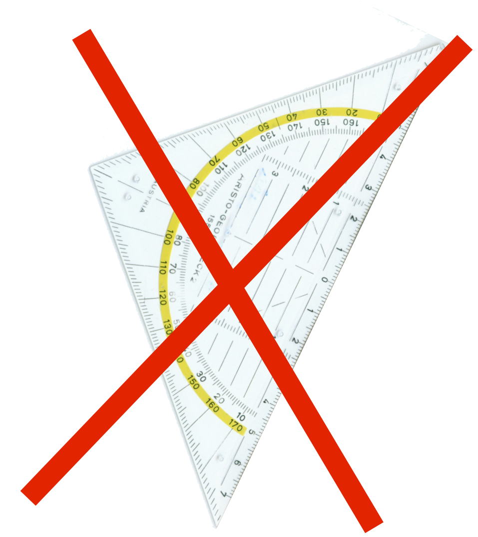
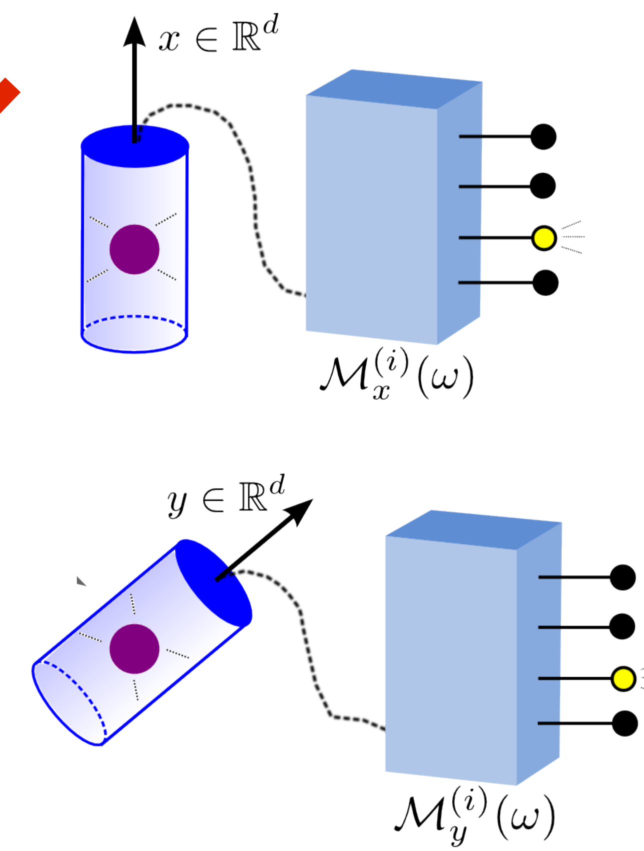
Problem: She doesn't have rulers, protractors etc. (maybe her laboratory space doesn't even *have* a metric!)





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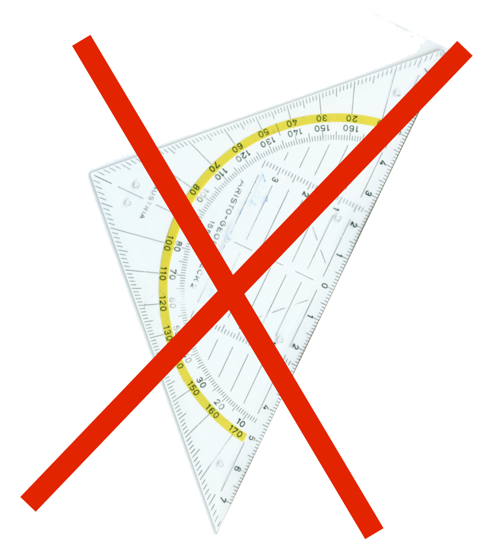

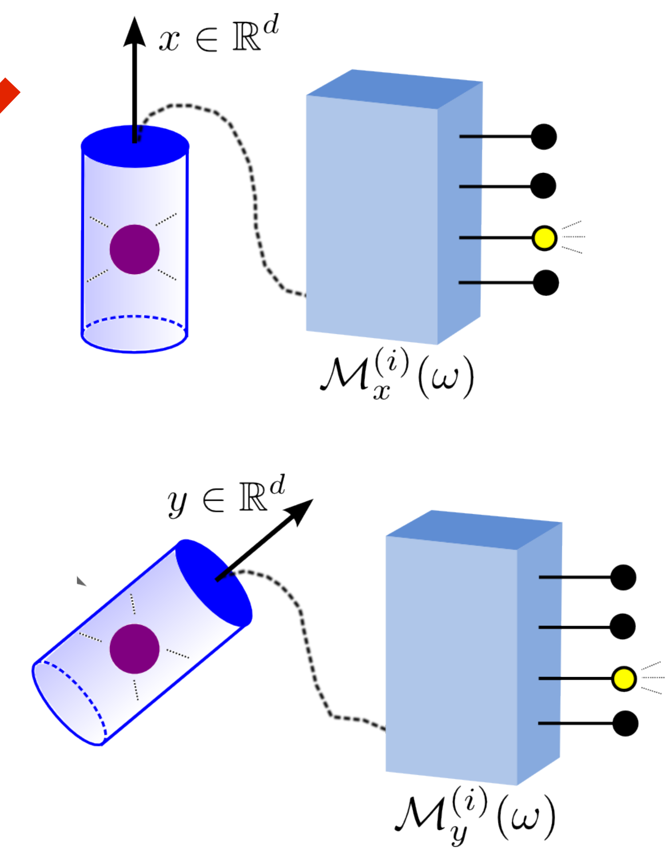

Solution: There is a protocol to determine the angle from comparing measurement outcome probabilities on (unknown) states.

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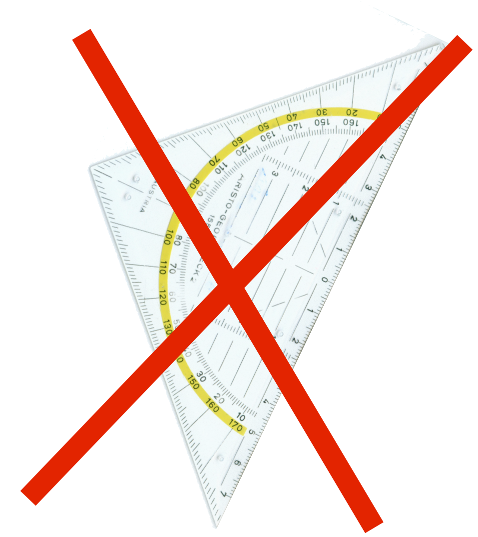

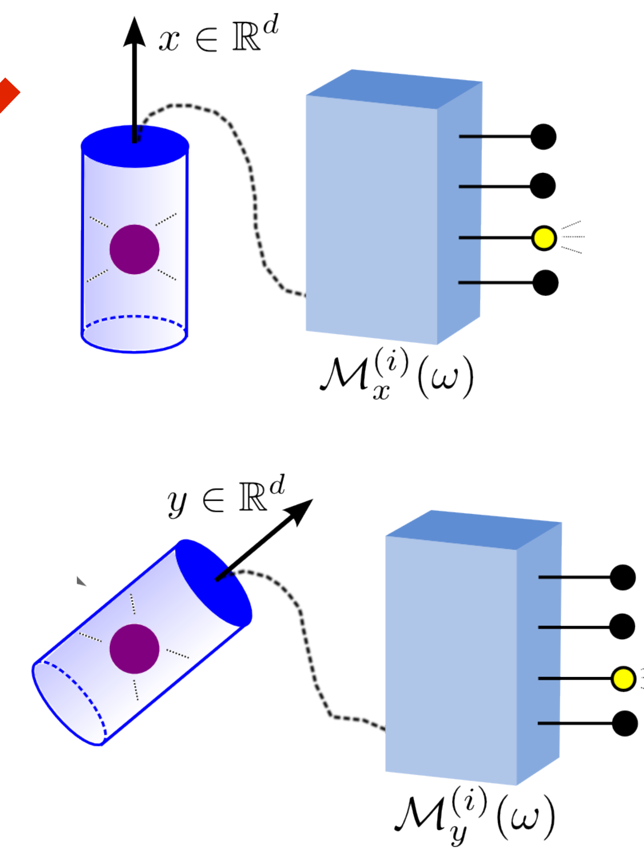

Problem: She doesn't have rulers, protractors etc. (maybe her laboratory space doesn't even *have* a metric!)

Solution: There is a protocol to determine the angle from comparing measurement outcome probabilities on (unknown) states.

⇒ Probabilities deliver linearity structure for free.

A glimpse on the "architecture" of physics

The Bloch ball is 3-dimensional because of...

- ... relativity of simultaneity on interferometers?
- ... possibility of tomographically-local continuous interaction?

And this allows for Stern-Gerlach-like behavior if space is 3D.

These facts constrain each other,
and are thus somehow fundamentally related.

- Reconstructions of QT only first step in broader research program:

  Study how QT and spacetime constrain each other.

arXiv:1206.0630

Thanks to:
Lluís Masanes, Andrew Garner, Oscar Dahlsten

Thank you!