AUSTRIAN ACADEMY OF SCIENCES



IQOQI - INSTITUTE FOR QUANTUM OPTICS AND QUANTUM INFORMATION VIENNA

A first-person approach to quantum paradoxes and beyond

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1. Probabilistic theories beyond quantum theory

2. Quantum theory from simple principles

3. Reproducing Wigner's Friend phenomena classically

4. Restriction A

When physics fails to allow agents to predict their future experiences

5. Consequence: fragmentalism/idealism



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John A. Wheeler, New York Times, Dec. 12 2000:

"Quantum physics […] has explained the structure of atoms and molecules, […] the behavior of semiconductors […] and the comings and goings of particles from neutrinos to quarks.

Successful, yes, but mysterious, too. Why does the quantum exist?"



The New York Times

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Role model: Einstein's derivation of the Lorentz transformations from the Light Principle and the Relativity Principle.

$$t' = \gamma \left(t - v \frac{x}{c^2} \right)$$

$$x' = \gamma (x - vt) \qquad \text{where} \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

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Need some intuition first: how could (probabilities in) physics be different from quantum and classical?





• In **classical** physics / prob. theory:

$$P(a, b|x, y) = \sum_{\lambda \in \Lambda} P_A(a|x, \lambda) P_B(b|y, \lambda) P_{\Lambda}(\lambda)$$



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 $P(a, b|x, y) = \operatorname{tr}\left[\rho_{AB}(E_x^a \otimes F_y^b)\right]$



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P(a|x, y) is independent of y, P(b|x, y) is independent of x.

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Quantum theory admits more general *P*'s due to the **violation of Bell inequalities**.

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CHSH := $|C_{00} + C_{01} + C_{10} - C_{11}| \le 2$ where $C_{xy} := \mathbb{E}(a \cdot b|x, y)$.

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No! Counterexample: the PR-box correlations $P(+1,+1|x,y) = P(-1,-1|x,y) = \frac{1}{2}$ if $(x,y) \in \{(0,0), (0,1), (1,0)\}$ $P(+1,-1|1,1) = P(-1,+1|1,1) = \frac{1}{2}$

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Correlations in **C** come from **classical prob. theory**, correlations in **Q** from **quantum theory**, correlations in **NS** from a theory called "**boxworld**".

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3 examples of a "generalized probabilistic theory".



Example: classical coin toss.



• On every push of button, the preparation device performs a biased coin toss.



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- The transformation device, for example, inverts the coin (if heads then tails, and vice versa).



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- The transformation device, for example, inverts the coin (if heads then tails, and vice versa).
- The measurement outcome is "heads" or "tails".



Example: classical coin toss.



 The preparation device prepares a physical system in a state ω. Here

$$\omega = \begin{pmatrix} \operatorname{Prob}(\operatorname{heads}) \\ \operatorname{Prob}(\operatorname{tails}) \end{pmatrix} = \begin{pmatrix} p \\ 1-p \end{pmatrix}.$$



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Maps states to states and is linear.



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• Every measurement outcome has a probability, depending linearly on the state:

Prob(heads
$$|\omega) = p = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} p \\ 1-p \end{pmatrix} = e \cdot \omega.$$




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 \bullet The preparation device prepares a qubit in some quantum state $\,\rho=|\psi\rangle\langle\psi|,\,$

 $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

More generally: ρ is (mixed-state-)2x2 density matrix.





Example: quantum bit.

Unitary transformation of the density matrix:

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- Unitary transformation of the density matrix: $\rho\mapsto U\rho U^{\dagger}$
- Measurement in arbitrary spin direction d:

$$\operatorname{Prob}(\uparrow | \rho) = \operatorname{tr}(P_d \rho).$$







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even:
$$\omega$$

odd: τ \longrightarrow $\sigma = \frac{1}{2}\omega + \frac{1}{2}\tau$

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QT:
$$\Omega = \{ \rho \in \mathbf{H}_N(\mathbb{C}) \mid \operatorname{tr}(\rho) = 1, \ \rho \ge 0 \}.$$



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CPT: $\Omega = \{(p_1, \dots, p_N) \mid p_i \ge 0, \sum_i p_i = 1\}.$

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- How to describe **measurements**? By a collection of linear functionals e_1, e_2, \ldots, e_n such that the probability of outcome *i* is $e_i(\omega)$.

QT: POVMs (positive operator-valued measures),

 $e_i(\omega) = \operatorname{tr}(E_i\omega).$













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Role model: Einstein's Relativity Principle and Light Postulate





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- Prehistory: Birkhoff & von Neumann (1936); quantum logic (Piron, ...), Ludwig (1954); Alfsen&Shultz (≈1980);
- Quantum information revolution:

L. Hardy 2001: Quantum Theory From Five Reasonable Axioms. But needs "simplicity axiom"...

Clifton, Bub, and Halvorson 2002.
But assumed C*-algebras.

Dakić+Brukner 2009; Masanes+MM 2009 Chiribella, d'Ariano, Perinotti 2010; Hardy 2011 the one I'll present now 2013; Barnum, MM, Ududec 2014; Hoehn 2015; Wilce 2016, ...







Ll. Masanes, MM, R. Augusiak, and D. Pérez-García, PNAS 110(4), 16373 (2013).



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• **Postulate 1**: Continuous reversibility.

Reversible transformations can (in principle) map every pure state continuously to every other.





Ll. Masanes, MM, R. Augusiak, and D. Pérez-García, PNAS 110(4), 16373 (2013).

- Postulate 1: Continuous reversibility.
- **Postulate 2**: Tomographic locality.

The state of a composite system is completely characterized by the correlations of measurements on the individual components.





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- **Postulate 1**: Continuous reversibility.
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There is a type of system (the "ubit") such that every system can be encoded into a sufficiently large number of ubits.

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There is a type of system (the "ubit") such that every system can be encoded into a sufficiently large number of ubits. Pairs of ubits can continuously reversibly interact.



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- Postulate 1: Continuous reversibility.
- **Postulate 2**: Tomographic locality.
- Postulate 3: Existence of an information unit.
- Postulate 4: No simultaneous encoding.

If a ubit is used to perfectly encode one classical bit, it cannot simultaneously encode any further information.



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Theorem. If Postulates 1-4 hold, then the state space of *n* ubits is $\Omega = \{ \rho \in \mathbf{H}_{2^n}(\mathbb{C}) \mid \operatorname{tr}(\rho) = 1, \rho \ge 0 \},$ and the reversible transformations are the unitaries, $\rho \mapsto U\rho U^{\dagger}$.

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and the reversible transformations are the unitaries, $\rho \mapsto U \rho U^{\dagger}$.

- No assumptions on complex numbers, operators, algebras, ... were made.
- Drop "continuous" in Postulate 1 -> obtain classical probability theory.

- **Postulate 1**: Continuous reversibility.
- **Postulate 4**: No simultaneous encoding.
Example: why are ubits balls?

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Group rep. theory: can reparametrize space such that transformations are rotations. Then, pure states lie on unit sphere (of some dim. *d*).



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Violates Postulate 4.



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Two ubits: some composite state space of two *d*-balls, $\mathcal{G}_A = \mathcal{G}_B$ transitive on ∂B^d . **Tomographic locality** $\Leftrightarrow d_{AB} = d^2 + 2d$

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Theorem. Among all dimensions d and all groups \mathcal{G}_A , there are only the following possibilities:

• The trivial solution: $\mathcal{G}_{AB} = \mathcal{G}_A \otimes \mathcal{G}_B$.

• d = 3, $\mathcal{G}_A = SO(3)$ (i.e. the quantum bit), $\mathcal{G}_{AB} \simeq PU(4)$, and Ω_{AB} is equivalent to the two-qubit quantum state space.

In particular, continuous reversible interaction is only possible for d = 3, in standard complex two-qubit quantum theory.

Ll. Masanes, MM, R. Augusiak, and D. Pérez-García, J. Math. Phys. 55, 122203 (2014).

Generator X of global reversible transformation (no idea what it is...)



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We must obtain valid probabilities. For example,

$$0 \leq (e_{-\vec{a}_1} \otimes e_{\vec{b}_2}) e^{\varepsilon X} (\omega_{\vec{a}_1} \otimes \omega_{\vec{a}_2}) \leq 1.$$

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$$\Rightarrow \begin{cases} \text{ if } d \neq 3 : X = X_A + X_B \\ \text{ if } d = 3 : \exp(\varepsilon X) = U_{AB}(\varepsilon) \bullet U_{AB}^{\dagger}(\varepsilon) \end{cases} \text{ no interaction.} \\ \text{unitary conjugation!} \end{cases}$$

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Main reason: SO(d-1) is only non-trivial and **commutative** for d = 3.

What does this tell us now?

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- Collapse: Bayesian updating.
- **Time evolution**: correlation with idealized clock variables.
- Superposition principle: not a principle, but a mathematical accident $|\psi\rangle\langle\psi|\mapsto U|\psi\rangle\langle\psi|U^{\dagger}.$

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Challenge to Everettians: start with a landscape of "theories of many worlds", write down a few simple principles of some kind, and prove that QT is the unique many-worlds-like theory that satisfies those.

A. Koberinski and MM, arXiv:1707.05602

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Picture credits: Nurya Nurgalieva





Wigner's description:

$$\frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_R + |1\rangle_A|1\rangle_R)$$

Friend's description: $|0\rangle_A |0\rangle_R$ or $|1\rangle_A |1\rangle_R$

Picture credits: Nurya Nurgalieva





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The quantum state is a "catalog of probabilities" of future observations / experiences of some agent (not necessarily human or conscious).

These will typically not "fit together" into a joint description. The "world" is a mosaic of the (more fundamental) first-person perspectives.

D. Frauchiger and R. Renner, Nat. Commun. 9, 3711 (2018).



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Incompatibility of three assumptions:

- (Q): Quantum theory is universally valid.
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Box 3: Assumption (C)

Suppose that agent A has established that

Statement A⁽ⁱ⁾: "I am certain that agent A', upon reasoning within the same theory as the one I am using, is certain that $x = \xi$ at time t." Then agent A can conclude that

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Thought Experiment 2. Imagine Freya and Wigner are to be put to sleep, and multiplied into N copies. Each couple is distributed to one of N identical laboratories. In each laboratory, a fair coin is tossed, and if the outcome is Heads, the copy of Freya (but not the copy of Wigner) is duplicated again. Then, all participants are woken and asked to give their credence that the outcome of their lab's coin toss was Tails. (We assume that they cannot notice the presence/absence of an identical copy of Freya in the lab). This scenario is sketched in Eigenver 2

In fact, all participants are offered a bet: they can buy a ticket from a bookie for $(2/3 - \varepsilon)$ \$, where $\varepsilon > 0$ is small, (say, for 66 cents) that wagers on the coin toss having shown Heads. It is natural to argue (see below) that the credence that Freya should assign to Tails (which directly determines the maximum price 1 - p she rationally ought to be prepared to pay) is 1/3, whilst for Wigner it is 1/2: Freya should buy the ticket, but Wigner should not.

Finally, all copies survive the experiment and are released. Everyone who has bought the ticket now receives 1\$ if the outcome of their lab's coin toss was indeed Tails. Freya and Wigner have been initially informed about all the details of the experiment.





Elga's Principle of Indifference:

Similar centered worlds deserve equal credence.

Adam Elga (Princeton)





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 $Z = +\frac{1}{2}$

According to ⁻

According to T

IIII

According to 7

000

000

Adam Elga (Princeton)

Principle CP (probabilistic consistency): Suppose that an agent A has established that *"I am pretty sure that agent A', upon reasoning within the same theory as the one I am using, and having the exact same knowledge of the world as I, is pretty sure that x=X at time t"*. Then agent A can conclude that *"I am pretty sure that x=X at time t"*.











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N

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Elga's Principle of Indifference:

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Principle CP (probabilistic consistency)

NUI (no update on irrelevant information):

If Freya knows that all her future copies will undergo the exact same procedure until the experiment is completed, then her predictions for her future observations are unchanged if she is told which copy she is.
A classical thought experiment







Elga's Principle of Indifference (or milder versions!) Similar centered worlds deserve equal credence.

T)

Adam Elga (Princeton)

Principle CP (probabilistic consistency)

NUI (no update on irrelevant information)

Theorem: These three principles cannot hold jointly in this experiment.





 We do not claim that FR-WF and this duplication scenario are ontologically similar!



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 Structural insight: there is <u>no joint probability distribution</u> of the observations of Freya and Wigner, if Elga's principle holds (or if Freya's credence depends on *M* in any other way).



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- Everettians might see an ontological similarity. But we can turn this around: since branching/dupl. messes up the Kolmogorovian probability space, it admits Everettians to tell their story.
- Isn't quantum theory natural, but duplication=science fiction?
 No. WF-scenarios are extremely invasive on the Friend! Resource requirements for classical duplication are ~5 orders of magnitude smaller than Quantum-WF (classical vs. quantum computation).

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If the statistics violates a so-called "local friendliness inequality", then the following three propositions cannot all be true:



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If the statistics violates a so-called "local friendliness inequality", then the following three propositions cannot all be true:

Locality, No Superdeterminism, Absoluteness of Observed Events:

For every x,y, there is a joint prob. distr. P(a,b,c,d|x,y) reproducing the observed distribution p(a,b|x,y) as its marginal.



From the perspective of A and B (say, using the quantum formalism), **there is no random variable c**, stable over the course of the experiment, describing Charlie's observations.

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- Structural interpretation: This is ultimately the reason for the non-existence of the joint distributions P(a,b,c|x,y).
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We can simulate this behavior via duplication. To do so, let us look at a reformulation of this WF thought experiment.

H. Wiseman, E. G. Cavalcanti, and E. G. Rieffel, Quantum 7, 1112 (2023).

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Conceived implementation of the previous thought experiment on a **quantum computer**, proving that the following cannot all hold:

- 1. Local Agency: Any [random] intervention [...] is uncorrelated with any set of physical events that are relevant to that phenomenon and outside the future light-cone of that intervention.
- 2. Physical Supervenience: Any thought supervenes upon some physical process in the brain (or other information-processing unit as appropriate) which can thus be located within a bounded region in space-time.
- 3. Ego Absolutism: My communicable thoughts are absolutely real.
- 4. Friendliness: If [...] an independent party displays cognitive ability at least on par with my own, then they have thoughts, and any thought they communicate is as real as any communicable thought of my own.

H. Wiseman, E. G. Cavalcanti, and E. G. Rieffel, Quantum 7, 1112 (2023).

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Another duplication thought experiment (fission & fusion):

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3. Ego Absolutism: My communicable thoughts are absolutely real.

Another duplication thought experiment (fission & fusion):

Naively understood, there exists an ambiguity regarding what "my" indexes for branching scenarios. Returning to the example of Freya, who is yet to be duplicated, she reads Ego Absolutism to say that her communicable thoughts are absolutely real. This includes her thoughts in that instance, such as "I am hungry". It may also be understood to include thoughts she had this morning, such as "It is raining". Does it include her future thoughts though? This afternoon, she will be duplicated, whereupon her future copies will have separate experiences. Thus, in describing any future thought she may have, there is an inherent ambiguity as to the meaning of such statements, and whether or not we should take their referent as "absolute". That is, it is unclear what the words "my (future) thoughts", if uttered by Freya before the start of the experiment, would refer to, and in disregarding this indexical ambiguity, we will typically be led to mathematical formulations of Ego Absolutism that tacitly involve additional assumptions. In particular, it will lead to the formal assumption that there is always, at every time, a single variable describing a single thought of some person called Freya, while in this branching scenario there are actually two. Indeed, this assumption is part of the mathematical formulation of Bong et al. [7].

H. Wiseman, E. G. Cavalcanti, and E. G. Rieffel, Quantum 7, 1112 (2023).

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2. Quantum theory from simple principles

3. Reproducing Wigner's Friend phenomena classically

4. Restriction A

When physics fails to allow agents to predict their future experiences

5. Consequence: fragementalism/idealism

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A common structural core: Restriction A

We interpret the probabilities in the thought experiments as answers to the question of **what the agent should believe to experience next** — essentially how we interpreted the probabilities implied by a quantum state. We interpret the probabilities in the thought experiments as answers to the question of **what the agent should believe to experience next** — essentially how we interpreted the probabilities implied by a quantum state.

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Sometimes even for *single* agents (n=1).

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This is relative to some theory T and background assumptions.

Observation: Unless our physical theory T is empirically incomplete, Restriction A can only apply to situations where it is **impossible** to repeat the scenario identically many times, record the observations of the *n* agents, and estimate the probabilities via frequencies.

Examples of Restriction A

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 Bong et al., Absoluteness of Observed Events: Under the background assumptions of Locality and No Superdeterminism, Restriction A applies to Quantum Theory (in this scenario). Indeed, assuming that an LF-inequality will be experimentally violated, Restriction A applies to all empirically adequate future physical theories for 2≤n agents.

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There is no P(a,b,c|x,y).

Classical fission&fusion: n=1.

Our physical theories have nothing at all to say about what Freya should believe about the color of room she will see.

Restriction A applies here to all current physical theories.

Friend measuring the spin, and Wigner in the entangled basis: They can use the Born rule to compute probabilities P(f) and P(w), and the joint distribution is simply the product distribution.

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• Example: violation of probabilistic consistency via classical duplication

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• Example: violation of probabilistic consistency via classical duplication

Restriction A applies to this scenario as described by, say, classical physics (because it does not tell us what Freya should believe about her future observations — i.e., to n=1 observer). Moreover, even if we supplement classical physics with any probability rule whatsoever (not necessarily Elga's Principle of Indifference), which informs Freya about what she should believe about her future observations *in a way that is not completely ignoring her subsequent multiplicity M*, then the resulting theory will be subject to Restriction A for $2 \le n$ agents.

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Restriction A for 2≤*n* agents is unavoidable, for *n*=1 agent **unacceptable**

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In 2048, you are terminally ill, but the doctor promises to simulate you on a computer when you fall asleep next time (eliminating the original).

God at His computer
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You: Great, but will I *really* wake up in the simulation? Damn, I *really, really want to know!* I'm so afraid! What should I believe will happen to me? **Doctor:** Hahaha, you fool! You are asking a non-question! All there is to say is that there is a human being here now, and a computer running a simulation of that thing later. This is all there is to know about the facts of the world. What is the proposition that you are even uncertain *about*?

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My claim: This is unacceptable. The first-person perspective is real. There always exists some (degree of) belief that a single agent *should* have — the agent can perform a **private** experiment, and the world will kick back.

What about our to-be-simulated and duplicated friends?





God at His computer



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For every **single** agent, there should be a mathematical expression for what they should believe about future experiences, *in all circumstances*. In ordinary physics, this is exactly the quantum state.

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This would be a theory that explains, starting with private probabilities,

- how a notion of "external world" emerges for N=1 agents, and
- how an approximate notion of objectivity emerges for N>1 agents.
- Since the **personalist** probabilities do not typically fit together into a joint distribution, the hope is that aspects of quantum theory arise.

• Then we can tell our simulated & duplicated friends what to expect.

An approach of this sort already exists, and is under further construction. If you are interested: <u>mpmueller.net/ai</u>

Coming soon: Adversarial collaboration with Kelvin McQueen.

Technical paper:

M. P. Müller, Law without law: from observer states to physics via algorithmic information theory, Quantum **4**, 3301 (2020).

Recent essay:

M. P. Müller, Algorithmic idealism: what should you believe to experience next?, arXiv:2412.02826.

- Quantum theory is just one of many probabilistic theories.
- Since 2011, we know how to derive QT from simple principles, without preassuming any of the usual mathematical machinery (algebras, complex numbers, Hilbert spaces...)
- Have shown one such "reconstruction" of QT:

Ll. Masanes, M. P. Müller, R. Augusiak, and D. Pérez-García, *Existence of an information unit as a postulate of quantum theory*, Proc. Natl. Acad. Sci. USA **110**, 16737 (2013).

but there are many others, e.g.

Ll. Masanes and M. P. Müller, New J. Phys. 13, 063001 (2011).

 The view that a quantum state is a "catalog of probabilities" for what some agent should believe to observe/experience next has the power to explain why QT has the very mathematical structure that it does.

- Have shown how to reproduce certain structural aspects of Wigner's friend scenarios classically via duplication.
- Restriction A as a common core: physical theories do not always give us joint probability distributions for the future observations of all agents (or even a *single* agent).
 Have argued that this is what Wigner's friend is ultimately about.
- This is at the core of several other puzzles in physics and philosophy, including **classical duplication** or the Boltzmann brain problem, see:

Caroline Jones and MM, arXiv:2402.08727 (and <u>mpmueller.net/ai</u>)

• Have argued that this motivates idealist/fragmentalist approaches where "reality" is a mosaic of the fundamental first-person pieces.

Thank you!