A derivation of quantum theory from physical requirements

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Joint work with Lluis Masanes

arXiv: 1004.1483

Outline

1. Motivation Why??

2. General Probabilistic Theories

3. The Axioms What do they mean?

4. Derivation of the Hilbert space formalism Why are qubits 3D-balls??

5. What‘s beyond QT?

1. Motivation

John A. Wheeler, New York Times, Dec. 12 2000:

„Quantum physics [...] has explained the structure of atoms and molecules, [...] the behavior of semiconductors [...] and the comings and goings of particles from neutrinos to quarks.

Successful, yes, but mysterious, too.
Why does the quantum exist?"

The New York Times

1. Motivation

ANNALS OF PHYSICS **194**, 336–386 (1989)

Testing Quantum Mechanics

STEVEN WEINBERG*

Theory Group, Department of Physics,
University of Texas, Austin, Texas 78712

Received March 6, 1989

This paper presents a general framework for introducing nonlinear corrections into ordinary quantum mechanics, that can serve as a guide to experiments that would be sensitive to such corrections. In the class of generalized theories described here, the equations that determine the time-dependence of the wave function are no longer linear, but are of Hamiltonian type. Also, wave functions that differ by a constant factor represent the same physical state and satisfy the same time-dependence equations. As a result, there is no difficulty in combining separated subsystems. Prescriptions are given for determining the states in which observables have definite values and for calculating the expectation values of observables for general states, but the calculation of probabilities requires detailed analysis of the method of measurement. A study is presented of

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Volume 143, number 1,2 PHYSICS LETTERS A 1 January 1990

WEINBERG'S NON-LINEAR QUANTUM MECHANICS AND SUPRALUMINAL COMMUNICATIONS

N. GISIN

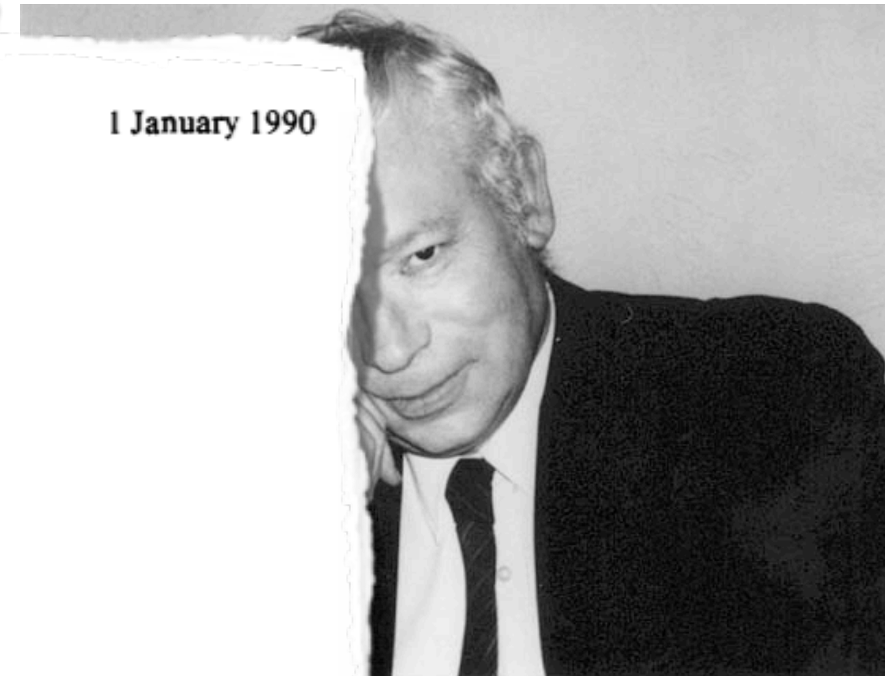
Group on Applied Physics, University of Geneva, 1211 Geneva 4, Switzerland

Received 16 October 1989; accepted for publication 3 November 1989
Communicated by J.P. Vigier

We show with an example that Weinberg's general framework for introducing non-linear corrections into quantum mechanics allows for arbitrarily fast communications.

Recently Weinberg has proposed a general framework for introducing non-linear corrections into ordinary quantum mechanics [1,2]. Although we fully support his emphasis on the importance of testing quantum mechanics, we would like in this Letter to draw attention to the difficulty of modifying quantum mechanics without introducing arbitrarily fast actions at a distance. Below we show how to construct, within Weinberg's framework, an arbitrarily fast telephone line. In ordinary quantum mechanics

to know what such an apparatus is... do you know what is inside your phone?) In order to simplify we consider only a single-bit message. The two directions z and u are in the xz-plane orthogonal to the incoming flow of particles, and are 45° from each other. The way the inhomogeneous magnetic field acts on the particles is well-known from experimental evidence. After the apparatus there are two counters. For each particle one of the counters will click. This click will be amplified until all readers of



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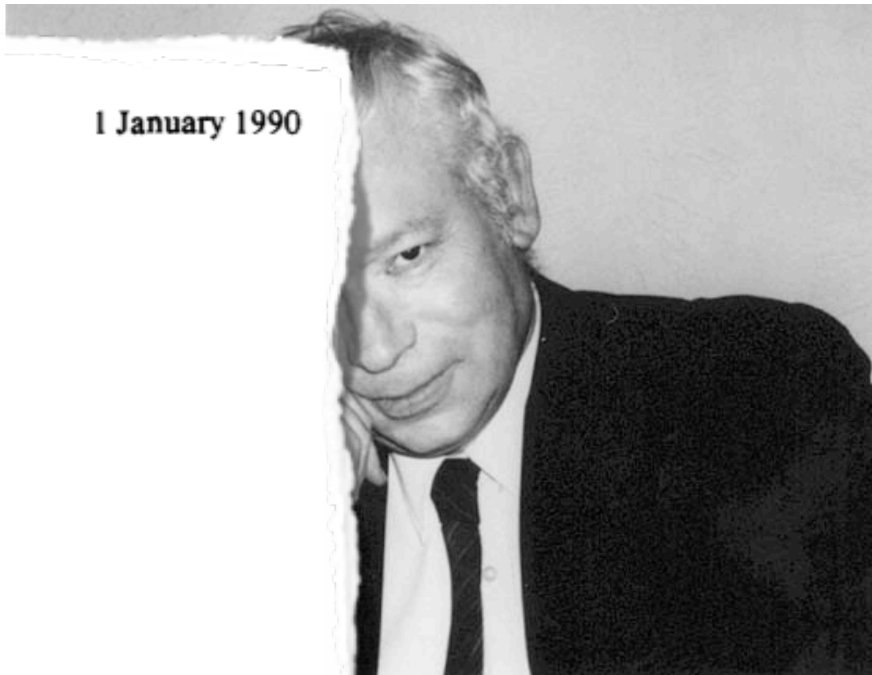

It is difficult to modify quantum theory.

Our results:

- A derivation of the full quantum formalism from operational / physical axioms.
- Methods to construct natural consistent modifications of quantum theory.

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• Methods to construct natural consistent modifications of quantum theory.

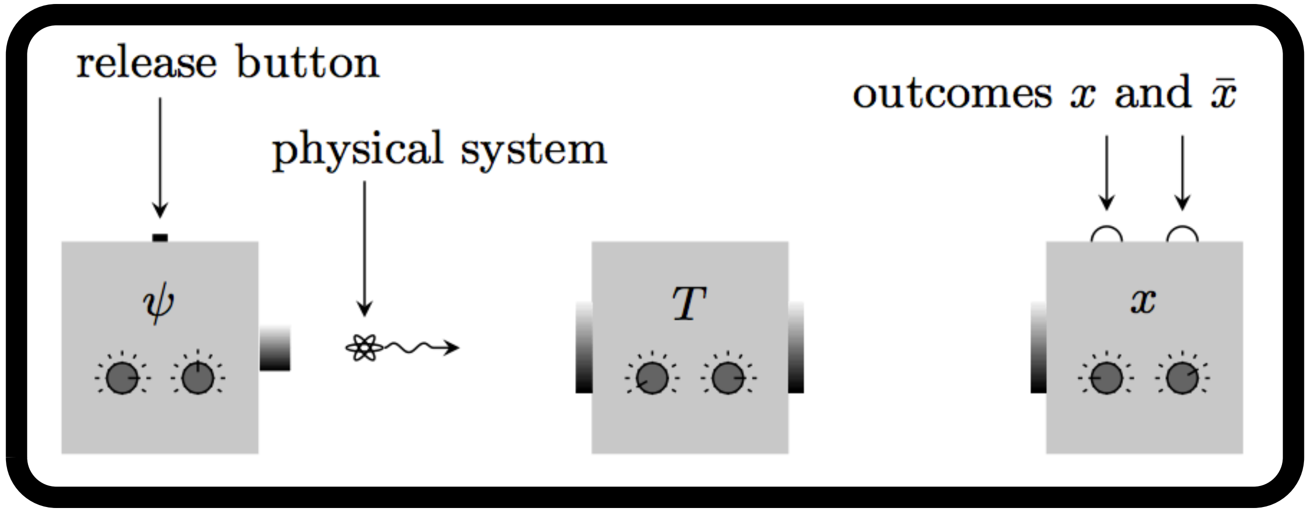
Builds on:

• L. Hardy, *Quantum Theory From Five Reasonable Axioms*, 2001
• B. Dakić and Č. Brukner, *Quantum Theory and Beyond: Is Entanglement Special?*, 2009

See also:

• G. Chiribella et al., *Informational derivation of Q.T.*, 2010
• L. Hardy, *Reformulating and Reconstructing Q.T.*, 2011

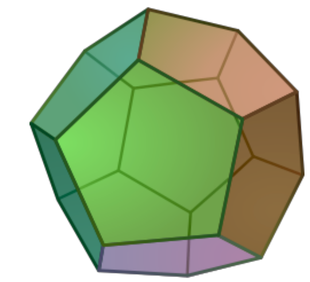
Basic physical / operational assumptions

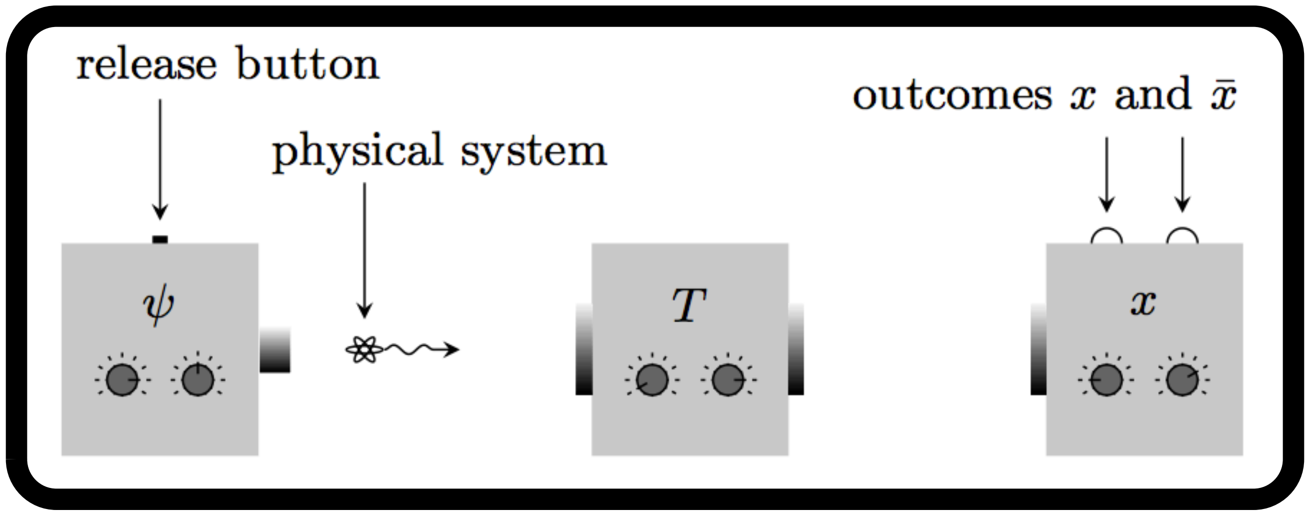


- States, transformations, and measurements with **outcome probabilities**.
- Combined systems: **no-signalling**.

Basic physical / operational assumptions

General probabilistic theories





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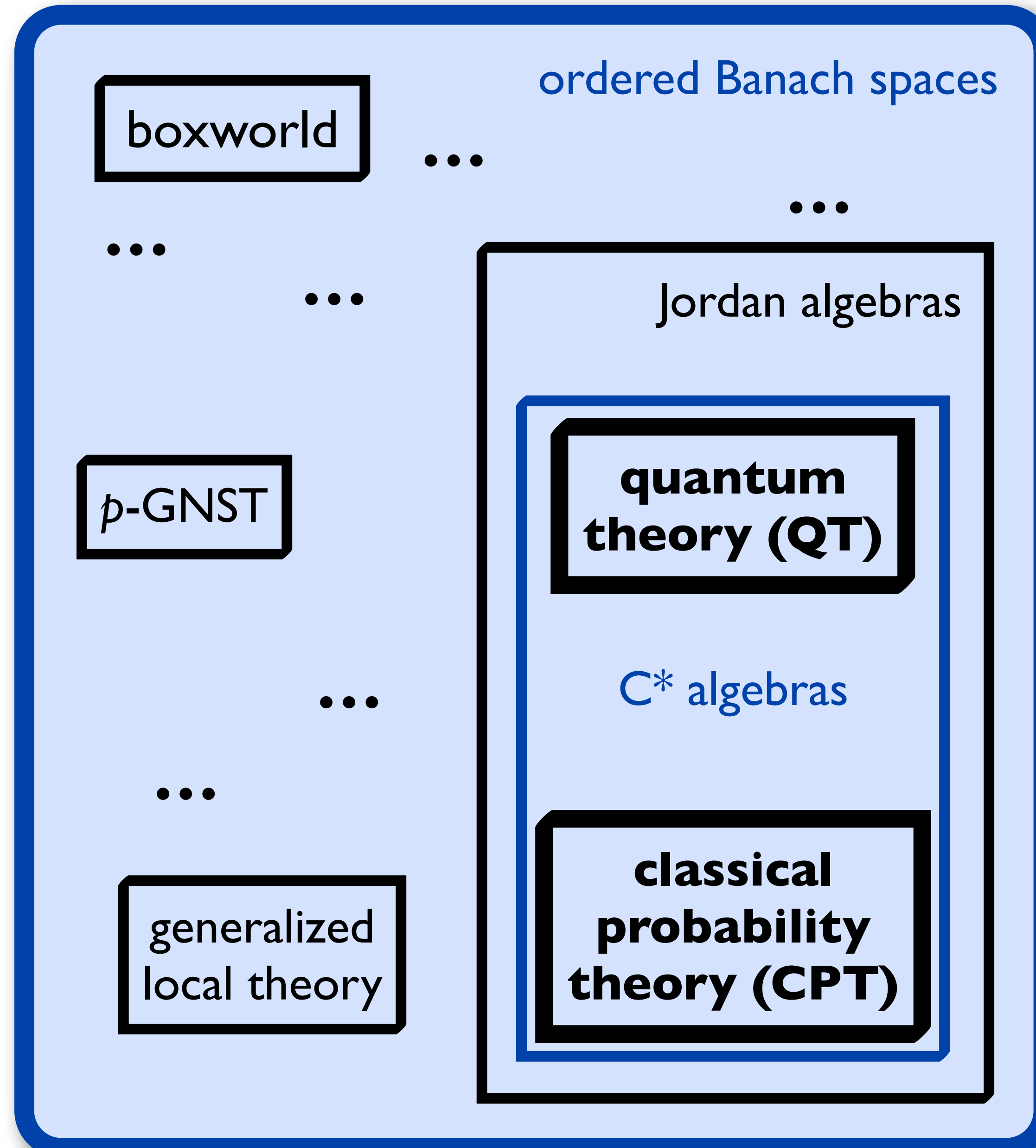

- **No** Hilbert spaces, complex numbers,...
- State spaces: **arbitrary convex sets**.
- Many ways to **combine systems**.

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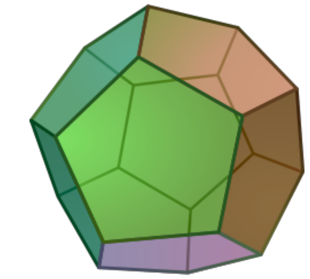

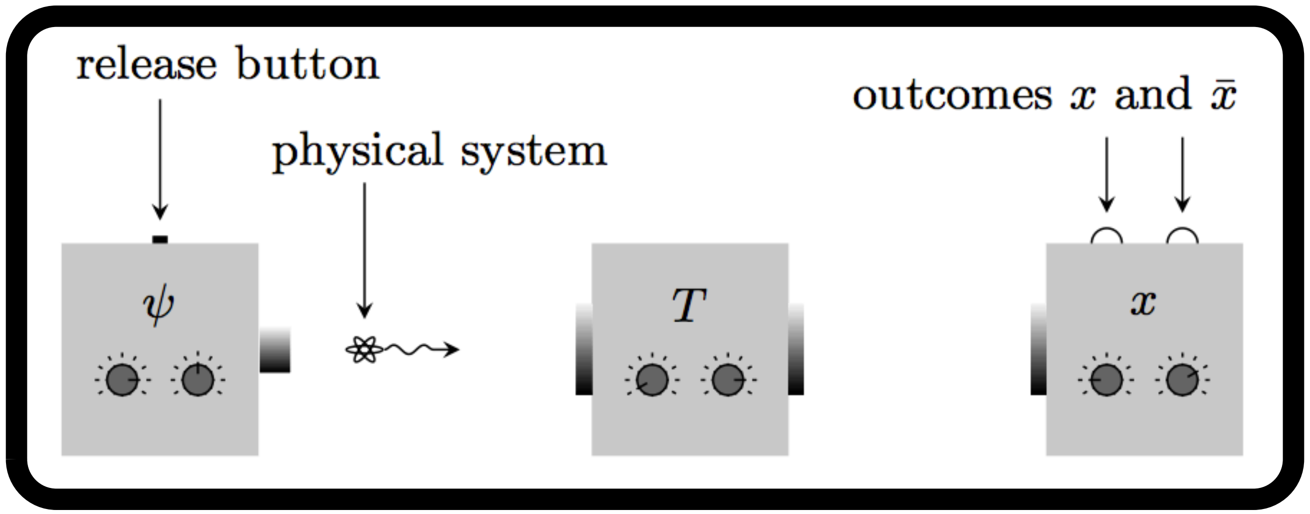

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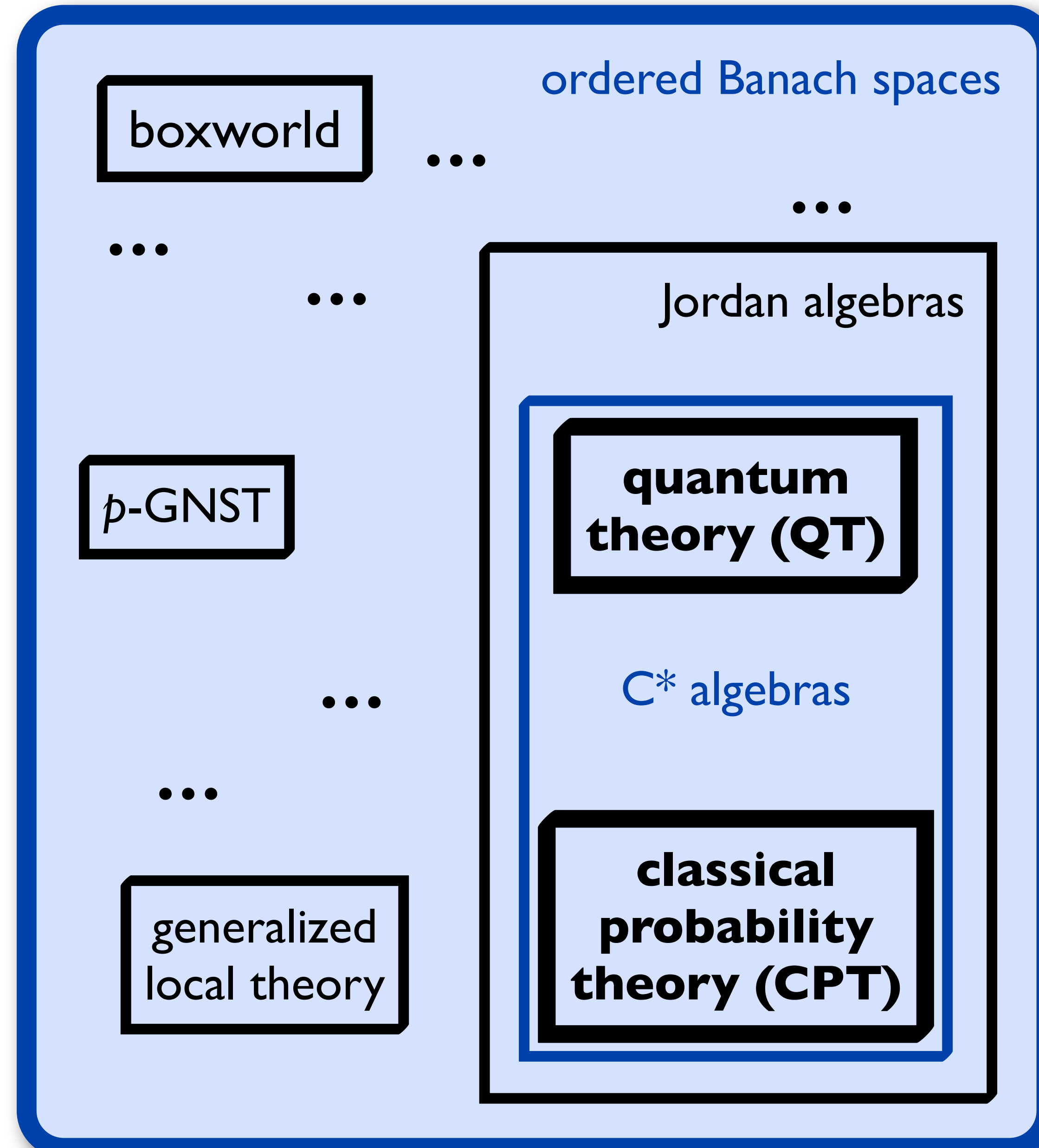
The Axioms:

I. Local tomography
II. Reversibility
III. Subspace axiom
IV. Finite-dimensionality
V. All measurements allowed



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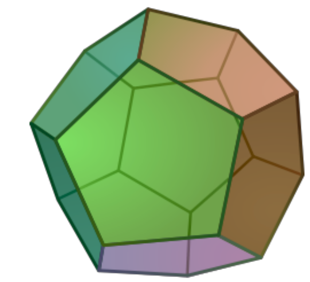

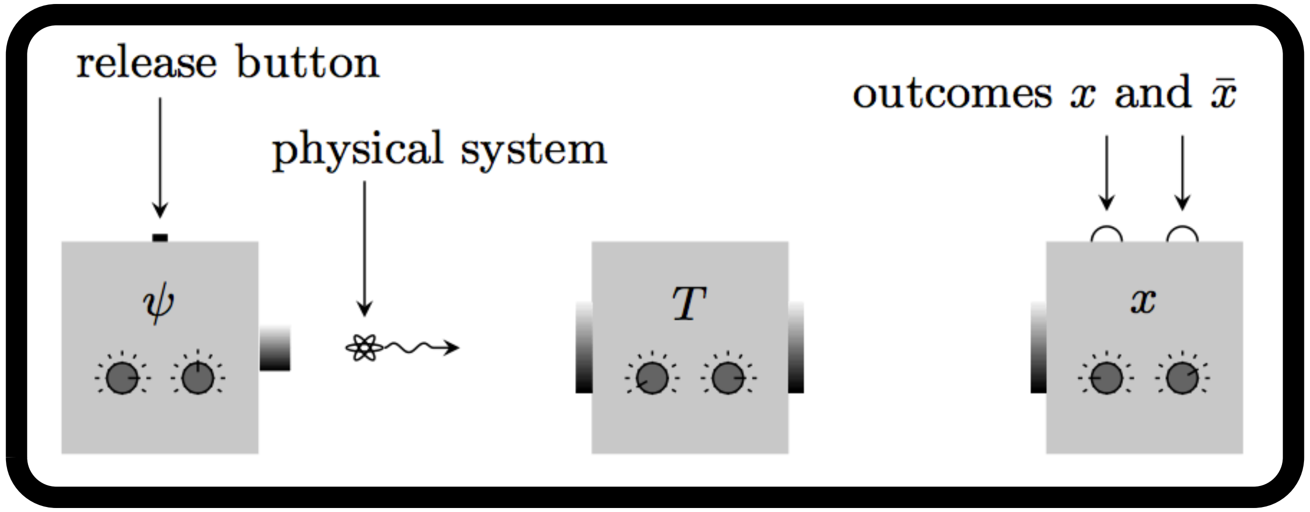

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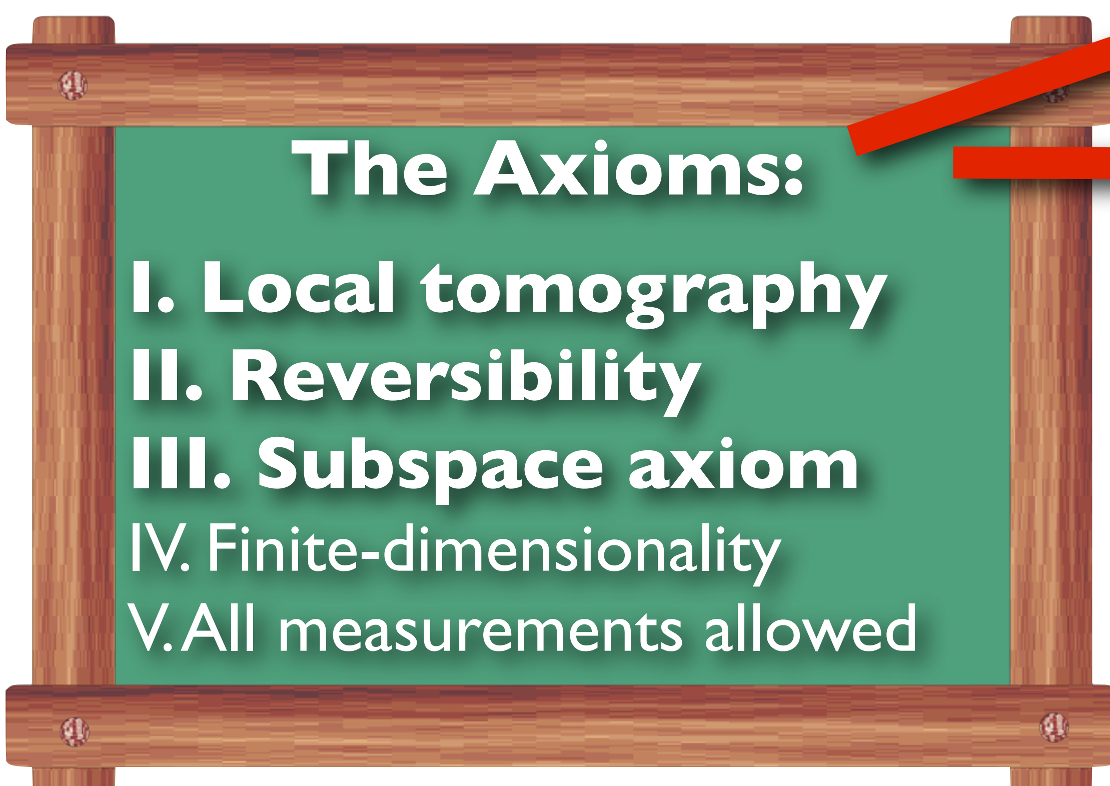
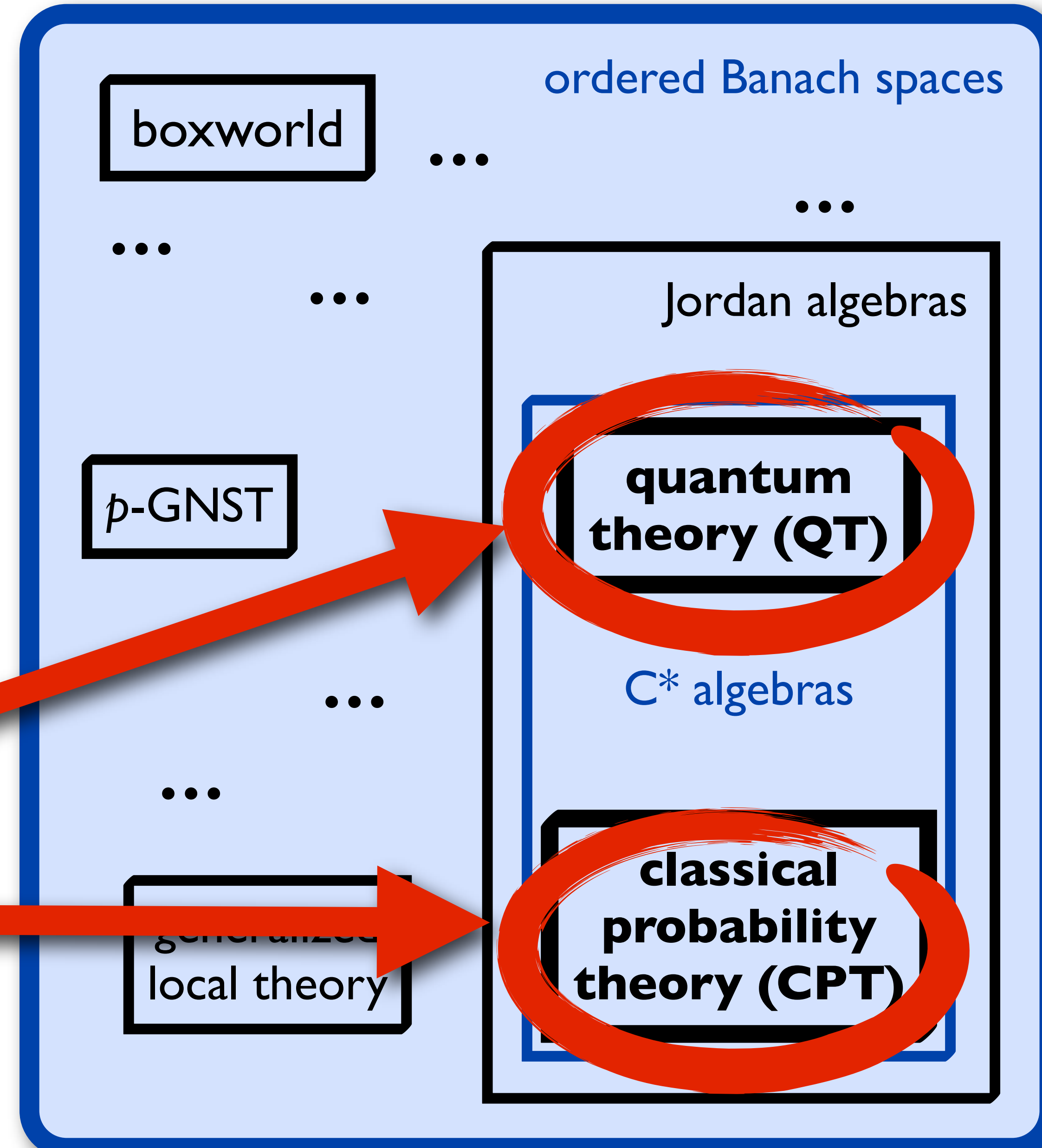


Determine CPT+QT **uniquely!**



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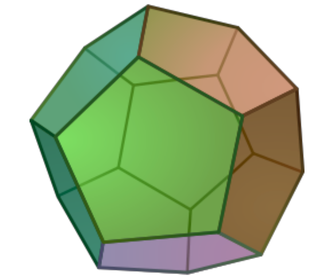

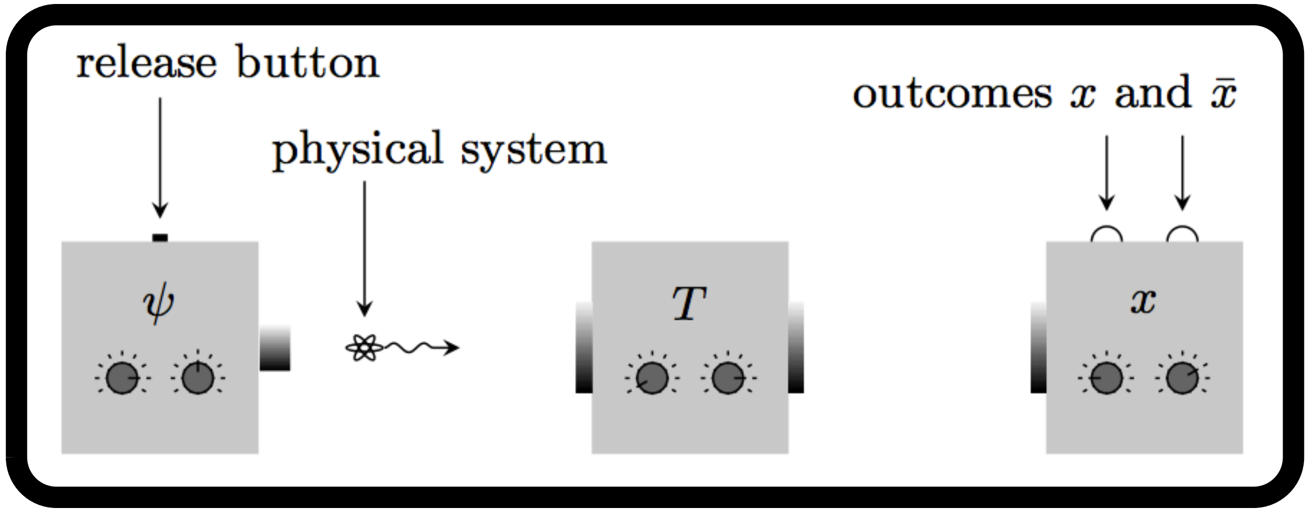

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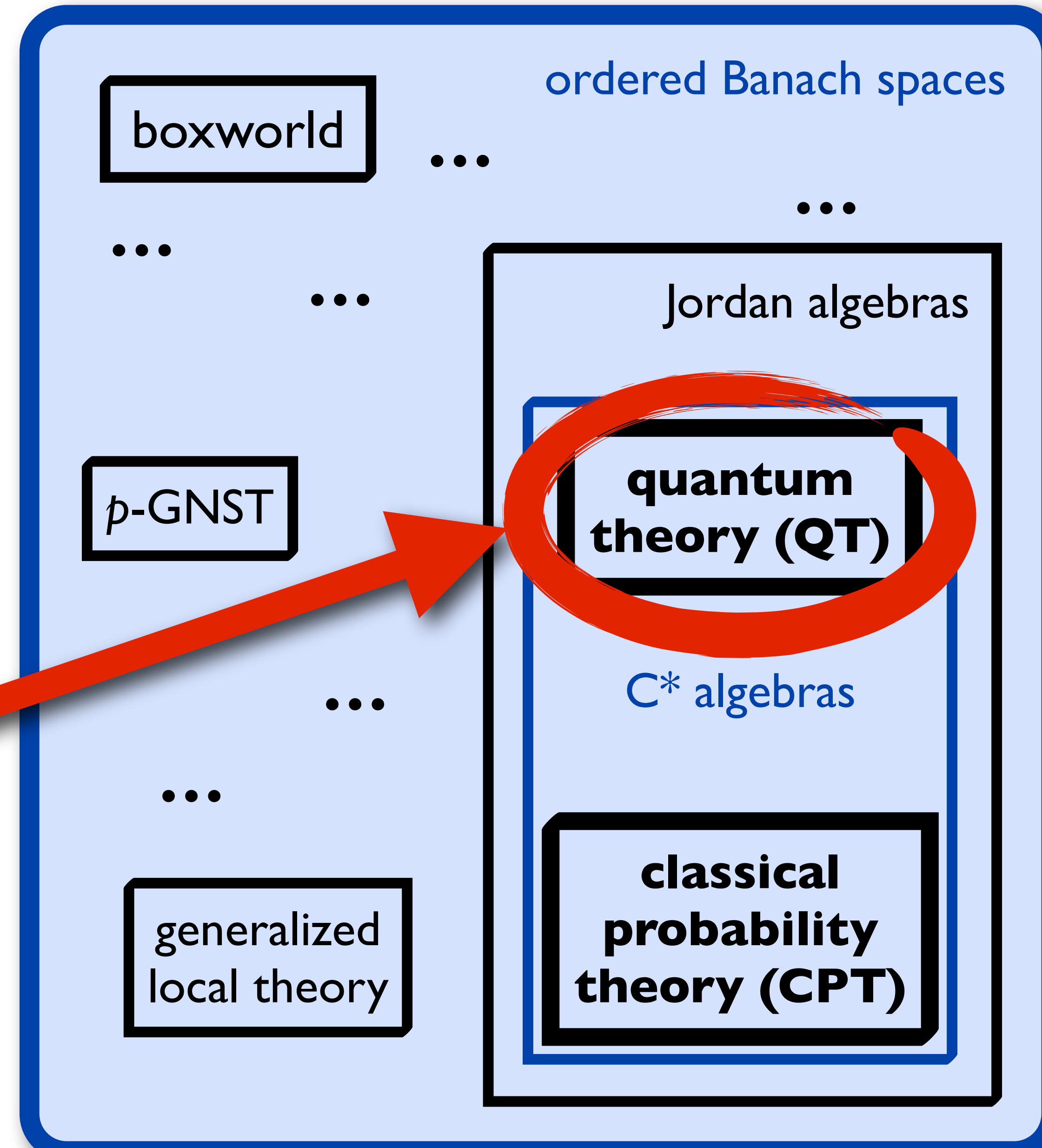

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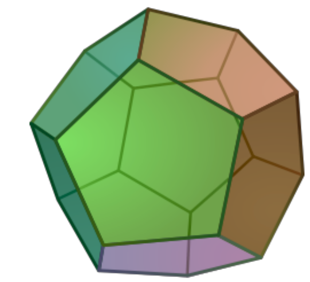

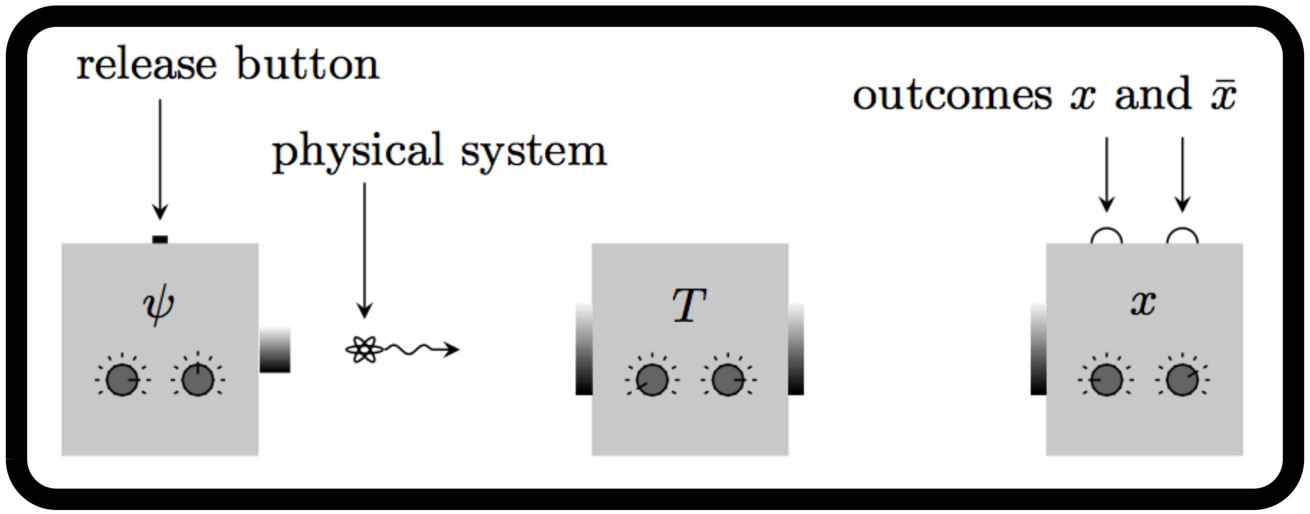

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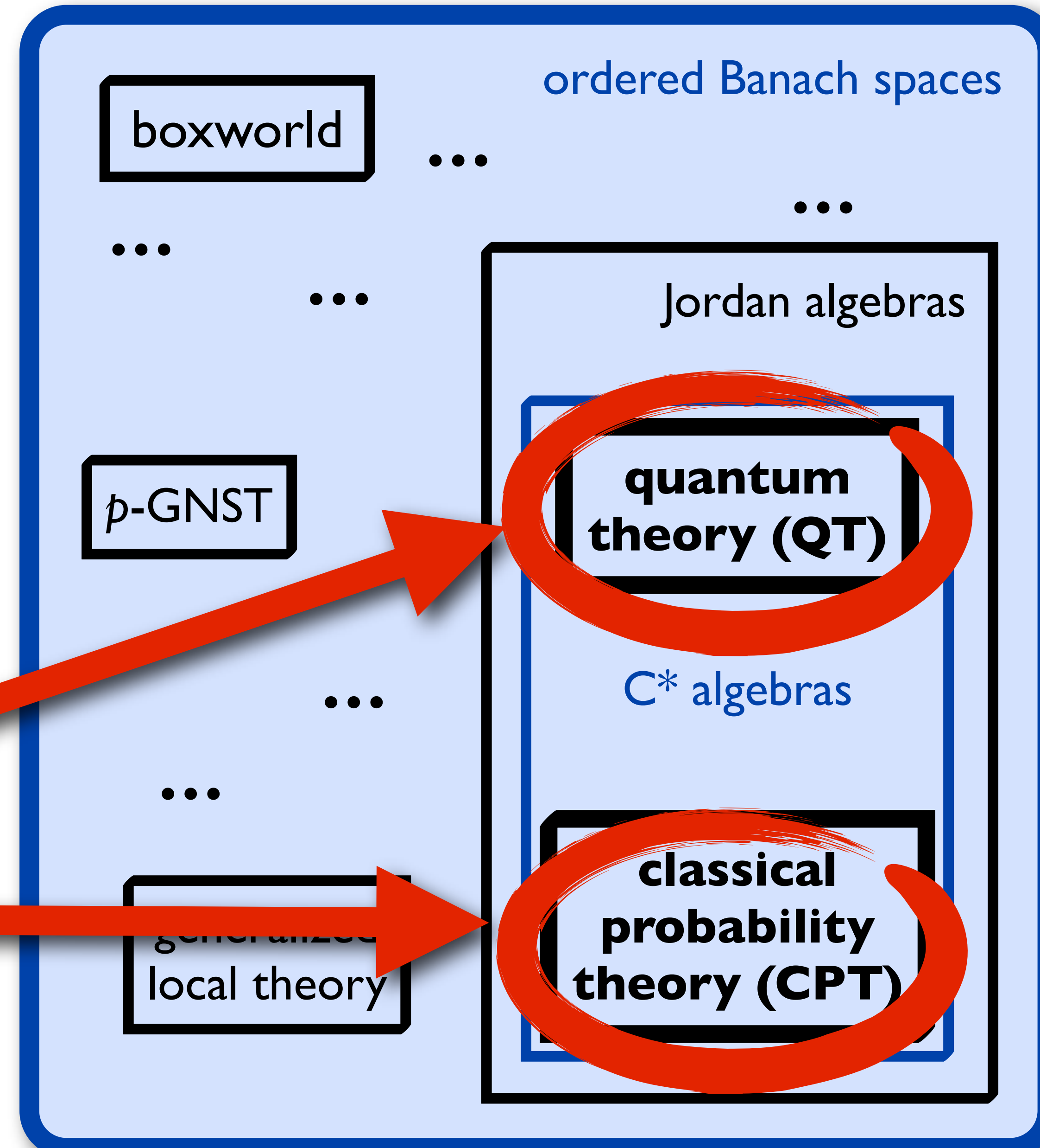

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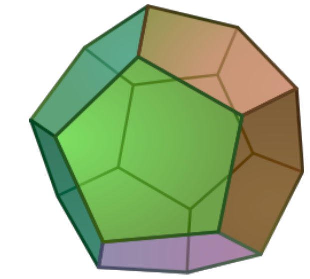

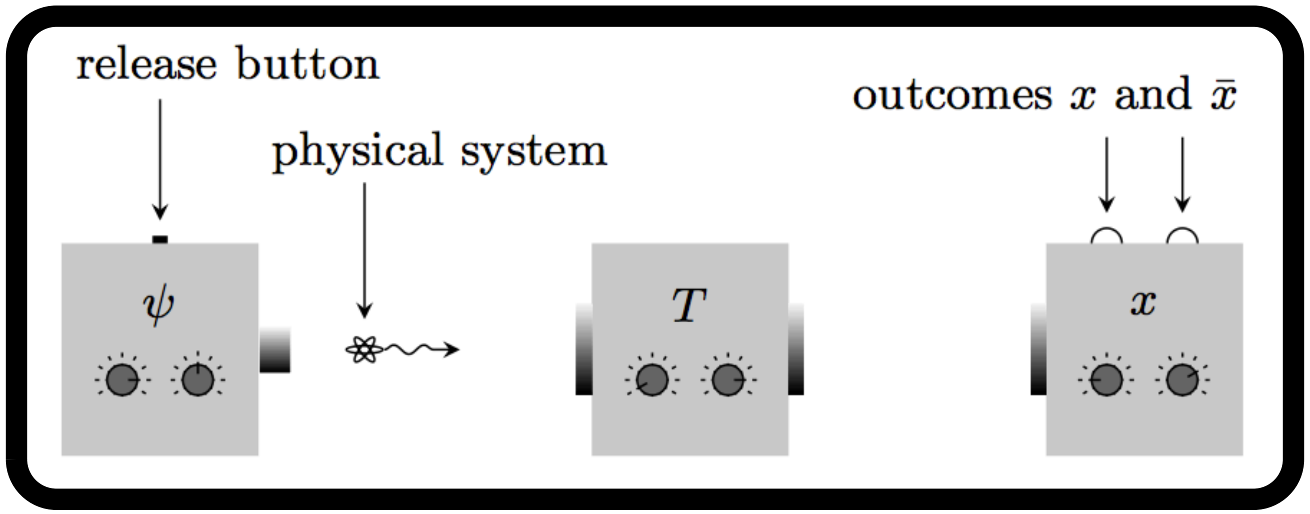

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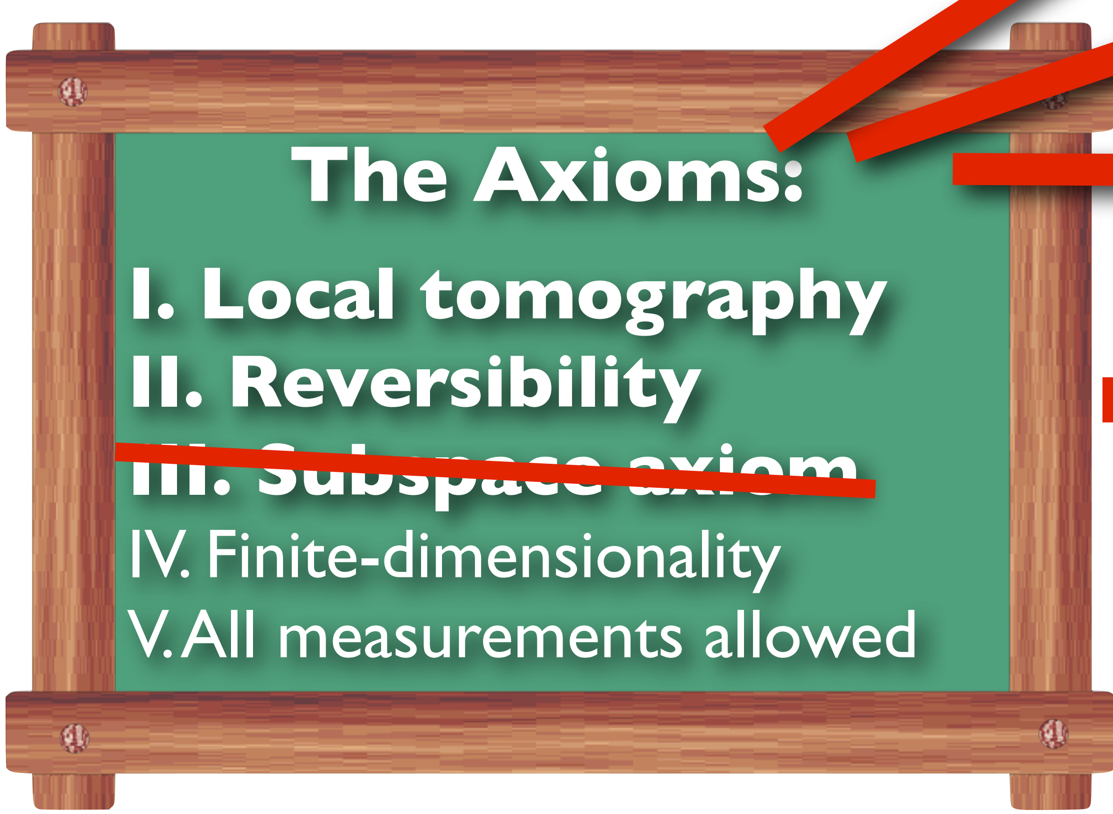
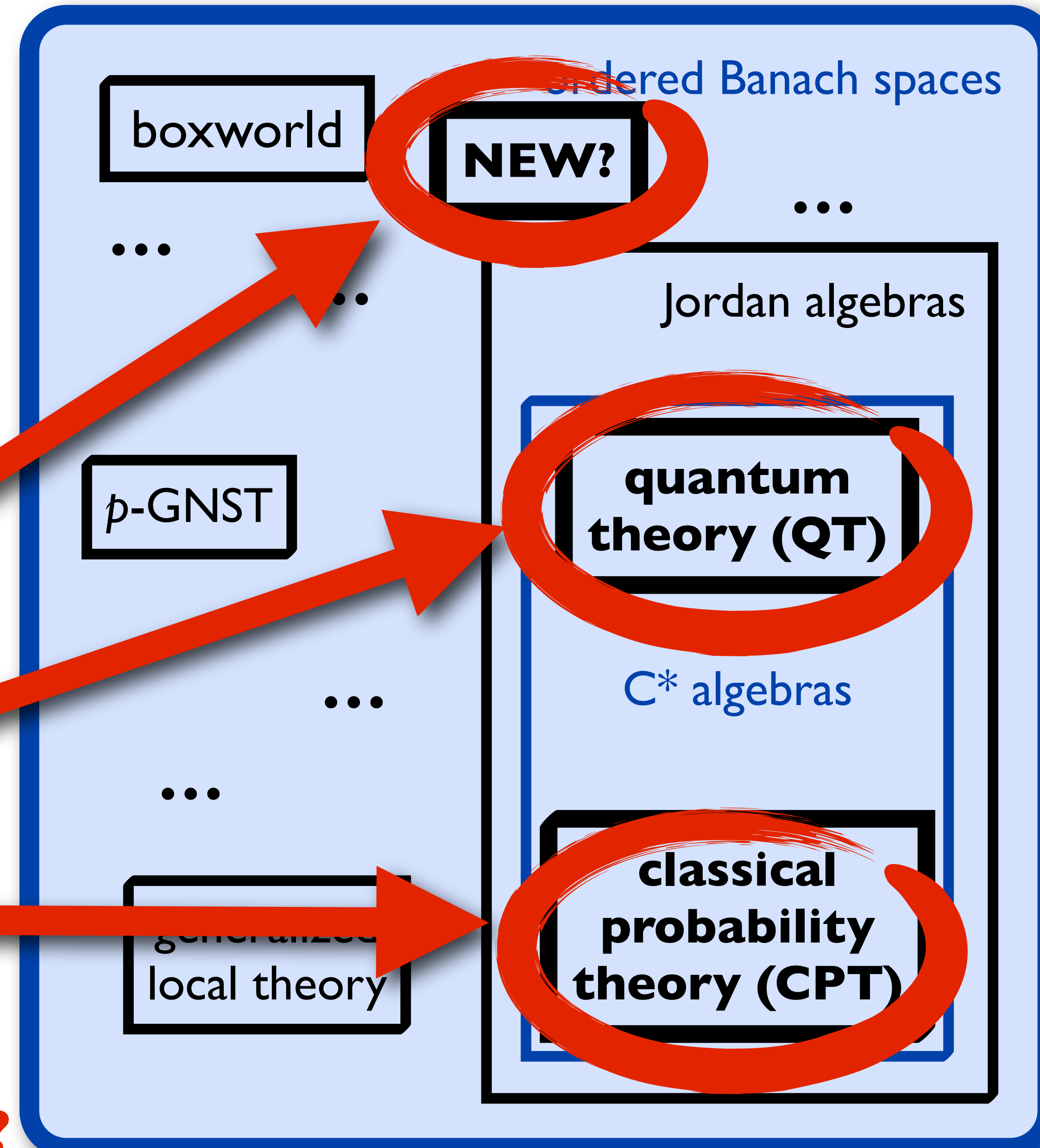


Natural theories **beyond QT?**



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Mechanics (Hamiltonian, phase space,...) + Probability Theory

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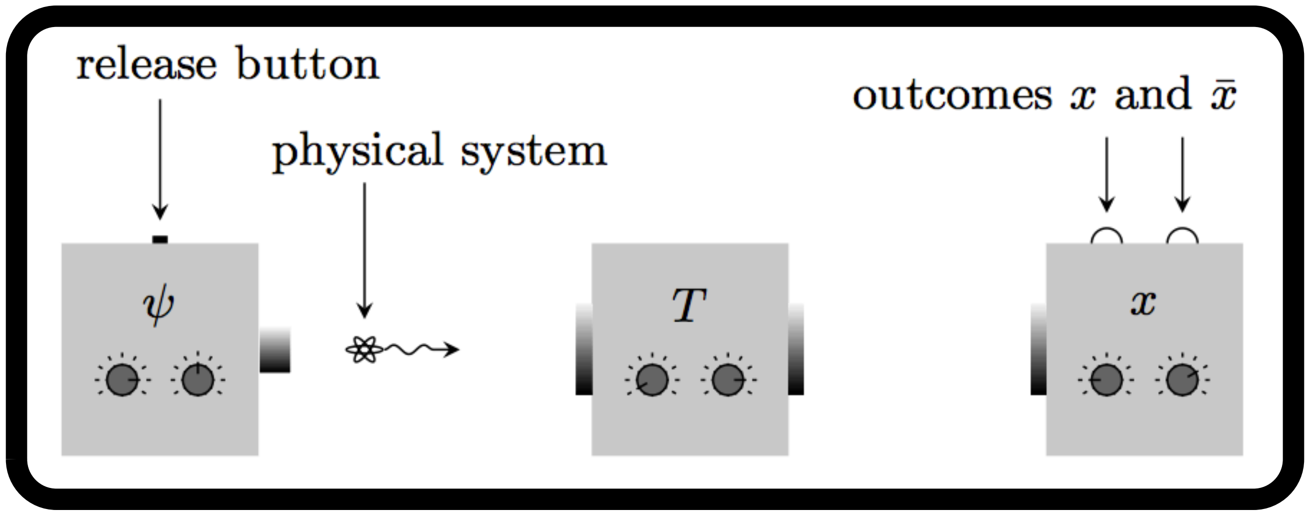

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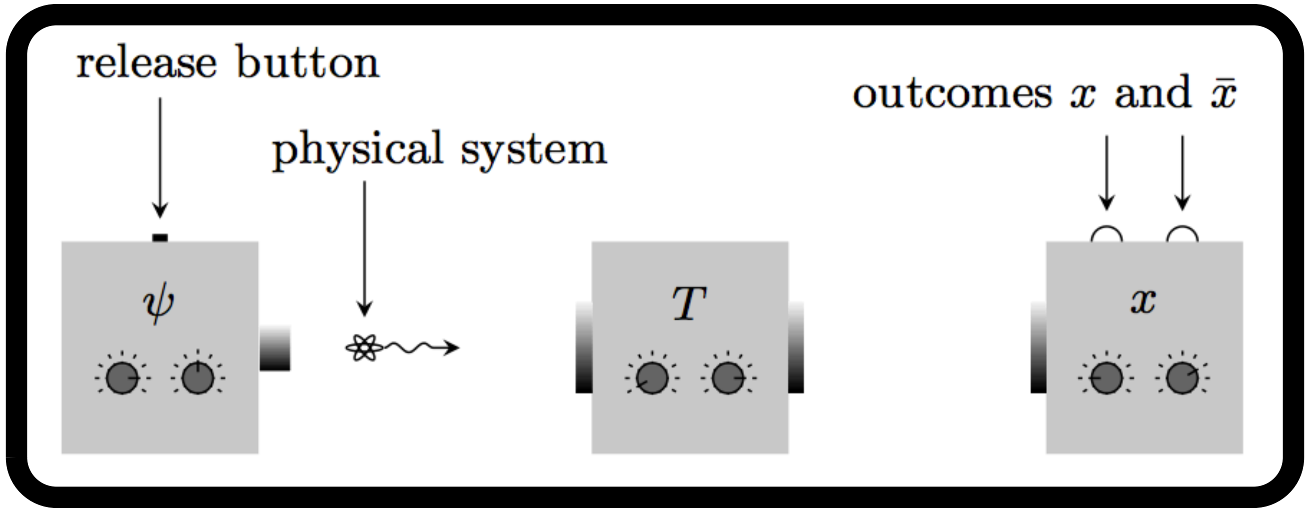

(Unnormalized) state ω =
list of all probabilities of „yes“-outcomes of all possible measurements.

$$\omega = (p_1, p_2, p_3, p_4, p_5, p_6, \ldots)$$

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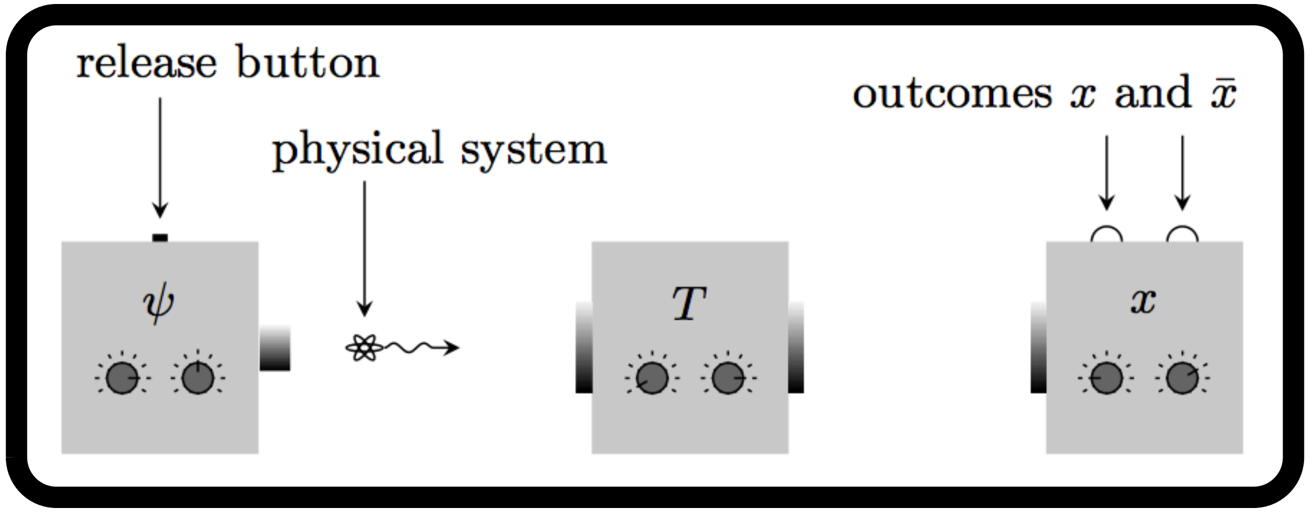

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Sometimes, all ω span a finite-dimensional subspace. Ex.: Qubit

- What‘s the prob. of „spin up“ in X-direction?
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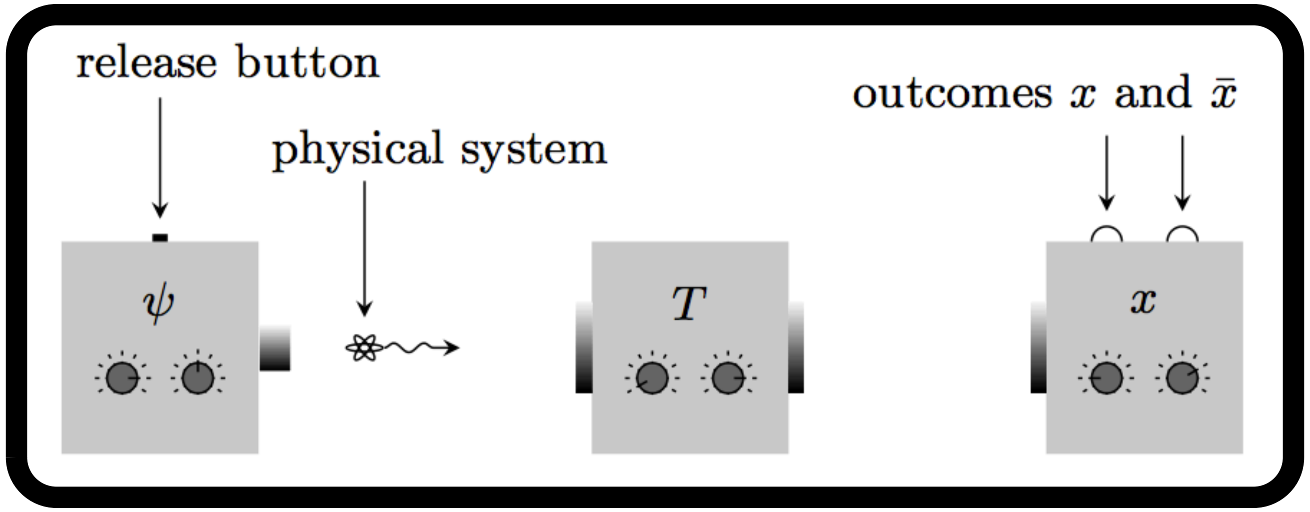

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Axiom IV: All state spaces are finite-dimensional.

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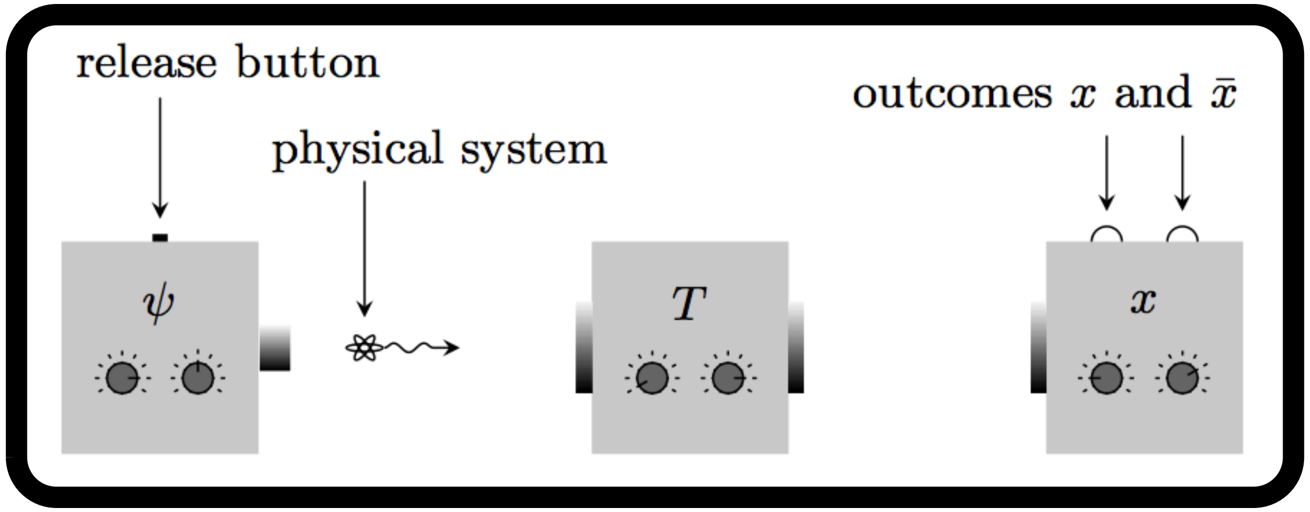

Prepare state ω or φ with prob. ½. Result: $\frac{1}{2}\omega + \frac{1}{2}\varphi$

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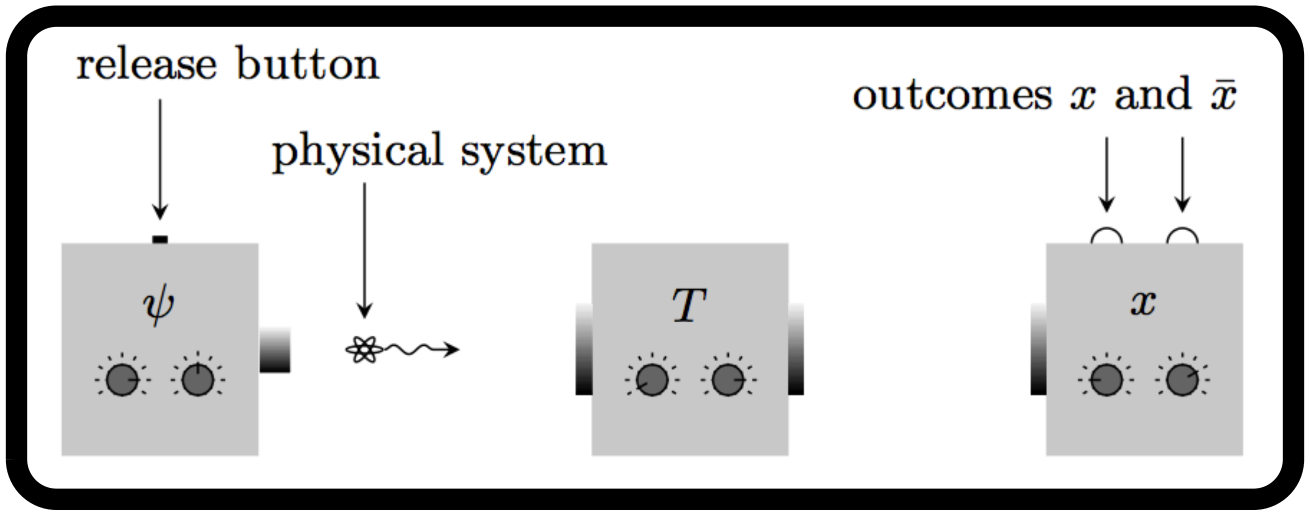

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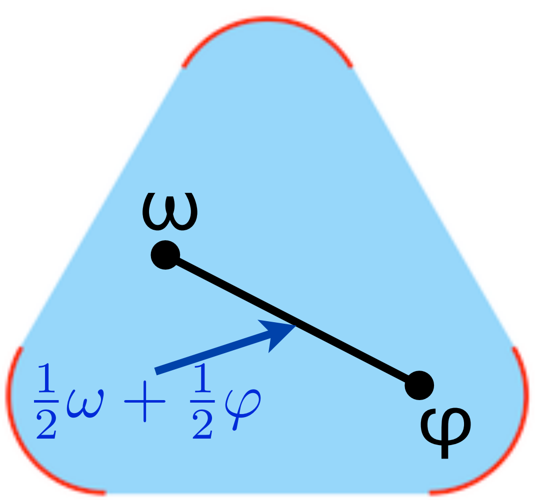

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Extremal points are pure states, others mixed.

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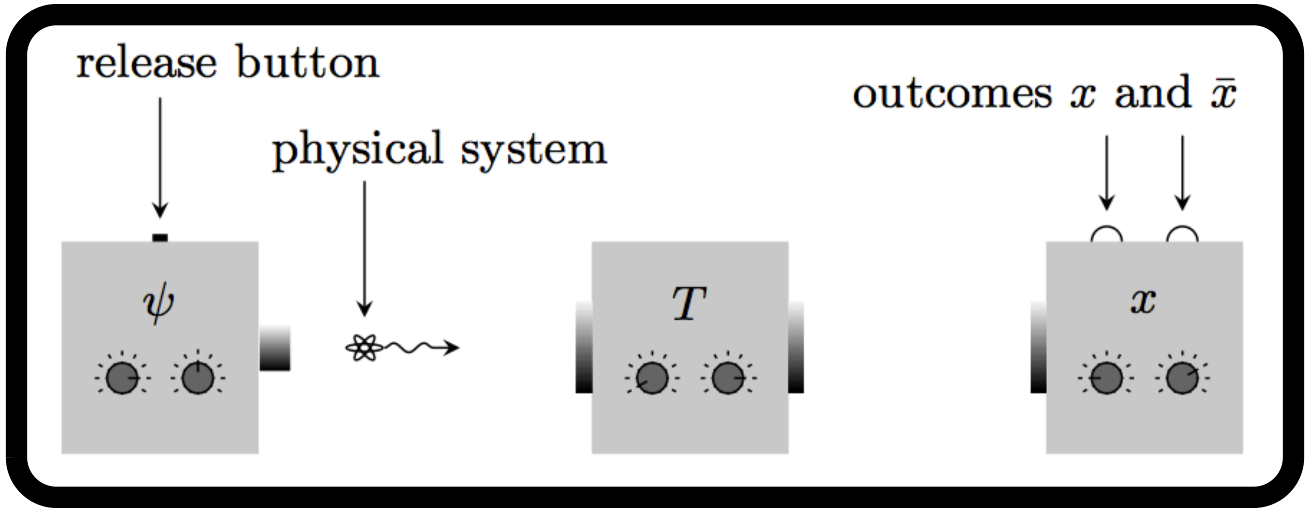

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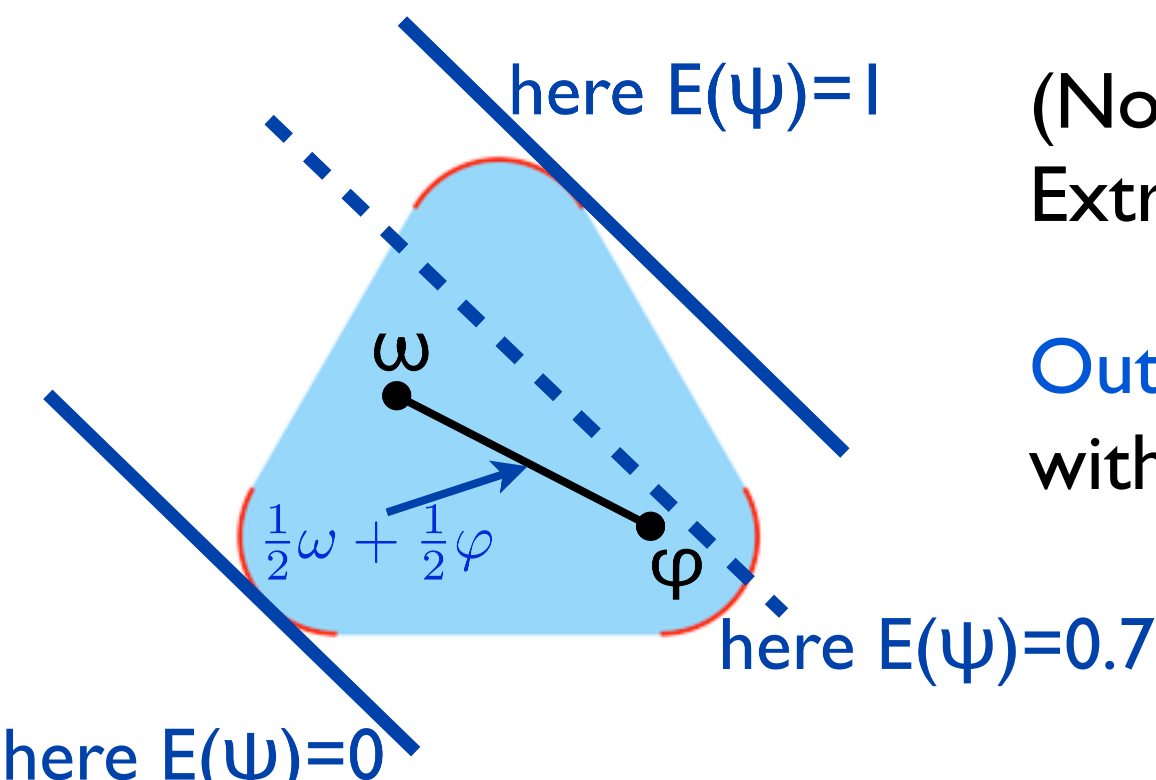

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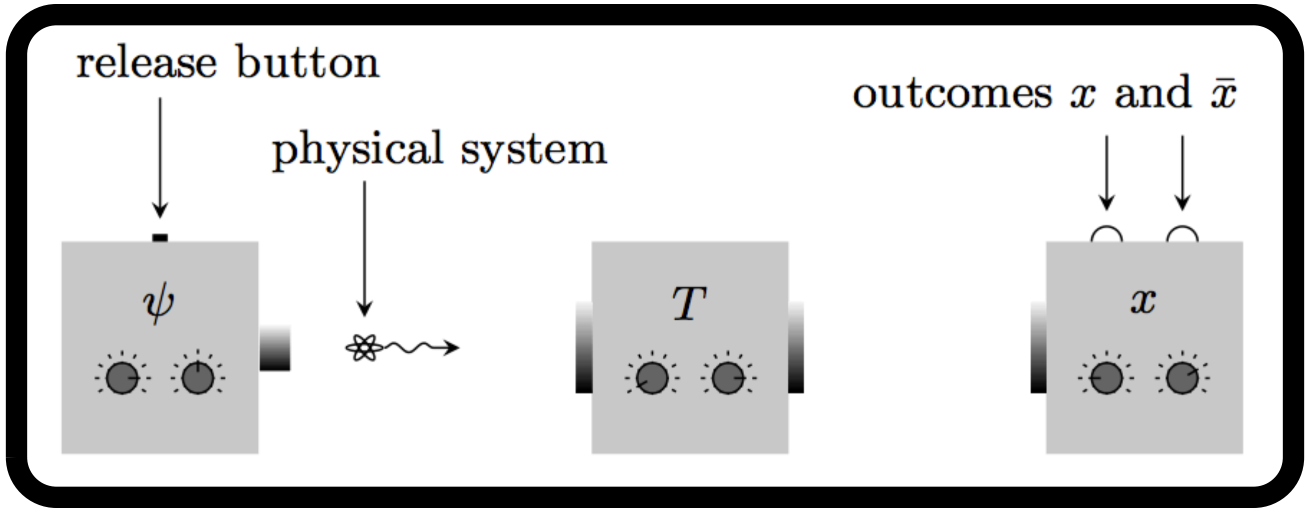

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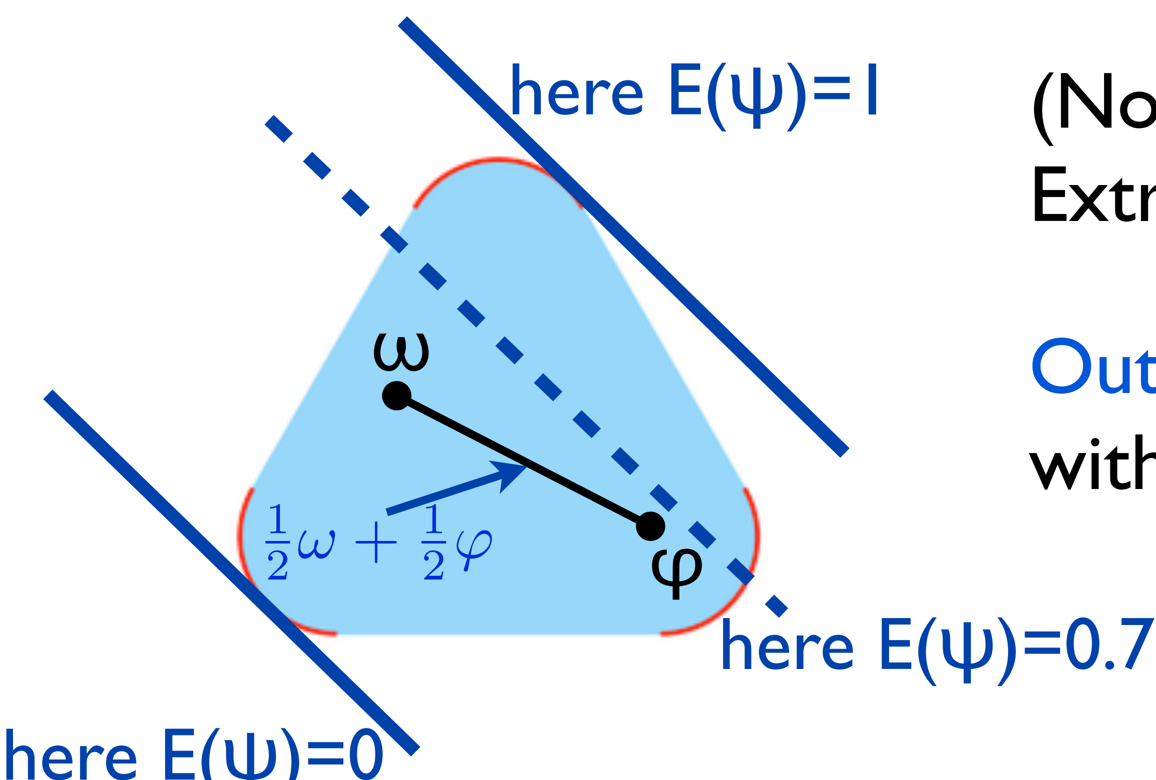

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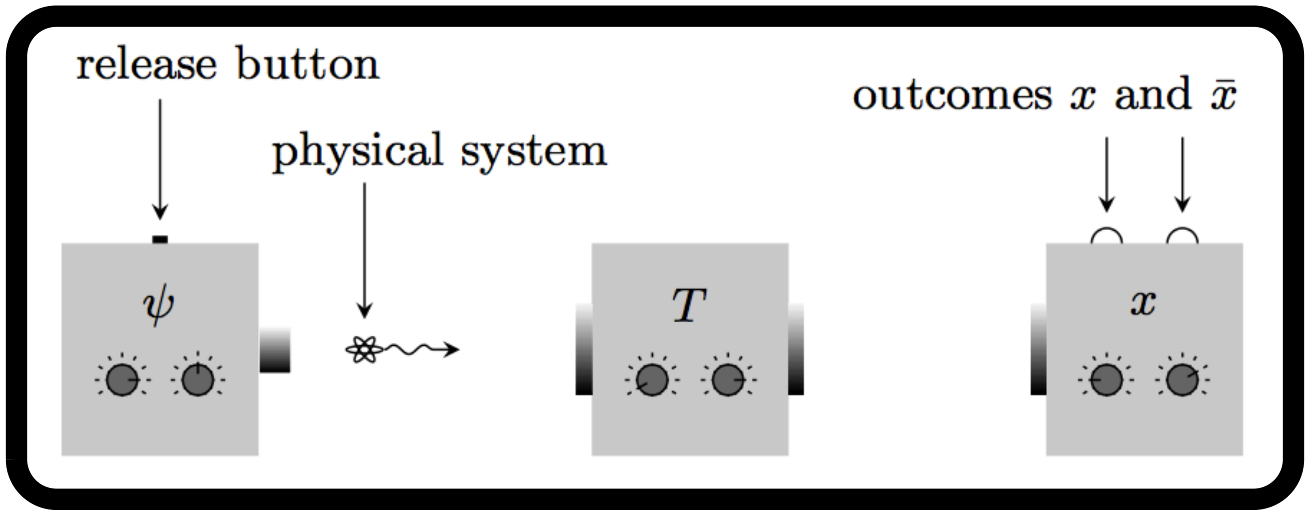

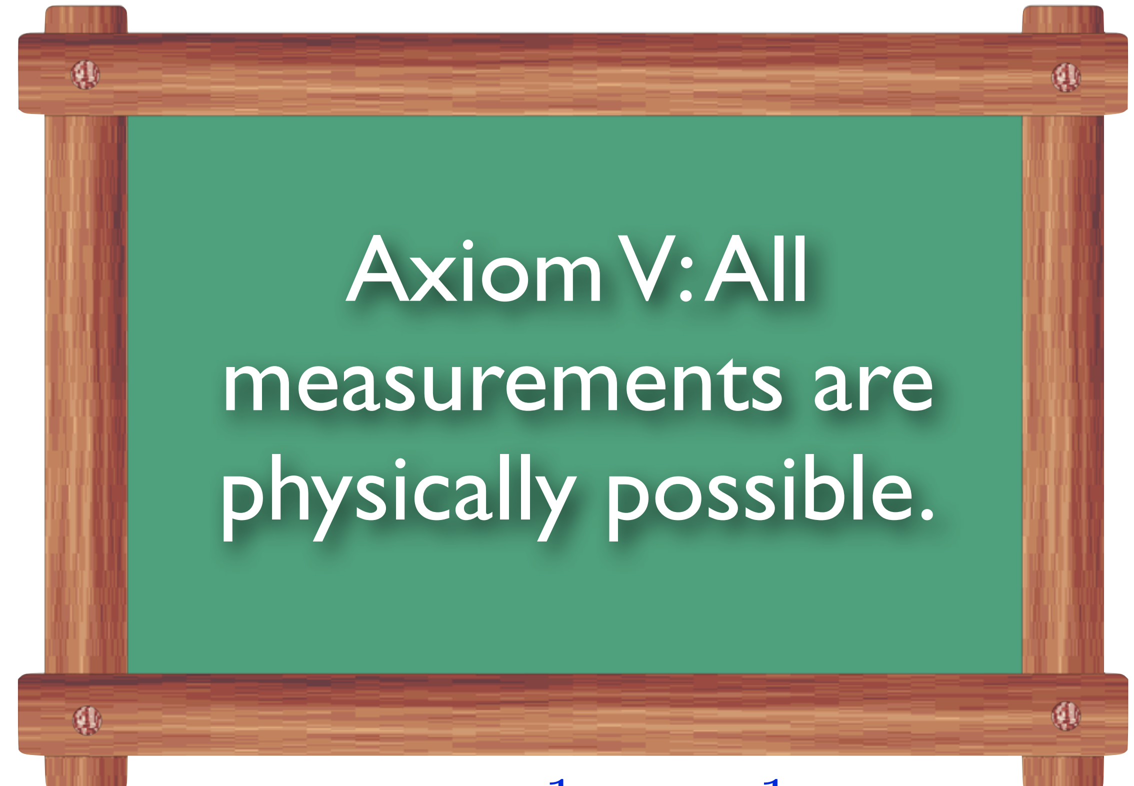

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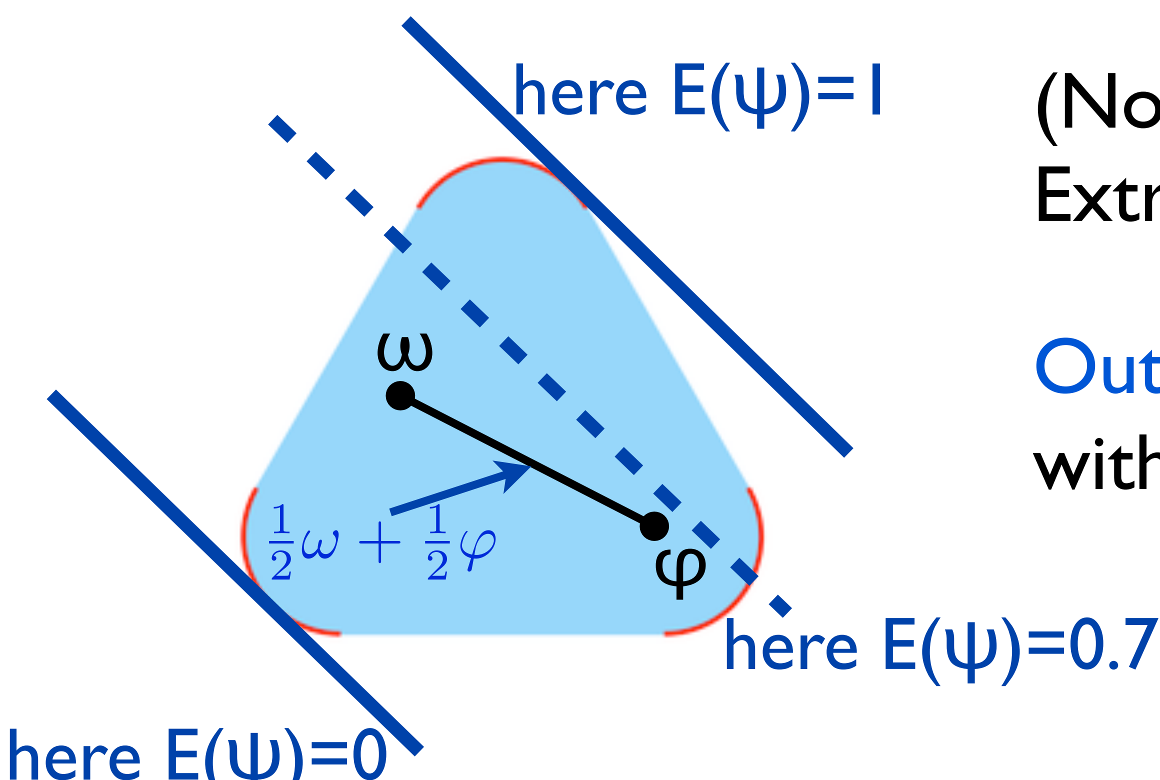

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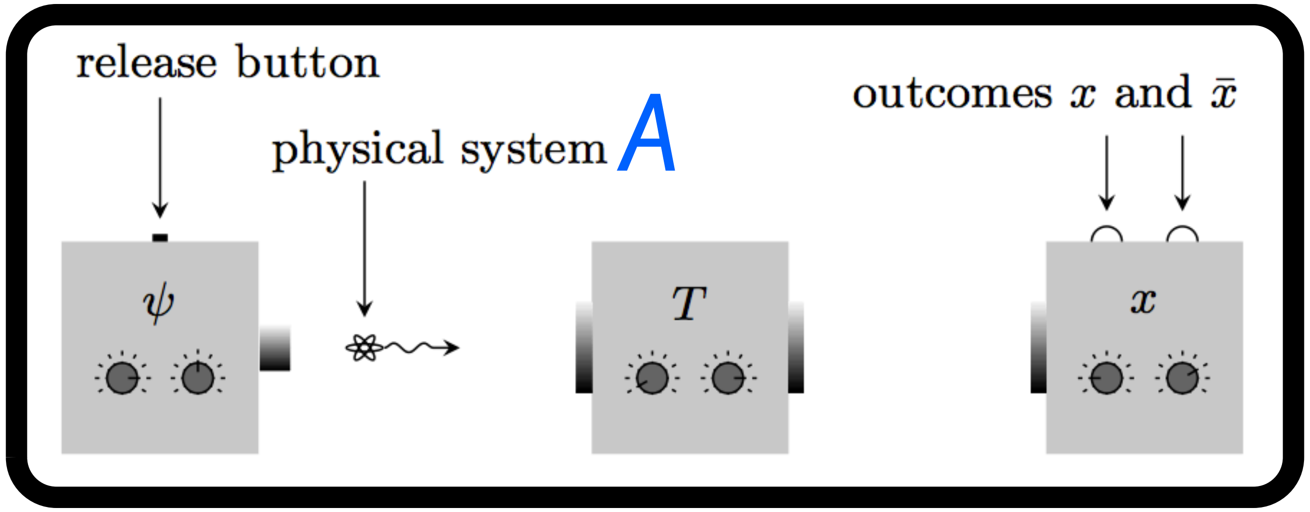

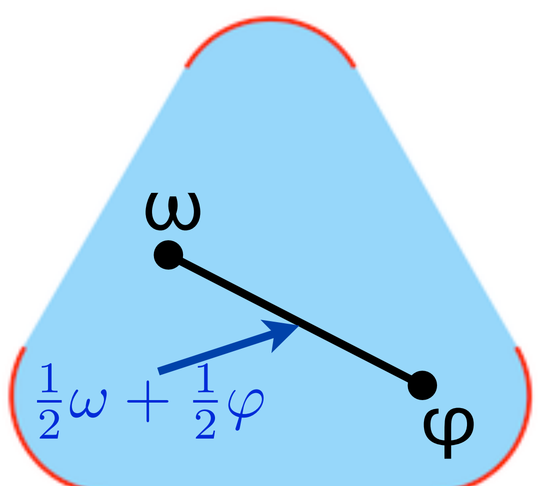

Normalized state space Ω_A

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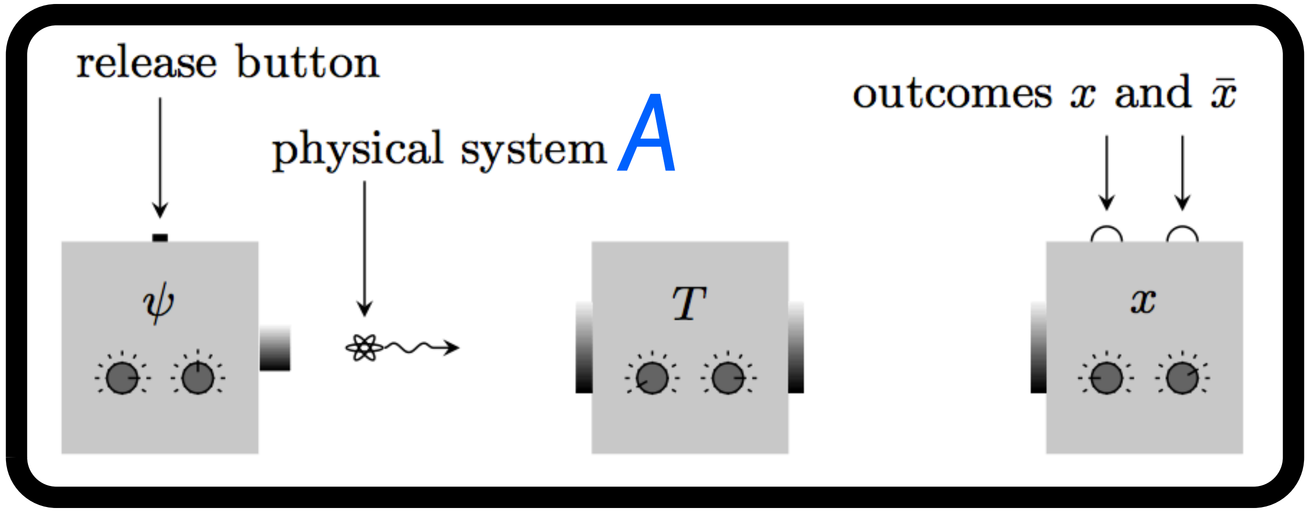
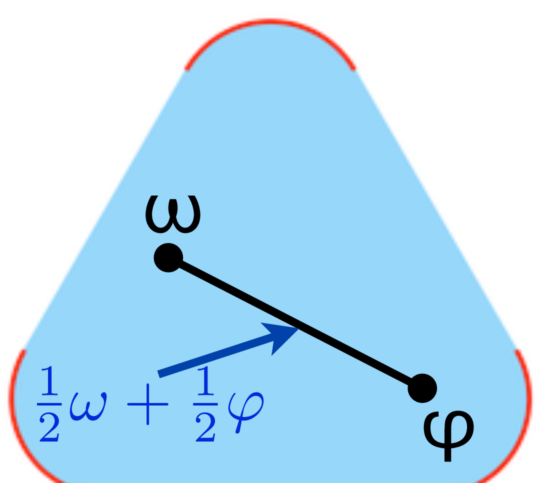


Transformations T map (unnormalized) states to states, and are linear.



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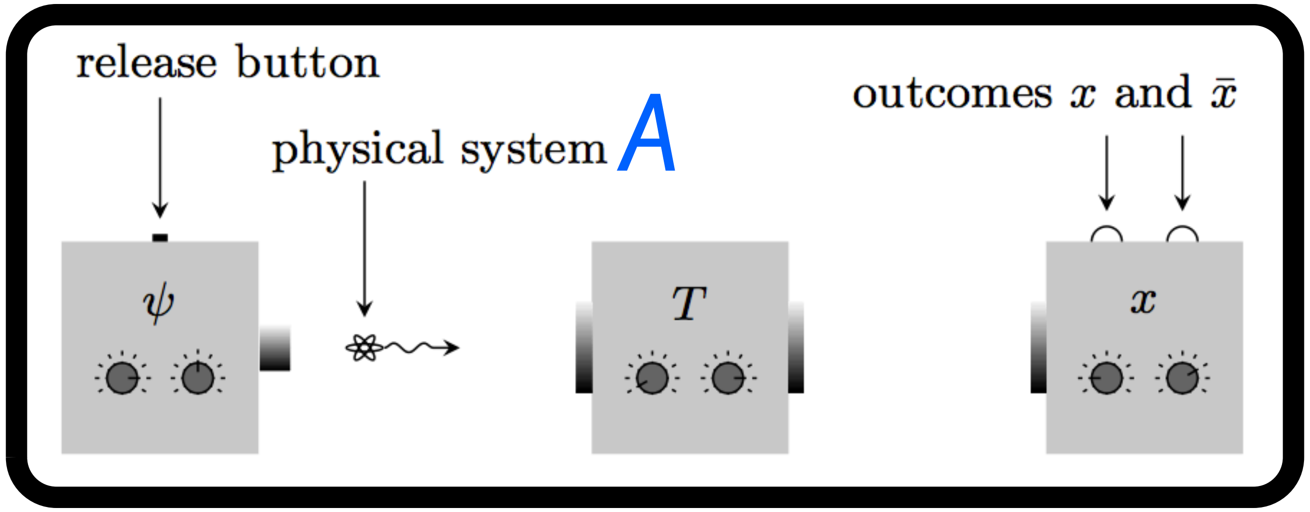

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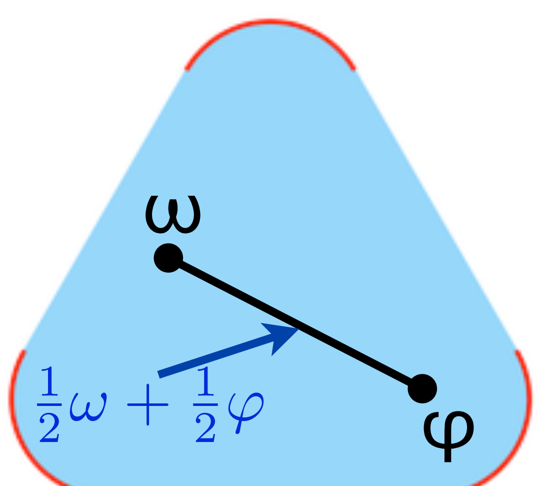
They are symmetries of state space: $T(\Omega_A) = \Omega_A$



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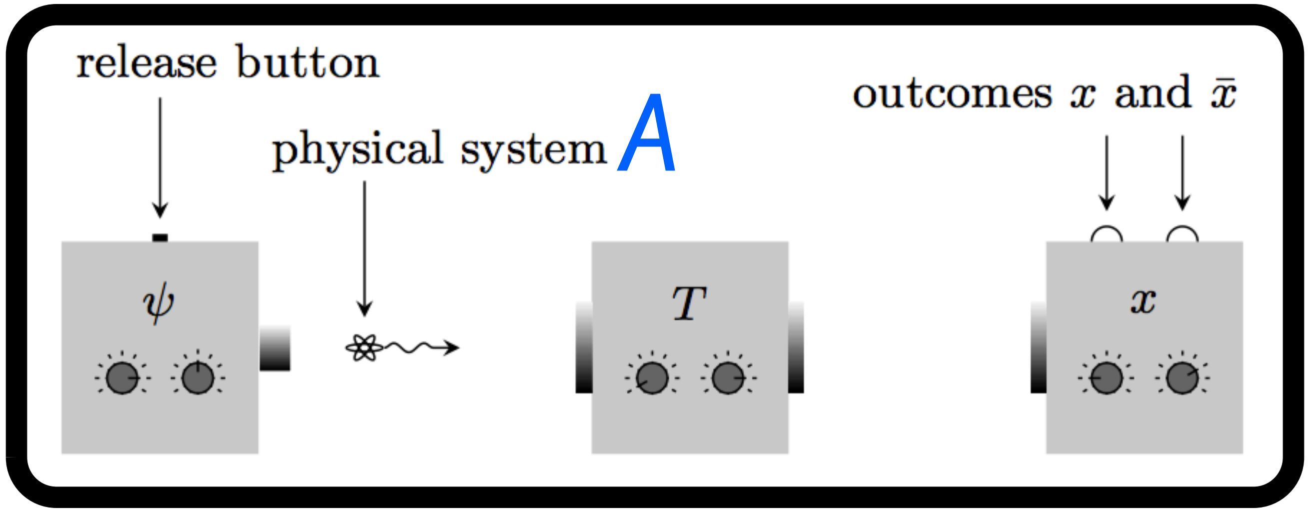

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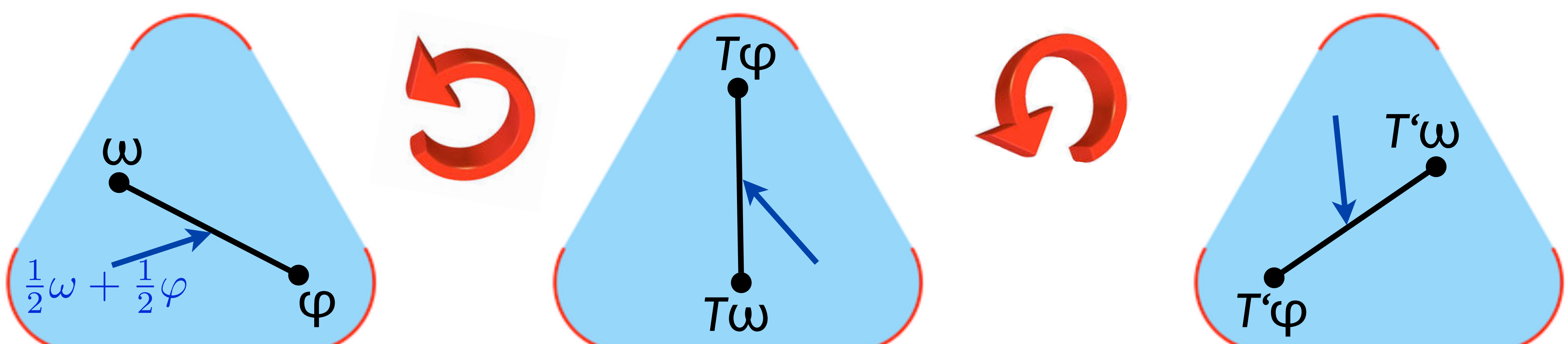

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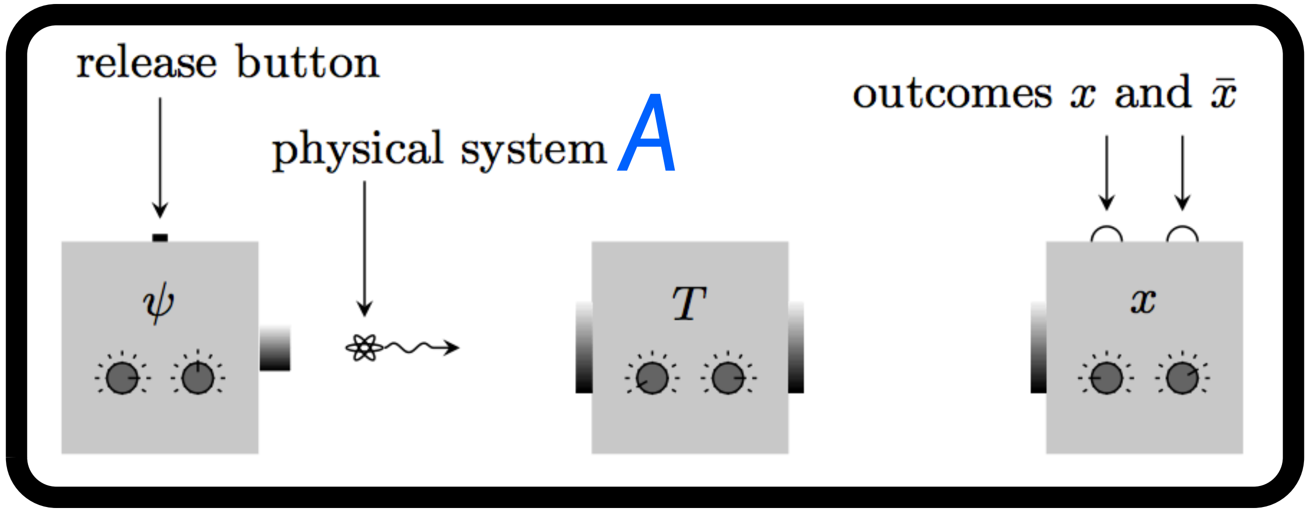

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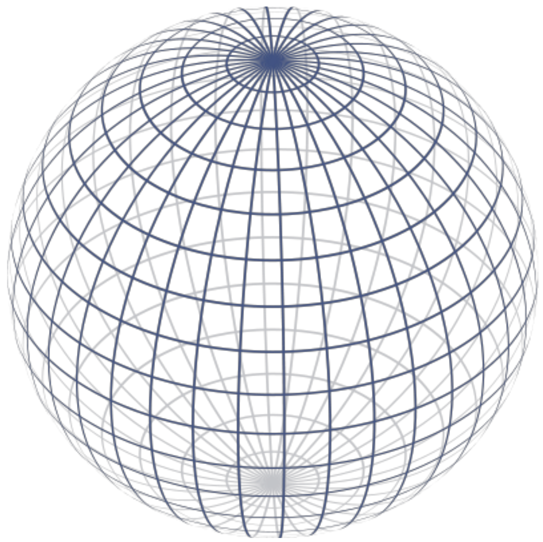

Qubit: Ω_A is the 3D unit ball,

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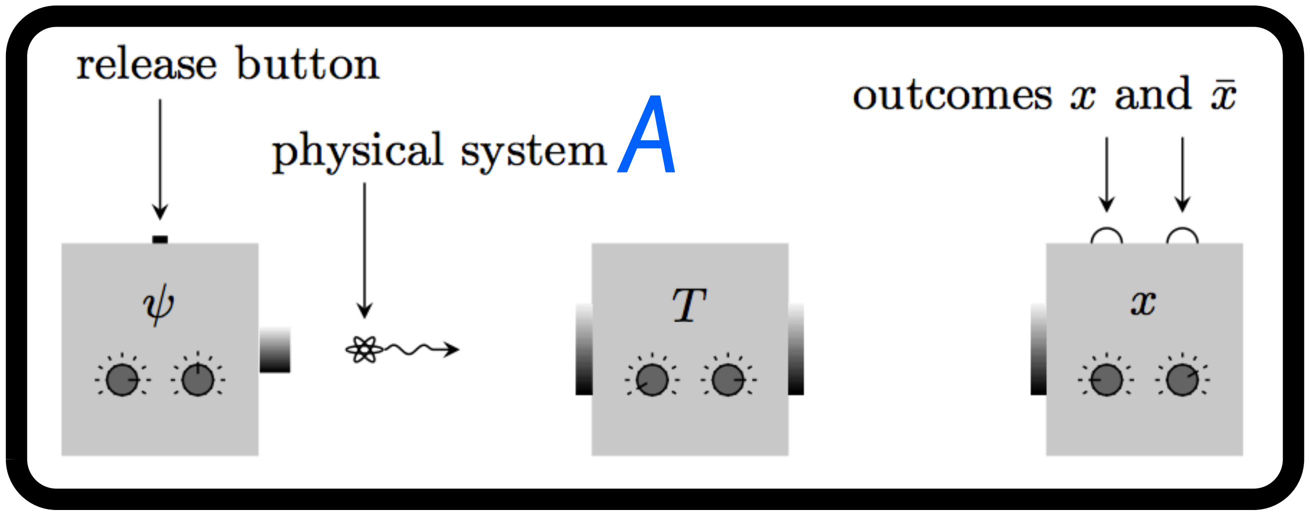

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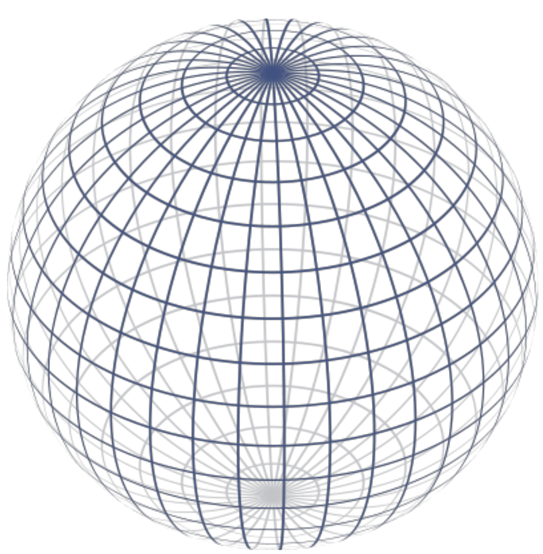

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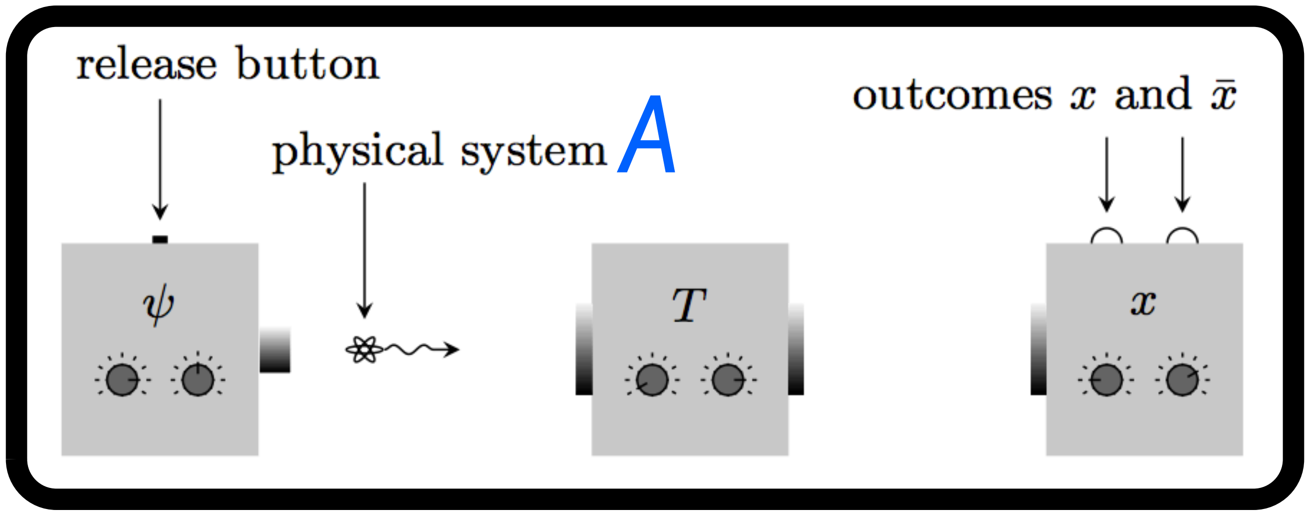
state space

reversible transformations

2. General Probabilistic Theories



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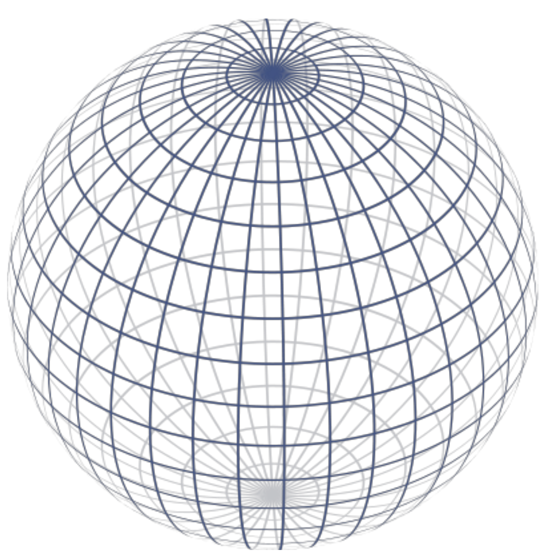

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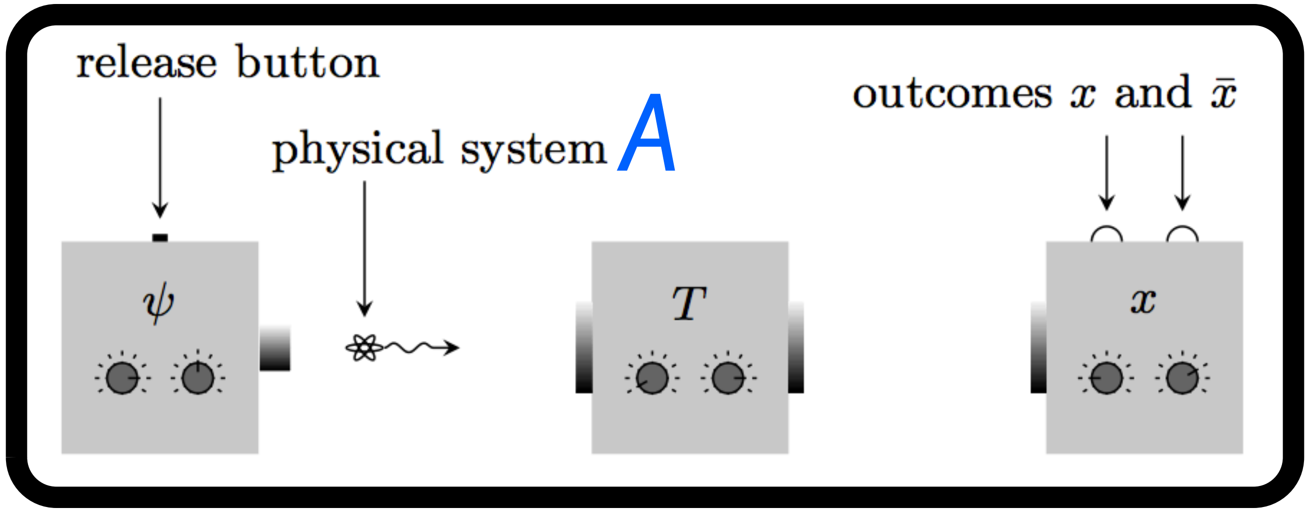

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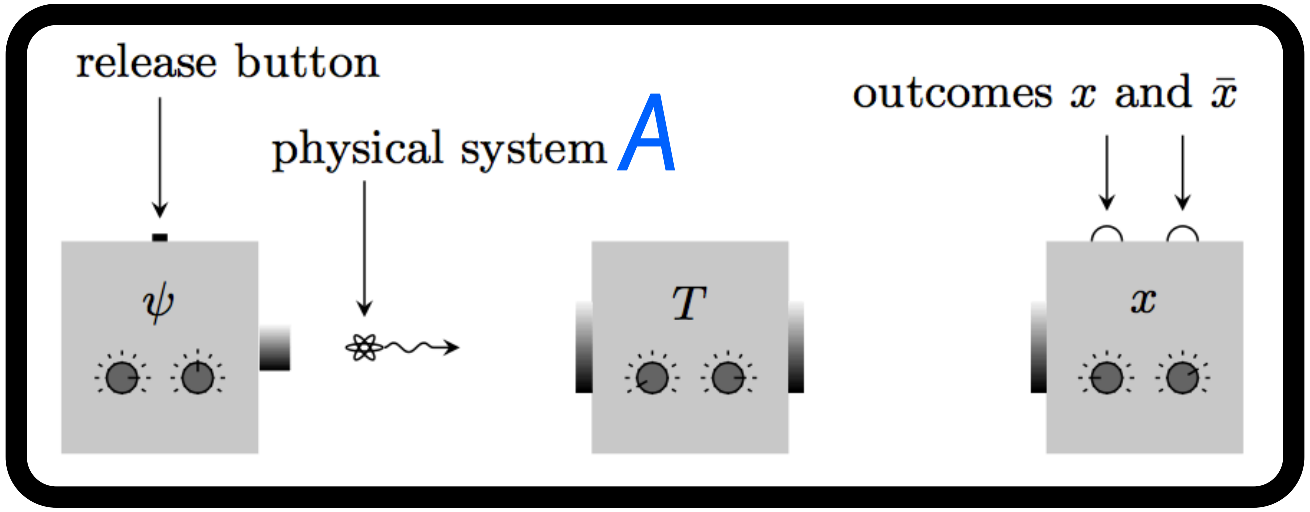

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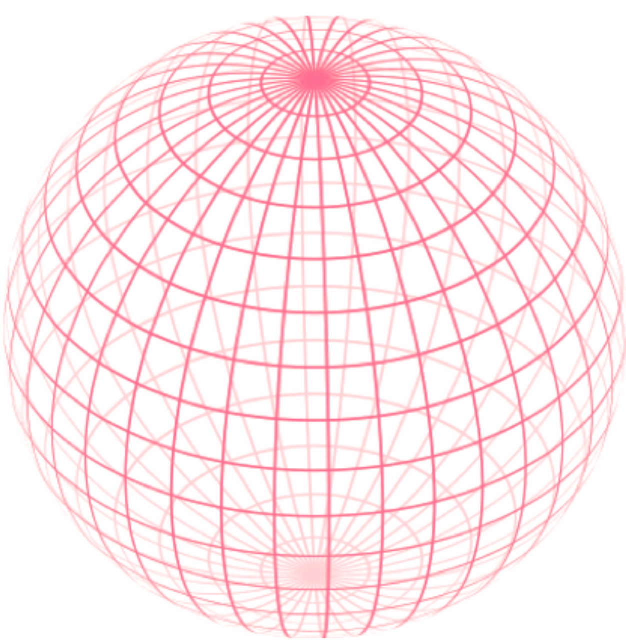

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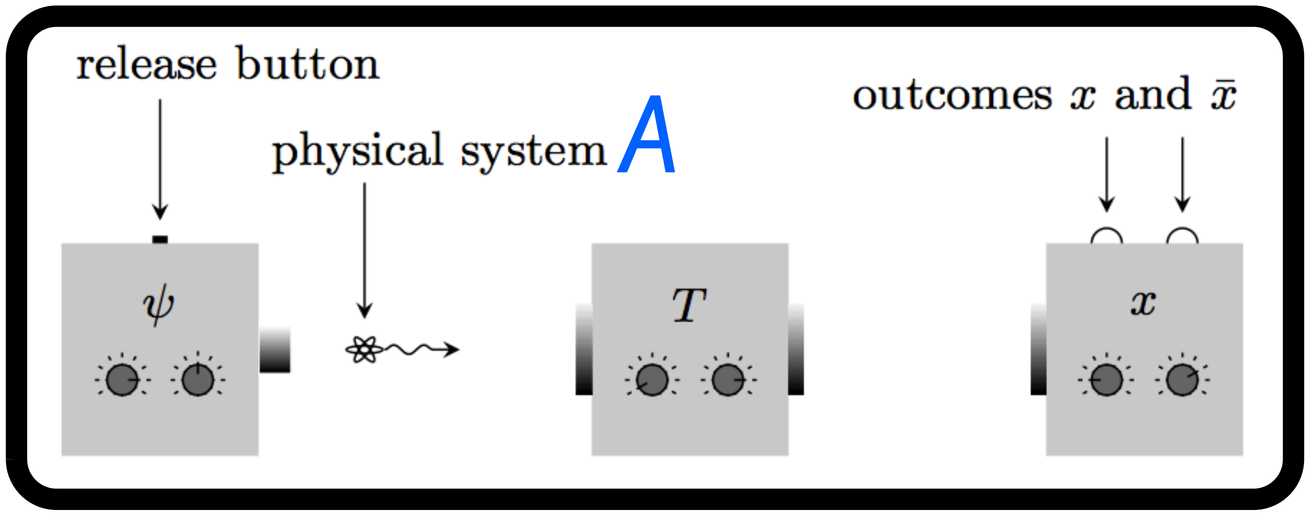

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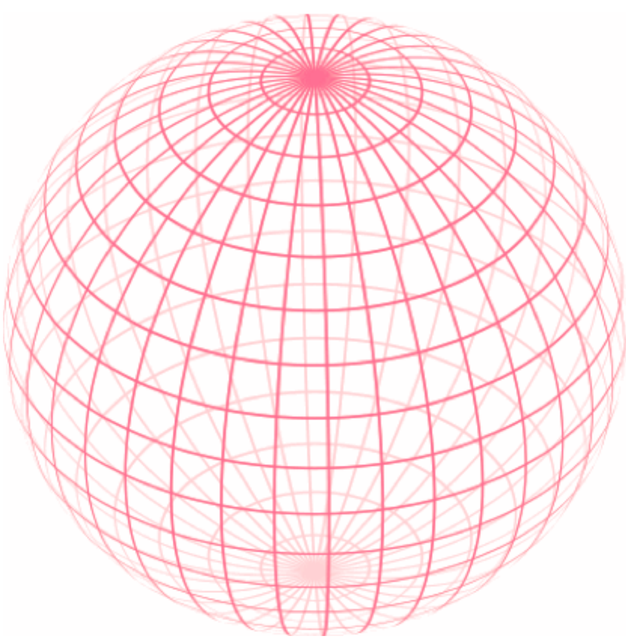

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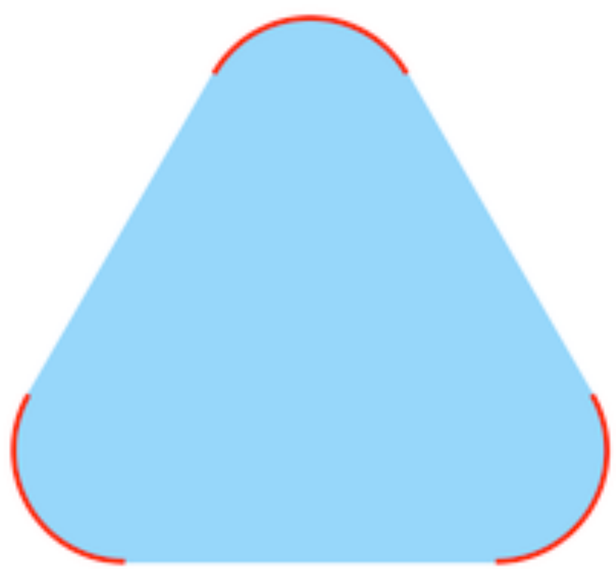

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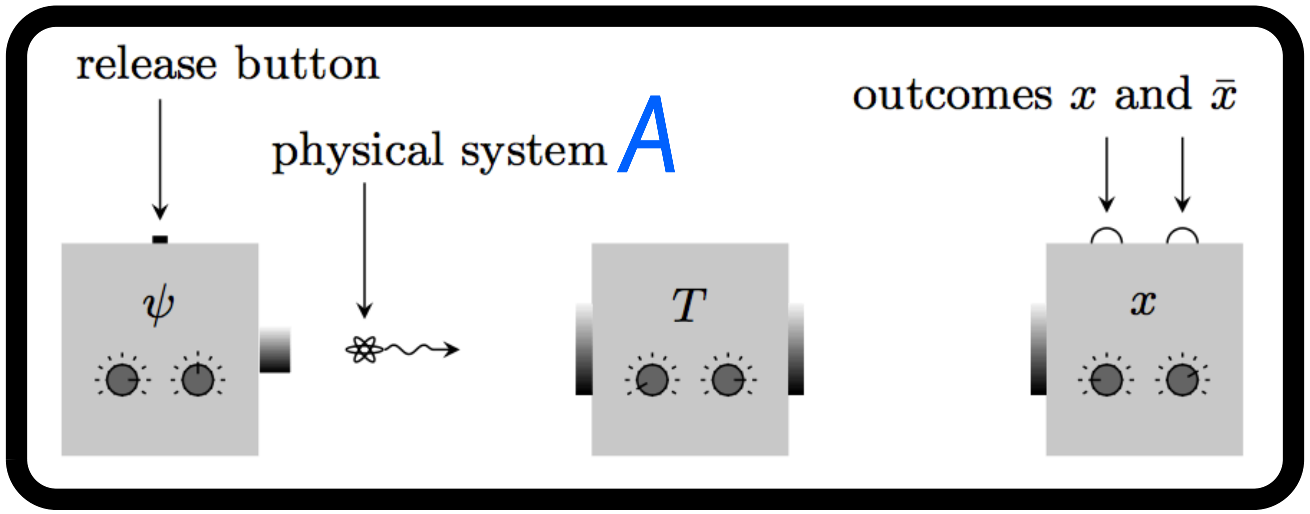

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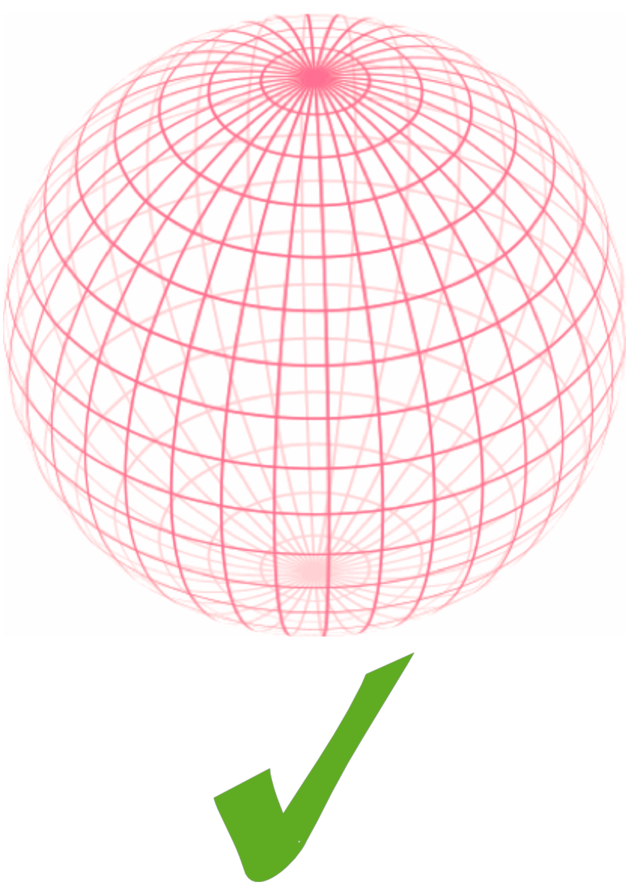

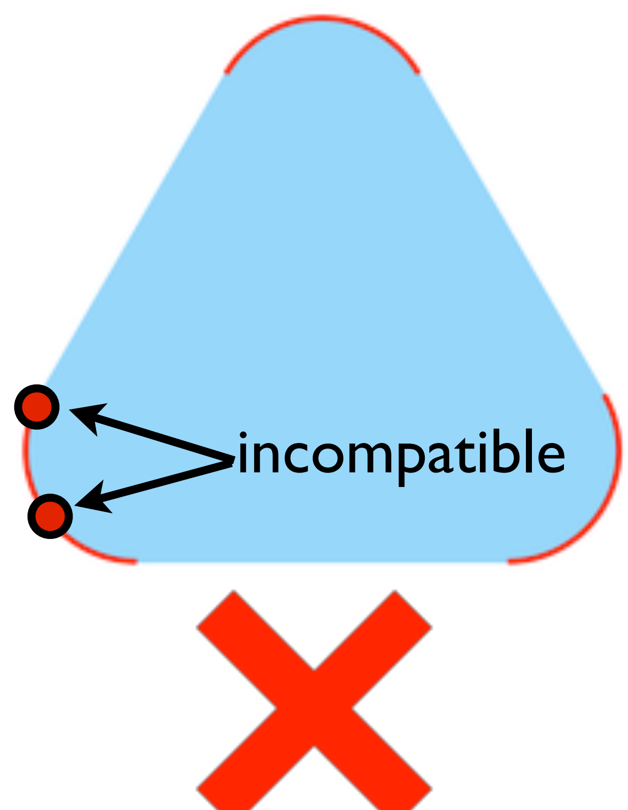

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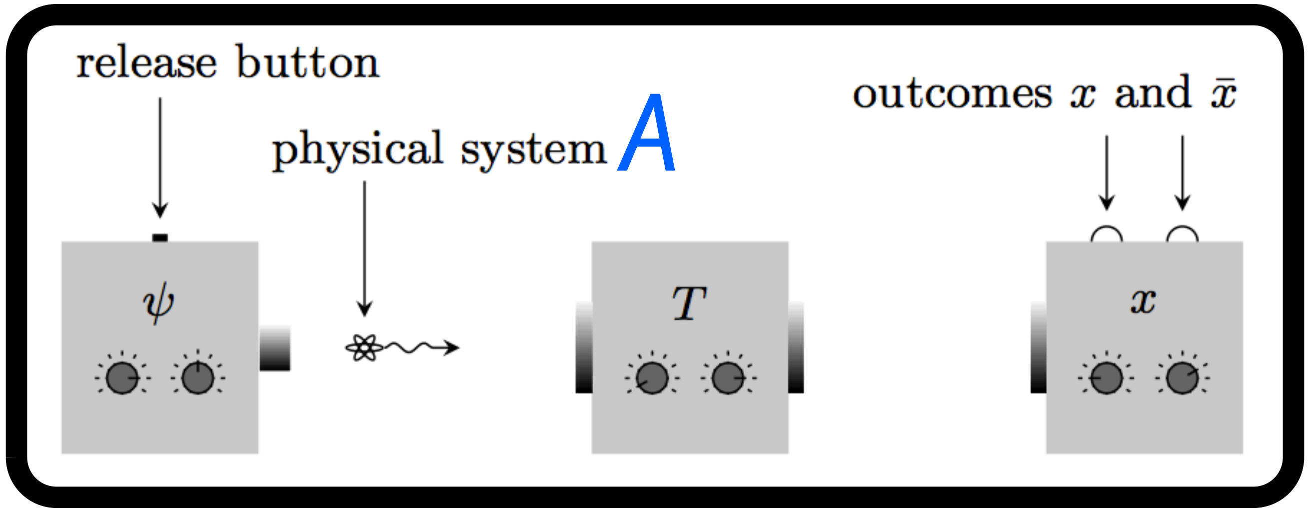

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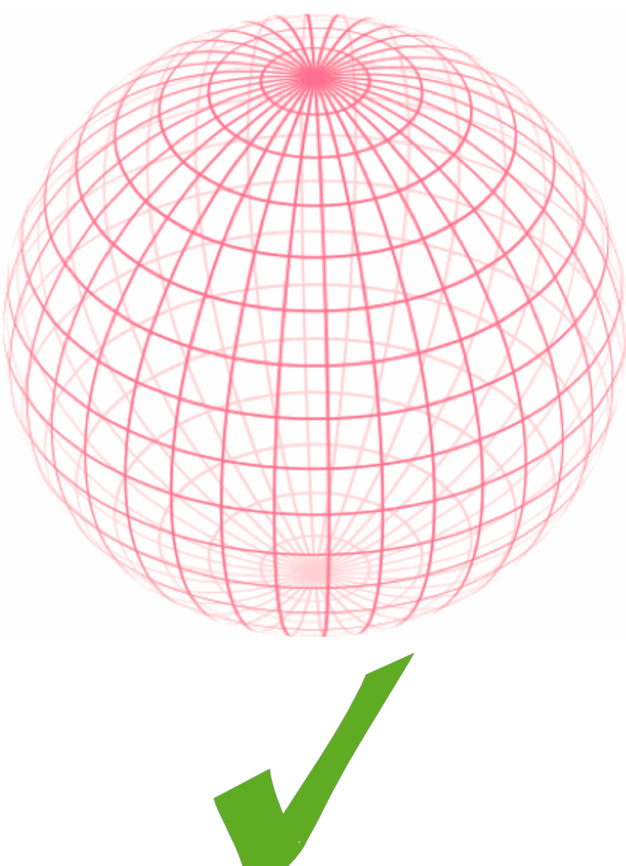

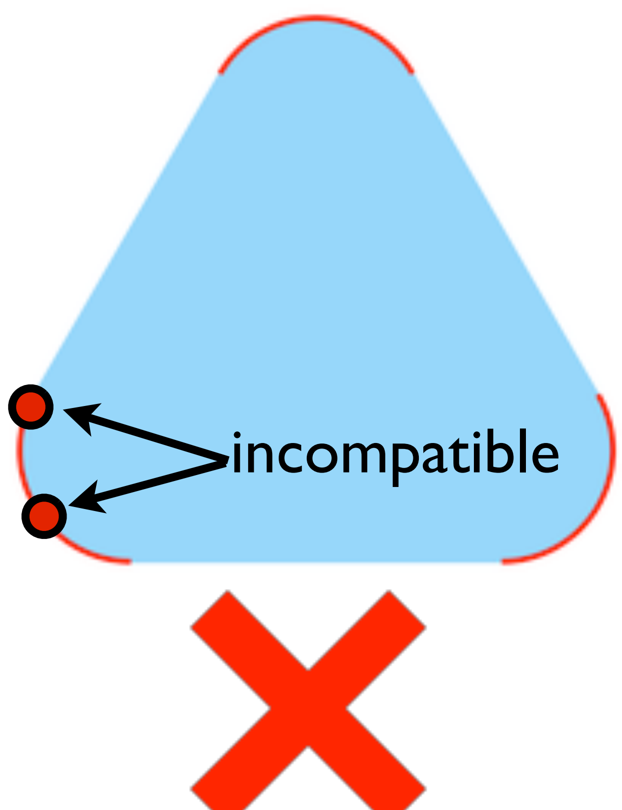

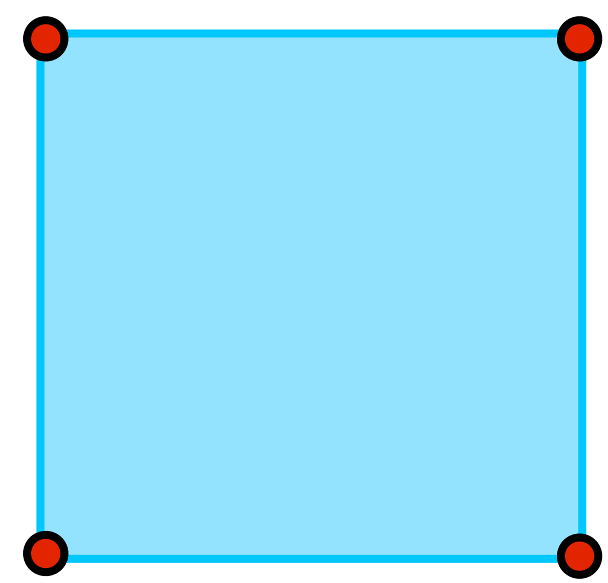

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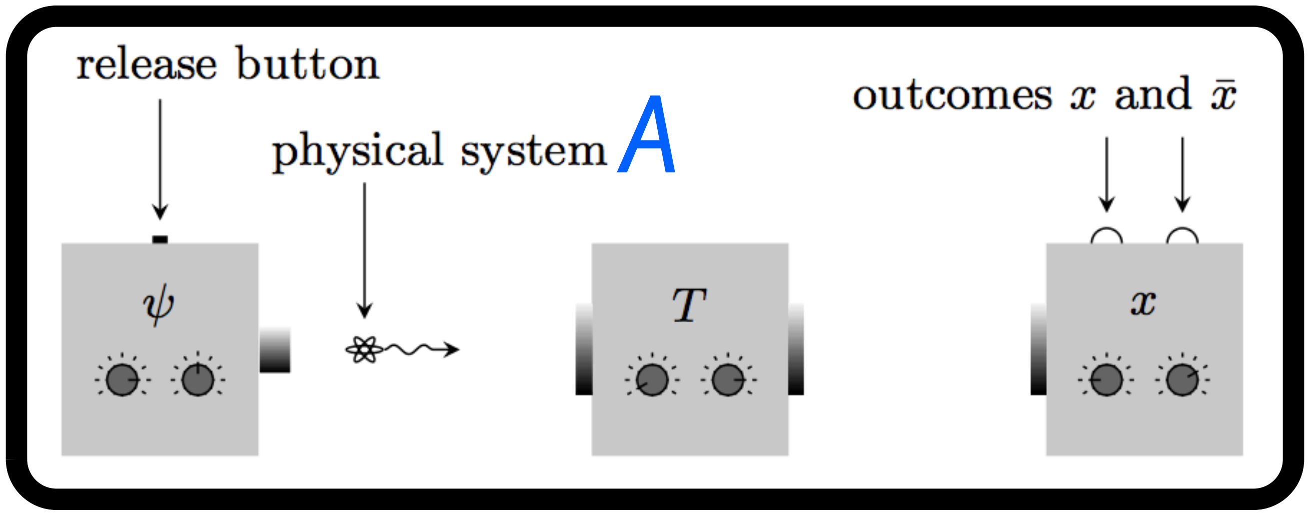

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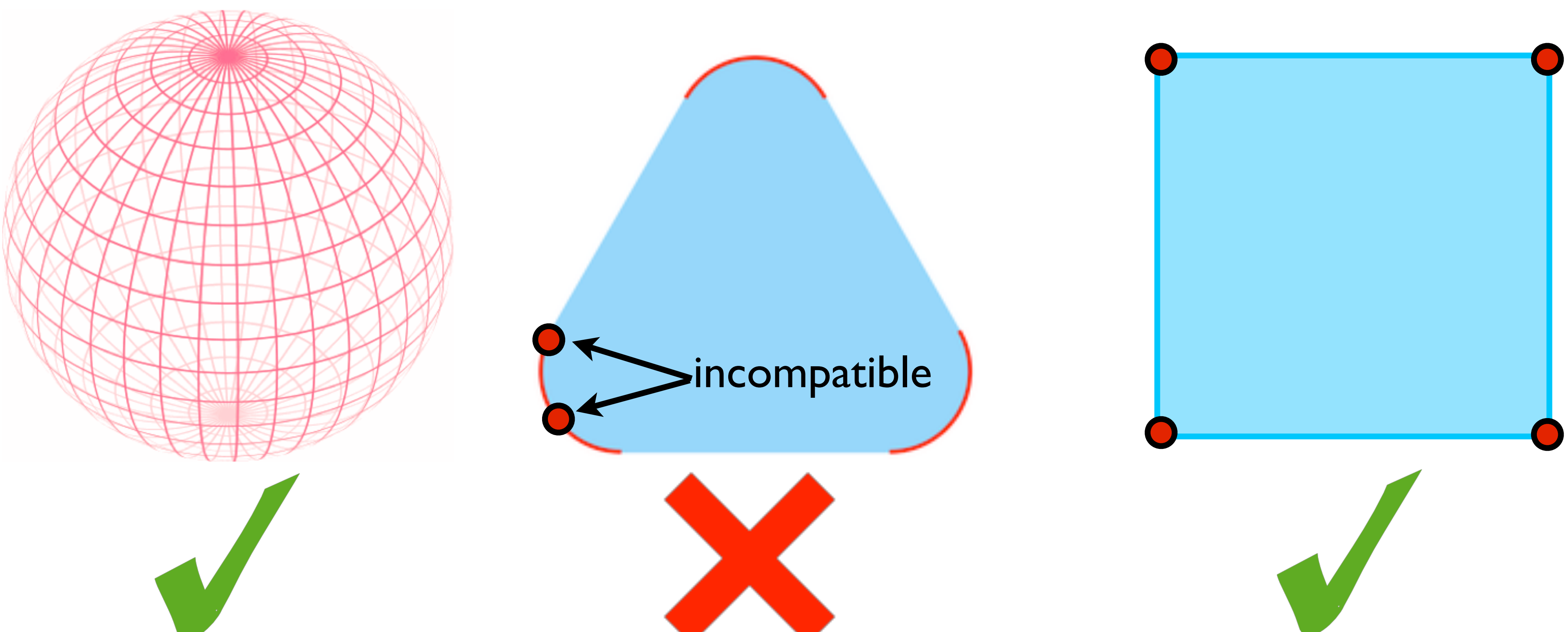

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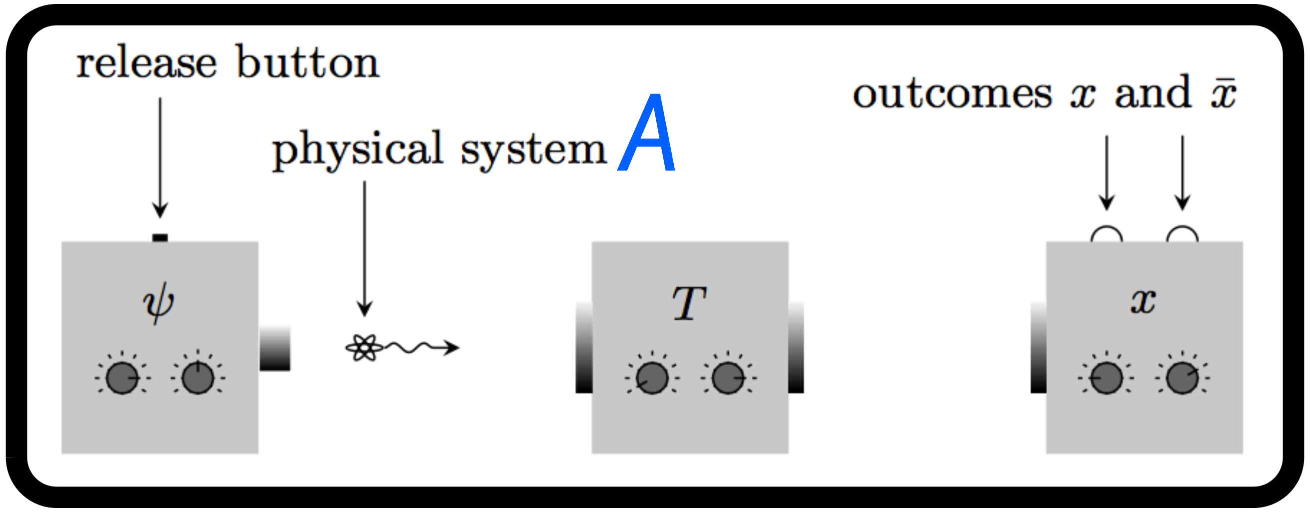

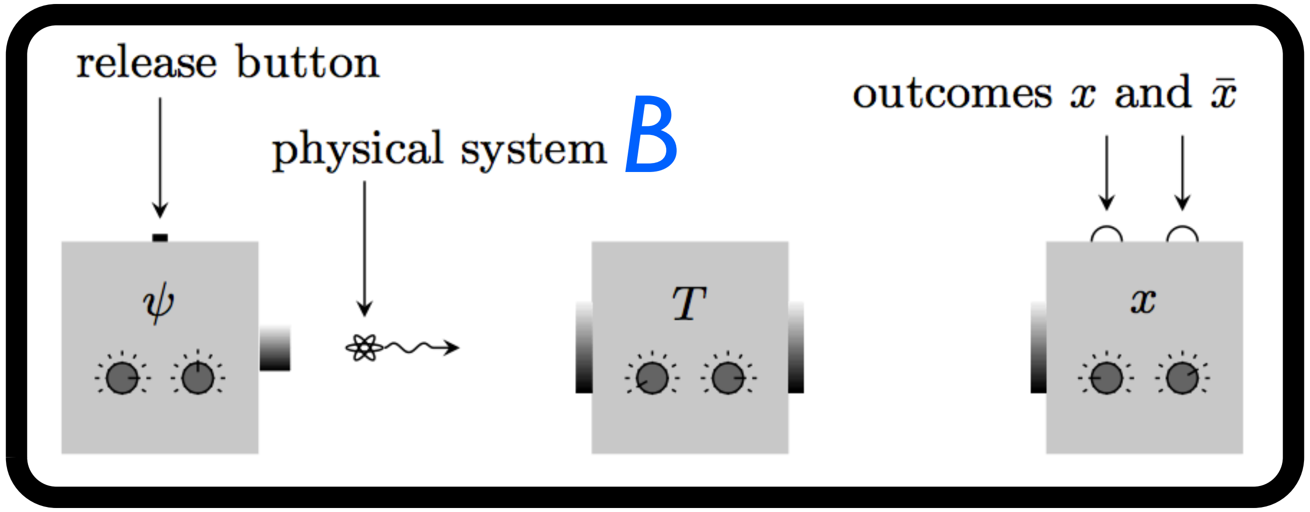

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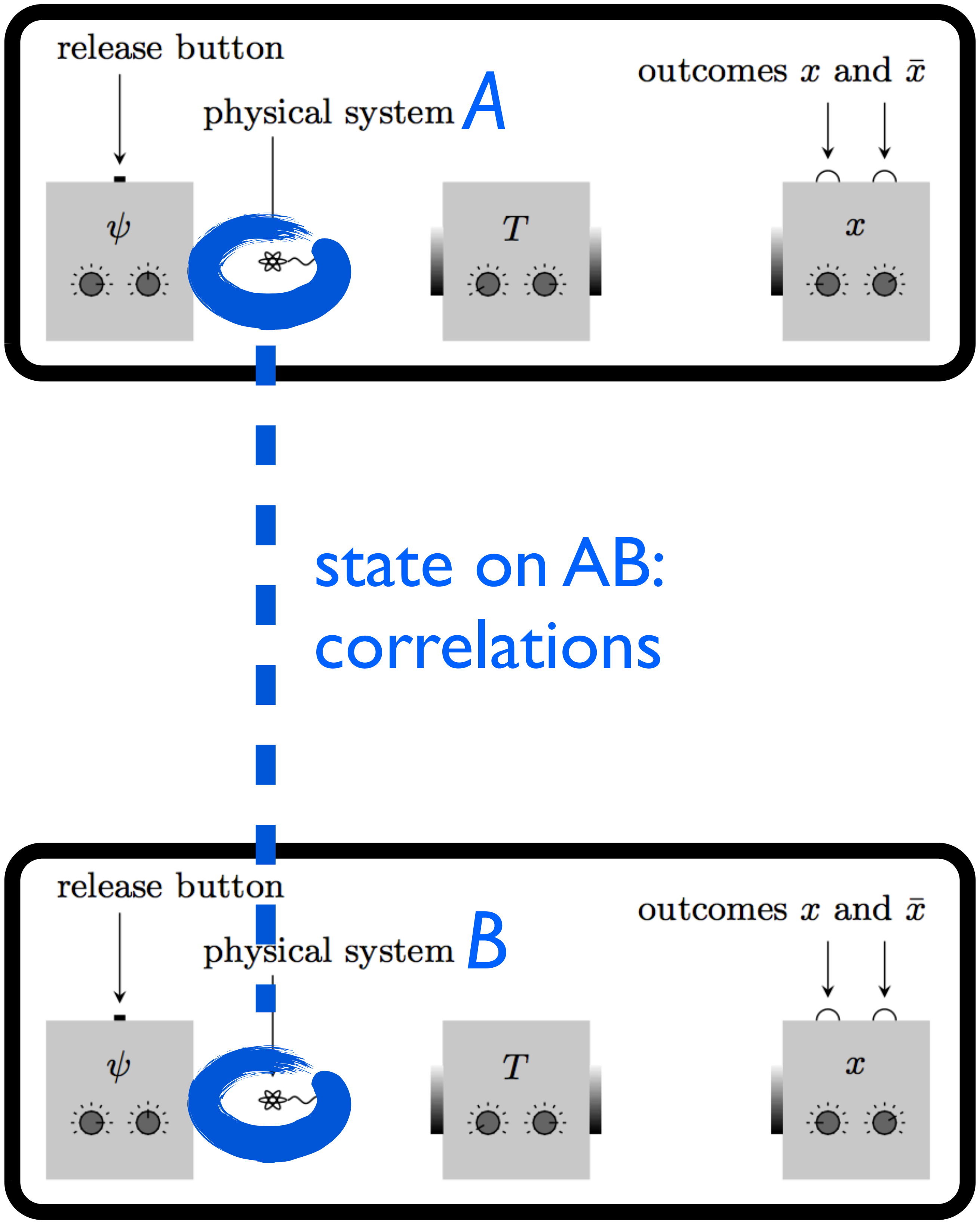

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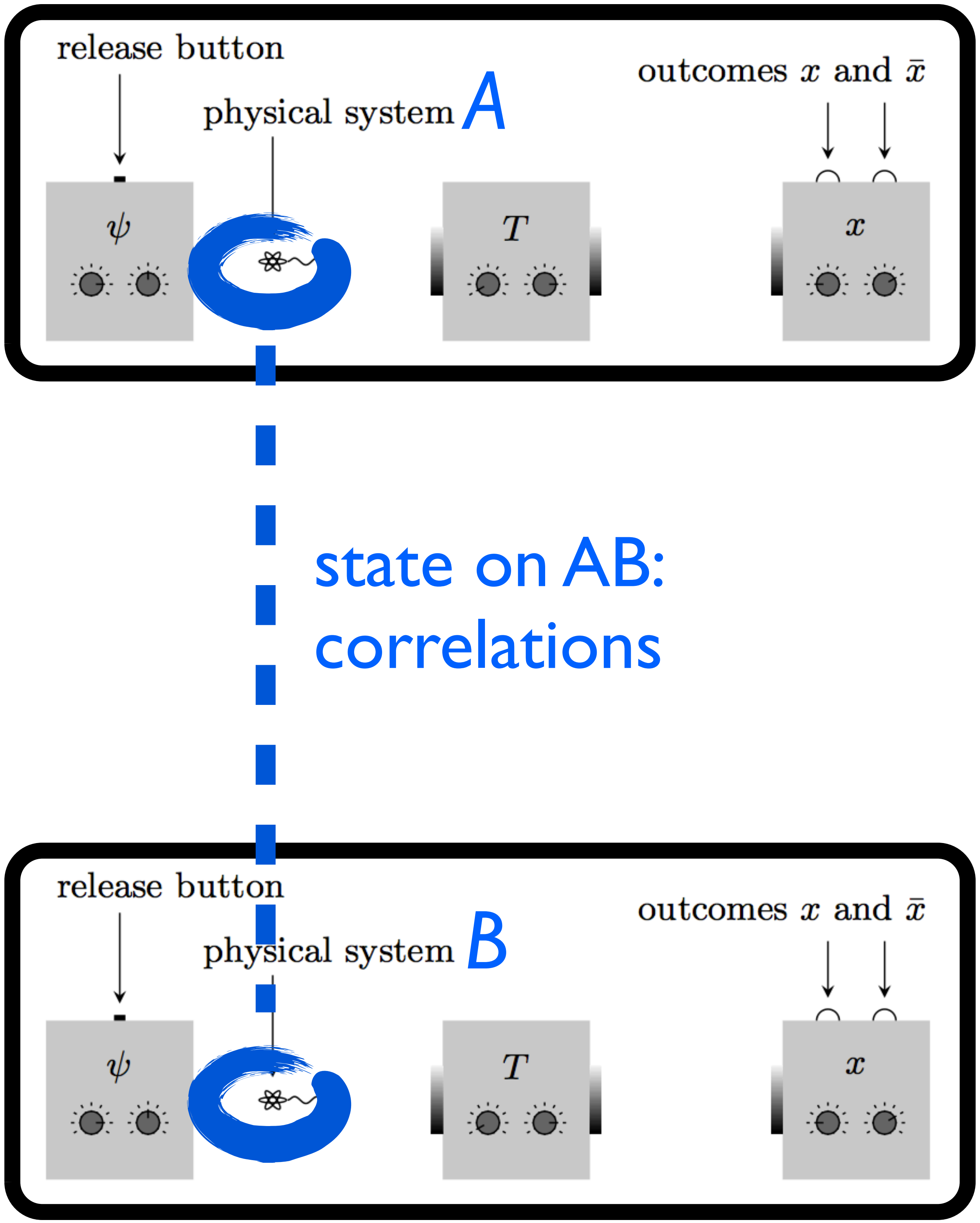

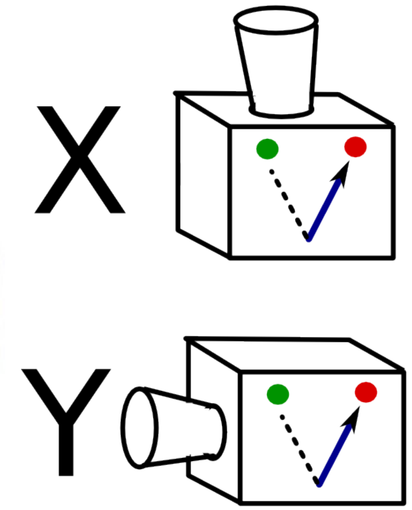

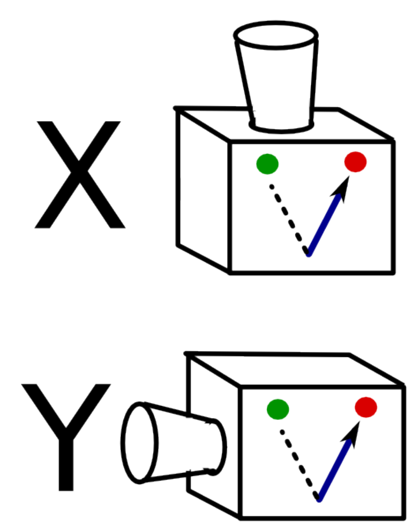

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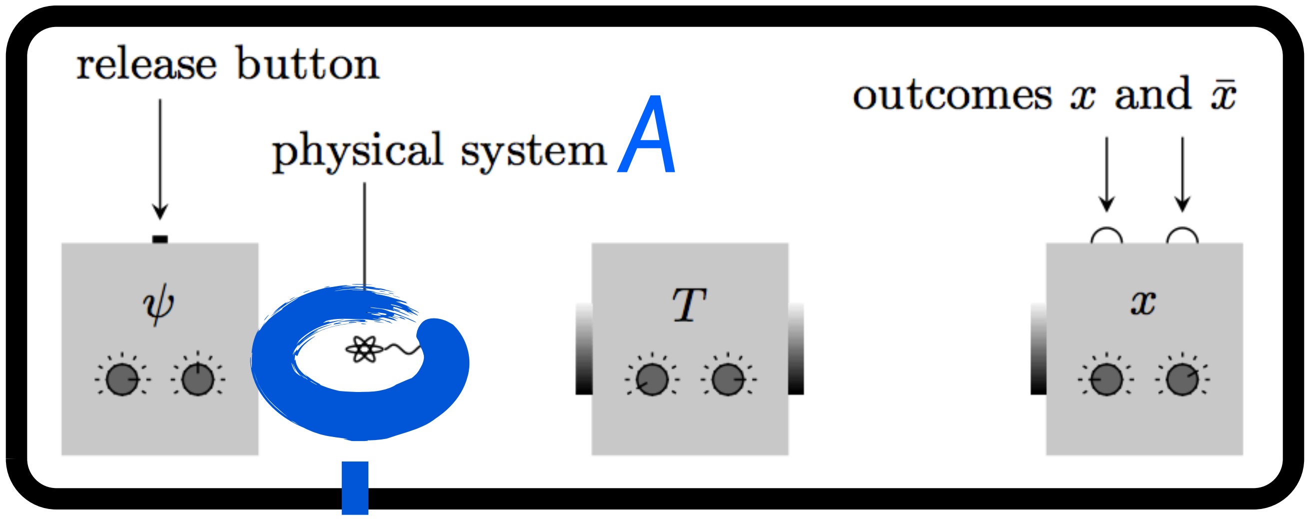
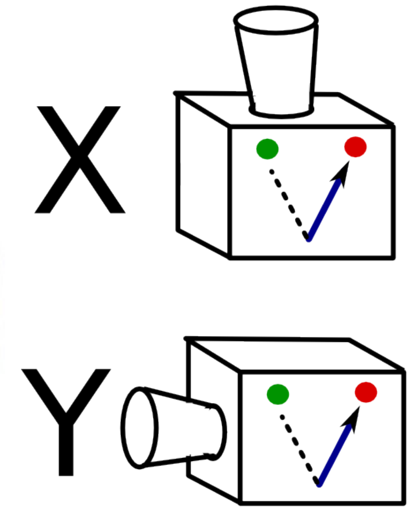
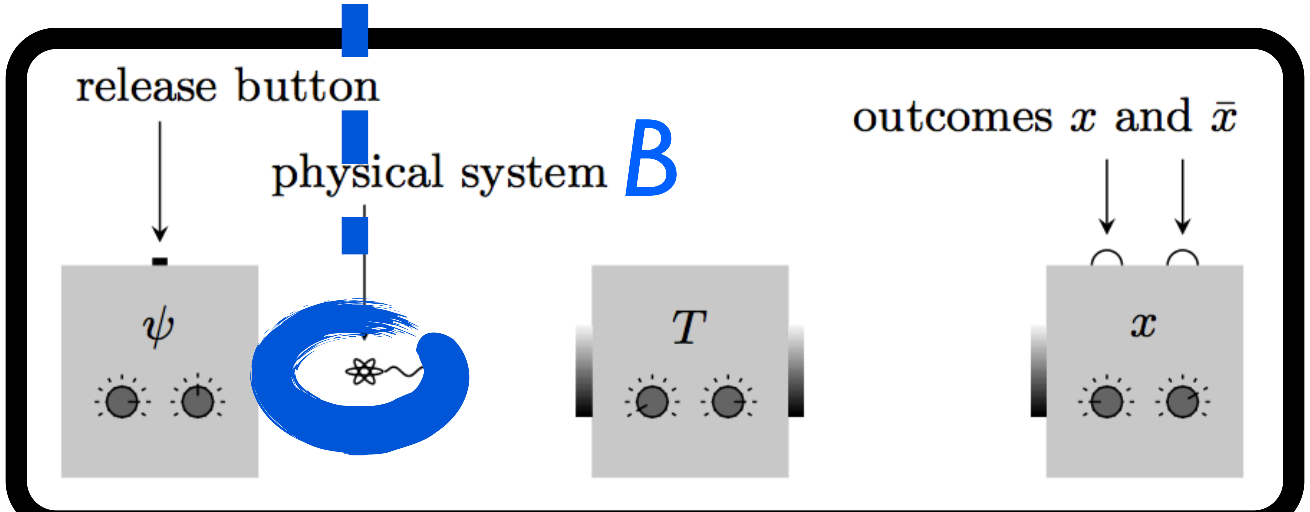
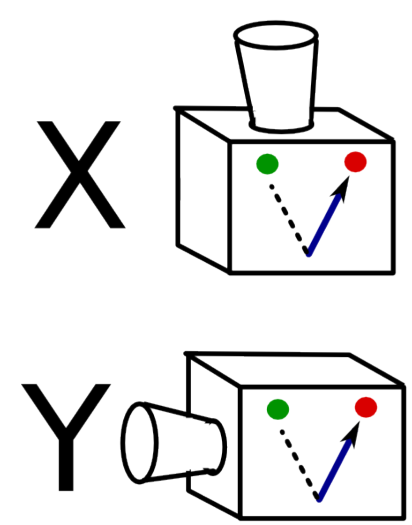




state on AB:
correlations

No-signalling condition:
Alice‘s probabilities do not depend on Bob‘s choice of measurement.





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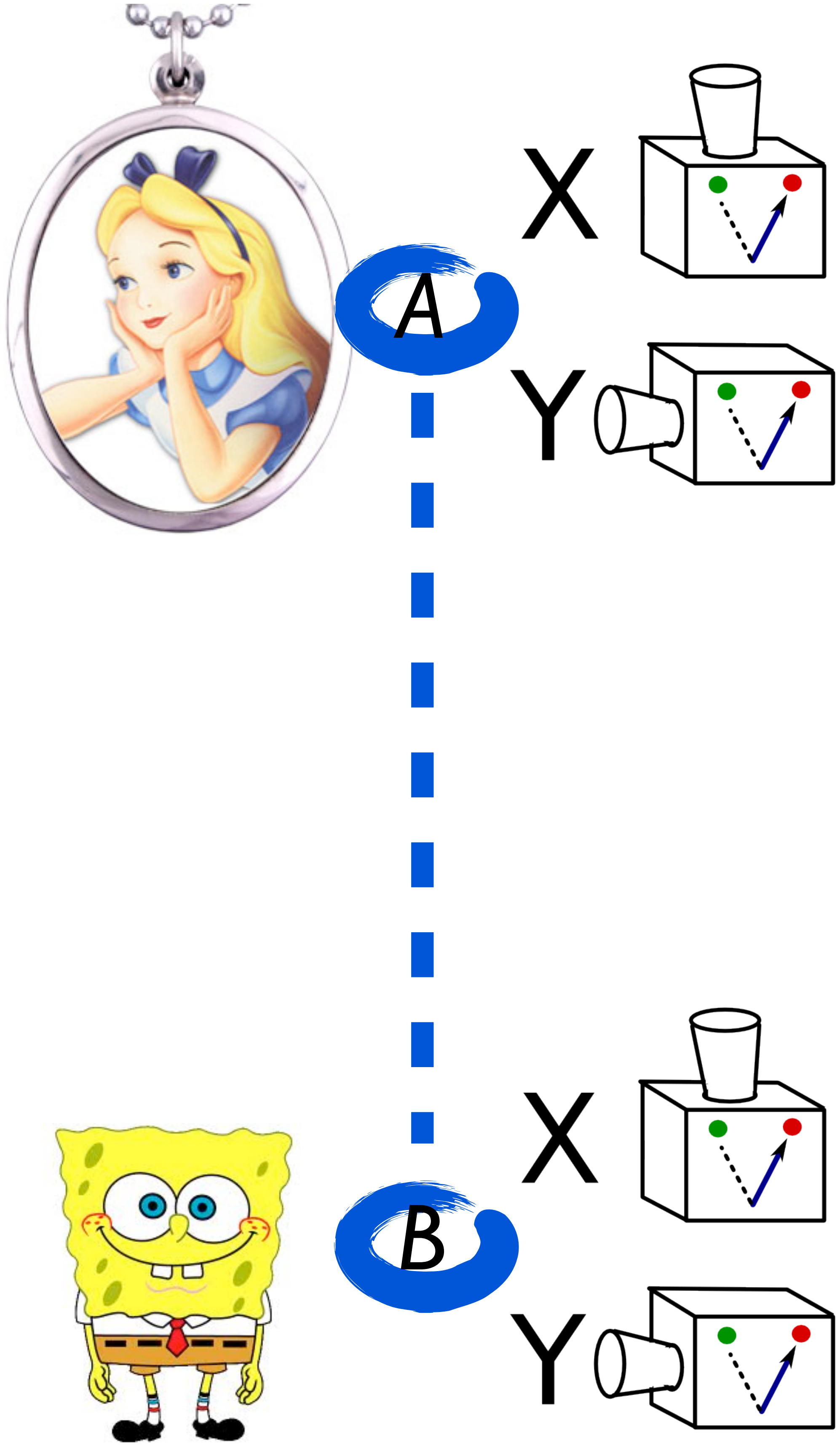

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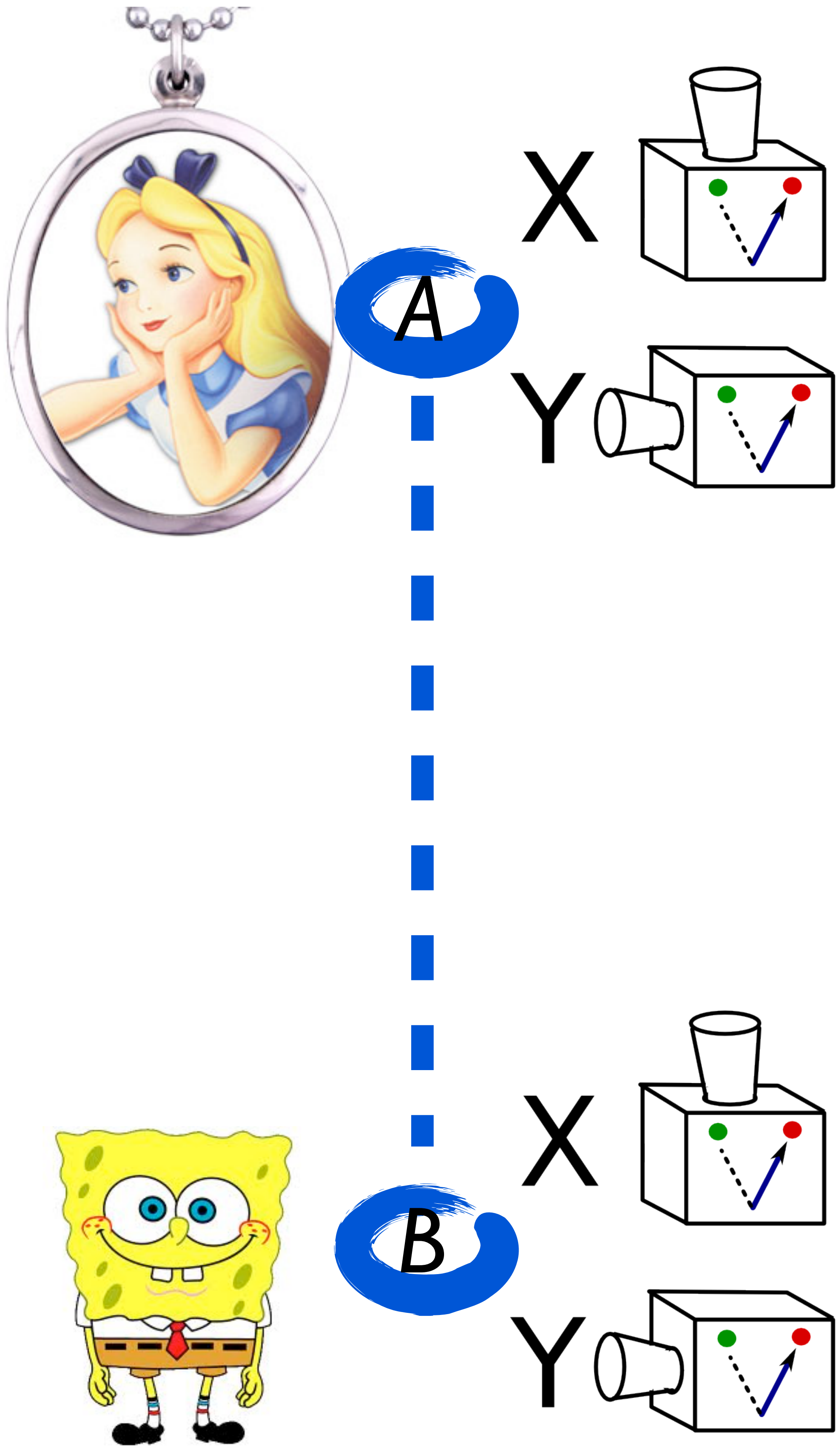

Axiom I: States on AB are uniquely determined by correlations of local measurements on A,B.

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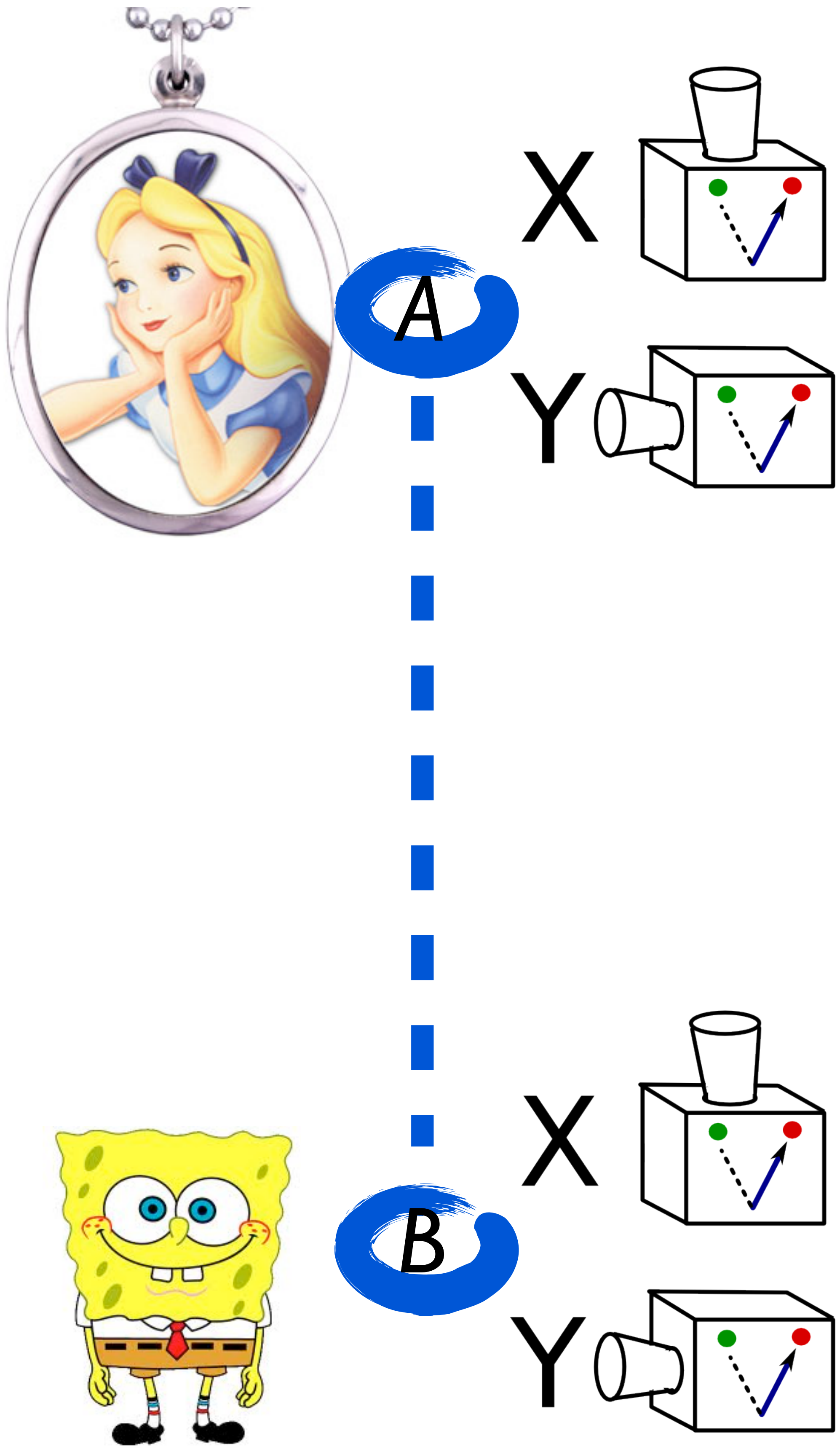

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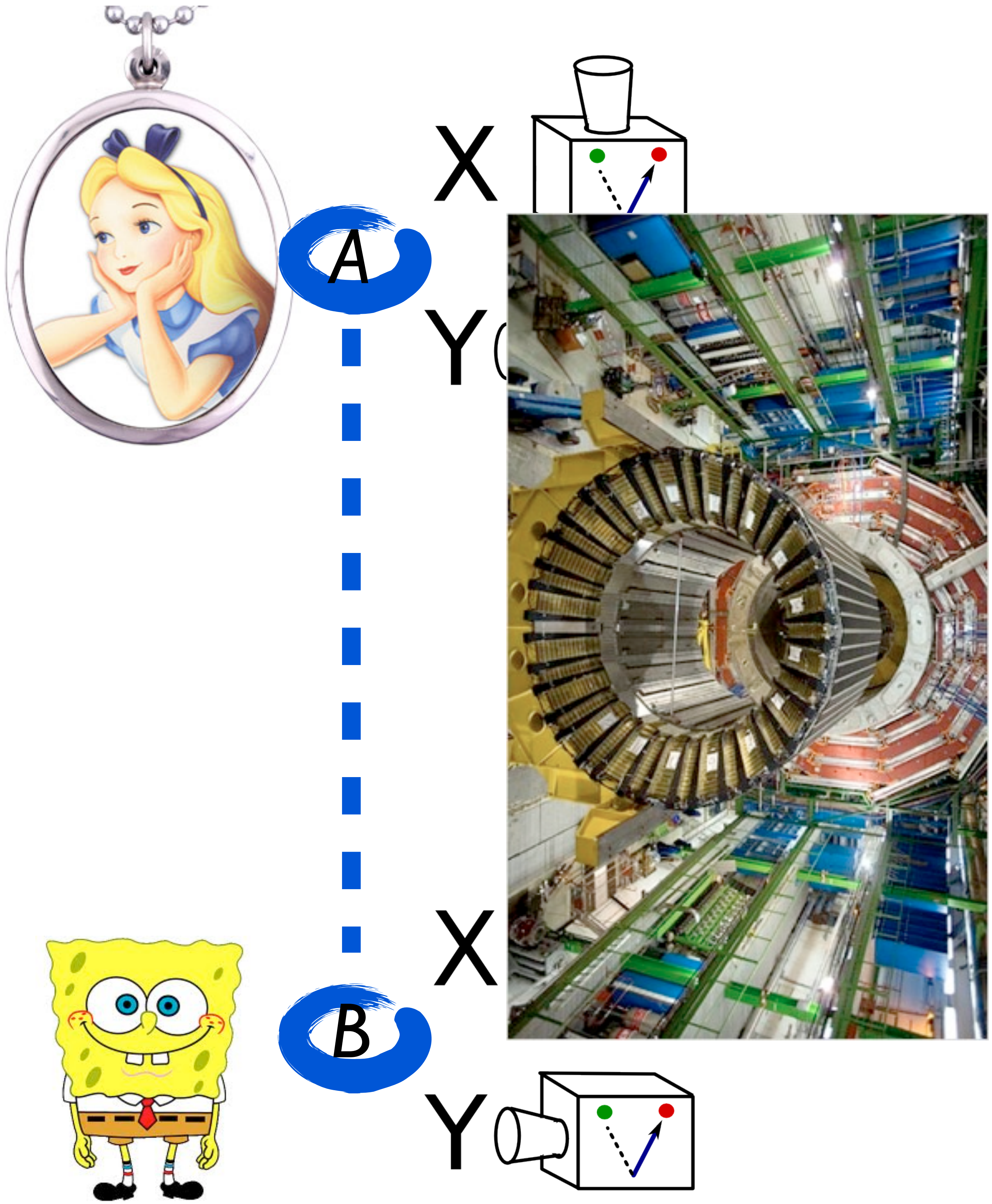

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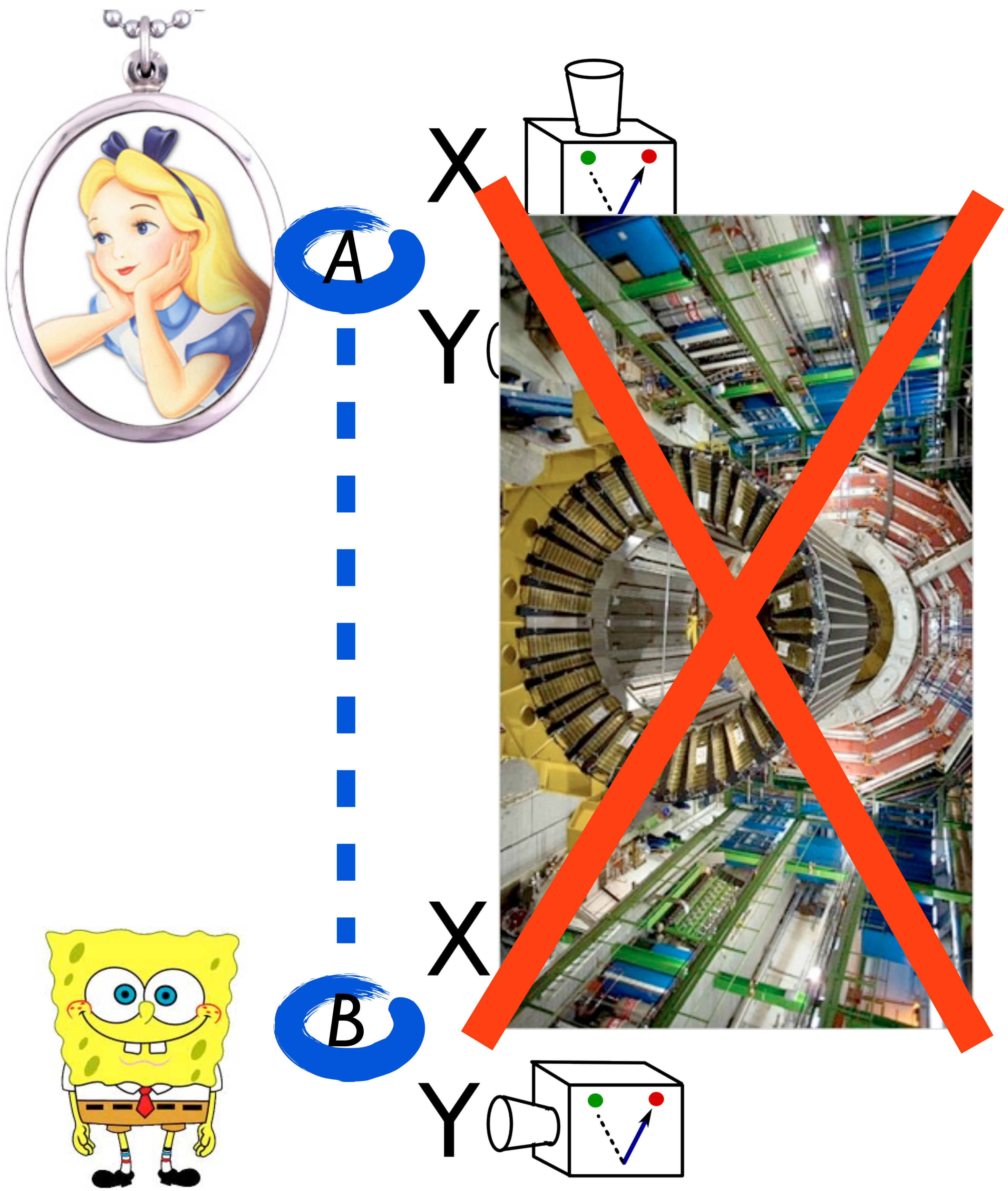

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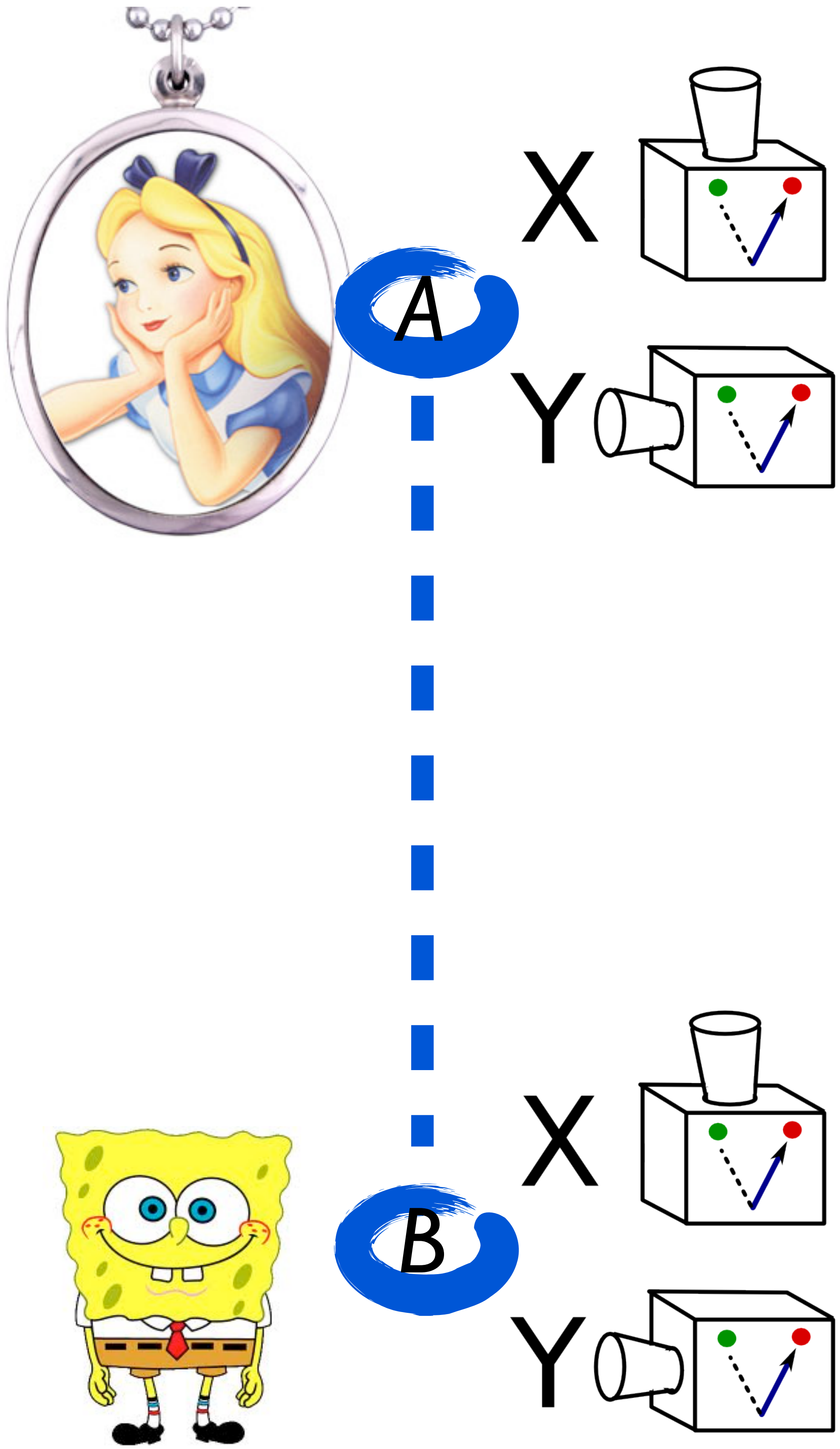

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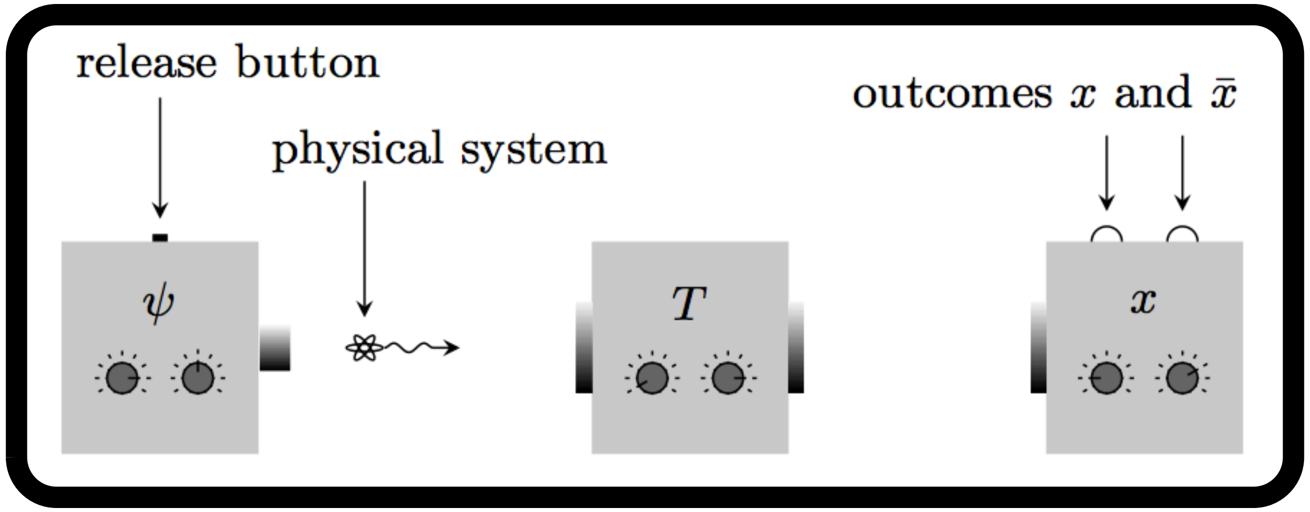
Global state space $\Omega_{AB} \subset A \otimes B$
but not uniquely fixed!

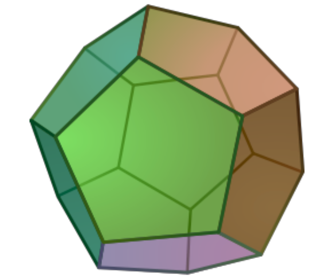
Basic physical / operational assumptions

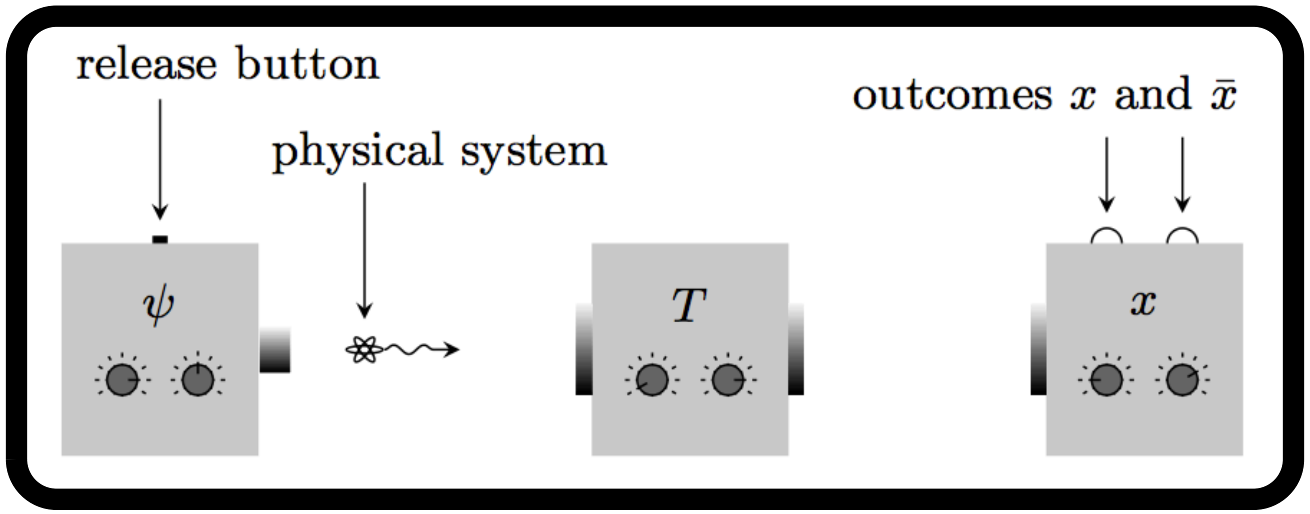


- States, transformations, and measurements with **outcome probabilities**.
- Combined systems: **no-signalling**.

Basic physical / operational assumptions

General probabilistic theories





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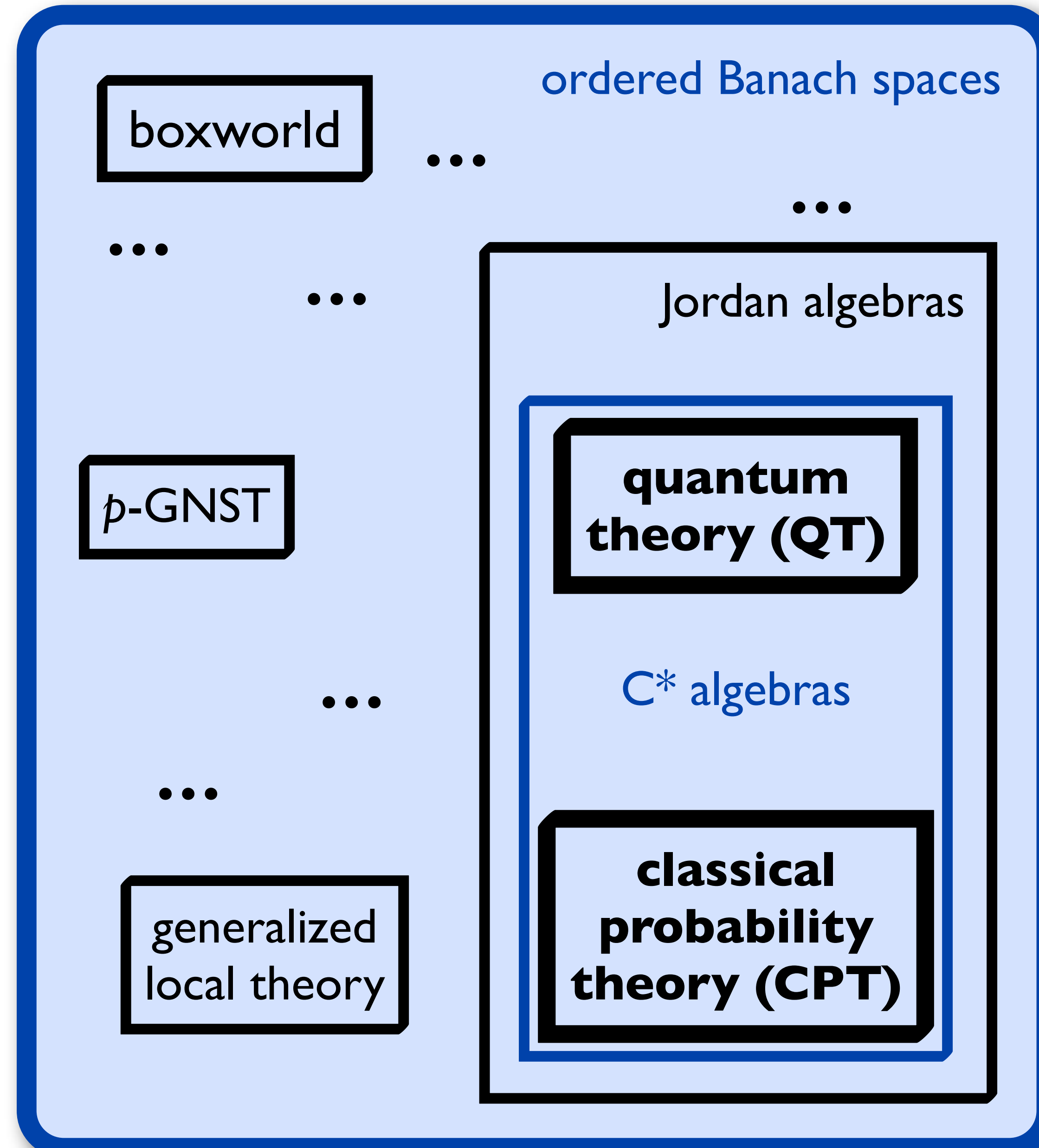

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- State spaces: **arbitrary convex sets**.
- Many ways to **combine systems**.

Basic physical / operational assumptions

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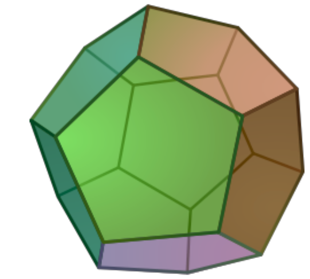

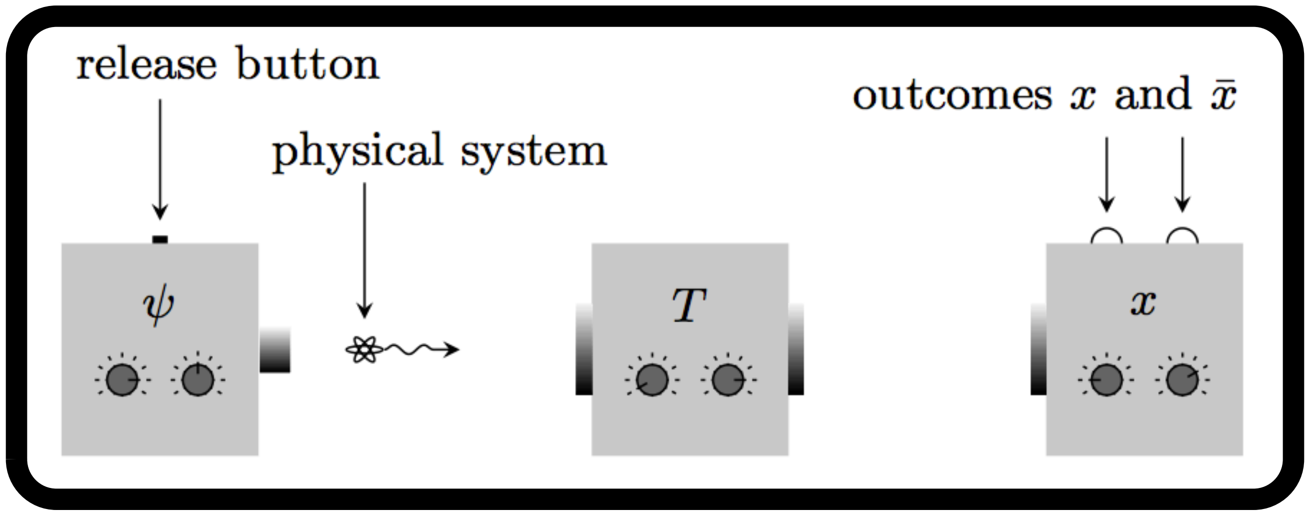

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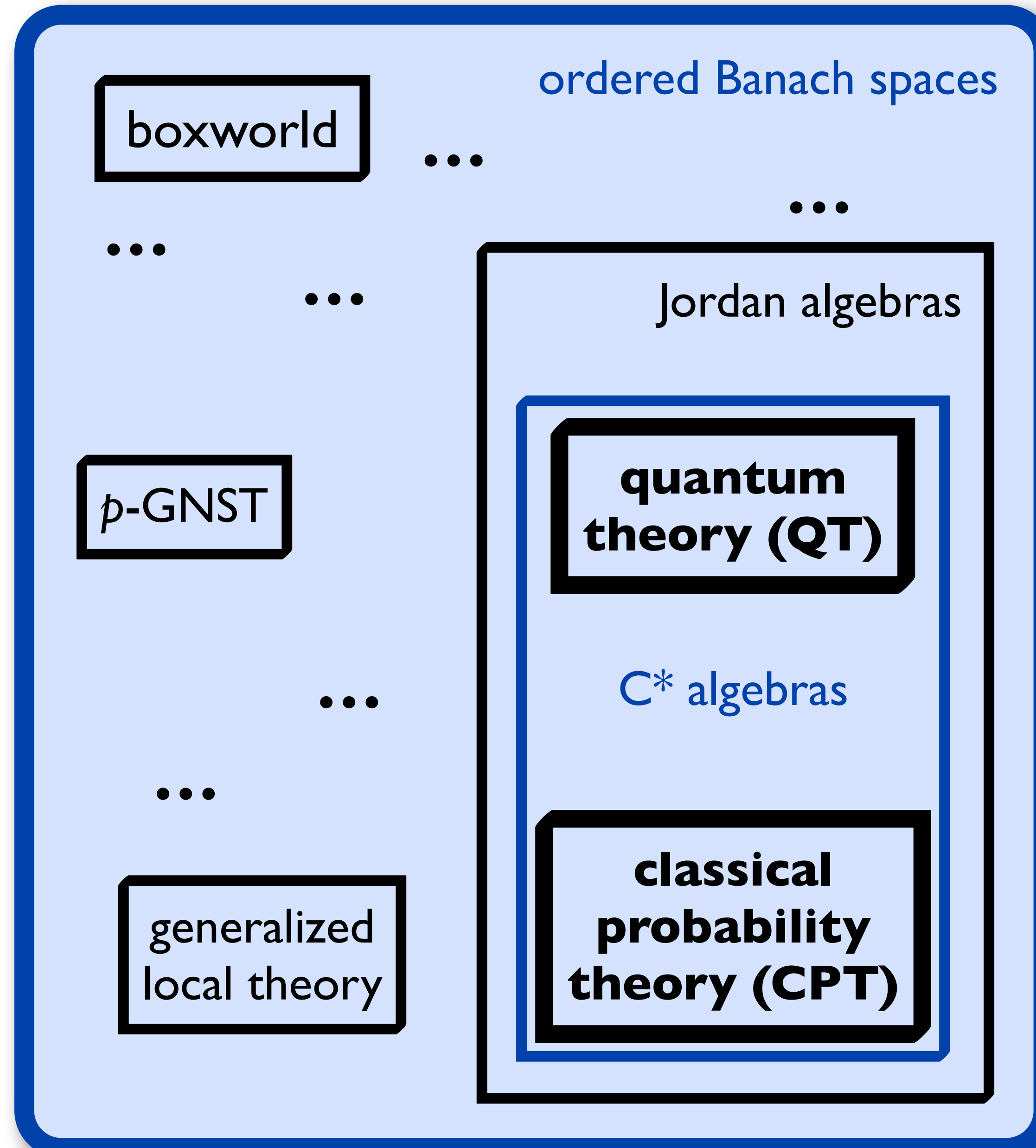
I. Local tomography
II. Reversibility
III. Subspace axiom
IV. Finite-dimensionality
V. All measurements allowed



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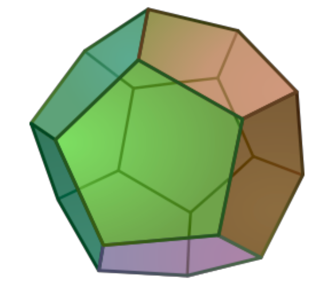
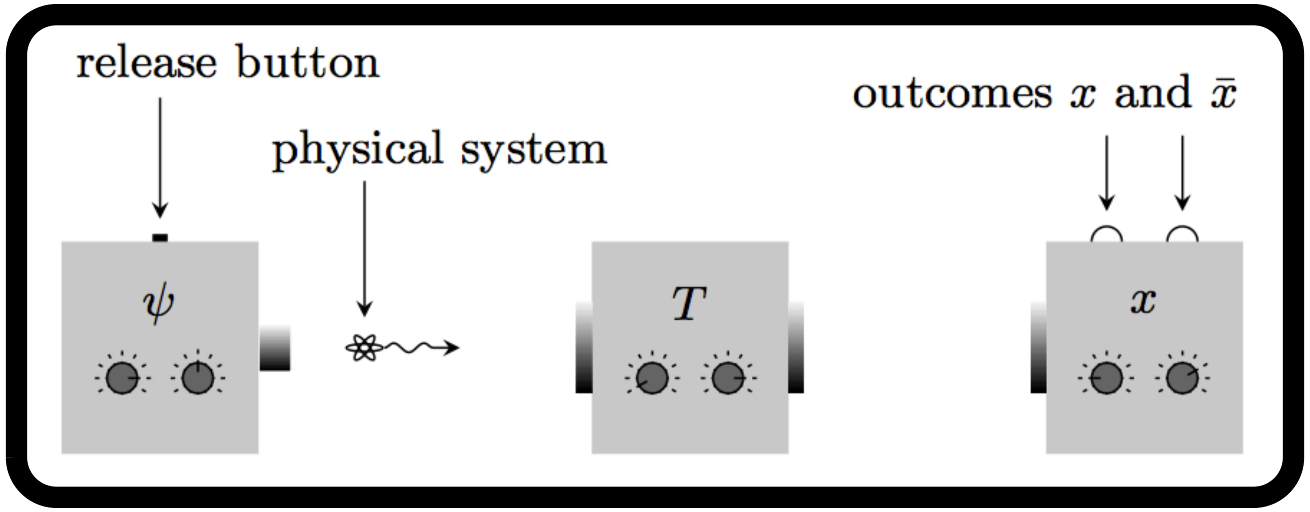
Basic physical / operational assumptions → **General probabilistic theories**





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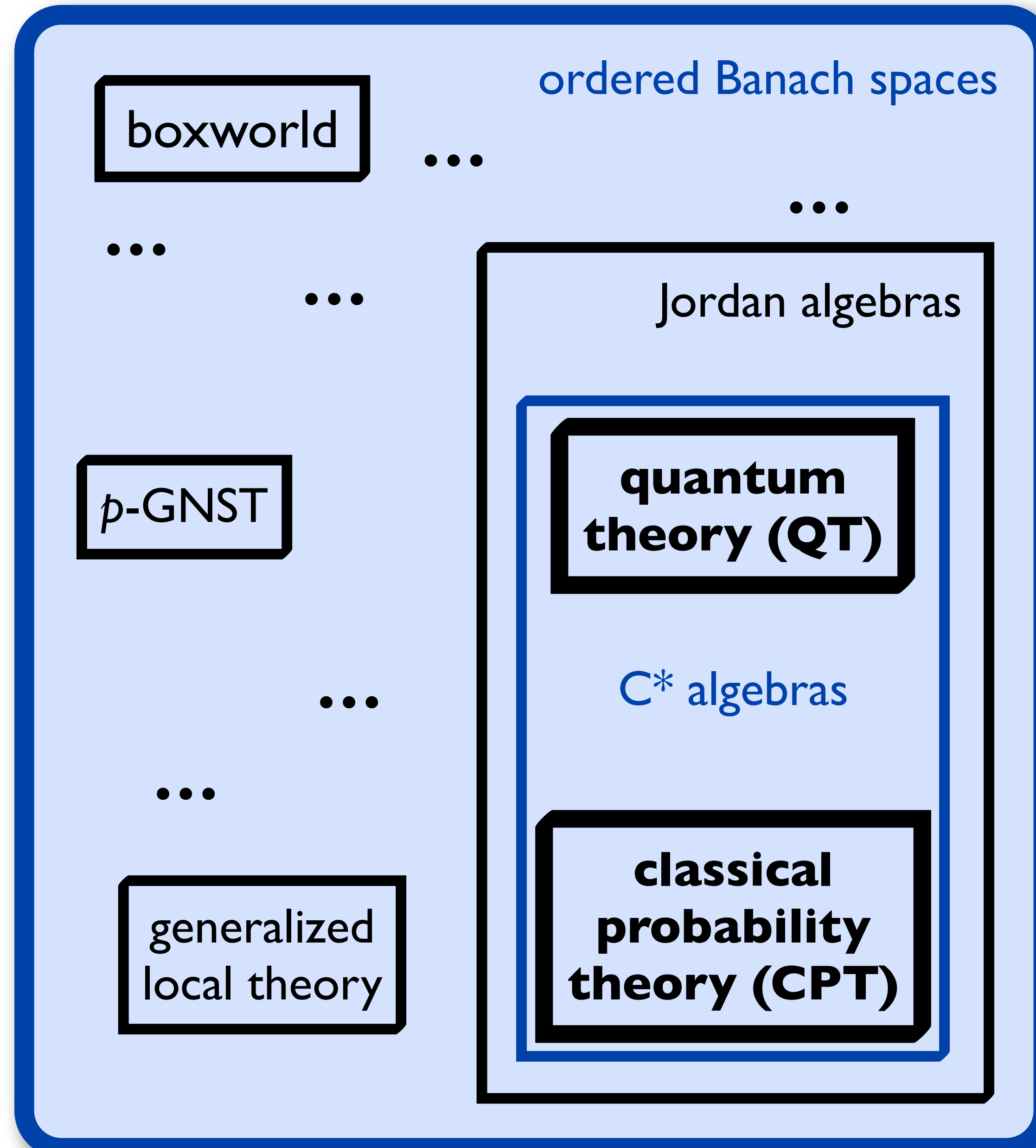

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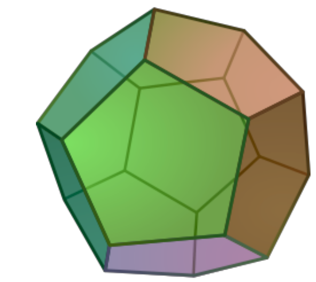

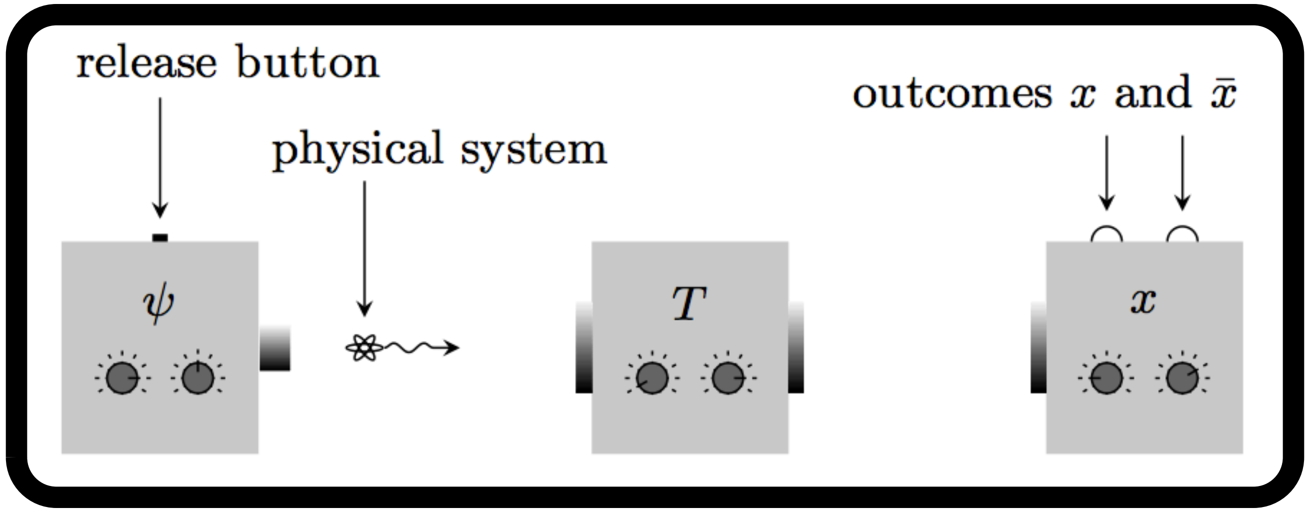

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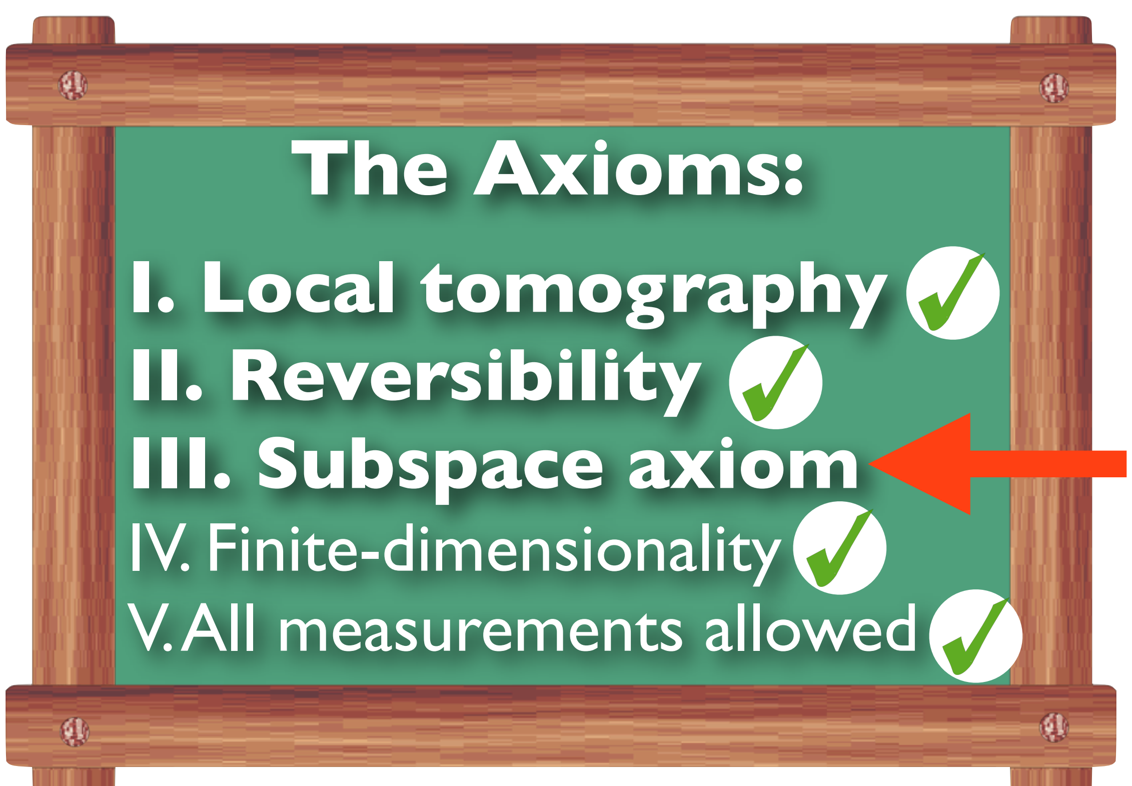

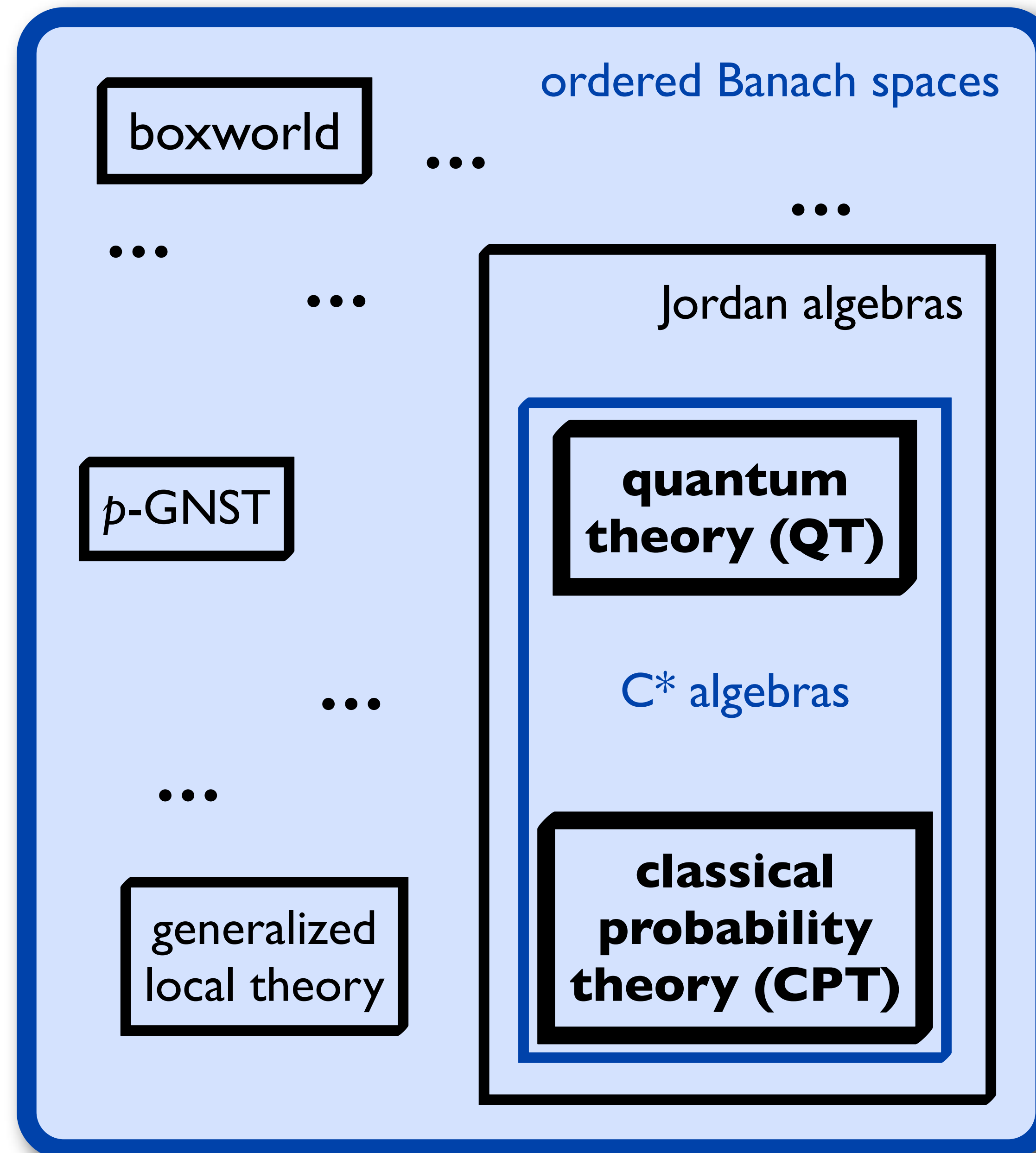

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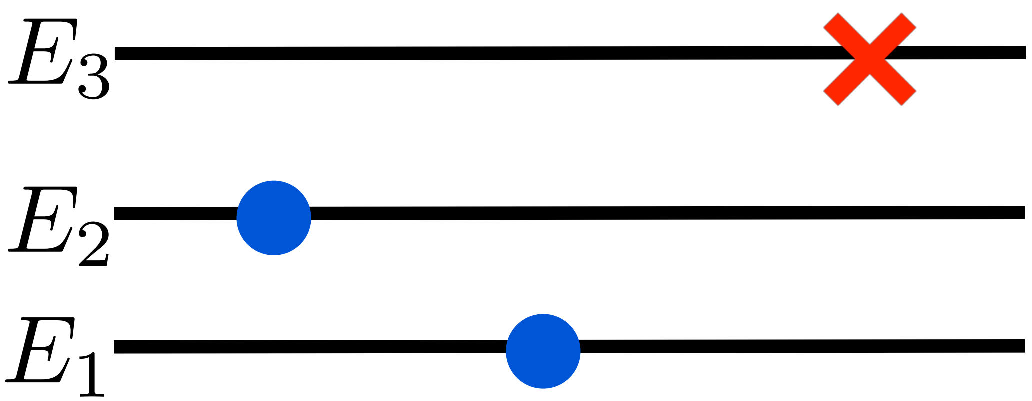
Some 3-level system:



Impossible to have system in 3rd level
$\Rightarrow$ find particle there with probab. 0

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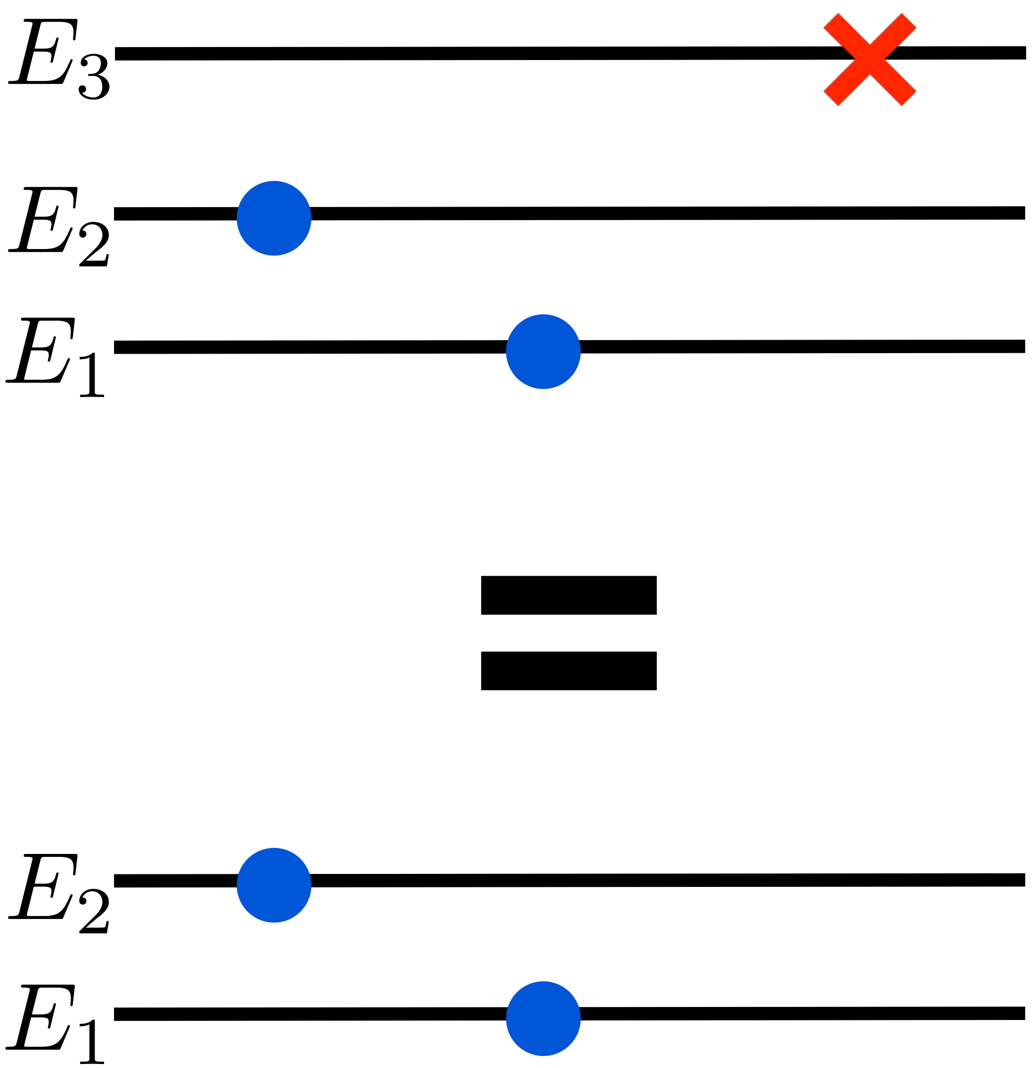

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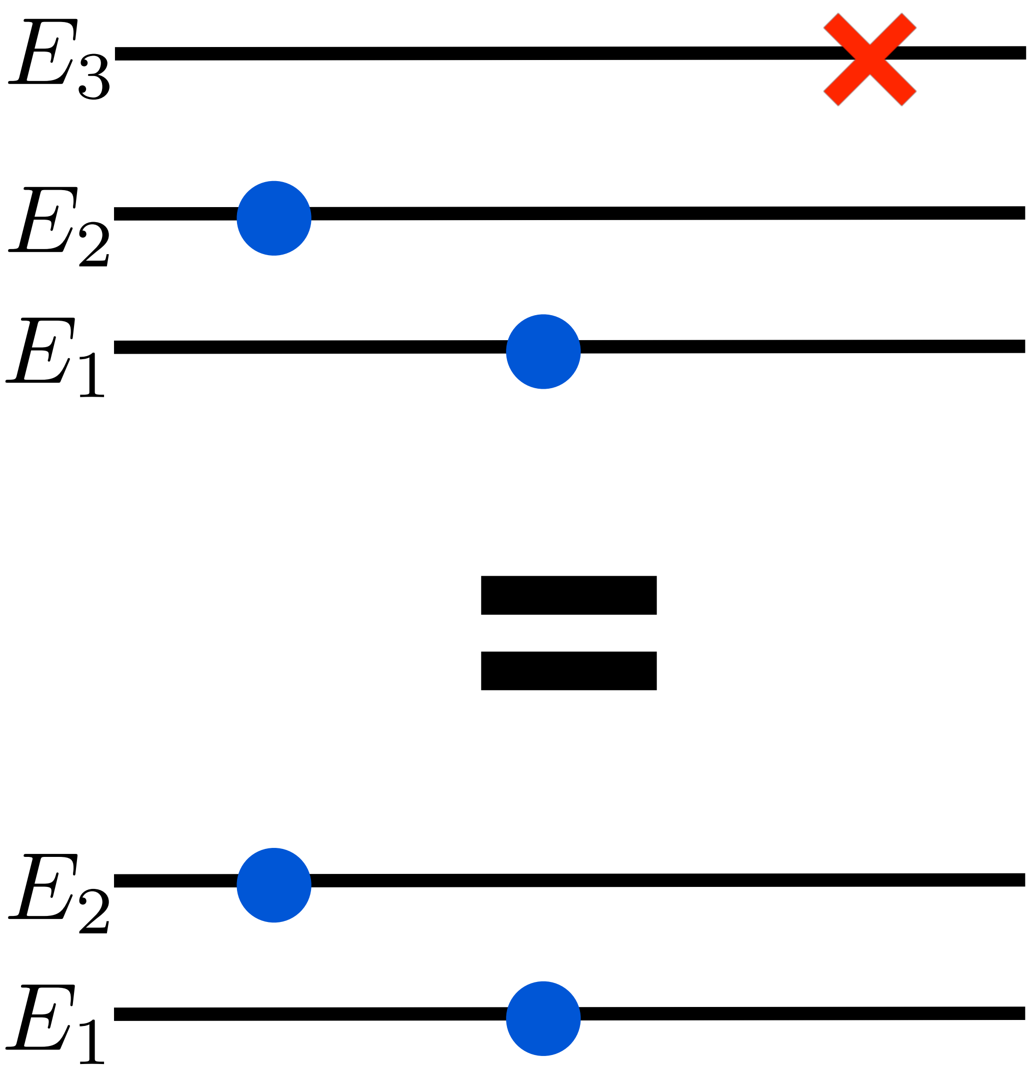

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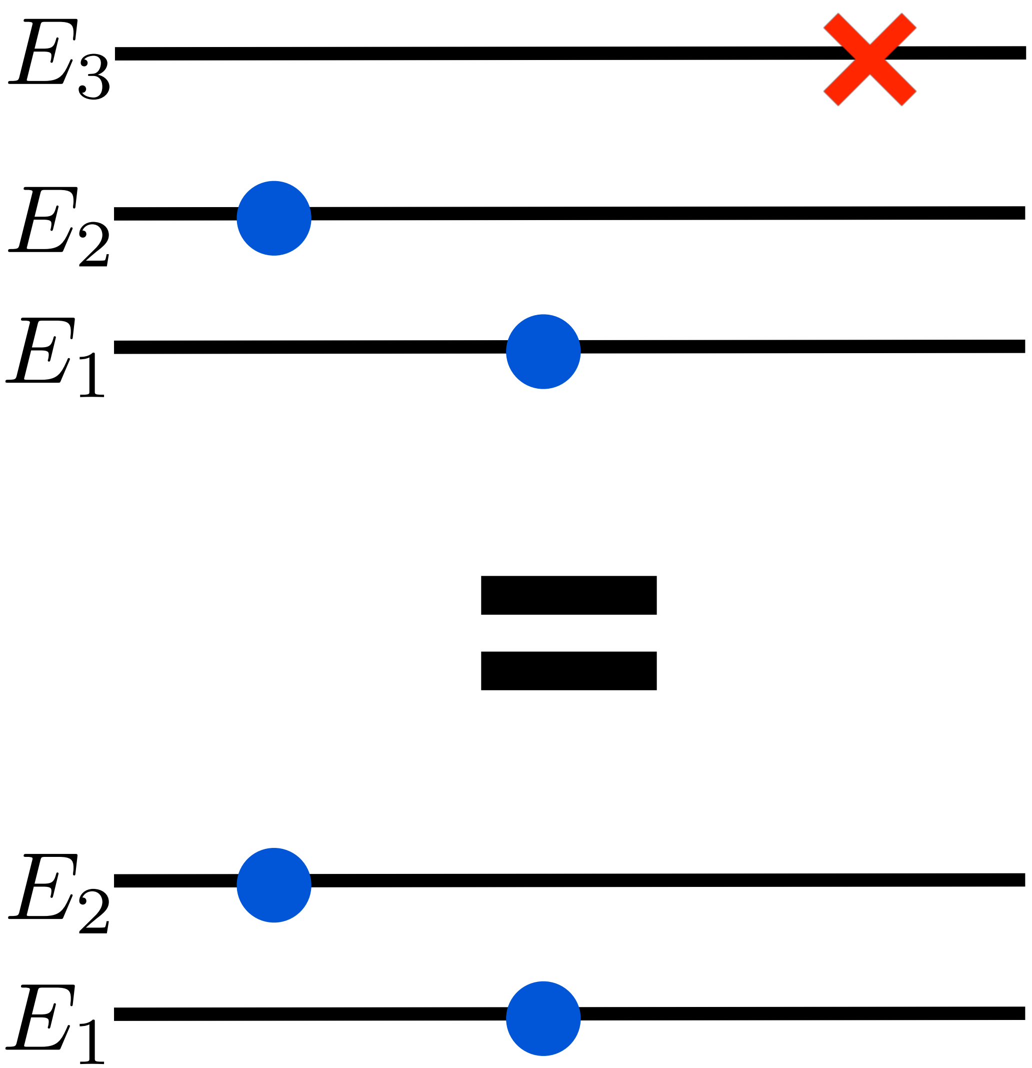

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2-level system.

Otherwise, physics would be affected by impossible potentialities.

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Axiom III: Let Ω_N and Ω_{N-1} be systems with capacities *N* and *N-1*. If $(E_1, \ldots, E_N)$ is a complete measurement on Ω_N, then the set of states ω with $E_N(\omega) = 0$ is equivalent to Ω_{N-1}.

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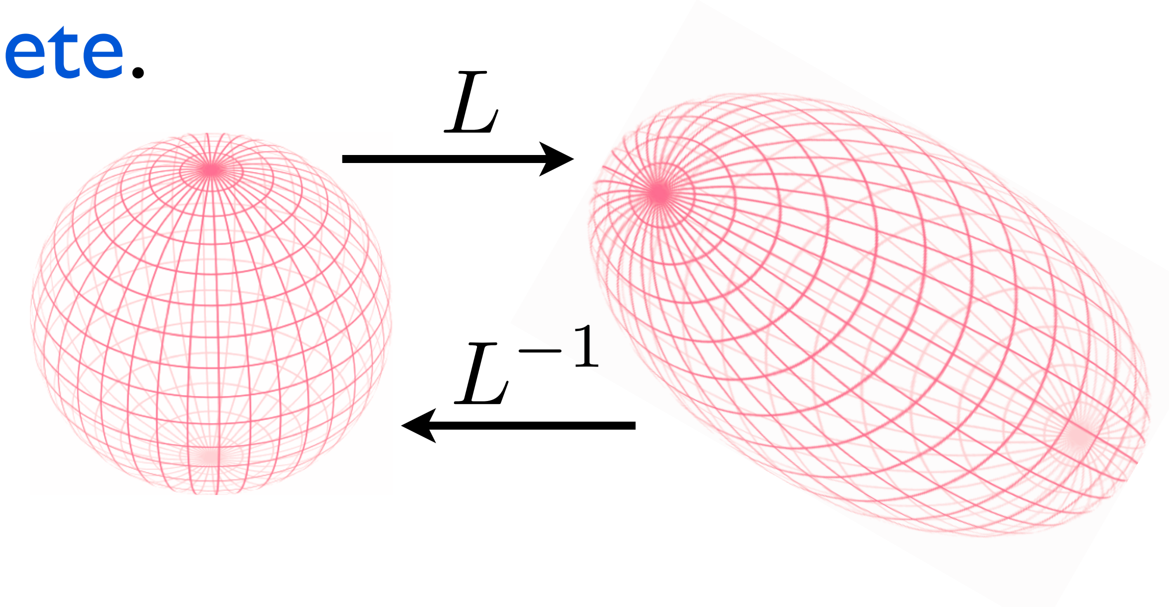
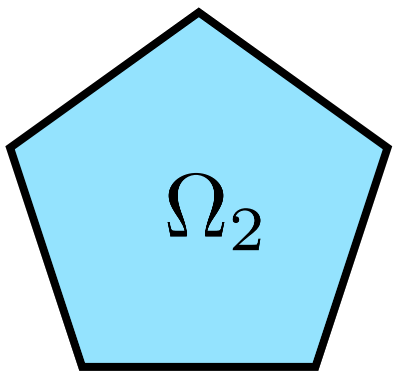
If $n = N$ then $(E_1, \ldots, E_n)$ is complete.

Equivalent = same state spaces up to a linear map (physically the same!)



4. Derivation of the Hilbert space formalism

Why a bit is described by a ball:



capacity 2 (bit)

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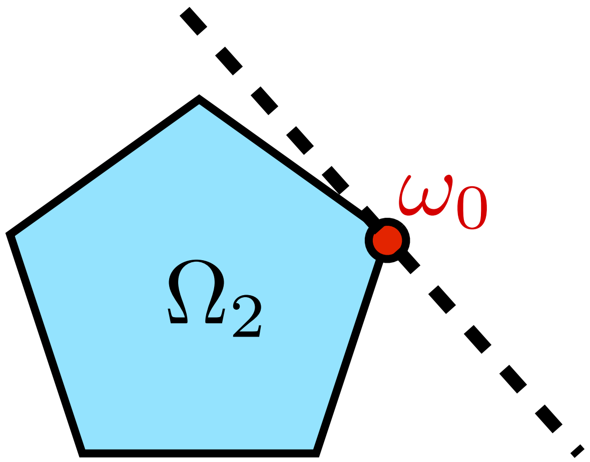

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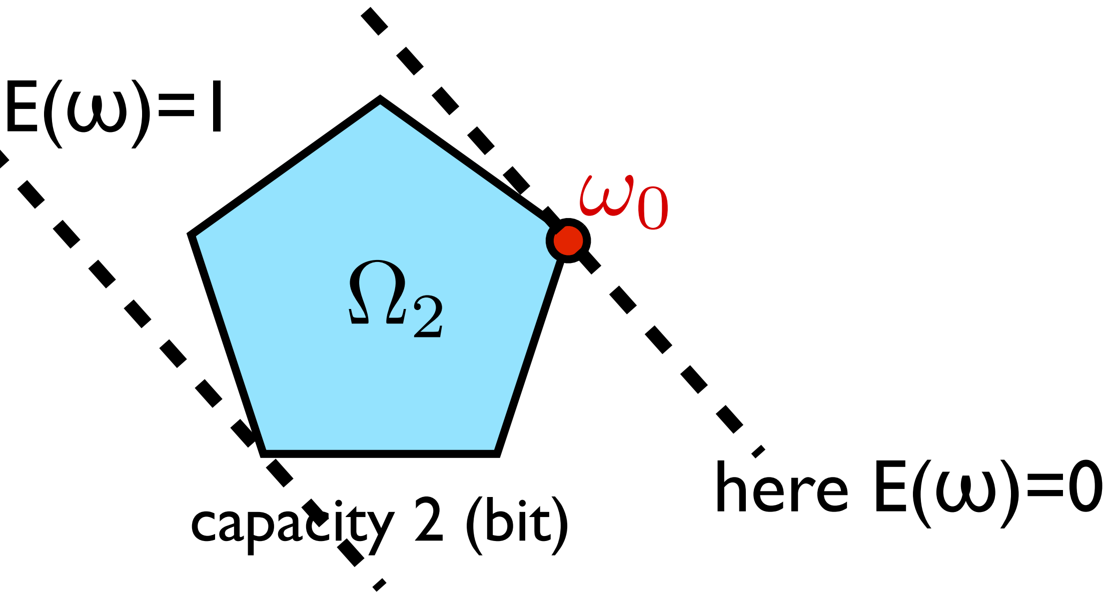

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$$\Rightarrow \{\omega : E(\omega) = 0\} = \{\omega_0\} \sim \Omega_1 .$$

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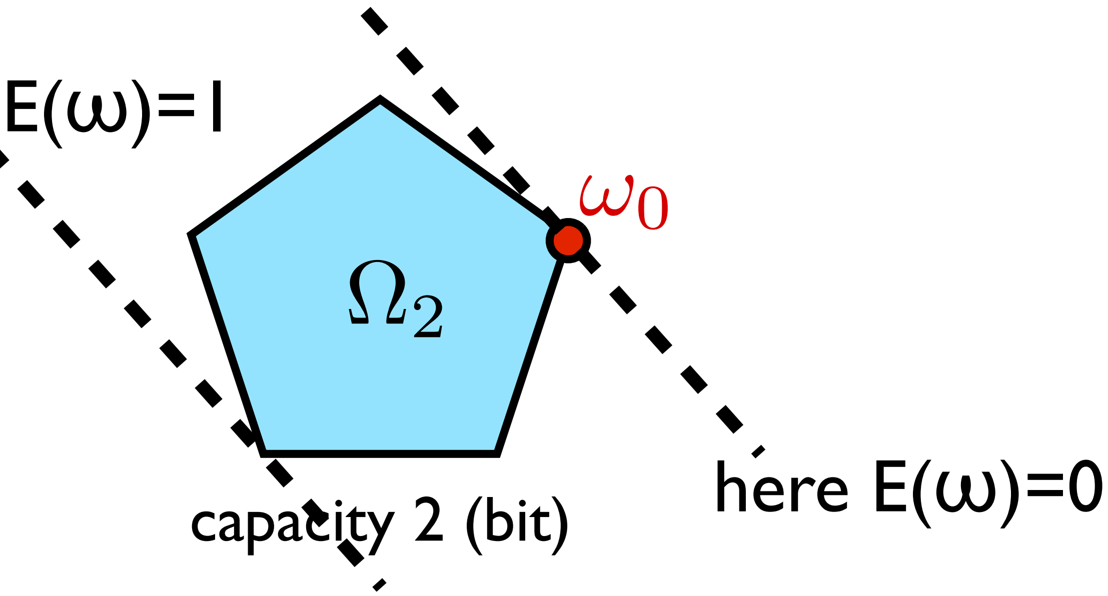

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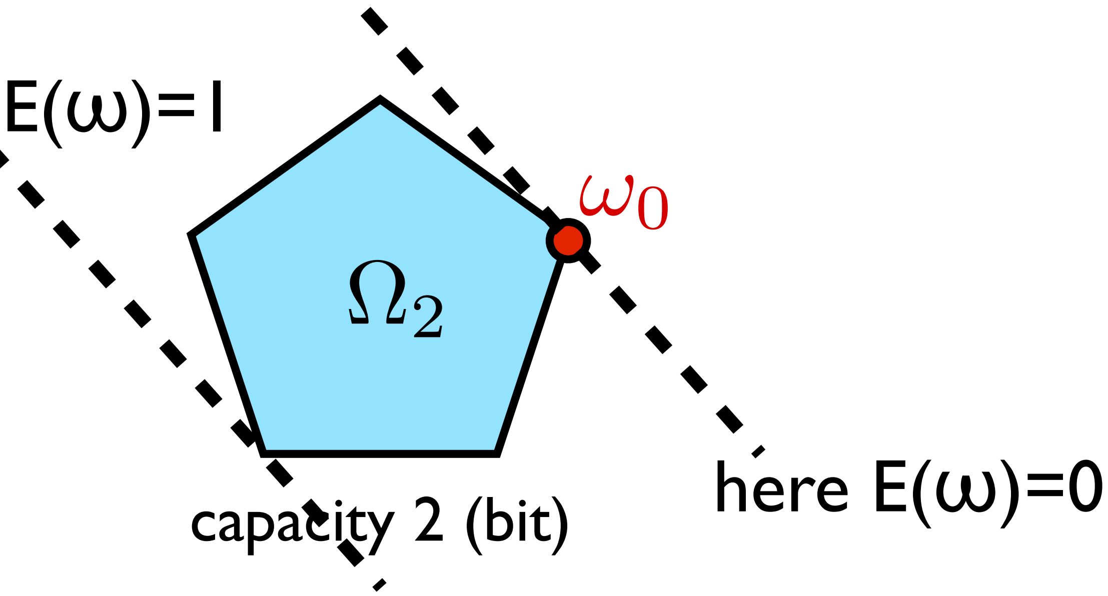

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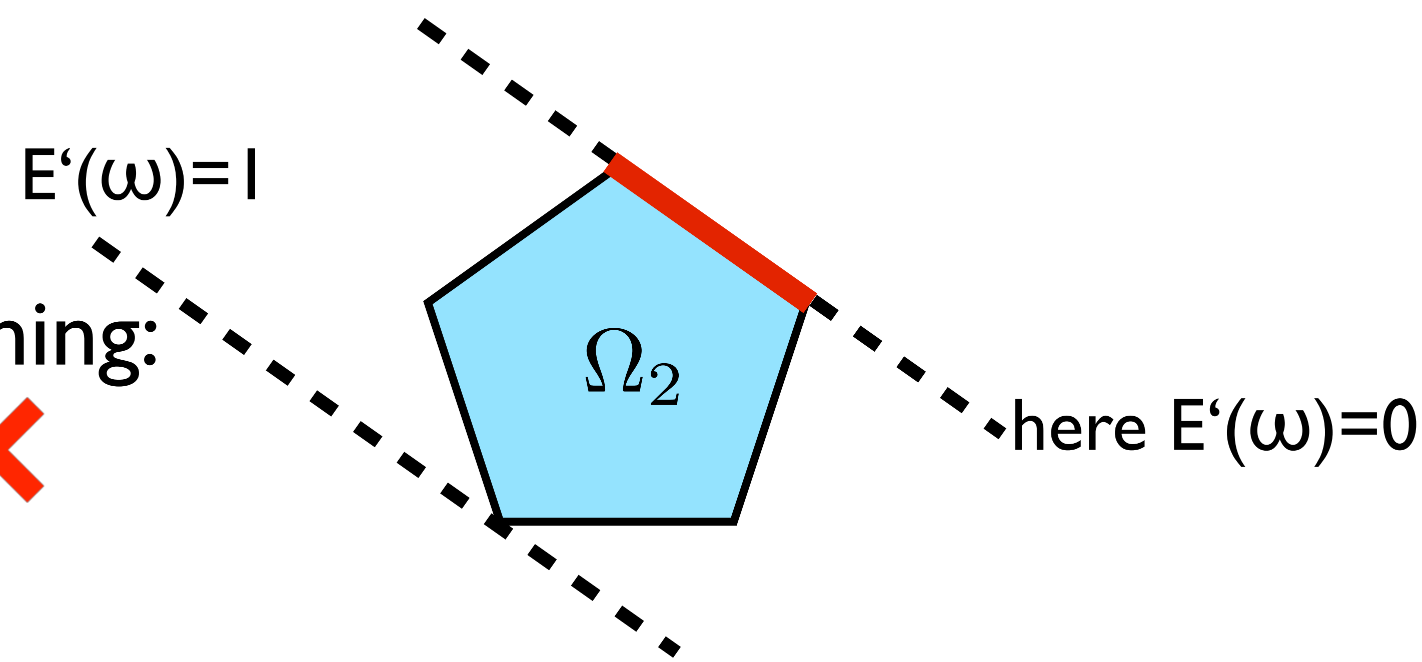

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Ω_1 contains ∞ many states. ✗

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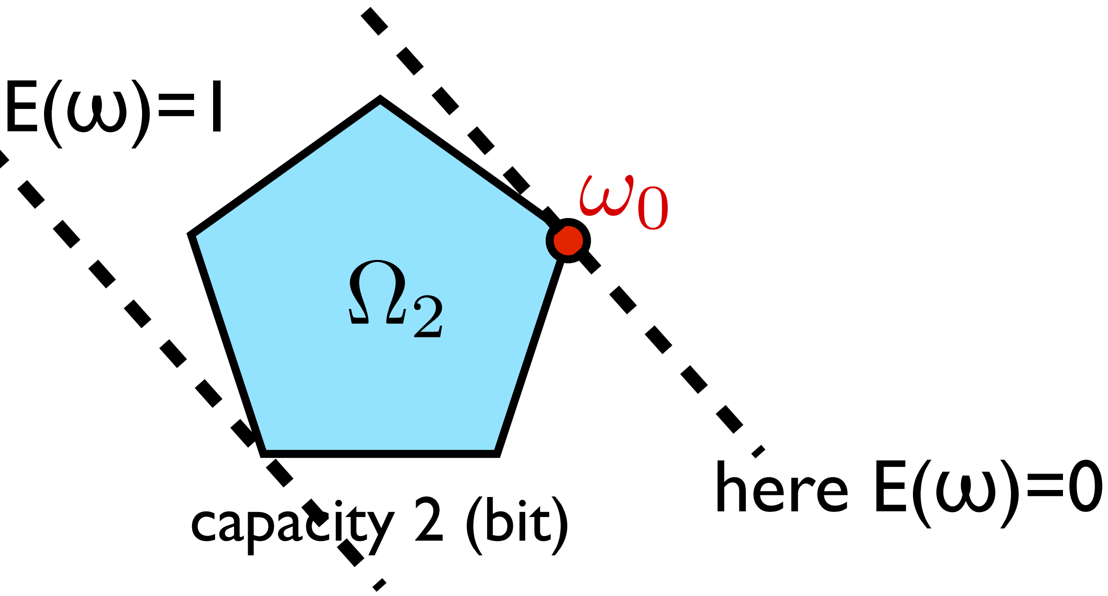

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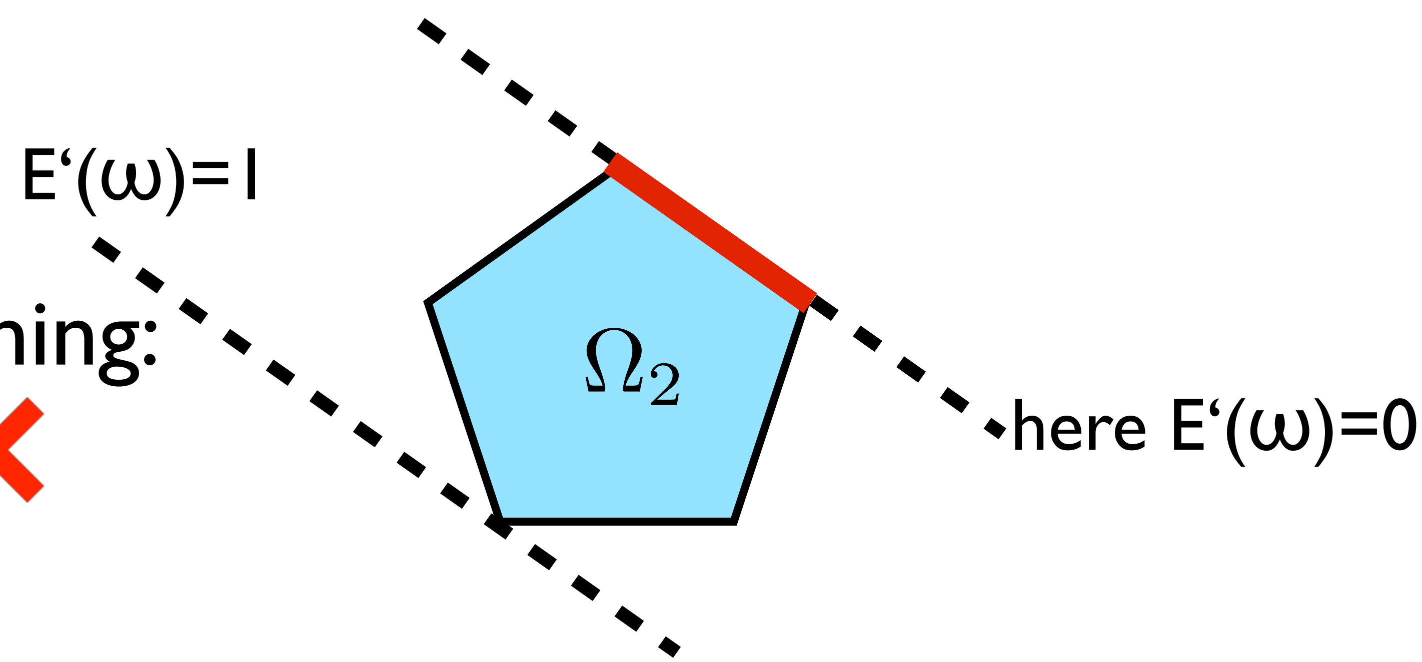

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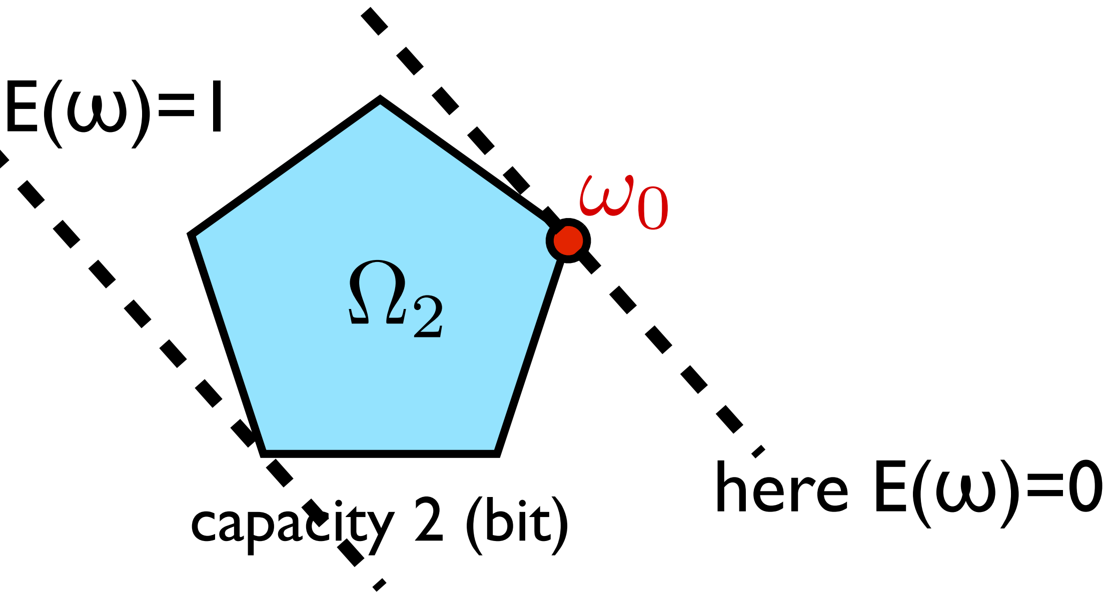

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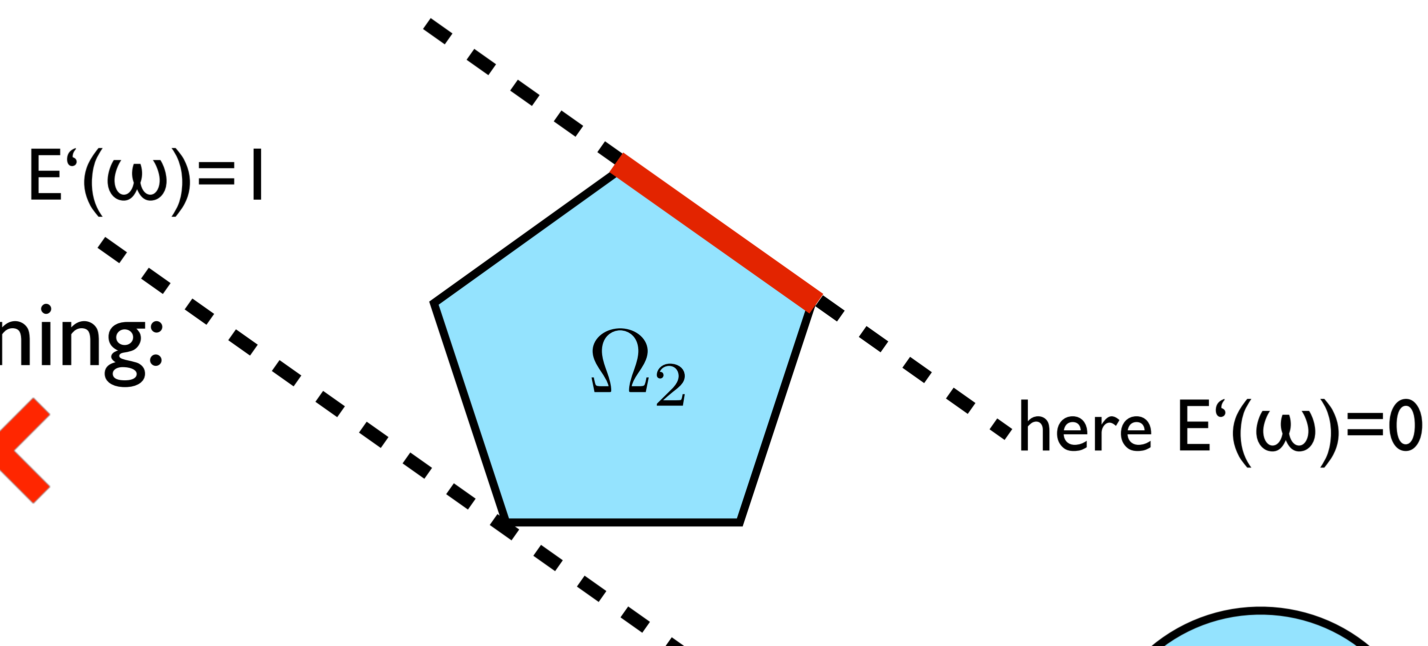

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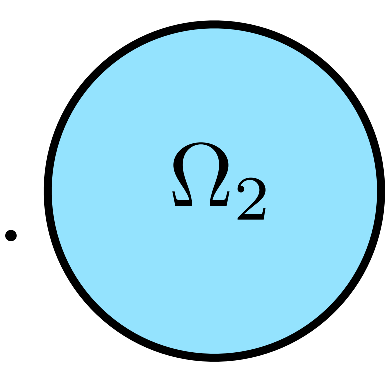
Reversibility axiom ⇒ Ω_2 is a ball.



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- capacity $N_{AB} = N_A N_B$ and bit ball dimension

$$\dim(\Omega_2) = 2^r - 1 \in \{1, 3, 7, 15, 31, \ldots\}.$$

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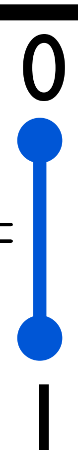
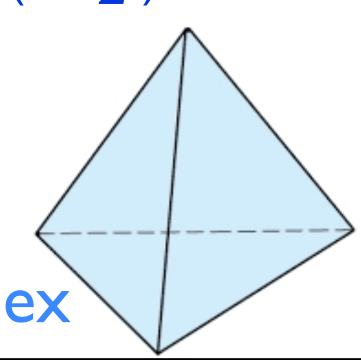
$\Omega_2 =$ 

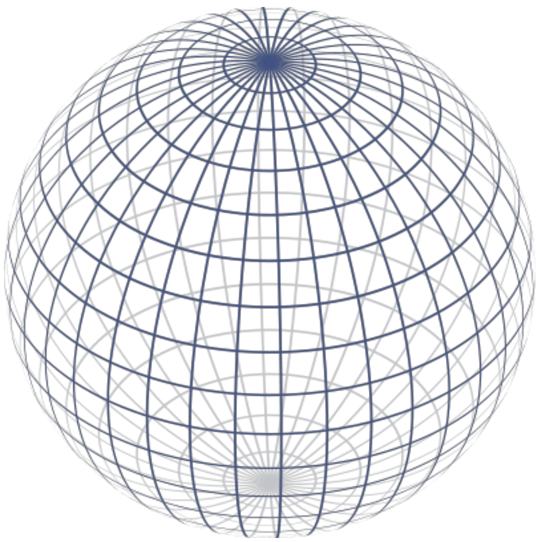
If $\dim(\Omega_2) = 1$ then the theory is CPT (easy):

$\Omega_N =$ 

N-simplex

$\mathcal{G}_N$=permutation group.

4. Derivation of the Hilbert space formalism



Generalized bit Ω_2

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CPT

By reversibility axiom, $\mathcal{G}_2$ is transitive on the sphere.

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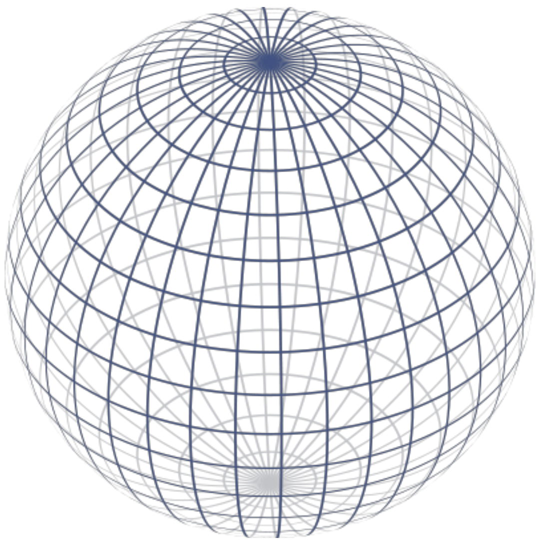

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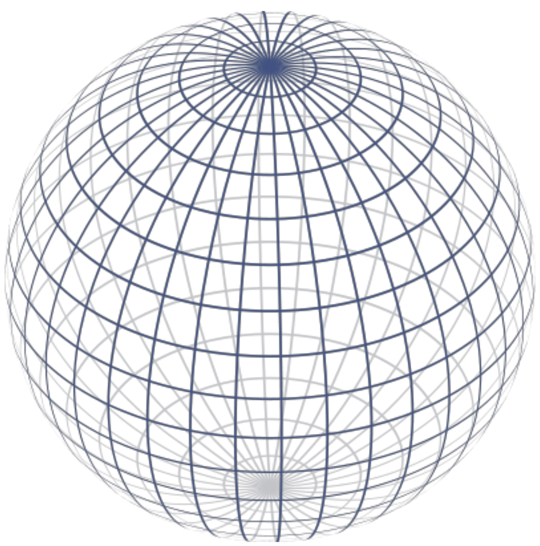

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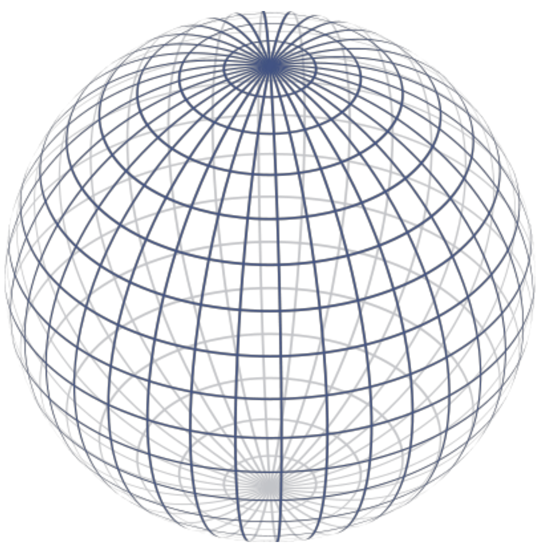

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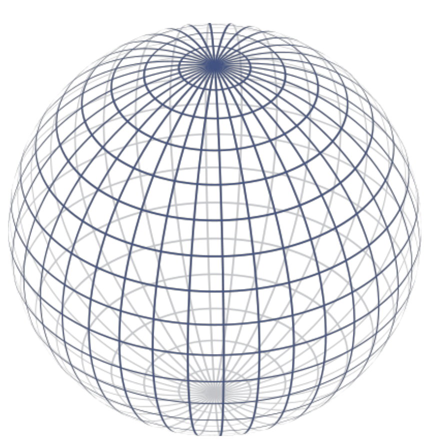
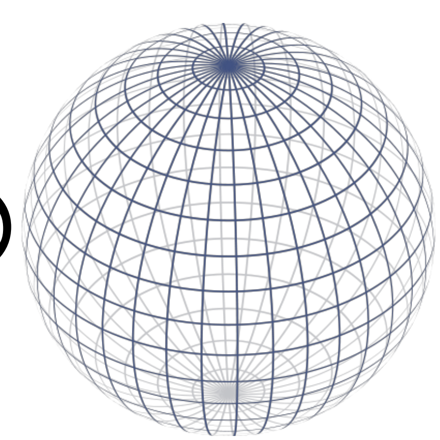
Two bits:  $\otimes$ 

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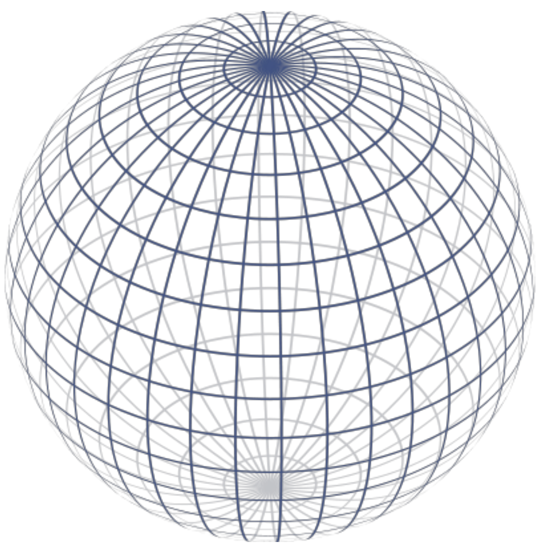

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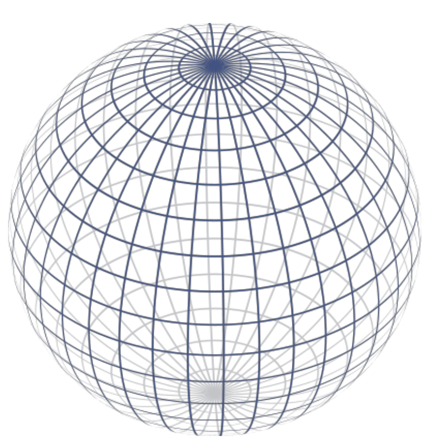
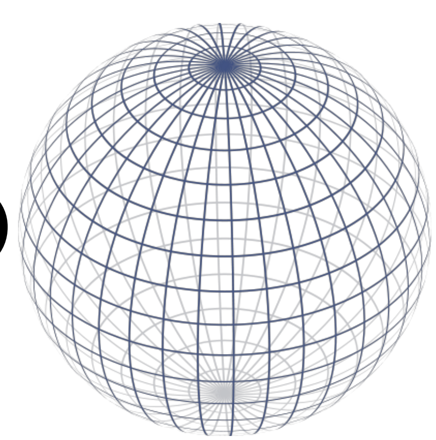
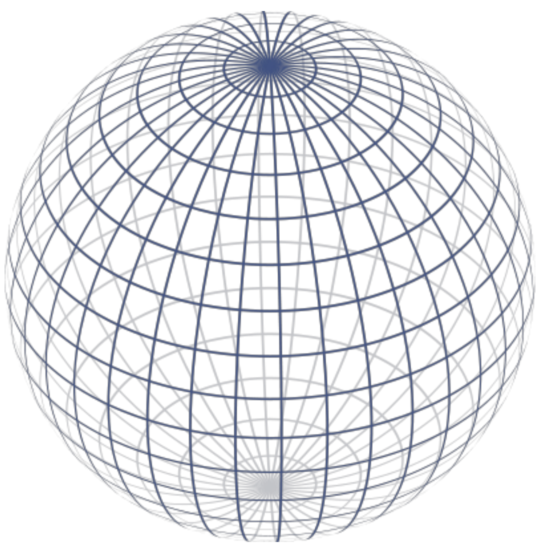
Two bits:  $\otimes$ 

Local transformations contain $\mathcal{G}_2 \otimes \mathcal{G}_2$.

4. Derivation of the Hilbert space formalism



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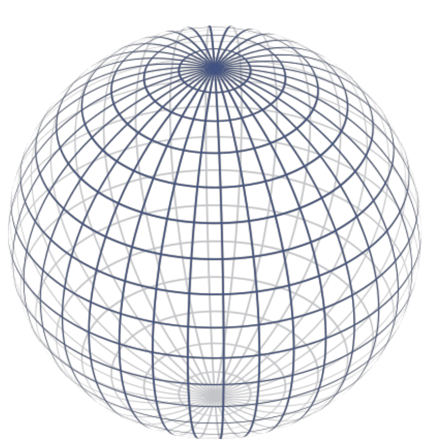
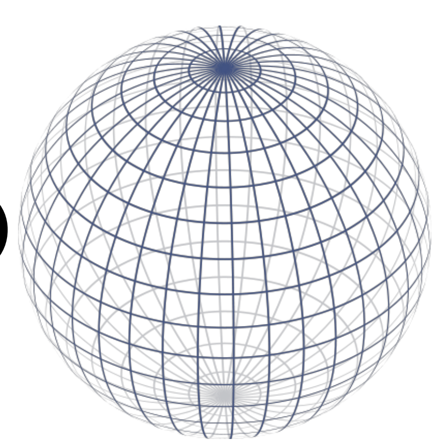
Two bits:  $\otimes$ 

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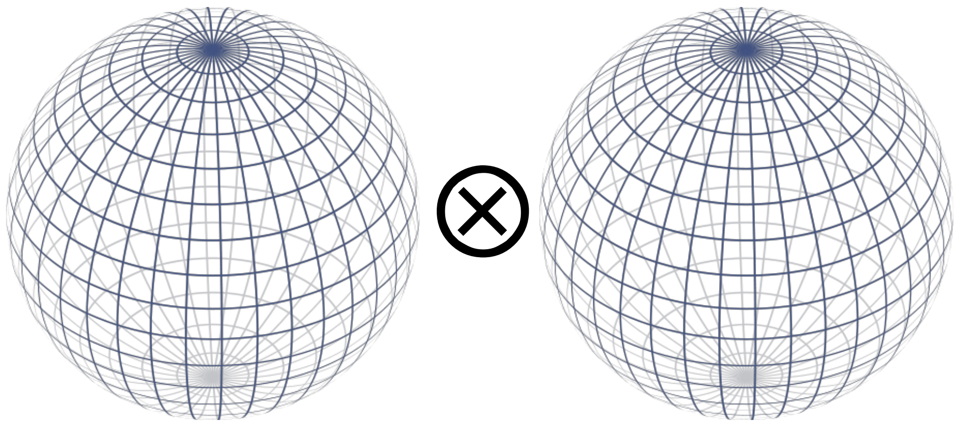
Two bits: 

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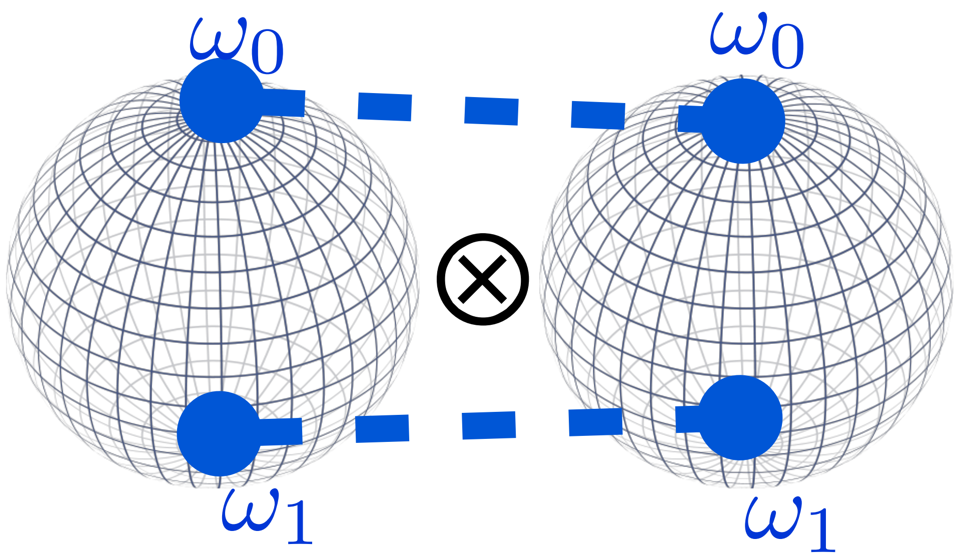

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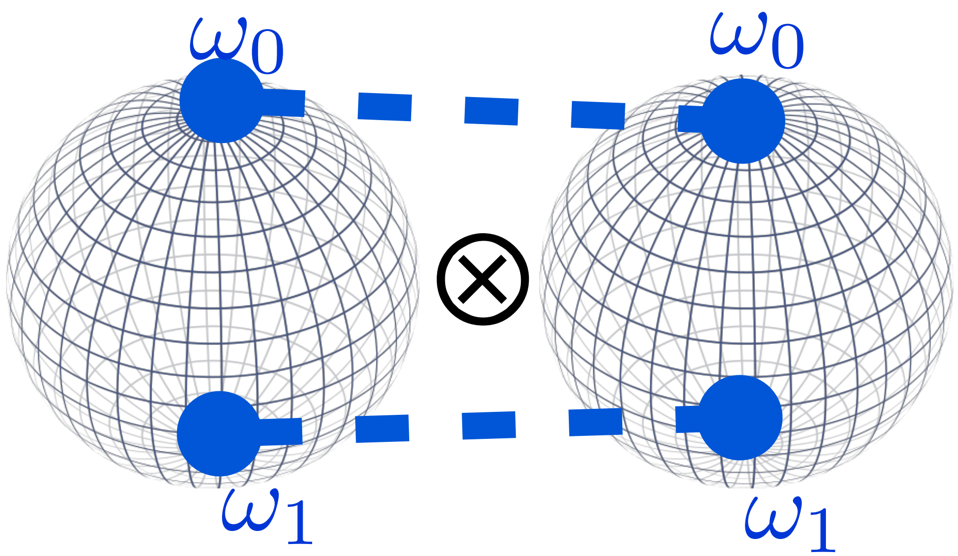
Two bits: 

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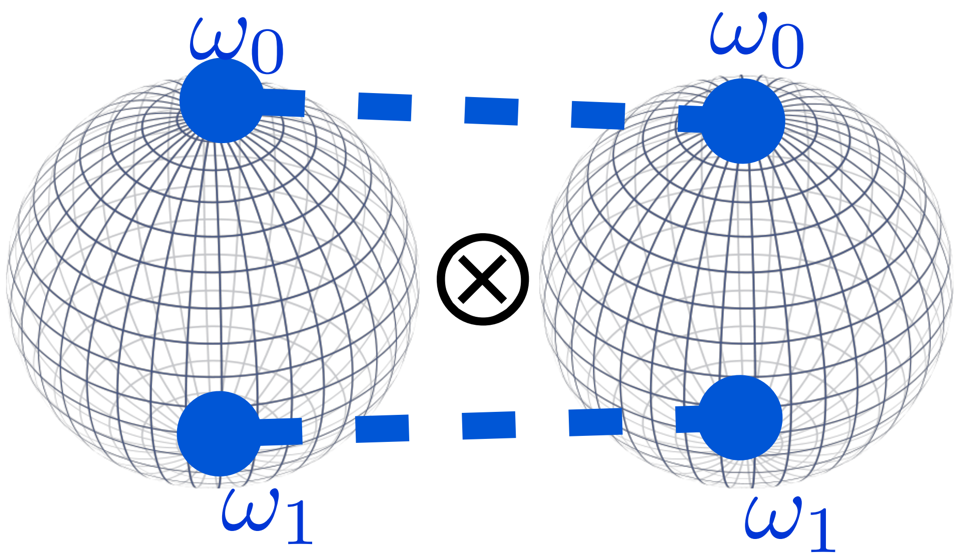

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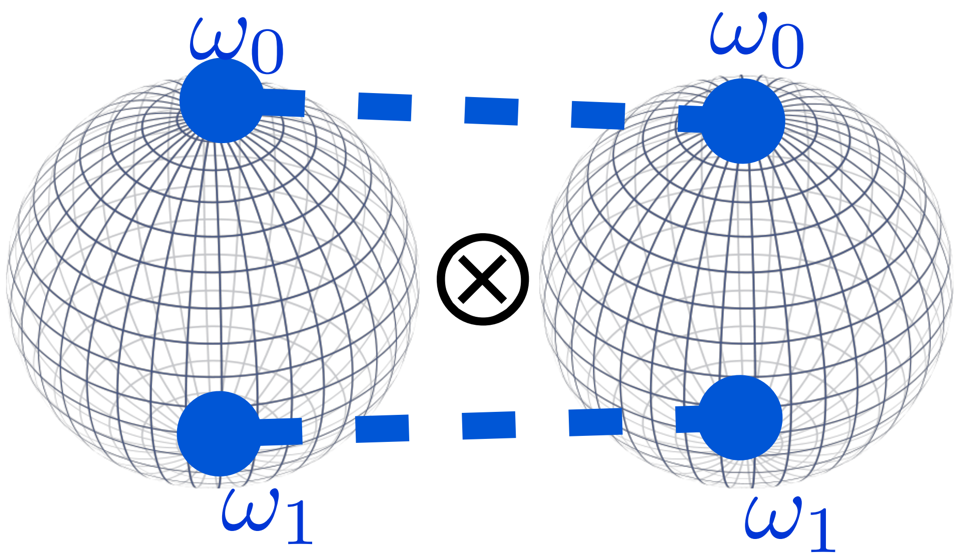

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Map 3-vectors to Hermitian matrices: $L(\omega) := \frac{1}{2}\left(\mathbf{1} + \sum_{i=1}^{3} \omega_i \sigma_i\right)$

- Facts on universal quantum computation,
- Wigner's theorem
- some other tricks

prove:

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prove:

Theorem: Every theory satisfying Axioms I-V (rather than CPT) is equivalent to $(\Omega_N, \mathcal{G}_N)$, where

- Ω_N are the density matrices on $\mathbb{C}^N$,
- $\mathcal{G}_N$ is the group of unitaries, acting by conjugation,
- the measurements are exactly the POVMs.

5. Some new developments

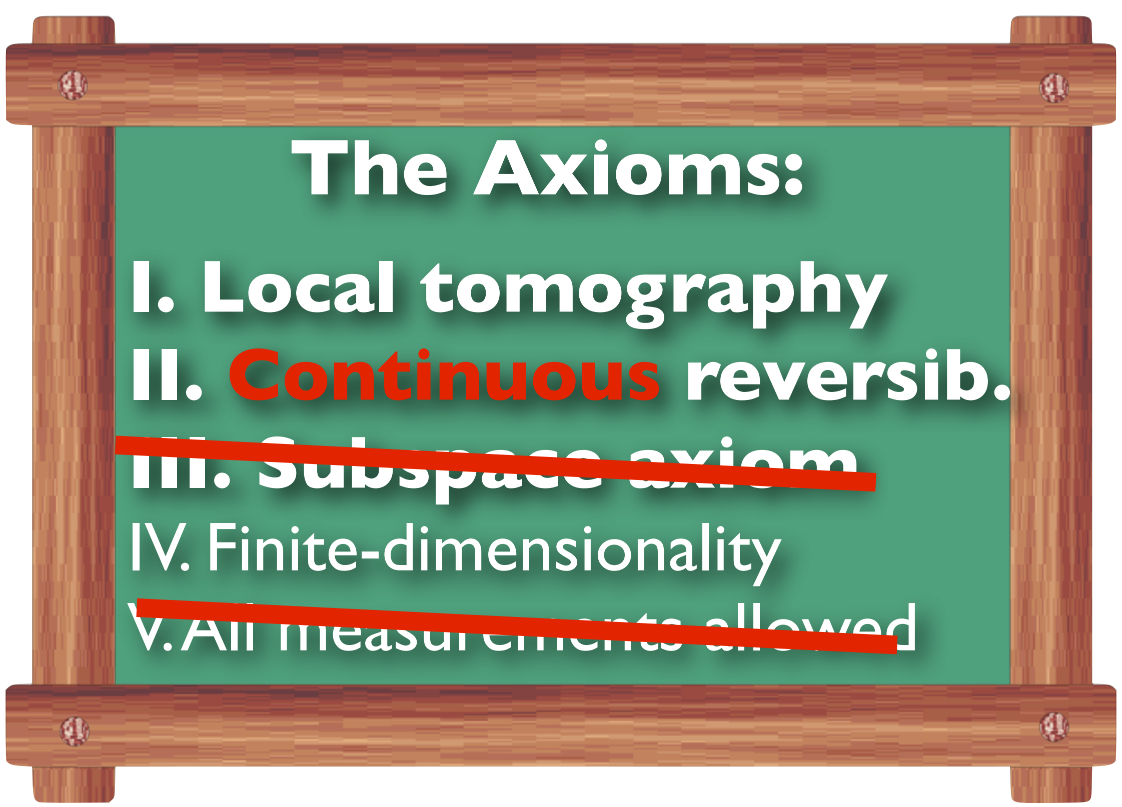
1 2 3

5. Some new developments

The Axioms:

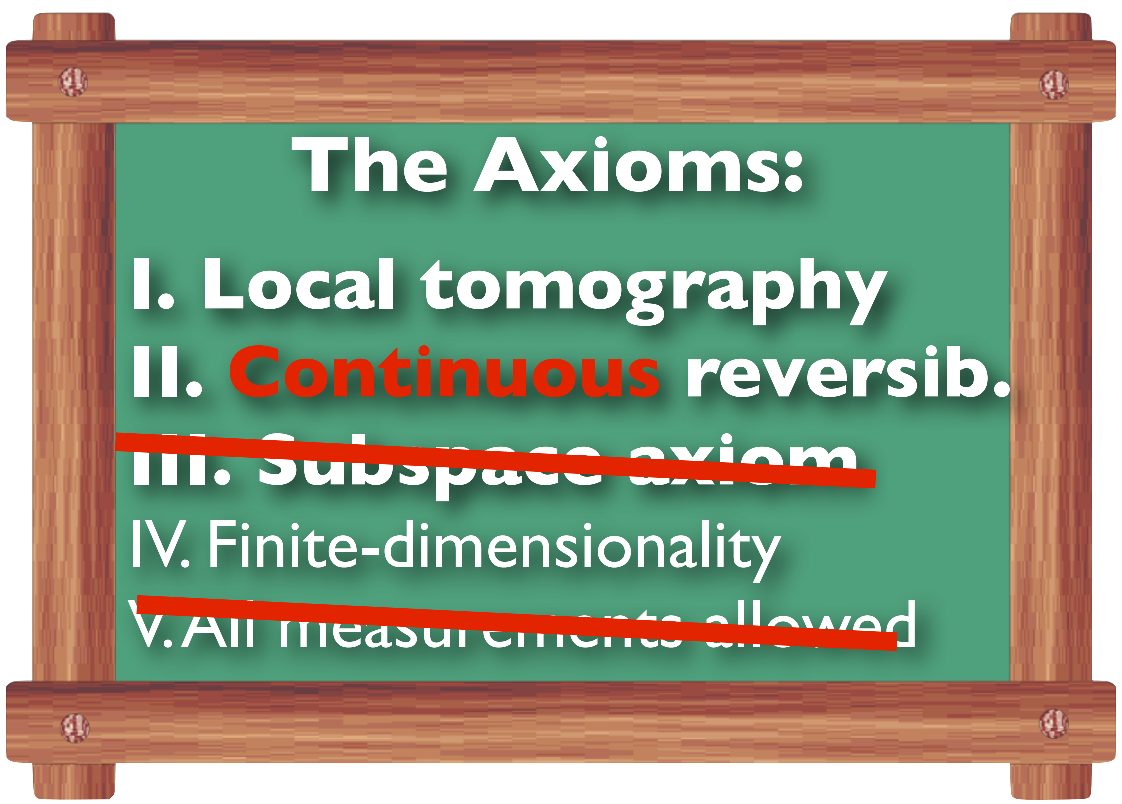
I. Local tomography
II. Reversibility
III. Subspace axiom
IV. Finite-dimensionality
V. All measurements allowed

5. Some new developments



5. Some new developments

Conjecture: All state spaces satisfying I,II,IV are quantum systems.



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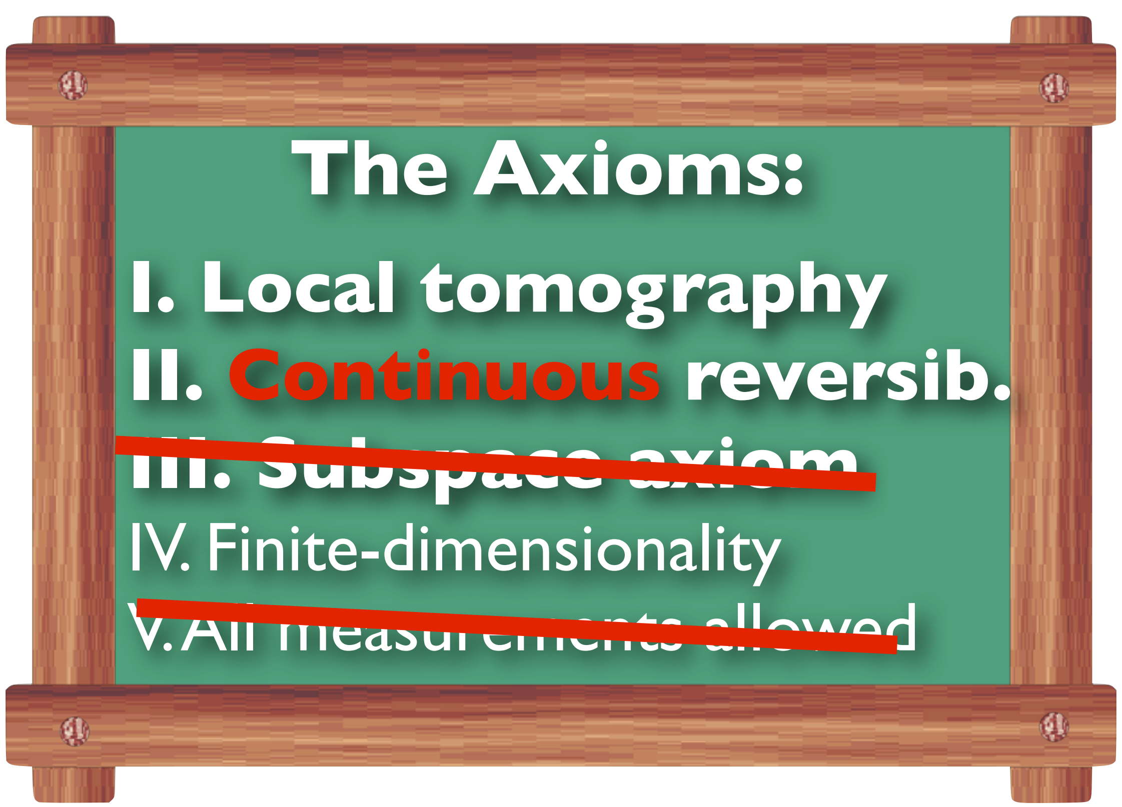
Probably wrong. Task: find counterexamples.



True if two local systems are balls:
Ll. Masanes, MM, D. Pérez-García, and R. Augusiak, arXiv:1111.4060

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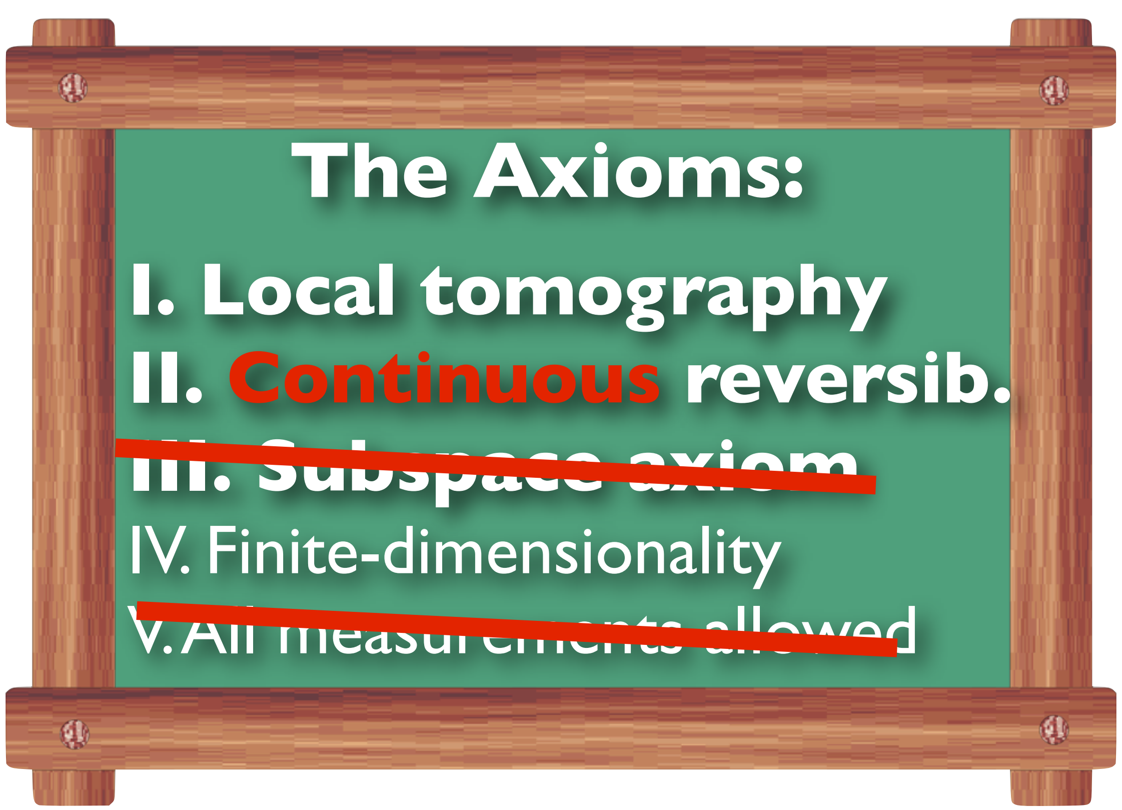

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$\Rightarrow$

5. Some new developments

Ll. Masanes, MM, R. Augusiak, and D. Pérez-García, *A digital approach to quantum theory*, arXiv:1208.0493

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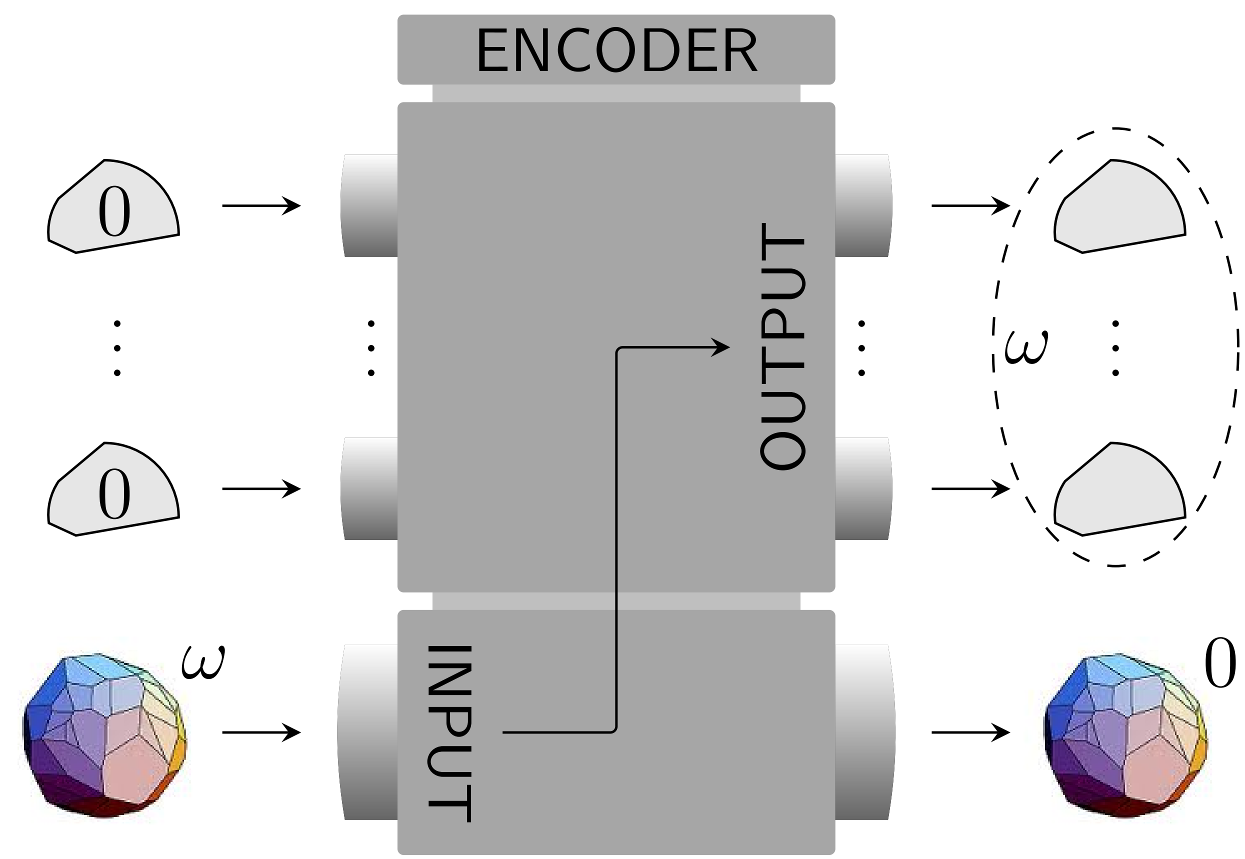
Quantum theory follows from

- Local tomography,
- Continuous reversibility,
- Existence of an information unit:

  there is "nice" binary system ("gbit") such that the state of any system can be reversibly encoded in a sufficiently large number of gbits.



5. Some new developments

1 2 3

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1 **2** 3

Science 23 July 2010:
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REPORT

Ruling Out Multi–Order Interference in Quantum Mechanics

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± Author Affiliations

*To whom correspondence should be addressed. E-mail: usinha@iqc.ca, gregor.weihs@uibk.ac.at

ABSTRACT

Quantum mechanics and gravitation are two pillars of modern physics. Despite their success in describing the physical world around us, they seem to be incompatible theories. There are suggestions that one of these theories must be generalized to achieve unification. For example, Born's rule—one of the axioms of quantum mechanics—could be violated. Born's rule predicts that quantum interference, as shown by a double–slit diffraction experiment, occurs from pairs of paths. A generalized version of quantum mechanics might allow multipath (i.e., higher–order) interference, thus leading to a deviation from the theory. We performed a three–slit experiment with photons and bounded the magnitude of three–path interference to less than 10^{-2} of the expected two–path interference, thus ruling out third– and higher–order interference and providing a bound on the

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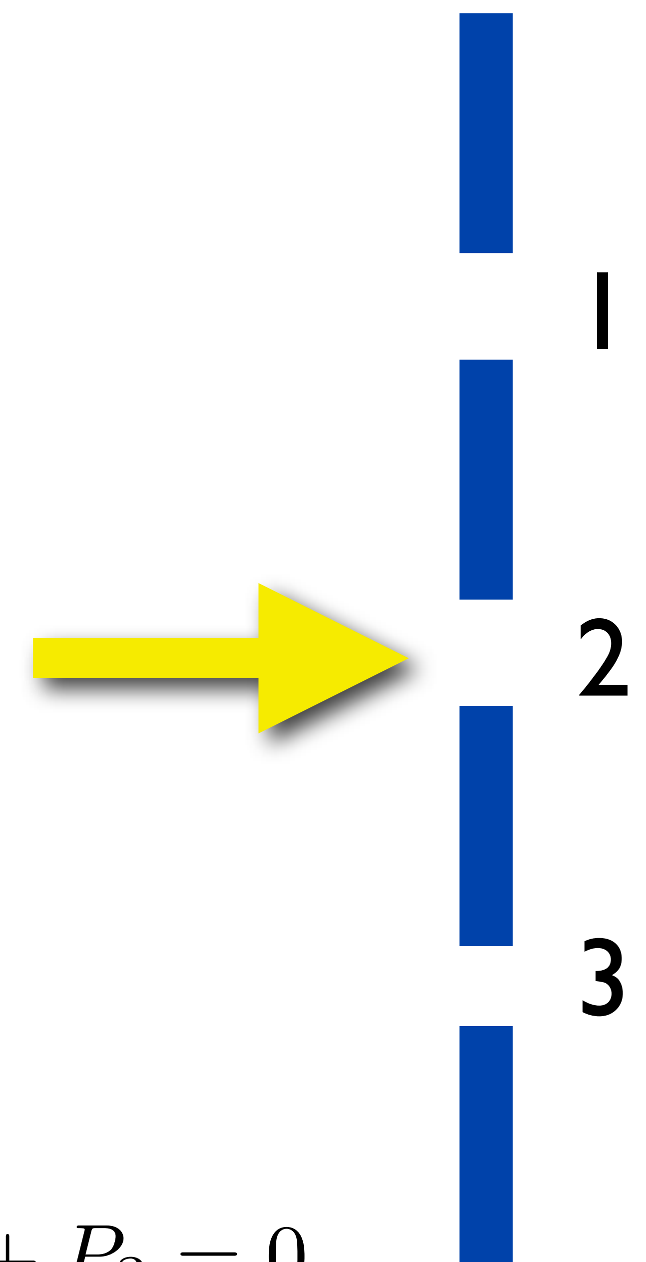
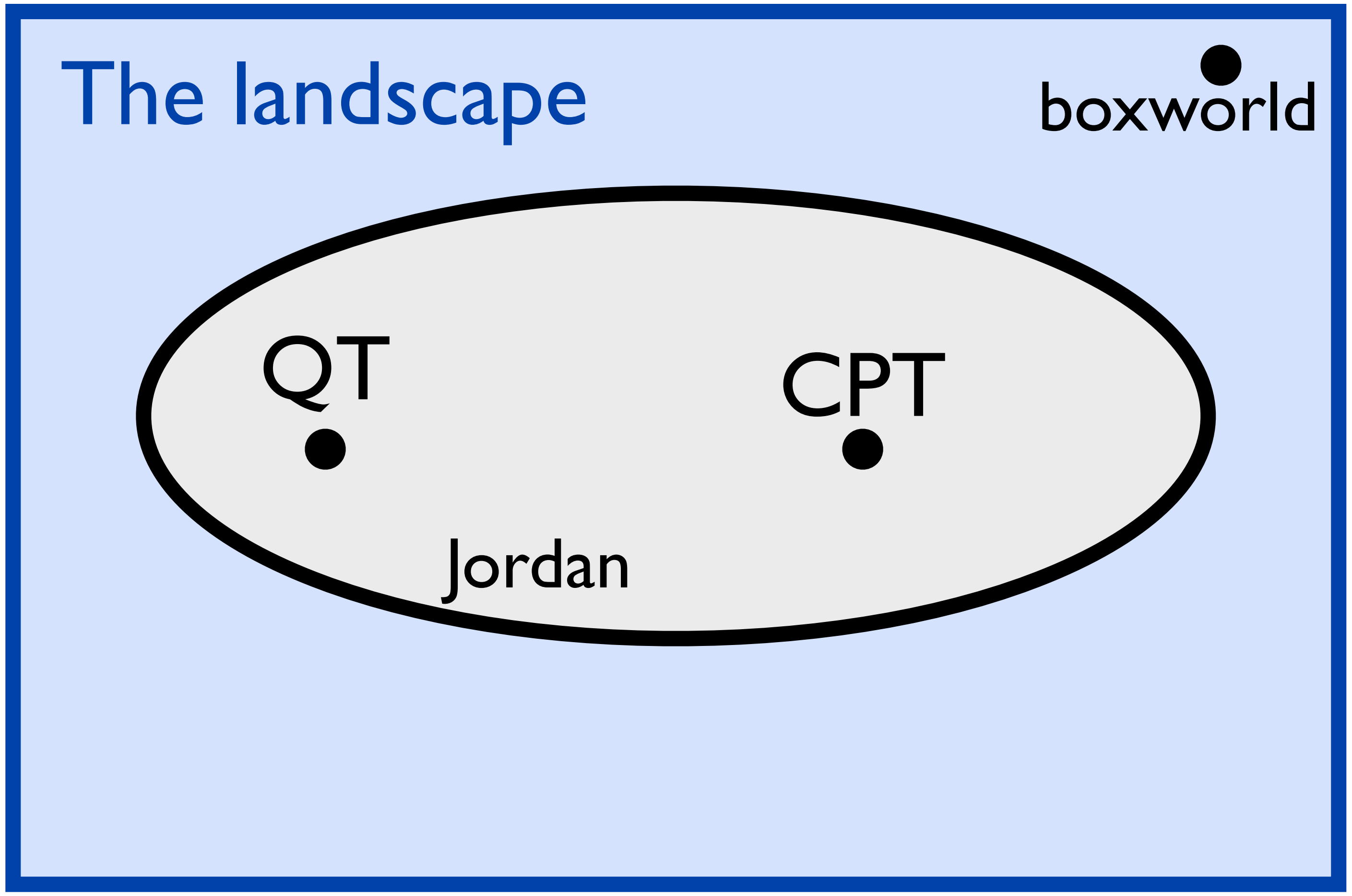


Quantum theory: $P_{123} - P_{12} - P_{23} - P_{13} + P_1 + P_2 + P_3 = 0$

⇒ no 3rd-order interference (R. Sorkin, Mod. Phys. Lett. **A9**, 3119 (1994))

5. Some new developments

1 **2** 3



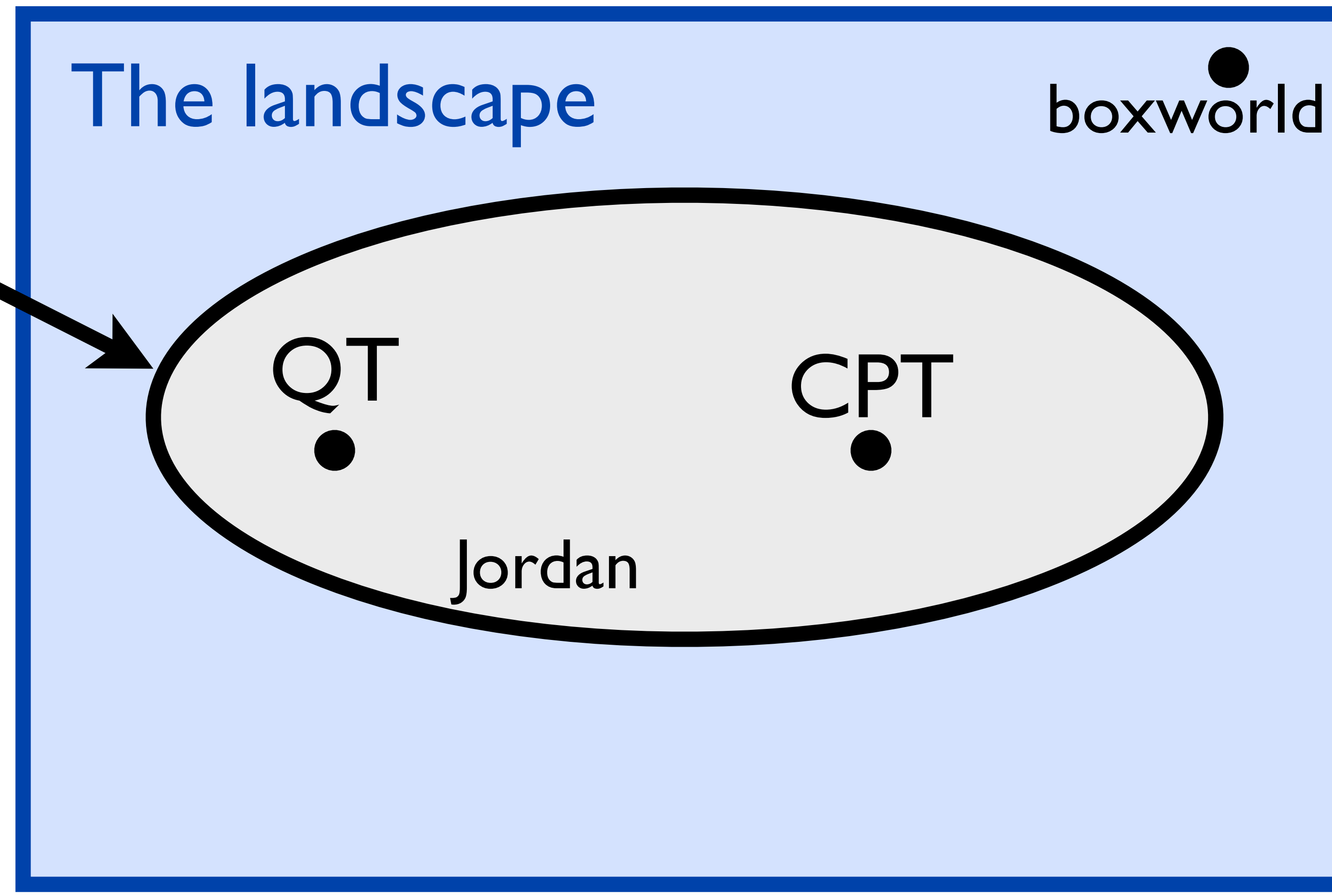
5. Some new developments

1 **2** 3

Jordan algebra state spaces

(real, complex, quaternionic QM, octonionic 3-level QM, ball state spaces)

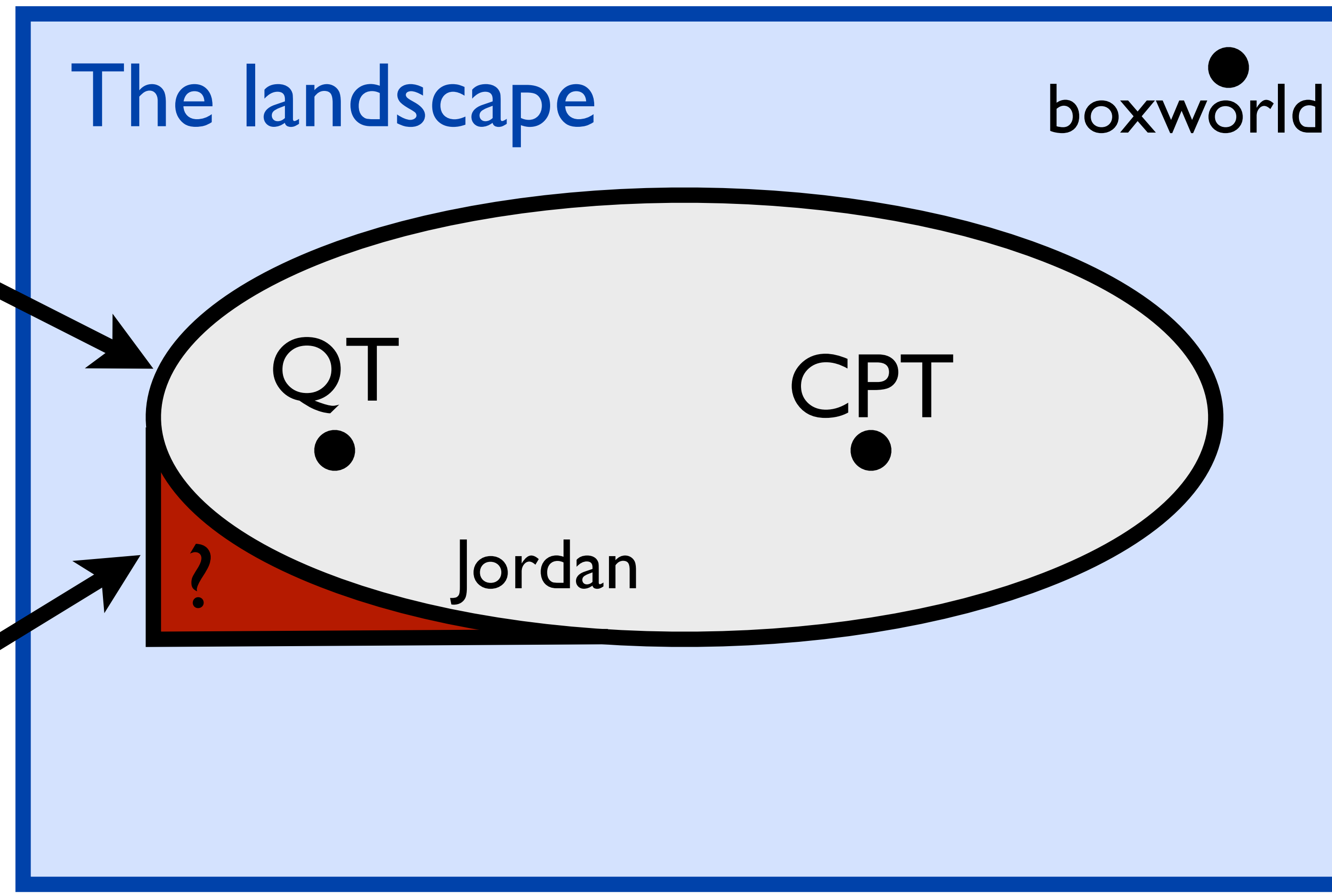
2nd order, but no 3rd-order interference



5. Some new developments

Jordan algebra state spaces

(real, complex, quaternionic QM, octonionic 3-level QM, ball state spaces)
2nd order, but no 3rd-order interference



What are QT's closest cousins
that show 3rd order interference?

5. Some new developments

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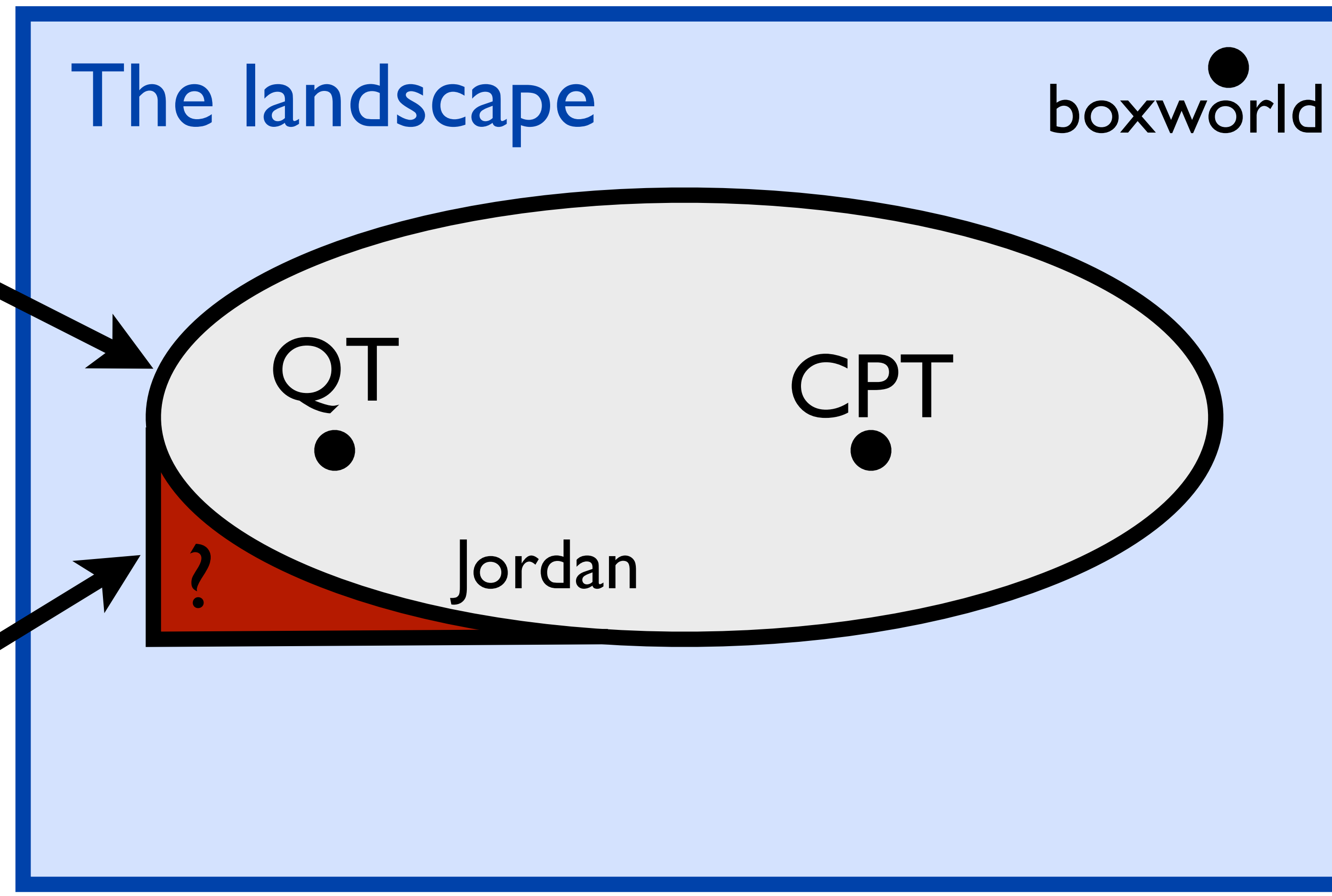


What are QT's closest cousins
that show 3rd order interference?

<u>Def.</u>: $\omega_1, \ldots, \omega_n$ pure & perfectly distinguishable states are called a frame.

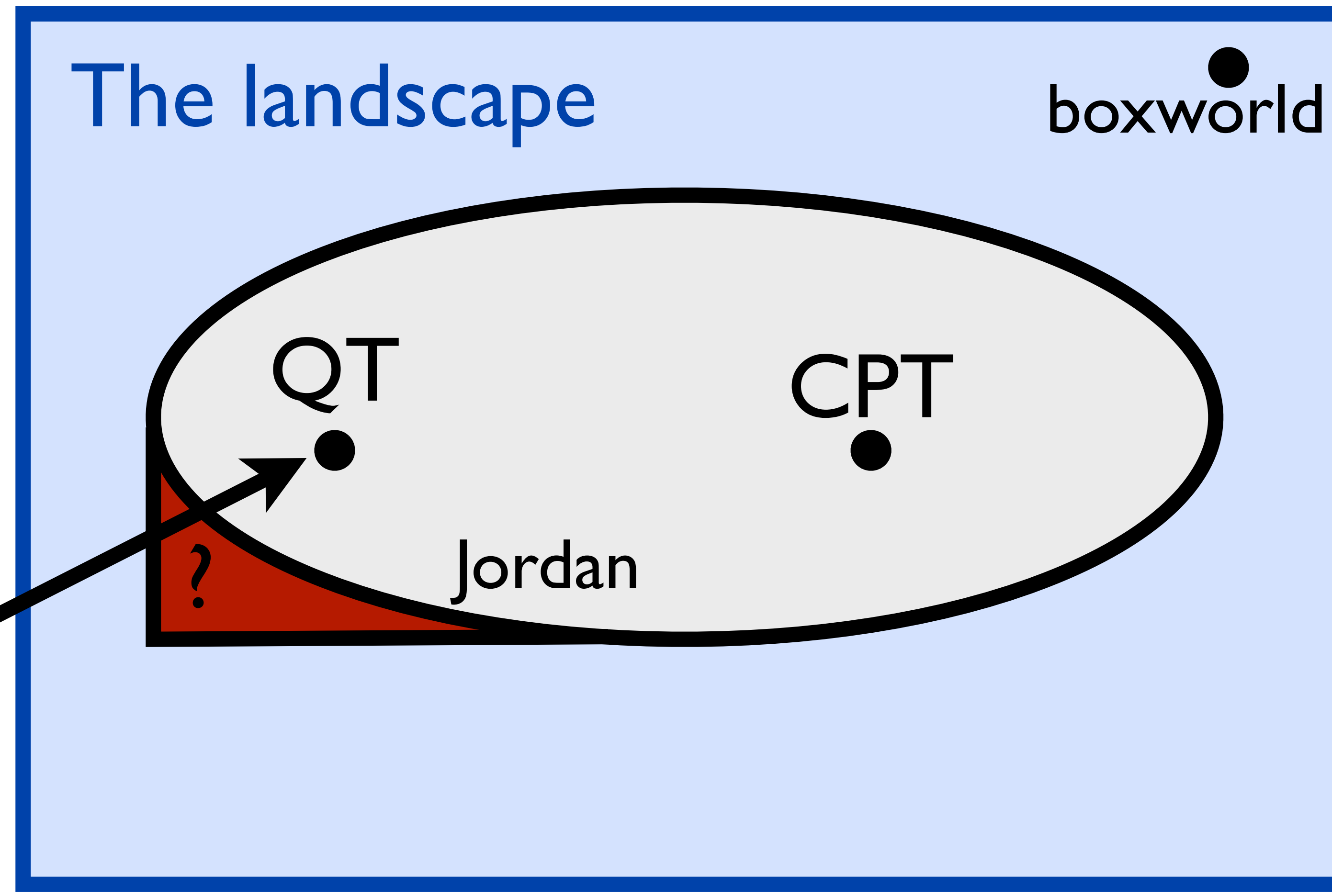
5. Some new developments

1 **2** 3

Joint work with Howard Barnum & Cozmin Ududec:

1. Every state is in the convex hull of some frame.
2. All frames of the same size are related by reversible transformations.
3. Local tomography

$\Rightarrow$ QT + CPT



5. Some new developments

1 **2** 3

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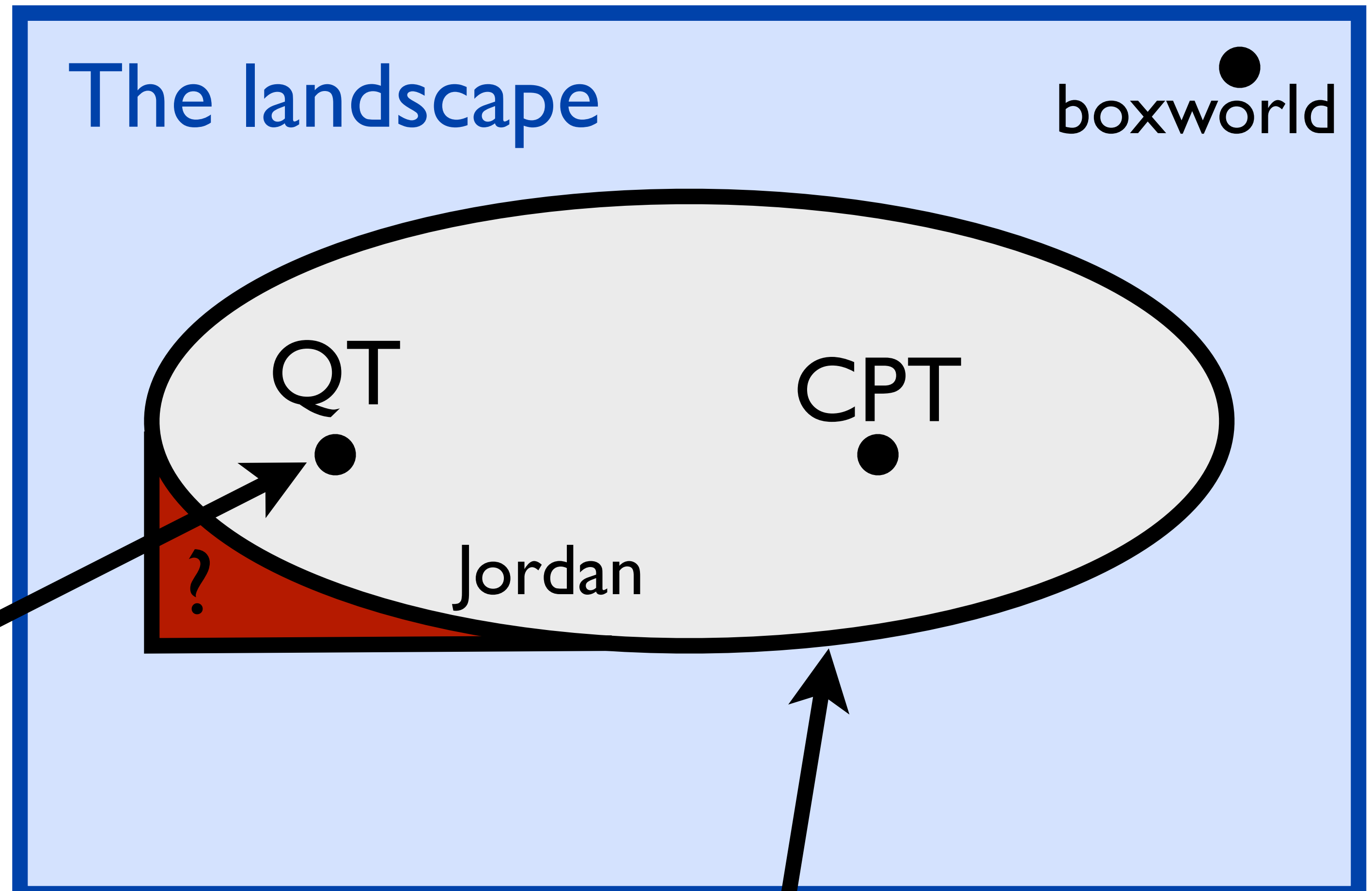

1. (as above)
2. (as above)

3'. absence of 3rd order interference

$\Rightarrow$ Jordan

5. Some new developments

Joint work with Howard Barnum & Cozmin Ududec:

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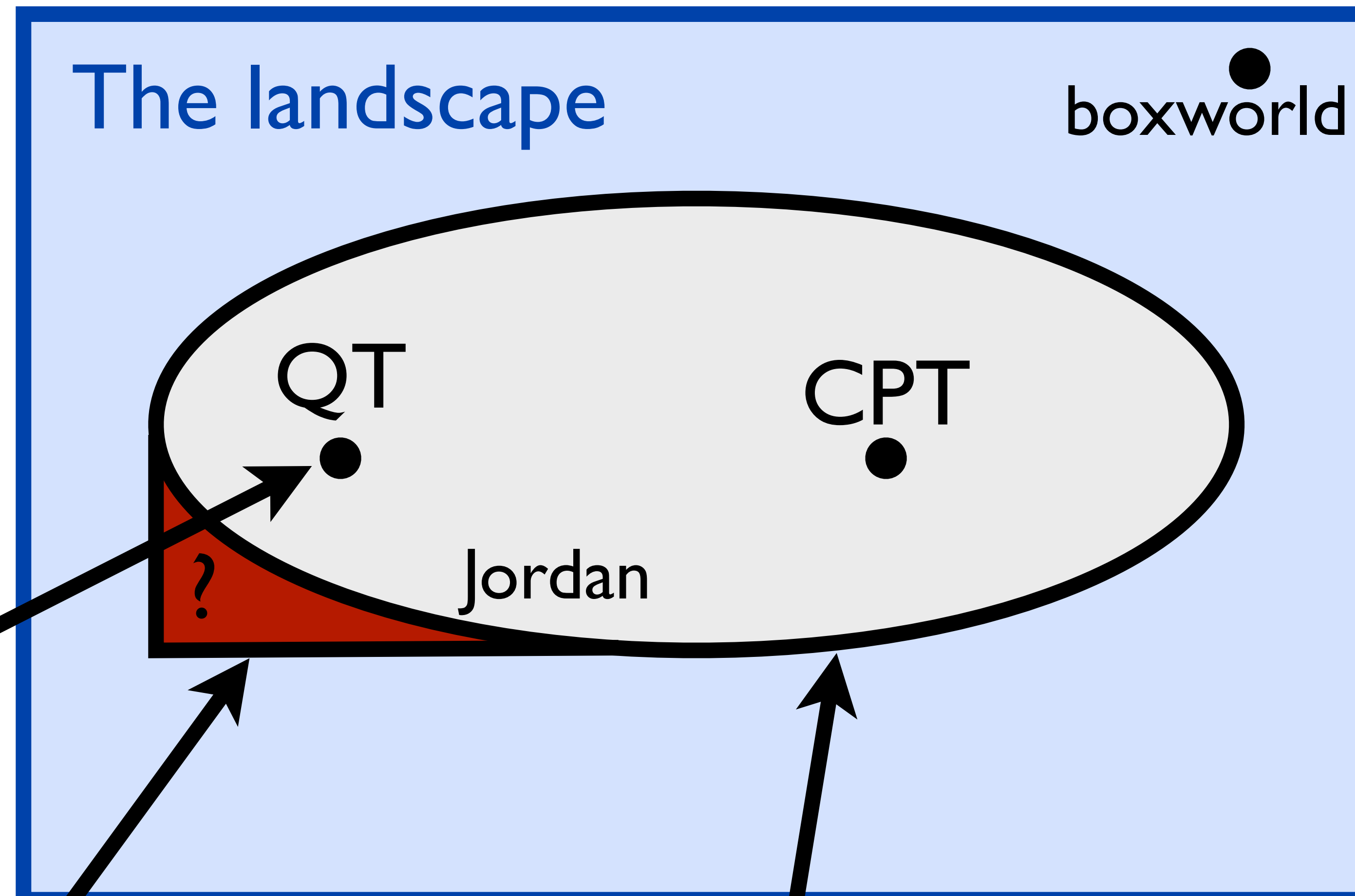
3. Local tomography

⇒ QT + CPT



1. (as above)
2. (as above)

⇒ ???

1. (as above)
2. (as above)

3'. absence of 3rd order interference

⇒ Jordan

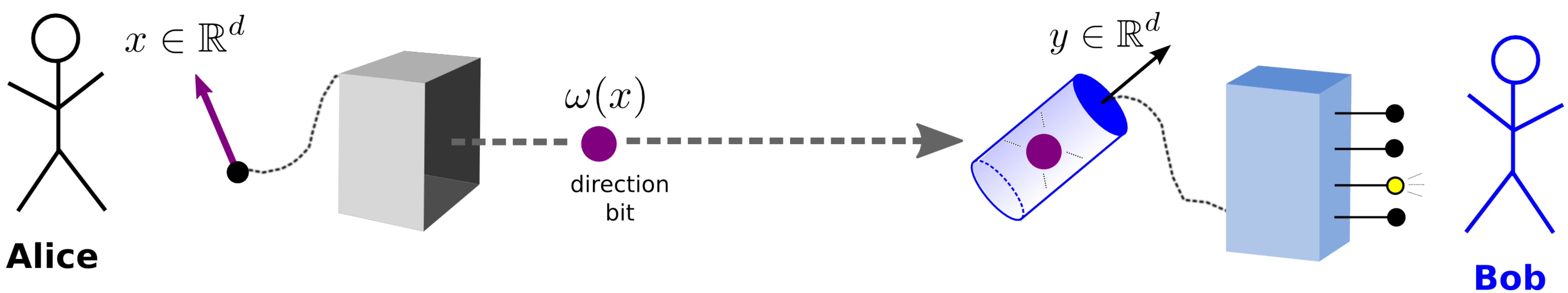
5. Some new developments

1 2 3

5. Some new developments

1 2 **3**

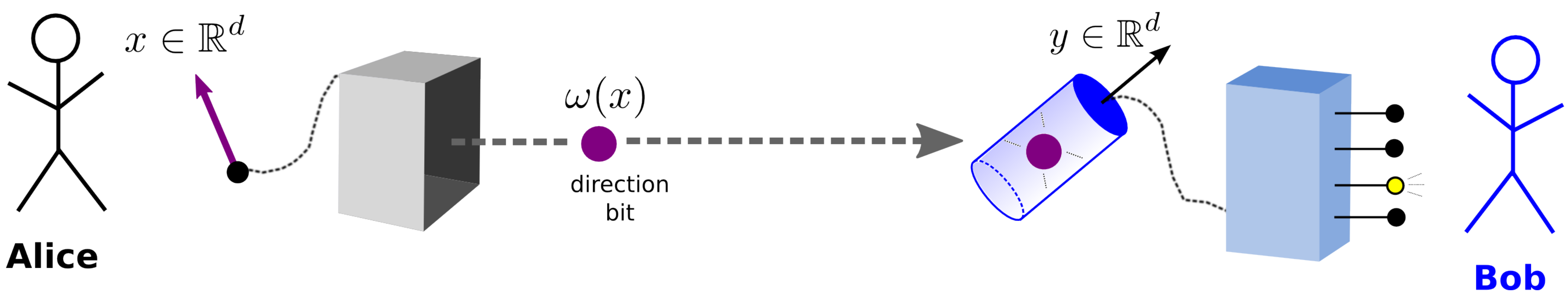
MM and Ll. Masanes, *Three-dimensionality of space and the quantum bit: how to derive both from information-theoretic postulates*, arXiv:1206.0630



5. Some new developments

1 2 **3**

MM and Ll. Masanes, *Three-dimensionality of space and the quantum bit: how to derive both from information-theoretic postulates*, arXiv:1206.0630



Invitation: Q+ Hangout Talk (online), Tuesday, Oct 23.
More info later at: mattleifer.info

Reference:
arXiv:1004.1483

Book chapter summary:
arXiv:1203.4516

More references:
mpmueller.net

Thank you!