Reversibility as an axiom for quantum theory
and the search for its closest cousins

Markus P. Müller
Perimeter Institute for Theoretical Physics, Waterloo (Canada)

Joint work with Lluís Masanes, Tony Short, ...

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Outline

1. Motivation

2. Axiomatization of QT

3. Reversibility as a strong axiom

4. QT‘s closest cousins

5. Geometry and probability

1. Motivation

John A. Wheeler, New York Times, Dec. 12 2000:

„Quantum physics [...] has explained the structure of atoms and molecules, [...] the behavior of semiconductors [...] and the comings and goings of particles from neutrinos to quarks.

Successful, yes, but mysterious, too.
Why does the quantum exist?“

The New York Times

ANNALS OF PHYSICS **194**, 336–386 (1989)

Testing Quantum Mechanics

STEVEN WEINBERG*

Theory Group, Department of Physics,
University of Texas, Austin, Texas 78712

Received March 6, 1989

This paper presents a general framework for introducing nonlinear corrections into ordinary quantum mechanics, that can serve as a guide to experiments that would be sensitive to such corrections. In the class of generalized theories described here, the equations that determine the time-dependence of the wave function are no longer linear, but are of Hamiltonian type. Also, wave functions that differ by a constant factor represent the same physical state and satisfy the same time-dependence equations. As a result, there is no difficulty in combining separated subsystems. Prescriptions are given for determining the states in which observables have definite values and for calculating the expectation values of observables for general states, but the calculation of probabilities requires detailed analysis ... A study is presented of ...

ANNALS OF PHYSICS **194**, 336–386 (1989)

Volume 143, number 1,2 PHYSICS LETTERS A 1 January 1990

WEINBERG'S NON-LINEAR QUANTUM MECHANICS AND SUPRALUMINAL COMMUNICATIONS

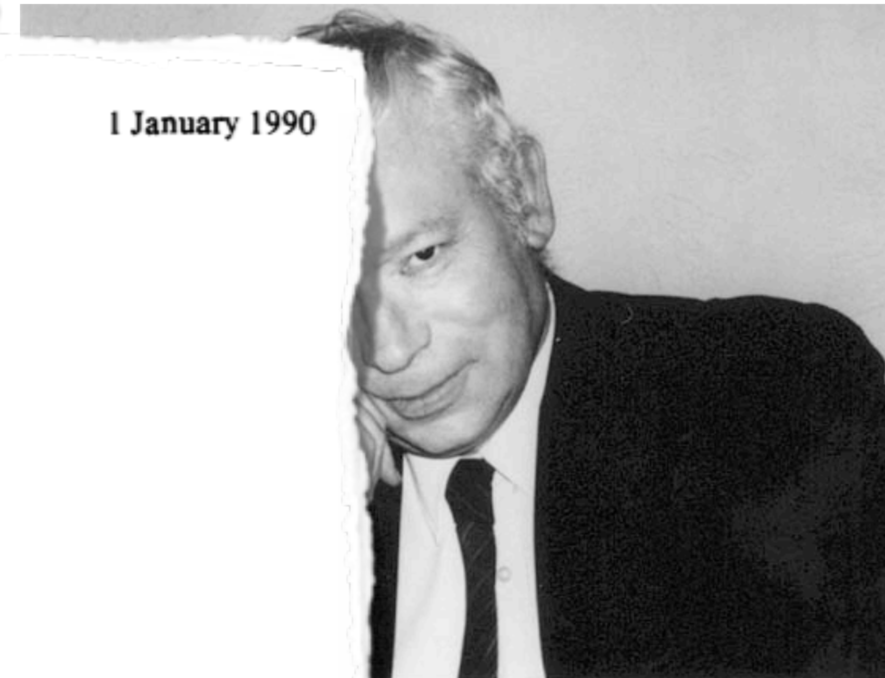
N. GISIN
Group on Applied Physics, University of Geneva, 1211 Geneva 4, Switzerland

Received 16 October 1989; accepted for publication 3 November 1989
Communicated by J.P. Vigier

We show with an example that Weinberg's general framework for introducing non-linear corrections into quantum mechanics allows for arbitrarily fast communications.

Recently Weinberg has proposed a general framework for introducing non-linear corrections into ordinary quantum mechanics [1,2]. Although we fully support his emphasis on the importance of testing quantum mechanics, we would like in this Letter to draw attention to the difficulty of modifying quantum mechanics without introducing arbitrarily fast actions at a distance. Below we show how to construct, within Weinberg's framework, an arbitrarily fast telephone line. In ordinary quantum mechanics

to know what such an apparatus is... do you know what is inside your phone?) In order to simplify we consider only a single-bit message. The two directions z and u are in the xz-plane orthogonal to the incoming flow of particles, and are 45° from each other. The way the inhomogeneous magnetic field acts on the particles is well-known from experimental evidence. After the apparatus there are two counters. For each particle one of the counters will click. This click will be amplified until all readers of



ANNALS OF PHYSICS **194**, 336–386 (1989)

Volume 143, number 1,2 PHYSICS LETTERS A 1 January 1990

WEINBERG'S NON-LINEAR QUANTUM MECHANICS AND SUPRALUMINAL COMMUNICATIONS

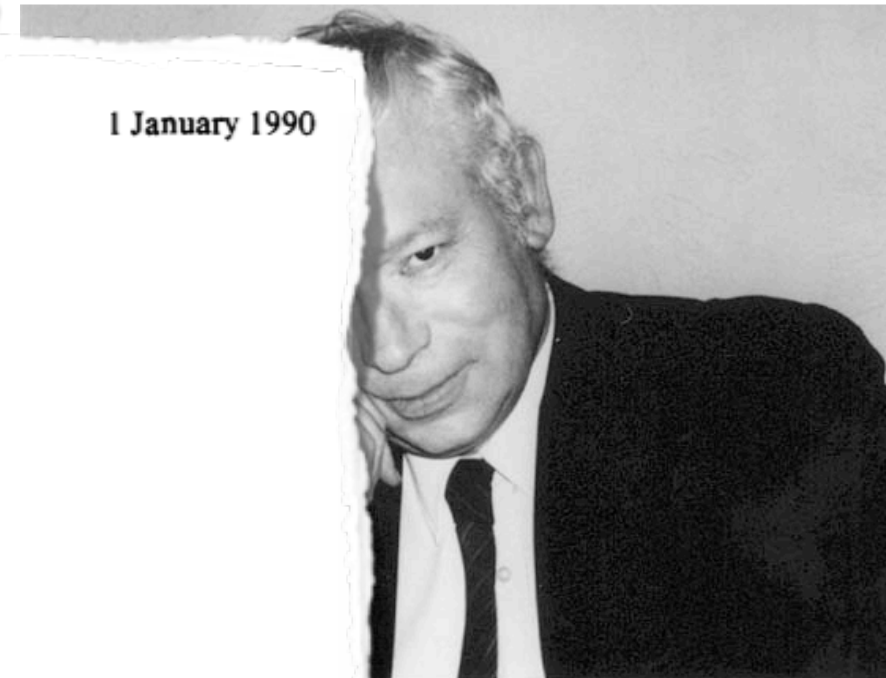
N. GISIN
Group on Applied Physics, University of Geneva, 1211 Geneva 4, Switzerland

Received 16 October 1989; accepted for publication 3 November 1989
Communicated by J.P. Vigier

Modifying quantum theory is difficult.

Recently … work for introducing non-linear corrections into ordinary quantum mechanics [1,2]. Although we fully support his emphasis on the importance of testing quantum mechanics, we would like in this Letter to draw attention to the difficulty of modifying quantum mechanics without introducing arbitrarily fast actions at a distance. Below we show how to construct, within Weinberg's framework, an arbitrarily fast telephone line. In ordinary quantum mechanics

… know what is inside your phone?) In order to simplify we consider only a single-bit message. The two directions z and u are in the xz-plane orthogonal to the incoming flow of particles, and are 45° from each other. The way the inhomogeneous magnetic field acts on the particles is well-known from experimental evidence. After the apparatus there are two counters. For each particle one of the counters will click. This click will be amplified until all readers of



In this talk:

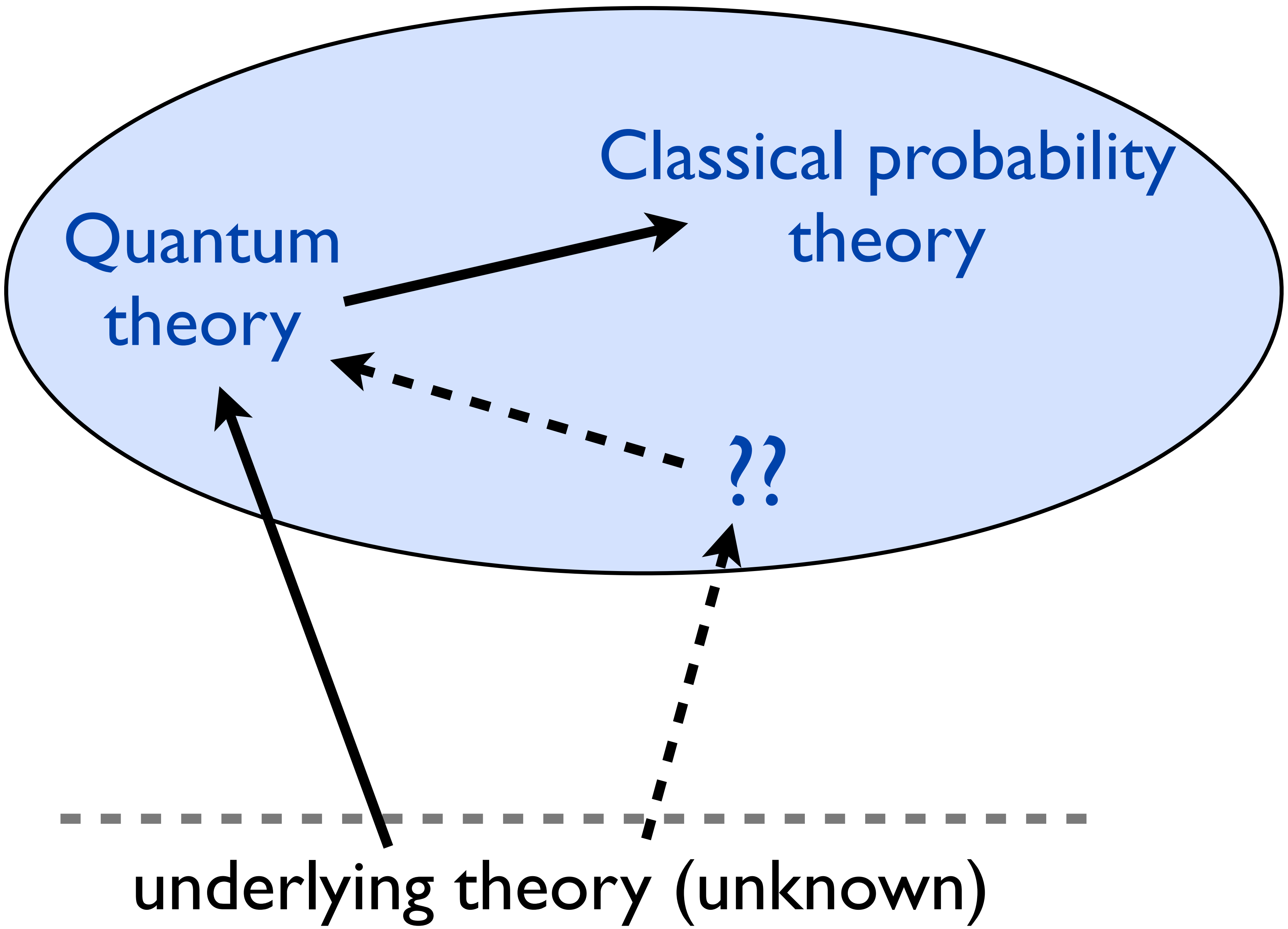
Reversible Dynamics

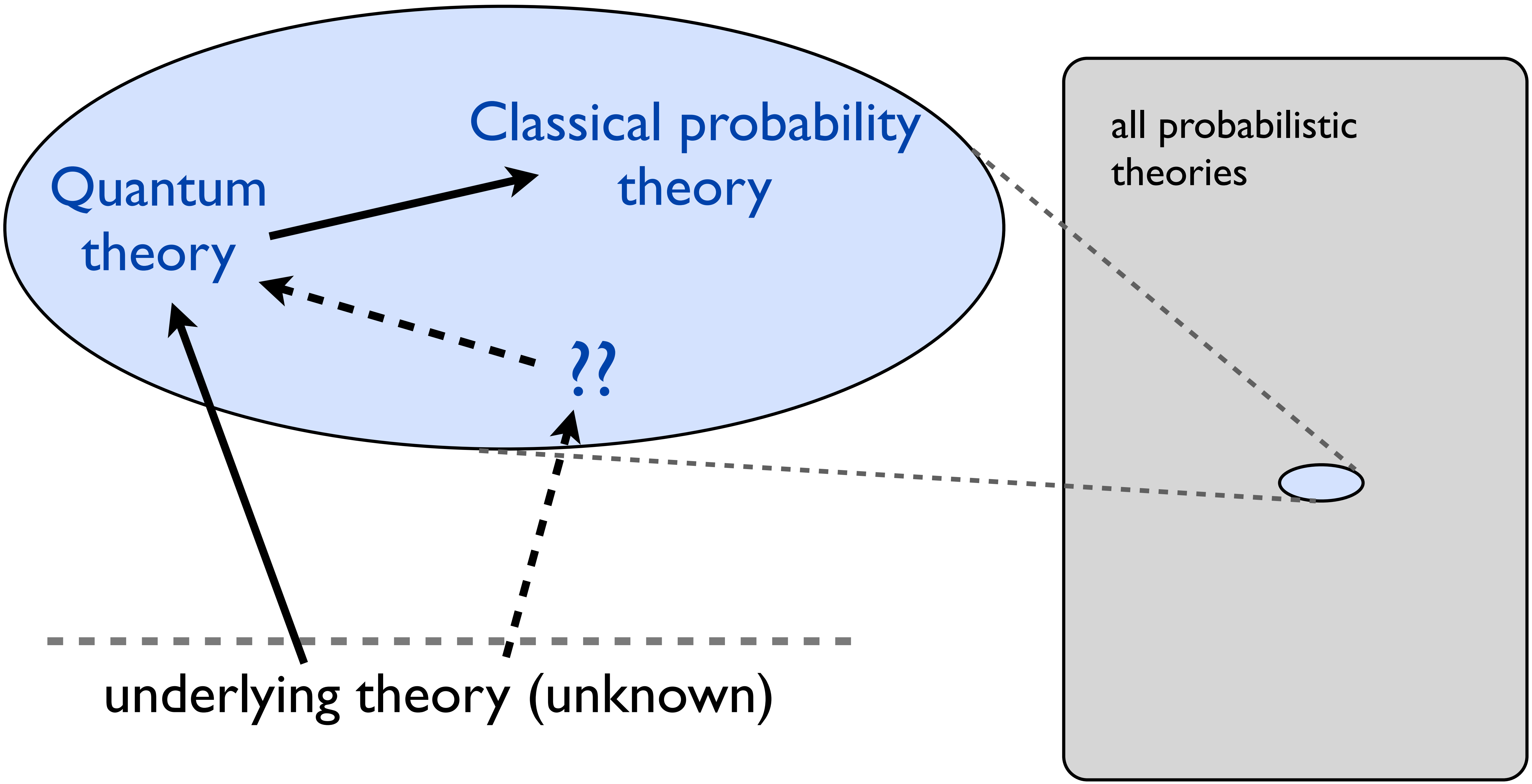
• determines to large extent the structure of QT,

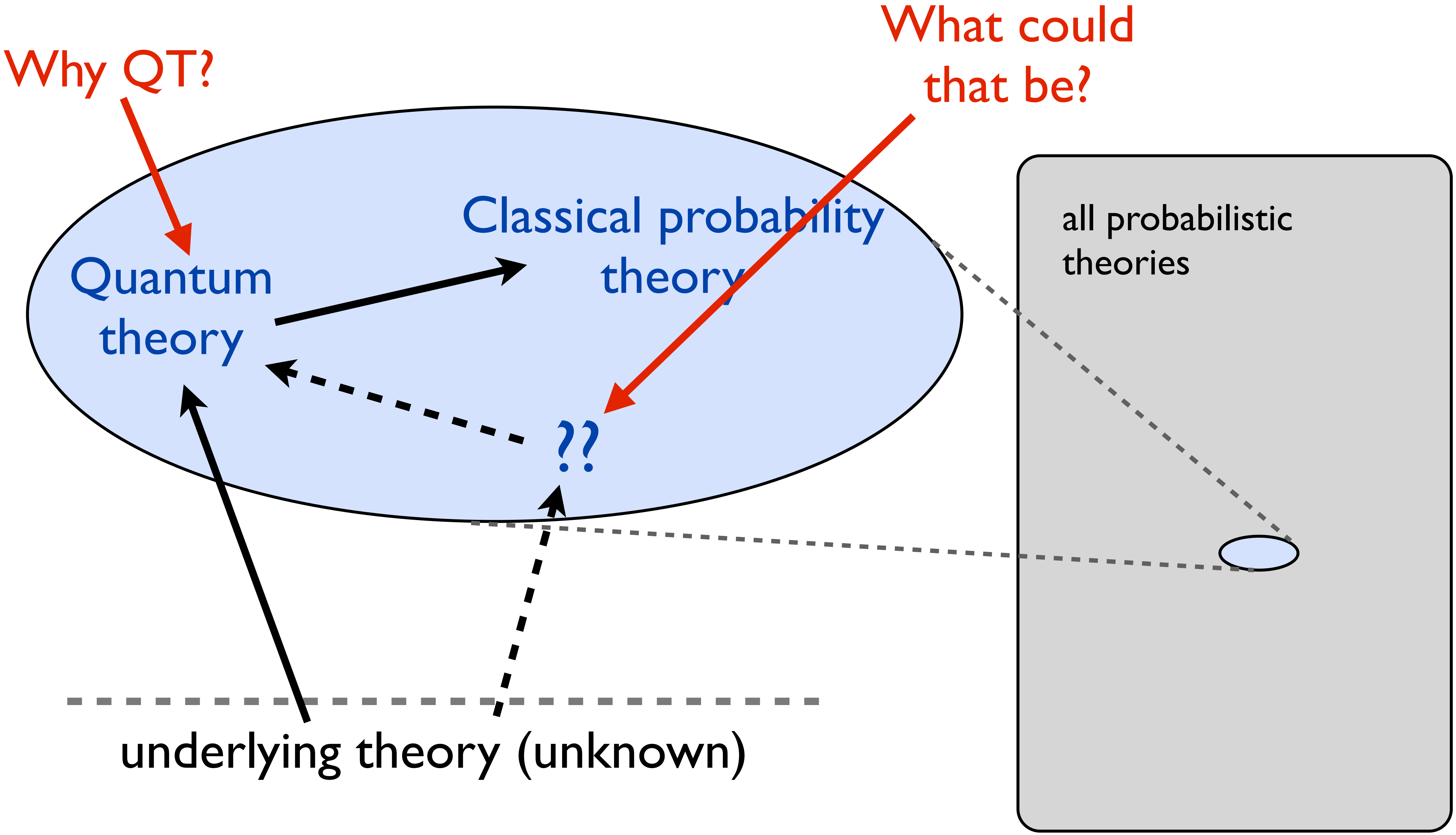
• allows to explore physically natural modifications of QT,

• suggests that geometry and probability might be fundamentally related.

Philosophy:


Classical probability theory
Quantum theory
??
underlying theory (unknown)


Classical probability theory
Quantum theory
??
underlying theory (unknown)
all probabilistic theories



Why QT?
What could
that be?
Classical probability
theory
Quantum
theory
??
underlying theory (unknown)
all probabilistic
theories

2. Axiomatization of QT

earlier work
(quantum logic; Birkhoff, von Neumann, Mackey, Ludwig, ...)

quantum information theory

2001: Lucien Hardy, *Quantum Theory from Five Reasonable Axioms*

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

2. Axiomatization of QT

earlier work
(quantum logic; Birkhoff, von Neumann, Mackey, Ludwig, ...)

quantum information theory

2001: Lucien Hardy, *Quantum Theory from Five Reasonable Axioms*

2009 Dakic, Brukner

2010 Masanes, MM

2010 Chiribella et al.

2011 Hardy

2. Axiomatization of QT

earlier work
(quantum logic; Birkhoff, von Neumann, Mackey, Ludwig, ...)

quantum information theory

2001: Lucien Hardy, *Quantum Theory from Five Reasonable Axioms*

2009 Dakic, Brukner

2010 Masanes, MM
reversible dynamics

2010 Chiribella et al.
purification

2011 Hardy
permutability

2. Axiomatization of QT

earlier work
(quantum logic; Birkhoff, von Neumann, Mackey, Ludwig, ...)

quantum information theory

2001: Lucien Hardy, *Quantum Theory from Five Reasonable Axioms*

2009 Dakic, Brukner

- close to physics
- power of group theory

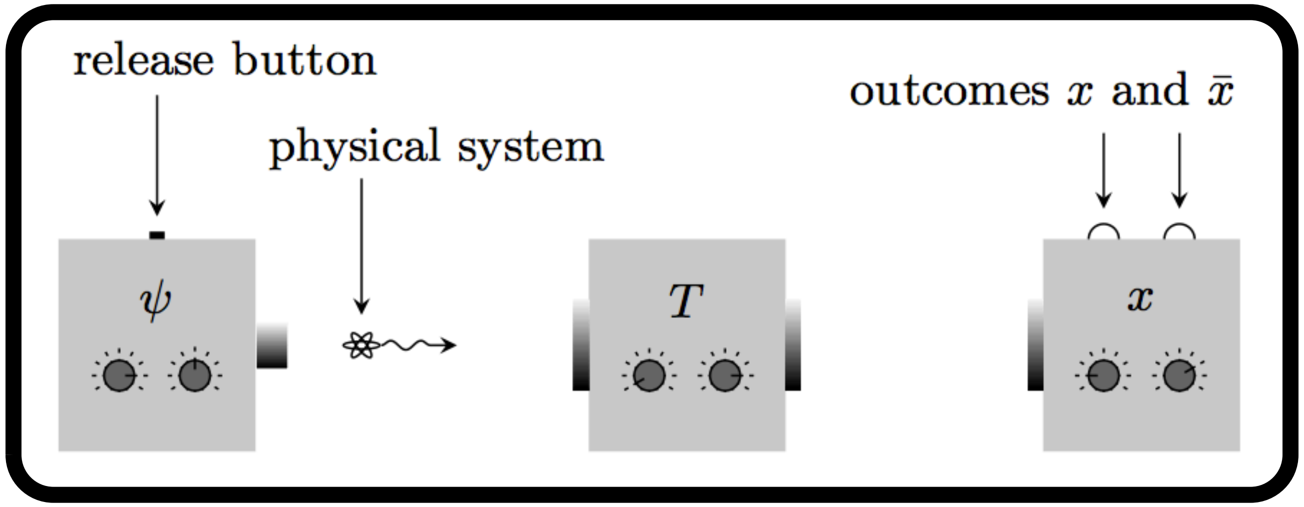
2010 Masanes, MM
reversible dynamics

2010 Chiribella et al.
purification

2011 Hardy
permutability

Setting the stage: probabilistic theories

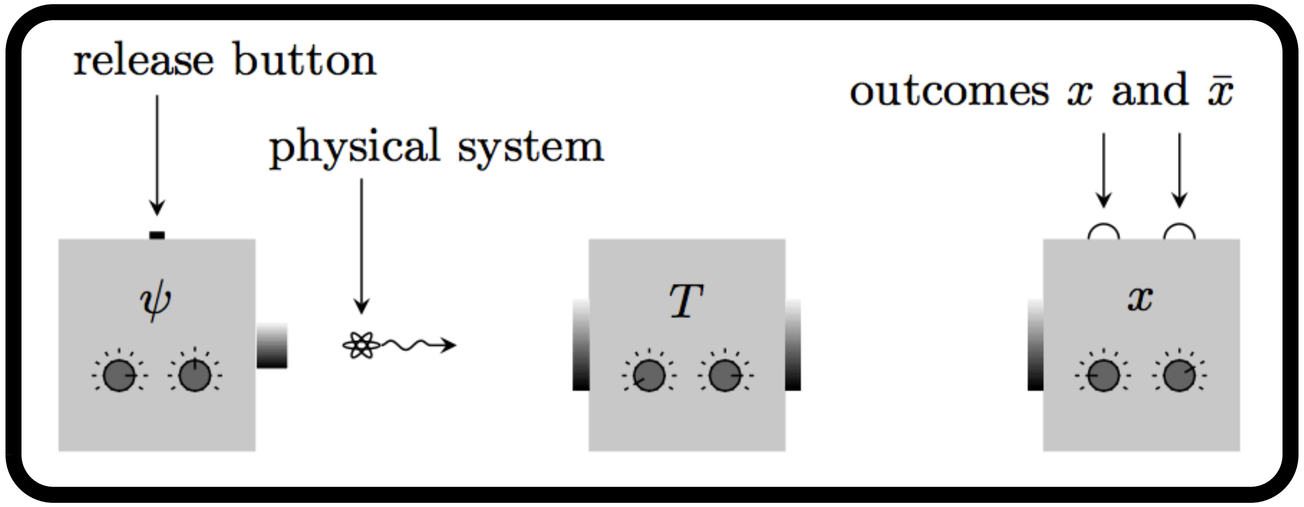
Setting the stage: probabilistic theories



(Unnormalized) state ω = list of all probabilities of „yes“-outcomes of all possible measurements.

$$\omega = (p_1, p_2, p_3, p_4, p_5, p_6, \ldots)$$

Setting the stage: probabilistic theories



(Unnormalized) state ω =
list of all probabilities of „yes“-outcomes of all possible measurements.

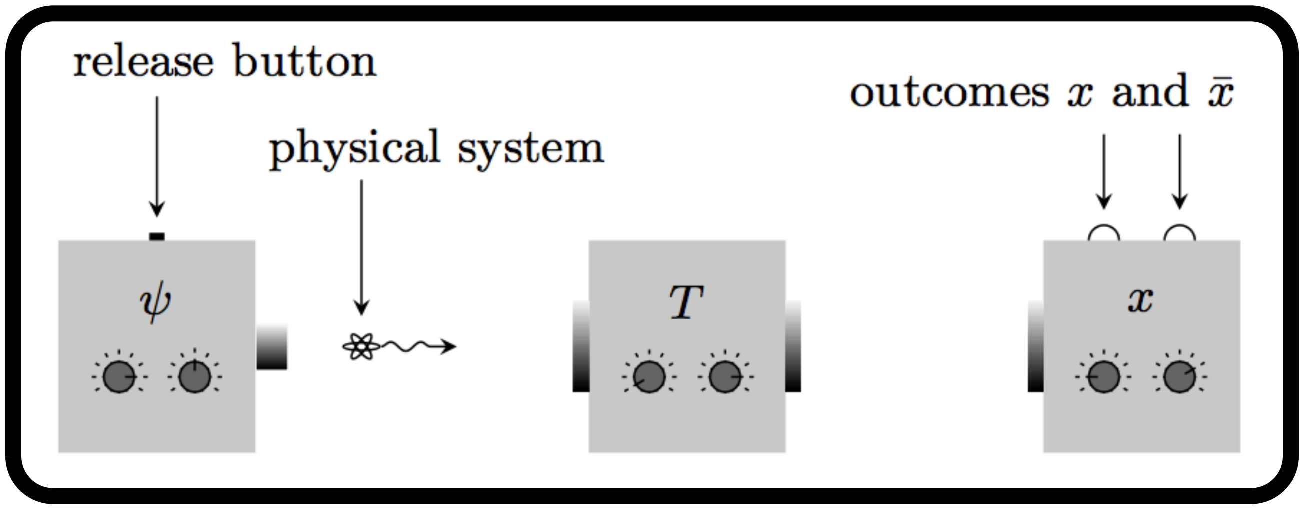
$$\omega = (p_1, p_2, p_3, p_4, p_5, p_6, \ldots)$$

Sometimes, all ω span a finite-dimensional subspace. Ex.: Qubit.

- What‘s the prob. of „spin up“ in X-direction?
- What‘s the prob. of „spin up“ in Y-direction?
- What‘s the prob. of „spin up“ in Z-direction?
- Is the particle there at all?

$$\omega = (p_1, p_2, p_3, p_4) \in \mathbb{R}^4$$

Setting the stage: probabilistic theories



(Unnormalized) state ω = list of all probabilities of „yes"-outcomes of all possible measurements.

$$\omega = (p_1, p_2, p_3, p_4, p_5, p_6, \ldots)$$

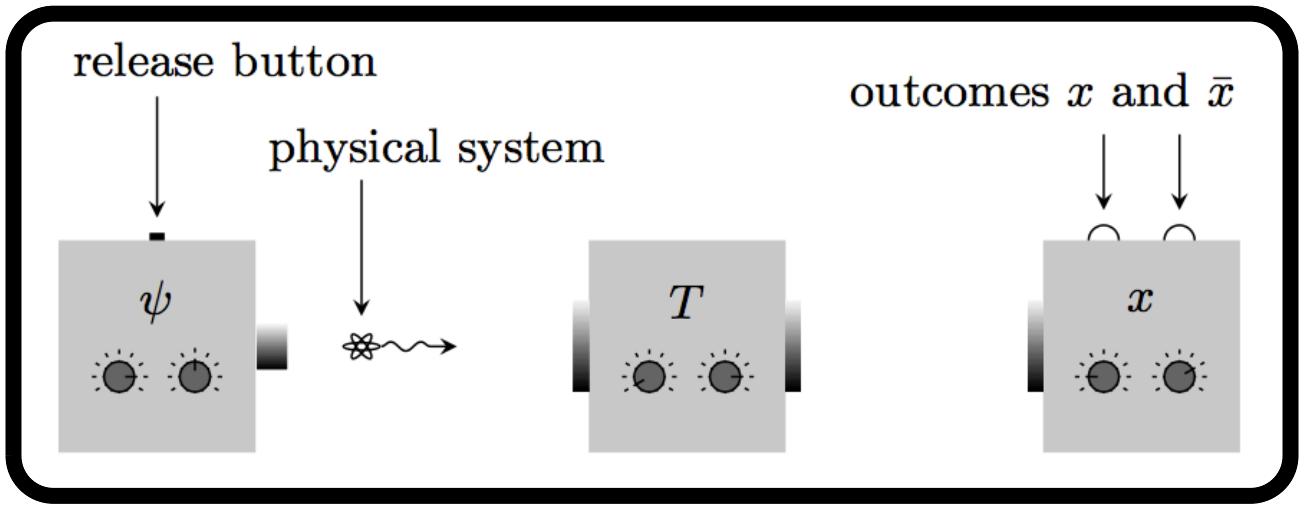
Sometimes, all ω span a finite-dimensional subspace. Ex.: Qubit.

- What's the prob. of „spin up" in X-direction?
- What's the prob. of „spin up" in Y-direction?
- What's the prob. of „spin up" in Z-direction?
- Is the particle there at all?

$$\omega = (p_1, p_2, p_3, p_4) \in \mathbb{R}^4$$

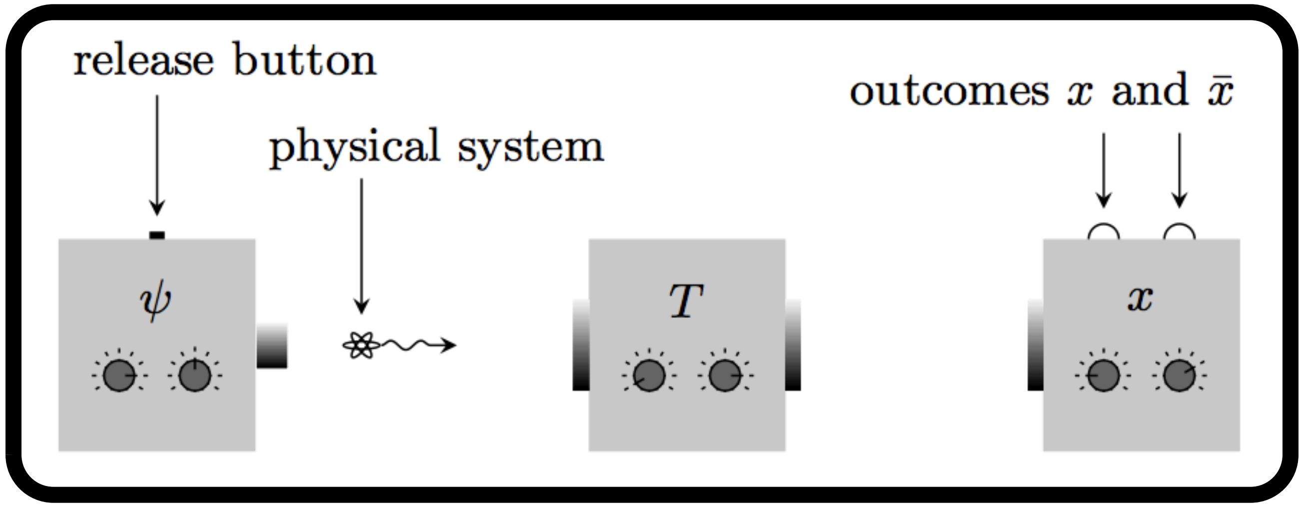
Axiom I: All state spaces are finite-dimensional.

Setting the stage: probabilistic theories



Prepare state ω or φ with prob. ½. Result: $\frac{1}{2}\omega + \frac{1}{2}\varphi$

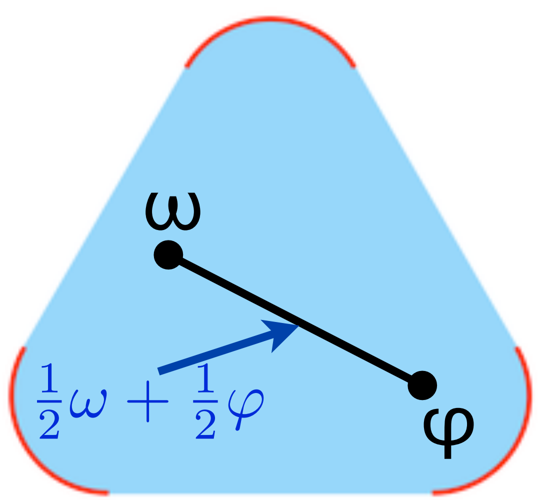
Setting the stage: probabilistic theories



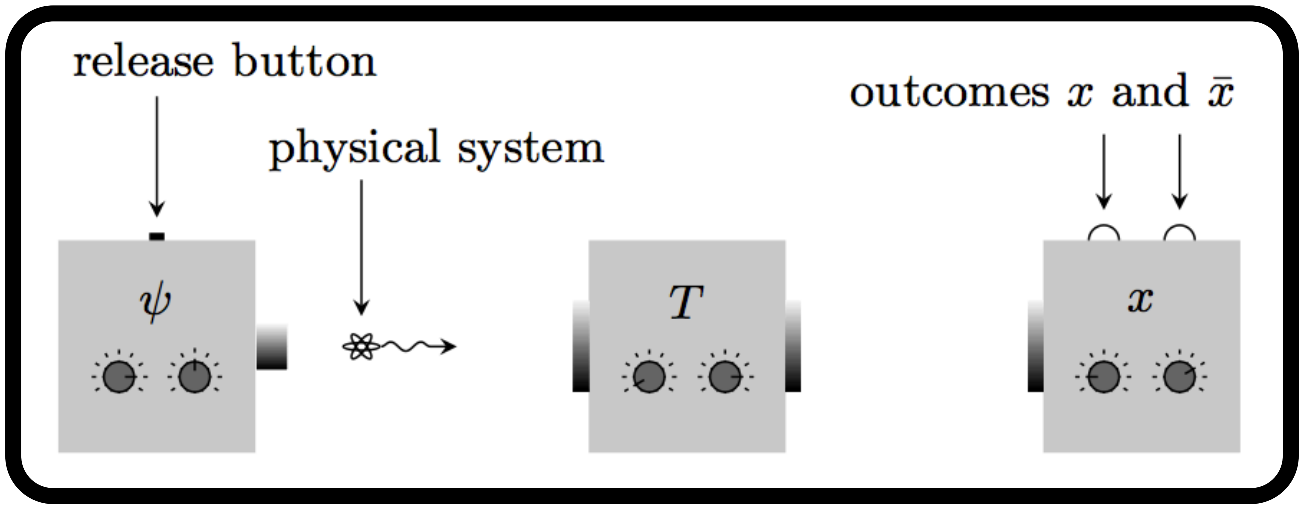
Prepare state ω or φ with prob. ½. Result: $\frac{1}{2}\omega + \frac{1}{2}\varphi$

State spaces are convex sets.

Extreme points are pure, others mixed.



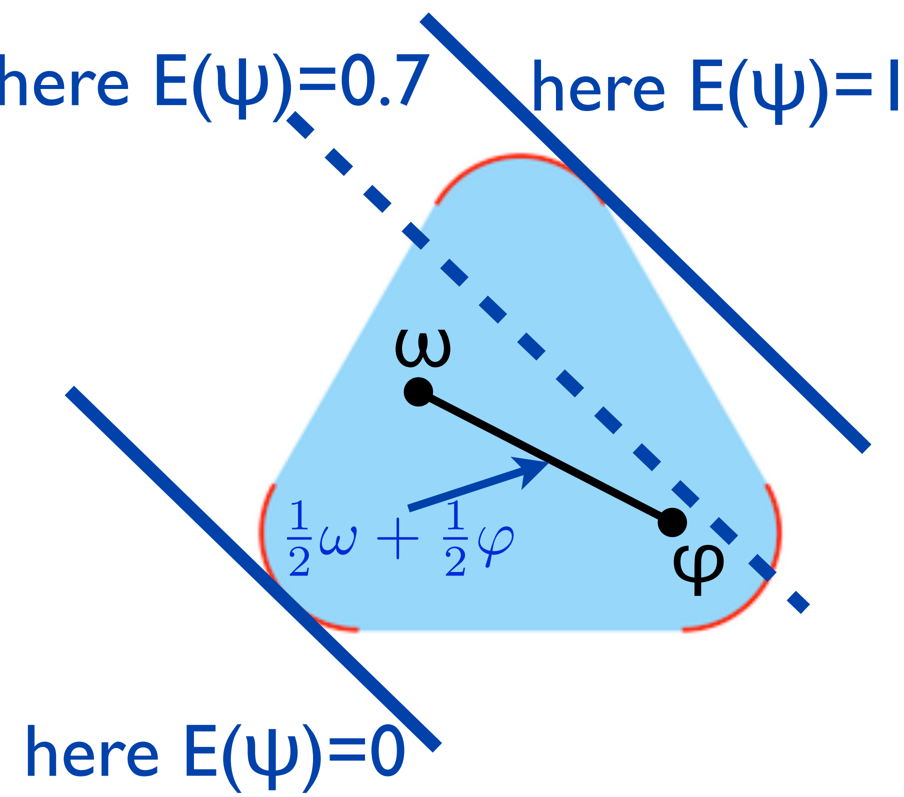
Setting the stage: probabilistic theories



Prepare state ω or φ with prob. ½. Result: $\frac{1}{2}\omega + \frac{1}{2}\varphi$

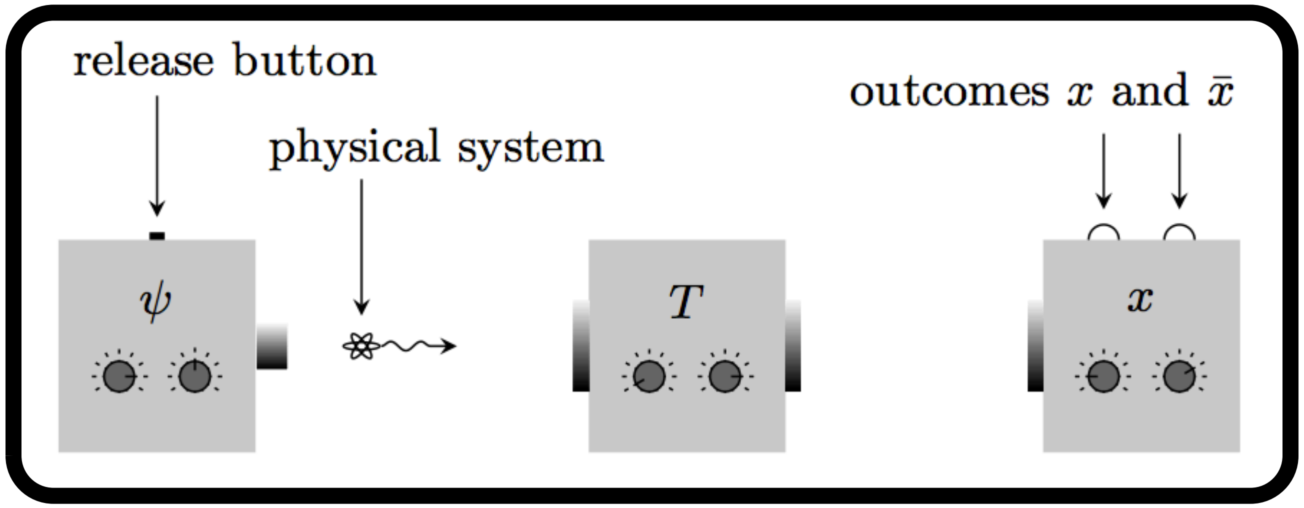
State spaces are convex sets.

Extreme points are pure, others mixed.



Outcome probabilities are linear functionals *E* with $0 \leq E(\psi) \leq 1$ for all ψ.

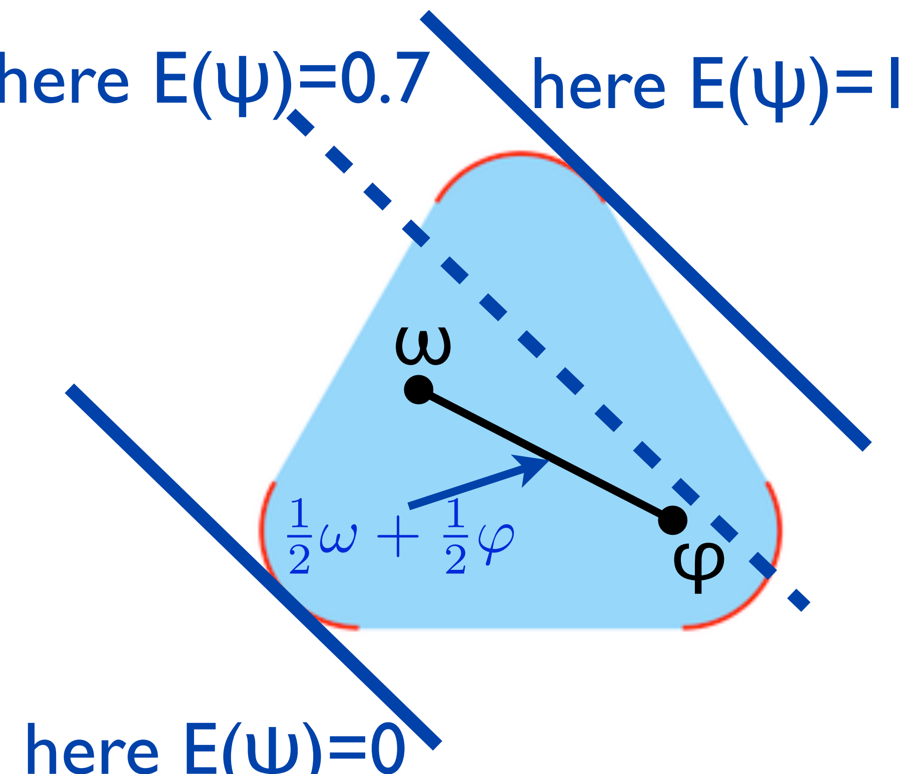
Setting the stage: probabilistic theories



Prepare state ω or φ with prob. ½. Result: $\frac{1}{2}\omega + \frac{1}{2}\varphi$

State spaces are convex sets.

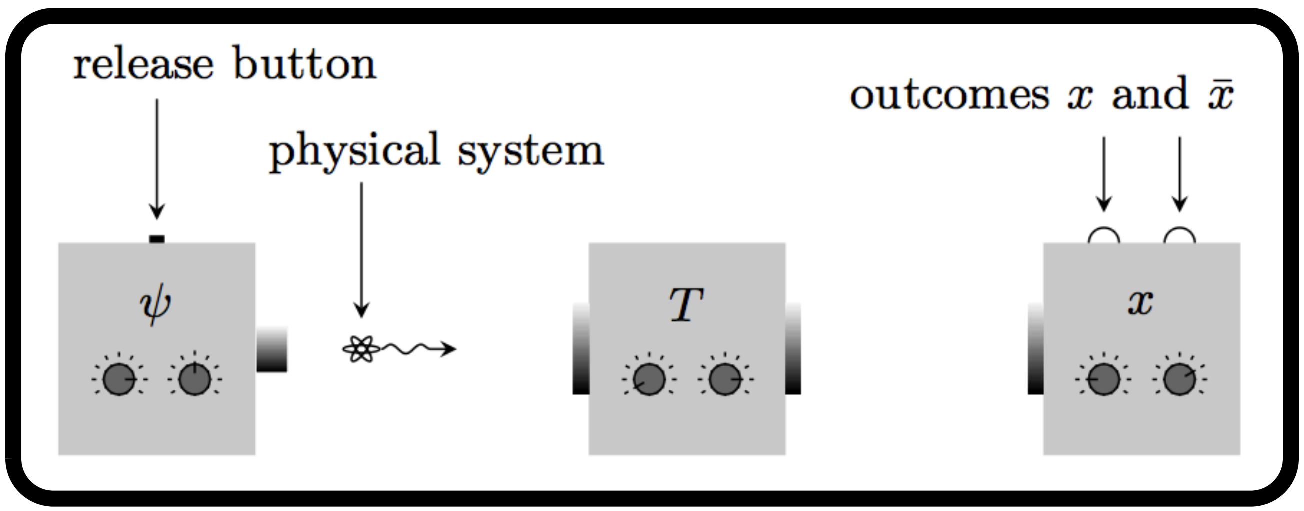
Extreme points are pure, others mixed.



Outcome probabilities are linear functionals *E* with $0 \leq E(\psi) \leq 1$ for all ψ.

Measurements are $(E_1, E_2, \ldots, E_k)$ with $\sum_i E_i(\psi) = 1$ for all ψ.

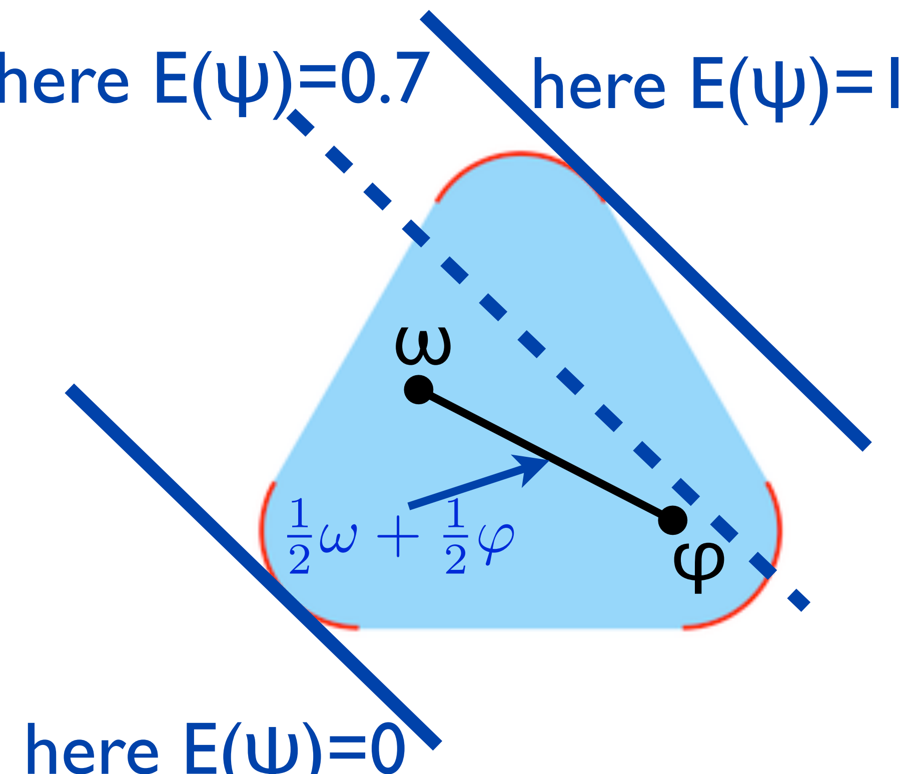
Setting the stage: probabilistic theories



Prepare state ω or φ with prob. ½. Result: $\frac{1}{2}\omega + \frac{1}{2}\varphi$

State spaces are convex sets.

Extreme points are pure, others mixed.



Outcome probabilities are linear functionals *E* with $0 \leq E(\psi) \leq 1$ for all ψ.

Measurements are $(E_1, E_2, \ldots, E_k)$ with $\sum_i E_i(\psi) = 1$ for all ψ.

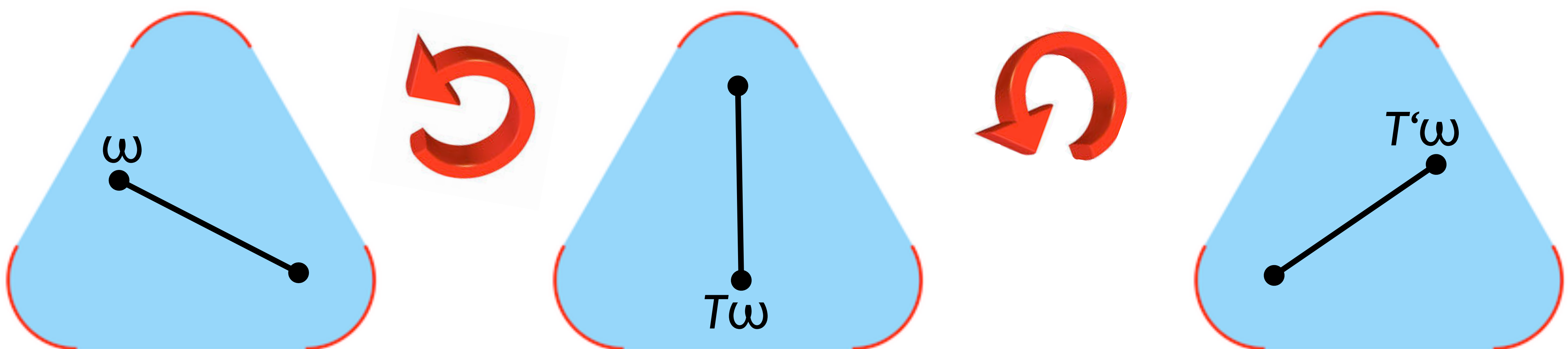
Axiom II: No further restrictions on the set of possible measurements.

Transformations T map states to states, and are linear.

Transformations T map states to states, and are linear.

Reversible transformations form a group $\mathcal{G}_A$. In quantum theory: $\rho \mapsto U\rho U^\dagger$

They are symmetries of state space: $T(\Omega_A) = \Omega_A$

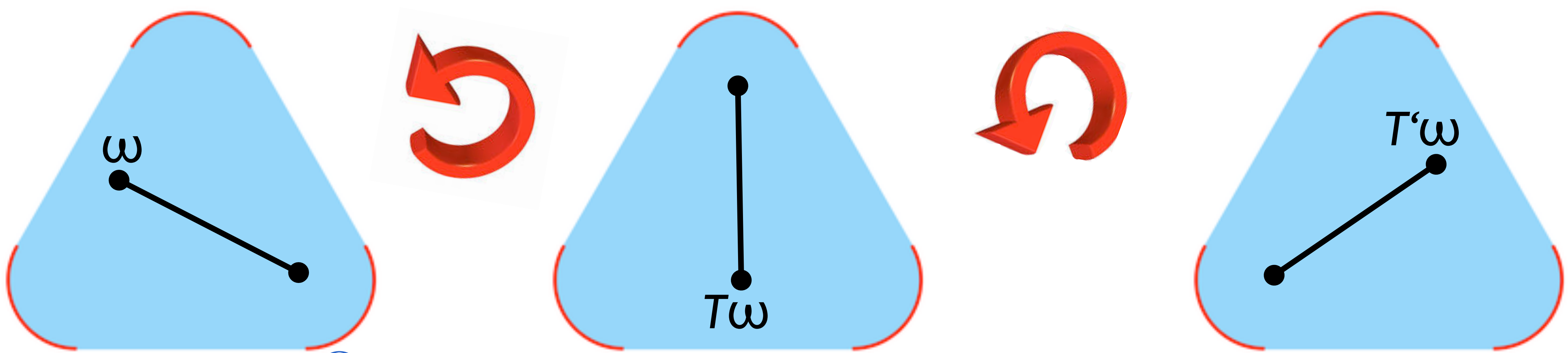


Normalized state space Ω_A

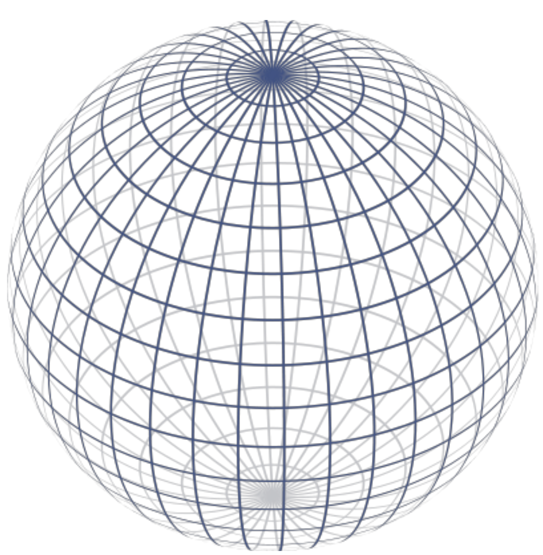
Transformations T map states to states, and are linear.

Reversible transformations form a group $\mathcal{G}_A$. In quantum theory: $\rho \mapsto U\rho U^\dagger$

They are symmetries of state space: $T(\Omega_A) = \Omega_A$



Normalized state space Ω_A



Qubit: Ω_A is the 3D unit ball,

$\mathcal{G}_A = SO(3)$ (no reflections!)

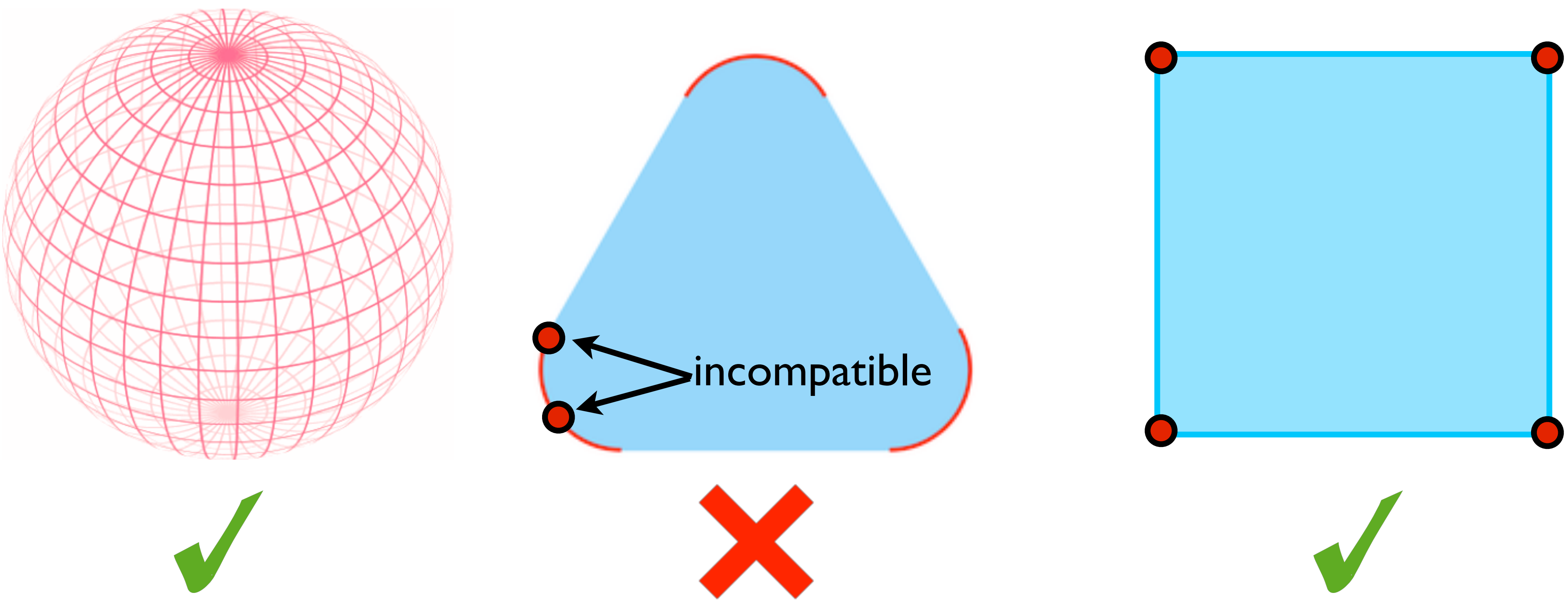
$\Rightarrow$ A system is a pair $(\Omega_A, \mathcal{G}_A)$.

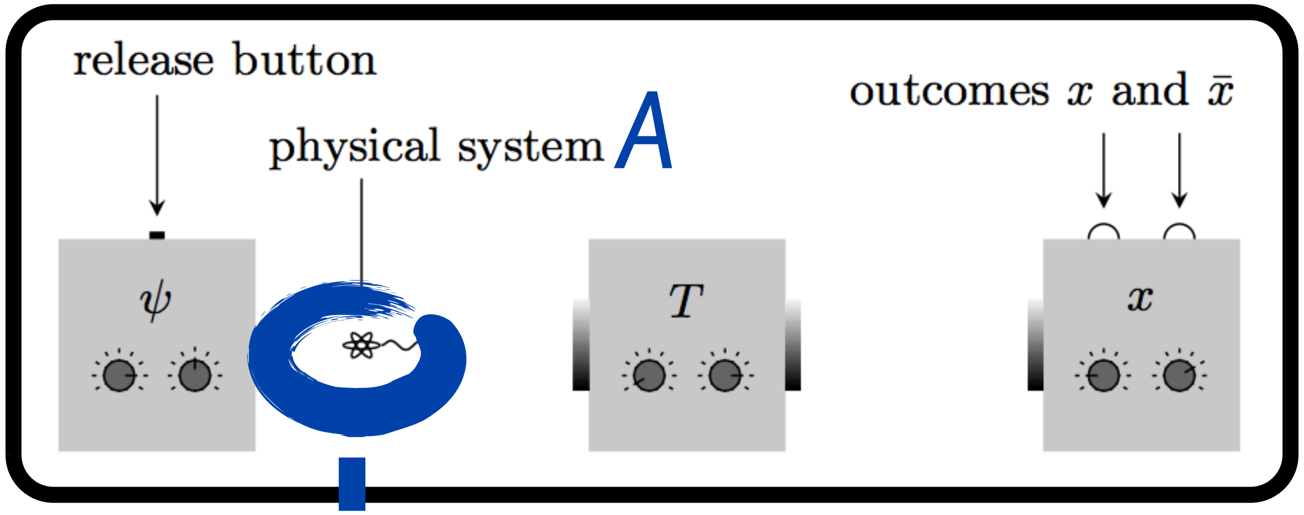
state space — reversible transformations

Axiom III: For every pair of pure states φ, ω, there is a reversible transformation $T \in \mathcal{G}_A$ such that $T\varphi = \omega$.

Axiom III: For every pair of pure states φ, ω, there is a reversible transformation $T \in \mathcal{G}_A$ such that $T\varphi = \omega$.

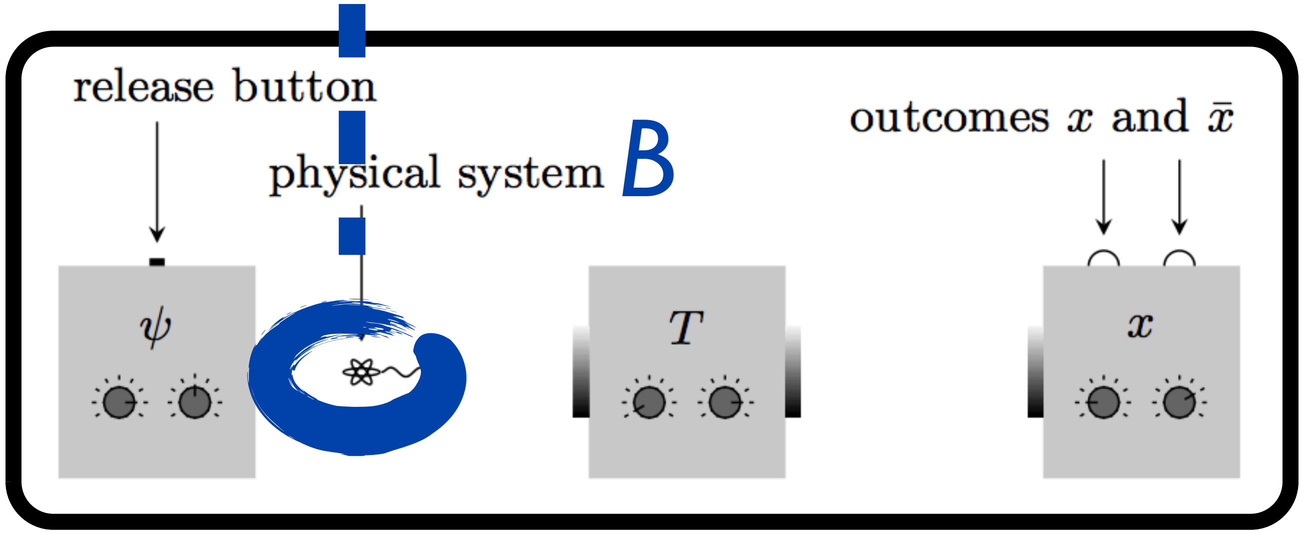
Enforces symmetry in state space:



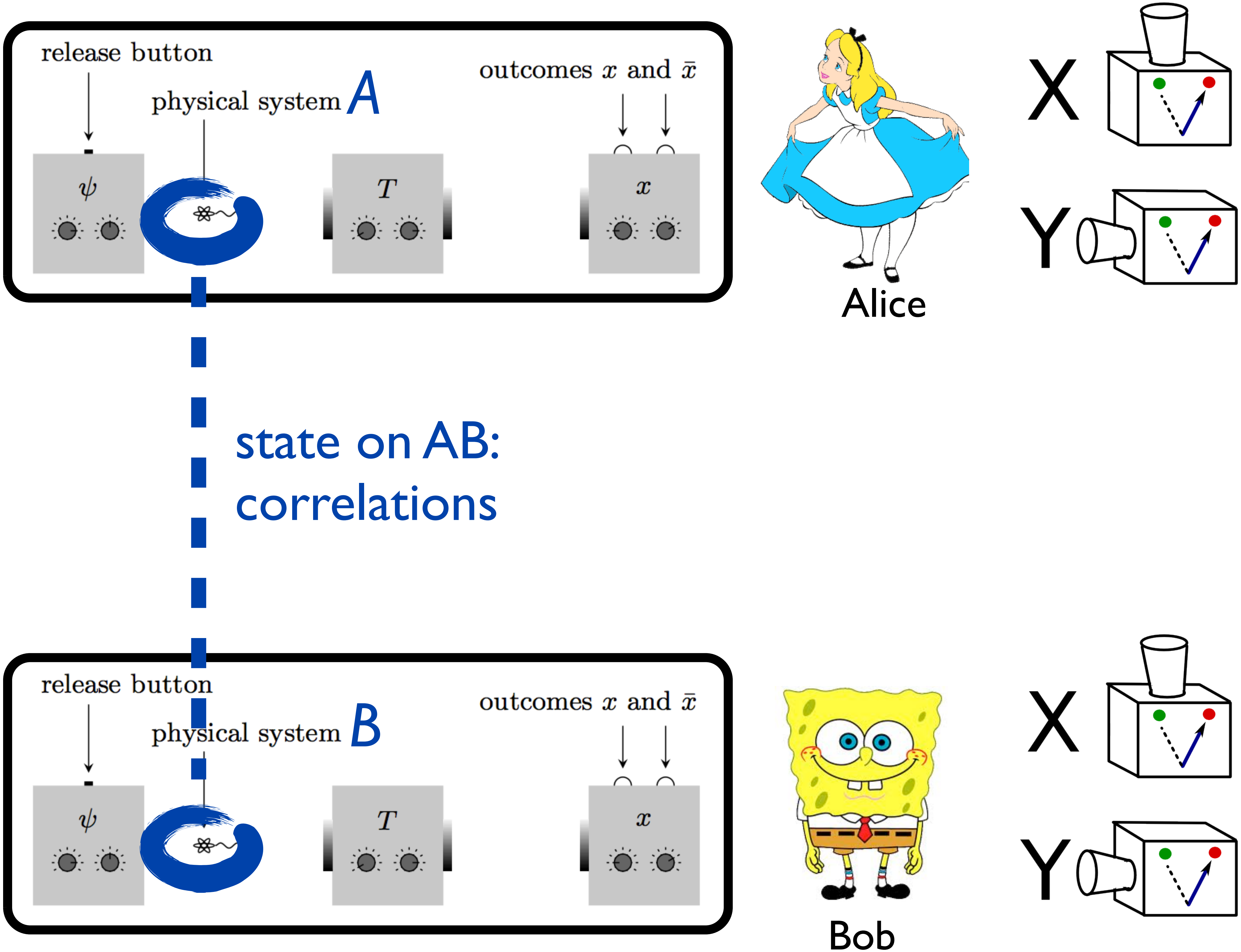


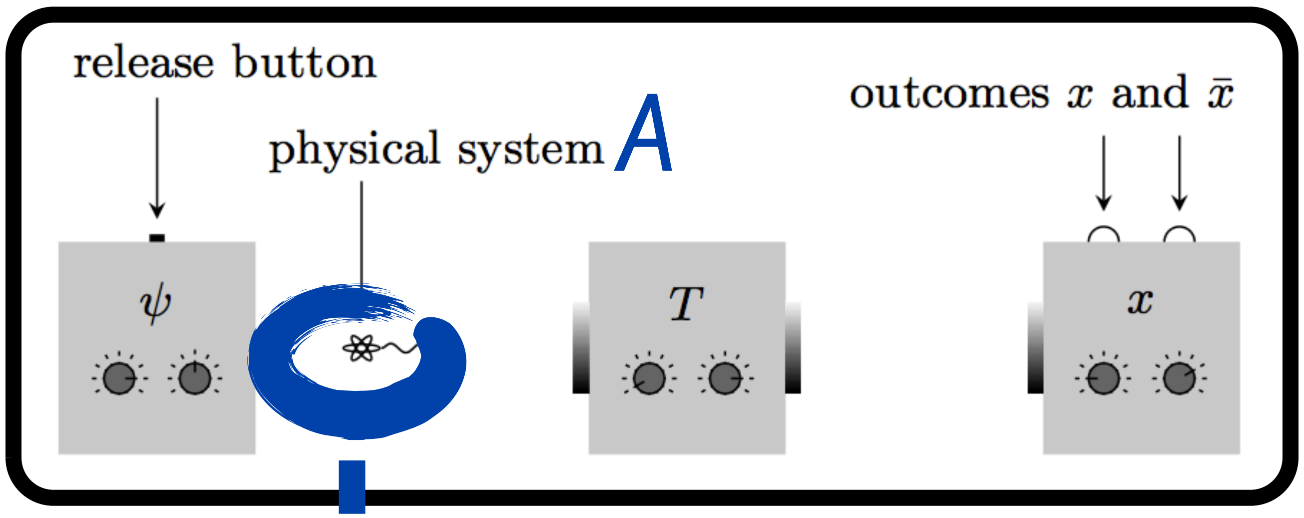
Alice

state on AB:
correlations

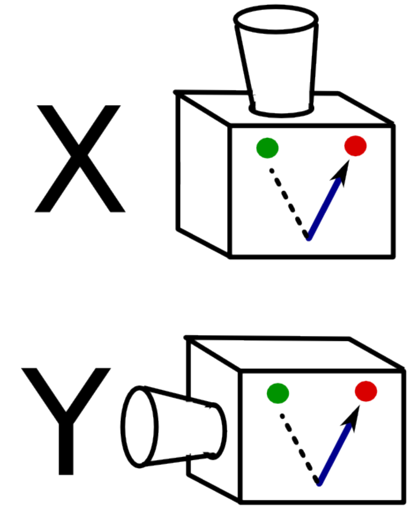


Bob


release button
physical system A
outcomes x and $\bar{x}$
ψ
T
x
Alice
X
Y
state on AB:
correlations
release button
physical system B
outcomes x and $\bar{x}$
ψ
T
x
Bob
X
Y

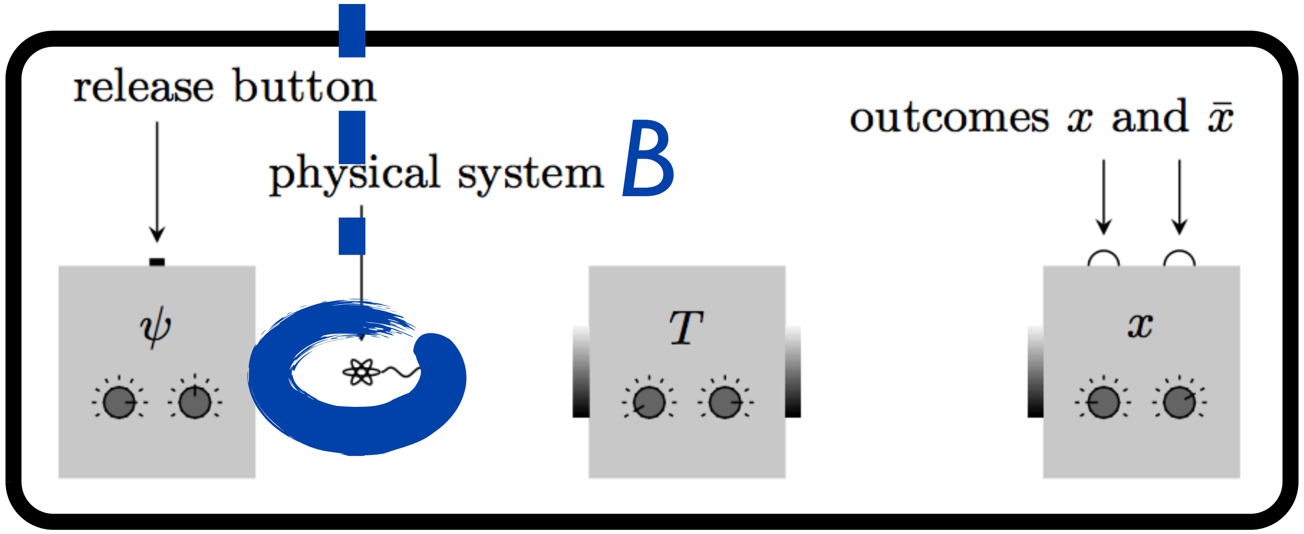


Alice

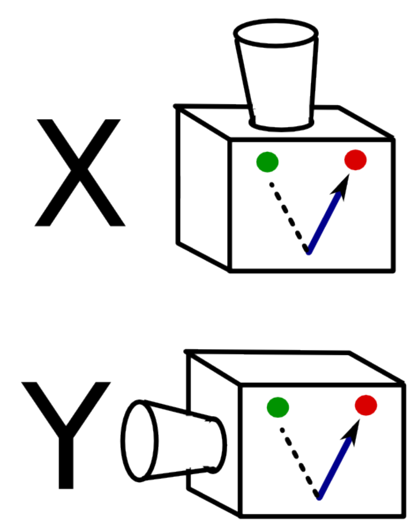


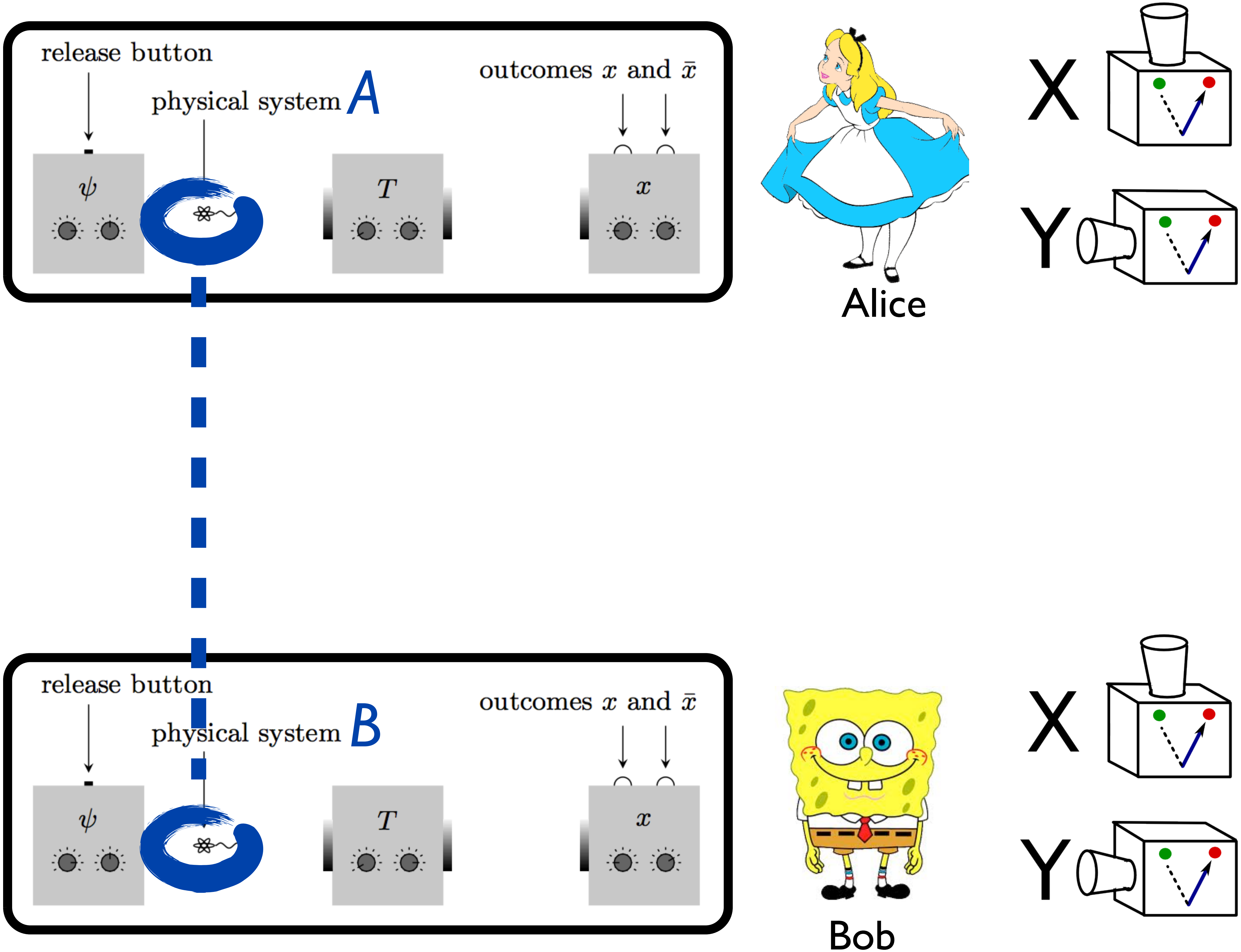
state on AB:
correlations

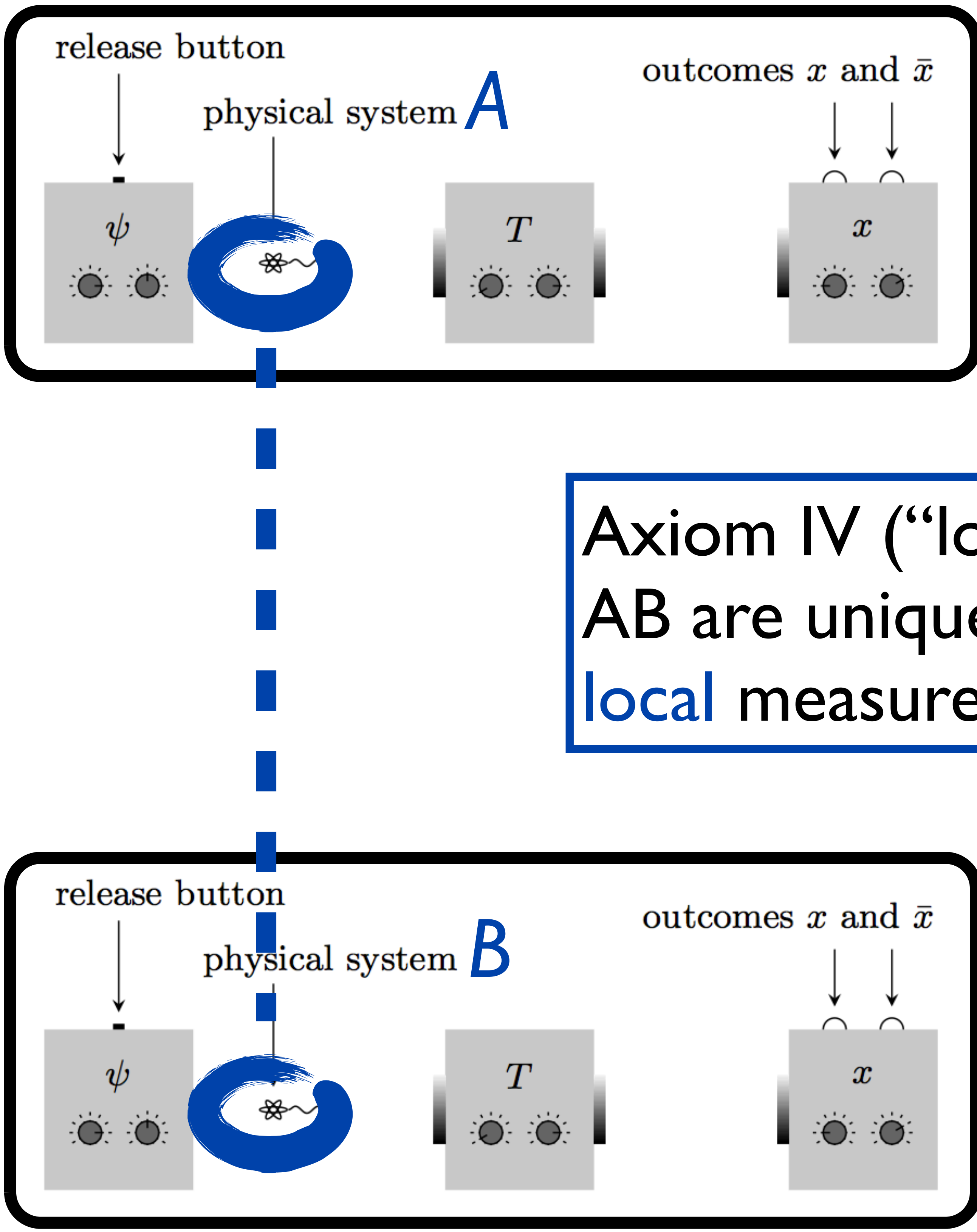
No-signalling condition:
Alice's probabilities do not depend on Bob's choice of measurement.



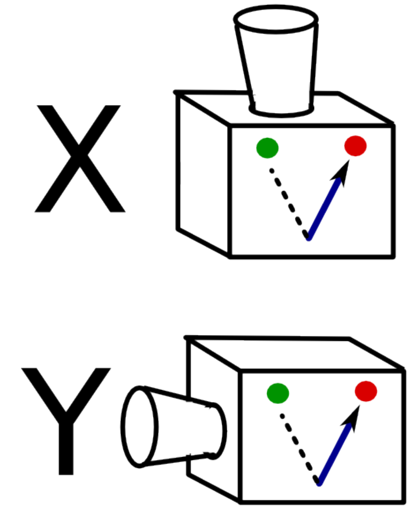
Bob




release button
physical system A
outcomes x and $\bar{x}$
ψ
T
x
Alice
X
Y
release button
physical system B
outcomes x and $\bar{x}$
ψ
T
x
Bob
X
Y

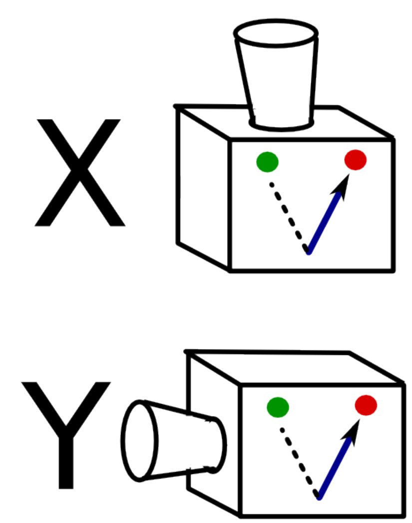


Alice

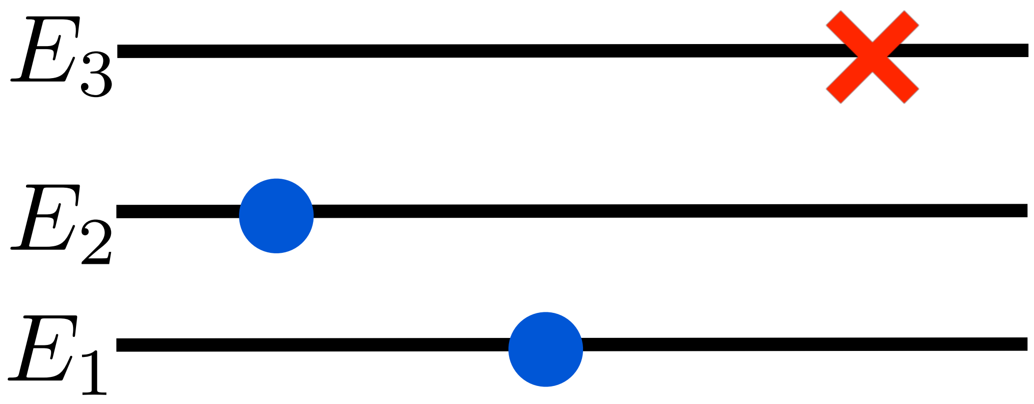


Axiom IV (“local tomography“): States on AB are uniquely determined by statistics of local measurements.

Bob

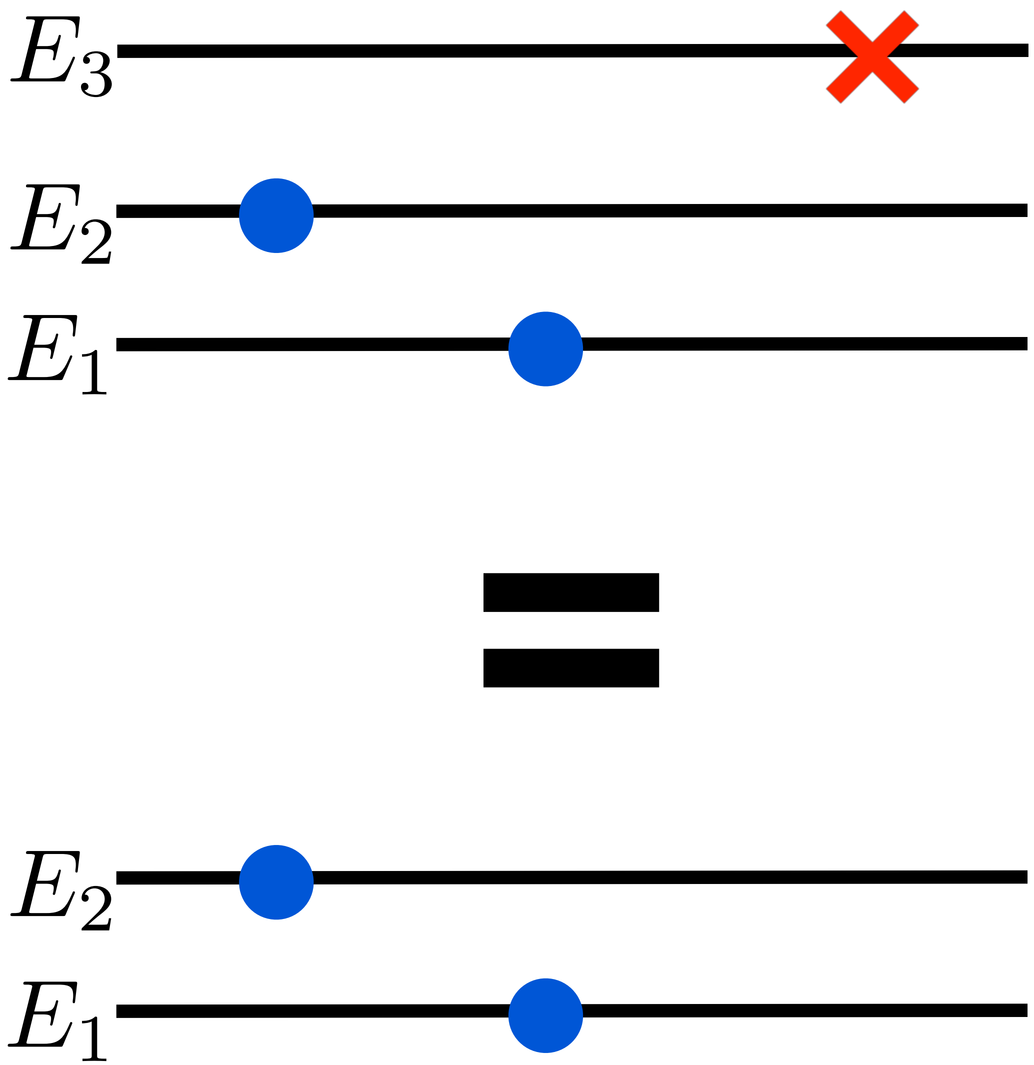


Some 3-level system:



Impossible to put system in 3rd level
⇒ find particle there with probab. 0

Some 3-level system:



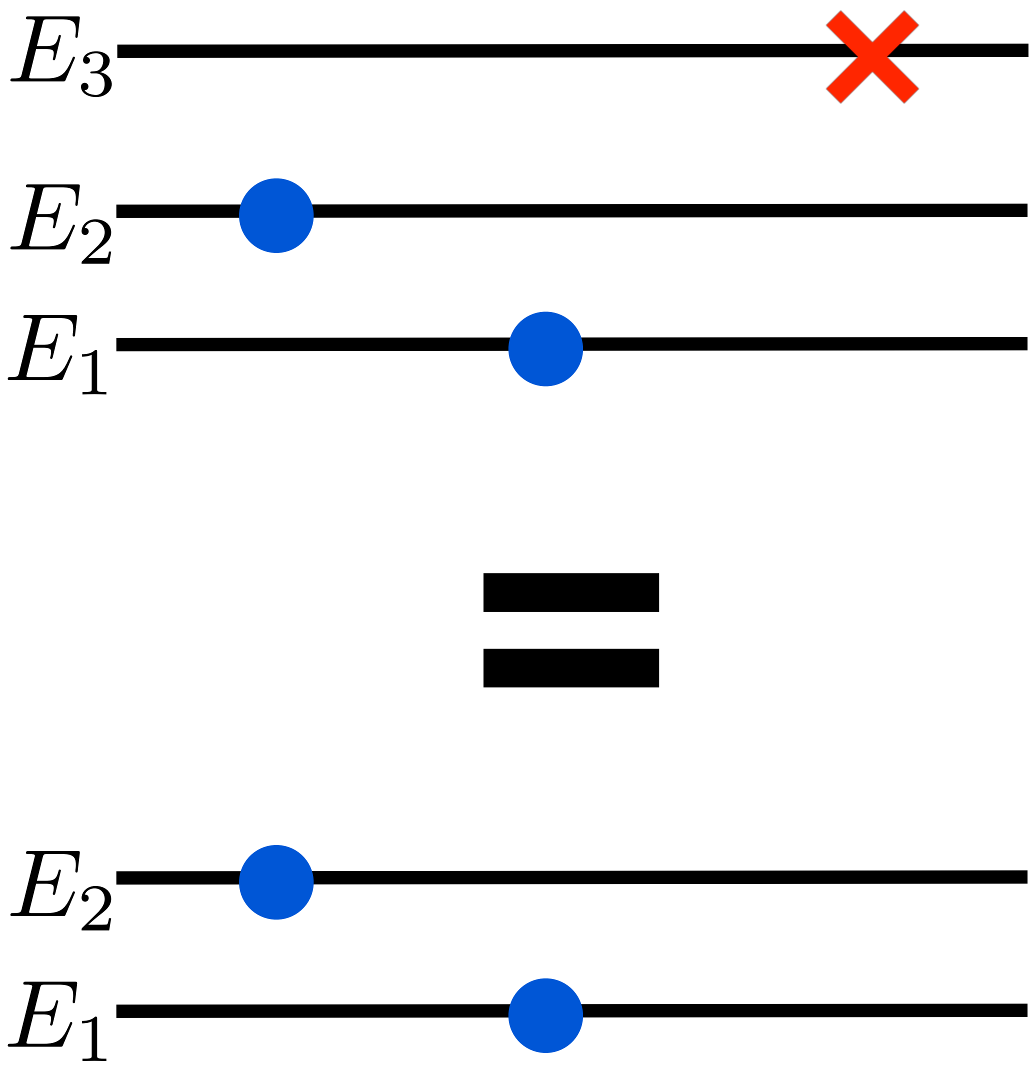
2-level system.

Impossible to put system in 3rd level
$\Rightarrow$ find particle there with probab. 0

QT: $\rho^{(3)} = \begin{pmatrix} \bullet & \bullet & 0 \\ \bullet & \bullet & 0 \\ 0 & 0 & 0 \end{pmatrix} \longrightarrow \rho^{(2)} = \begin{pmatrix} \bullet & \bullet \\ \bullet & \bullet \end{pmatrix}$

CPT: $P^{(3)} = (P_1, P_2, 0) \longrightarrow P^{(2)} = (P_1, P_2)$

Some 3-level system:



2-level system.

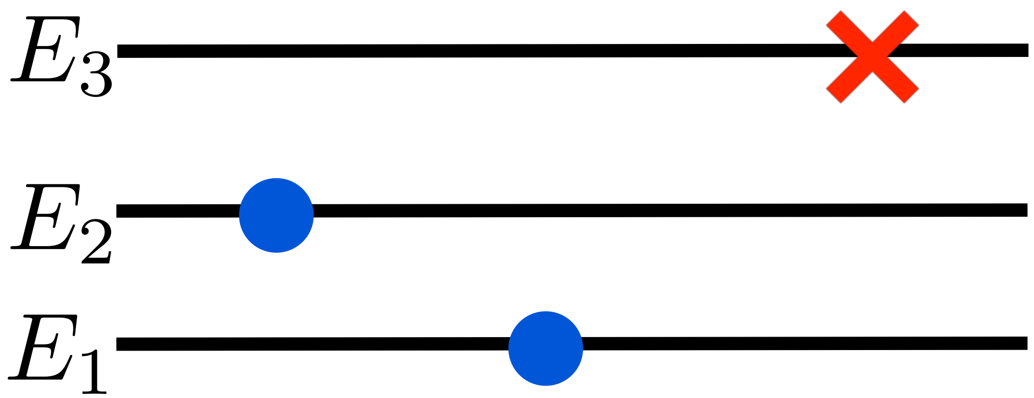
Impossible to put system in 3rd level
⇒ find particle there with probab. 0

QT: $\rho^{(3)} = \begin{pmatrix} \bullet & \bullet & 0 \\ \bullet & \bullet & 0 \\ 0 & 0 & 0 \end{pmatrix} \longrightarrow \rho^{(2)} = \begin{pmatrix} \bullet & \bullet \\ \bullet & \bullet \end{pmatrix}$

CPT: $P^{(3)} = (P_1, P_2, 0) \longrightarrow P^{(2)} = (P_1, P_2)$

Axiom V: Subset of an *N*-outcome state space with $P_N=0$ is equivalent to *(N-1)*-outcome state space.

Some 3-level system:

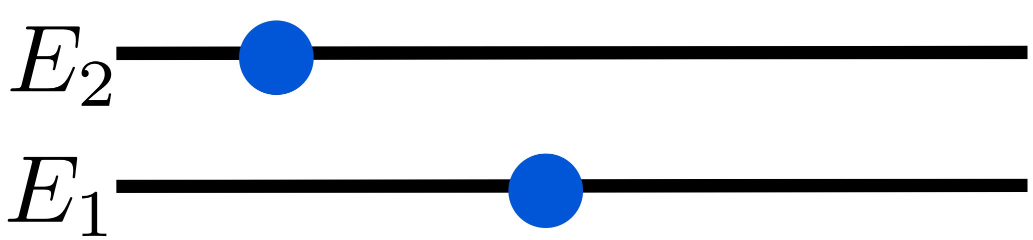


Impossible to put system in 3rd level
⇒ find particle there with probab. 0

=

QT: $\rho^{(3)} = \begin{pmatrix} \bullet & \bullet & 0 \\ \bullet & \bullet & 0 \\ 0 & 0 & 0 \end{pmatrix} \longrightarrow \rho^{(2)} = \begin{pmatrix} \bullet & \bullet \\ \bullet & \bullet \end{pmatrix}$

CPT: $P^{(3)} = (P_1, P_2, 0) \longrightarrow P^{(2)} = (P_1, P_2)$



2-level system.

"Subspace Axiom"

Axiom V: Subset of an *N*-outcome state space with $P_N=0$ is equivalent to *(N-1)*-outcome state space.

Ll. Masanes, MM, New J Phys. 13, 063001 (2011):

I. All state spaces finite-dimensional
II. No additional restrictions on measurements
III. Reversibility
IV. Local tomography
V. Subspace axiom

Thm.: CPT and QT are the only probabilistic theories satisfying Axioms I-V.

Ll. Masanes, MM, New J Phys. 13, 063001 (2011):

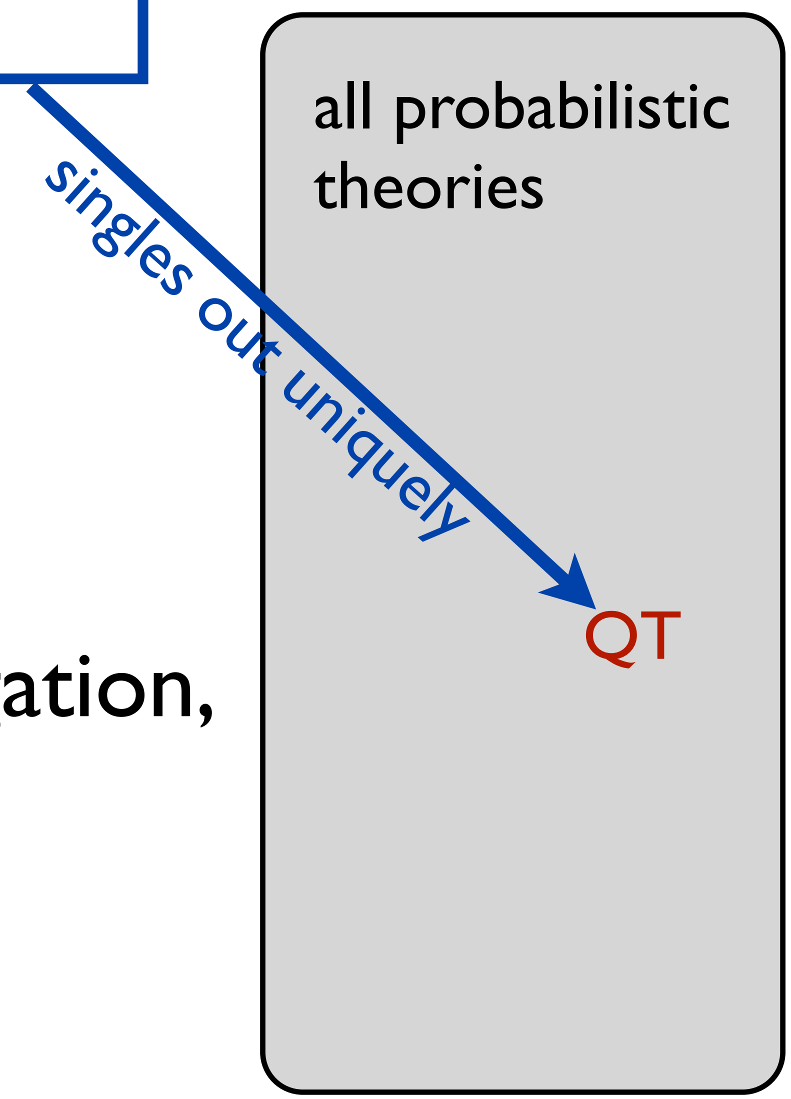
I. All state spaces finite-dimensional
II. No additional restrictions on measurements
III. Continuous reversibility
IV. Local tomography
V. Subspace axiom

Theorem: Every theory satisfying Axioms I-V is equivalent to $(\Omega_N, \mathcal{G}_N)$, where

- Ω_N are the density matrices on $\mathbb{C}^N$,
- $\mathcal{G}_N$ is the group of unitaries, acting by conjugation,
- the measurements are exactly the POVMs.

Ll. Masanes, MM, New J Phys. 13, 063001 (2011):

I. All state spaces finite-dimensional
II. No additional restrictions on measurements
III. Continuous reversibility
IV. Local tomography
V. Subspace axiom



Theorem: Every theory satisfying Axioms I-V is equivalent to $(\Omega_N, \mathcal{G}_N)$, where

- Ω_N are the density matrices on $\mathbb{C}^N$,
- $\mathcal{G}_N$ is the group of unitaries, acting by conjugation,
- the measurements are exactly the POVMs.

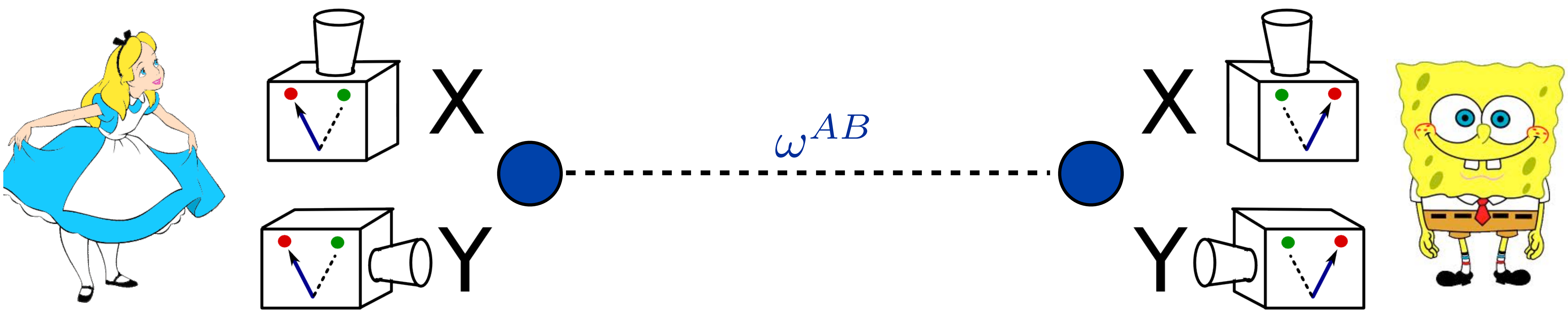
Reversibility alone as a powerful axiom

Reversibility alone as a powerful axiom

No reversible dynamics in boxworld:

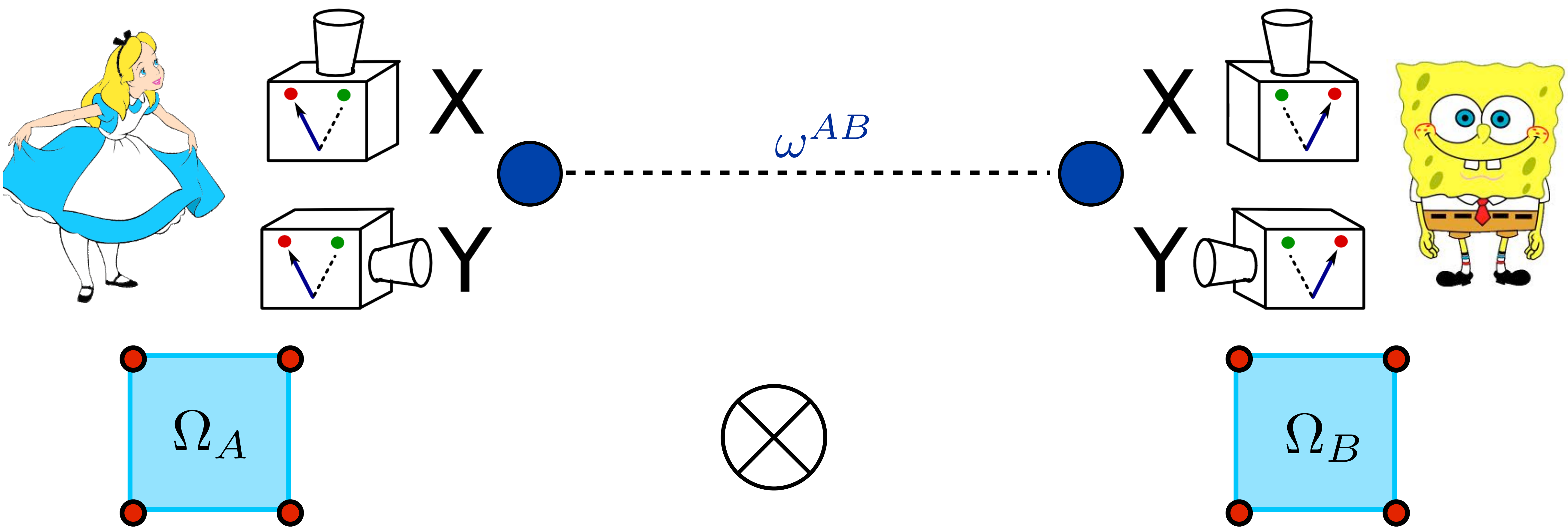
Reversibility alone as a powerful axiom

No reversible dynamics in boxworld:



Reversibility alone as a powerful axiom

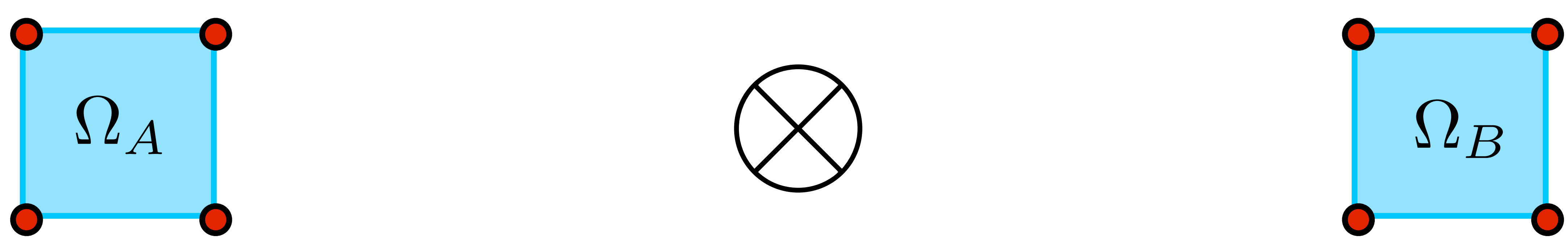
$$\omega^{AB} = \begin{pmatrix} \vdots \\ \text{Prob}(\text{green}\bullet\,\text{red}\bullet \mid XX) \\ \vdots \\ \text{Prob}(\text{red}\bullet\,\text{red}\bullet \mid YX) \\ \vdots \end{pmatrix}$$



Reversibility alone as a powerful axiom

$$\omega^{AB} = \begin{pmatrix} \vdots \\ \mathrm{Prob}(\bullet\bullet | XX) \\ \vdots \\ \mathrm{Prob}(\bullet\bullet | YX) \\ \vdots \end{pmatrix}$$

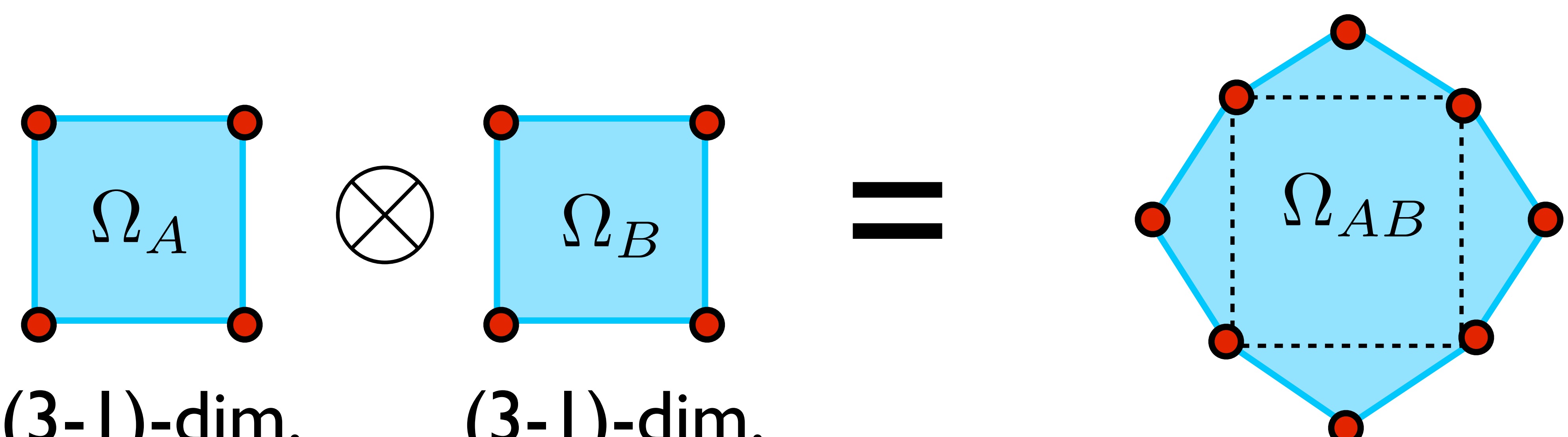
$\Omega_{AB} :=$ all distributions satisfying no-signalling ("boxworld")



Reversibility alone as a powerful axiom

$$\omega^{AB} = \begin{pmatrix} \vdots \\ \mathrm{Prob}(\bullet\bullet|XX) \\ \vdots \\ \mathrm{Prob}(\bullet\bullet|YX) \\ \vdots \end{pmatrix}$$

$\Omega_{AB} :=$ all distributions satisfying no-signalling ("boxworld")

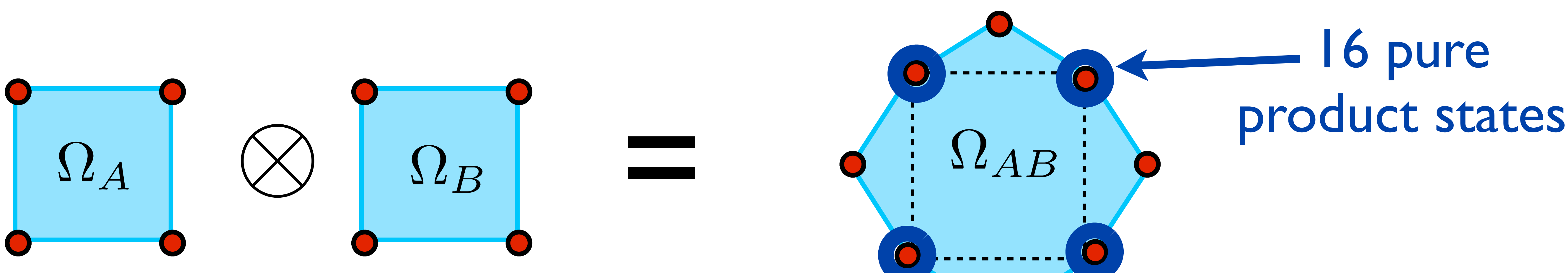


(3-1)-dim. (3-1)-dim. (9-1)-dim. no-signalling polytope

Reversibility alone as a powerful axiom

$$\omega^{AB} = \begin{pmatrix} \vdots \\ \mathrm{Prob}(\bullet\bullet|XX) \\ \vdots \\ \mathrm{Prob}(\bullet\bullet|YX) \\ \vdots \end{pmatrix}$$

Ω_{AB} := all distributions satisfying no-signalling ("boxworld")



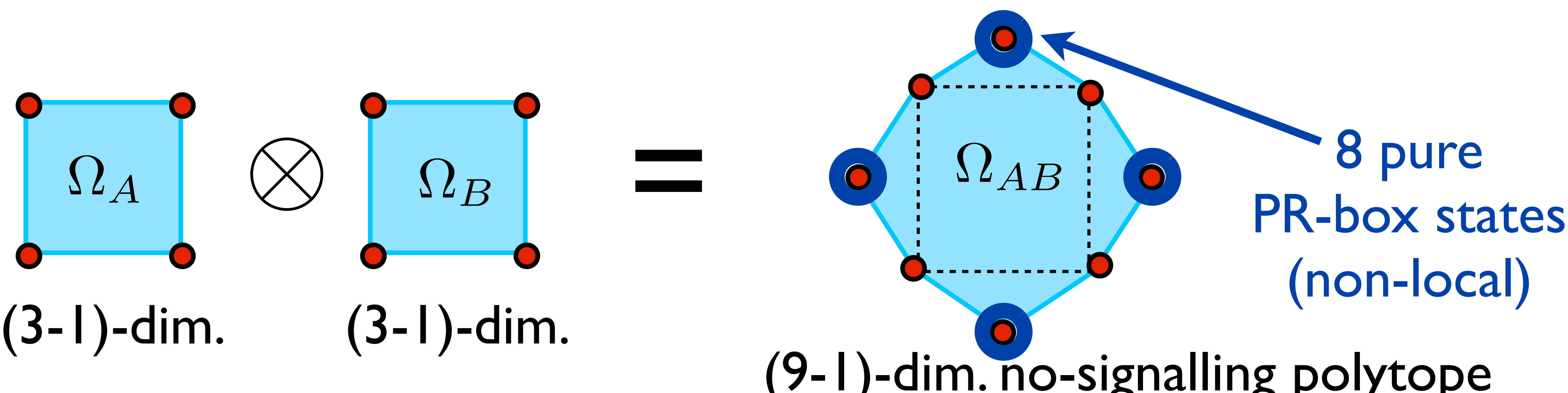
(3-1)-dim. (3-1)-dim.

(9-1)-dim. no-signalling polytope

Reversibility alone as a powerful axiom

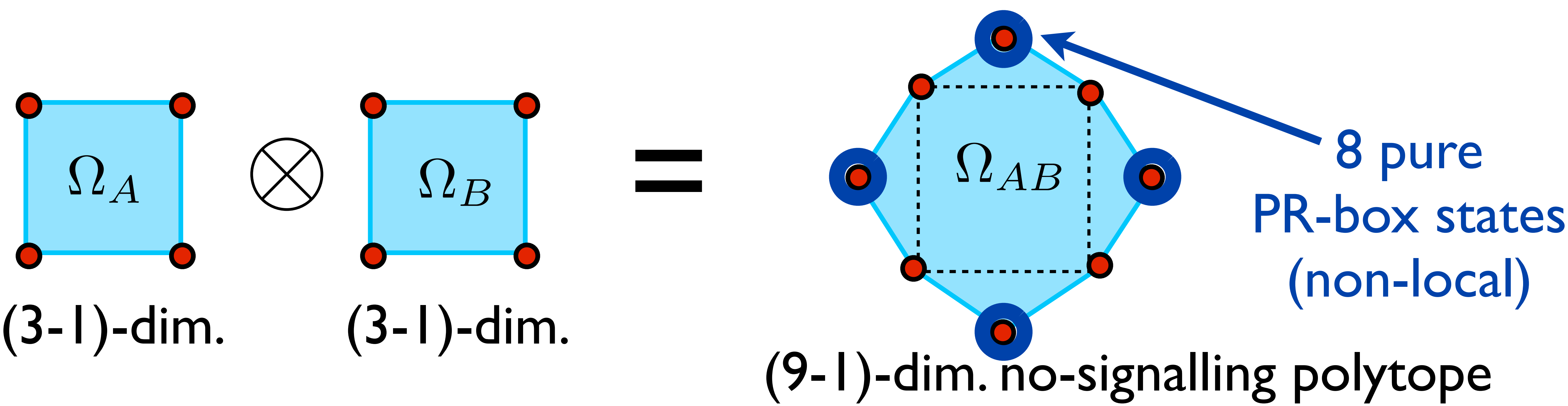
$$\omega^{AB} = \begin{pmatrix} \vdots \\ \text{Prob}(\bullet\bullet|XX) \\ \vdots \\ \text{Prob}(\bullet\bullet|YX) \\ \vdots \end{pmatrix}$$

$\Omega_{AB} :=$ all distributions satisfying no-signalling ("boxworld")



Reversibility alone as a powerful axiom

No reversible transformation can map a product state to a PR-box state.



Reversibility alone as a powerful axiom

D. Gross, MM, R. Colbeck, O. Dahlsten, PRL 104, 080402 (2010):

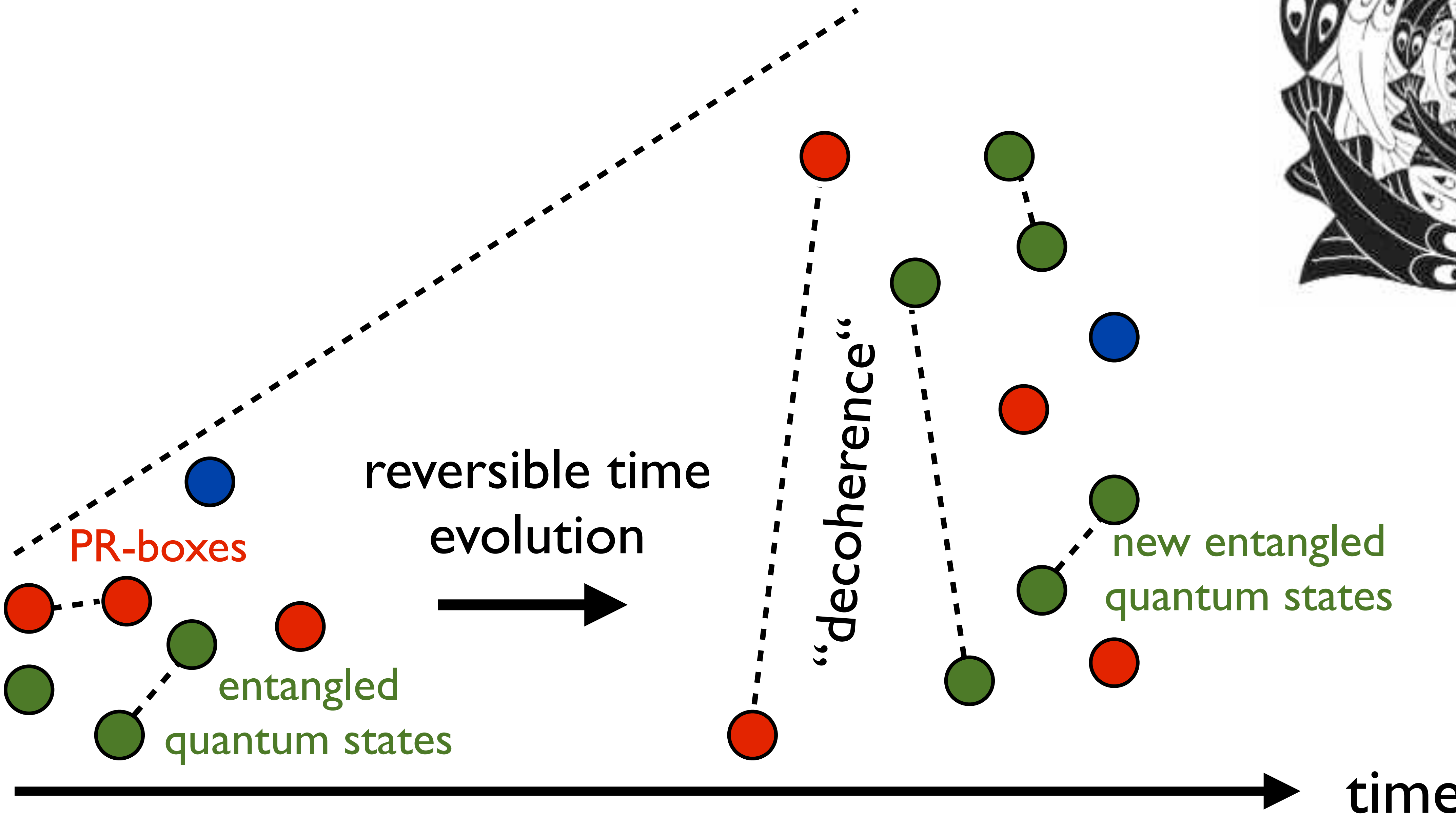
Reversibility alone as a powerful axiom

D. Gross, MM, R. Colbeck, O. Dahlsten, PRL 104, 080402 (2010):

Thm.: For any number of parties, $M \geq 2$ measurements, and outcomes, the only reversible transformations in boxworld are
- local relabellings, and
- permutations of subsystems.

Thm.: Even if all parameters vary arbitrarily from site to site, no reversible transformation can map product states to entangled states.

Reversibility alone as a powerful axiom



PR-boxes get lost over time...

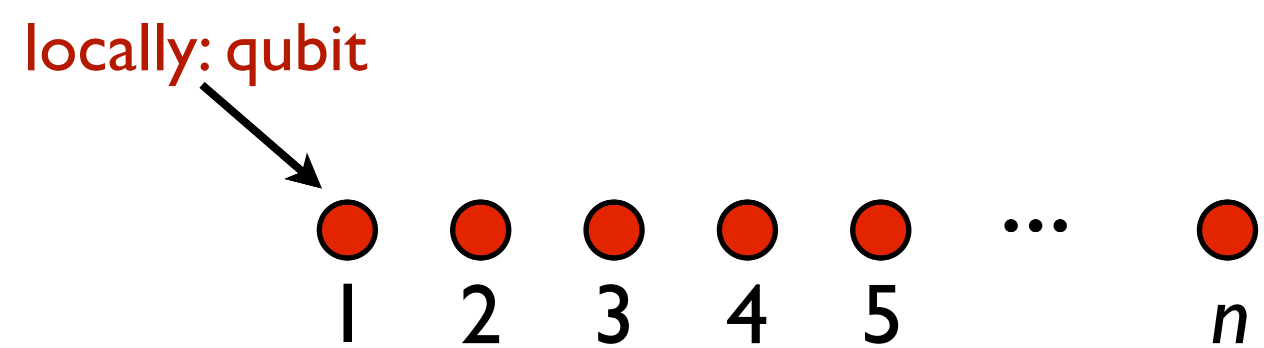
Does locally quantum $\overset{?}{\Longrightarrow}$ globally quantum?

$$\text{Does locally quantum} \stackrel{?}{\Longrightarrow} \text{globally quantum?}$$

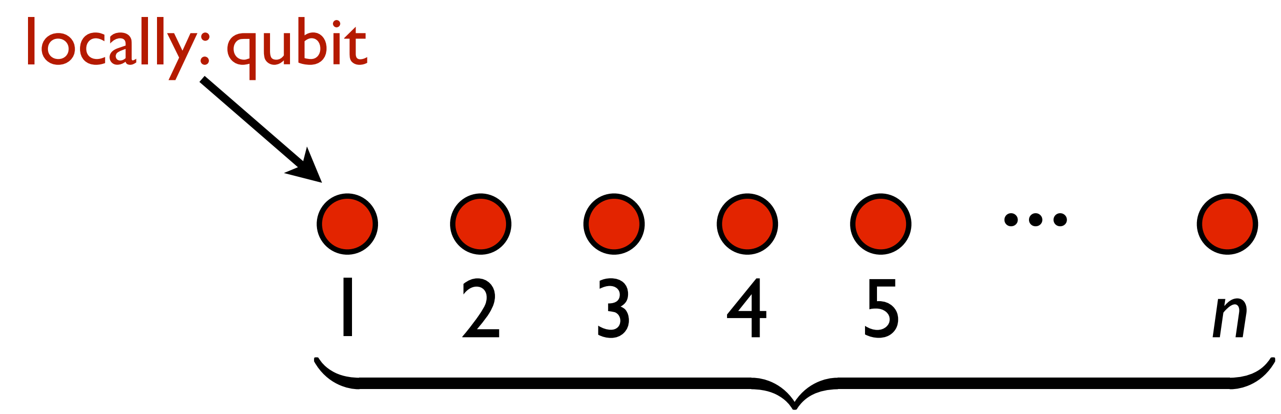
- Barnum, Beigi, Boixo, Elliott, Wehner, PRL **104**, 140401 (2010):
If *A* and *B* are quantum systems, and *AB* any composition, then all correlations on *AB* are quantum correlations.

- Acin, Augusiak, Cavalcanti, Hadley, Korbidcz, Lewenstein, Masanes, Piani, PRL **104**, 140404 (2010):
There are quantum systems *A*, *B*, *C* and a composition *ABC* which contains post-quantum correlations (not allowed in QT).

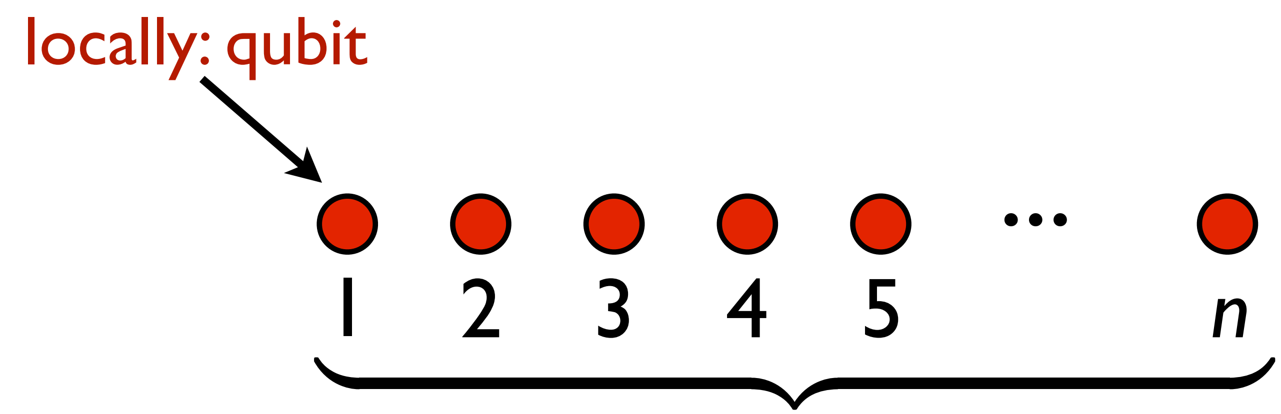
Does locally quantum $\stackrel{?}{\Longrightarrow}$ globally quantum?



Does locally quantum $\stackrel{?}{\Longrightarrow}$ globally quantum?



“all systems the same“:
state space is permutation-invariant

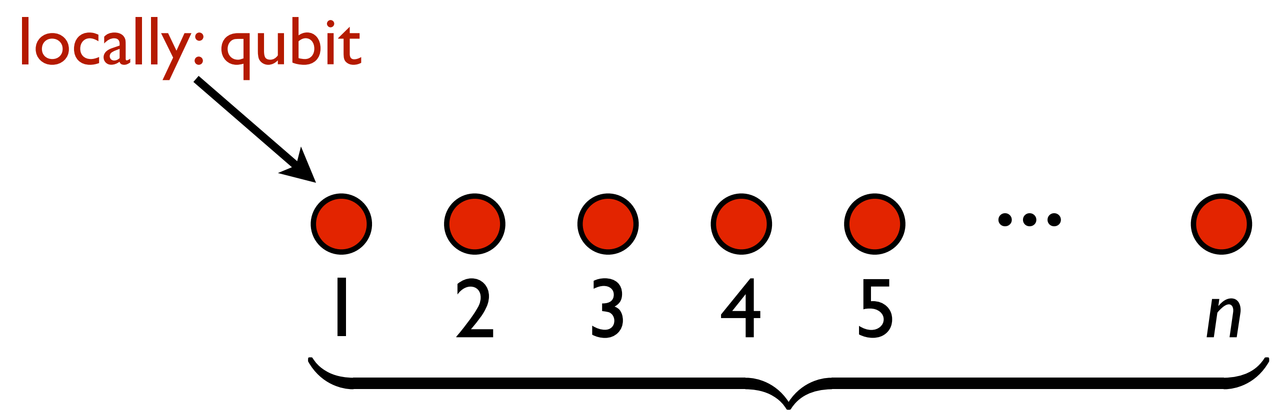


"all systems the same":
state space is permutation-invariant

de la Torre, Masanes, Short, MM, arXiv: 1110.5482:

Thm.: Consider any locally-tomographic theory in which the individual systems are identical qubits. If the theory admits at least one continuous reversible interaction between systems, then the allowed states, measurements, and transformations must be exactly those of quantum theory.

Fundamental failure of QT on "large scales" a bit more unlikely.



"all systems the same":
state space is permutation-invariant

de la Torre, Masanes, Short, MM, arXiv: 1110.5482:

Thm.: Consider any locally-tomographic theory in which the individual systems are identical qubits. If the theory admits at least one continuous reversible interaction between systems, then the allowed states, measurements, and transformations must be exactly those of quantum theory.

Understanding strong self-duality from reversible dynamics:

Understanding strong self-duality from reversible dynamics:

- Usually, outcome probabilities are computed as

$$E(\omega) \in [0,1],$$

where $E : A \to \mathbb{R}$ is an "effect", and $\omega \in \Omega_A \subset A$ a state.

Understanding strong self-duality from reversible dynamics:

- Usually, outcome probabilities are computed as

$$E(\omega) \in [0,1],$$

where $E : A \to \mathbb{R}$ is an "effect", and $\omega \in \Omega_A \subset A$ a state.

- Dropping normalization:

$$\begin{aligned} S_A &= \{\lambda \cdot \omega \mid \omega \in \Omega_A, \lambda \geq 0\}, \\ E_A &= \{E : A \to \mathbb{R} \text{ linear} \mid E(\omega) \geq 0 \ \forall \omega\}. \end{aligned}$$

Understanding strong self-duality from reversible dynamics:

- Usually, outcome probabilities are computed as

$$E(\omega) \in [0,1],$$

where $E : A \to \mathbb{R}$ is an "effect", and $\omega \in \Omega_A \subset A$ a state.

- Dropping normalization:

$$\begin{aligned} S_A &= \{\lambda \cdot \omega \mid \omega \in \Omega_A, \lambda \geq 0\}, \\ E_A &= \{E : A \to \mathbb{R} \text{ linear} \mid E(\omega) \geq 0 \;\; \forall \omega\}. \end{aligned}$$

- In QT: $$\begin{aligned} S_A &= \{\rho \mid \rho \geq 0\}, \\ E_A &= \{\rho \mapsto \mathrm{Tr}(\rho P) \mid P \geq 0\} \simeq S_A. \end{aligned}$$

QT is self-dual!

Understanding strong self-duality from reversible dynamics:

- Usually, outcome probabilities are computed as

$$E(\omega) \in [0,1],$$

where $E : A \to \mathbb{R}$ is an "effect", and $\omega \in \Omega_A \subset A$ a state.

- Dropping normalization:

$$\begin{aligned} S_A &= \{\lambda \cdot \omega \mid \omega \in \Omega_A, \lambda \geq 0\}, \\ E_A &= \{E : A \to \mathbb{R} \text{ linear} \mid E(\omega) \geq 0 \ \forall \omega\}. \end{aligned}$$

- In QT: $$\begin{aligned} S_A &= \{\rho \mid \rho \geq 0\}, \\ E_A &= \{\rho \mapsto \langle \rho, P \rangle \mid P \geq 0\} \simeq S_A. \end{aligned}$$

QT is self-dual! $\langle \rho, P \rangle := \mathrm{Tr}(\rho P).$

Def.: If there is an inner product such that the effects are

$$E = \{\omega \mapsto \langle \varphi, \omega \rangle \mid \varphi \in S\} \simeq S,$$

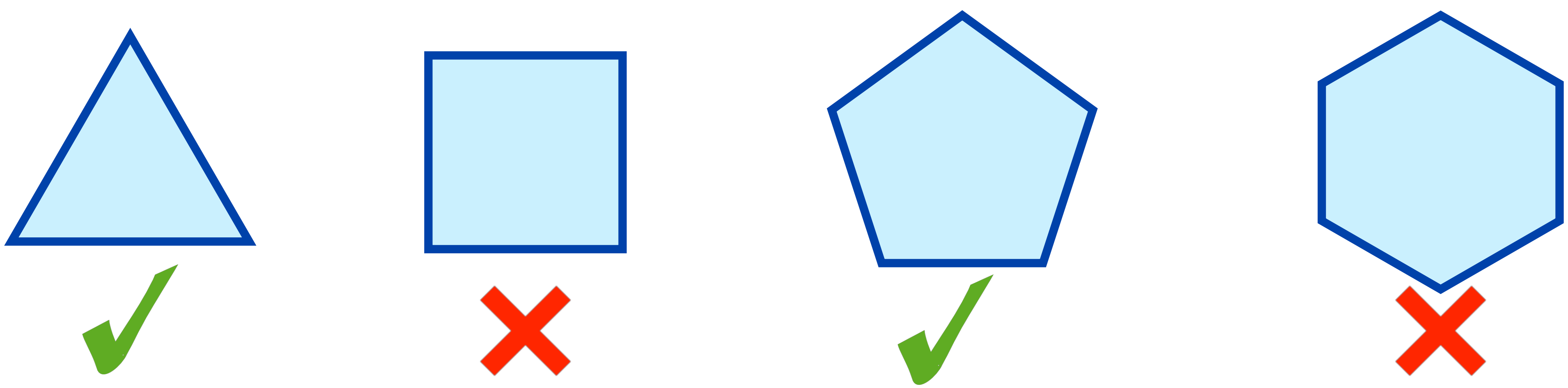
then the state space is strongly self-dual.

Def.: If there is an inner product such that the effects are

$$E = \{\omega \mapsto \langle \varphi, \omega \rangle \mid \varphi \in S\} \simeq S,$$

then the state space is strongly self-dual.

- "Most" state spaces are *not* strongly self-dual. In 2D, regular *n*-gons are strongly self-dual iff *n* is odd:

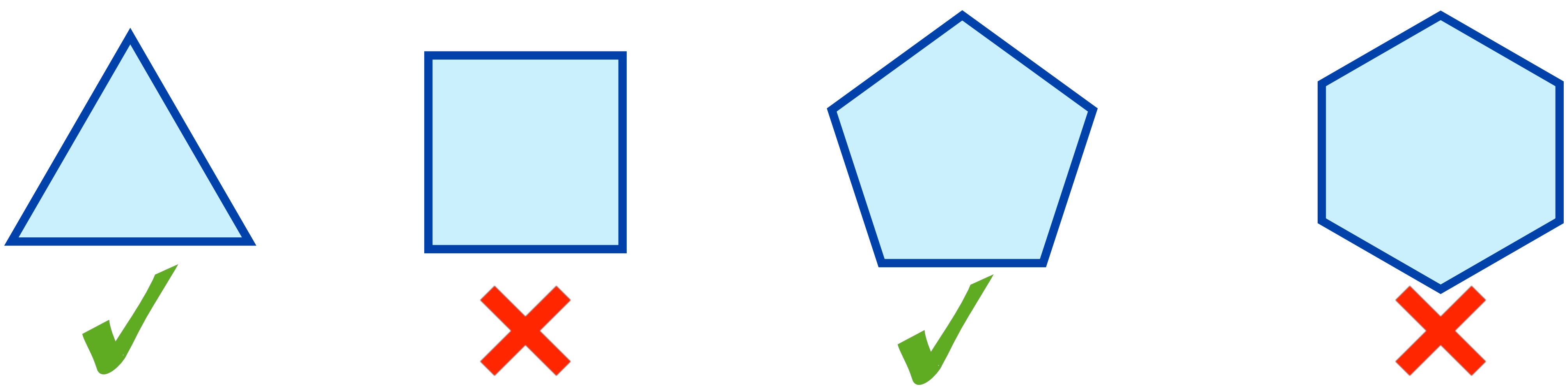


Def.: If there is an inner product such that the effects are

$$E = \{\omega \mapsto \langle \varphi, \omega \rangle \mid \varphi \in S\} \simeq S,$$

then the state space is strongly self-dual.

- "Most" state spaces are *not* strongly self-dual. In 2D, regular *n*-gons are strongly self-dual iff *n* is odd:



- H. Barnum and A. Wilce: several operational approaches to strong self-duality, cf. A. Wilce, arXiv:1110.6607
  H. Barnum, R. Duncan, A. Wilce, arXiv:1004.2920.

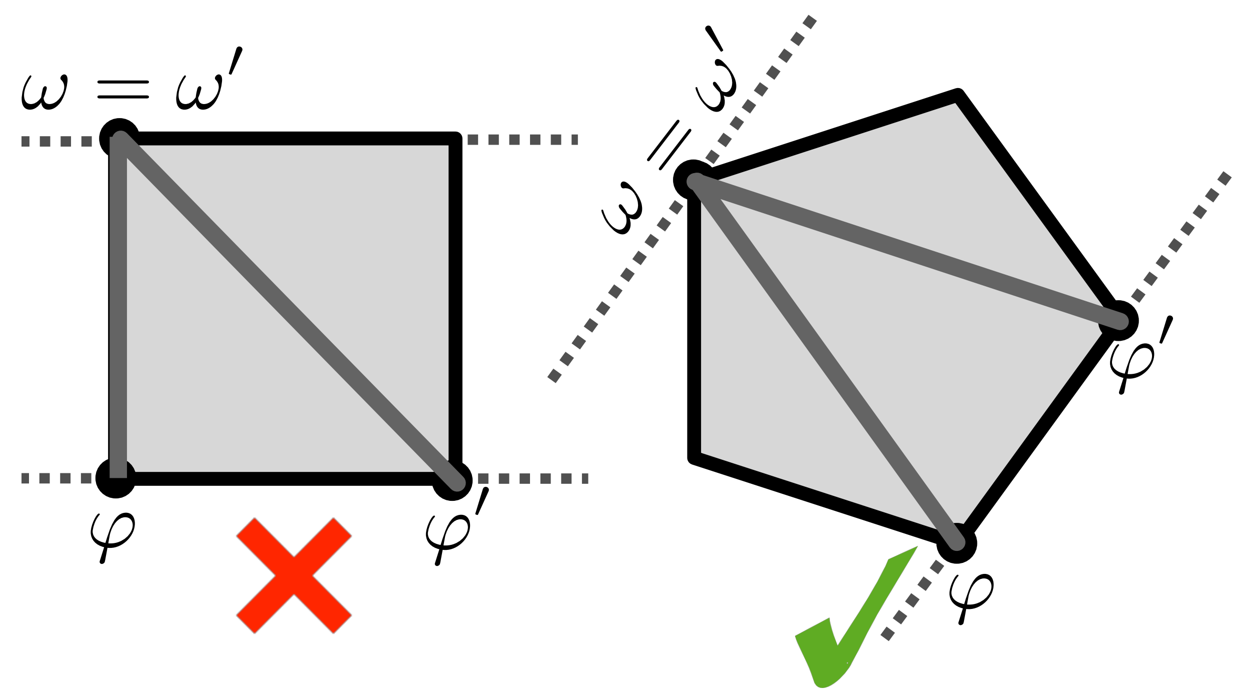
MM, C. Ududec, arXiv: 1110.3516:

Thm.: If a theory is bit-symmetric, then it is strongly self-dual. Moreover, inner product can be chosen invariant, non-negative on states, $\langle\omega,\omega\rangle=1$ iff ω is pure, and $\langle\varphi,\omega\rangle=0$ if φ and ω are perfectly distinguishable.

MM, C. Ududec, arXiv: 1110.3516:

Thm.: If a theory is bit-symmetric, then it is strongly self-dual. Moreover, inner product can be chosen invariant, non-negative on states, $\langle\omega,\omega\rangle = 1$ iff ω is pure, and $\langle\varphi,\omega\rangle = 0$ if φ and ω are perfectly distinguishable.

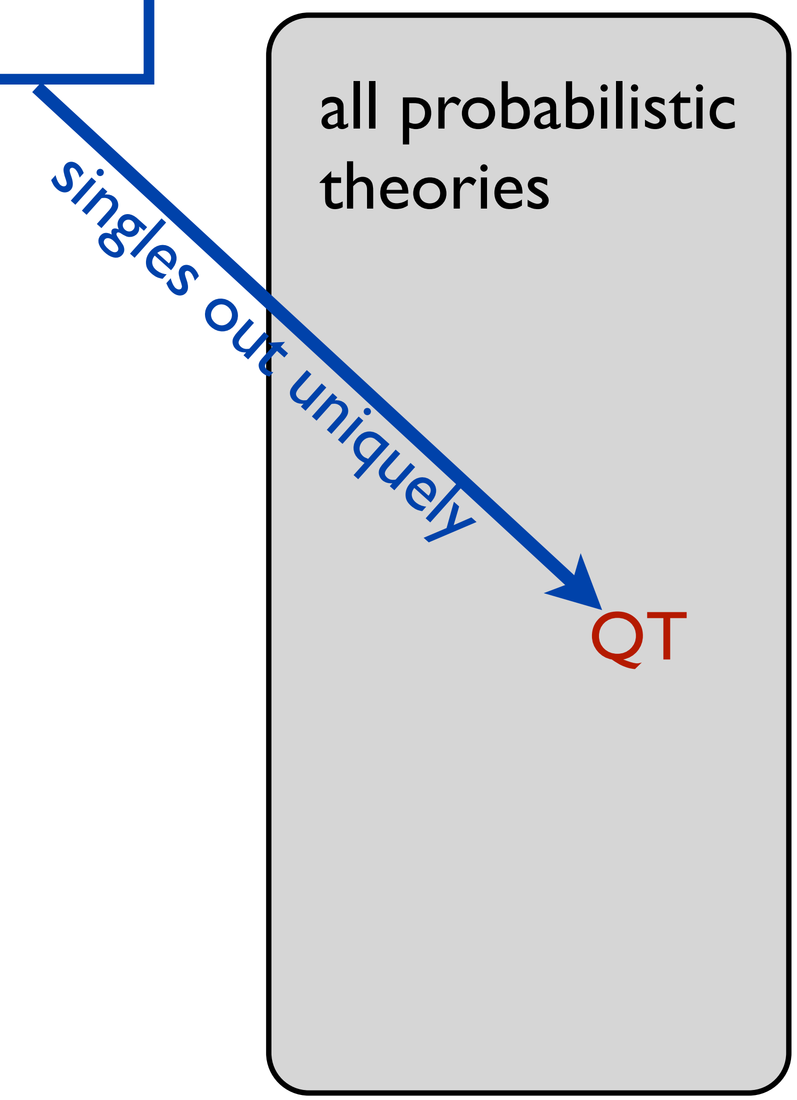
Bit symmetry: If ω,φ are perfectly distinguishable pure states, and so are ω',φ', then there is a reversible transformation T such that $T\omega=\omega'$ and $T\varphi=\varphi'$.



QT's closest cousins?

Ll. Masanes, MM, New J Phys. 13, 063001 (2011):

I. All state spaces finite-dimensional
II. No additional restrictions on measurements
III. Continuous reversibility
IV. Local tomography
V. Subspace axiom



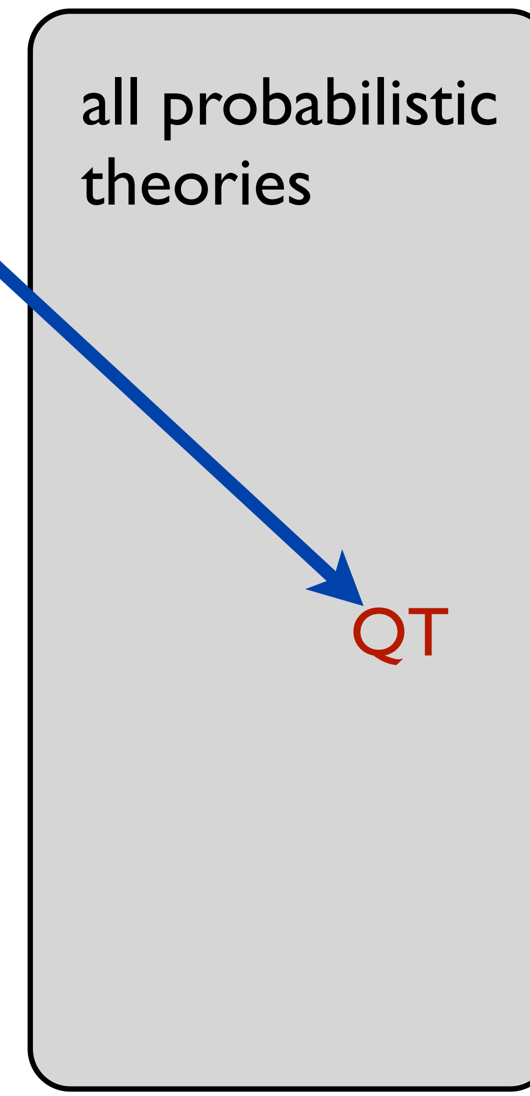
I. All state spaces finite-dimensional

II. No additional restrictions on measurements

III. Continuous reversibility

IV. Local tomography

~~V. Subspace axiom~~



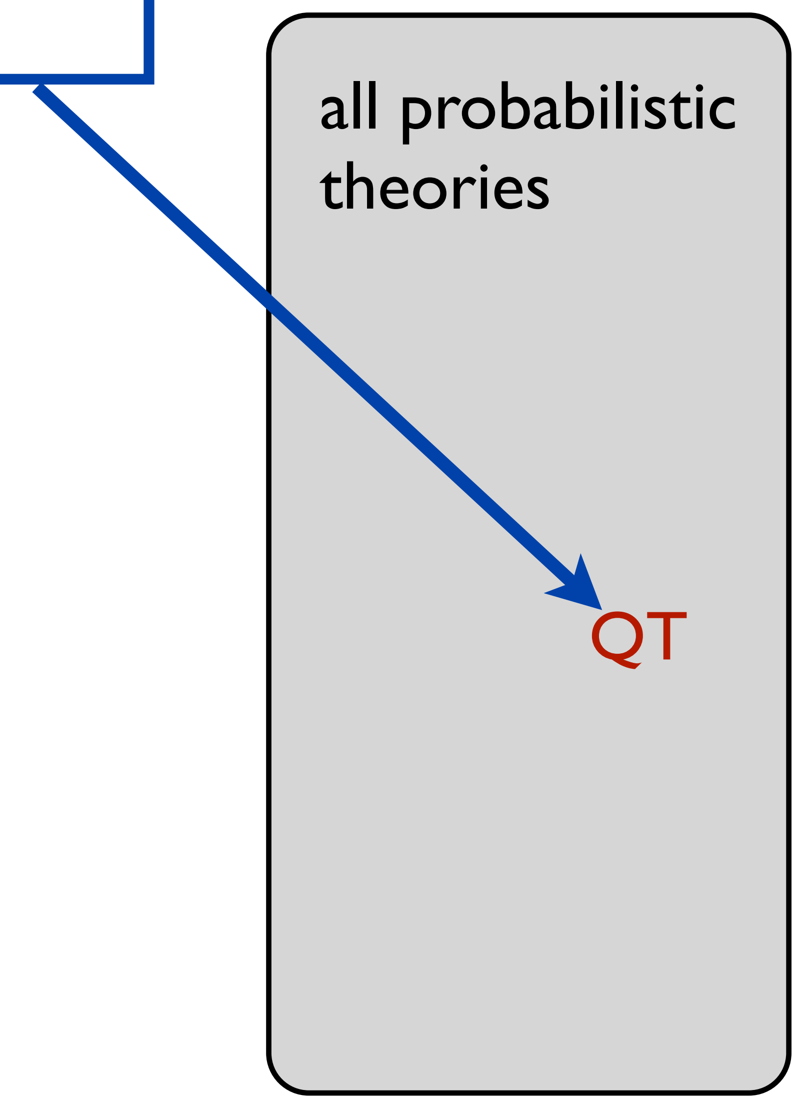
I. All state spaces finite-dimensional

~~II. No additional restrictions on measurements~~

III. Continuous reversibility

IV. Local tomography

~~V. Subspace axiom~~



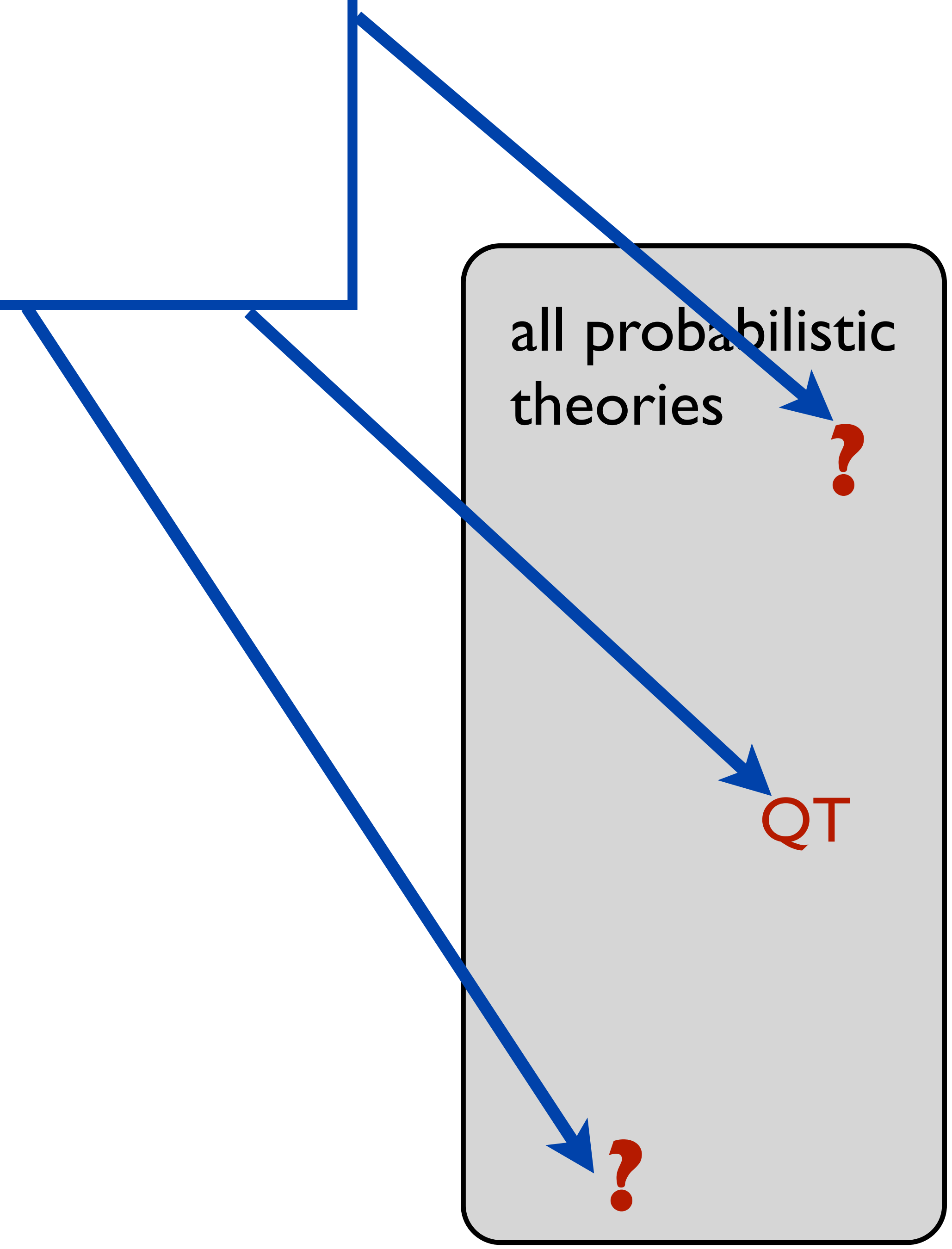
I. All state spaces finite-dimensional

~~II. No additional restrictions on measurements~~

III. Continuous reversibility

IV. Local tomography

~~V. Subspace axiom~~



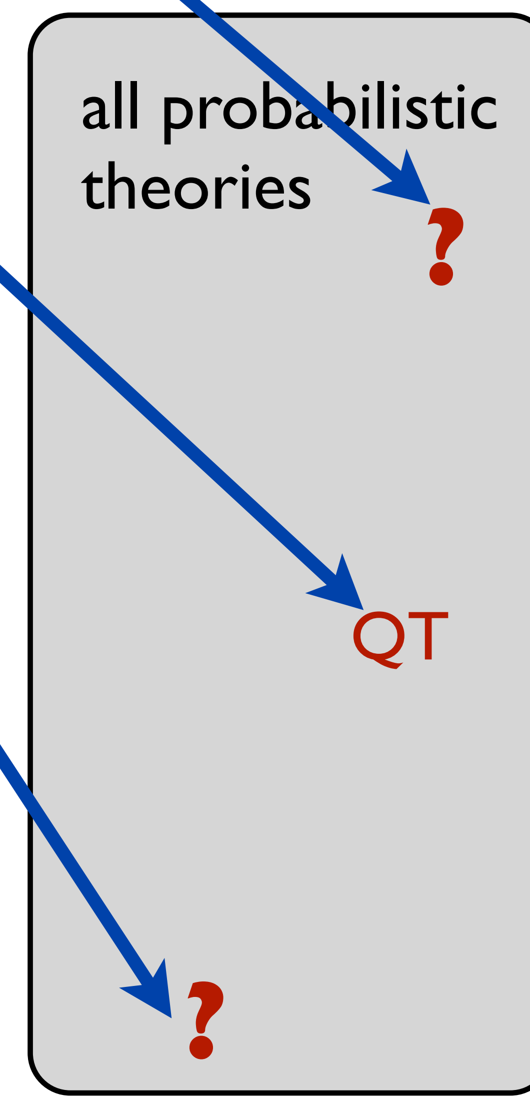
I. All state spaces finite-dimensional

~~II. No additional restrictions on measurements~~

III. Continuous reversibility

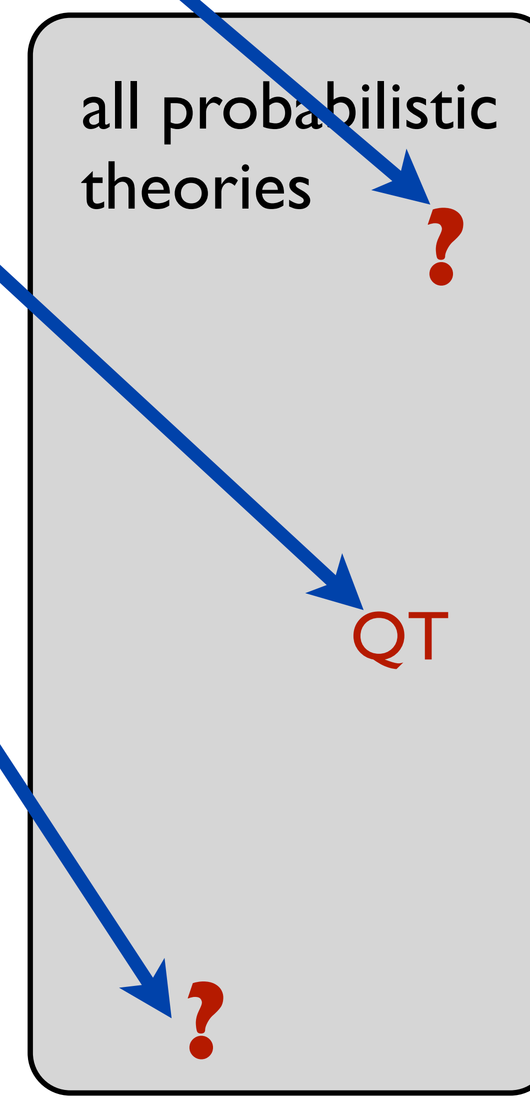
IV. Local tomography

~~V. Subspace axiom~~



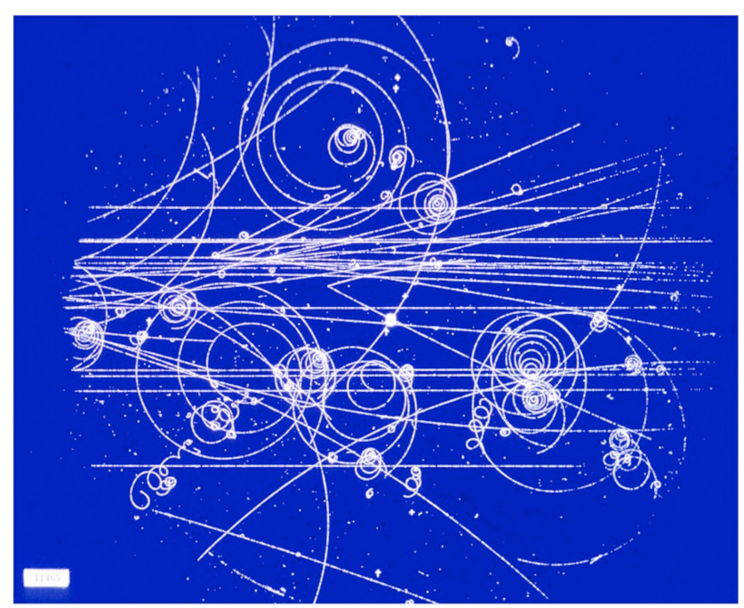
- If QT remains the only solution: exciting!
- Otherwise: find a few “closest cousins“ of QT.
  Even more exciting!

I. All state spaces finite-dimensional
~~II. No additional restrictions on measurements~~
III. Continuous reversibility
IV. Local tomography
~~V. Subspace axiom~~



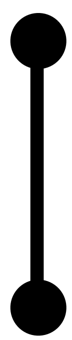
- If QT remains the only solution: exciting!
- Otherwise: find a few “closest cousins“ of QT.
  Even more exciting!

- Like in particle physics, group theory leaves only “few“ possibilities.

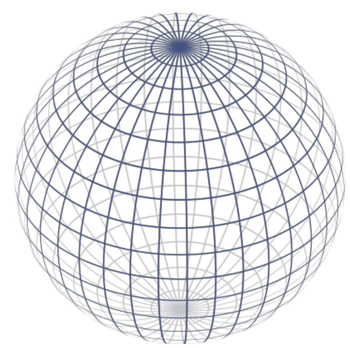


Ll. Masanes, MM, D. Pérez-García, R. Augusiak, *Entangling dynamics beyond quantum theory*, arXiv:1111.4060.

Consider two (generalized) bits, described by ball state spaces.



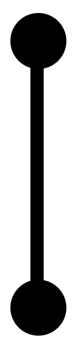
class. bit



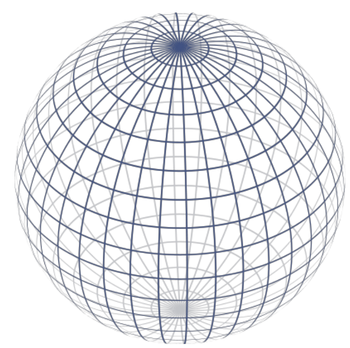
qubit

Ll. Masanes, MM, D. Pérez-García, R. Augusiak, *Entangling dynamics beyond quantum theory*, arXiv:1111.4060.

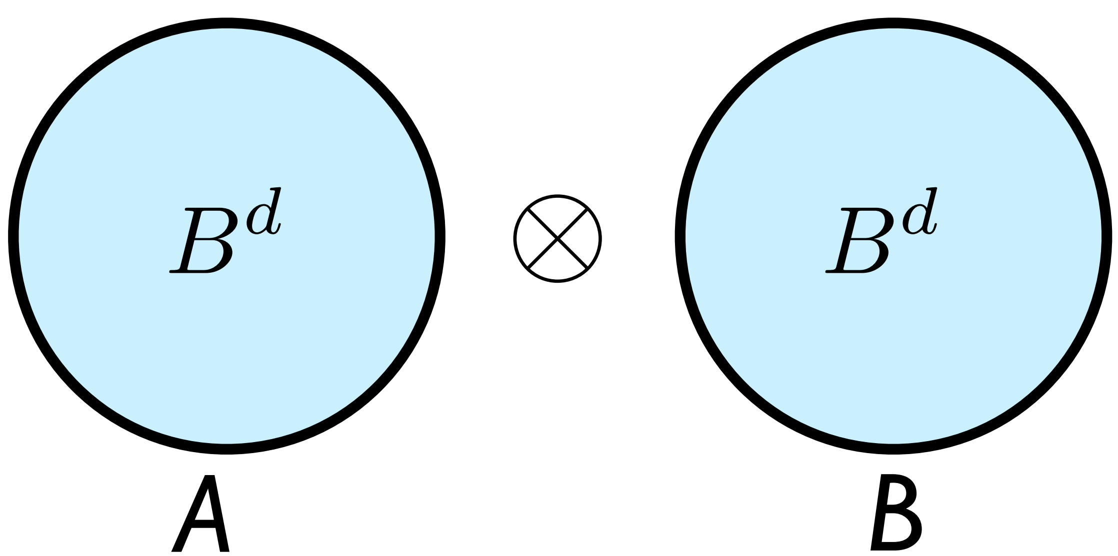
Consider two (generalized) bits, described by ball state spaces.



class. bit (*d*=*1*)

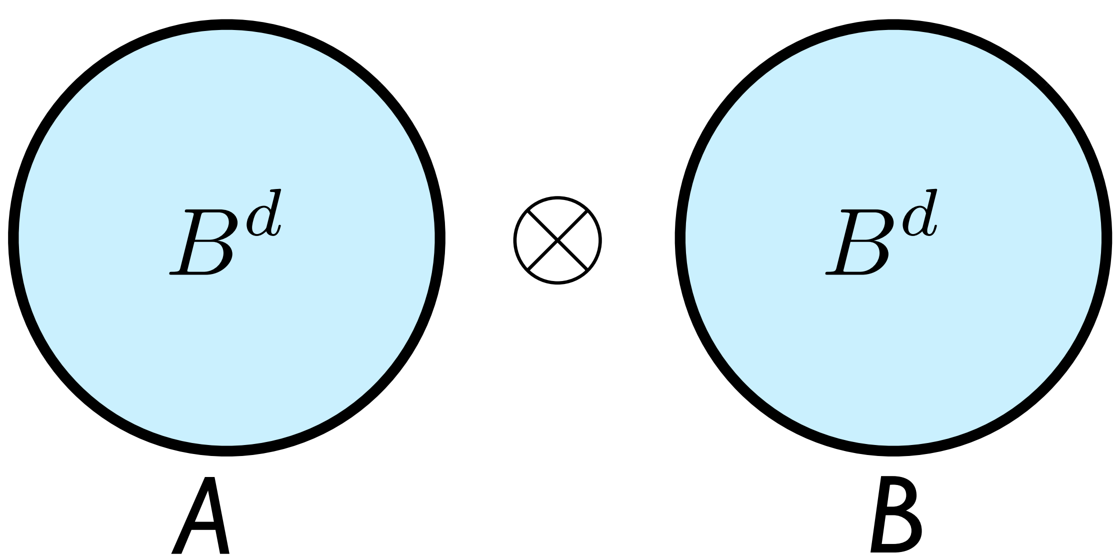


qubit (*d*=*3*)



Ll. Masanes, MM, D. Pérez-García, R. Augusiak, *Entangling dynamics beyond quantum theory*, arXiv:1111.4060.

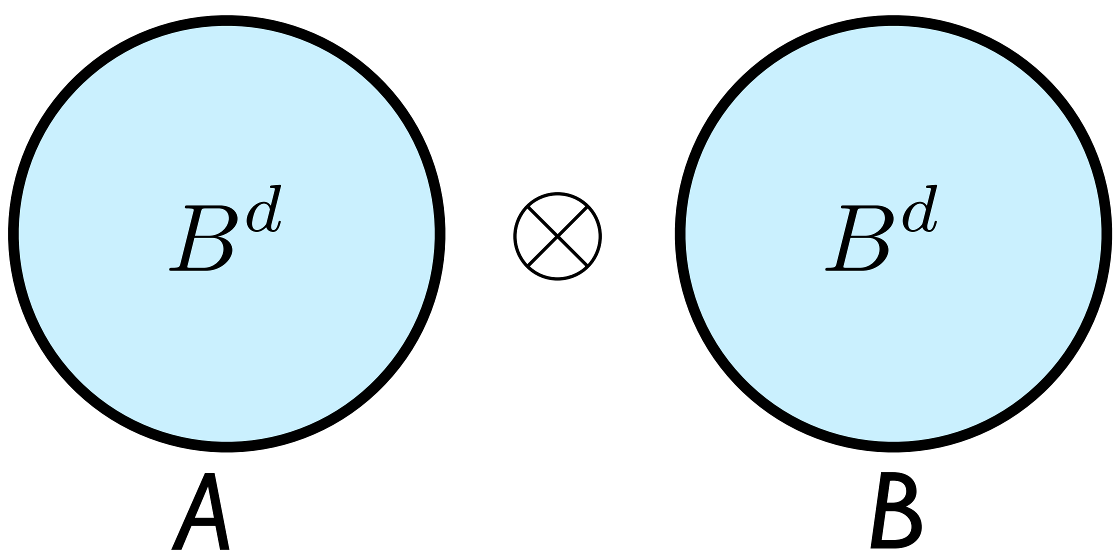
Consider two (generalized) bits, described by ball state spaces.
We ask for joint state spaces *AB* that satisfy the following:



Ll. Masanes, MM, D. Pérez-García, R. Augusiak, *Entangling dynamics beyond quantum theory*, arXiv:1111.4060.

Consider two (generalized) bits, described by ball state spaces. We ask for joint state spaces *AB* that satisfy the following:

1. Local tomography
2. Continuous reversibility: In every system, for every pair of pure states there is a continuous reversible transformation that maps one state to the other.
3. No restrictions on local measurements.

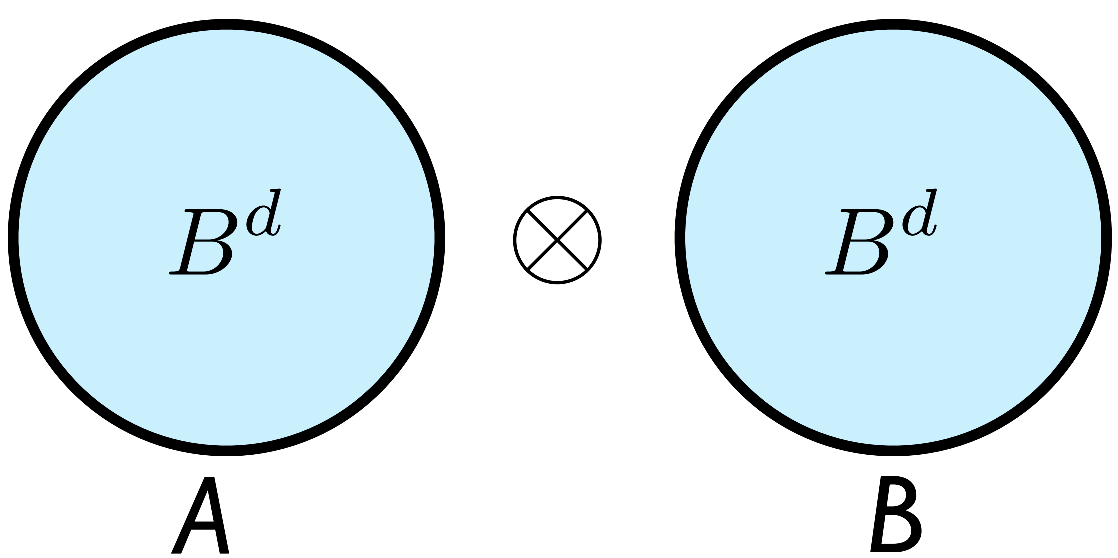


Ll. Masanes, MM, D. Pérez-García, R. Augusiak, *Entangling dynamics beyond quantum theory,* arXiv:1111.4060.

Consider two (generalized) bits, described by ball state spaces. We ask for joint state spaces *AB* that satisfy the following:

1. Local tomography
2. Continuous reversibility: In every system, for every pair of pure states there is a continuous reversible transformation that maps one state to the other.
3. ~~No restrictions on local measurements.~~

Not needed.

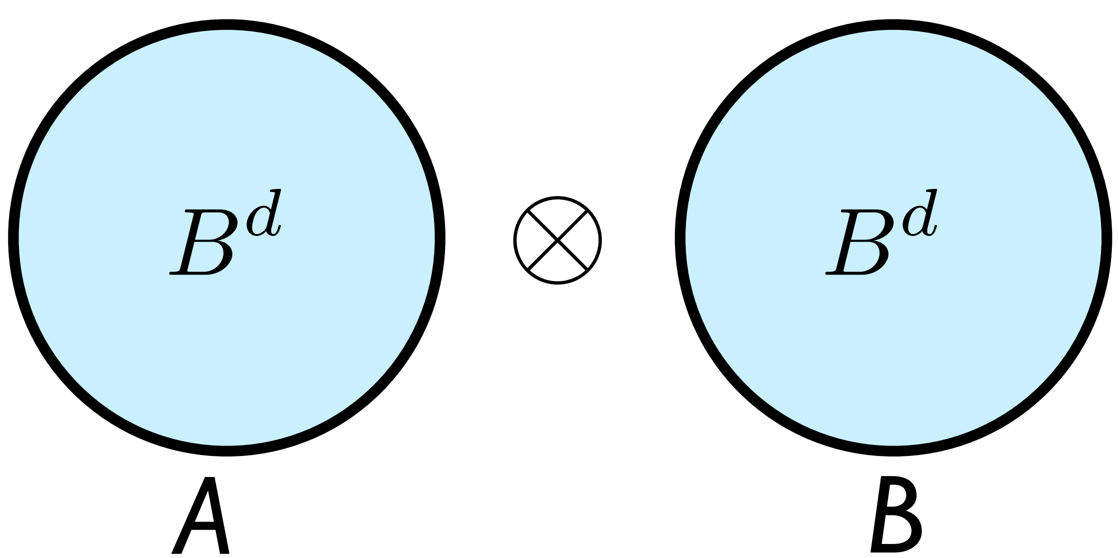


Ll. Masanes, MM, D. Pérez-García, R. Augusiak, *Entangling dynamics beyond quantum theory*, arXiv:1111.4060.

Consider two (generalized) bits, described by ball state spaces.
We ask for joint state spaces *AB* that satisfy the following:

1. Local tomography
2. Continuous reversibility: In every system, for every pair of pure states there is a continuous reversible transformation that maps one state to the other.
3. ~~No restrictions on local measurements.~~

Not needed.

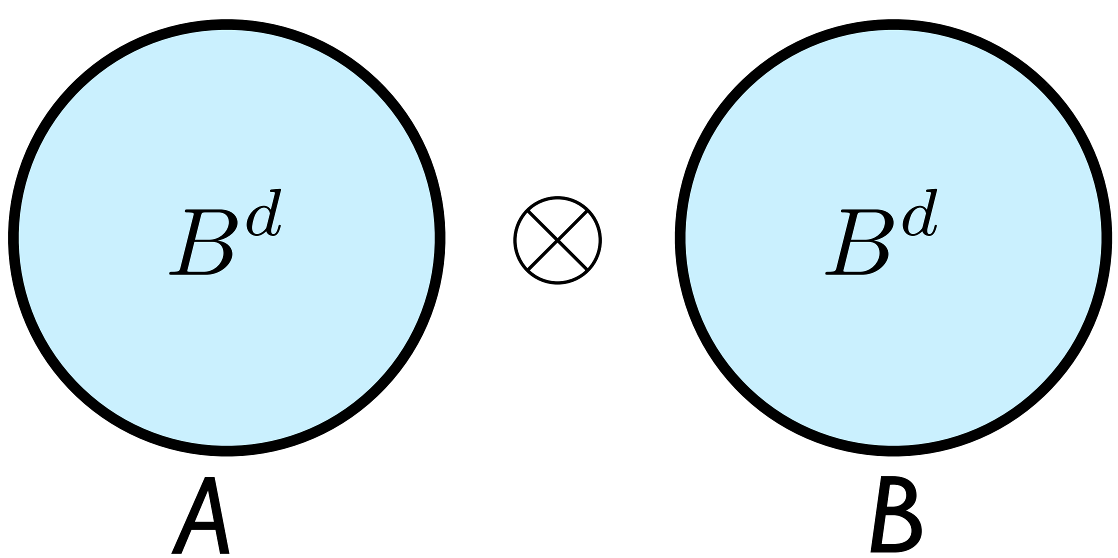


Actually, only need to assume "strict convexity" (see paper).

Ll. Masanes, MM, D. Pérez-García, R. Augusiak, *Entangling dynamics beyond quantum theory*, arXiv:1111.4060.

Thm.: The only interacting theory in this family is QT:

- If $d \neq 3$, then all reversible transformations on *AB* are of the form $T_{AB} = T_A \otimes T_B$, and all states on *AB* are unentangled.
- If $d = 3$, then we have either QT, or partially-transposed QT, or the unentangled states of QT.

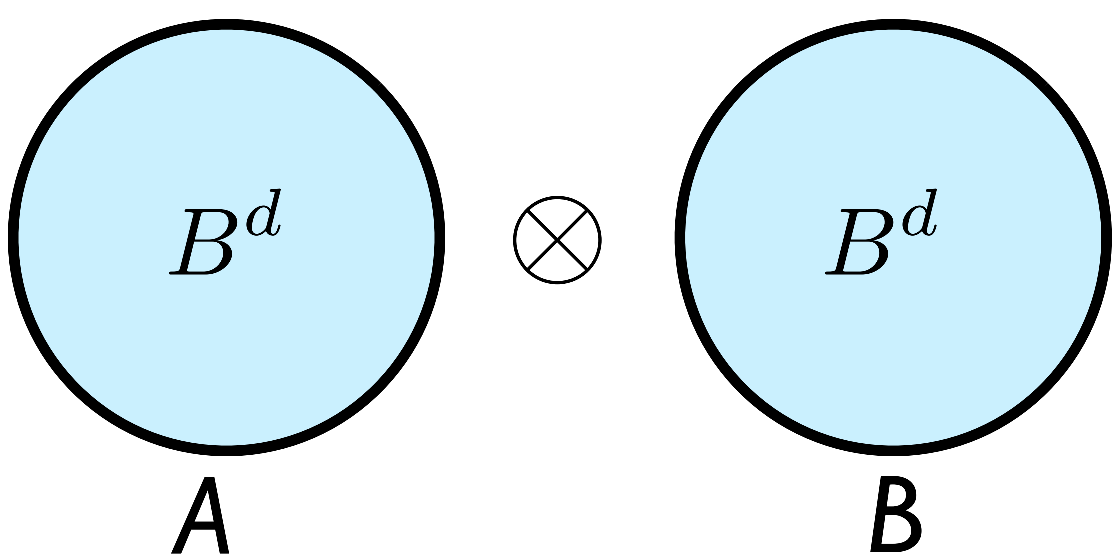


Ll. Masanes, MM, D. Pérez-García, R. Augusiak, *Entangling dynamics beyond quantum theory*, arXiv:1111.4060.

Thm.: The only interacting theory in this family is QT:

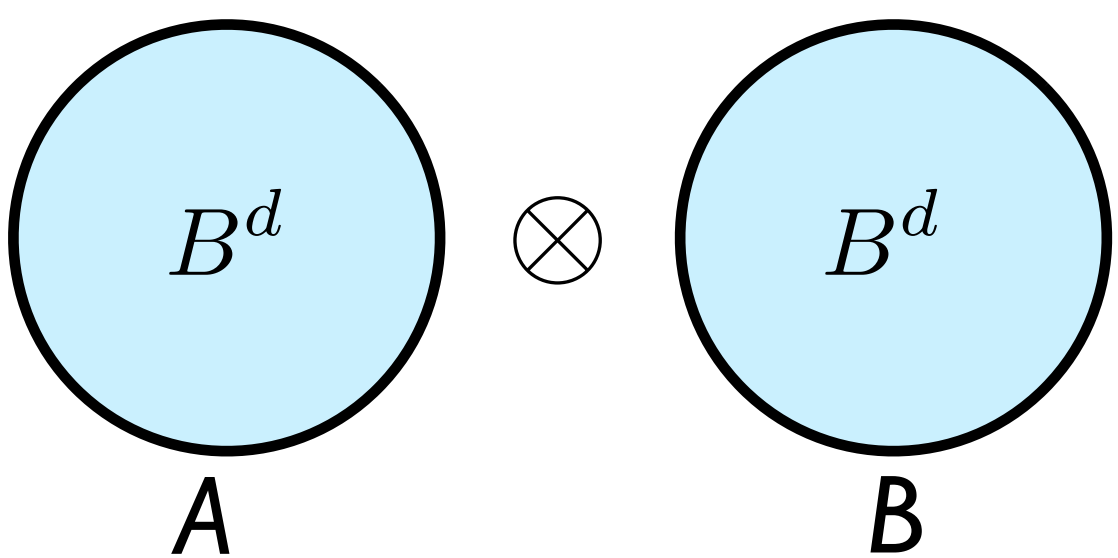
- If $d \neq 3$, then all reversible transformations on *AB* are of the form $T_{AB} = T_A \otimes T_B$, and all states on *AB* are unentangled.
- If $d = 3$, then we have either QT, or partially-transposed QT, or the unentangled states of QT.

First, guess the *local* transformation group:



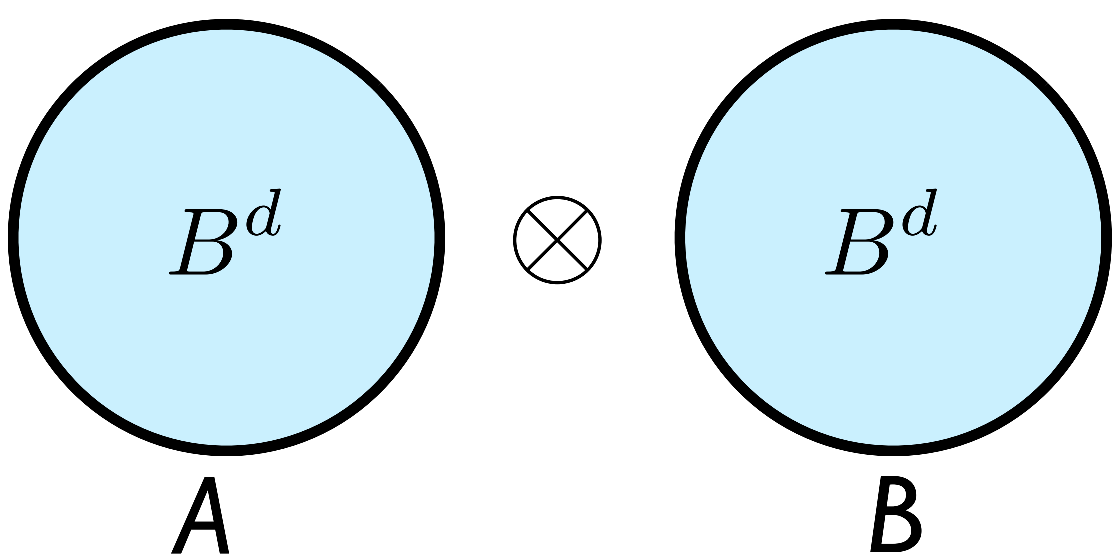
abstract groups	d
$\mathrm{SO}(d)$	$3, 4, 5 \ldots$
$\mathrm{SU}(d/2)$	$4, 6, 8 \ldots$
$\mathrm{U}(d/2)$	$2, 4, 6, 8 \ldots$
$\mathrm{Sp}(d/4)$	$8, 12, 16 \ldots$
$\mathrm{Sp}(d/4) \times \mathrm{U}(1)$	$8, 12, 16 \ldots$
$\mathrm{Sp}(d/4) \times \mathrm{SU}(2)$	$4, 8, 12 \ldots$
G_2	7
$\mathrm{Spin}(7)$	8
$\mathrm{Spin}(9)$	16

In the following, restrict to *SO(d)*.

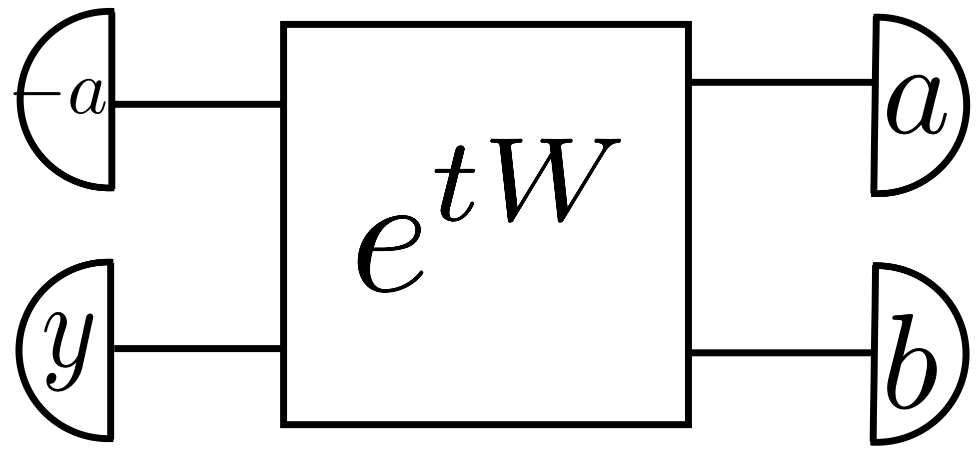


In the following, restrict to *SO(d)*.

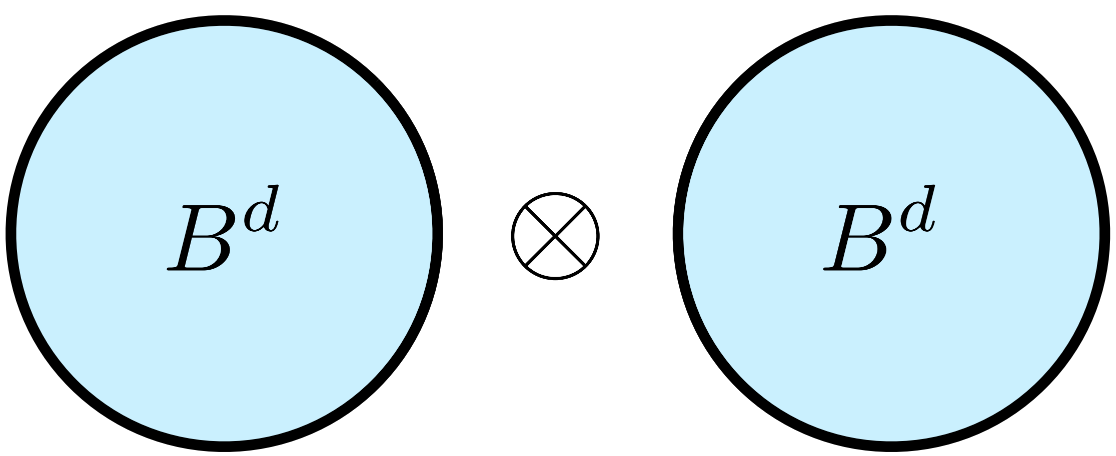
- local states: $\omega = \begin{pmatrix} 1 \\ a \end{pmatrix}, \; a \in \mathbb{R}^d, \; |a| \leq 1.$
- local effects: $E(\omega) = E \cdot \omega, \; E = \frac{1}{2}\begin{pmatrix} 1 \\ x \end{pmatrix}, \; x \in \mathbb{R}^d, \; |x| \leq 1.$
- product states: $\omega^A \otimes \omega^B = \begin{pmatrix} 1 \\ a \end{pmatrix} \otimes \begin{pmatrix} 1 \\ b \end{pmatrix} = \begin{pmatrix} 1 \\ b \\ a \\ a \otimes b \end{pmatrix}.$



Consider $G(t) = e^{tW}$ transformations in global Lie group $\mathcal{G}^{AB}$.

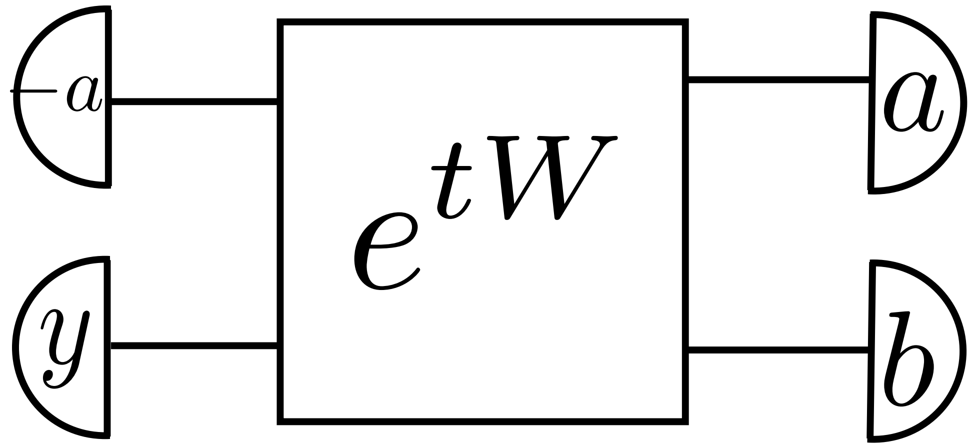


$$|a| = 1.$$



A *B*

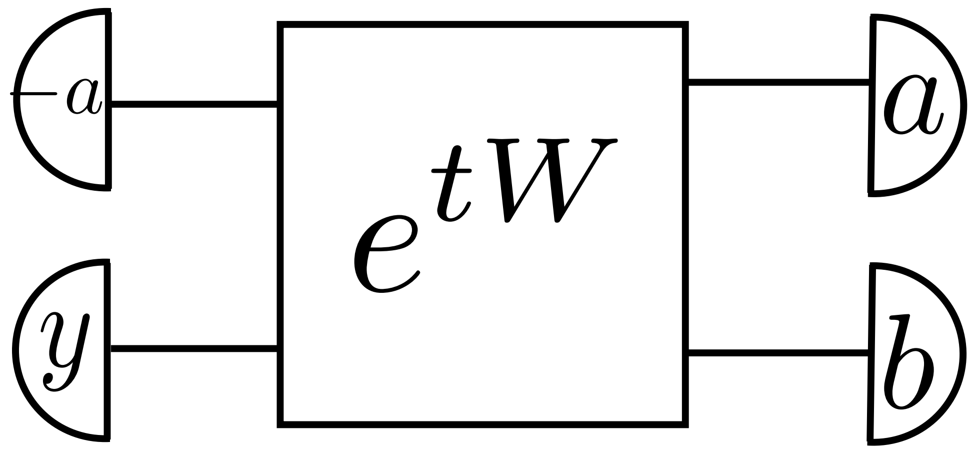
Consider $G(t)=e^{tW}$ transformations in global Lie group $\mathcal{G}^{AB}$.



$|a|=1.$

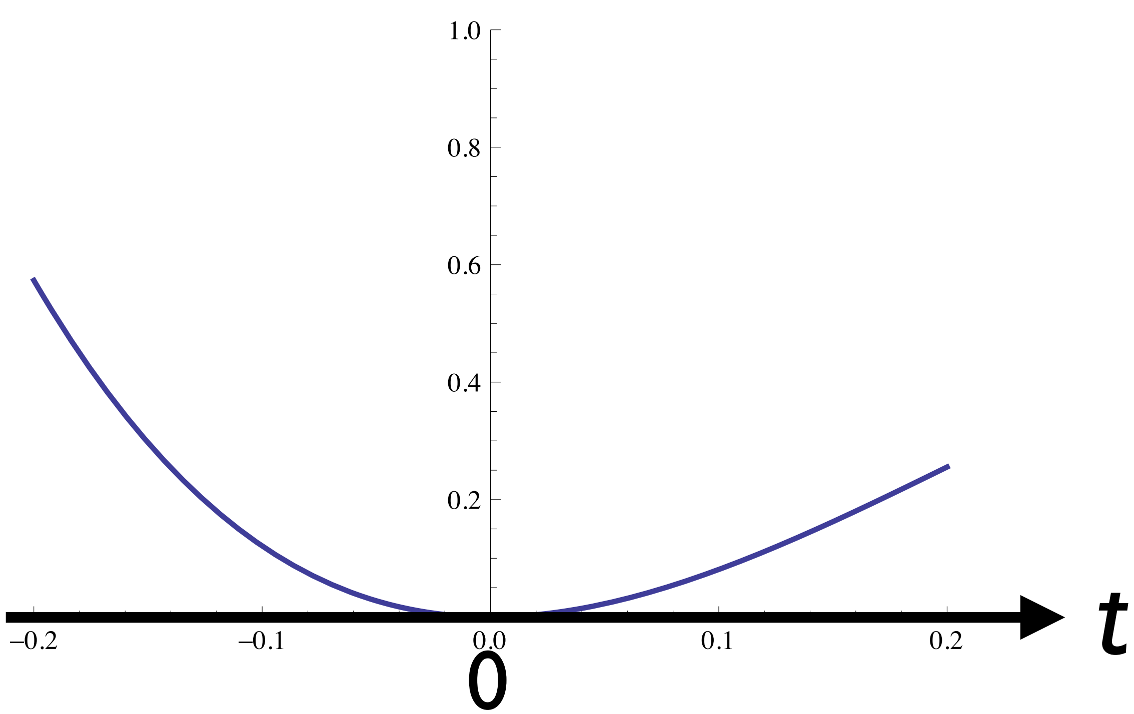
$$f(t):=\frac{1}{4}\left(\begin{array}{c}1\\a\end{array}\right)\otimes\left(\begin{array}{c}1\\b\end{array}\right)e^{tW}\left(\begin{array}{c}1\\-a\end{array}\right)\otimes\left(\begin{array}{c}1\\y\end{array}\right)\in[0,1]\ \ \forall t.$$

Consider $G(t)=e^{tW}$ transformations in global Lie group $\mathcal{G}^{AB}$.



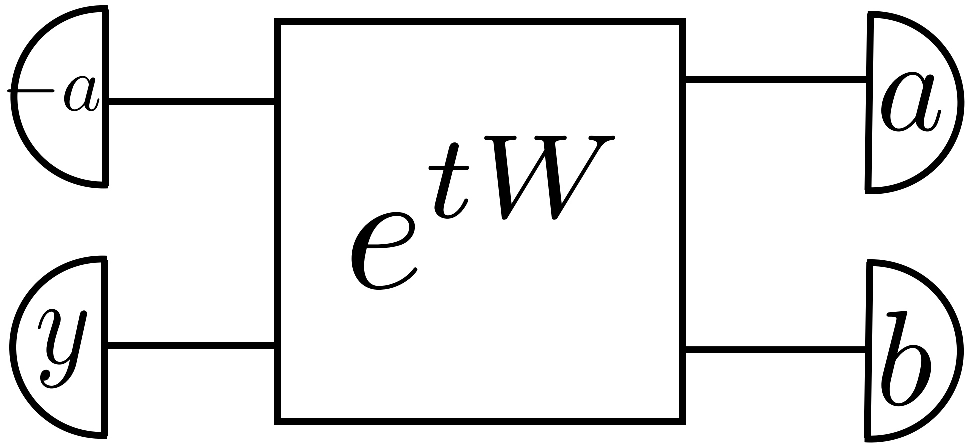
$|a|=1.$

$$f(t):=\frac{1}{4}\left(\begin{array}{c}1\\ a\end{array}\right)\otimes\left(\begin{array}{c}1\\ b\end{array}\right)e^{tW}\left(\begin{array}{c}1\\ -a\end{array}\right)\otimes\left(\begin{array}{c}1\\ y\end{array}\right)\in[0,1]\ \forall t.$$



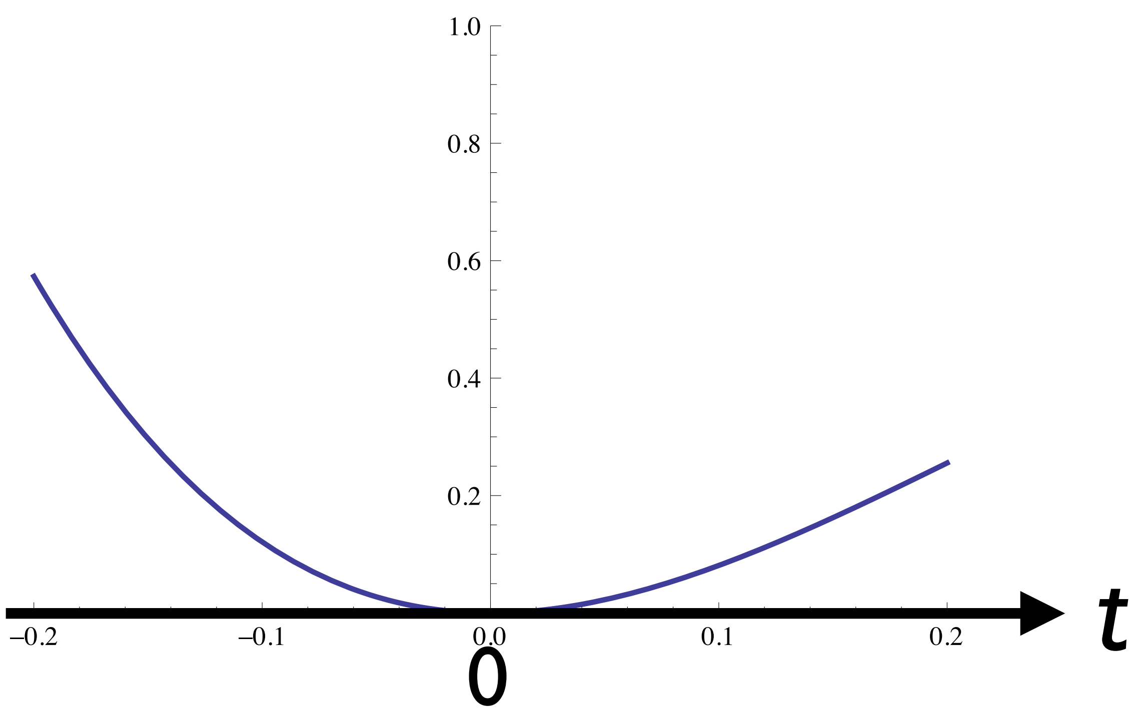
We have $f(0)=0$.
Thus $f'(0)=0$ and $f''(0)\geq 0$.

Consider $G(t) = e^{tW}$ transformations in global Lie group $\mathcal{G}^{AB}$.



$|a| = 1.$

$$f(t) := \frac{1}{4}\begin{pmatrix} 1 \\ a \end{pmatrix} \otimes \begin{pmatrix} 1 \\ b \end{pmatrix} e^{tW} \begin{pmatrix} 1 \\ -a \end{pmatrix} \otimes \begin{pmatrix} 1 \\ y \end{pmatrix} \in [0,1] \;\; \forall t.$$



We have $f(0) = 0$.
Thus $f'(0) = 0$ and $f''(0) \geq 0$.

$$\begin{pmatrix} 1 \\ a \end{pmatrix} \otimes \begin{pmatrix} 1 \\ b \end{pmatrix} W \begin{pmatrix} 1 \\ -a \end{pmatrix} \otimes \begin{pmatrix} 1 \\ y \end{pmatrix} = 0$$

$$\begin{pmatrix} 1 \\ a \end{pmatrix} \otimes \begin{pmatrix} 1 \\ b \end{pmatrix} W^2 \begin{pmatrix} 1 \\ -a \end{pmatrix} \otimes \begin{pmatrix} 1 \\ y \end{pmatrix} \geq 0.$$

For $d \geq 4$, it turns out that the only global Lie algebra elements W that satisfy all constraints are of the form

$$W = W_A \otimes \mathbf{1}_B + \mathbf{1}_A \otimes W_B.$$

They generate non-interacting dynamics.

For $d \geq 4$, it turns out that the only global Lie algebra elements *W* that satisfy all constraints are of the form

$$W = W_A \otimes \mathbf{1}_B + \mathbf{1}_A \otimes W_B.$$

They generate non-interacting dynamics.

Main group-theoretic reasons:

Rotations in *(d-1)* dimensions commute only if *d=3*.

For $d \geq 4$, it turns out that the only global Lie algebra elements *W* that satisfy all constraints are of the form

$$W = W_A \otimes \mathbf{1}_B + \mathbf{1}_A \otimes W_B.$$

They generate non-interacting dynamics.

Main group-theoretic reasons:

Rotations in *(d-1)* dimensions commute only if *d=3*.

d=7 with G_2 works *almost*!

Geometry and probability?

G. Mauro D‘Ariano:

“We want to start talking about *real* stuff !“

Geometry and probability?

G. Mauro D'Ariano:

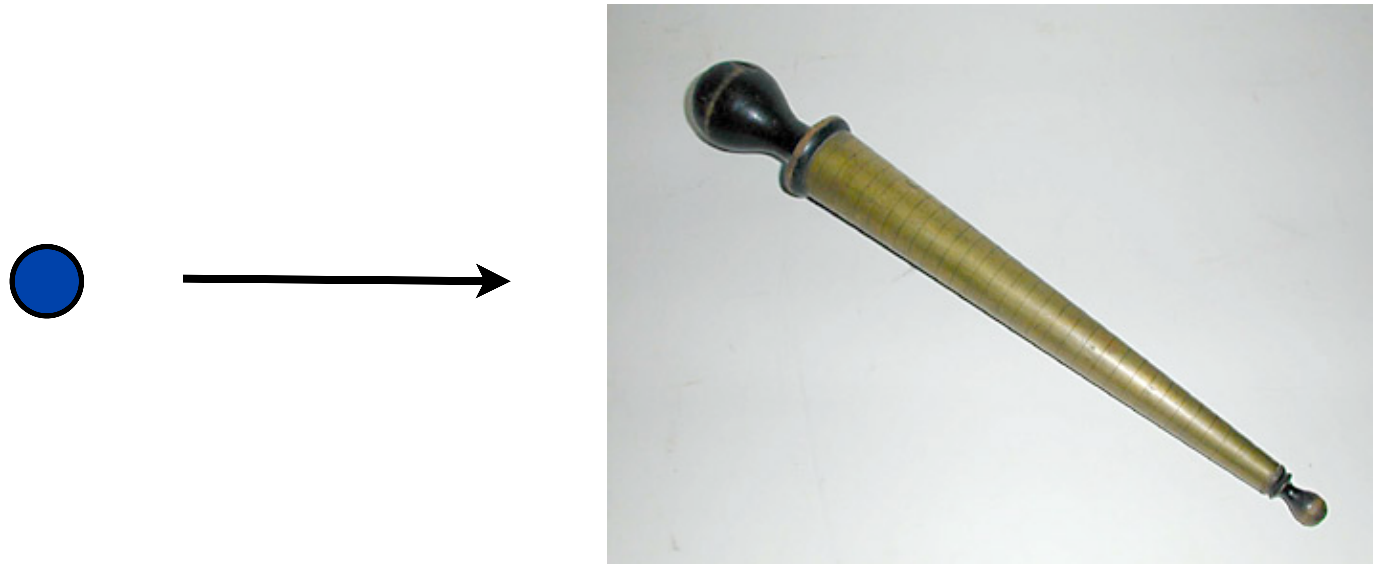
"We want to start talking about *real* stuff !"

Wikipedia on Weizsäcker's "ur-alternatives" (1966+):

"Physicist Carl Friedrich von Weizsäcker's theory of ur-alternatives... is a kind of digital physics as it axiomatically constructs quantum physics from the distinction between empirically observable, binary alternatives.
Weizsäcker used his theory to derive the 3-dimensionality of space [...]"

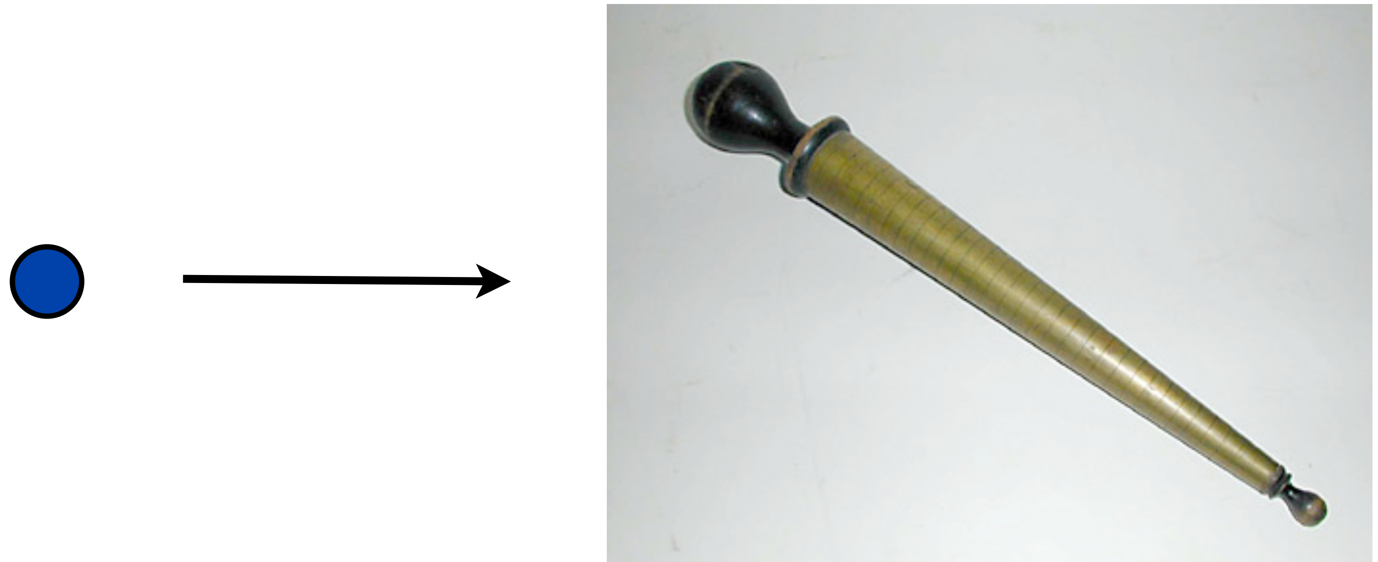
Assume we live in *(d+1)* dimensions, and there exist certain systems ("direction units") and devices measuring them.

Assume we live in $(d+1)$ dimensions, and there exist certain systems ("direction units") and devices measuring them.



2 outcomes:
yes / no.

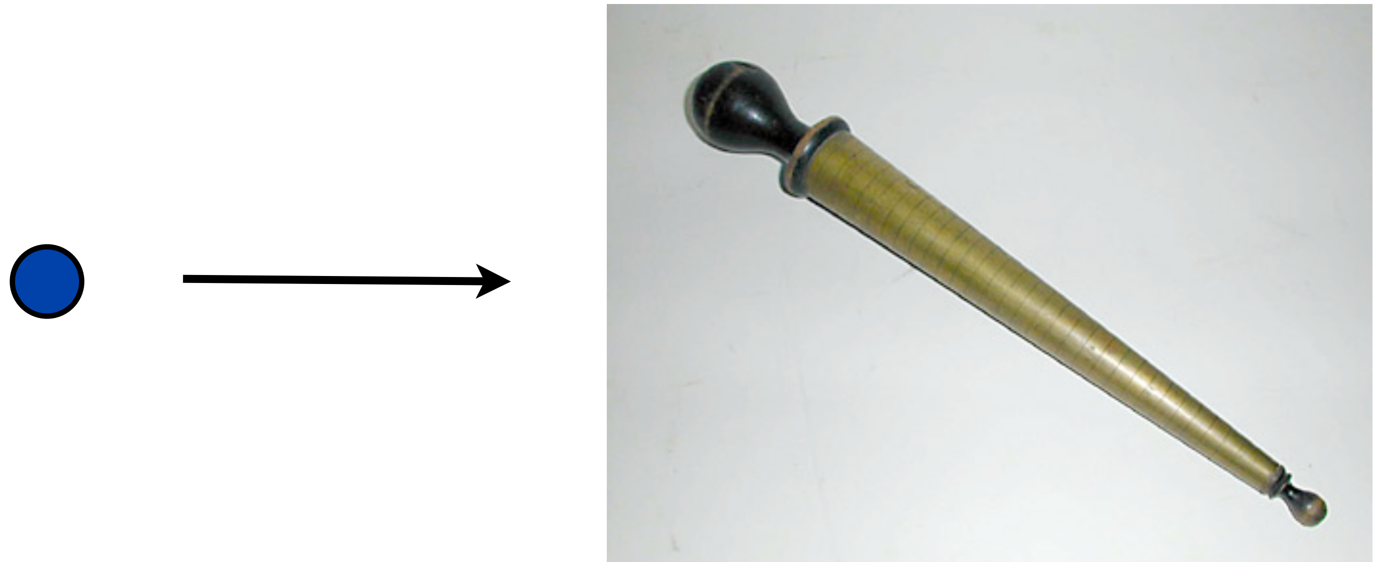
Assume we live in *(d+1)* dimensions, and there exist certain systems ("direction units") and devices measuring them.



2 outcomes:
yes / no.

Device can be rotated in space. Direction: $v \in S^{d-1} \subset \mathbb{R}^d$.

Assume we live in *(d+1)* dimensions, and there exist certain systems ("direction units") and devices measuring them.

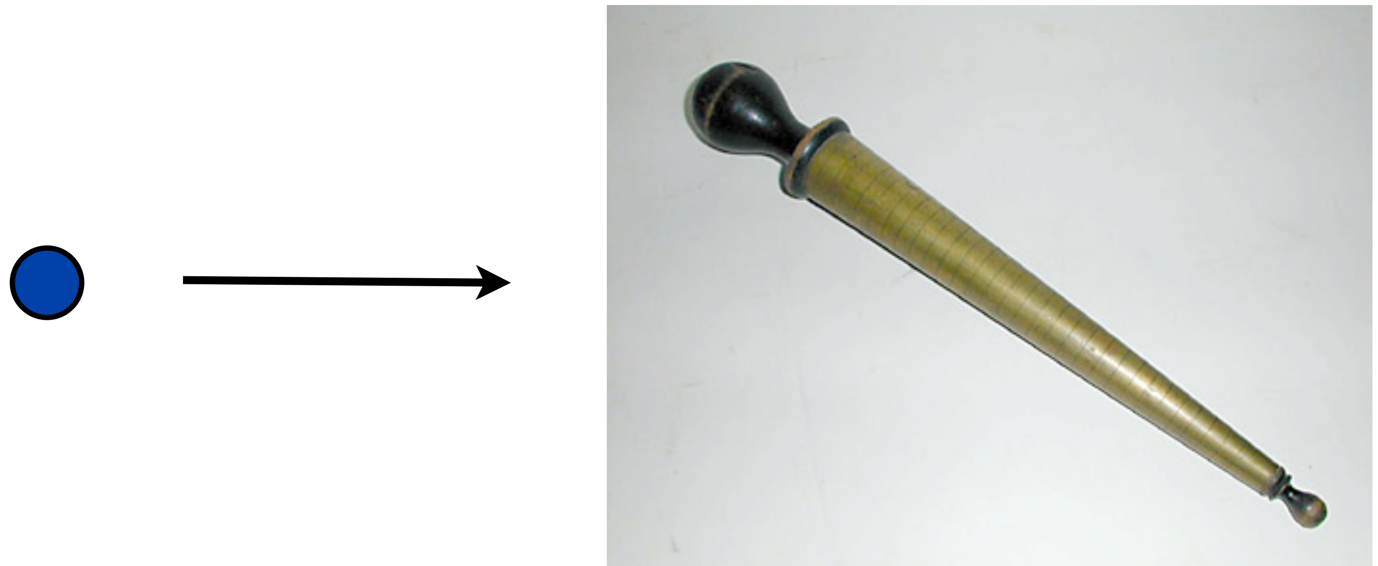


2 outcomes:
yes / no.

Device can be rotated in space. Direction: $v \in S^{d-1} \subset \mathbb{R}^d$.

System is described by finite-dim. convex state space.
In principle, effect measured by device can depend on *v* (somehow).
Notation: E_v.
Measurements take place locally and at rest.

Assume we live in *(d+1)* dimensions, and there exist certain systems ("direction units") and devices measuring them.

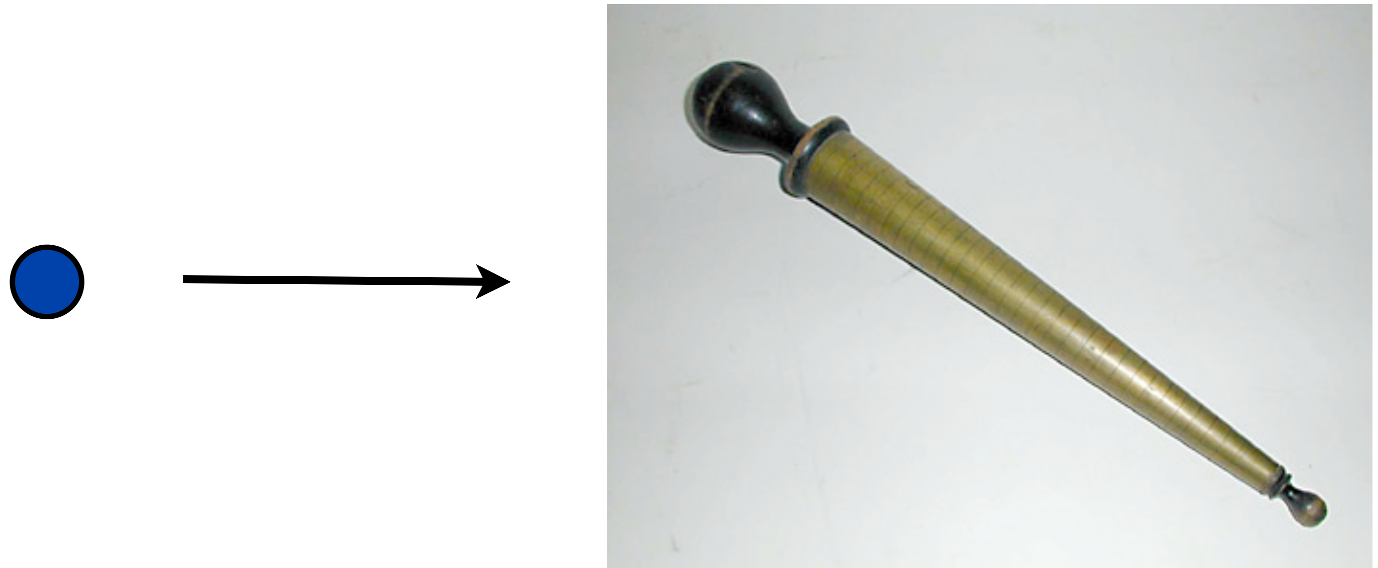


2 outcomes:
yes / no.

Device can be rotated in space. Direction: $v \in S^{d-1} \subset \mathbb{R}^d$.

System is described by finite-dim. convex state space.
In principle, effect measured by device can depend on *v* (somehow).
Notation: E_v.
Measurements take place locally and at rest.

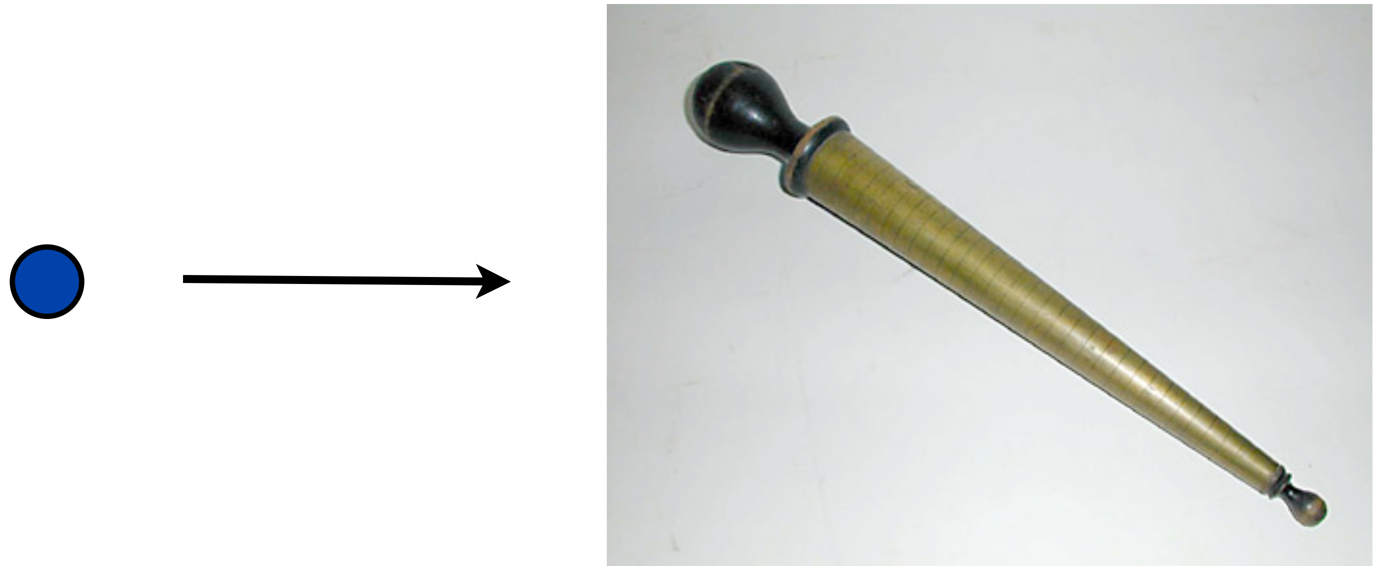
Now assume 3 operational postulates:



2 outcomes:
yes / no.

1. Rotation of device makes *some* difference:

For every $v \in S^{d-1}$ there is a state ω such that $E_v(\omega) = 1$ and $E_{v'}(\omega) < 1$ for all $v' \neq v$.



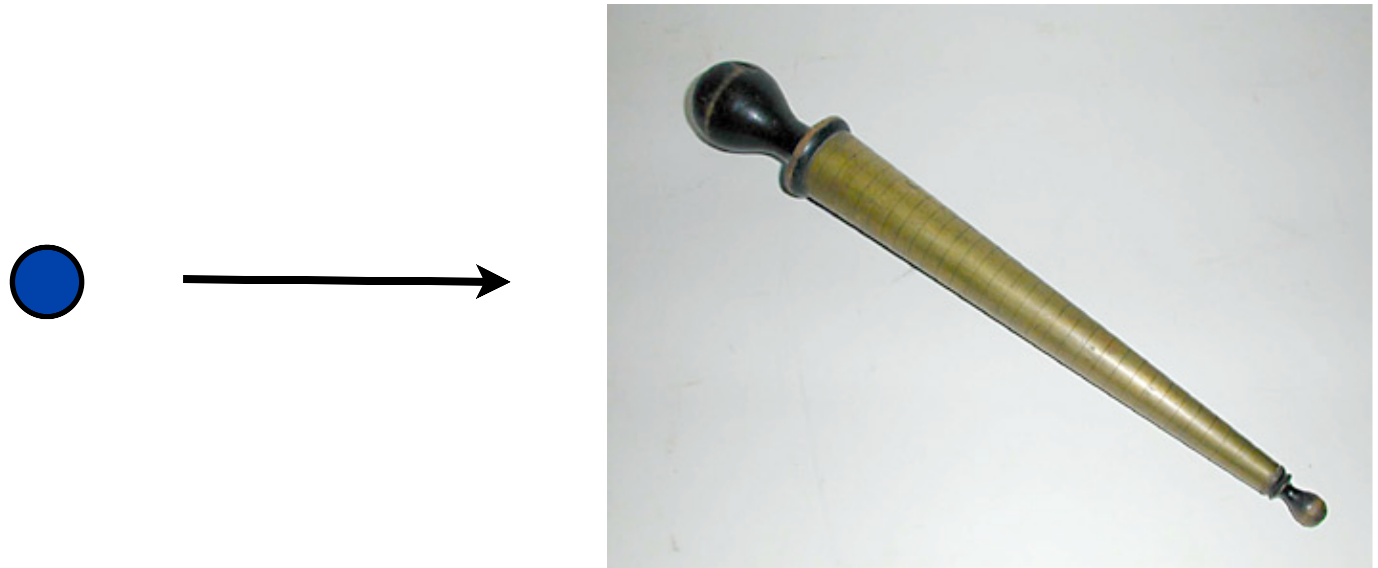
2 outcomes:
yes / no.

1. Rotation of device makes *some* difference:

For every $v \in S^{d-1}$ there is a state ω such that $E_v(\omega) = 1$ and $E_{v'}(\omega) < 1$ for all $v' \neq v$.

2. Direction units carry *only* direction information:

If ω and ω' are states that have the *same* maximal yes-probability $\max_v E_v(\omega)$ in the *same* direction v, then $\omega = \omega'$.



2 outcomes:
yes / no.

1. Rotation of device makes *some* difference:

For every $v \in S^{d-1}$ there is a state ω such that $E_v(\omega) = 1$ and $E_{v'}(\omega) < 1$ for all $v' \neq v$.

2. Direction units carry *only* direction information:

If ω and ω' are states that have the *same* maximal yes-probability $\max_v E_v(\omega)$ in the *same* direction v, then $\omega = \omega'$.

3. Two uncorrelated direction units can become correlated by continuous reversible time evolution, and bipartite states are uniquely determined by local measurements.

1. Rotation of device makes *some* difference:
For every $v \in S^{d-1}$ there is a state ω such that $E_v(\omega) = 1$ and $E_{v'}(\omega) < 1$ for all $v' \neq v$.

2. Direction units carry *only* direction information:
If ω and ω' are states that have the *same* maximal yes-probability $\max_v E_v(\omega)$ in the *same* direction v, then $\omega = \omega'$.

3. Two uncorrelated direction units can become correlated by continuous reversible time evolution, and bipartite states are uniquely determined by local measurements.

1. Rotation of device makes *some* difference:
For every $v \in S^{d-1}$ there is a state ω such that $E_v(\omega) = 1$ and $E_{v'}(\omega) < 1$ for all $v' \neq v$.

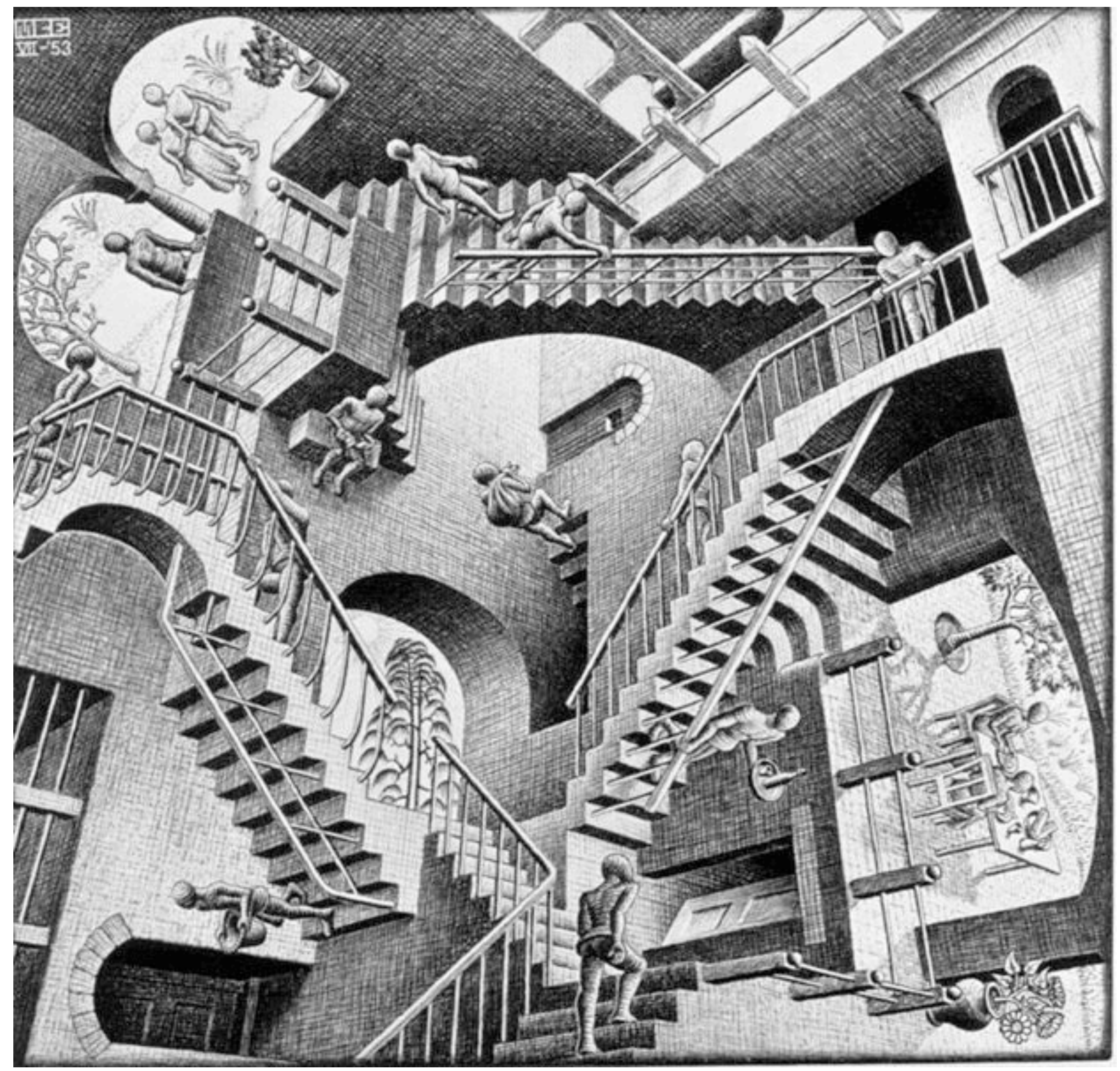
2. Direction units carry *only* direction information:
If ω and ω' are states that have the *same* maximal yes-probability $\max_v E_v(\omega)$ in the *same* direction v, then $\omega = \omega'$.

3. Two uncorrelated direction units can become correlated by continuous reversible time evolution, and bipartite states are uniquely determined by local measurements.

Thm.: If 1.-3. hold, then necessarily *d=3*, direction units are qubits, devices are (possibly noisy) spin measurements, two direction units combine to quantum state space, reversible time evolution is unitary & generated by some Hamiltonian.

A challenge:

Are there other aspects of space-time geometry that can be derived operationally / on information-theoretic grounds?



Conclusions

- Axiomatization of QT

- Reversibility as a strong axiom for QT:
    - Rules out boxworld
    - Locally qubits $\Rightarrow$ globally quantum
    - bit symmetry $\Rightarrow$strong self-duality
    - singles out *d*=*3* balls

- Geometry and probability?

Thank you!