ÖAW AUSTRIAN ACADEMY OF SCIENCES

IQOQI – INSTITUTE FOR QUANTUM OPTICS AND QUANTUM INFORMATION VIENNA

Why quantum theory?

Markus P. Müller

Institute for Quantum Optics and Quantum Information (IQOQI), Vienna

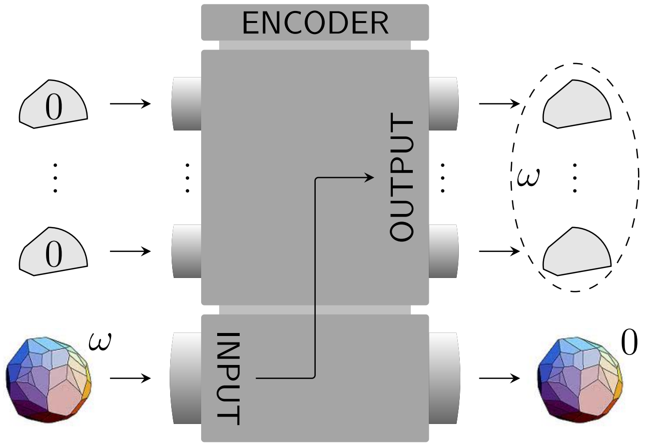
Perimeter Institute for Theoretical Physics (PI), Waterloo, Canada

PERIMETER INSTITUTE

Overview

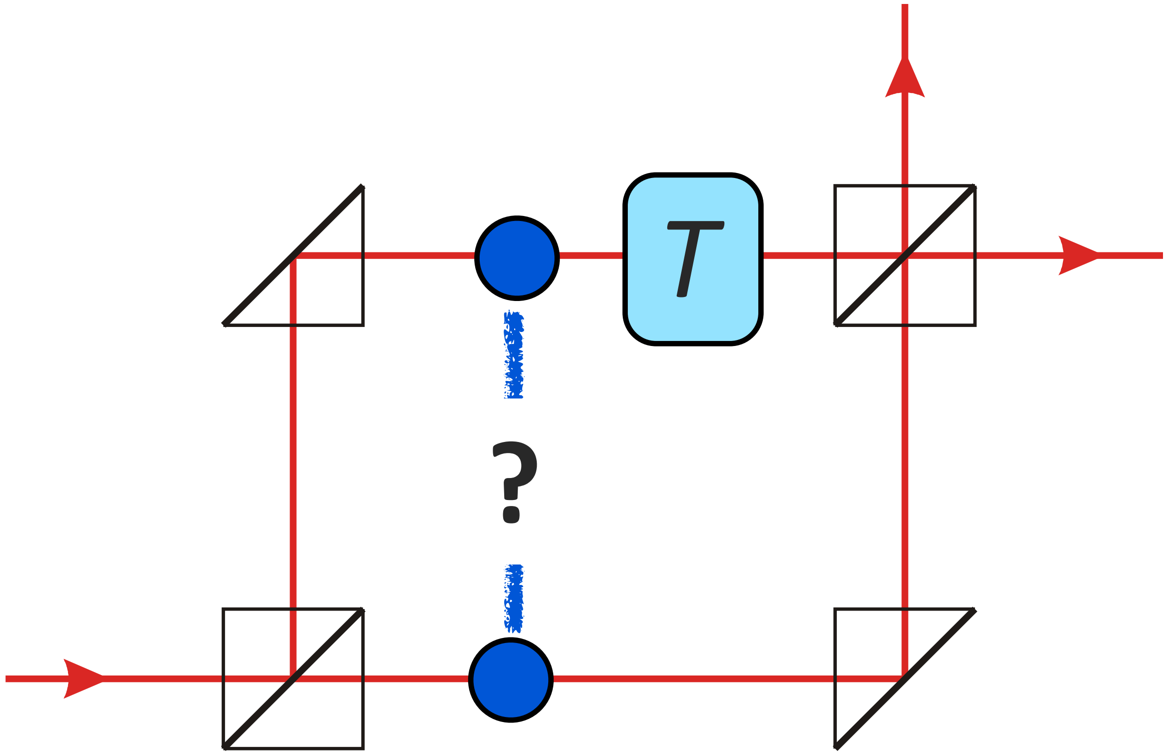
1. Probabilistic theories beyond quantum theory

2. Quantum theory from simple principles



3. The quest for higher-order interference

4. QT and spacetime



5. Conclusion

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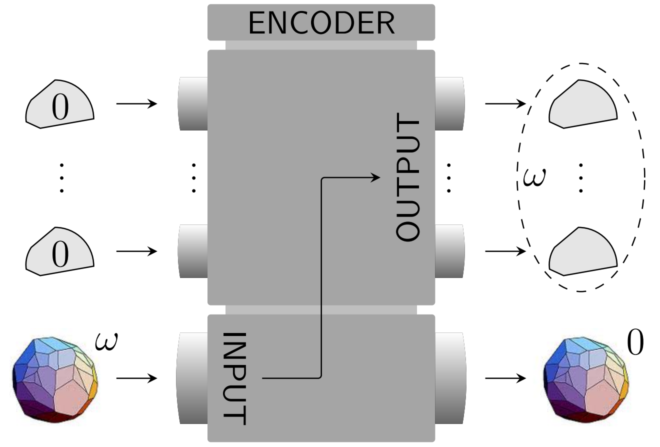

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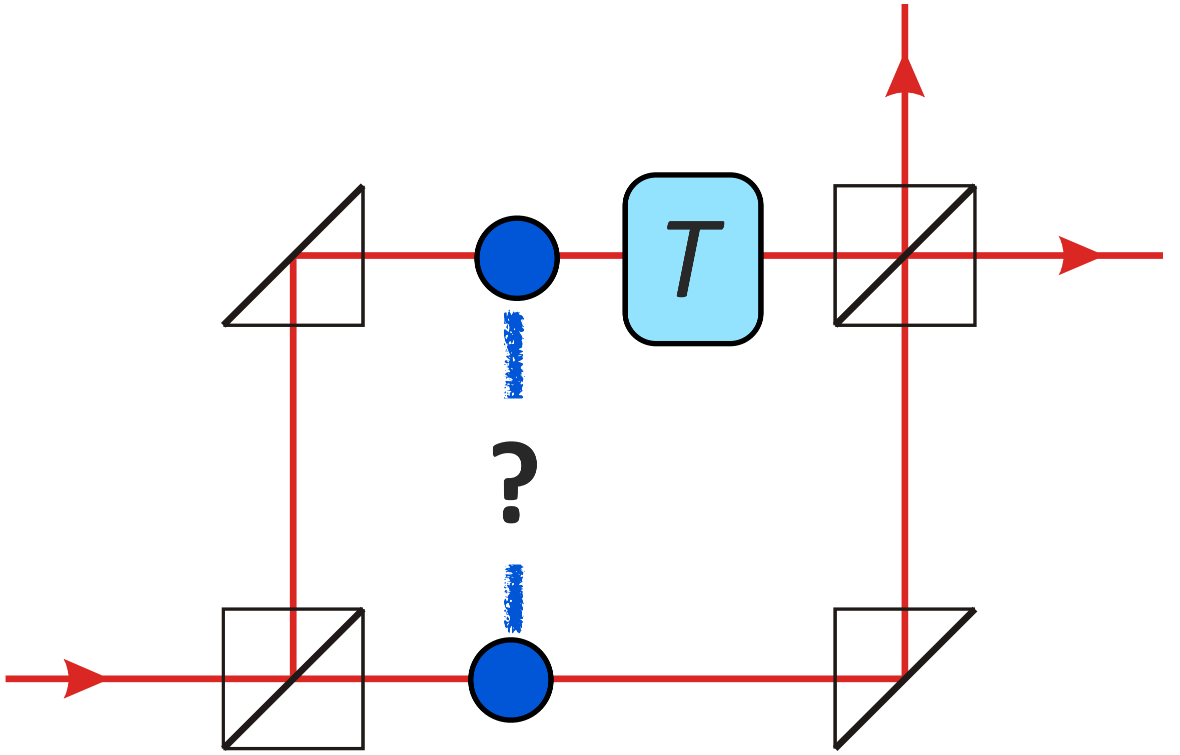

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More general than quantum?

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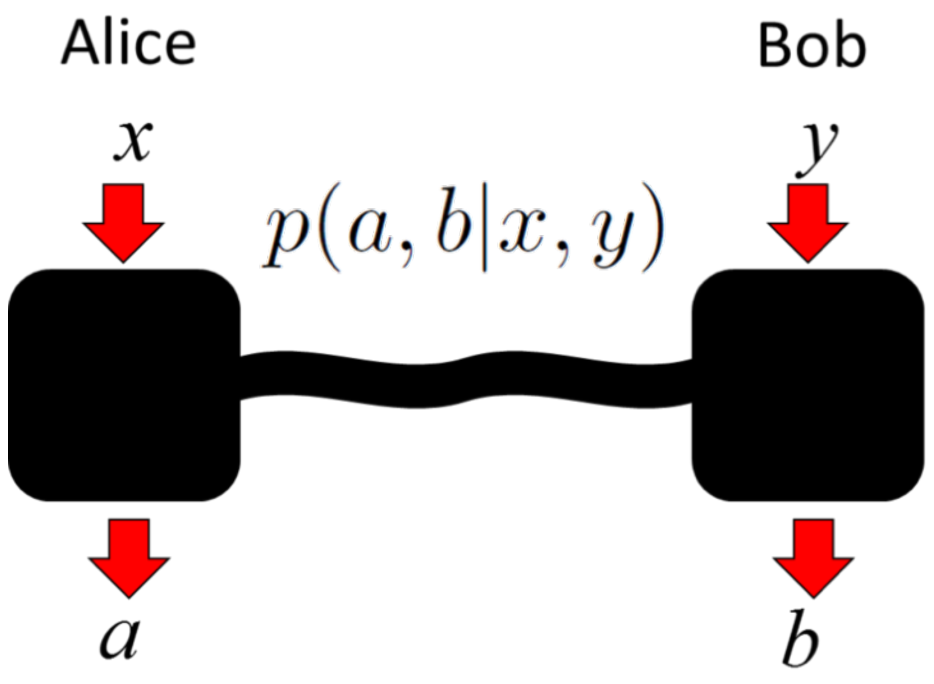

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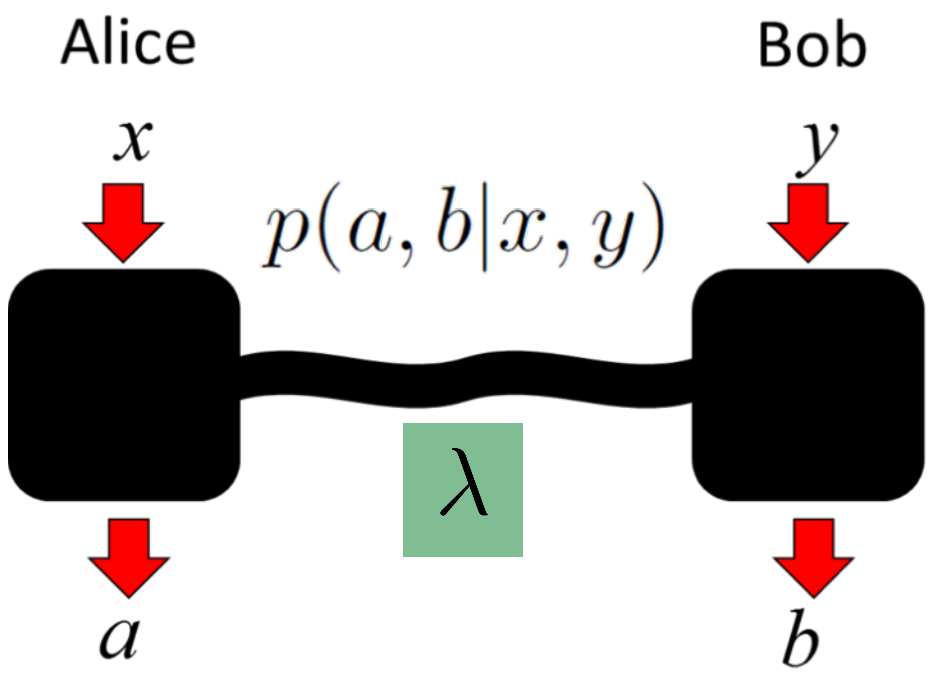

- In **classical** physics / prob. theory:

$$P(a, b|x, y) = \sum_{\lambda \in \Lambda} P_A(a|x, \lambda) P_B(b|y, \lambda) P_\Lambda(\lambda)$$

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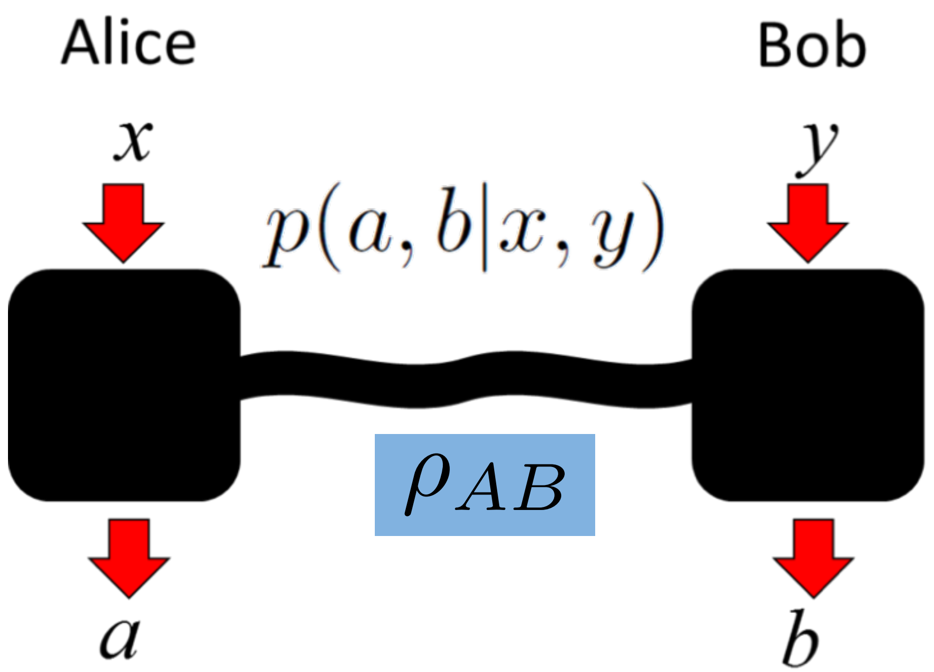

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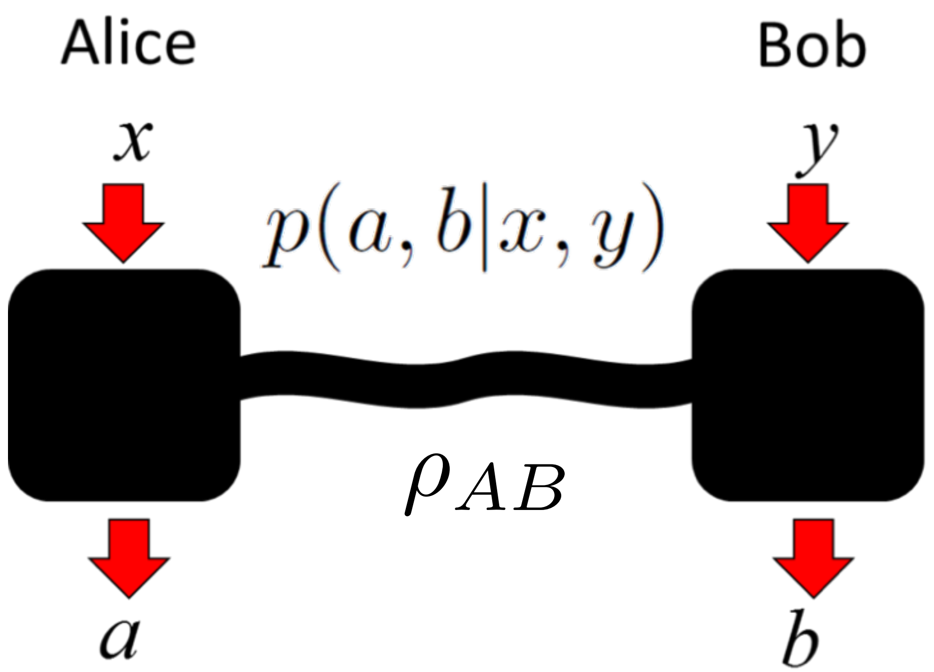

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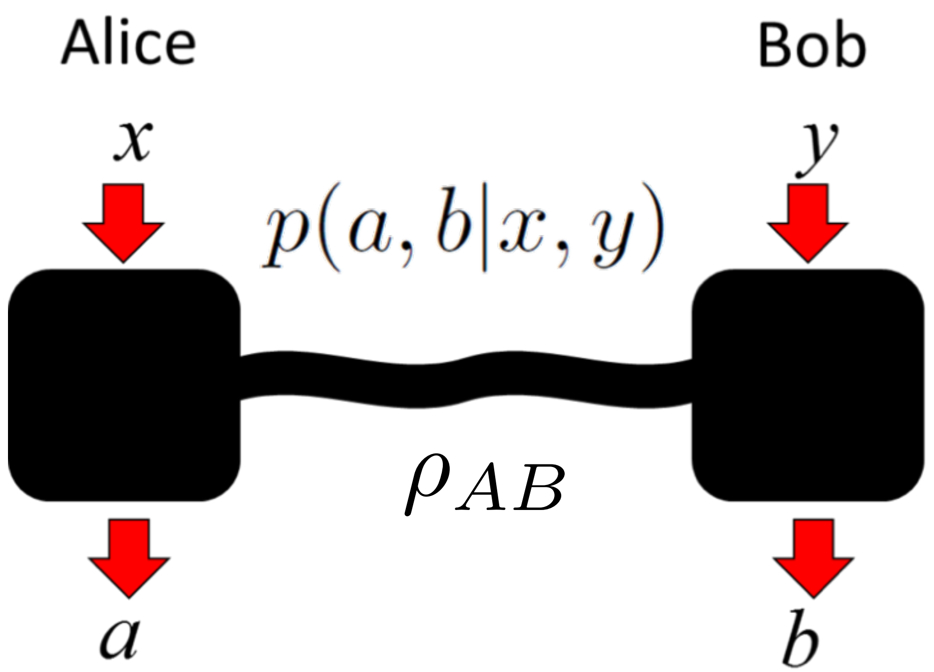

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Quantum admits more general *P*'s due to the **violation of Bell inequalities**.

The Bell-CHSH inequality

Classical probability distributions satisfy Bell inequality:

$$\mathrm{CHSH} := |C_{00} + C_{01} + C_{10} - C_{11}| \leq 2 \quad \text{where} \quad C_{xy} := \mathbb{E}(a \cdot b | x, y).$$

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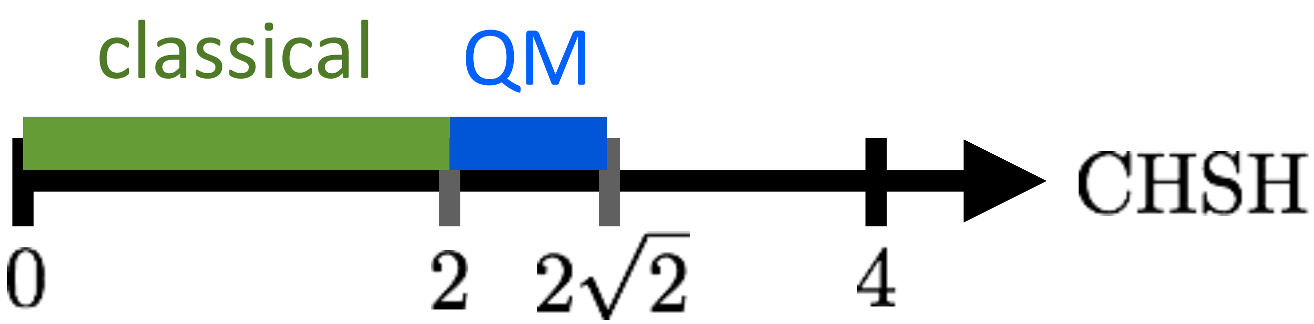

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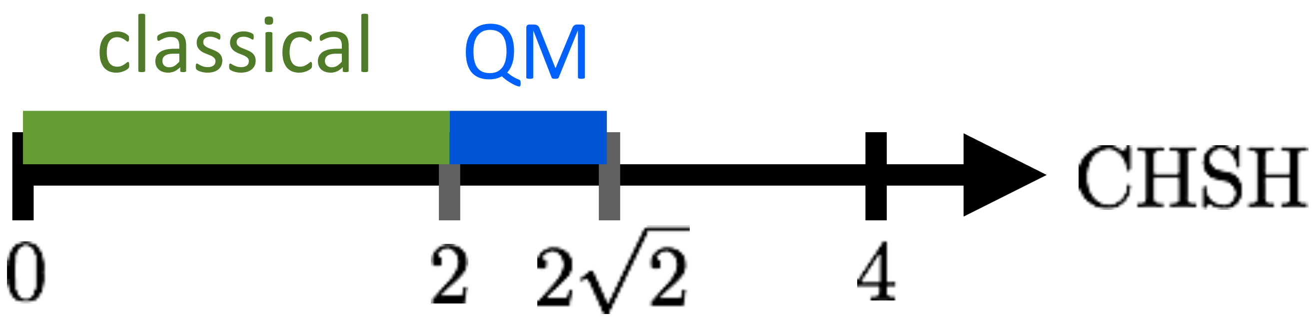

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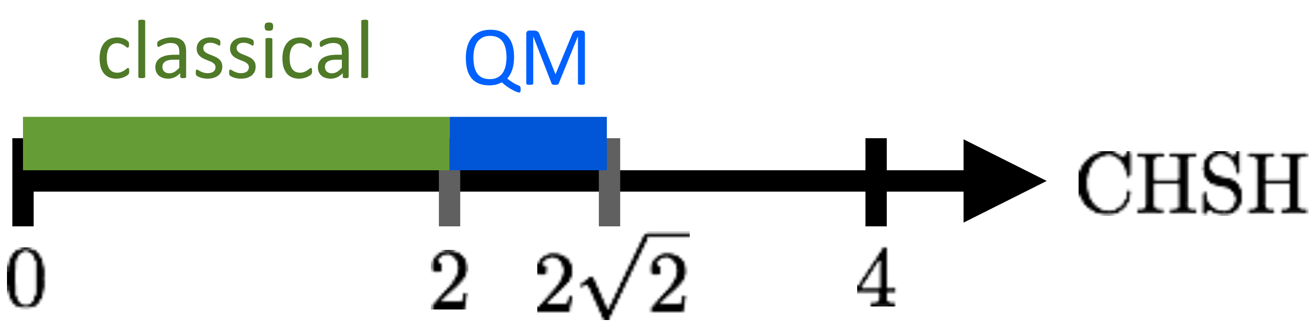

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No! Counterexample: the PR-box correlations

$$\begin{aligned} P(+1, +1|x, y) &= P(-1, -1|x, y) = \frac{1}{2} \\ &\quad \text{if } (x, y) \in \{(0,0), (0,1), (1,0)\} \\ P(+1, -1|1, 1) &= P(-1, +1|1, 1) = \frac{1}{2} \end{aligned}$$

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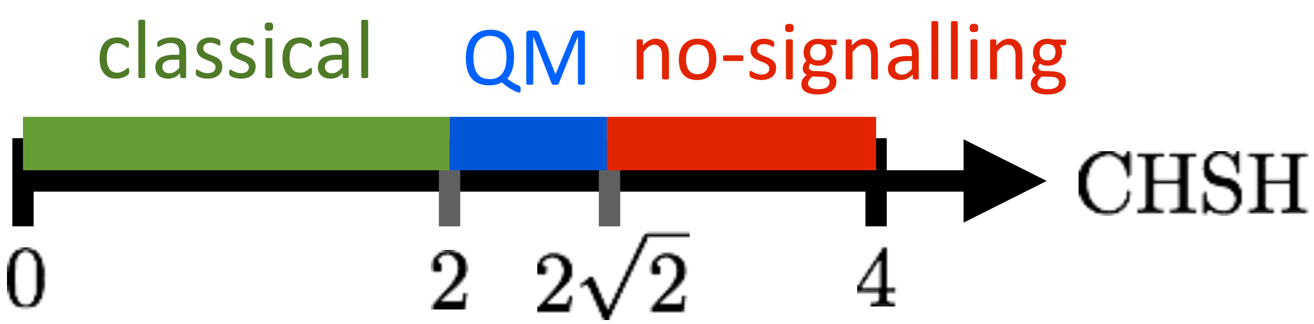

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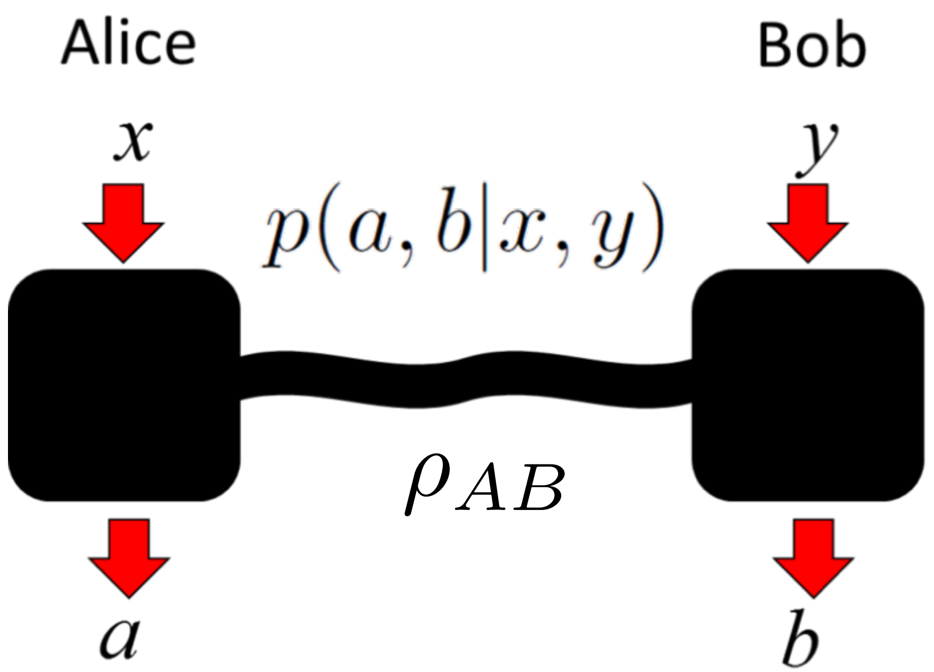

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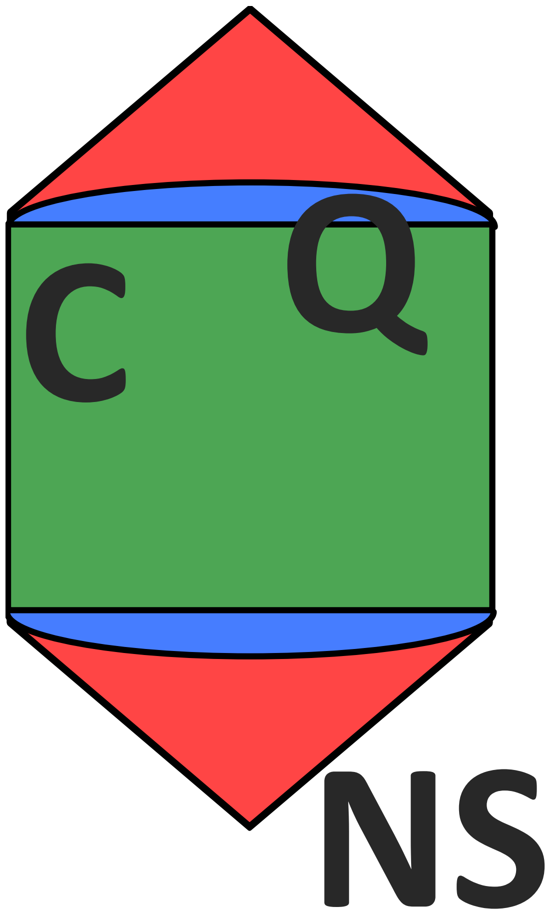

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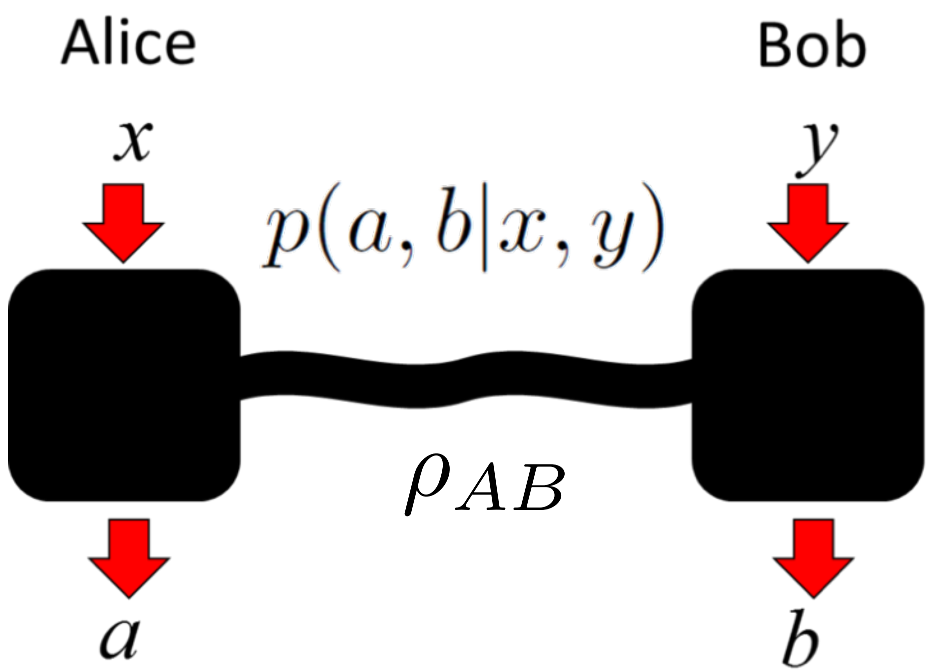

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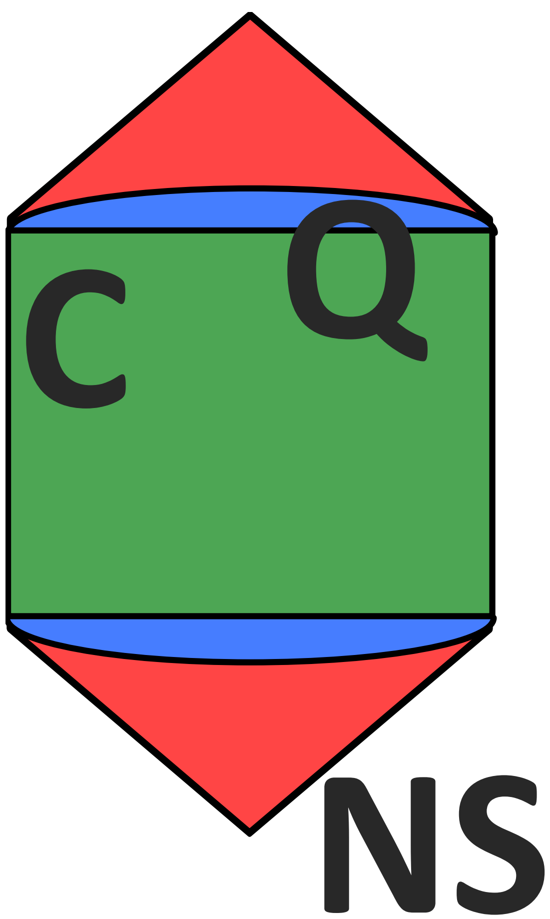

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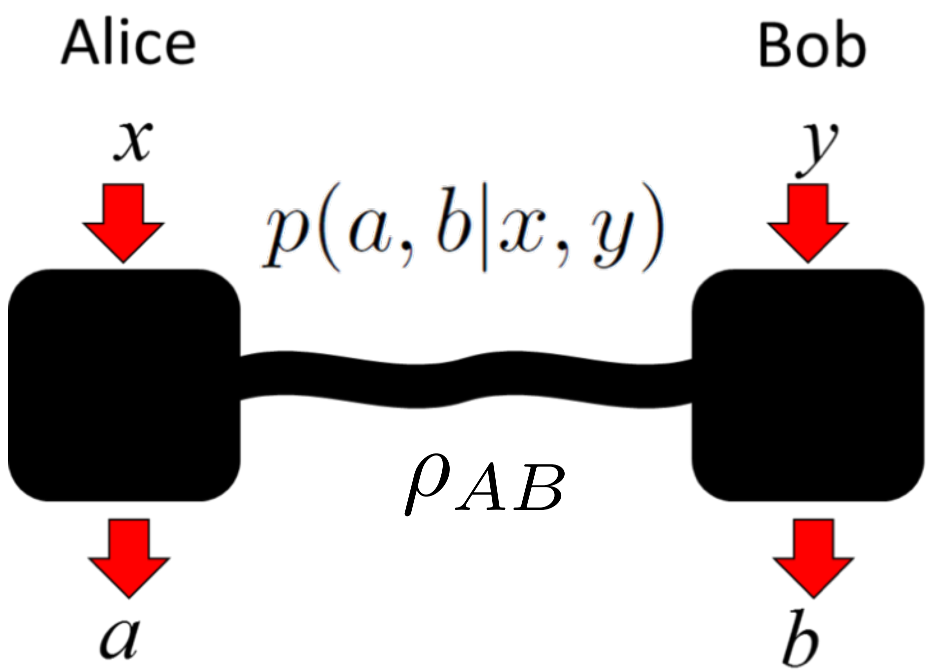

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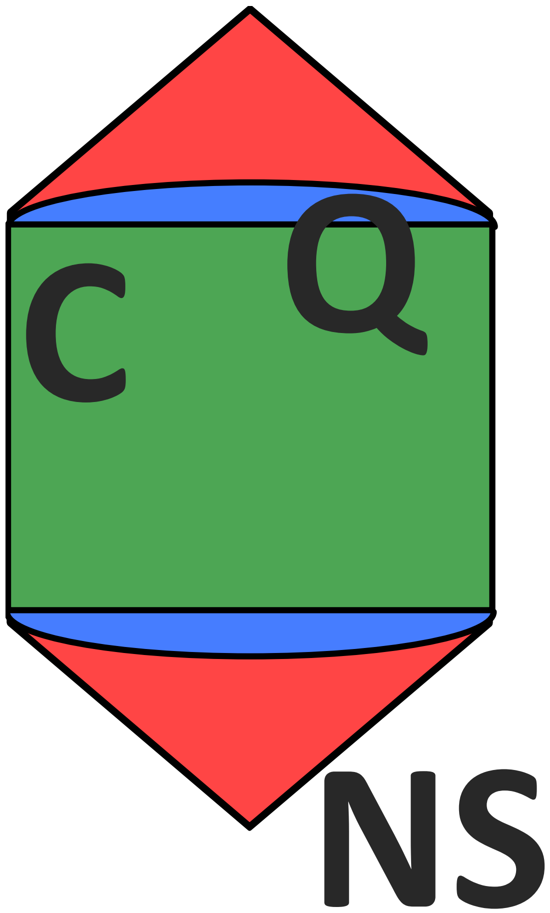

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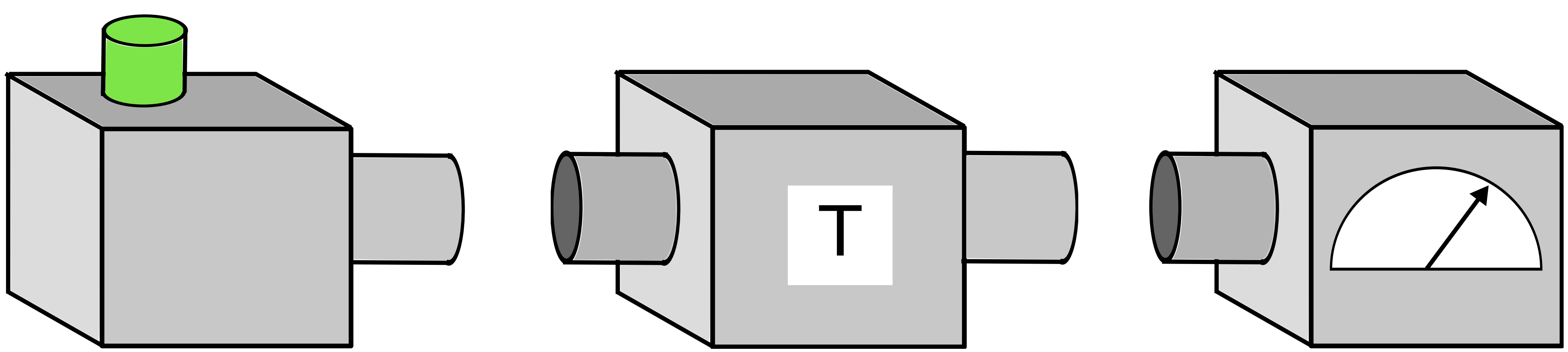
3 examples of a "**generalized probabilistic theory**".

Generalized probabilistic theories

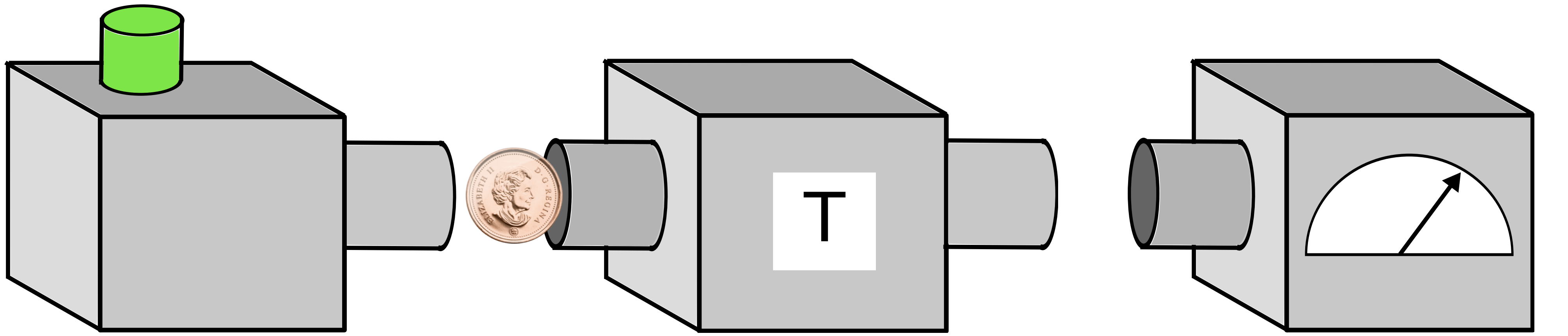


Preparation transformation measurement

Generalized probabilistic theories

Example: classical coin toss.

- On every push of button, the preparation device performs a biased coin toss.



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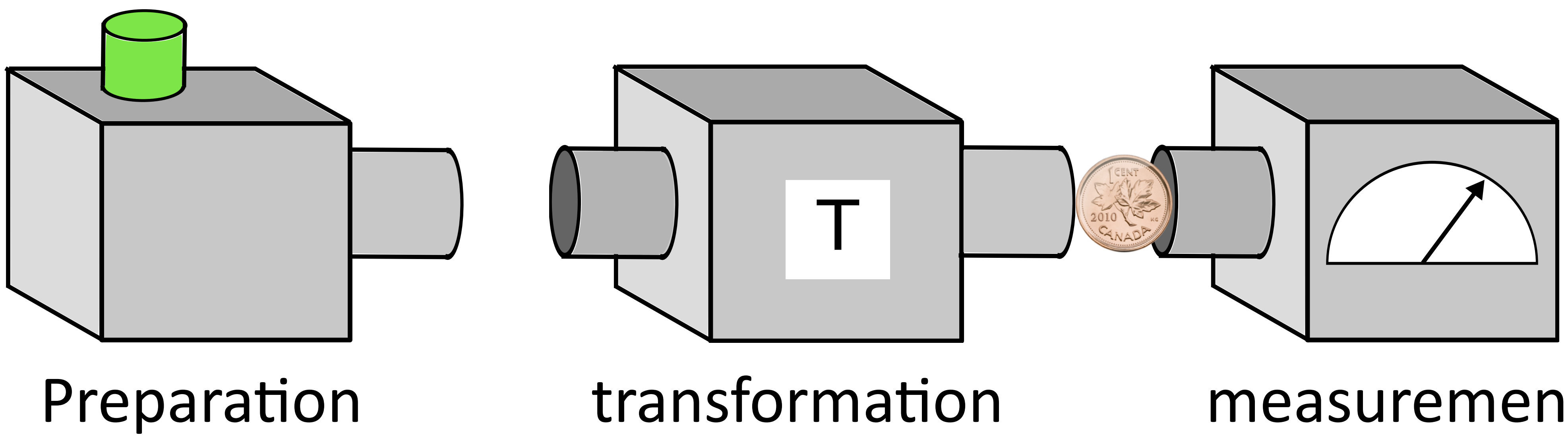

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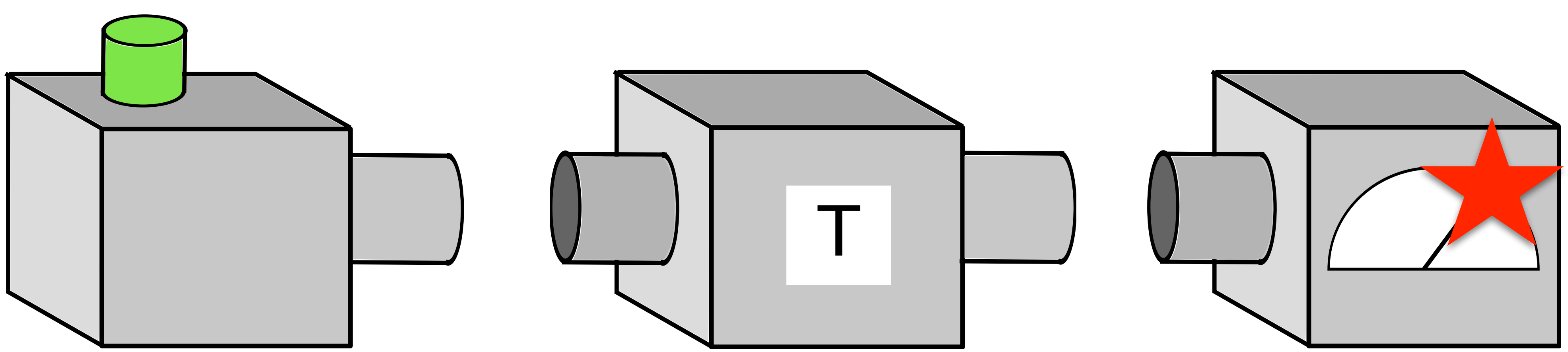
- The measurement outcome is "heads" or "tails".



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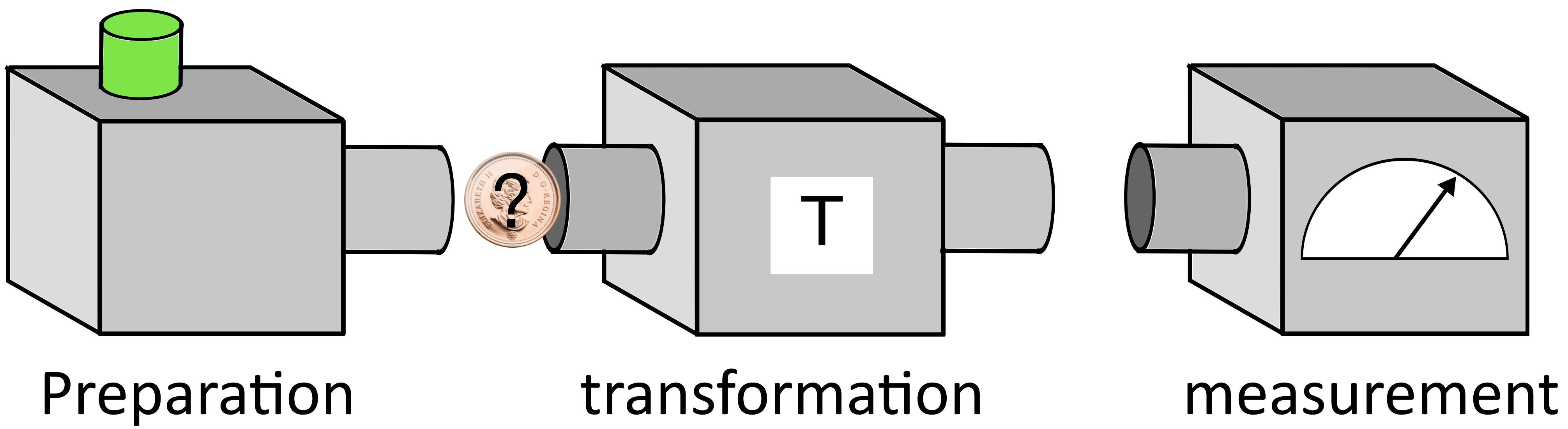
- The preparation device prepares a physical system in a state ω. Here

$$\omega = \begin{pmatrix} \text{Prob(heads)} \\ \text{Prob(tails)} \end{pmatrix} = \begin{pmatrix} p \\ 1-p \end{pmatrix}.$$



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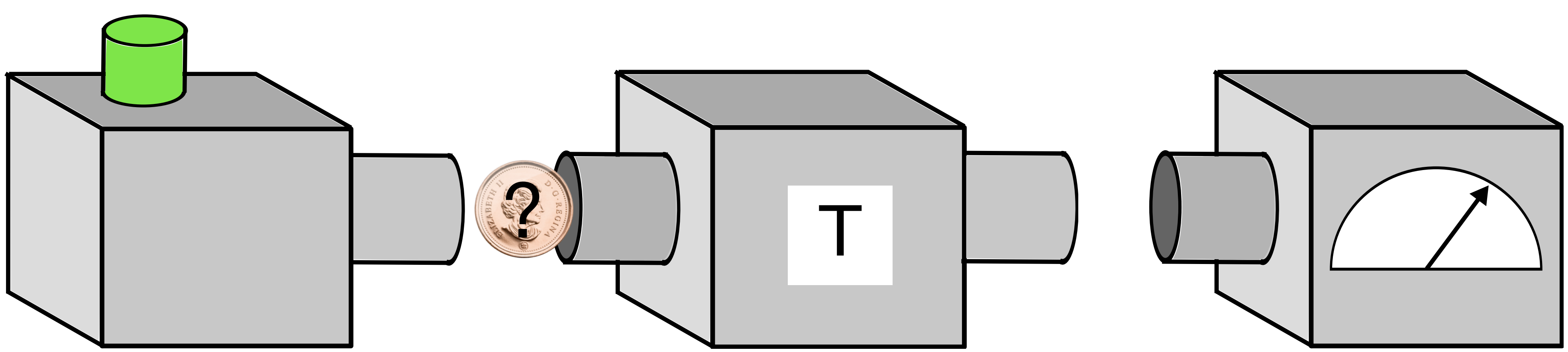

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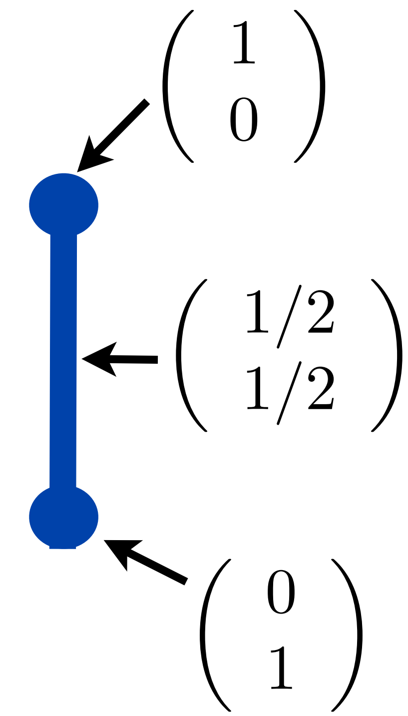

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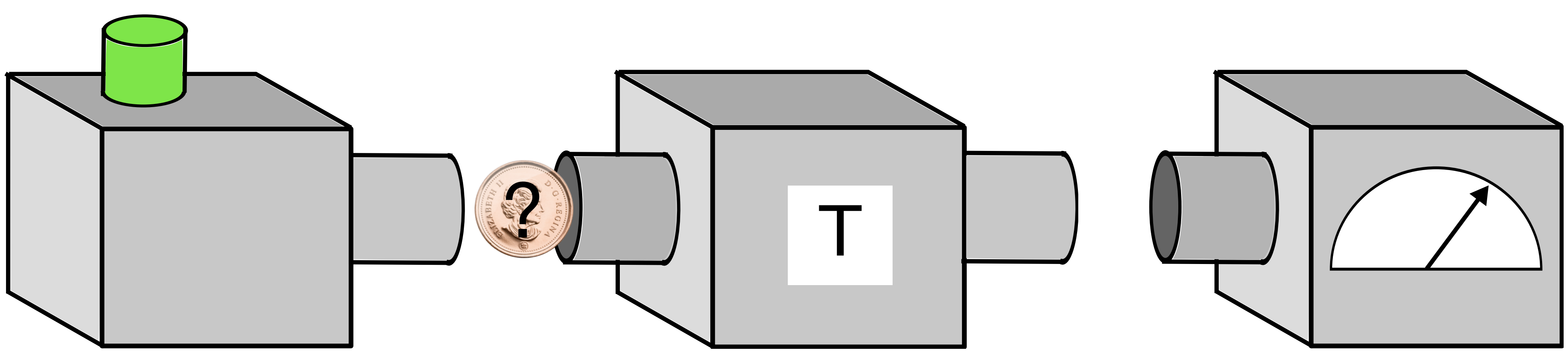

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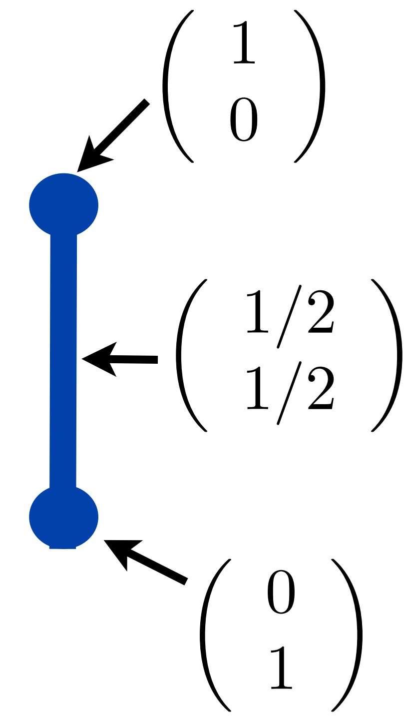

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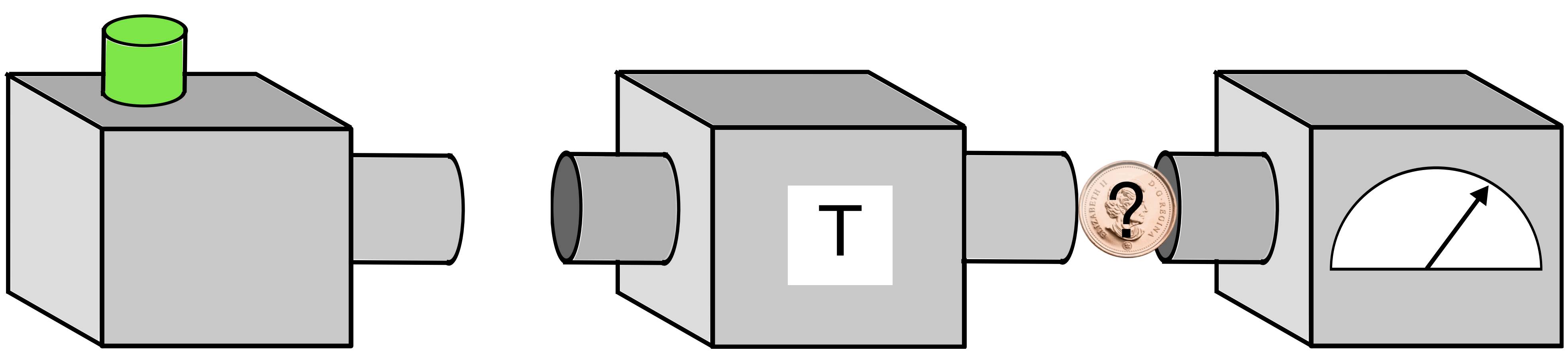

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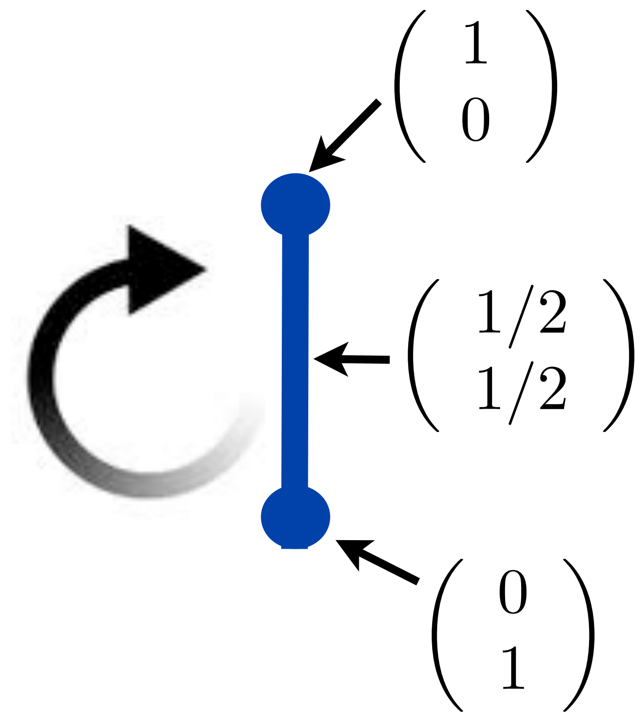

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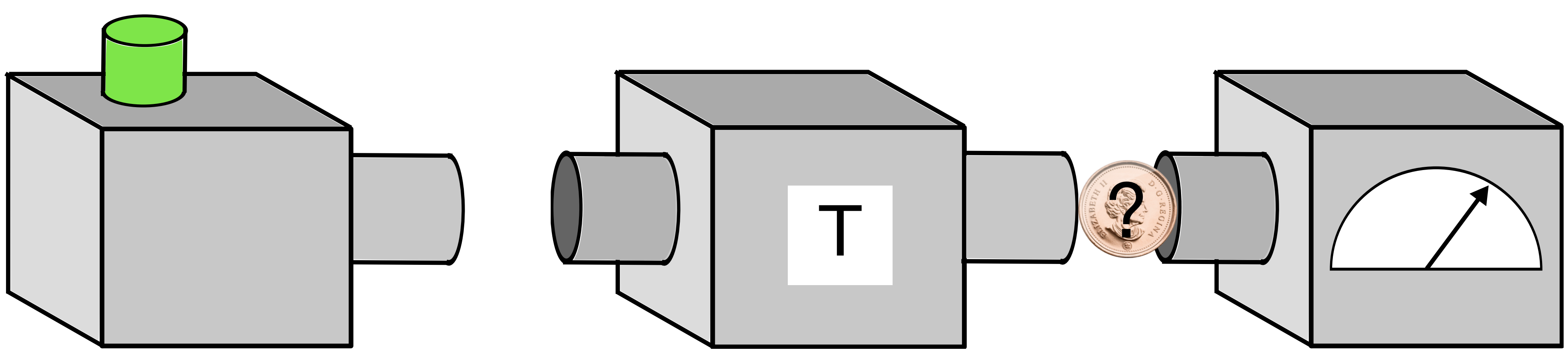
  Maps states to states and is linear.



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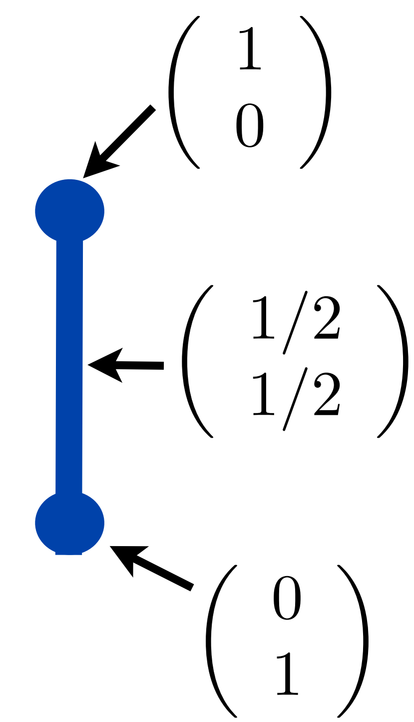

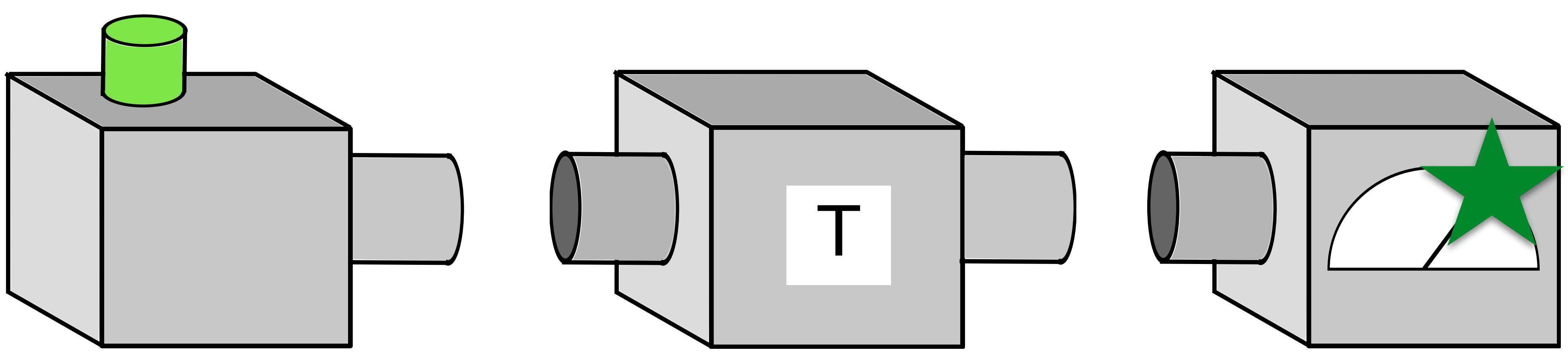

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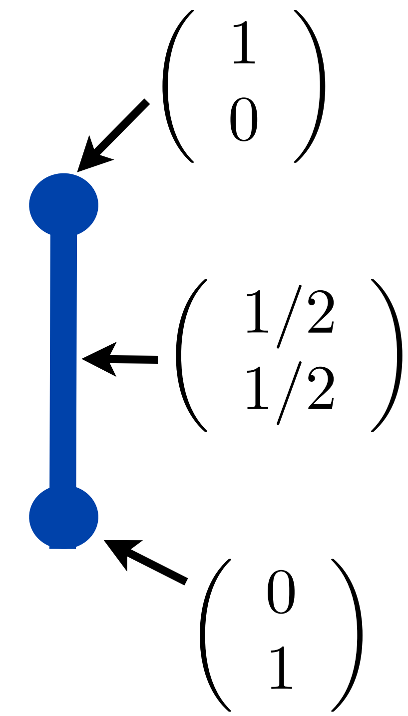
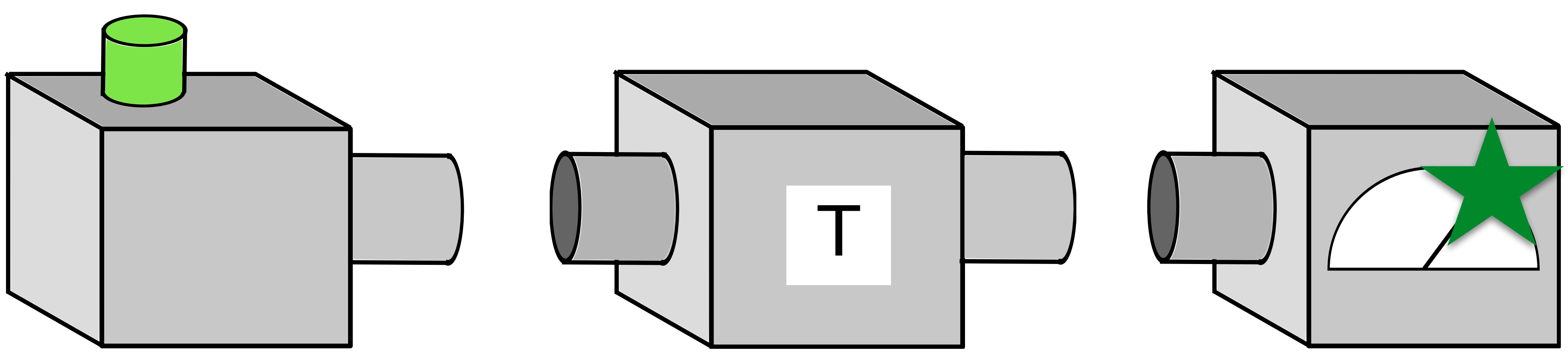
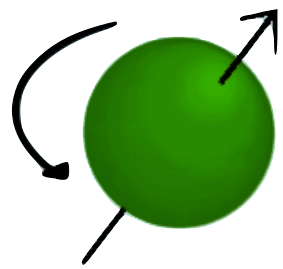
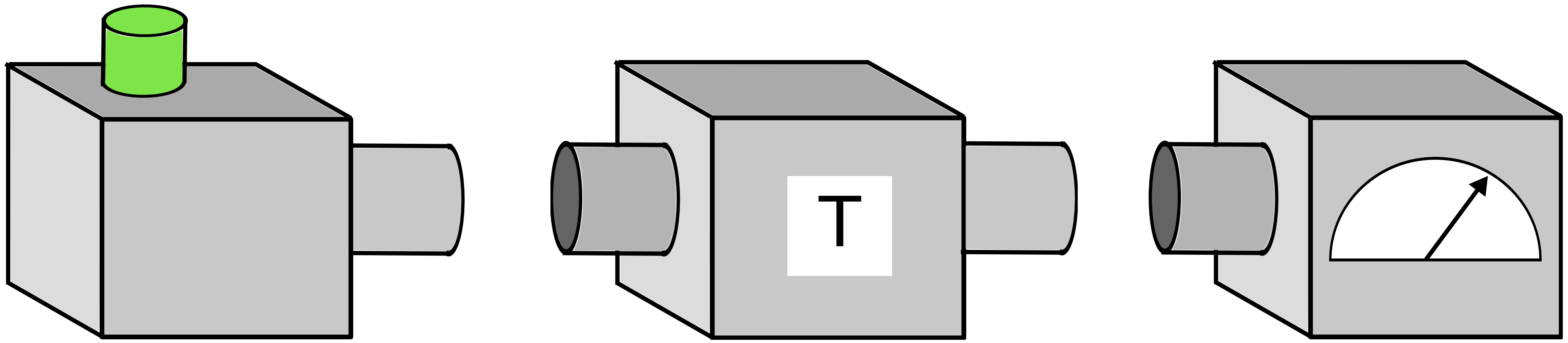




Generalized probabilistic theories

Example: quantum spin-1/2 particle.





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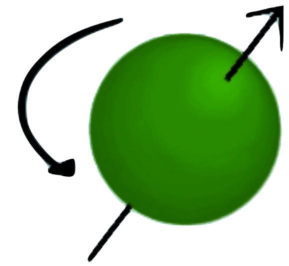

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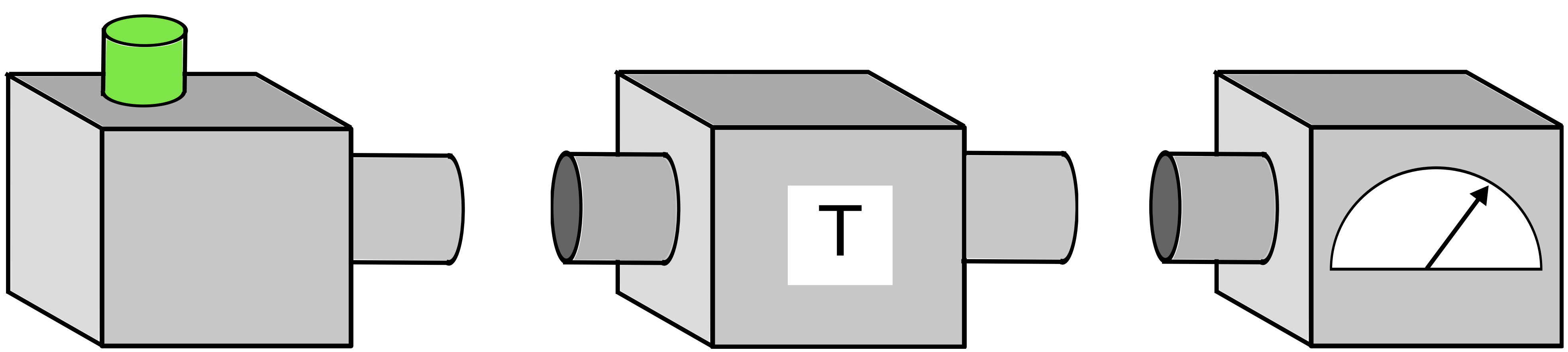
$$\alpha|\uparrow\rangle + \beta|\downarrow\rangle$$

More generally: ω is 2x2 density matrix.



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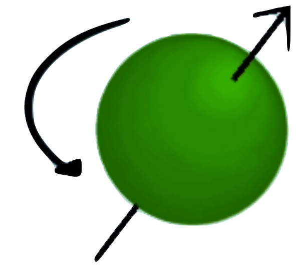

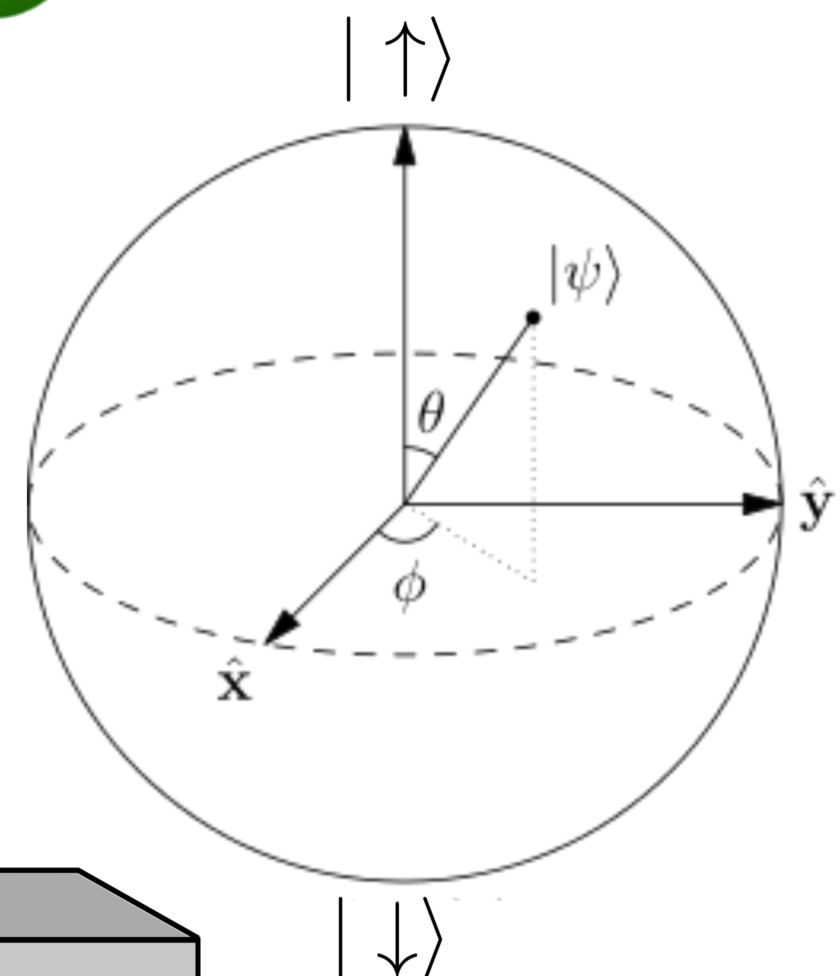

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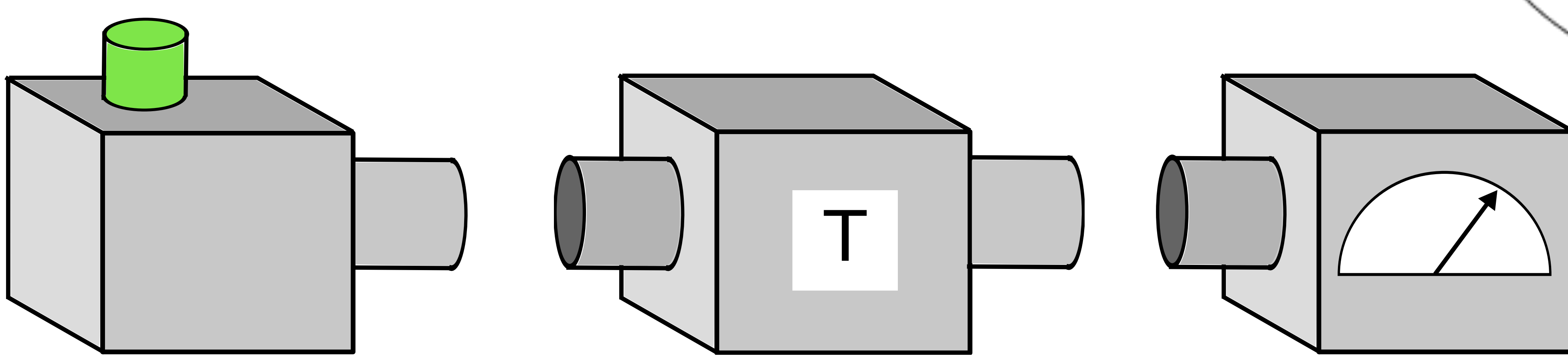

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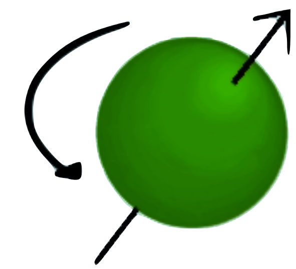

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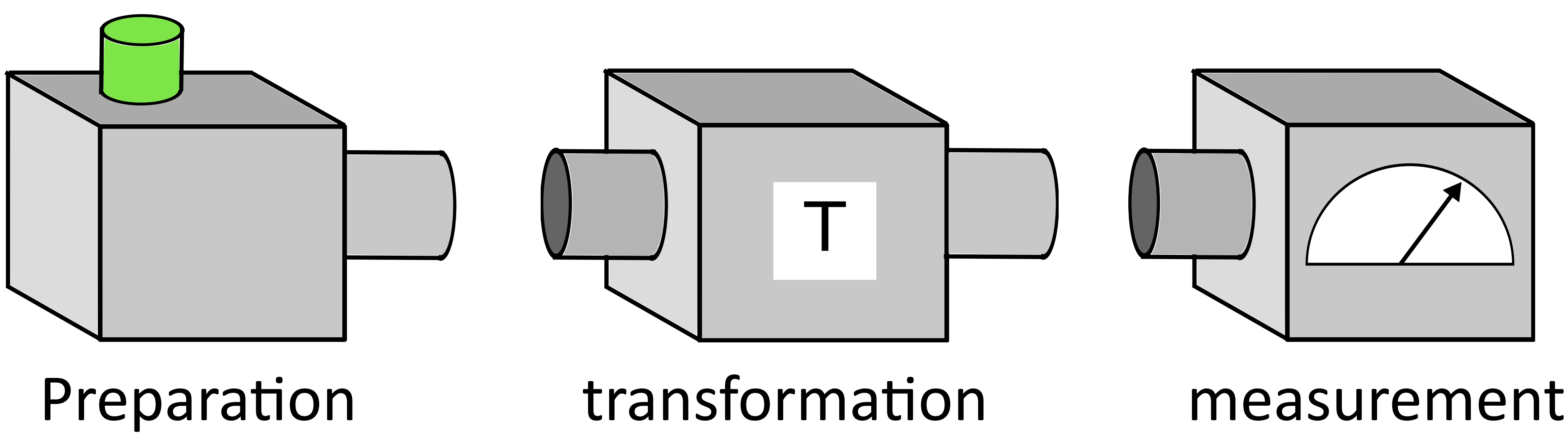

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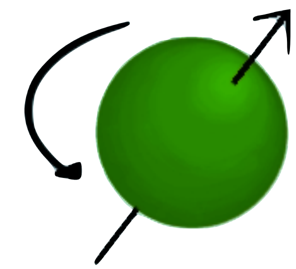

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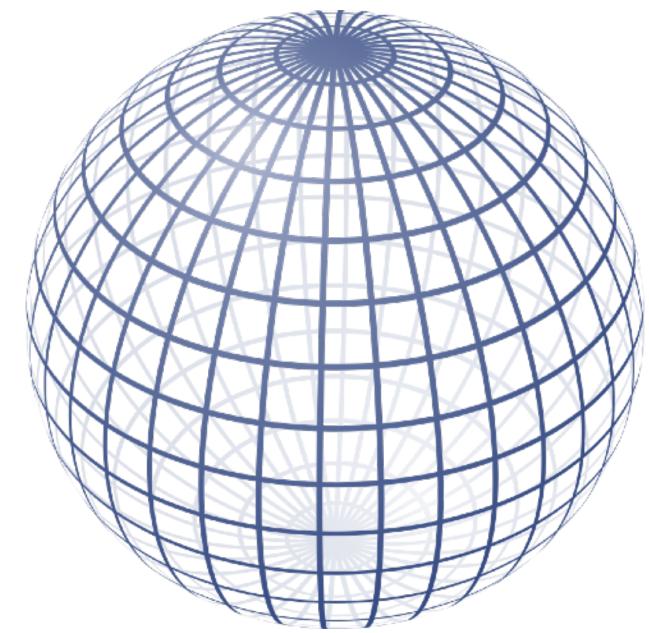
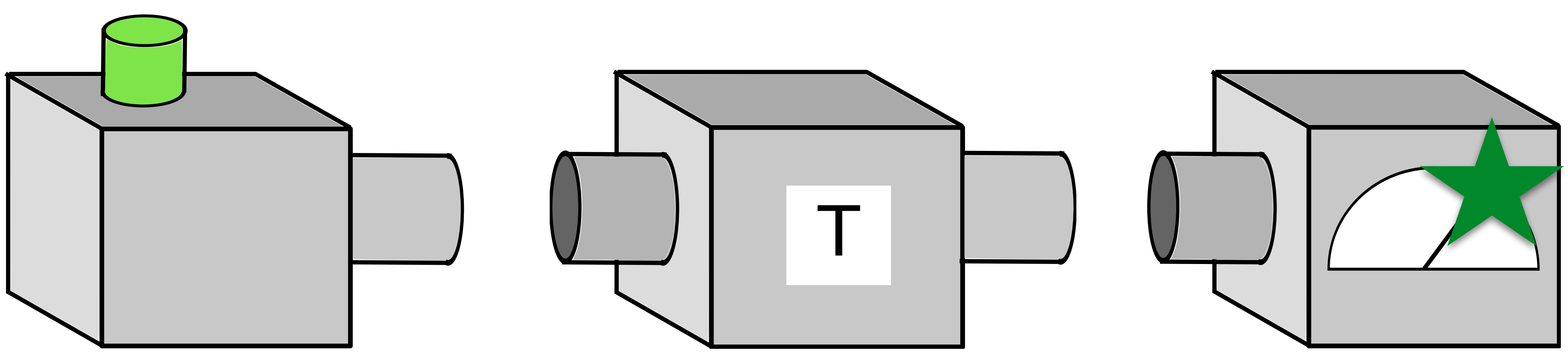
- Measurement in arbitrary spin direction *d*:

$$\mathrm{Prob}(\uparrow\,|\,\omega) = \mathrm{Tr}(P_d\,\omega)$$





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even: ω
odd: τ

$$\longrightarrow \quad \sigma = \frac{1}{2}\omega + \frac{1}{2}\tau$$

Generalized probabilistic theories

- What is a **state**?
  It is the thing that allows us to determine, *for all possible measurements,* the probabilities of the possible outcomes.

QT: Density matrix ρ.

Measure whether spin is up or down: $P(\mathrm{up}) = \mathrm{tr}(\rho |\uparrow\rangle\langle\uparrow|)$.

- What is a **state space**?
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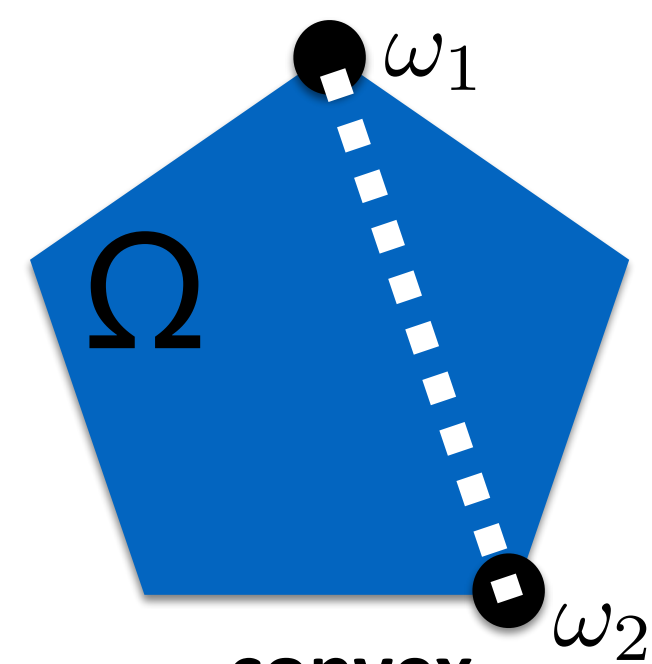

convex

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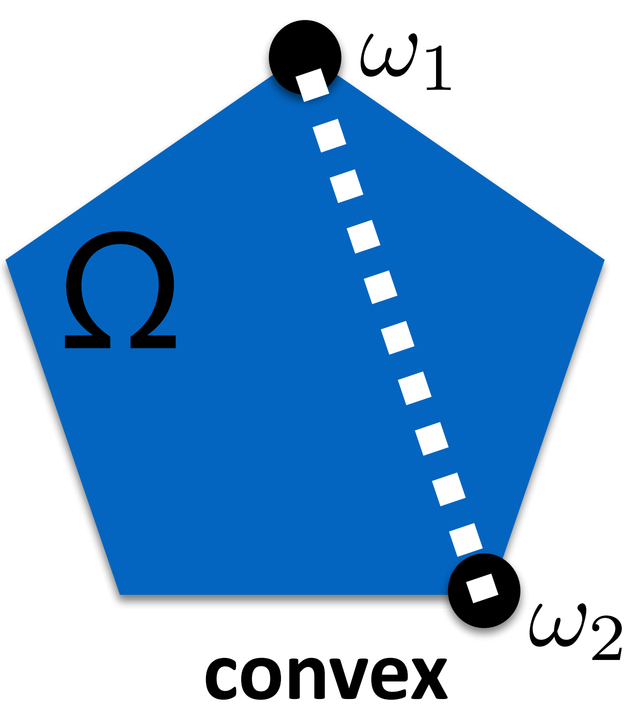

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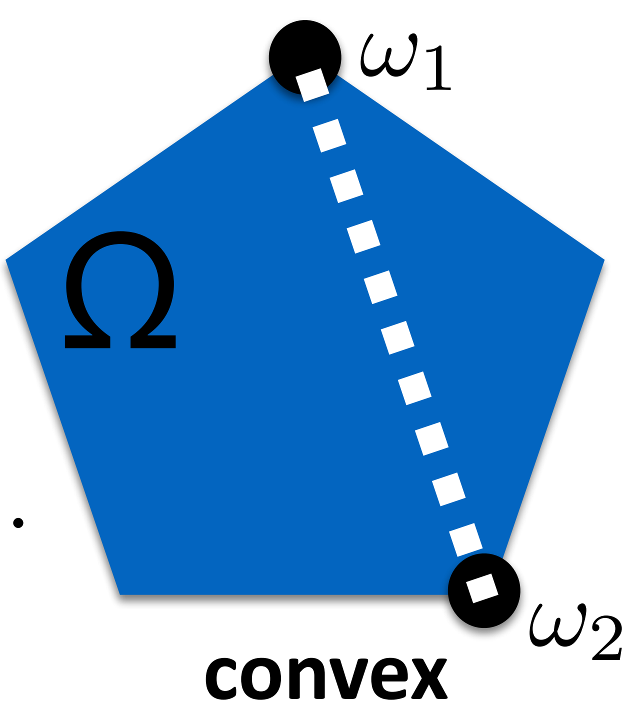
CPT: $\Omega = \{(p_1, \ldots, p_N) \mid p_i \geq 0, \ \sum_i p_i = 1\}.$



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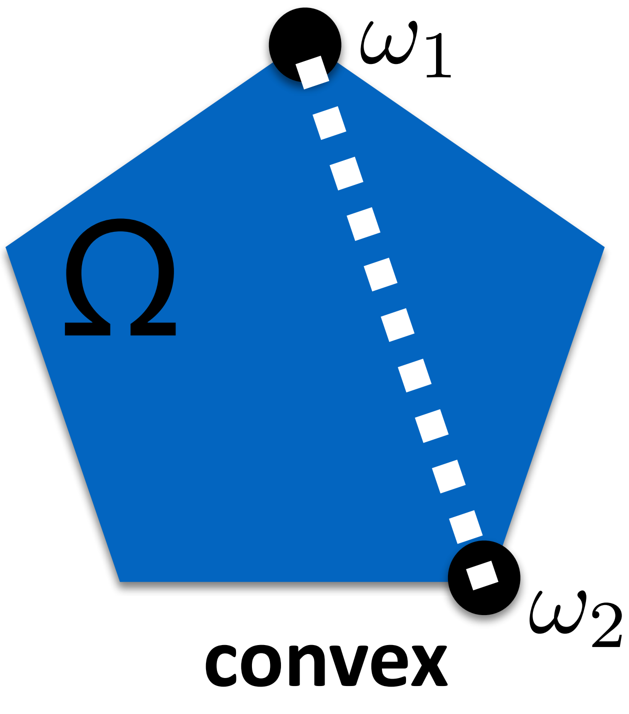

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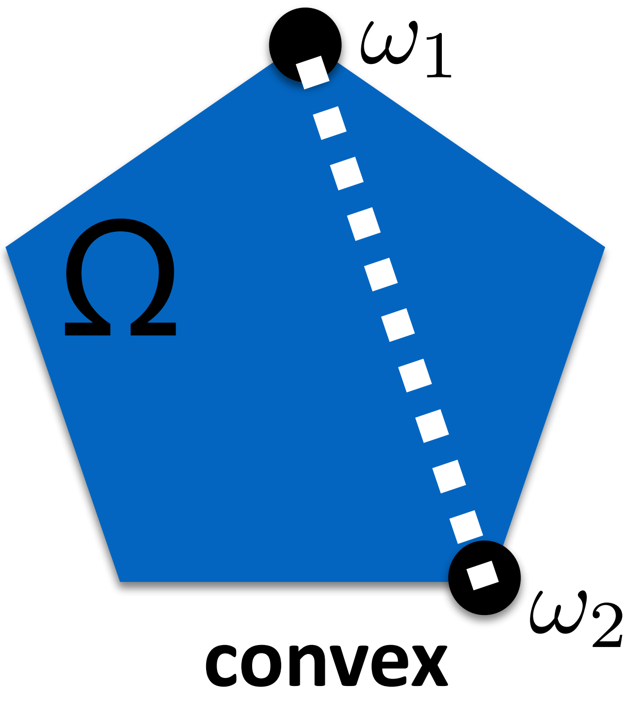

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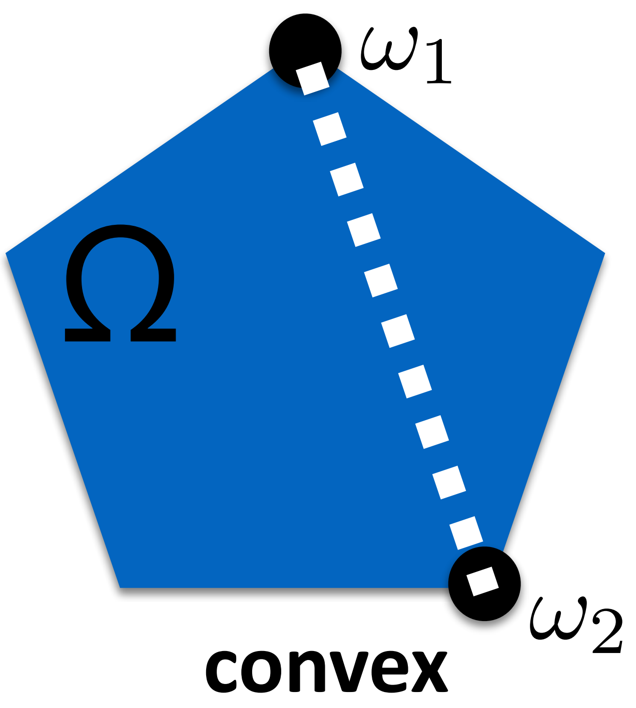

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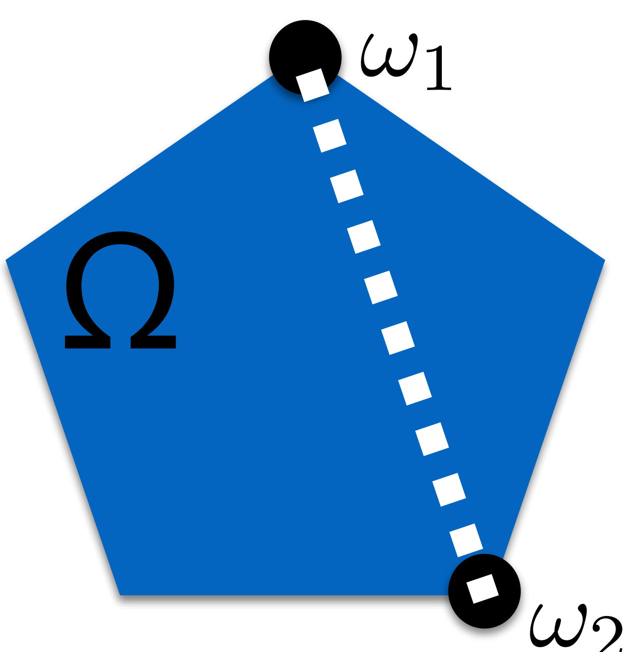
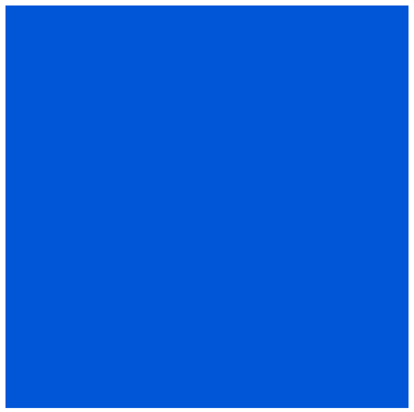


convex

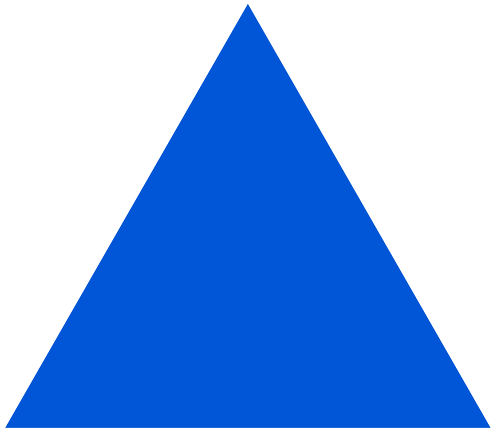
Generalized probabilistic theories

classical
bit

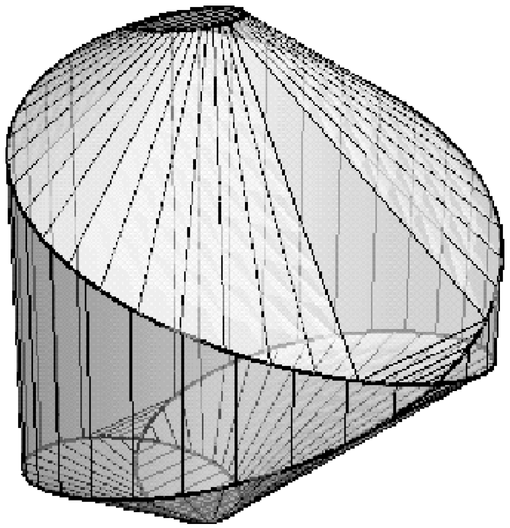
quantum
bit



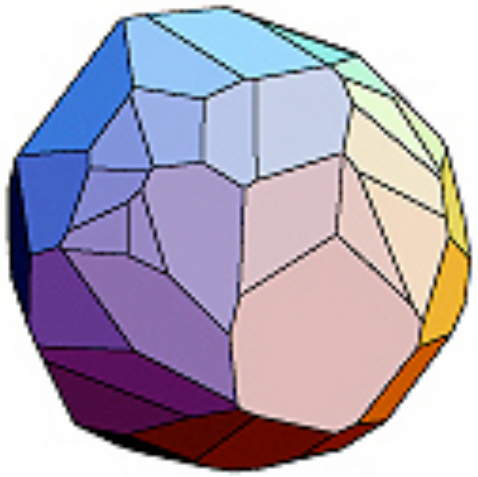
"gbit"



Classical trit
(3-level-system)



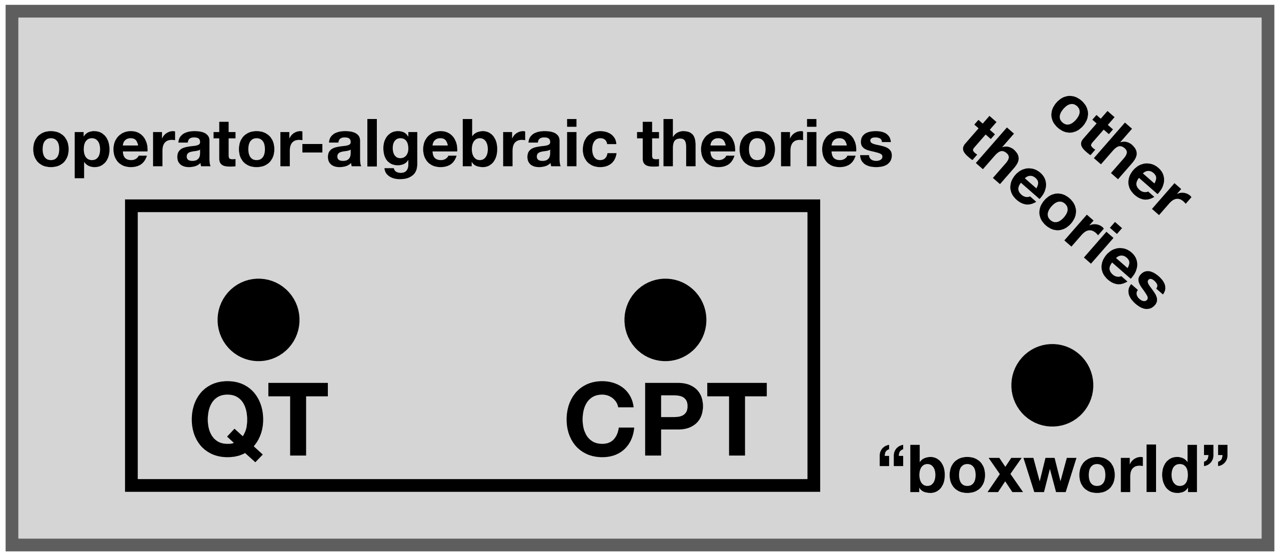
Quantum trit:
8D and complicated!



Arbitrary convex
state space

Generalized probabilistic theories

There is a large landscape of state spaces, or *theories* (collections of allowed state spaces):



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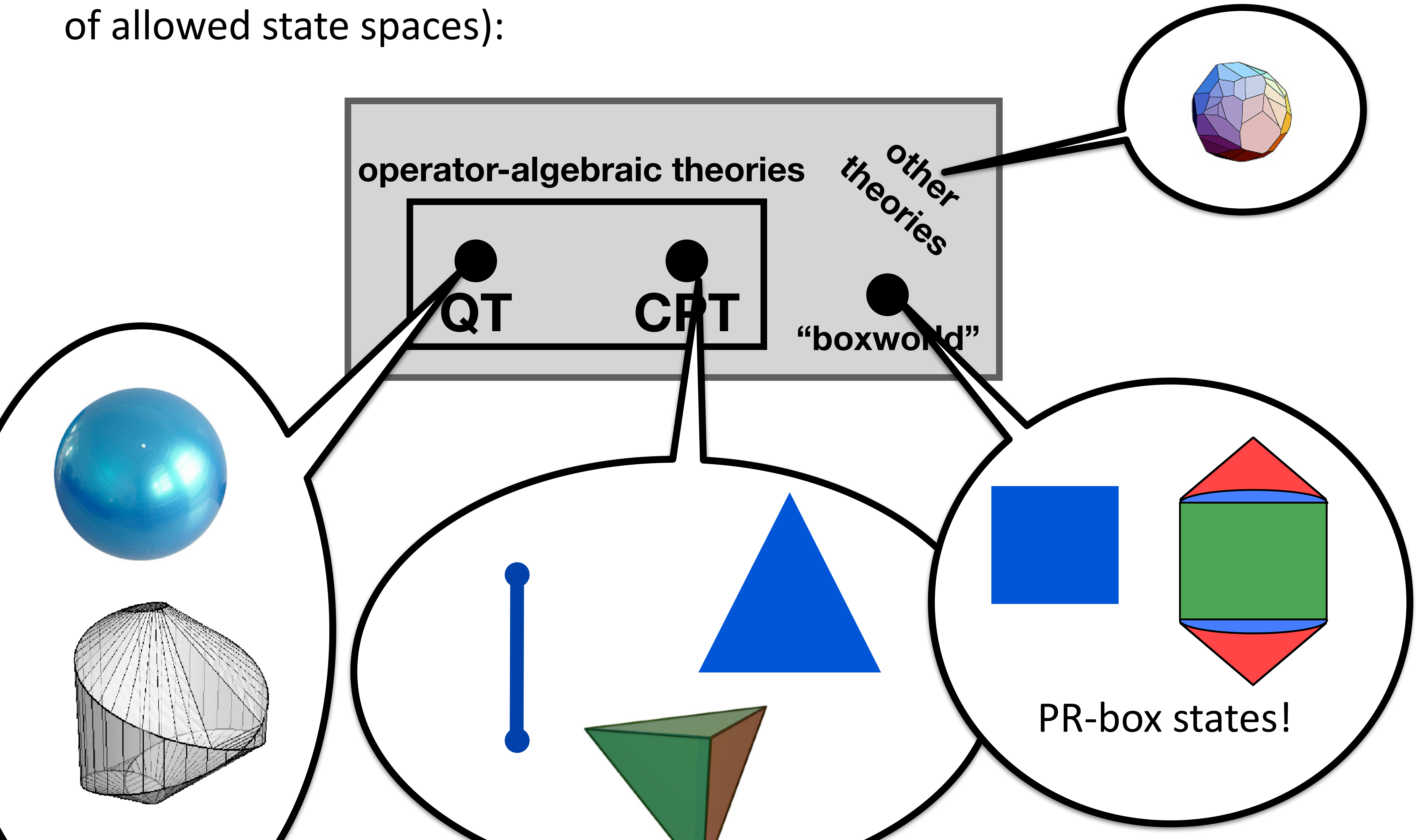

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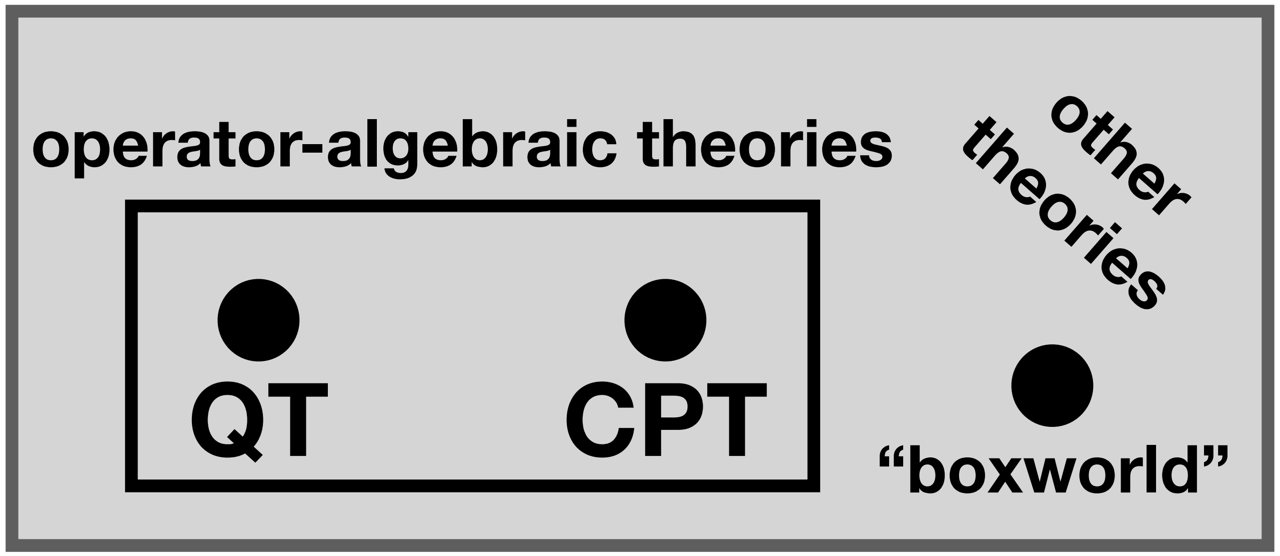

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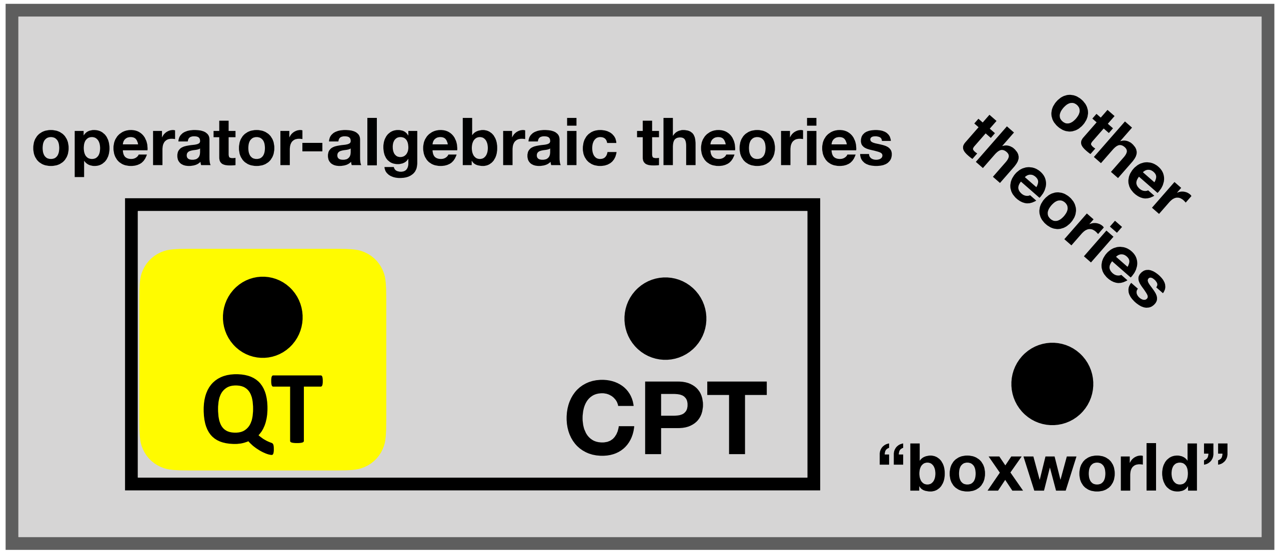

Goal: Find a small set of simple physical / information-theoretic **principles** that singles out QT uniquely.

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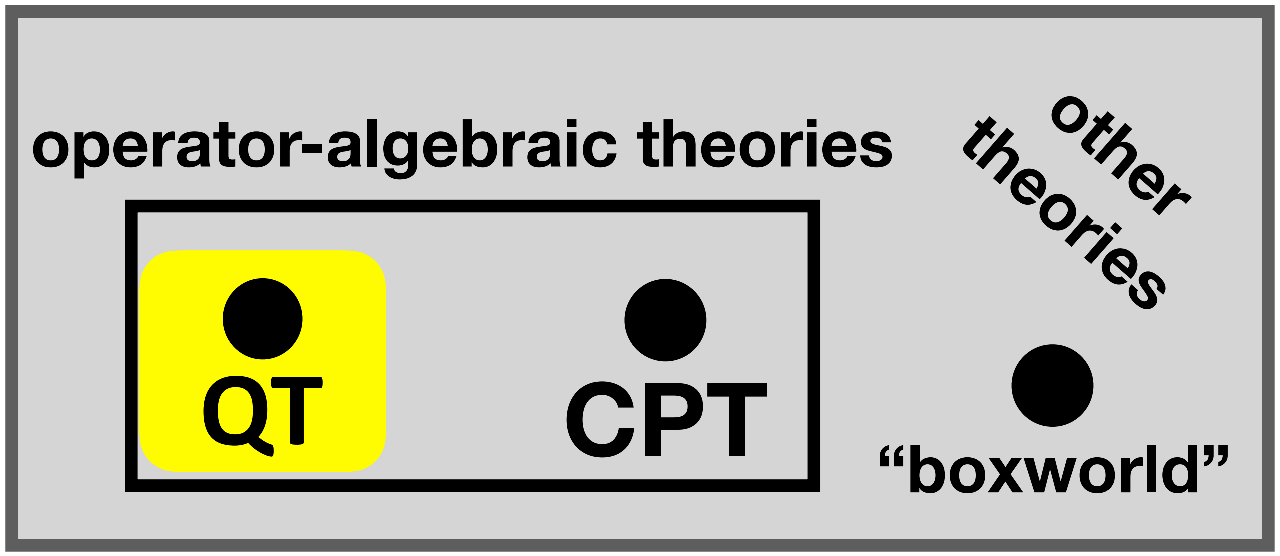
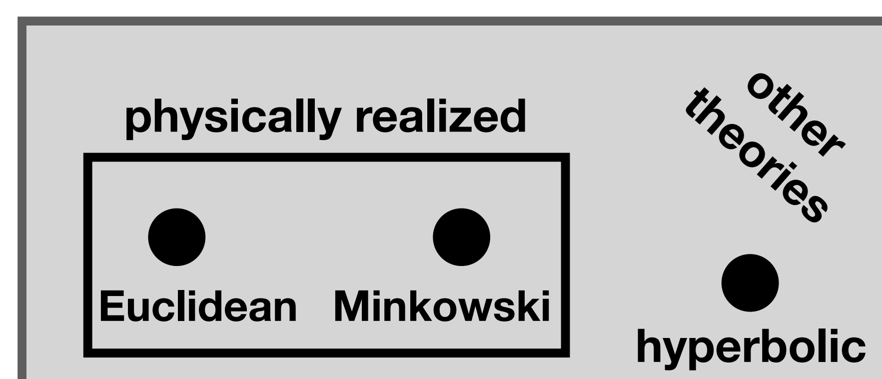


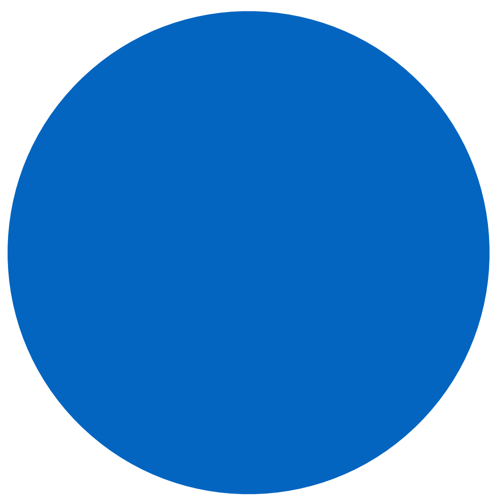
Goal: Find a small set of simple physical / information-theoretic **principles** that singles out QT uniquely.

Role model: Einstein's **Relativity Principle** and **Light Postulate** determine Minkowski spacetime.



Generalized probabilistic theories

Physical properties depend (strongly!) on the **shape** of the state space.



quantum
bit (over $\mathbb{R}$)

"gbit"

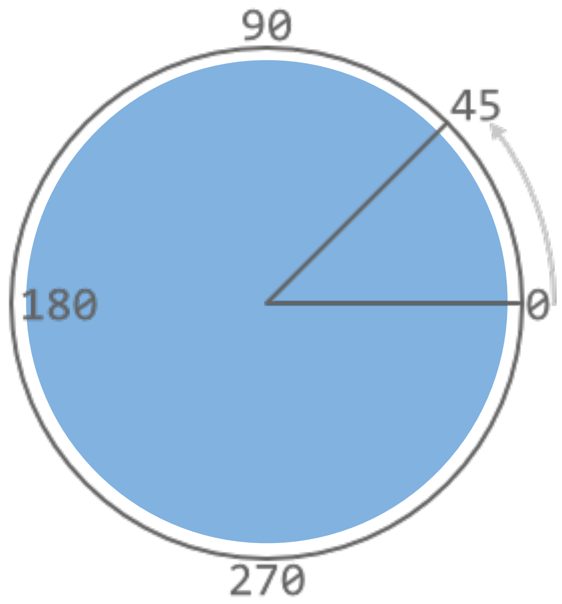
$$\rho = \frac{1}{2}\begin{pmatrix} 1+r_3 & r_1 - ir_2 \\ r_1 + ir_2 & 1 - r_3 \end{pmatrix},$$

$$r_2 := 0.$$

Generalized probabilistic theories

Physical properties depend (strongly!) on the **shape** of the state space.

The qubit admits "much more" reversible transformations than the gbit.



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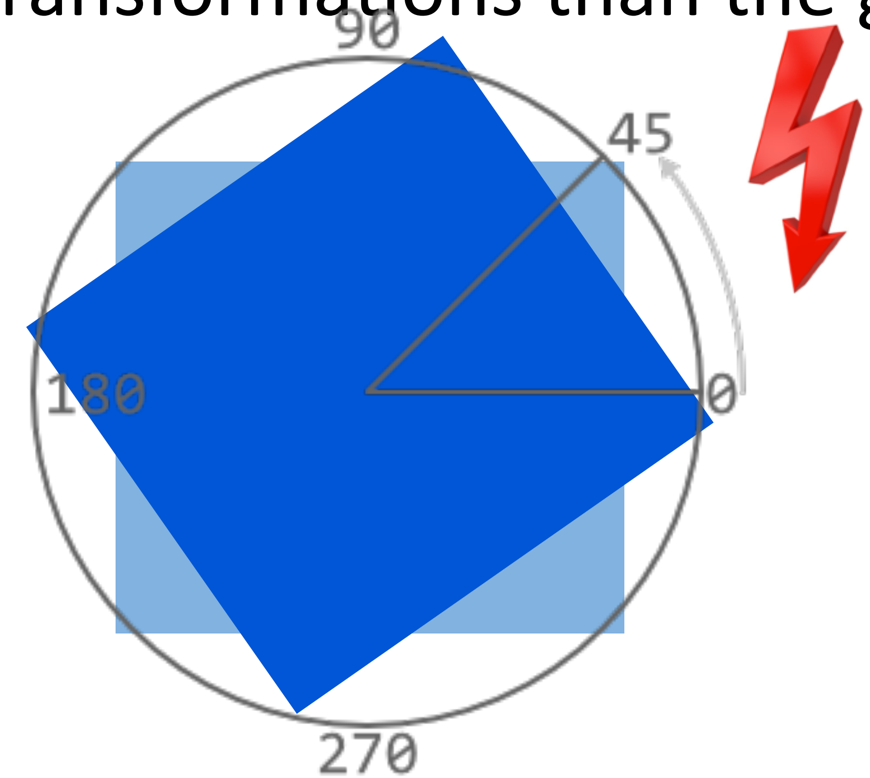

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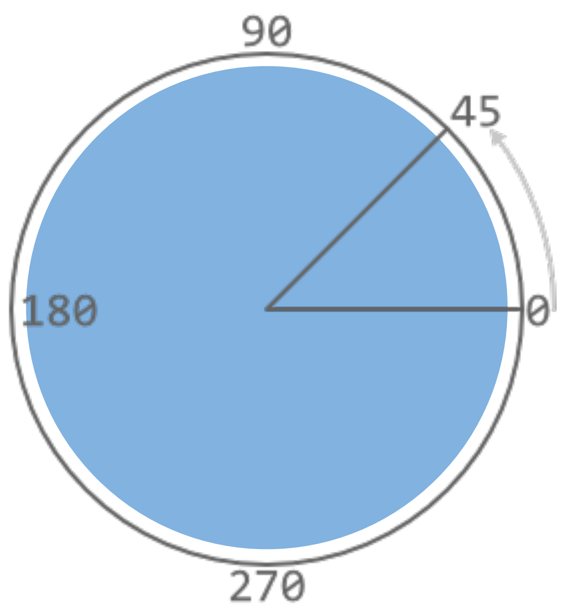

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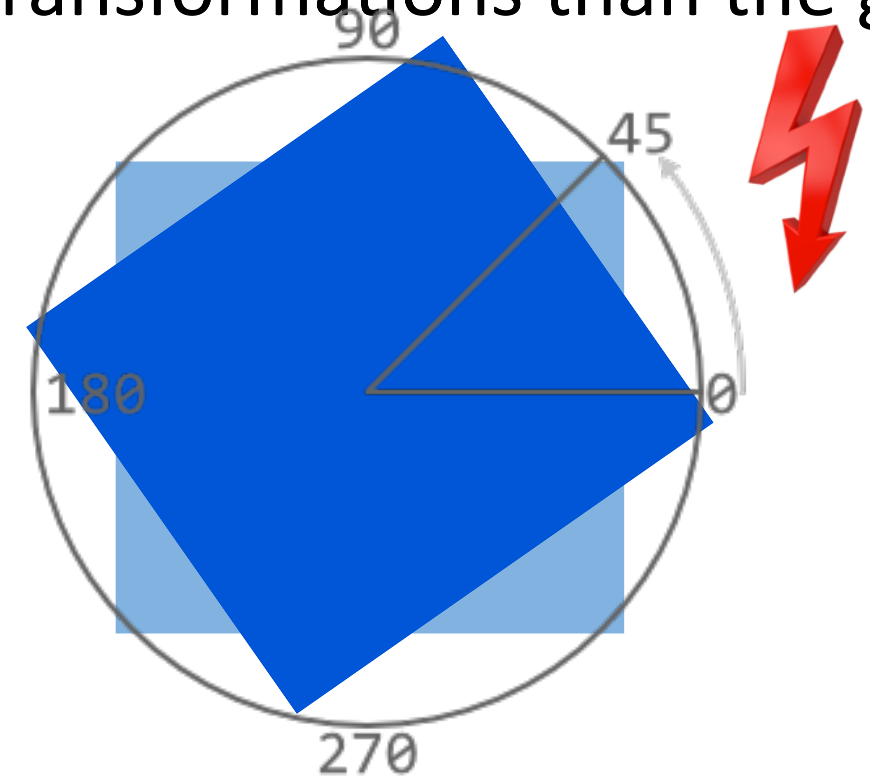

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$\mathcal{G} \simeq \mathrm{SO}(2)$

$\mathcal{G} \simeq D_4$ (dihedral group)

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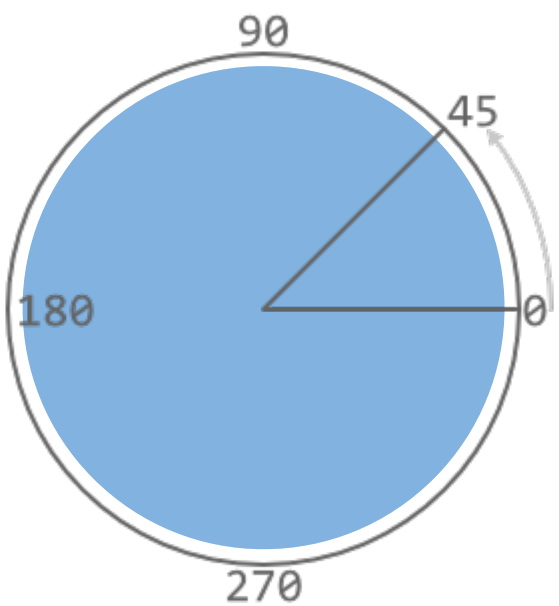

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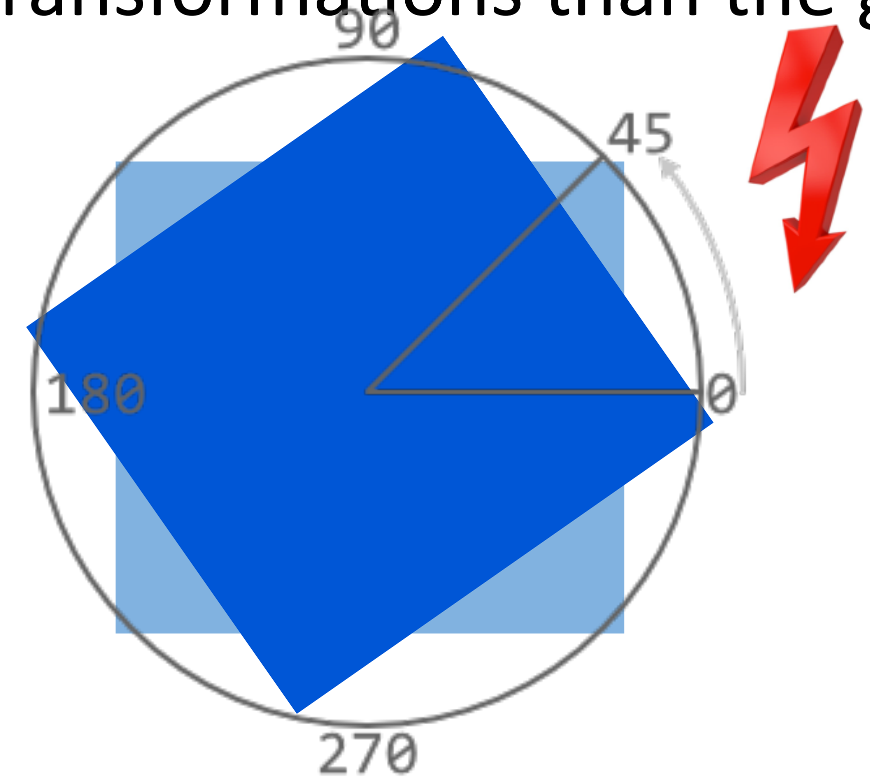

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In particular, the qubit admits **continuous reversible time evolution**, but the gbit does not.

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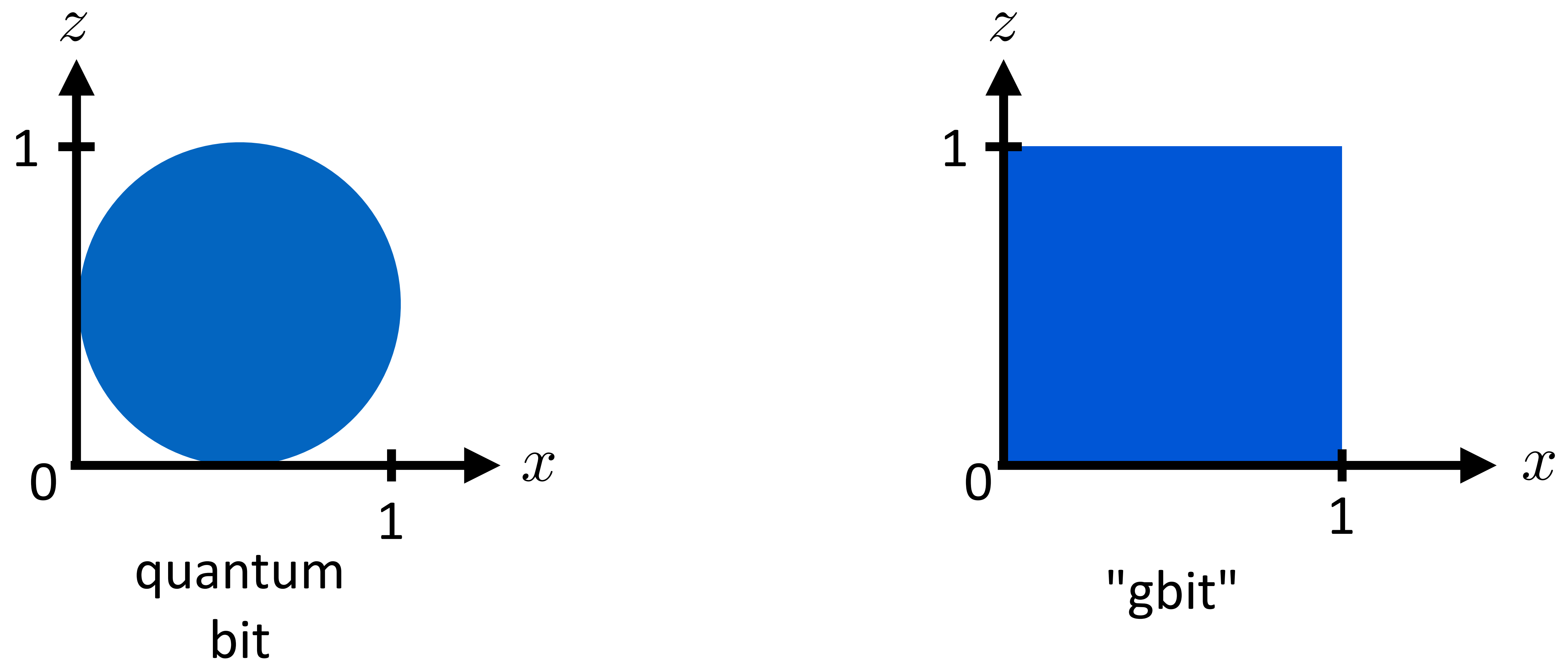

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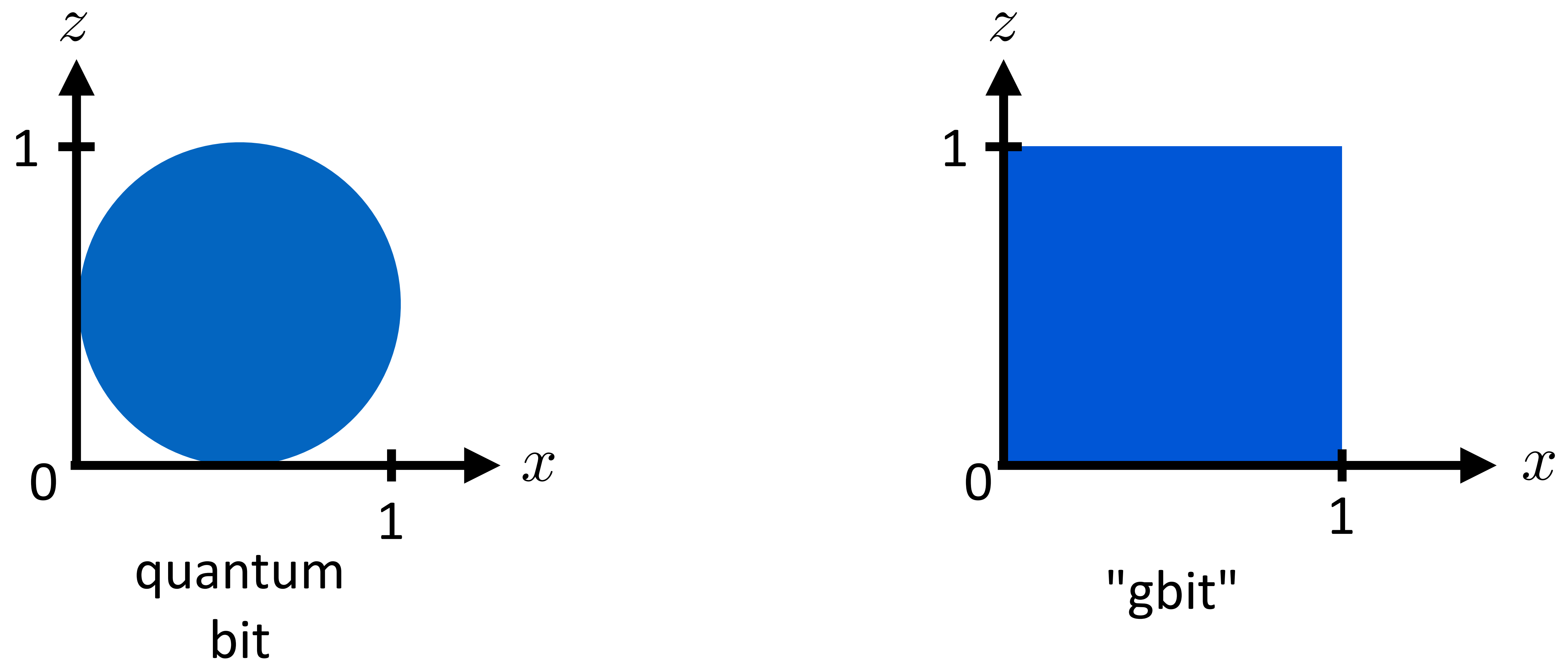

In both cases, can think of a 2-outcome "X"-measurement with

$$\mathrm{Prob}(\mathrm{yes}|X,\omega) = x, \quad \mathrm{Prob}(\mathrm{no}|X,\omega) = 1 - x.$$

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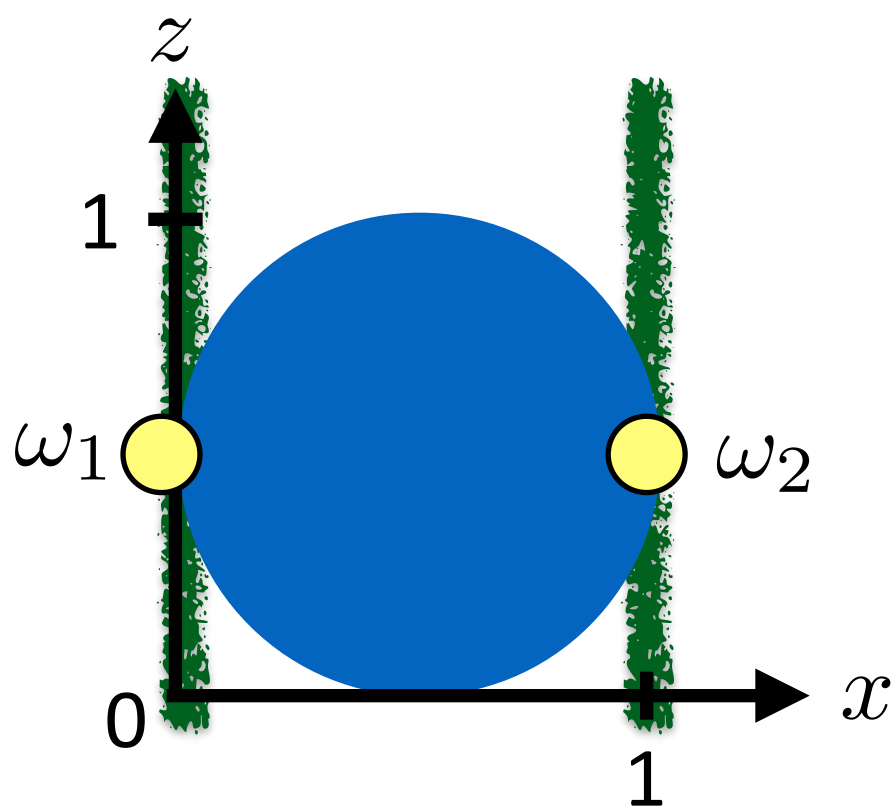
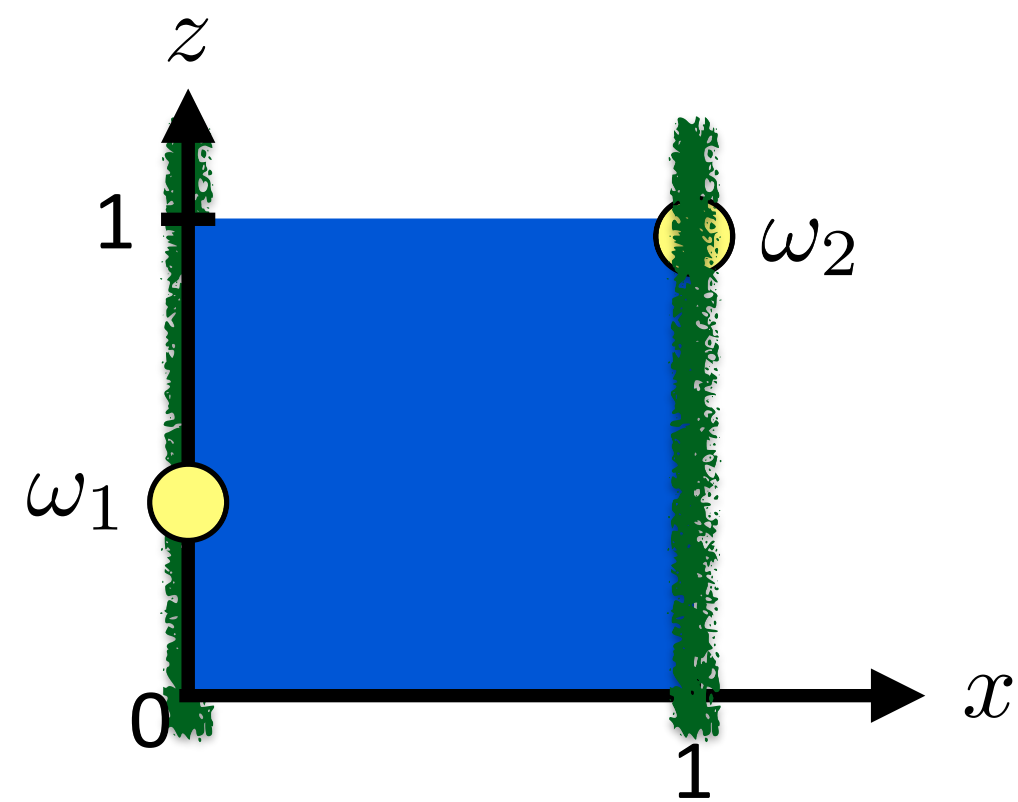


quantum bit



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Distinguishes two states ω_1, ω_2 perfectly (deterministically).

Generalized probabilistic theories

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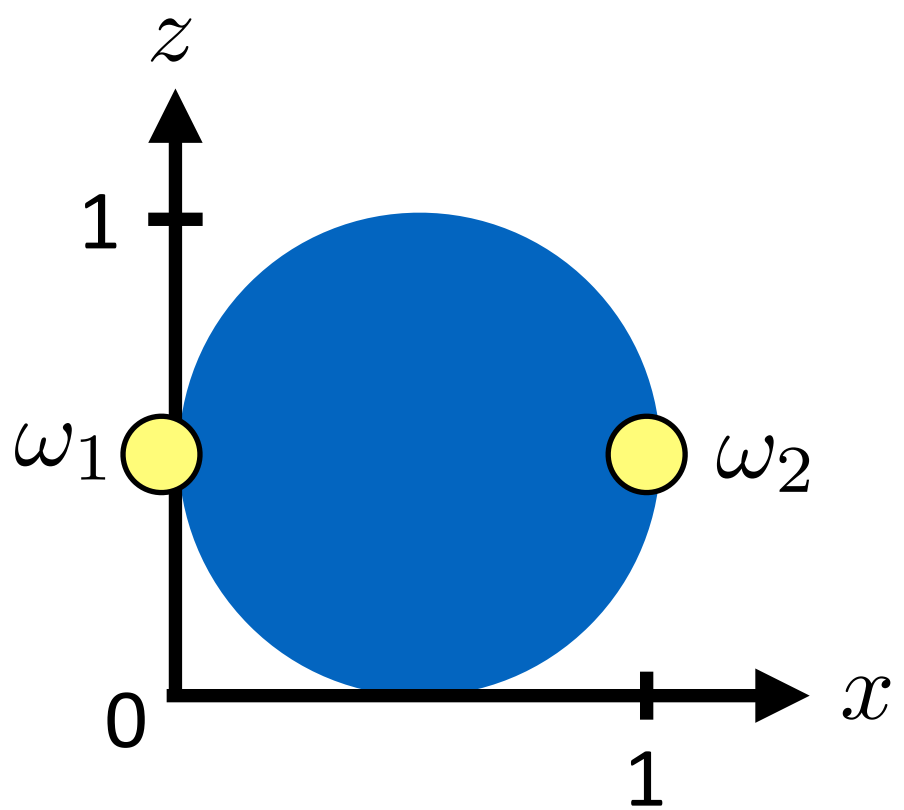
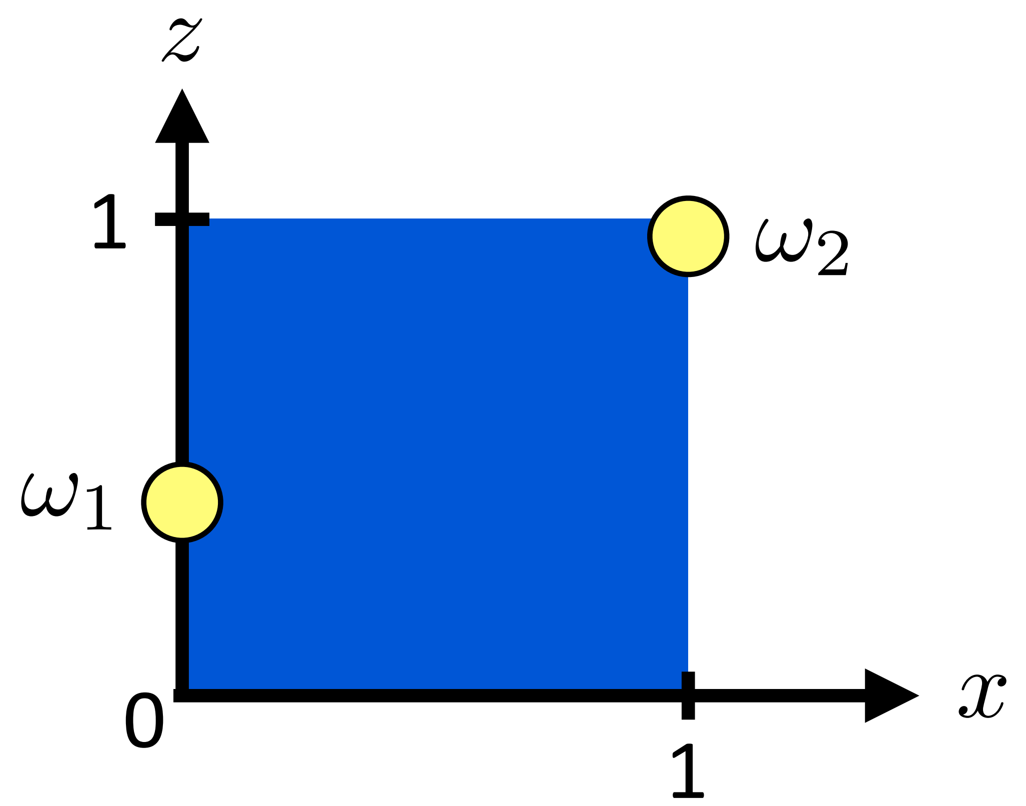


quantum bit



"gbit"

In both cases, the maximal number of perfectly distinguishable states is 2:

$\omega_1, \ldots, \omega_n$ states, $e_1, \ldots, e_n$ effects summing to unit effect,

$e_i(\omega_j) = \delta_{ij} \quad \Rightarrow n \leq 2.$

Thus, these are both "**bits**".

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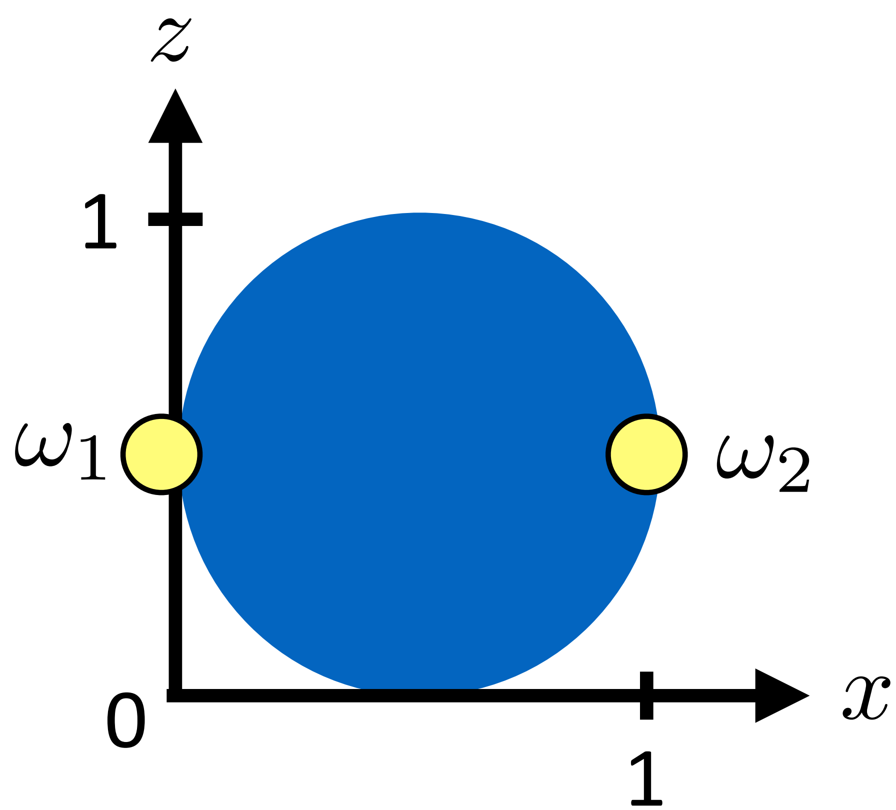

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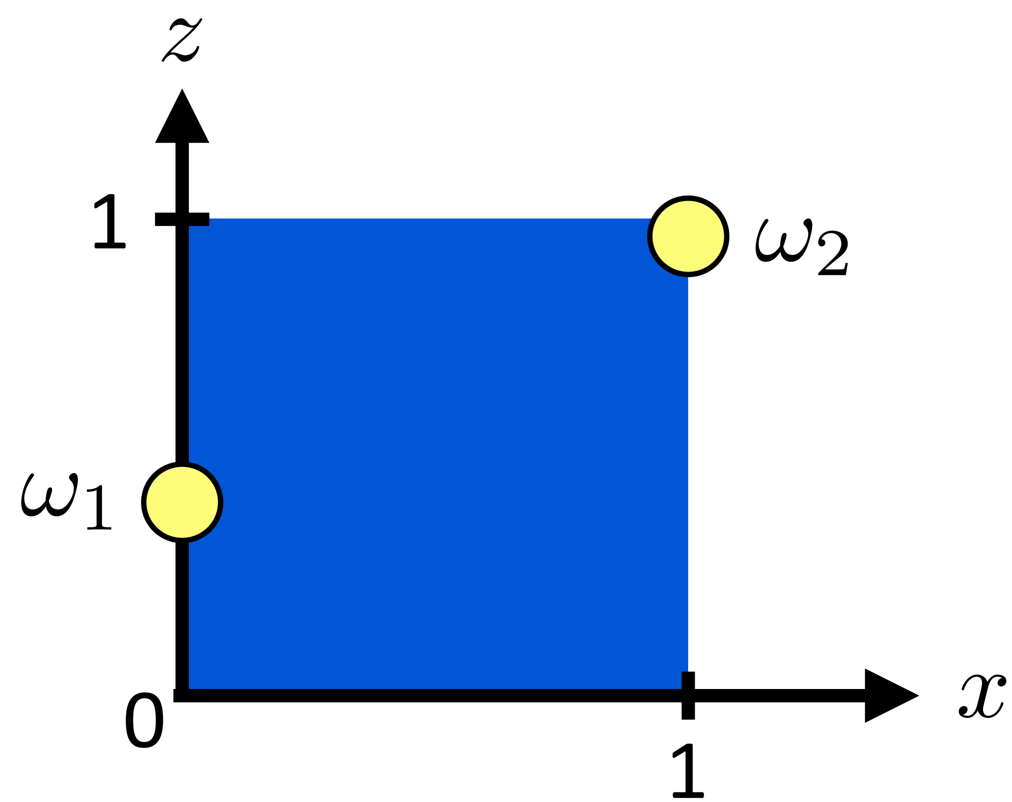

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Both feature "**complementarity**": cannot predict X and Z simultaneously.

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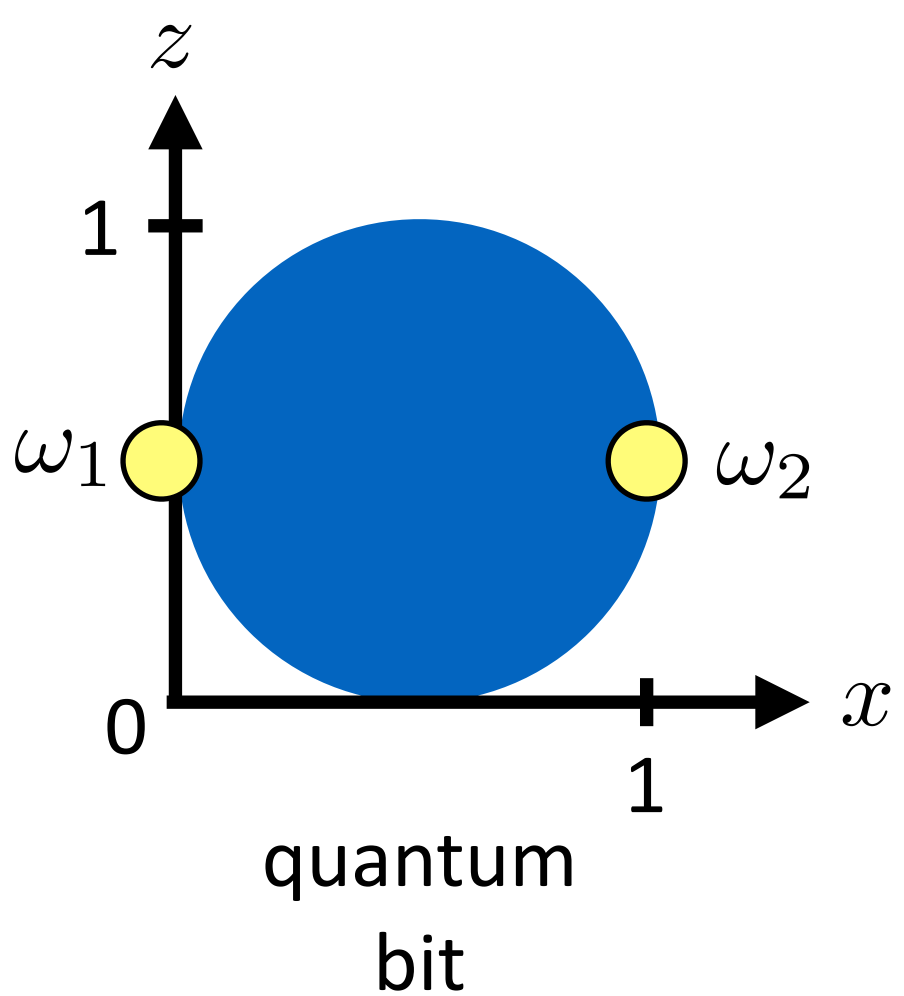

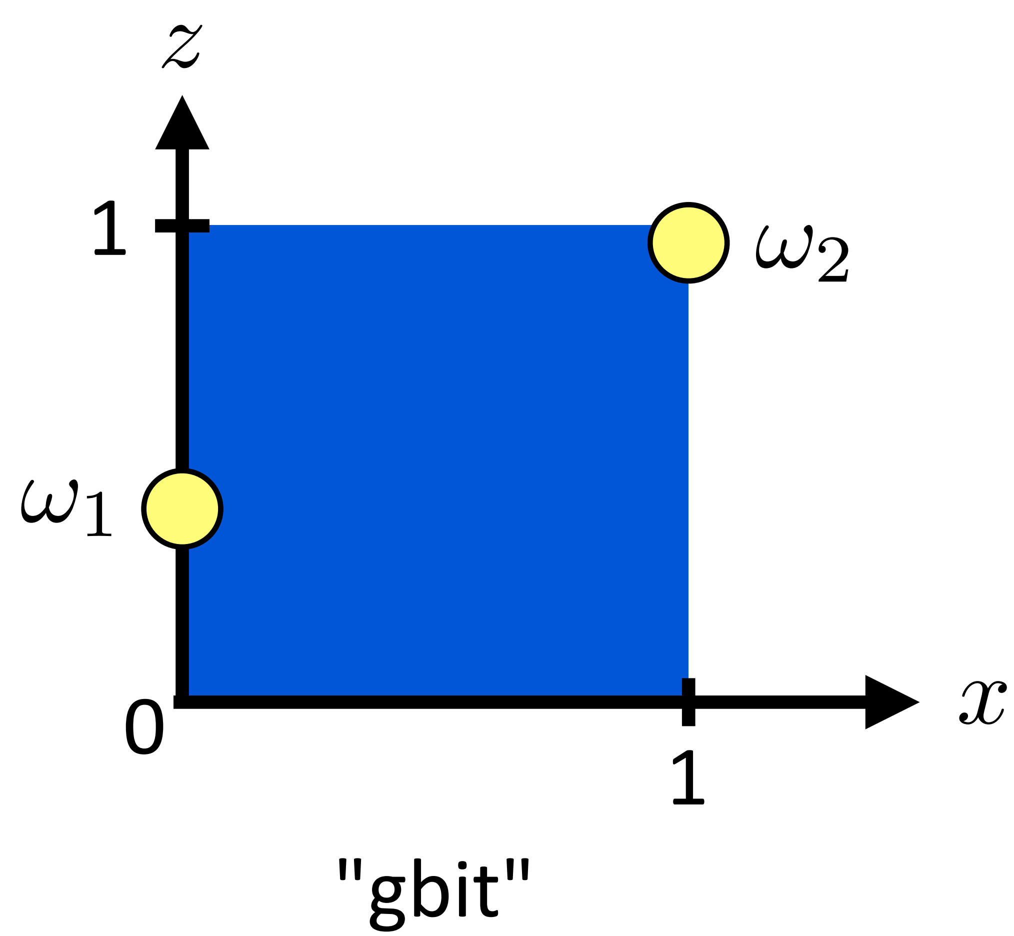

However, the qubit features **uncertainty relations**, but not the gbit.

Qubit: $\mathrm{Prob}(\mathrm{yes}|X,\omega) = 0 \text{ or } 1 \;\Rightarrow \mathrm{Prob}(\mathrm{yes}|Z,\omega) = \frac{1}{2}.$

Gbit: results of both measurements can be simultaneously predetermined.

Generalized probabilistic theories

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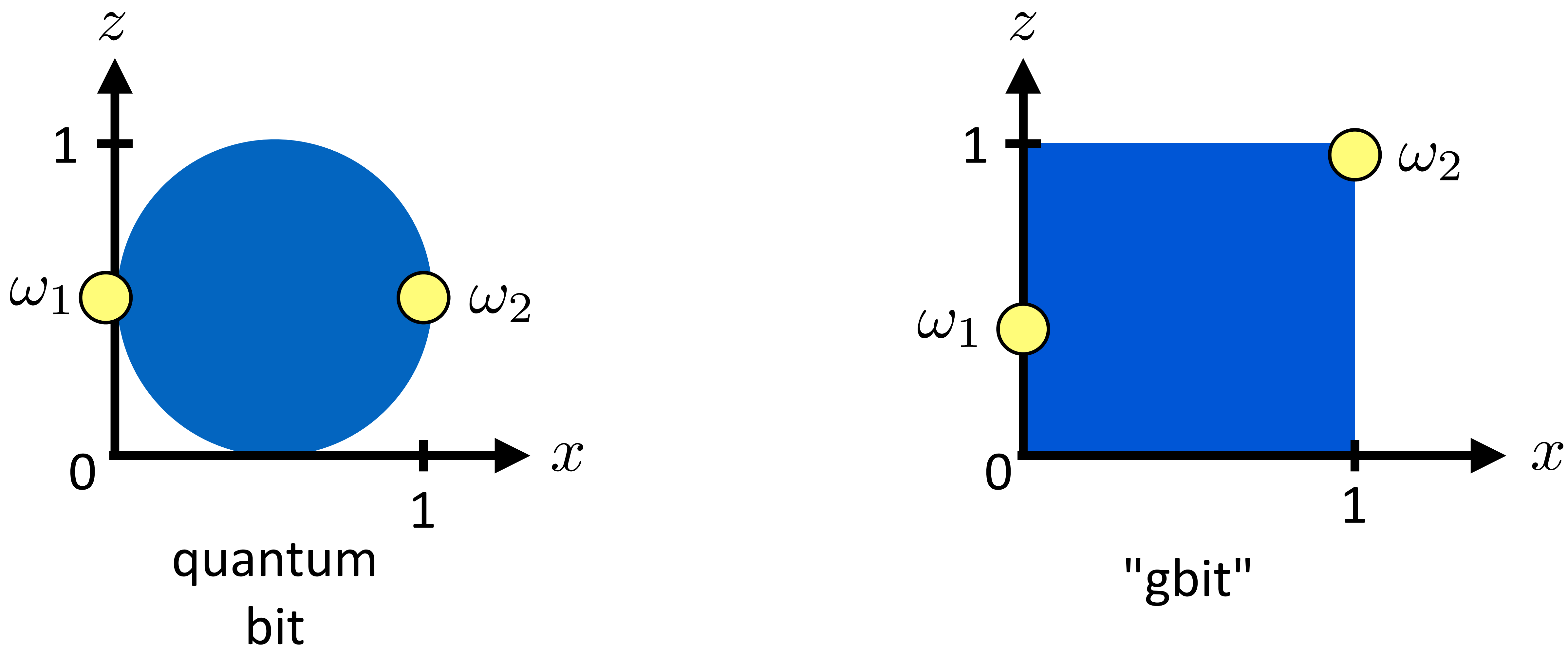
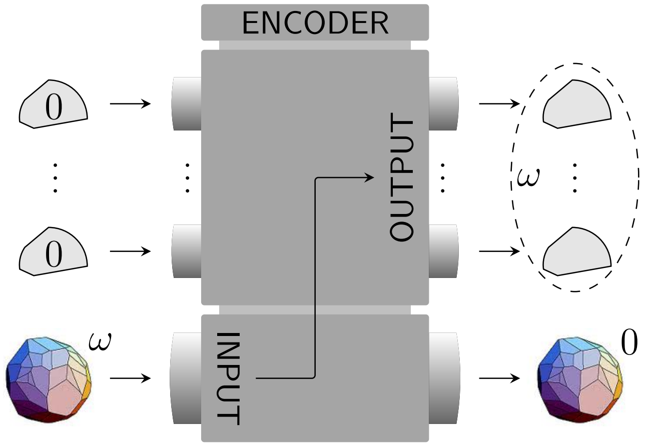


In some sense (to be made rigorous later), complementarity is "the prize to pay" for Nature to admit of continuous reversible time evolution!

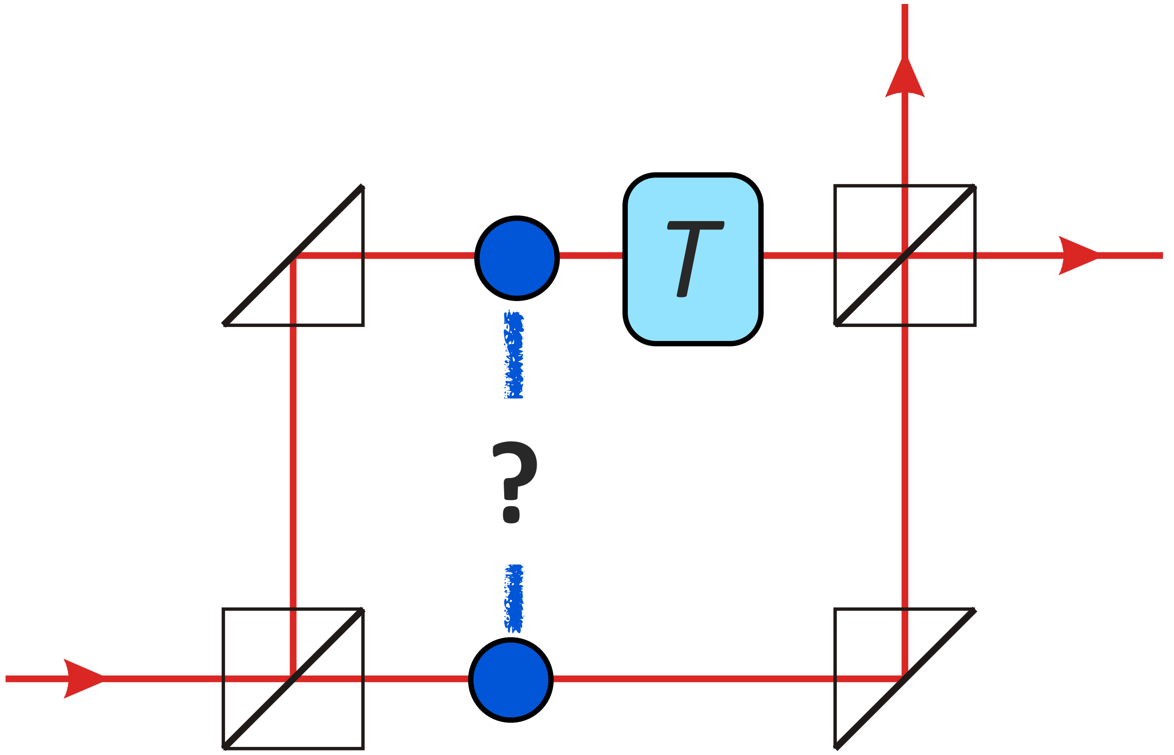
Overview

1. Probabilistic theories beyond quantum theory

2. Quantum theory from simple principles



3. The quest for higher-order interference



4. QT and spacetime

5. Conclusion

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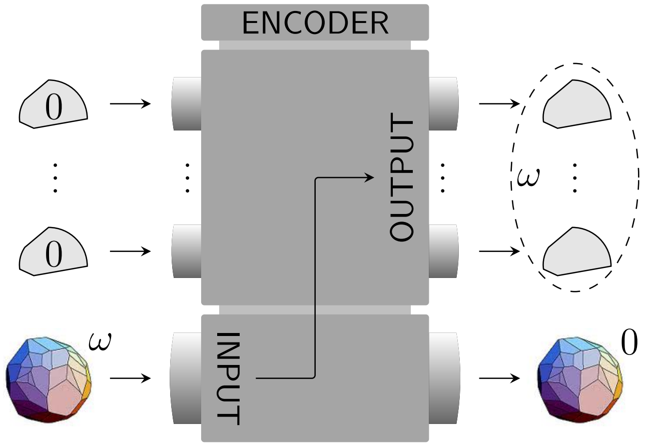

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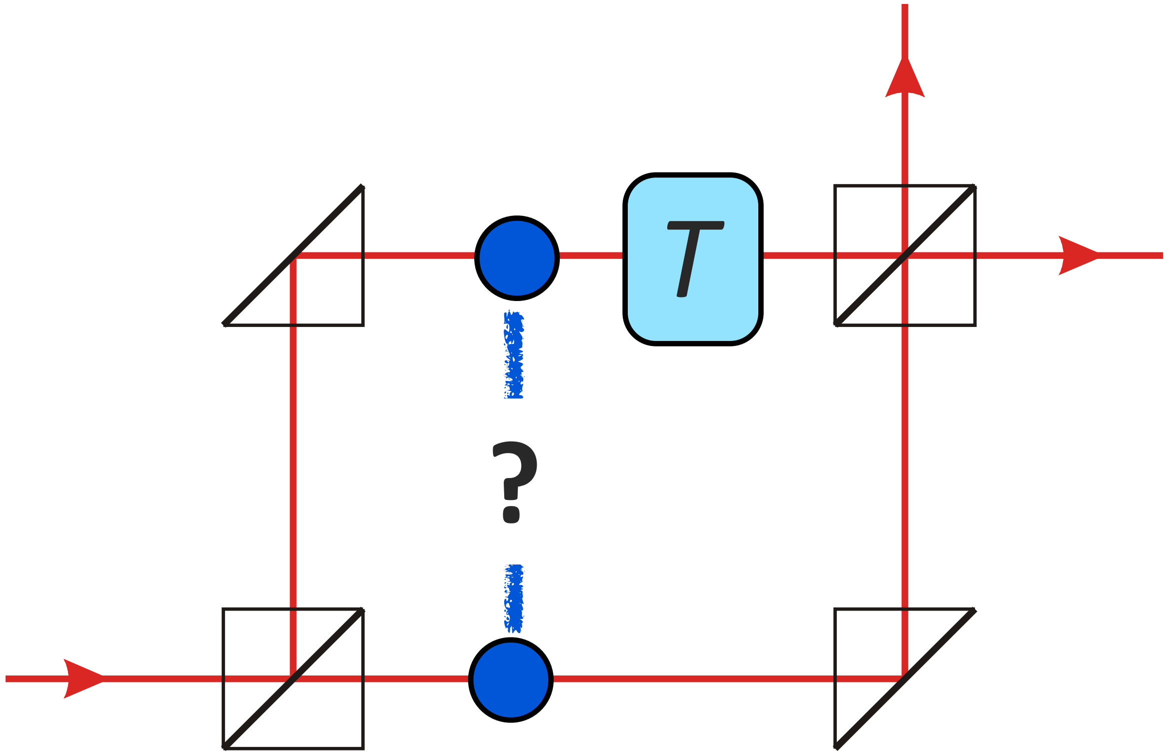


4. QT and spacetime

5. Conclusion

A reconstruction of quantum theory

- Prehistory:
  Birkhoff & von Neumann (1936); quantum logic (Piron, ...), Ludwig (1954); Alfsen&Shultz (≈1980); .....

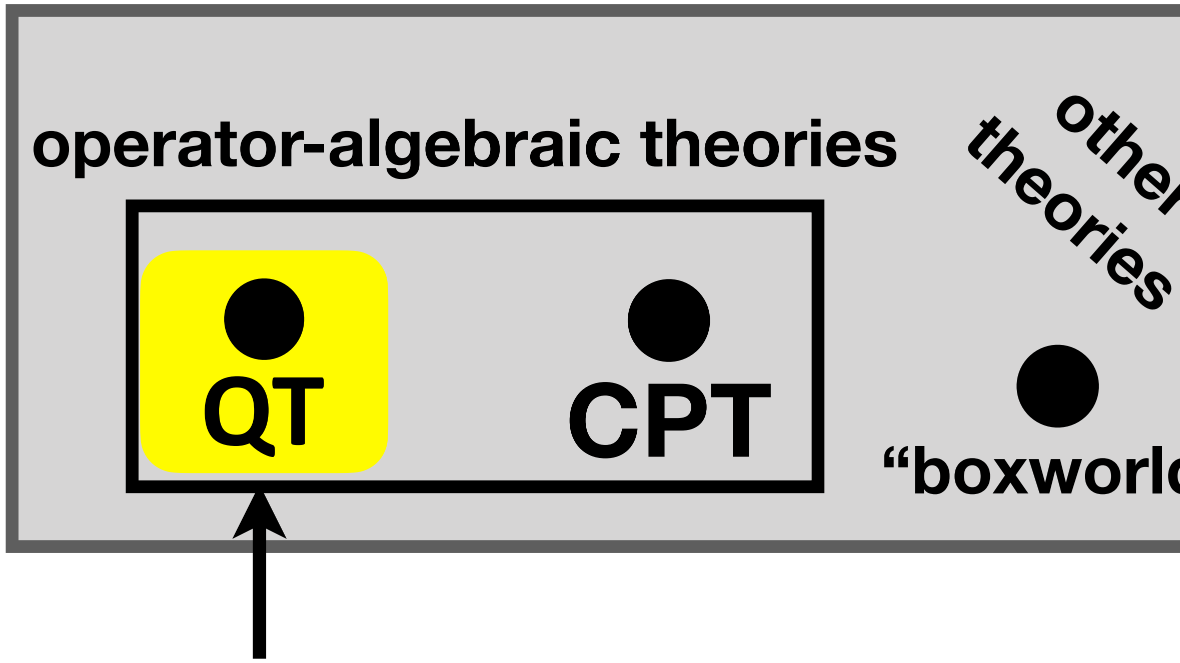
- Quantum information revolution:

  L. Hardy 2001: Quantum Theory From Five Reasonable Axioms. But needs "simplicity axiom"...

- Clifton, Bub, and Halvorson 2002.
  But assumed C*-algebras.

Dakić+Brukner 2009; Masanes+MM 2009
Chiribella, d'Ariano, Perinotti 2010; Hardy 2011
the one I'll present now 2013;
Barnum, MM, Ududec 2014; Hoehn 2015;
Wilce 2016, ...

A reconstruction of quantum theory



Which principles
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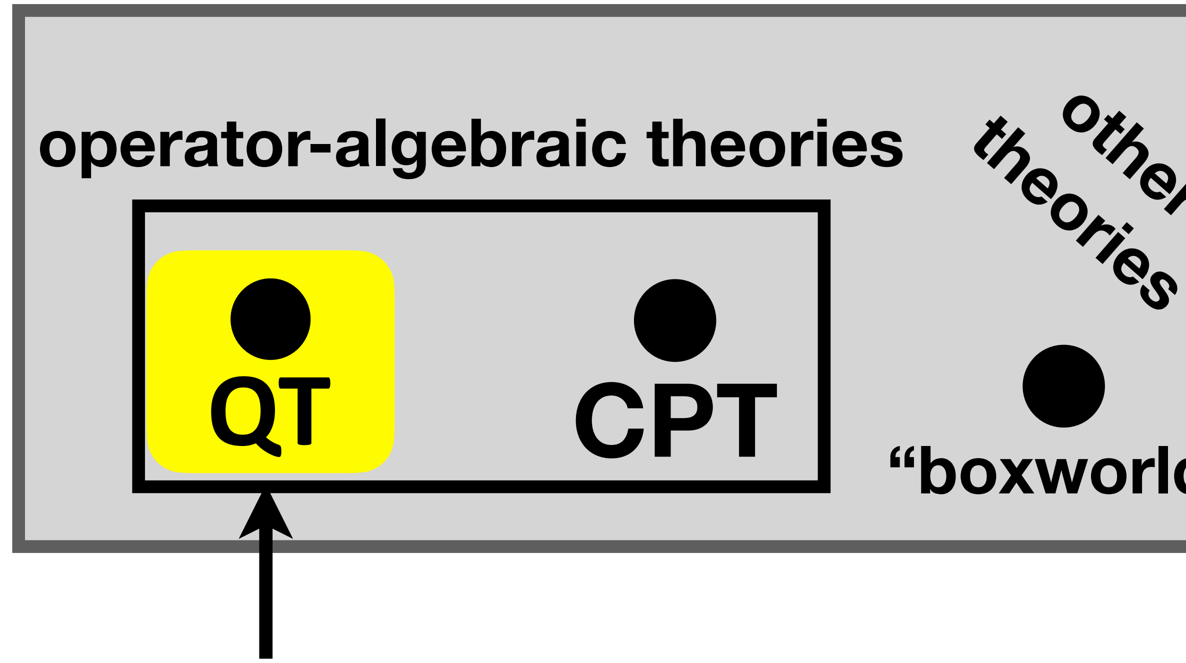

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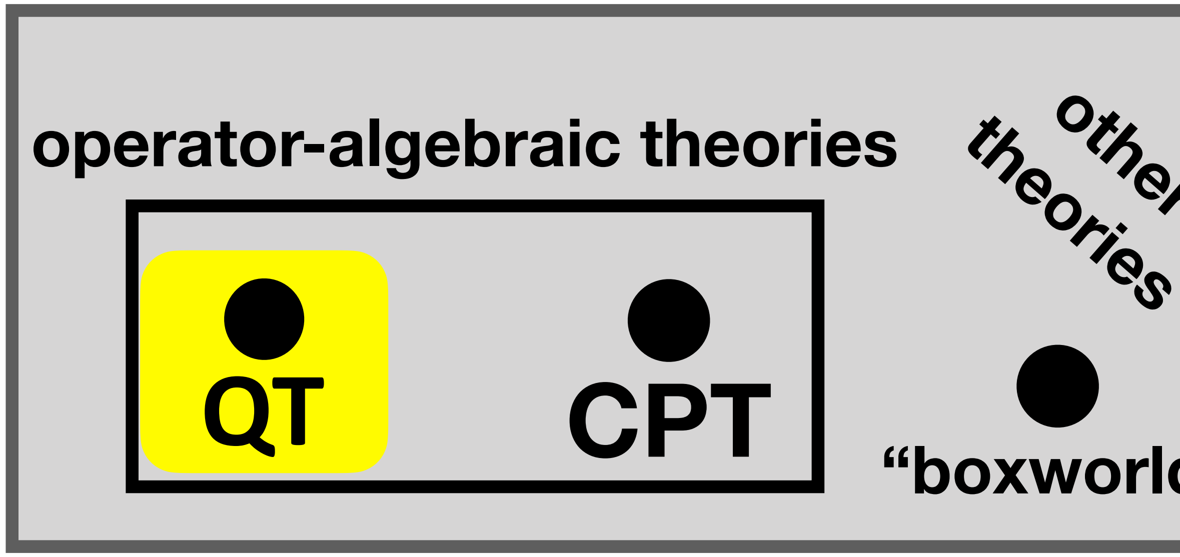
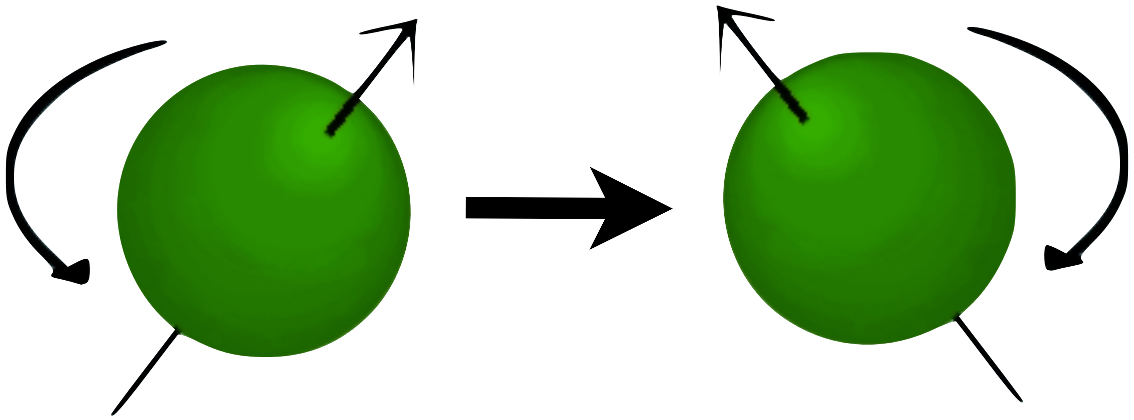


Reversible transformations can (in principle) map every pure state continuously to every other.



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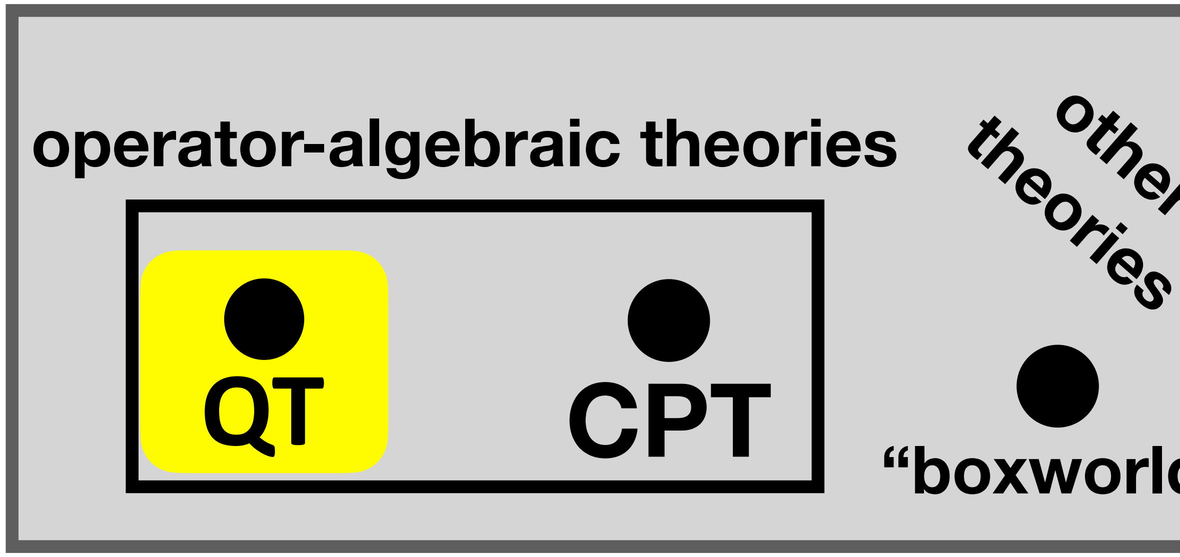
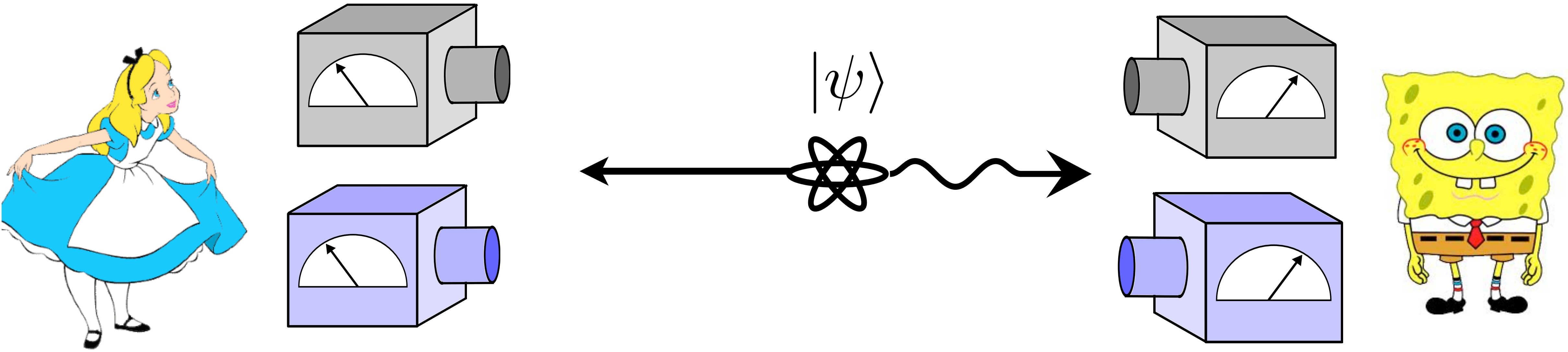


The state of a composite system is completely characterized by the correlations of measurements on the individual components.



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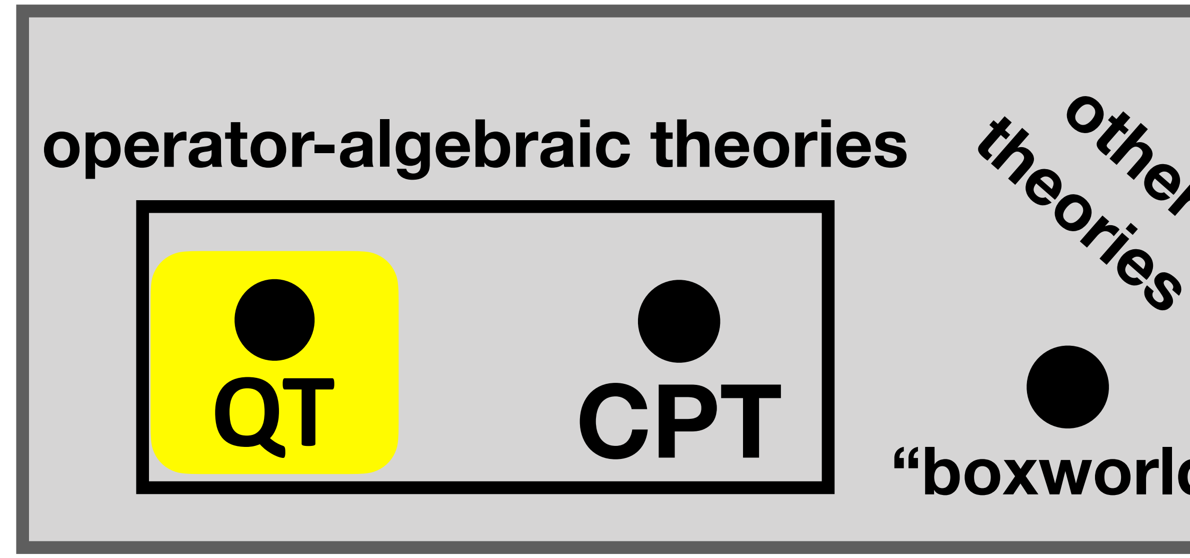

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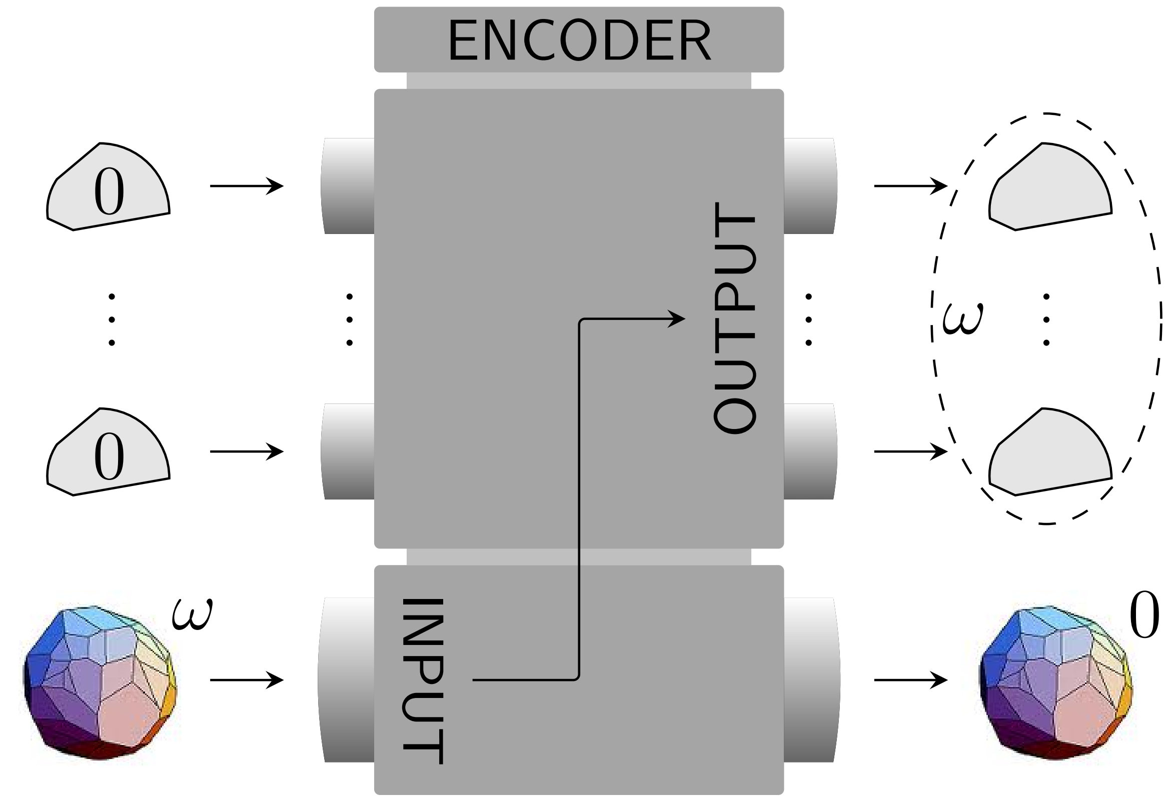

There is a type of system (the "ubit") such that every system can be encoded into a sufficiently large number of ubits.

Pairs of ubits can continuously reversibly interact.

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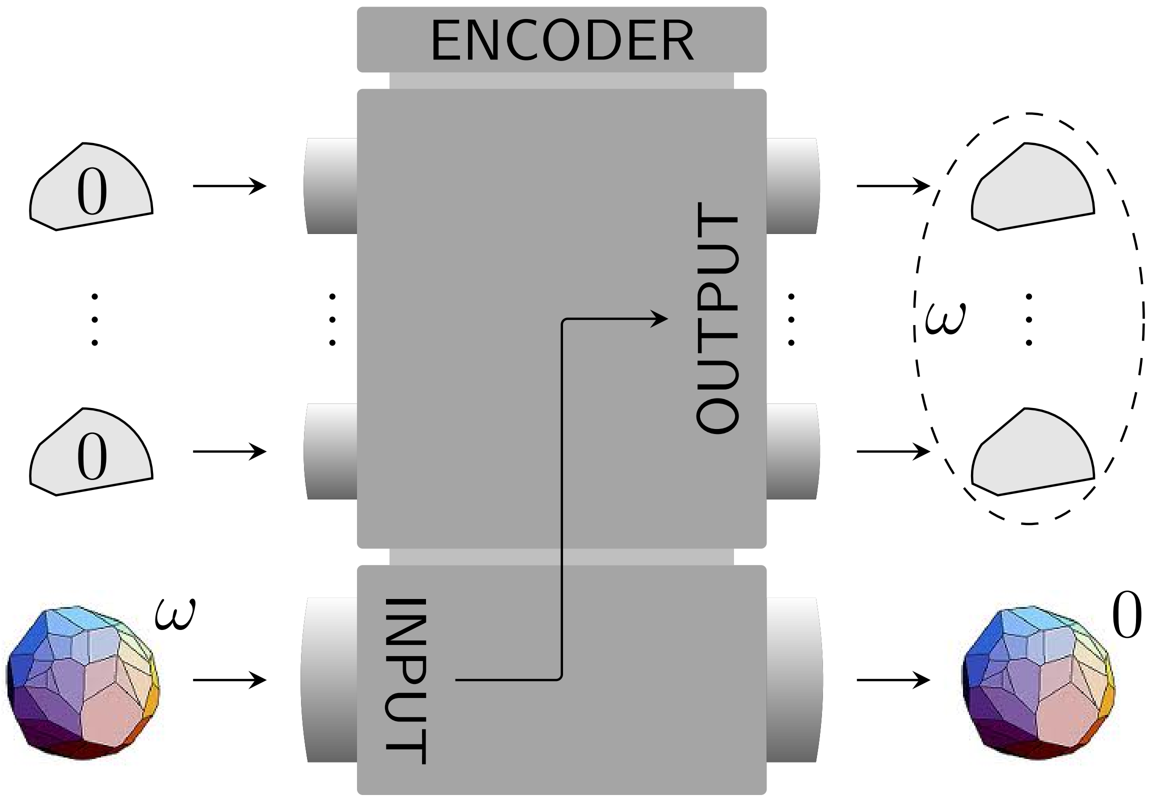
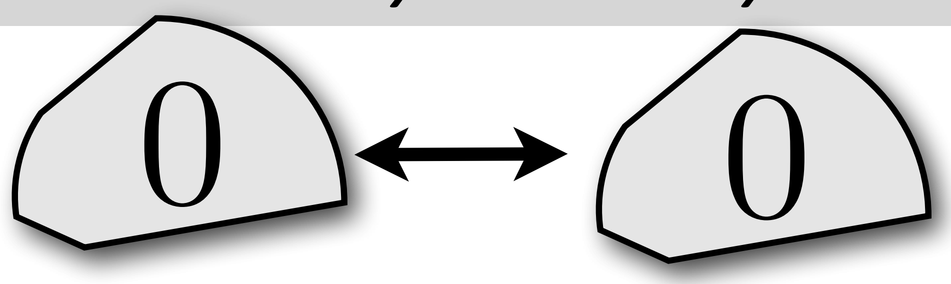


There is a type of system (the "ubit") such that every system can be encoded into a sufficiently large number of ubits. Pairs of ubits can continuously reversibly interact.



A reconstruction of quantum theory

Ll. Masanes, **MM**, R. Augusiak, and D. Pérez-García, PNAS **110**(4), 16373 (2013).

- **Postulate 1**: Continuous reversibility.
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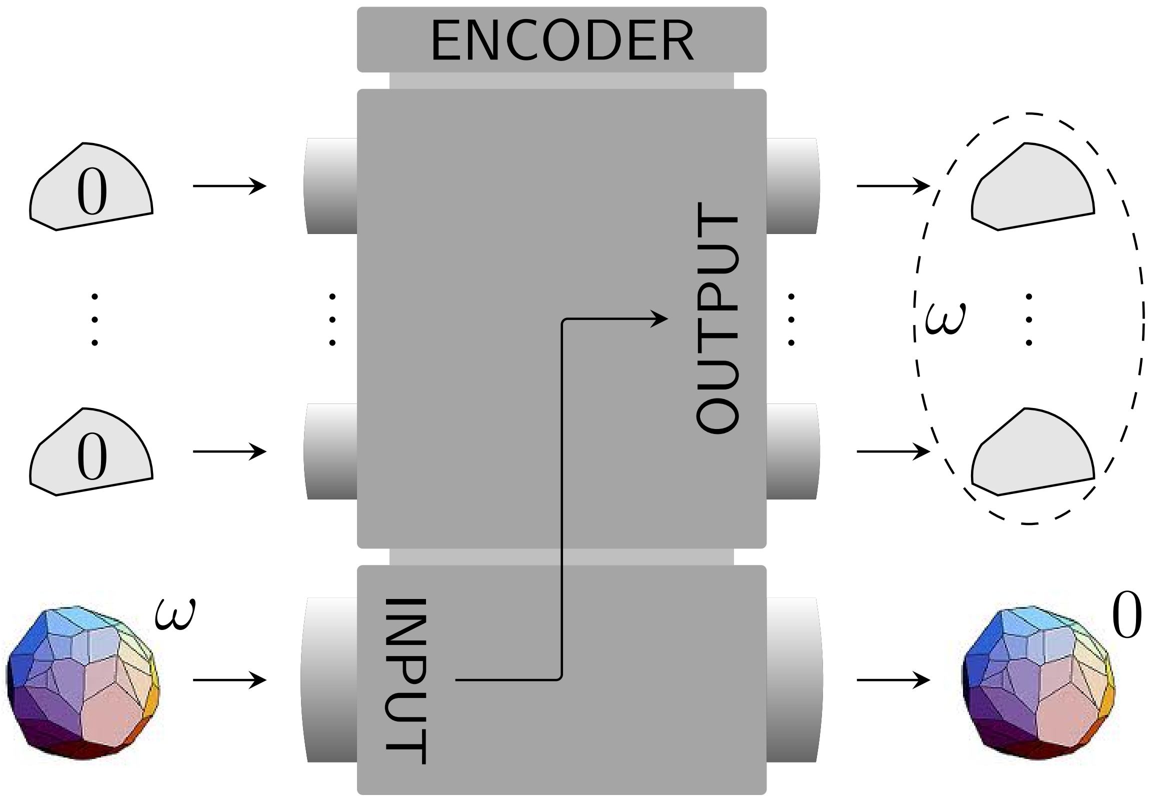

If a ubit is used to perfectly encode one classical bit, it cannot simultaneously encode any further information.

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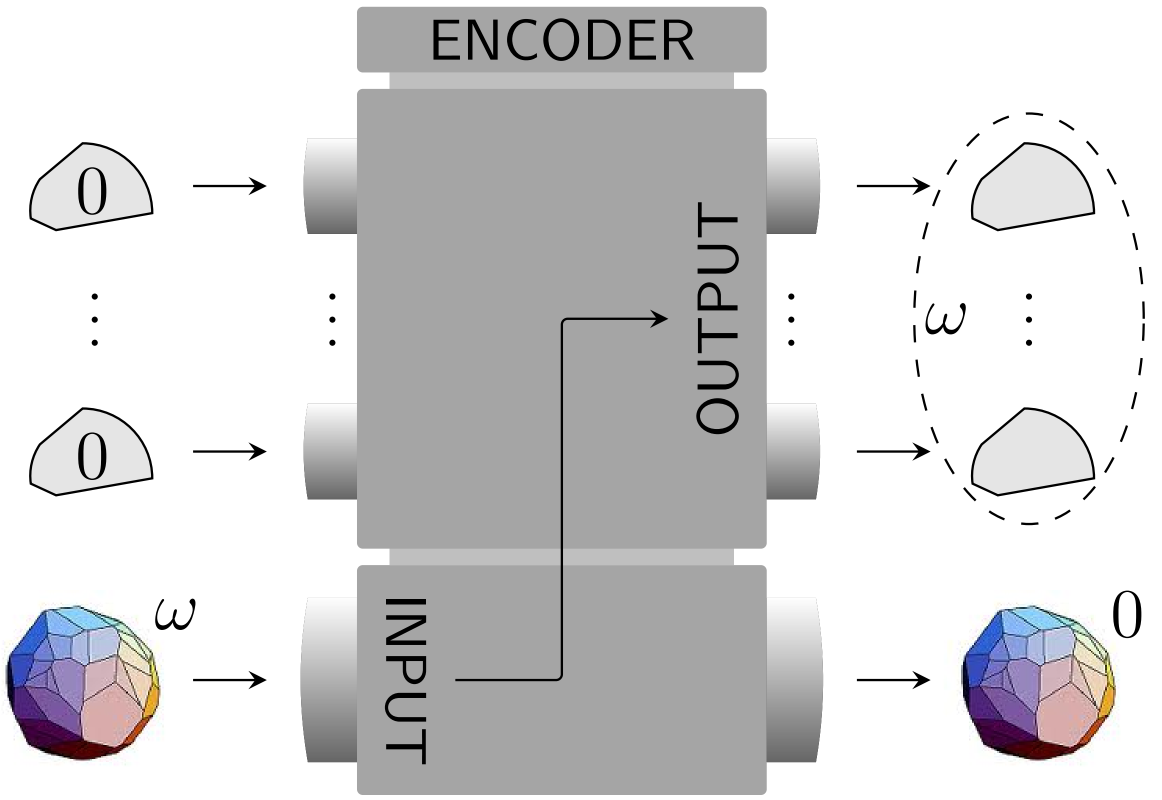

Theorem. If Postulates 1-4 hold, then the state space of *n* ubits is

$$\Omega = \{\rho \in \mathbf{H}_{2^n}(\mathbb{C}) \mid \mathrm{tr}(\rho) = 1, \rho \geq 0\},$$

and the reversible transformations are the unitaries, $\rho \mapsto U\rho U^\dagger$.

Example: why are ubits balls?

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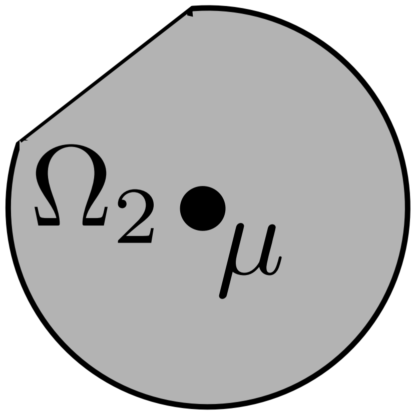

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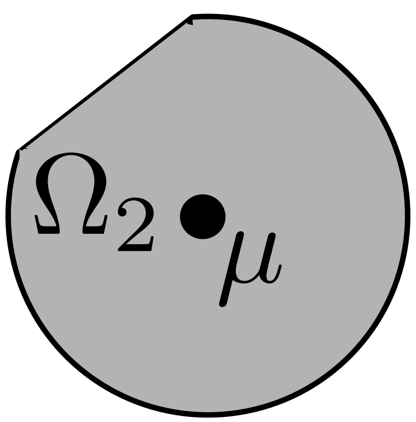
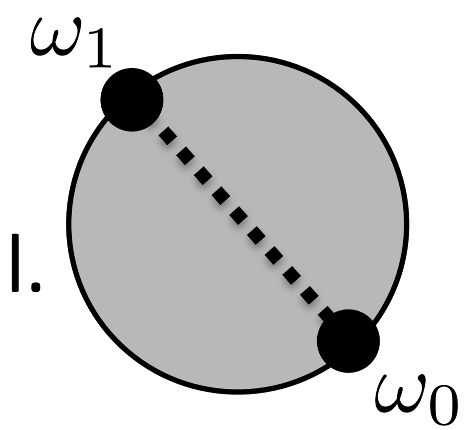


If **full** ball: can encode one bit by preparing state or antipodal state. That's all.



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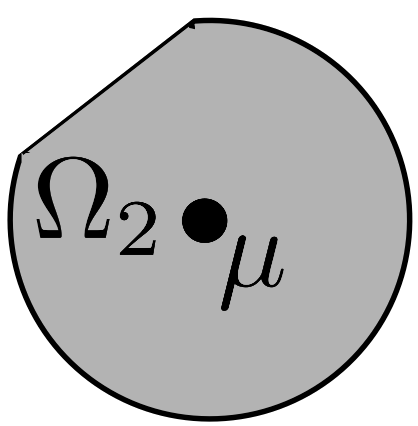

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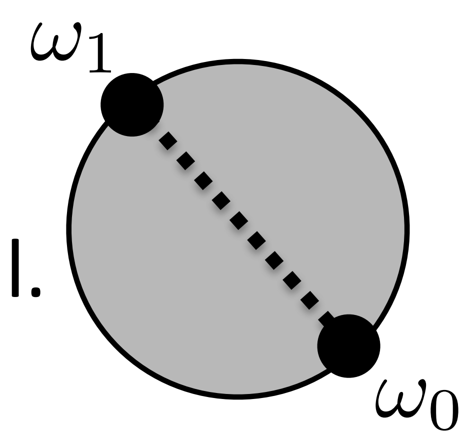
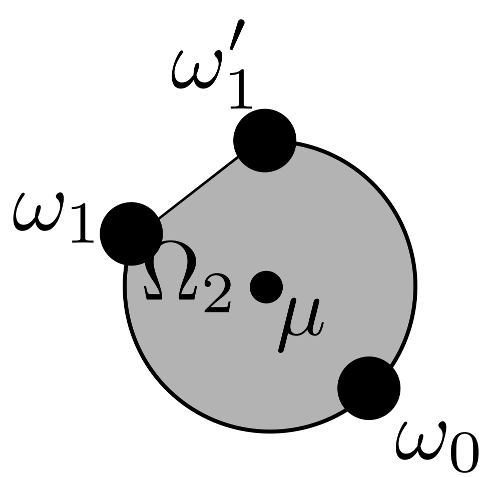


If **not** full ball: can encode one bit **and a little more** by preparing state or **one of** antipodal states.



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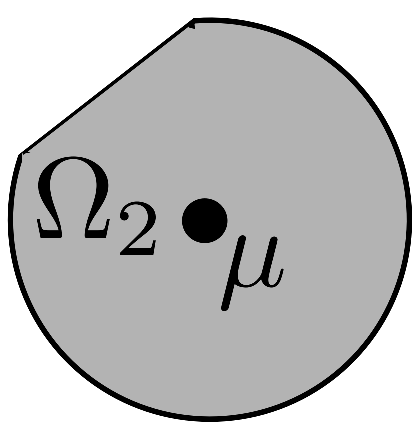

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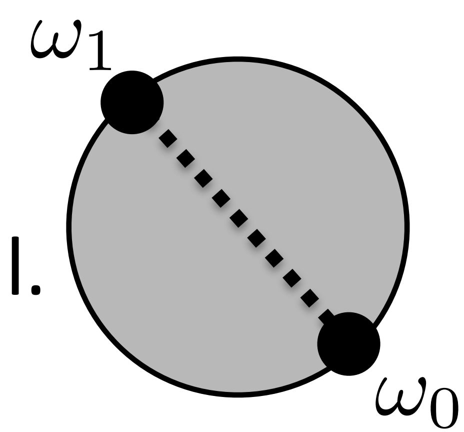

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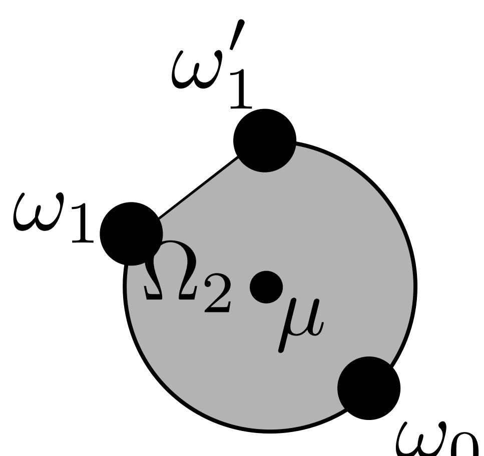

Violates Postulate 4.

Why is the ubit “Bloch ball” 3-dimensional?

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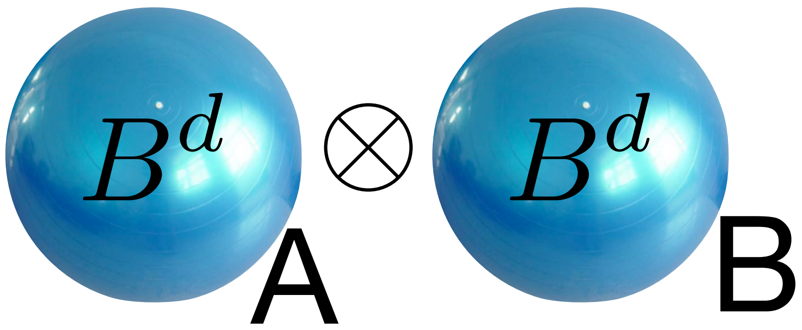

Two ubits: *some* composite state space of two *d*-balls, $\mathcal{G}_A = \mathcal{G}_B$ transitive on ∂B^d.

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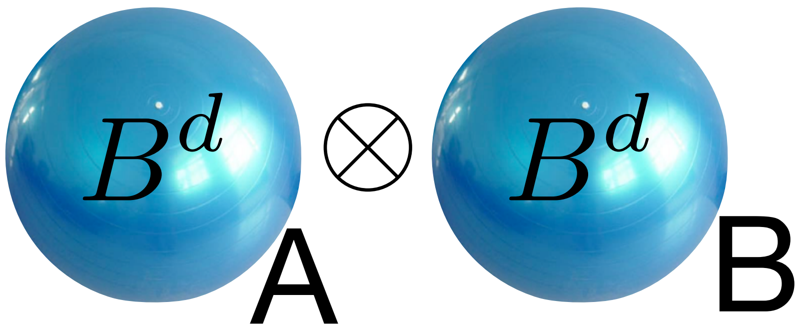

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Theorem. Among all dimensions d and all groups $\mathcal{G}_A$, there are only the following possibilities:

- The trivial solution: $\mathcal{G}_{AB} = \mathcal{G}_A \otimes \mathcal{G}_B$.
- $d = 3$, $\mathcal{G}_A = \mathrm{SO}(3)$ (i.e. the quantum bit), $\mathcal{G}_{AB} \simeq \mathrm{PU}(4)$, and Ω_{AB} is equivalent to the two-qubit quantum state space.

In particular, **continuous reversible interaction** is only possible for $d = 3$, in standard complex two-qubit quantum theory.

Ll. Masanes, **MM**, R. Augusiak, and D. Pérez-García, J. Math. Phys. **55**, 122203 (2014).

Proof idea

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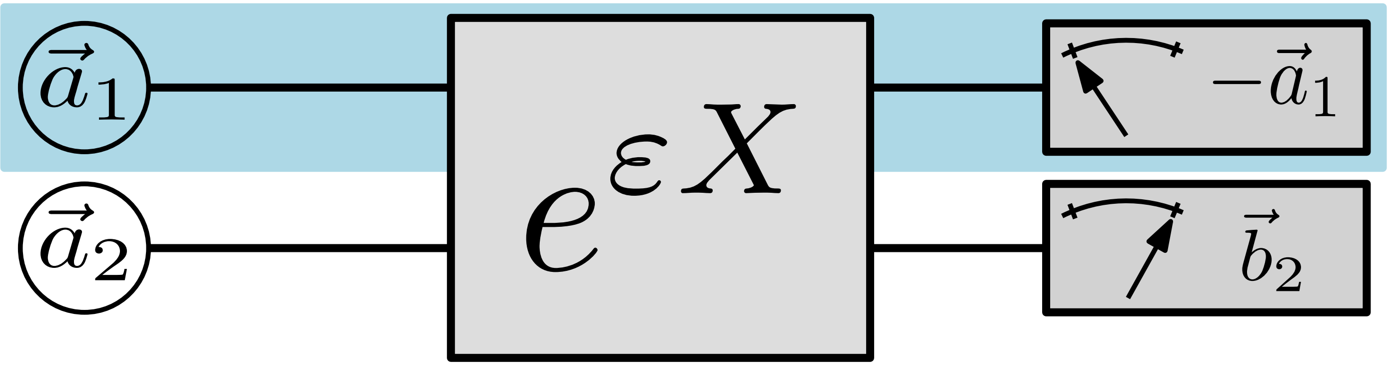
Generator *X* of global reversible transformation (no idea what it is…)



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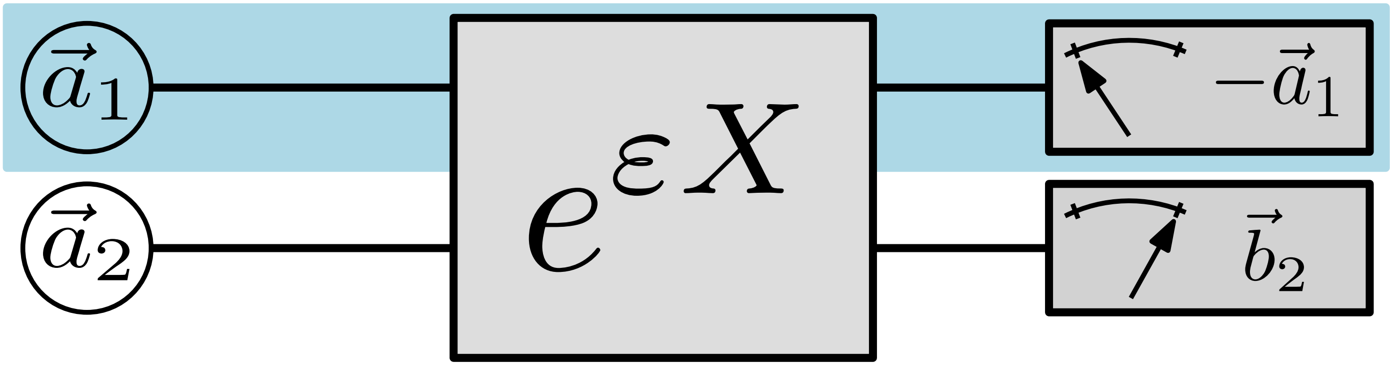

We must obtain **valid probabilities**. For example,

$$0 \leq (e_{-\vec{a}_1} \otimes e_{\vec{b}_2}) e^{\varepsilon X} (\omega_{\vec{a}_1} \otimes \omega_{\vec{a}_2}) \leq 1.$$

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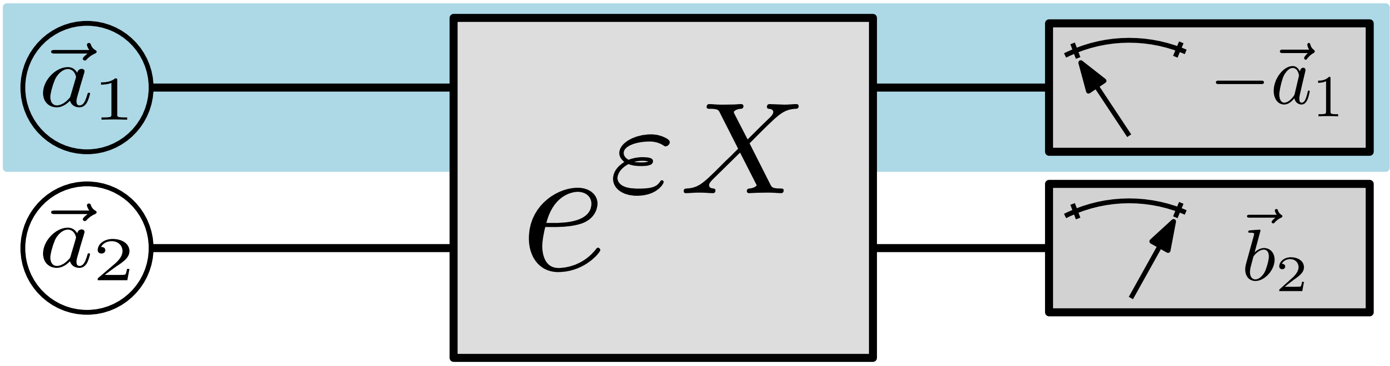

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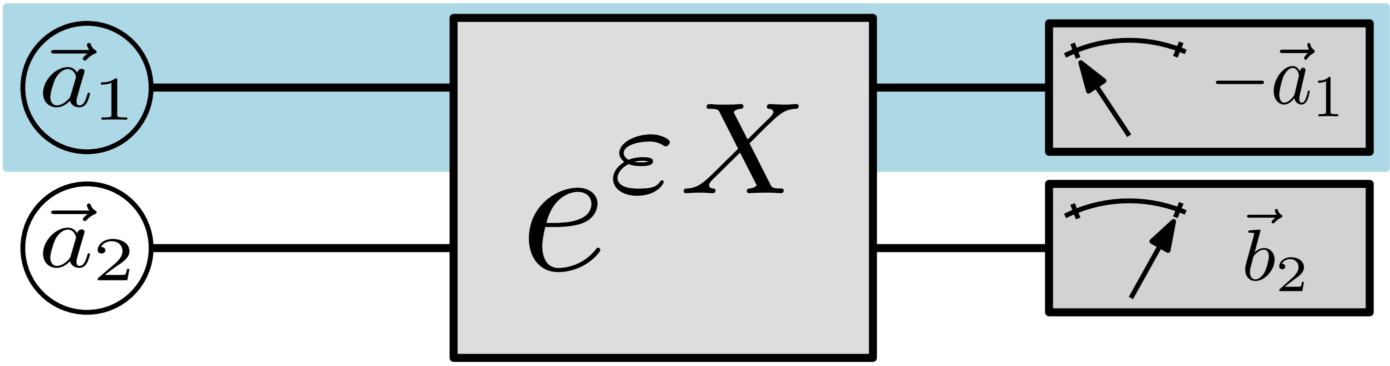

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A lot more work…

$$\Rightarrow \begin{cases} \text{if } d \neq 3: & X = X_A + X_B \\ \text{if } d = 3: & \exp(\varepsilon X) = U_{AB}(\varepsilon) \bullet U_{AB}^\dagger(\varepsilon) \end{cases}$$

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unitary conjugation!

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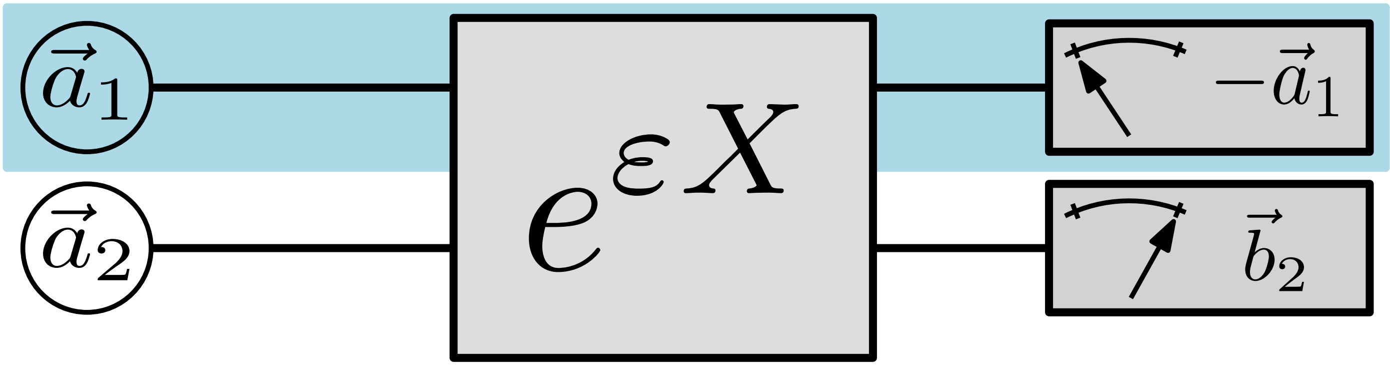

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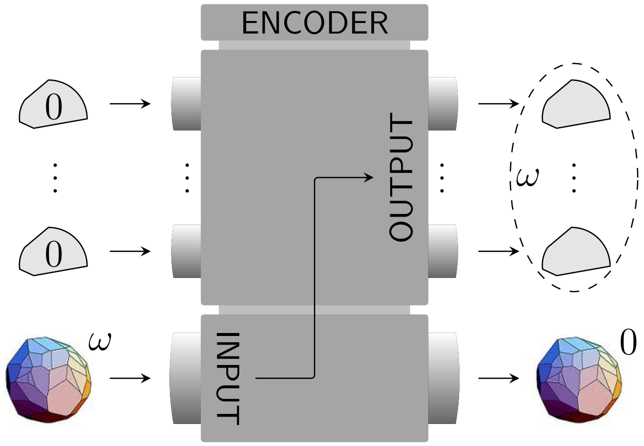
Main reason: $\mathrm{SO}(d-1)$ is only non-trivial and **commutative** for $d = 3$.

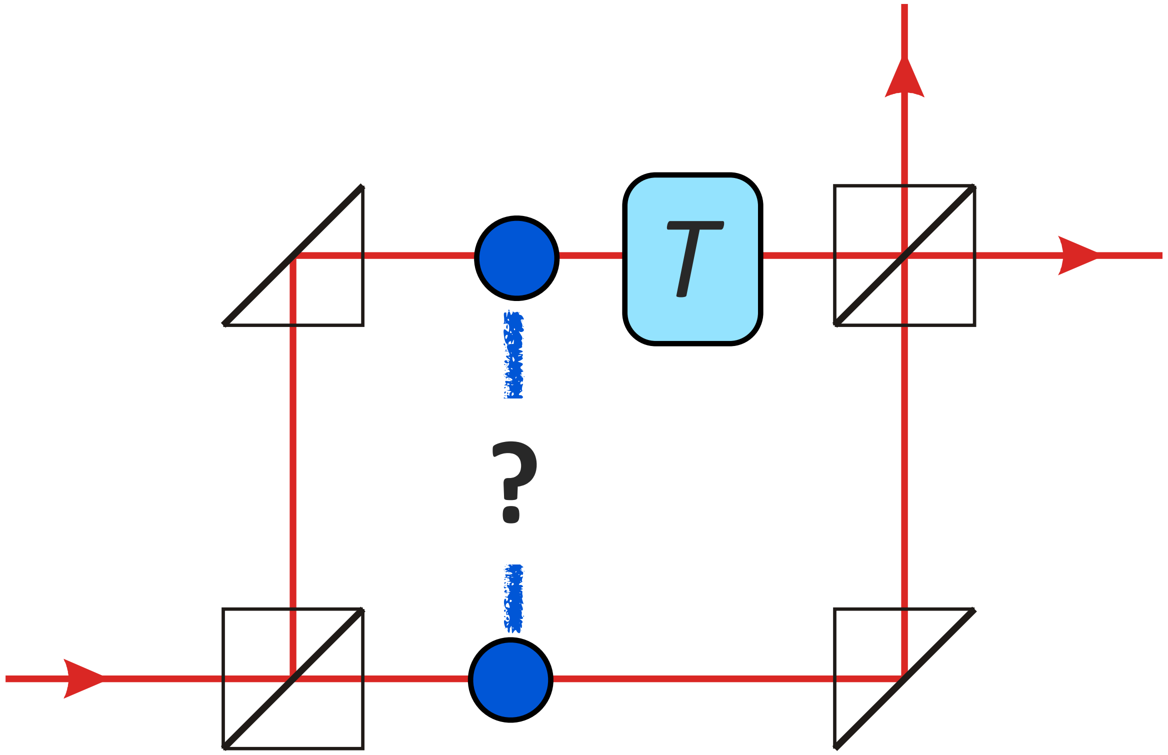
Overview

1. Probabilistic theories beyond quantum theory

2. Quantum theory from simple principles



3. The quest for higher-order interference



4. QT and spacetime

5. Conclusion

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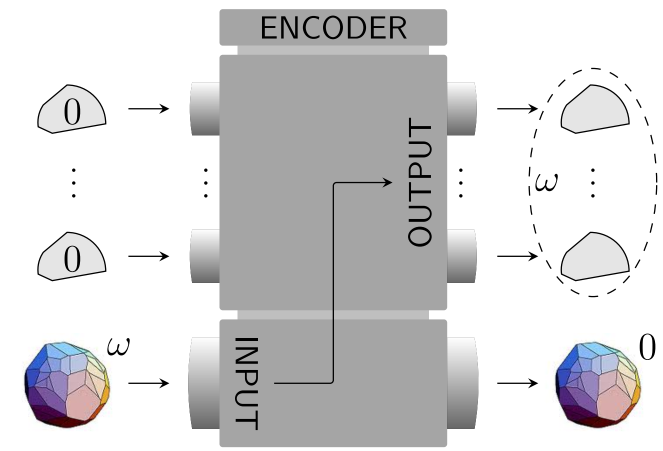

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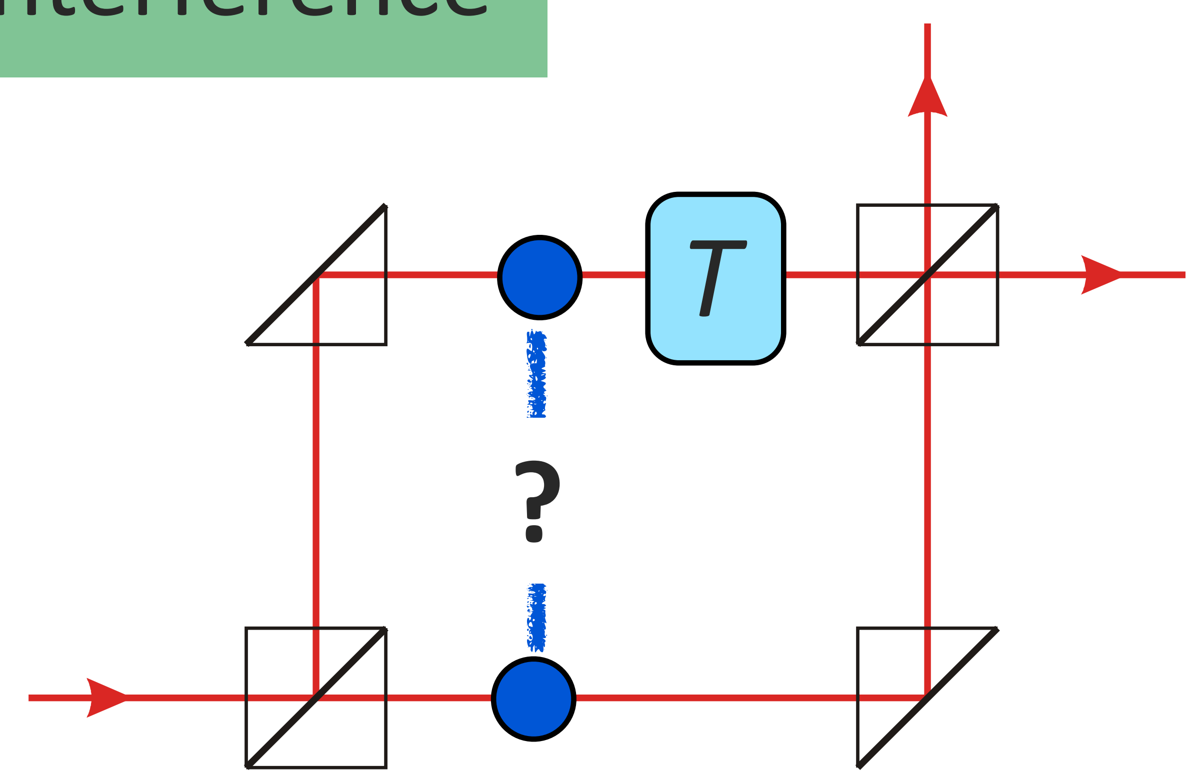

5. Conclusion

Higher-order interference

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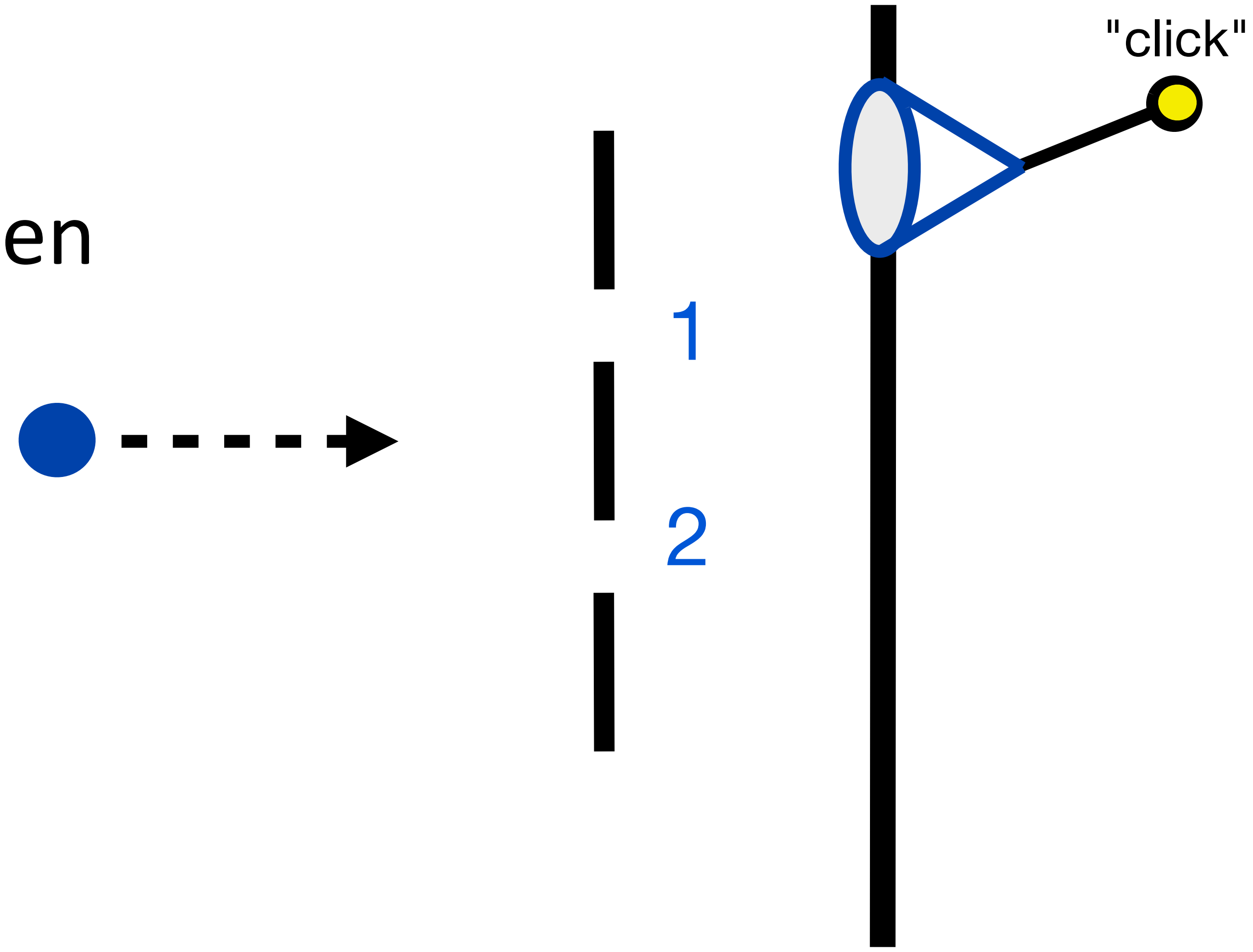

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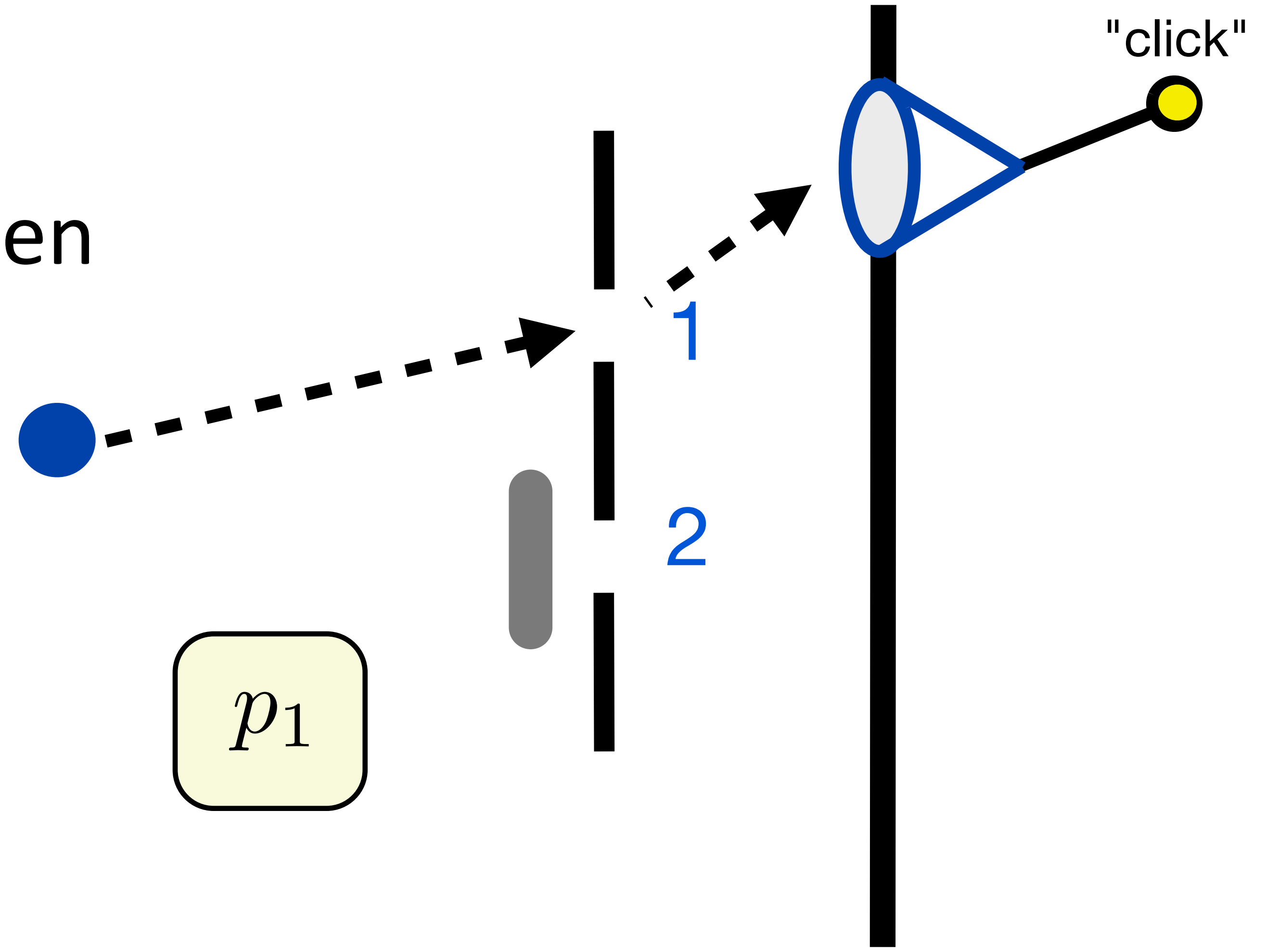

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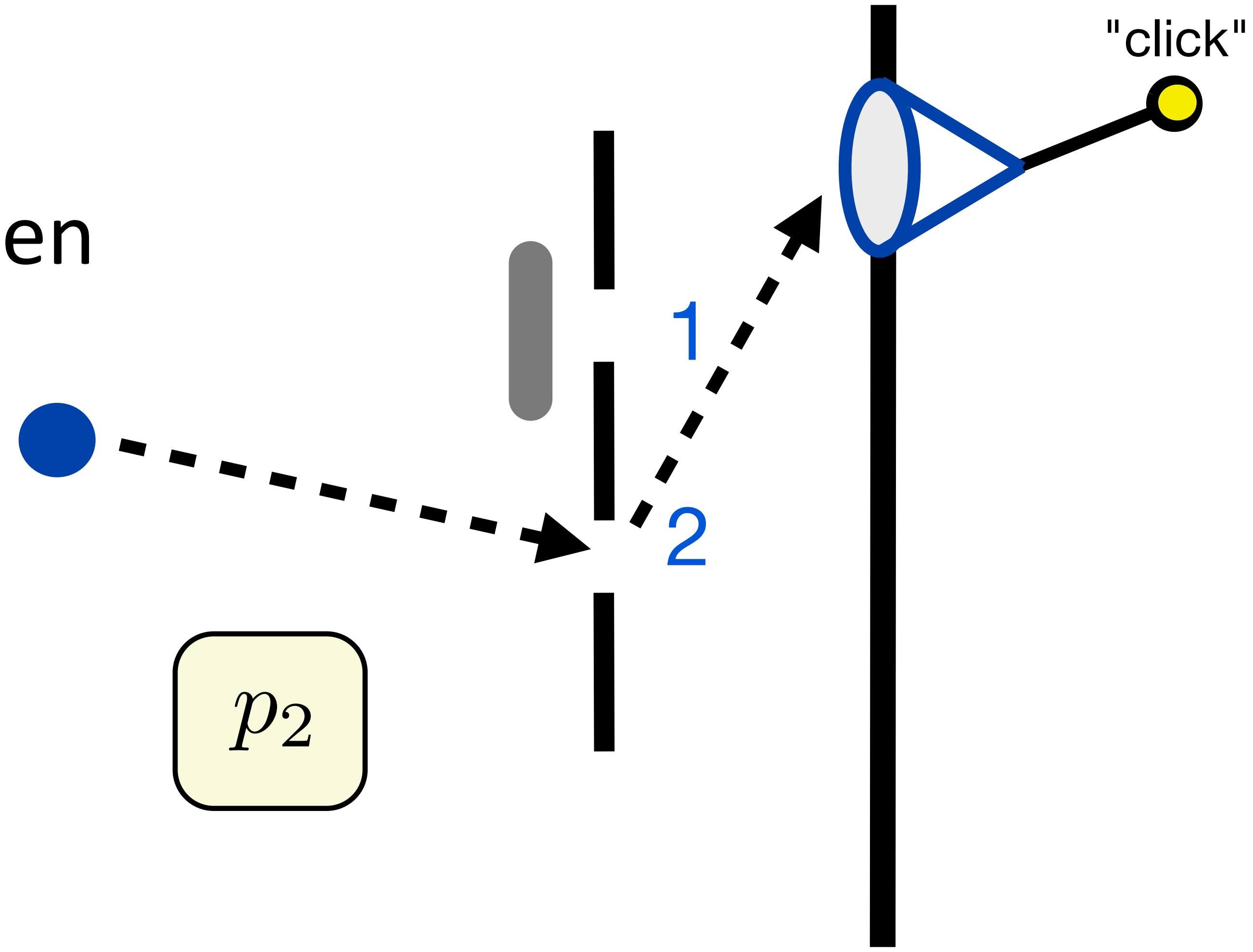

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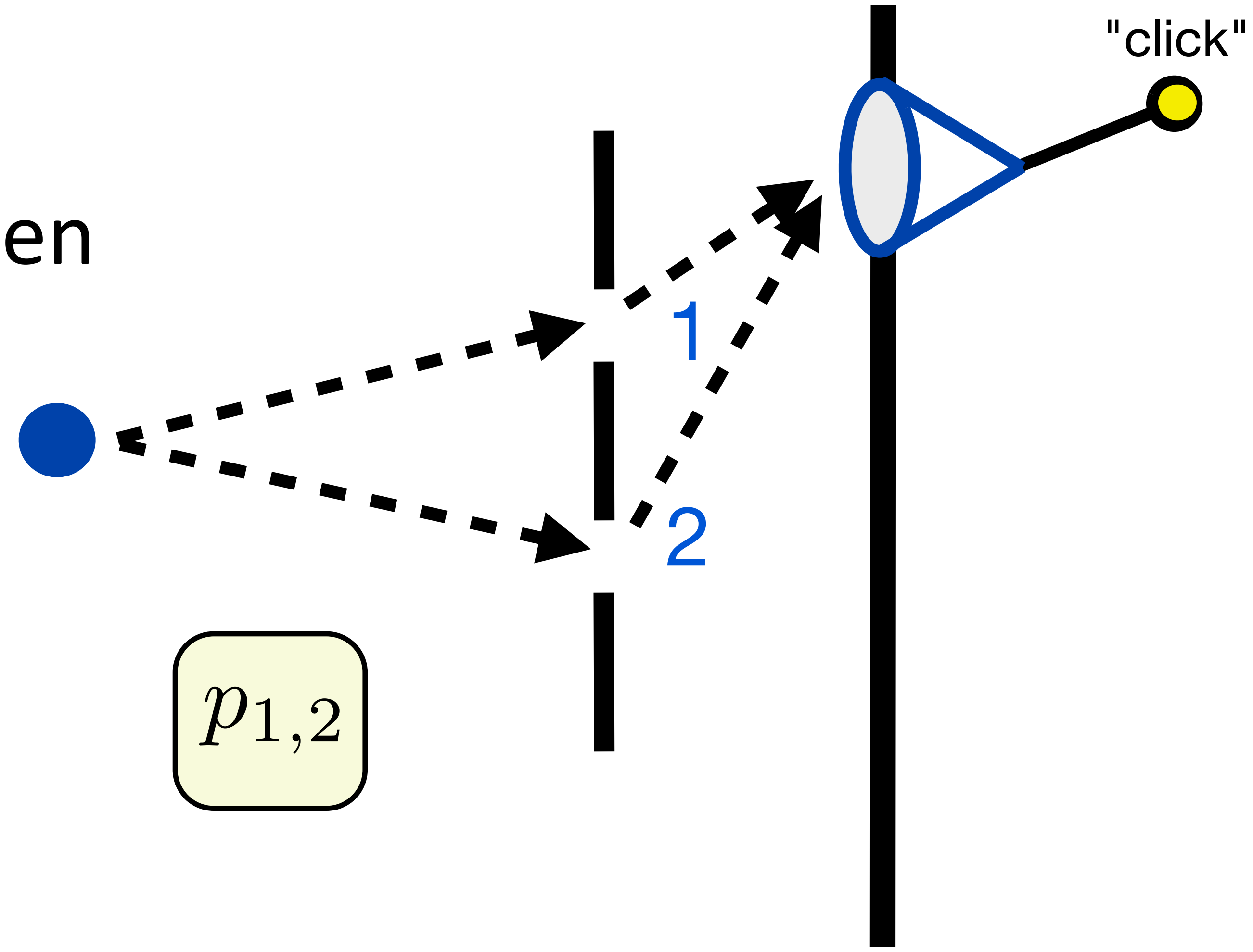

$p_{1,2}$

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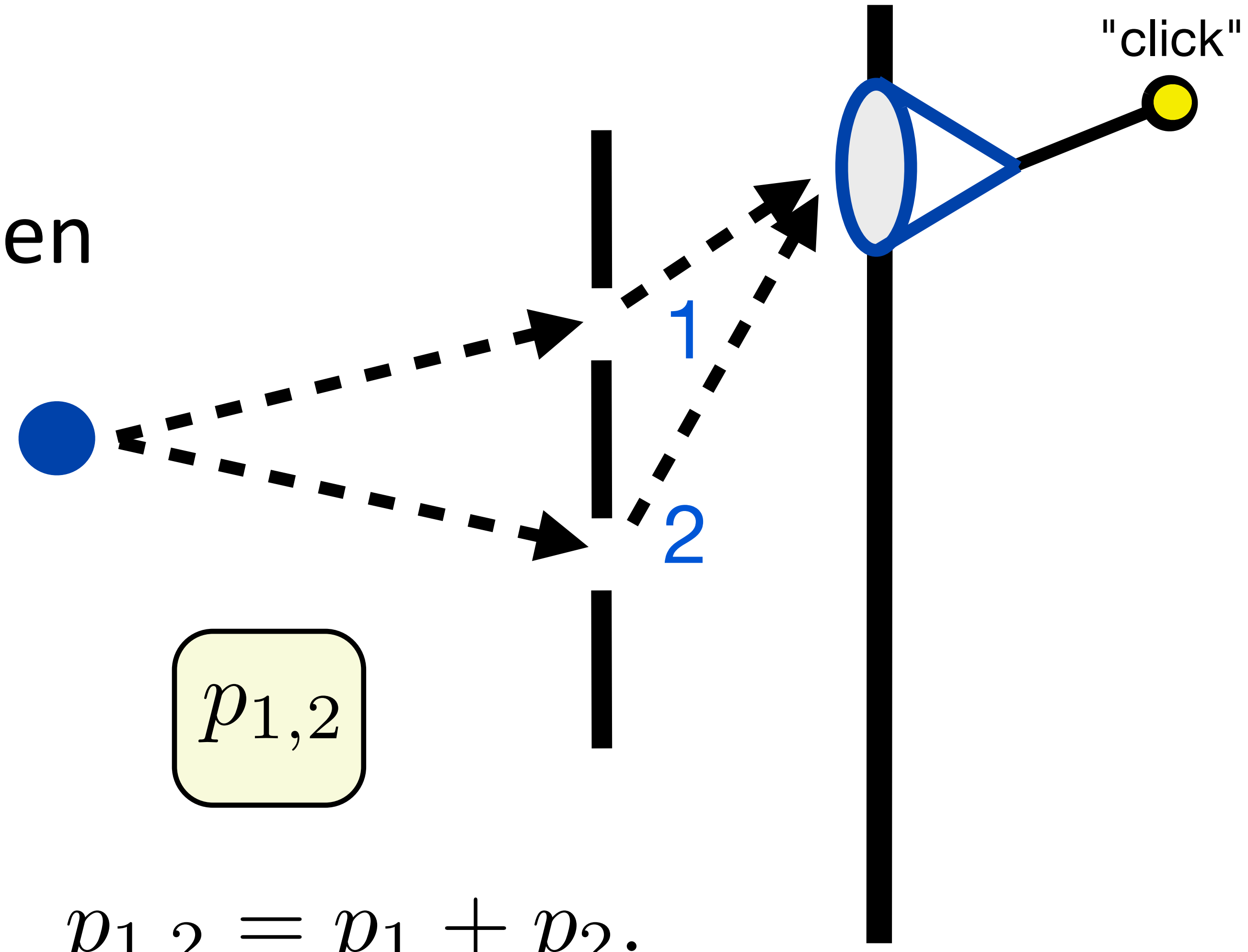

Classical probability theory: $p_{1,2} = p_1 + p_2.$

Quantum theory: $p_{1,2} \neq p_1 + p_2.$

(2nd order) **interference**!

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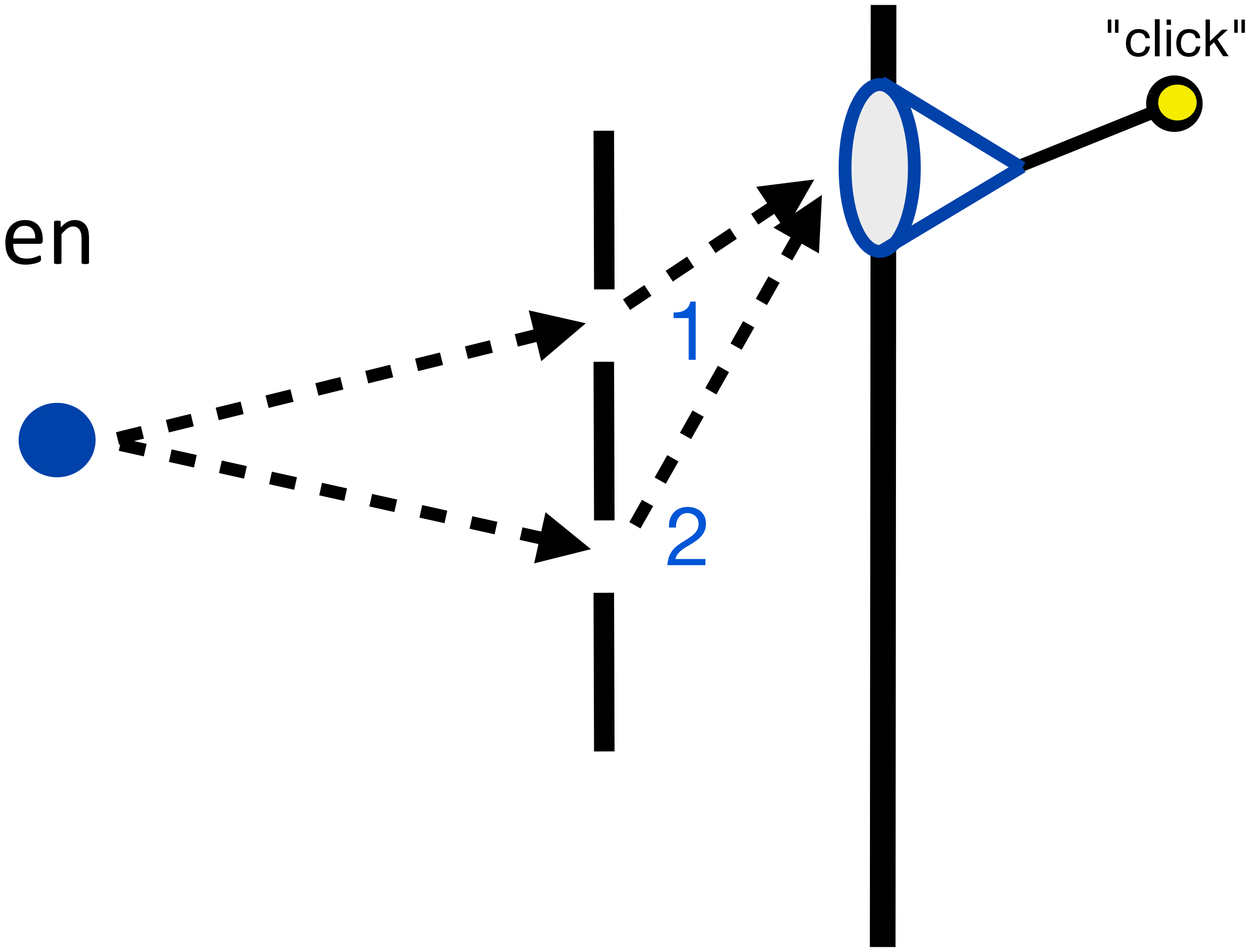

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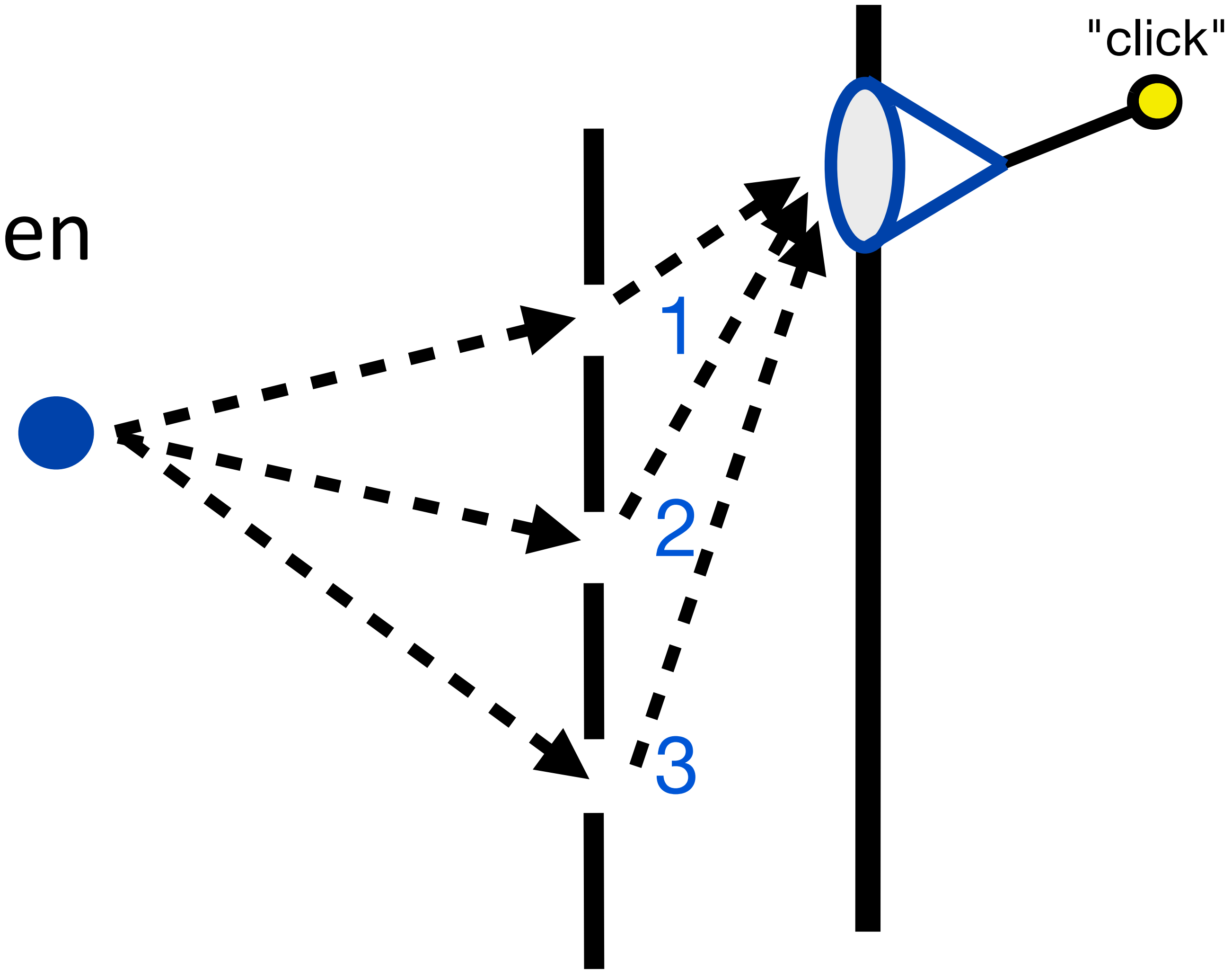

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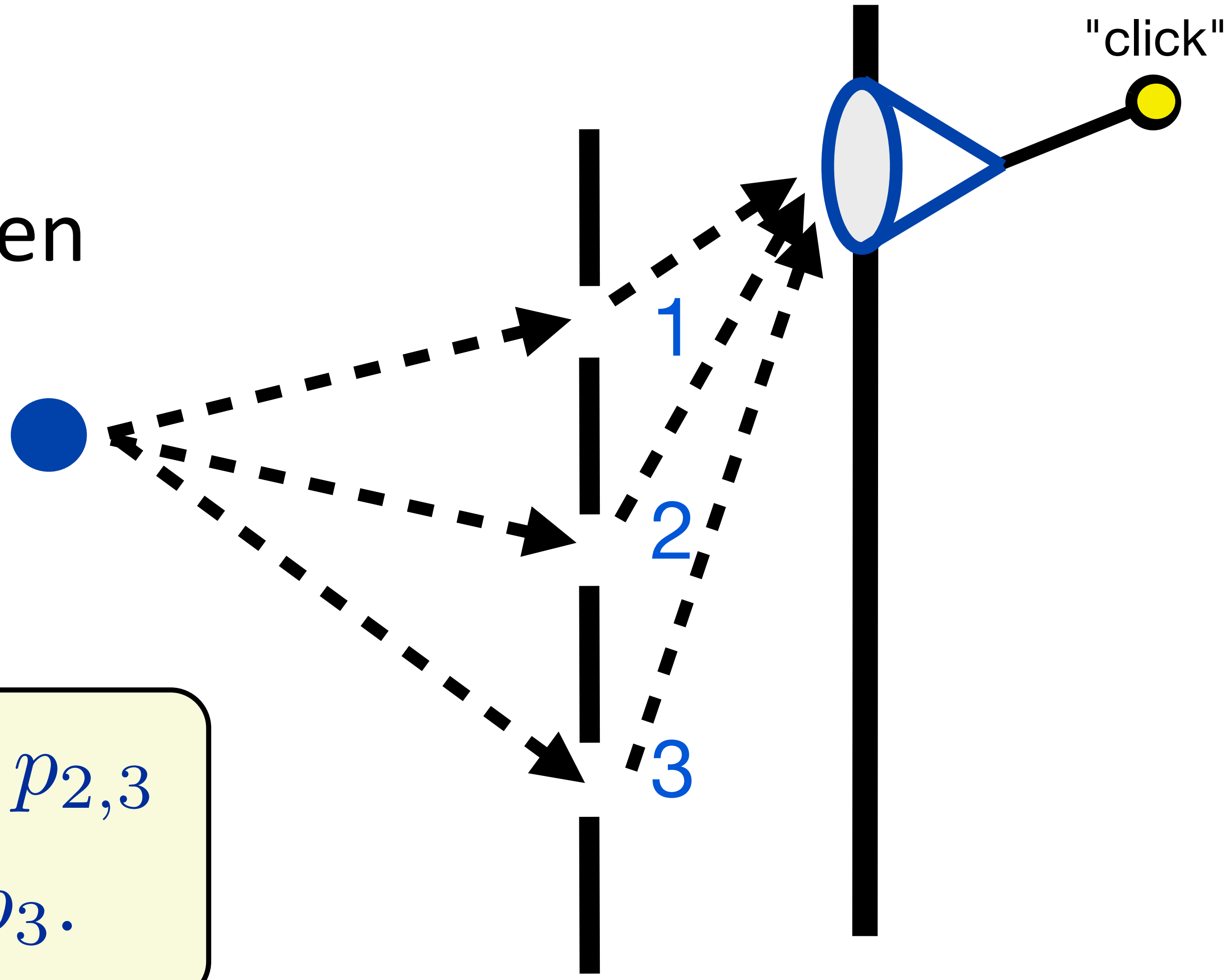

QT satisfies (like CPT!)

$$p_{1,2,3} = p_{1,2} + p_{1,3} + p_{2,3} - p_1 - p_2 - p_3.$$

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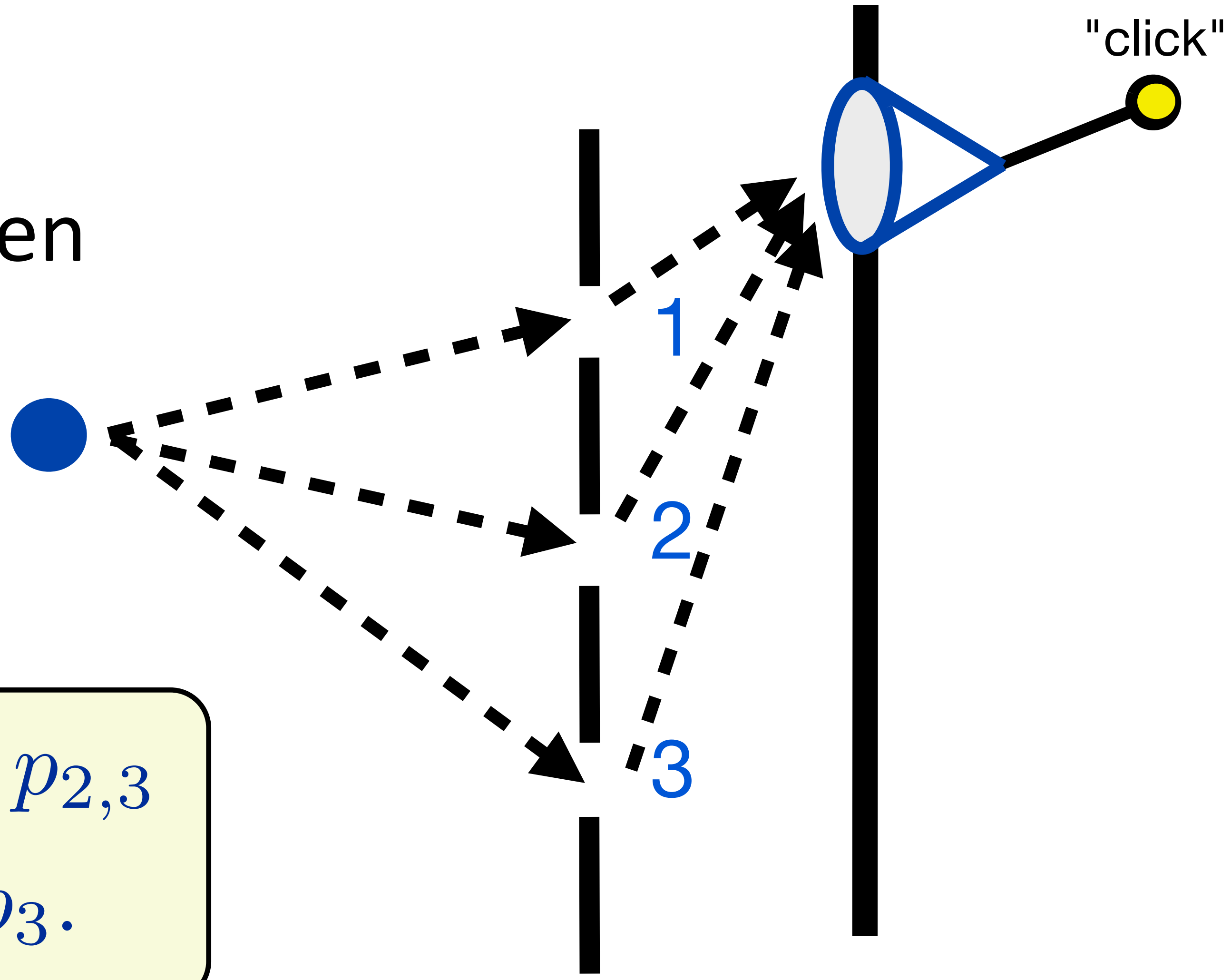

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No 3rd-order interference in QT.

Why does QT not have 3rd-order interference?

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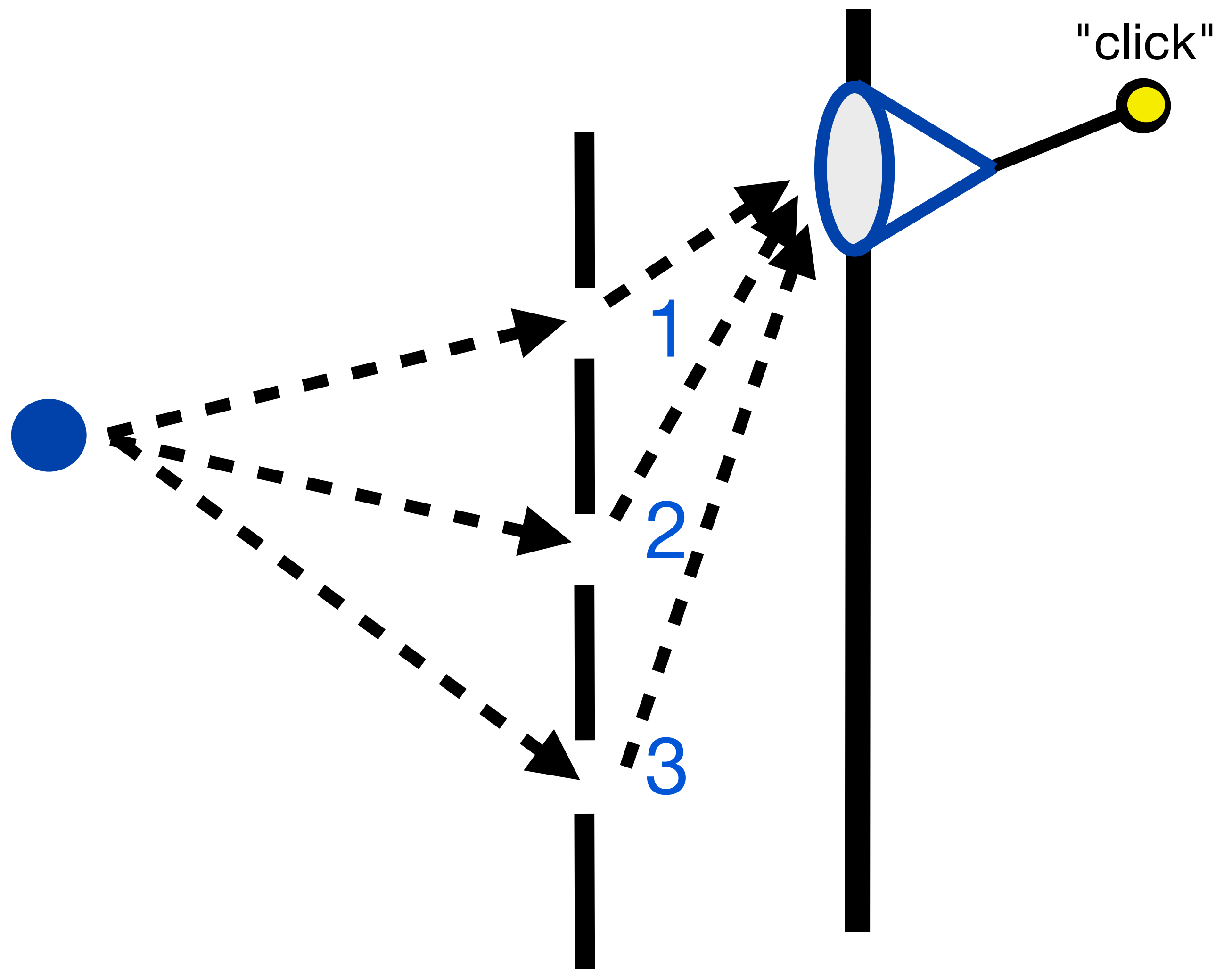

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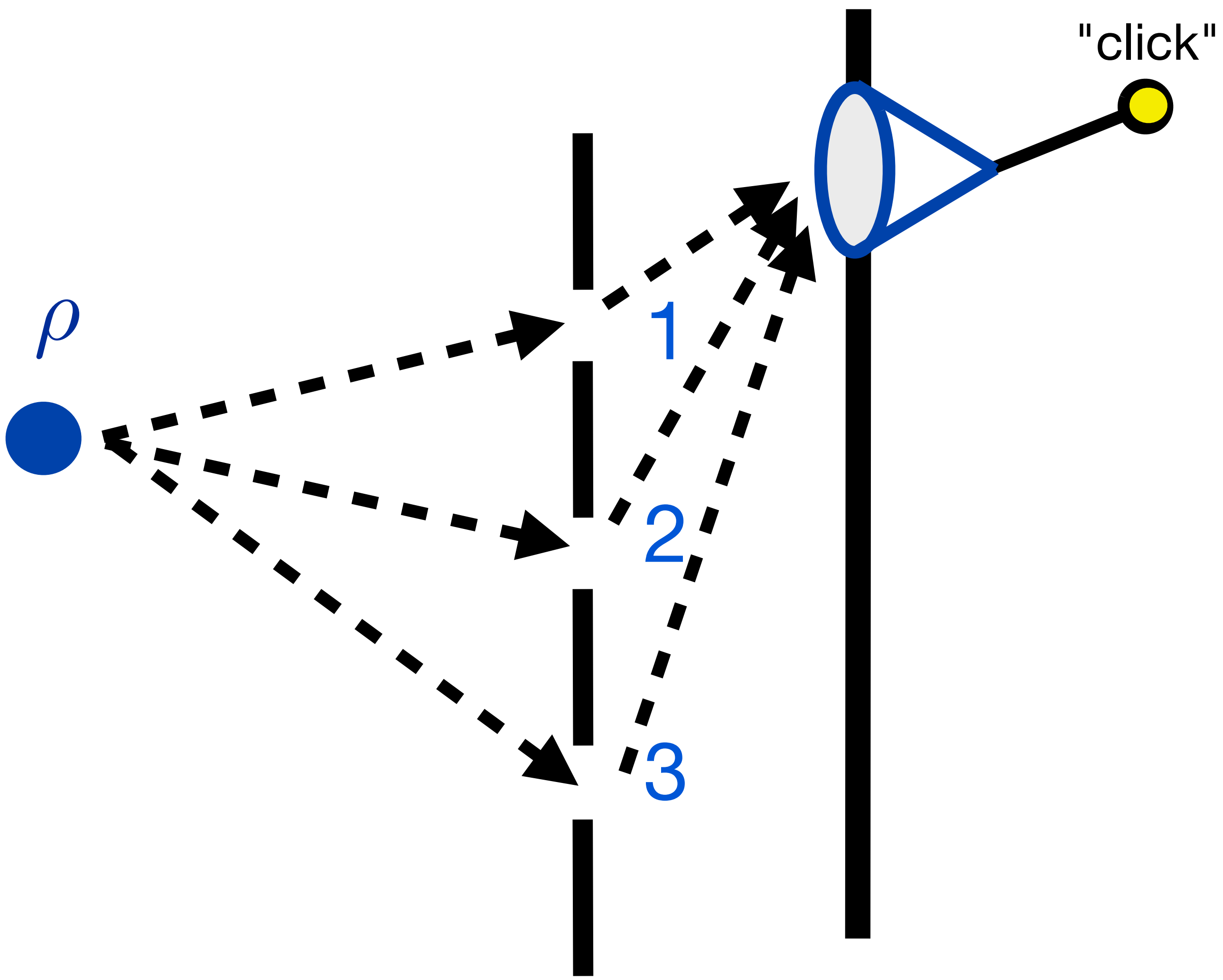
$$\rho = \begin{pmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{pmatrix}$$



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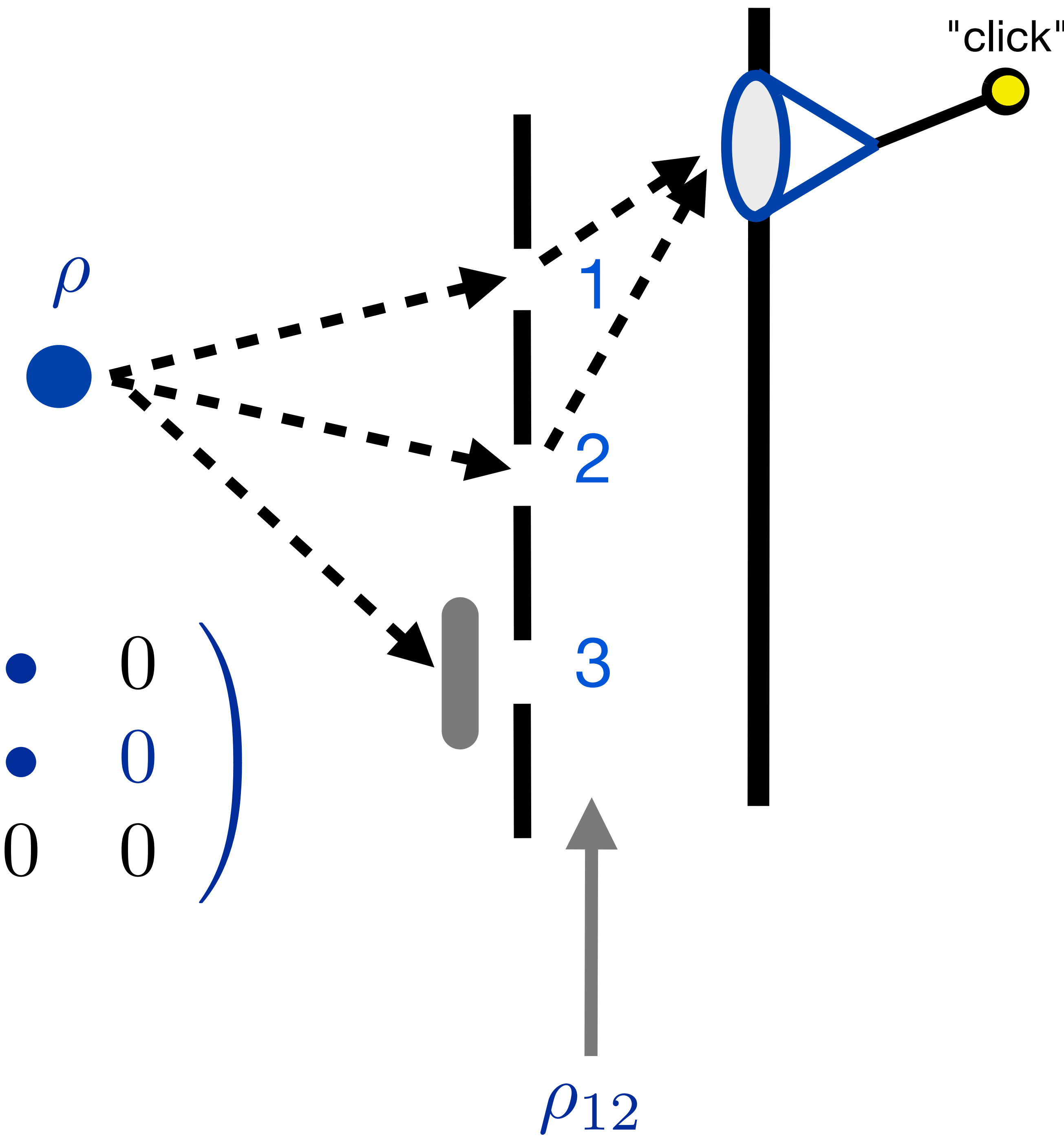

$$\rho \mapsto P_{12}\rho P_{12} =: \rho_{12} = \begin{pmatrix} \bullet & \bullet & 0 \\ \bullet & \bullet & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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$$- \begin{pmatrix} \bullet & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & \bullet & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \bullet \end{pmatrix}$$

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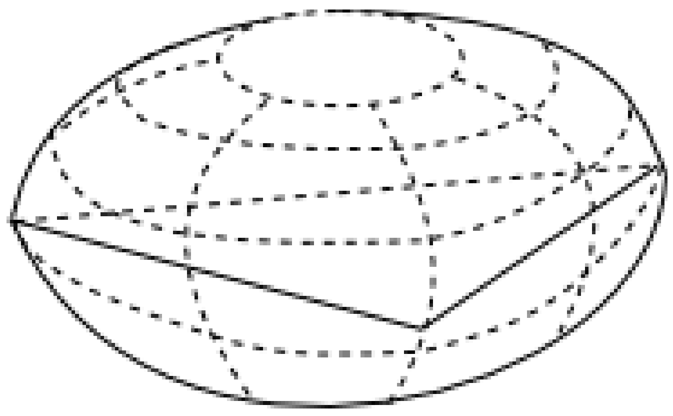
CPT:

$$\begin{pmatrix} \bullet \\ \bullet \\ \bullet \end{pmatrix} = \begin{pmatrix} \bullet \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \bullet \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \bullet \end{pmatrix}$$

$$p_{1,2,3} = p_1 + p_2 + p_3.$$

Why does CPT not have 2nd-order interference?

Some "artificial" GPTs exhibit order-3 interference:



C. Ududec, *Perspectives on the Formalism of Quantum Theory*, PhD thesis, University of Waterloo, 2012.

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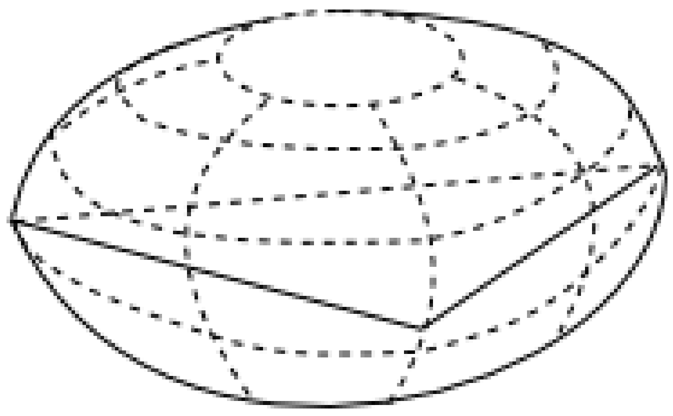

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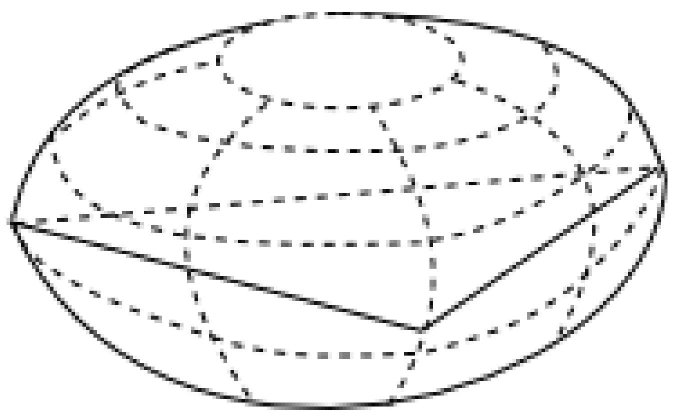

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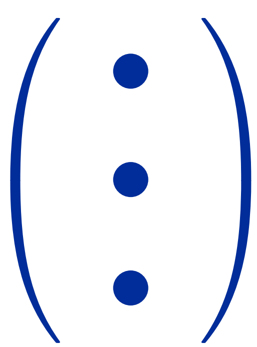

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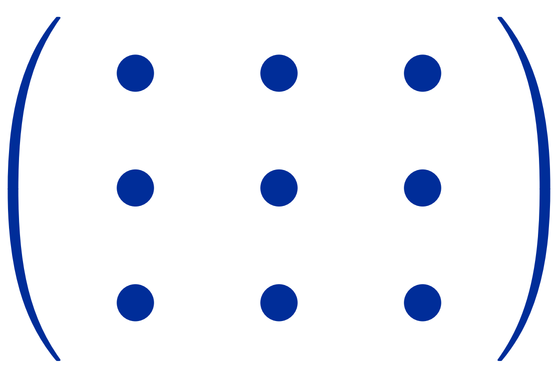

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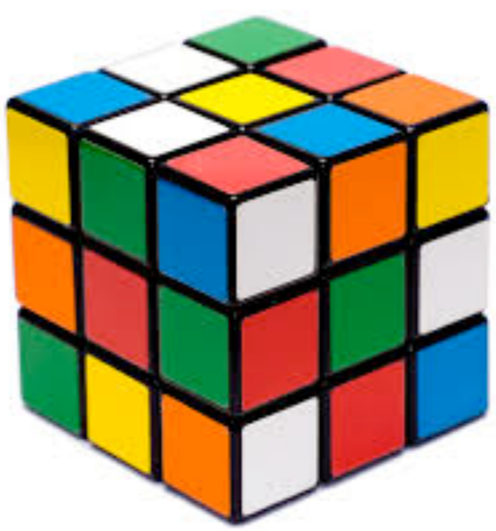

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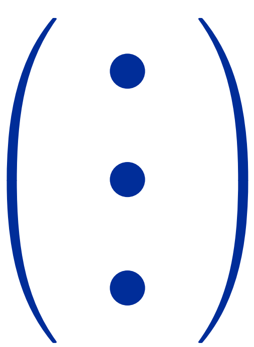
A quantum detective story



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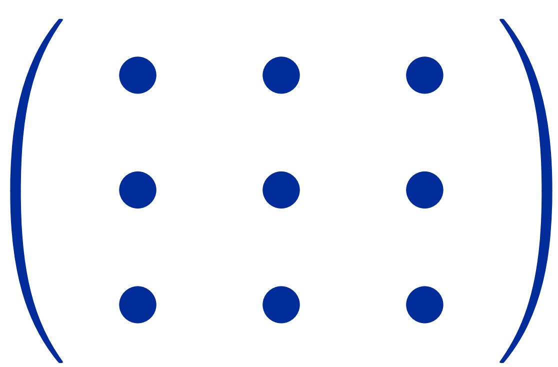

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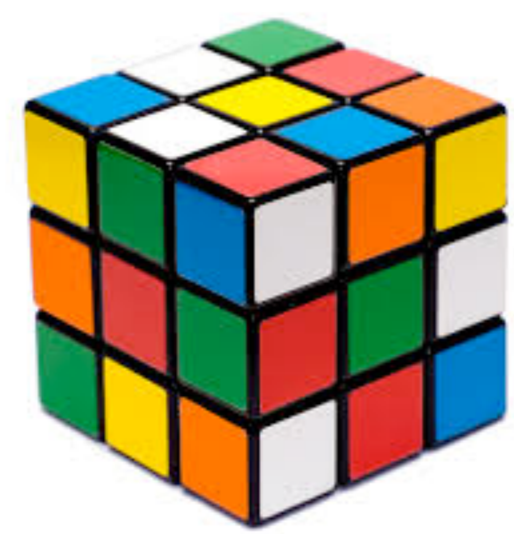

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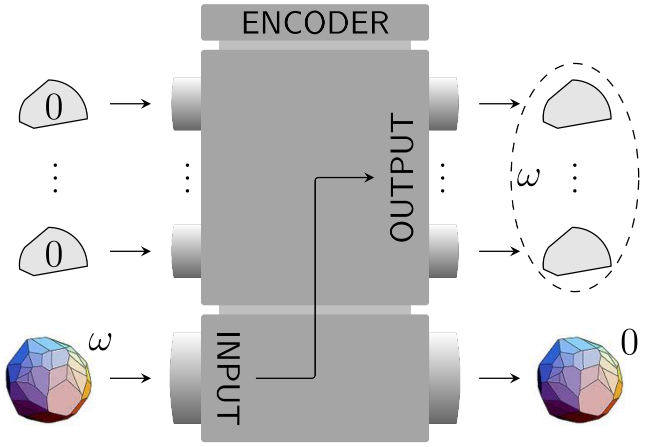
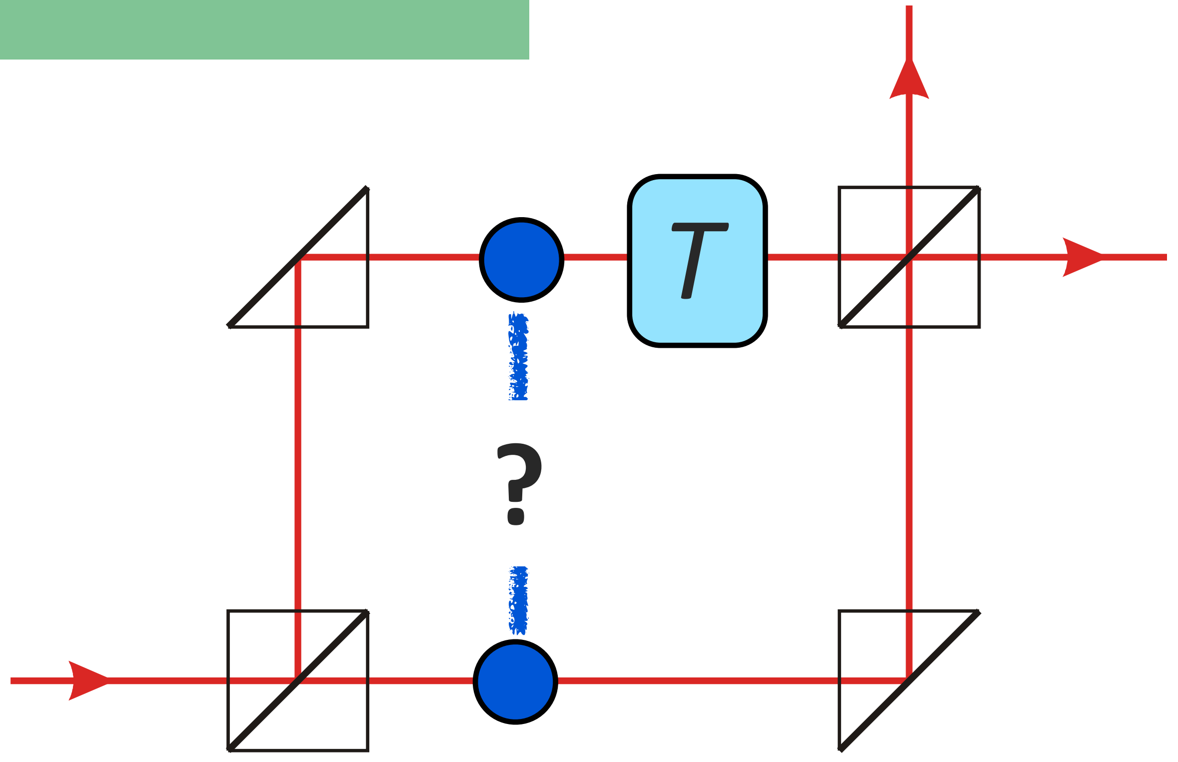
2. Quantum theory from simple principles



3. The quest for higher-order interference

4. QT and spacetime

5. Conclusion



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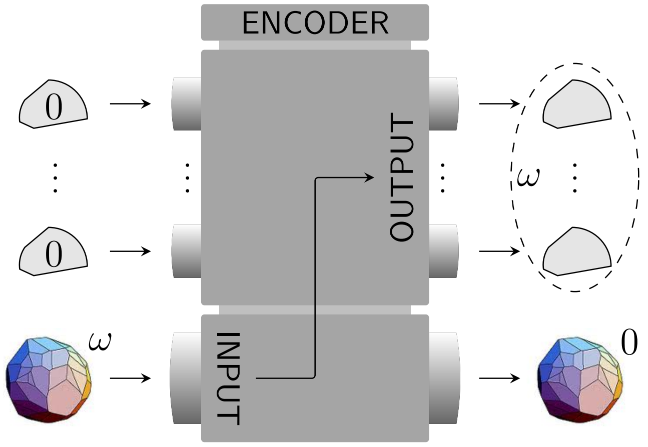

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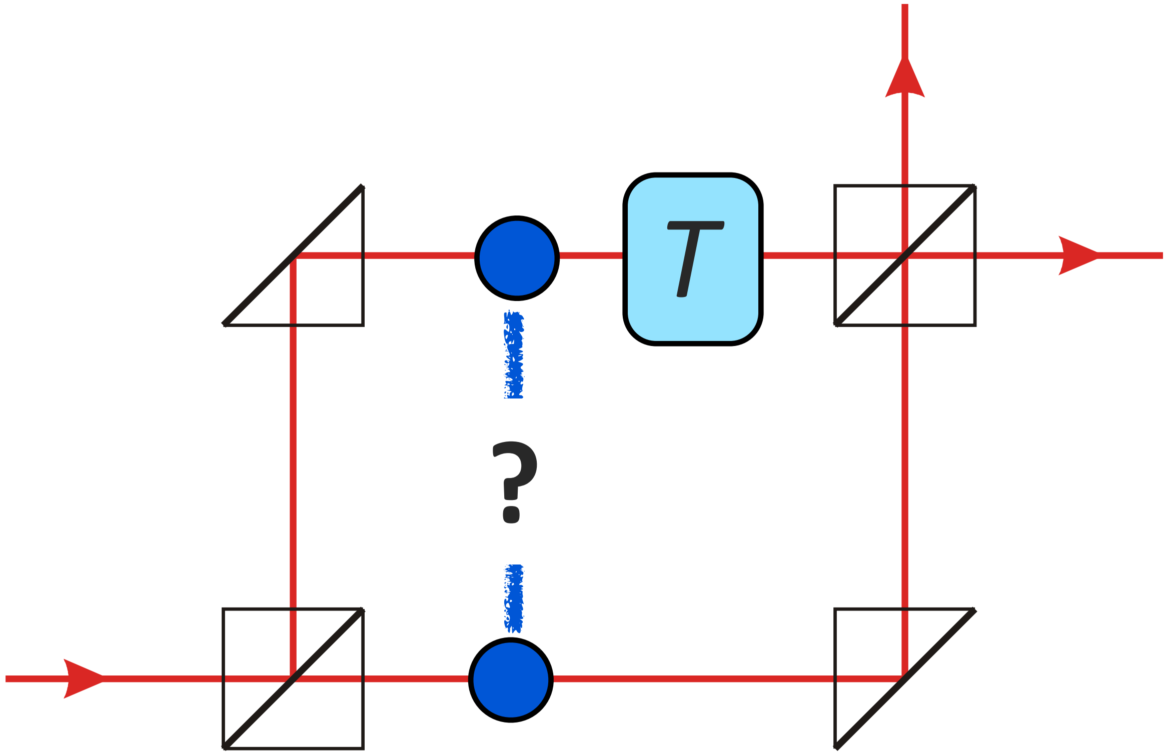

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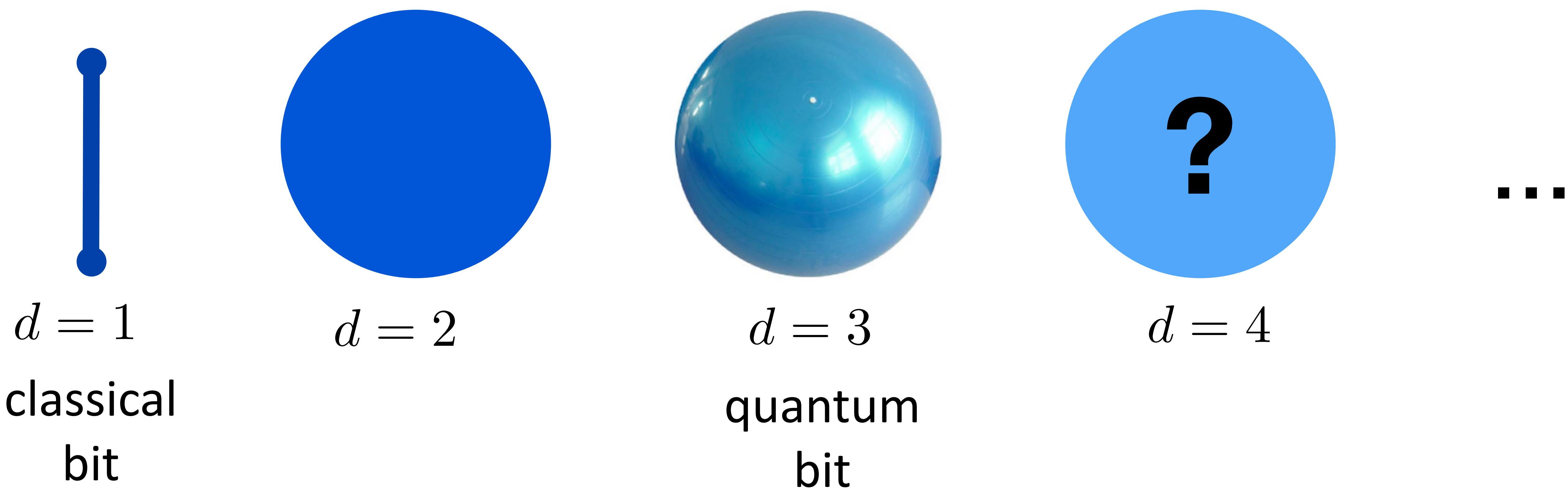

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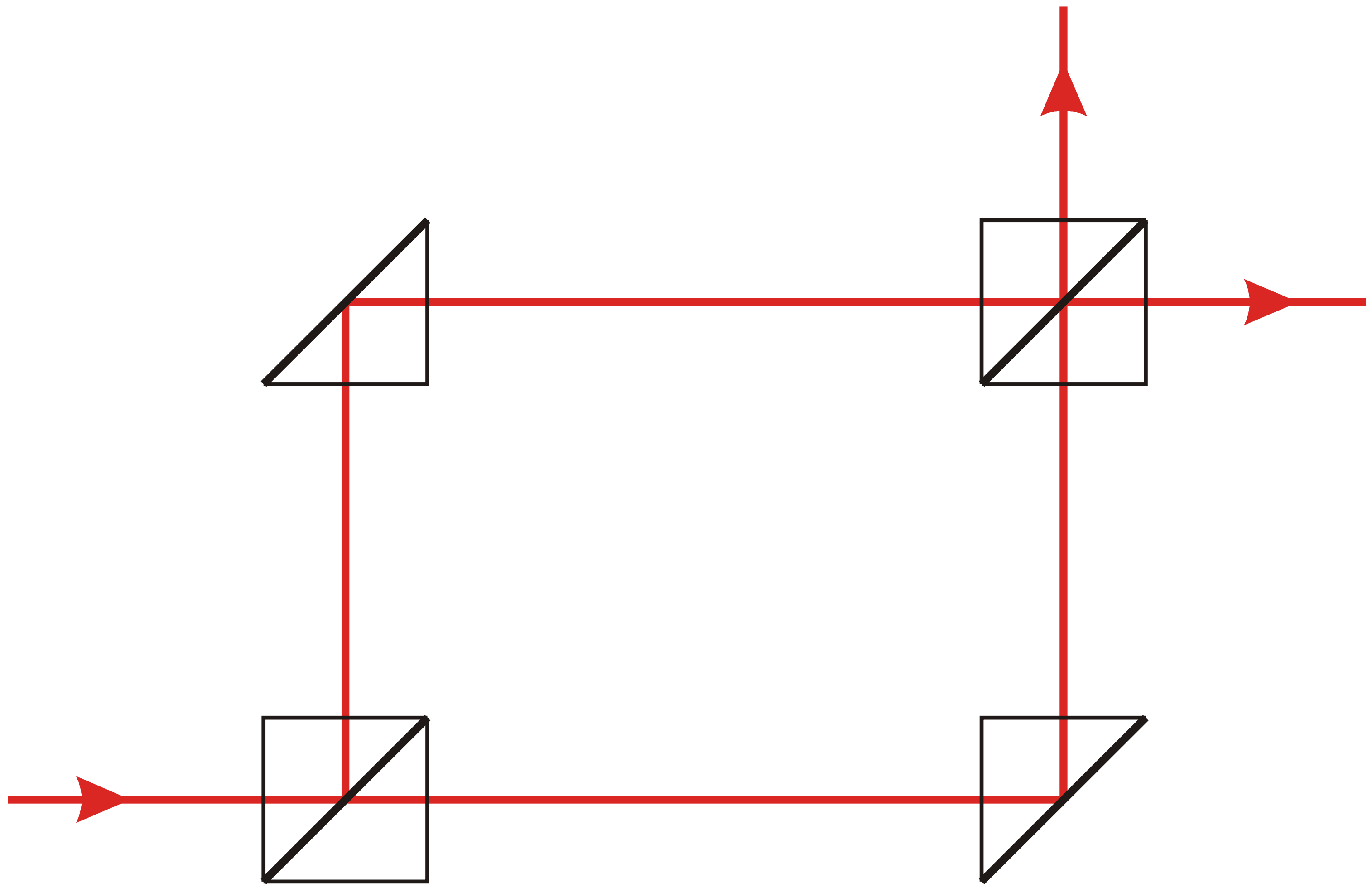
We have already seen an **information-theoretic** reason.
But there is also a “spacetime” reason!

Constraints from relativity

A. Garner, **MM**, O. C. O. Dahlsten, Proc. R. Soc. A **473**, 20170596 (2017).



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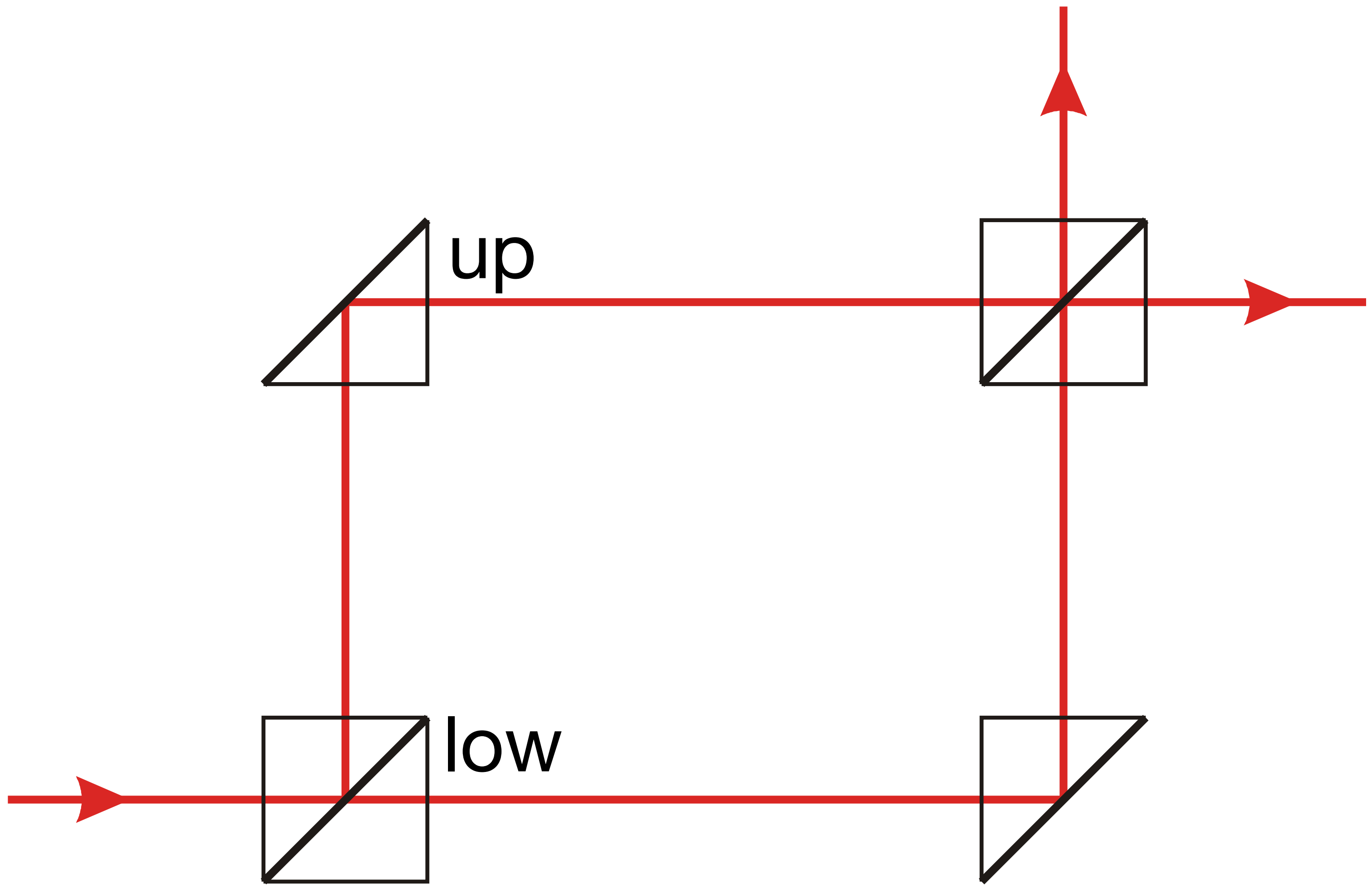

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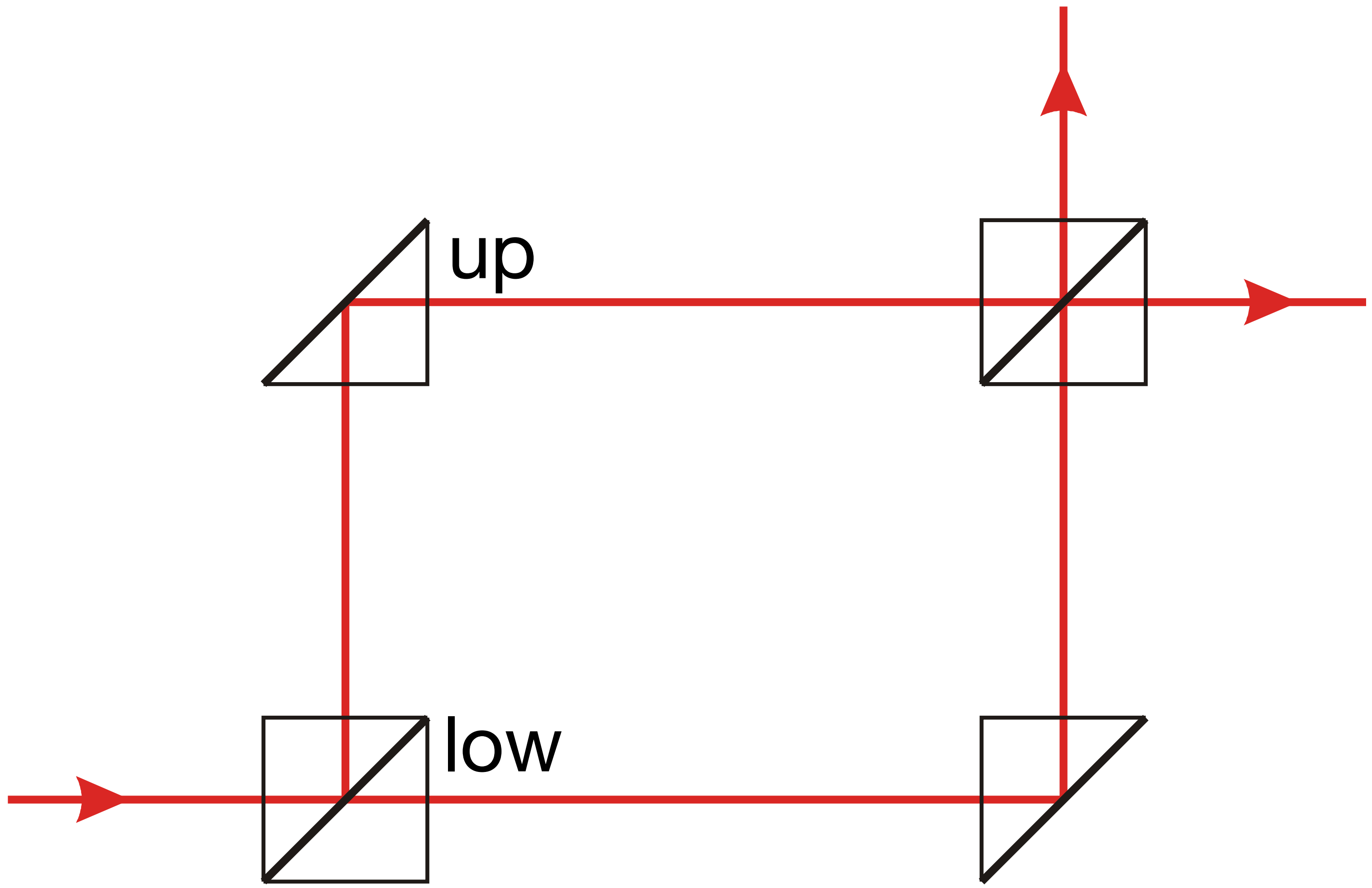

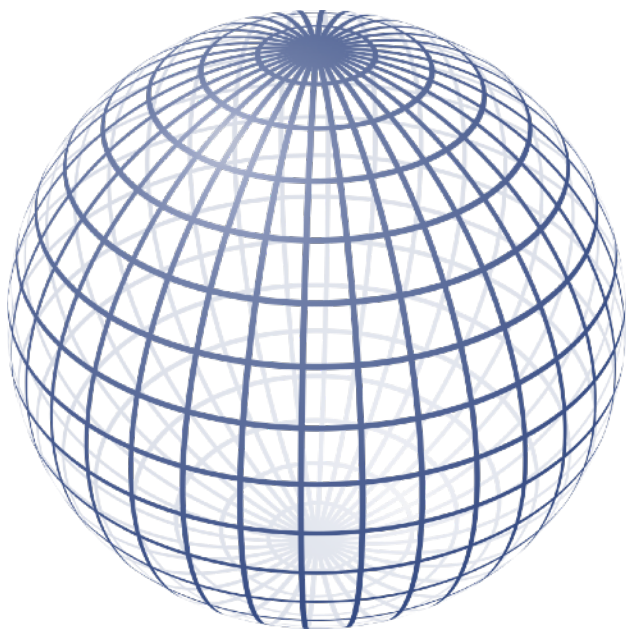

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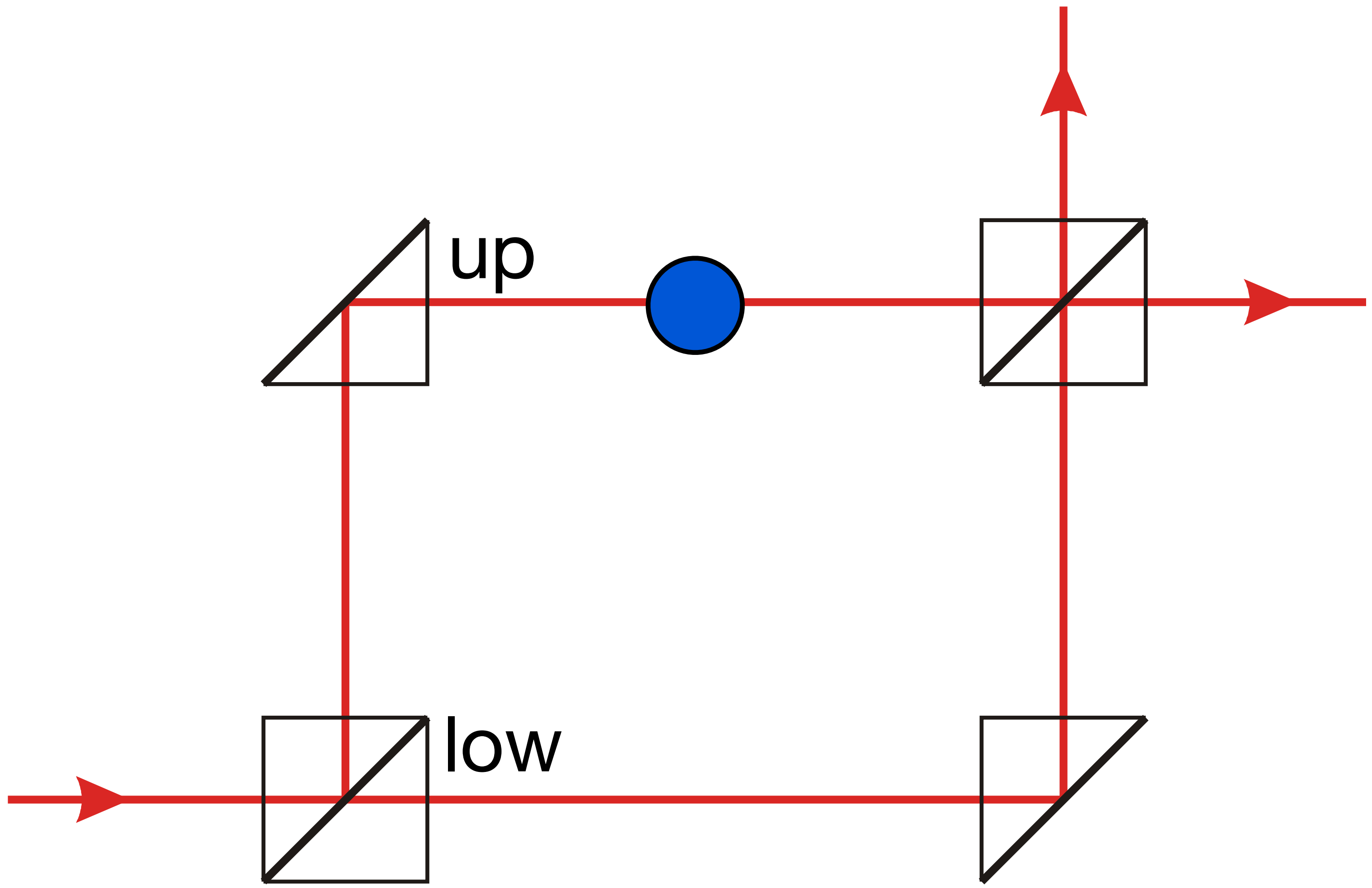

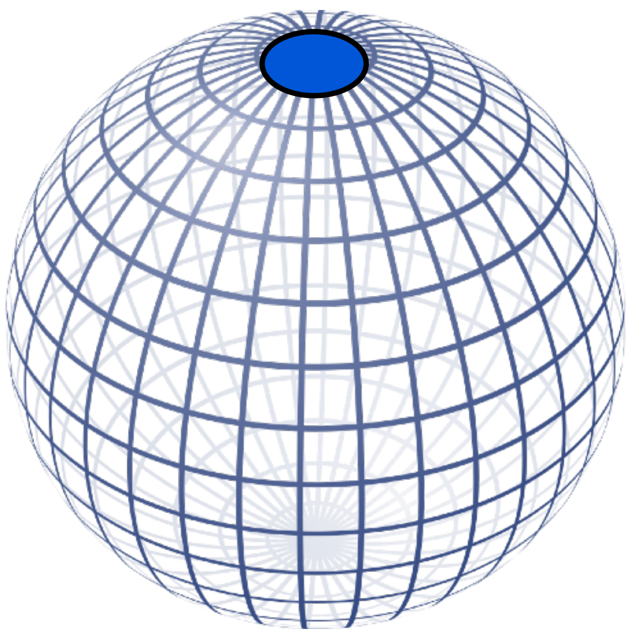

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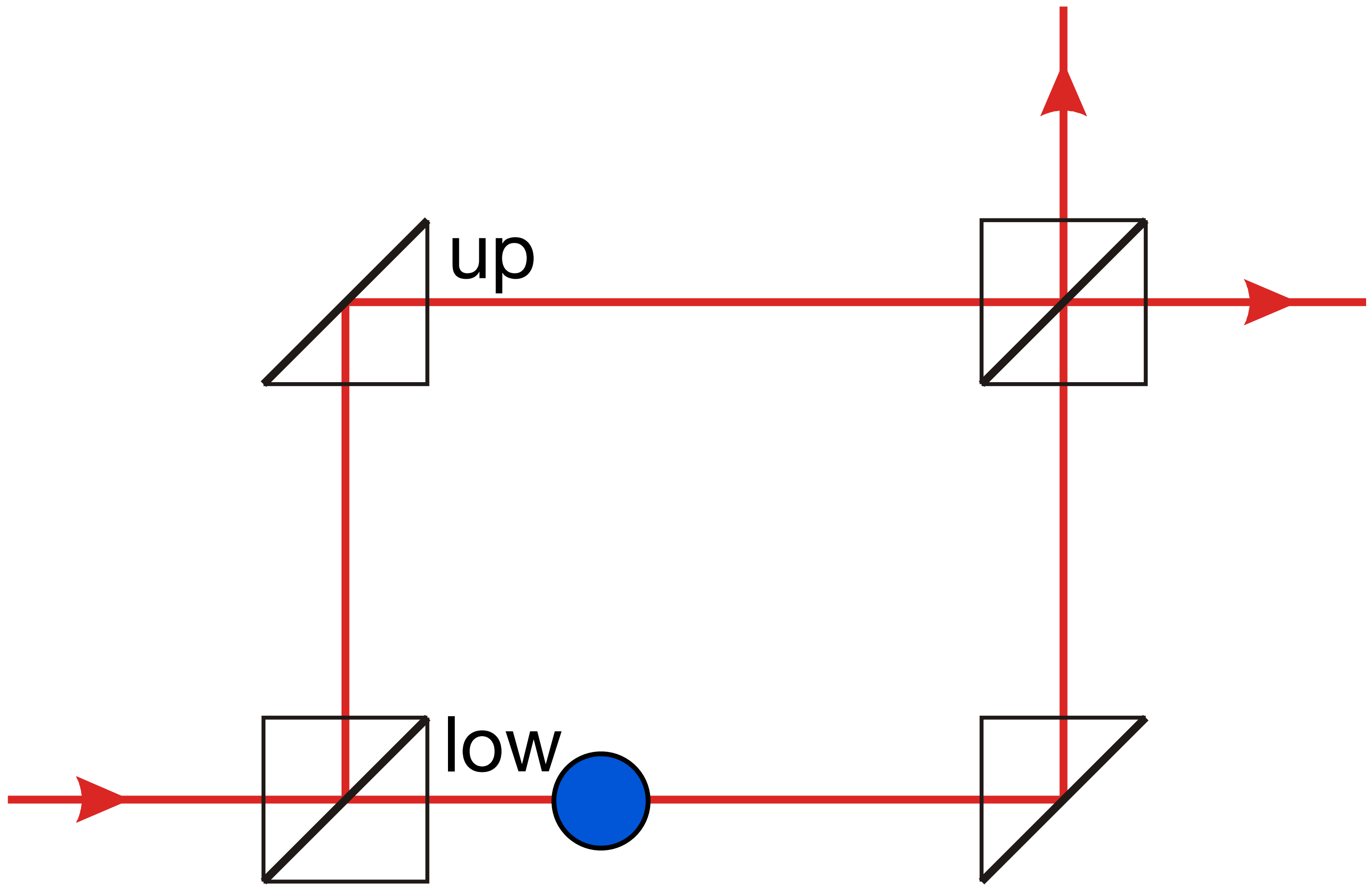

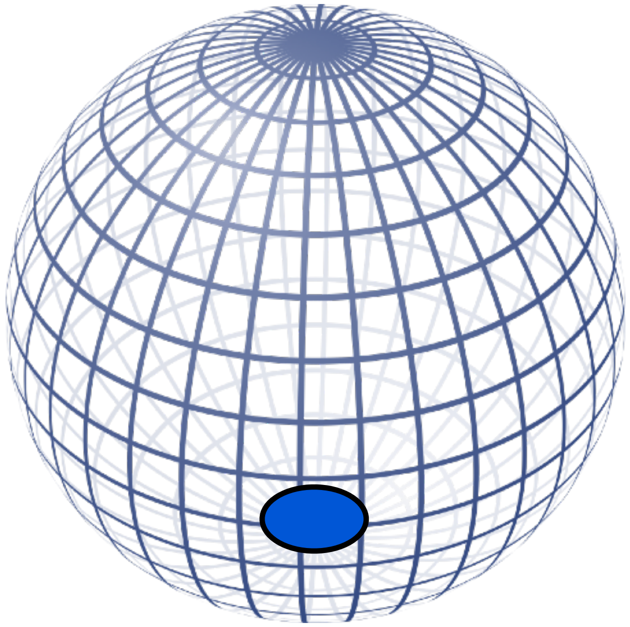

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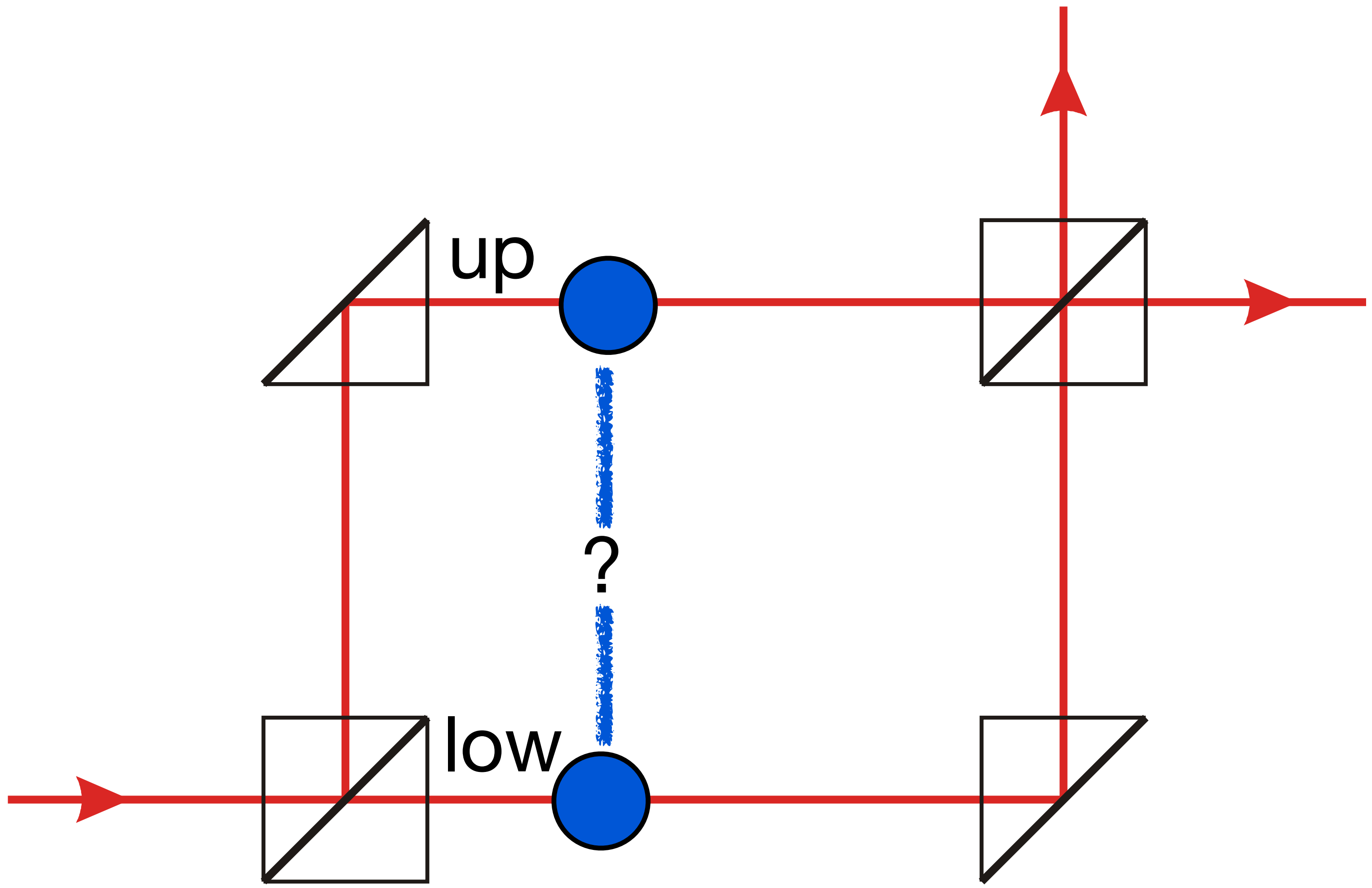

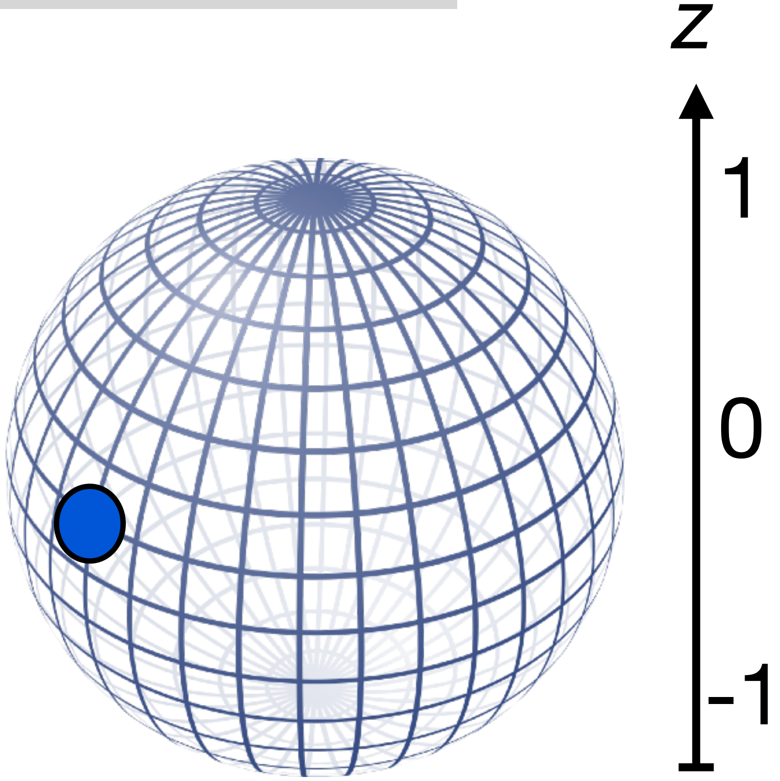

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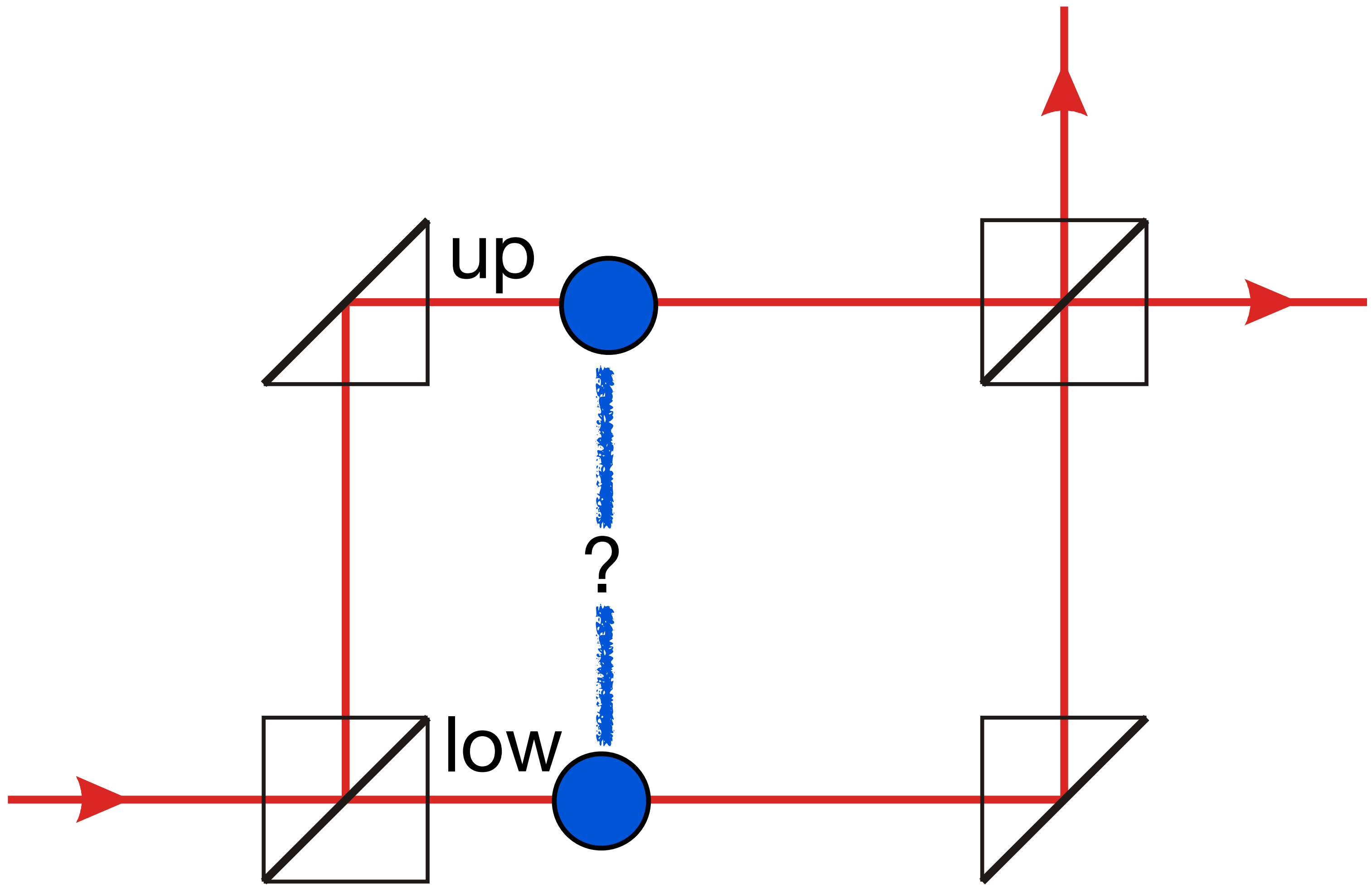

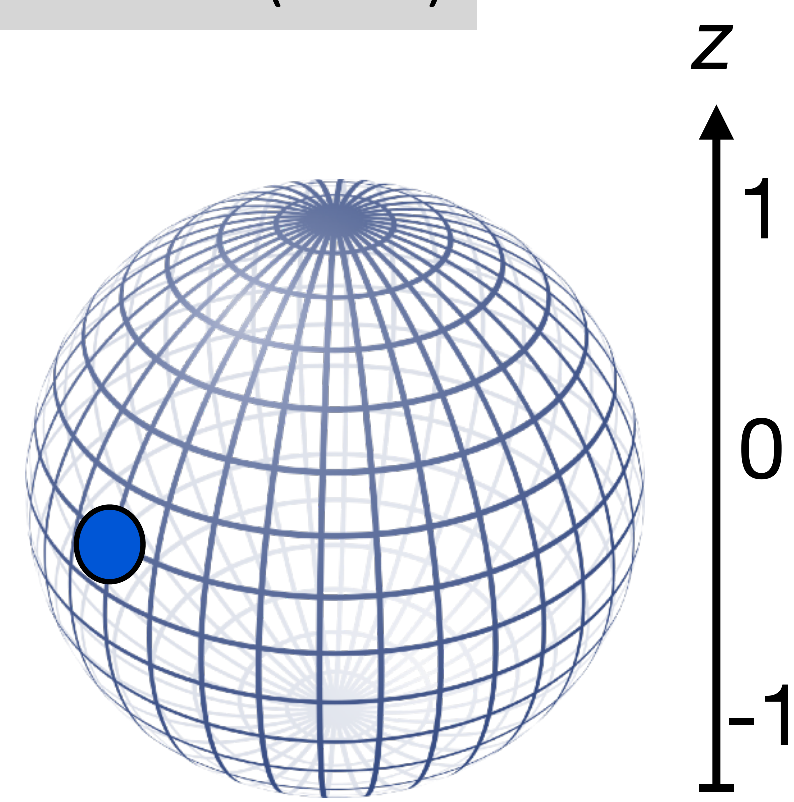

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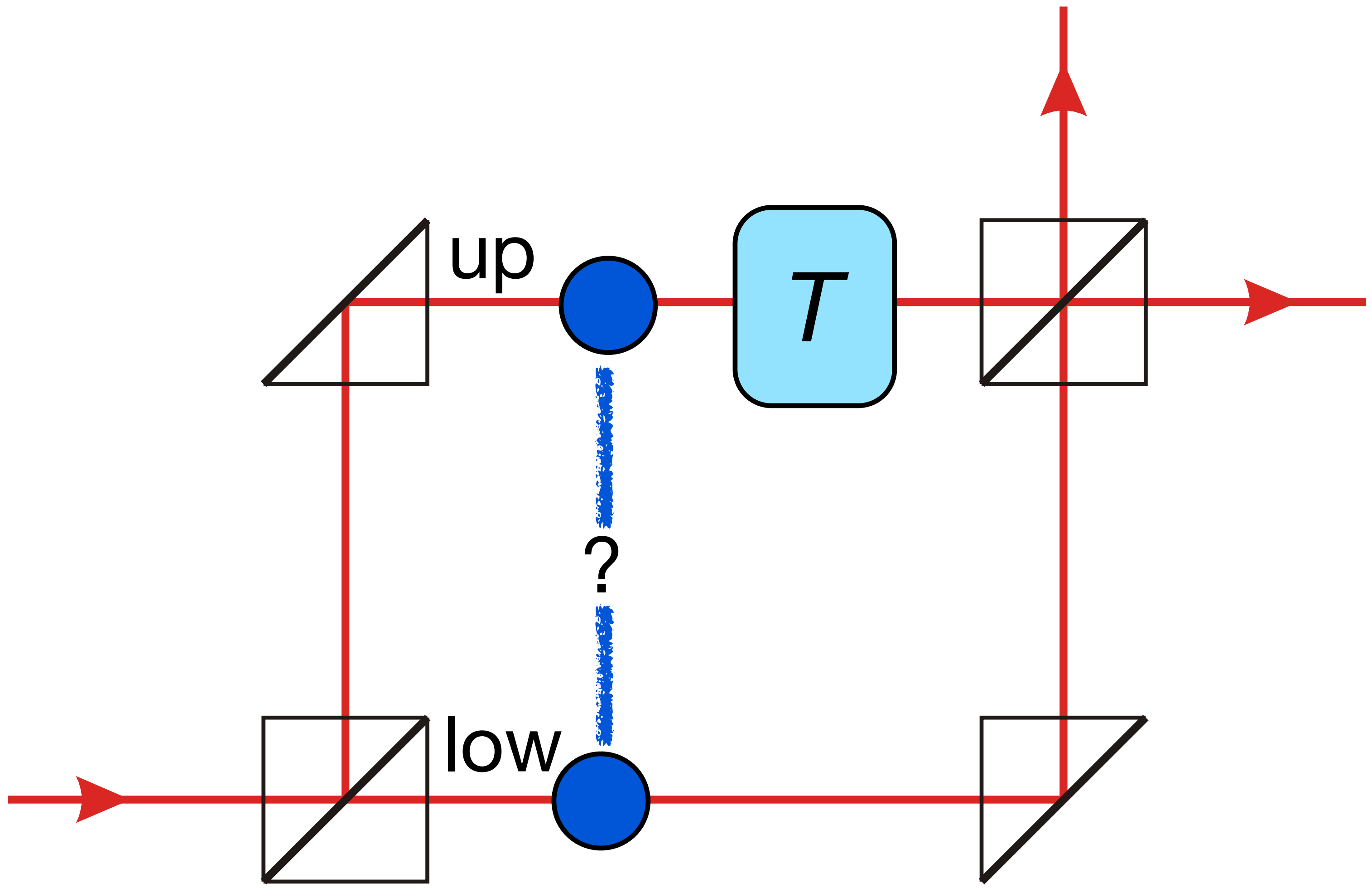

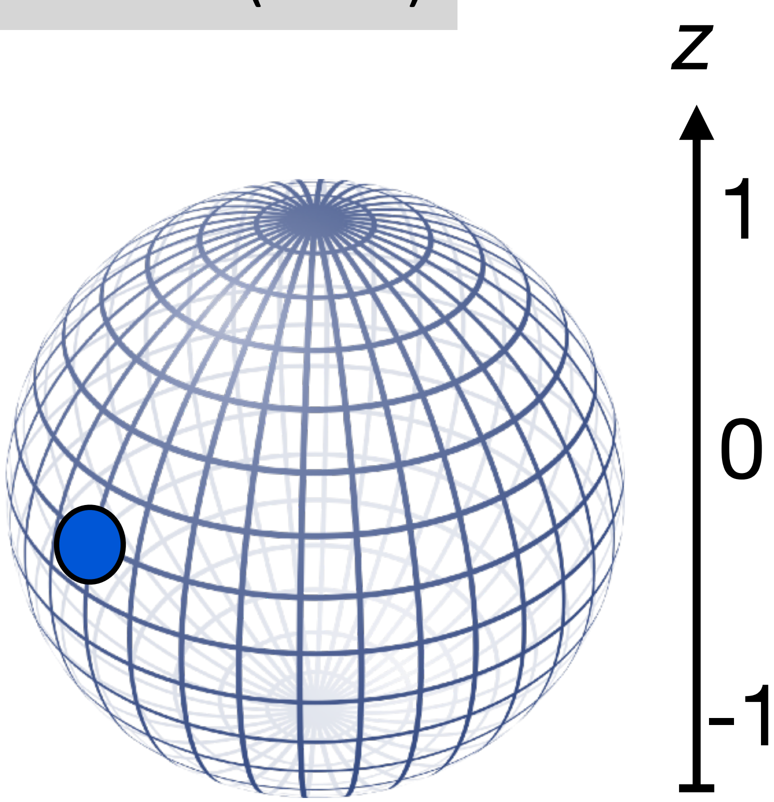

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What transformations *T* can we perform locally in one arm...
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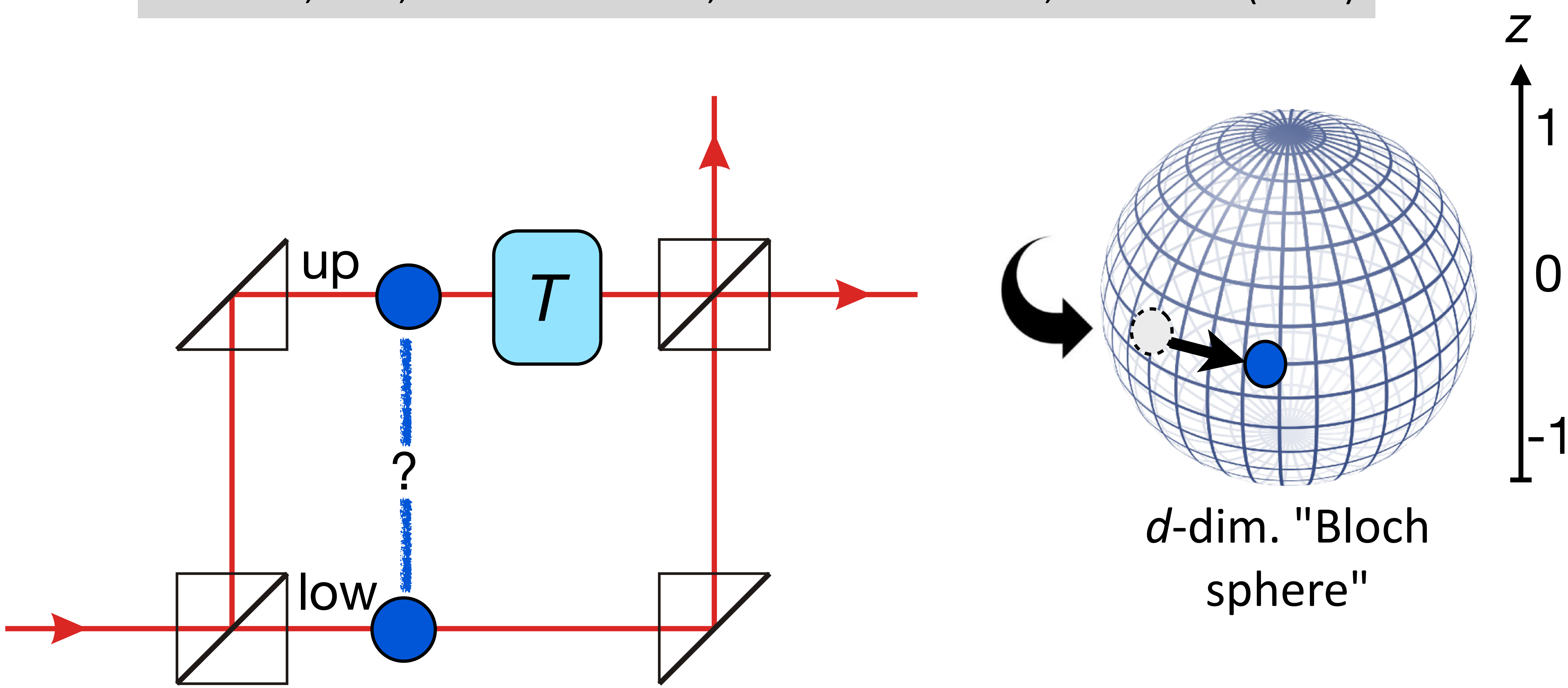

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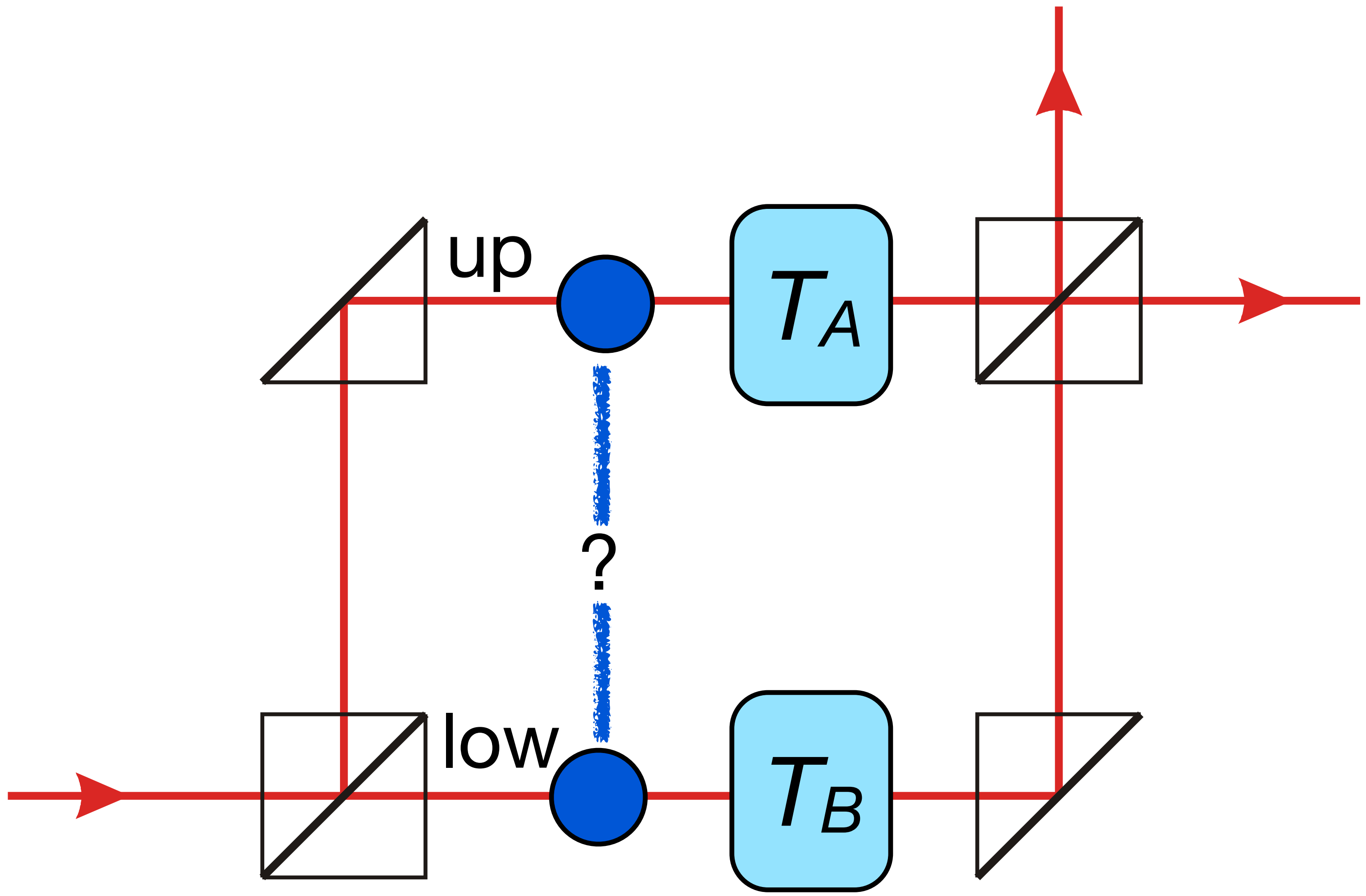

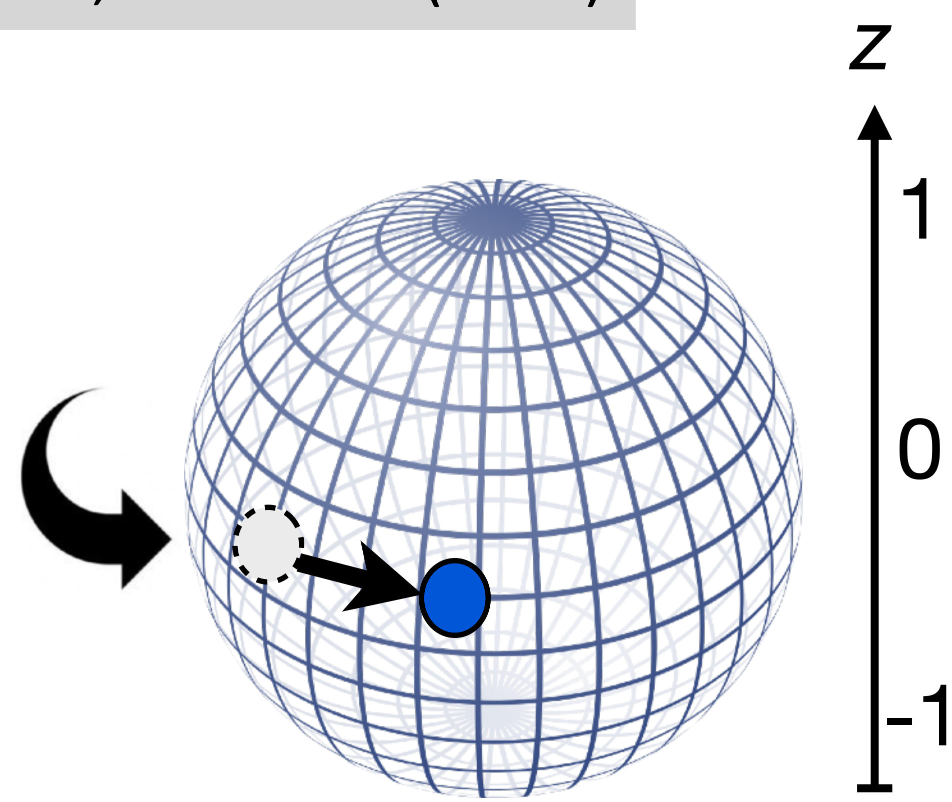

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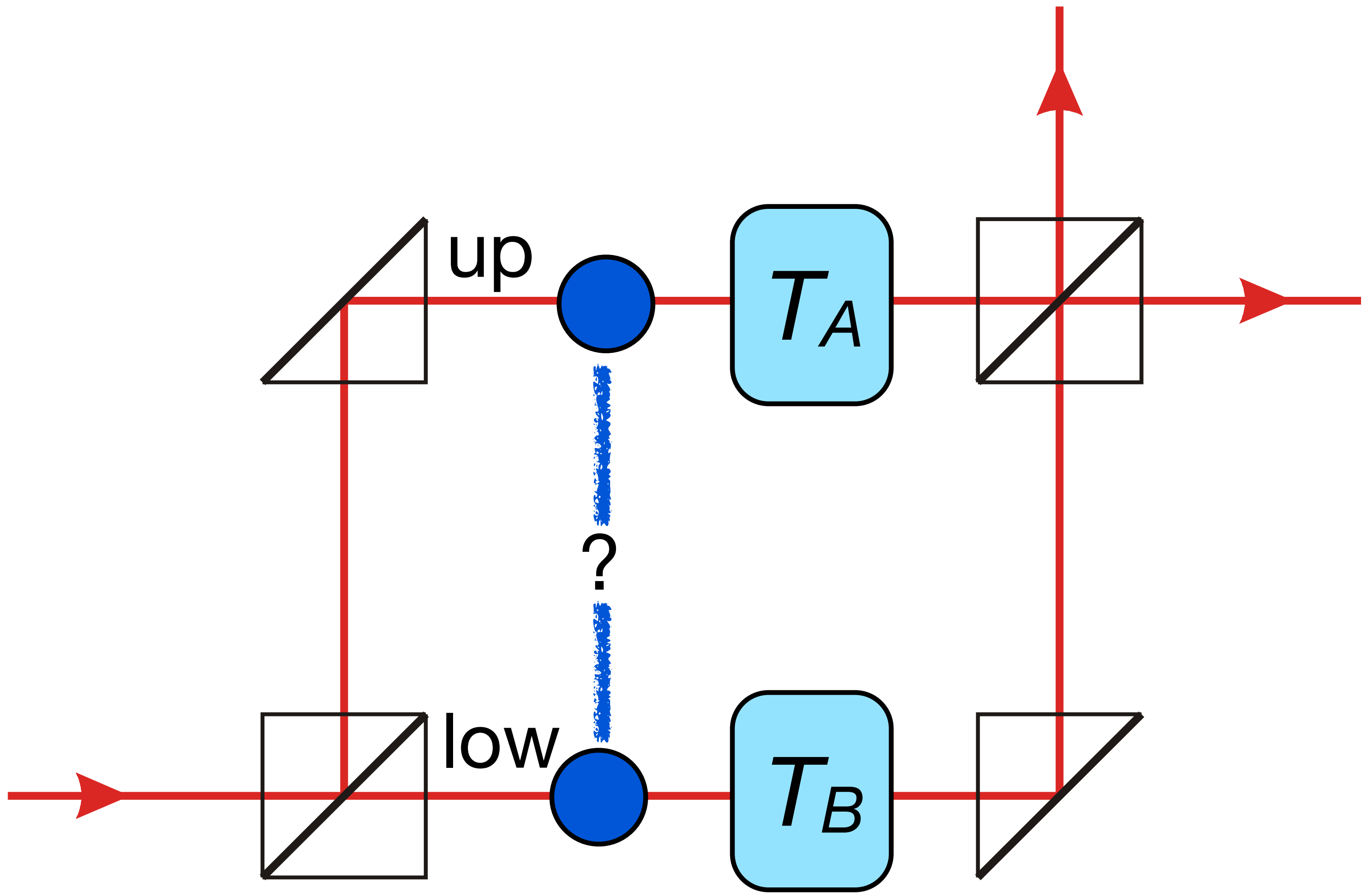

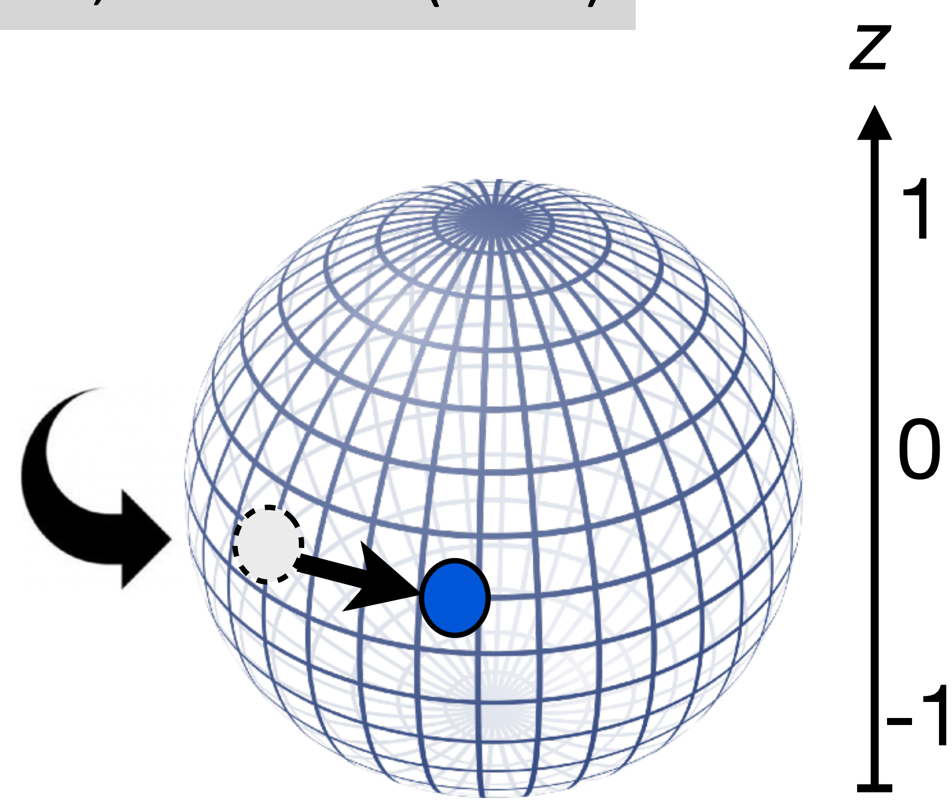

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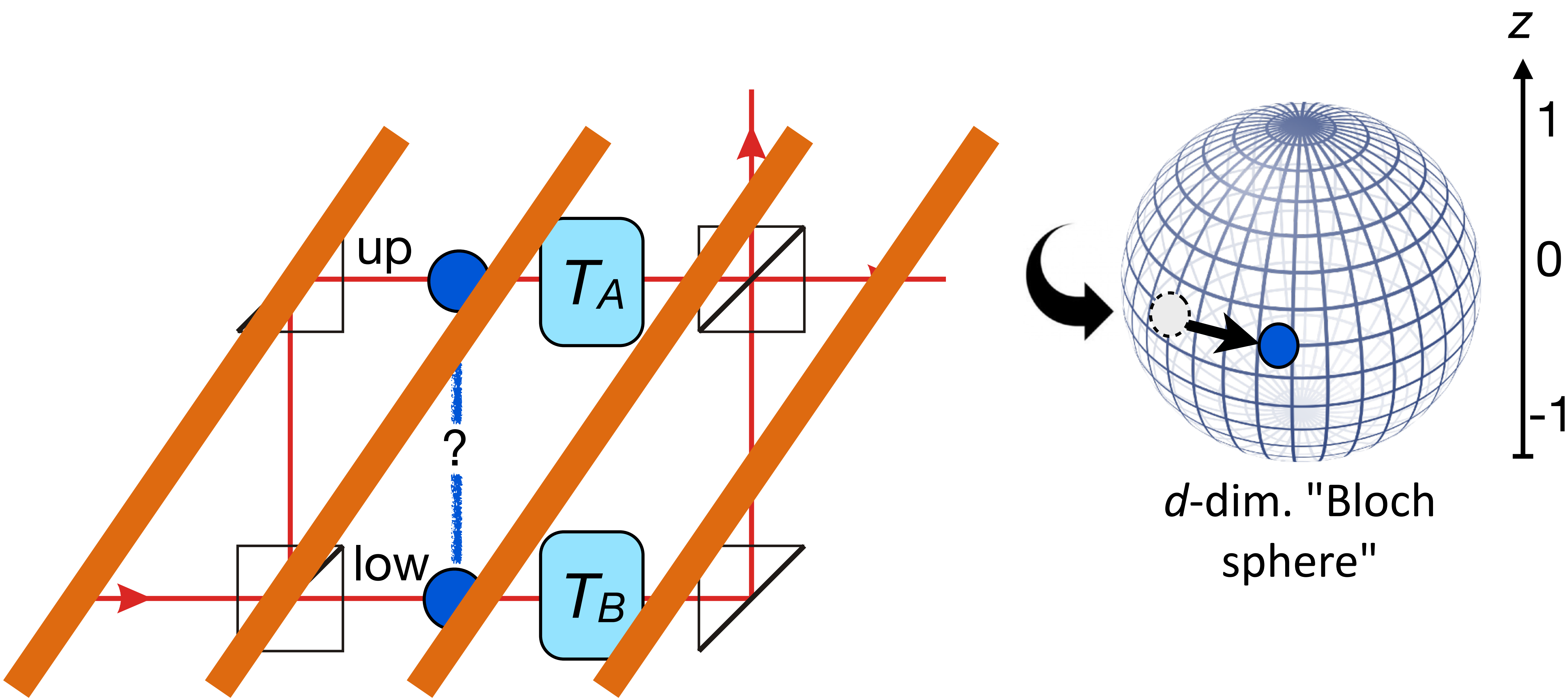

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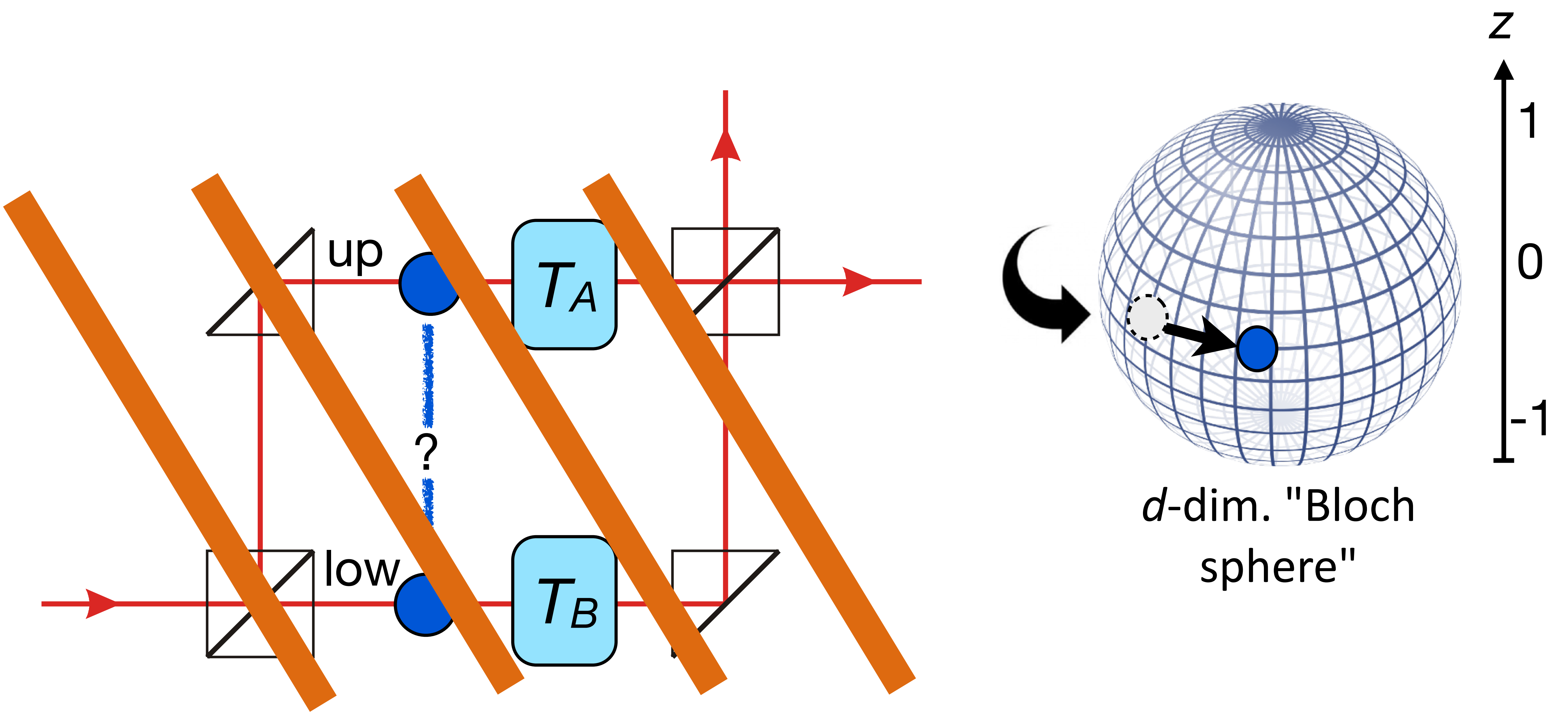

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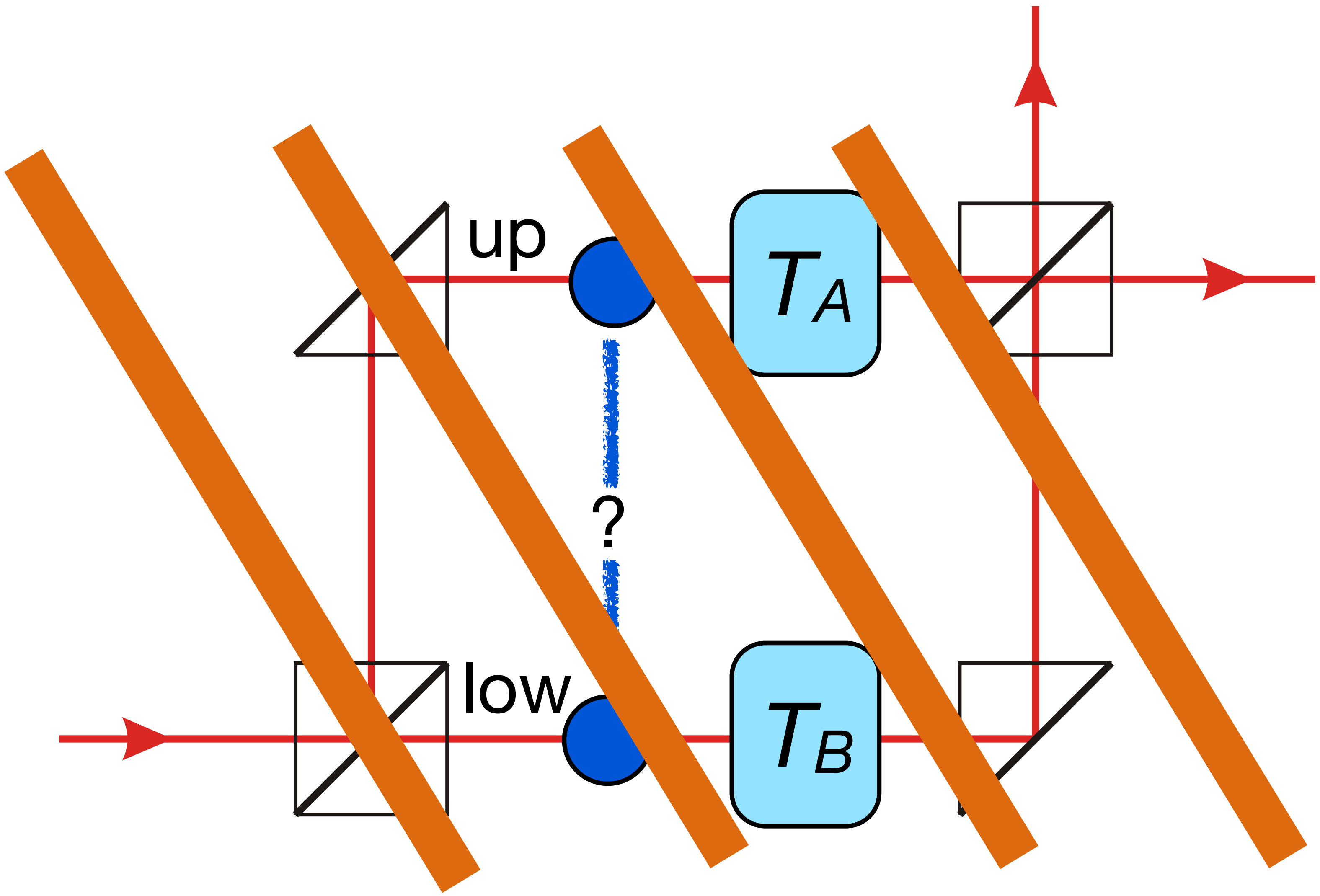

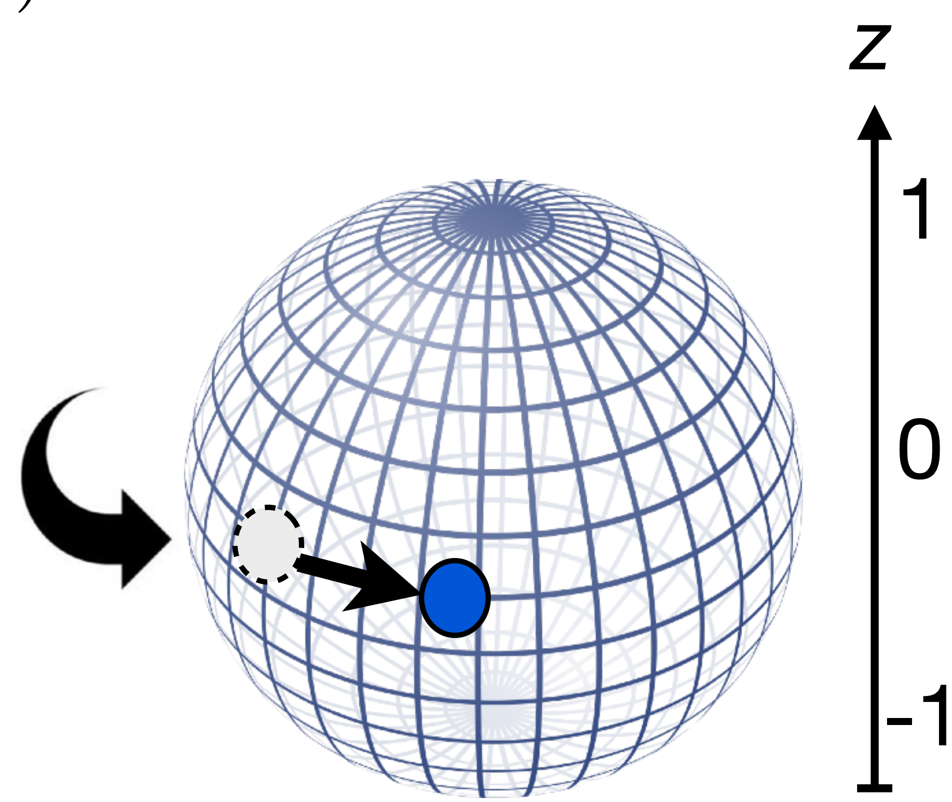

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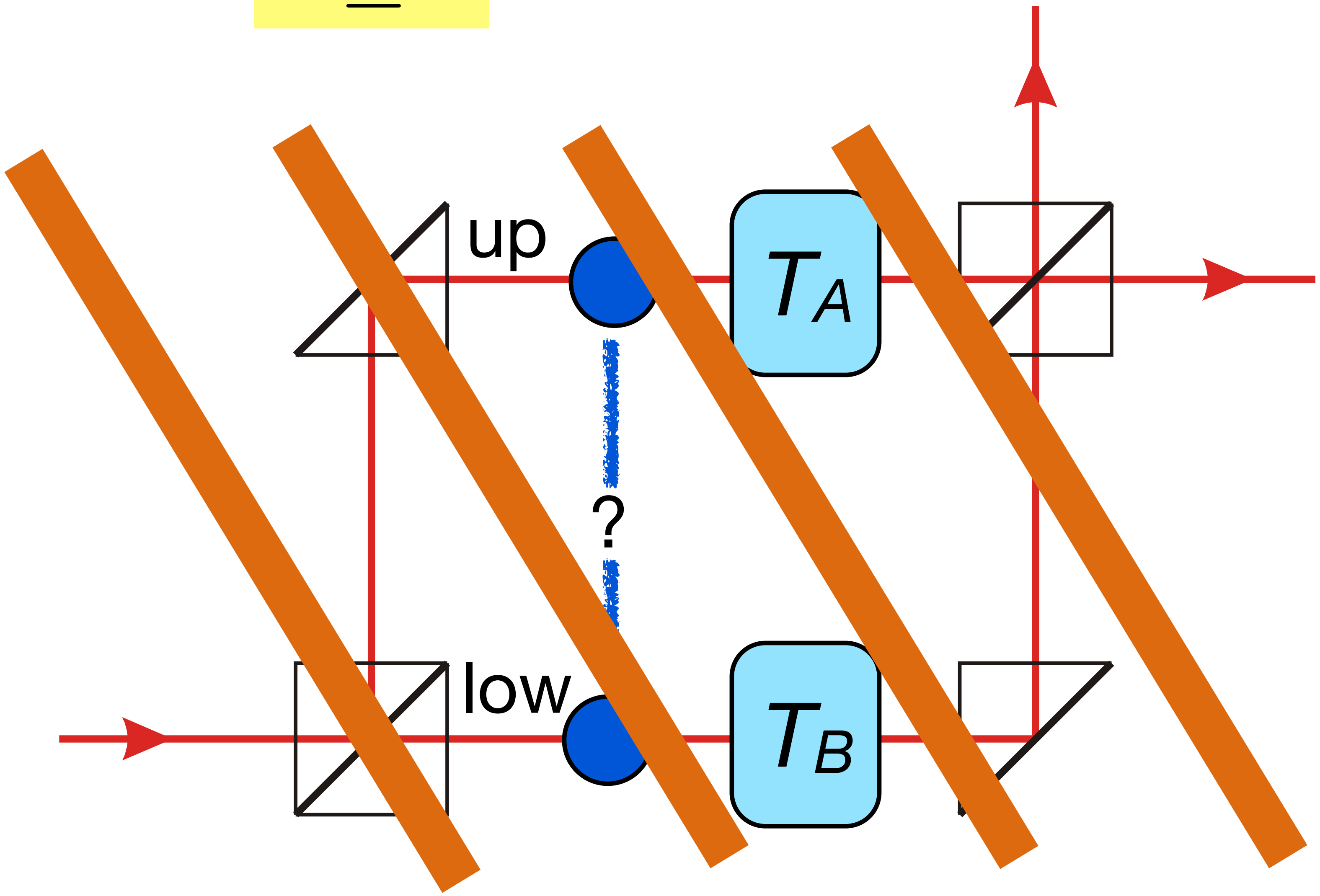

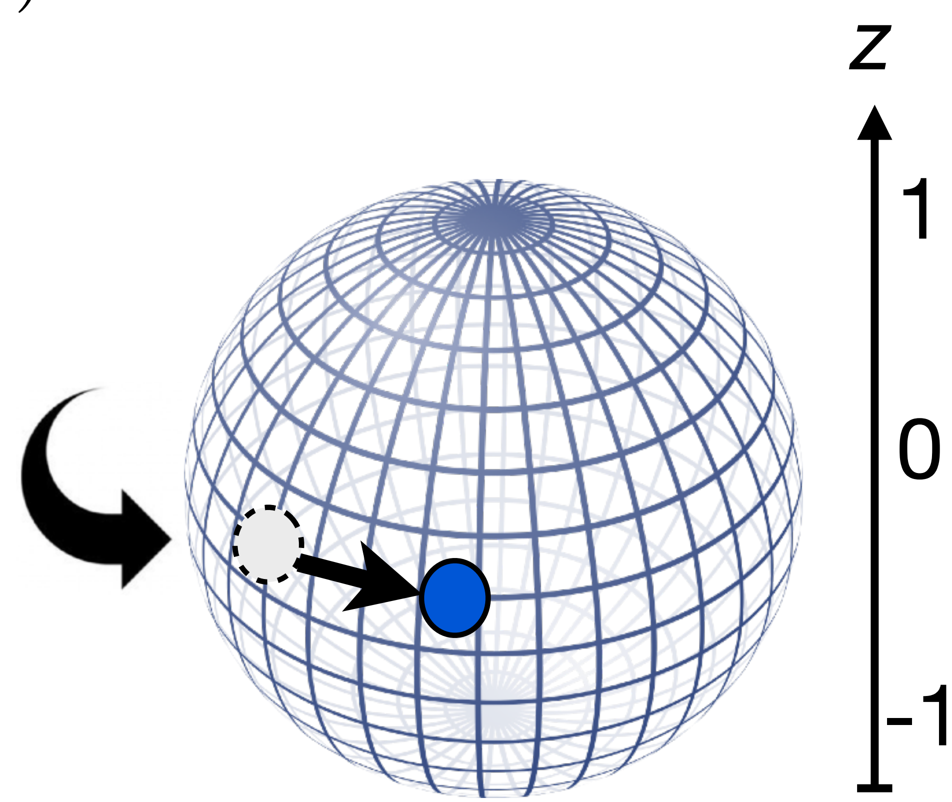

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Information theory

↕

spacetime

?!

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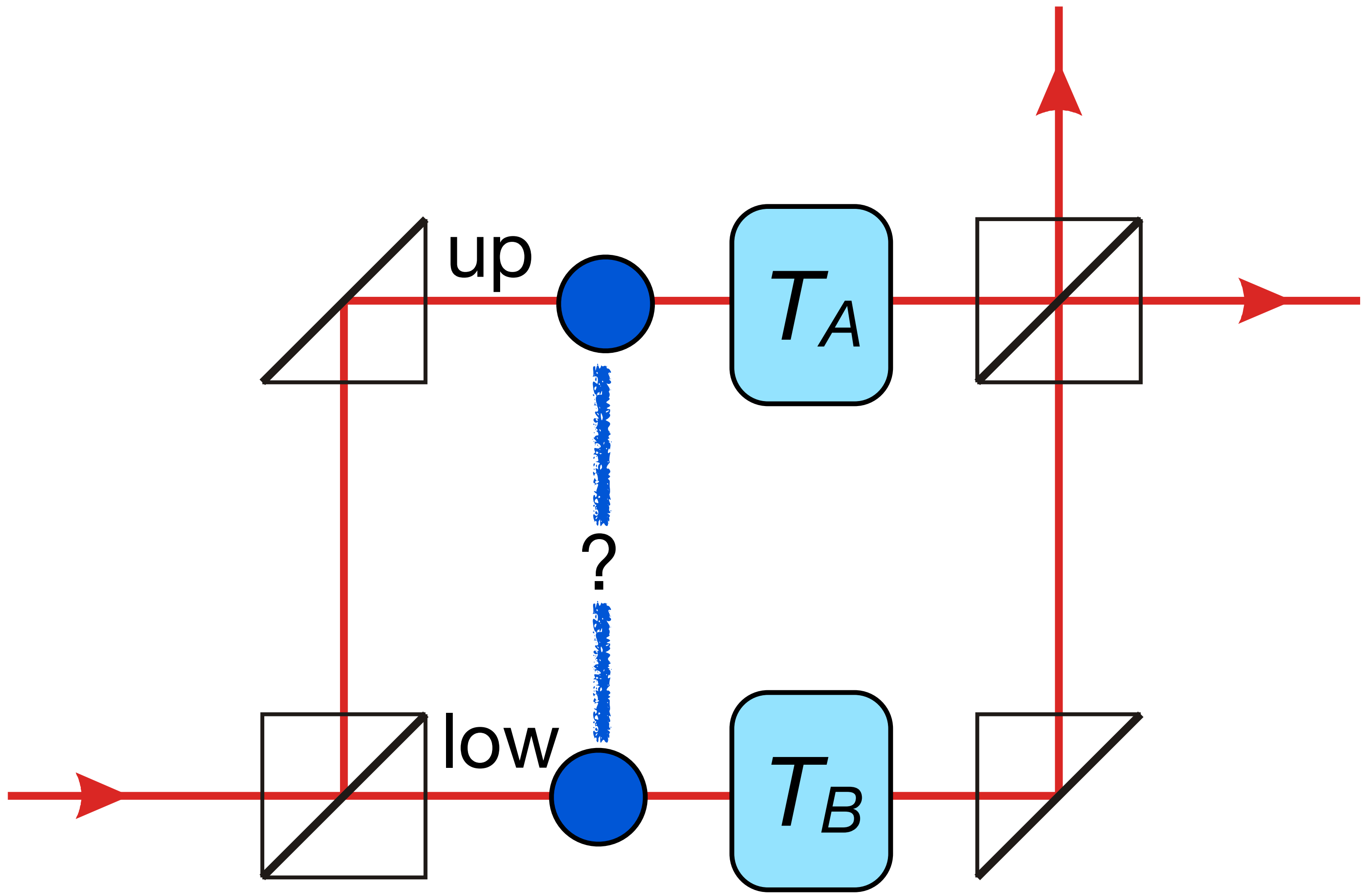
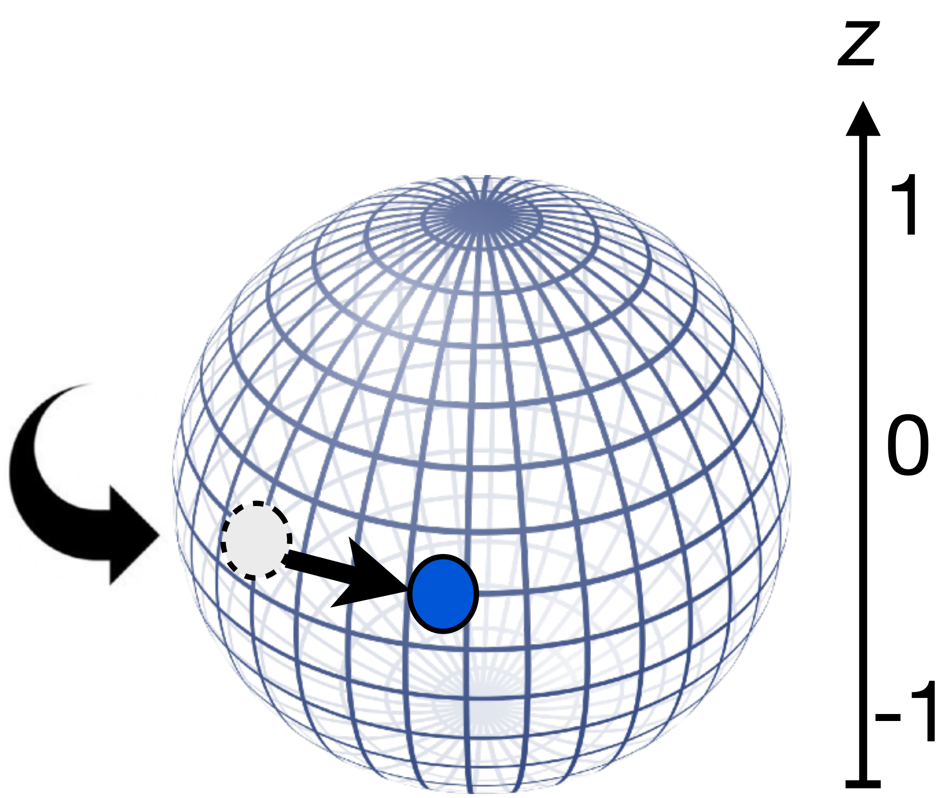
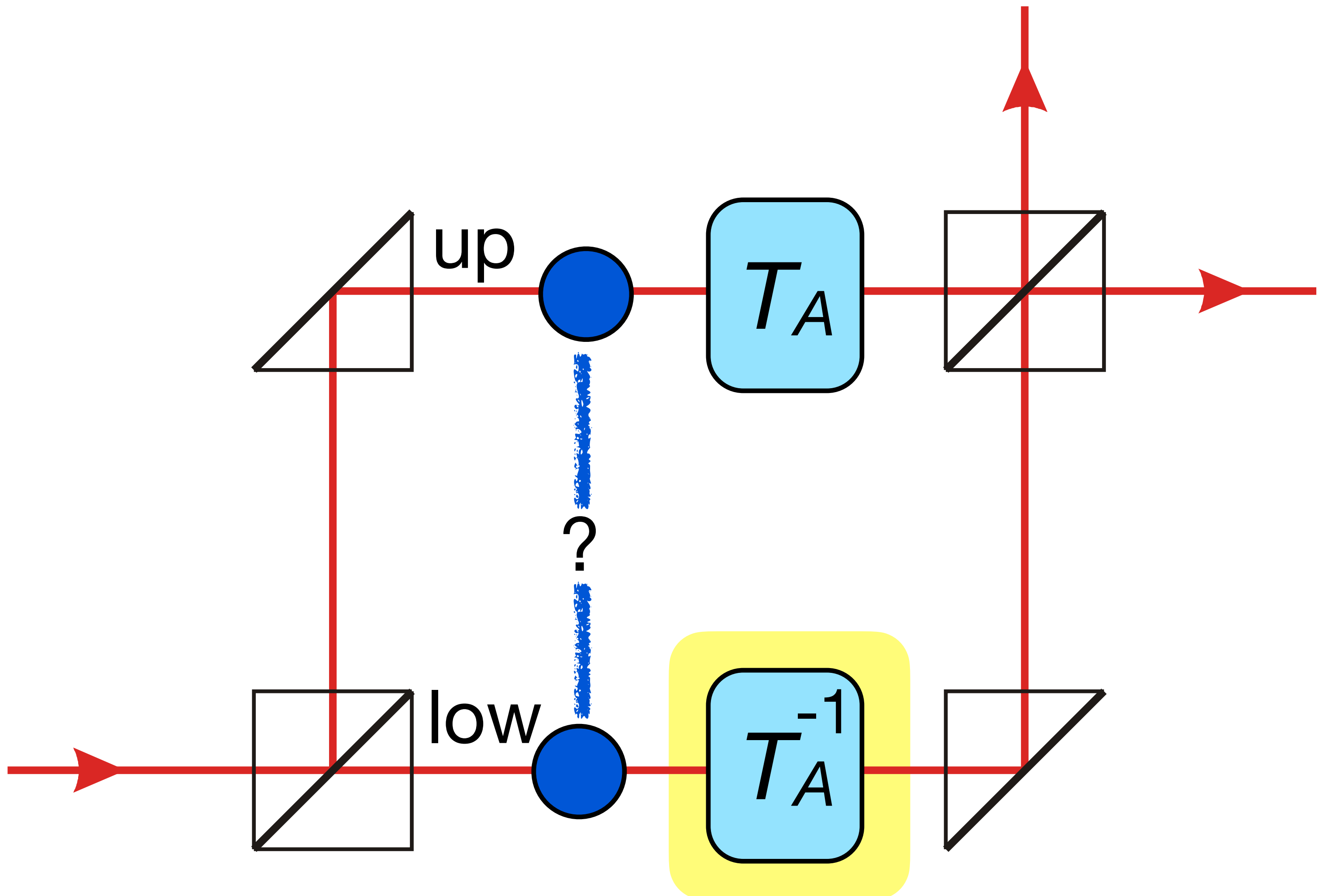
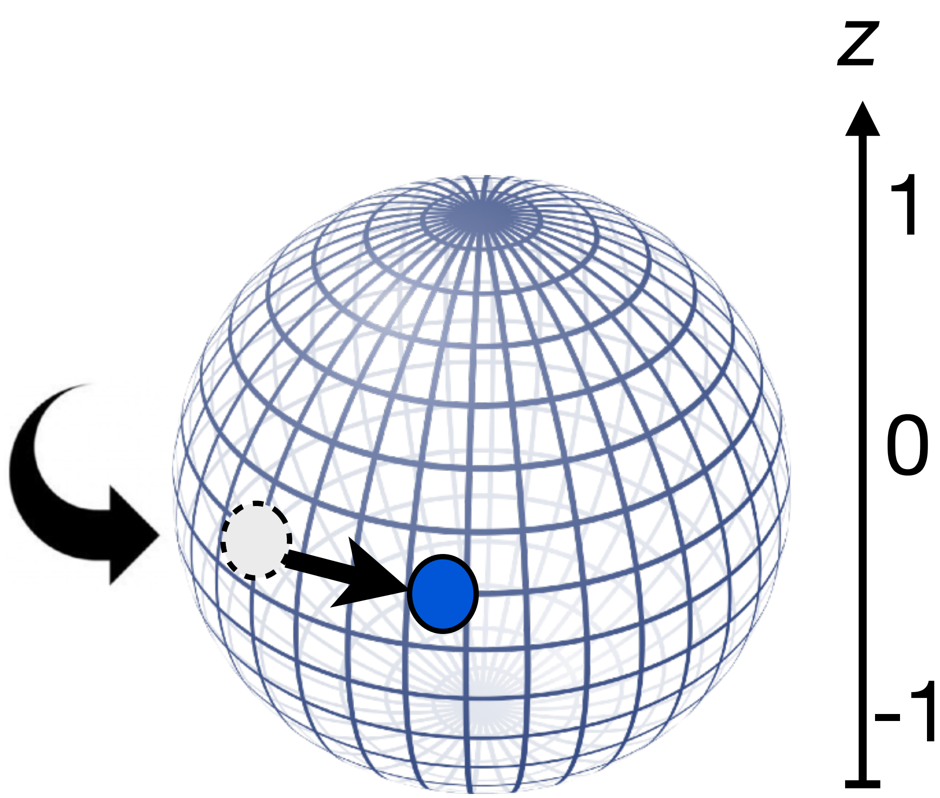




So far, we assumed: $\mathcal{G}_A = \mathcal{G}_B$. Assumption of **relationality!**

Constraints from relativity





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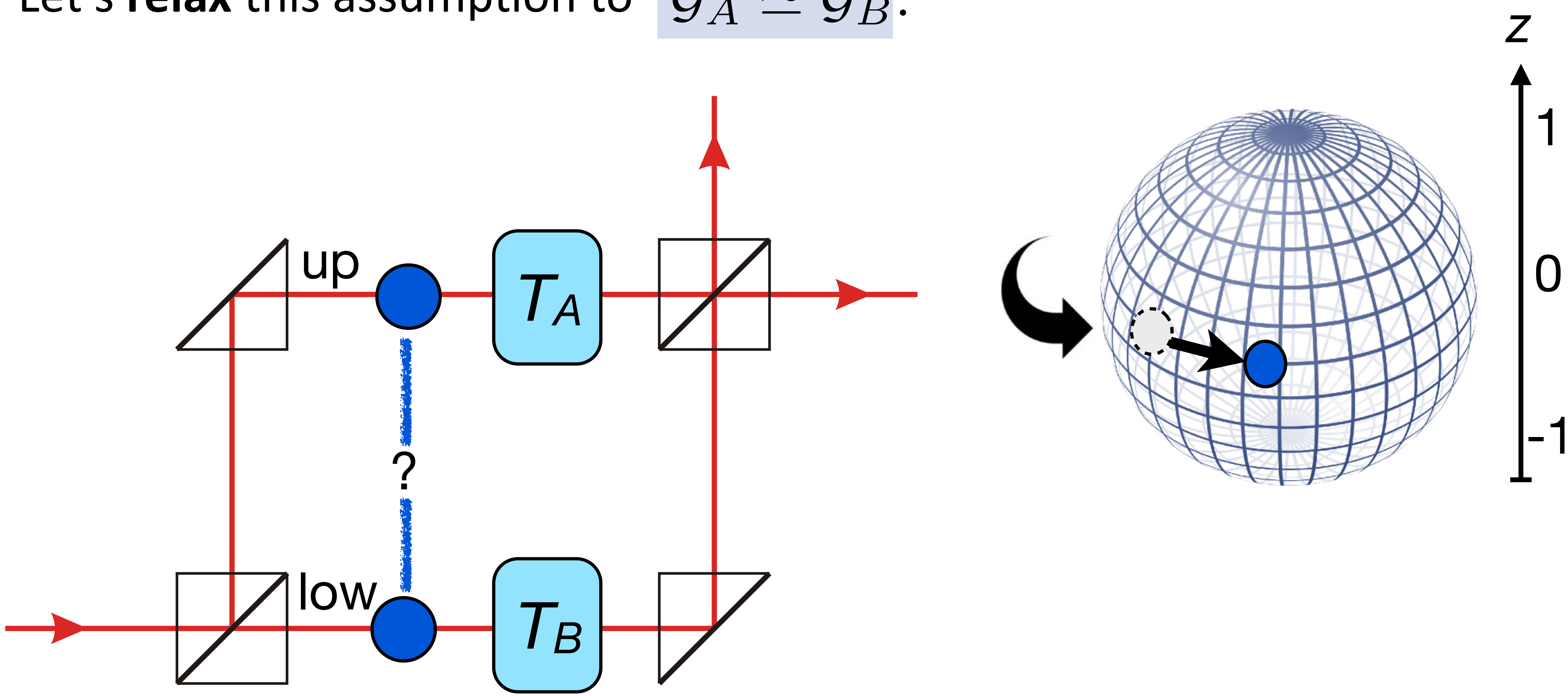

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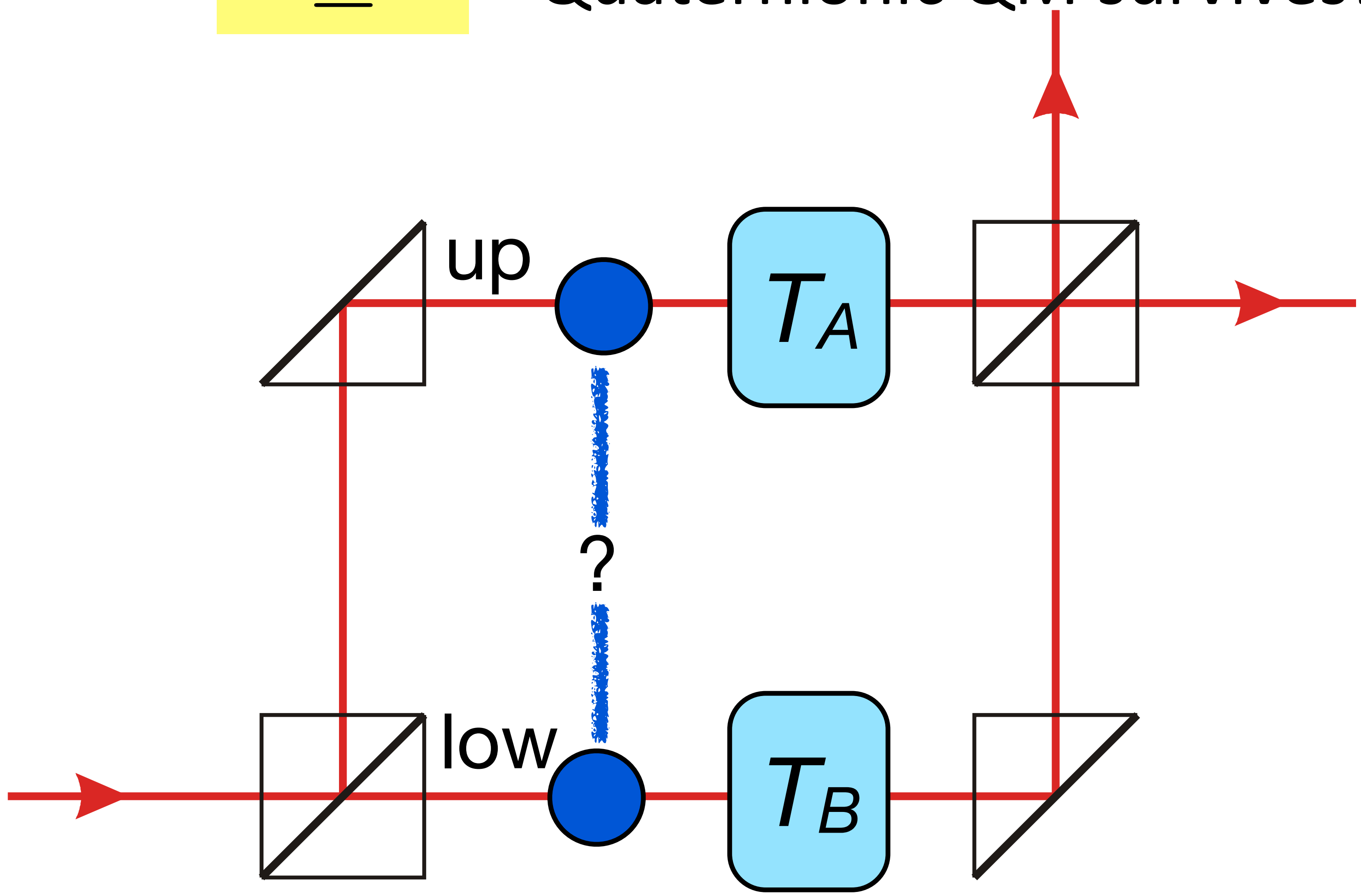
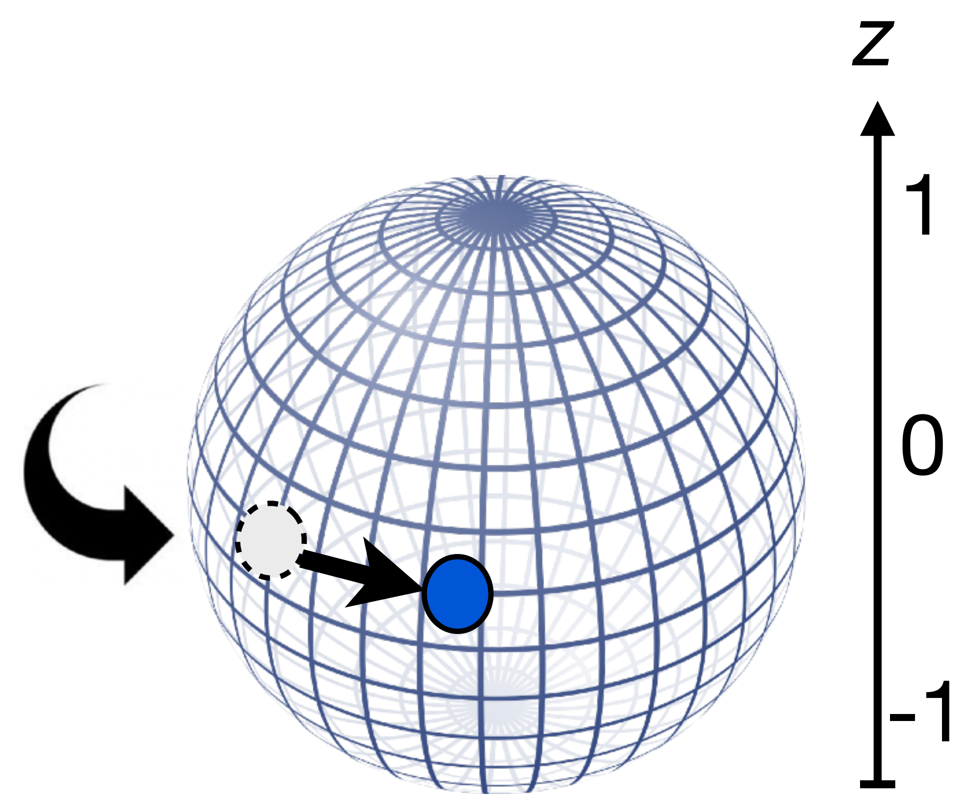
$\Rightarrow d \leq 5$. Quaternionic QM survives!





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Classification of possibilities

A. Garner, **MM**, O. C. O. Dahlsten, Proc. R. Soc. A **473**, 20170596 (2017).

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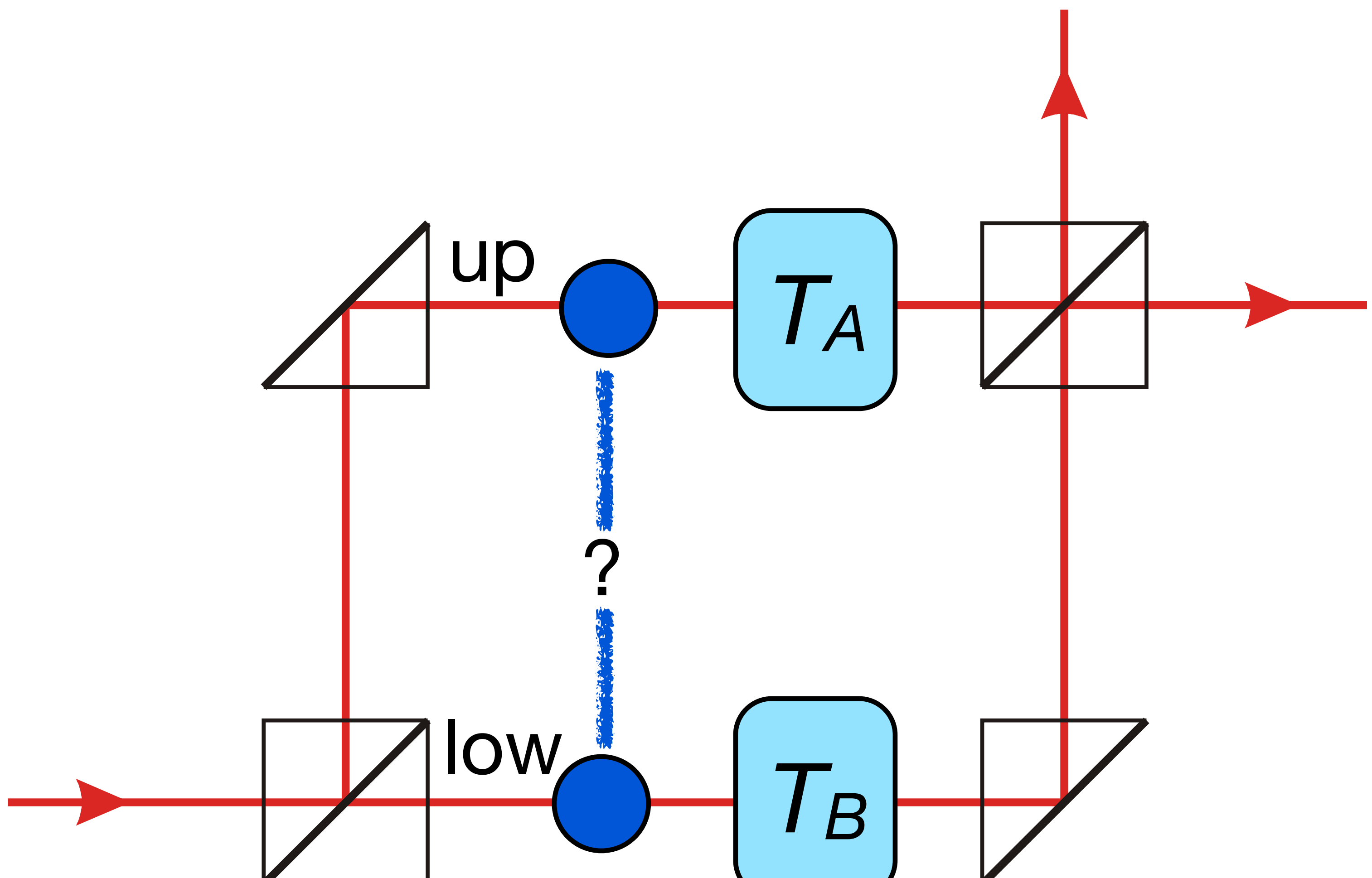

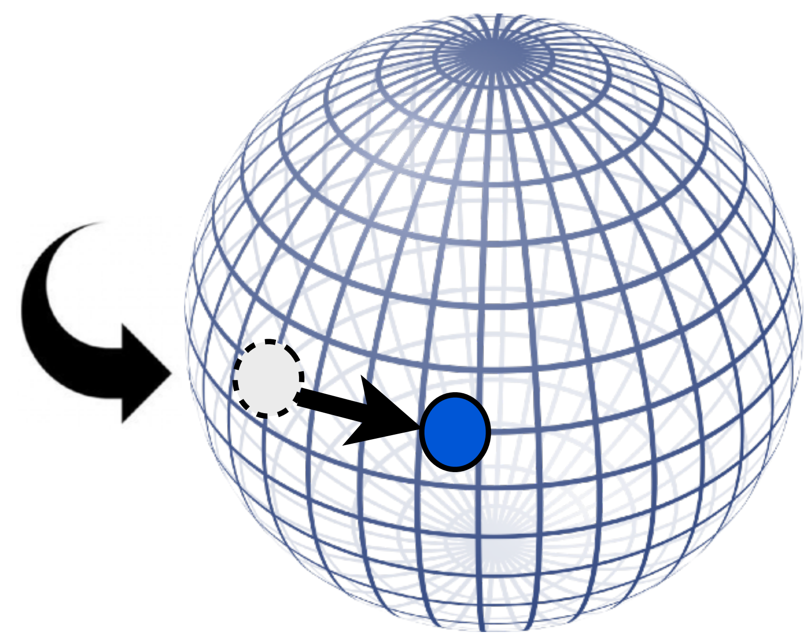

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Theorem 6.2. *Under the assumptions* A1, A2, A3, *relativity of simultaneity* (*REL*) *allows for the following possibilities and not more*:

— $d = 1$ (*the classical bit*), *with* $\mathcal{G}_{\rm A} = \mathcal{G}_{\rm B} = \{\mathbf{1}\}$ (*i.e. without any non-trivial local transformations*),

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— $d = 5$ (*the quaternionic quantum bit*), *with* $\mathcal{G}_{\rm AB} = {\rm SO}(4)$, $\mathcal{G}_{\rm A}$ *the left- and* $\mathcal{G}_{\rm B}$ *the right-isoclinic rotations in* SO(4) (*or vice versa*) *which are both isomorphic to* SU(2), *and* $\mathcal{G}_{\rm A} \cap \mathcal{G}_{\rm B} = \{+\mathbb{I}, -\mathbb{I}\}$.

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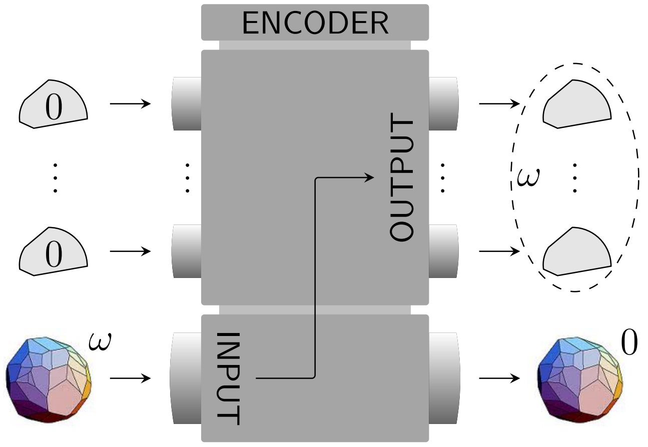
Relativity constrains the state space to $d = 1, 2, 3, 5$!

Overview

1. Probabilistic theories beyond quantum theory

2. Quantum theory from simple principles



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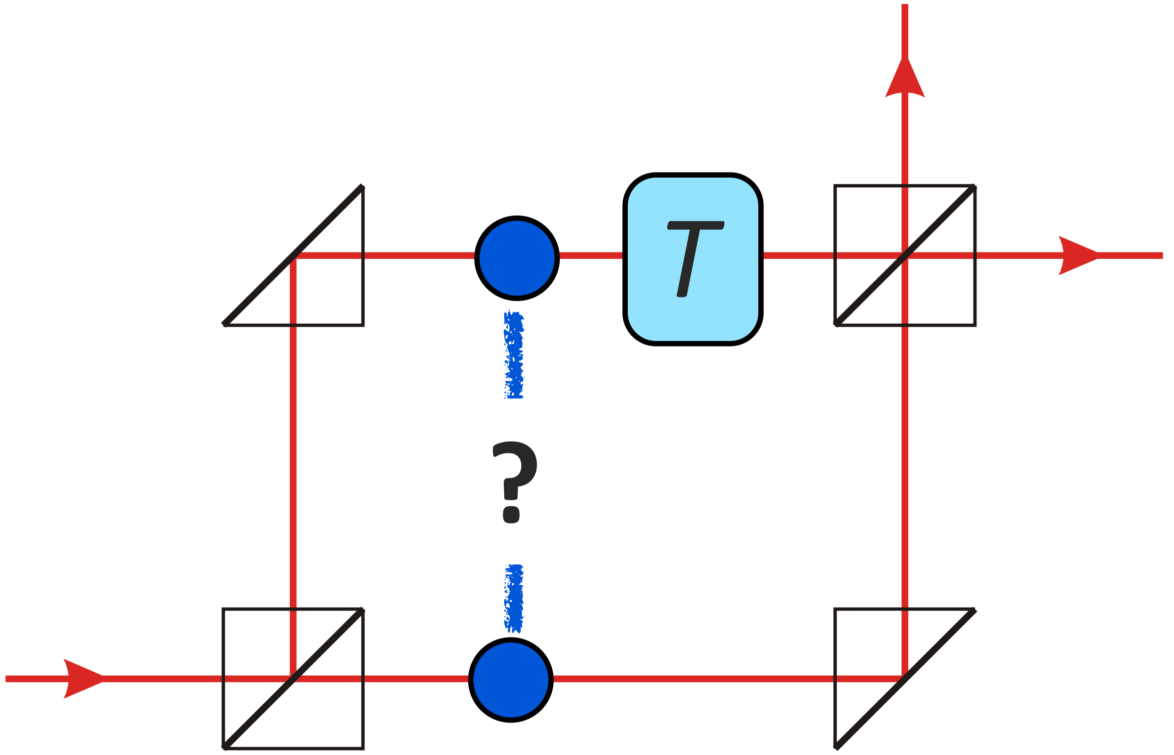

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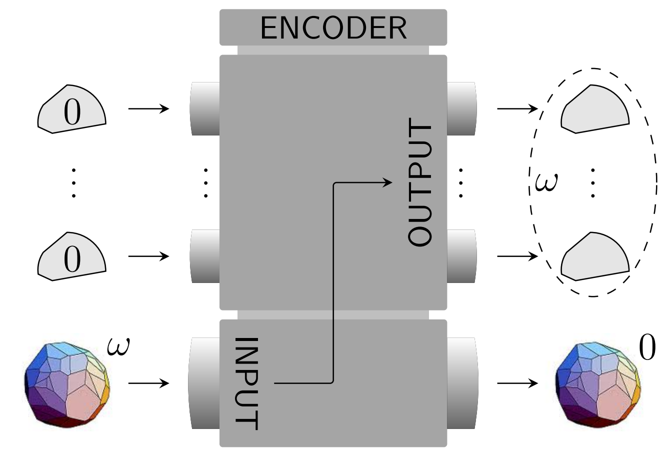

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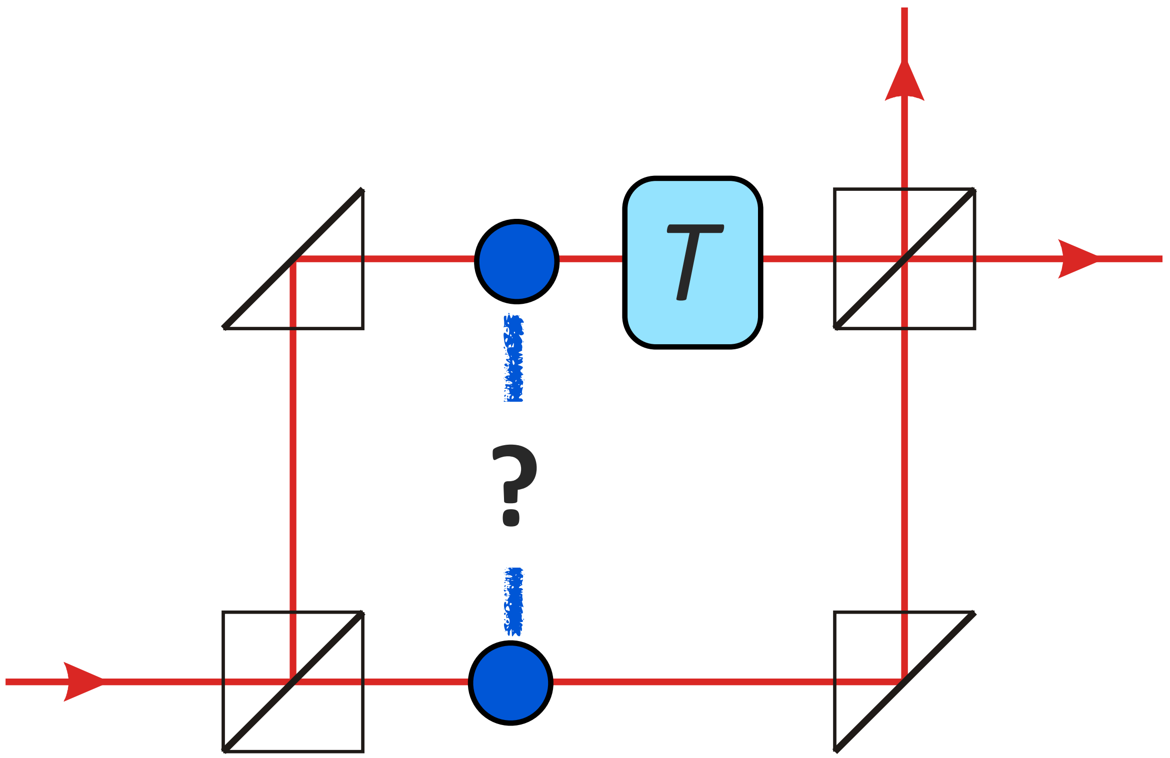

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- **Unitary evolution**: correlation with idealized clock variables.
- **Superposition principle**: not a principle, but a mathematical accident

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Challenge to Everettians: start with a landscape of "theories of many worlds", write down a few simple principles of some kind, and prove that QT is the unique many-worlds-like theory that satisfies those.

A. Koberinski and **MM**, arXiv:1707.05602

Summary

Quantum theory can be **derived from simple principles**, and this improves our understanding of its structure in several ways.

Thank you!

More info: **mpmueller.net**