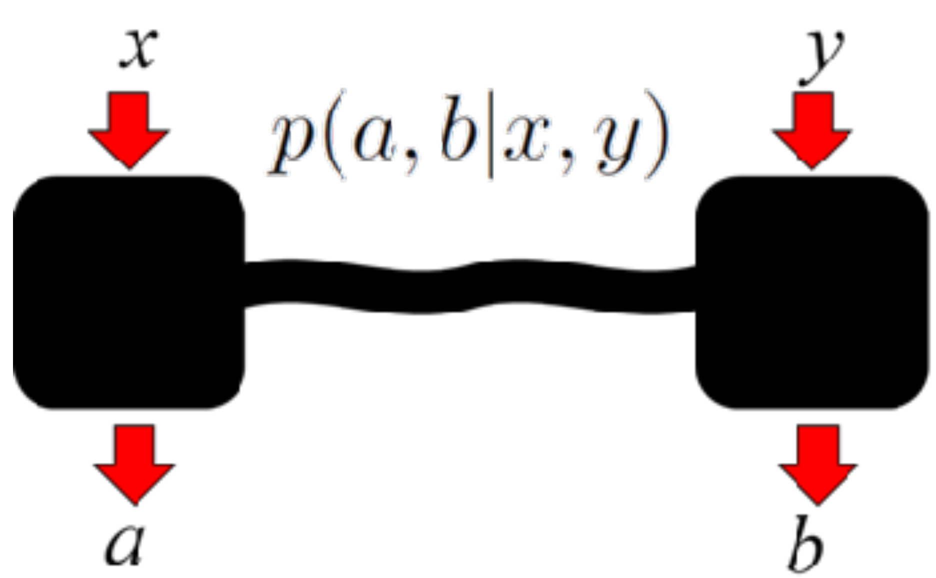
# Black boxes in space and time: from quantum reconstructions to protocols

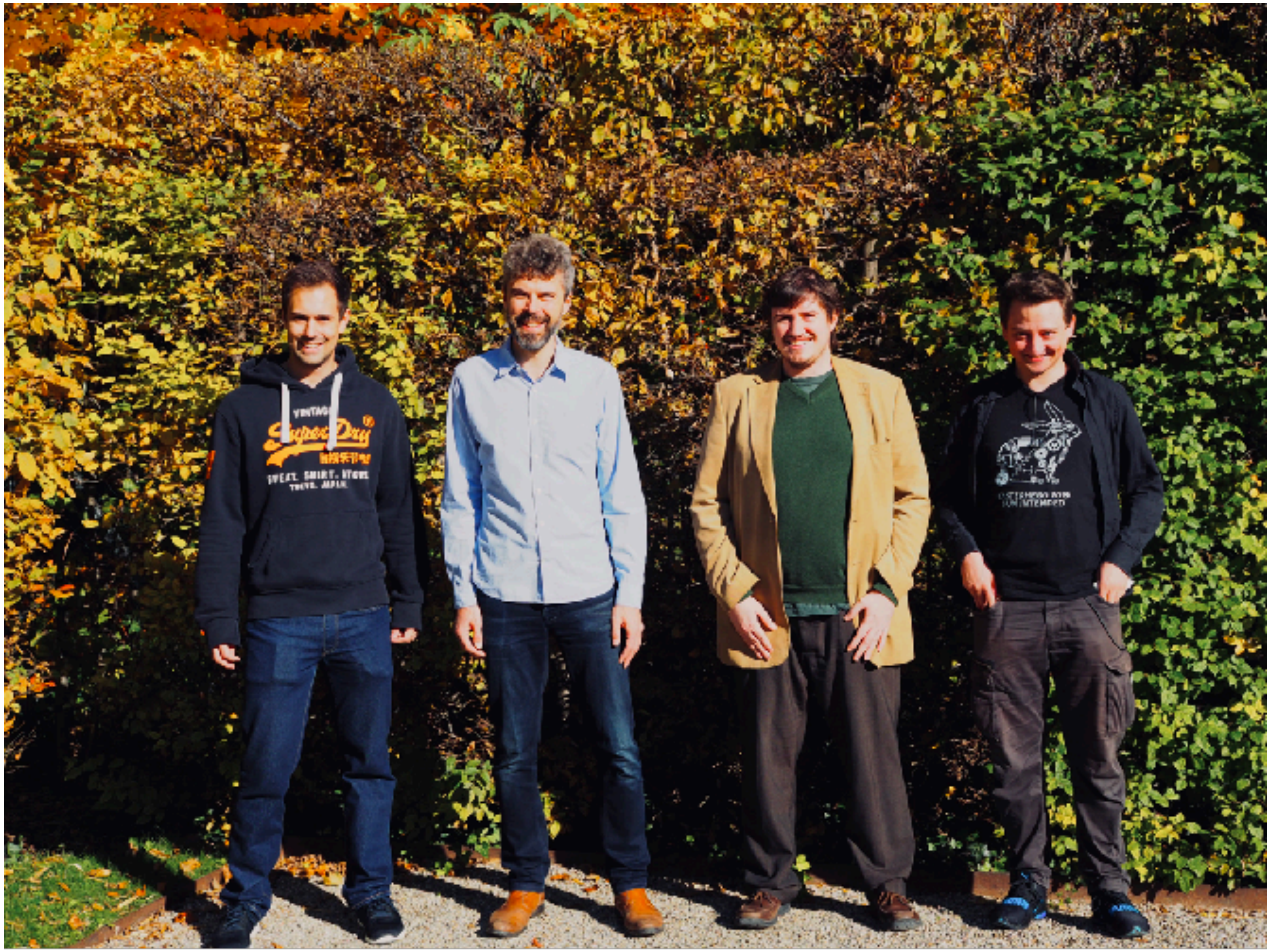

**Markus P. Müller**

Institute for Quantum Optics and Quantum Information (IQOQI), Vienna

Perimeter Institute for Theoretical Physics (PI), Waterloo, Canada

# Müller group at IQOQI



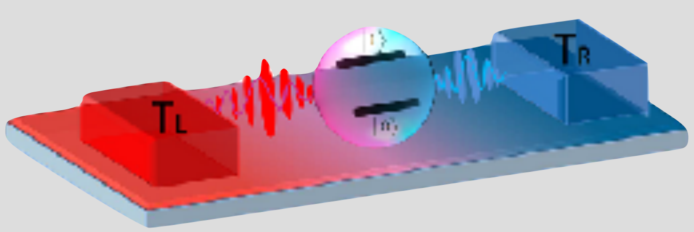
left to right:

**Stefan Ludescher** (PhD student), **Markus Müller** (group leader), **Andy Garner** (postdoc), **Marius Krumm** (PhD student).

# Research

Vision: **Information-theoretic / operational** perspective is the way forward in the Foundations of Physics.

Resource-theoretic approach to **Quantum Thermodynamics**

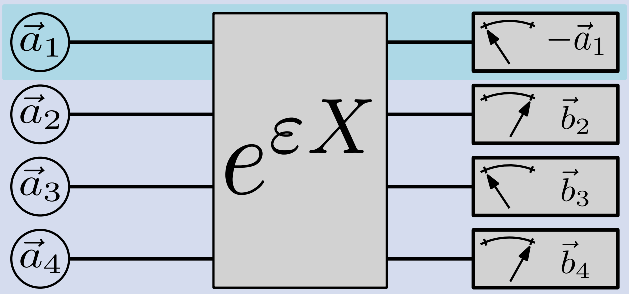



**Spacetime and** Relational **Quantum Information**

**Mathematical Q.I.T.**

**Philosophy of Physics:** structural realism

**Quantum Foundations** and reconstructions of QM



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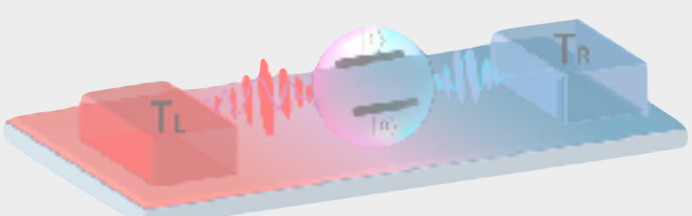

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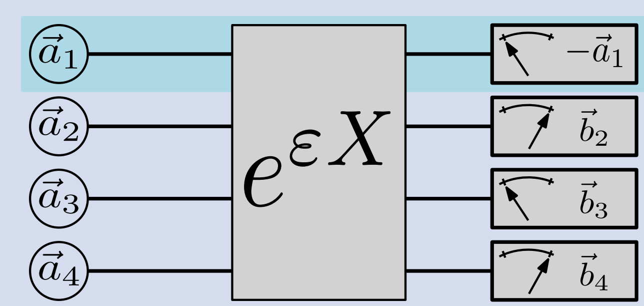
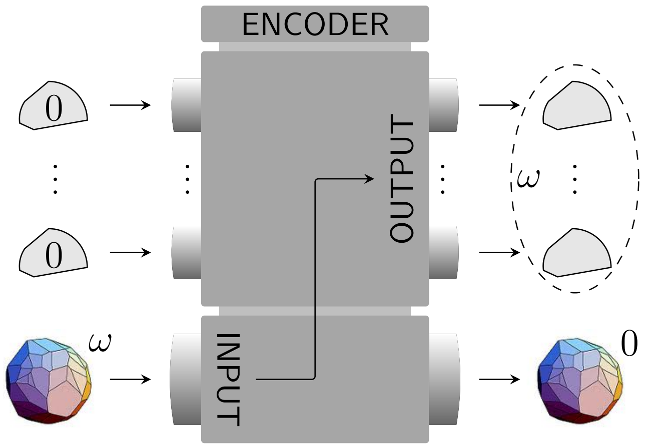


# Overview

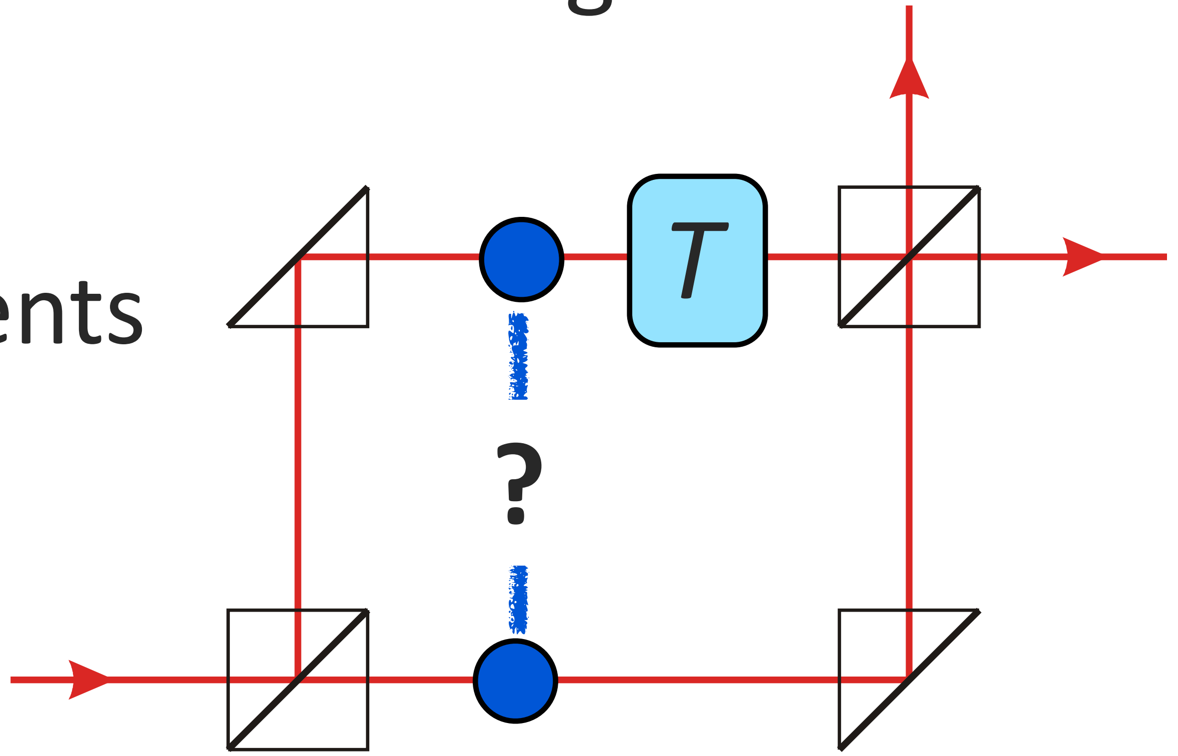
1. Boxes and theories beyond quantum theory

2. Quantum theory from simple principles




3. Spacetime and QT: from foundational insights…

4. … to protocols and experiments



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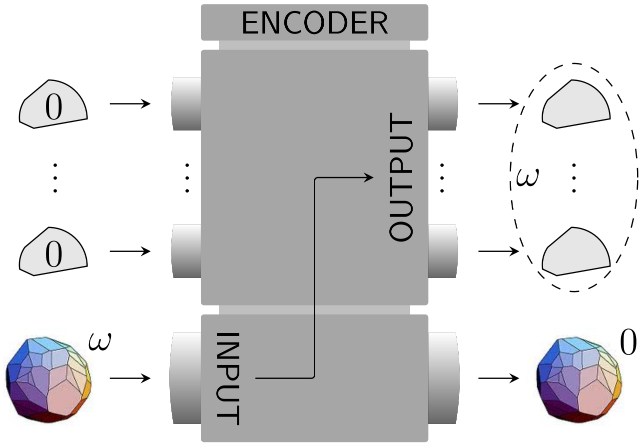


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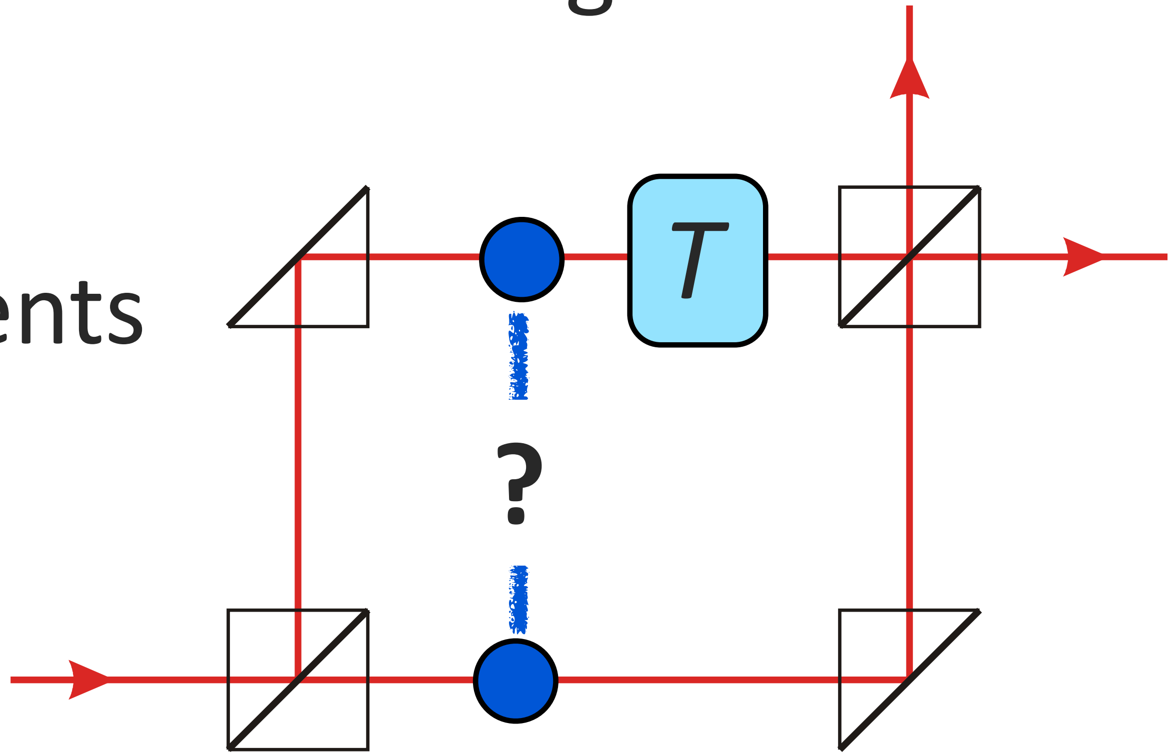

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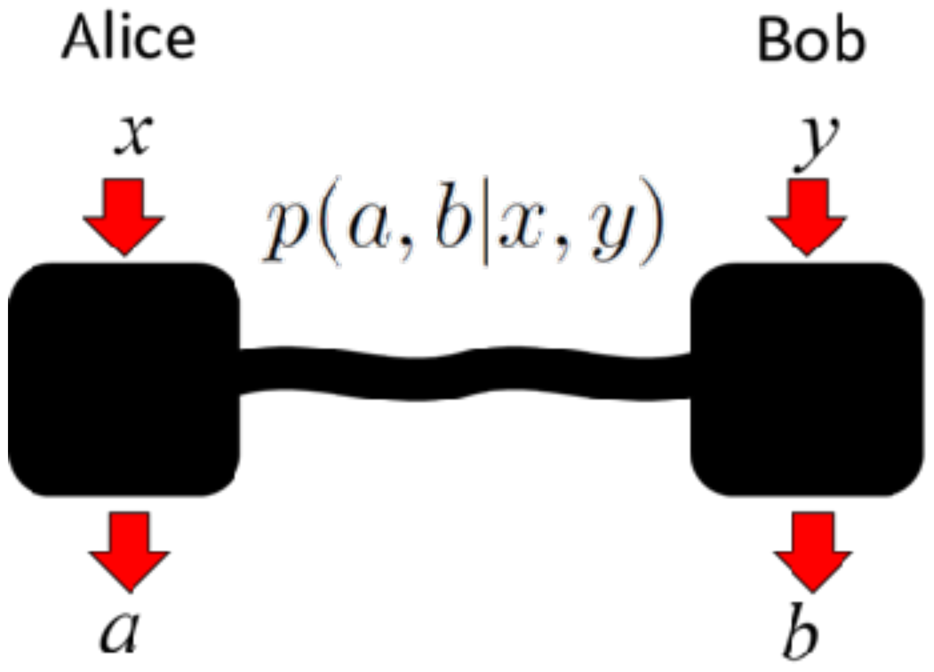

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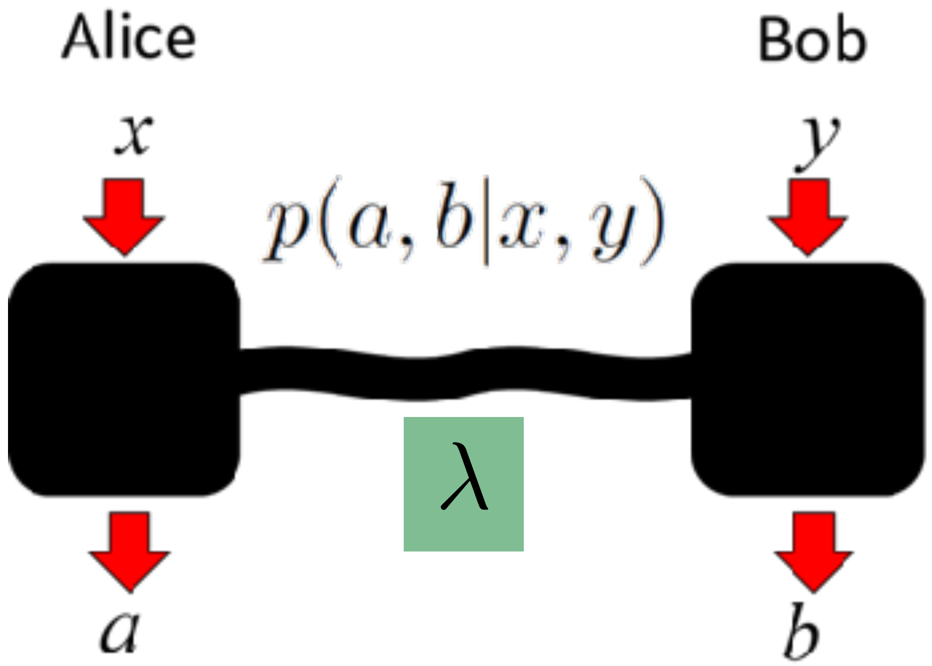


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$$P(a,b|x,y) = \sum_{\lambda\in\Lambda} P_A(a|x,\lambda)P_B(b|y,\lambda)P_\Lambda(\lambda)$$

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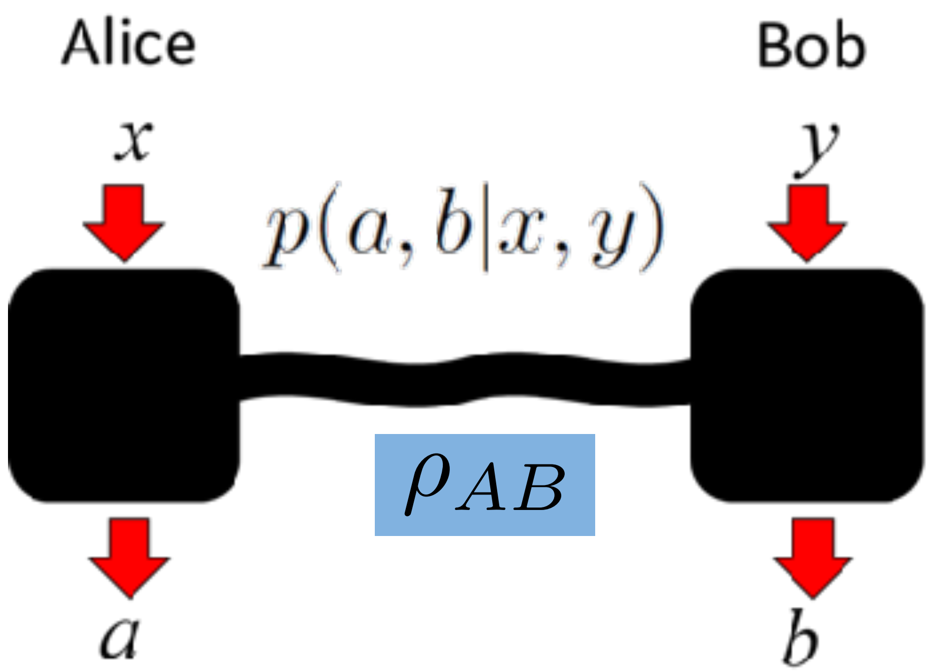


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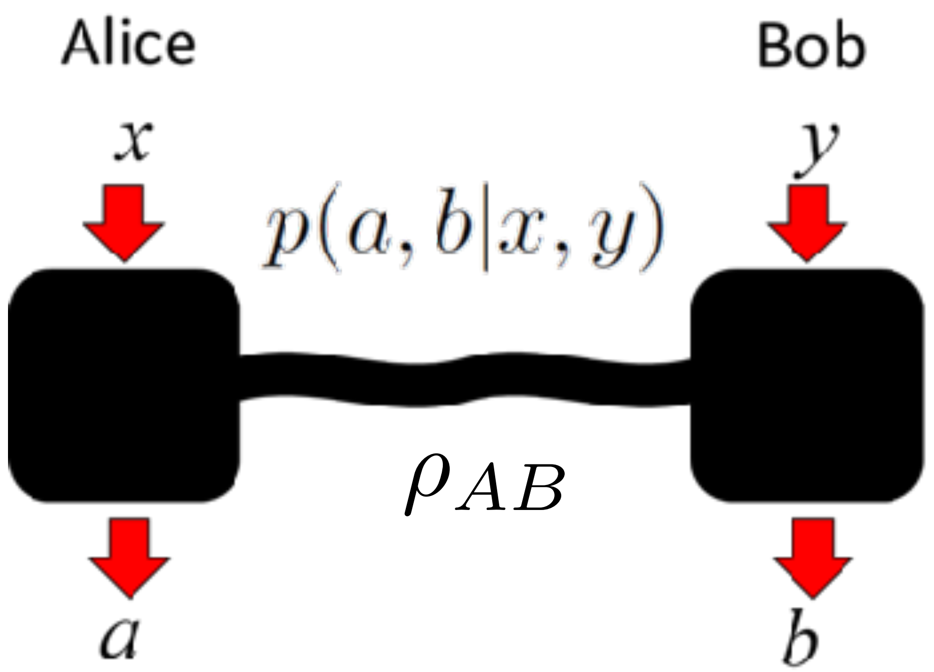


**No-signalling** conditions:

$P(a|x,y)$ is independent of $y$,

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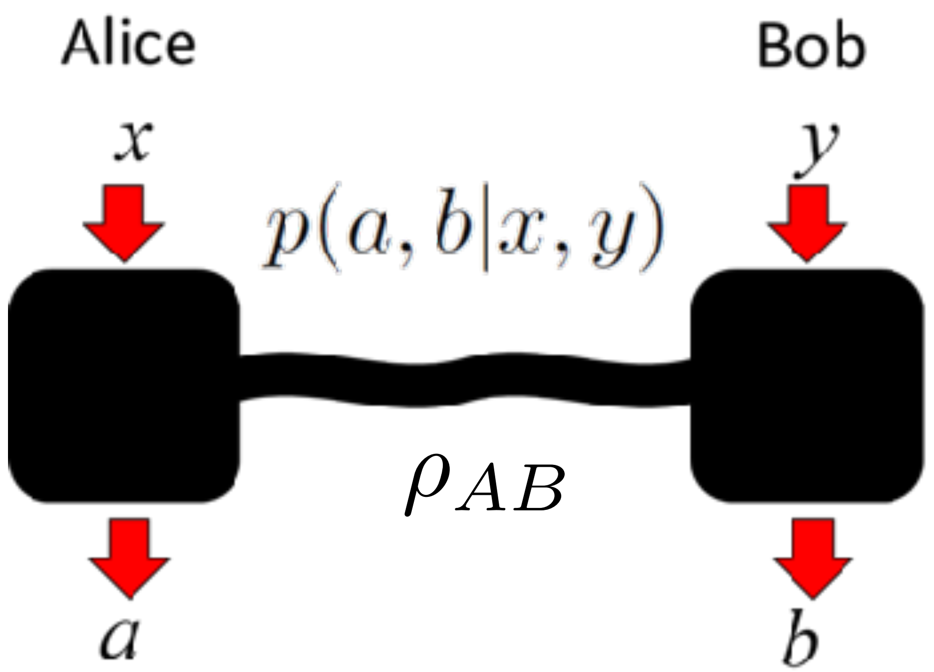


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Quantum admits more general *P*'s due to the **violation of Bell inequalities**.

# The Bell-CHSH inequality

Classical probability distributions satisfy Bell inequality:

$$\mathrm{CHSH} := |C_{00} + C_{01} + C_{10} - C_{11}| \leq 2 \quad \text{where} \quad C_{ab} := \mathbb{E}\left(x \cdot y | a, b\right).$$

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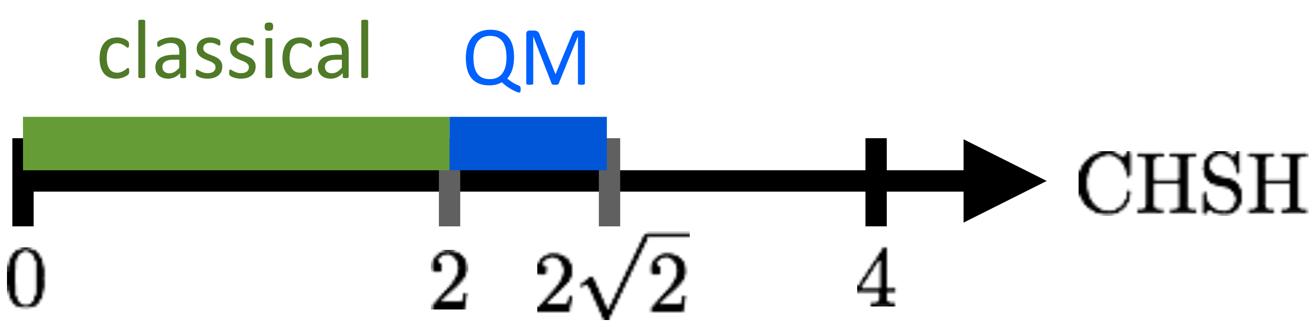


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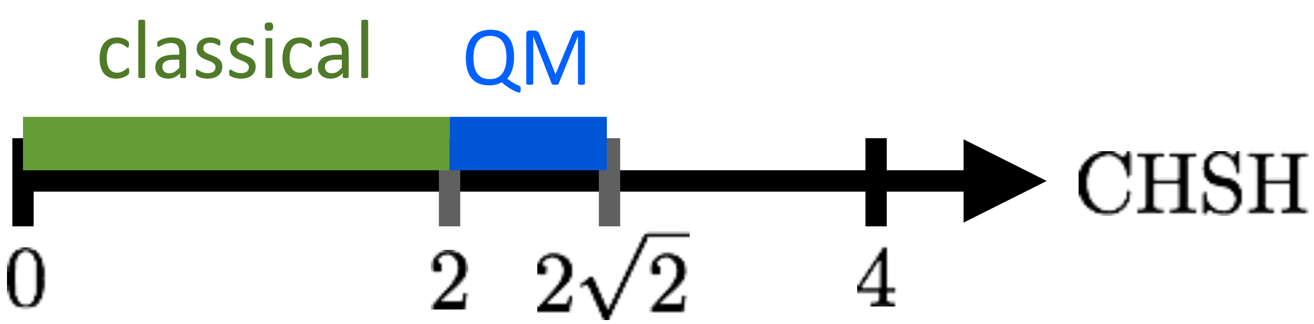


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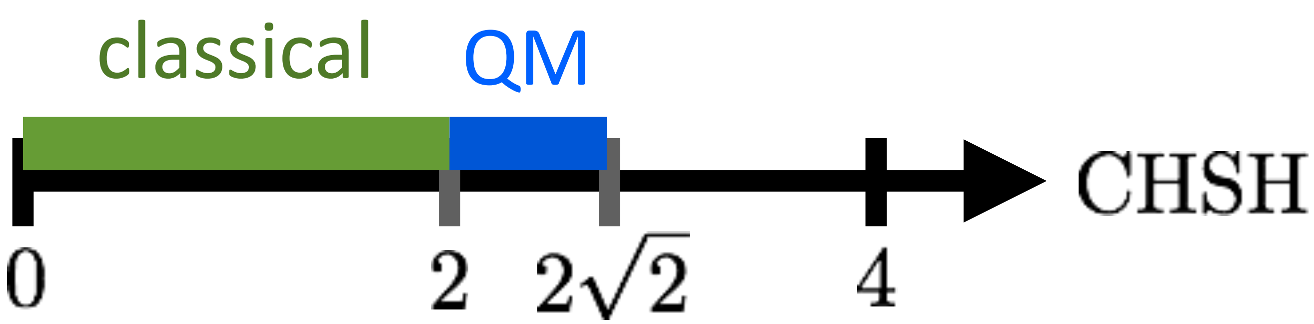


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No! Counterexample: the PR-box correlations

$$\begin{aligned} p(+1,+1|a,b) &= p(-1,-1|a,b) = \frac{1}{2} \\ &\quad \text{if } (a,b) \in \{(0,0),(0,1),(1,0)\} \\ p(+1,-1|1,1) &= p(-1,+1|1,1) = \frac{1}{2} \end{aligned}$$

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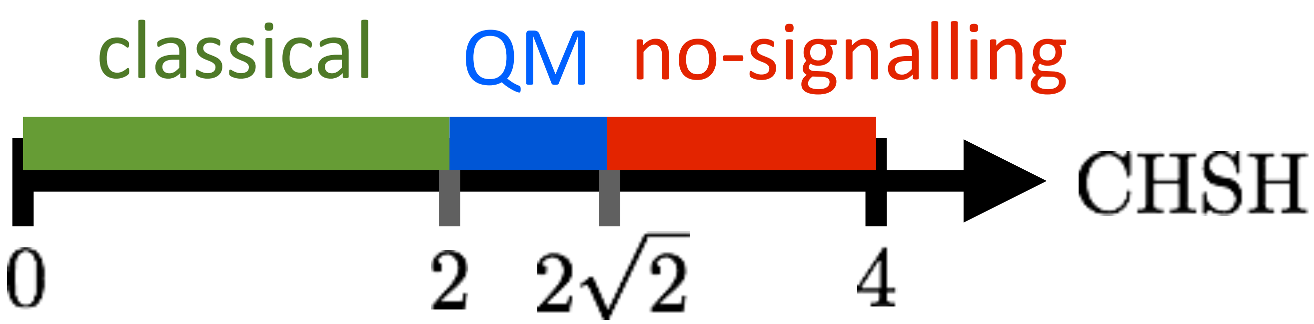


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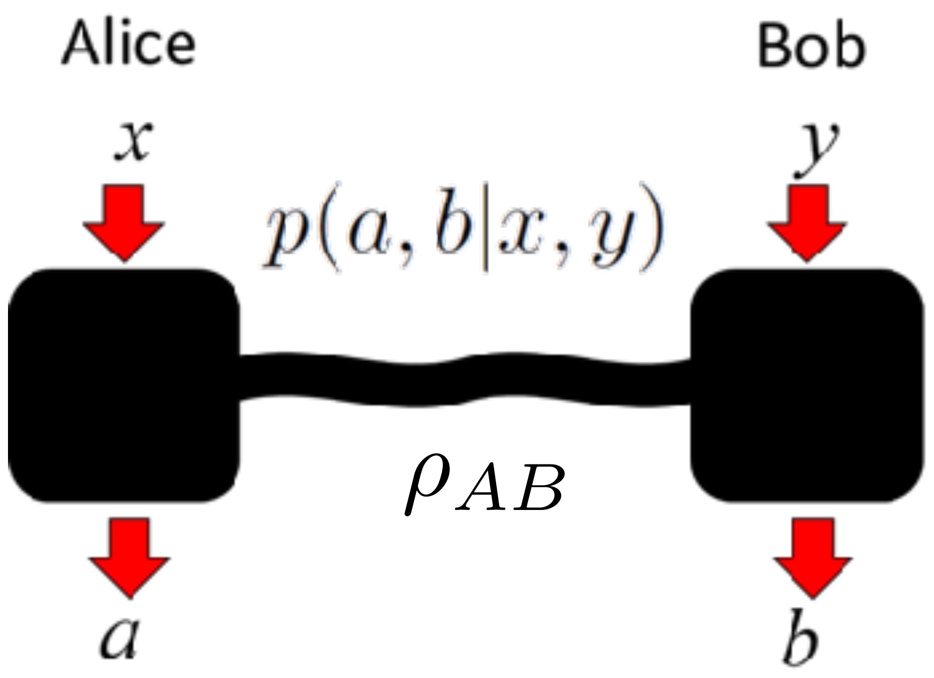


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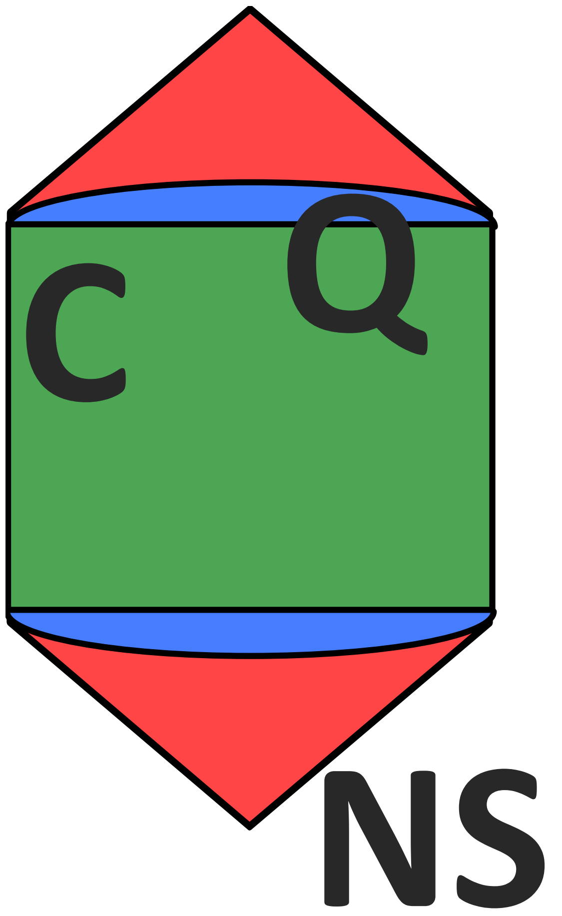

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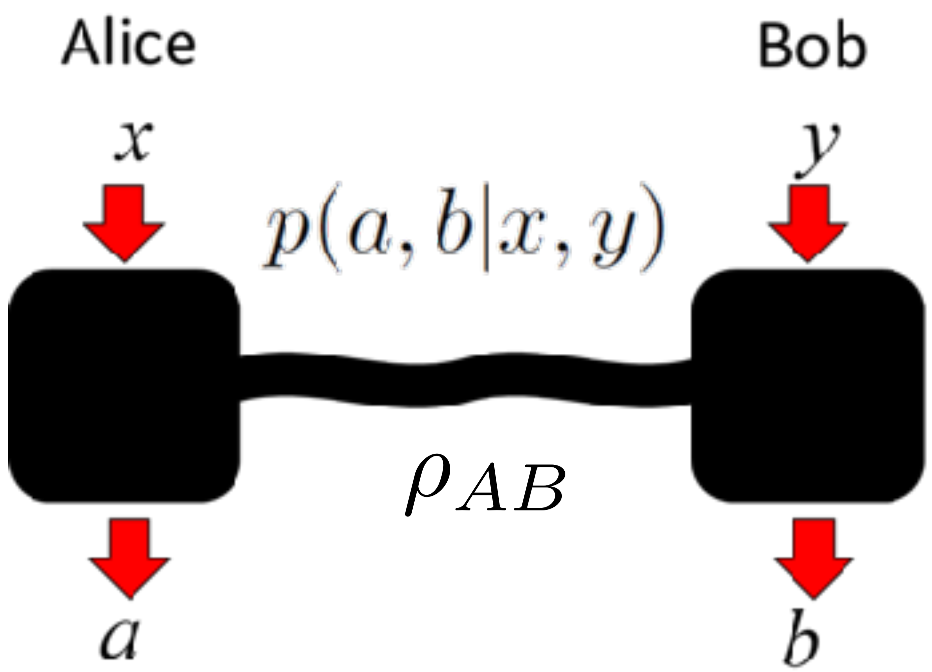


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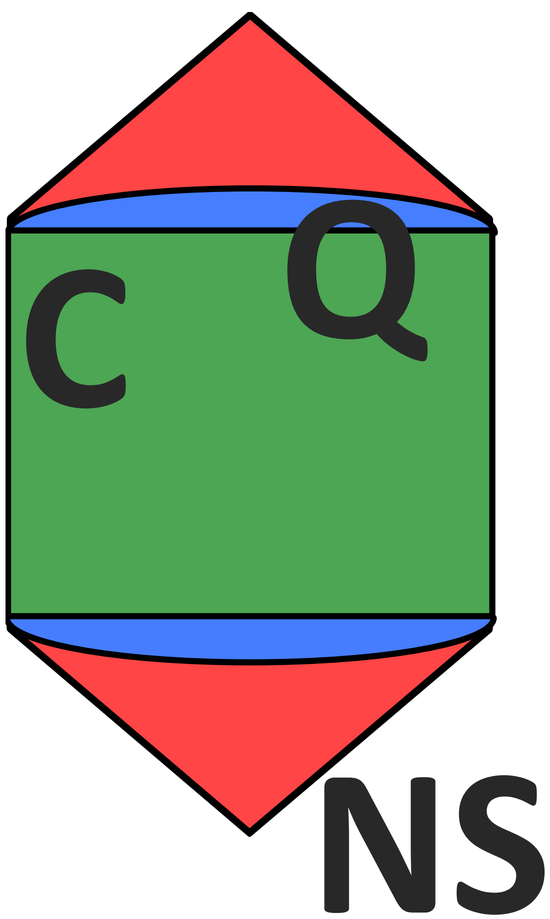


Correlations in **C** come from **classical prob. theory**, correlations in **Q** from **quantum theory**, correlations in **NS** from a theory called "**boxworld**".

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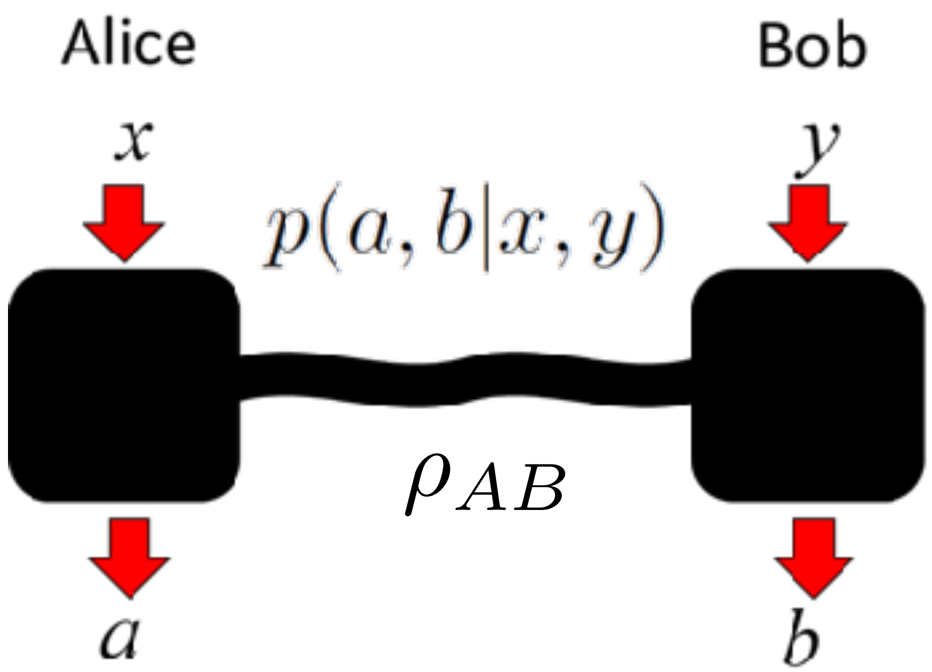


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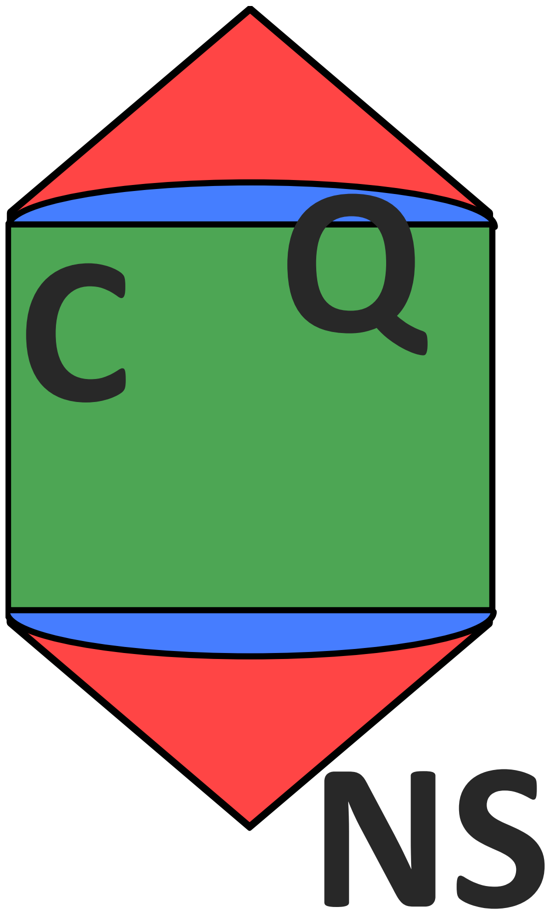


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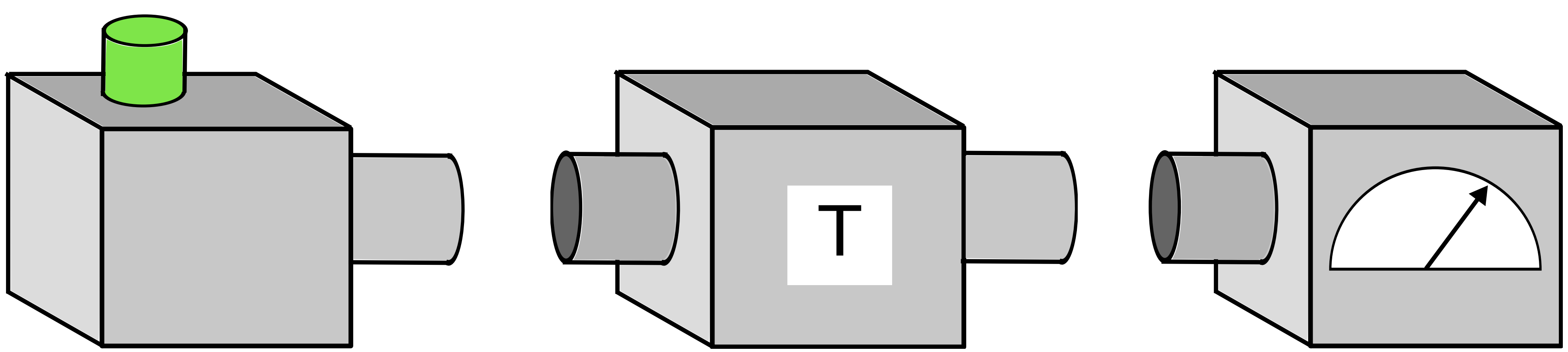
3 examples of a "**generalized probabilistic theory**".

# Generalized probabilistic theories

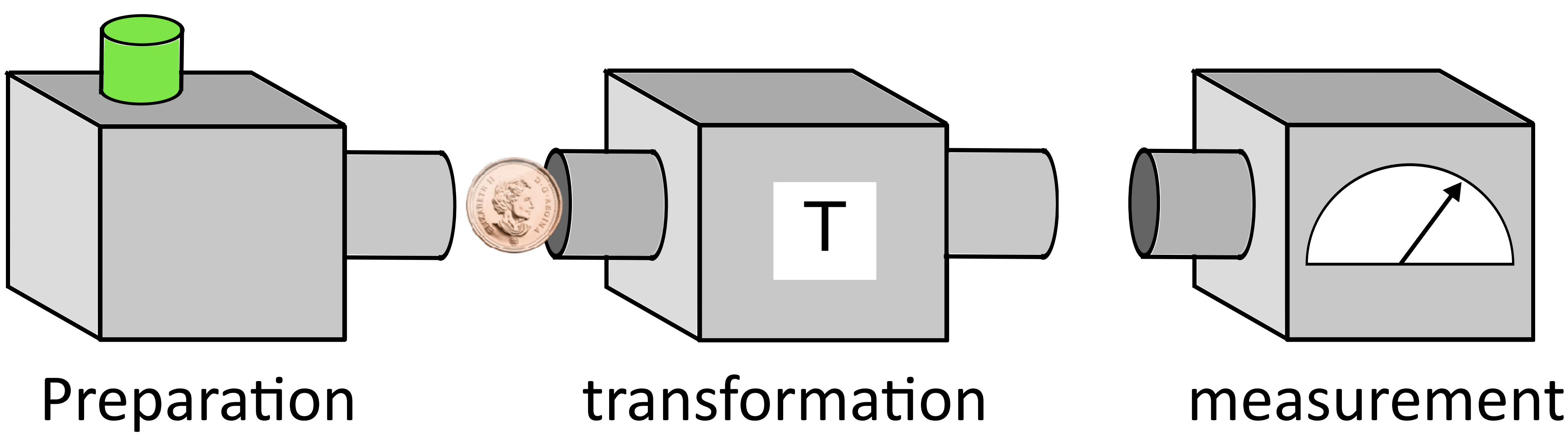



Preparation transformation measurement

# Generalized probabilistic theories

**Example**: classical coin toss.


- On every push of button, the preparation device performs a biased coin toss.



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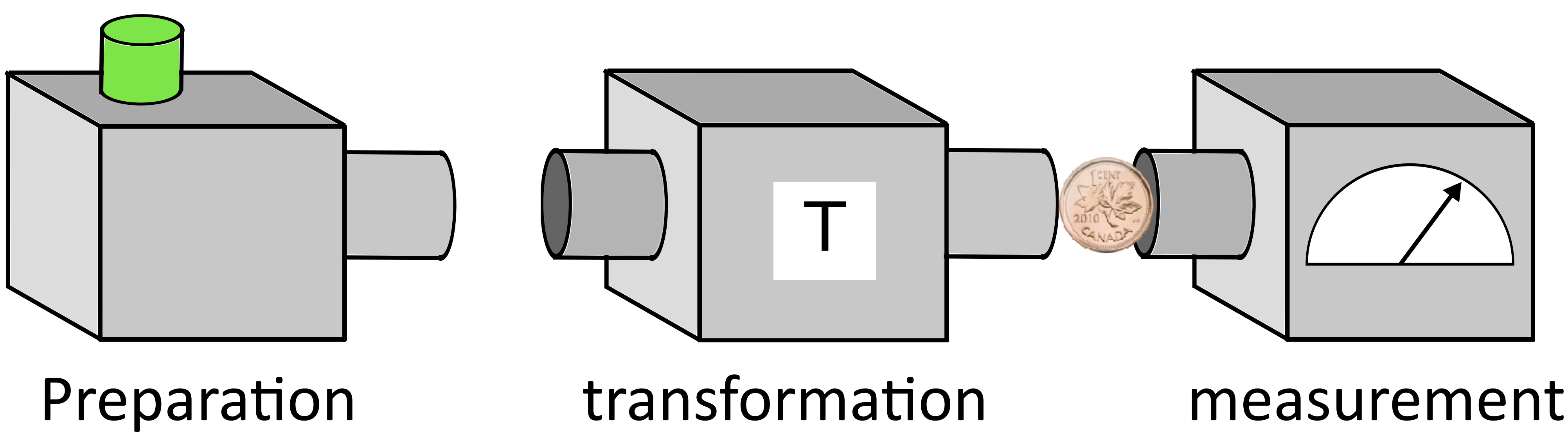

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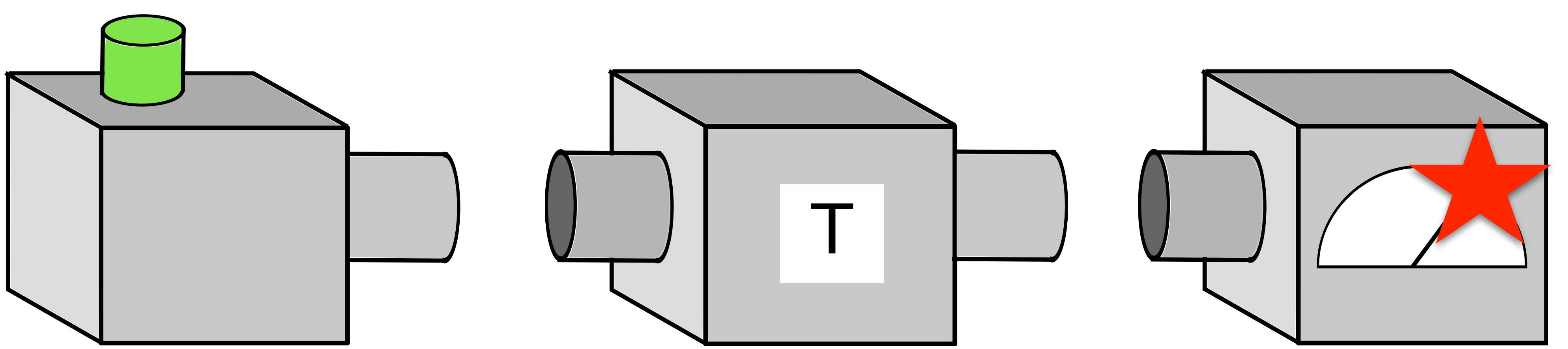
- The measurement outcome is "heads" or "tails".

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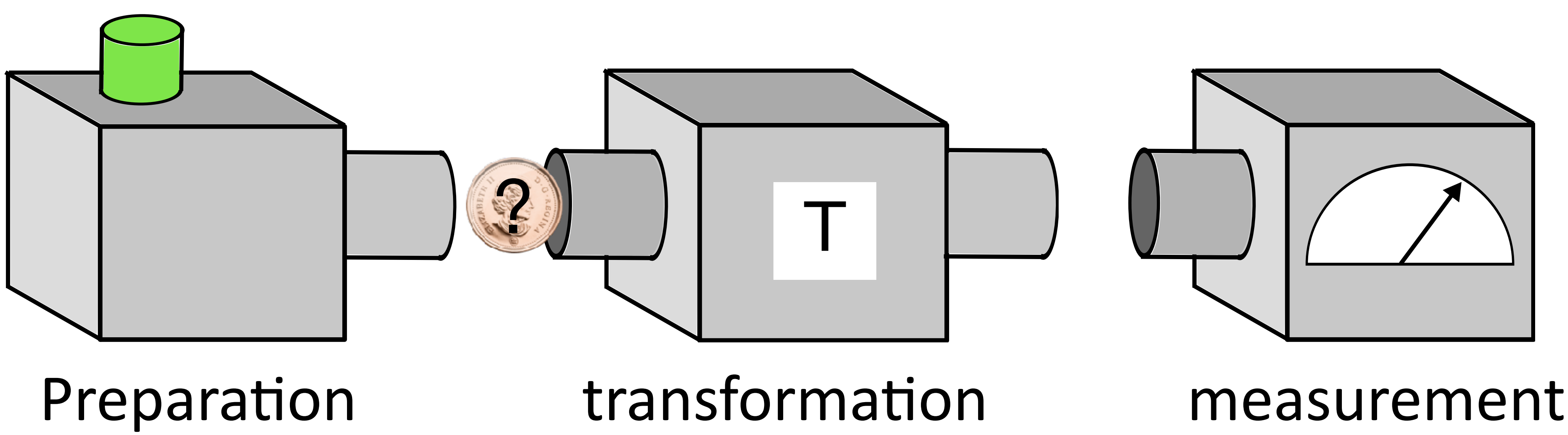
$$\omega = \begin{pmatrix} \mathrm{Prob(heads)} \\ \mathrm{Prob(tails)} \end{pmatrix} = \begin{pmatrix} p \\ 1-p \end{pmatrix}.$$



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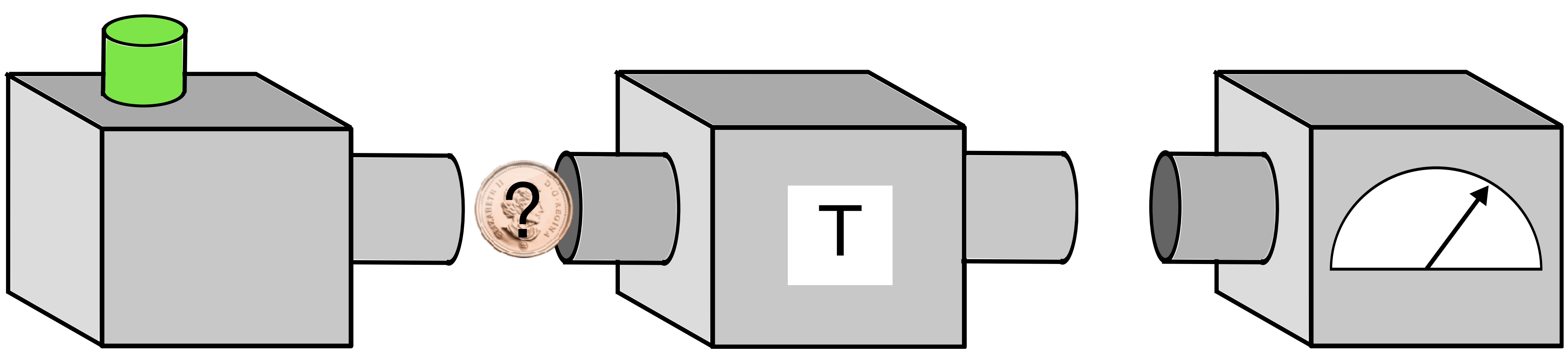


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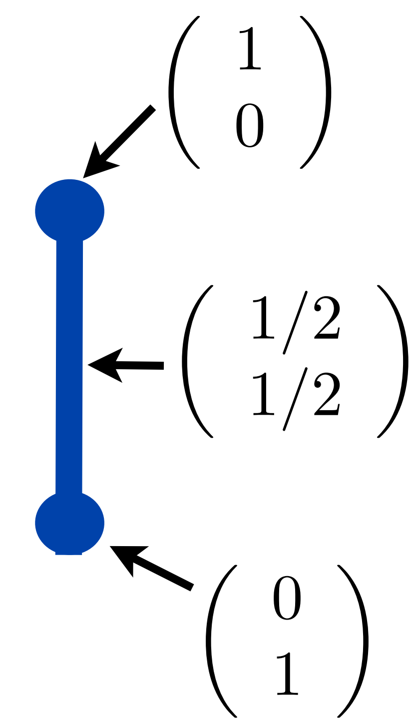


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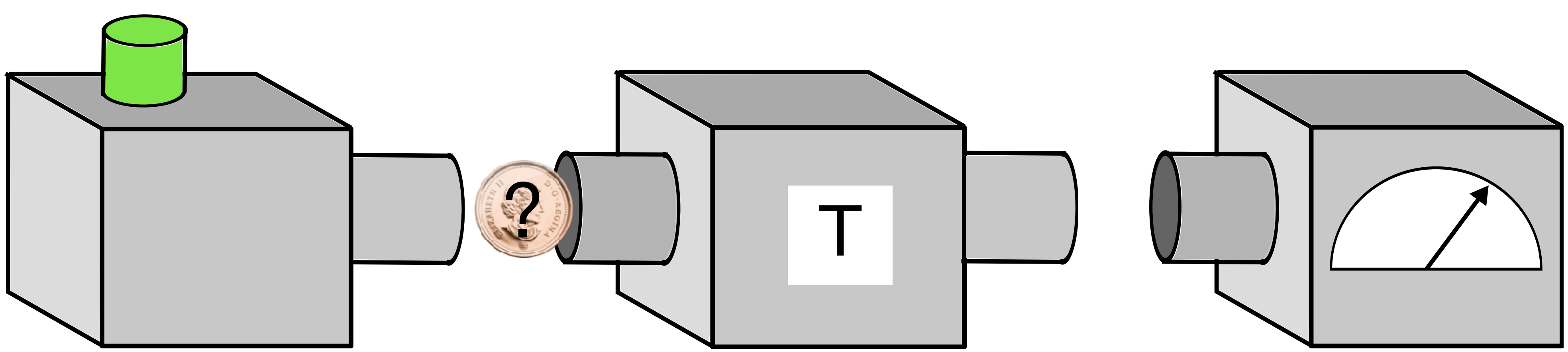


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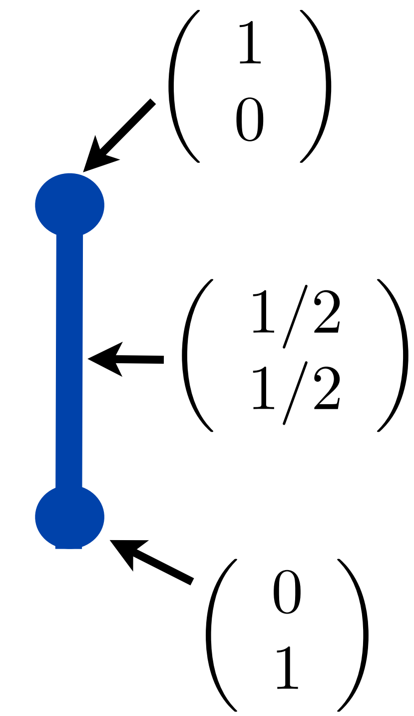


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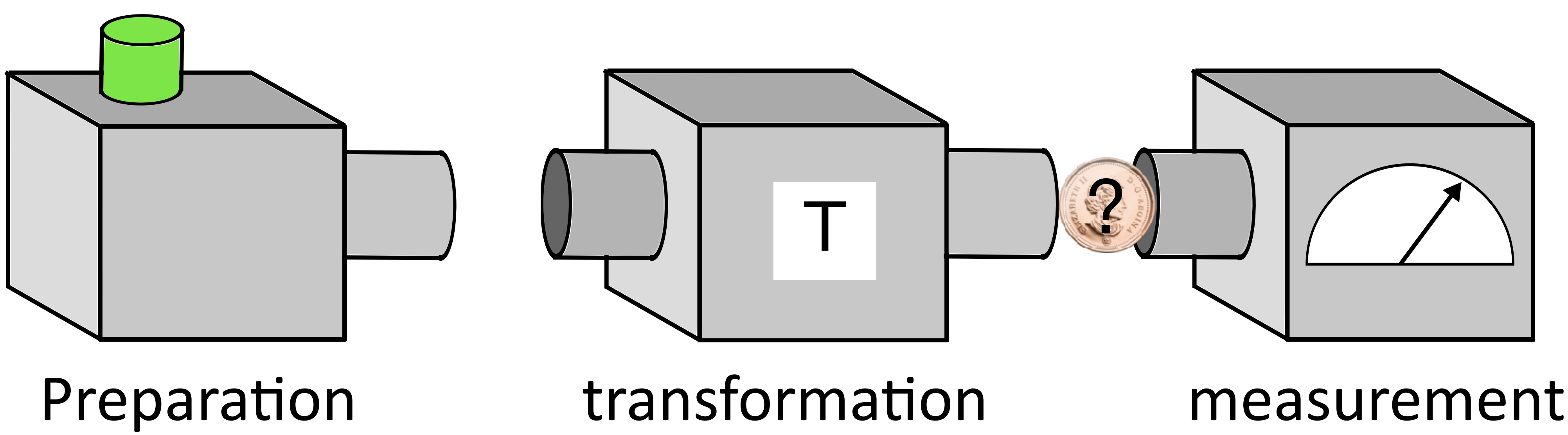

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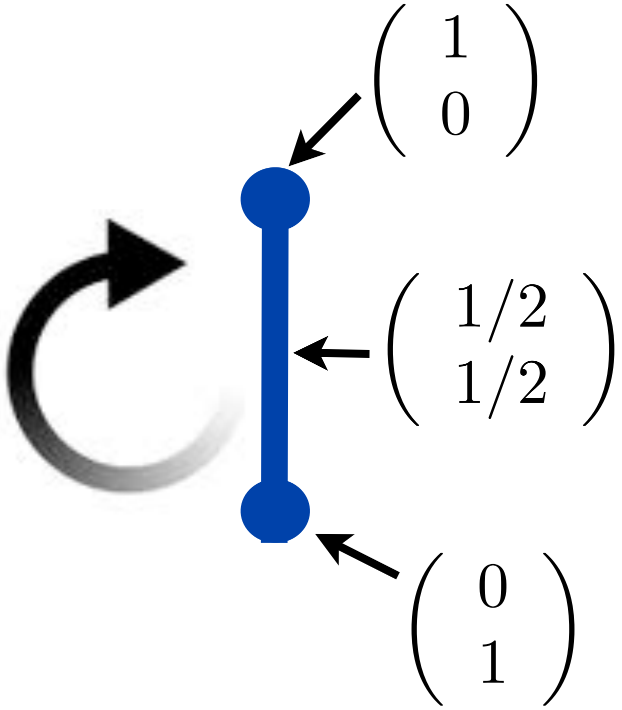
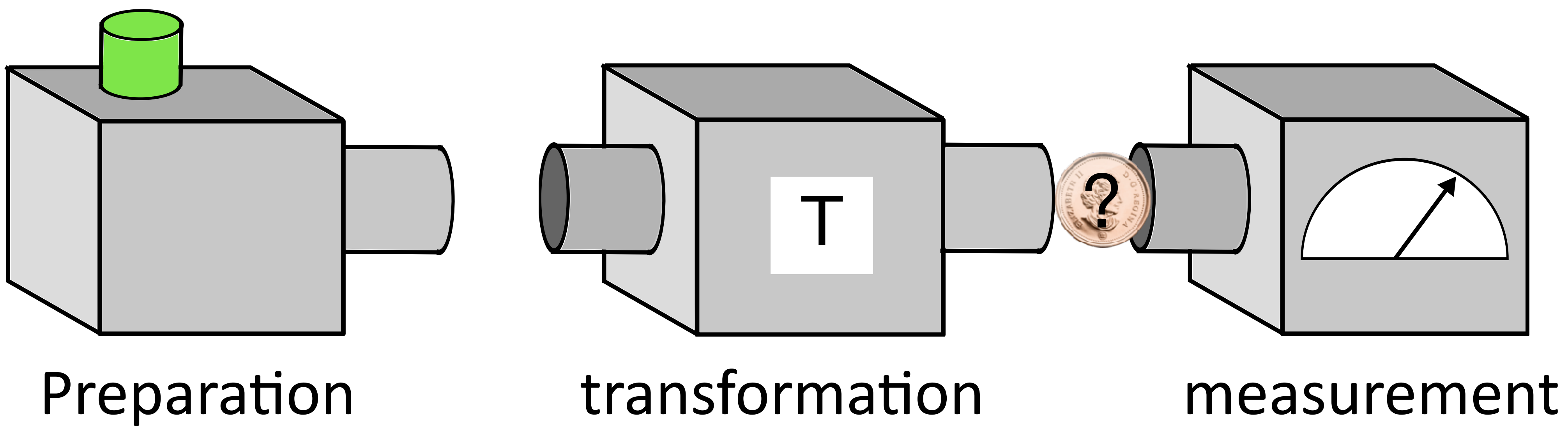
Maps states to states and is linear.



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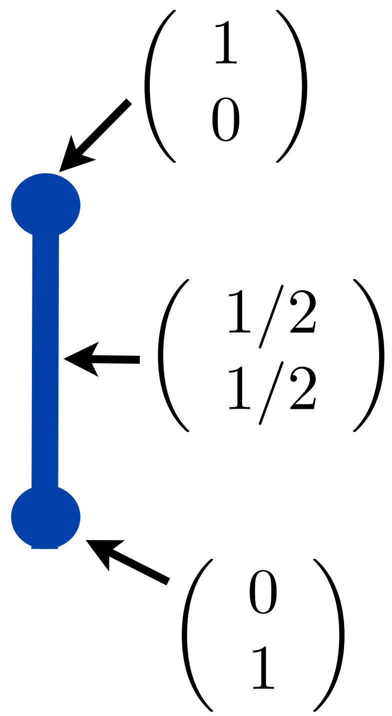


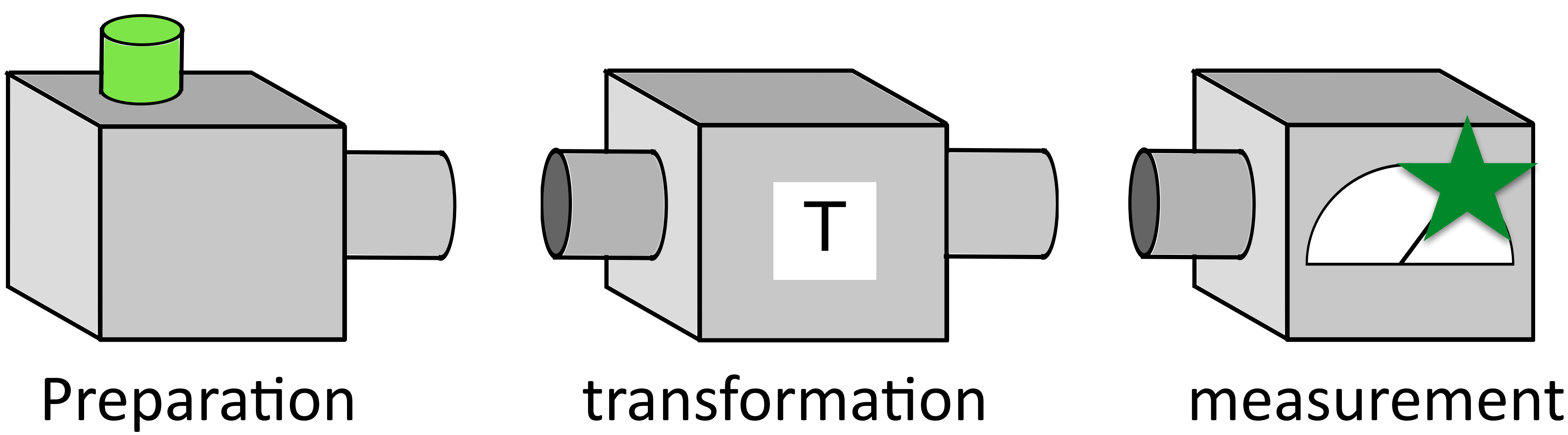

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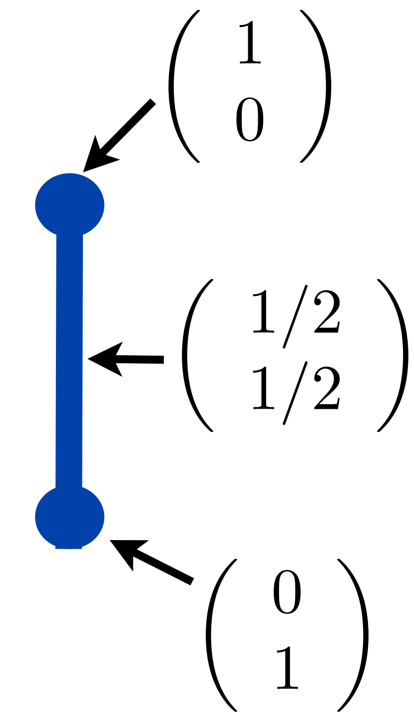
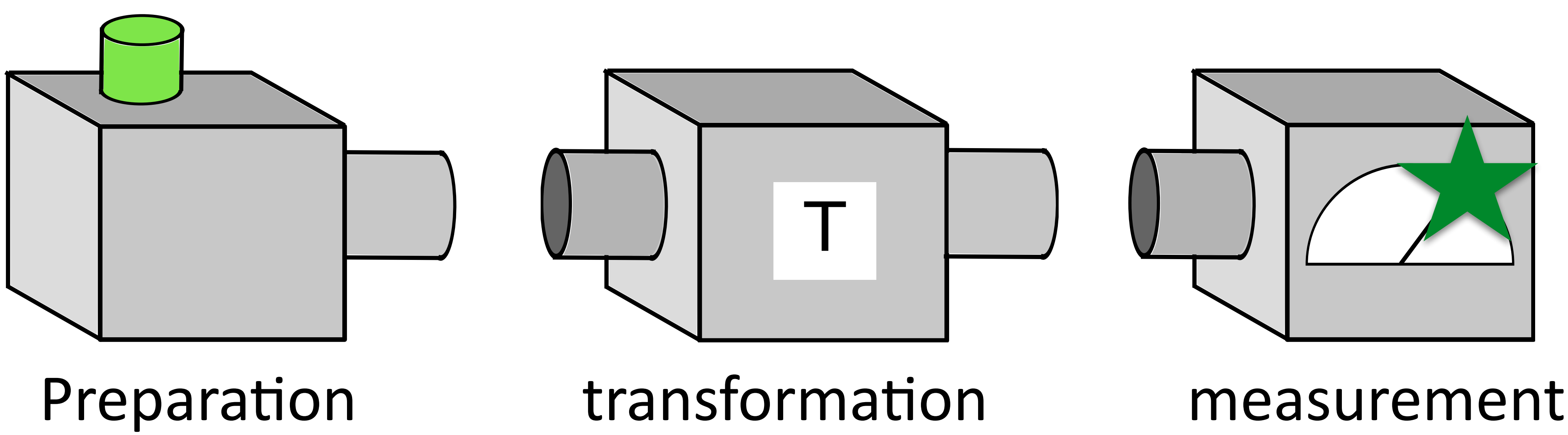
$$\mathrm{Prob}(\mathrm{heads}|\omega) = p = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} p \\ 1-p \end{pmatrix} = e \cdot \omega.$$



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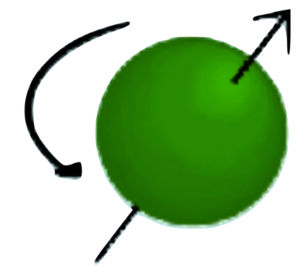
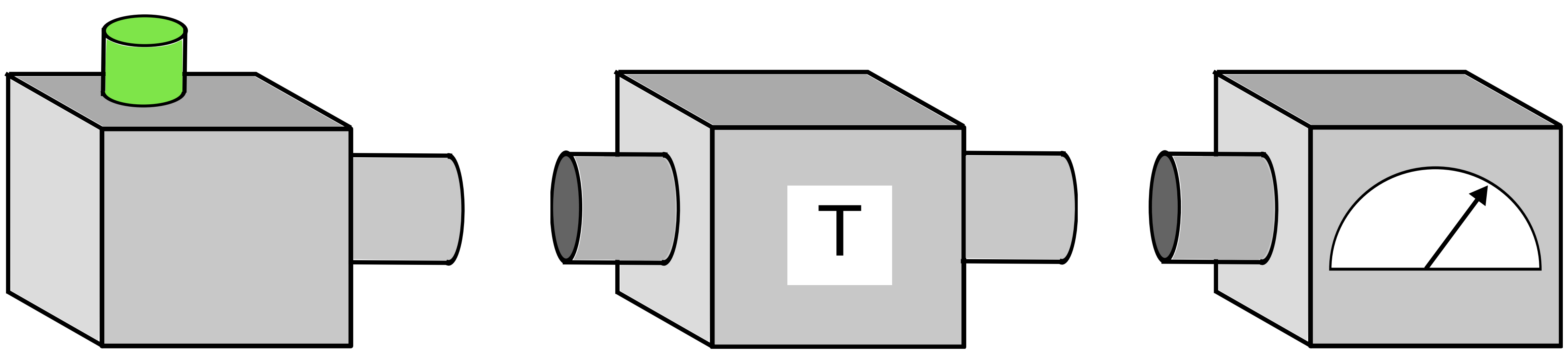
**Example**: quantum spin-1/2 particle.



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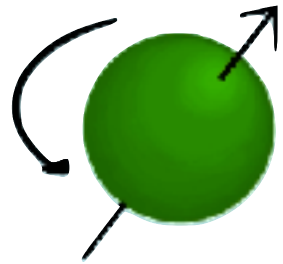
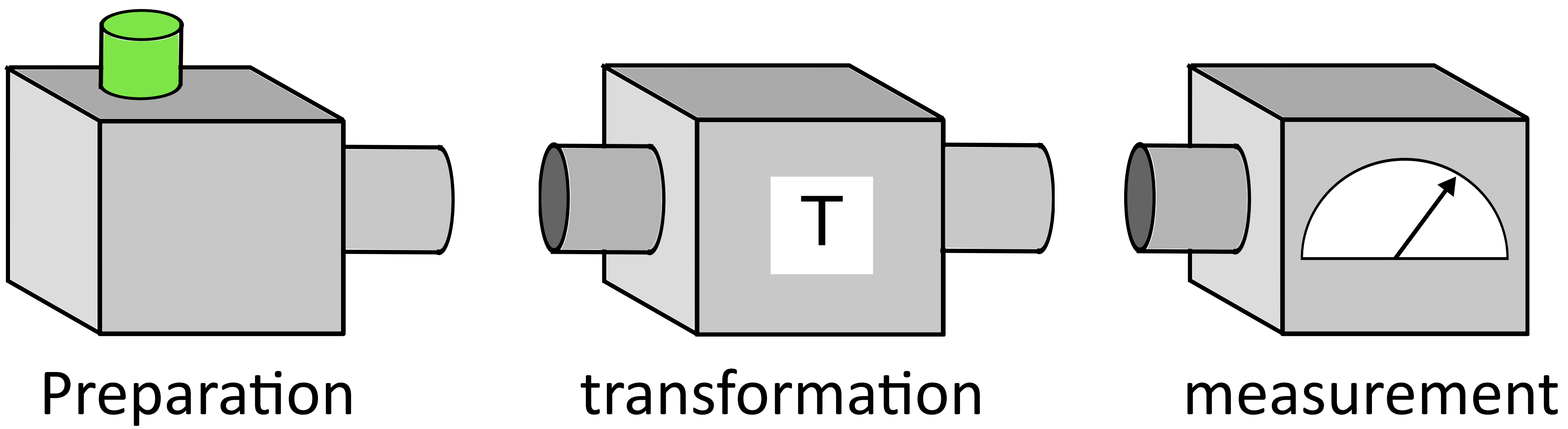


- The preparation device prepares a spin-1/2 particle in quantum state ω.

$$\alpha|\uparrow\rangle + \beta|\downarrow\rangle$$

More generally: $\omega$ is 2x2 density matrix.



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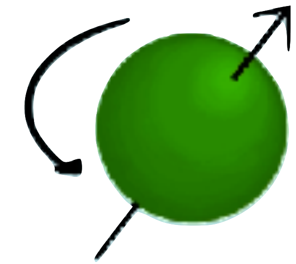

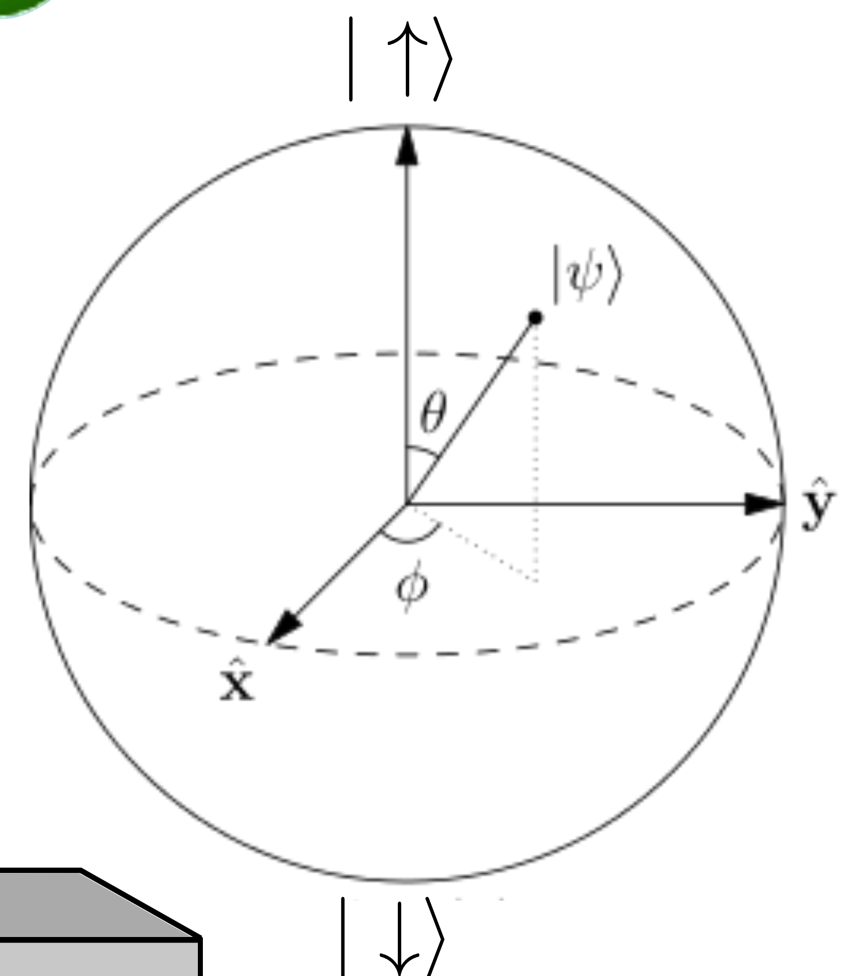


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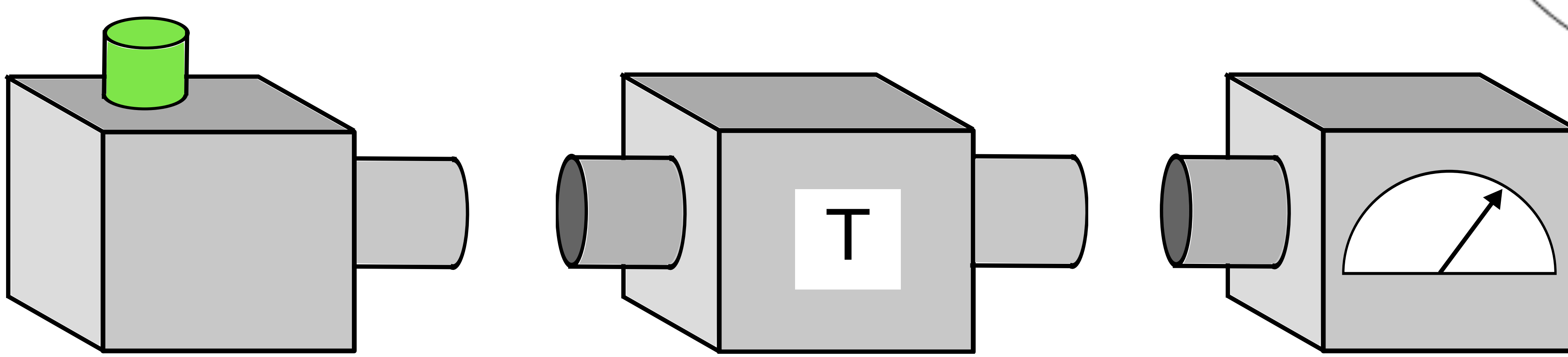


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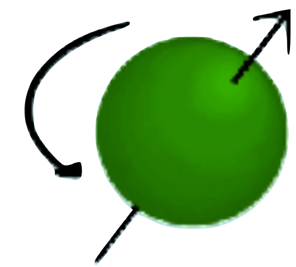
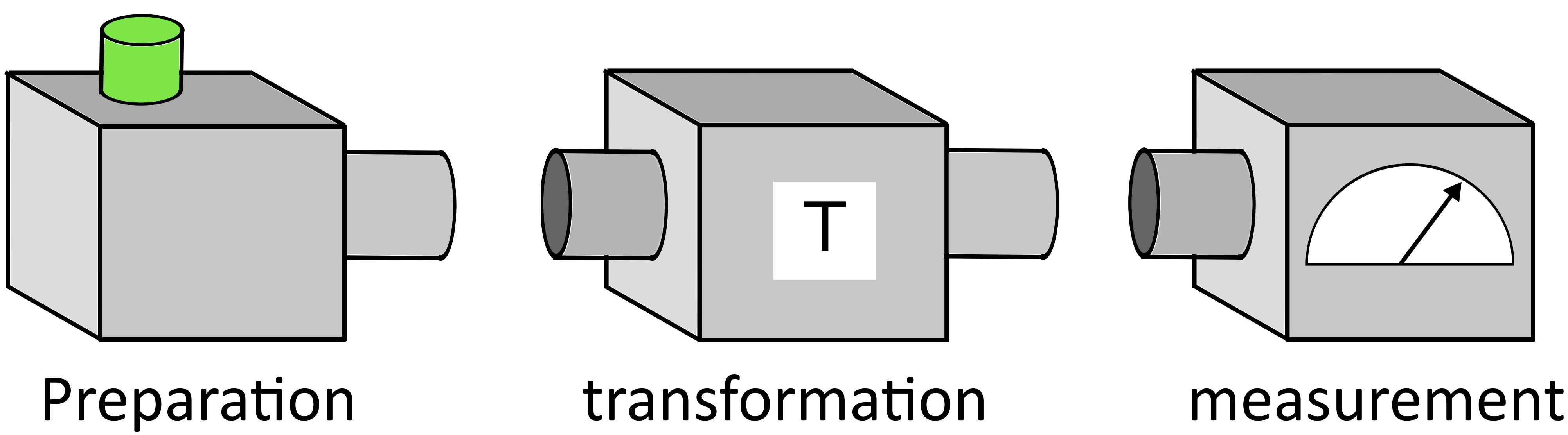


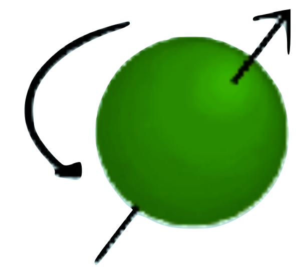
- Unitary transformation of the density matrix:

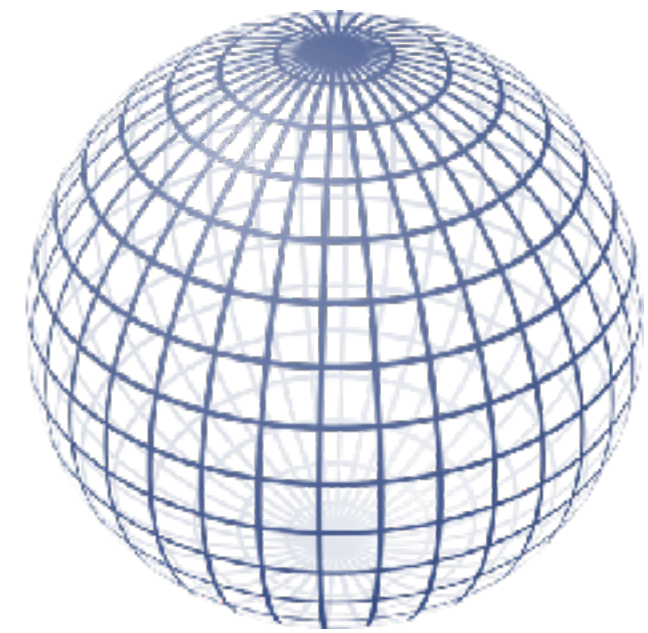
$$\omega \mapsto U\omega U^\dagger.$$



# Generalized probabilistic theories

**Example**: quantum spin-1/2 particle.





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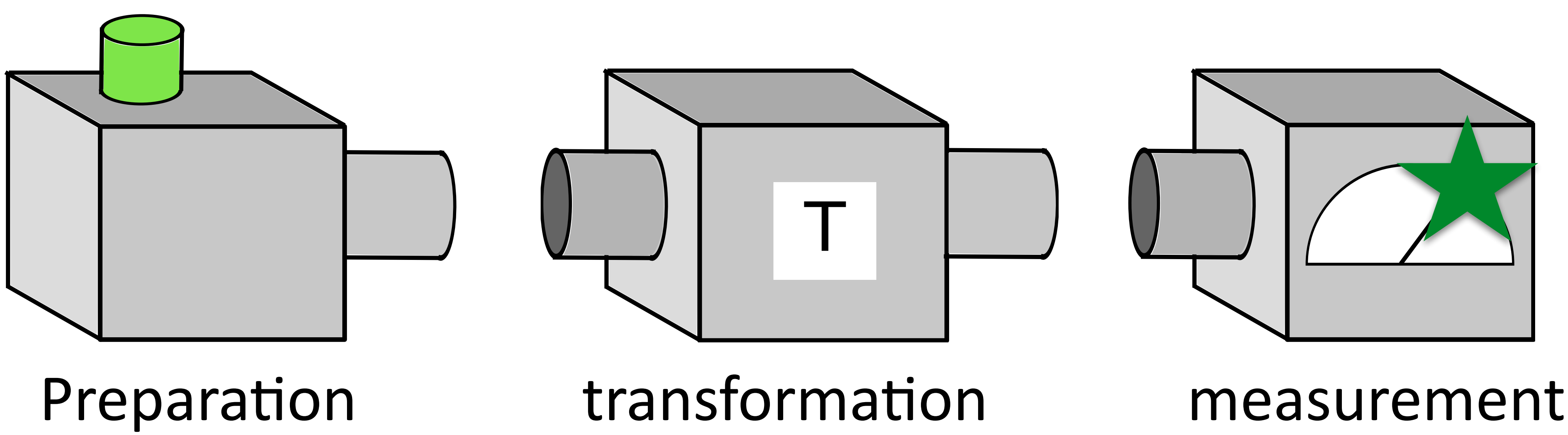
- Measurement in arbitrary spin direction *d*:

$$\mathrm{Prob}(\uparrow\,|\,\omega) = \mathrm{Tr}(P_d\,\omega)$$



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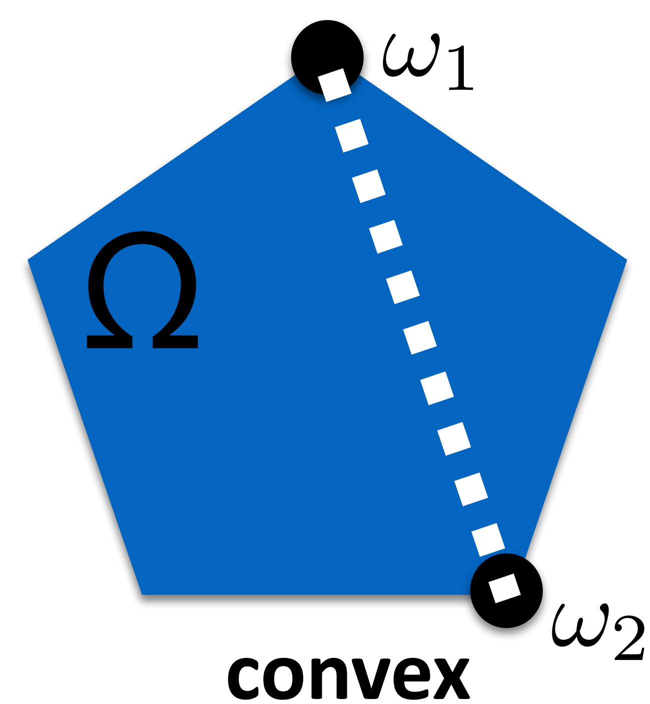

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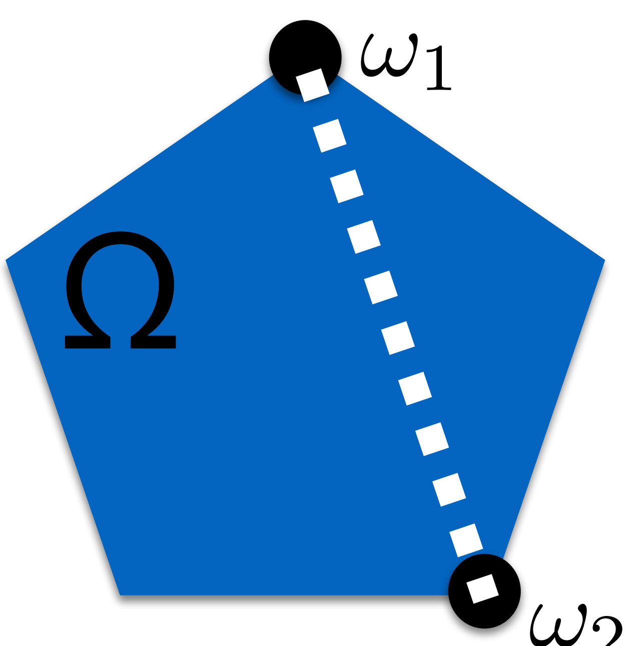

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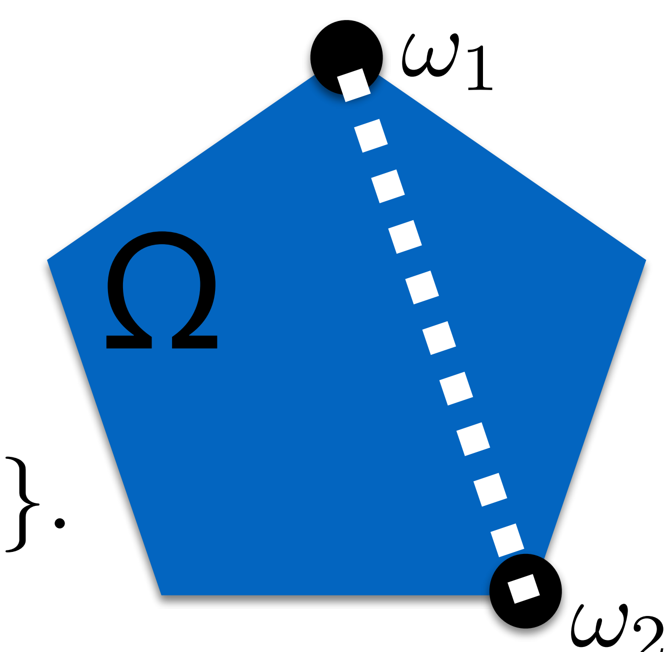
**CPT:** $\Omega = \{(p_1, \ldots, p_N) \mid p_i \geq 0,\ \ \sum_i p_i = 1\}.$




**convex**

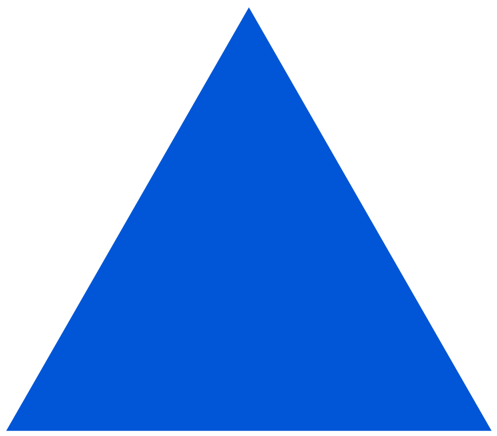
# Generalized probabilistic theories

(Almost) everything can be inferred from shape of state space.

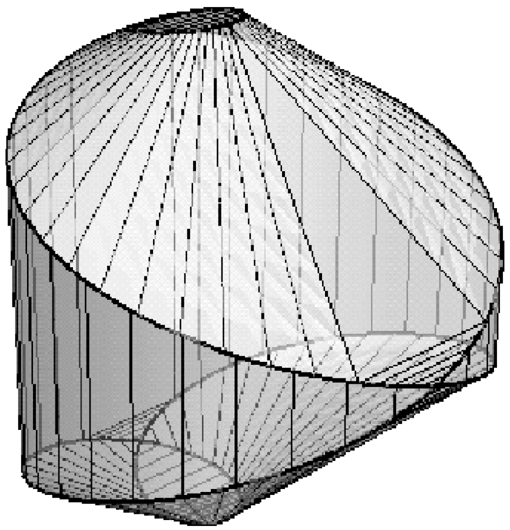
classical
bit

quantum
bit

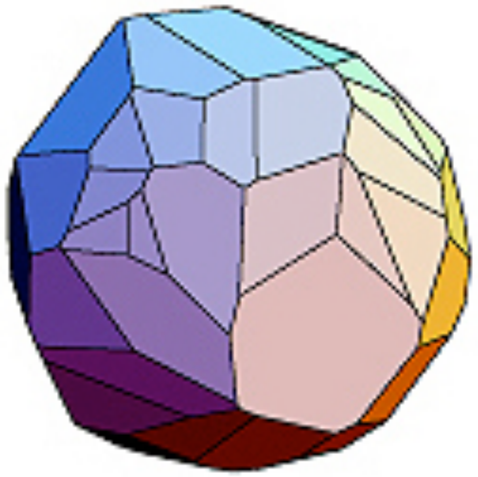
"gbit"



Classical trit
(3-level-system)



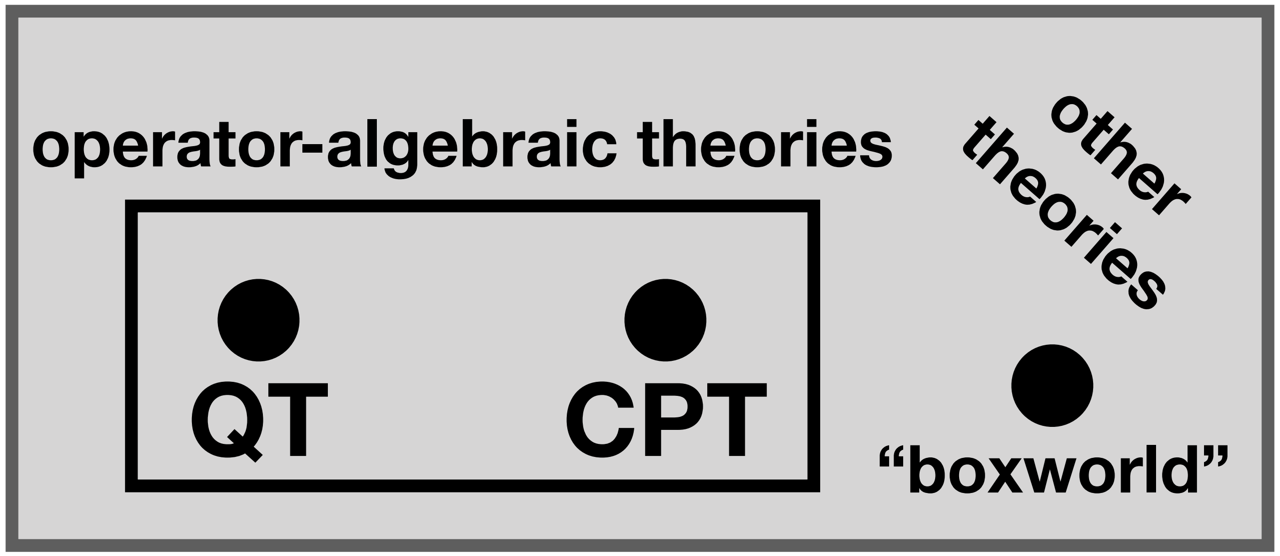
Quantum trit:
8D and complicated!



Arbitrary convex
state space

# Generalized probabilistic theories

There is a large landscape of state spaces, or *theories* (collections of allowed state spaces):



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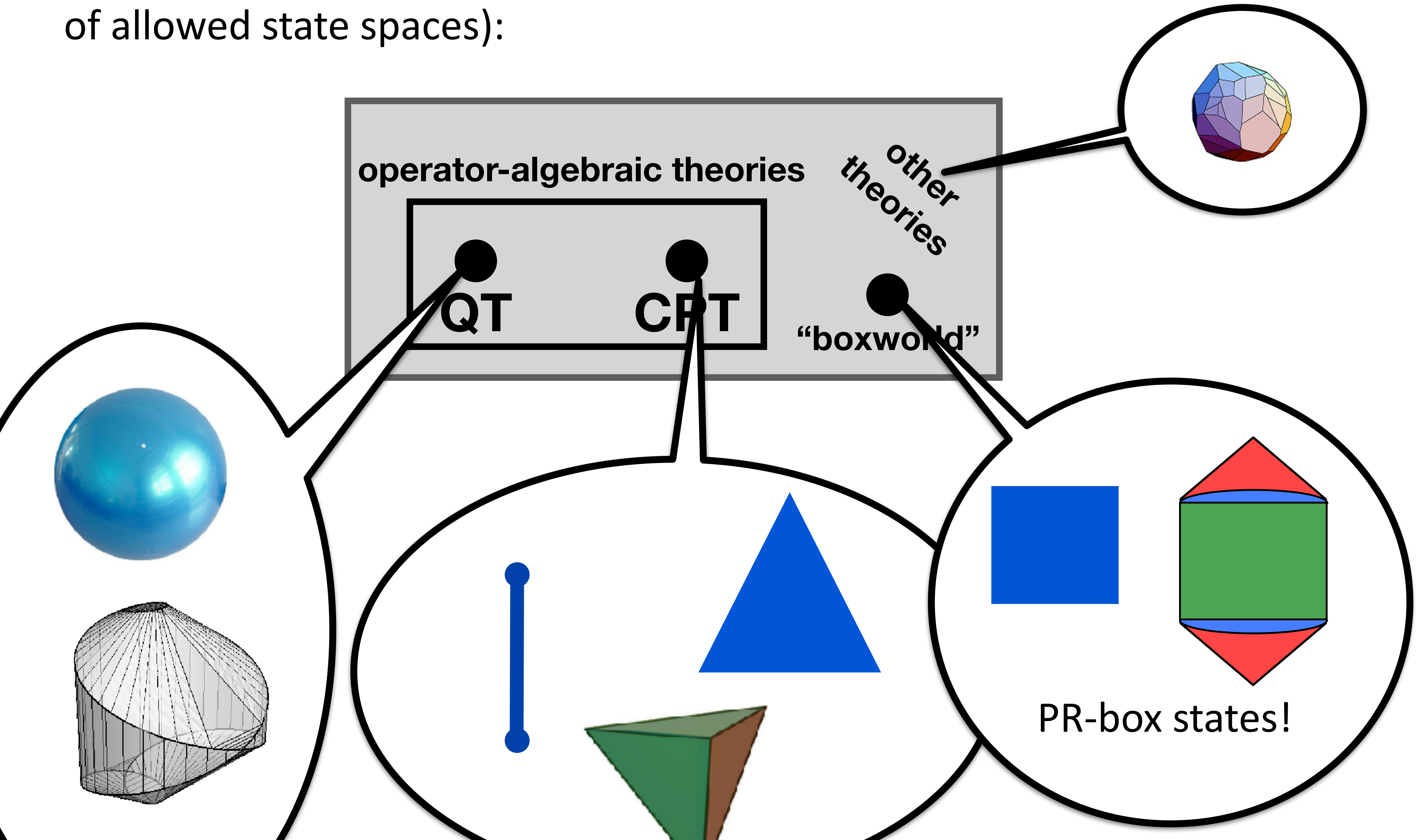

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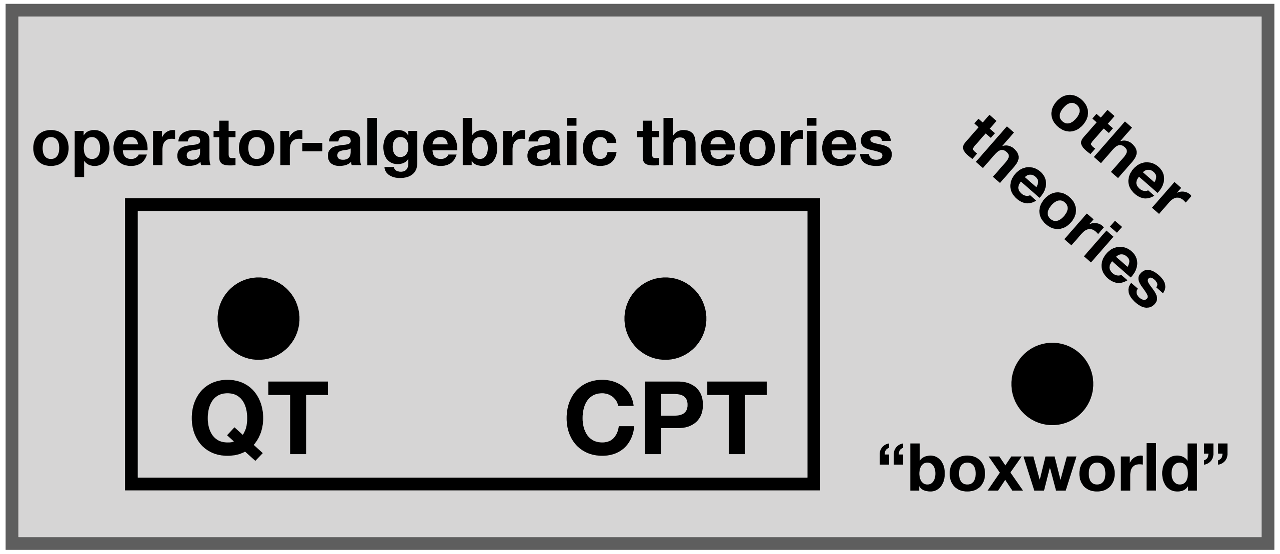

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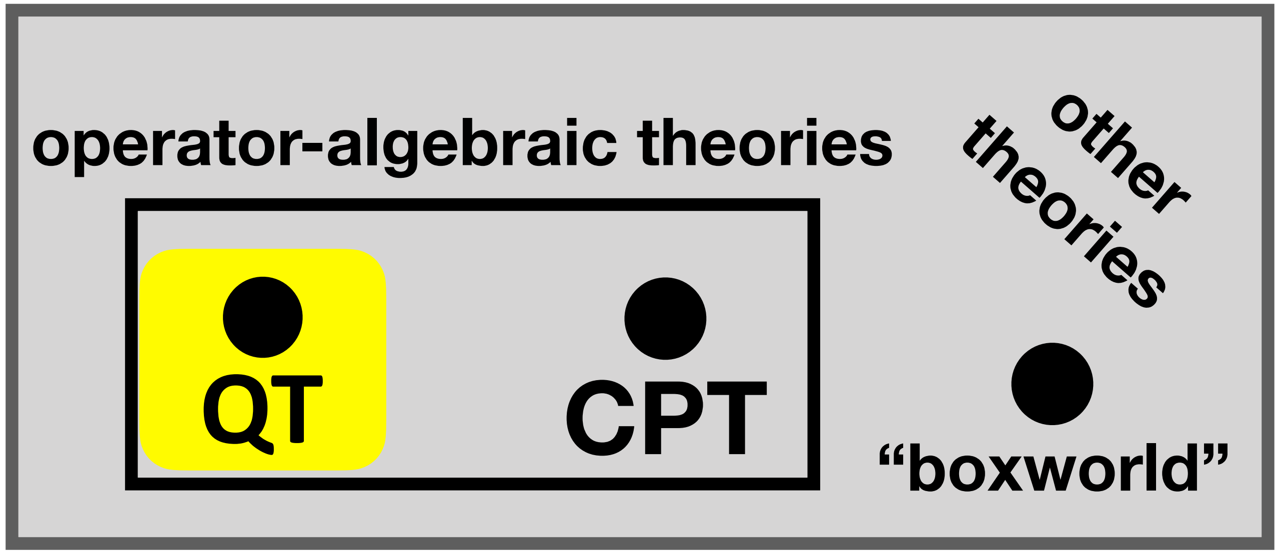


**Goal:** Find a small set of simple physical / information-theoretic **principles** that singles out QT uniquely.

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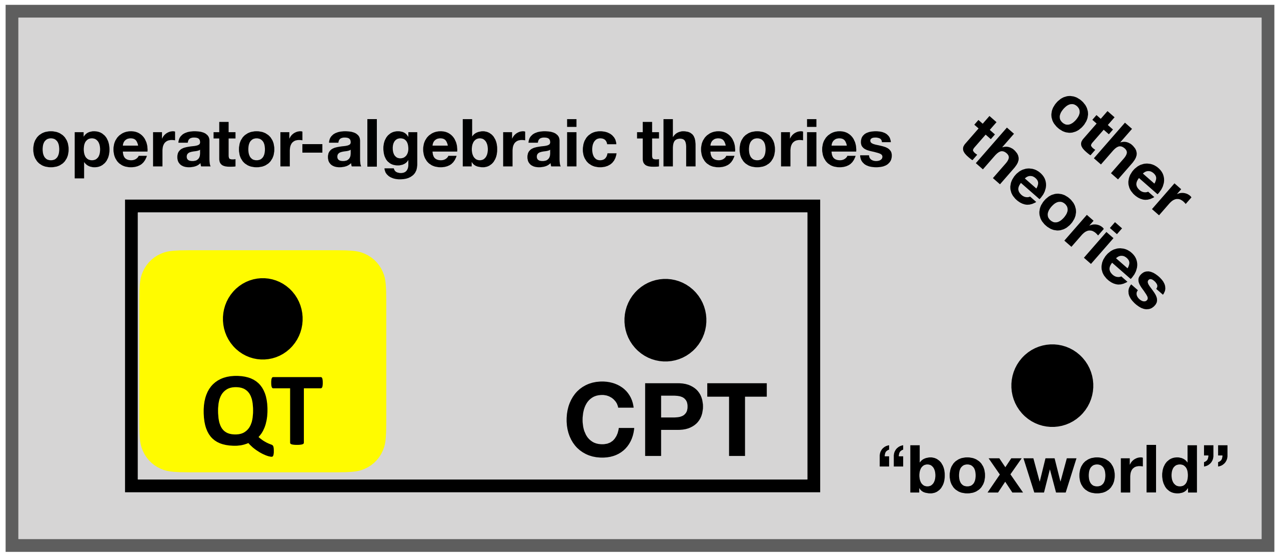


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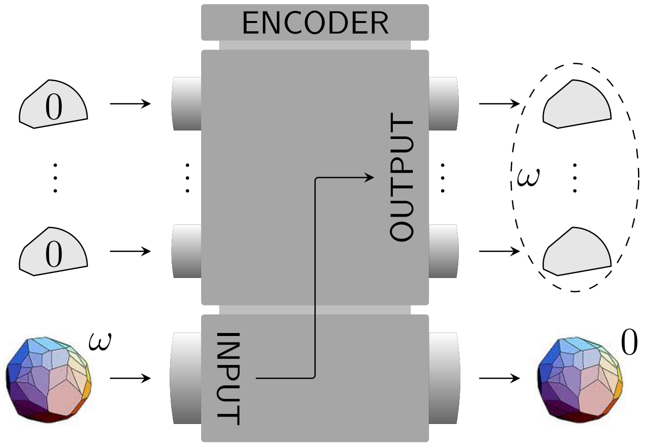
**Role model:** Einstein's **Relativity Principle** and **Light Postulate** determine Minkowski spacetime.

## Overview

1. Boxes and theories beyond quantum theory

2. Quantum theory from simple principles

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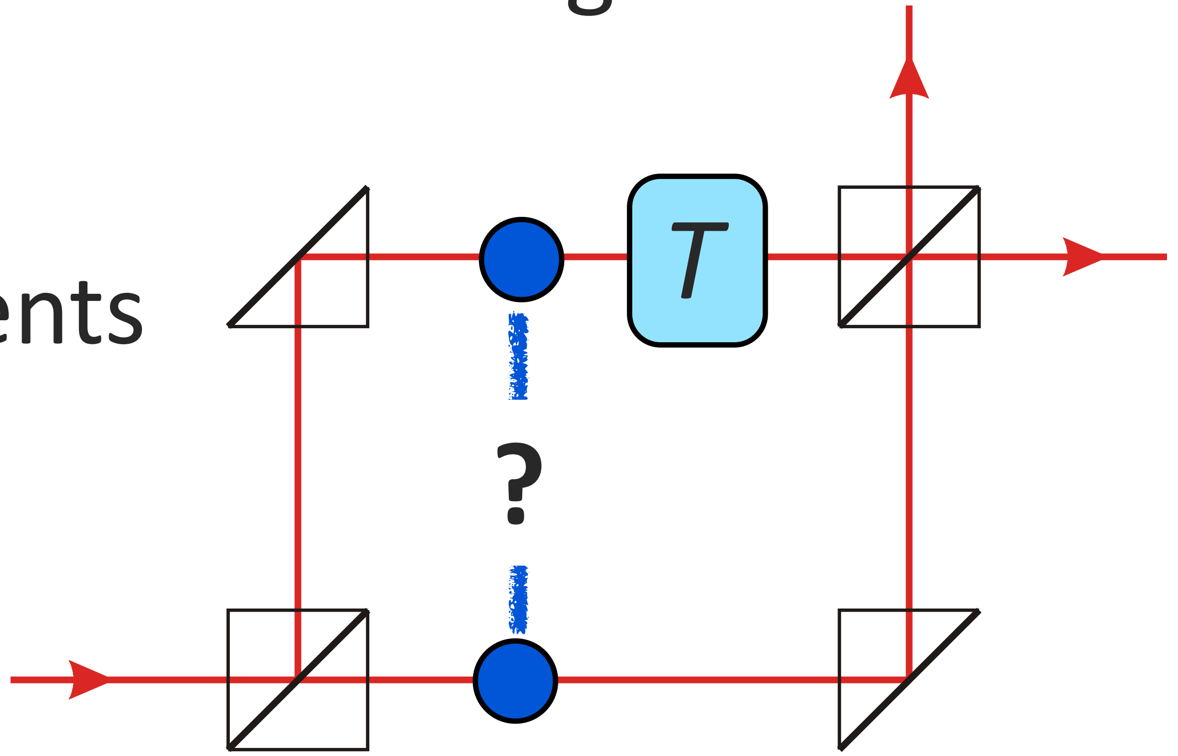

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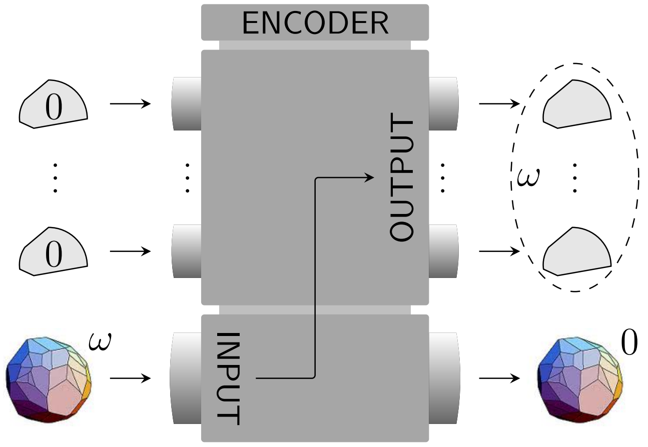


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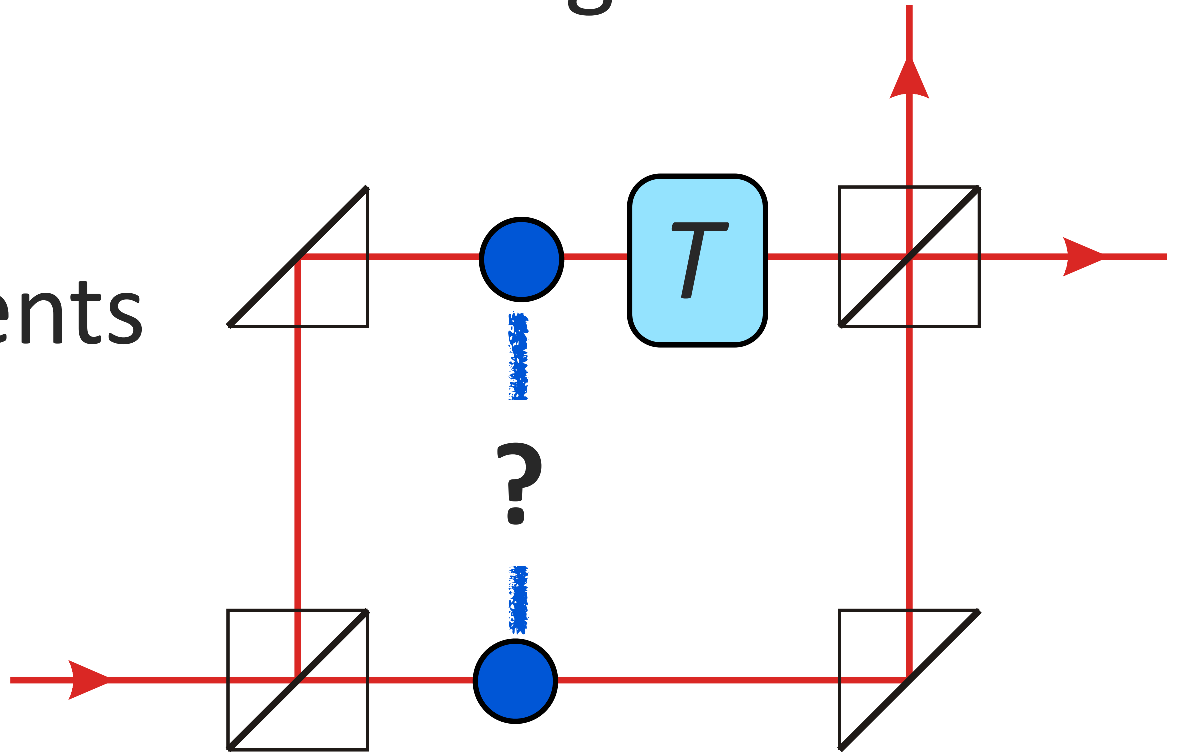

# A reconstruction of quantum theory

- Prehistory:

  Birkhoff & von Neumann (1936); quantum logic (Piron, ...), Ludwig (1954); Alfsen&Shultz (≈1980); .....

- Quantum information revolution:

  **L. Hardy 2001:** Quantum Theory From Five Reasonable Axioms. But needs "simplicity axiom"...

- Clifton, Bub, and Halvorson 2002.

  But assumed C*-algebras.

**Dakić+Brukner 2009; Masanes+MM 2009**
**Chiribella, d'Ariano, Perinotti 2010**; Hardy 2011
*the one I'll present now* 2013;
Barnum, MM, Ududec 2014; Hoehn 2015;
Wilce 2016, ...

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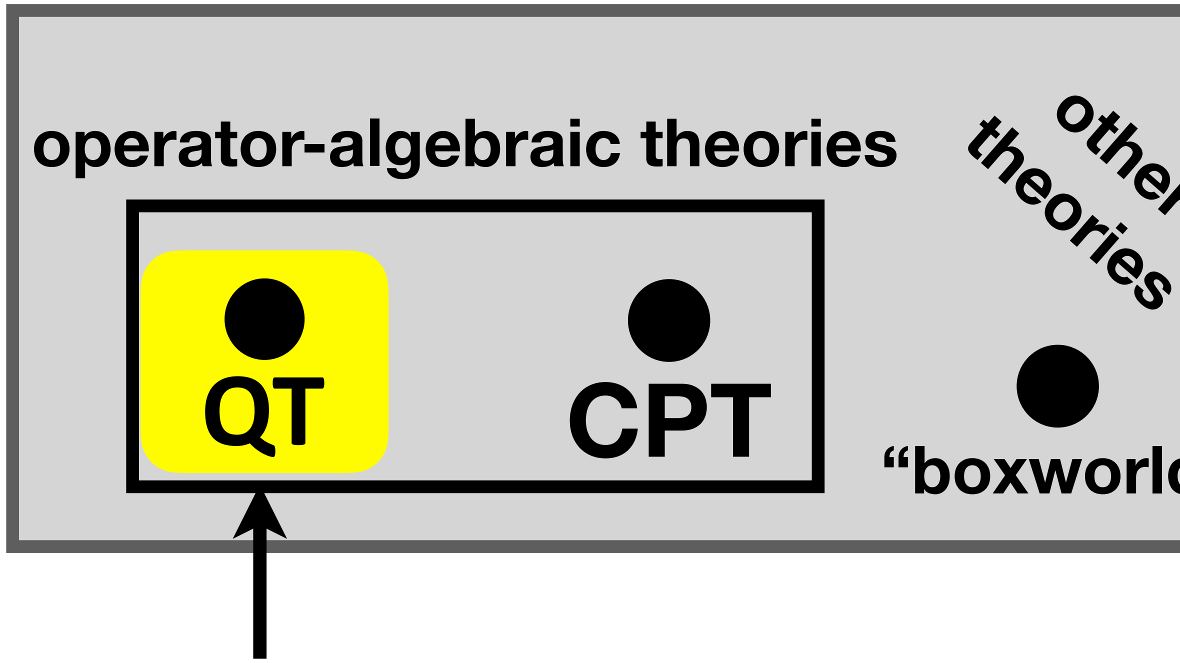


Which principles
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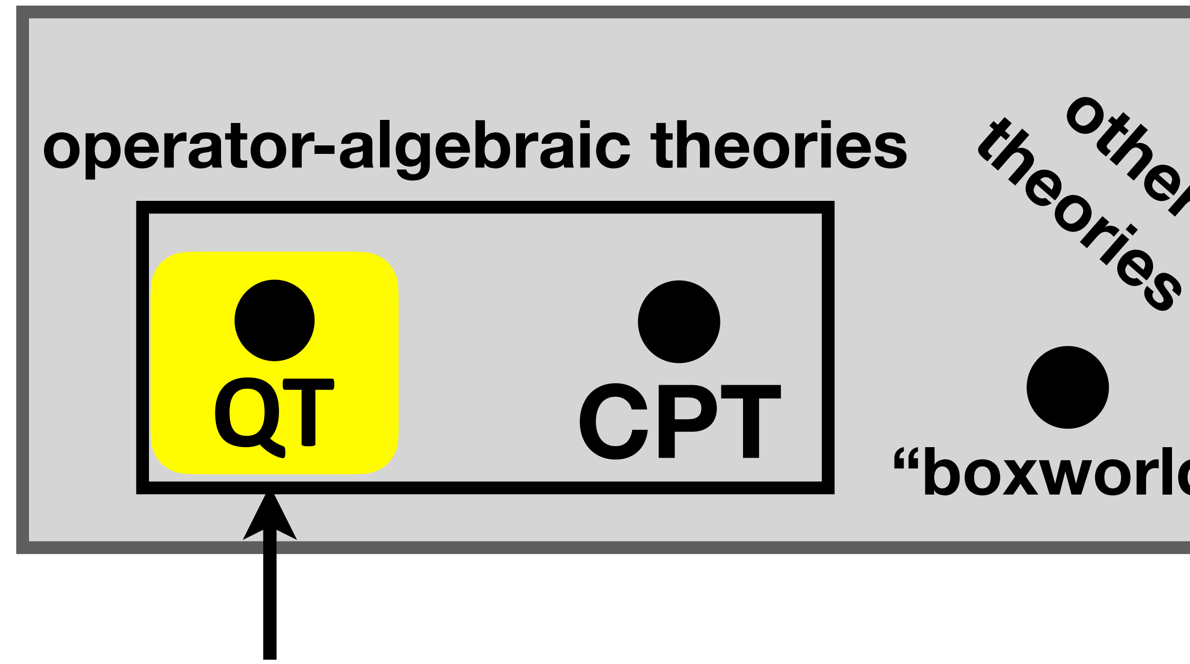


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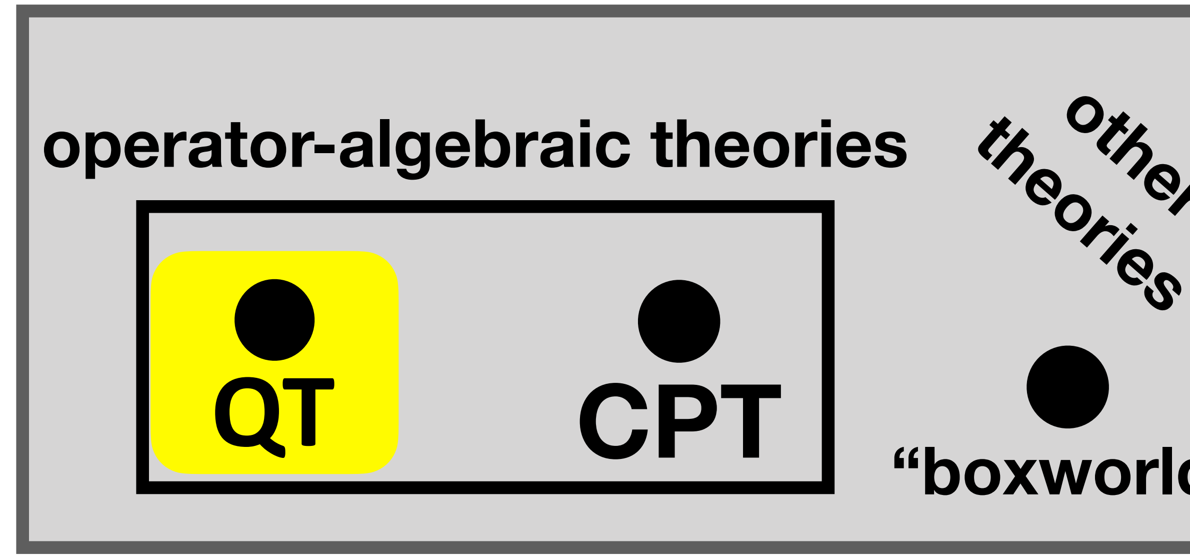
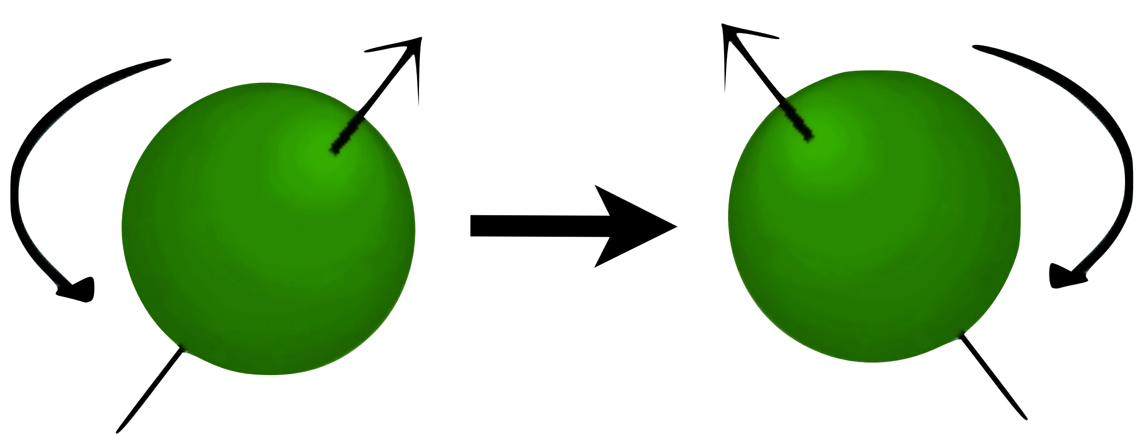



*Continuous reversible time evolution can (in principle) map every pure state to every other.*



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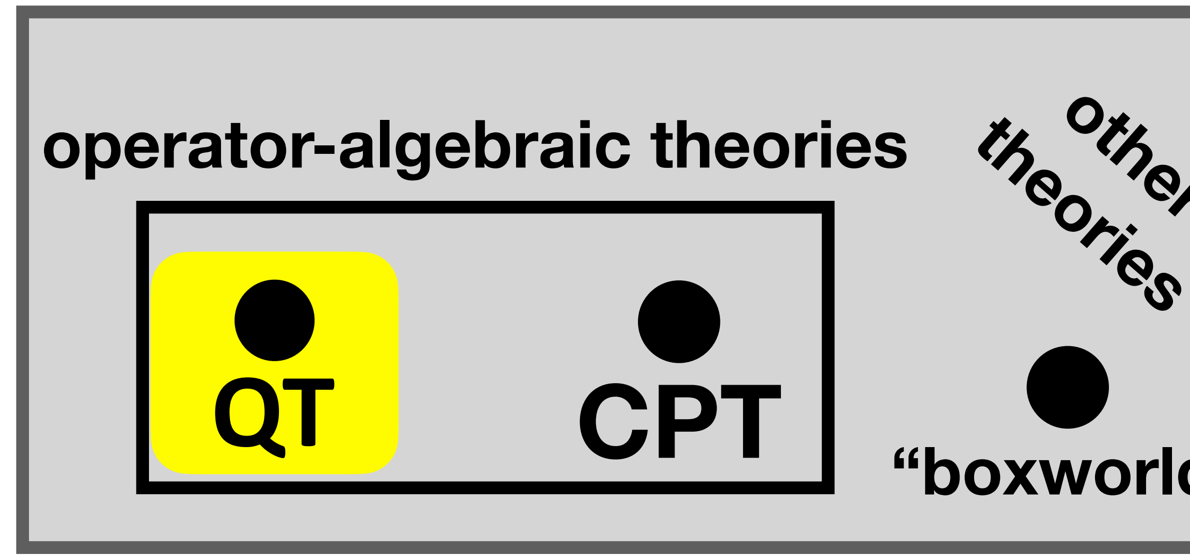
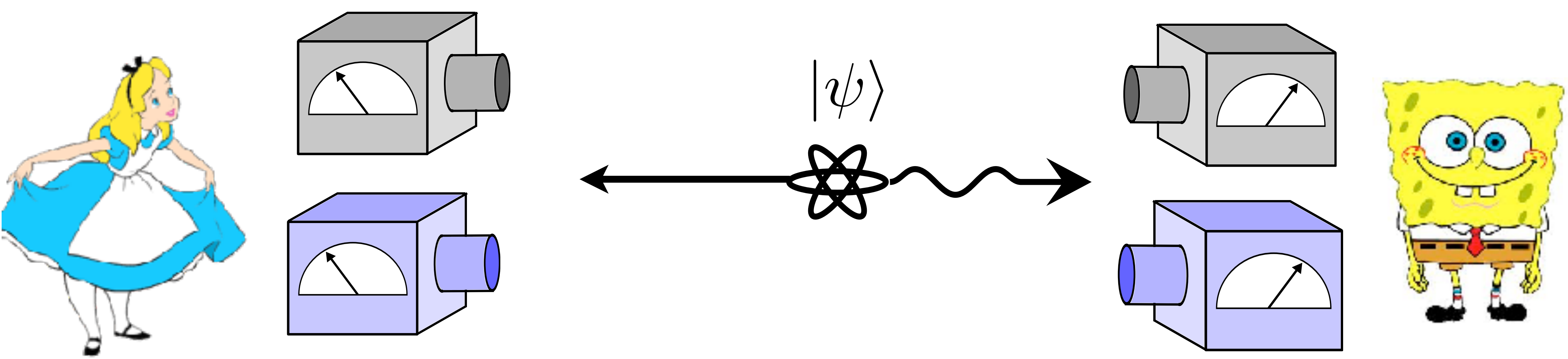



*The state of a composite system is completely characterized by the correlations of measurements on the individual components.*



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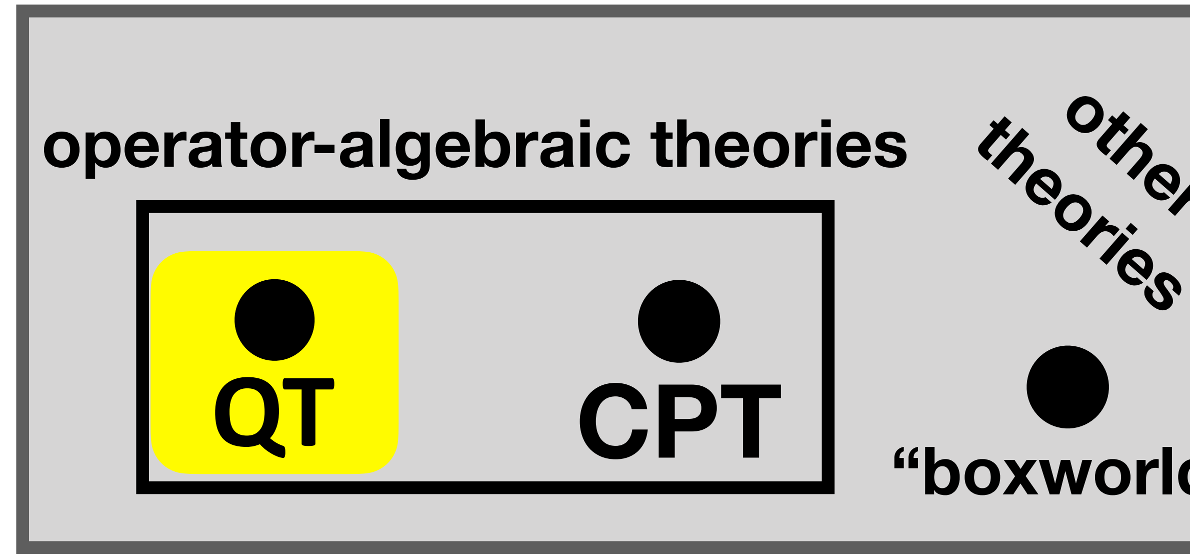

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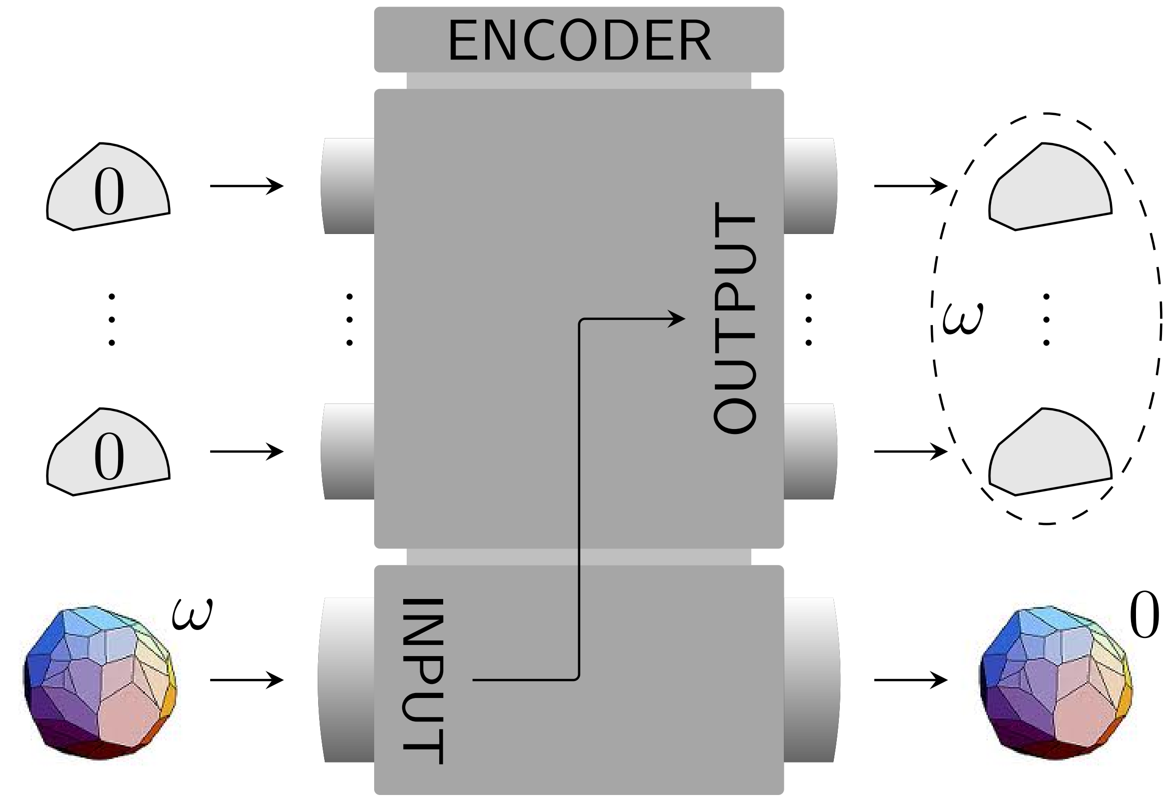


*There is a type of system (the "ubit") such that every system can be encoded into a sufficiently large number of ubits.*

*Pairs of ubits can continuously reversibly interact.*

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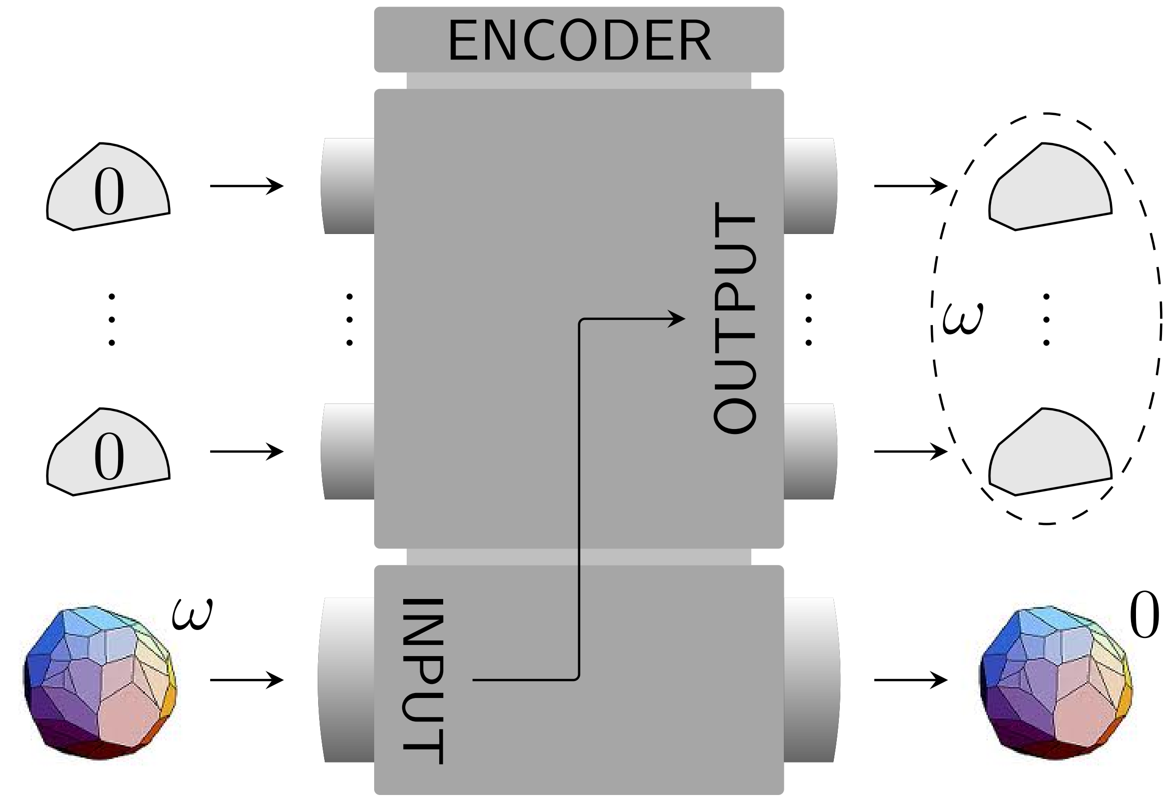


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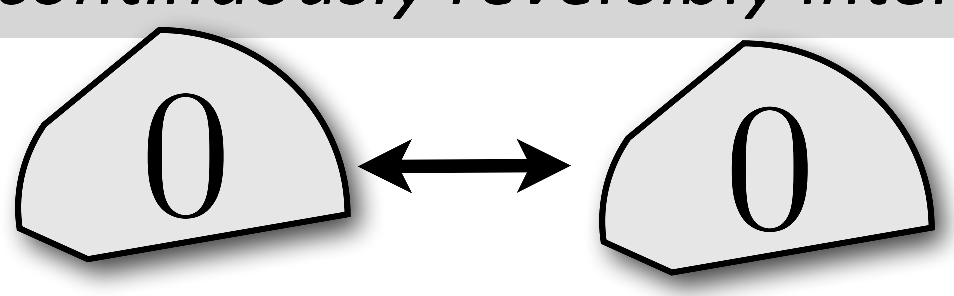

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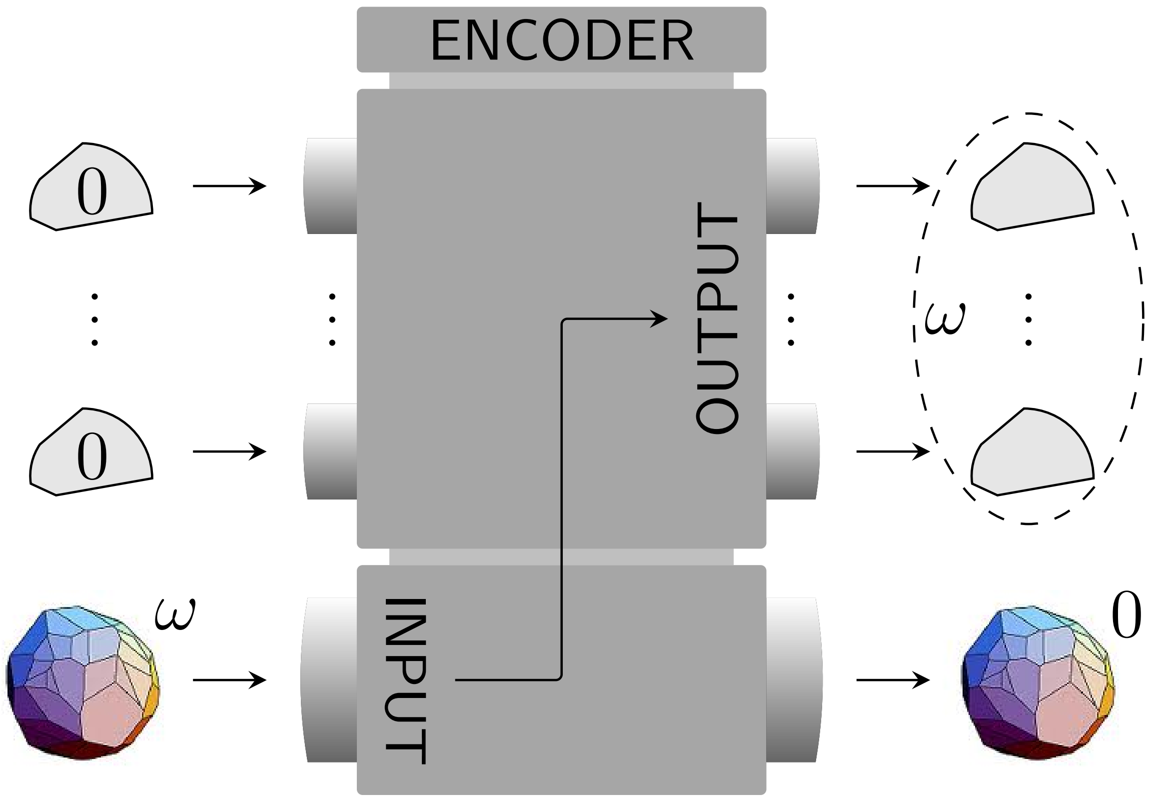


*If a ubit is used to perfectly encode one classical bit, it cannot simultaneously encode any further information.*

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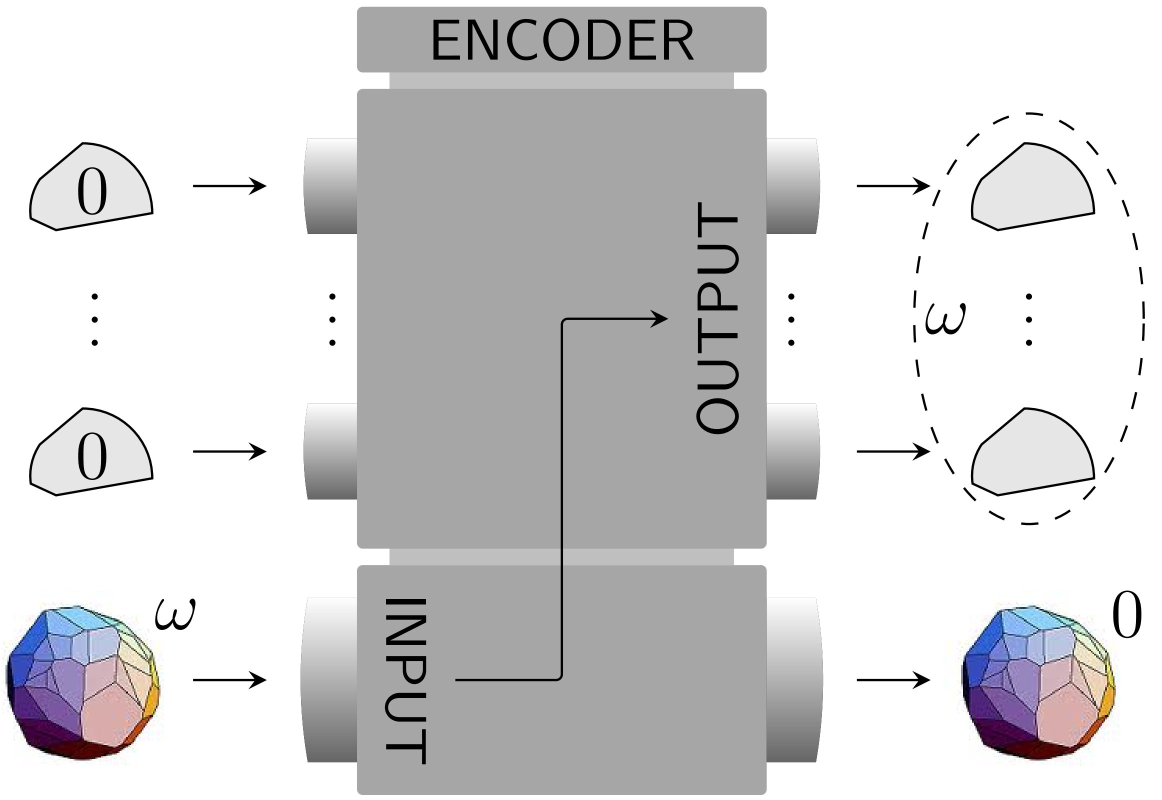


**Theorem.** If Postulates 1-4 hold, then the state space of *n* ubits is

$$\Omega = \{\rho \in \mathbf{H}_{2^n}(\mathbb{C}) \mid \mathrm{tr}(\rho) = 1, \rho \geq 0\},$$

and the reversible transformations are the unitaries, $\rho \mapsto U\rho U^\dagger$.

## Example: why are ubits balls?

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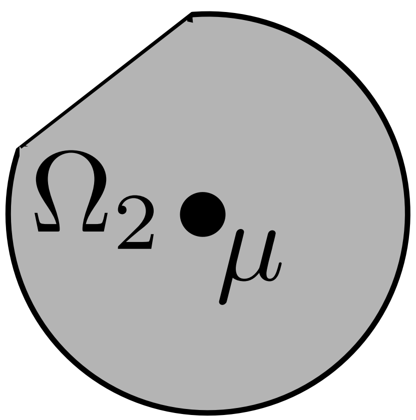
**Group rep. theory:** can reparametrize space such that transformations are rotations. Then, pure states lie on unit sphere (of some dim. *d*).

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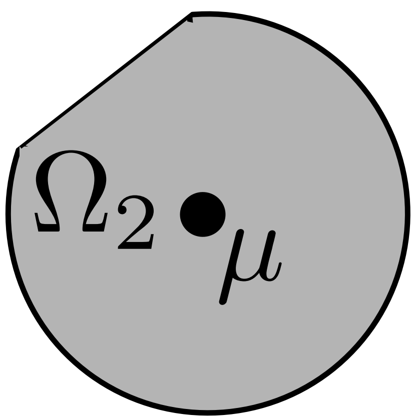
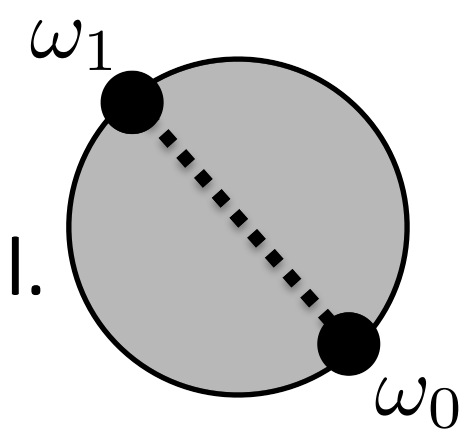



If **full** ball: can encode one bit by preparing state or antipodal state. That's all.



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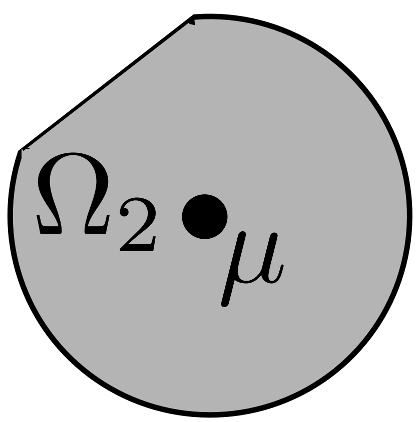


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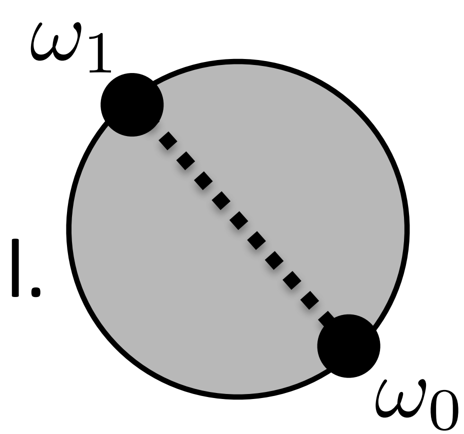
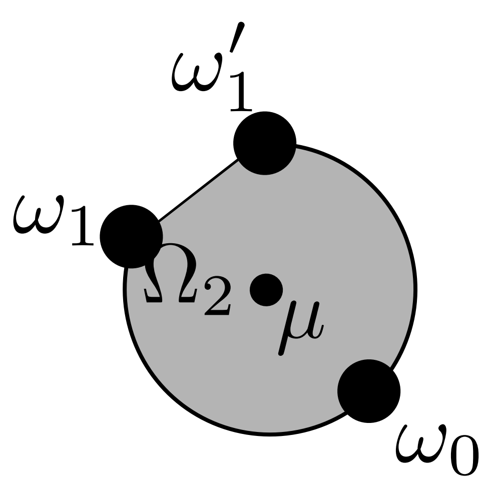



If **not** full ball: can encode one bit **and a little more** by preparing state or **one of** antipodal states.



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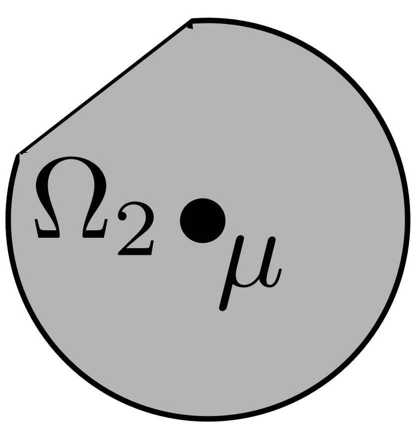


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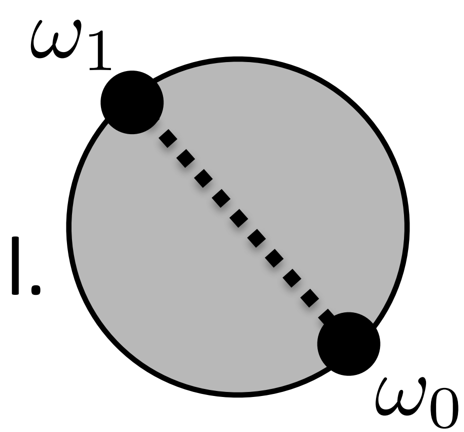


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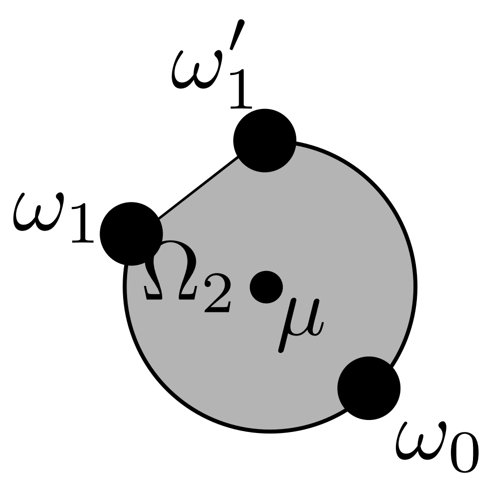


Violates Postulate 4.

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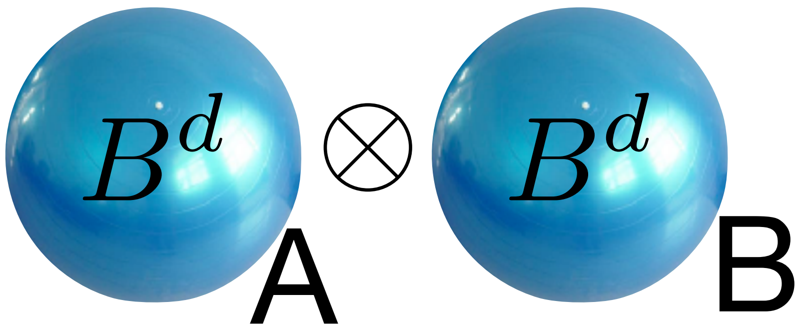


**Two ubits:** *some* composite state space of two *d*-balls, $\mathcal{G}_A = \mathcal{G}_B$ transitive on $\partial B^d$.

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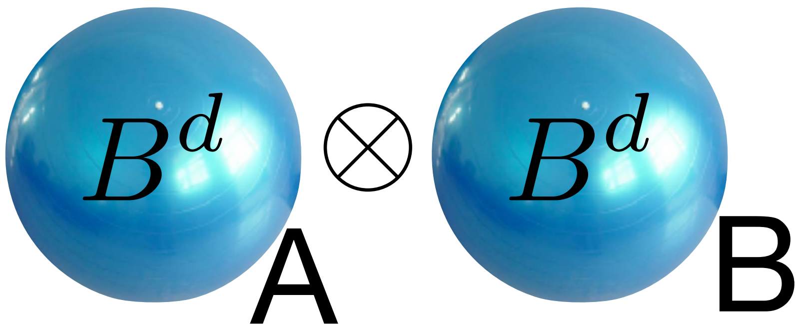


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**Theorem.** Among all dimensions $d$ and all groups $\mathcal{G}_A$, there are only the following possibilities:

- The trivial solution: $\mathcal{G}_{AB} = \mathcal{G}_A \otimes \mathcal{G}_B$.
- $d = 3$, $\mathcal{G}_A = \mathrm{SO}(3)$ (i.e. the quantum bit), $\mathcal{G}_{AB} \simeq \mathrm{PU}(4)$, and $\Omega_{AB}$ is equivalent to the two-qubit quantum state space.

In particular, **continuous reversible interaction** is only possible for $d = 3$, in standard complex two-qubit quantum theory.

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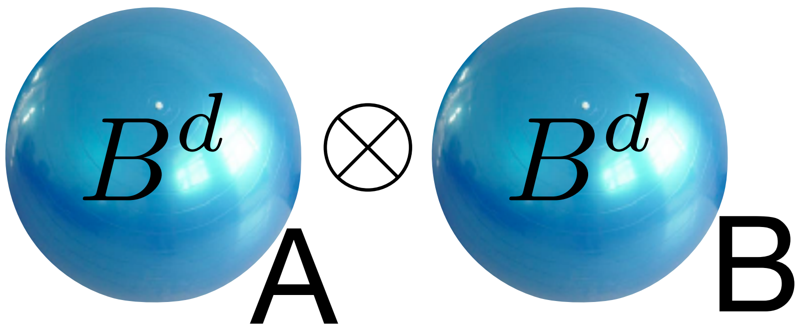


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In particular, **continuous reversible interaction** is only possible for $d = 3$, in standard complex two-qubit quantum theory.

Mathematical reason (at core of proof):

$\mathrm{SO}(d-1)$ is only non-trivial and **commutative** for $d = 3$.

# A possible “loophole” and its resolution

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We have assumed that bits can interact **pairwise**.

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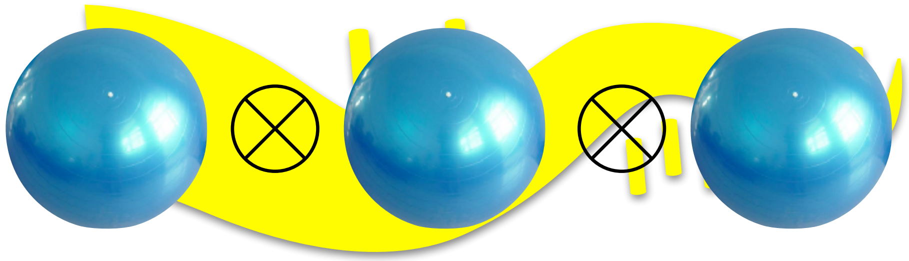

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**ARTICLE** OPEN

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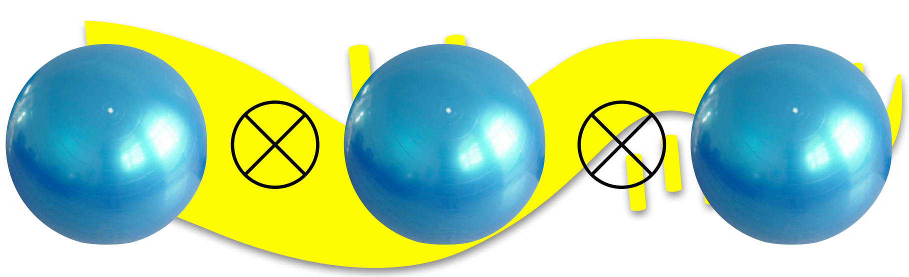
Marius Krumm[1,2] and Markus P. Müller[1,3]



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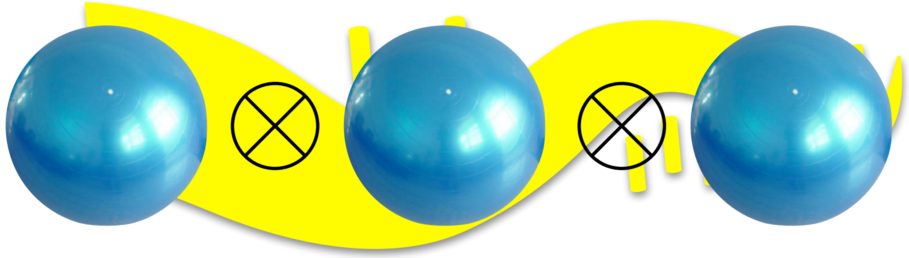
**DERSTANDARD**

Startseite › Wissen und Gesellschaft › **Technik**

GEDANKENEXPERIMENT

**In einer hypothetischen Welt wären Quantencomputer "langweilig"**

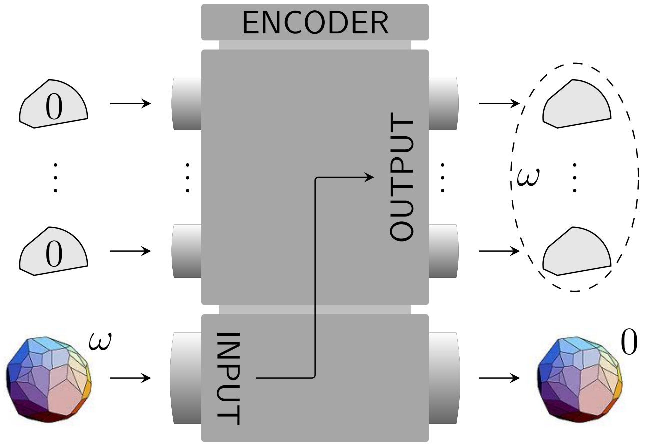
In mehr als drei Raumdimensionen und mit komplexeren Bits wären Quantencomputer nicht leistungsfähiger



# Overview

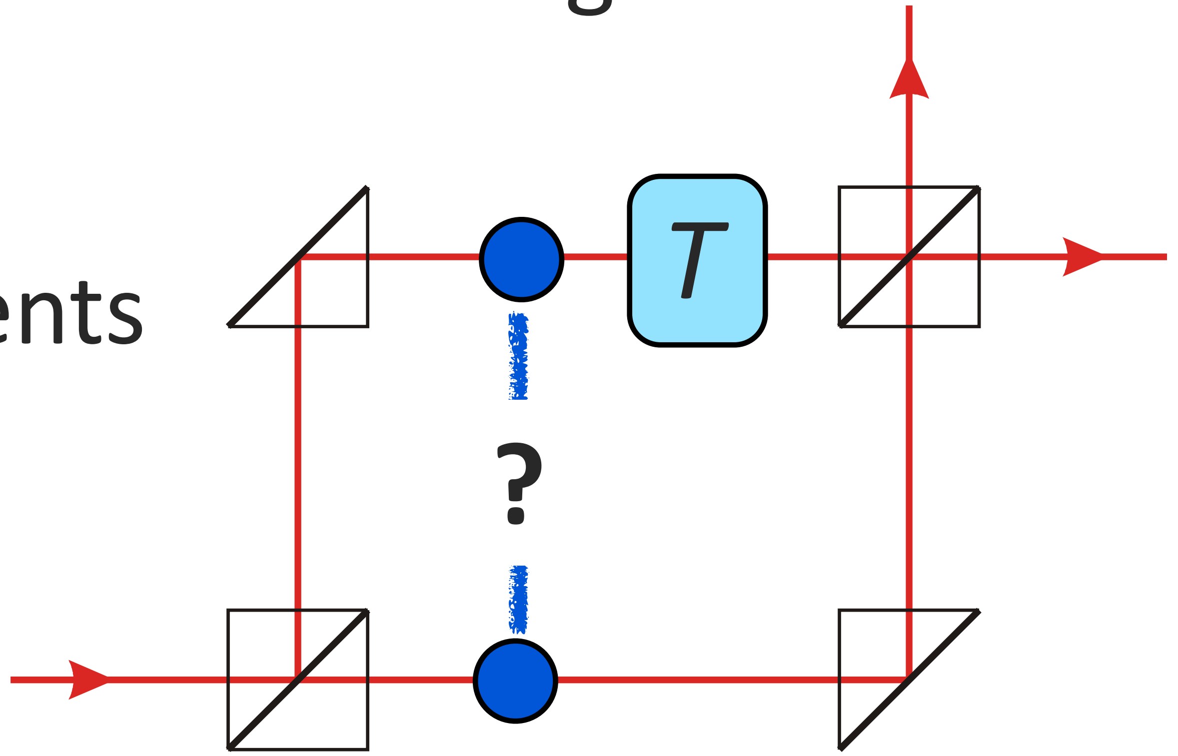
1. Boxes and theories beyond quantum theory

2. Quantum theory from simple principles




3. Spacetime and QT: from foundational insights…

4. … to protocols and experiments



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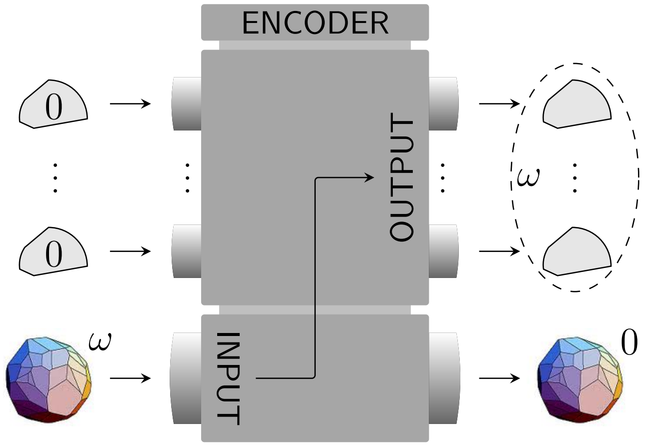


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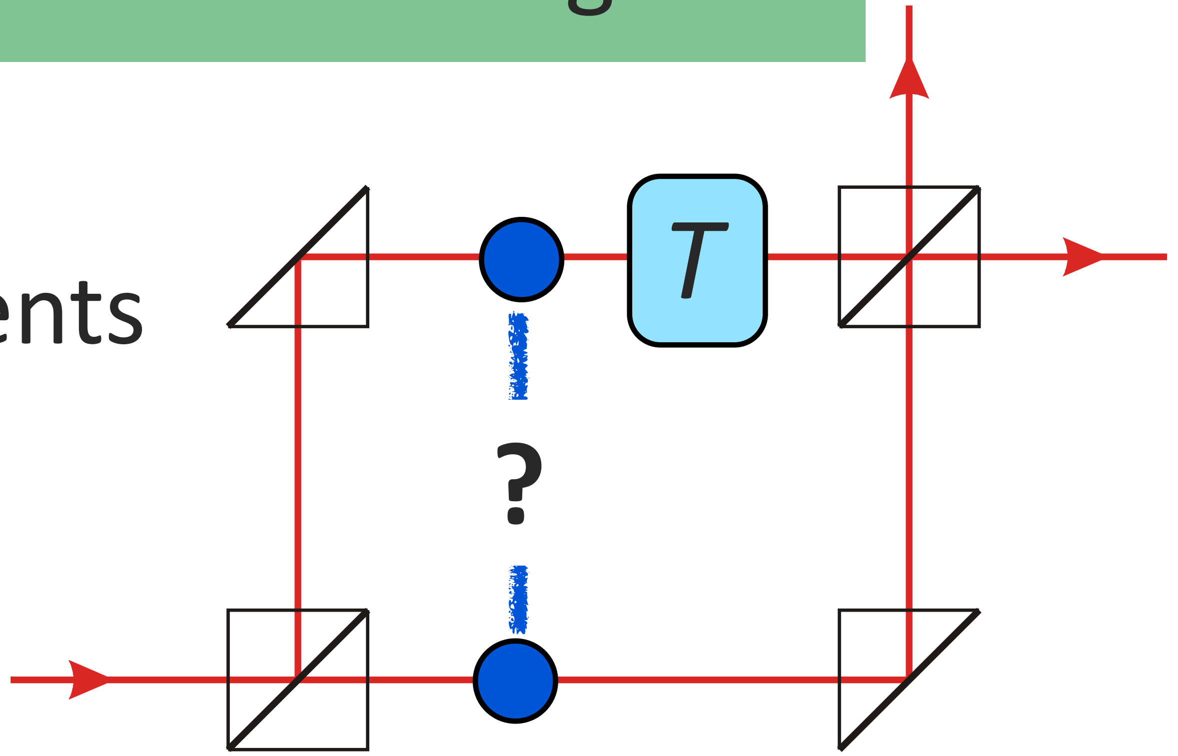

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We have already seen an **information-theoretic** reason.
But there is also a “spacetime” reason!

# Constraints from relativity

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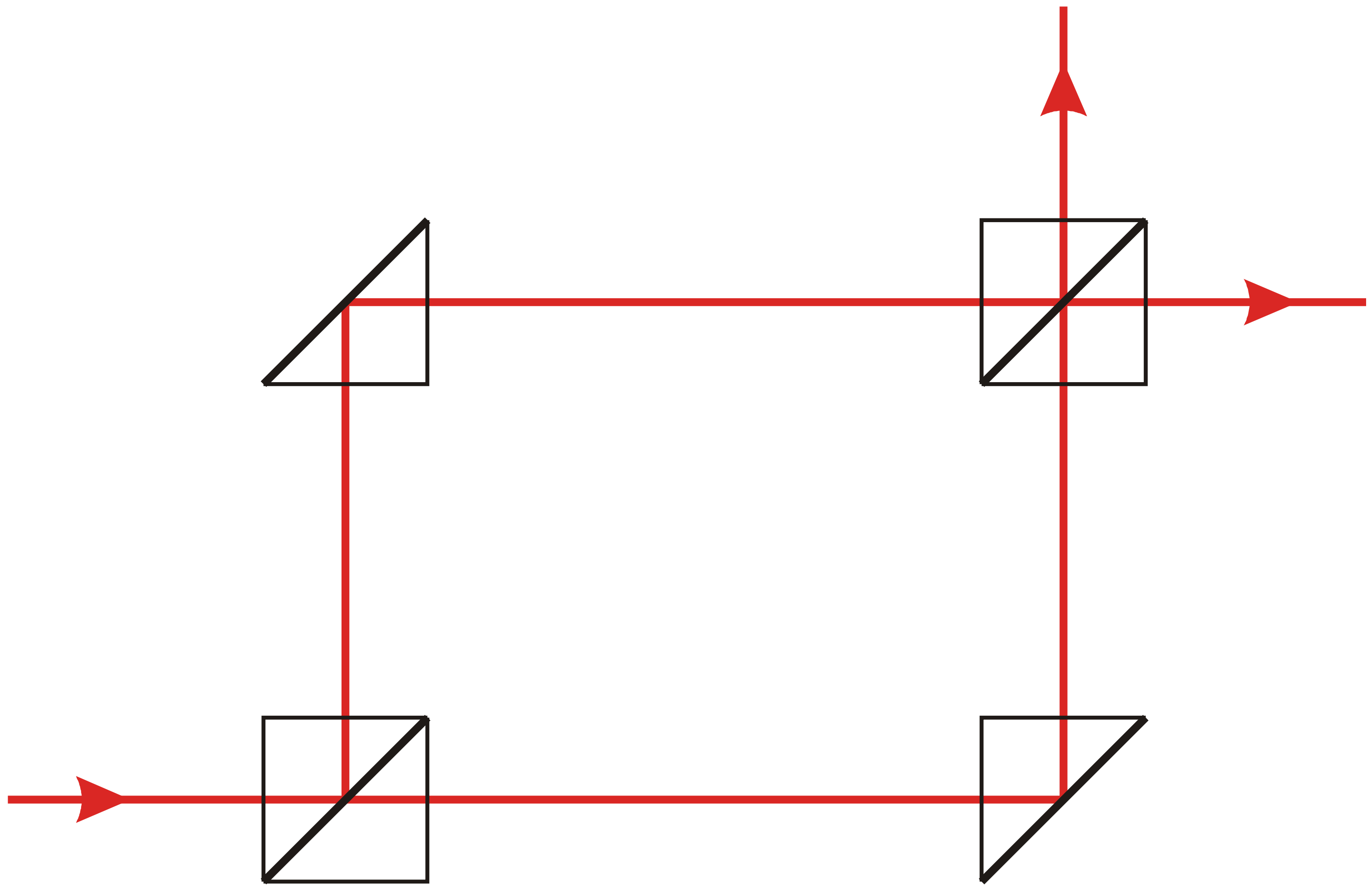

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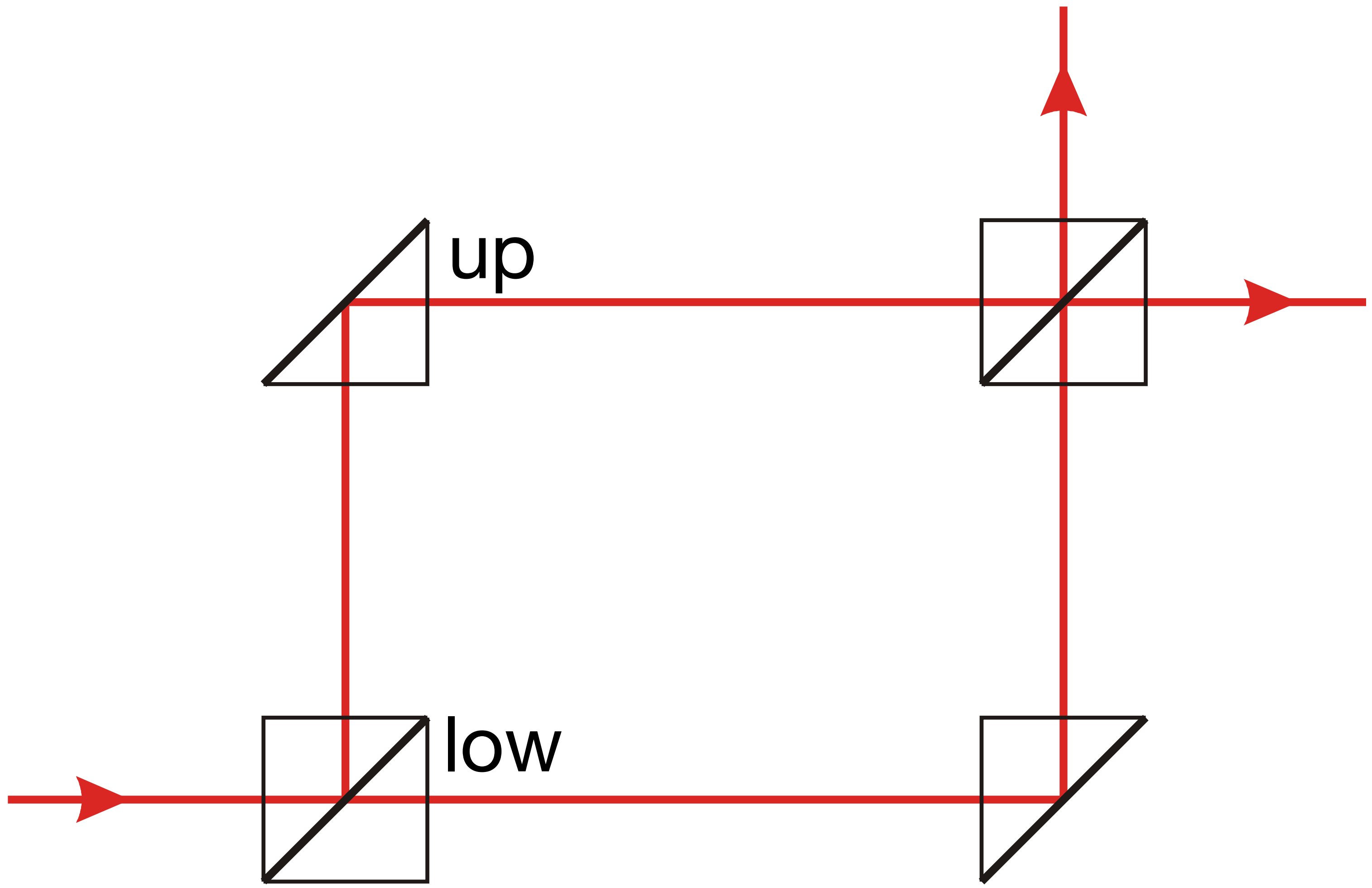

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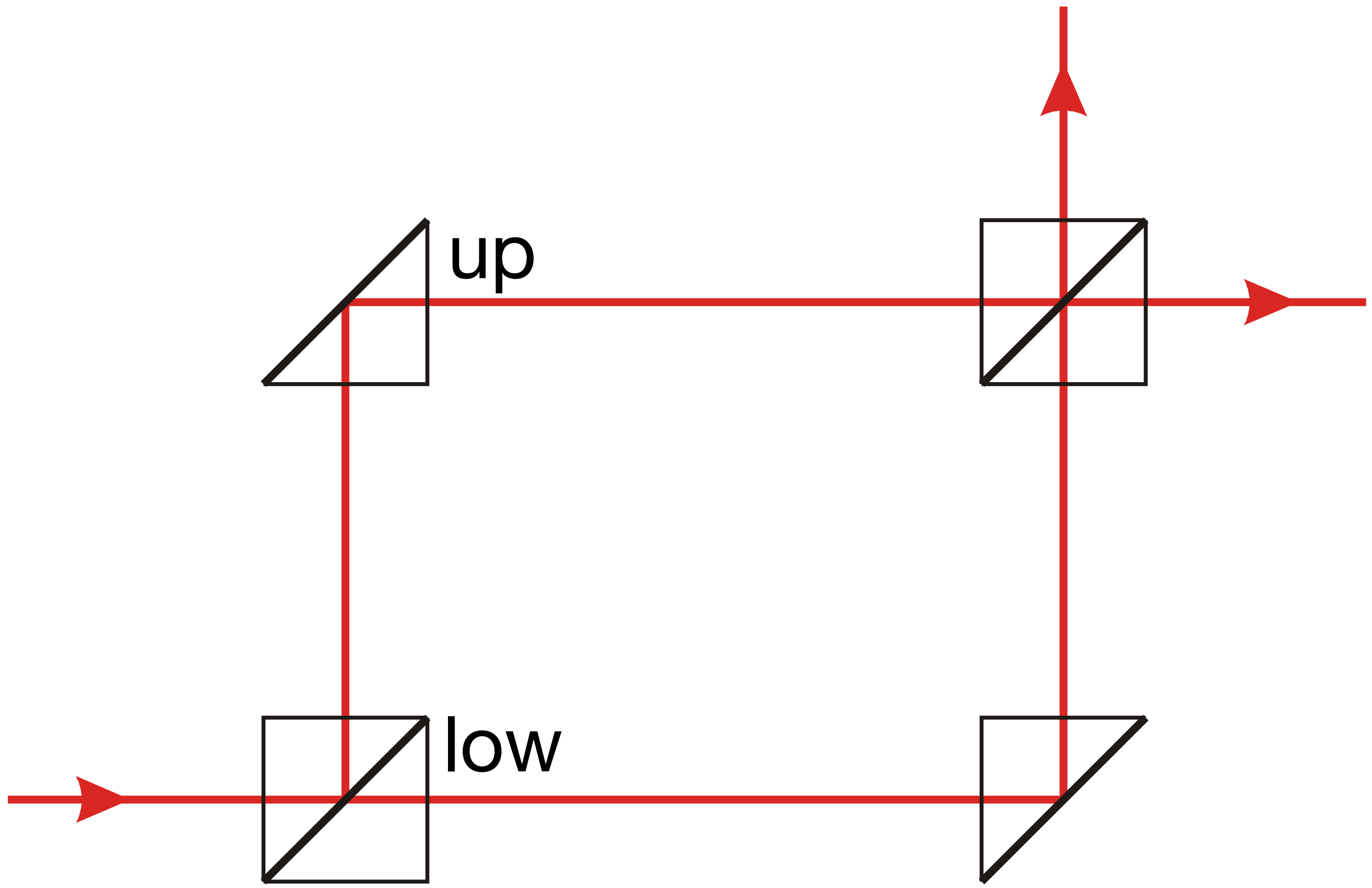


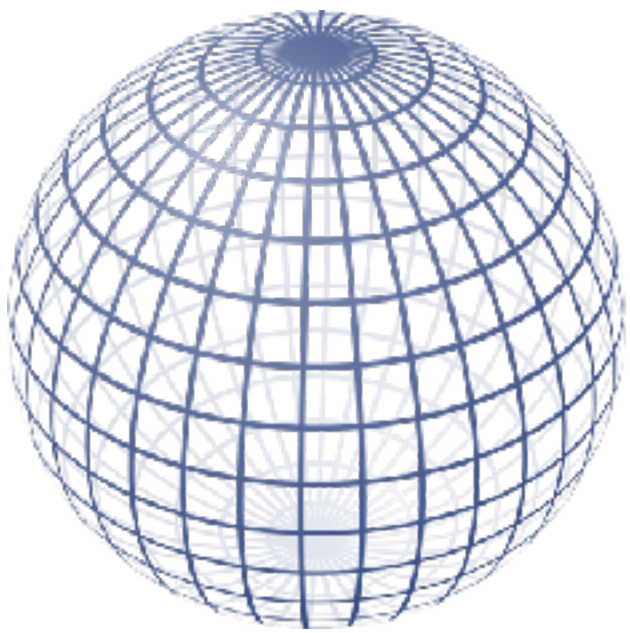

*d*-dim. "Bloch sphere"

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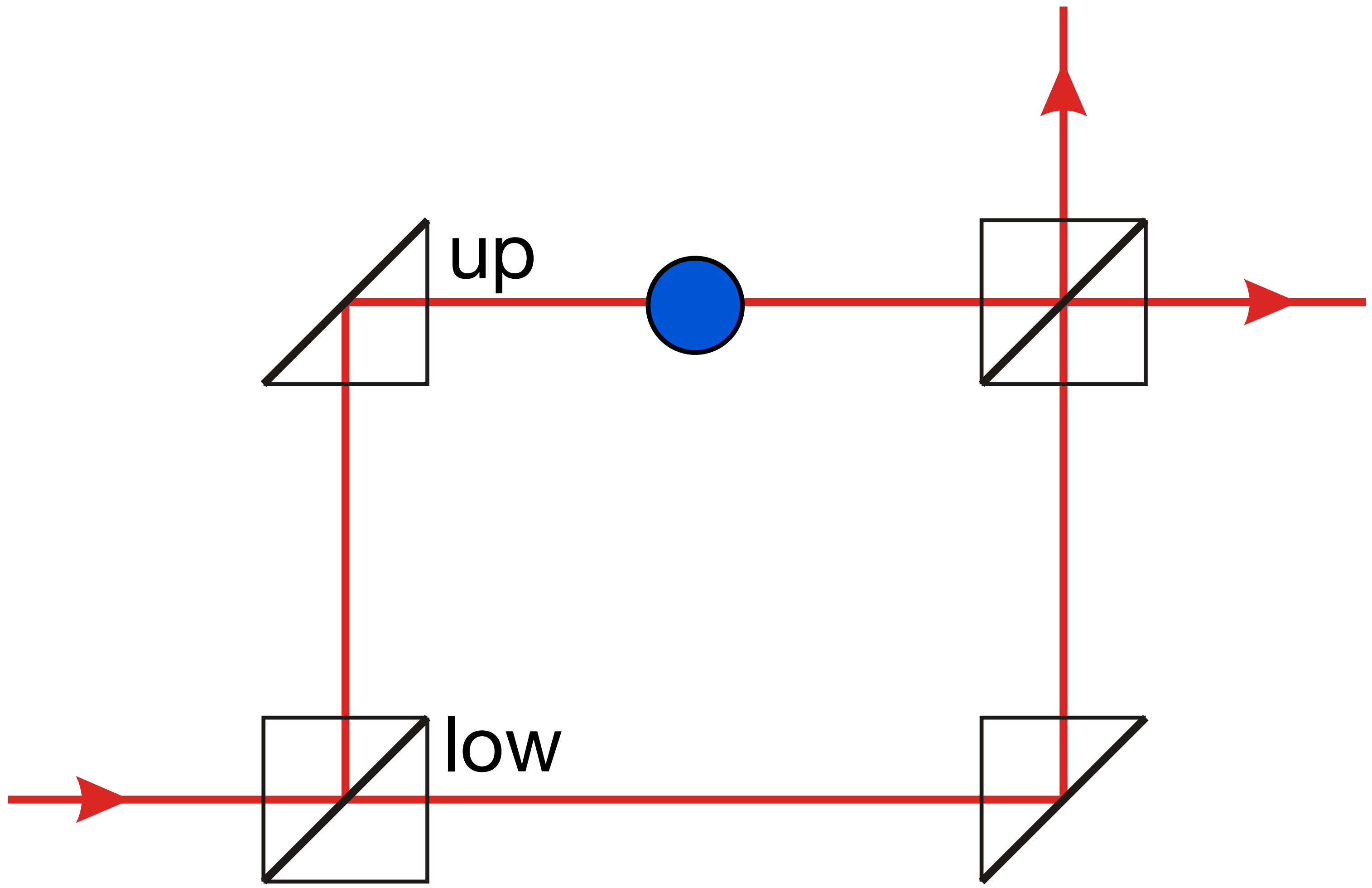


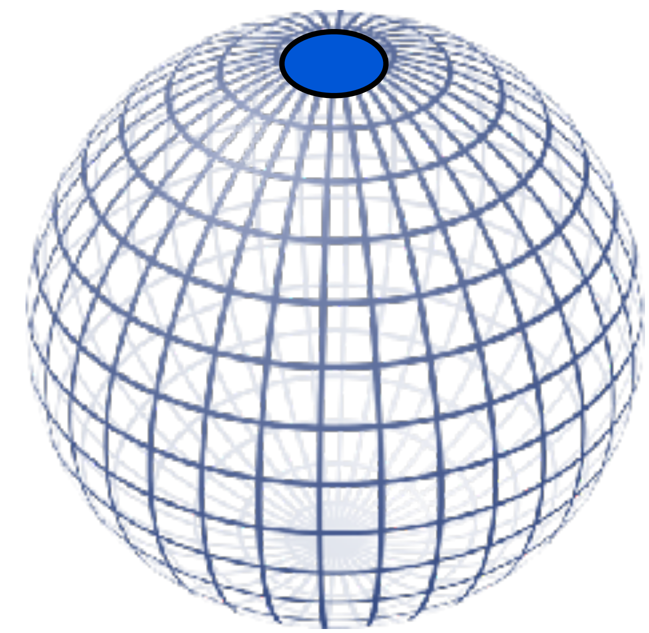

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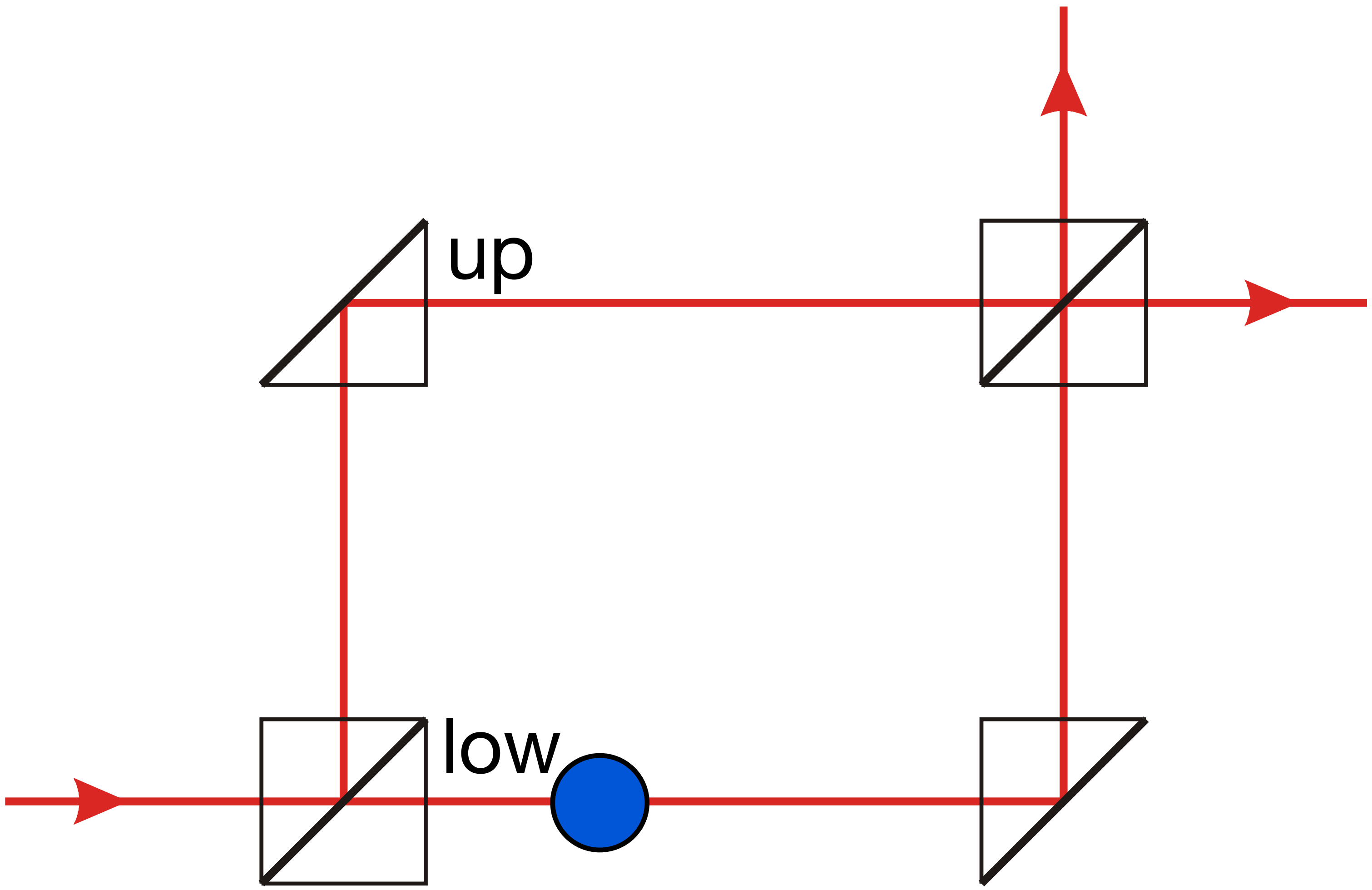


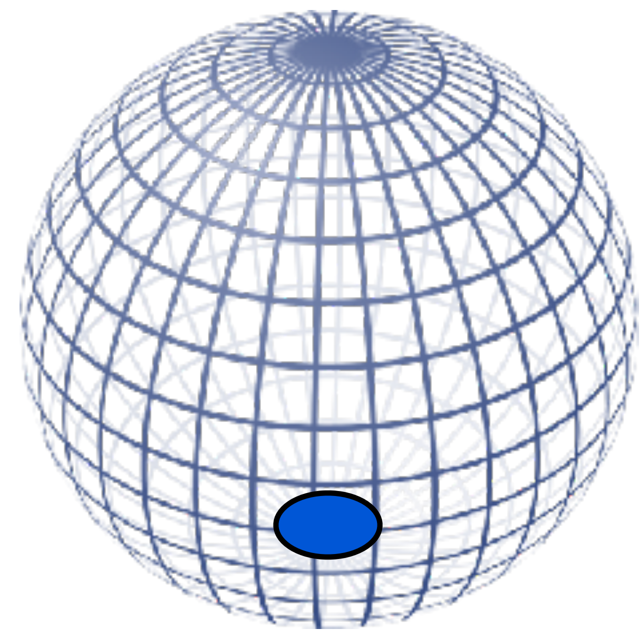

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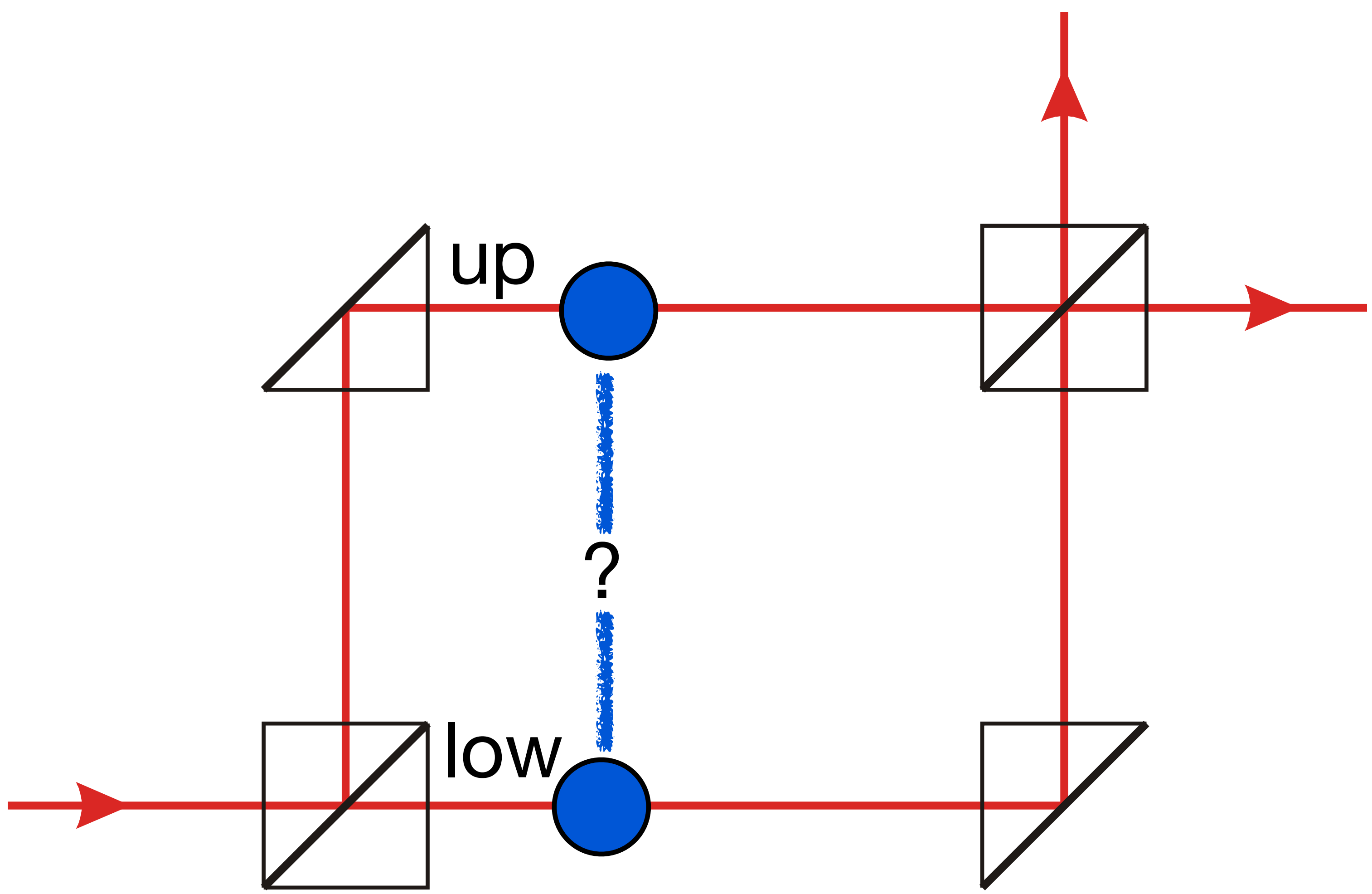


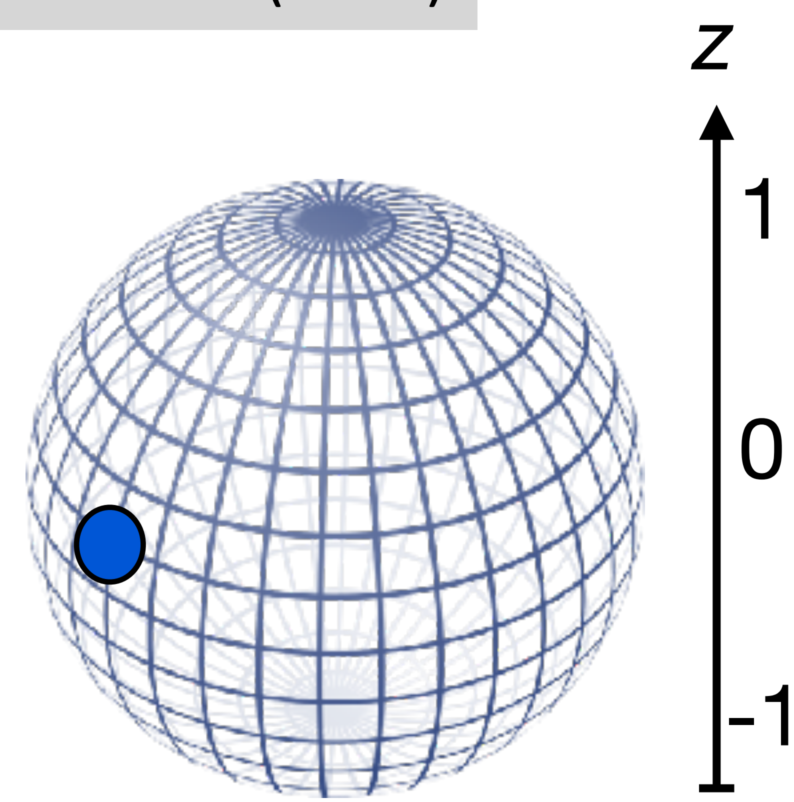


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State on equator *z=0*: probability 1/2 for each.

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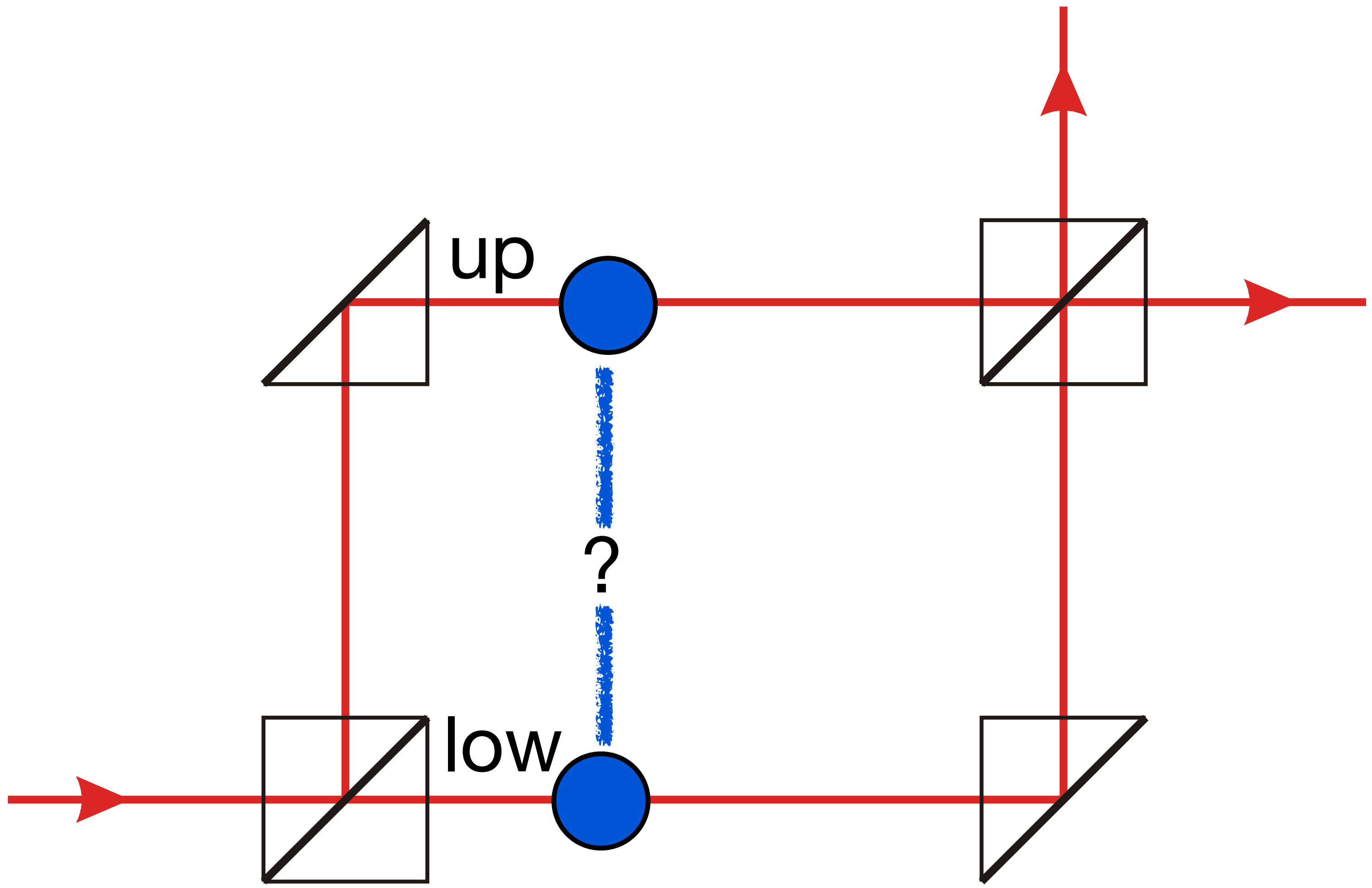


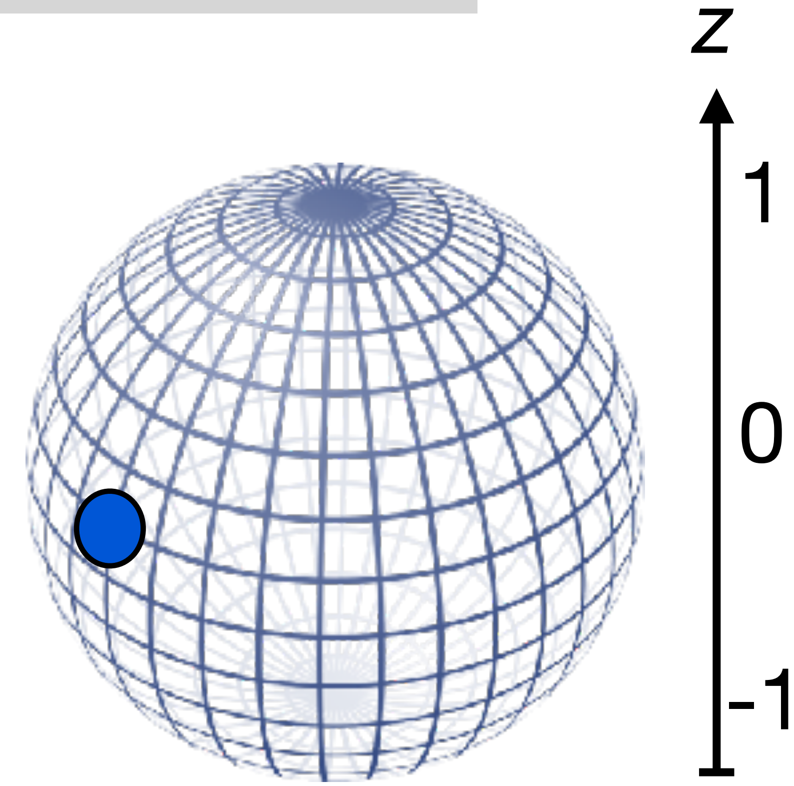


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$$p(\mathrm{up}) = \frac{1}{2}(z+1)$$

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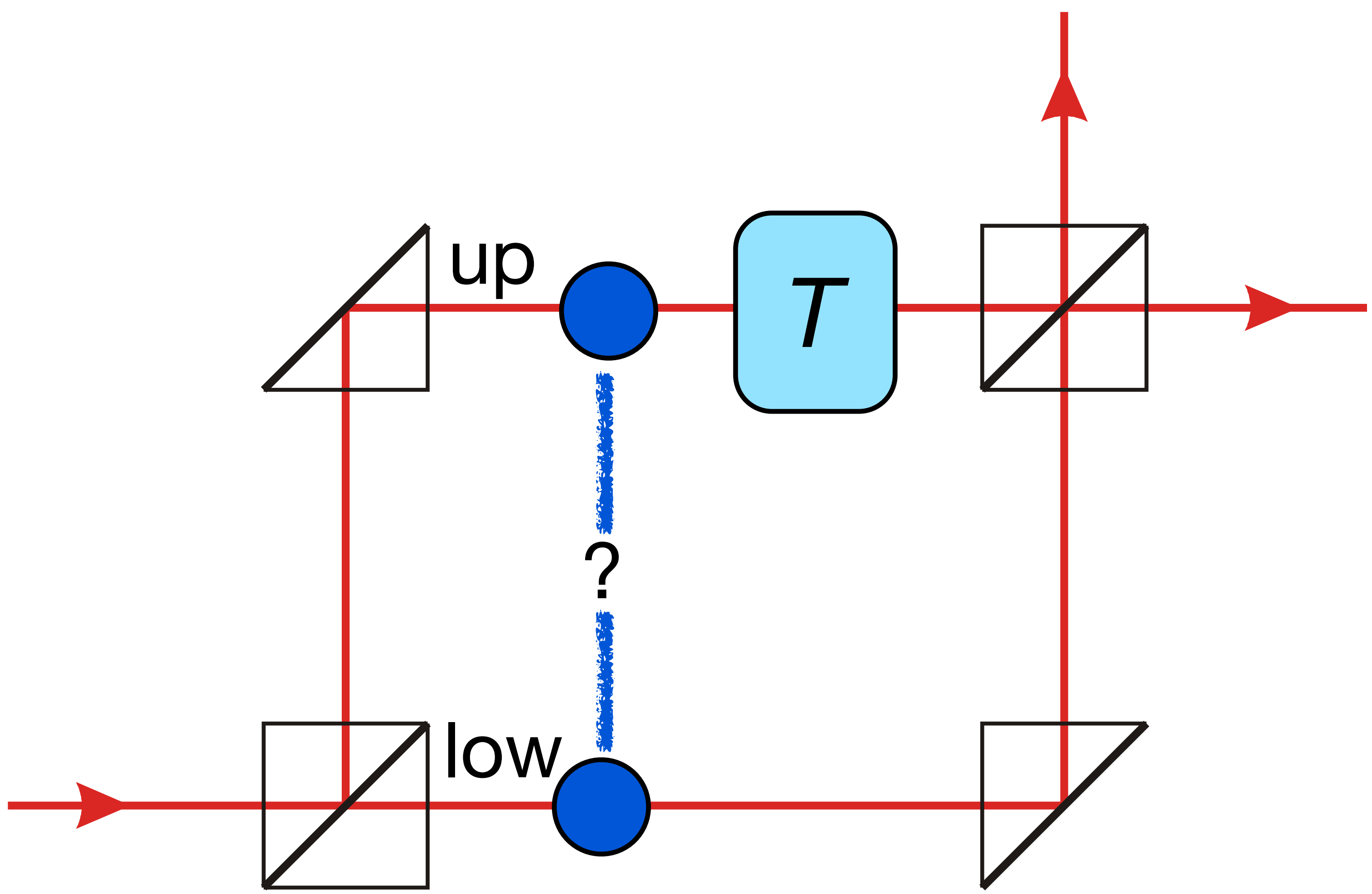


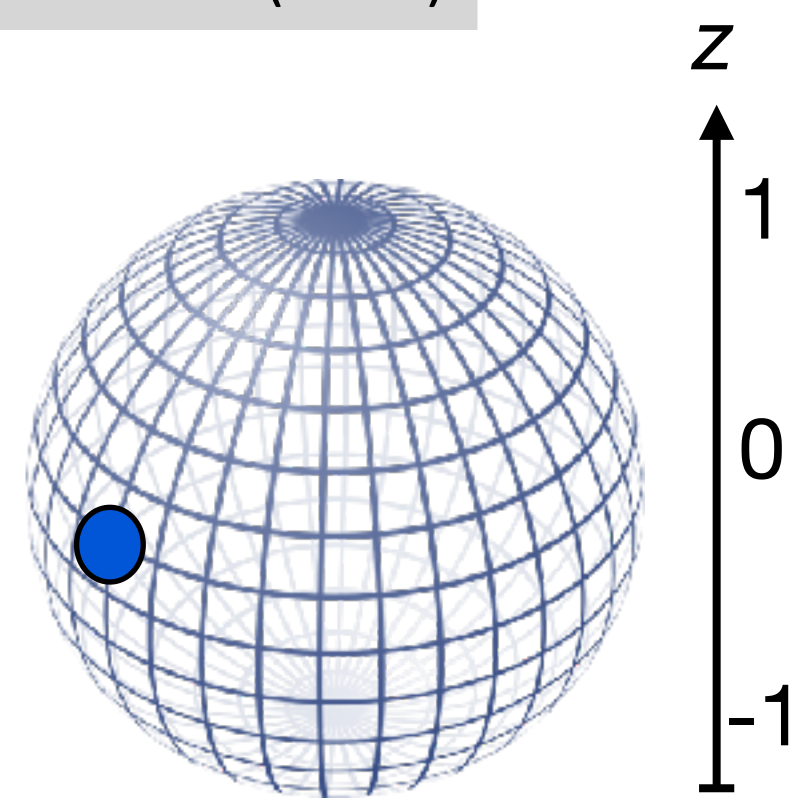


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What transformations *T* can we perform locally in one arm...
... without any information loss?

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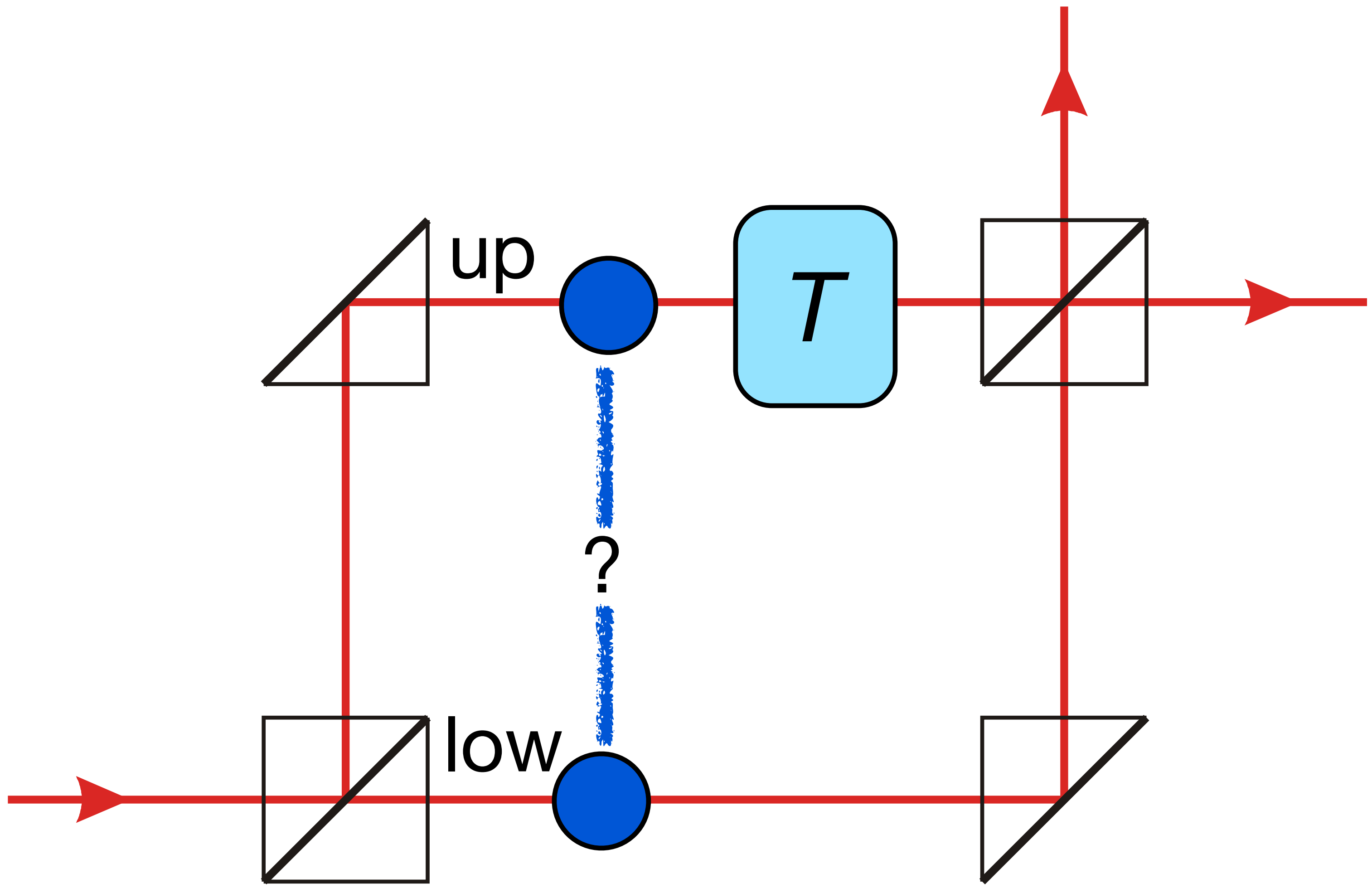


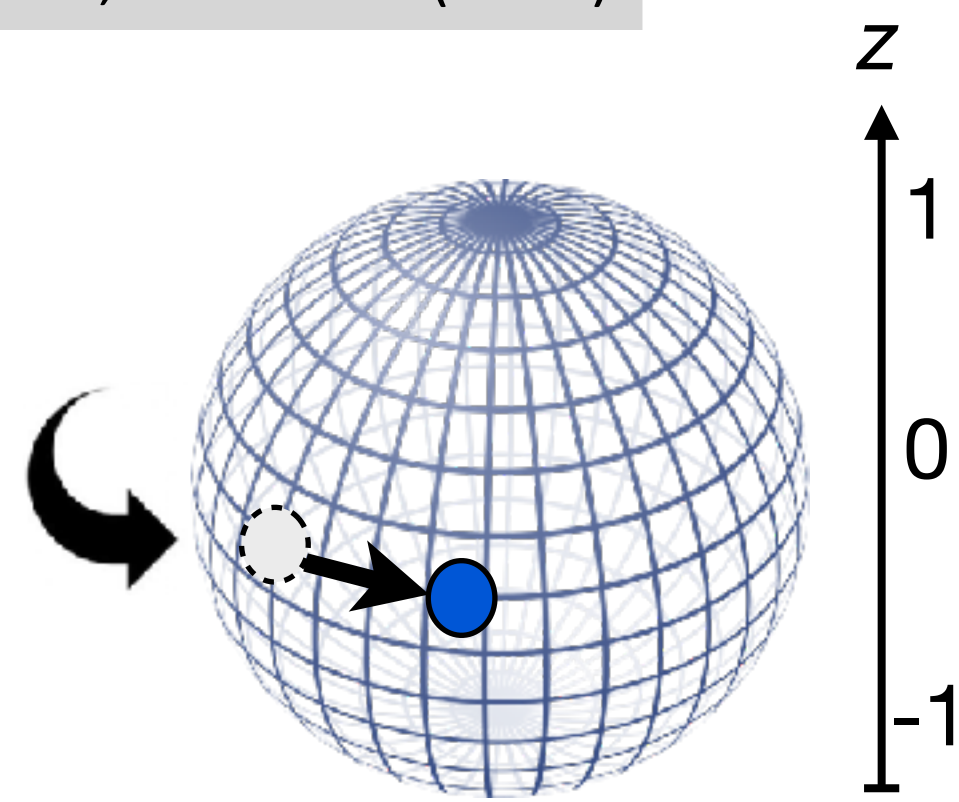


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*T* must be a rotation of the Bloch ball (reversible+linear)...
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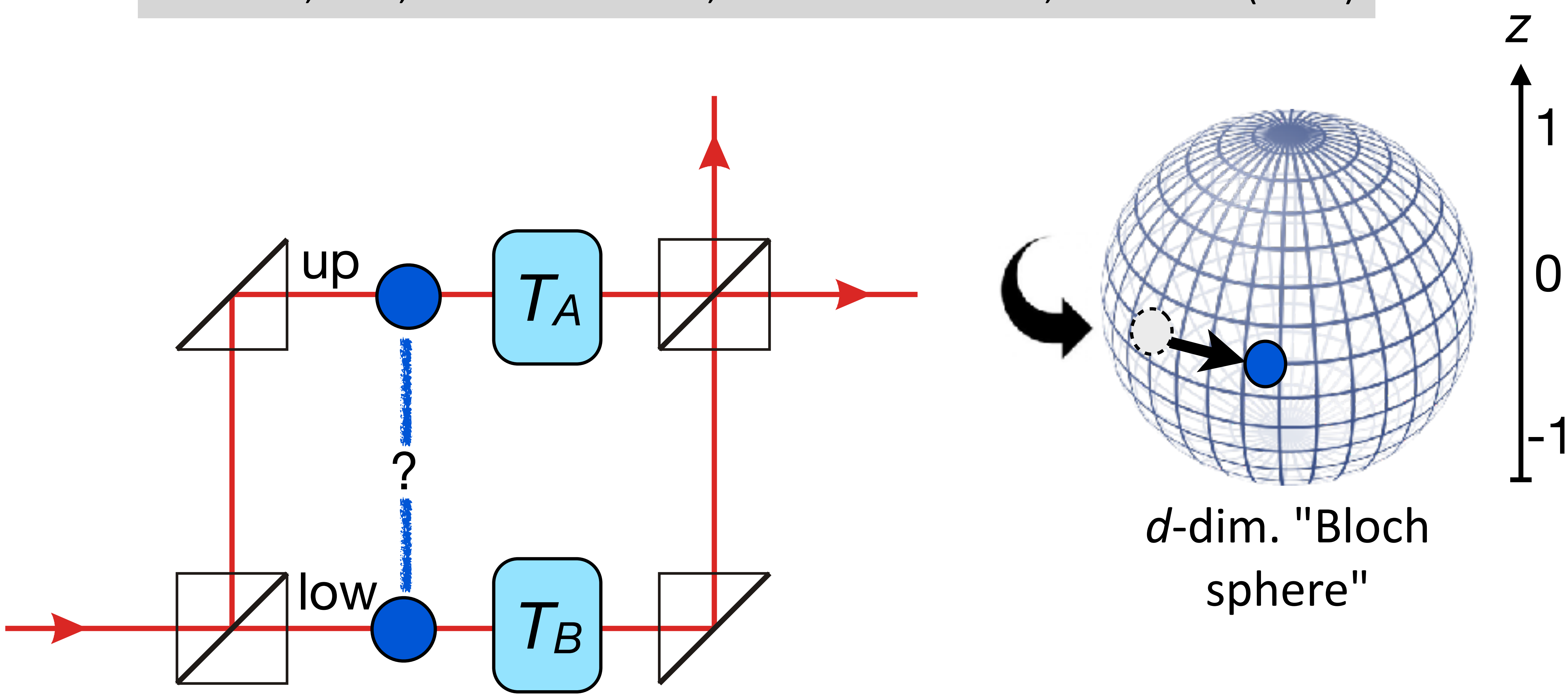


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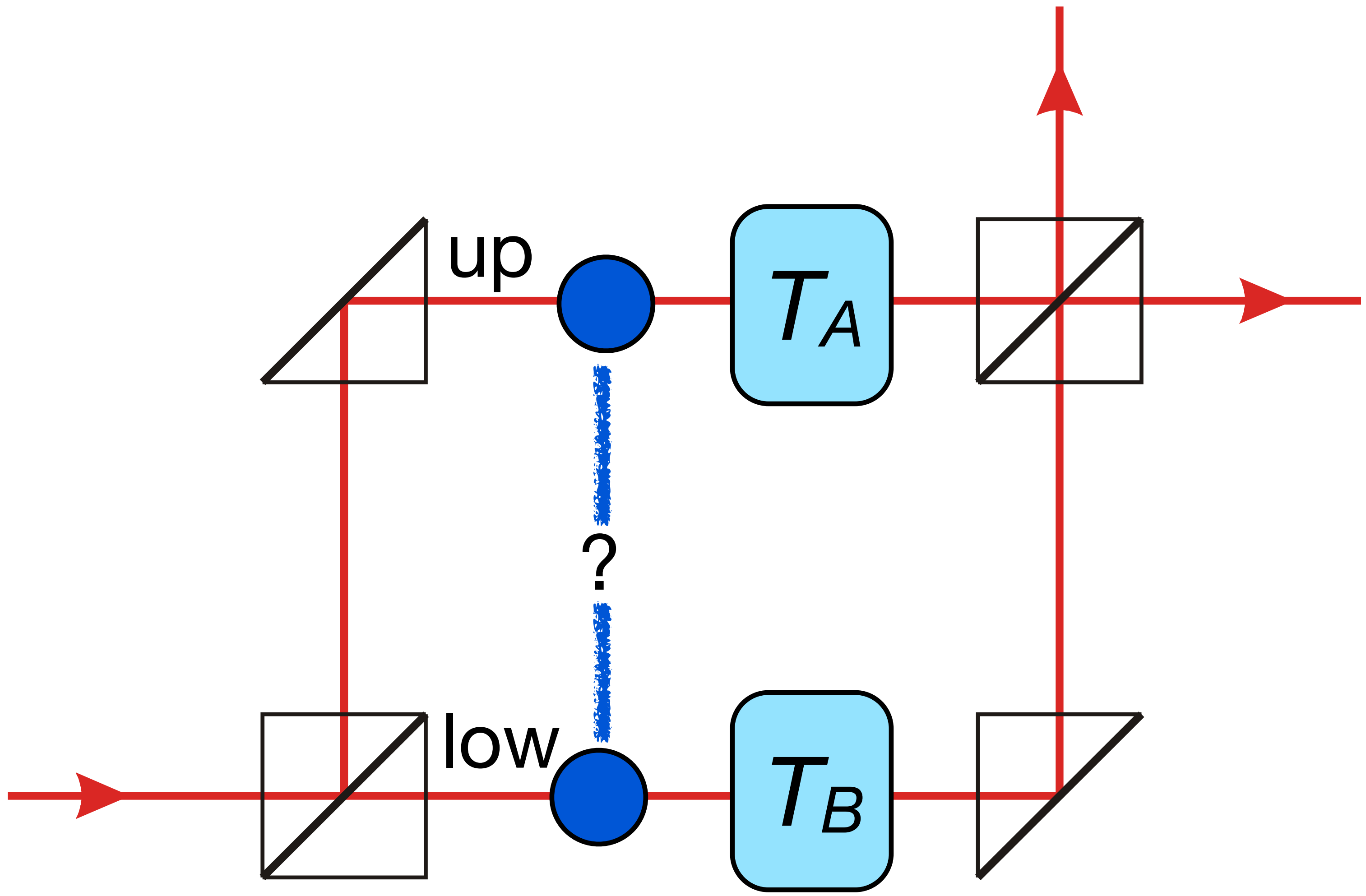


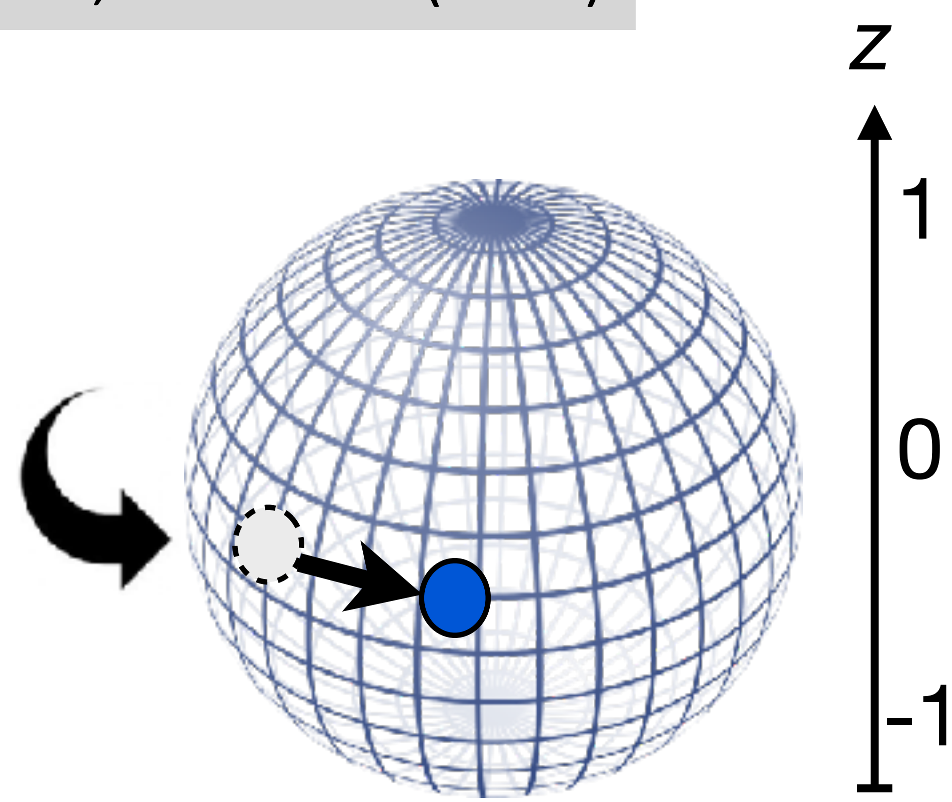


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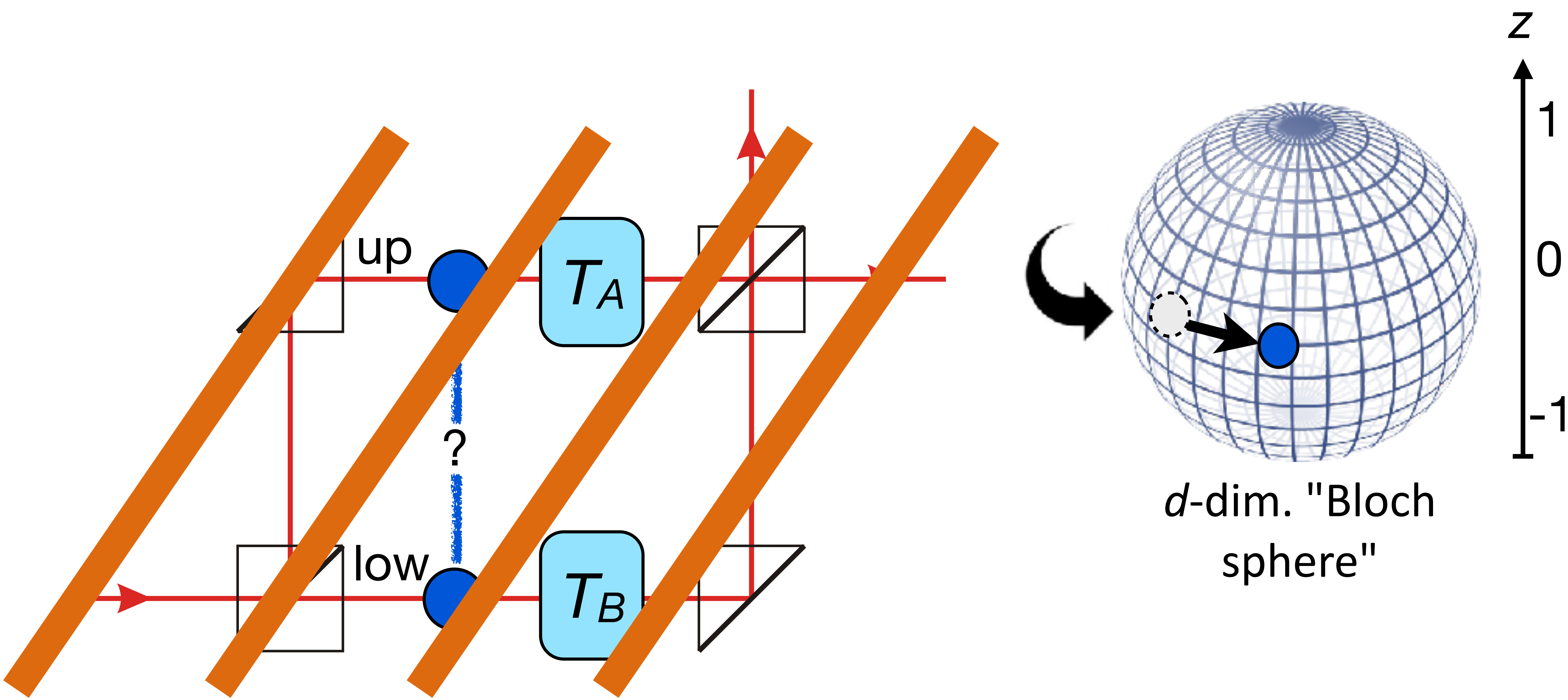


Relativity: there's a frame of reference in which $T_A$ happens before $T_B$...

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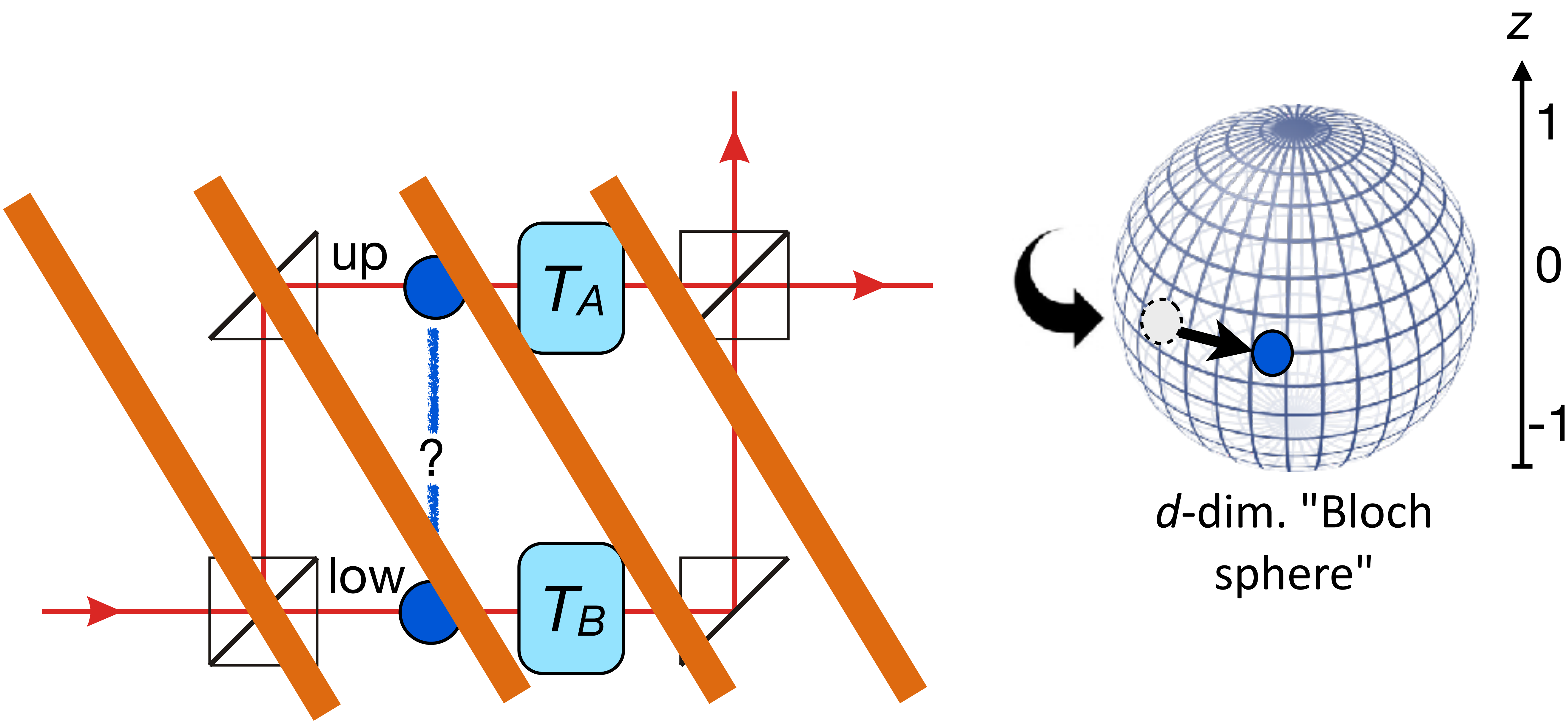


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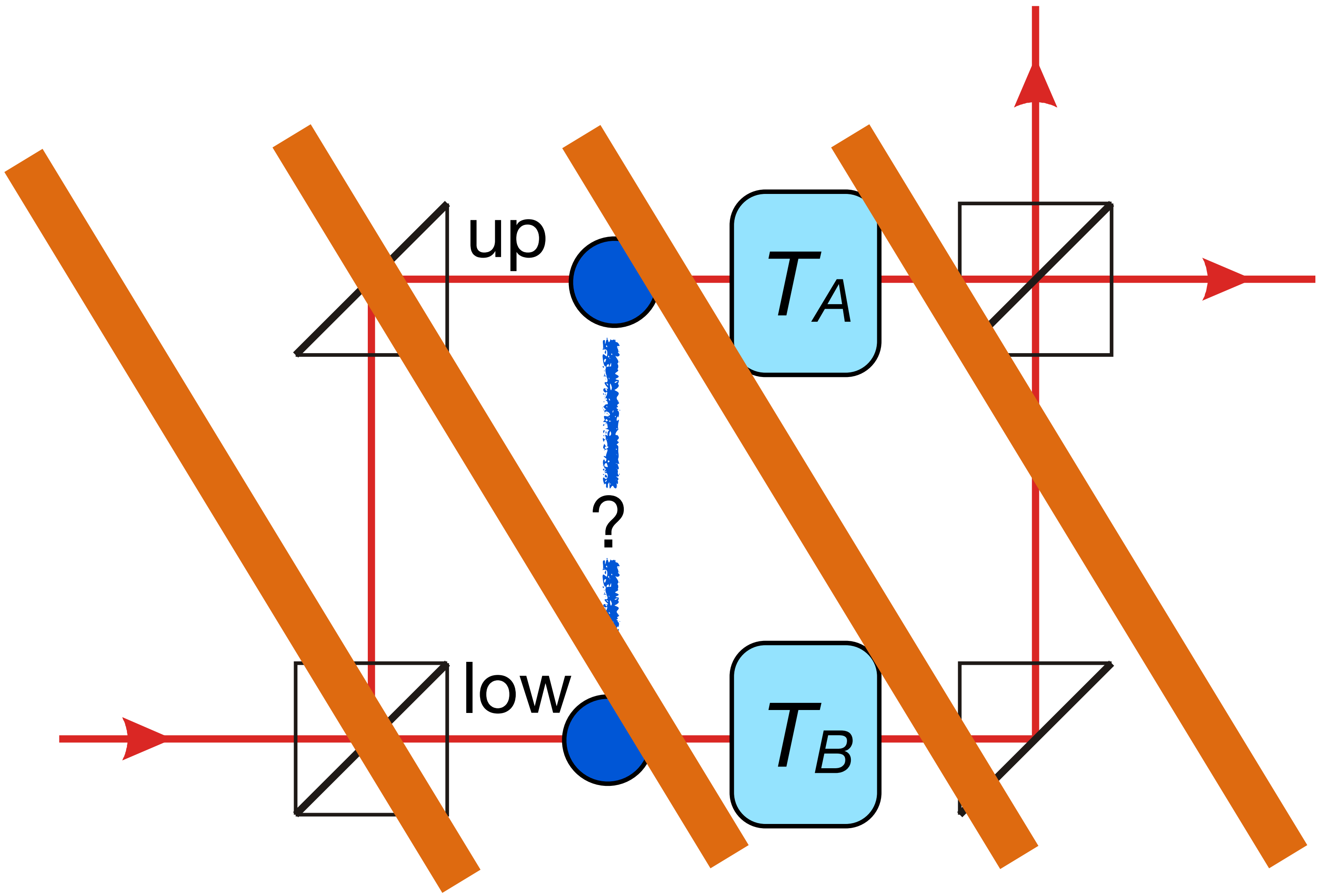


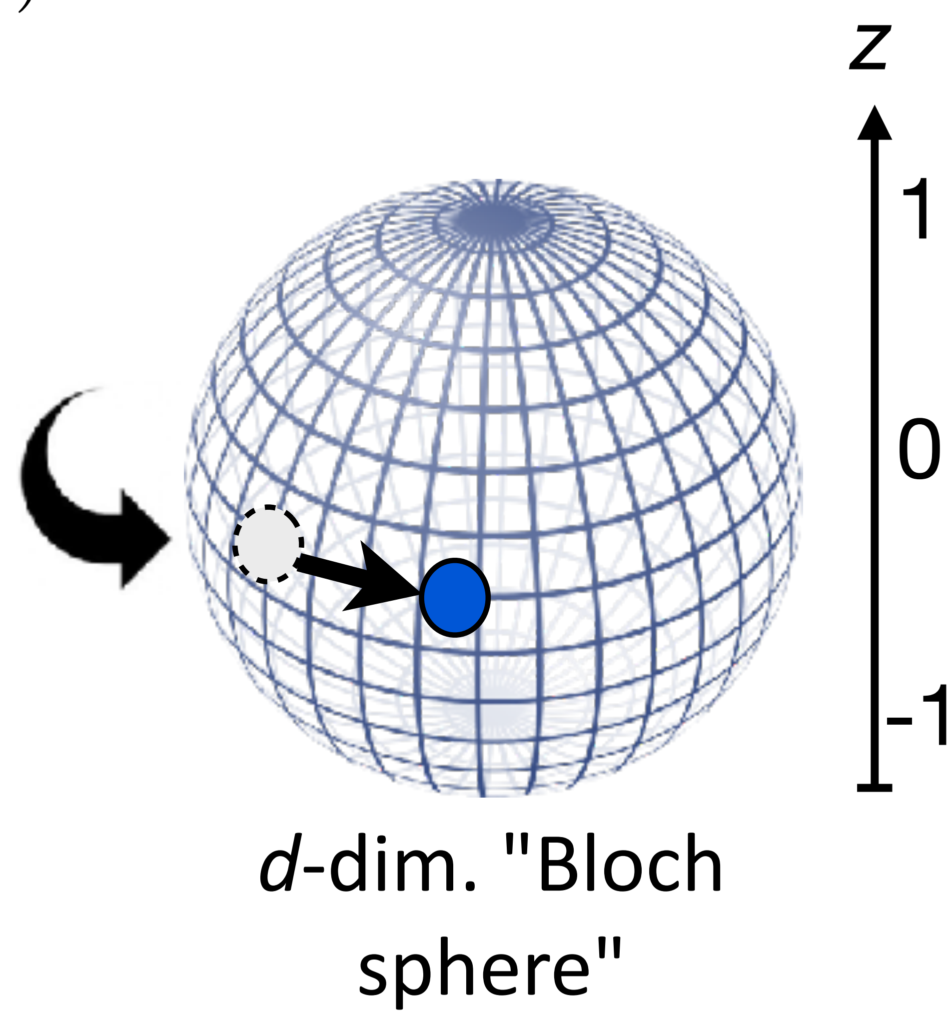


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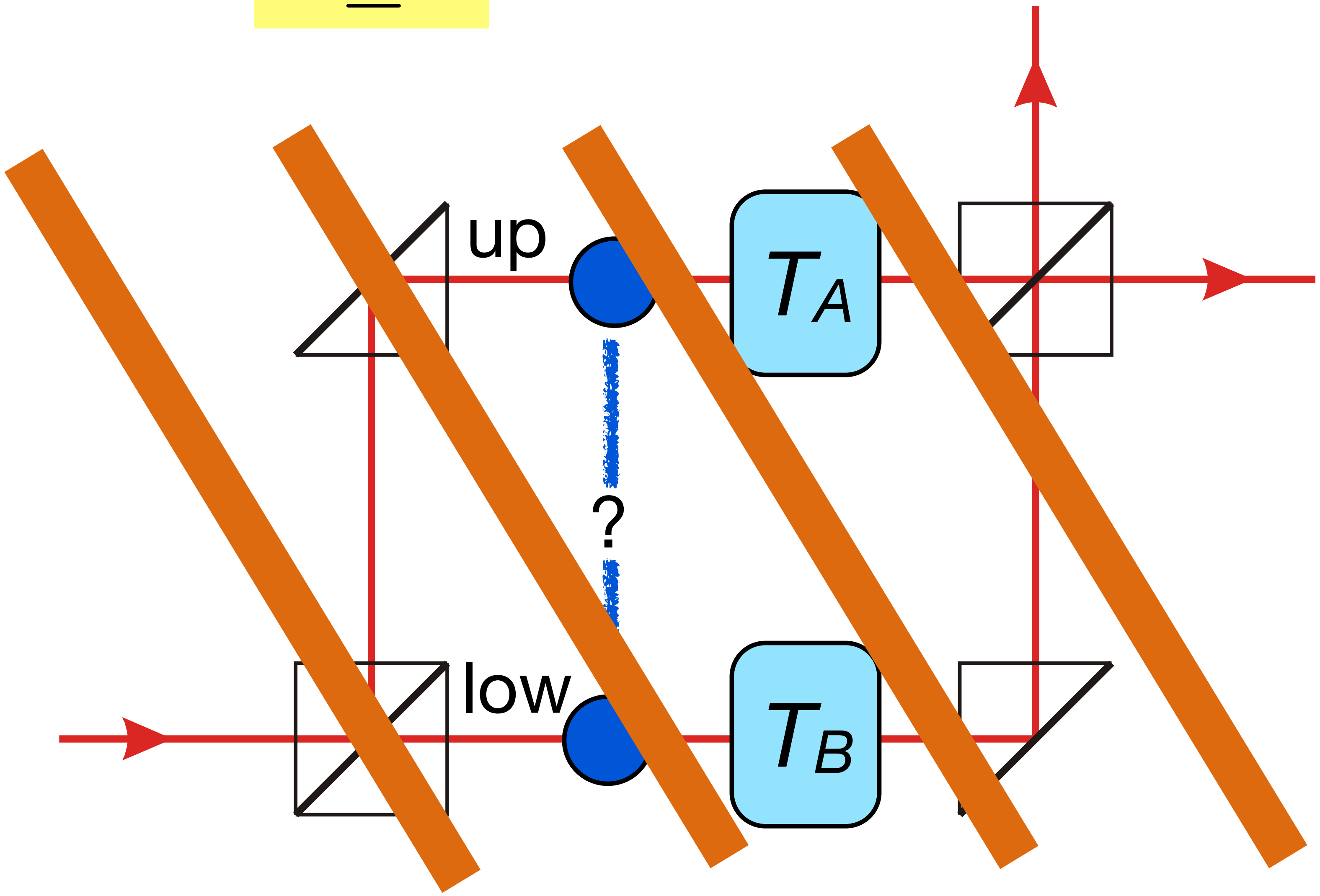


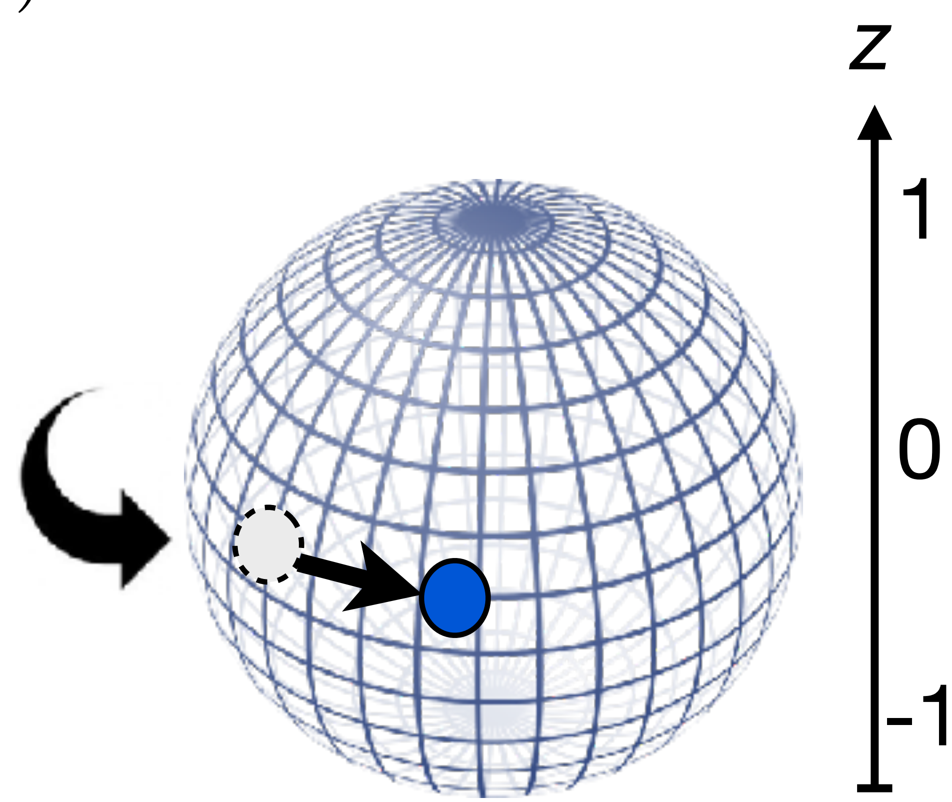


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Information theory

$\Updownarrow$

spacetime

?!

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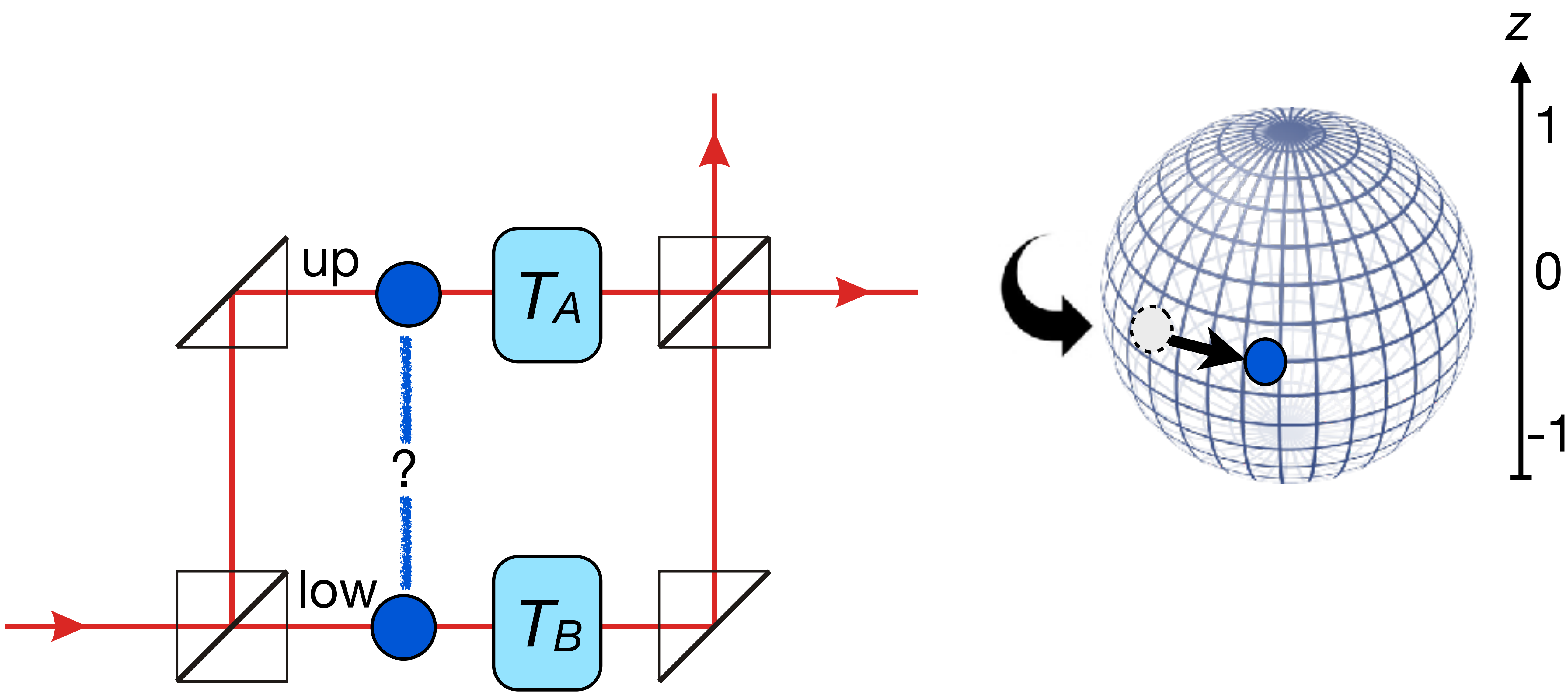


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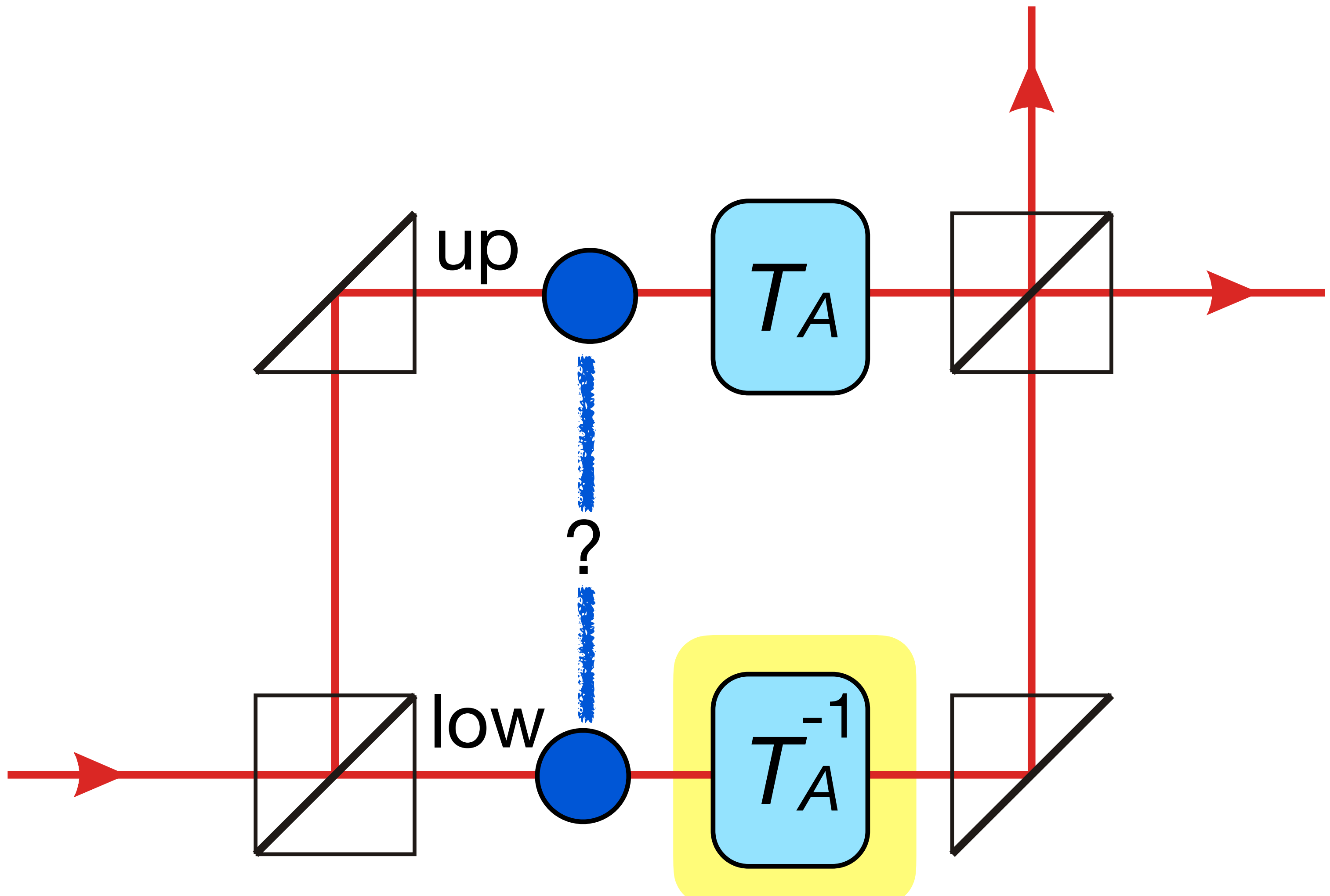


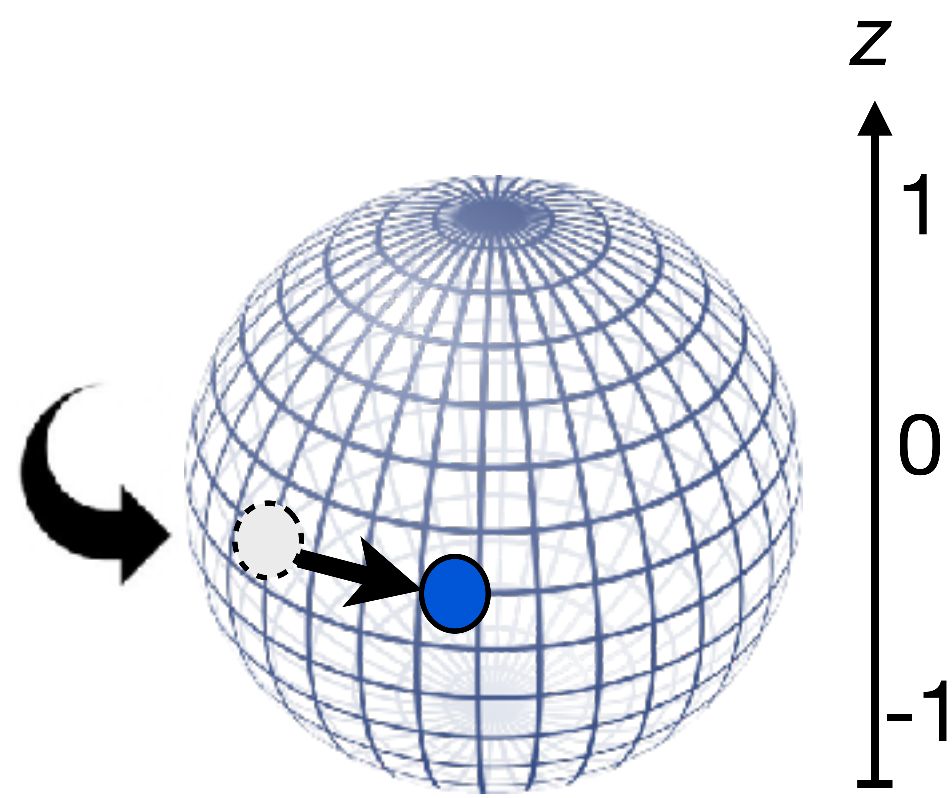


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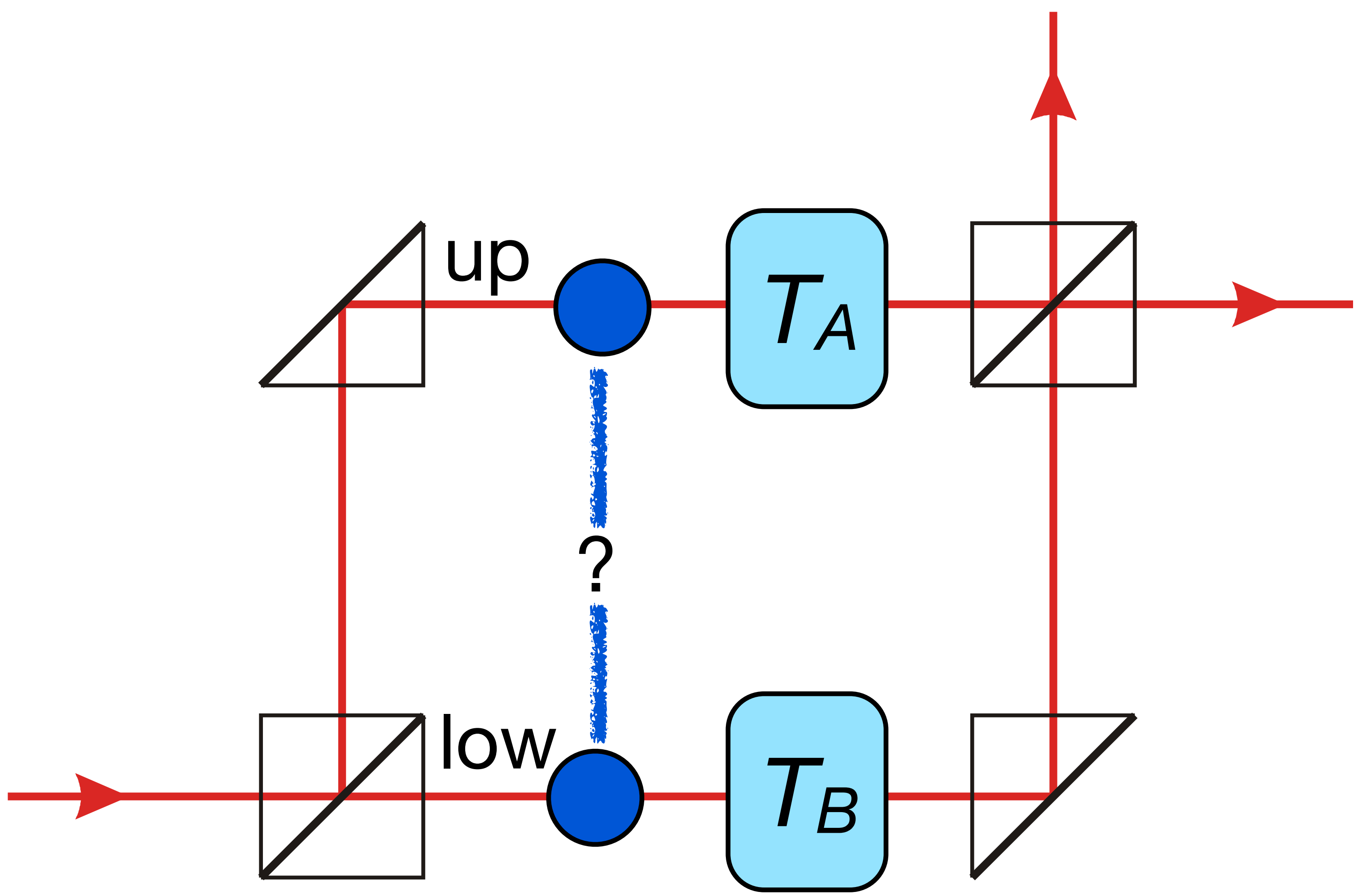


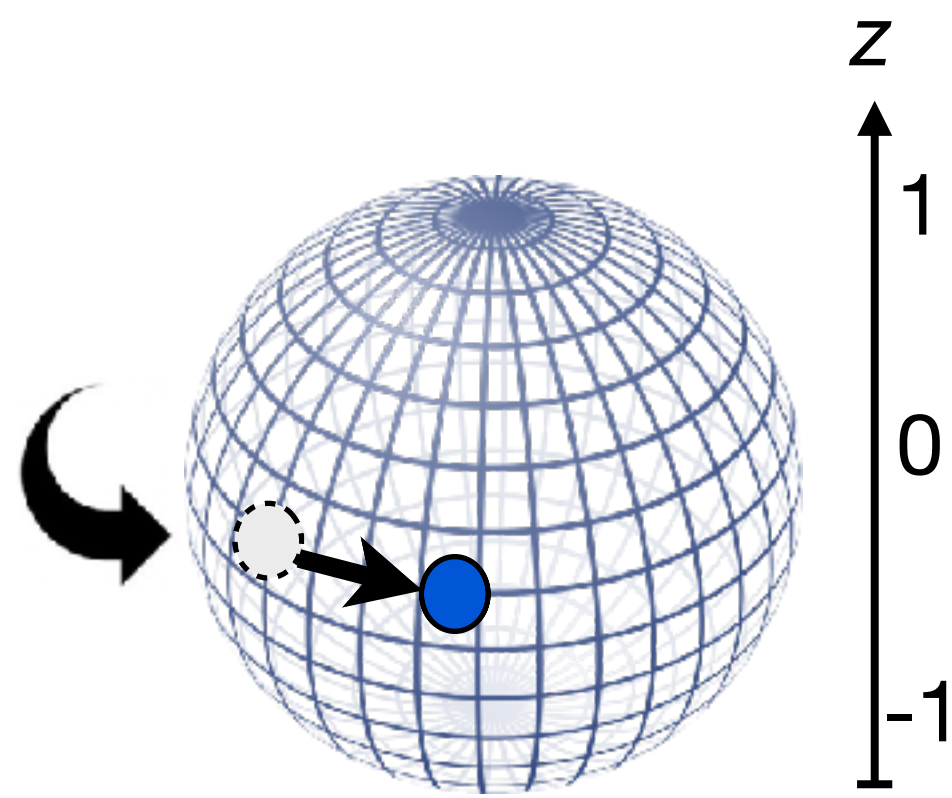


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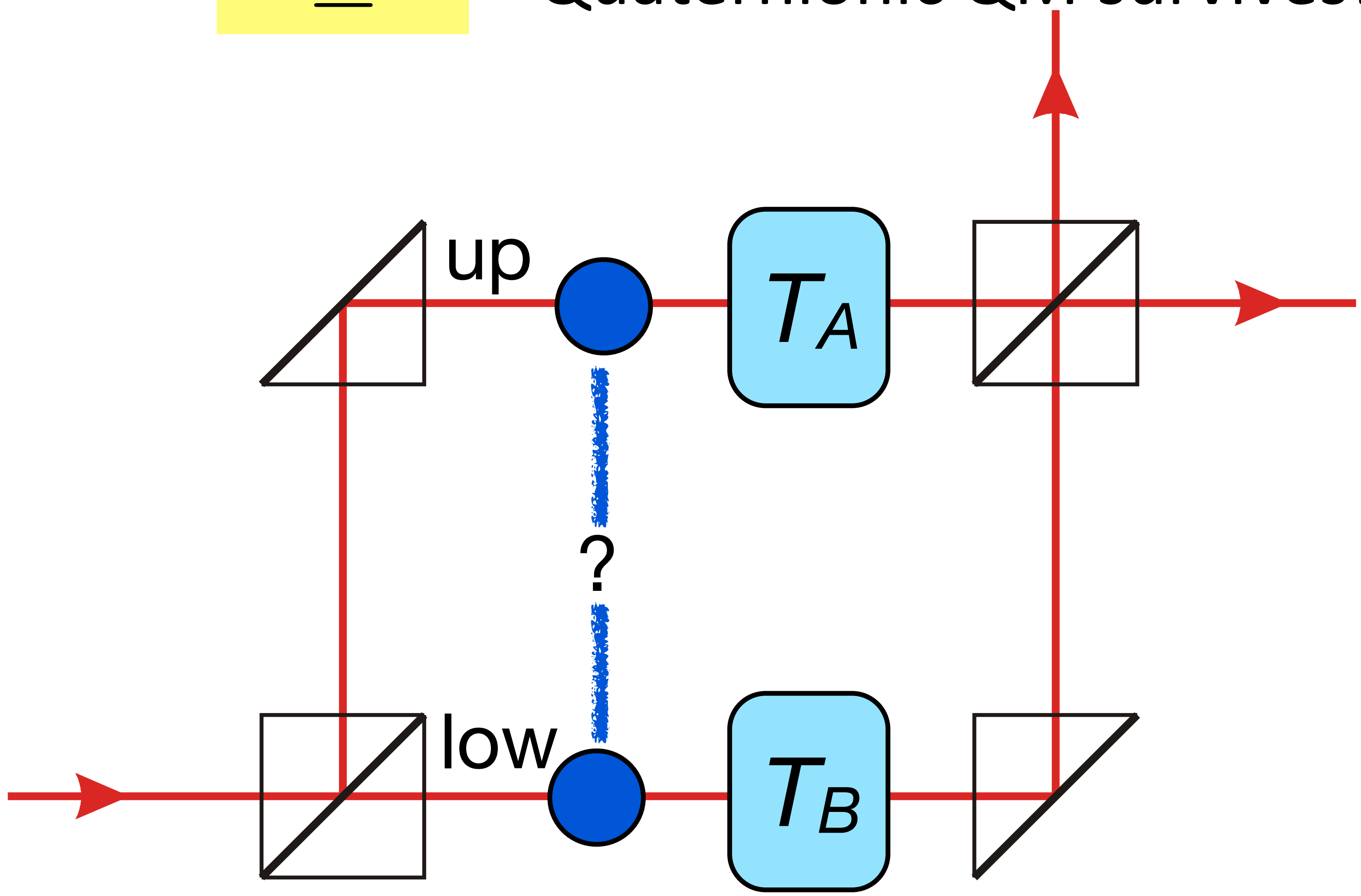
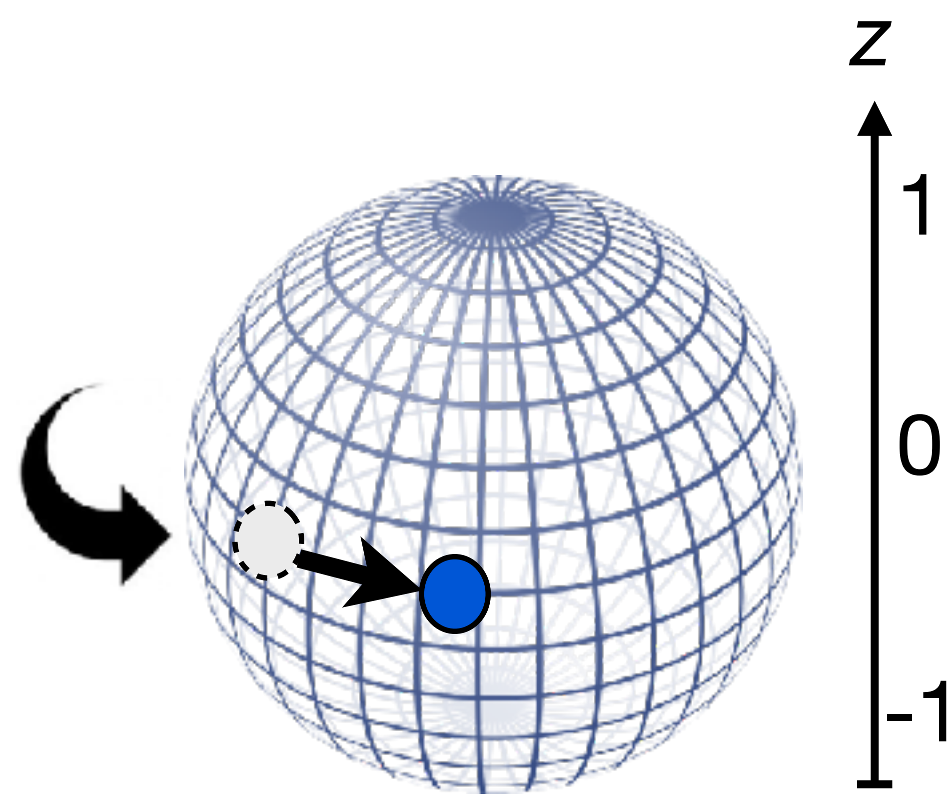
$\Rightarrow$ $d \leq 5$. Quaternionic QM survives!

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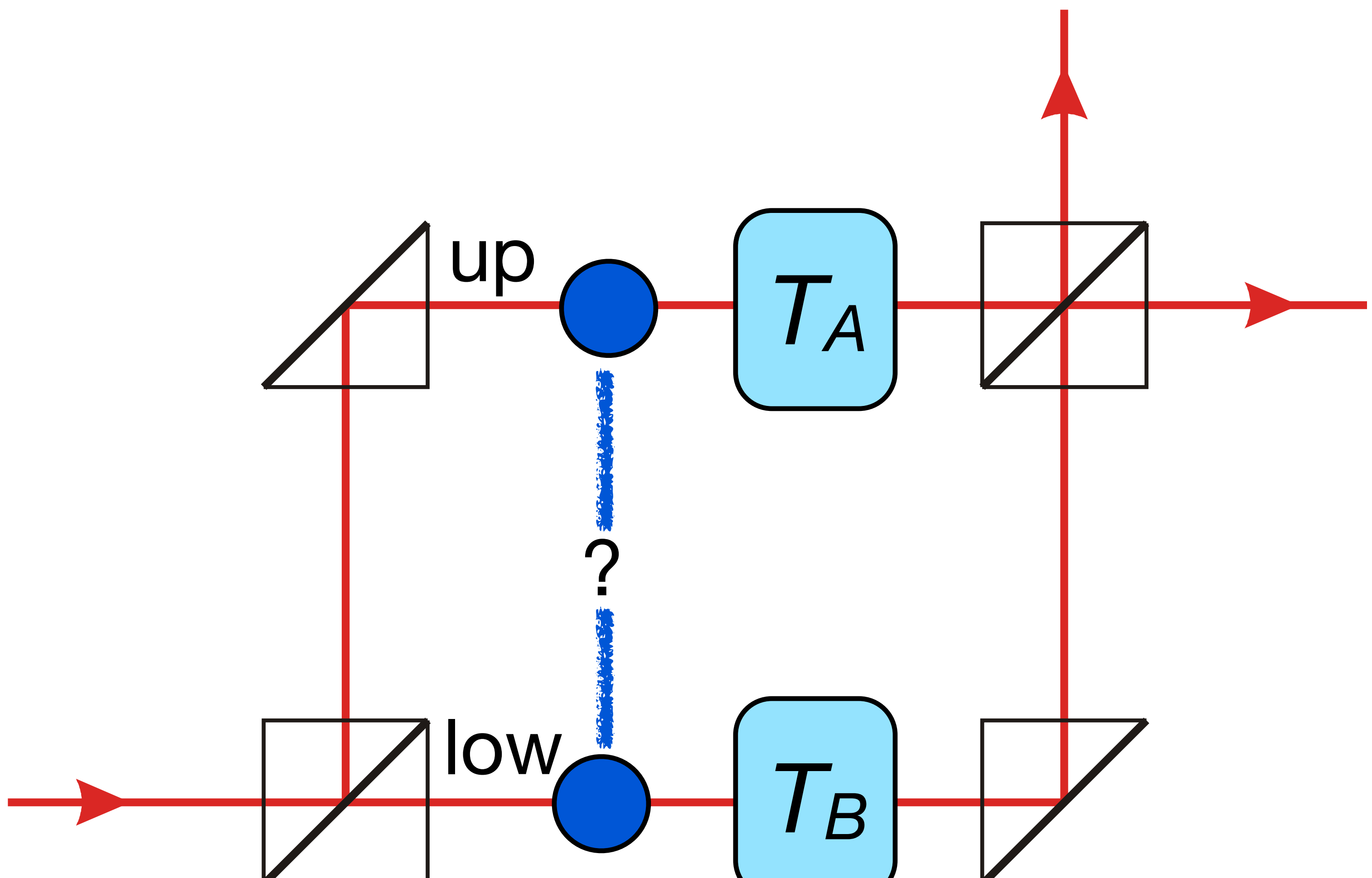


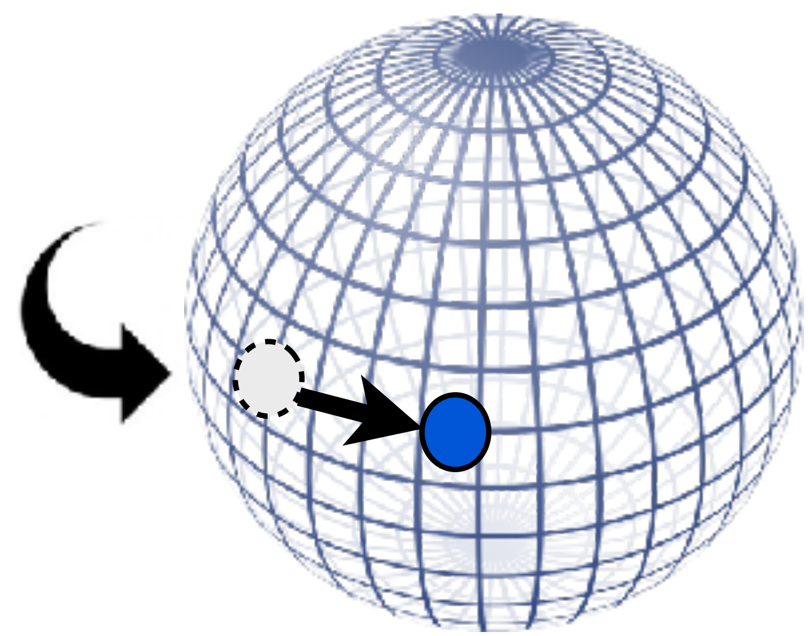

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**Theorem 6.2.** *Under the assumptions* A1, A2, A3, *relativity of simultaneity* (*REL*) *allows for the following possibilities and not more*:

— $d=1$ (*the classical bit*), *with* $\mathcal{G}_{\mathrm{A}}=\mathcal{G}_{\mathrm{B}}=\{\mathbf{1}\}$ (*i.e. without any non-trivial local transformations*),
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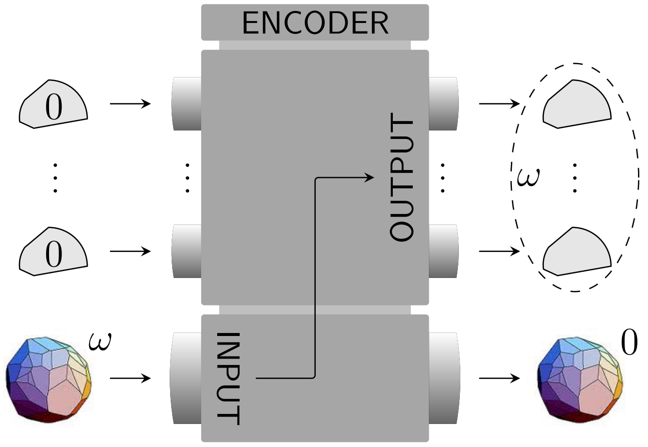
— $d = 5$ (*the quaternionic quantum bit*), *with* $\mathcal{G}_{\mathrm{AB}} = \mathrm{SO}(4)$, $\mathcal{G}_{\mathrm{A}}$ *the left- and* $\mathcal{G}_{\mathrm{B}}$ *the right-isoclinic rotations in* $\mathrm{SO}(4)$ (*or vice versa*) *which are both isomorphic to* $\mathrm{SU}(2)$, *and* $\mathcal{G}_{\mathrm{A}} \cap \mathcal{G}_{\mathrm{B}} = \{+\mathbb{I}, -\mathbb{I}\}$.

Relativity constrains the state space to $d = 1, 2, 3, 5$ !

# Overview

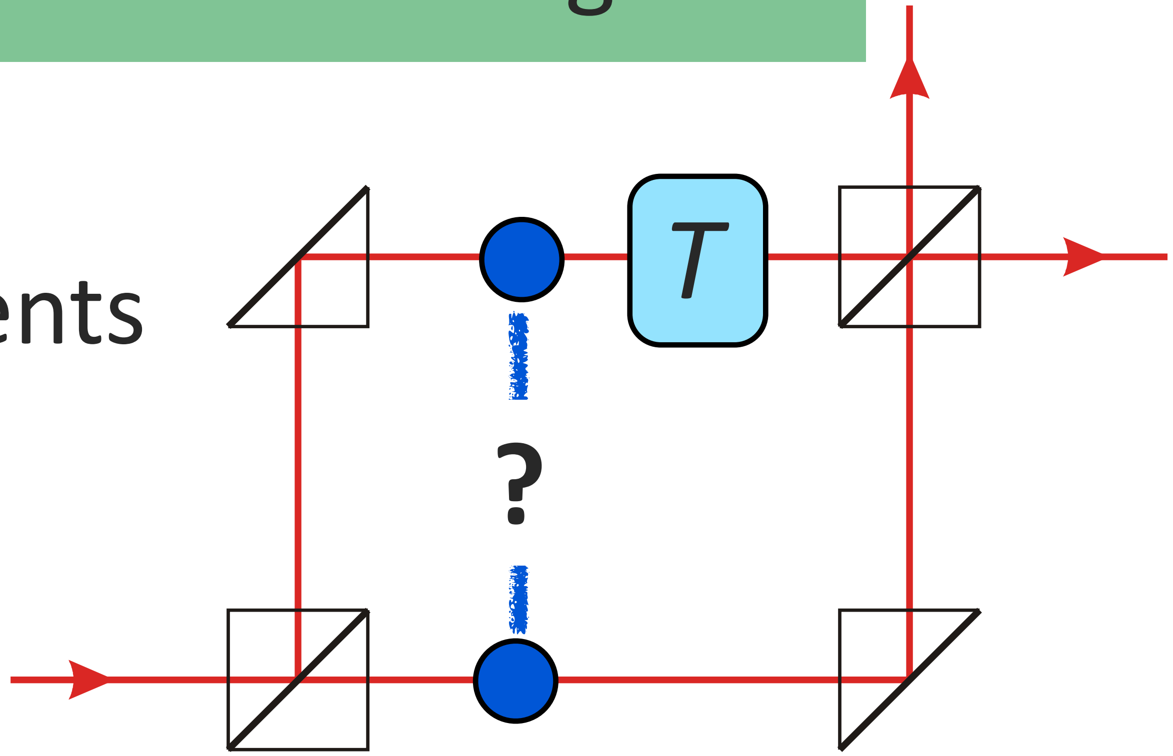
1. Boxes and theories beyond quantum theory

2. Quantum theory from simple principles




3. Spacetime and QT: from foundational insights…

4. … to protocols and experiments



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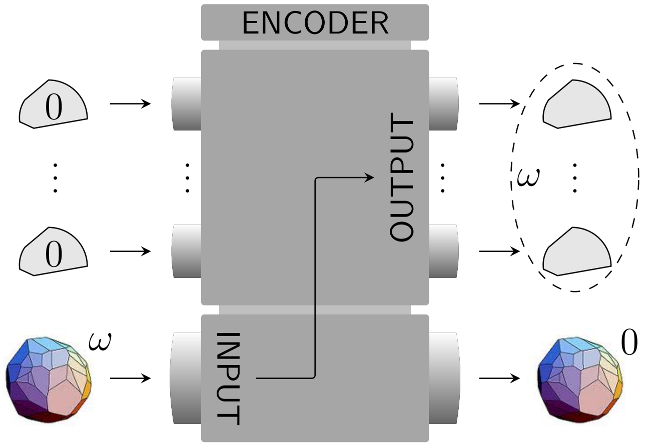


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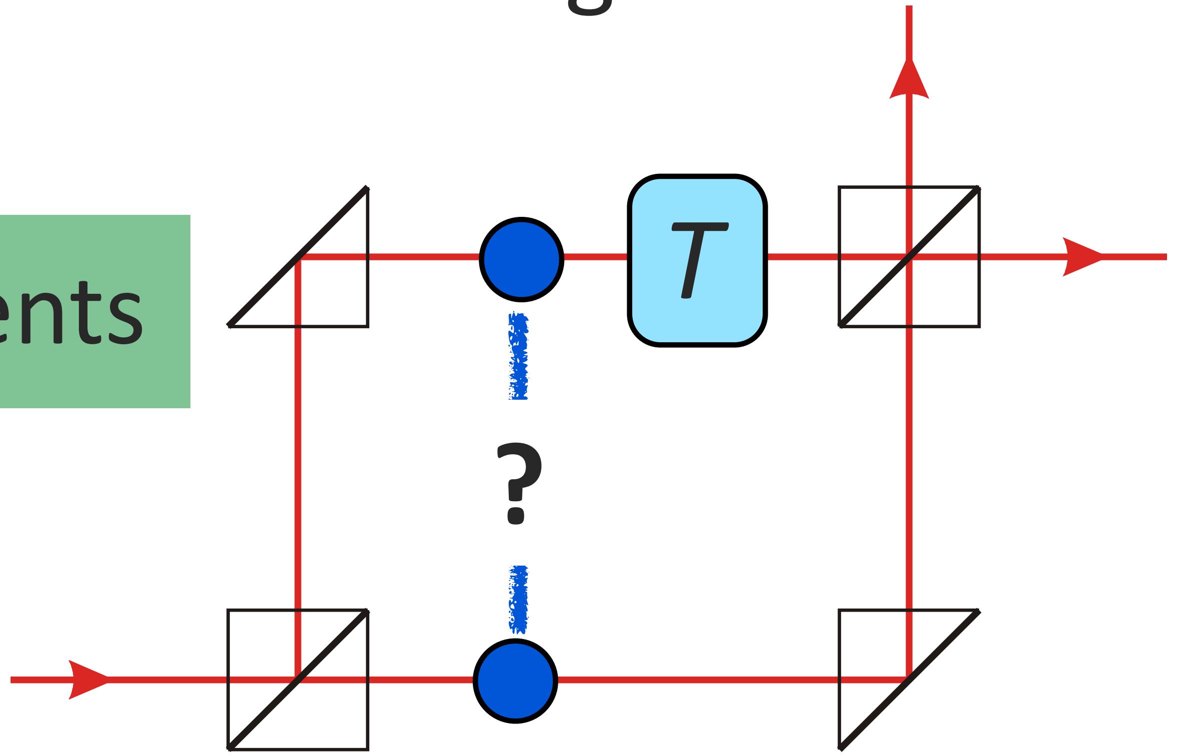

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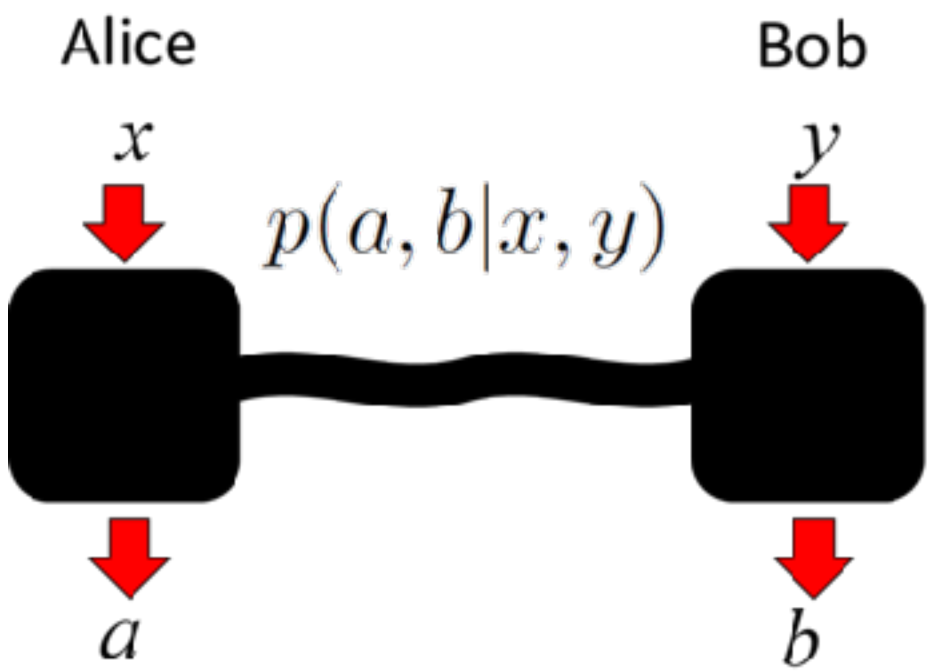

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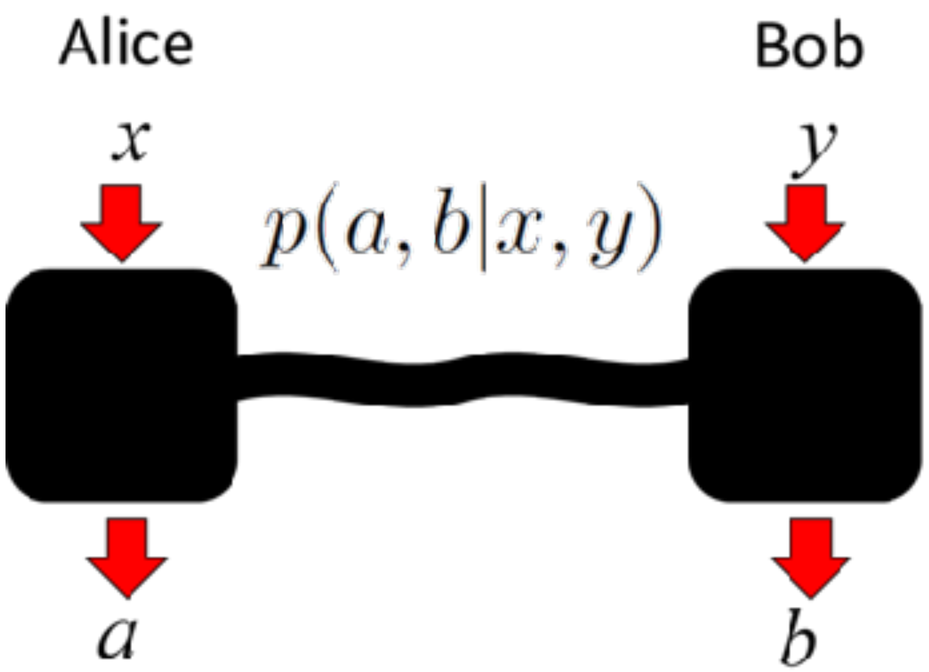


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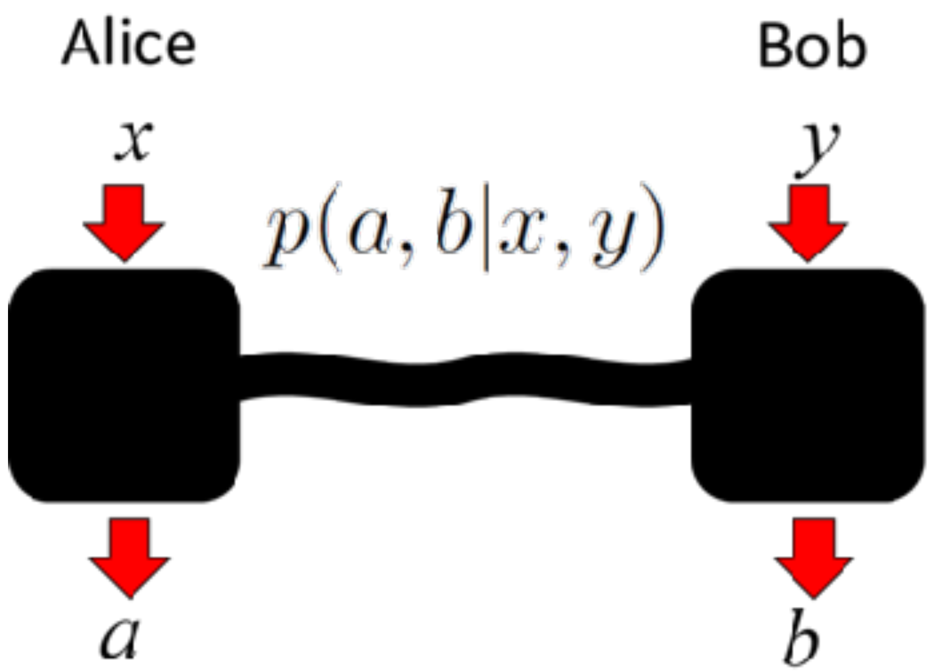


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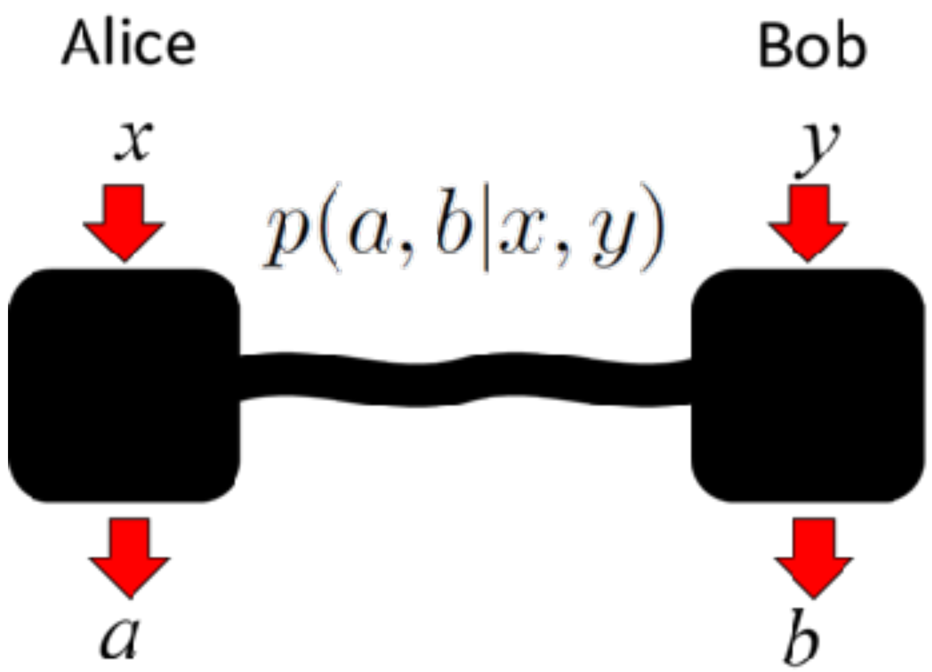


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A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

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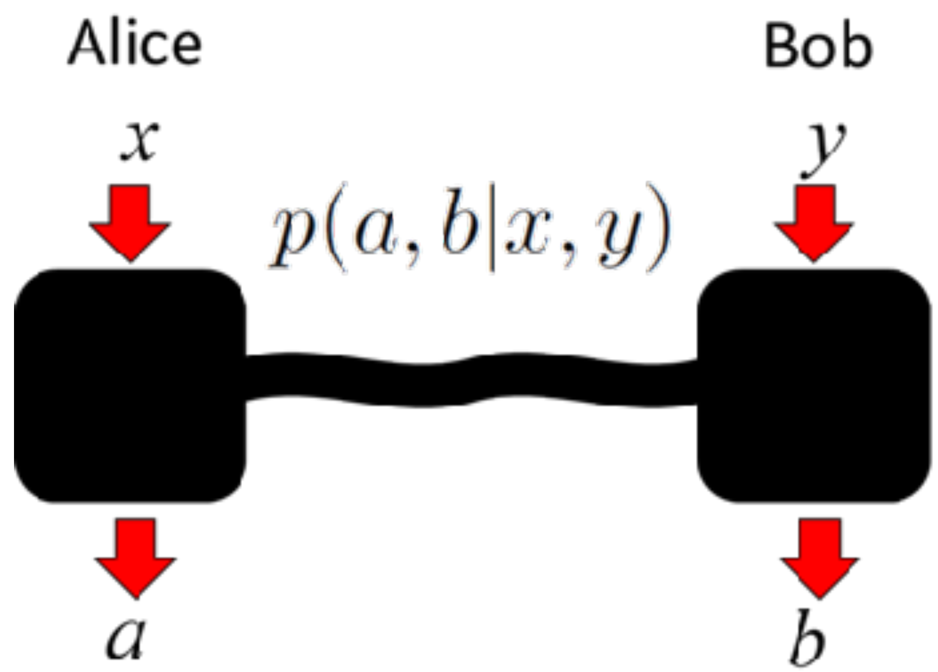


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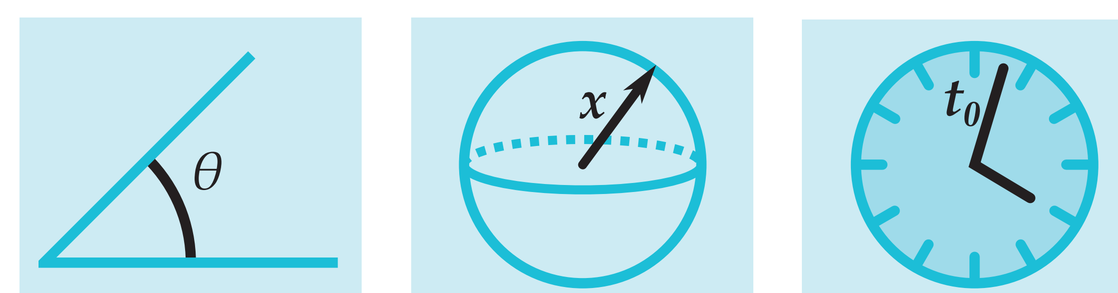
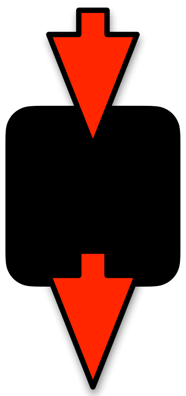



ANGLES DIRECTIONS DURATIONS

# Spacetime boxes

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

$x \in \{0, 1\}$



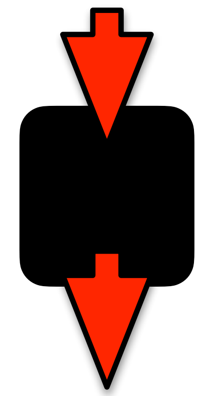
$P(a|x)$

$a \in \{-1, +1\}$

$\vec{x} \in \mathcal{G}/\mathcal{H}$

$\mathcal{G}$ : symmetry group

$\mathcal{H}$ : stabilizer subgroup



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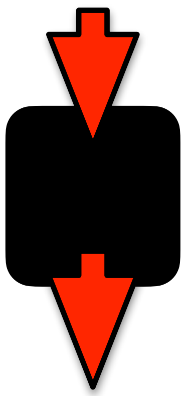

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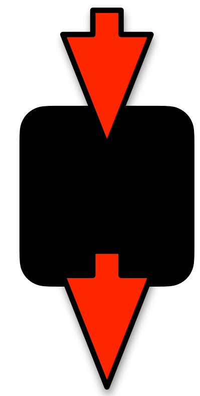

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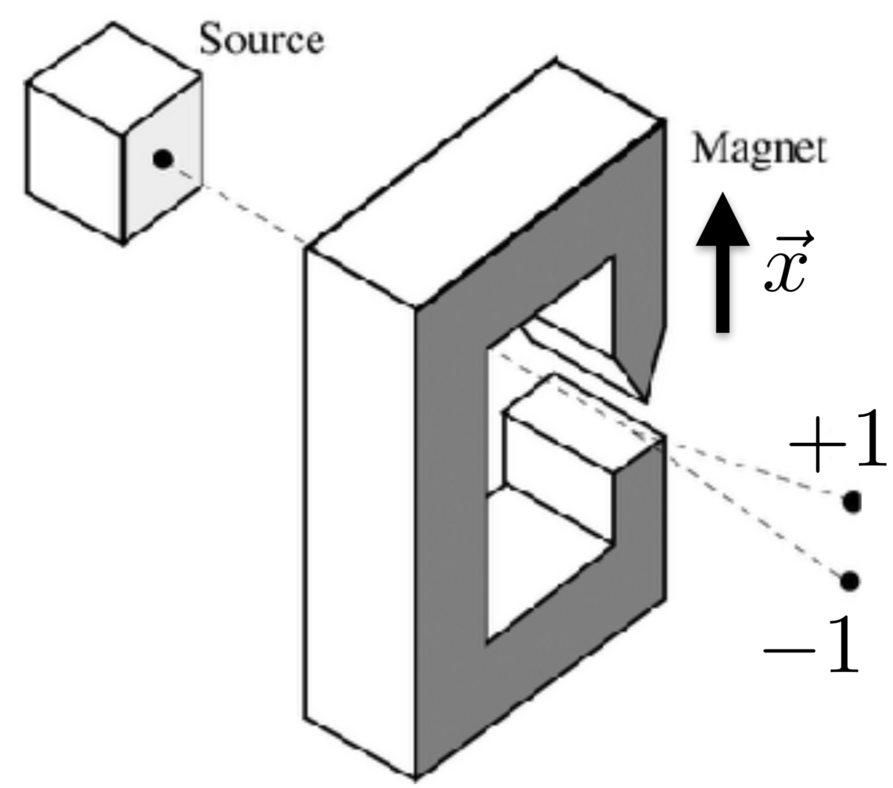
**Example:** Stern-Gerlach experiment

$\mathcal{G} = \mathrm{SO}(3)$ (spatial rotations)

$\mathcal{H} = \mathrm{SO}(2)$ (axial symmetry of magnetic field)

$\vec{x} \in \mathcal{G}/\mathcal{H} = S^2$ (unit vector: field direction)



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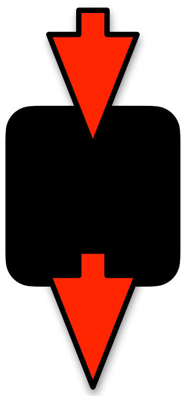

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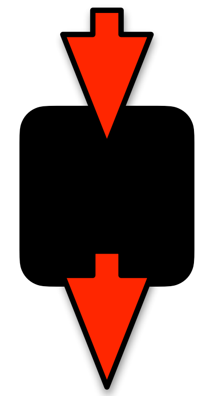

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**Example:** Input is evolution time *t*

$\mathcal{G} = (\mathbb{R}, +)$ (group of time translations)

$\mathcal{H} = \{\mathbf{1}\}$ (no additional symmetry)

$\vec{x} = t \in \mathbb{R}$

$P(a|t)$

# Symmetries constrain the correlations

A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

**Theorem.** Even *without assuming quantum mechanics*, $P(a|\vec{x})$ is a linear combination of matrix entries of a representation of $\mathcal{G}$.

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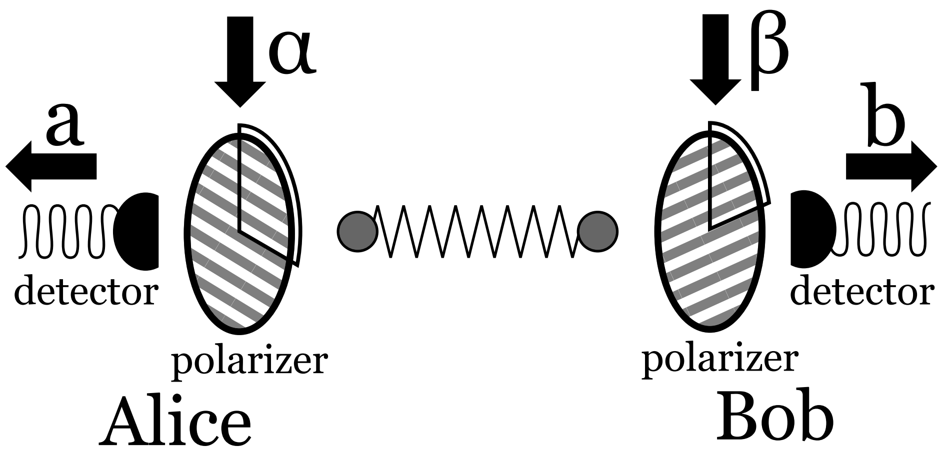


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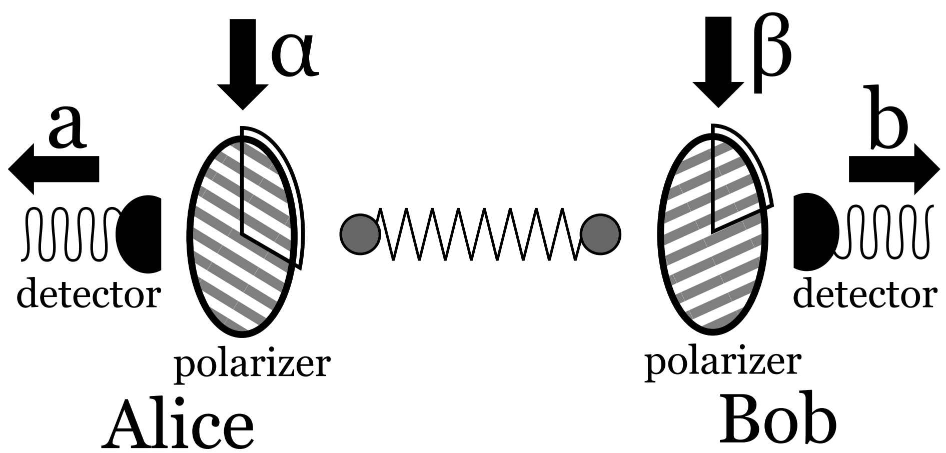


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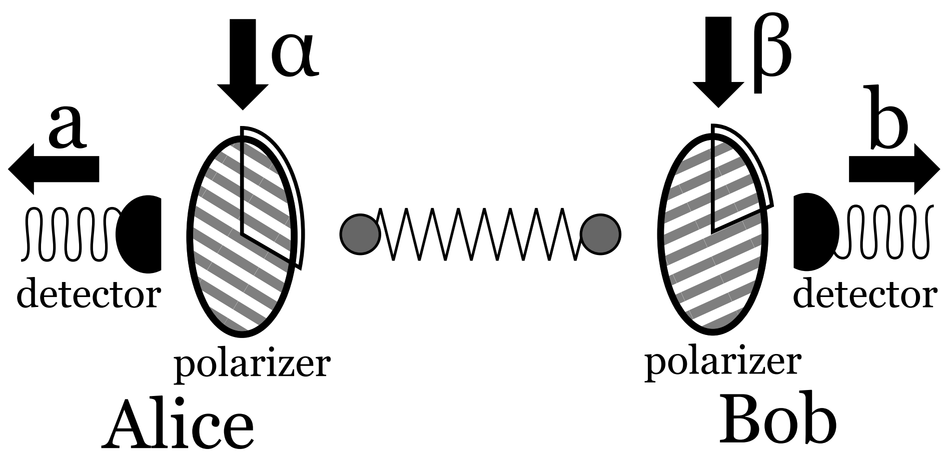


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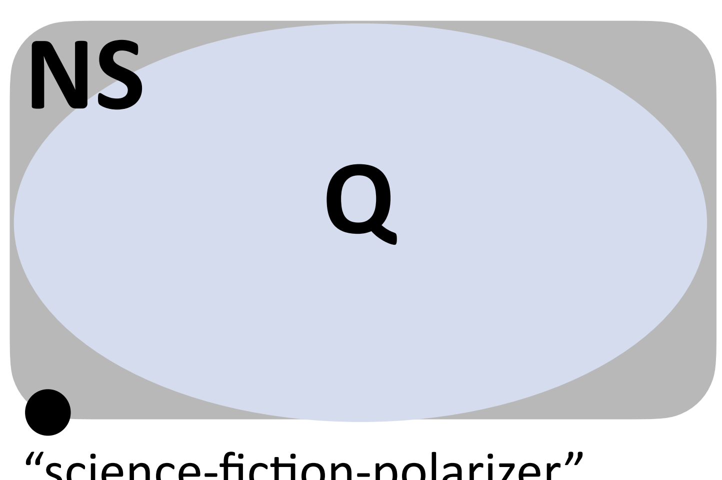

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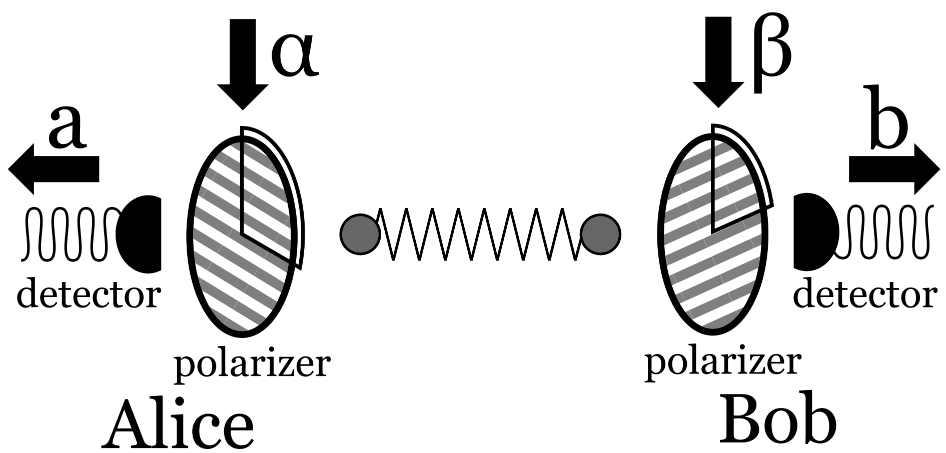


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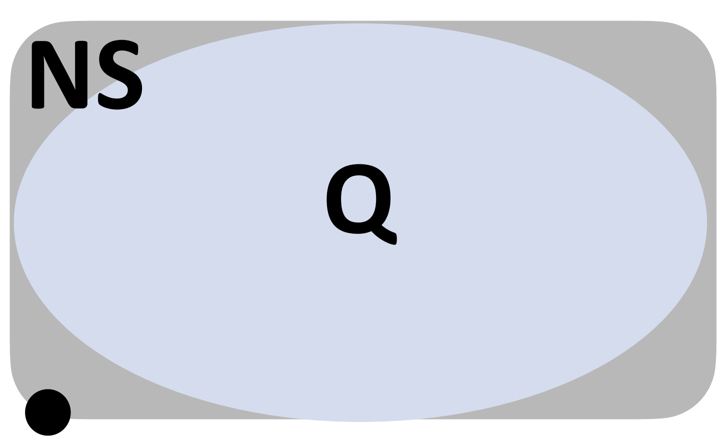


"science-fiction-polarizer"

Assumptions on the **device's response to spatiotemporal symmetries**, like

$$J \leq 2,$$

constrain the correlations severely.

# Foundational consequences

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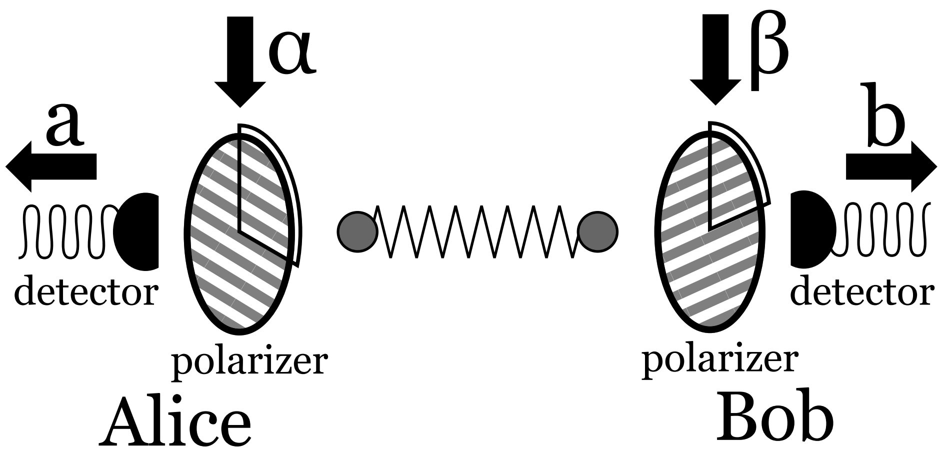


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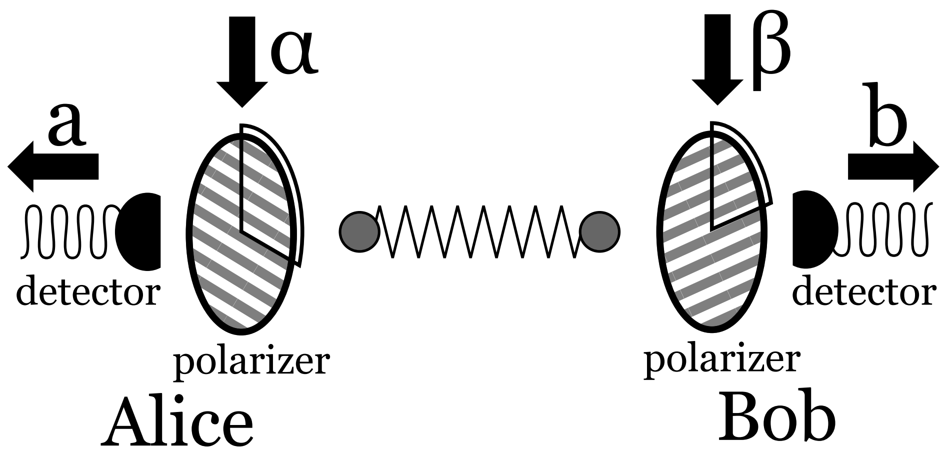


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Probabilities transform **locally fundamentally,**
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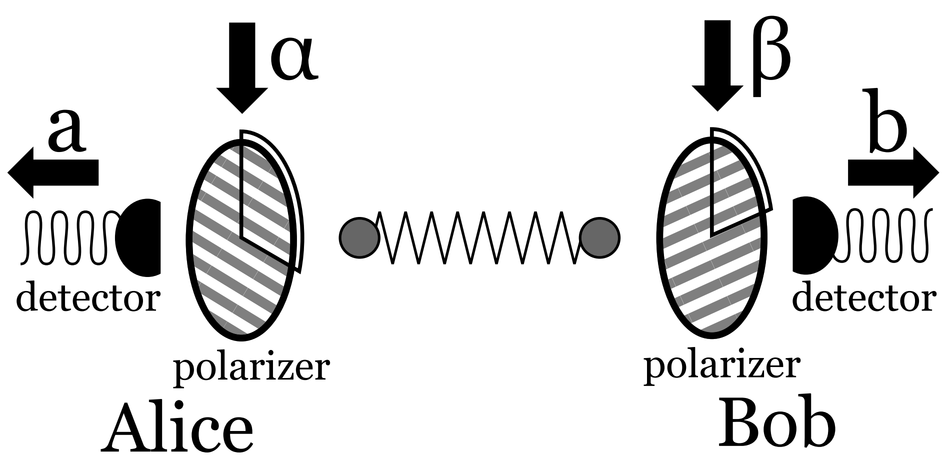


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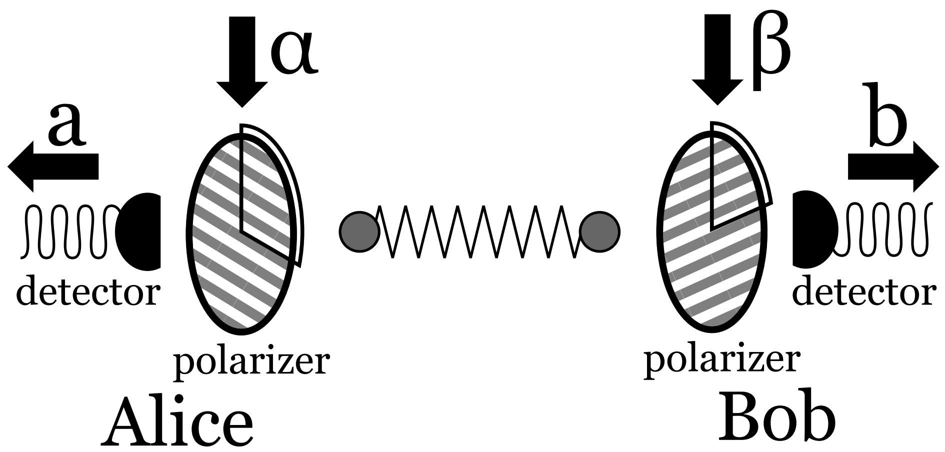


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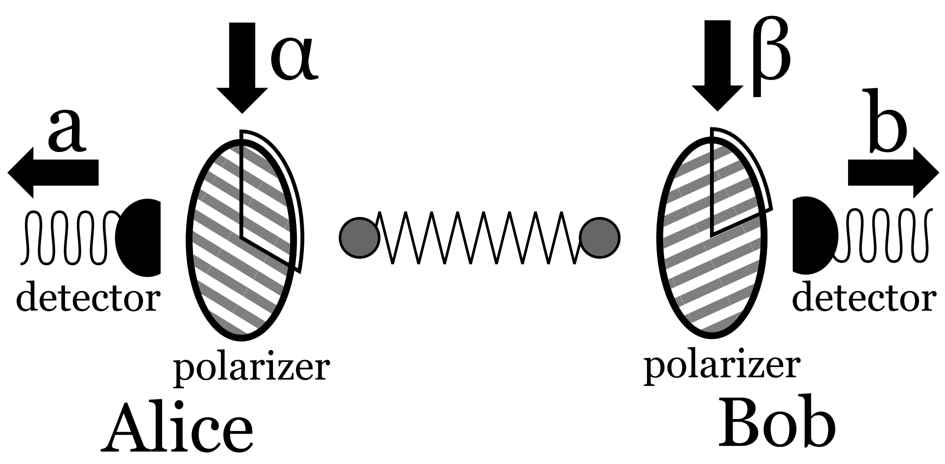


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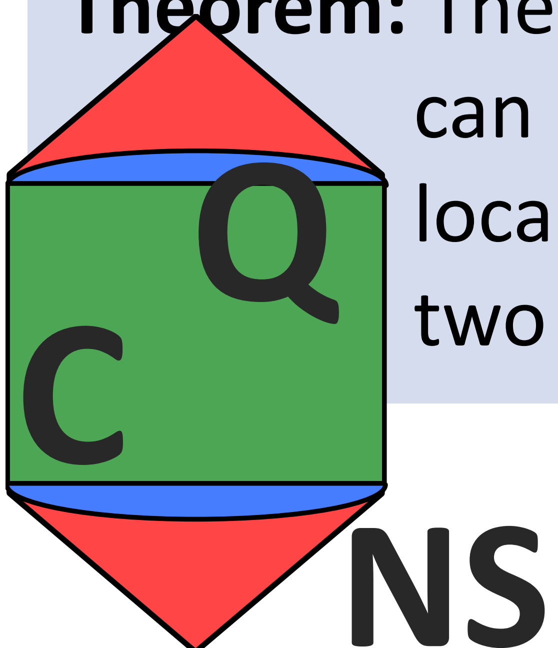


Fundamental relation between QT and space(time)?

# Experiments as “black boxes”

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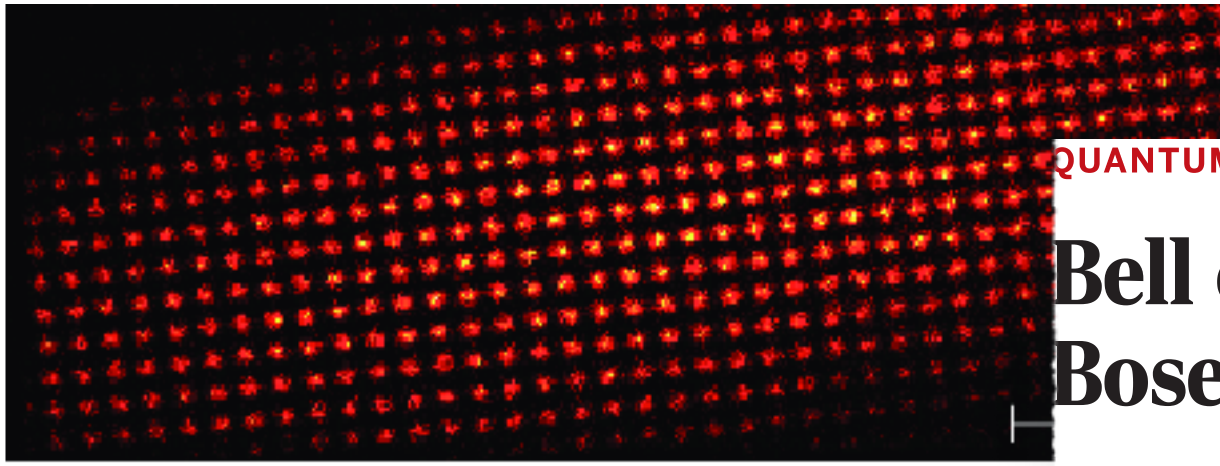

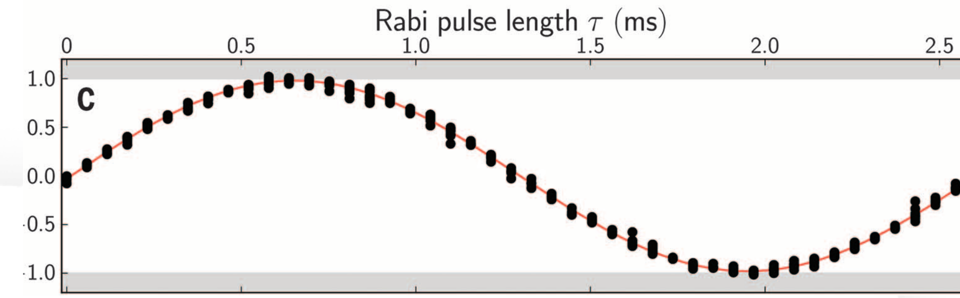


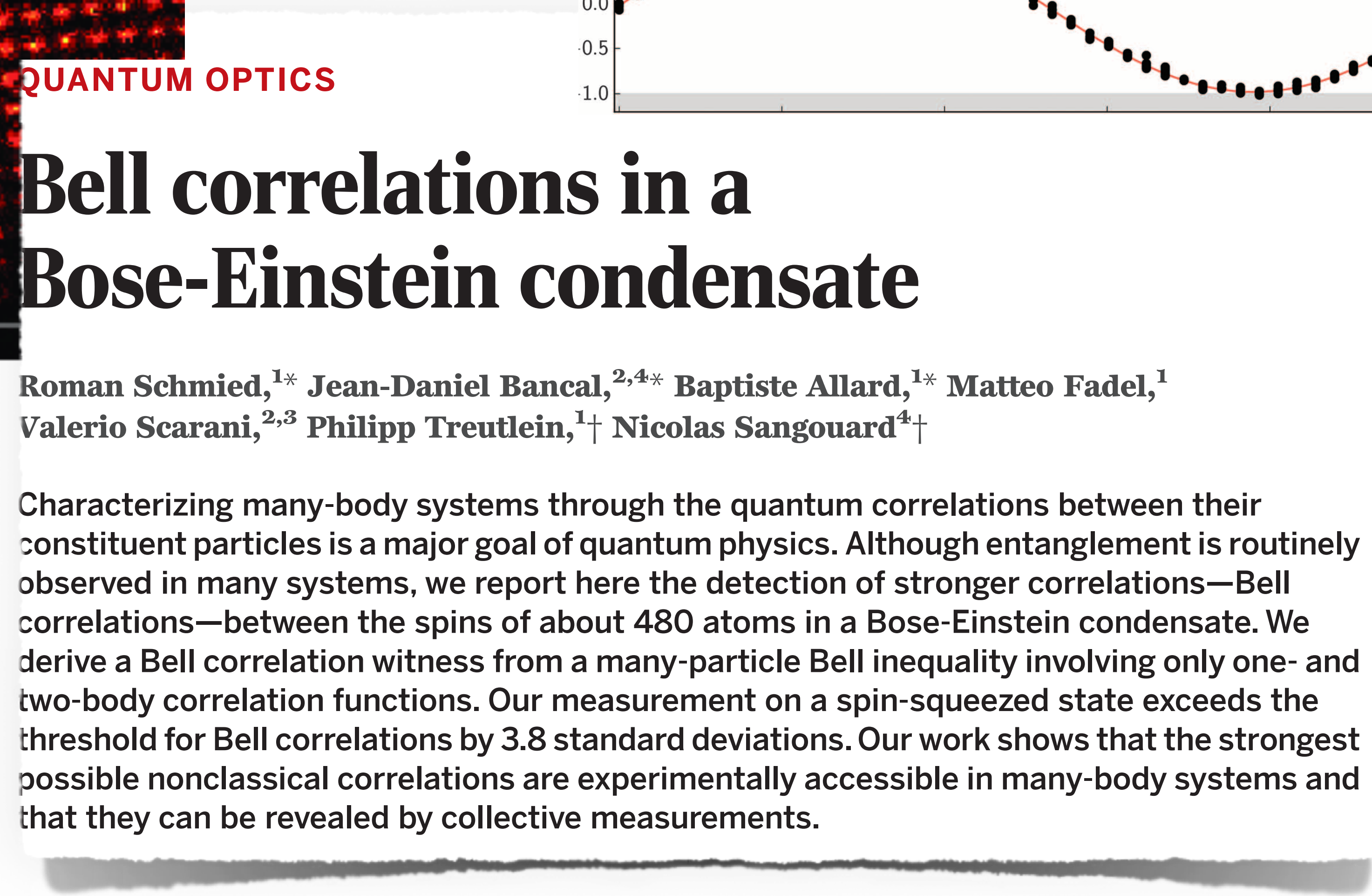

QUANTUM OPTICS

**Bell correlations in a Bose-Einstein condensate**

Roman Schmied,[1*] Jean-Daniel Bancal,[2,4*] Baptiste Allard,[1*] Matteo Fadel,[1] Valerio Scarani,[2,3] Philipp Treutlein,[1†] Nicolas Sangouard[4†]

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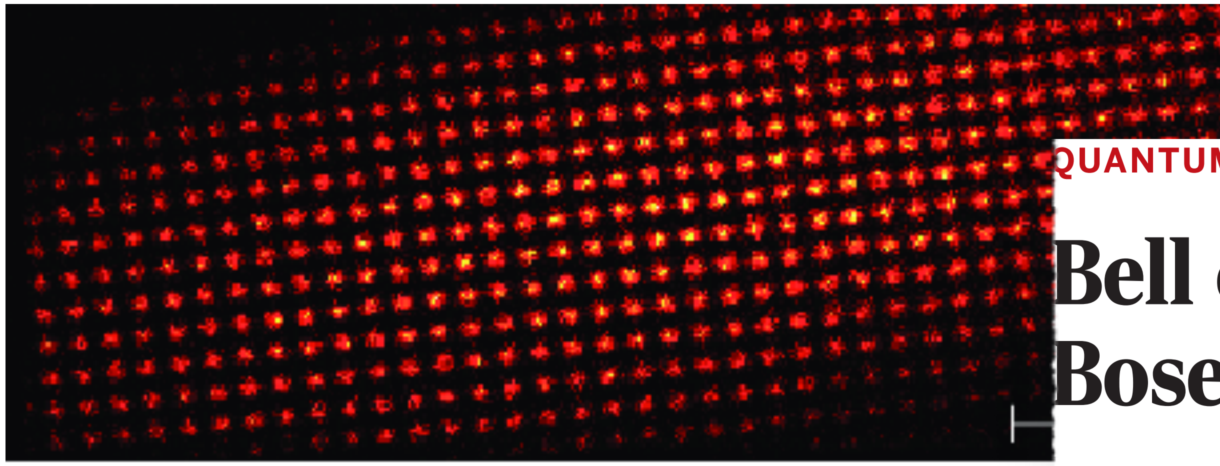

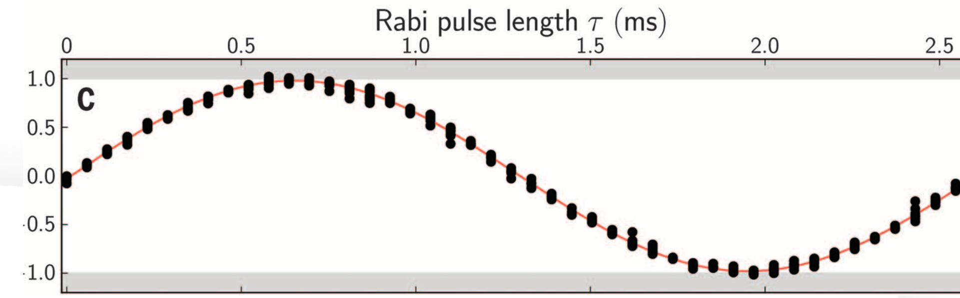


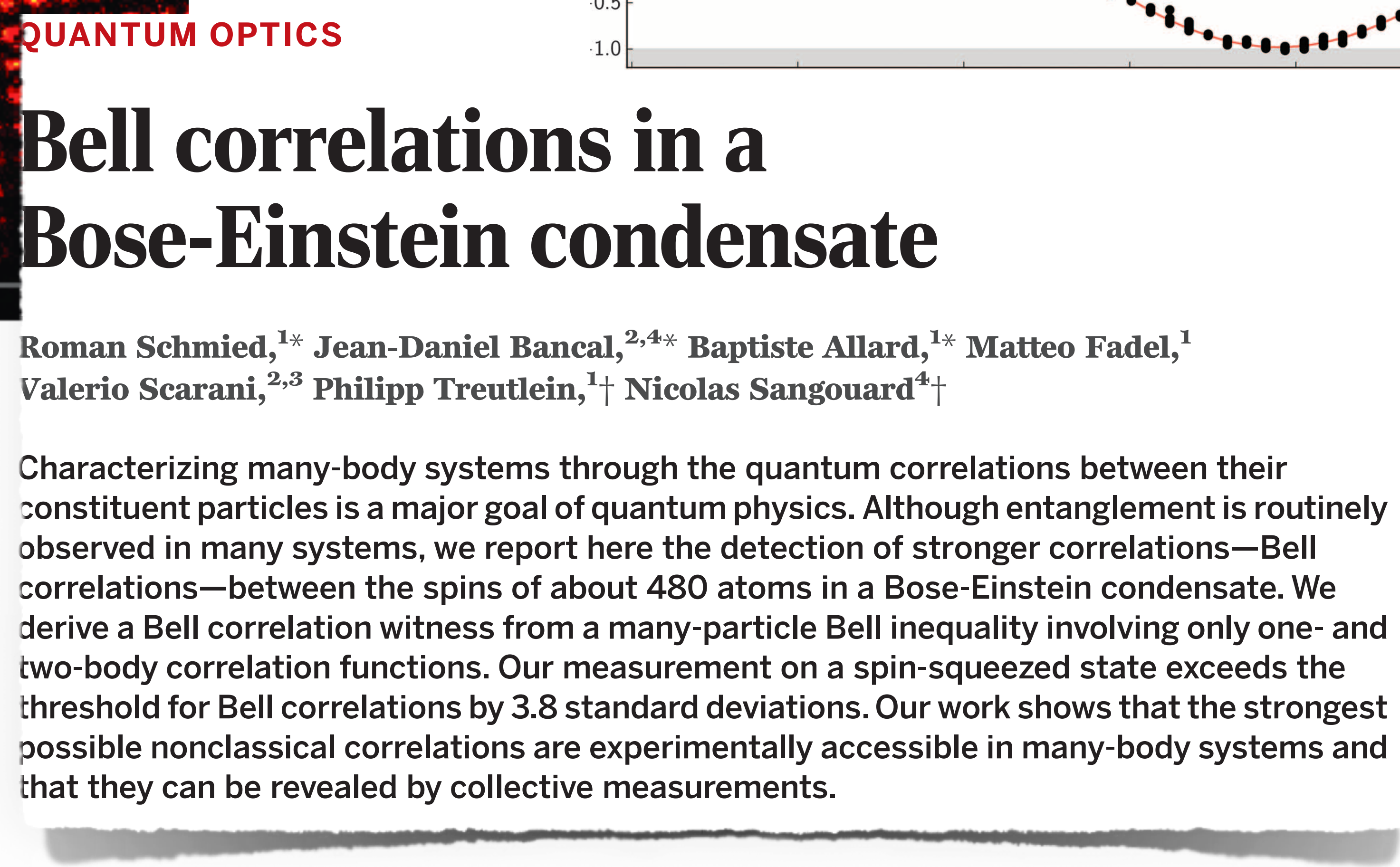

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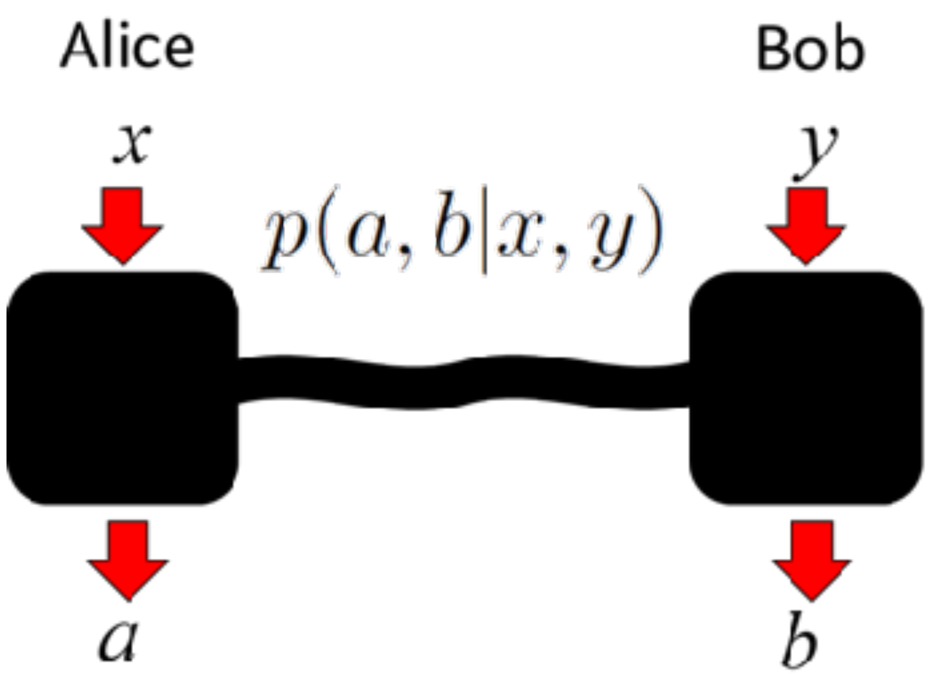
Sometimes, **all we know for sure** is that we’ve sent a pulse of a certain duration (or some other S.T.-quantity) and recorded an outcome.

What can we infer **from this alone?**
Or from **very few additional assumptions, incl. (or not) QM?**

# Towards protocols

**Device-independent QIT:**




Violation of a Bell inequality admits
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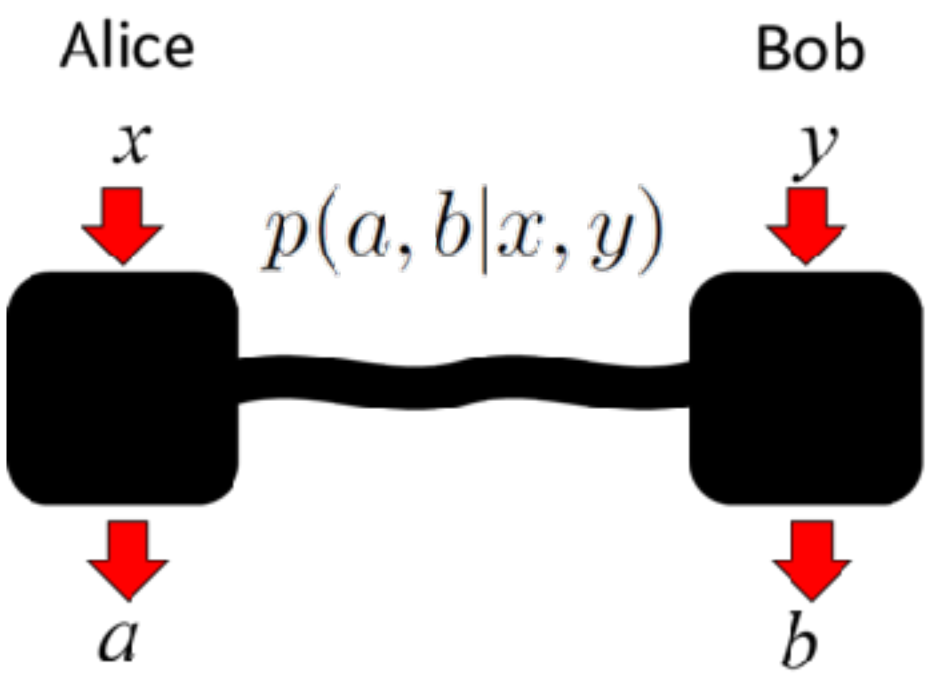


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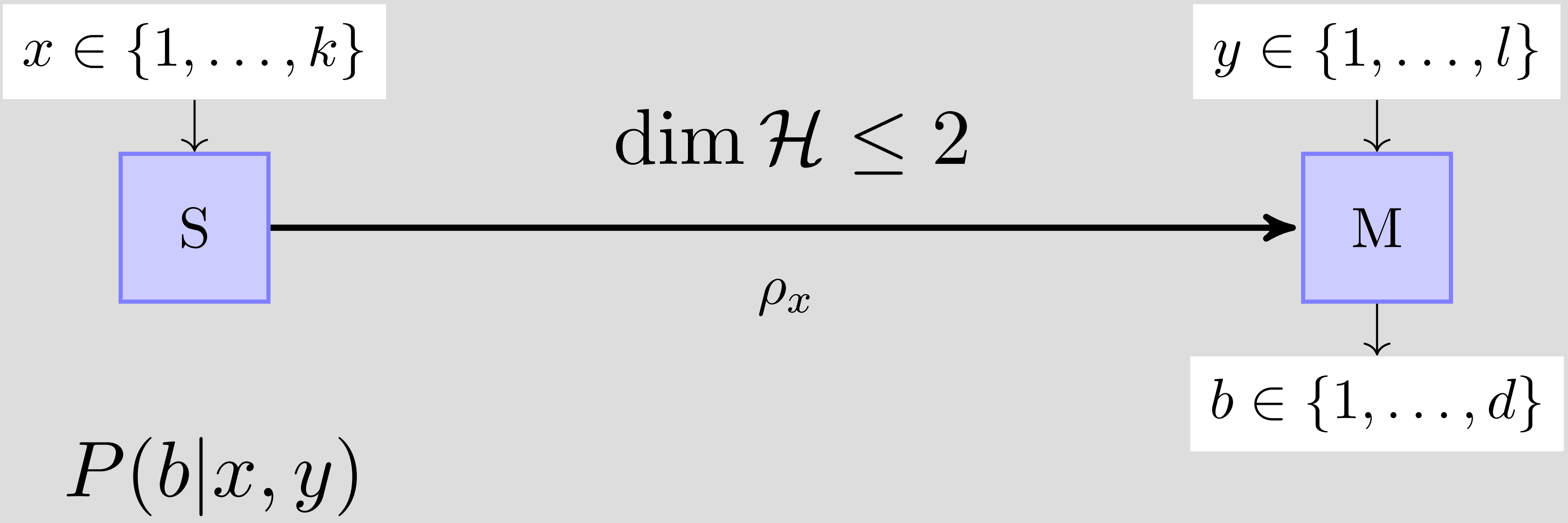


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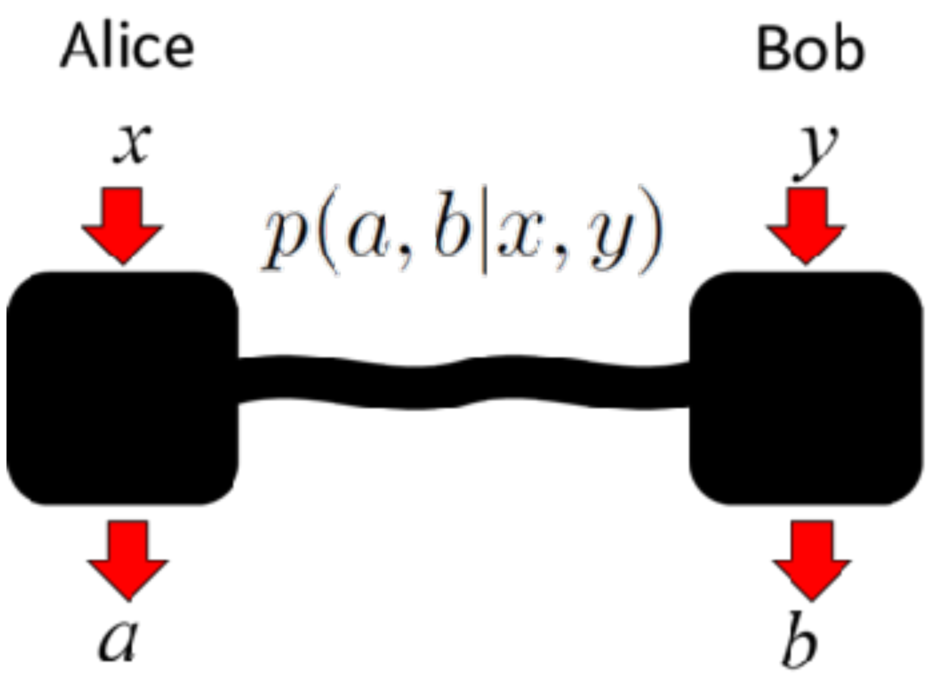


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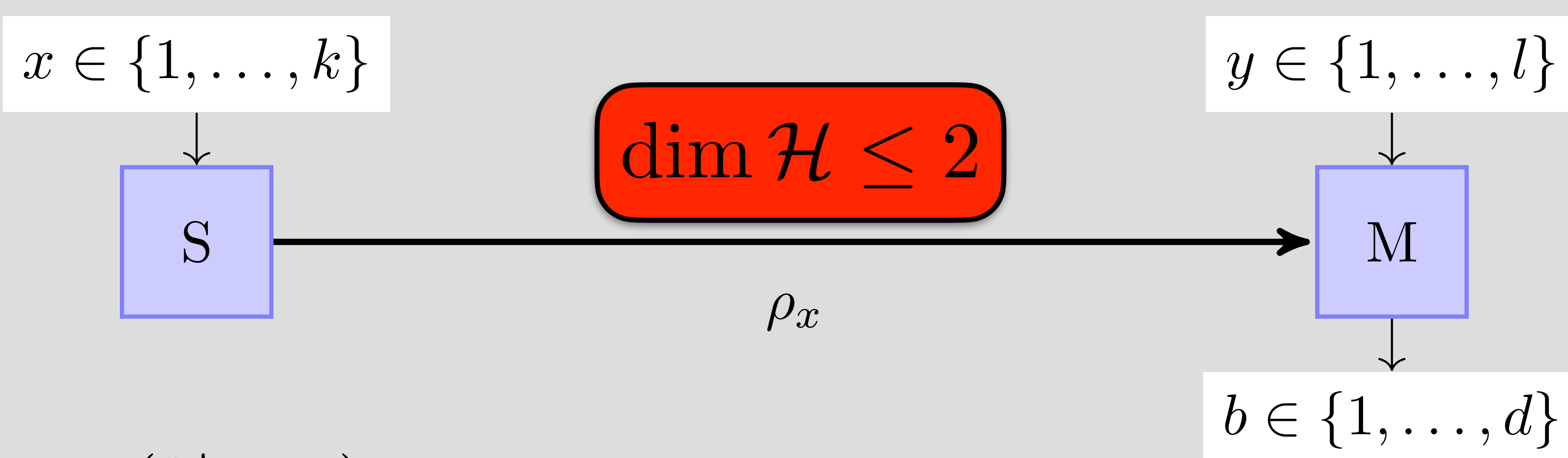


$P(b|x,y)$

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physical motivation?

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A. J. P. Garner, M. Krumm, **MM**, Phys. Rev. Research **2**, 013112 (2020).

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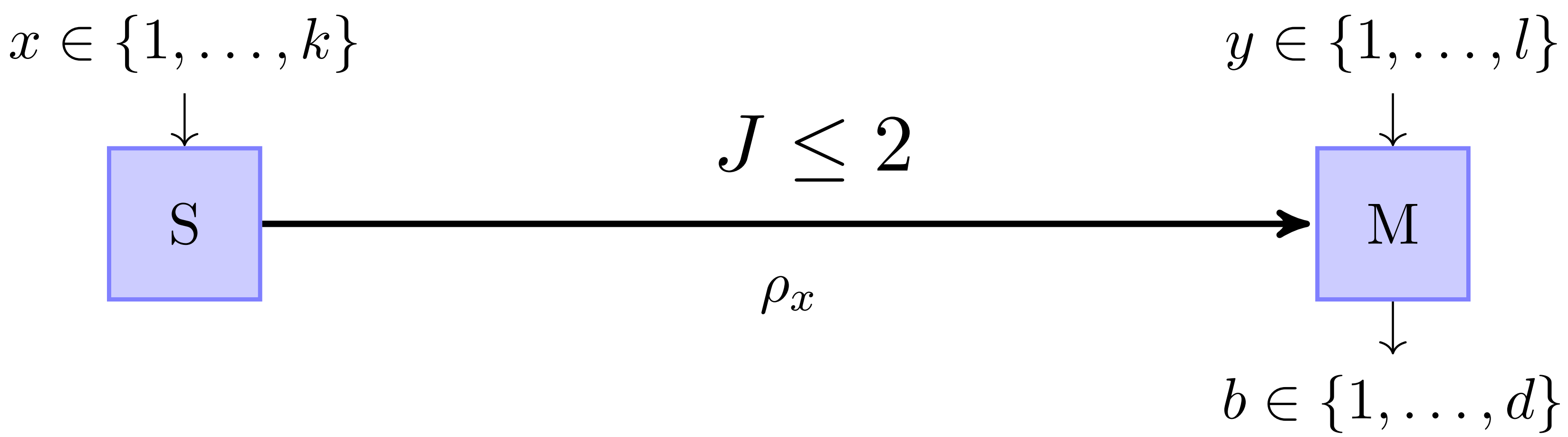

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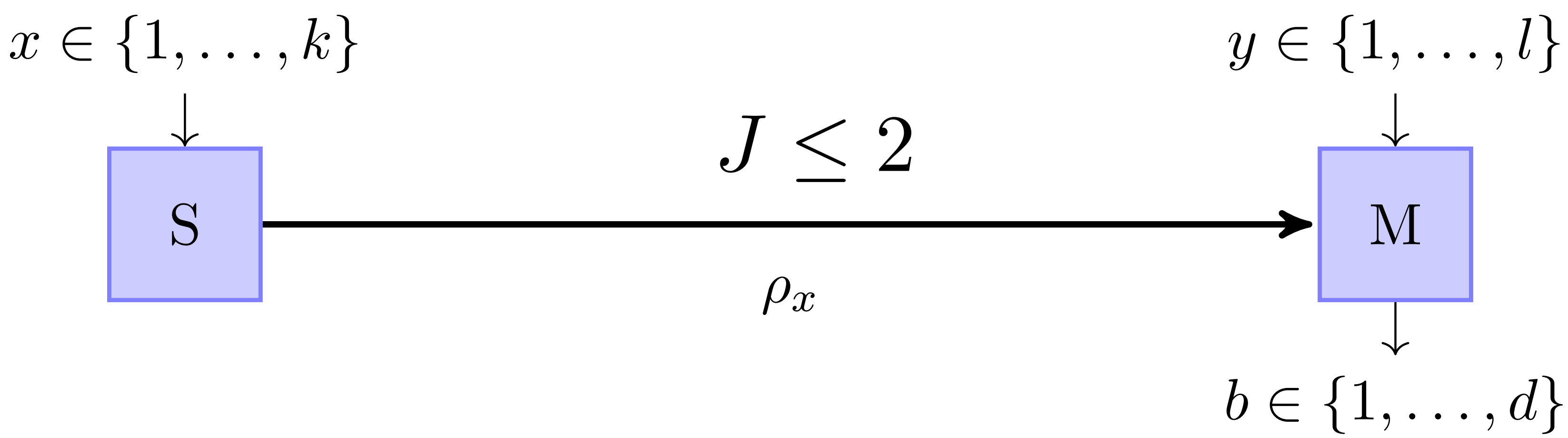


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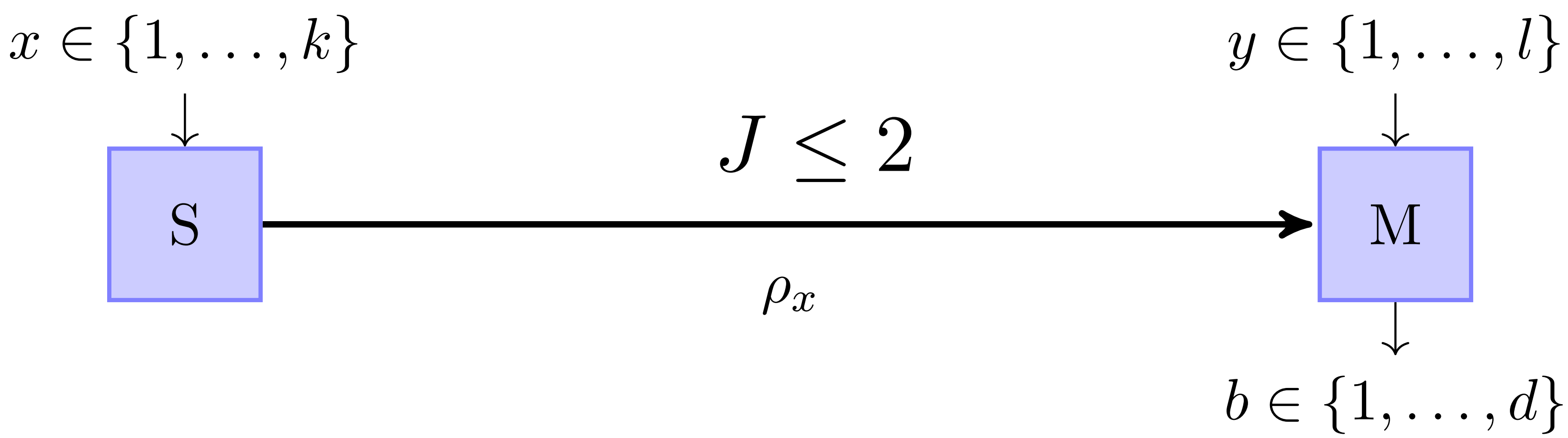


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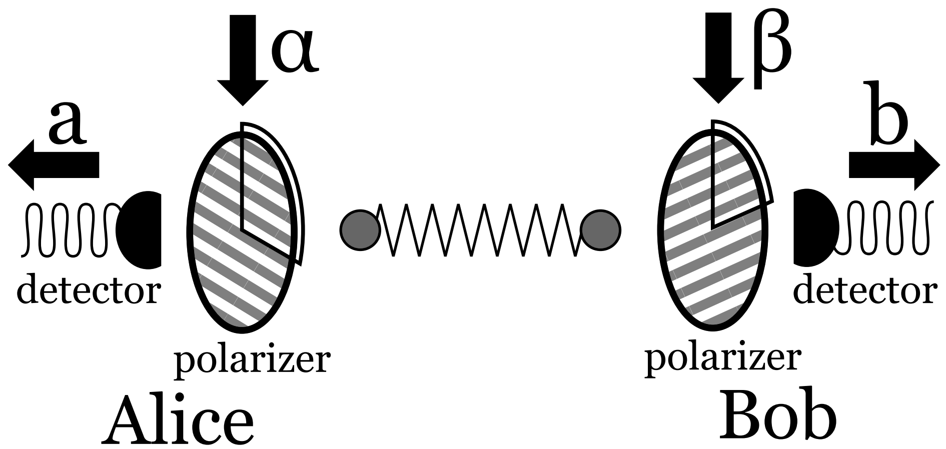


$$P(a, b|\alpha, \beta) := \sum_{m=0}^{2J} \sum_{n=-2J}^{2J} c_{mn}^{ab} \cos(m\alpha - n\beta) + s_{mn}^{ab} \sin(m\alpha - n\beta),$$

For $\mathcal{G}$=time translations, this corresponds to **energy bounds**.

Also, closer to particle physics intuition: don't count dimensions, but representation labels (of the Poincaré group).

Proof of principle: Bell scenario, assumption $J \leq J_0 < \infty$.
Can **witness nonlocality** with only **one-sided free choice**.

# Future work

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- Related to **Quantum Gravity** / constraint quantization:

  What is the most general **relational** (post)quantum theory?
  Wheeler-DeWitt-equation, or more? $\hat{H}|\psi\rangle = 0$

  1st step: M. Krumm, P.A. Höhn, **MM**, arXiv:2011.01951.

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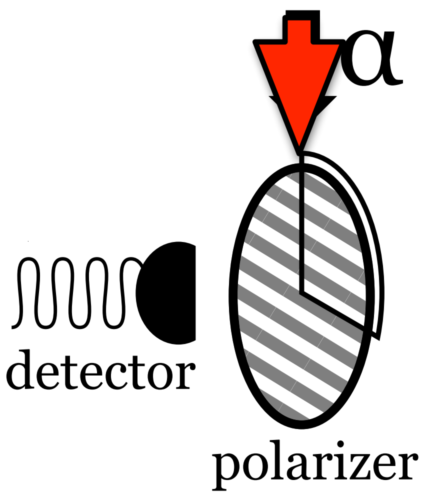
- With Stefan Ludescher:
  Does QT admit the **most general time evolutions / most general single-party "polarizers"?**




$P(a|\alpha)$

→ novel **experimental tests** of QT?

## Summary

- QT ist just **one probabilistic theory among many "GPTs"**.
- It can be reconstructed from **simple principles**.

Les Houches 2019 lecture notes: **MM**, arXiv:2011.01286.

- GPTs allow to study QT's structural relation to **spacetime**.
- Understanding this relation is of foundational importance, but also promises **new protocols** and experimental tests of QT.

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