Three-dimensionality of space and the quantum bit: an information-theoretic approach

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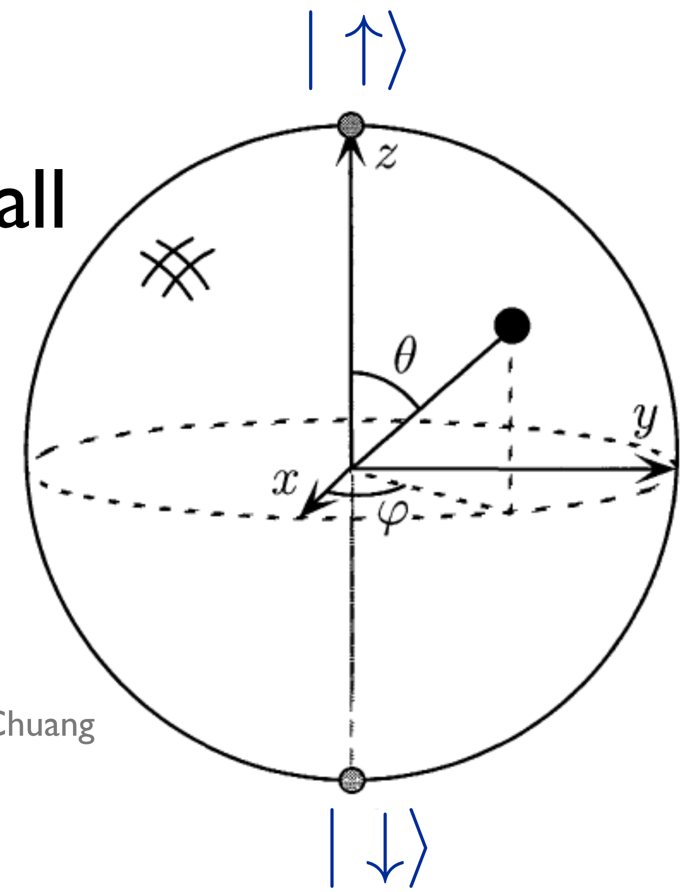
PERIMETER INSTITUTE
FOR THEORETICAL PHYSICS

A surprising coincidence

State space of quantum 2-level system is a 3D Euclidean ball:



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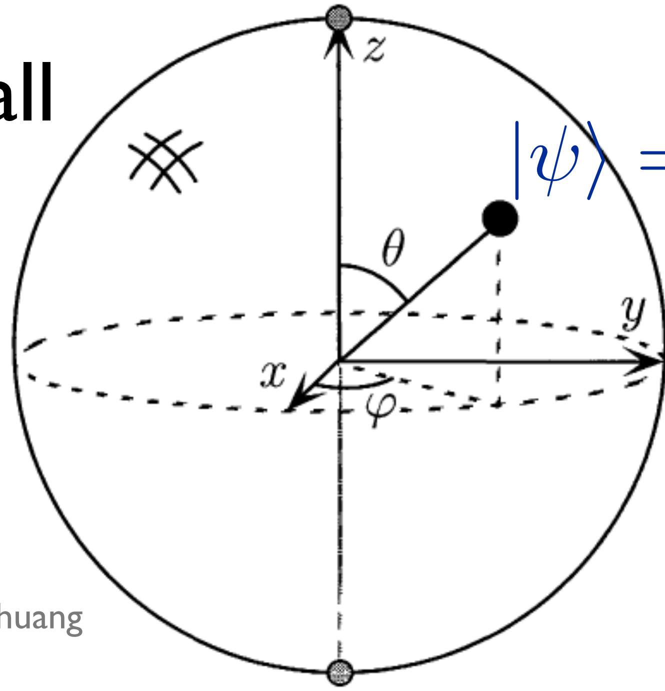

Bloch ball

$|\uparrow\rangle$

$|\downarrow\rangle$

$$|\psi\rangle = \cos\frac{\theta}{2}|\uparrow\rangle + e^{i\varphi}\sin\frac{\theta}{2}|\downarrow\rangle$$

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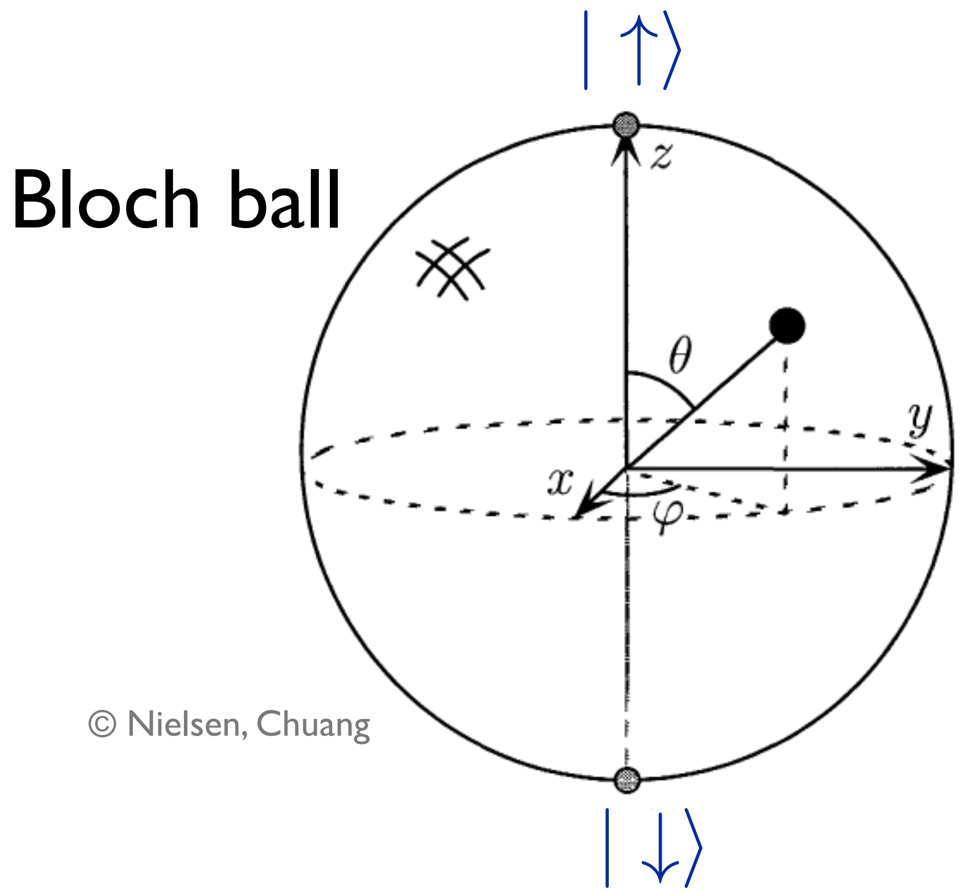

Space is also 3-dimensional! Is there some deeper reason for this?

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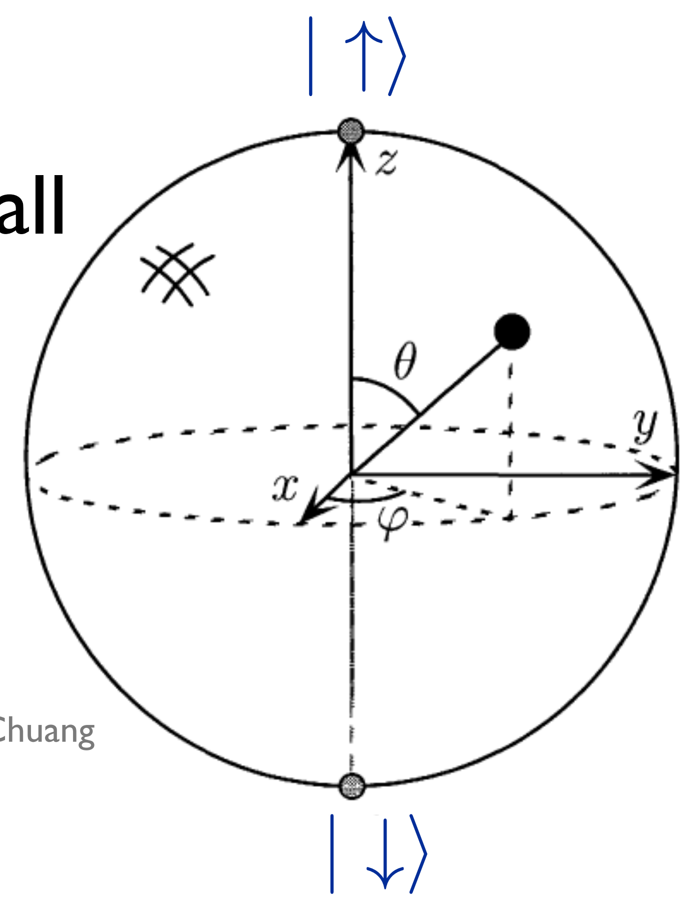

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Probability ⟷ (Spacetime) geometry

Our approach

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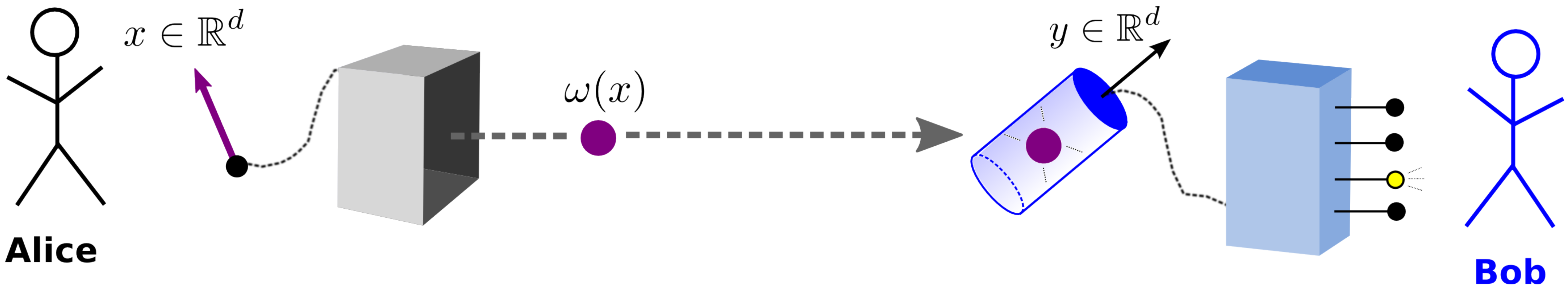
Information-theoretic task in d-dimensional space:



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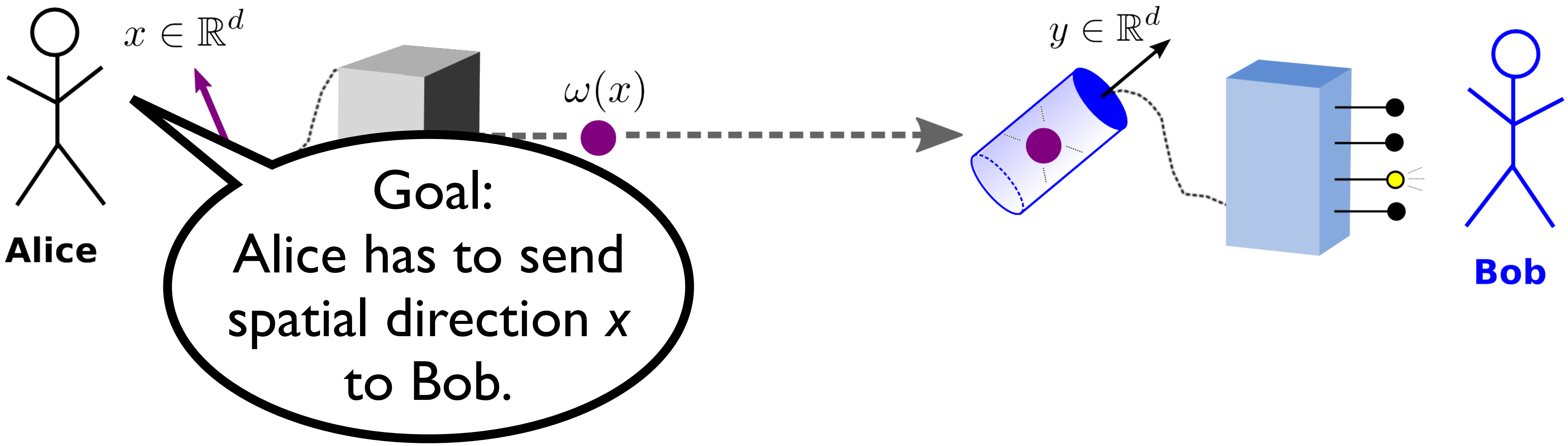

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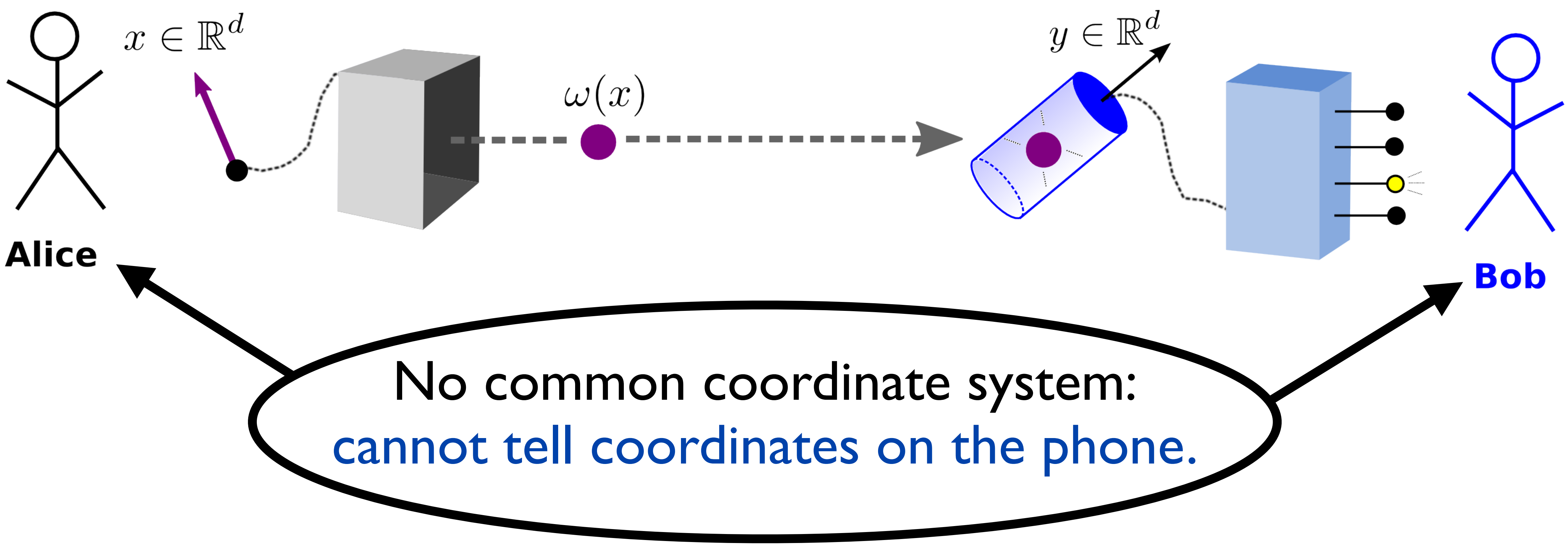

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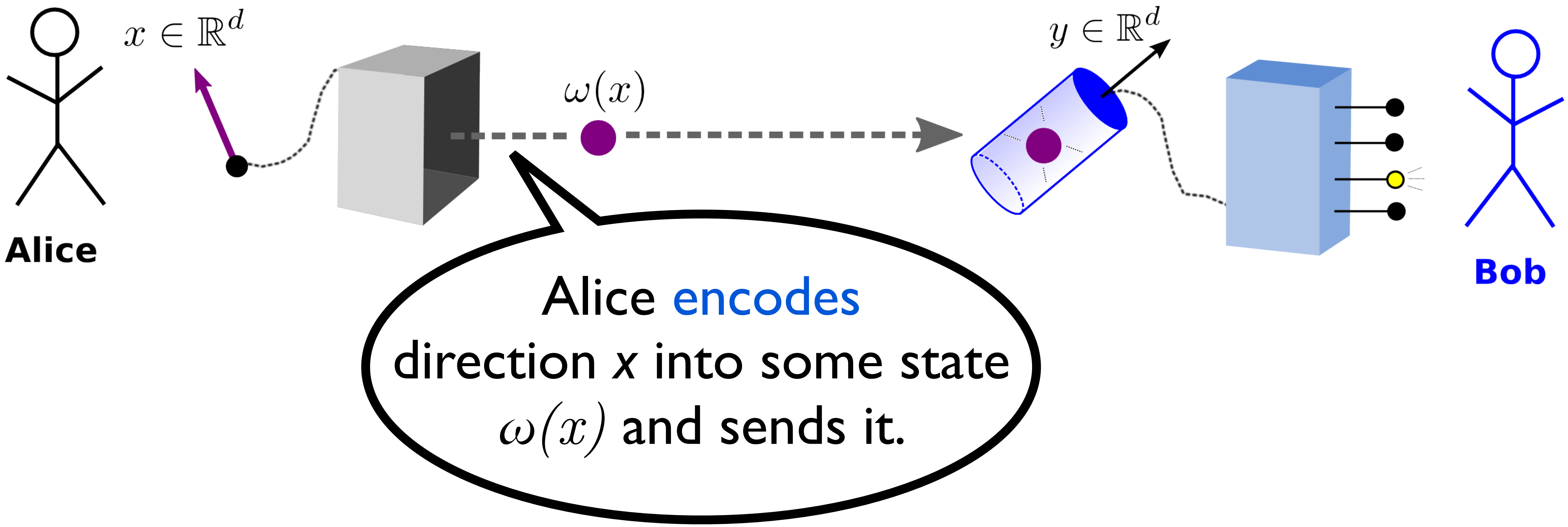

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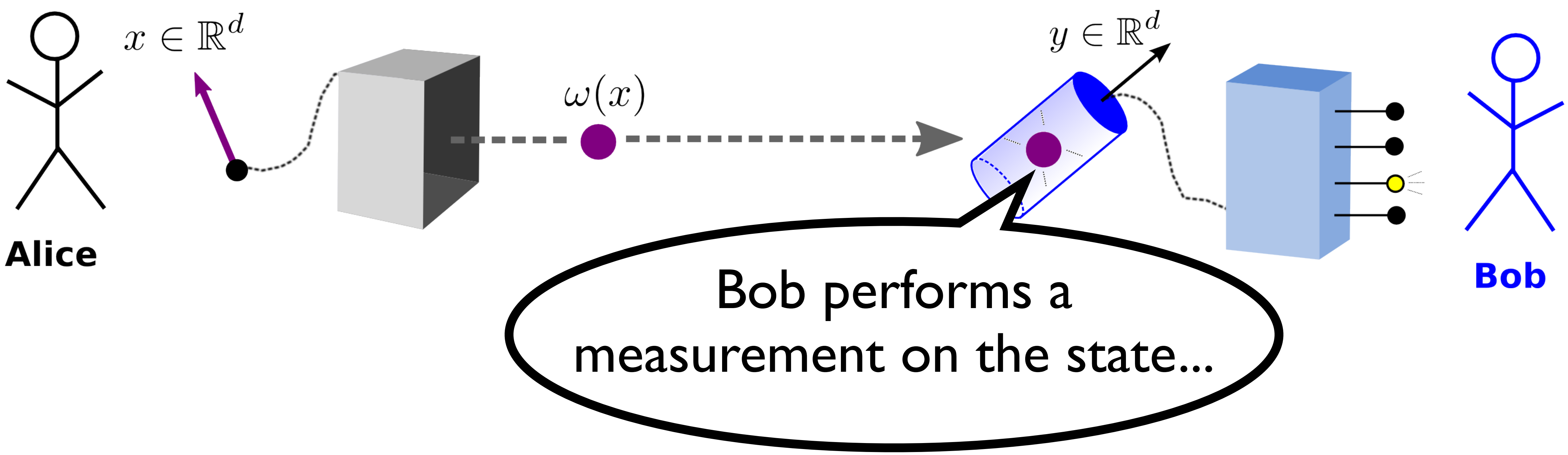

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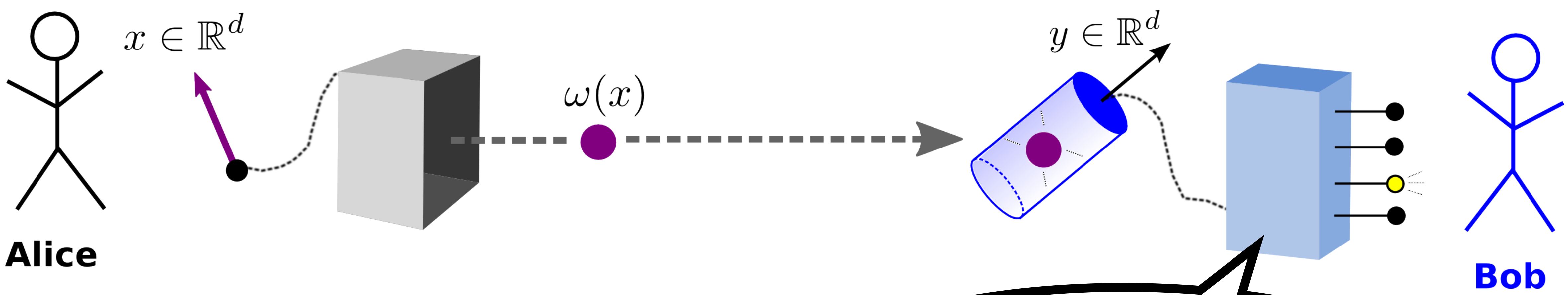

... and obtains one of several outcomes with some probability.

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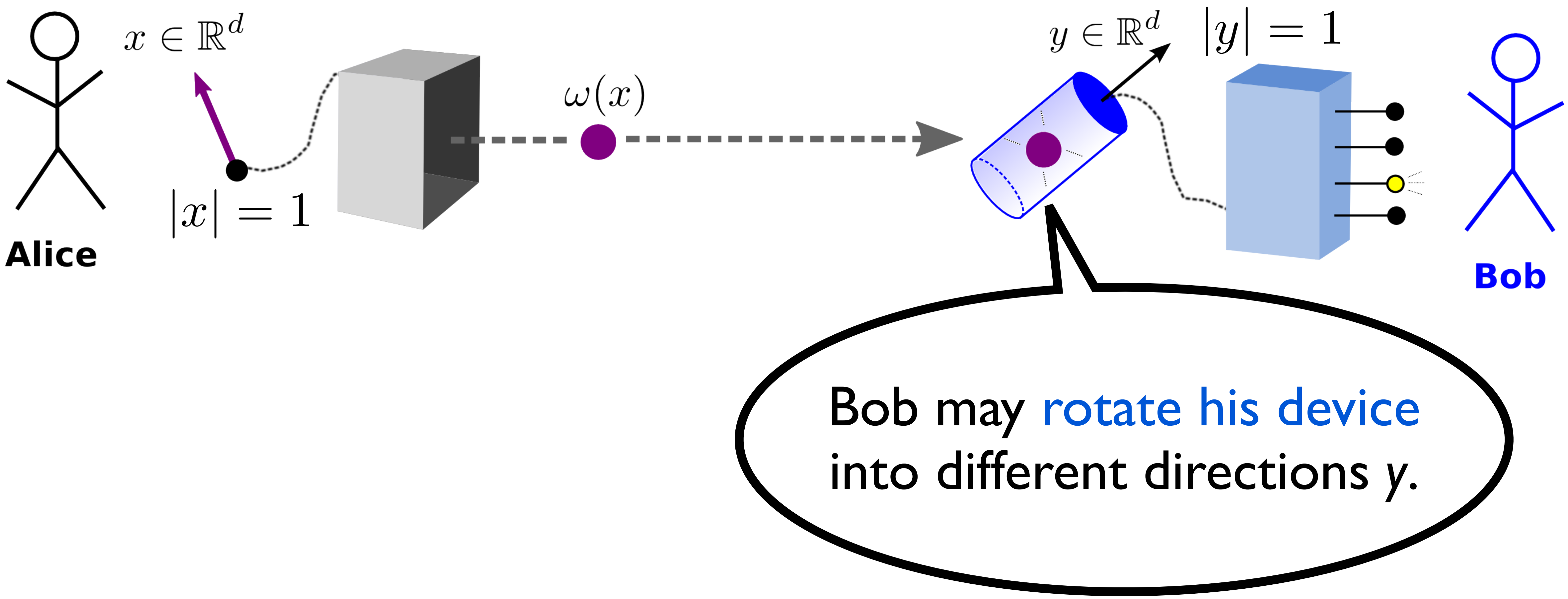

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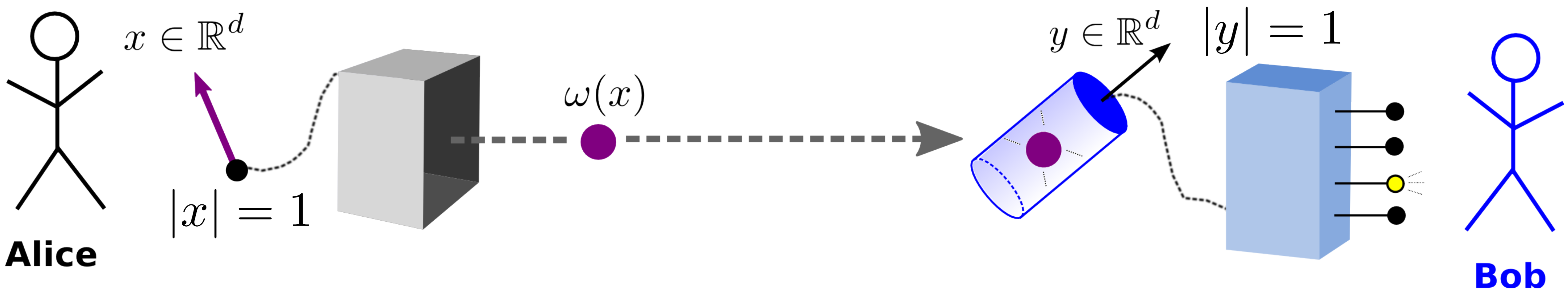
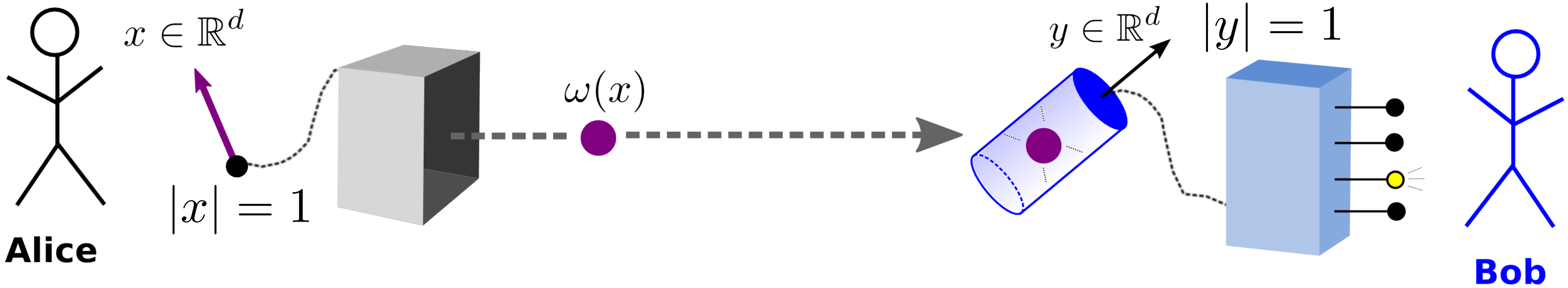


Example within quantum theory:

- spin-1/2 particle with "spin up" in *x*-direction,
- Stern-Gerlach measurement device.

Our approach

Information-theoretic task in d-dimensional space:



Here we do **not** assume quantum theory.

Our approach

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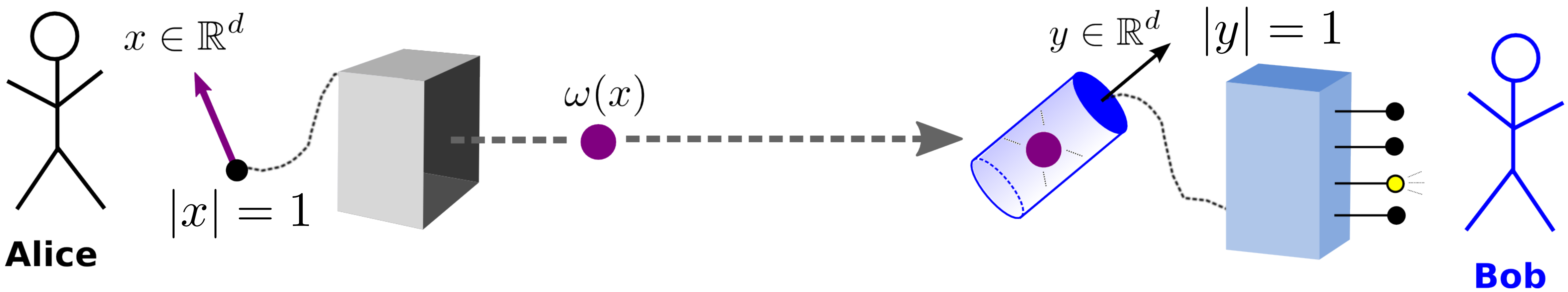

IF

- Bob can determine *x* in the limit of many copies, but
- Alice cannot encode any additional information, and
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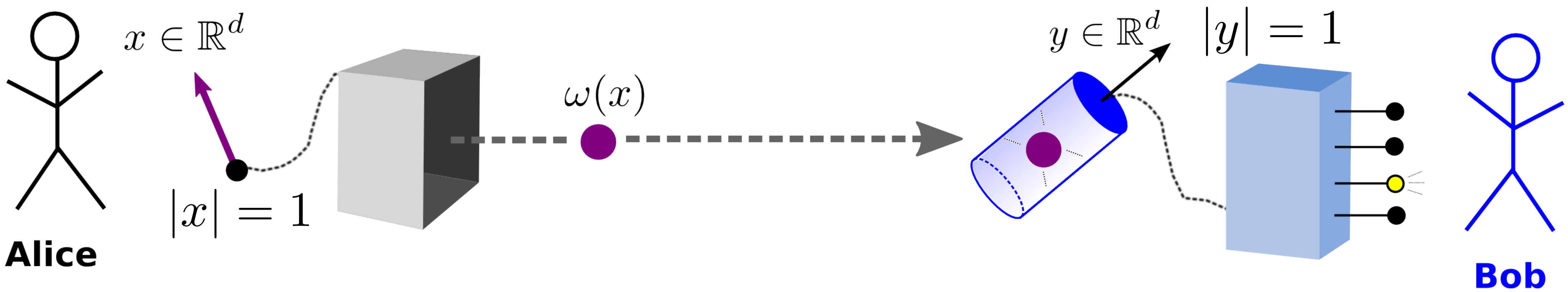

IF

- Bob can determine *x* in the limit of many copies, but
- Alice cannot encode any additional information, and
- the information carriers can interact continuously and reversibly in time,

THEN

- necessarily *d=3* and
- quantum theory holds for information carriers

(we get unitary time evolution, entanglement, QT state space).

Overview

1. Overview

2. Convex state spaces

3. The postulates

4. Deriving *d=3* and quantum theory

5. What does all this tell us?

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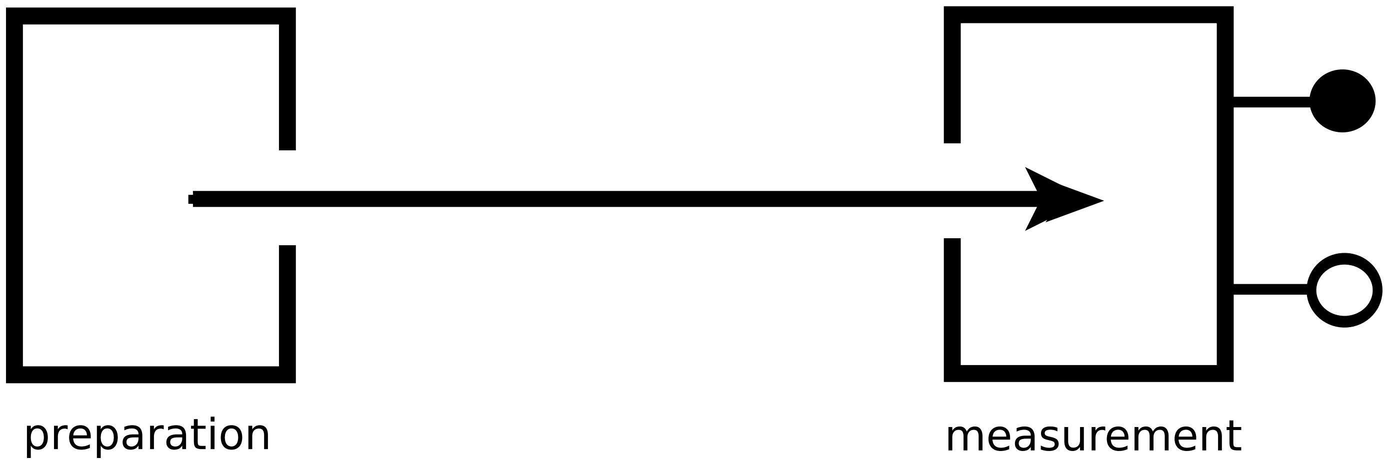
2. Convex state spaces

Assumption: there are some events that happen probabilistically.



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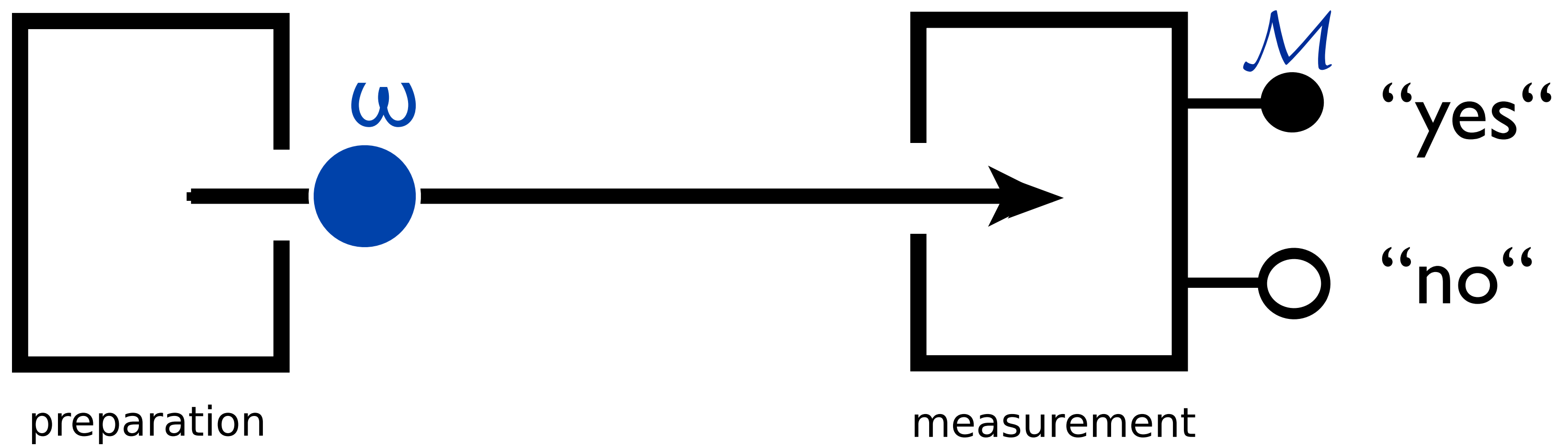

- Physical systems can be in some state ω. From this, probabilities of outcomes of all possible measurements can be computed:

$$\mathrm{Prob}(\text{outcome ''yes'' in this measurement} \mid \text{input state } \omega) =: \mathcal{M}(\omega).$$

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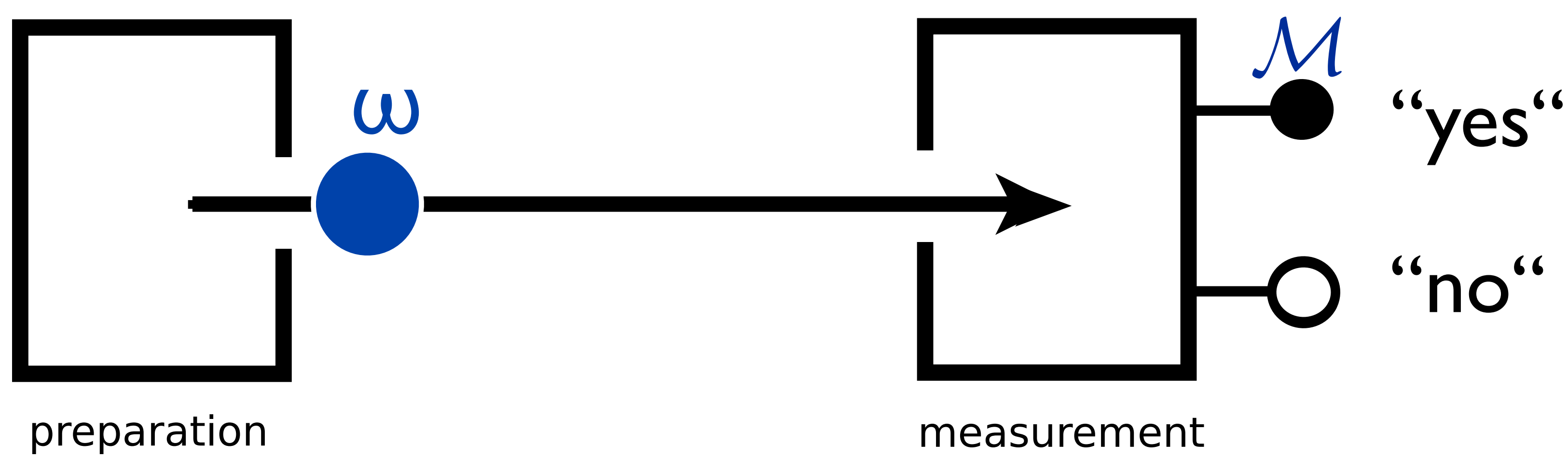

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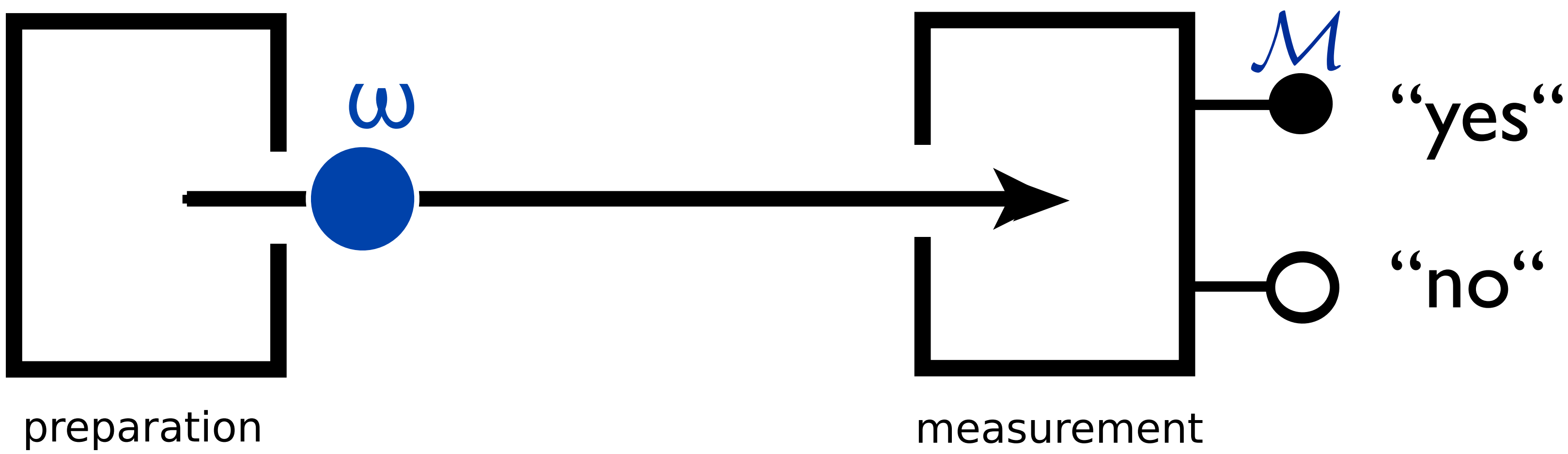

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- Statistical mixtures are described by convex combinations: prepare ω with prob. p and state φ with prob. $(1-p)$, result:

$$p\,\omega + (1-p)\varphi$$

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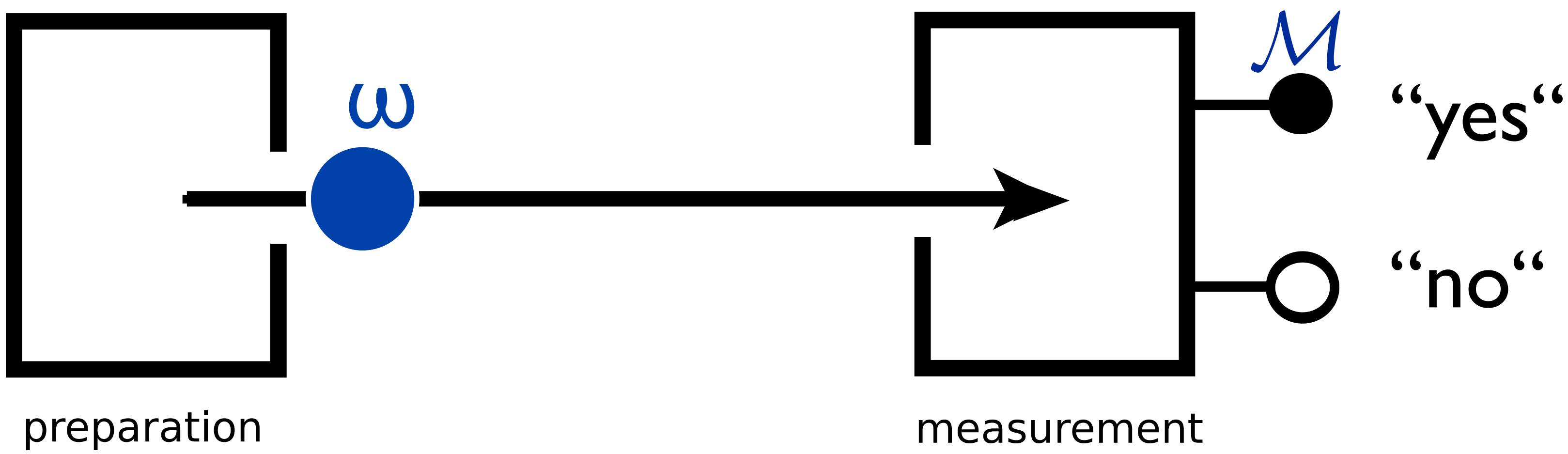

- Consequence: events $\mathcal{M}$ are affine-linear maps:

$$\mathcal{M}(p\omega + (1-p)\varphi) = p\mathcal{M}(\omega) + (1-p)\mathcal{M}(\varphi).$$

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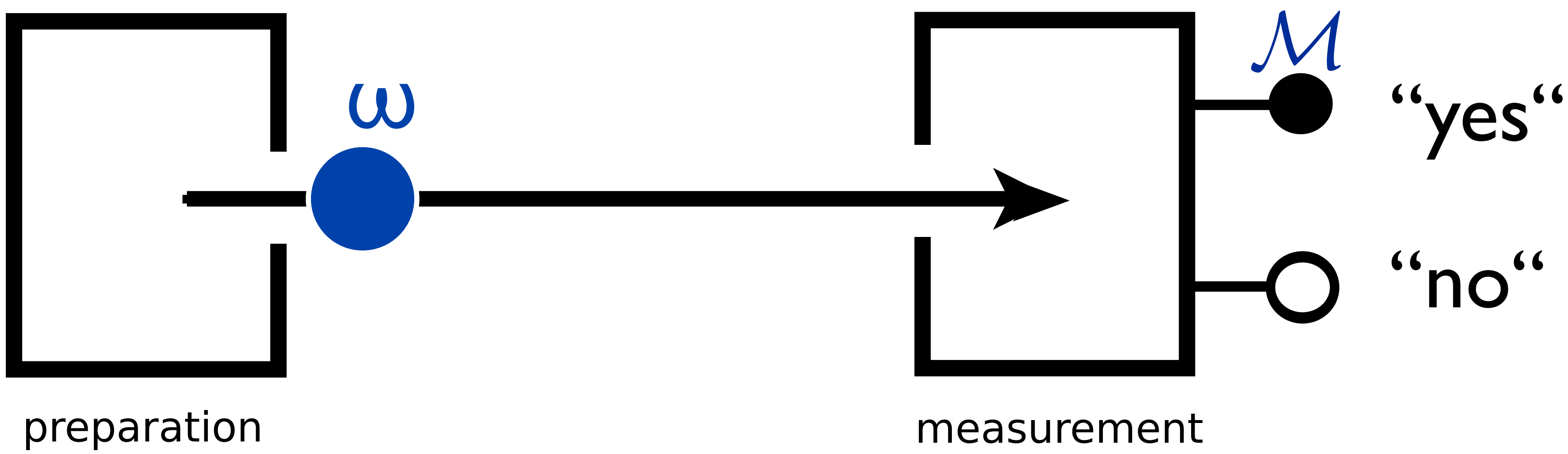

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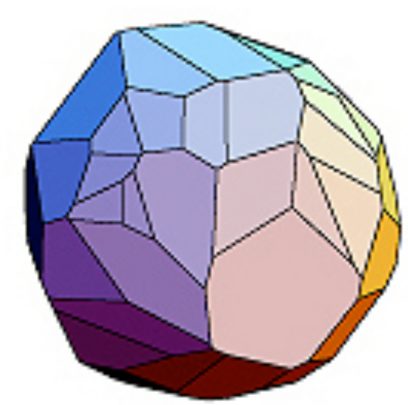
- State space Ω = set of all possible states ω.
  Convex, compact, finite-dimensional.
  Otherwise arbitrary.



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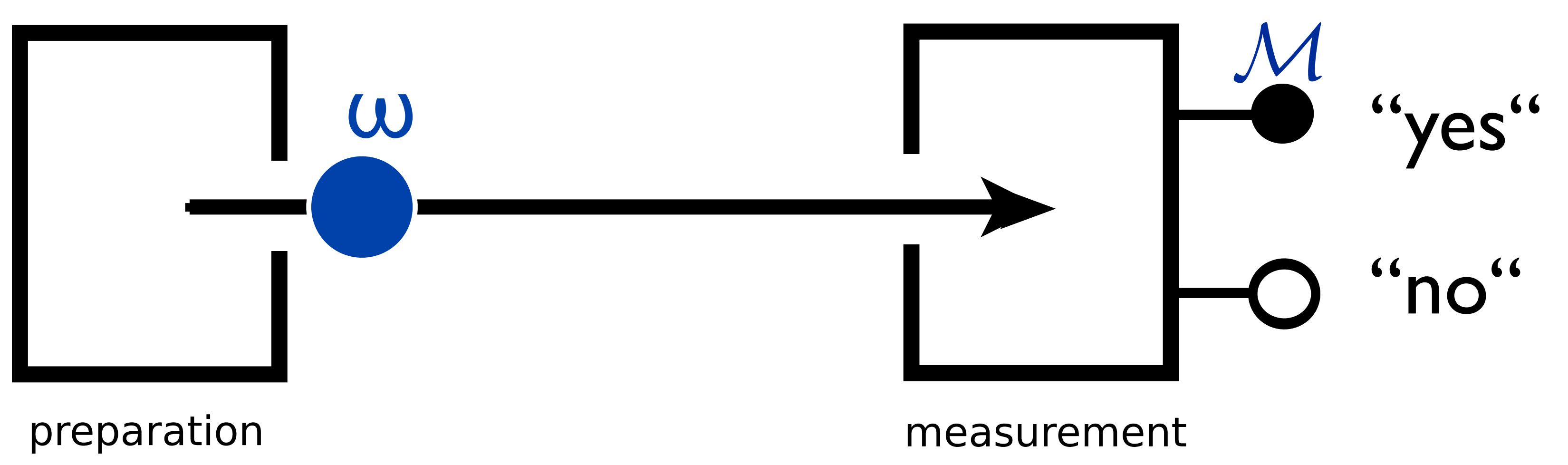

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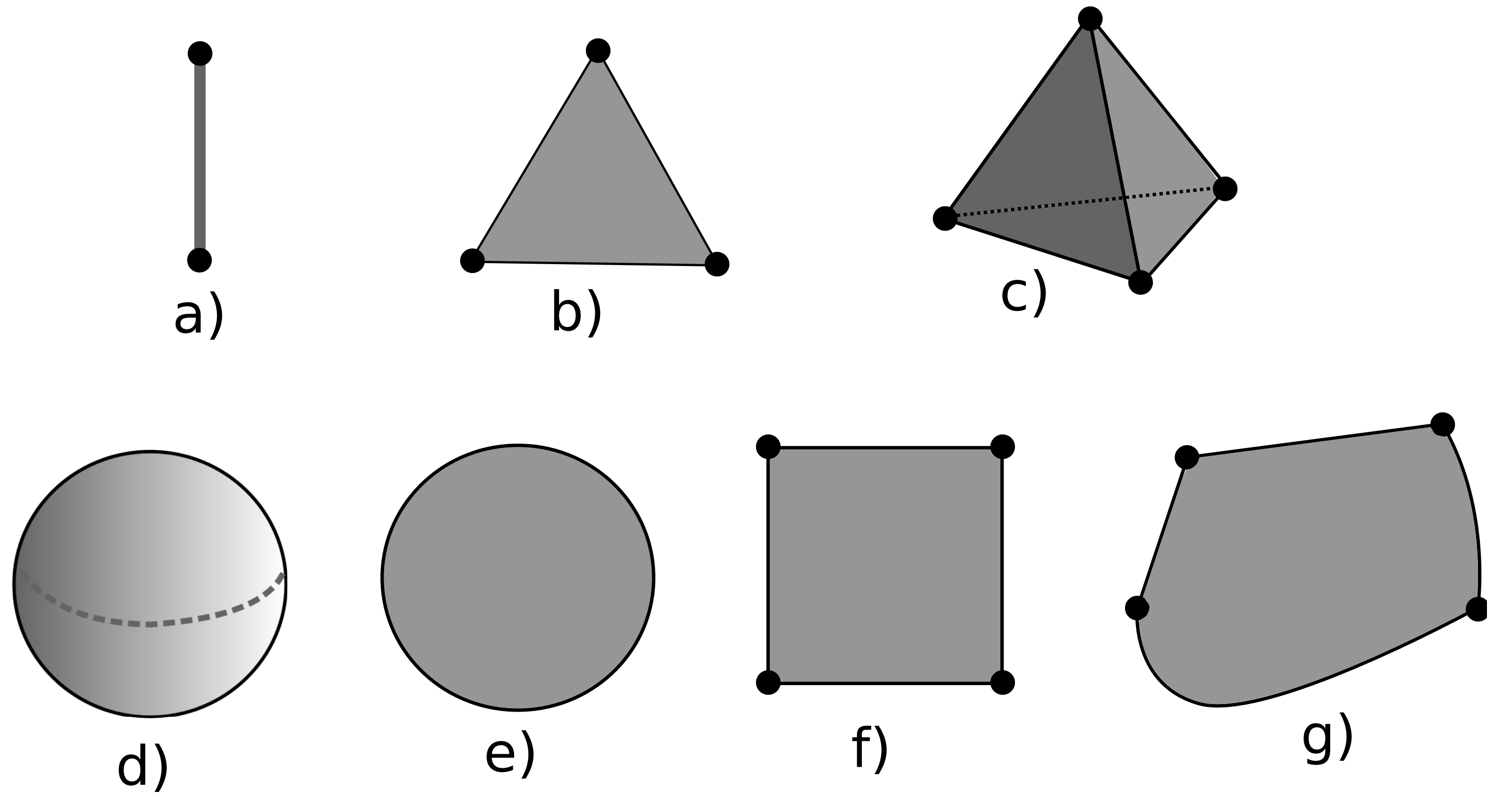
  Extremal points are pure states, others mixed.

2. Convex state spaces

Some examples:



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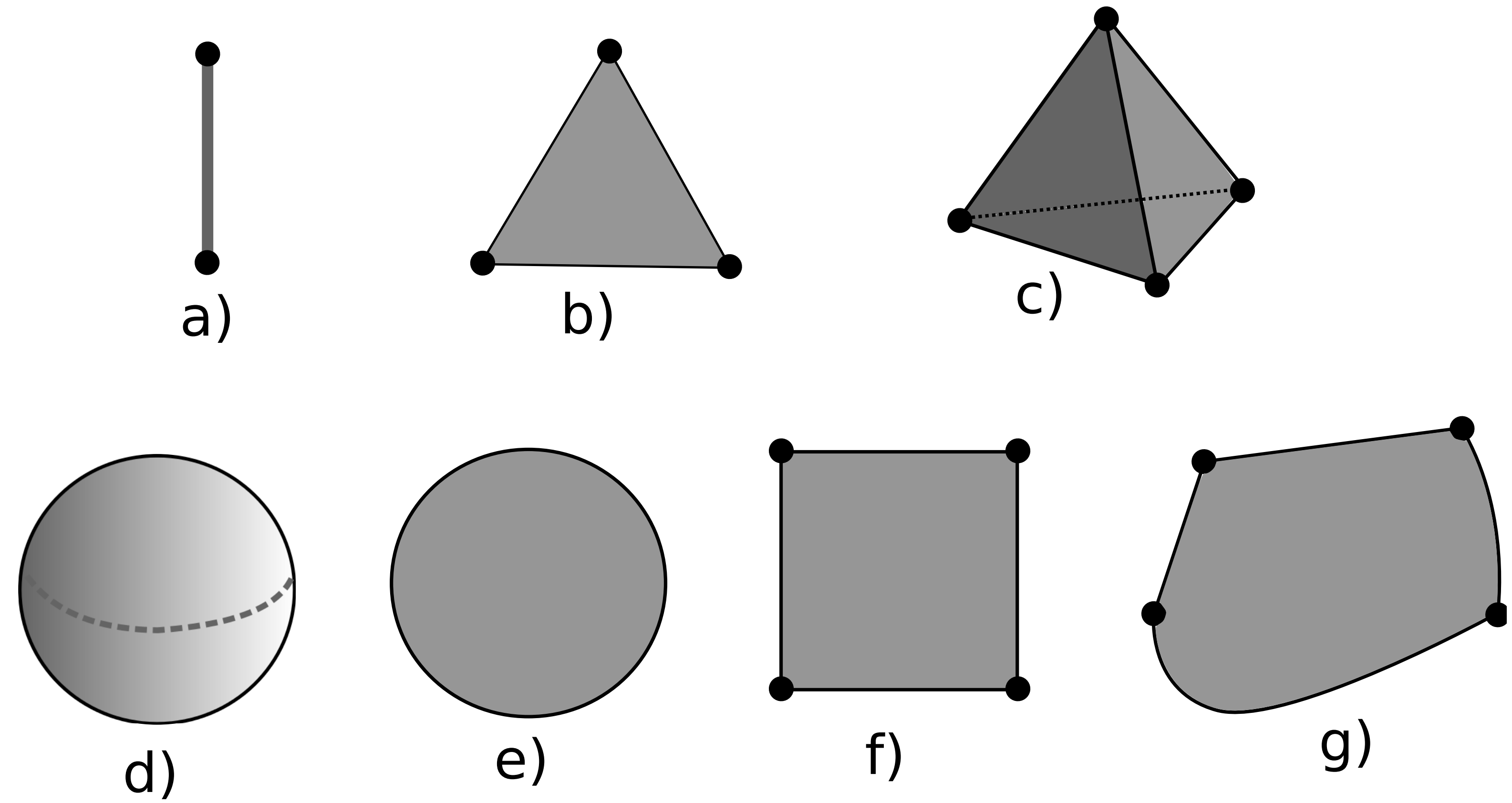

- Classical *n*-level system:

$$\Omega = \{\omega = (p_1, \ldots, p_n) \mid p_i \geq 0, \ \sum_i p_i = 1\}.$$

n pure states: $\omega_1 = (1, 0, \ldots, 0), \ldots, \omega_n = (0, \ldots, 0, 1).$

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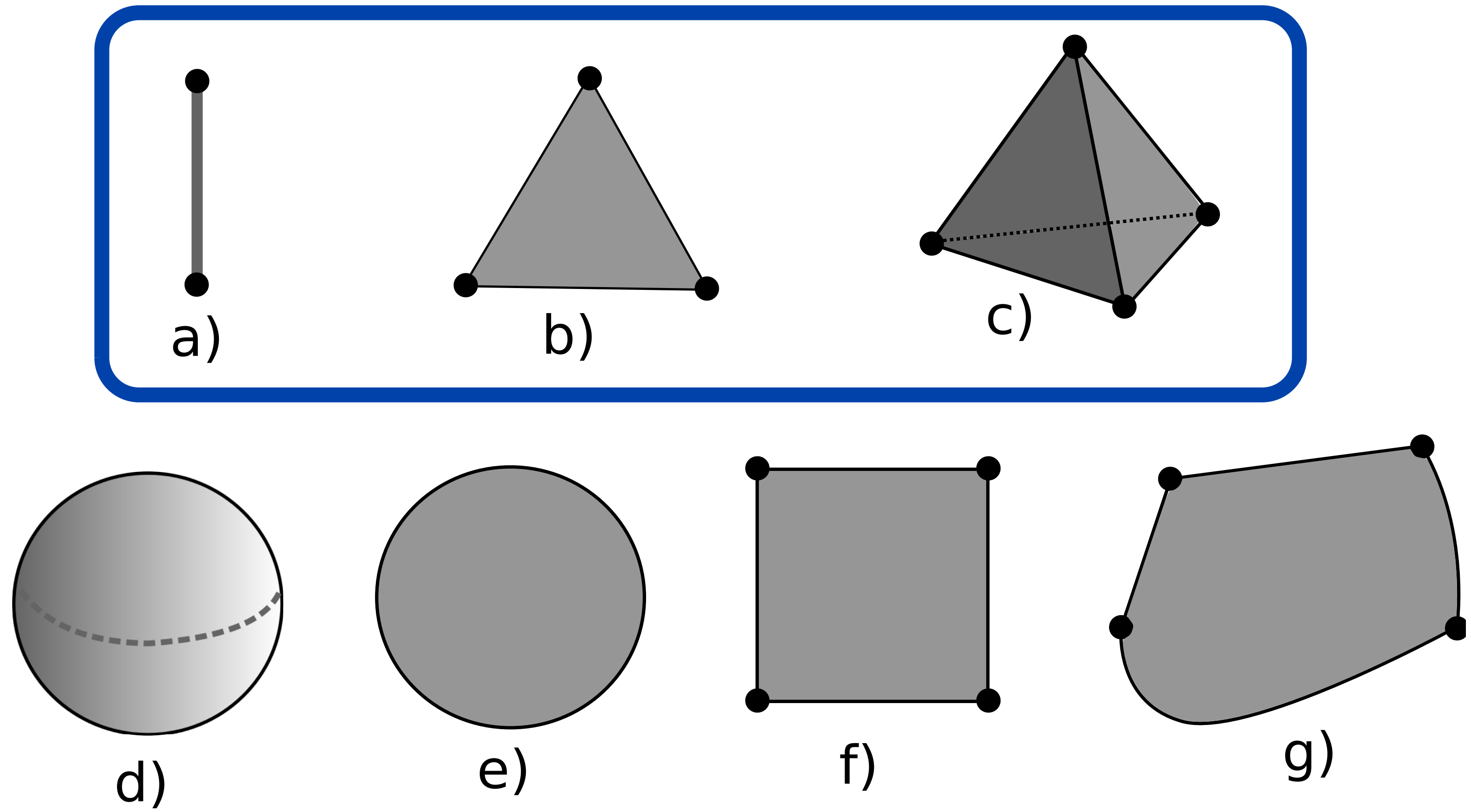

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a), b), c): classical 2-, 3-, 4-level systems.

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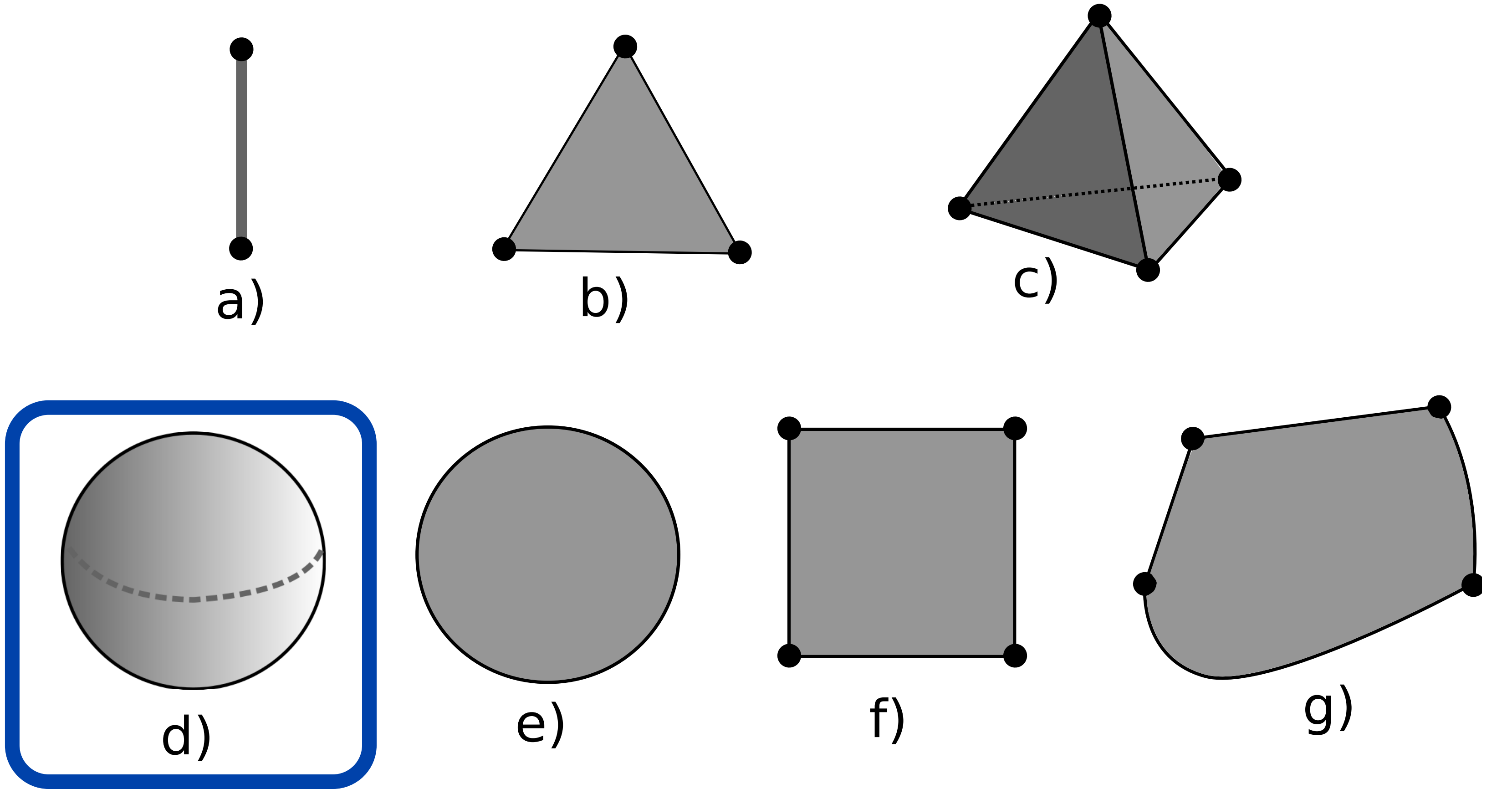

- d): quantum 2-level system (qubit)

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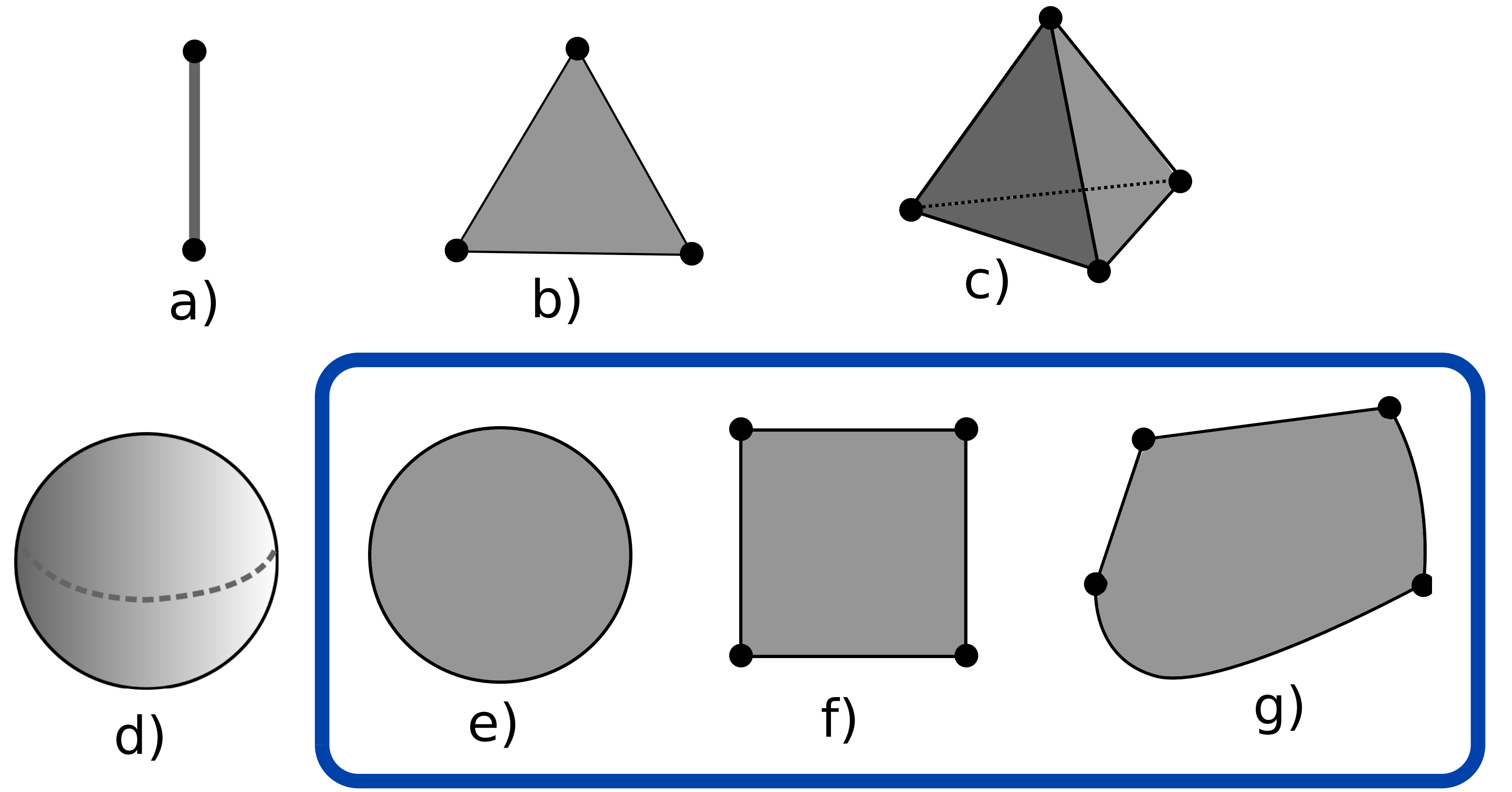
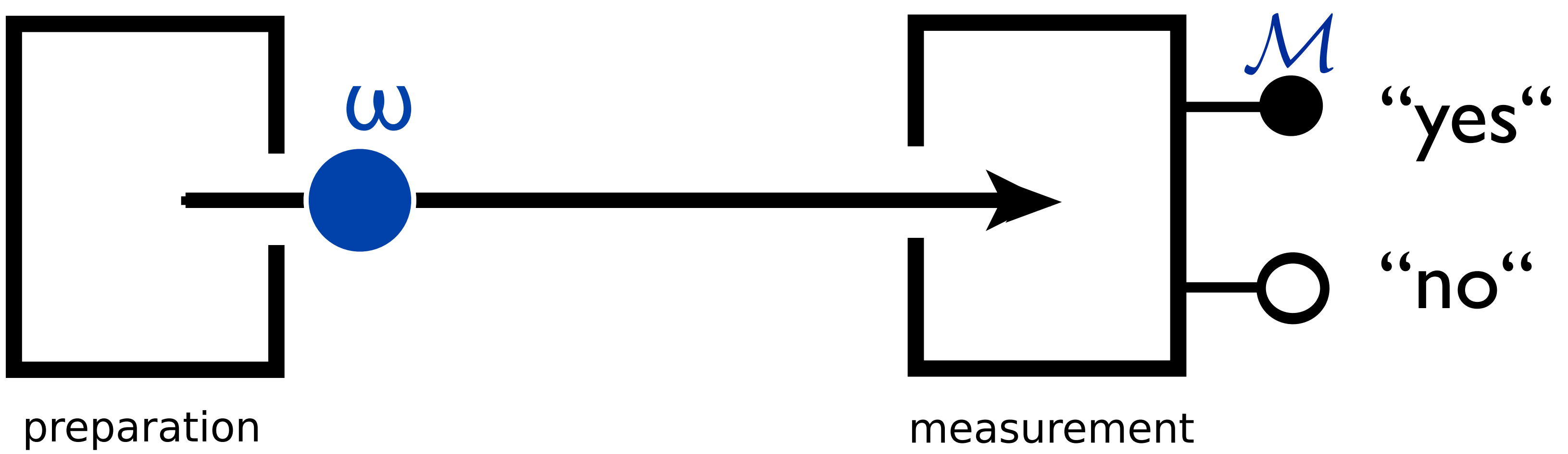


- d): quantum 2-level system (qubit)
- e), f), g): neither classical nor quantum.

2. Convex state spaces



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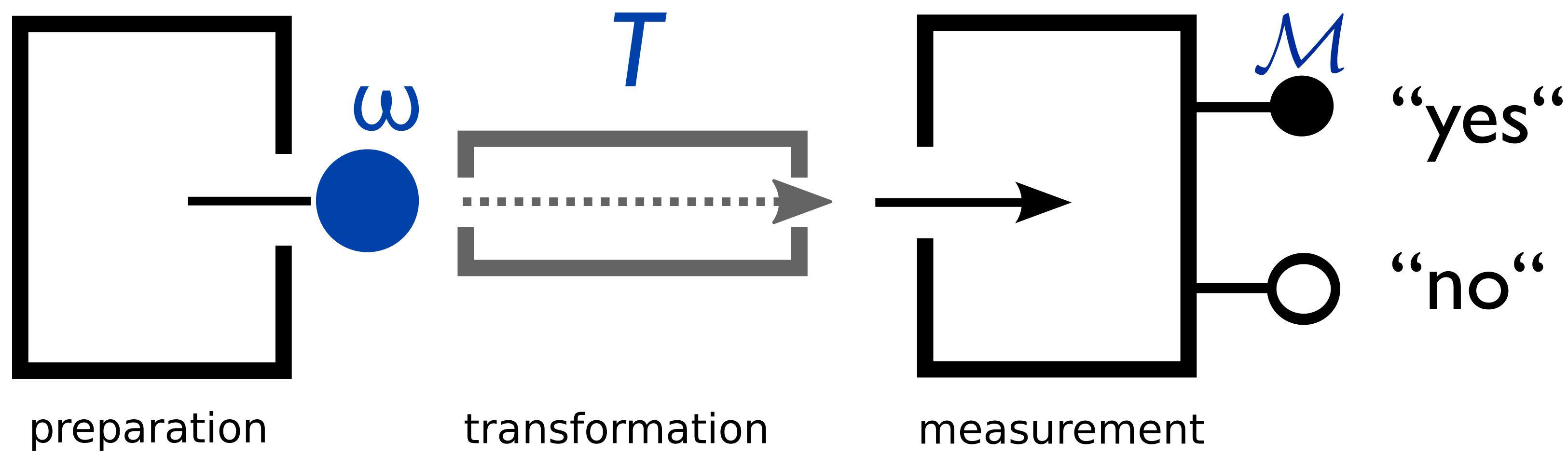

Reversible transformations *T* map states to states, are linear and invertible.

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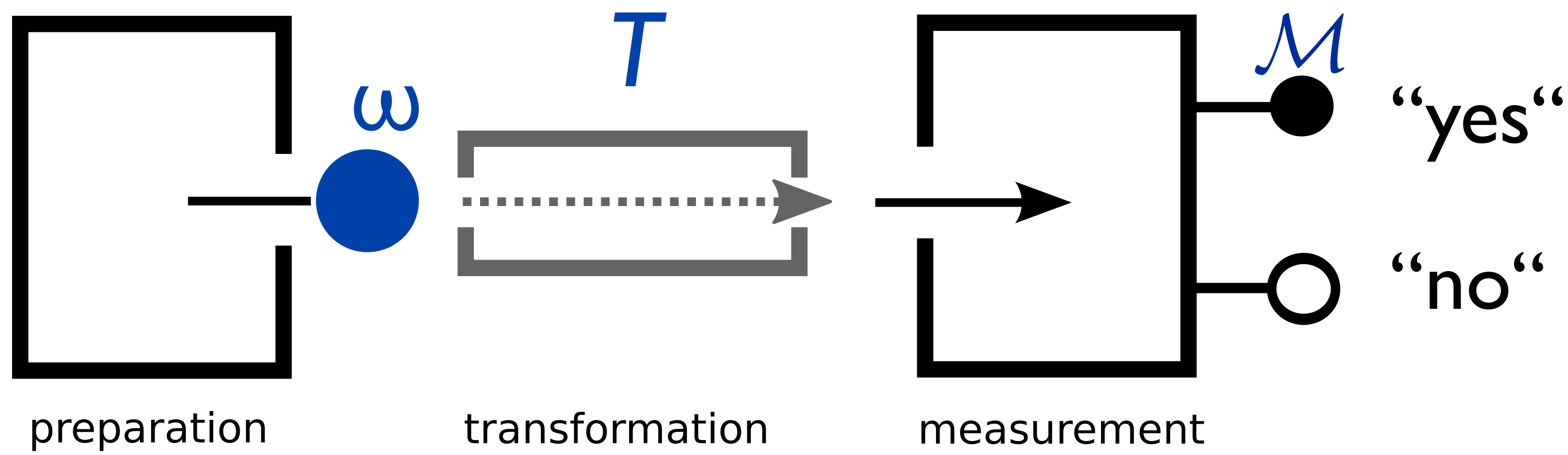

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$$\rho \mapsto U\rho U^{\dagger}.$$

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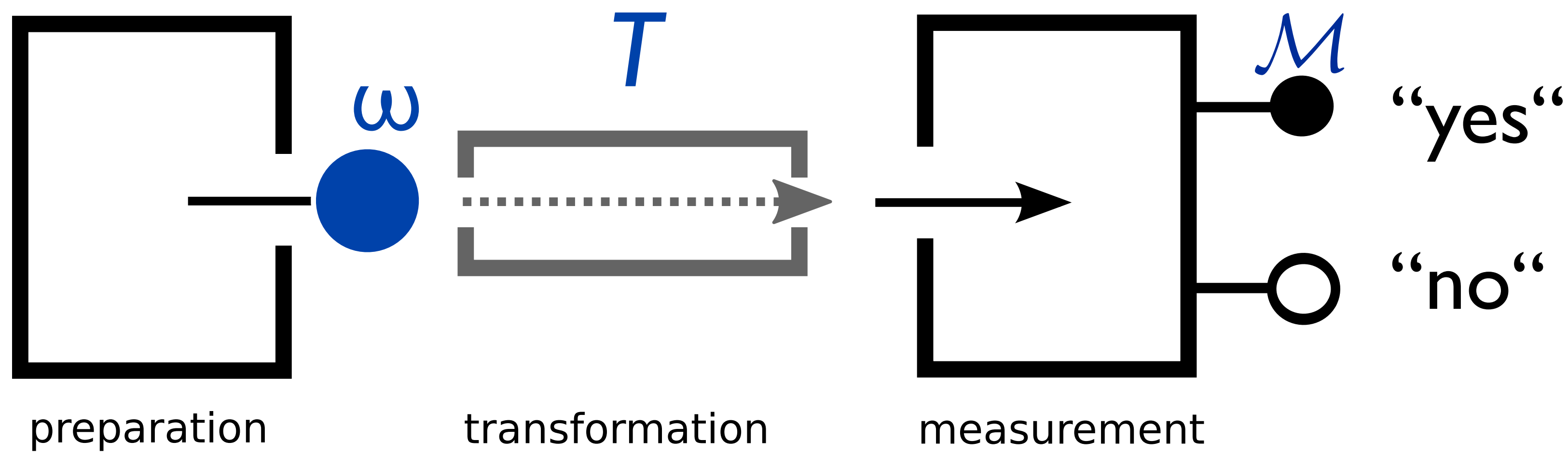

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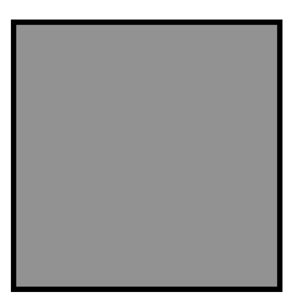

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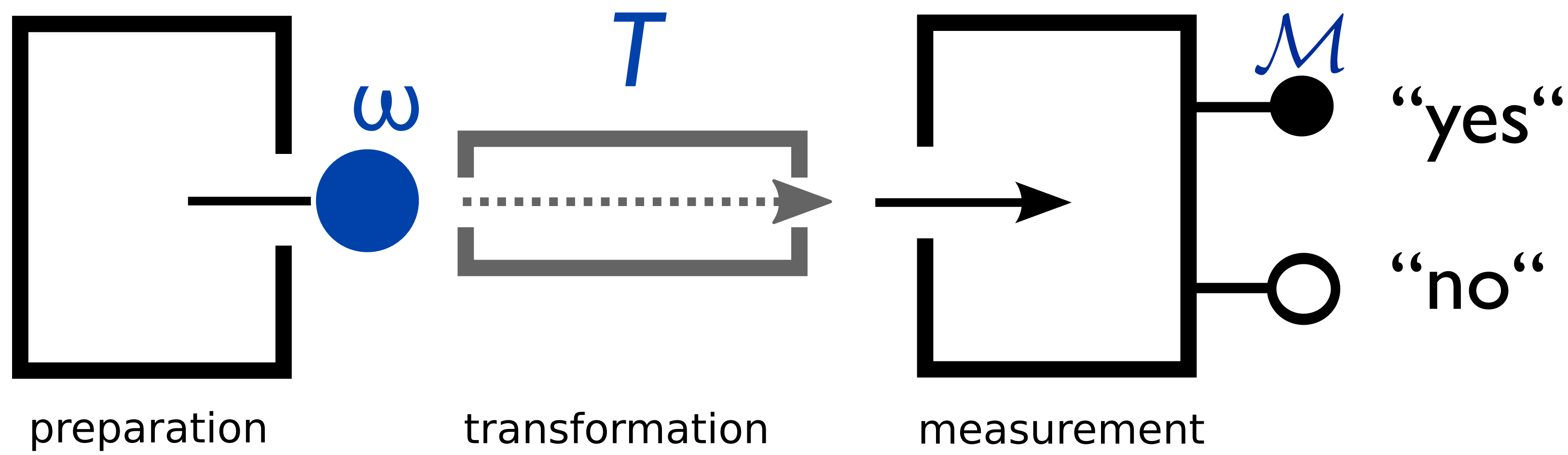

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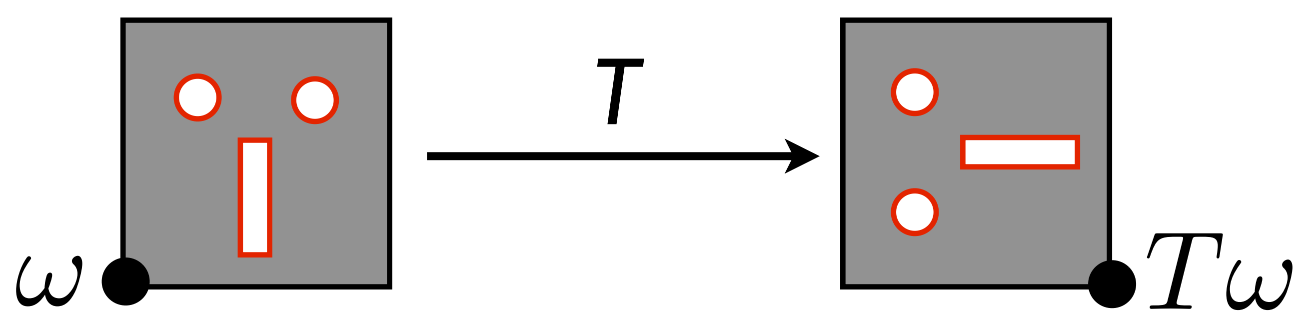
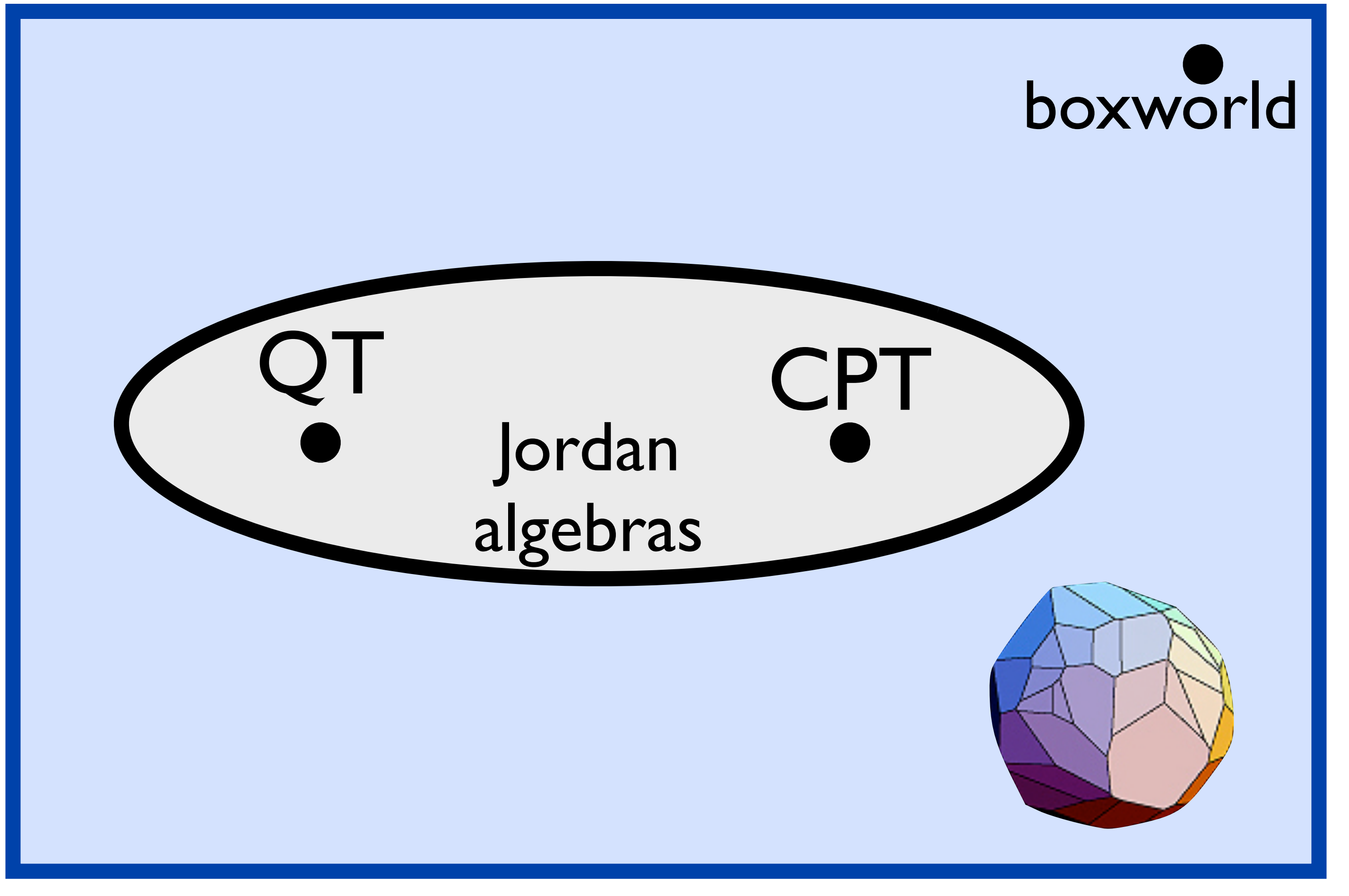


2. Convex state spaces

Contains vast landscape of all possible "probabilistic theories".



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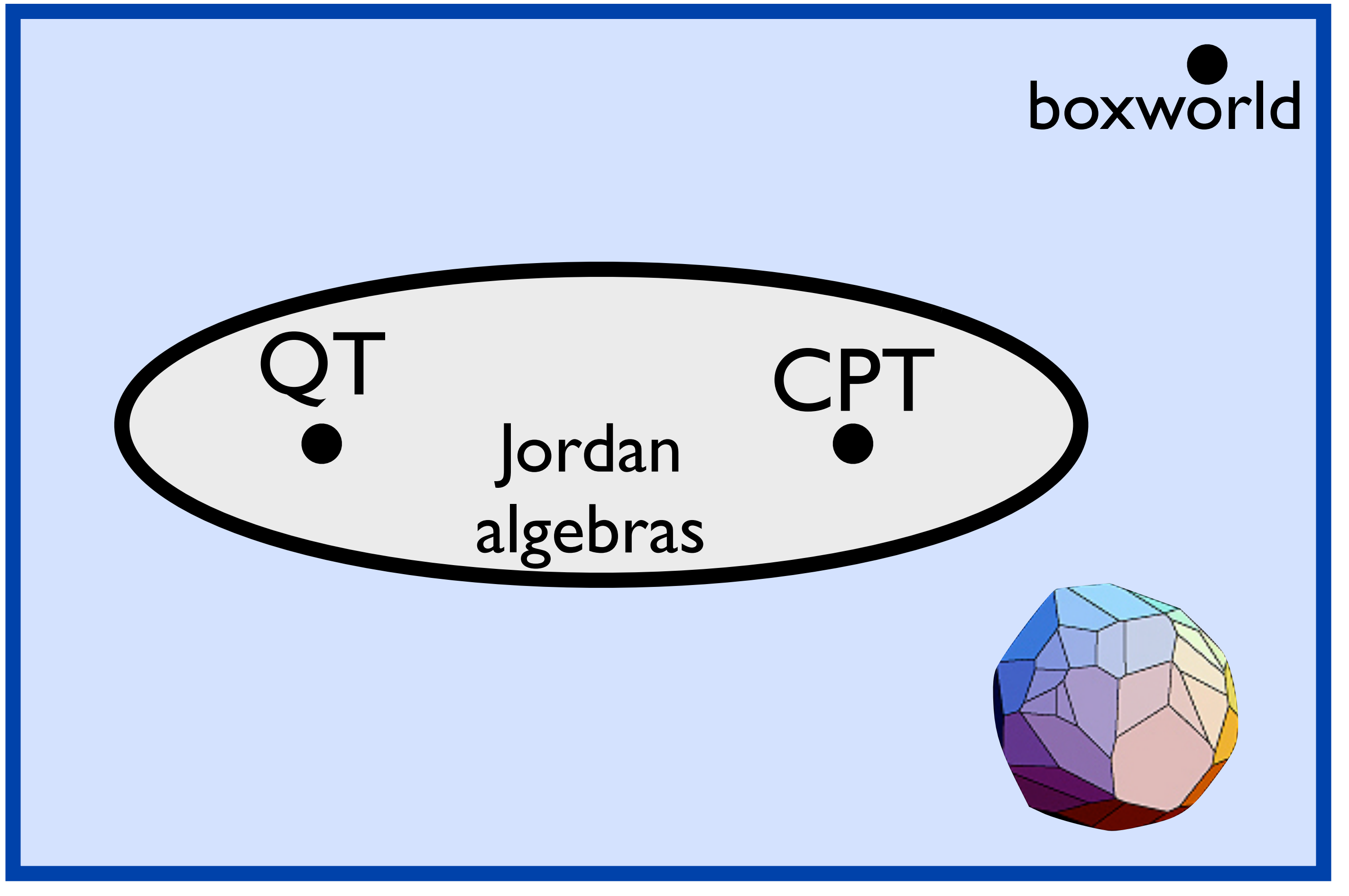

Many physical properties different from QT: superstrong non-locality etc.

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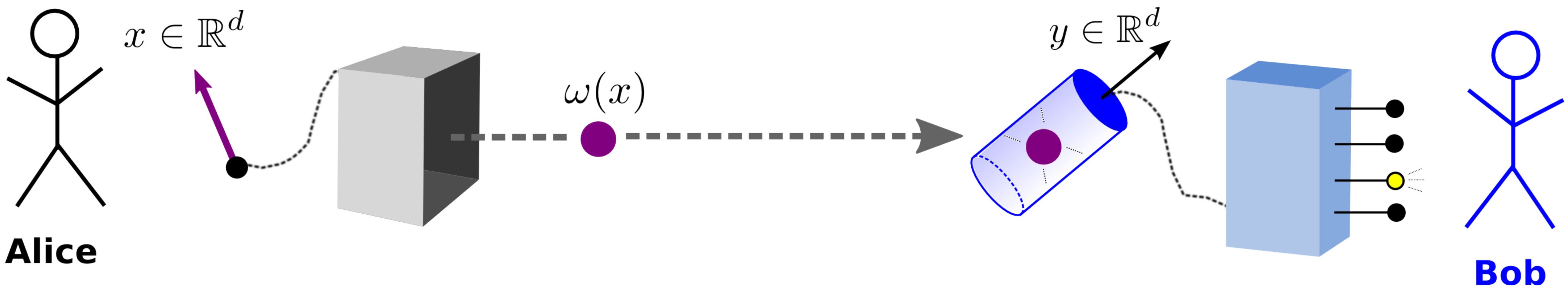
3. The postulates



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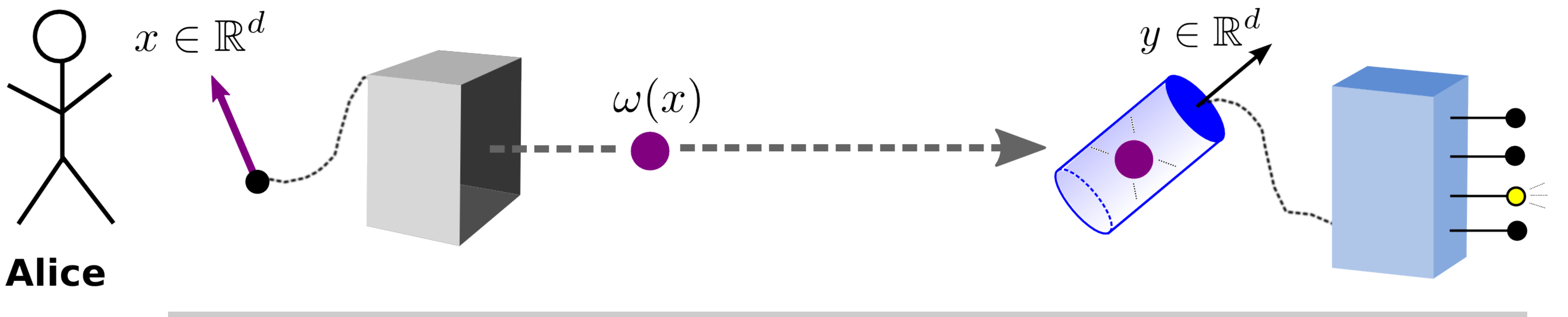

Postulate 1 (Achievability). There is a protocol which allows Alice to encode any spatial direction $x \in \mathbb{R}^d$, $|x| = 1$, into a state $\omega(x)$, such that Bob is able to retrieve x in the limit of many copies.

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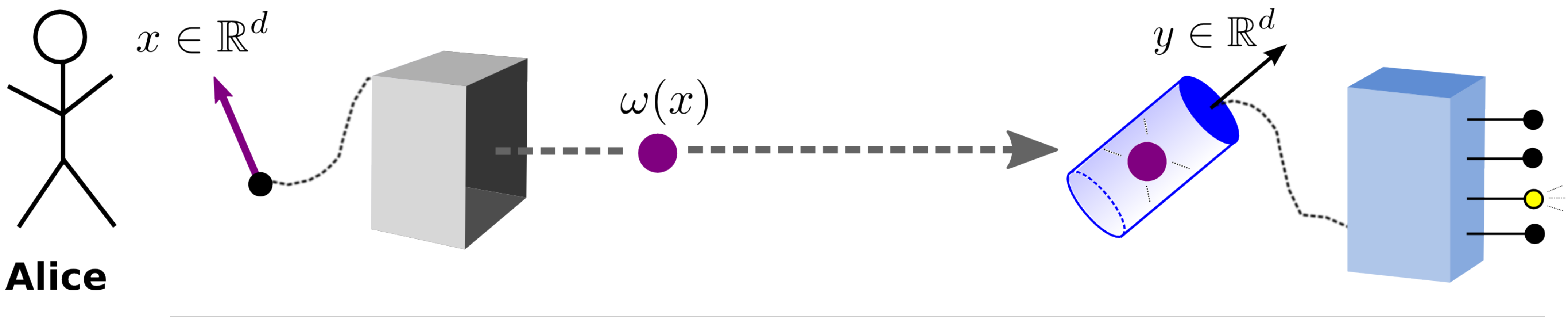

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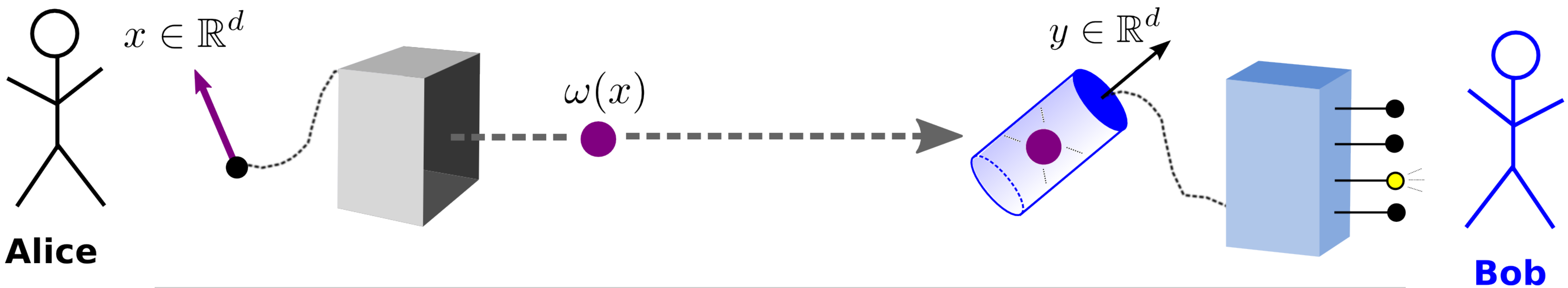

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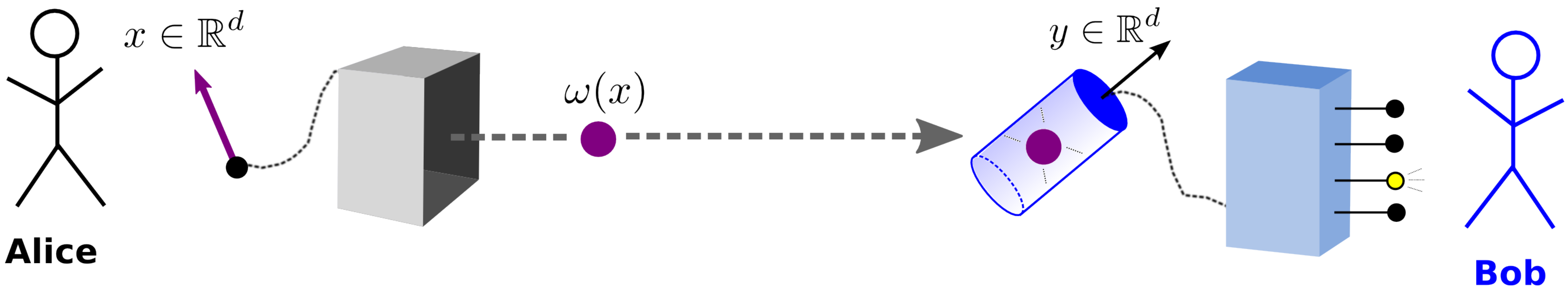
Would contain
huge amount
of information!
Want *minimality*.

3. The postulates



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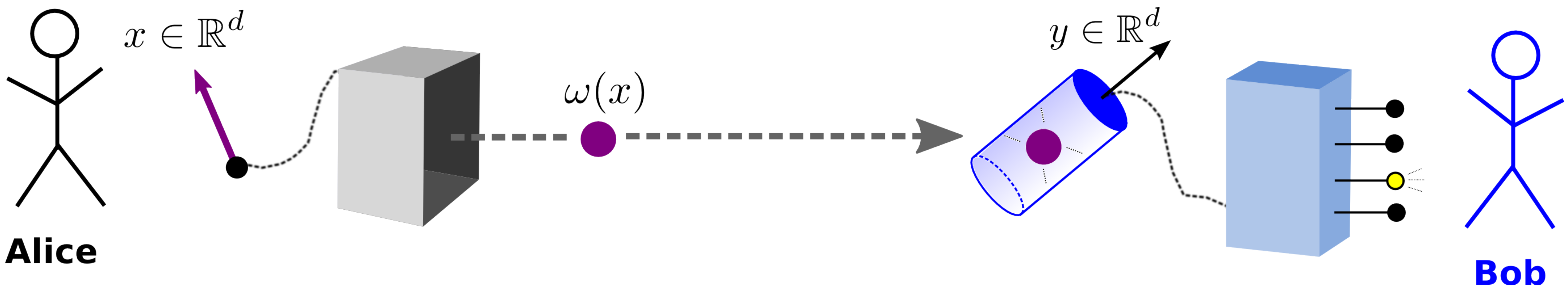

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→ by choosing to send ω or φ,

Alice can encode an additional bit

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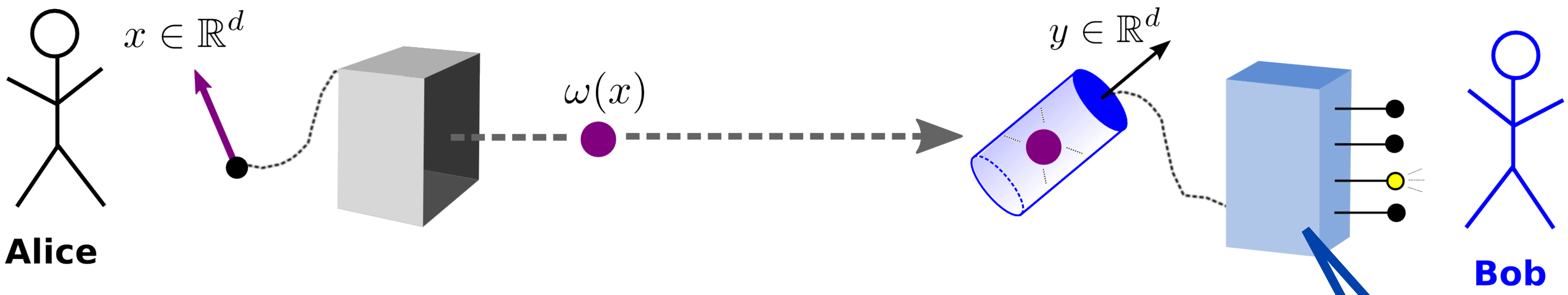

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probability of *i*-th outcome: $\mathcal{M}_y^{(i)}(\omega)$

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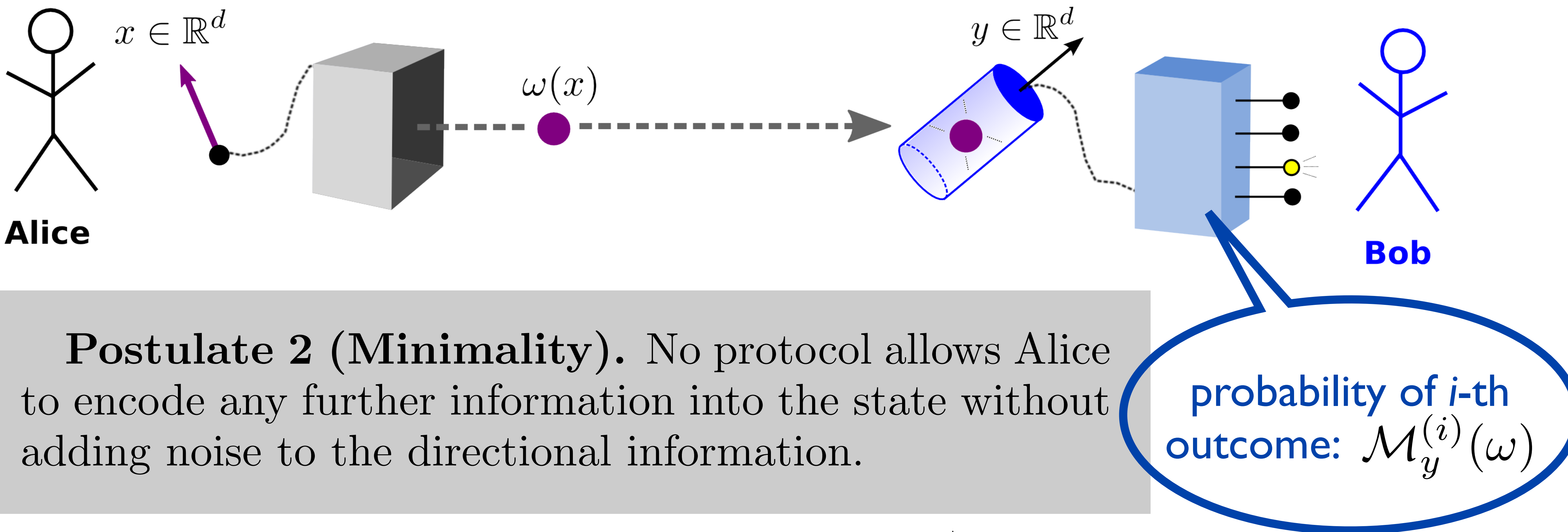

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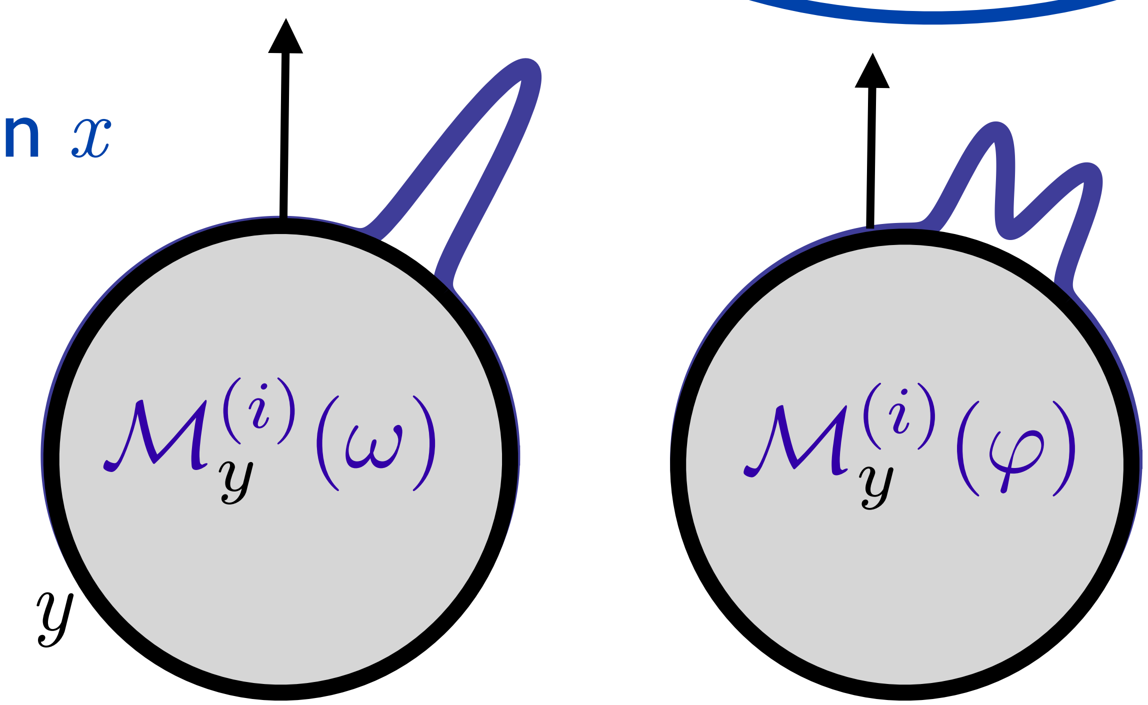
→ one directional profile more noisy than the other



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With some effort, one can prove from Postulates 1+2:

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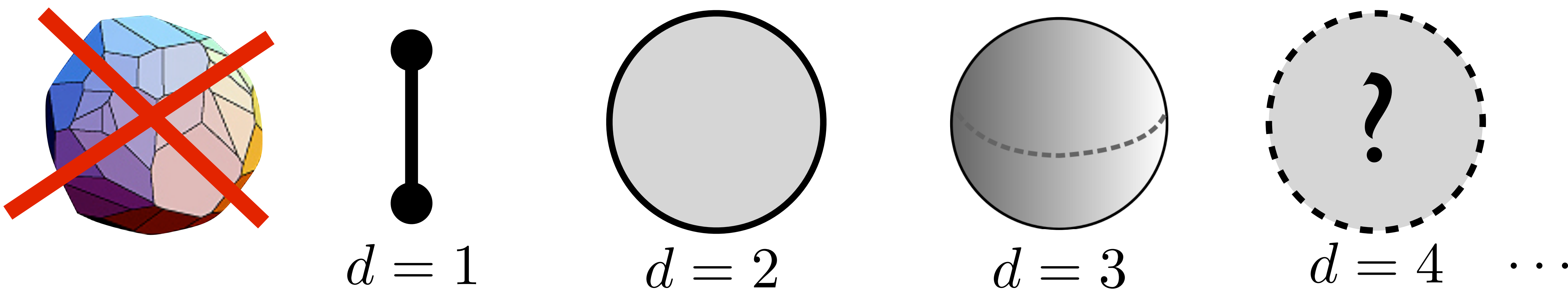

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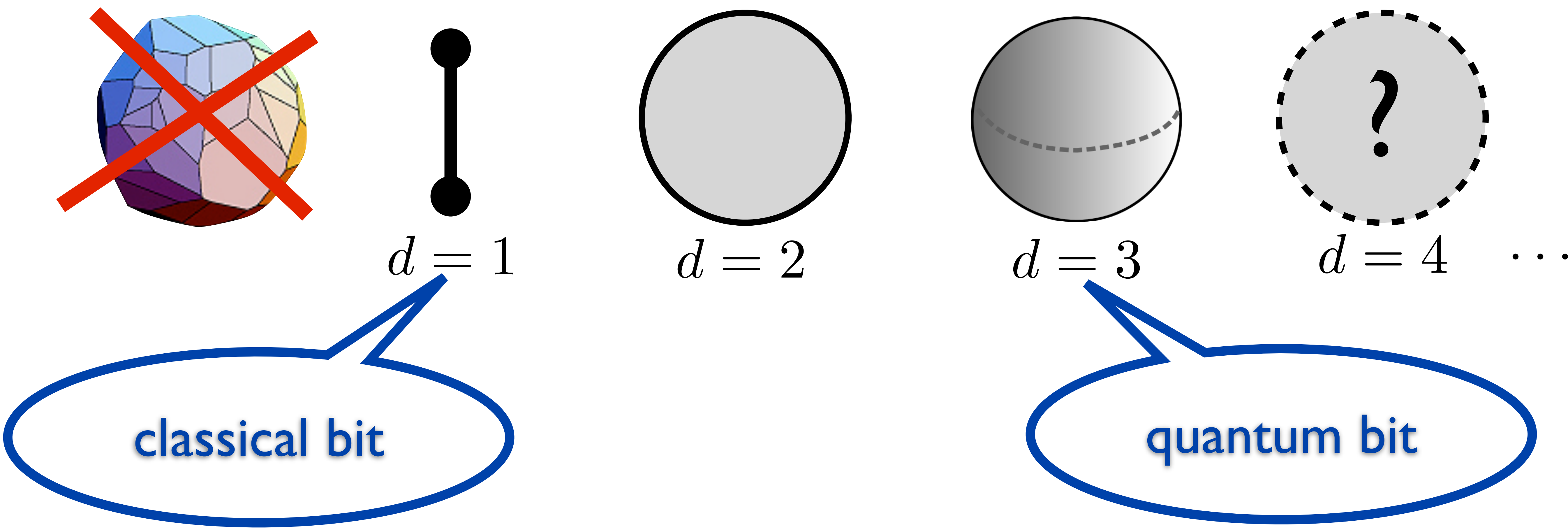

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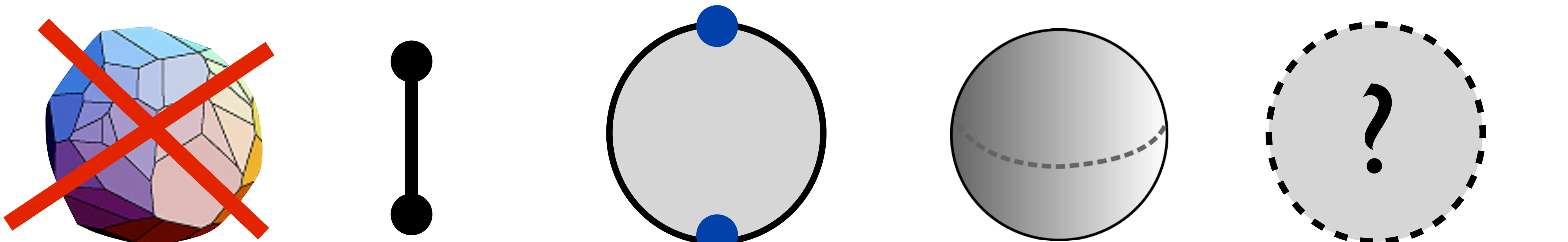

$d = 1$ $d = 2$ $d = 3$ $d = 4$ $\cdots$

classical bit

all balls are 2-level systems.

quantum bit

M. Müller*, Ll. Masanes

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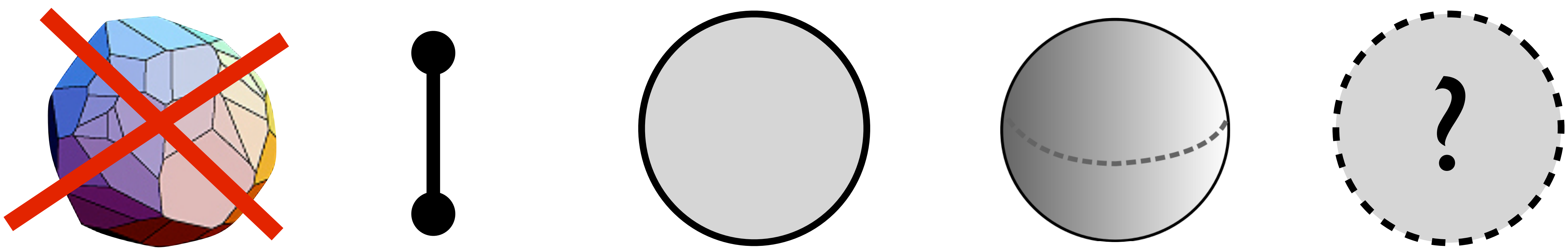

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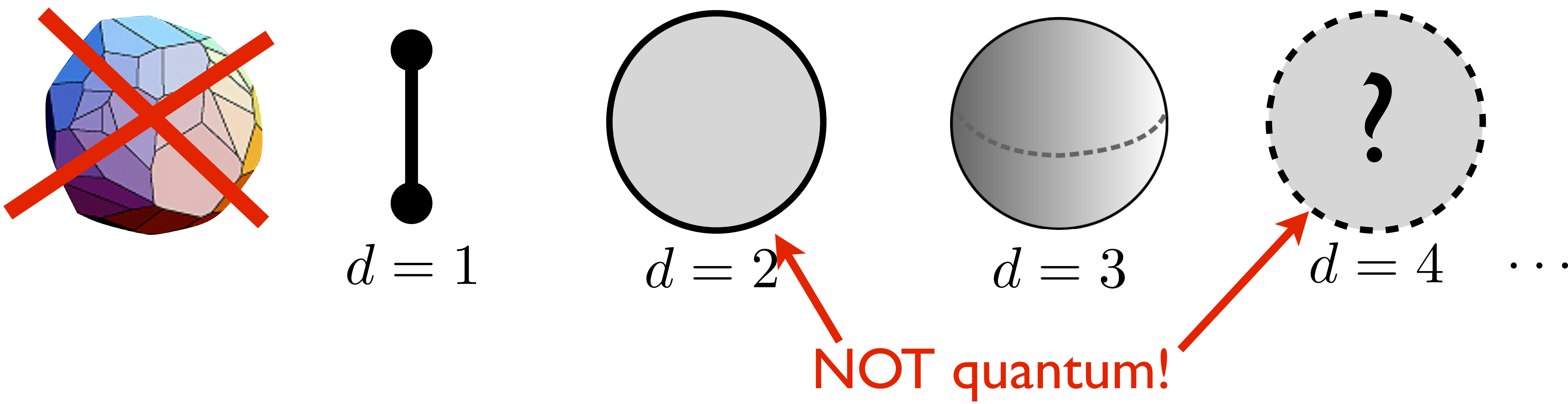

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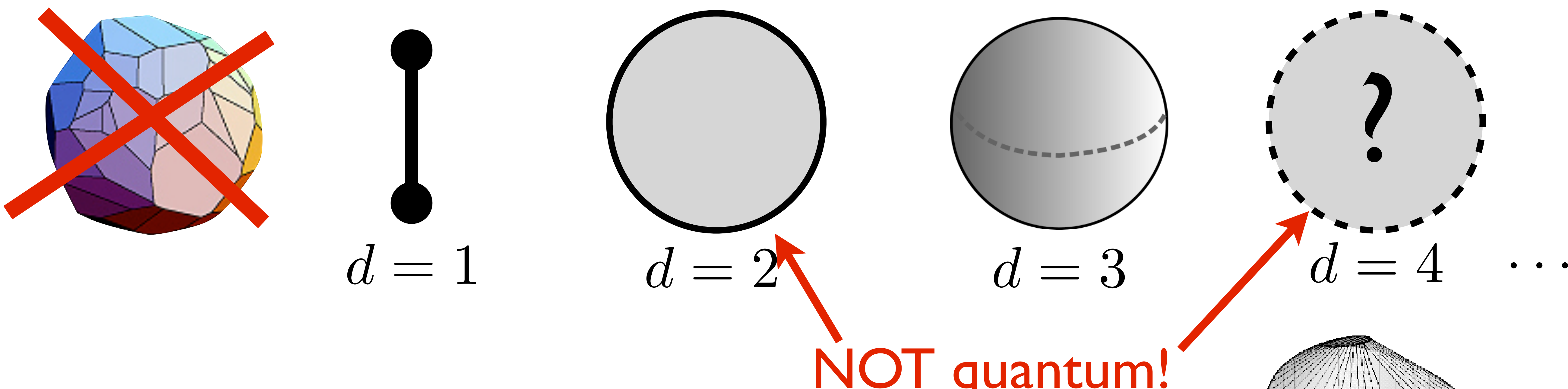
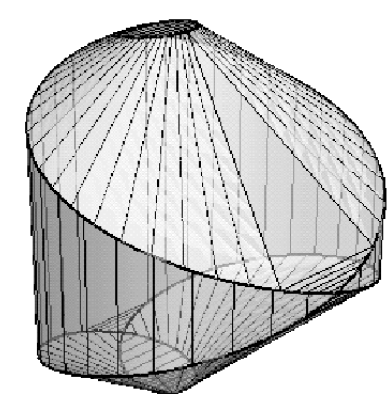


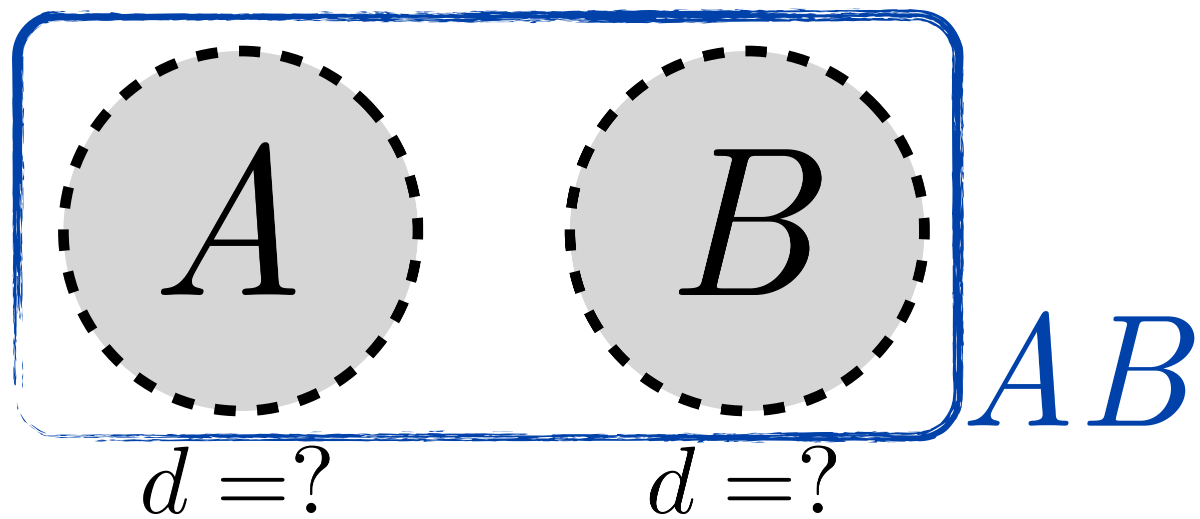
Quantum 3-level state space looks more like this:



Bengtsson, Weis, Zyczkowski, "Geometry of the set of mixed quantum states: An apophatic approach", arXiv:1112.2347

3. The postulates

To single out $d=3$: consider pairs of direction bits.



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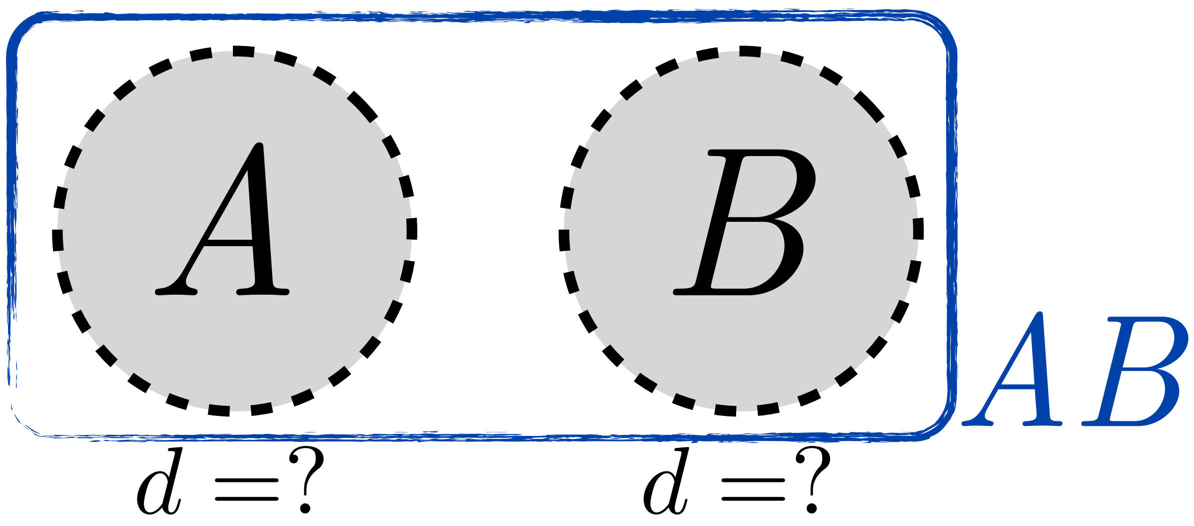
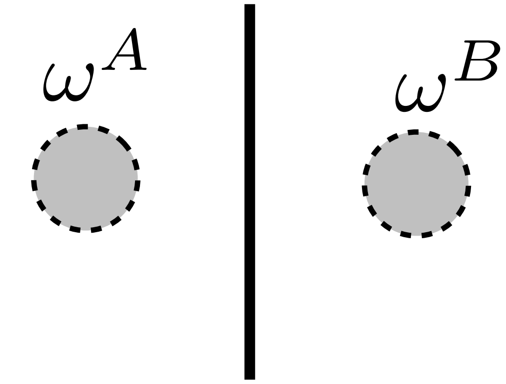


Basic assumptions on composite state space *AB*:

- Contains "product states" $\omega^A \omega^B$.
- Allows for "product measurements" $\mathcal{M}^A \mathcal{M}^B$:

$$\mathcal{M}^A \mathcal{M}^B (\omega^A \omega^B) = \mathcal{M}^A(\omega^A) \cdot \mathcal{M}^B(\omega^B).$$



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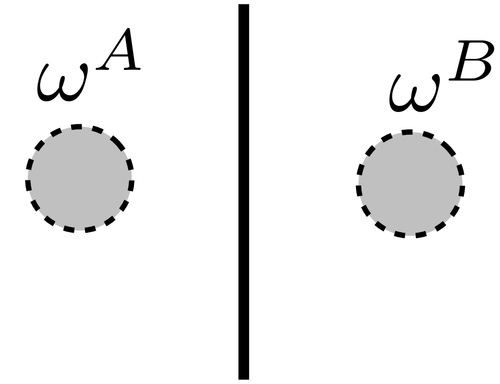

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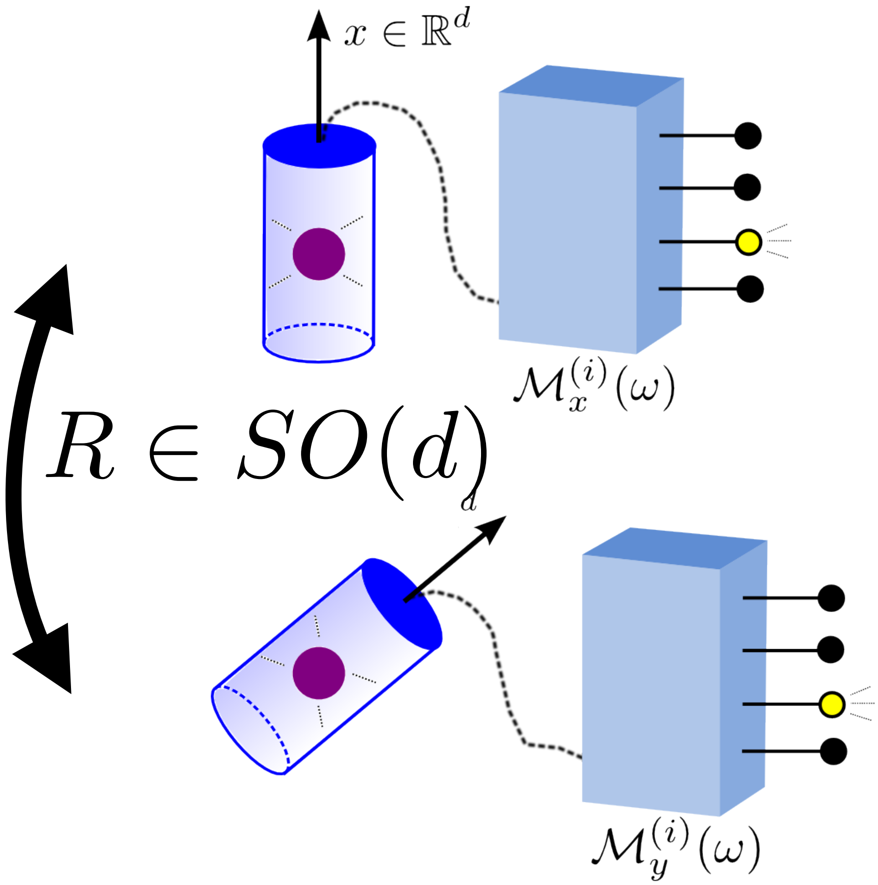

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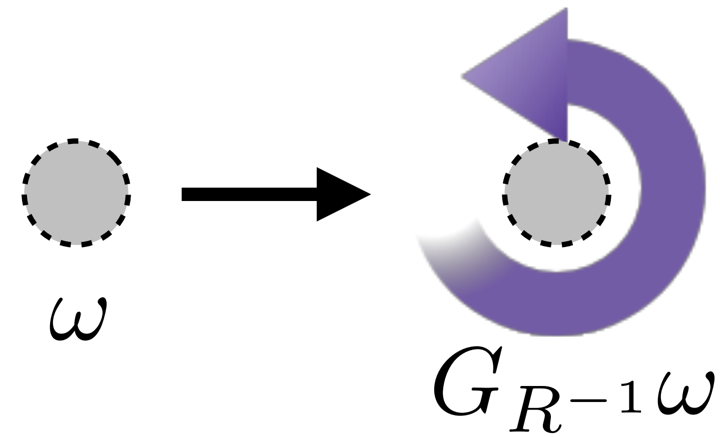

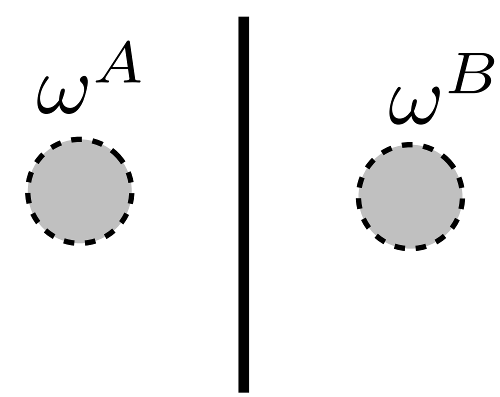

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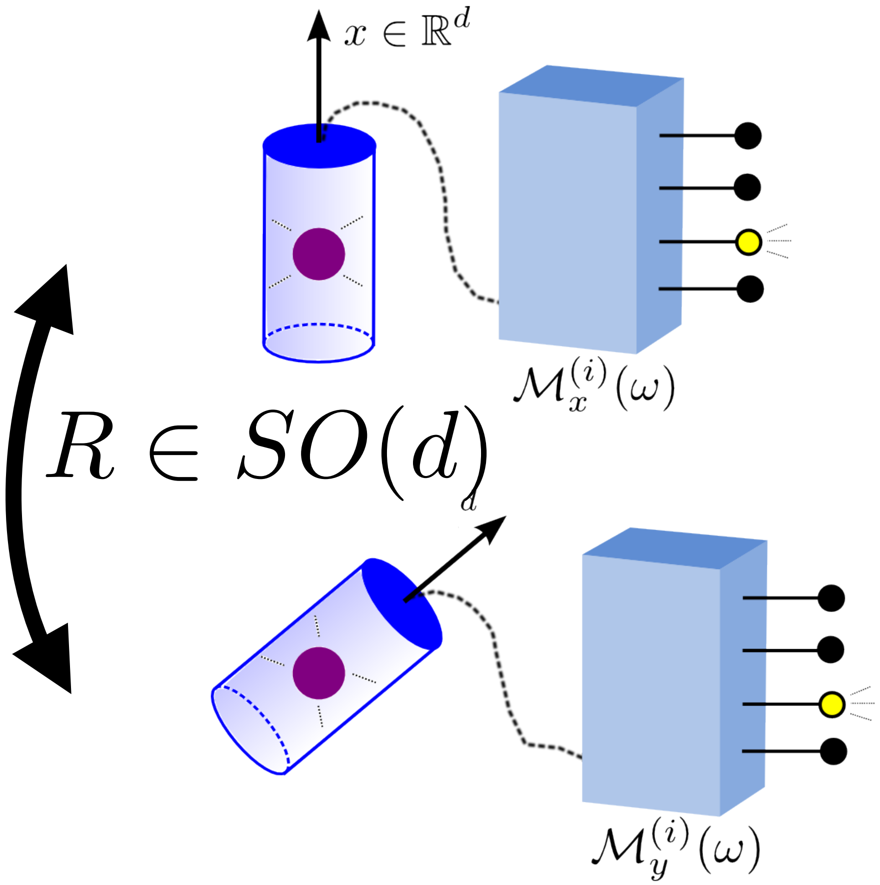

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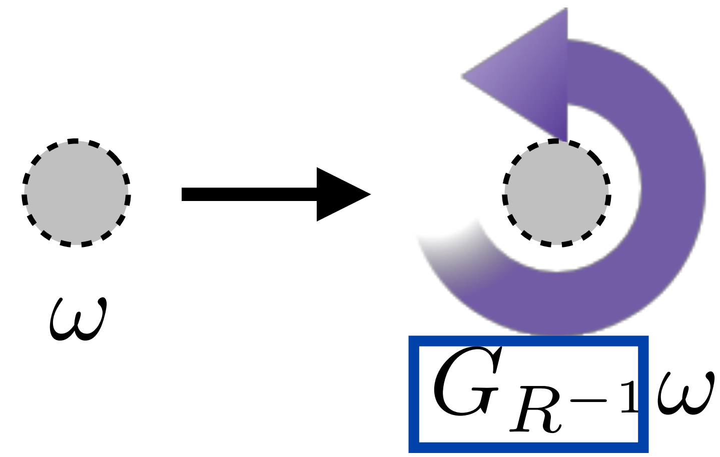
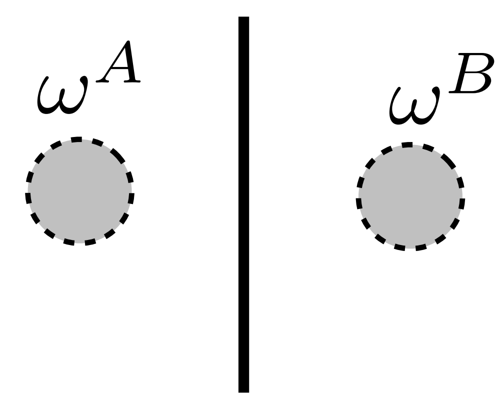


representation of SO(d) on state space



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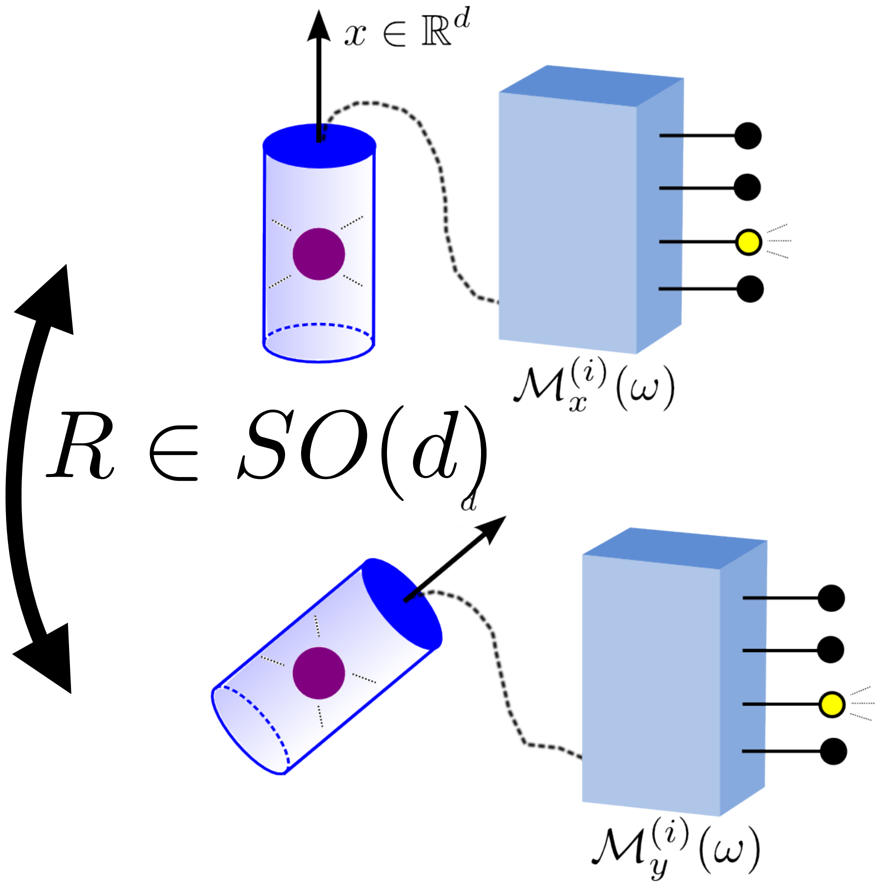

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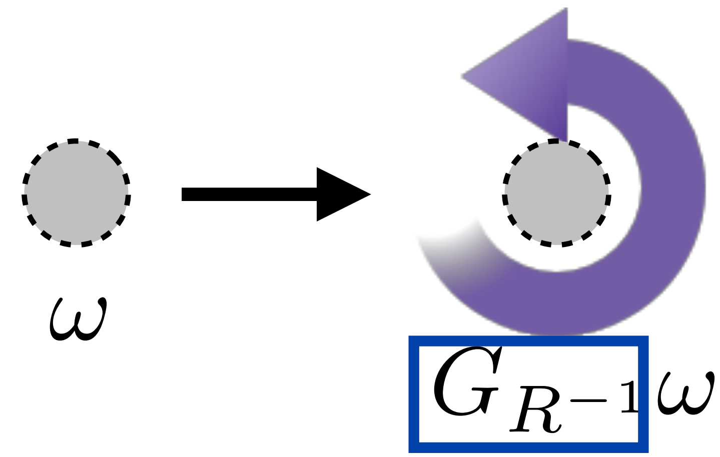

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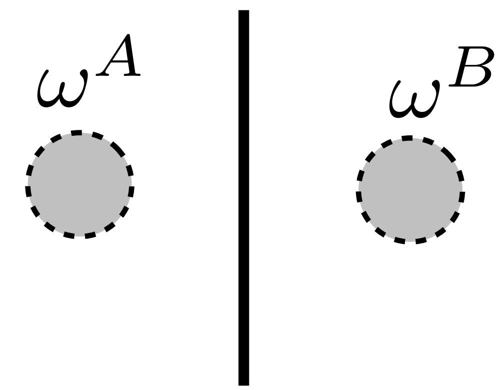
Postulate 3 (Global coordinate transformation). For any rotation $R \in SO(d)$, there is a *unique* linear map on AB which acts as R on both subsystems individually.



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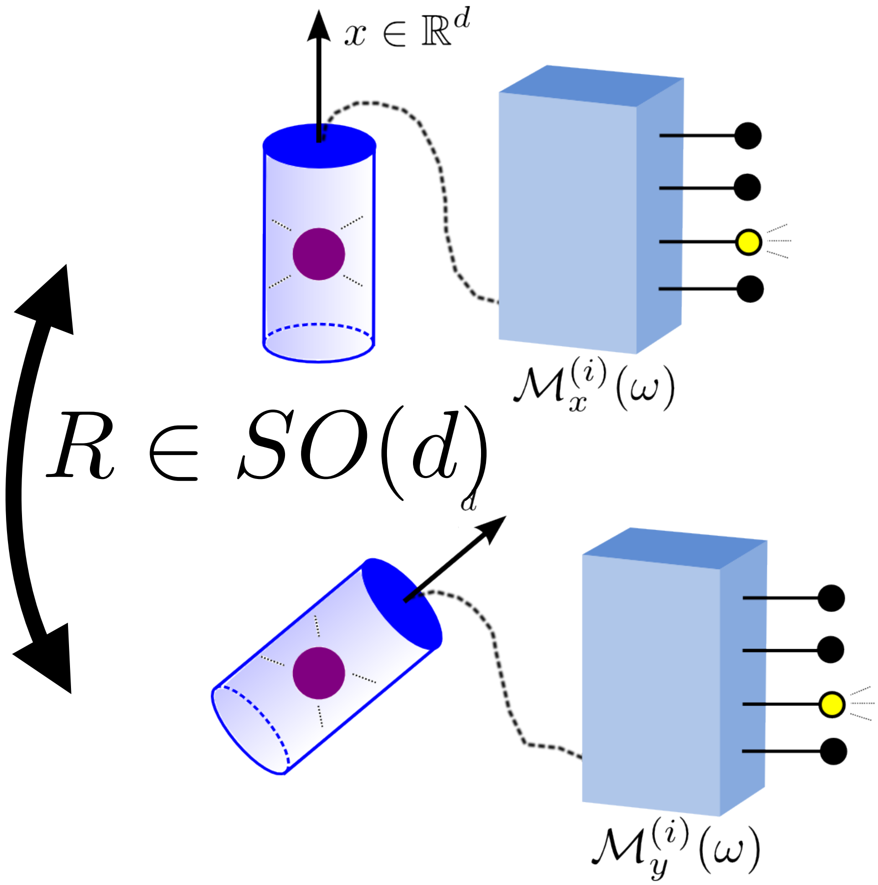

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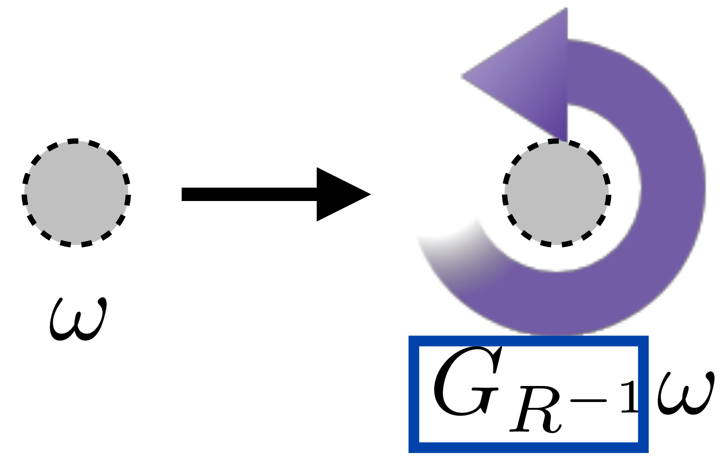

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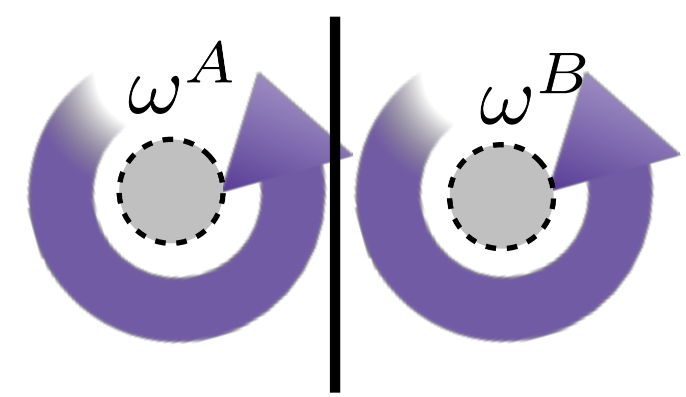

$$\omega^A \omega^B \mapsto (G_R \omega^A)(G_R \omega^B)$$

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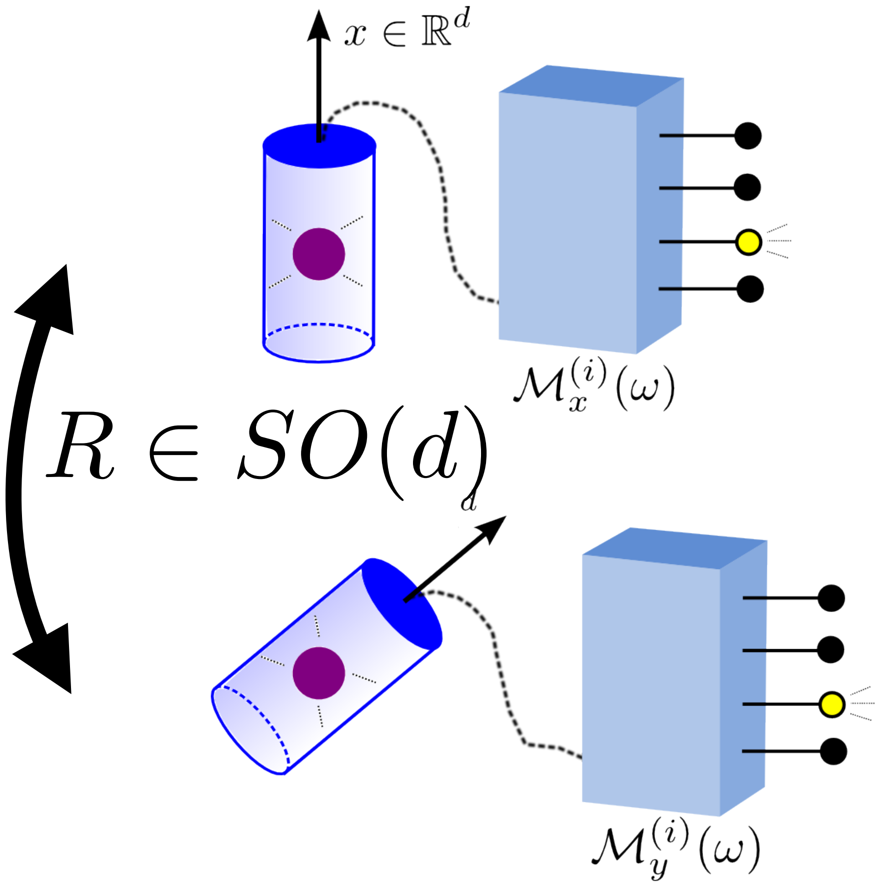

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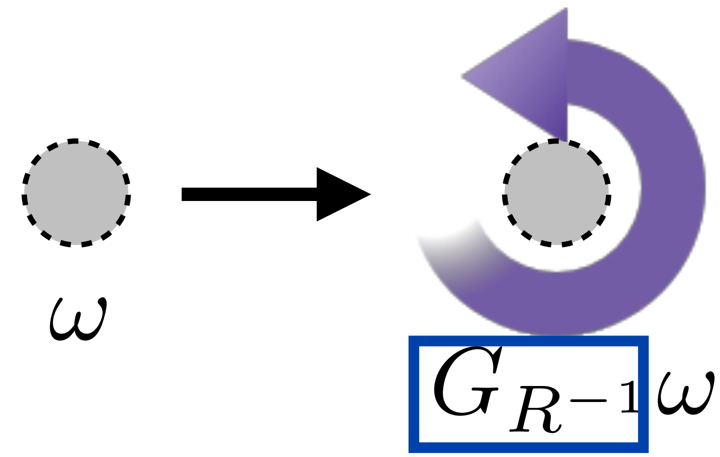

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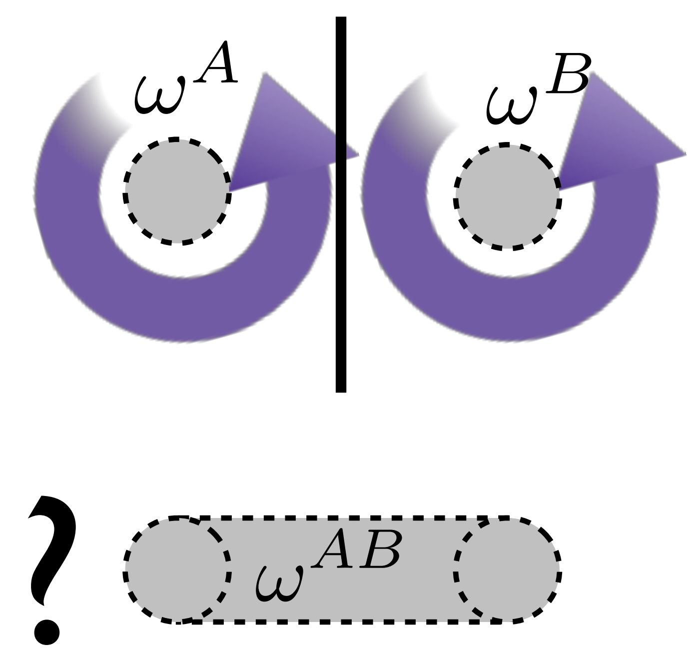

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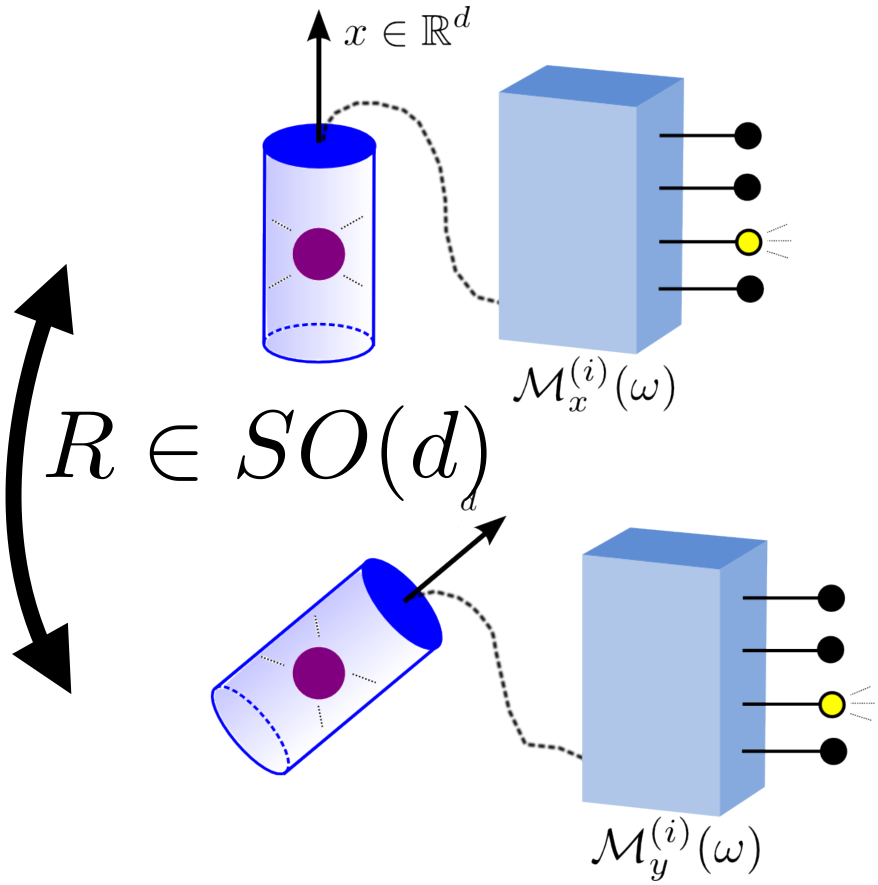

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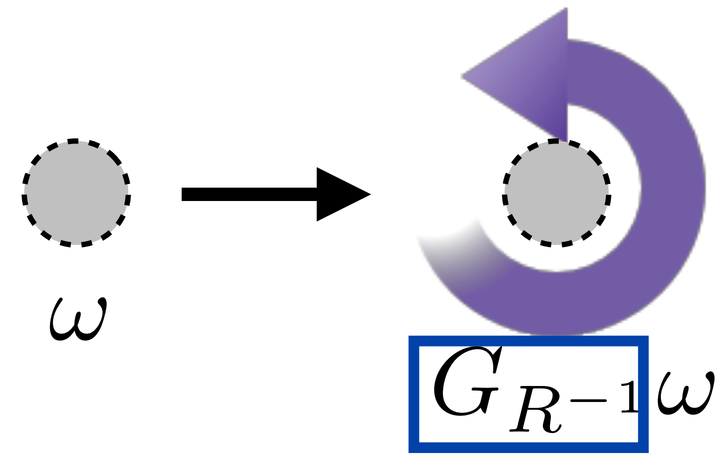

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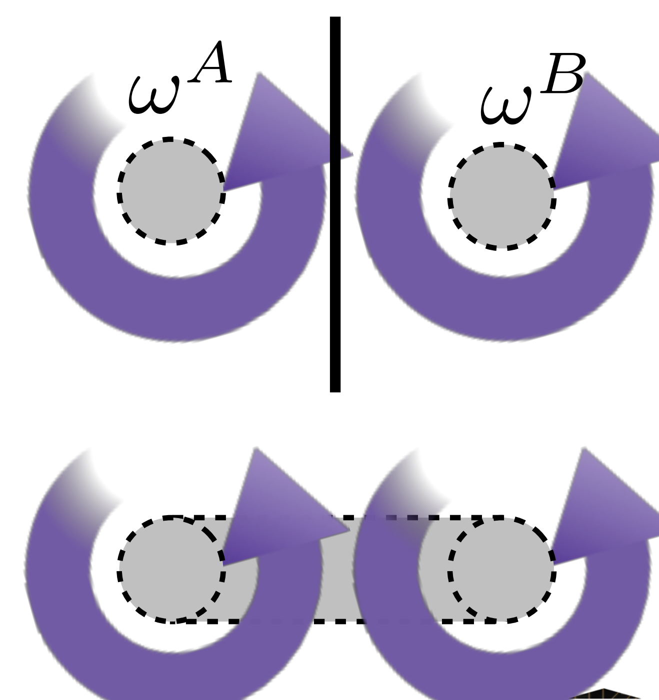

$$\omega^A \omega^B \mapsto (G_R \omega^A)(G_R \omega^B)$$

hence $\omega^{AB} \mapsto (G_R G_R)\omega^{AB}$.

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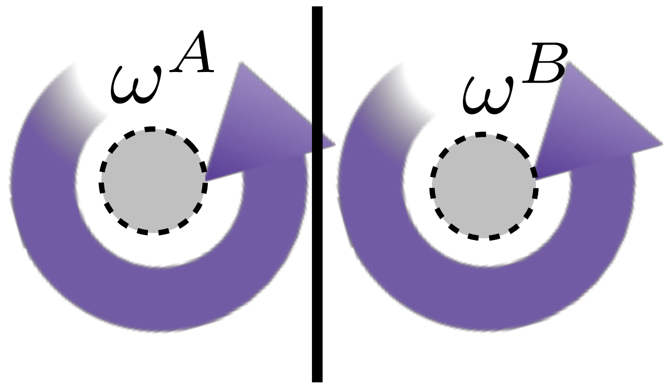

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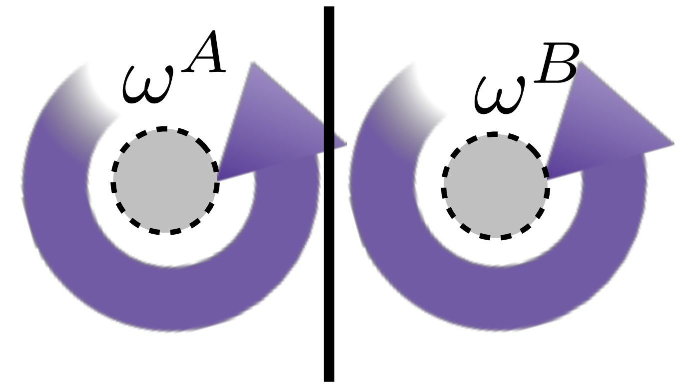

Equivalent: "The product states span AB".

"Local tomography"

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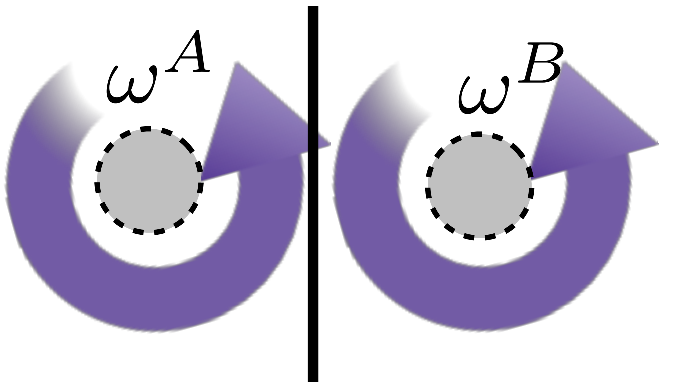

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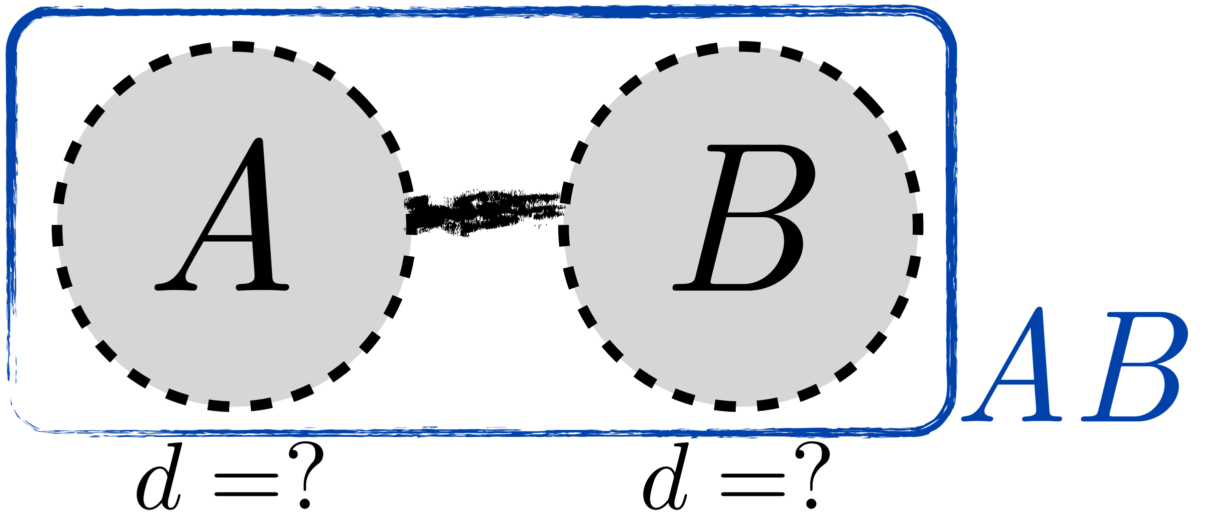

Still many possibilities in all dimensions d.

3. The postulates

Two direction bits should be able to interact via some continuous reversible time evolution:

M. Müller*, Ll. Masanes

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

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Postulate 4 (Interaction). On AB, there is a continuous one-parameter group of transformations $\{T_t^{AB}\}_{t\in\mathbb{R}}$ which is not a product of local transformations, $T_t^{AB} \neq T_t^A T_t^B$.

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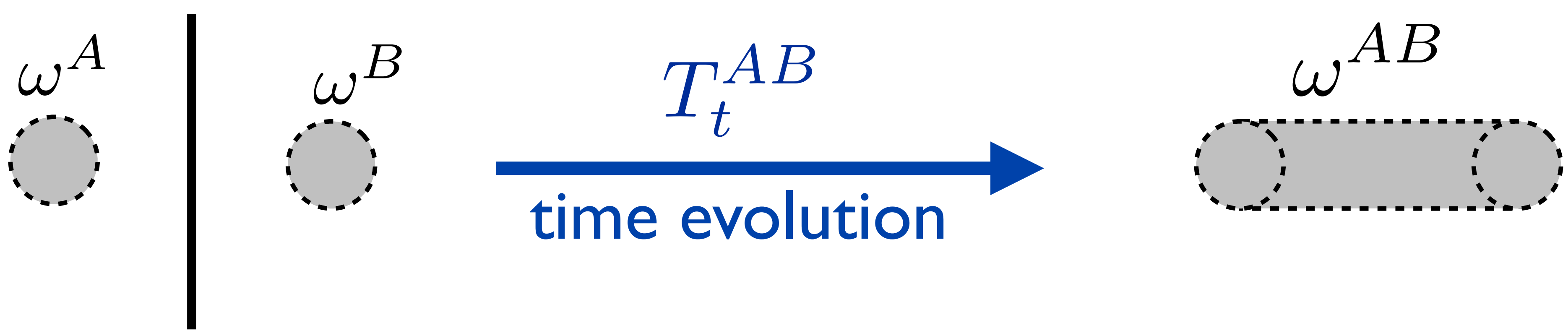
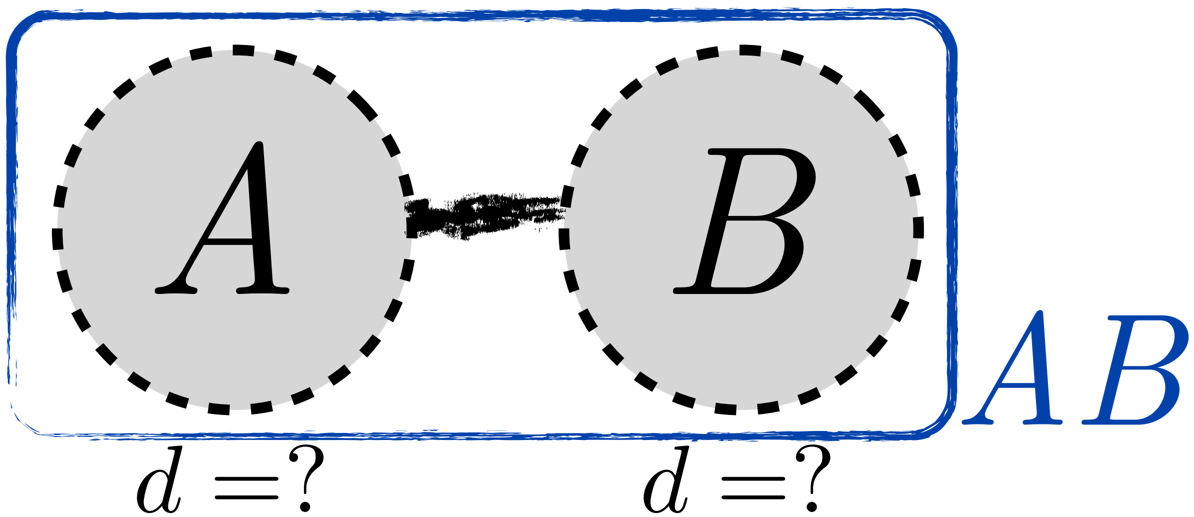


Otherwise no interaction, never!

Overview

1. Overview
2. Convex state spaces
3. The postulates
4. Deriving *d=3* and quantum theory
5. What does all this tell us?

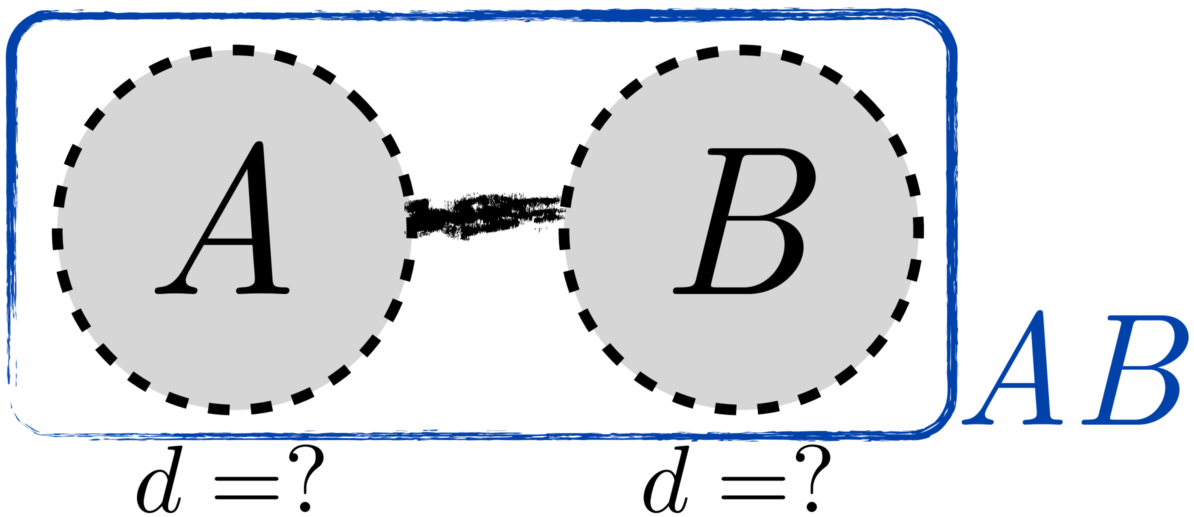
4. Deriving *d=3* and QT



Only **3**D-balls can "talk to each other":

Ll. Masanes, MM, R. Augusiak, and D. Pérez-García, arXiv:1111.4060

4. Deriving *d=3* and QT



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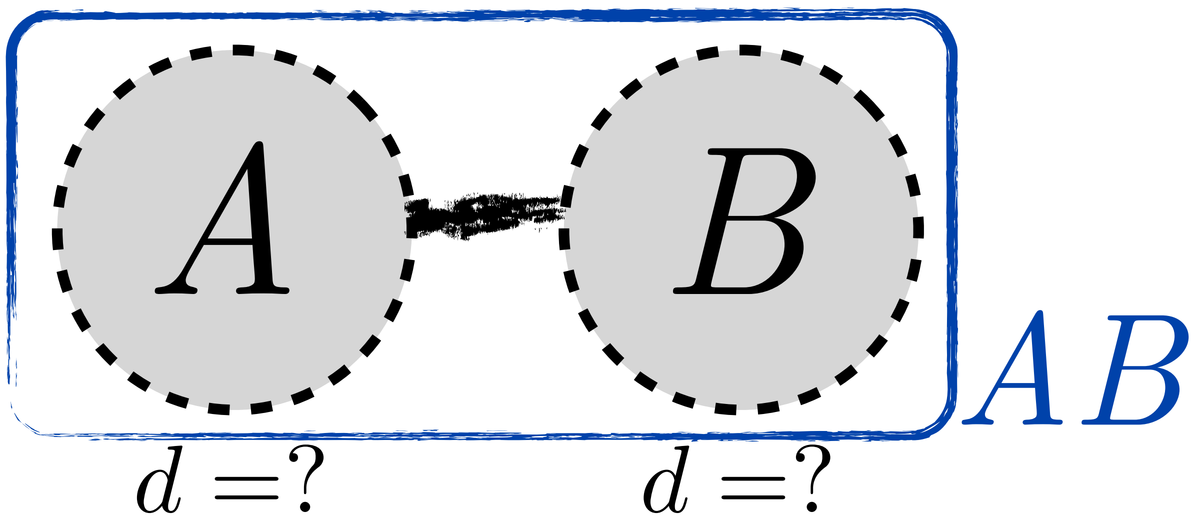

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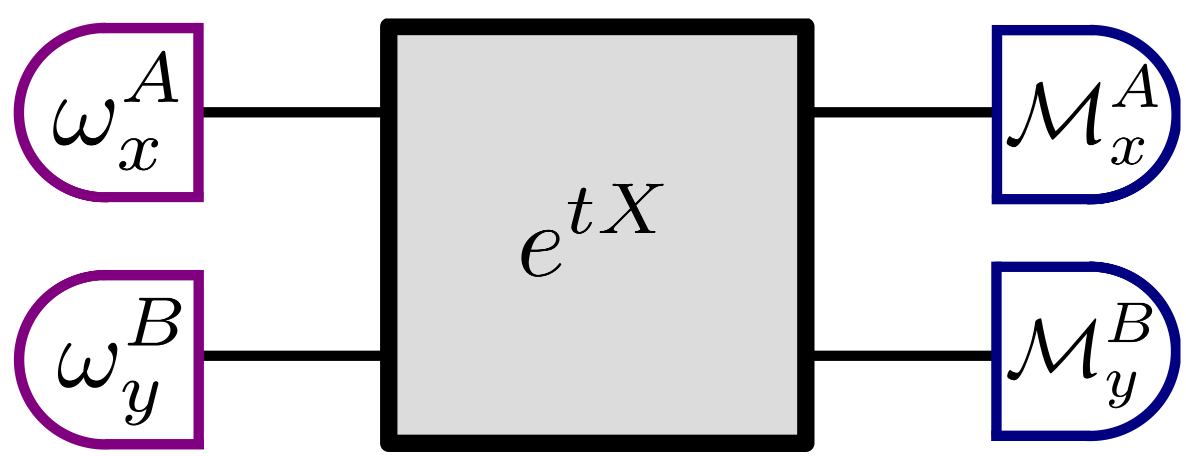

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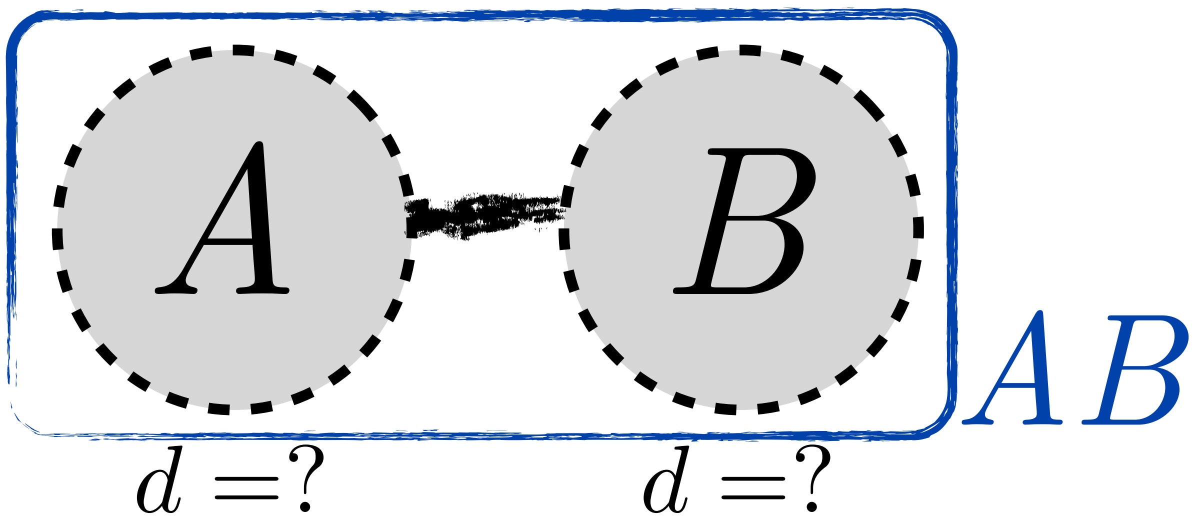

Only **3**D-balls can "talk to each other":

Prepare pure state on A with "Bloch vector" $x \in \mathbb{R}^d \ldots$

arcía, arXiv:1111.4060

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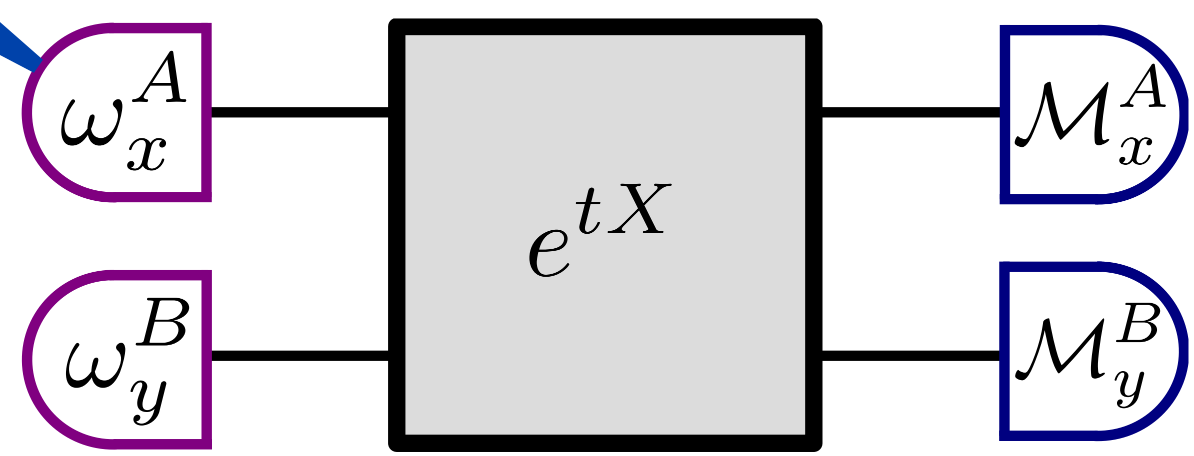

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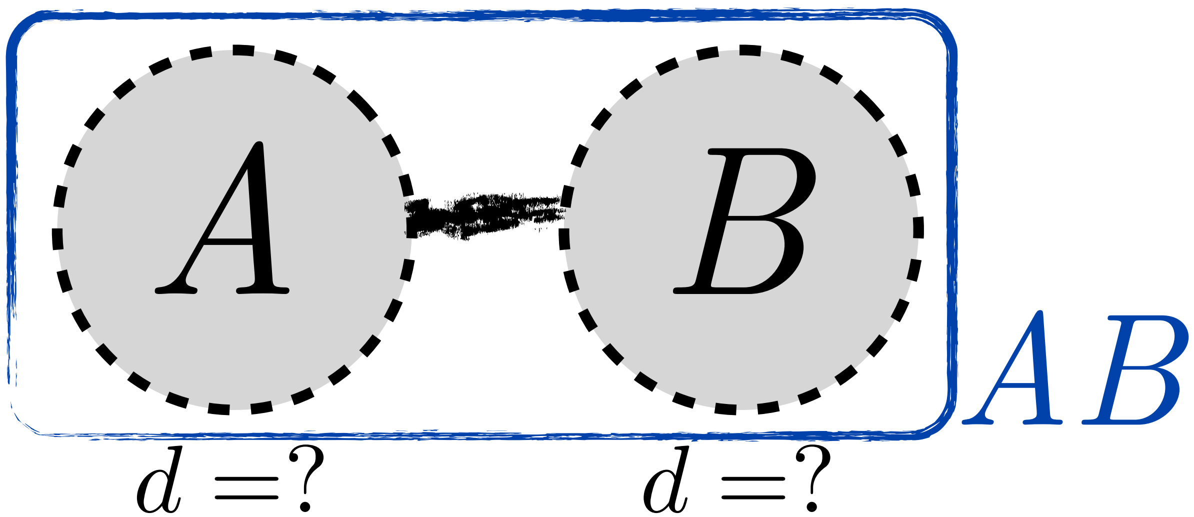

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...arcía, arXiv:1111.4060

... similarly on B...

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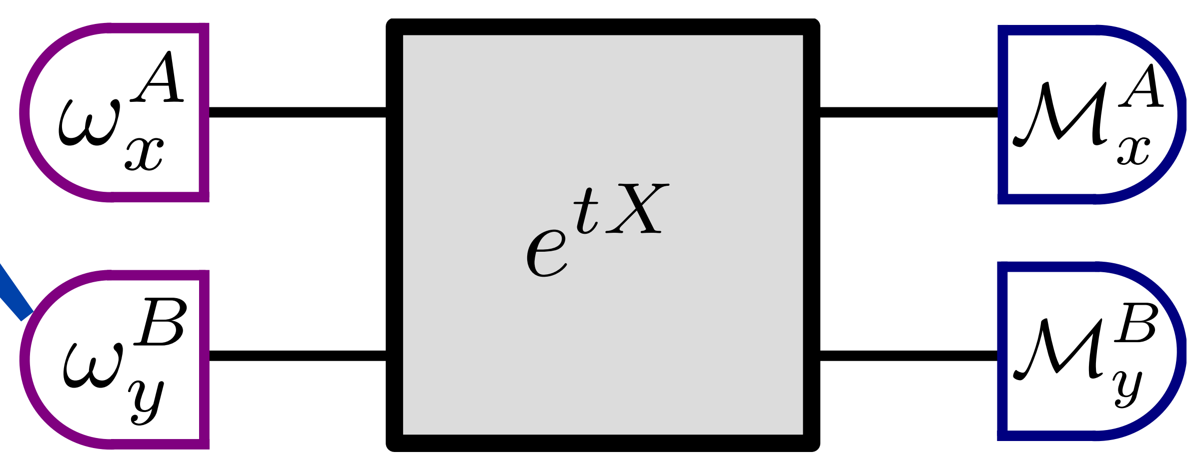

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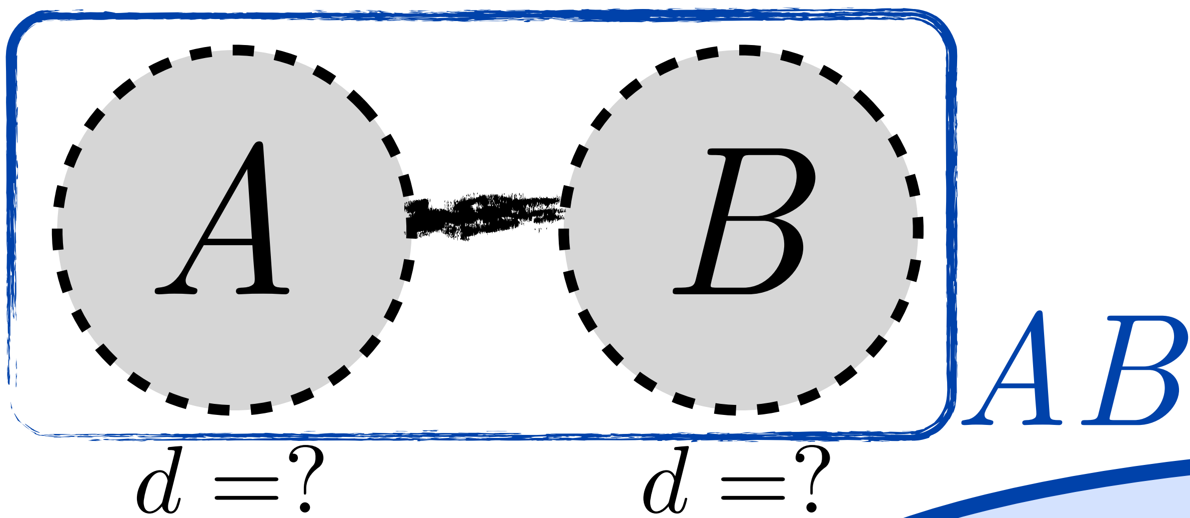

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Ll. Masanes, MM, R. Augu... ...060

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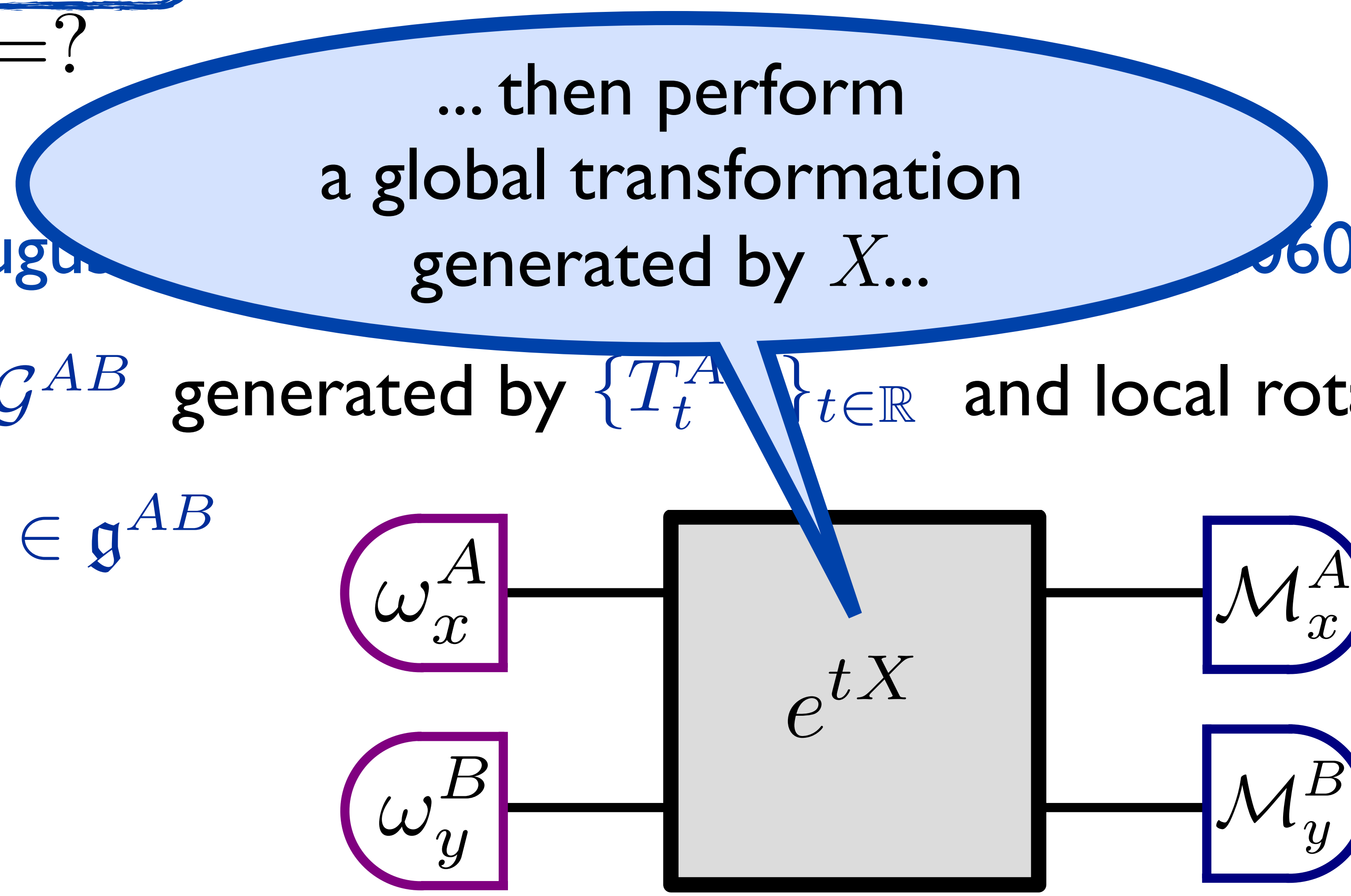

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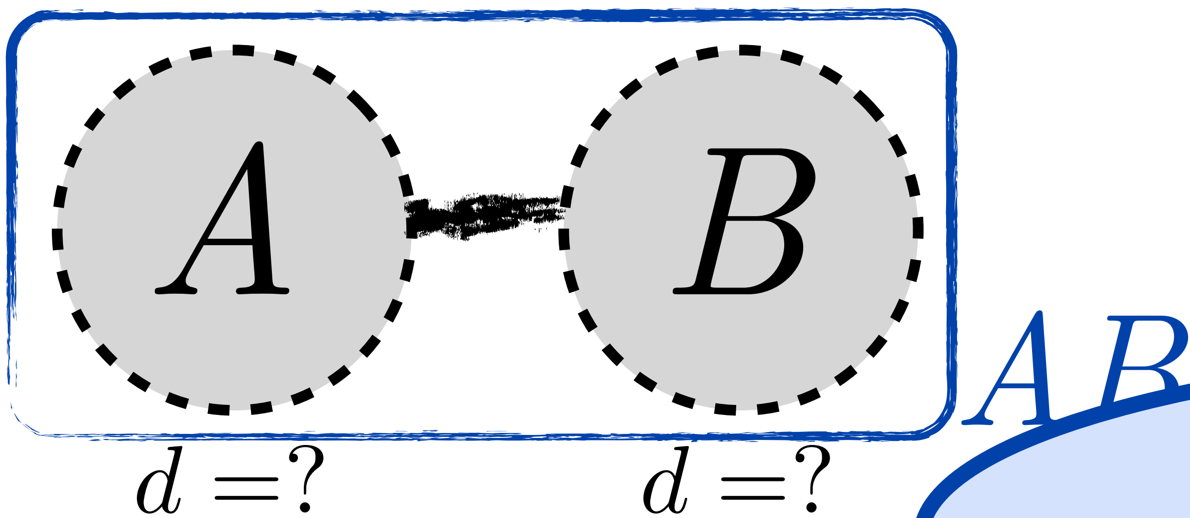

Only **3**D-balls can "talk to each other":

... and finally measure if the local state still points in direction x.

Ll. Masanes, MM, R. Augusiak, and D. Perez-Garcia, arXiv:1111.4060

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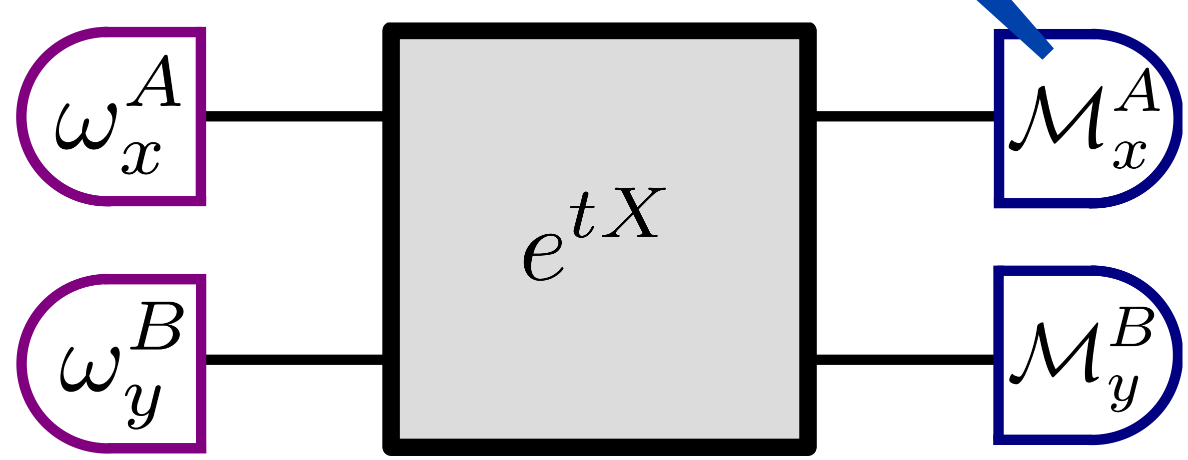

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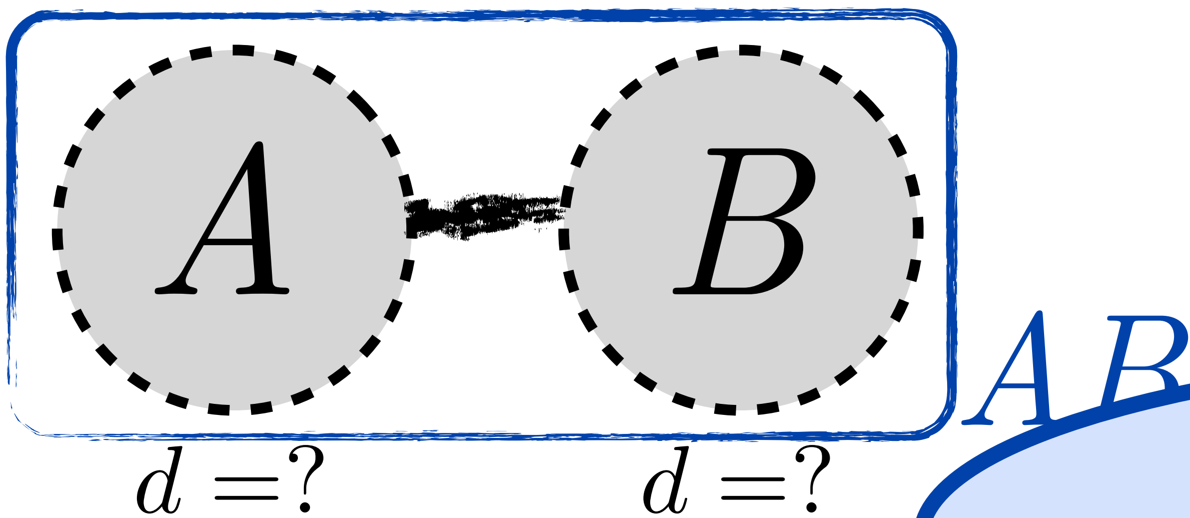

Only **3**D-balls can "talk to each other":

Same on system B.

Ll. Masanes, MM, R. Augusiak, and D. Perez-Garcia, arXiv:1111.4060

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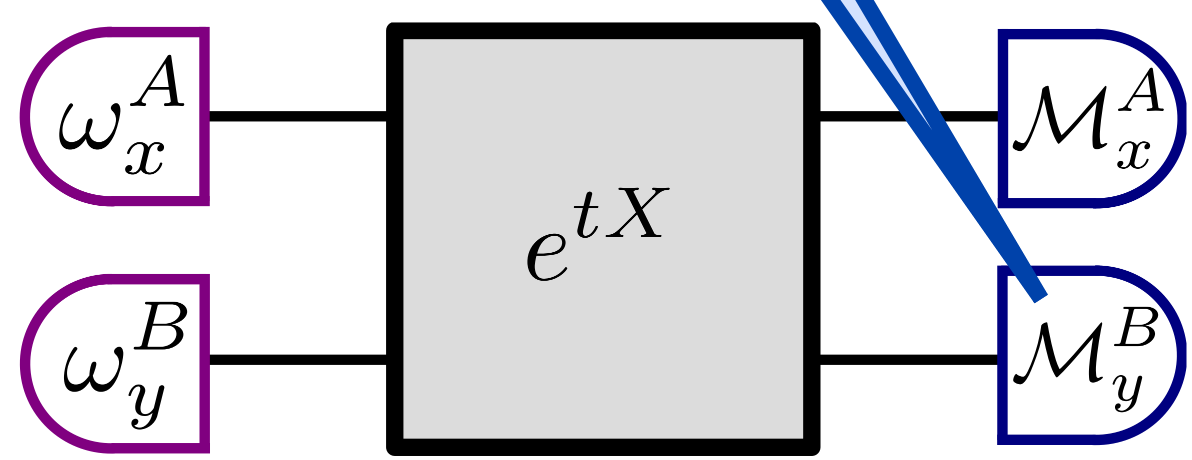

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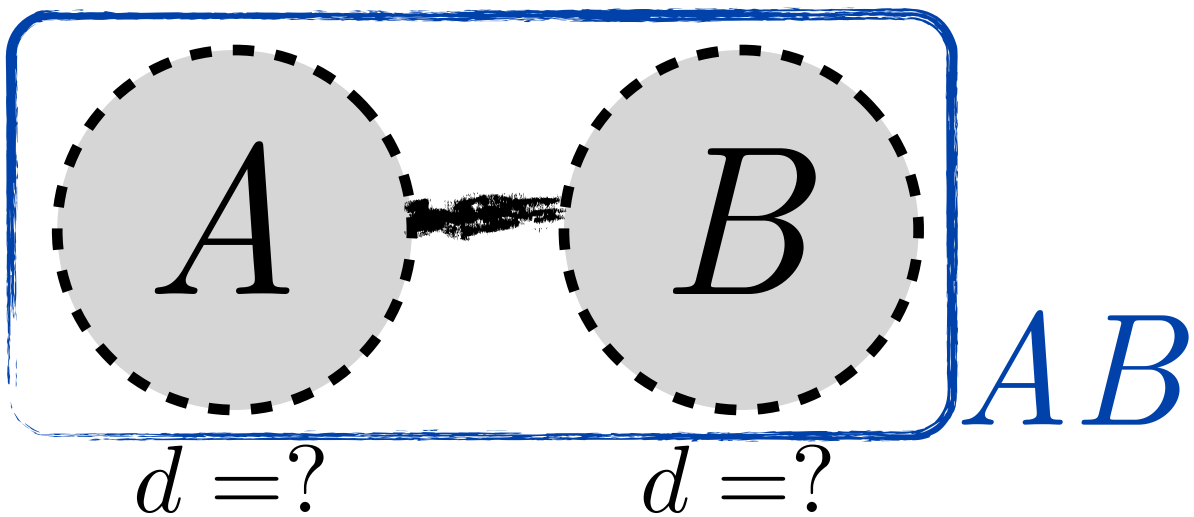

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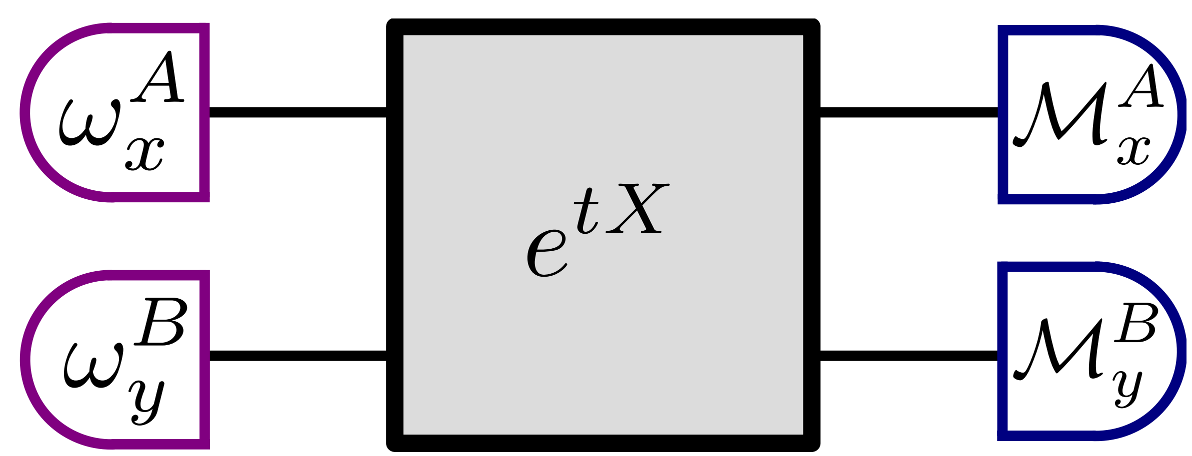
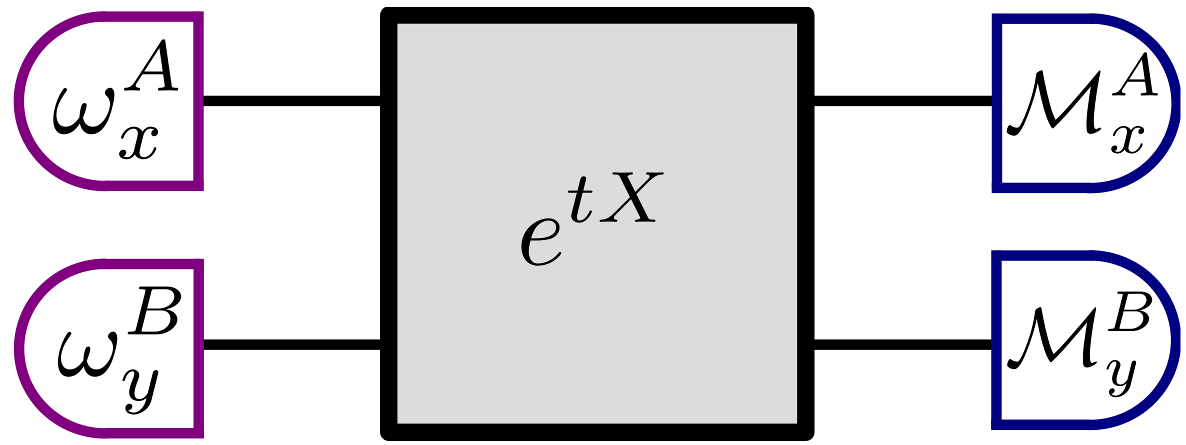


Probability for $t=0$ is

$p(t=0)=1.$

4. Deriving *d=3* and QT



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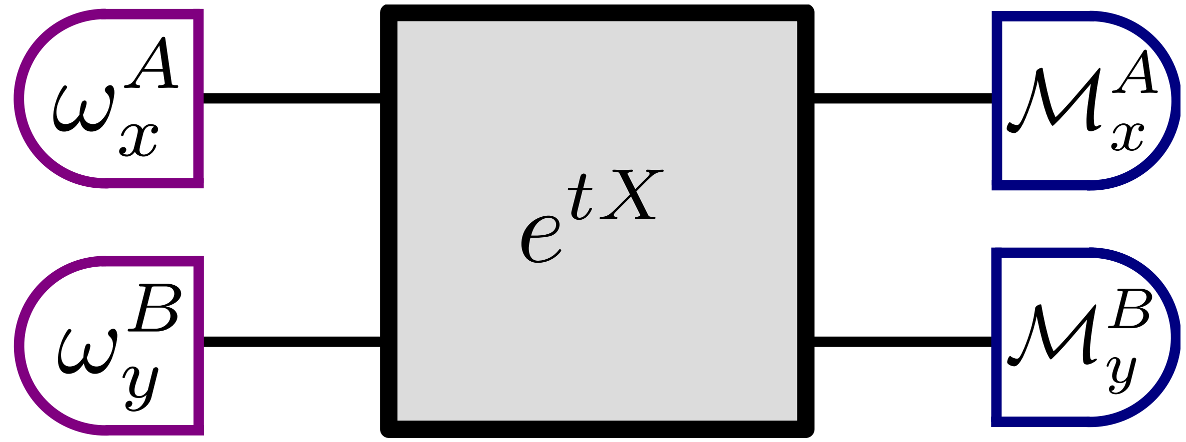

Probability for $t=0$ is

$p(t=0)=1.$ ← local maximum!

$\Rightarrow p'(0)=0,\ \ p''(0)\leq 0.$

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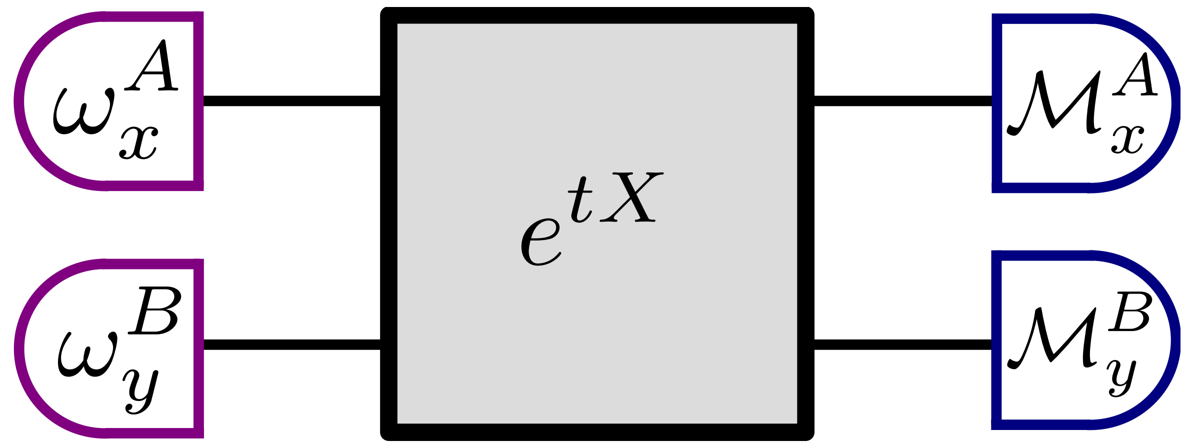

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⇒ Constraints on X:

$$\begin{aligned} \mathcal{M}_x^A \mathcal{M}_y^B X \omega_x^A \omega_y^B &= 0, \\ \mathcal{M}_x^A \mathcal{M}_y^B X^2 \omega_x^A \omega_y^B &\leq 0. \end{aligned}$$

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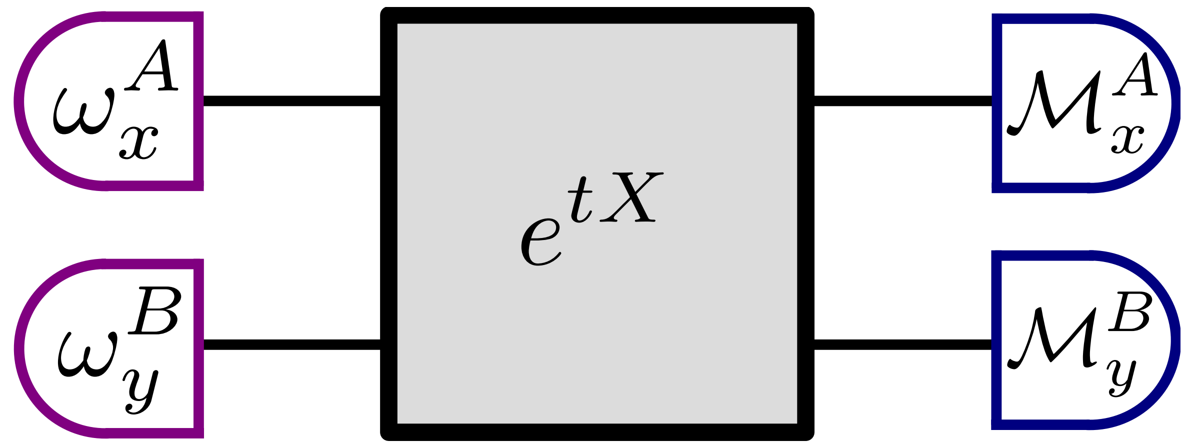

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With some effort, one proves:

- If $d\neq 3$, X satisfies all constraints only if $X=X^A+X^B$ (non-interacting).

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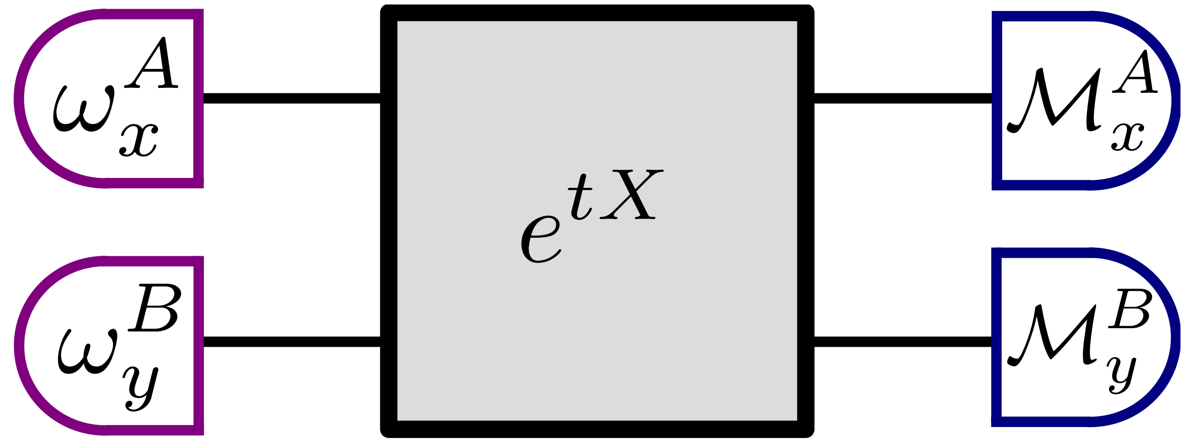

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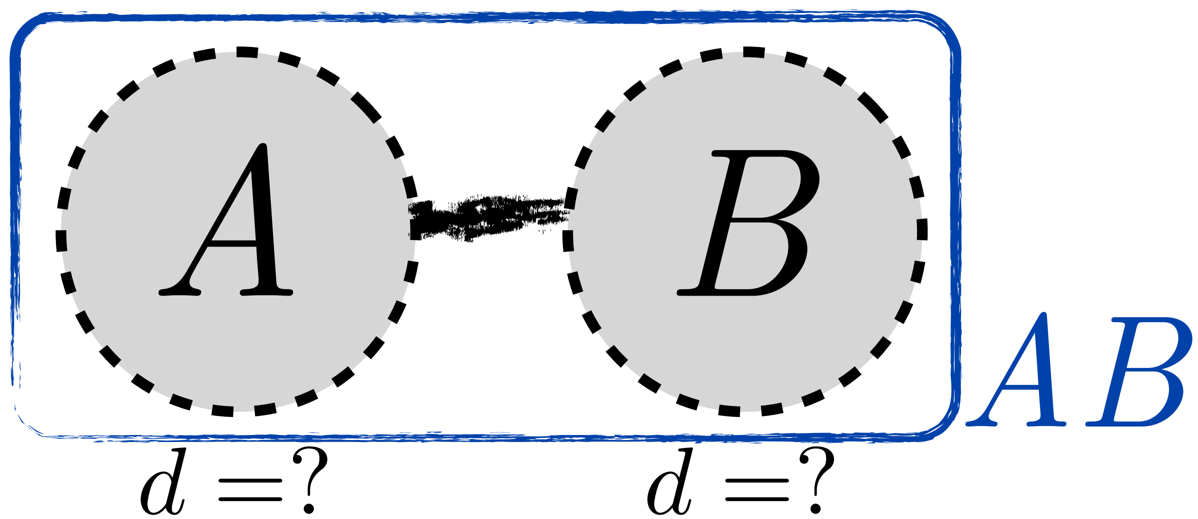
With some effort, one proves:

- If $d\neq 3$, X satisfies all constraints only if $X = X^A + X^B$ (non-interacting).
- If d=3, states ω can be parametrized as 4x4 Hermitian matrices, and X satisfies all constraints iff it generates conjugation by unitaries,

$$\rho \mapsto U\rho U^\dagger.$$

4. Deriving *d*=3 and QT

Theorem 2: From Postulates 1–4, it follows that the spatial dimension must be $d = 3$.



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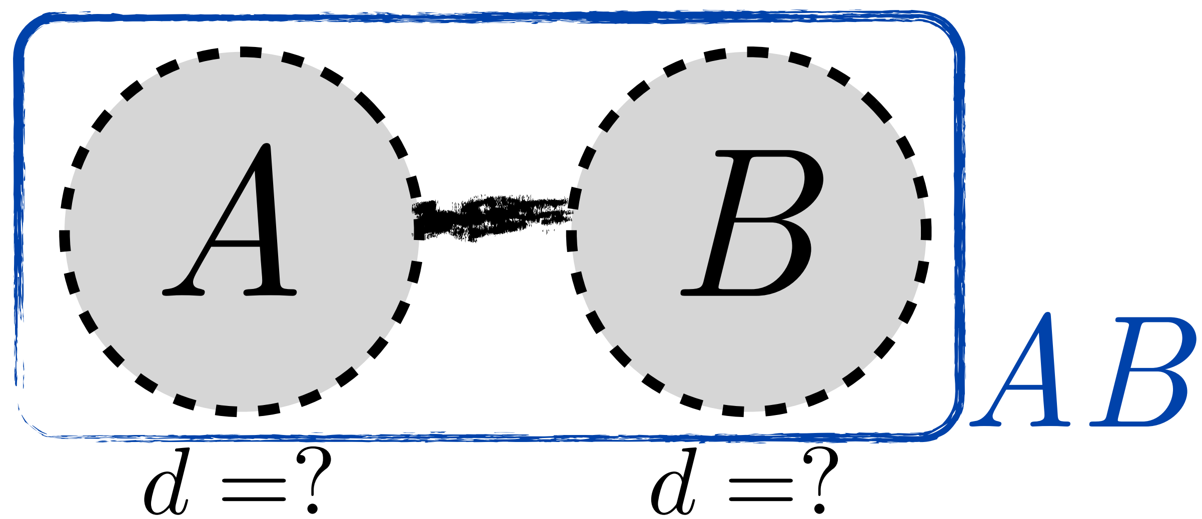

Theorem 3: From Postulates 1–4, it follows that the state space of two direction bits is two-qubit quantum state space (i.e. the set of 4×4 density matrices), and time evolution is given by a one-parameter group of unitaries, $\rho \mapsto U(t)\rho U(t)^\dagger$.

Overview

1. Overview

2. Convex state spaces

3. The postulates

4. Deriving *d=3* and quantum theory

5. What does all this tell us?

Conclusions

- We proved: natural interplay between space + probability is only possible if space is 3-dimensional, and quantum theory holds.

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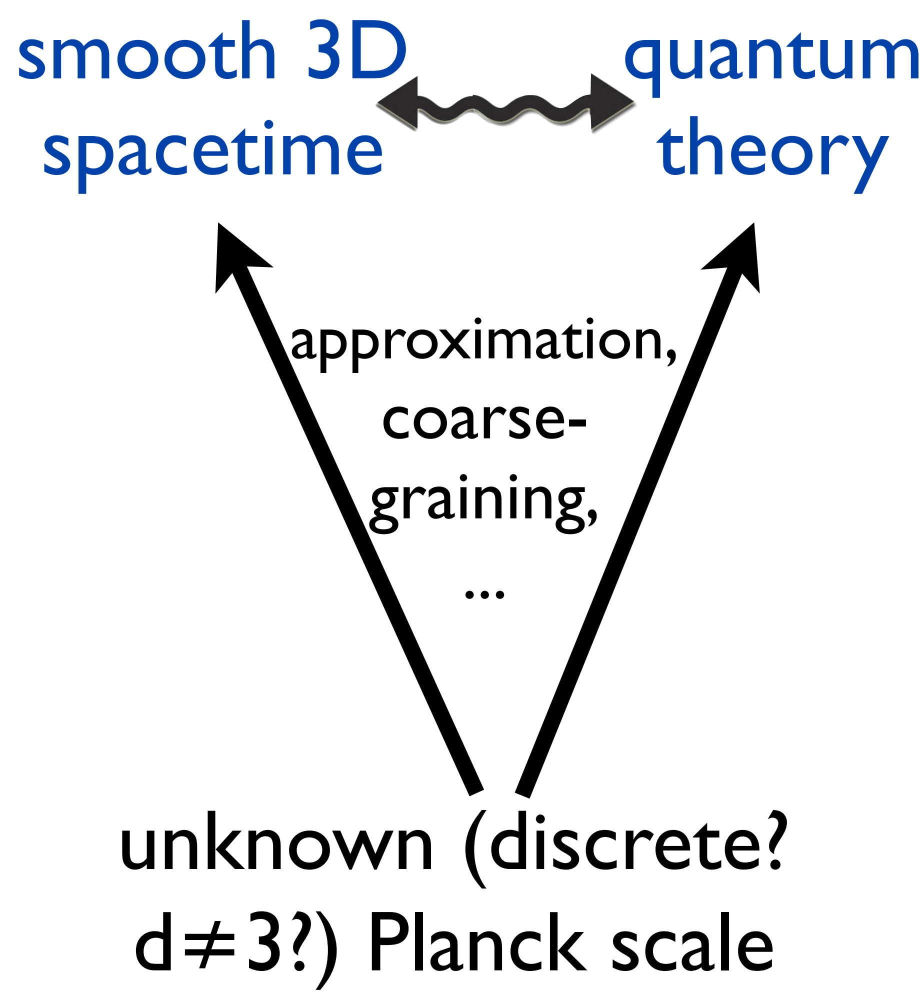

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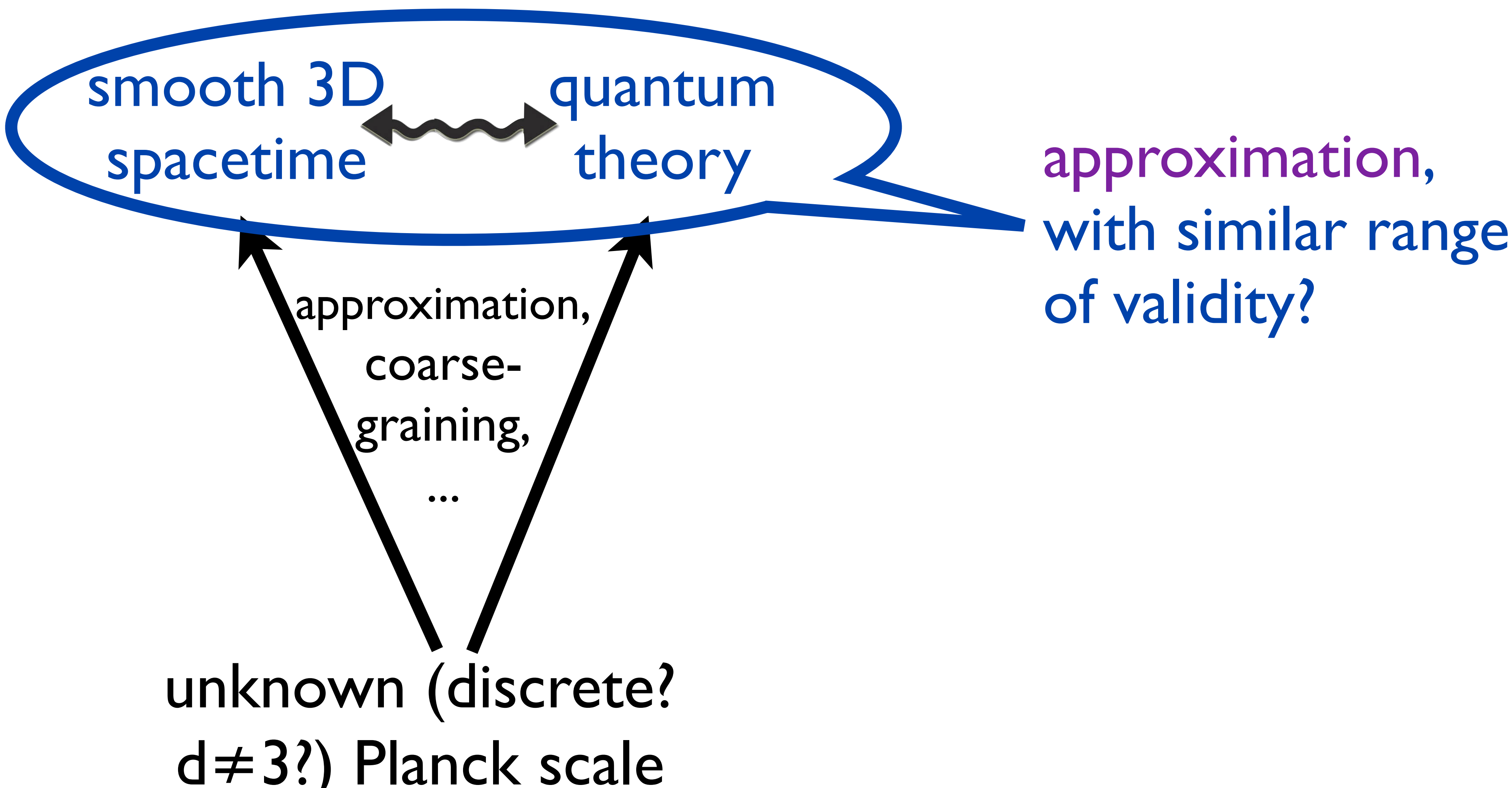

5. Conclusions

ruling out *d≠3*:

Ll. Masanes, MM, D. Pérez-García, R. Augusiak, arXiv:1111.4060

d=3 implies quantum theory:

G. de la Torre, Ll. Masanes, T. Short, MM, Phys. Rev. Lett. **109**, 090403 (2012) arXiv:1110.5482

this talk:

MM, Ll. Masanes, arXiv:1206.0630

Thank you!

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