ÖAW AUSTRIAN ACADEMY OF SCIENCES

IQOQI – INSTITUTE FOR QUANTUM OPTICS AND QUANTUM INFORMATION VIENNA

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Contextuality and Quantum Theory as a Probabilistic Theory

Markus P. Müller

Institute for Quantum Optics and Quantum Information (IQOQI), Vienna

Perimeter Institute for Theoretical Physics (PI), Waterloo, Canada

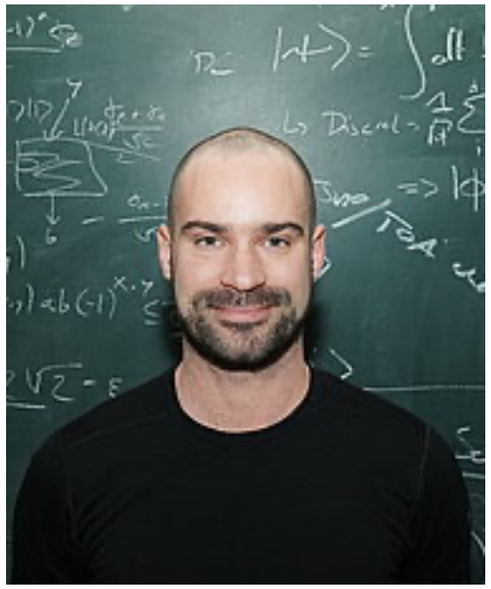




PI PERIMETER INSTITUTE

Müller Group @ IQOQI Vienna





Jan Glowacki (postdoc)

Manuel Mekonnen (PhD student), **Markus Müller** (PI), **Stefan Ludescher** (PhD student), **Thomas Galley** (postdoc), **Caroline Jones** (PhD student), **Albert Aloy** (postdoc).

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Some questions that we are interested in, and have suggested answers to:

- Does the **Second Law of Thermodynamics** also hold for single, small quantum systems without thermodynamic limit? (Yes.)
- How can **Quantum Theory be derived** from simple information-theoretic principles? (More soon.)
- Why are there **only Bosons and Fermions**, and no “paraparticles”?
- Could the world even be “**weirder than quantum**”, would this be compatible with spacetime physics, and how to test this experimentally?

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I've proposed an (ontologically) radical, but rigorous theory of reality — **Algorithmic Idealism** — that aspires to answer questions such as: *Why is there a lawlike "world" at all? How to think of the Boltzmann brain problem? What should you expect to observe when you are duplicated, simulated on a computer, or die?* "Idealism [monism…] without miracles". But not today. See mpmueller.net/ai

Overview

1. Contextuality: the Kochen-Specker theorem

2. Against the received view: QM as a theory of probability

3. Quantum theory from simple principles

4. Spekkens' contextuality in a nutshell (if time allows)

5. Conclusions

Goal: (Mostly agnostic) description of what our community believes is a good way to think about QM, and to say that something is "nonclassical".

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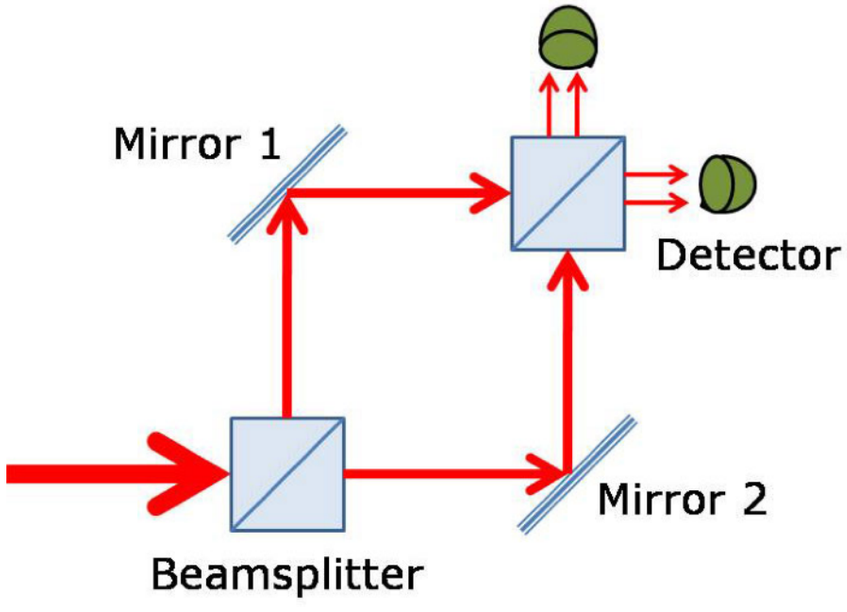
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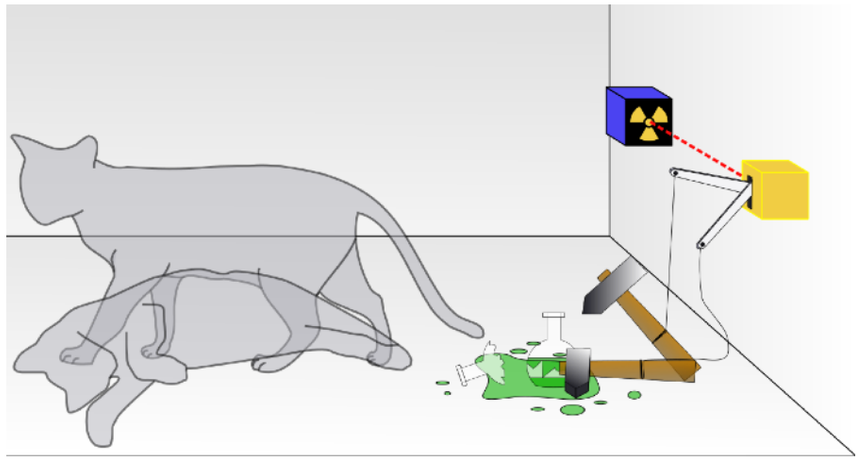
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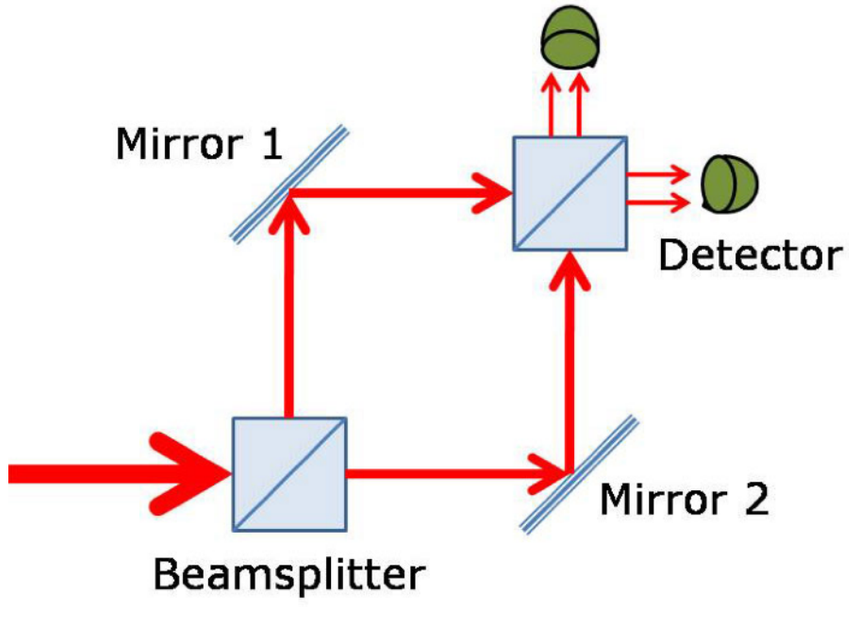


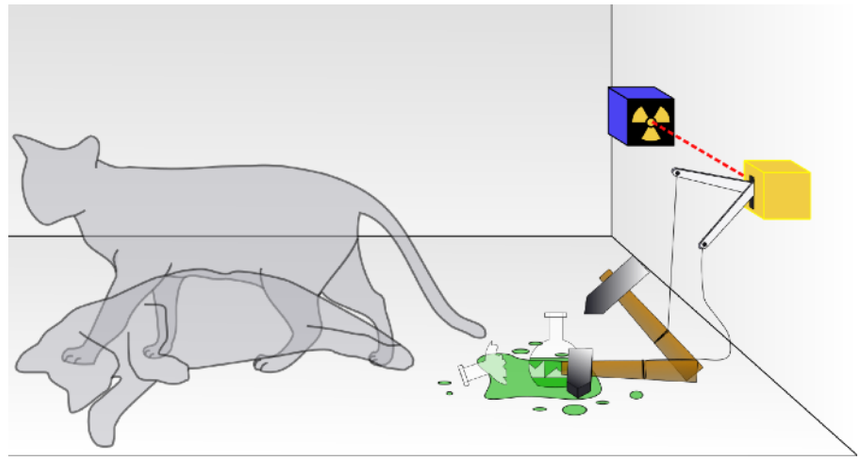


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The **state** of any quantum system is given by its wave function $|\psi\rangle$.
This is a vector in an abstract, complex vector space, normalized such that $\langle\psi|\psi\rangle = 1$.

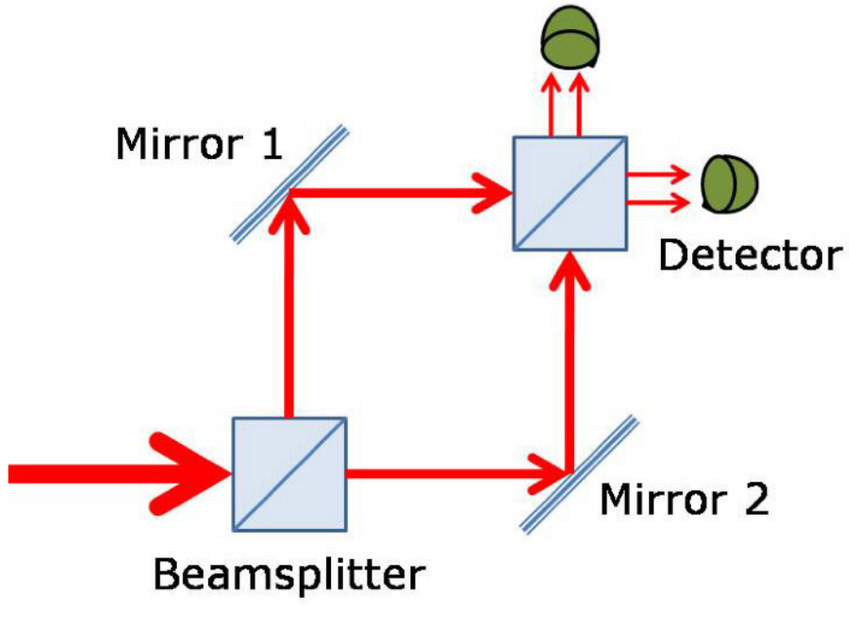




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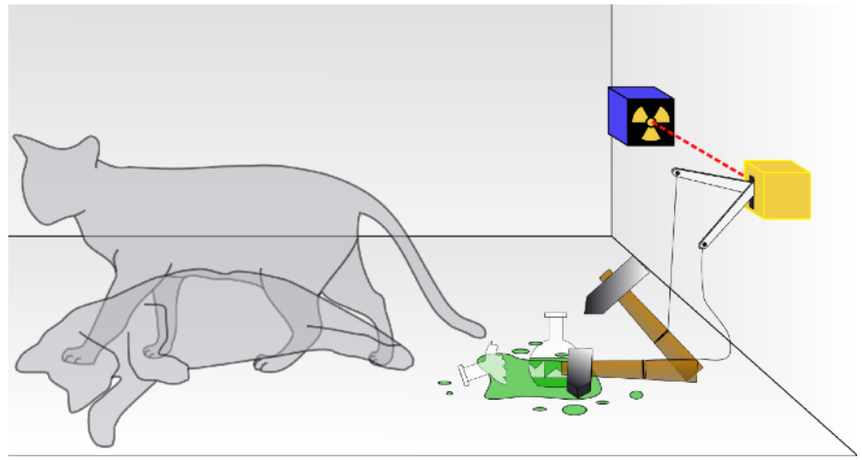
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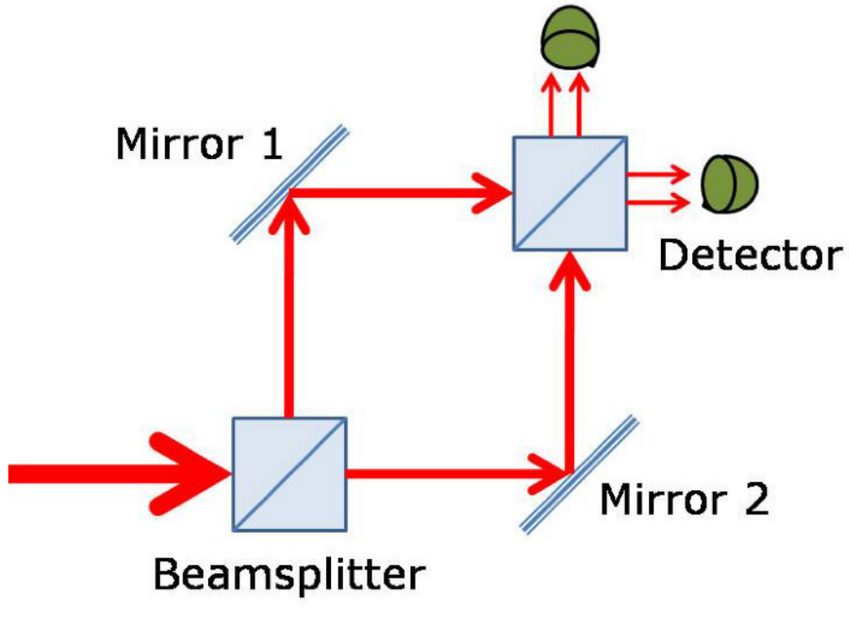
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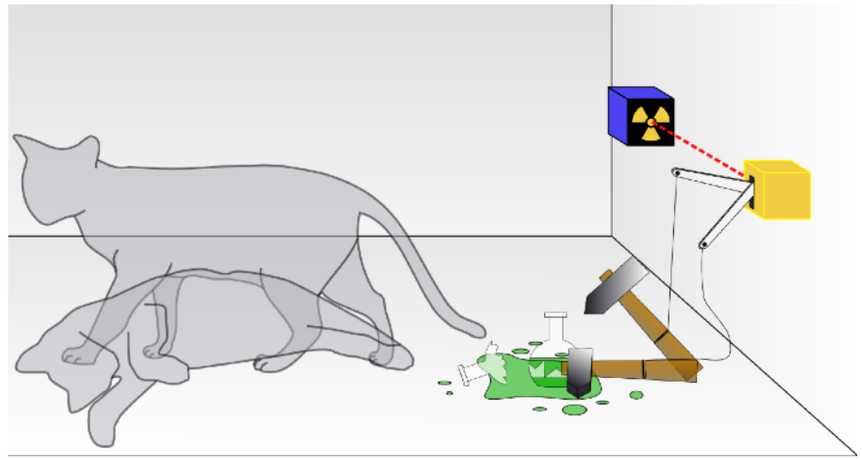
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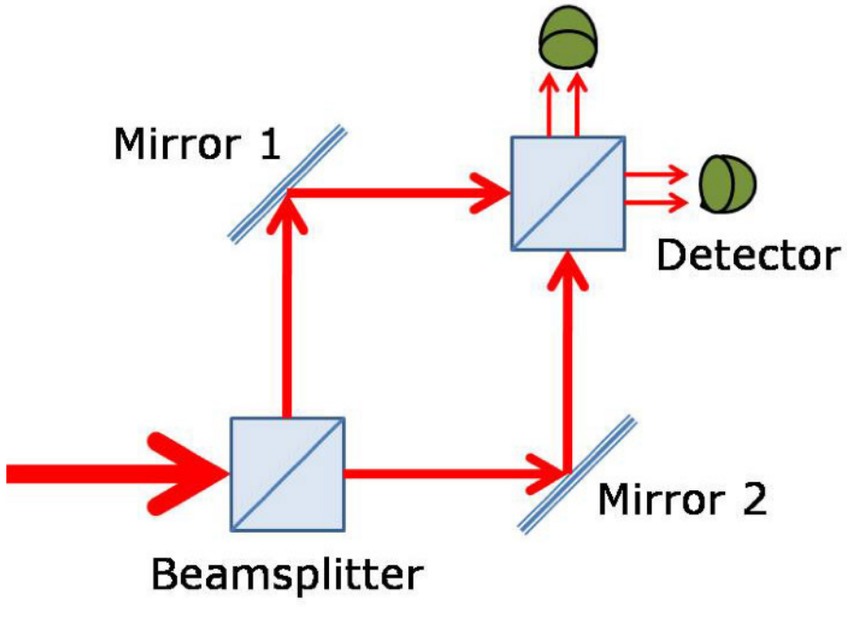
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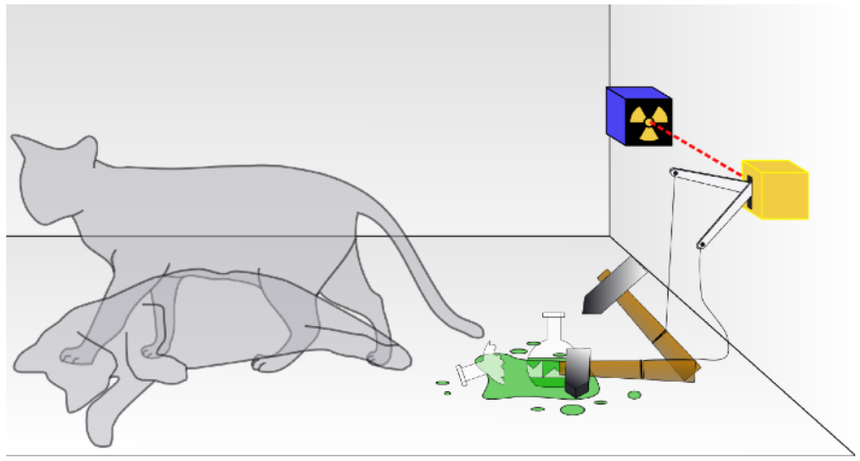
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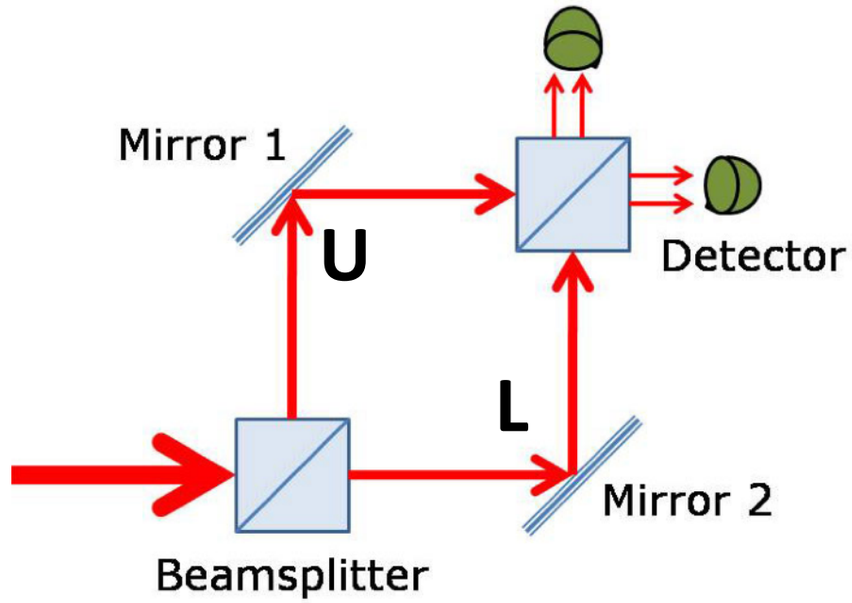
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$P_1' \simeq$ "cat is dead", $P_2' \simeq$ "cat alive and happy", $P_3' \simeq$ "cat alive and unhappy"

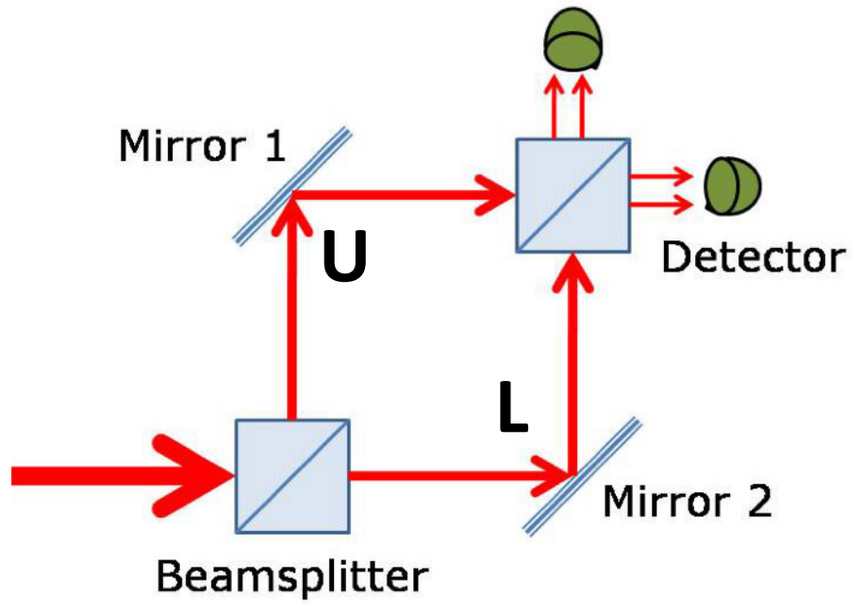
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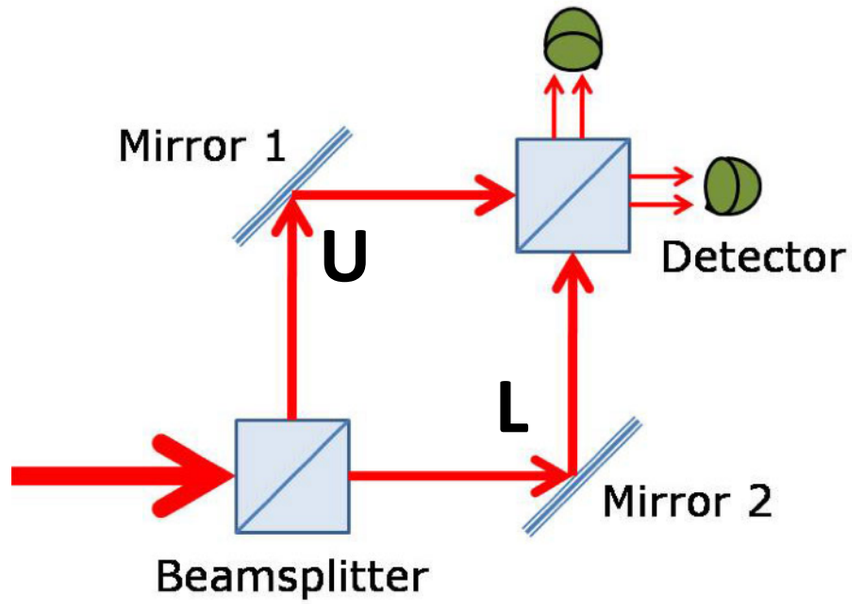
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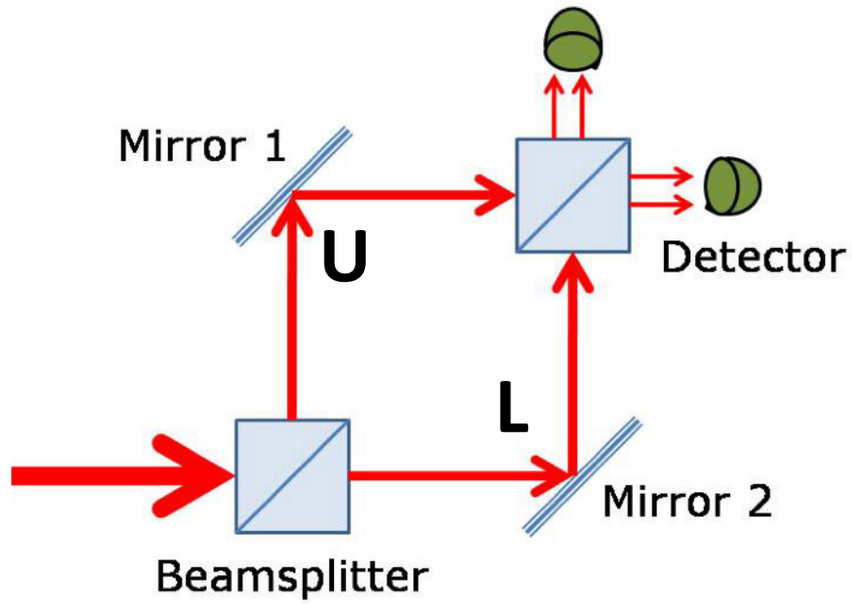


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$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|u\rangle + |l\rangle\right) = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}.$$

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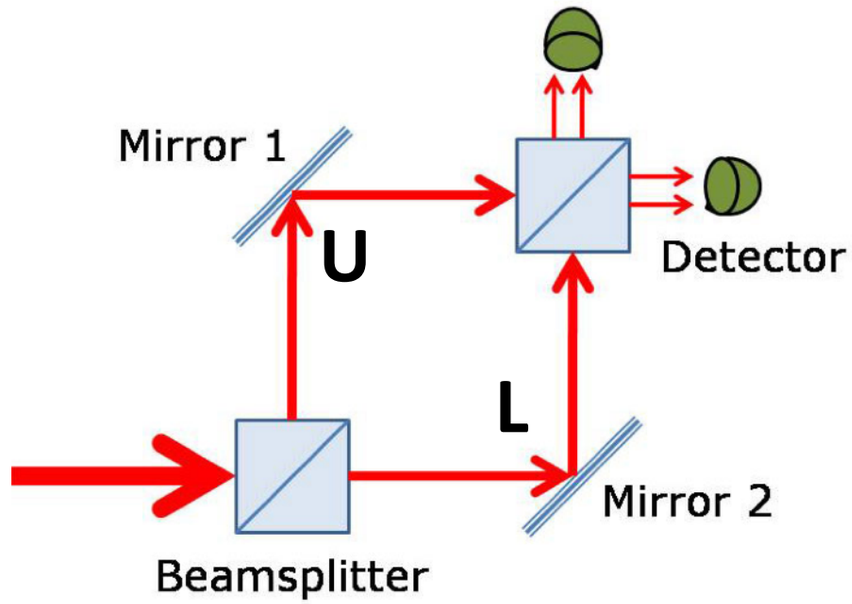
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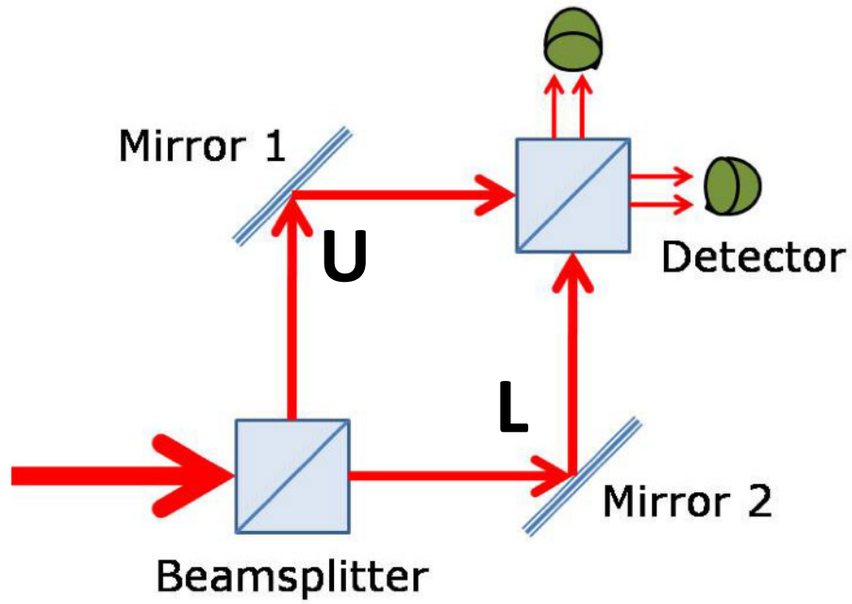
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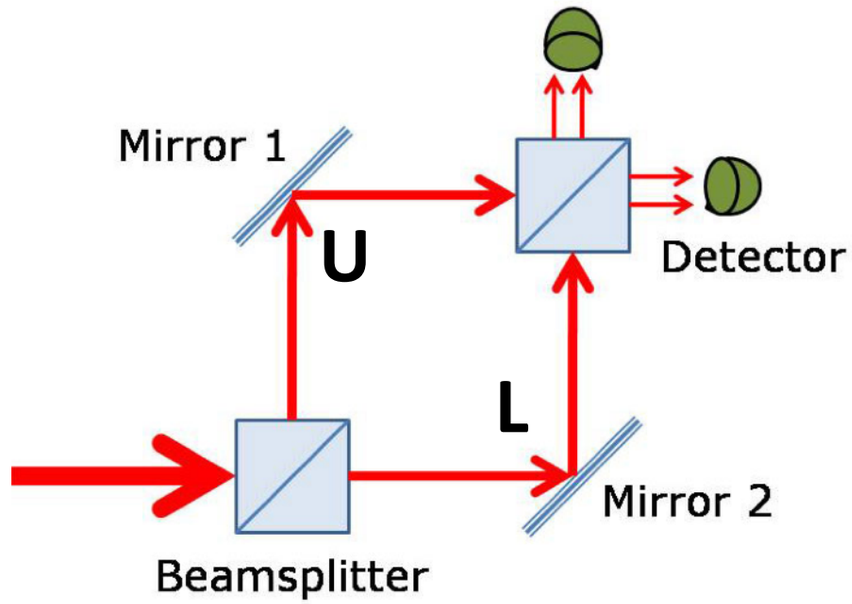
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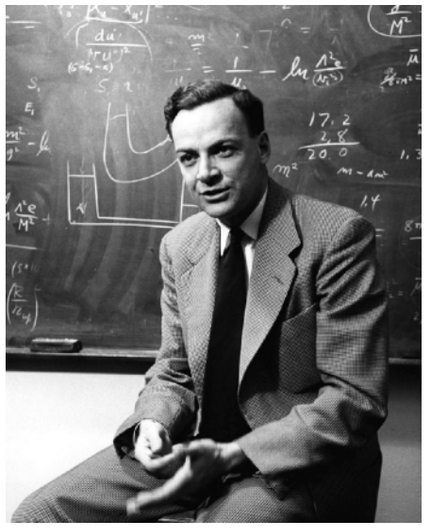
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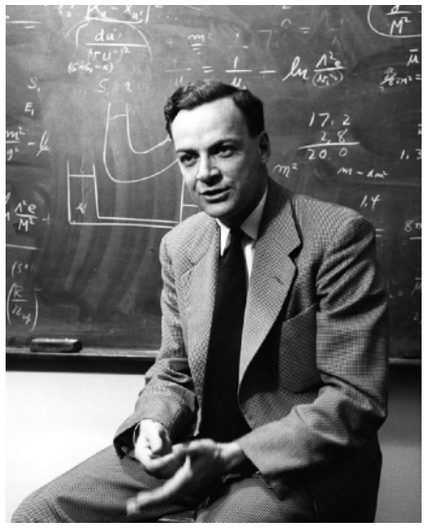
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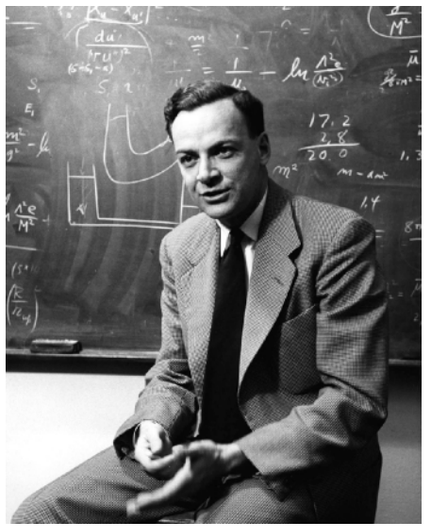
Ehm... no...

Why interference phenomena do not capture the essence of quantum theory **(Nov. 2021)**

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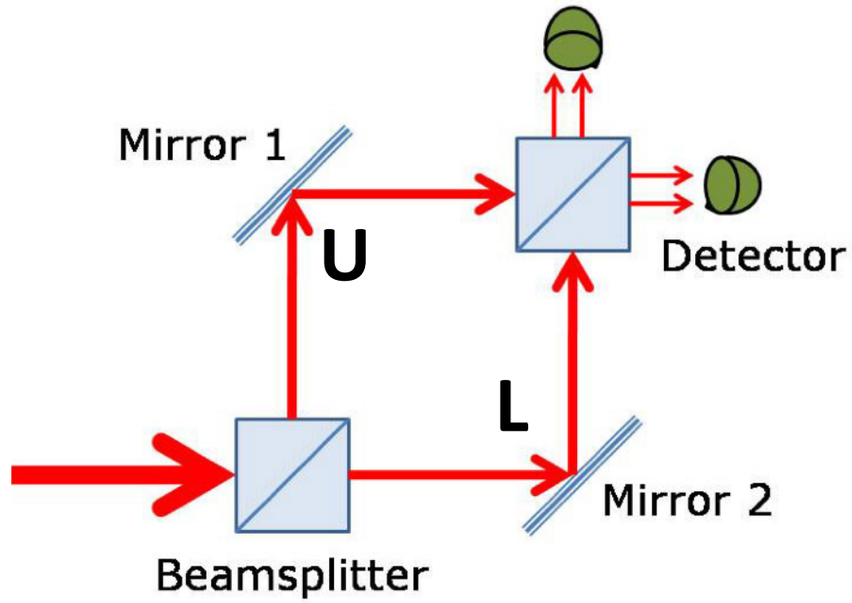
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Interference phenomenon *can* be simulated classically (in a locally-realistic manner). **Need no-go theorems**, saying *exactly what* is classically impossible.

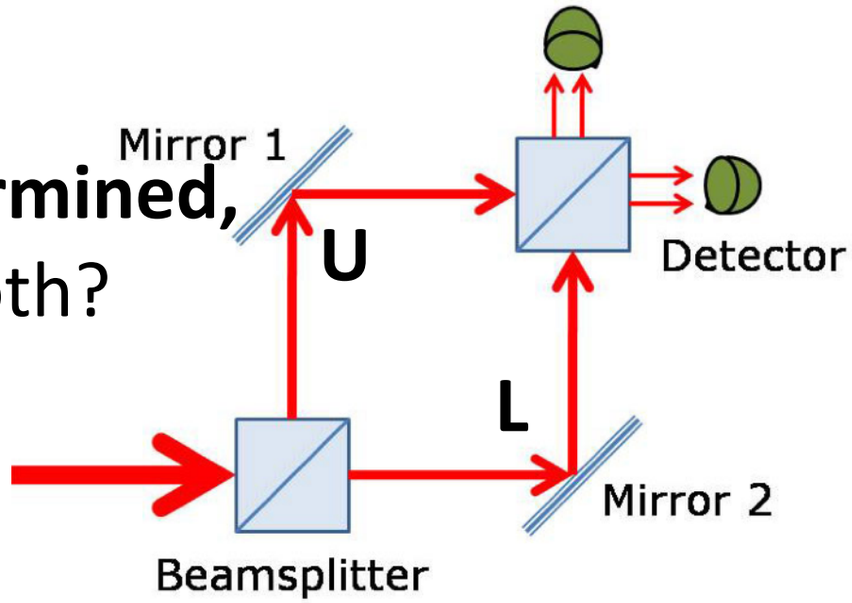
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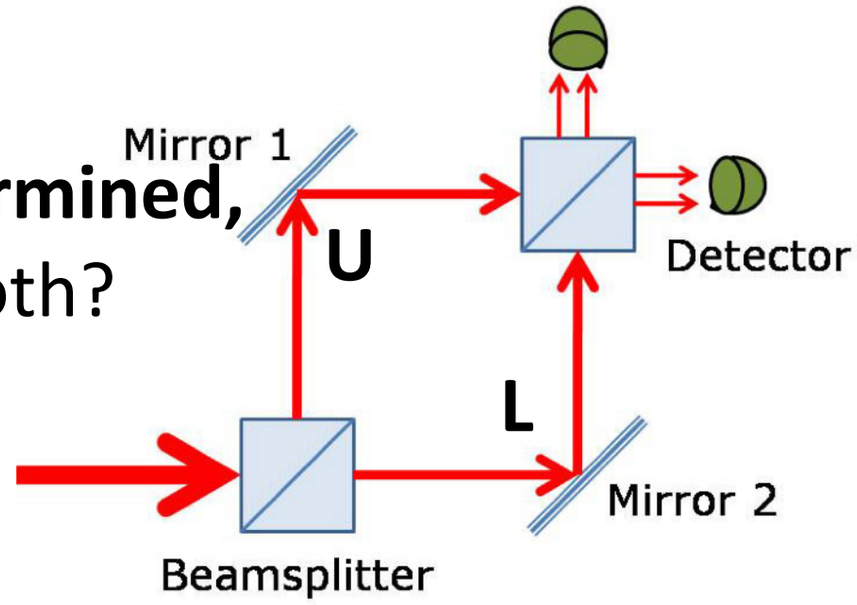
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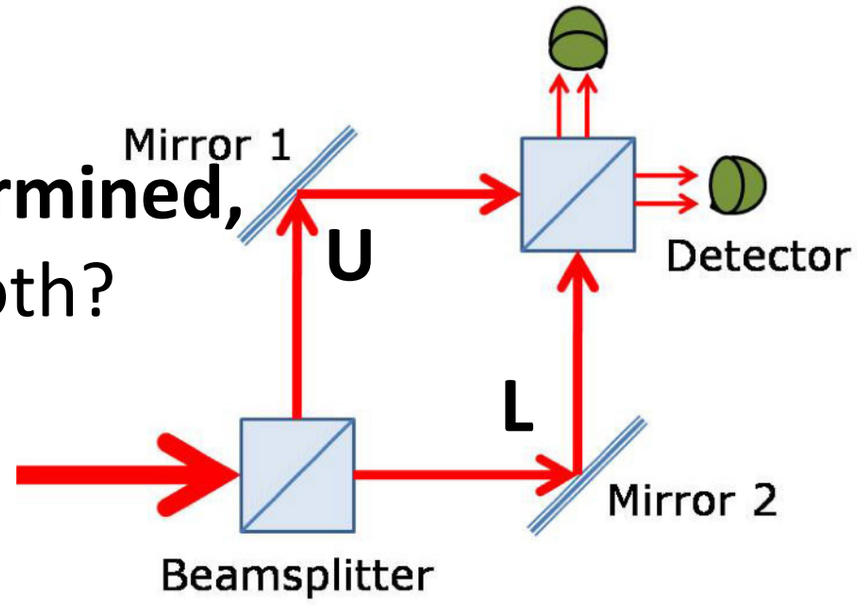
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If so, then every projector would have a hidden “truth value”: either

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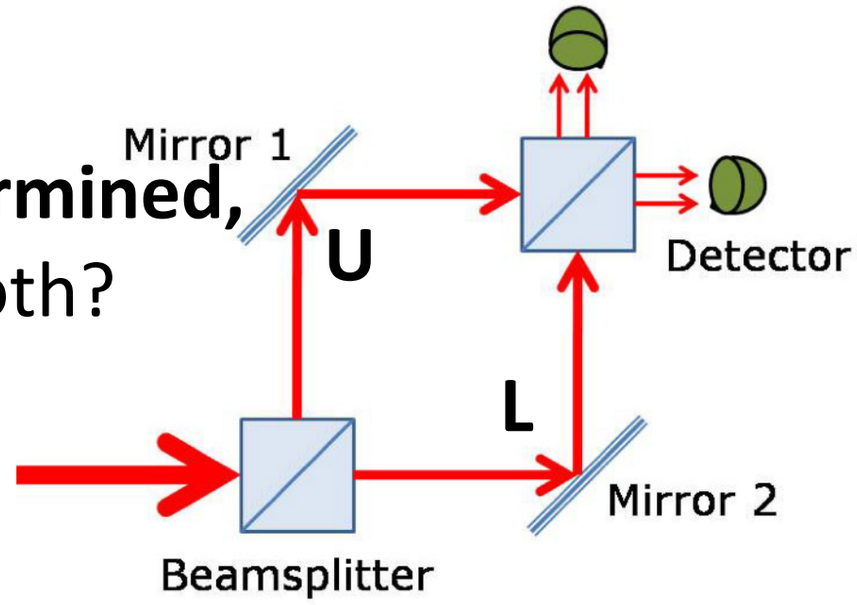
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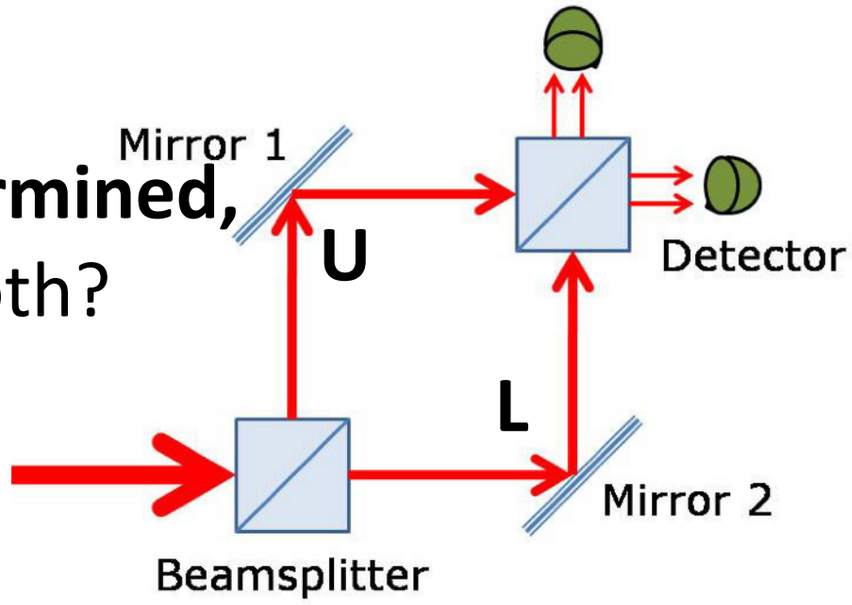
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But for three and more dimensions, it gets more interesting…

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Theorem. If the dimension is $d \geq 3,$ then there is **no** map from the (orthogonal) projection operators to {0,1} such that $\nu(P_1) + \nu(P_2) + \ldots + \nu(P_d) = 1$ whenever $P_1 + P_2 + \ldots + P_d = \mathbf{1}.$

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Hence, the truth values of all propositions **cannot all coexist** (say, before we decided which measurement we actually want to implement).

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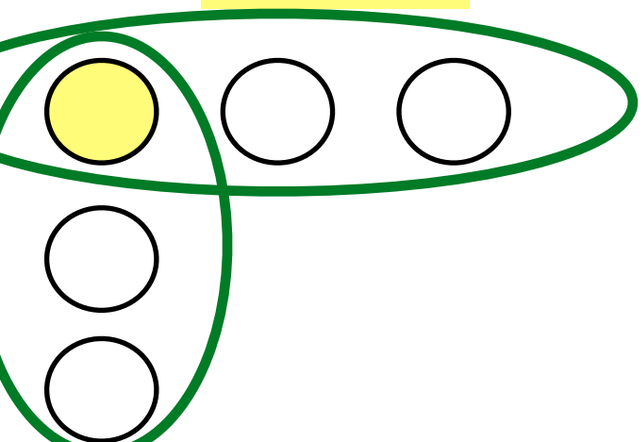
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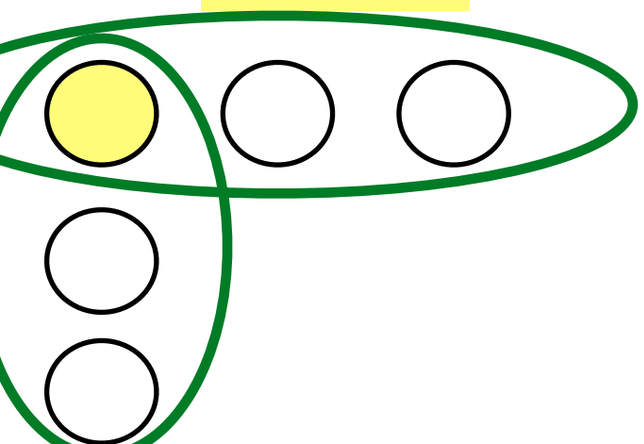
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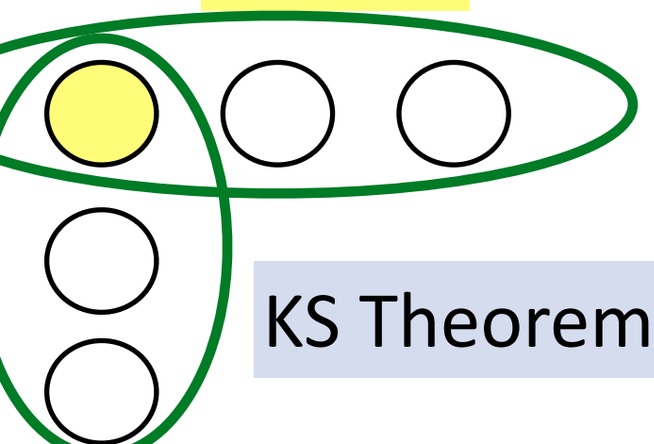
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KS Theorem rules out the existence of **noncontextual** hidden variables.

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Contradiction. Hence, no such truth value assignment can exist.

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2. Against the received view: QM as a theory of probability

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4. Spekkens' contextuality in a nutshell (if time allows)

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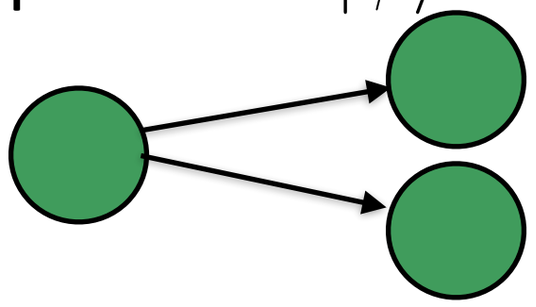
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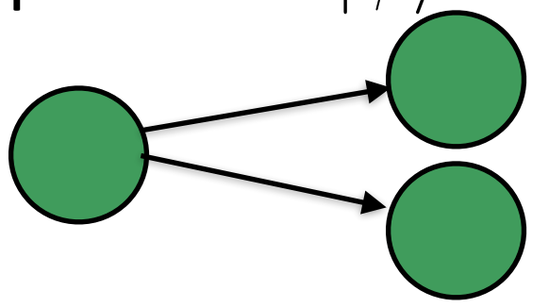
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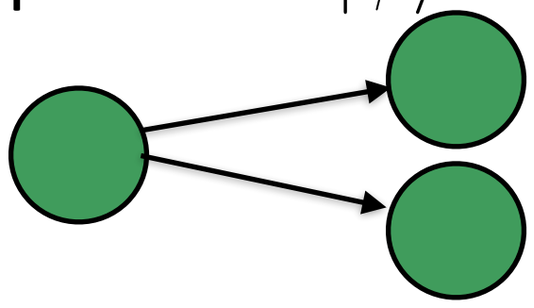
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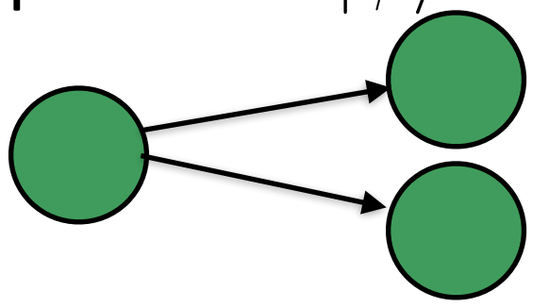
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All probabilistic theories have these (and other) properties in common.

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MM: later…

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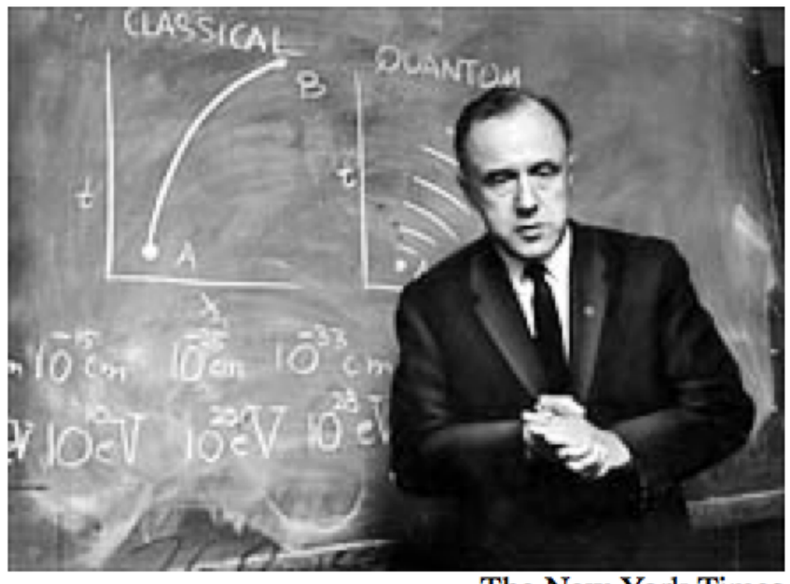
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Motivation

John A. Wheeler, New York Times, Dec. 12 2000:

„Quantum physics [...] has explained the structure of atoms and molecules, [...] the behavior of semiconductors [...] and the comings and goings of particles from neutrinos to quarks.

Successful, yes, but mysterious, too.
Why does the quantum exist?“



The New York Times

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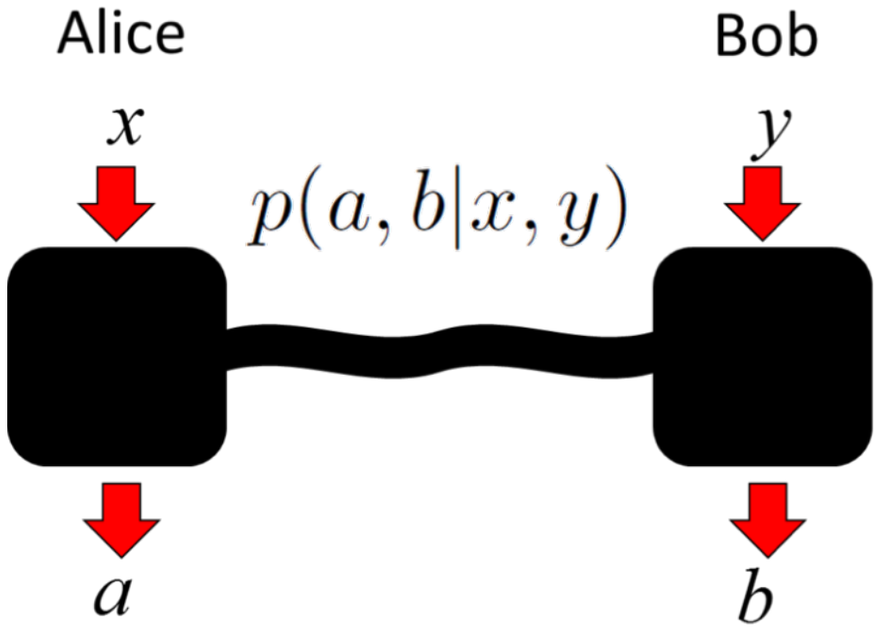
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Long prehistory of unsuccessful (but insightful) attempts:

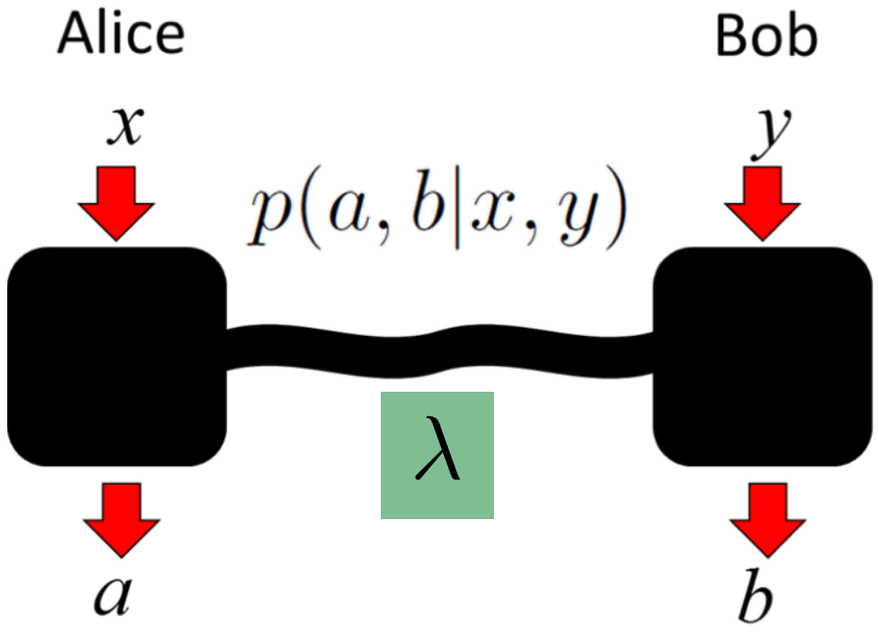
- Via **quantum logic:** Birkhoff & von Neumann (1936), Piron…
- As a **probabilistic** theory: Günther Ludwig (1970s/80s) (pre-QIT)
- Lucien Hardy 2001: "QM From Five Reasonable Axioms"
- Around 2011: three groups finally manage to do this.
  Dakić+Brukner, Masanes+MPM , Chiribella+Perinotti+d'Ariano.

More general than quantum?

More general than quantum?



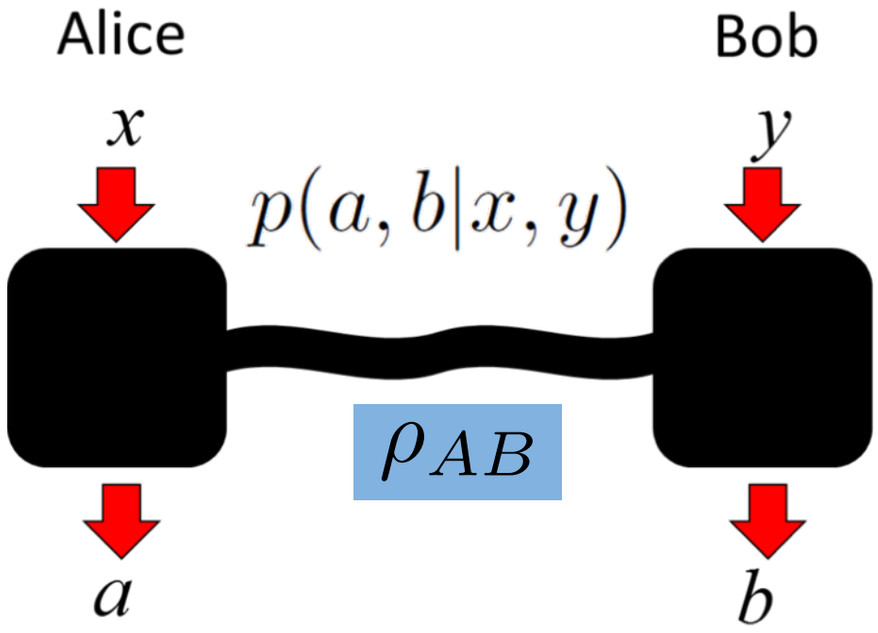
More general than quantum?



- In **classical** physics / prob. theory:

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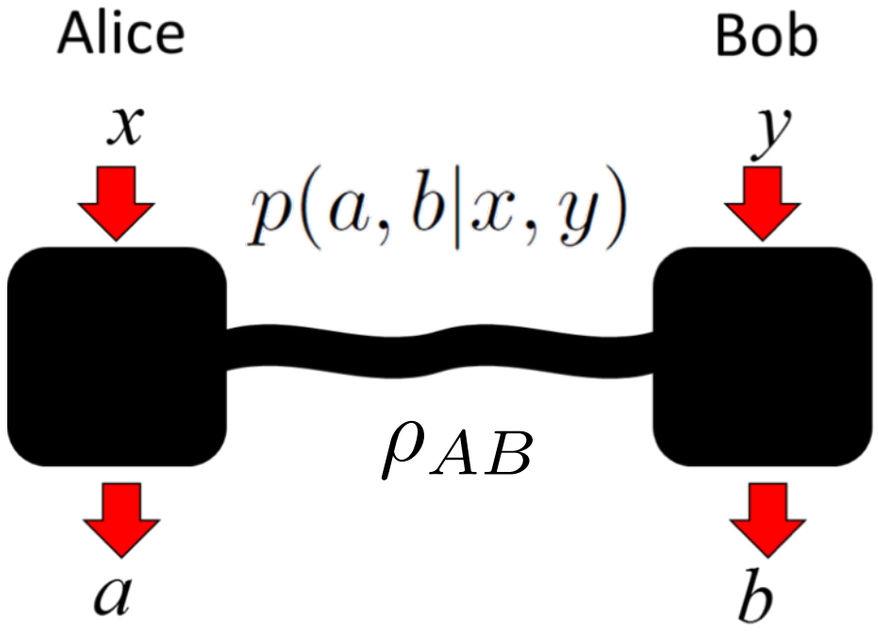
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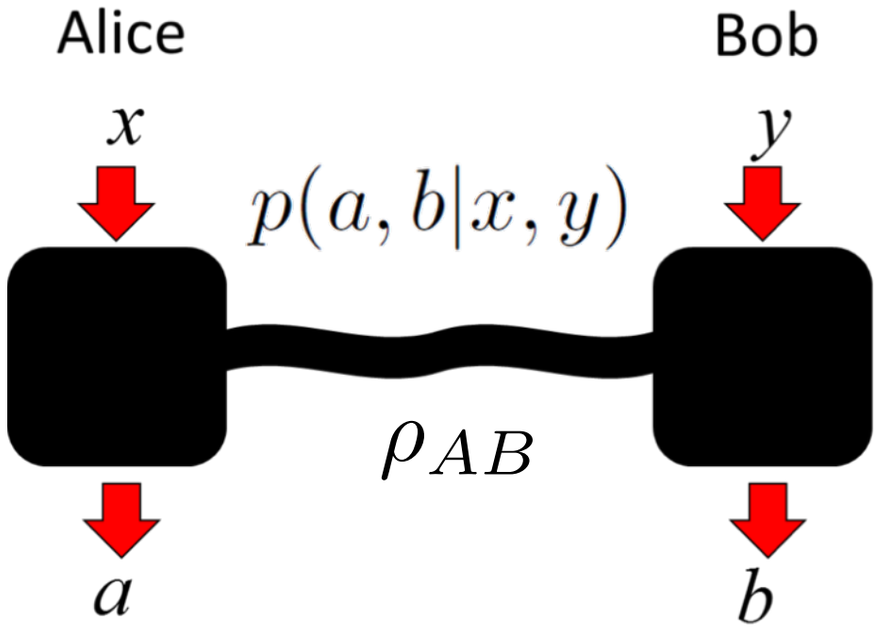
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Quantum theory admits more general *P*'s due to the **violation of Bell inequalities**.

The Bell-CHSH inequality

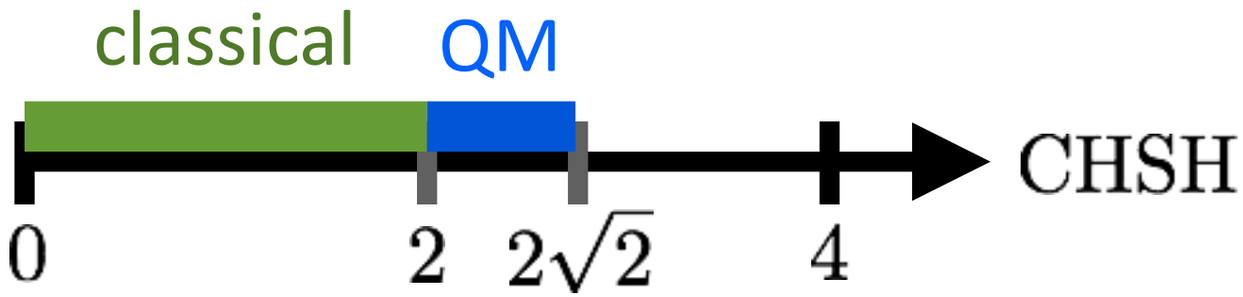
Classical probability distributions satisfy Bell inequality:

$$\mathrm{CHSH} := |C_{00} + C_{01} + C_{10} - C_{11}| \leq 2 \quad \text{where} \quad C_{xy} := \mathbb{E}(a \cdot b | x, y).$$

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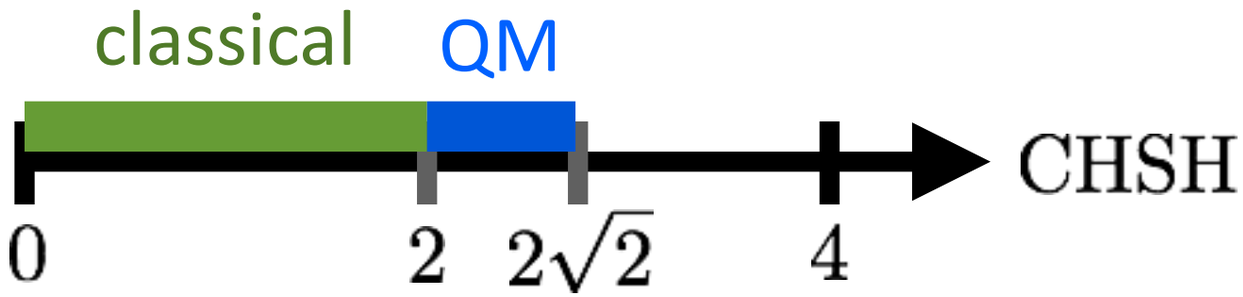
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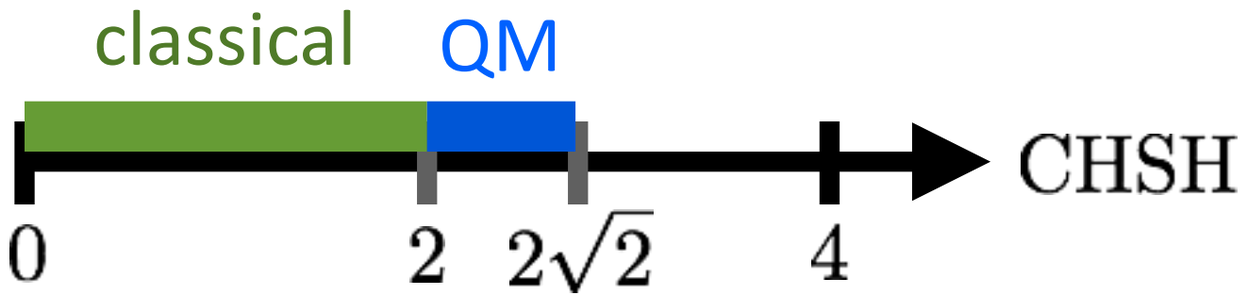
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Are **quantum** correlations the **most general** $P(a, b|x, y)$ that satisfy the no-signalling principle?

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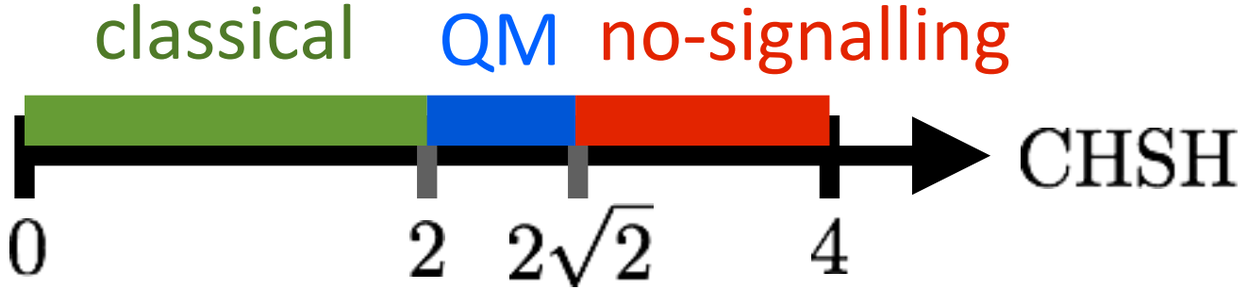
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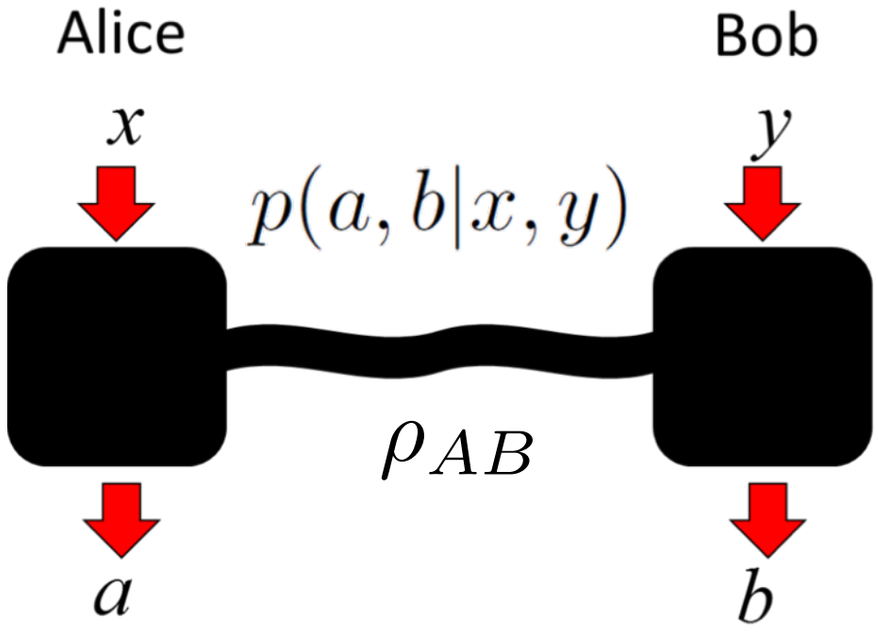
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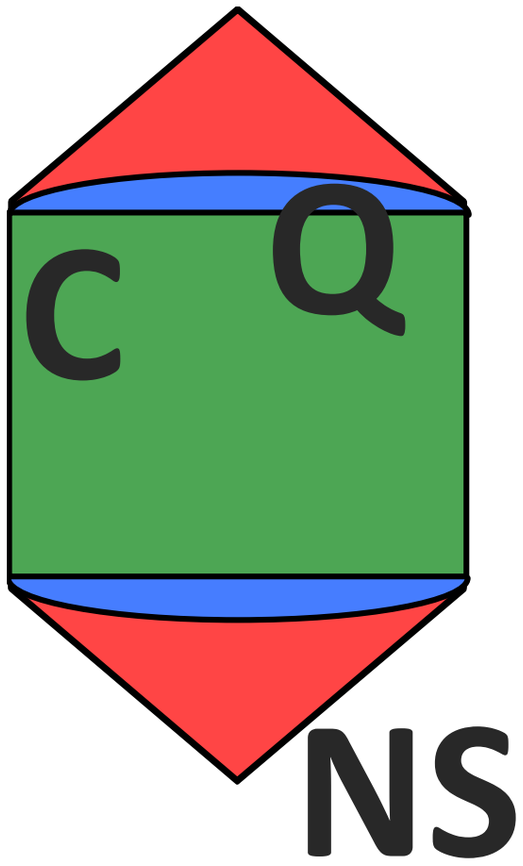
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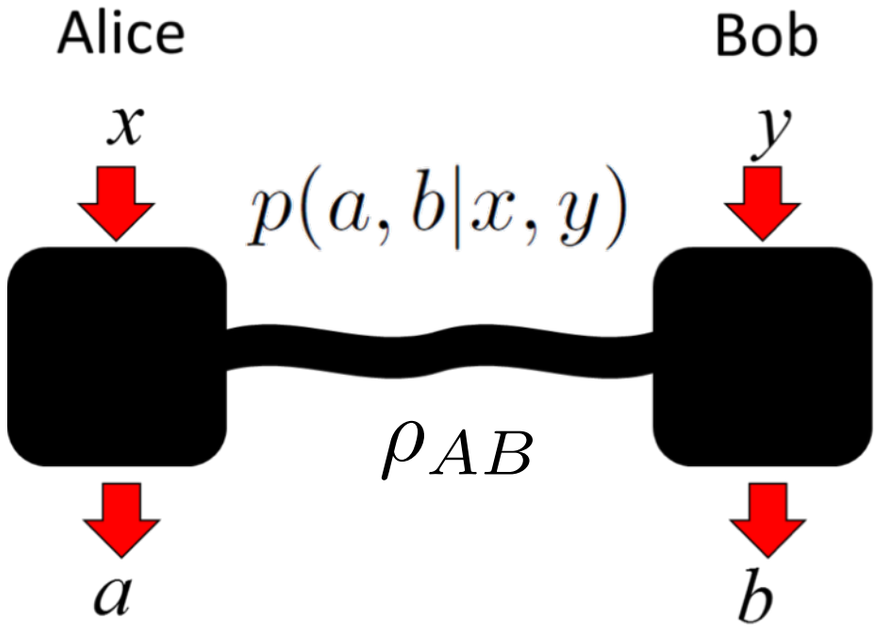
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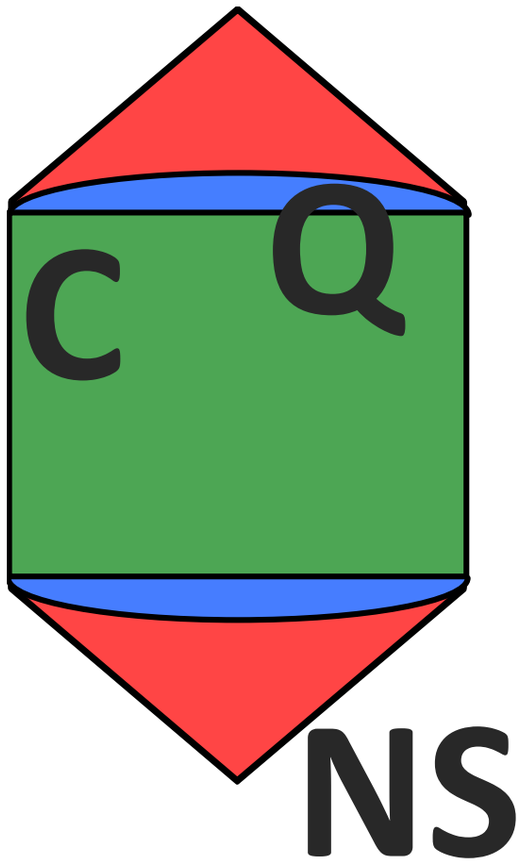
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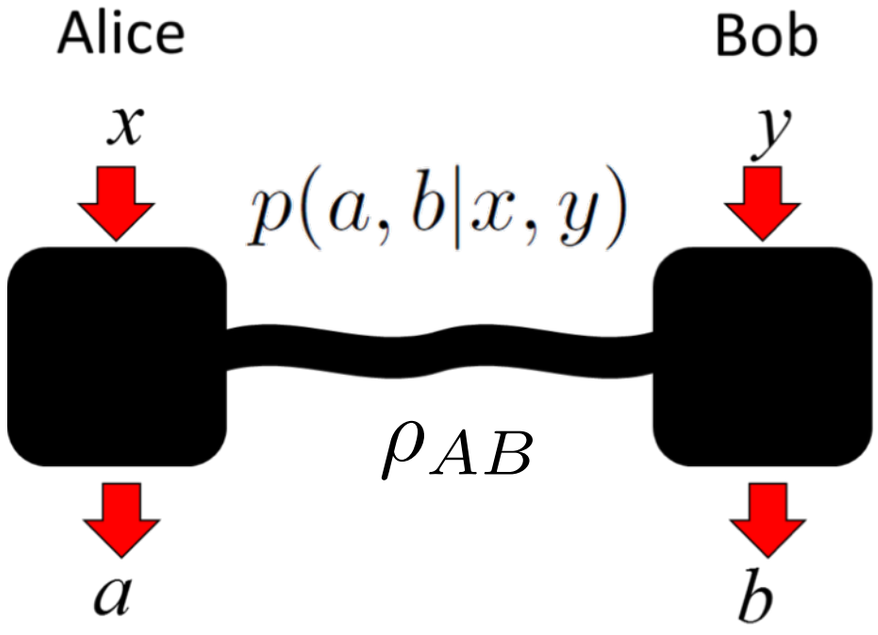
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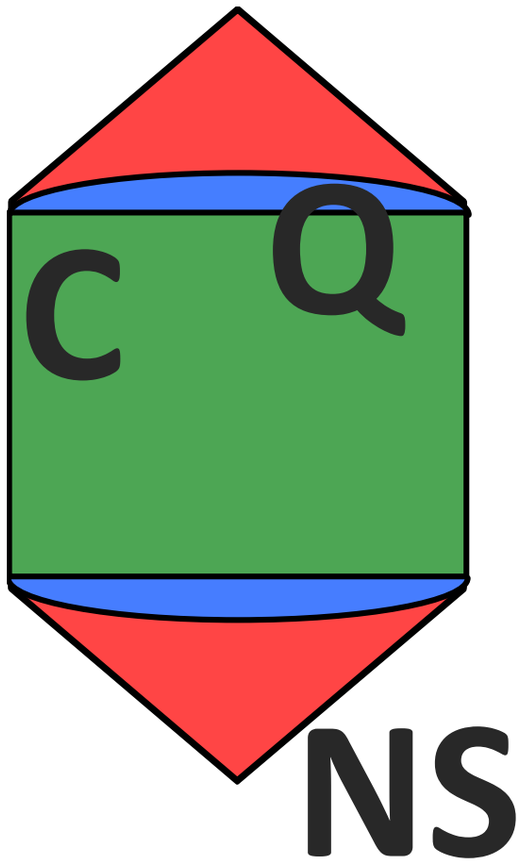
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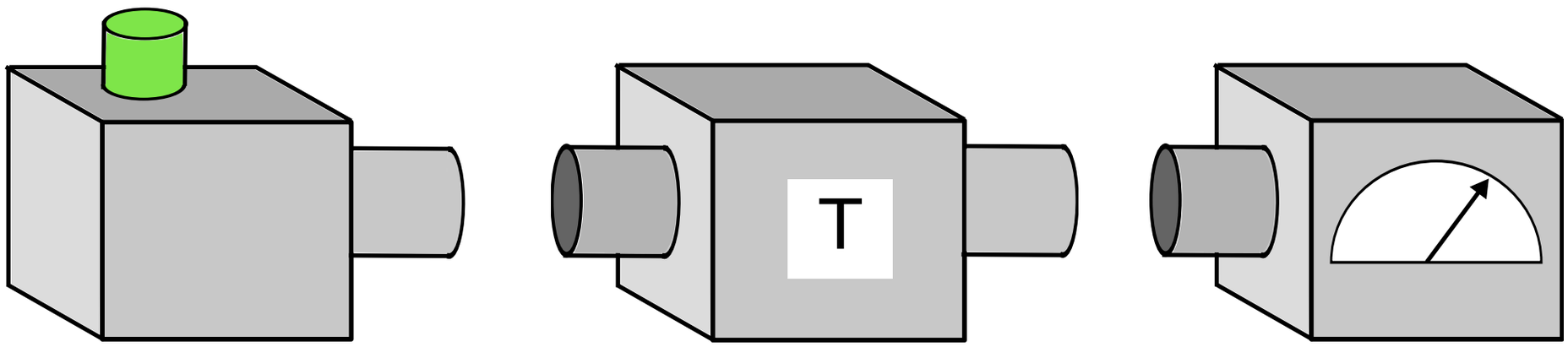
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3 examples of a "**generalized probabilistic theory**".

Generalized probabilistic theories



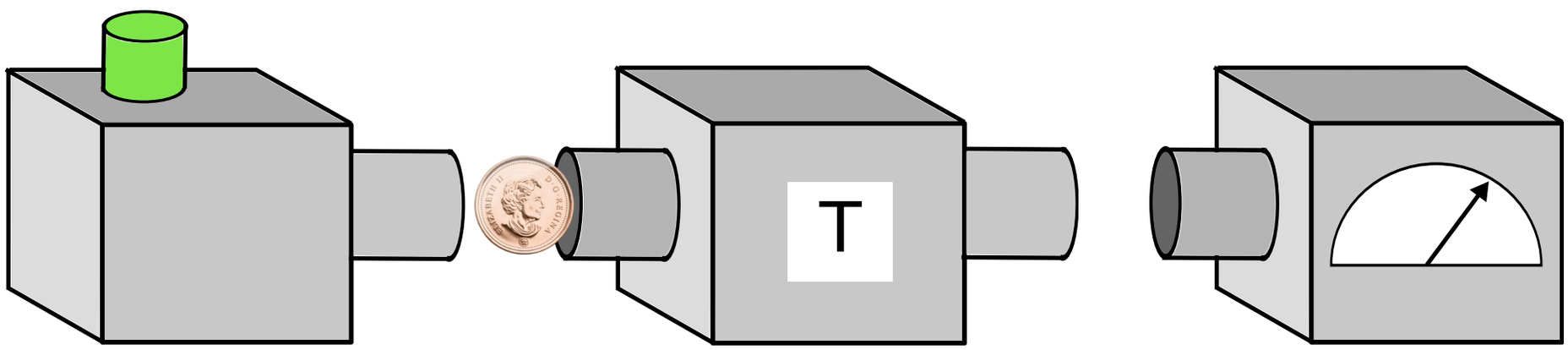
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Generalized probabilistic theories

Example: classical coin toss.



- On every push of button, the preparation device performs a biased coin toss.

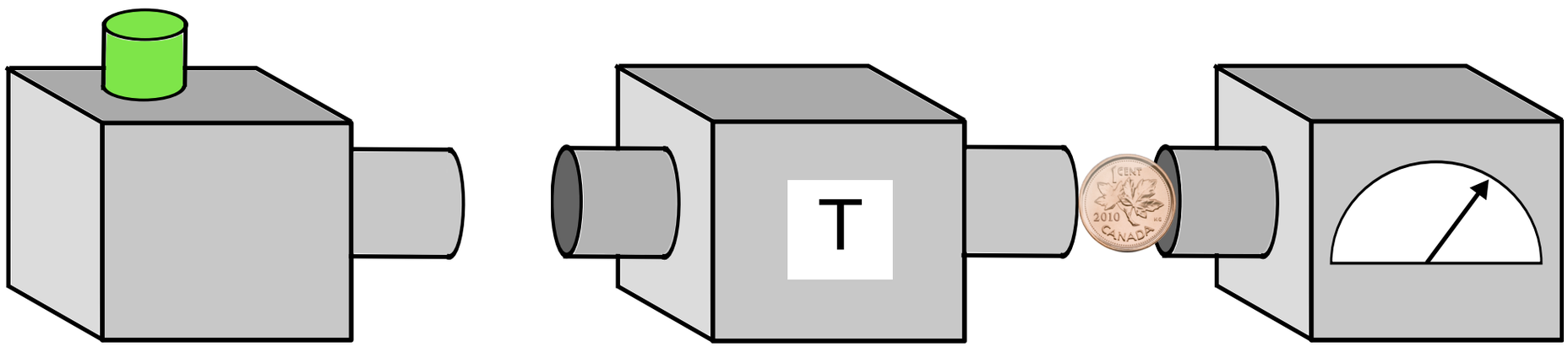


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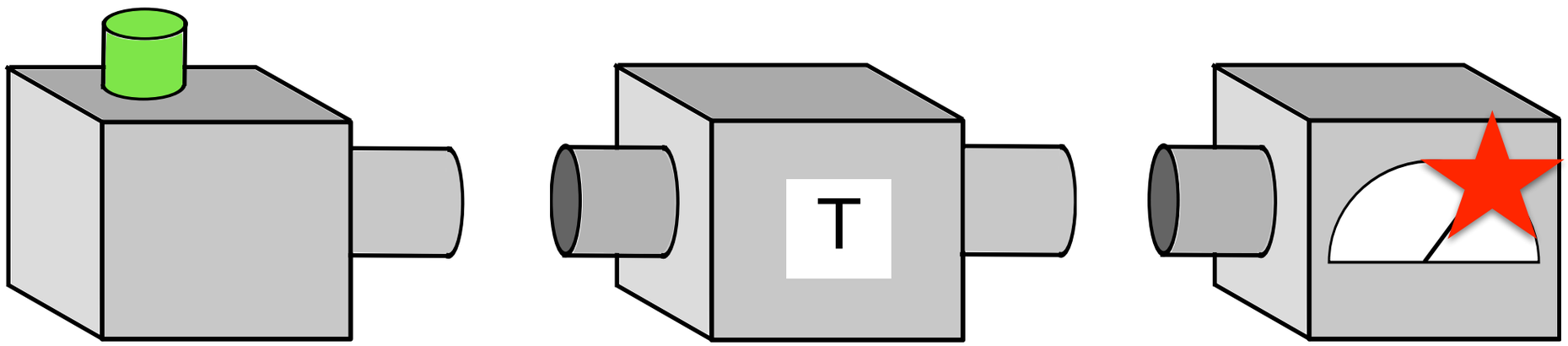


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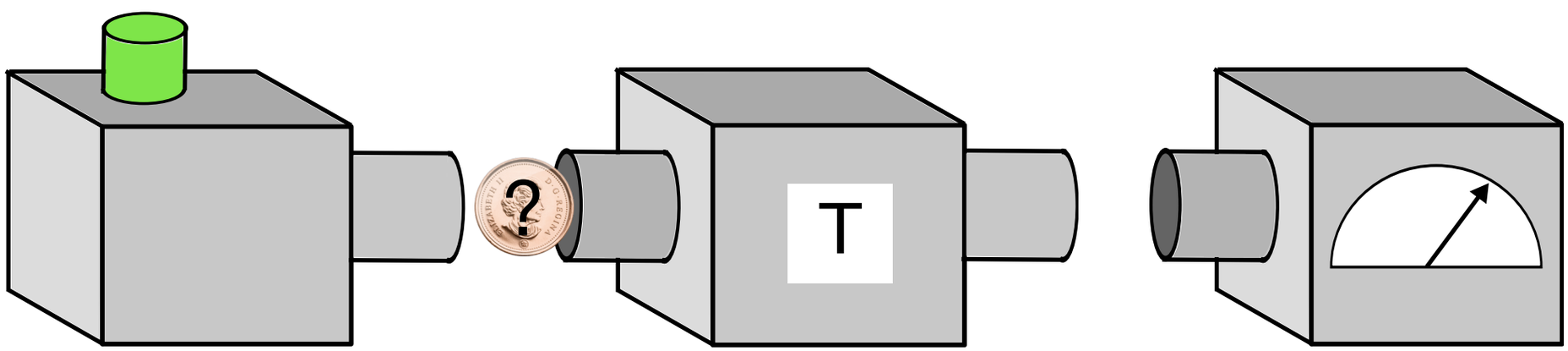
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$$\omega = \begin{pmatrix} \mathrm{Prob}(\mathrm{heads}) \\ \mathrm{Prob}(\mathrm{tails}) \end{pmatrix} = \begin{pmatrix} p \\ 1-p \end{pmatrix}.$$



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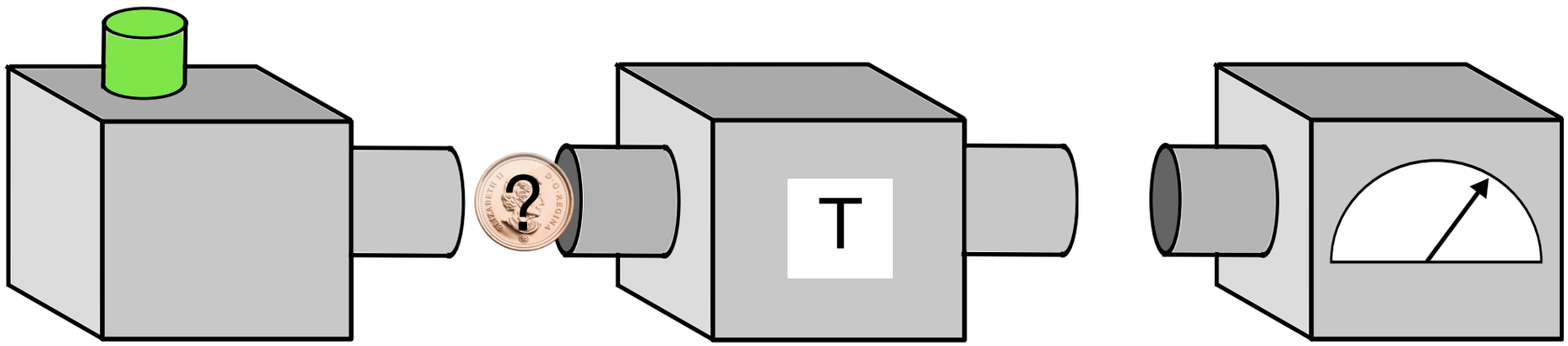
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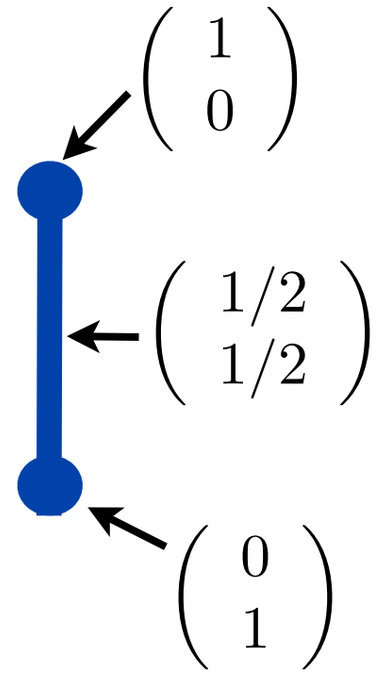
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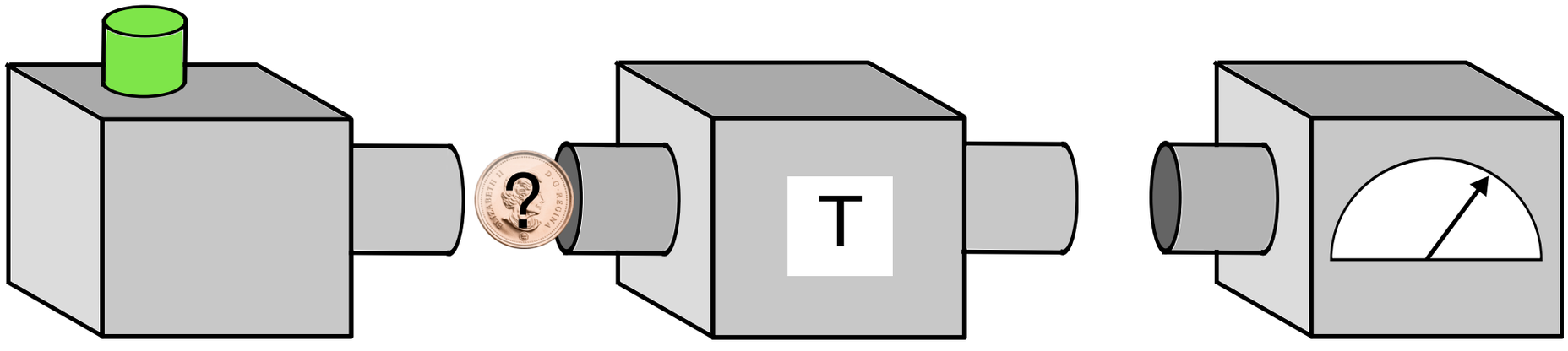


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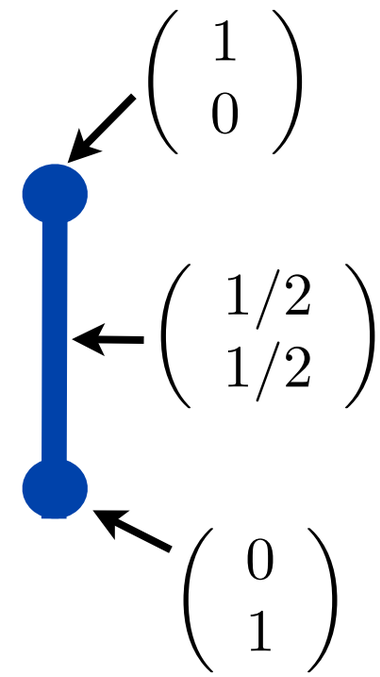
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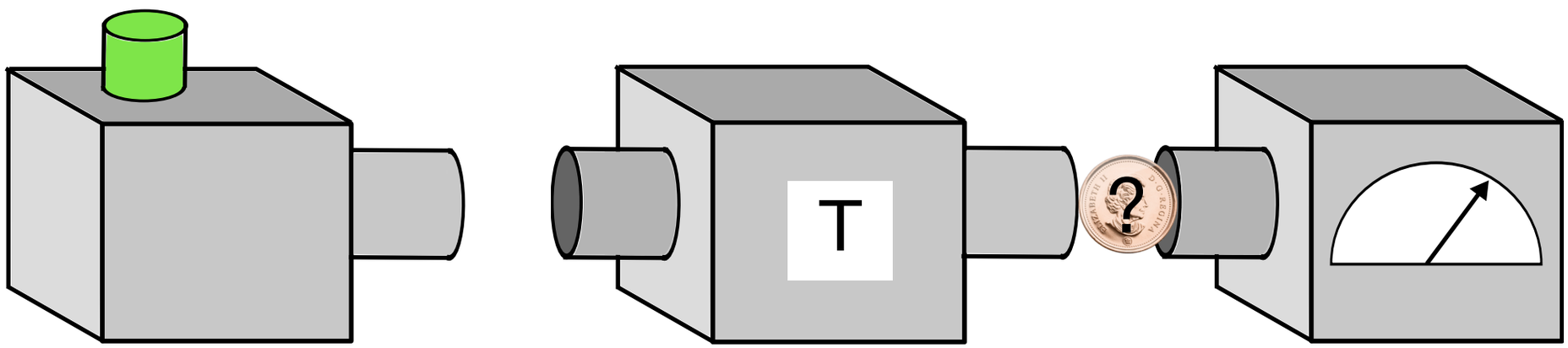
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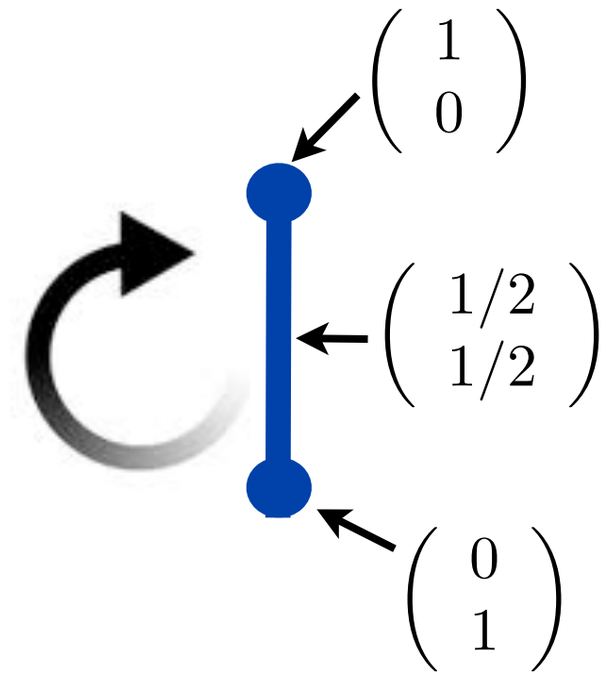
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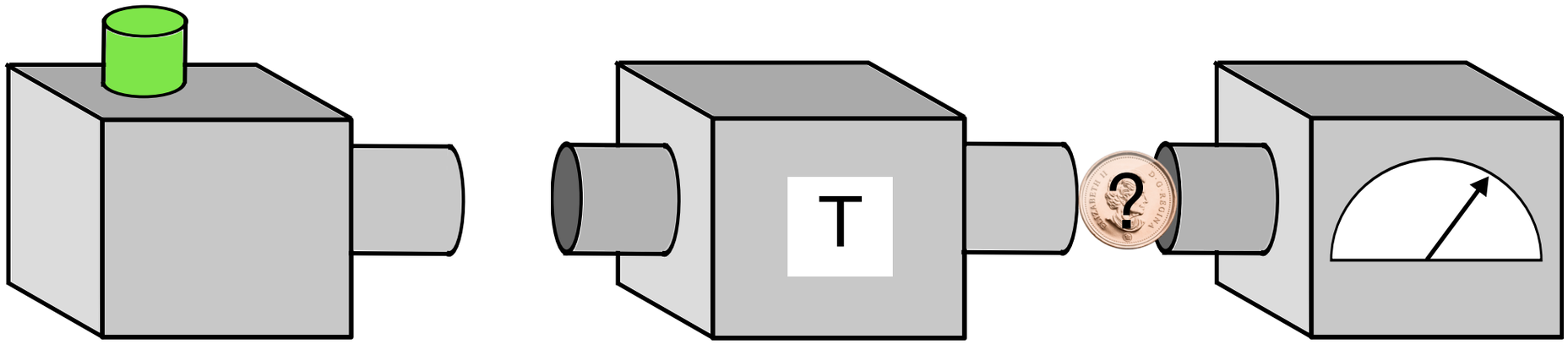
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Maps states to states and is linear.



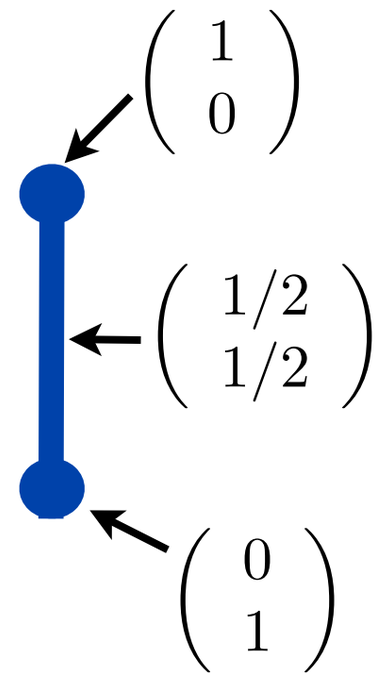
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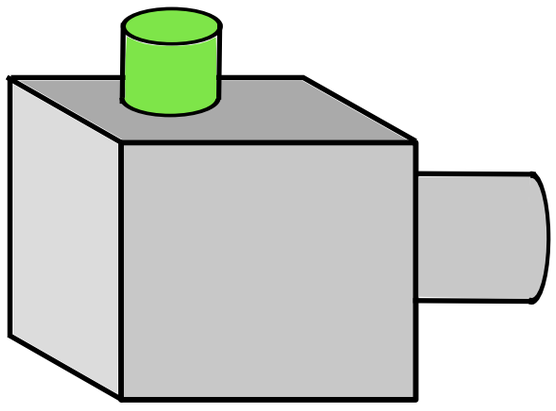
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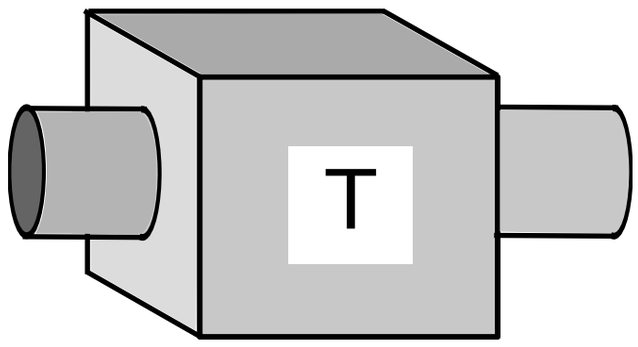


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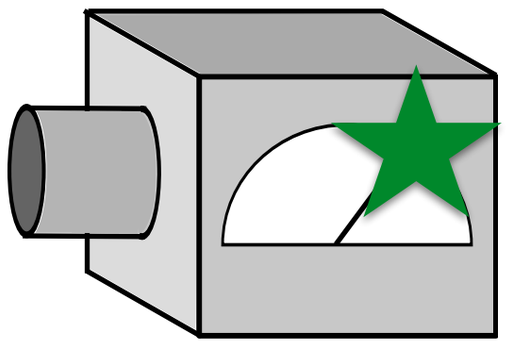




Preparation



transformation



measurement

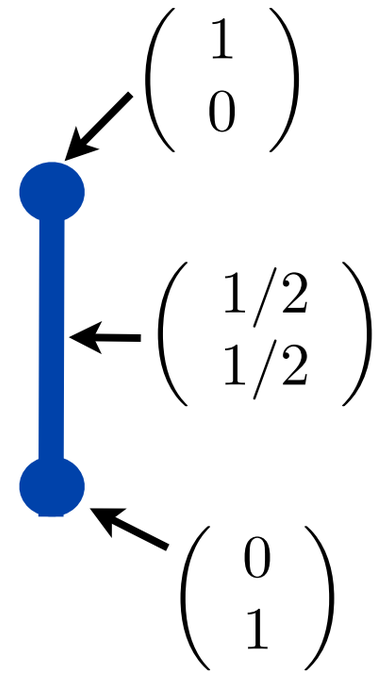
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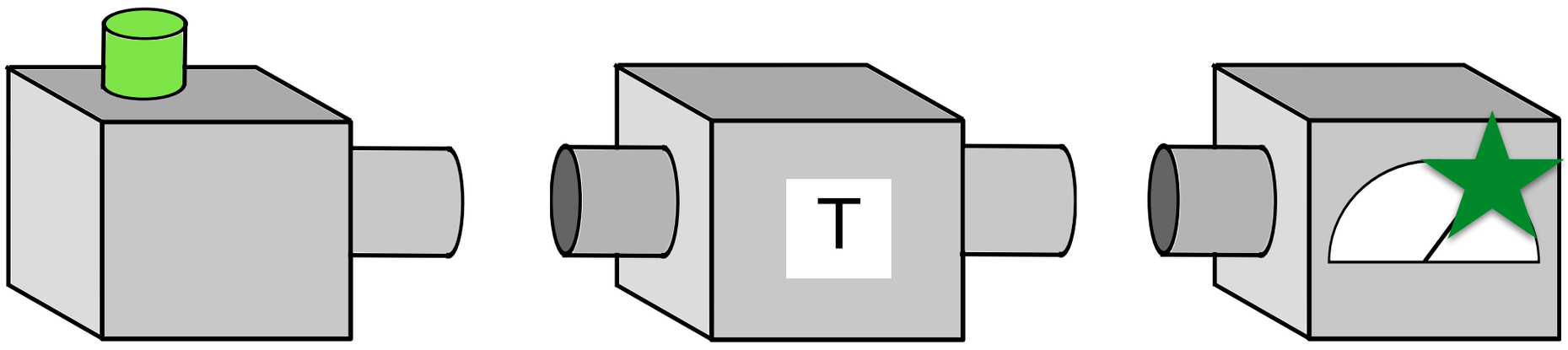
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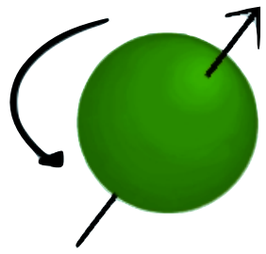


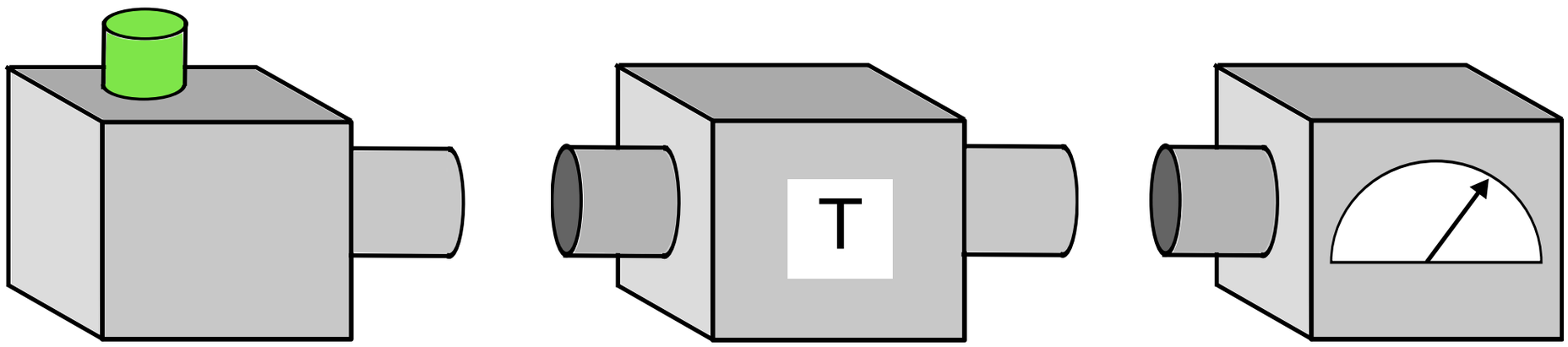


Preparation transformation measurement

Generalized probabilistic theories

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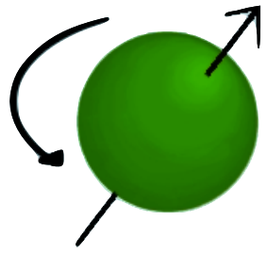




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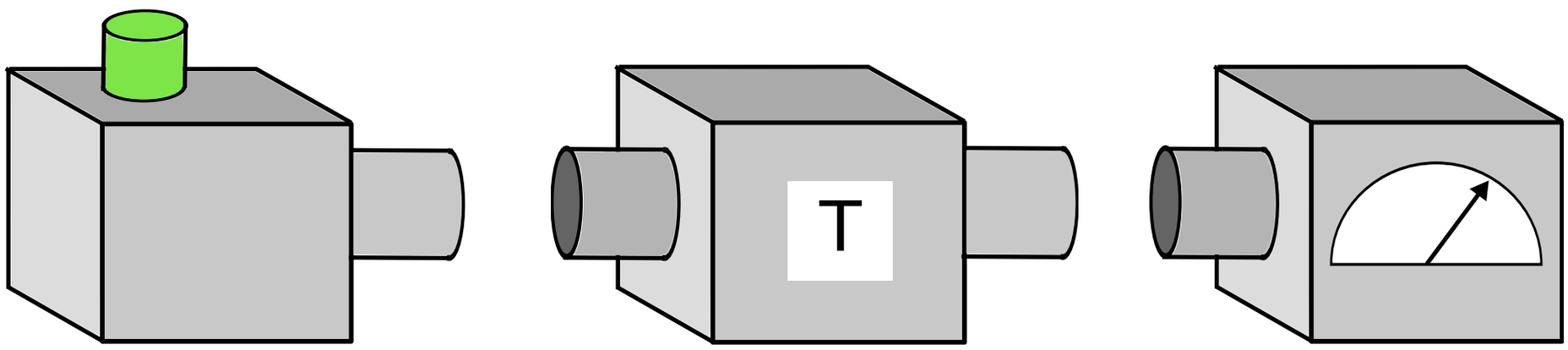
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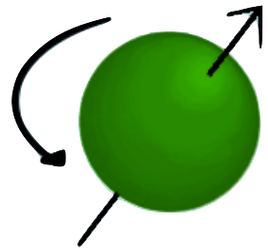
More generally: ρ is (mixed-state-)2x2 density matrix.

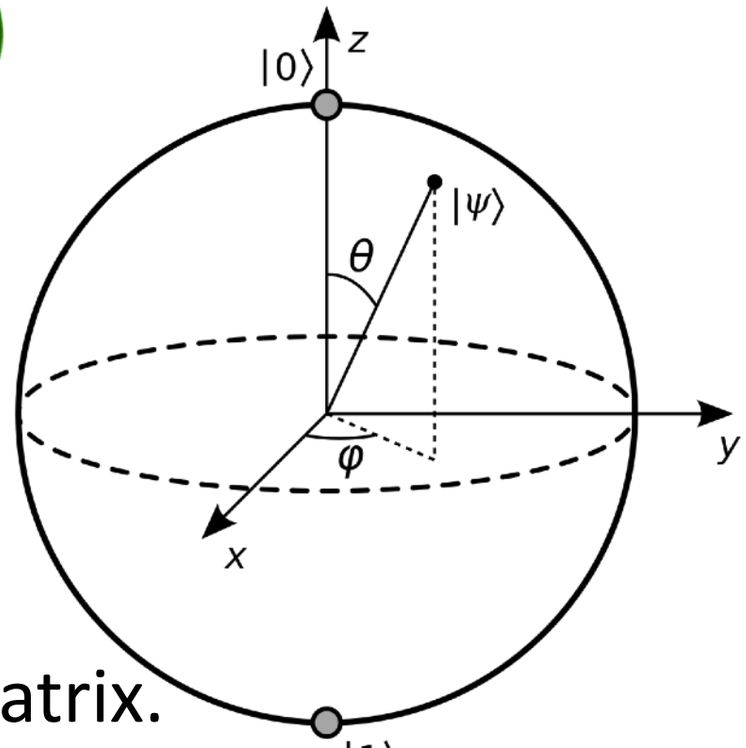


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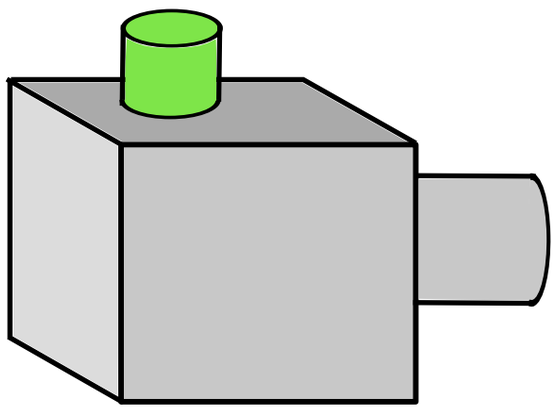


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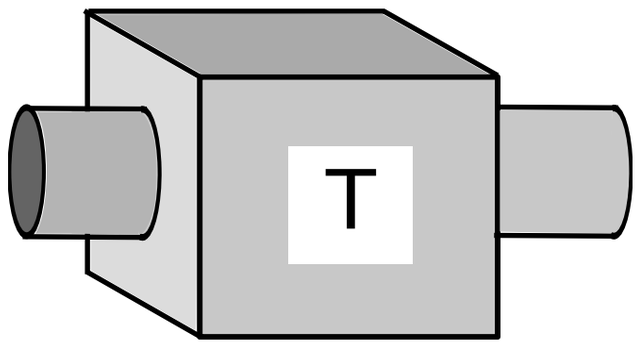
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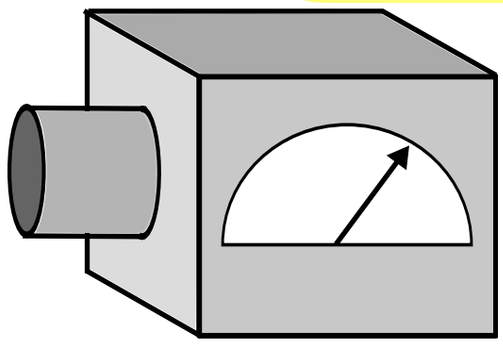
$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$



Preparation



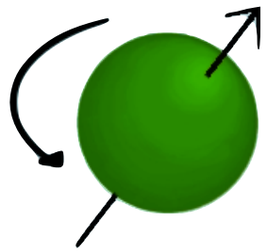
transformation



measurement

Generalized probabilistic theories

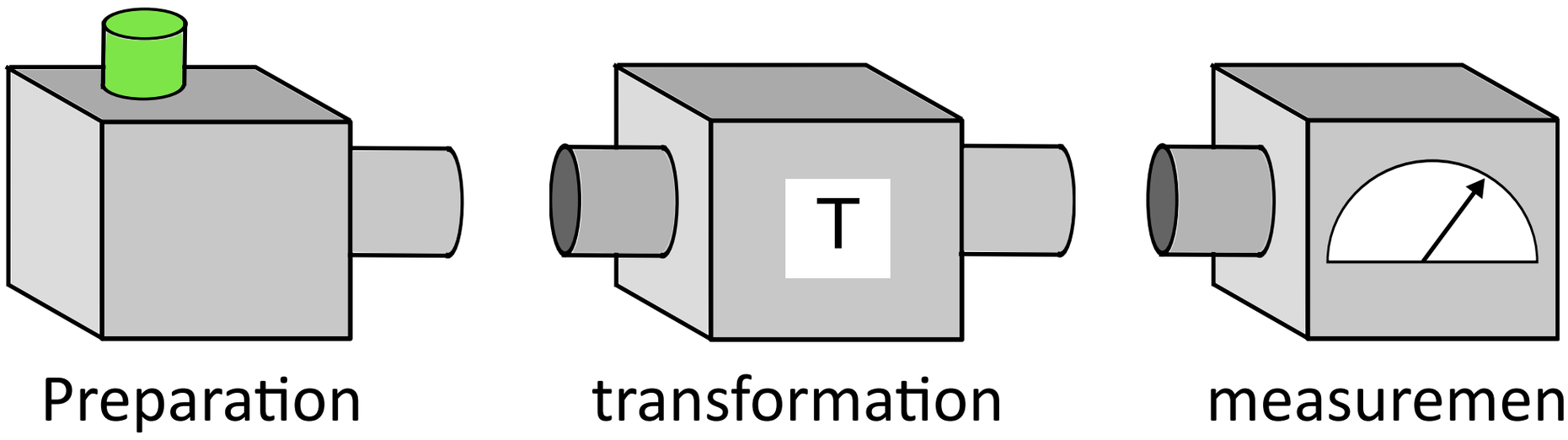
Example: quantum bit.





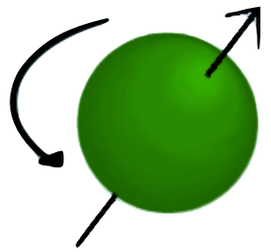
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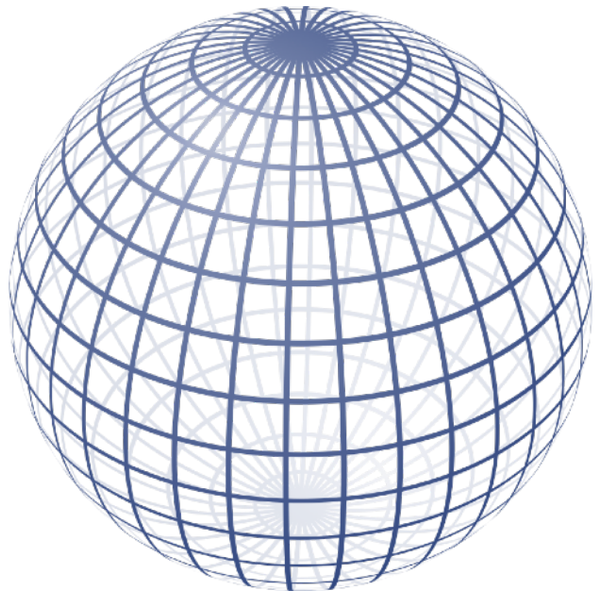
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Generalized probabilistic theories

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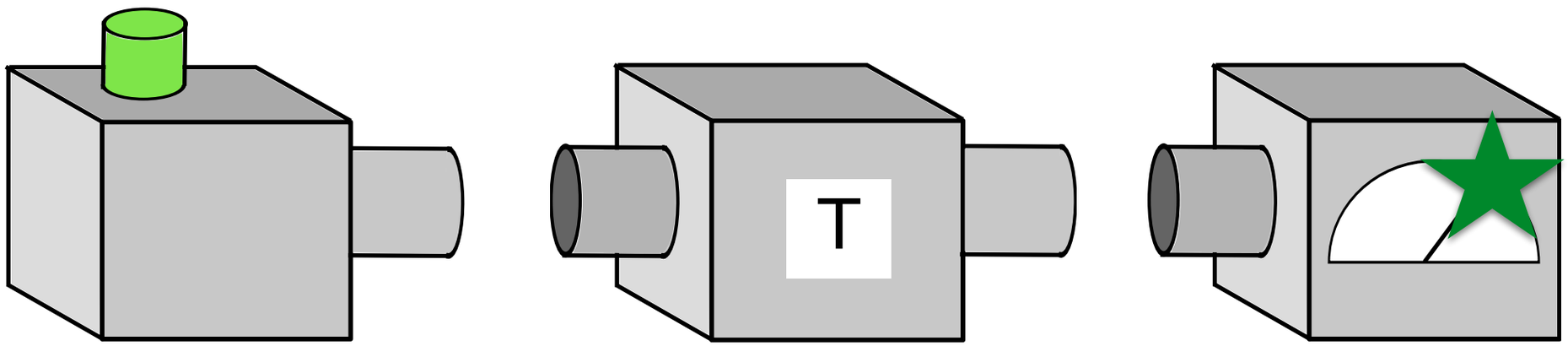


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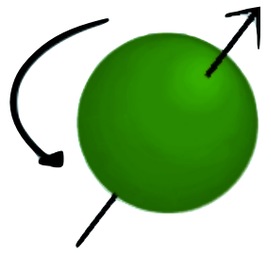
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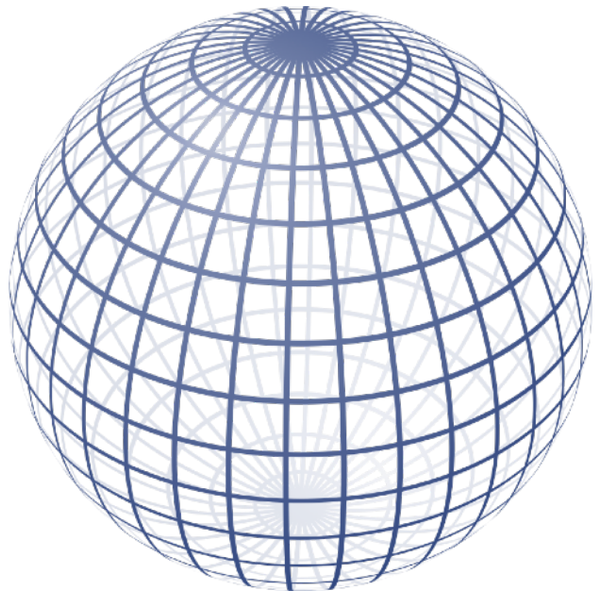


Preparation transformation measurement

Generalized probabilistic theories

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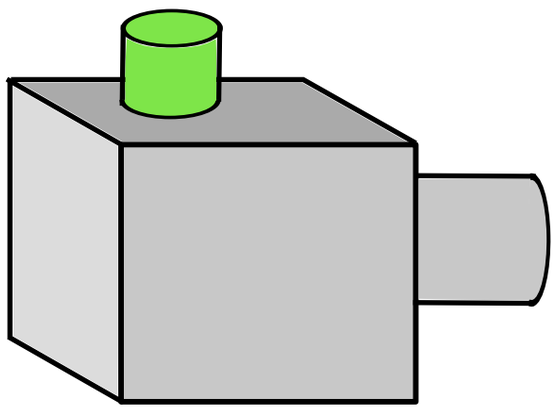


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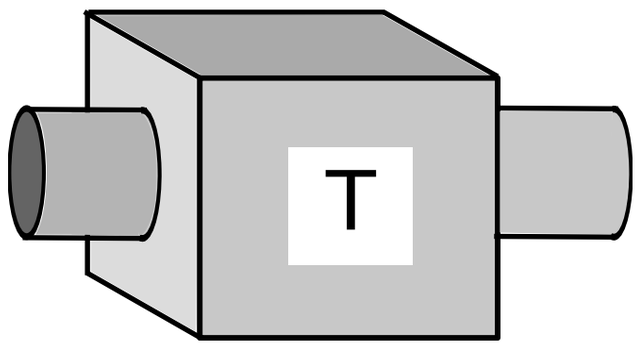
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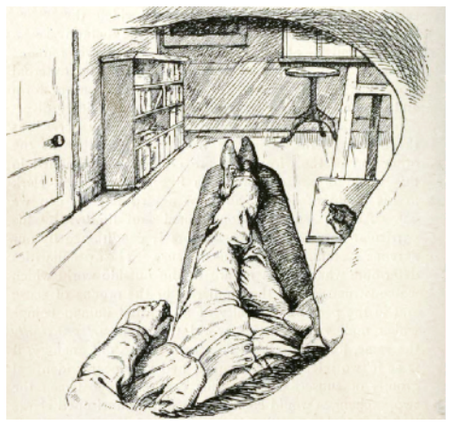
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Preparation



transformation



obeservation / experience?

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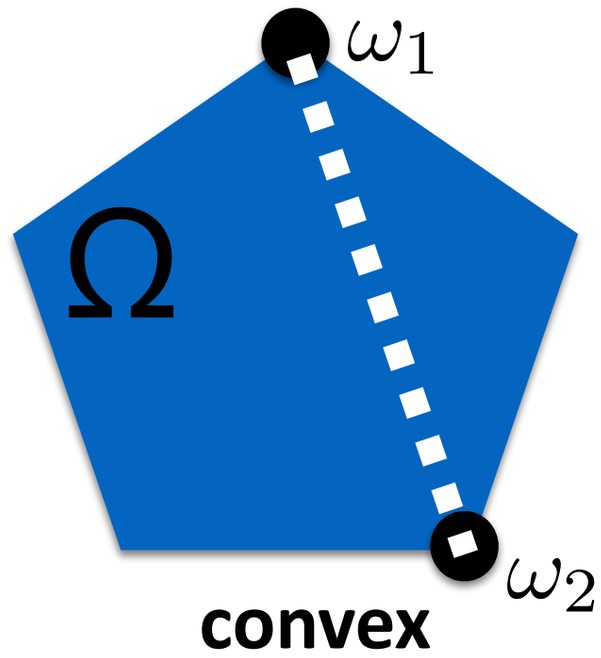
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convex

Generalized probabilistic theories

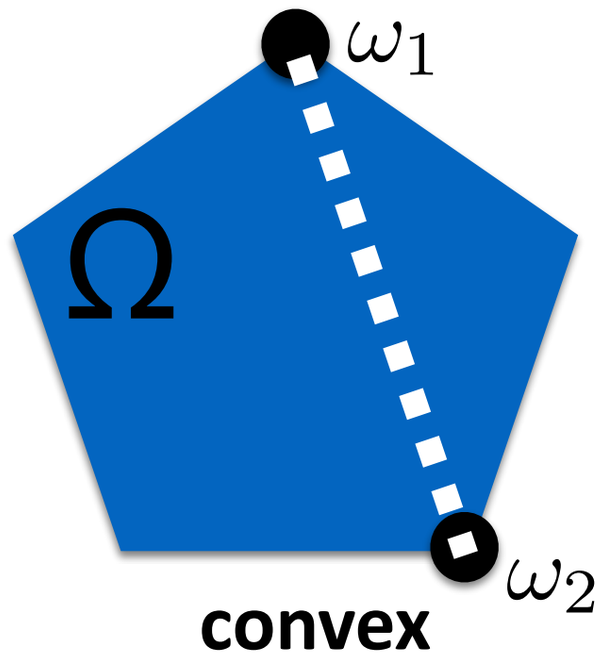
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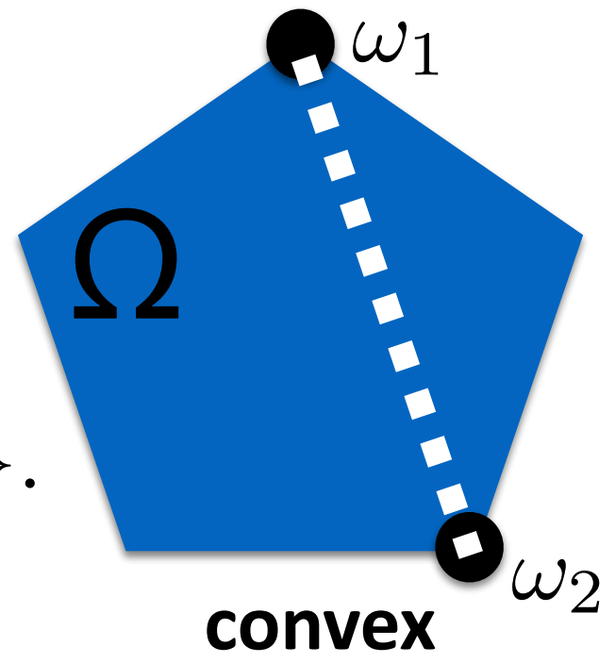
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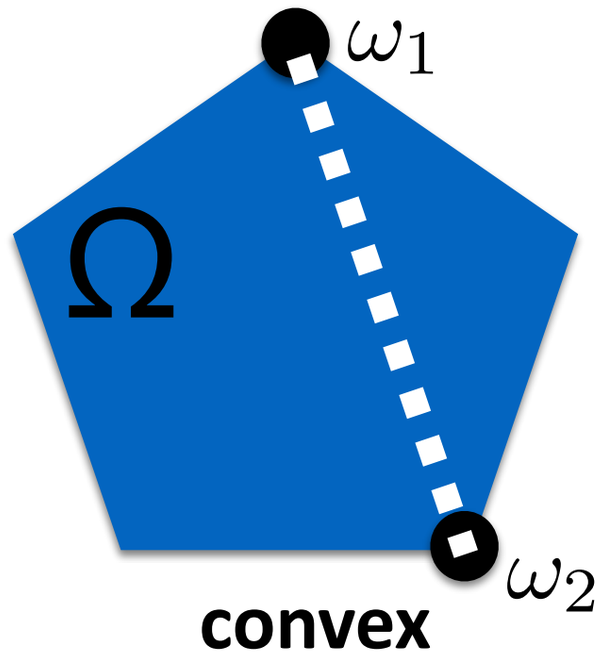
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Generalized probabilistic theories

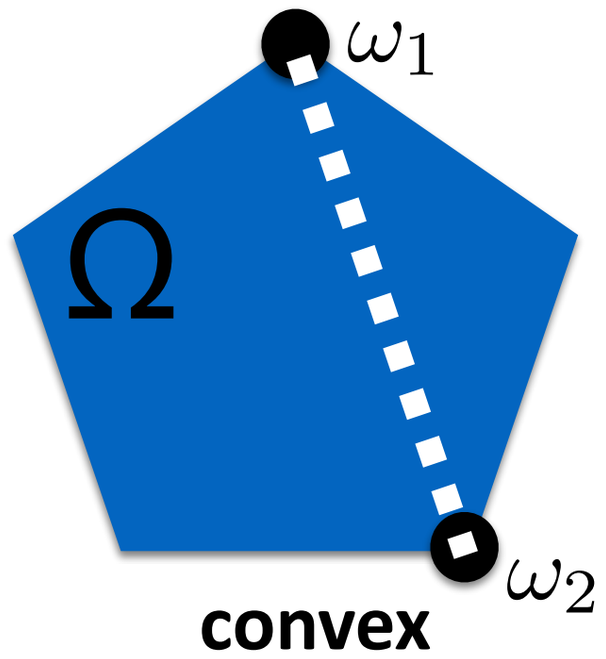
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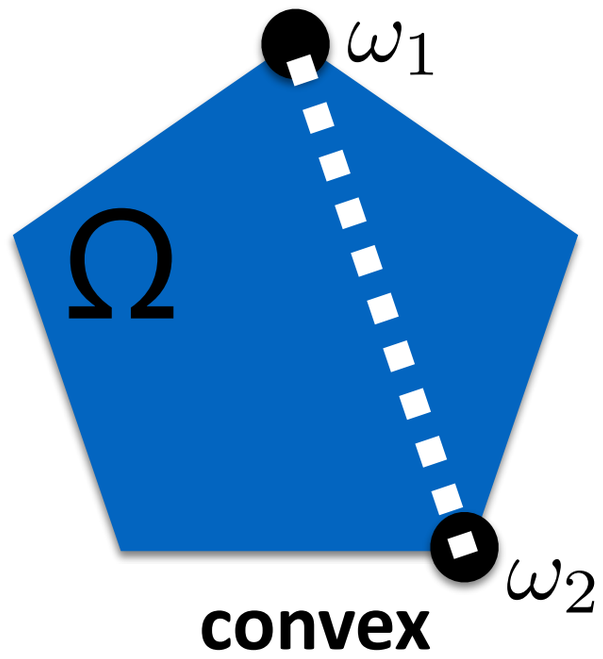


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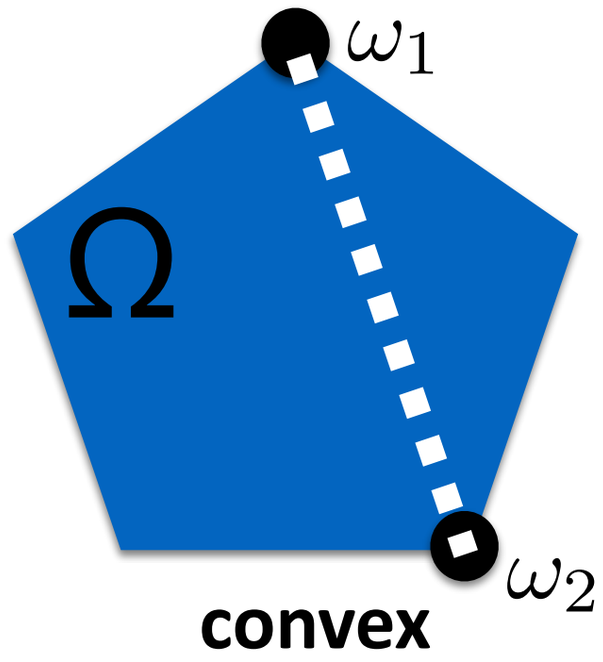
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Generalized probabilistic theories



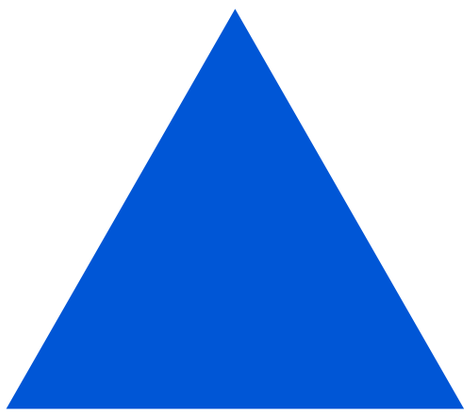
classical bit



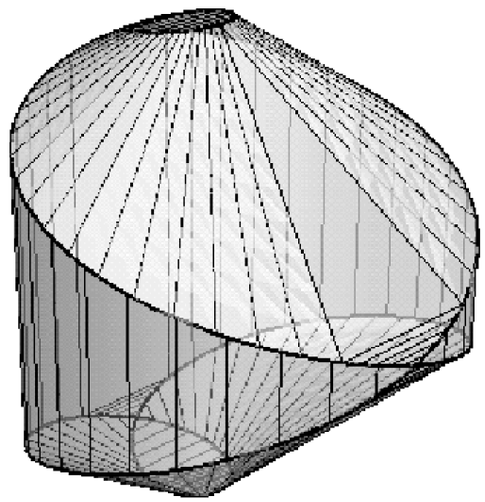
quantum bit



"gbit"



Classical trit
(3-level-system)



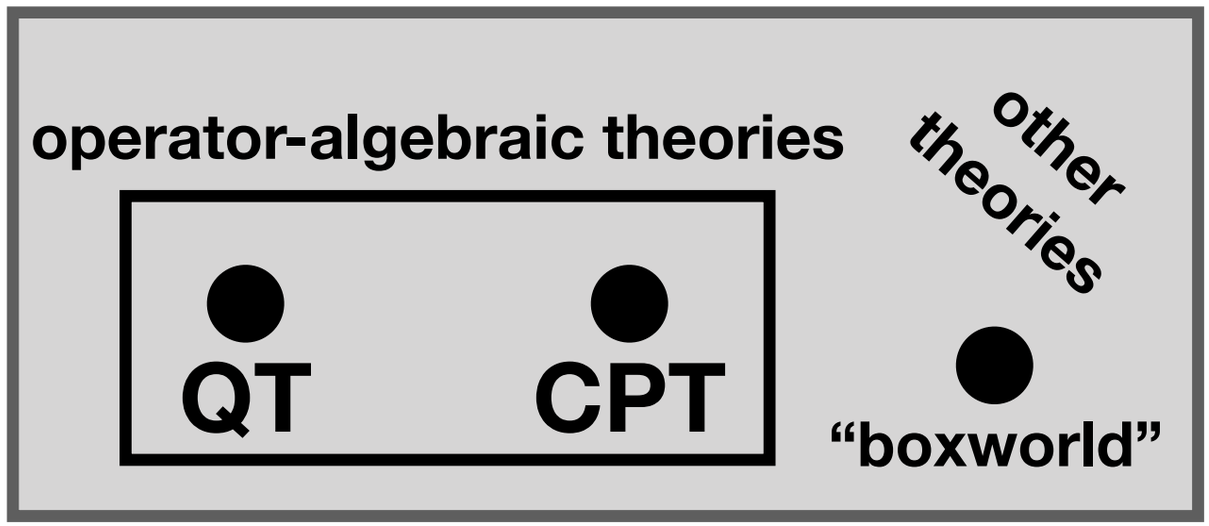
Quantum trit:
8D and complicated!



Arbitrary convex state space

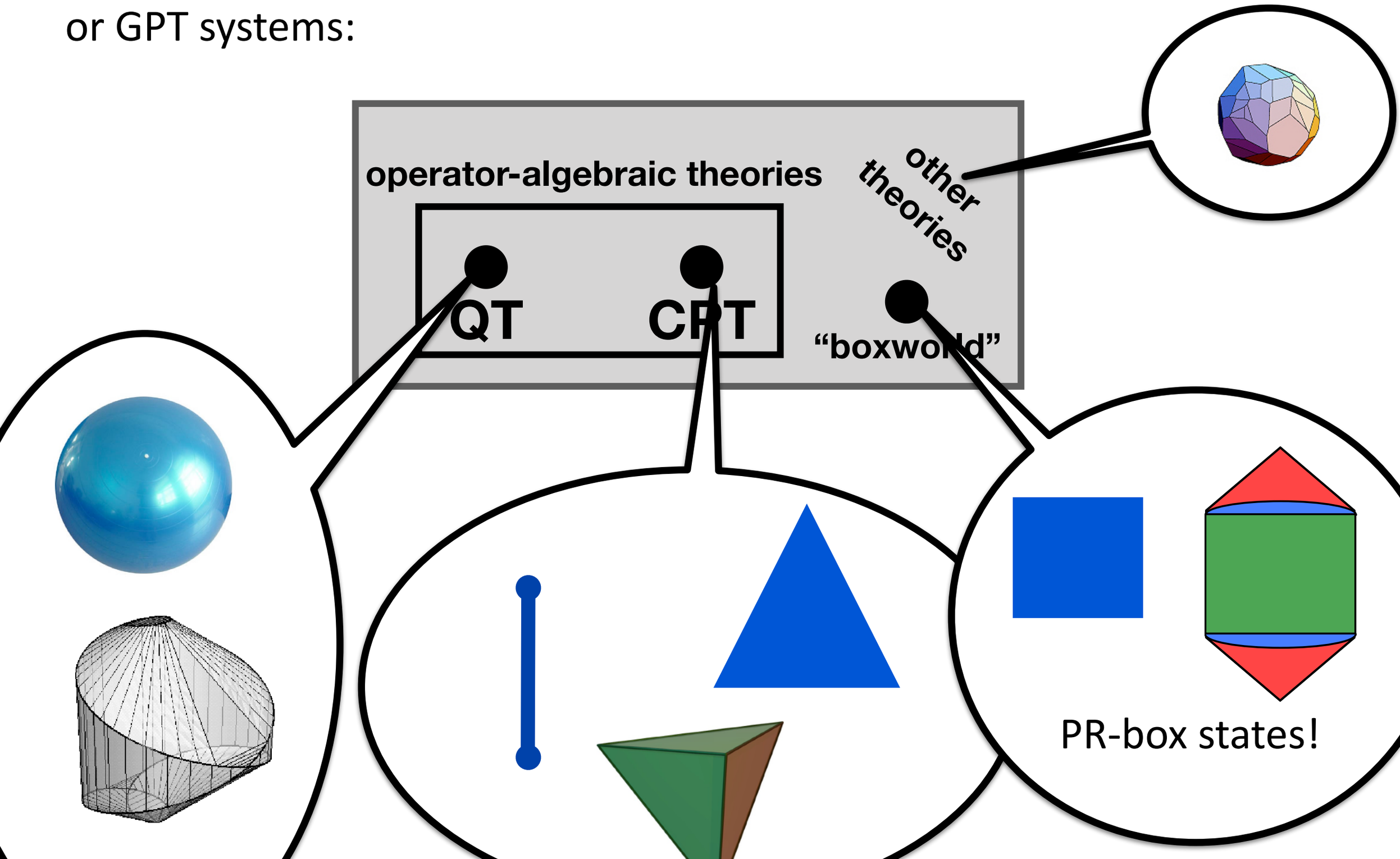
Generalized probabilistic theories

There is a large landscape of generalized probabilistic theories or GPT systems:



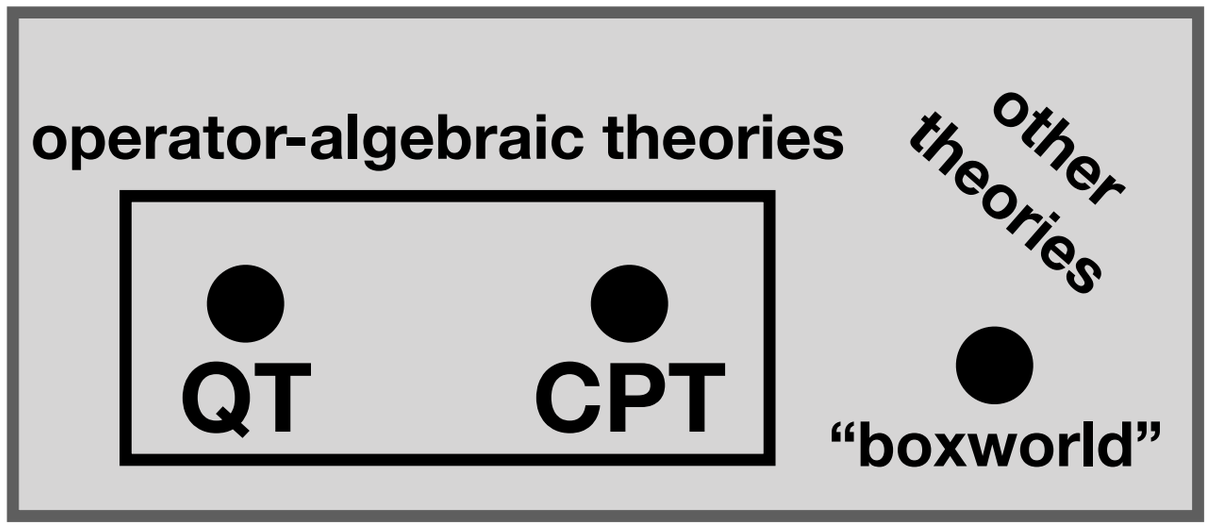
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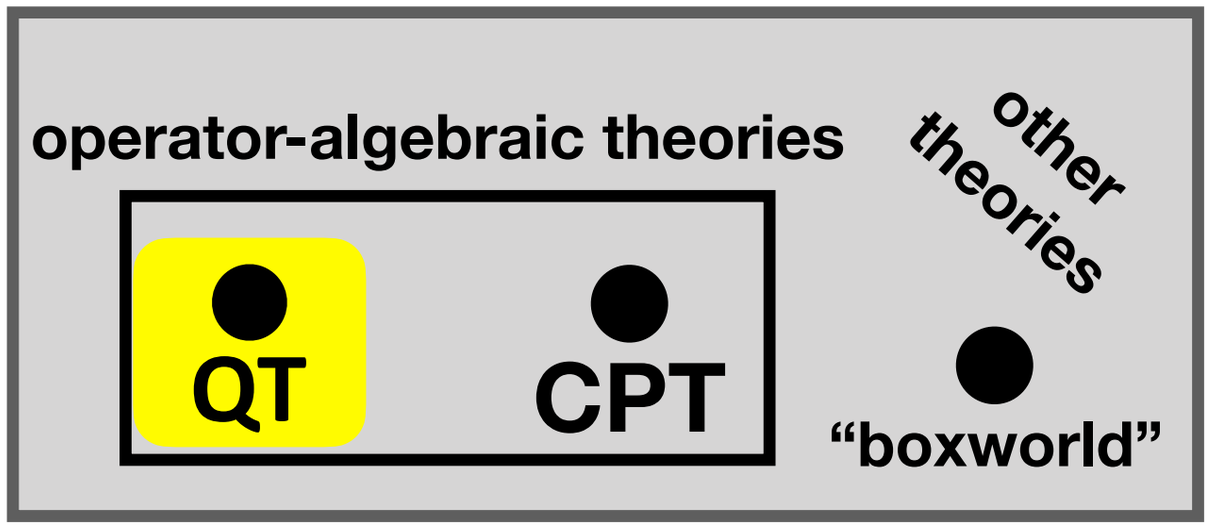
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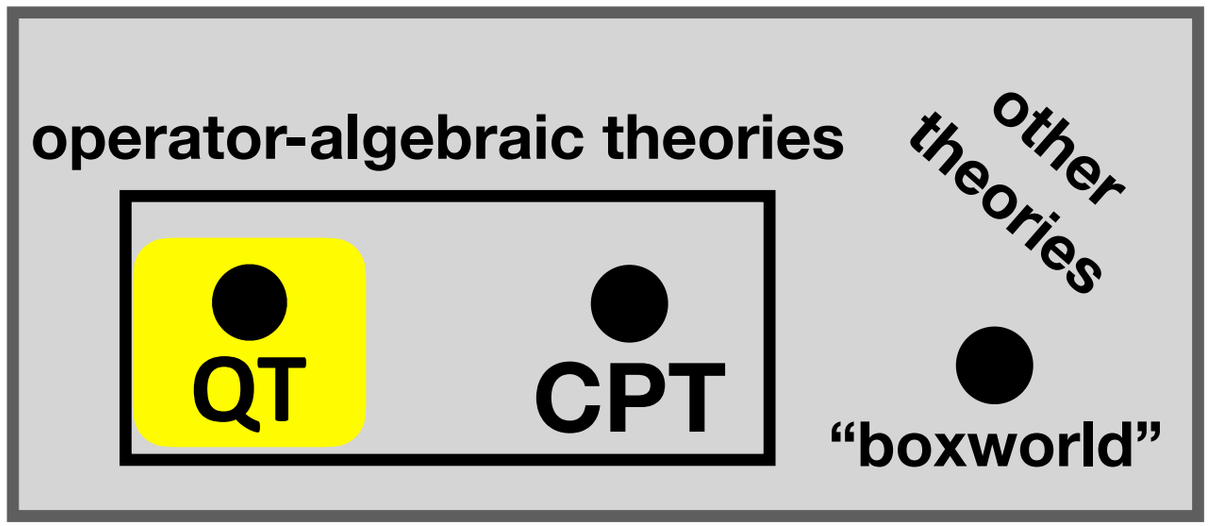
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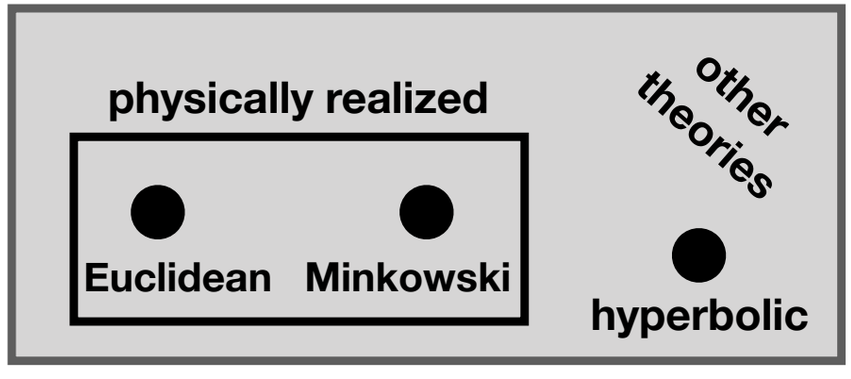
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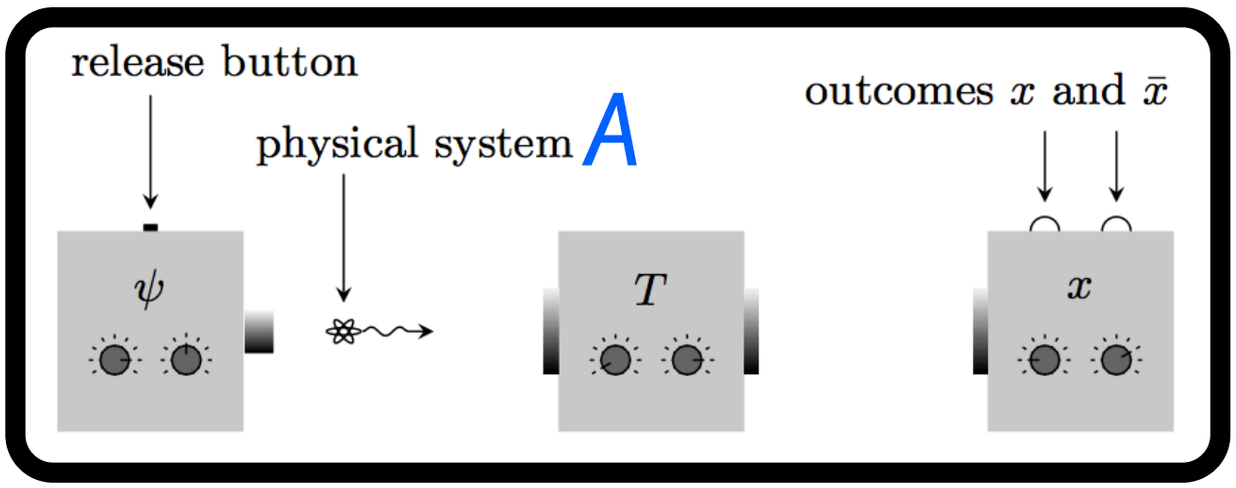


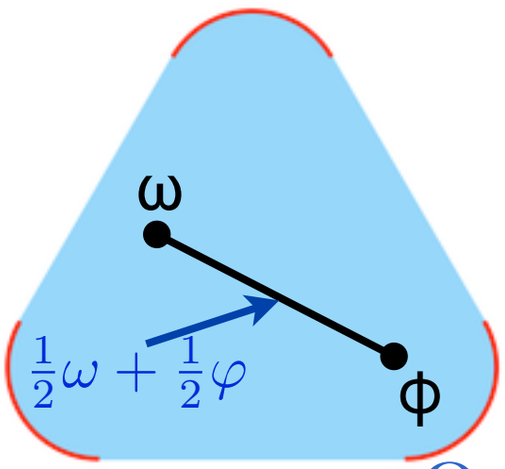
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Role model: Einstein's **Relativity Principle** and **Light Postulate** determine Minkowski spacetime.



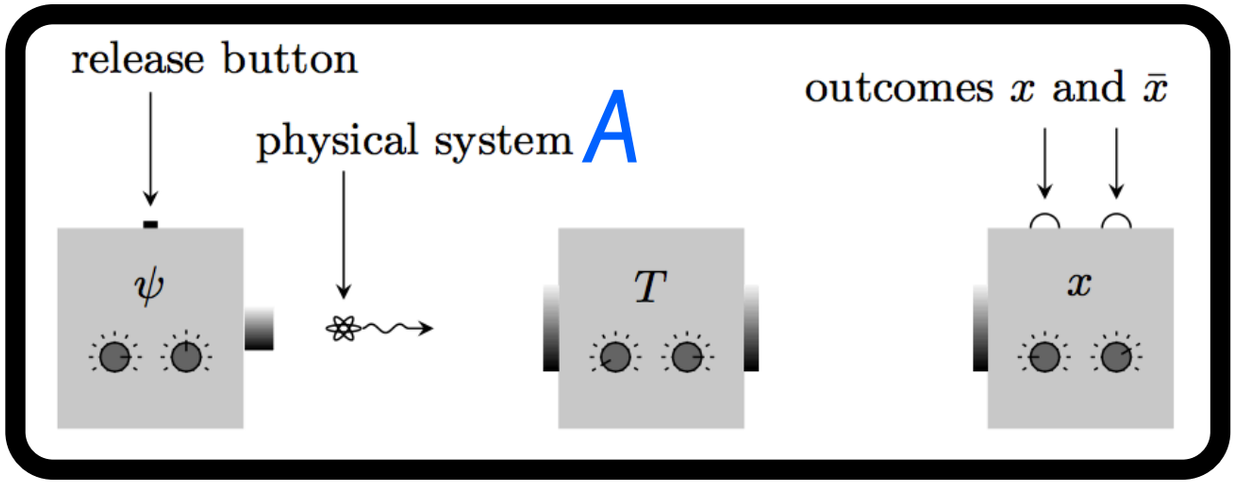
QT from three principles (“axioms”)



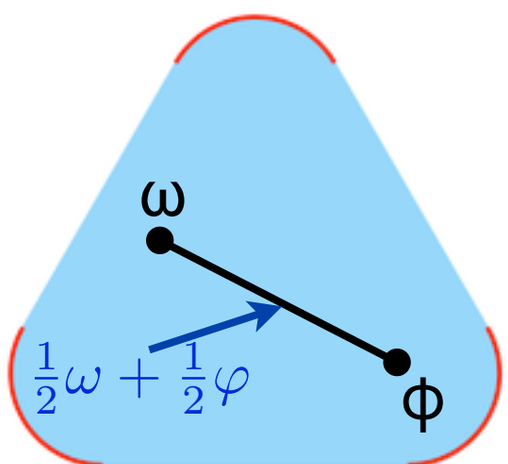


Normalized state space Ω_A

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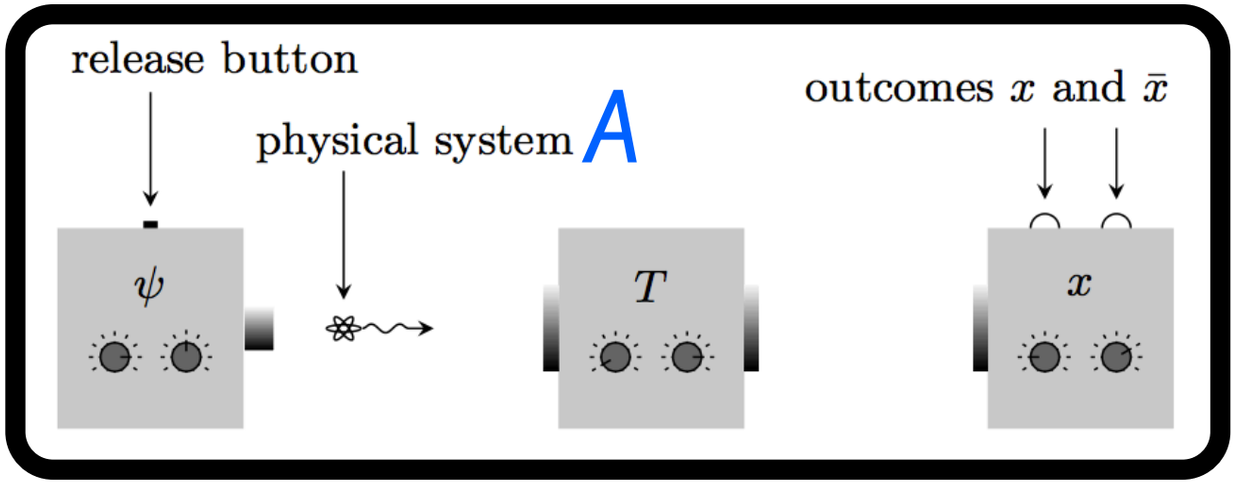


Transformations T map (unnormalized) states to states, and are linear.



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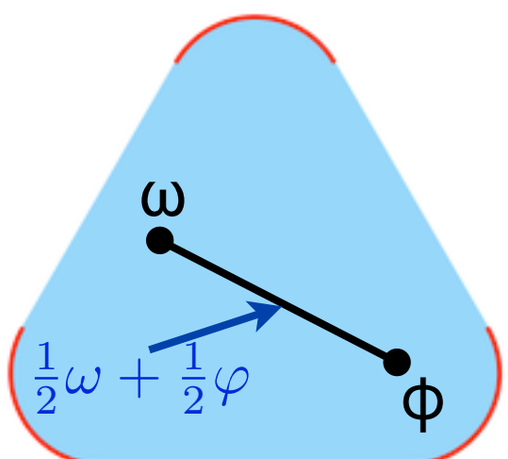
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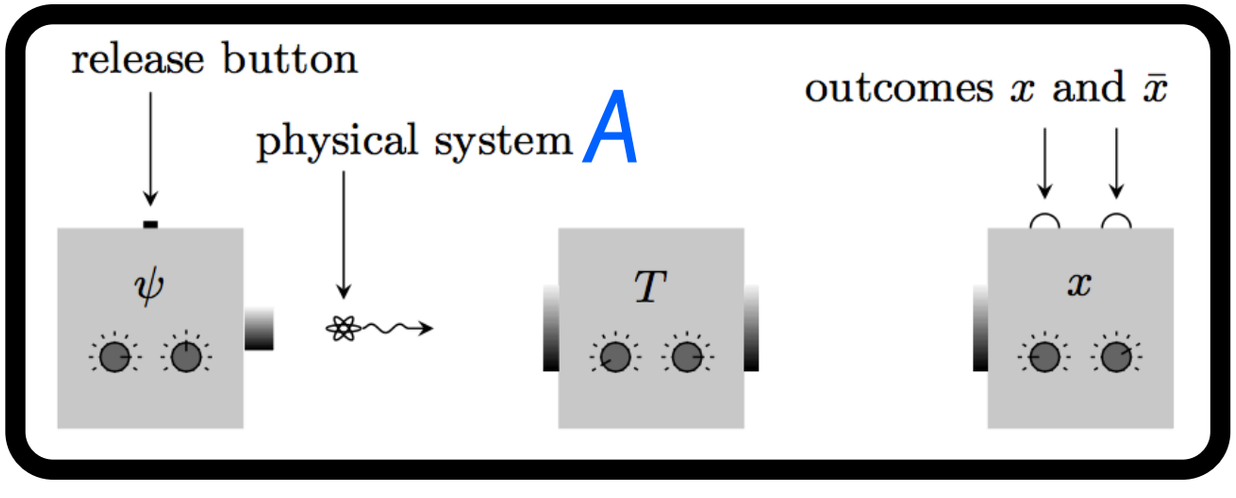
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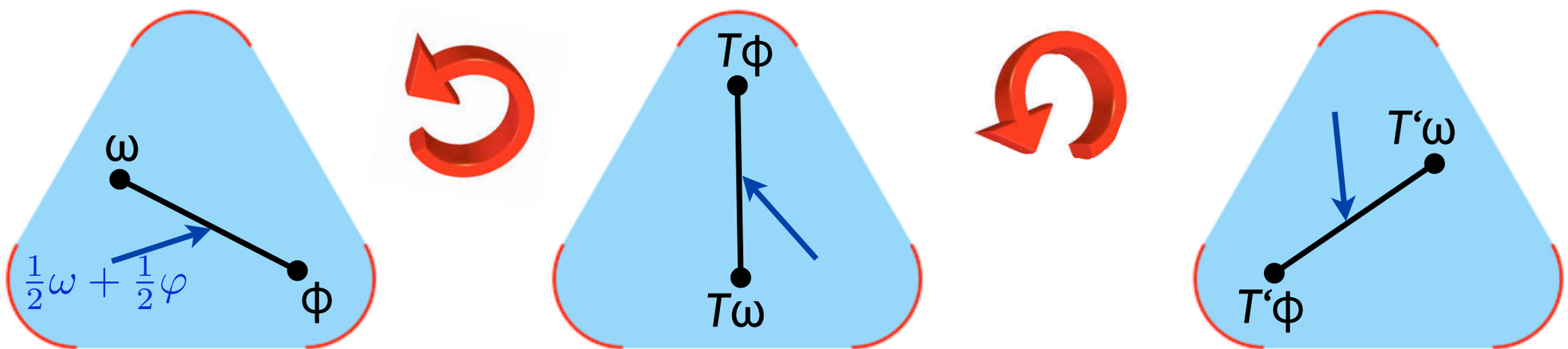
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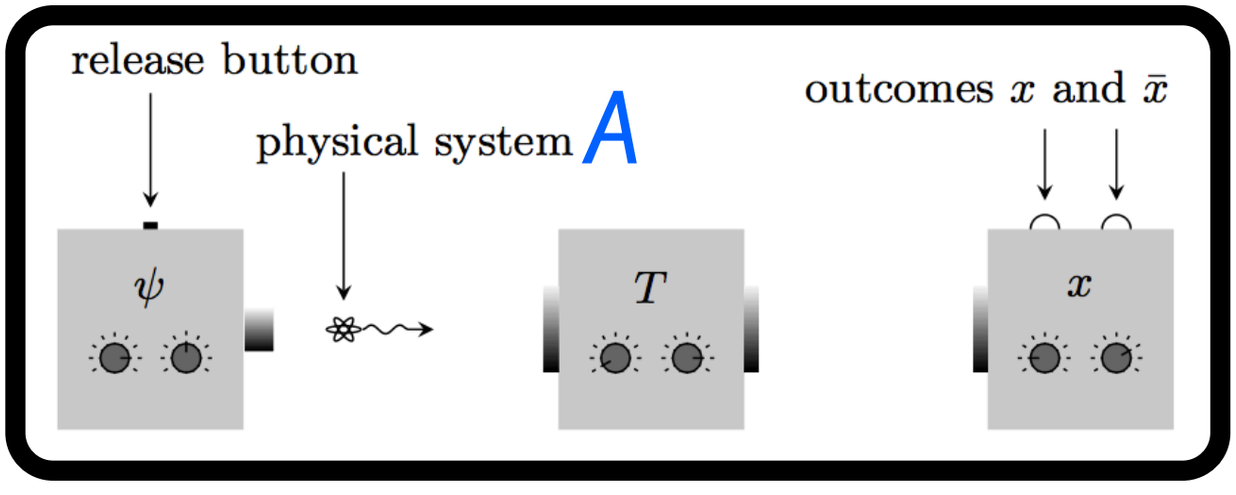
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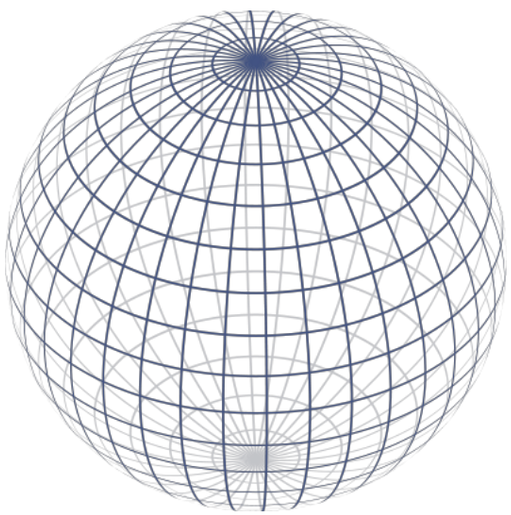


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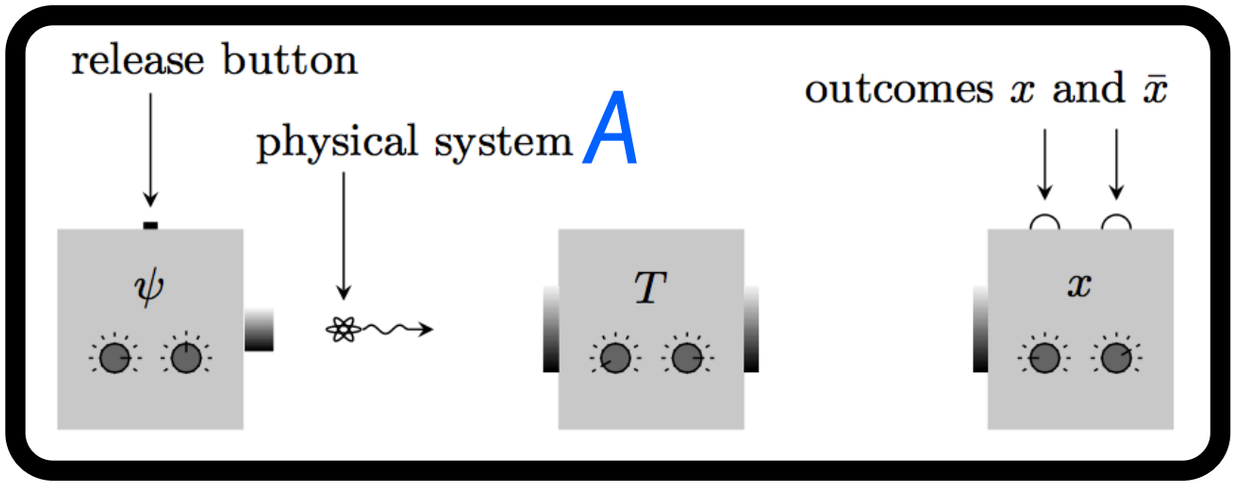
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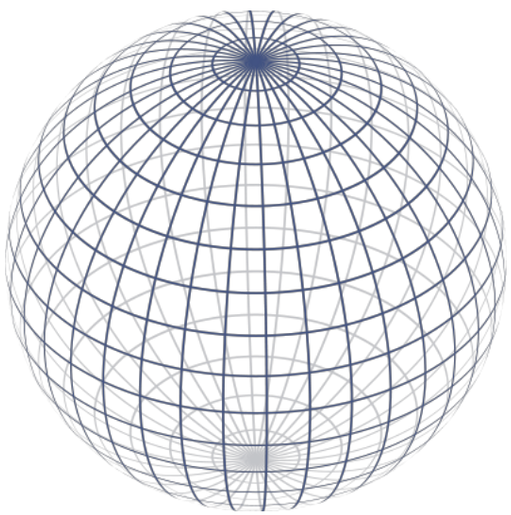
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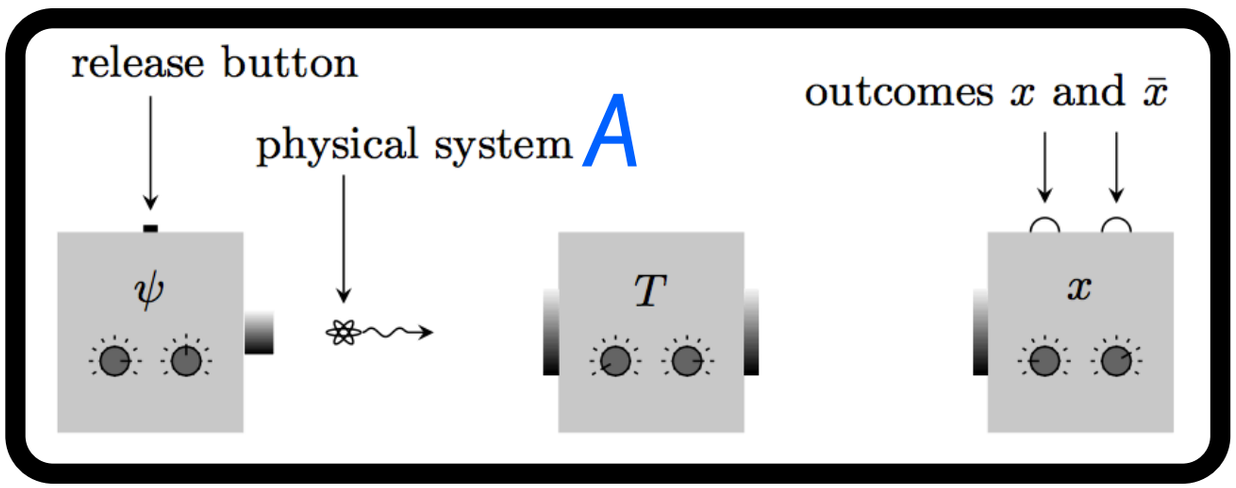
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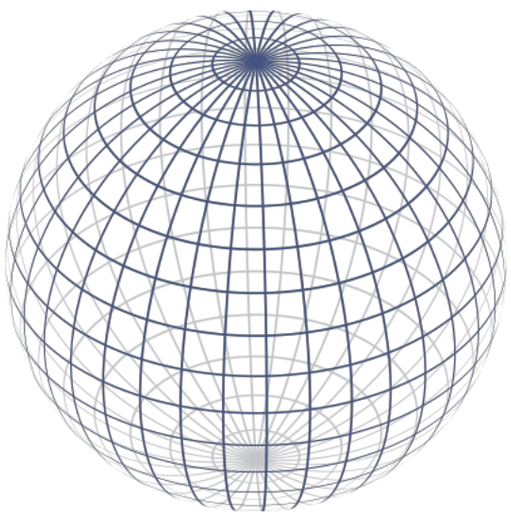
state space — reversible transformations

QT from three principles ("axioms")



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Axiom I (Reversibility):
If ϕ and ω are pure, then there is a reversible T with $T\phi=\omega$.



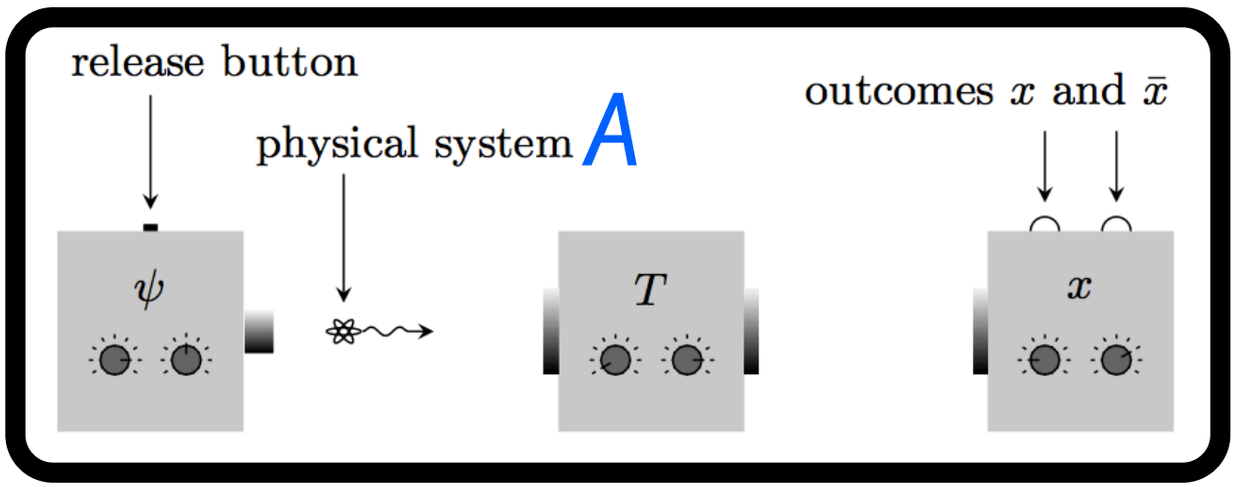
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state space — reversible transformations

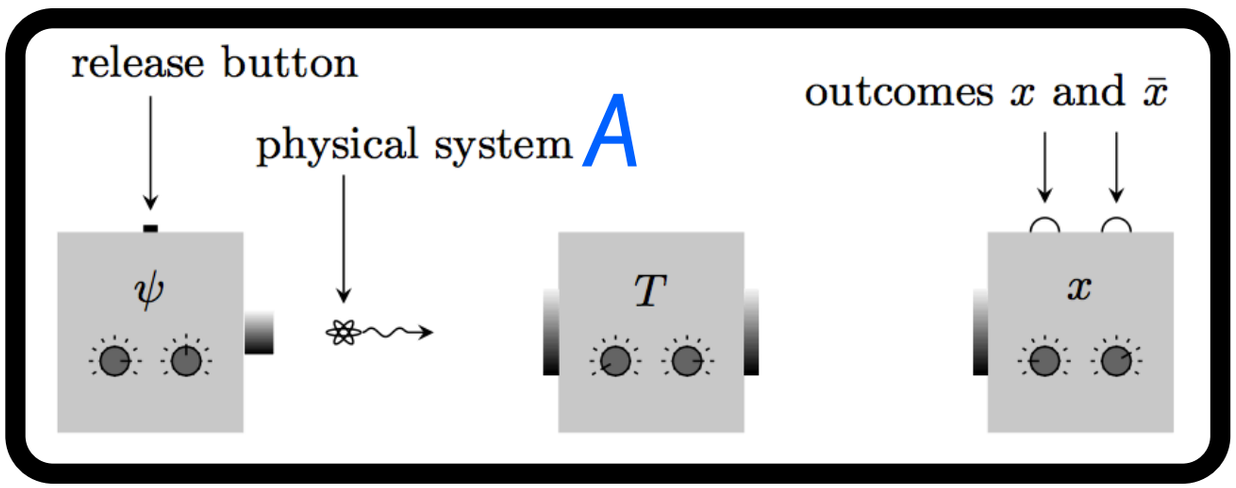
QT from three principles ("axioms")



Axiom I (Reversibility):
If ϕ and ω are pure, then there is a reversible T with $T\phi=\omega$.

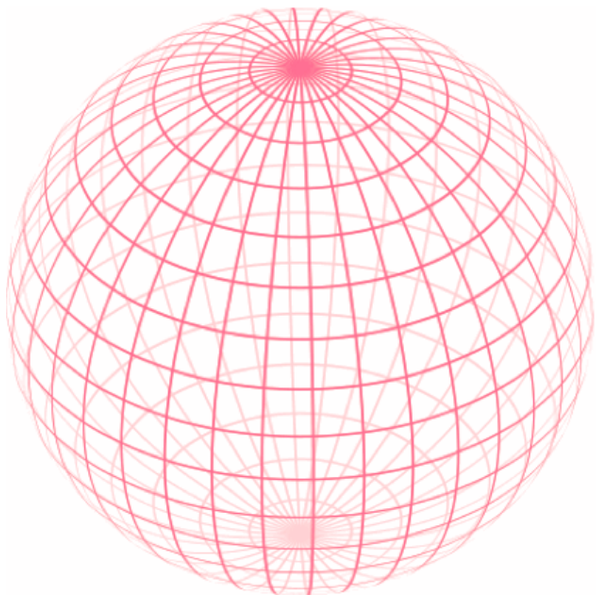
Enforces some symmetry in state space:

QT from three principles ("axioms")



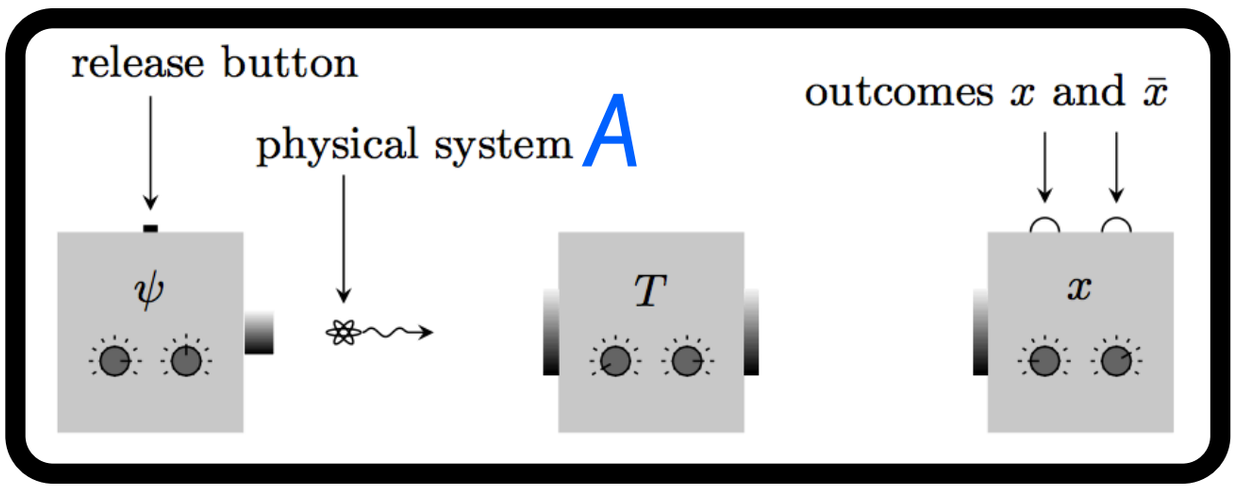
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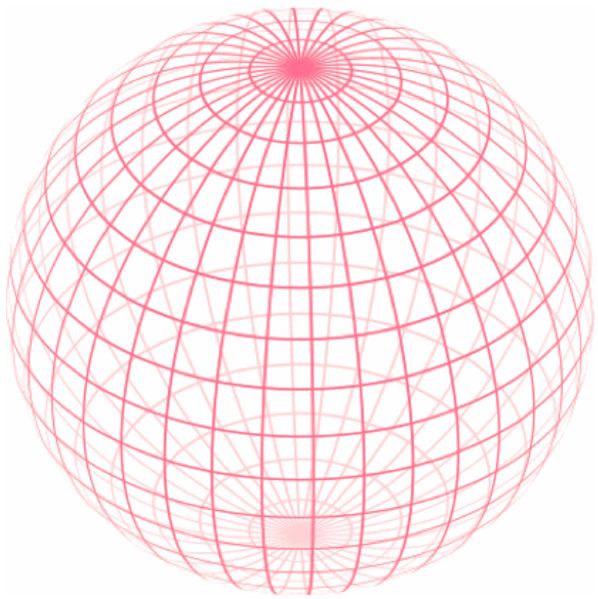


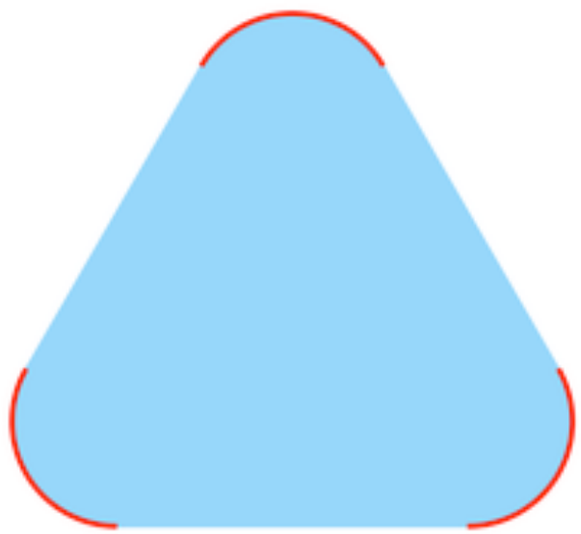
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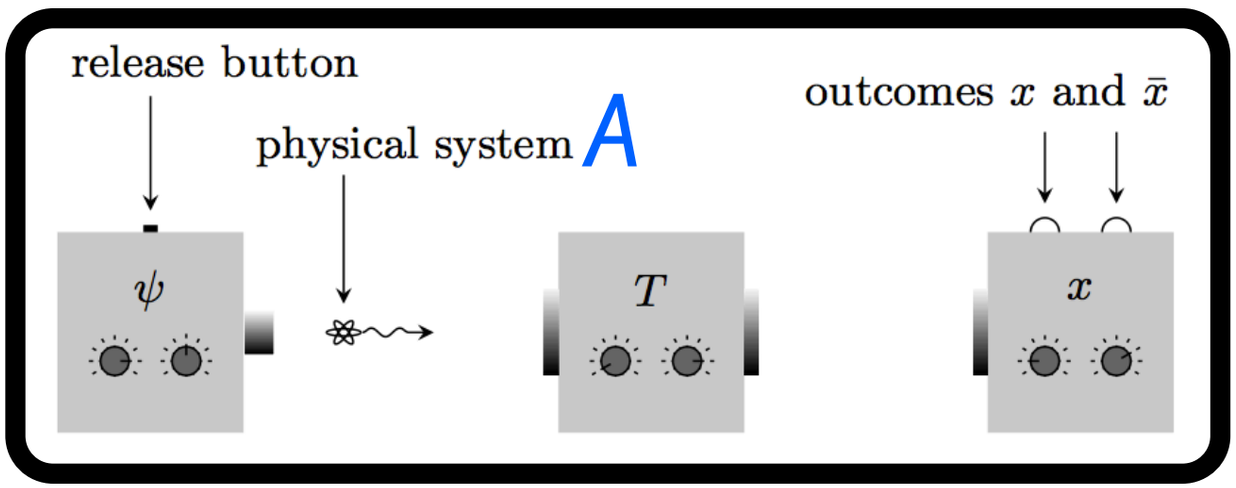
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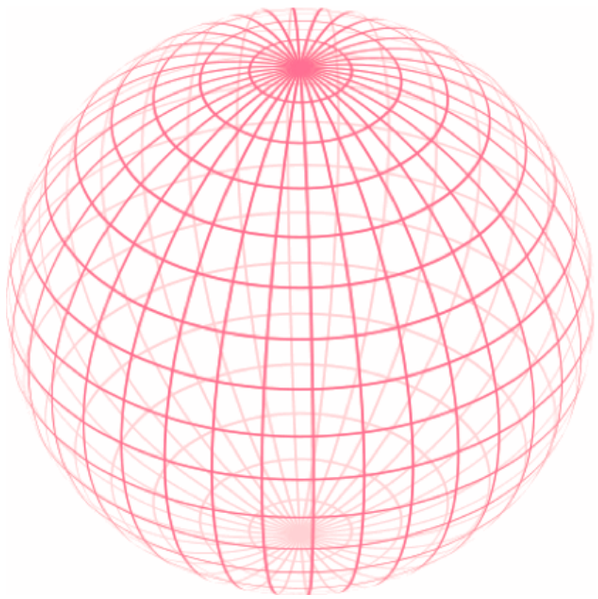


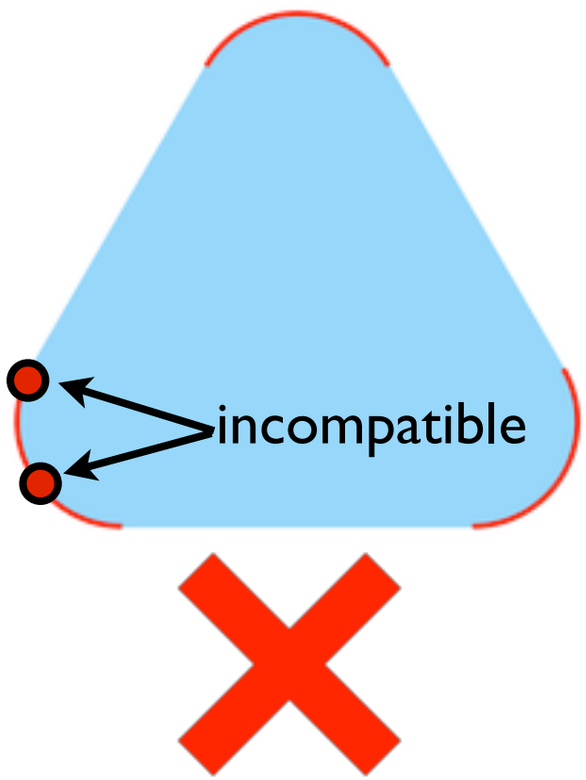
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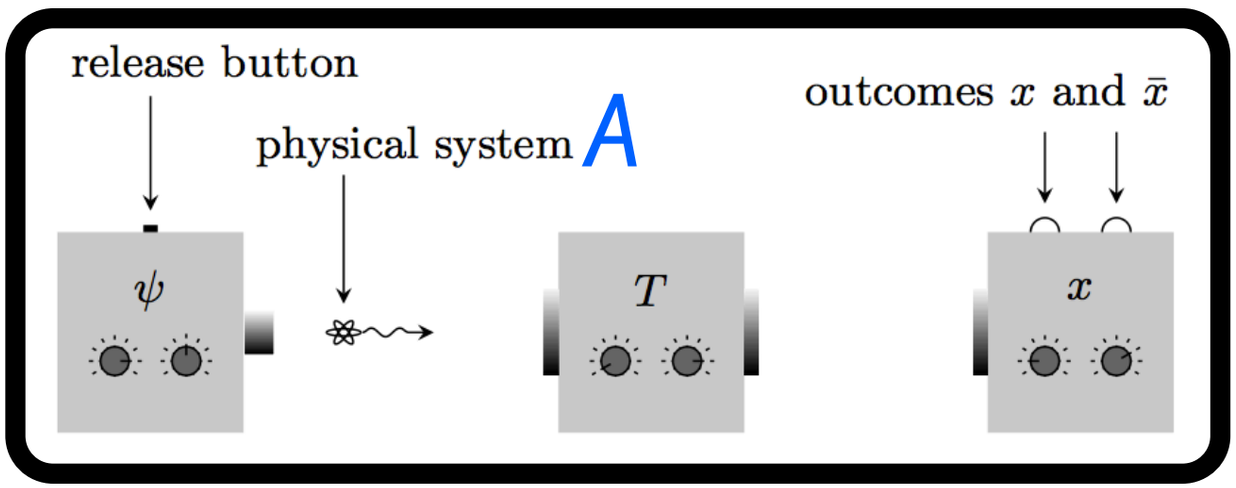
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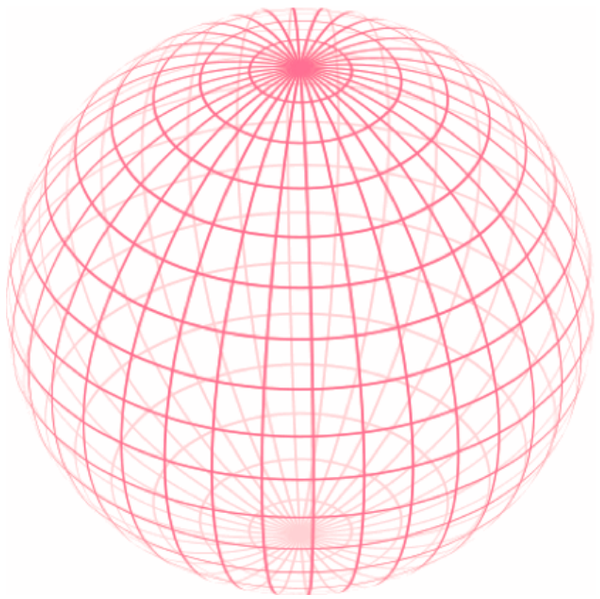


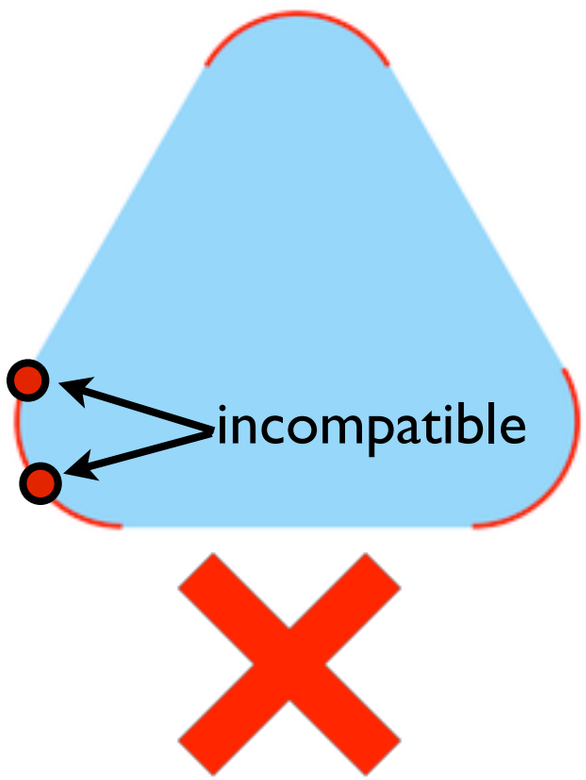
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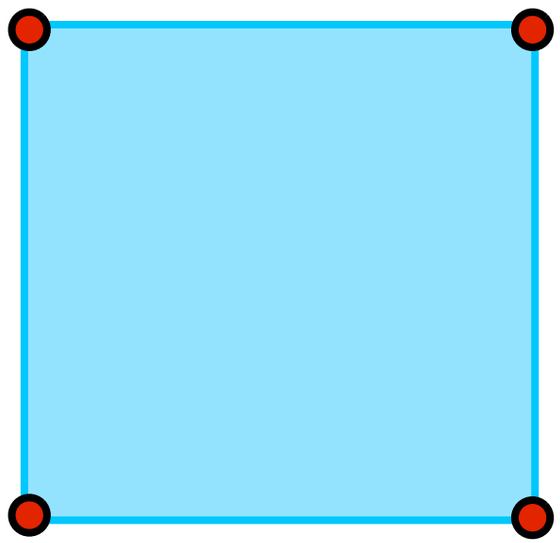


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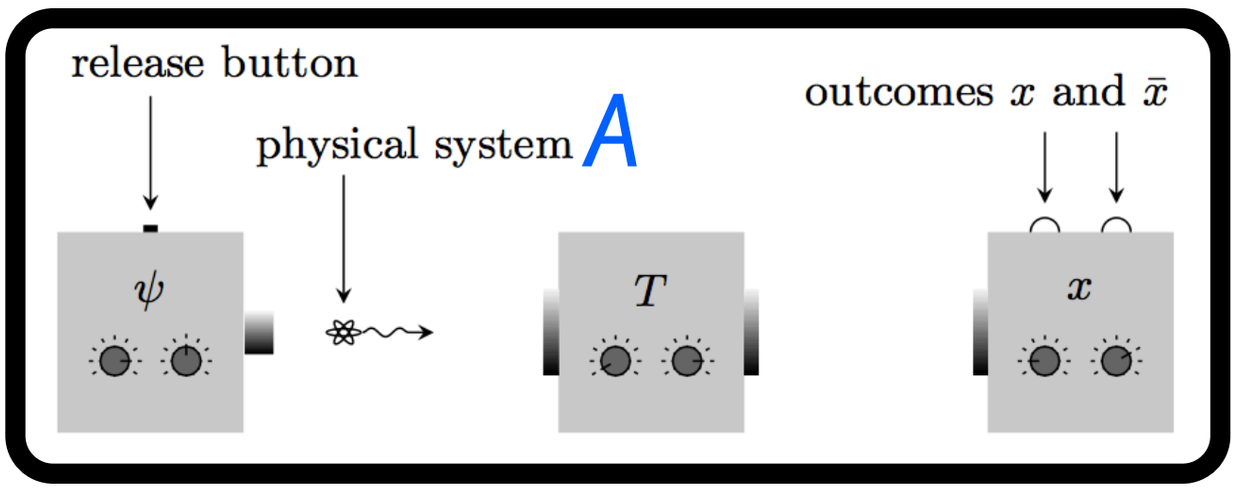
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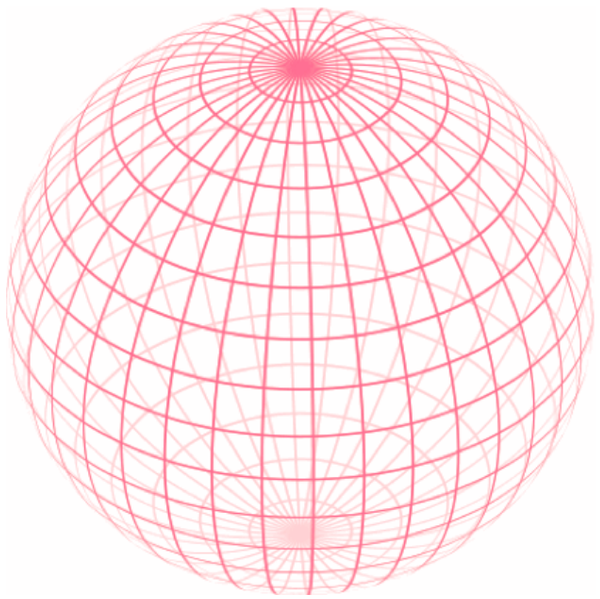


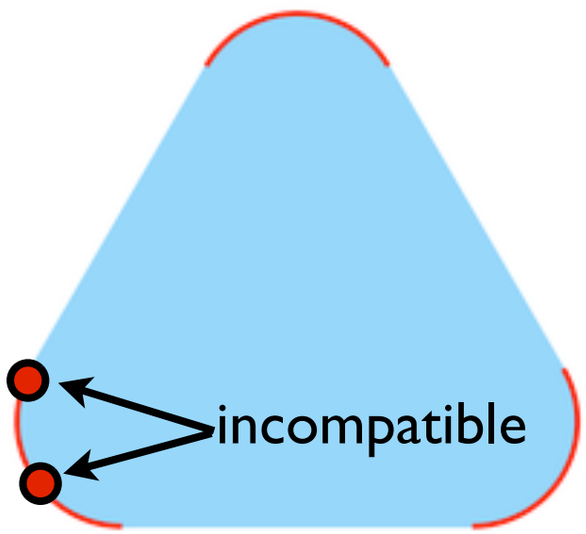
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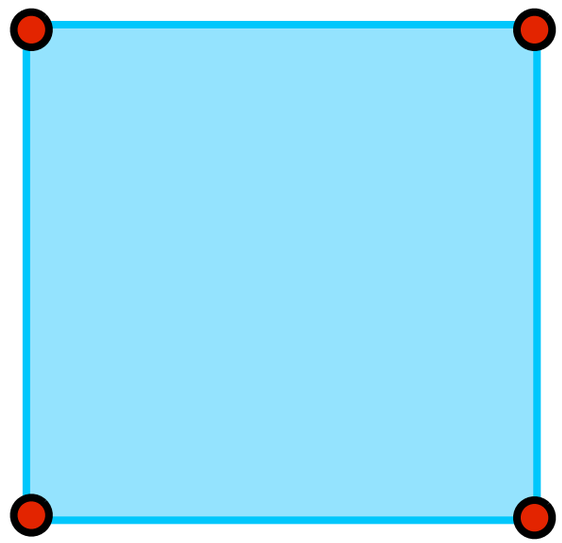


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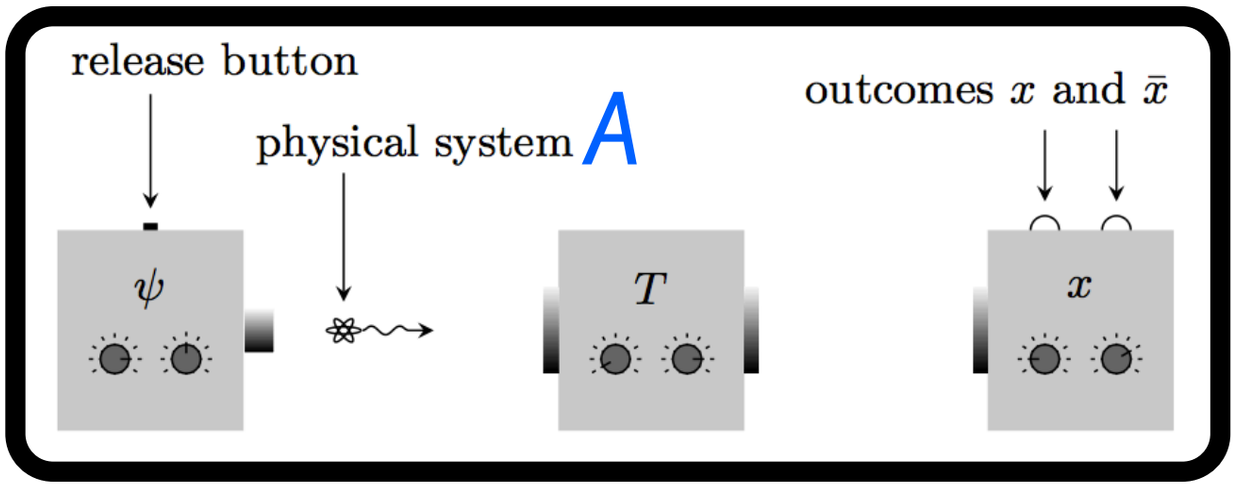
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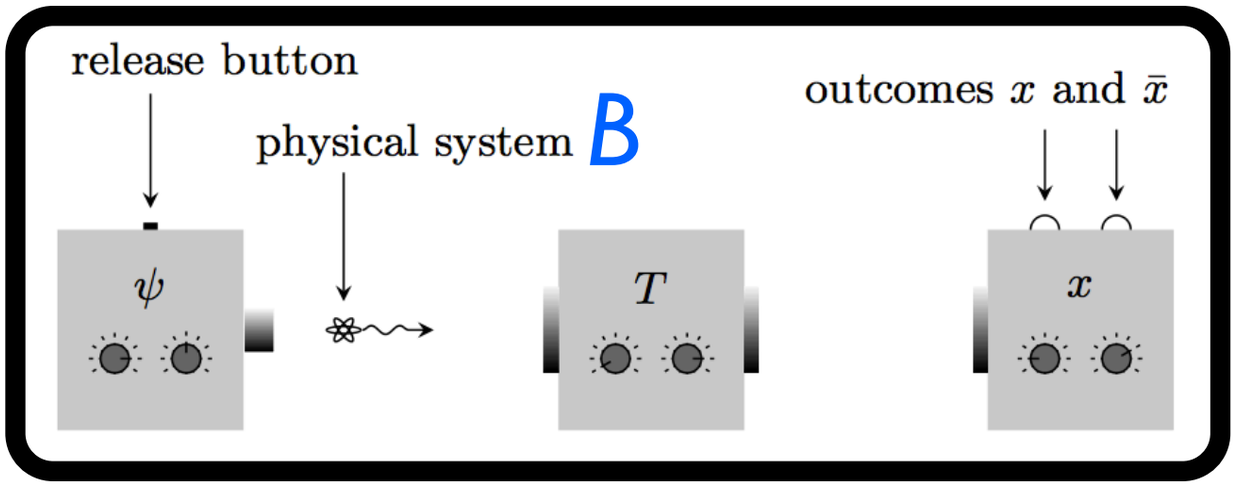




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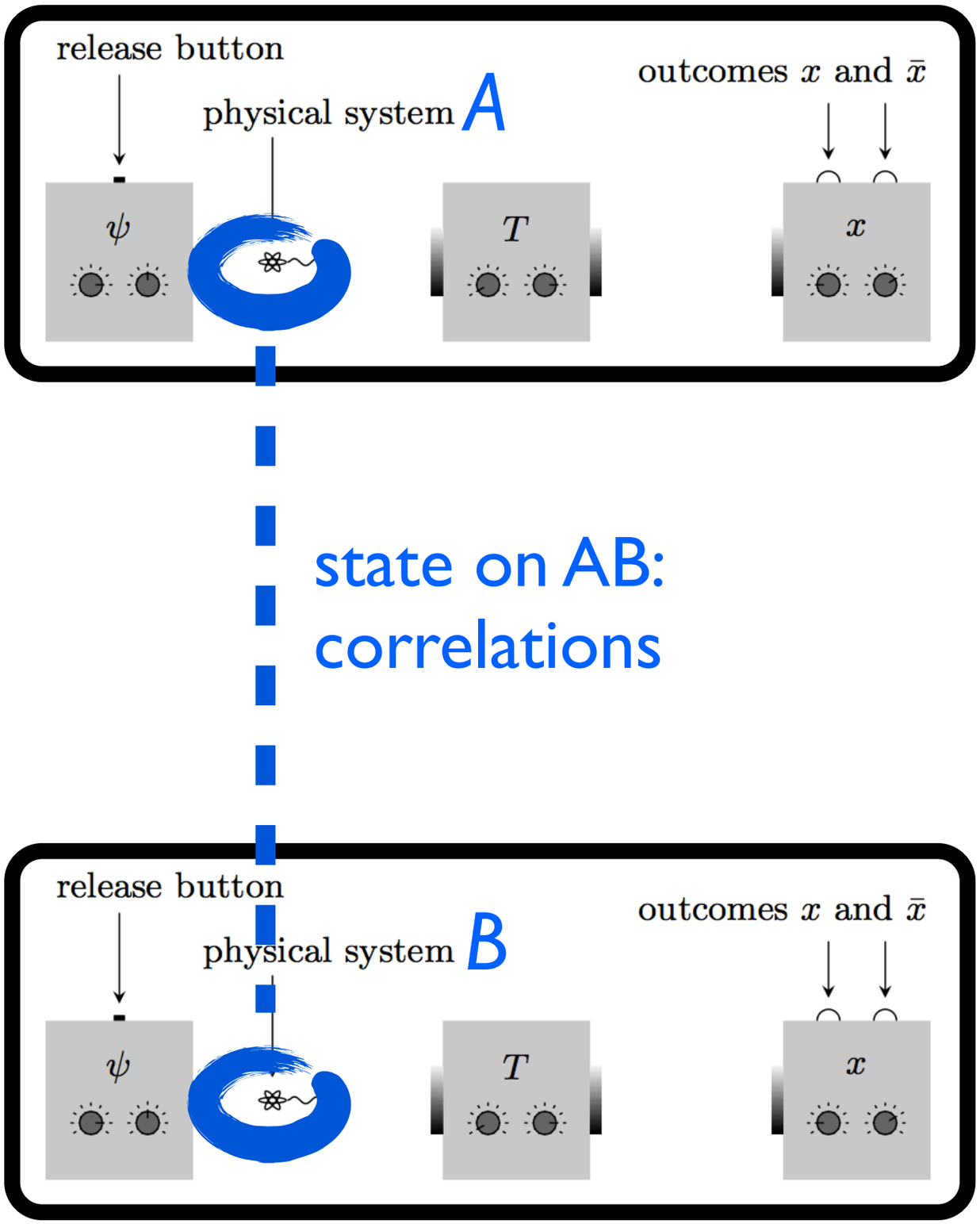








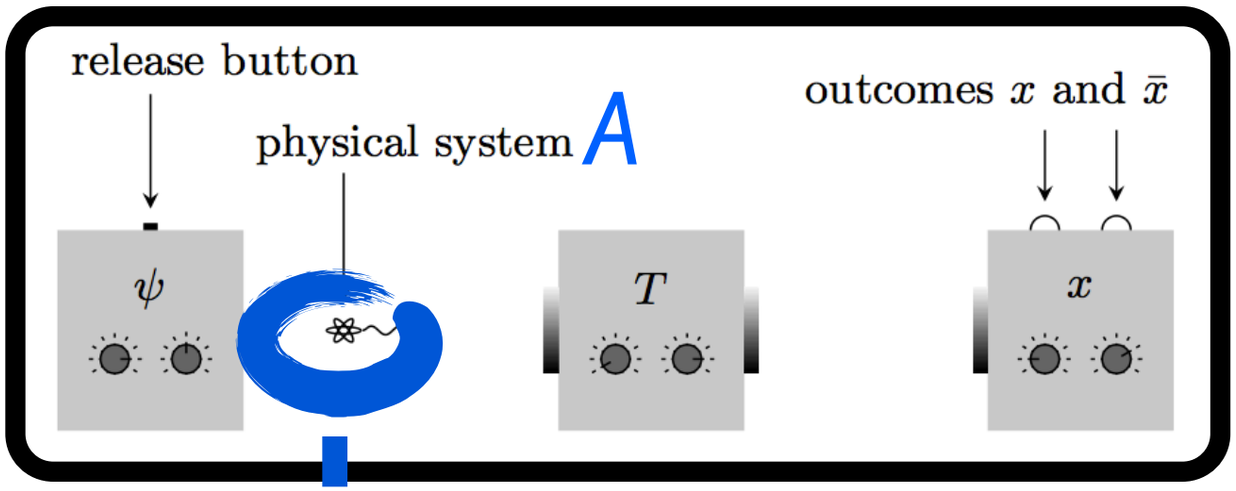
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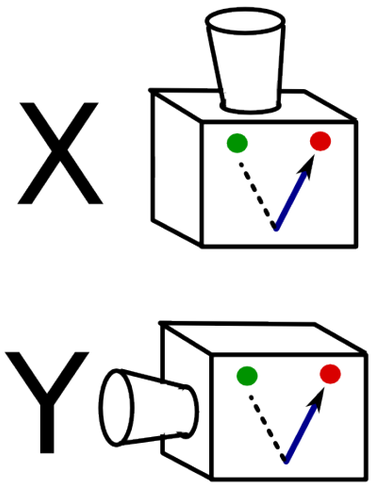




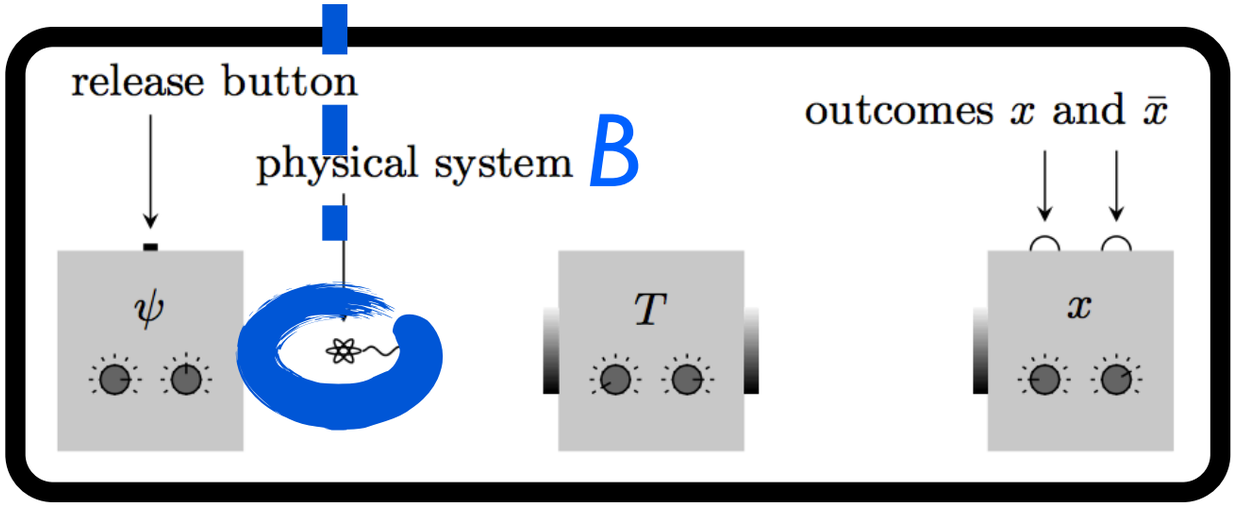
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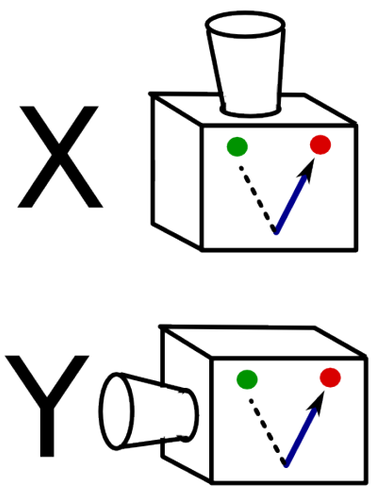




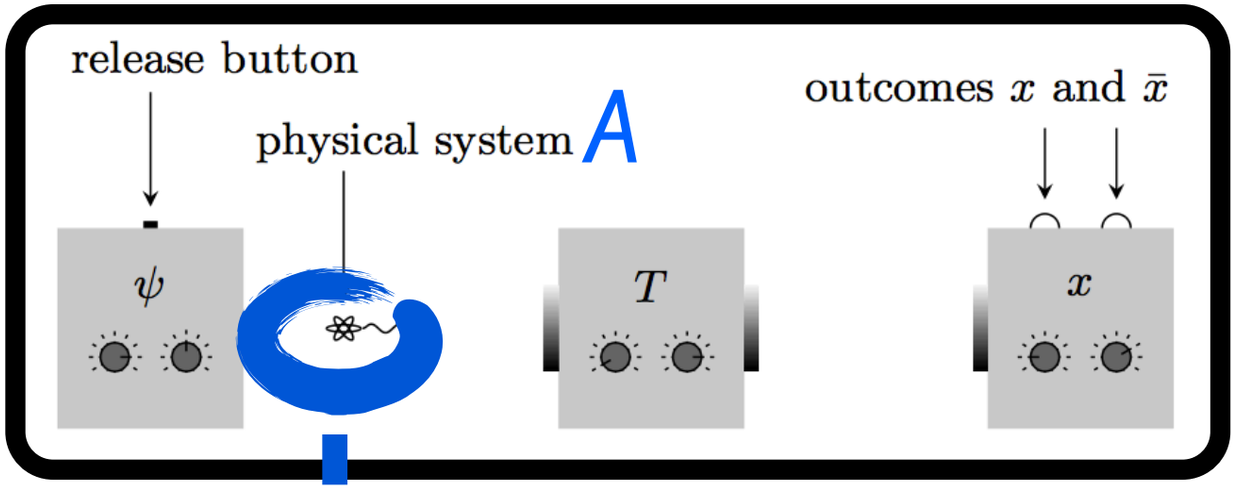
state on AB:
correlations



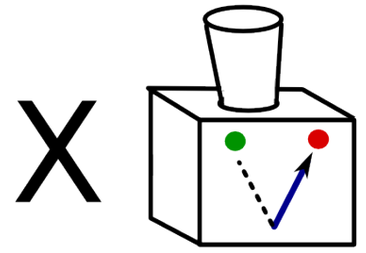




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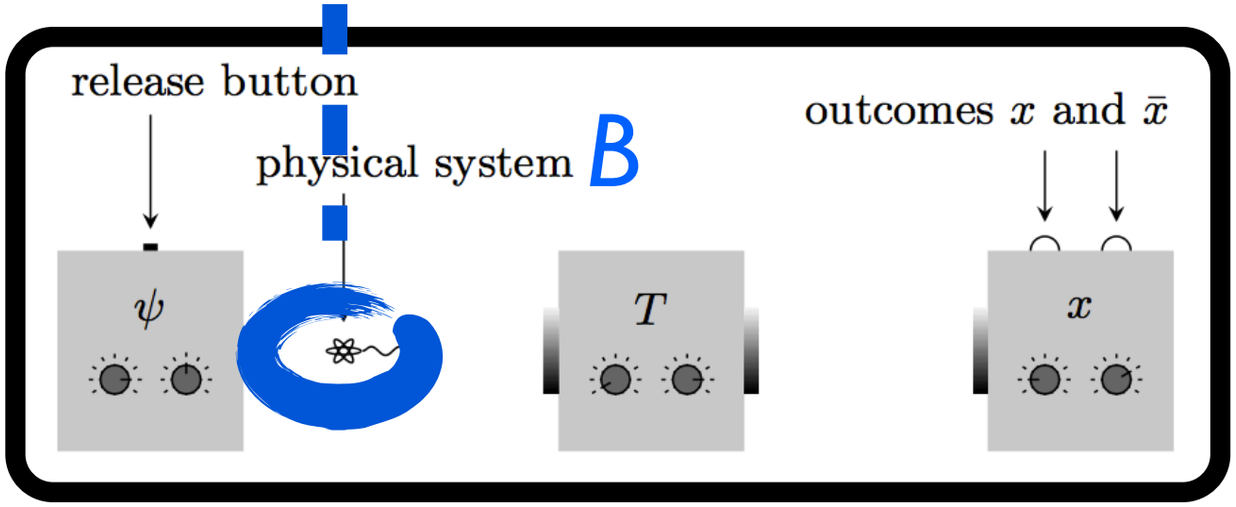




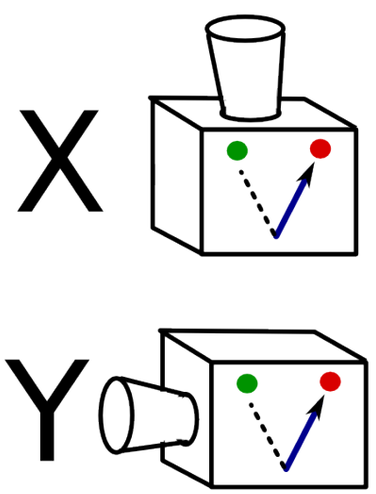


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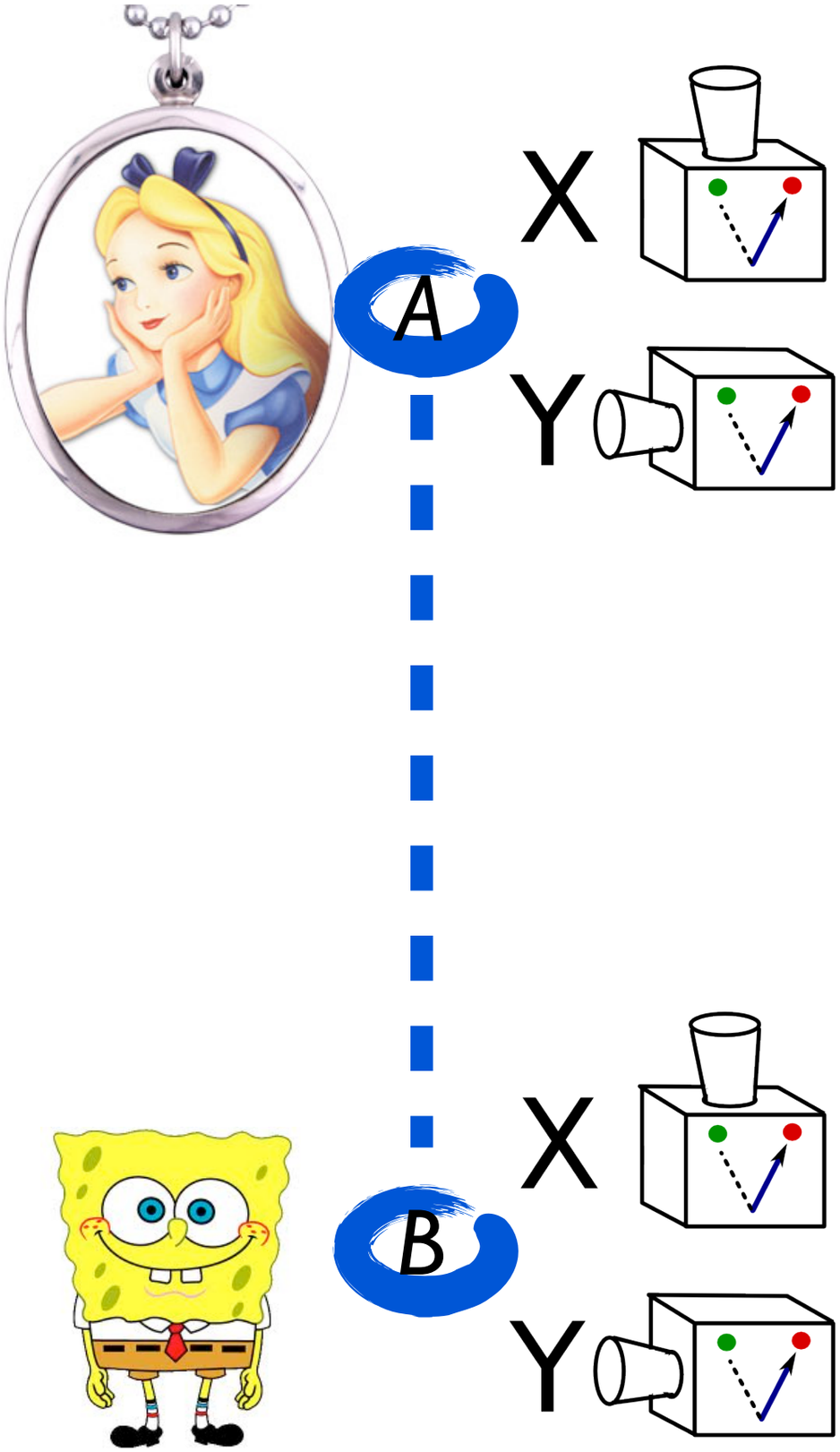
No-signalling condition:
Alice‘s probabilities do not depend on Bob‘s choice of measurement.



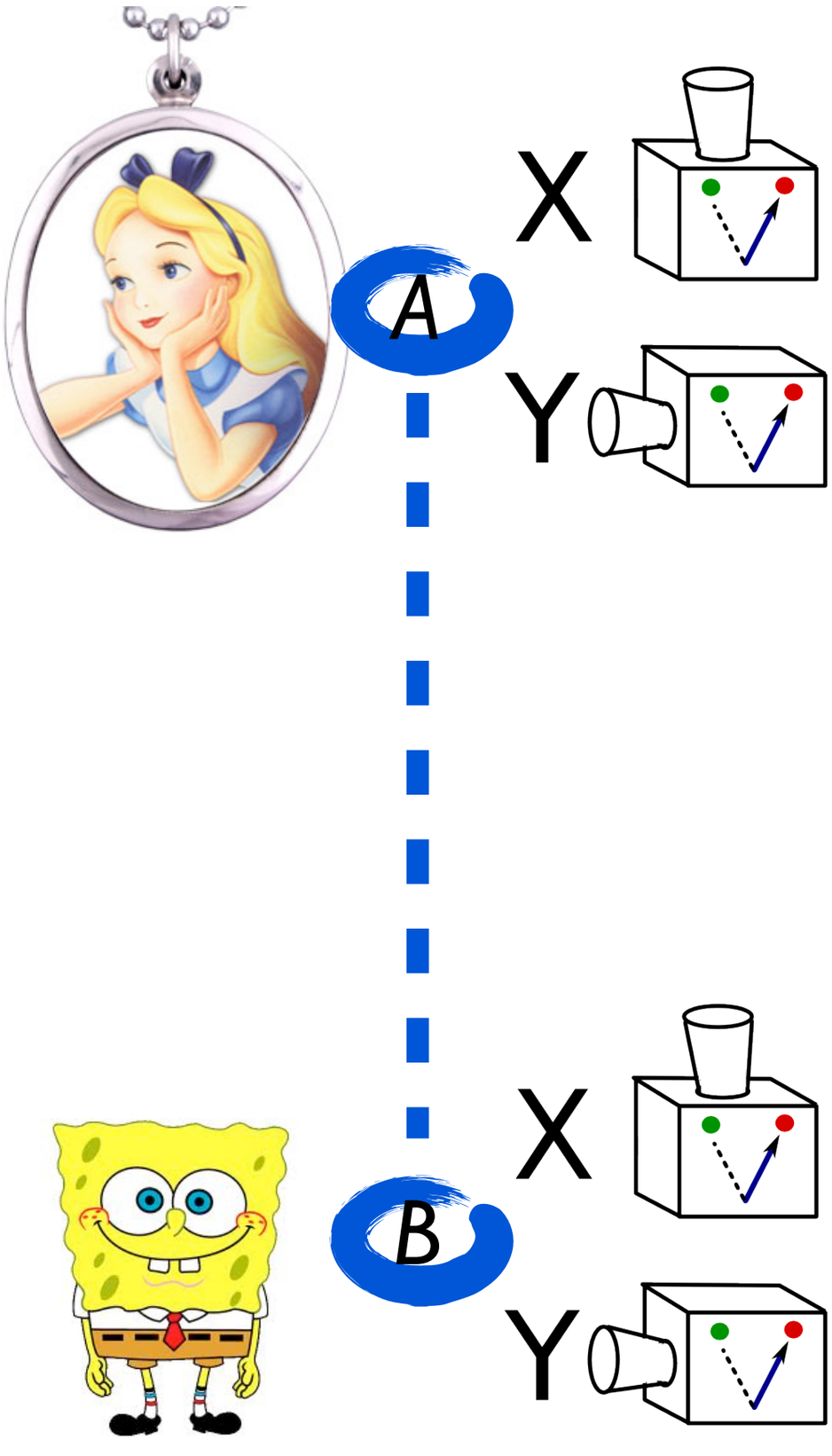




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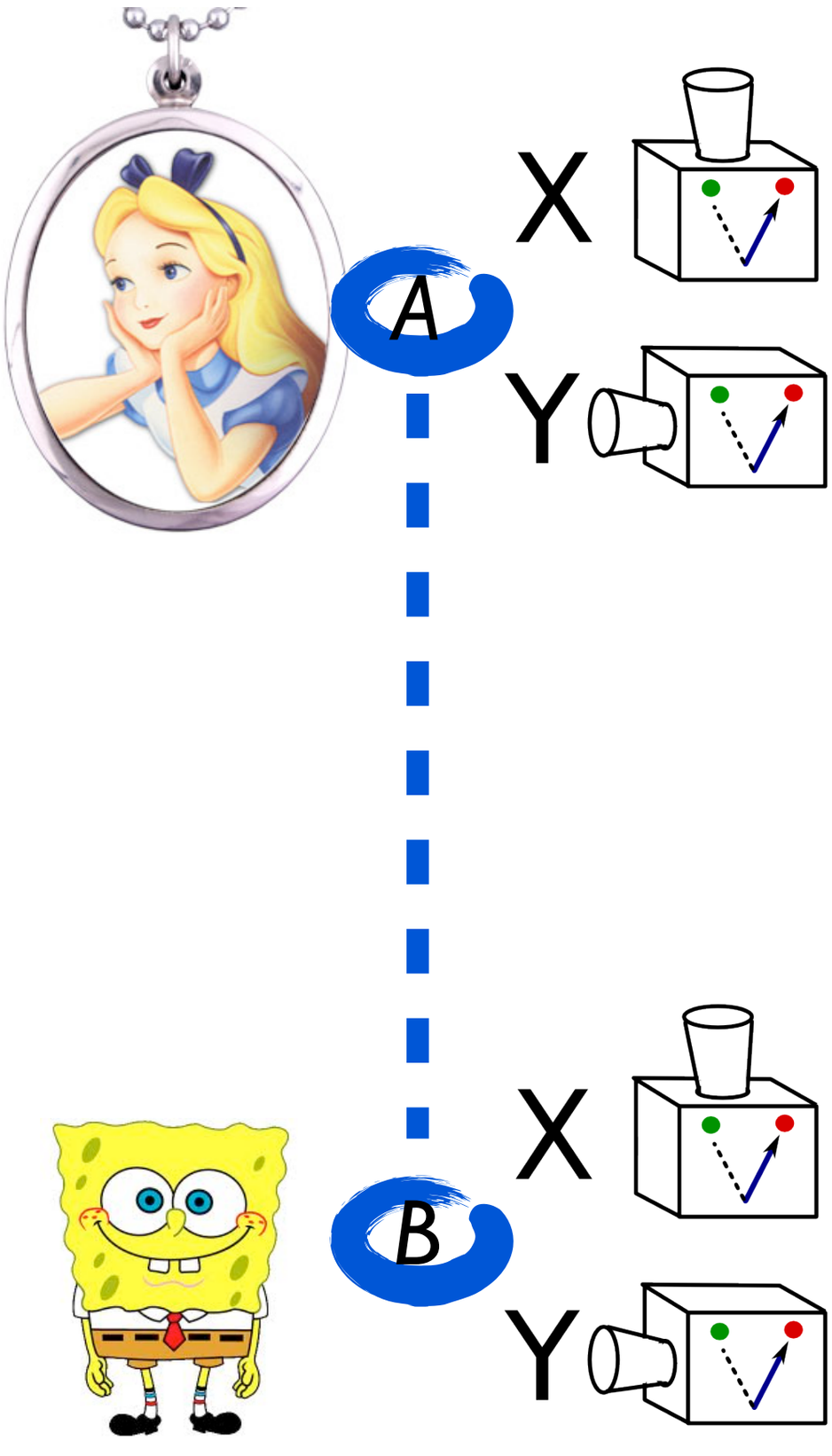


QT from three principles (“axioms”)



Axiom II: States on AB are uniquely determined by correlations of local measurements on A,B.

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= „Local tomography“:
No non-local measurements necessary.

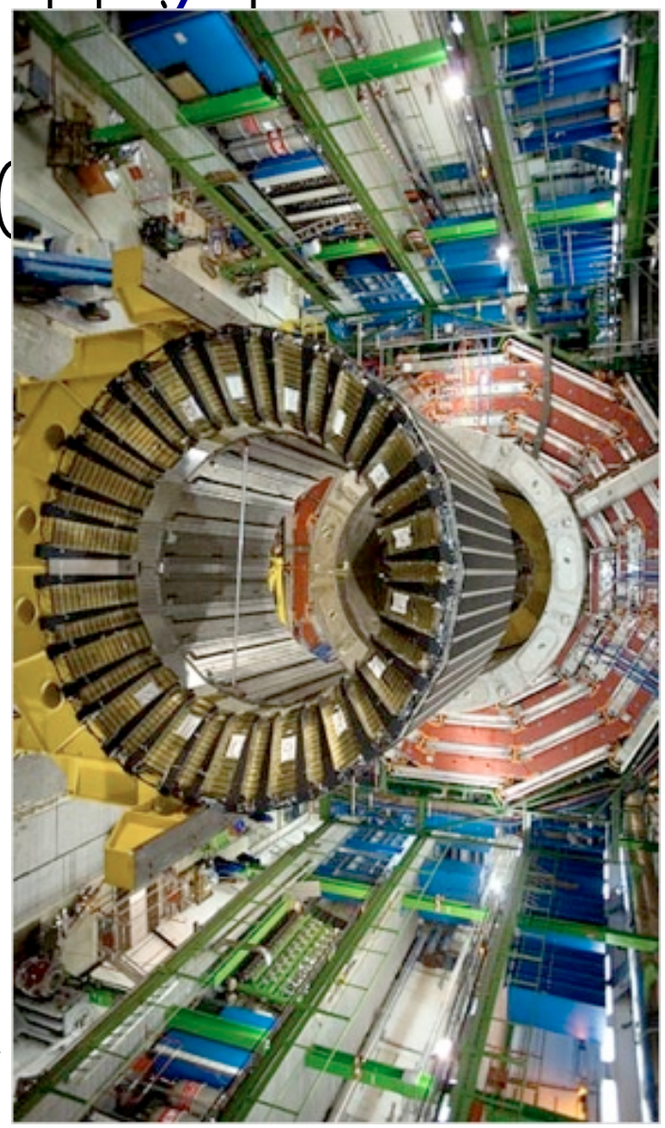
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A

X

Y



B

X

Y

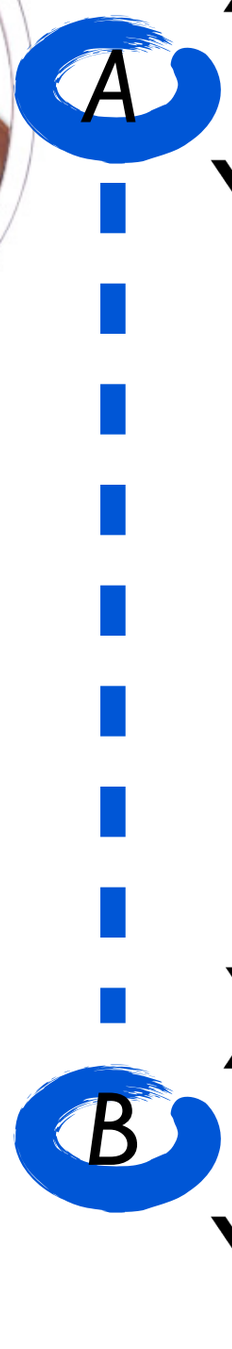


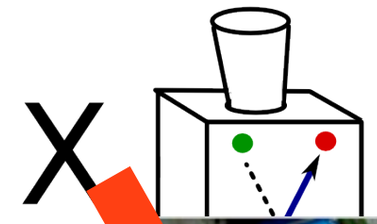
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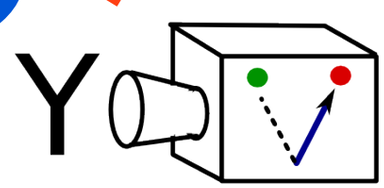










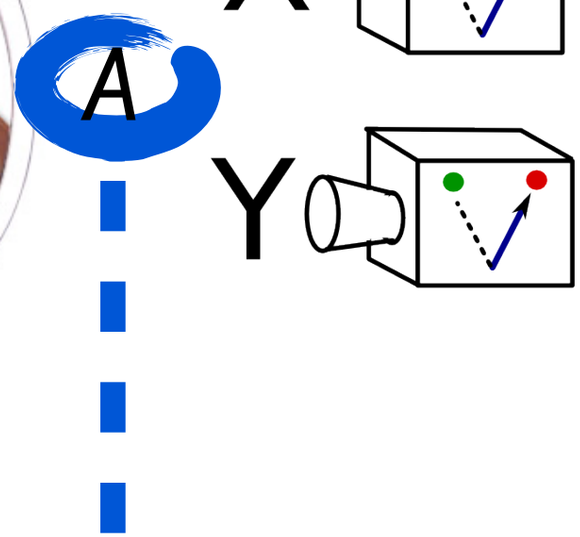


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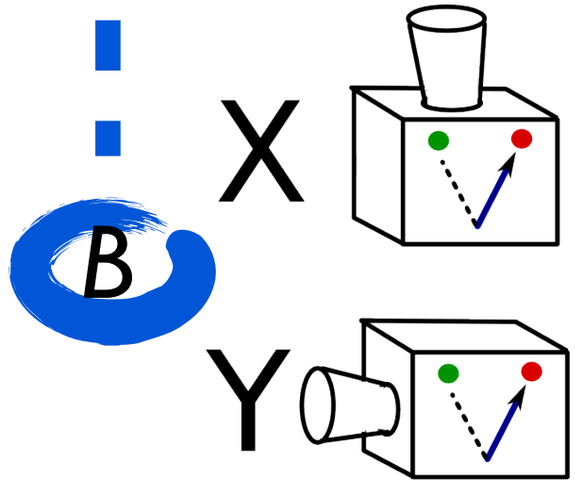




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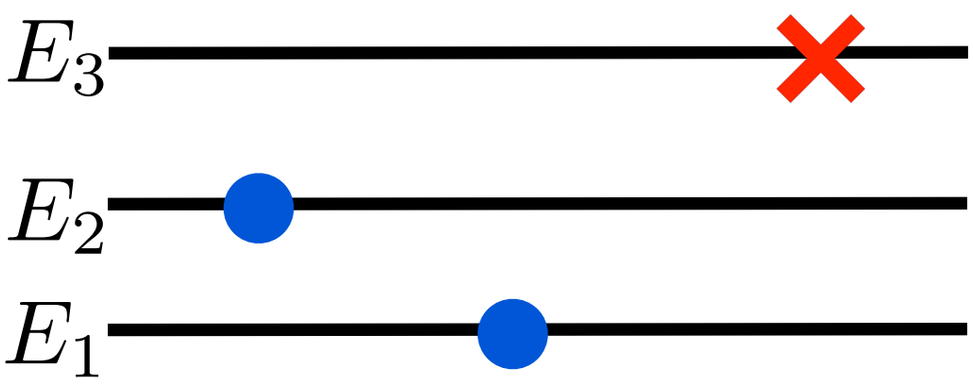




Global state space $\Omega_{AB} \subset A \otimes B$
but not uniquely fixed!

Axiom III: The Subspace Axiom

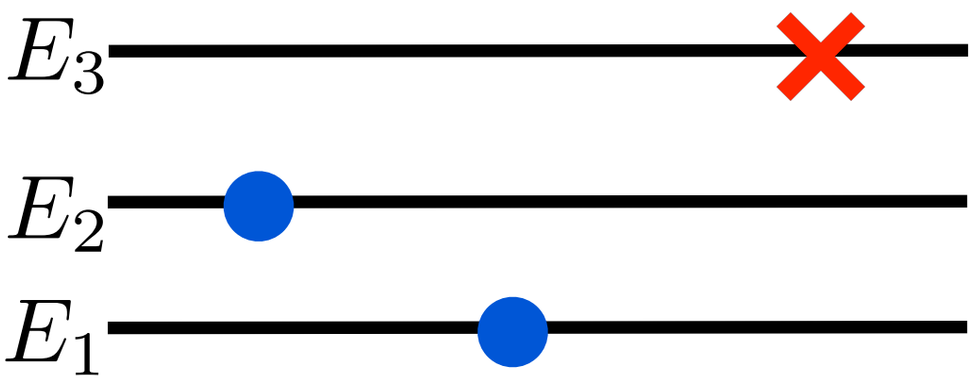
Some 3-level system:



Impossible to have system in 3rd level
⇒ find particle there with probab. 0

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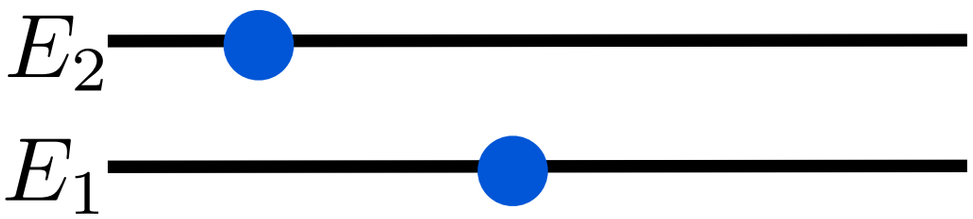
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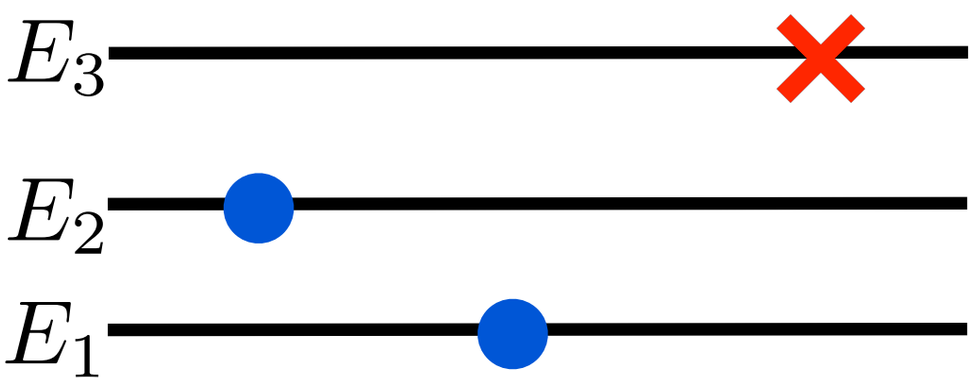
=



2-level system.

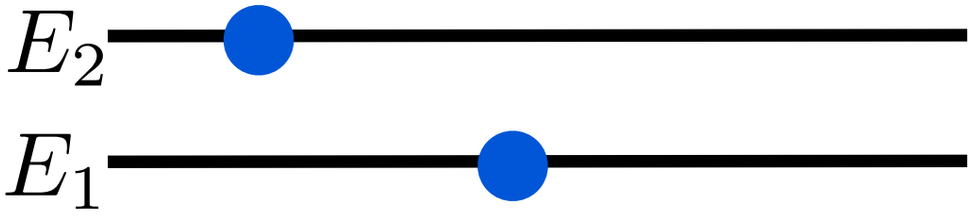
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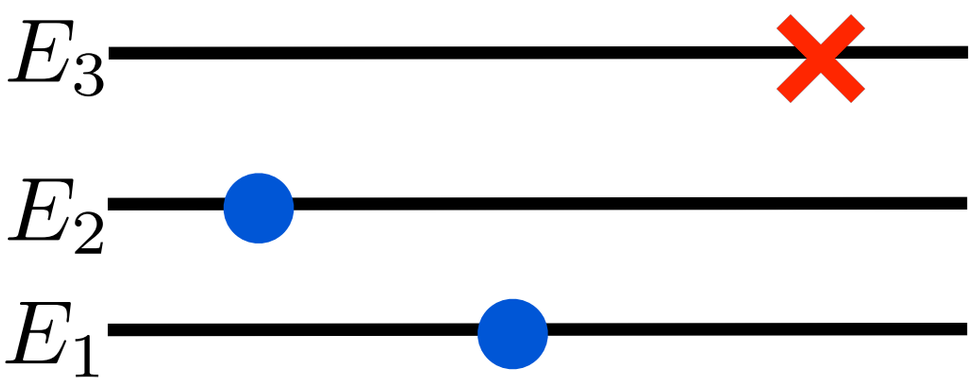
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QT: $\rho^{(3)} = \begin{pmatrix} \bullet & \bullet & 0 \\ \bullet & \bullet & 0 \\ 0 & 0 & 0 \end{pmatrix} \longrightarrow \rho^{(2)} = \begin{pmatrix} \bullet & \bullet \\ \bullet & \bullet \end{pmatrix}$

CPT: $P^{(3)} = (P_1, P_2, 0) \longrightarrow P^{(2)} = (P_1, P_2)$

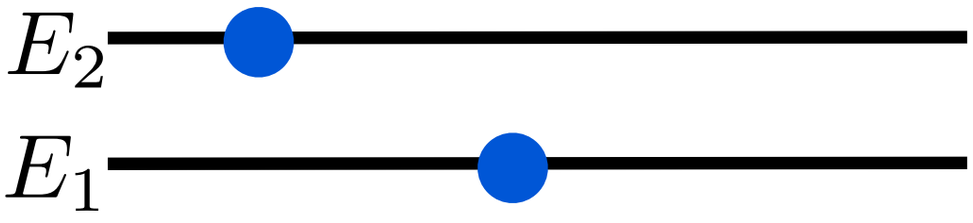
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Otherwise, physics would be affected by impossible counterfactuals.

Axiom III: The Subspace Axiom

Axiom III: Let Ω_N and Ω_{N-1} be systems with capacities *N* and *N-1*. If $(E_1, \ldots, E_N)$ is a complete measurement on Ω_N, then the set of states ω with $E_N(\omega) = 0$ is equivalent to Ω_{N-1}.

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Capacity N of Ω = maximal # of perfectly distinguishable states.

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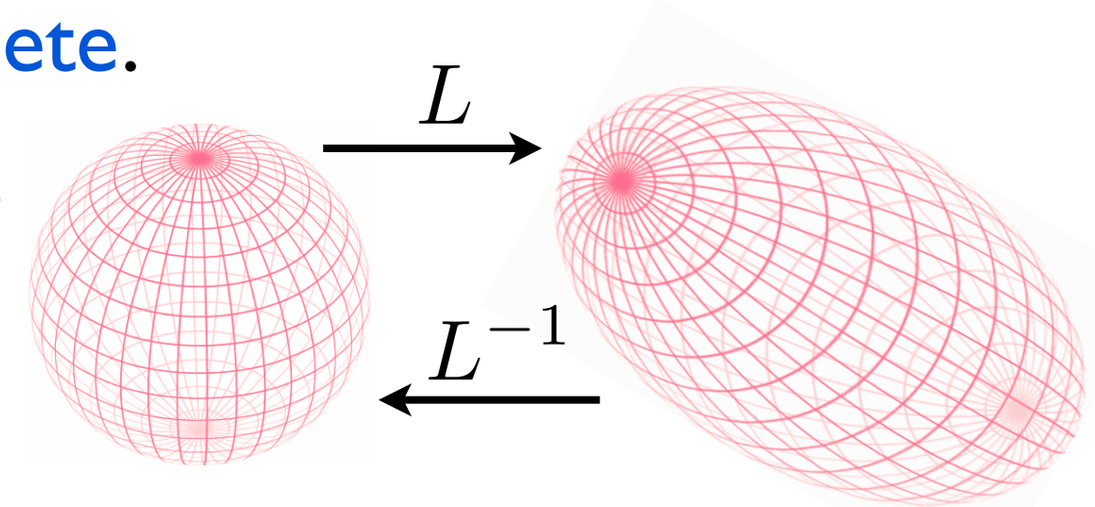
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Equivalent = same state spaces up to a linear map (physically the same!)



Result

Ll. Masanes and M. P. Müller, *A derivation of quantum theory from physical requirements*, New J. Phys. **13**, 063001 (2011).

Theorem. There are exactly two GPTs satisfying the following three axioms:

I. Reversibility

II. Local Tomography

III. Subspace Axiom

These are:

- **Classical Probability Theory**

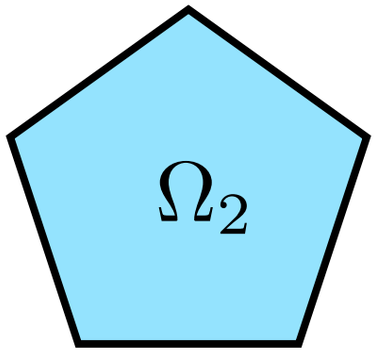
  Capacity-*n* states are the probability vectors $(p_1, \ldots, p_n)$;
  reversible transformations are permutations; the usual measurements.

- **Quantum Theory**

  Capacity-*n* states are the complex $n \times n$ density matrices;
  reversible transformations are unitary conjugations $\rho \mapsto U \rho U^\dagger$;
  the measurements are the POVMs.

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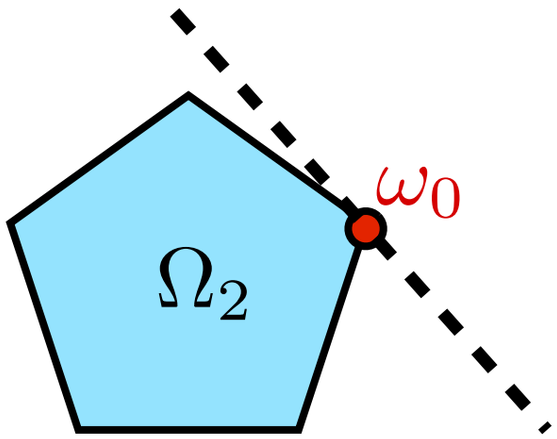
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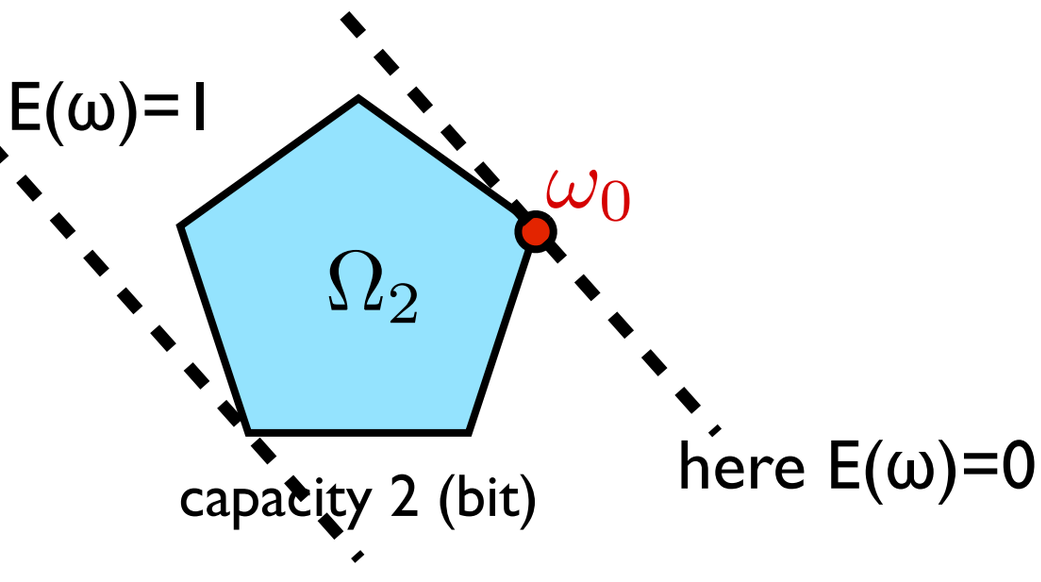
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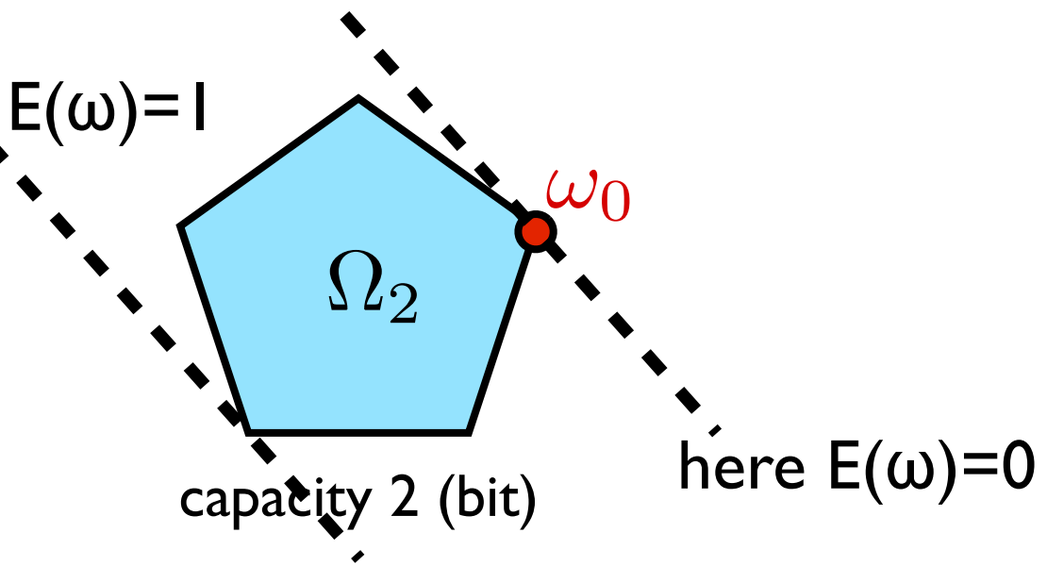


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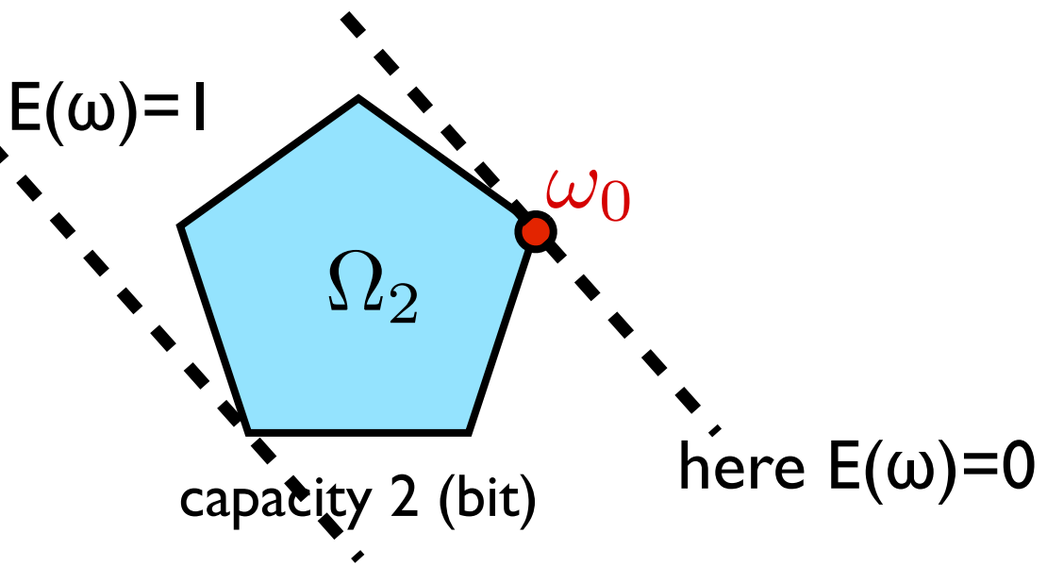
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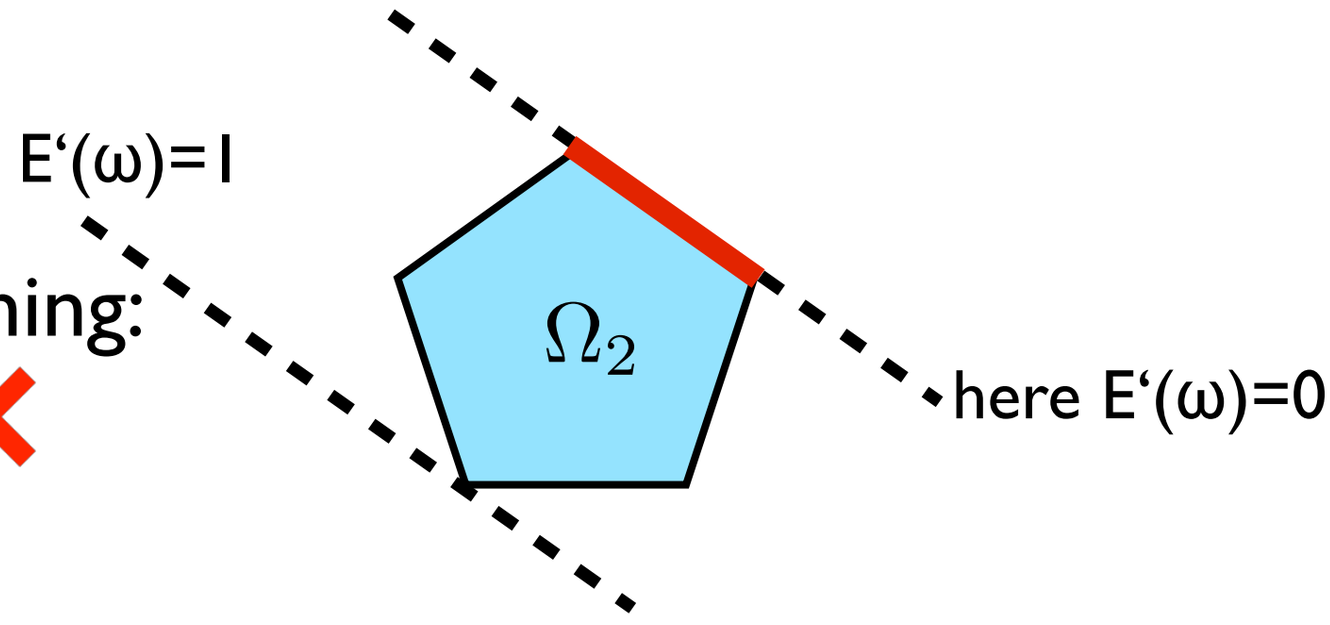
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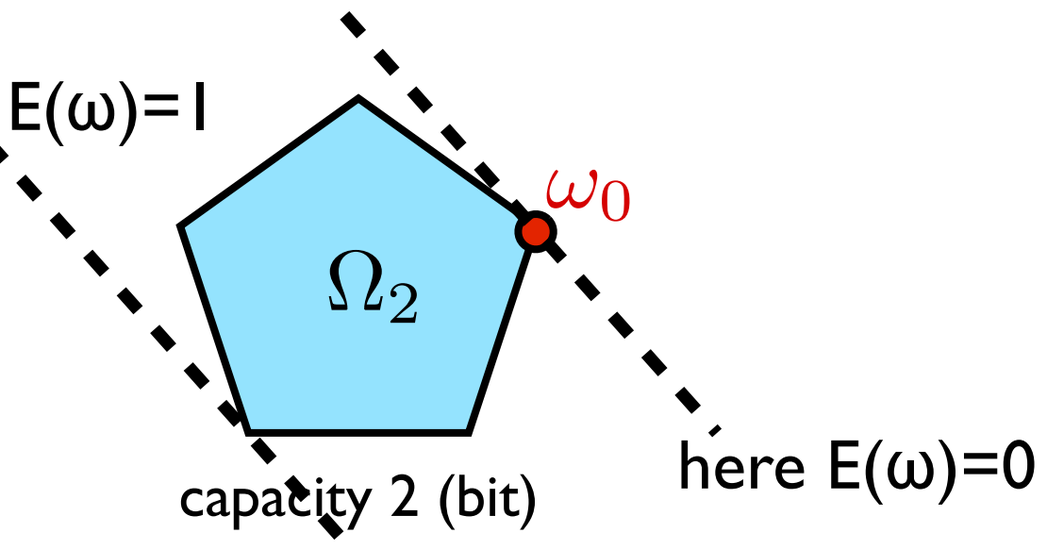
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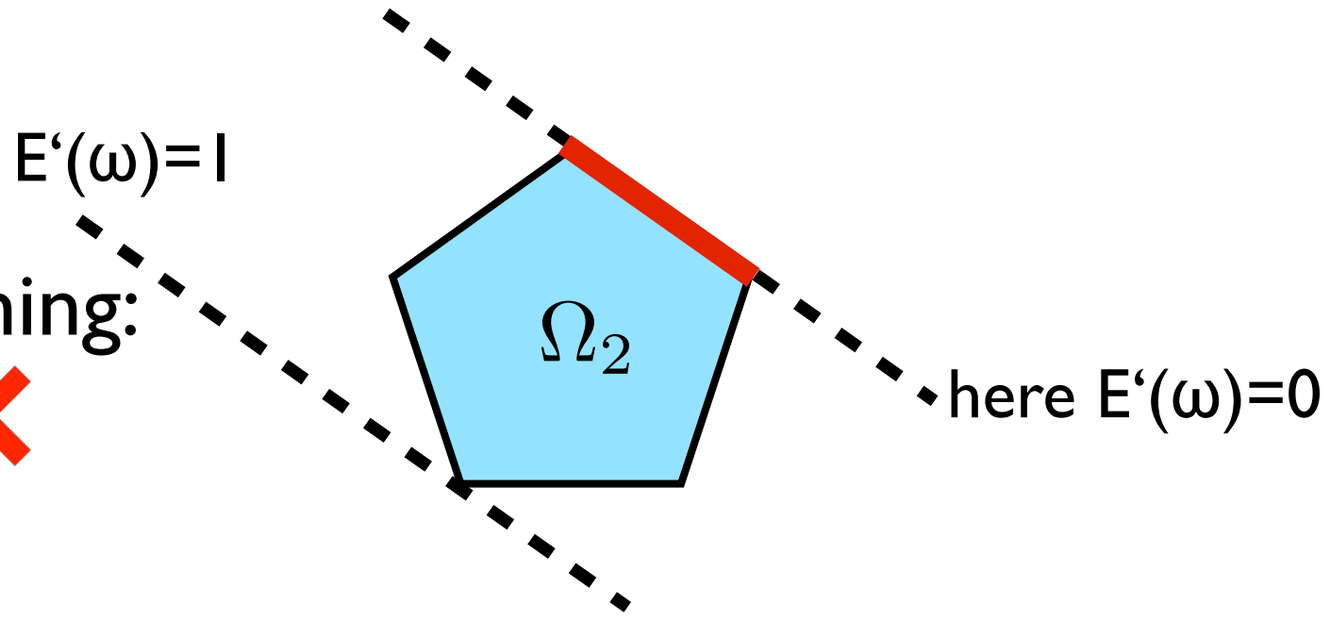
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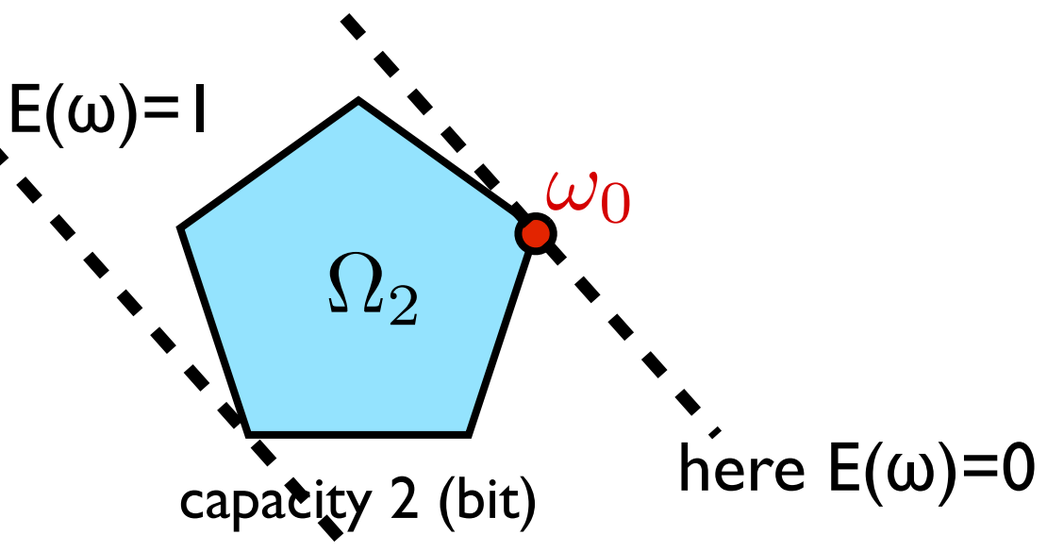


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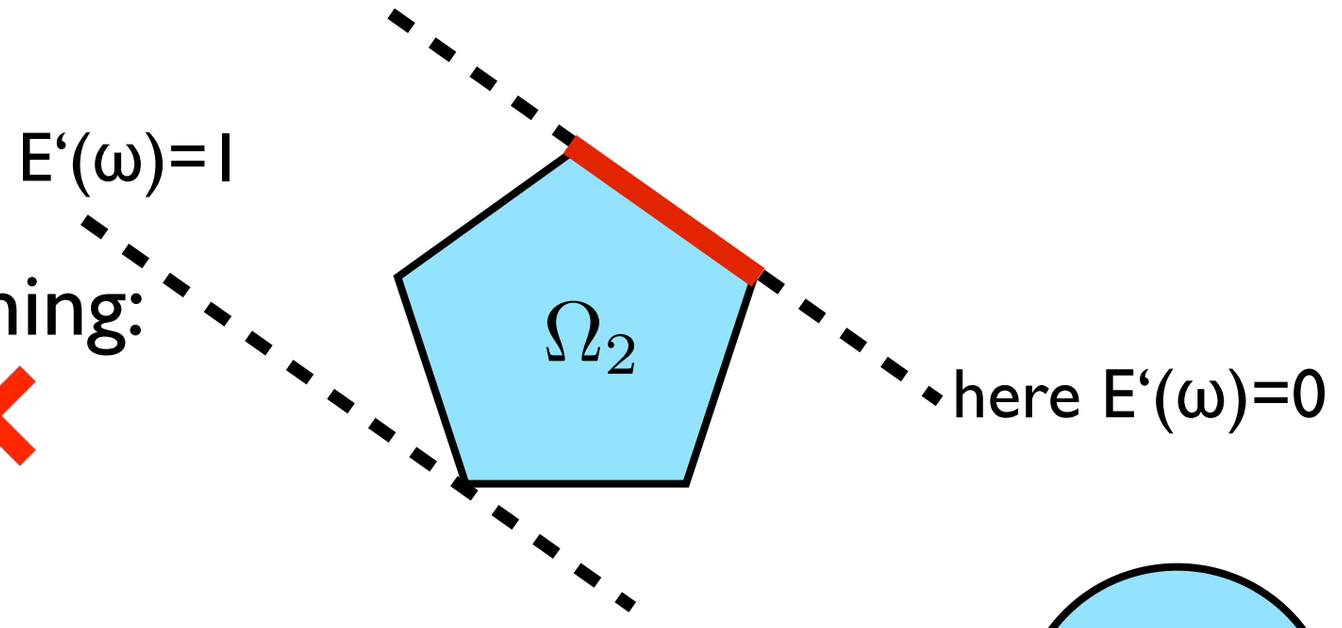
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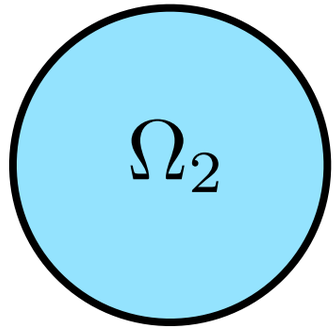
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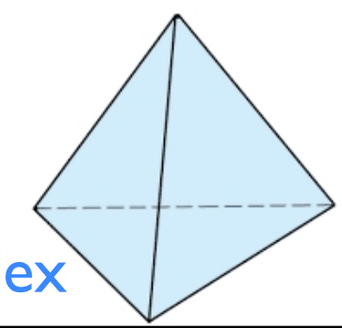
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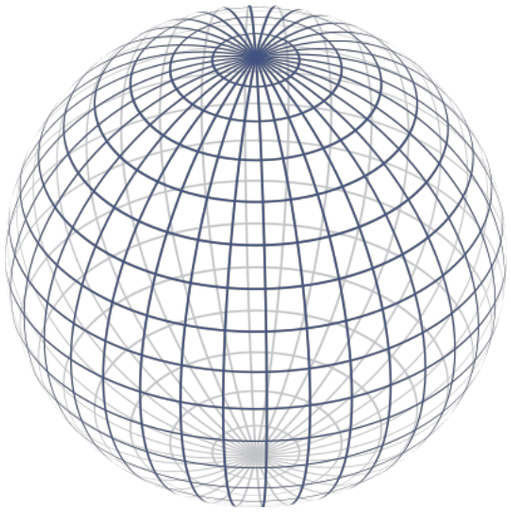
$\Omega_2 =$ 

If $\dim(\Omega_2) = 1$ then the theory is CPT (easy):

$\Omega_N =$ 

$\mathcal{G}_N$=permutation group.

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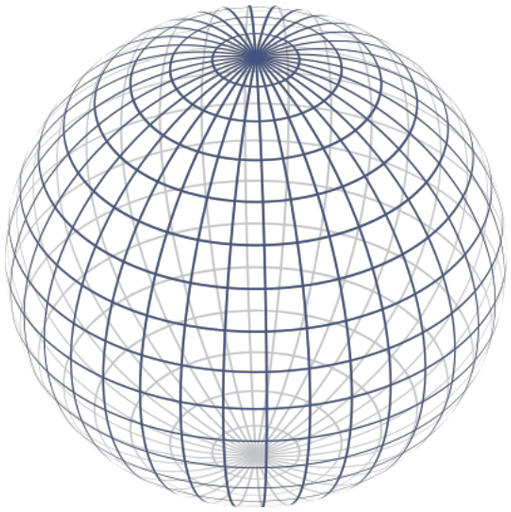
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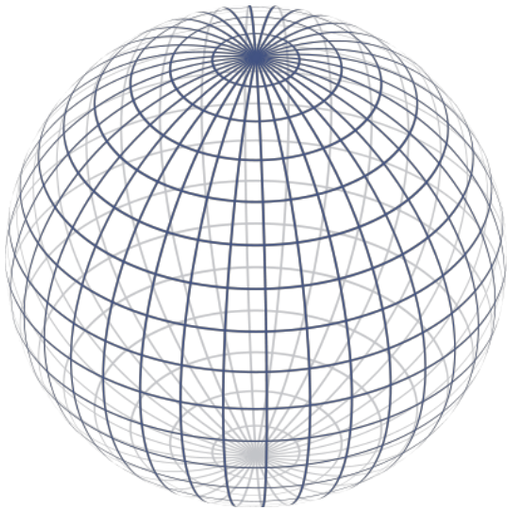
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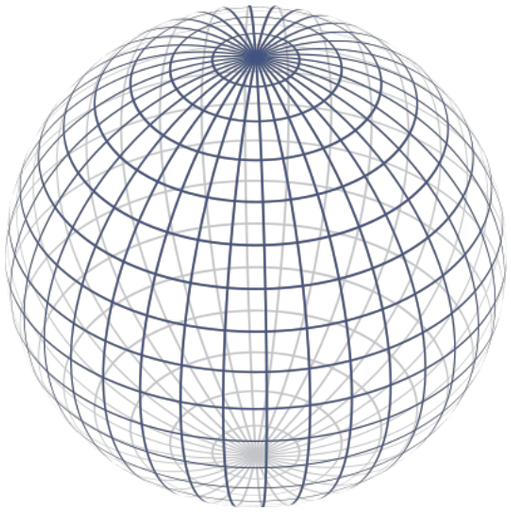
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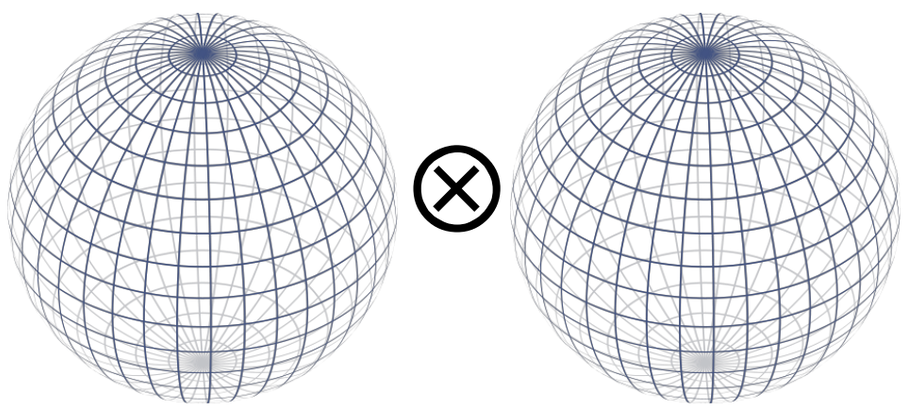
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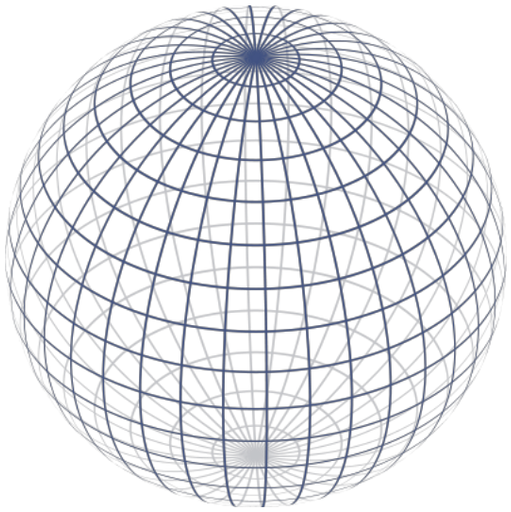
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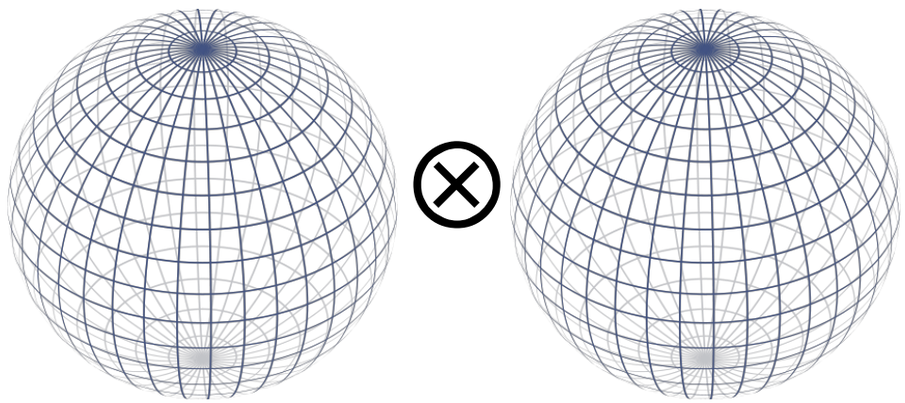
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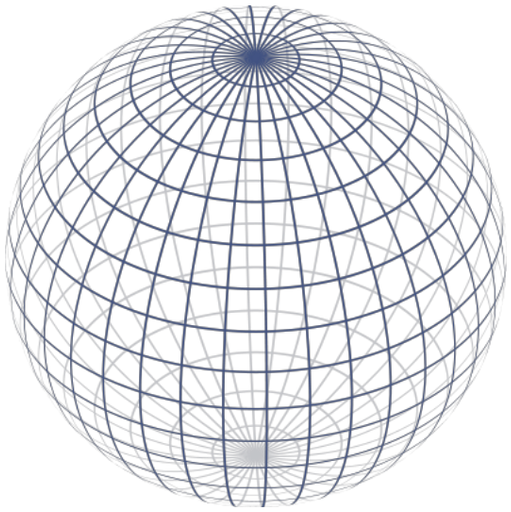
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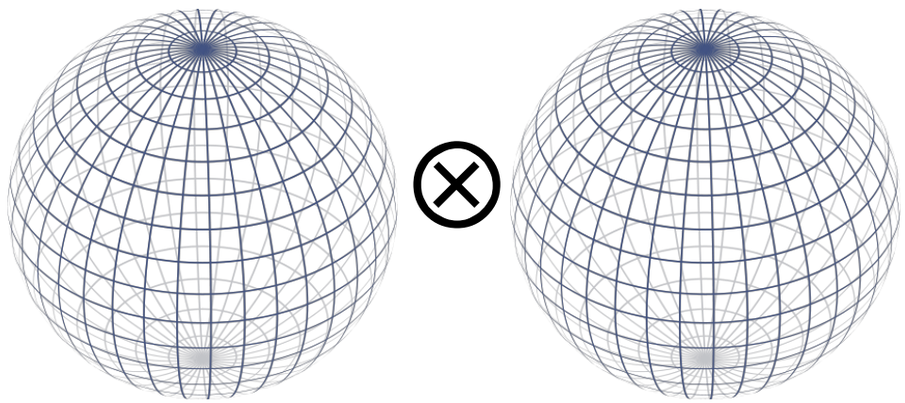
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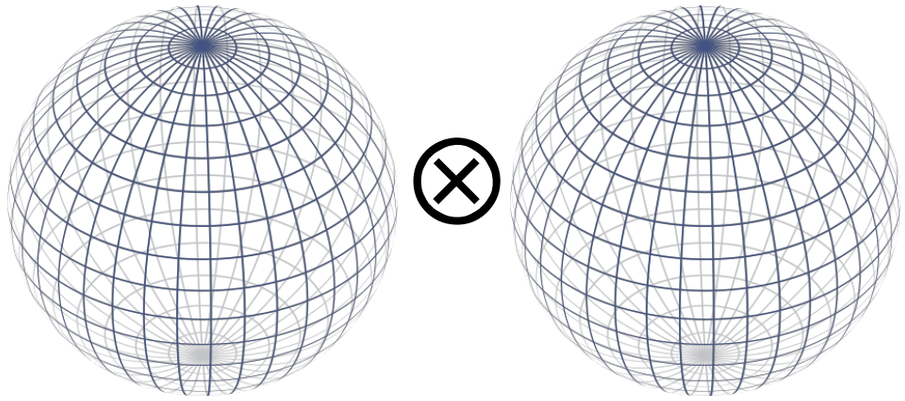
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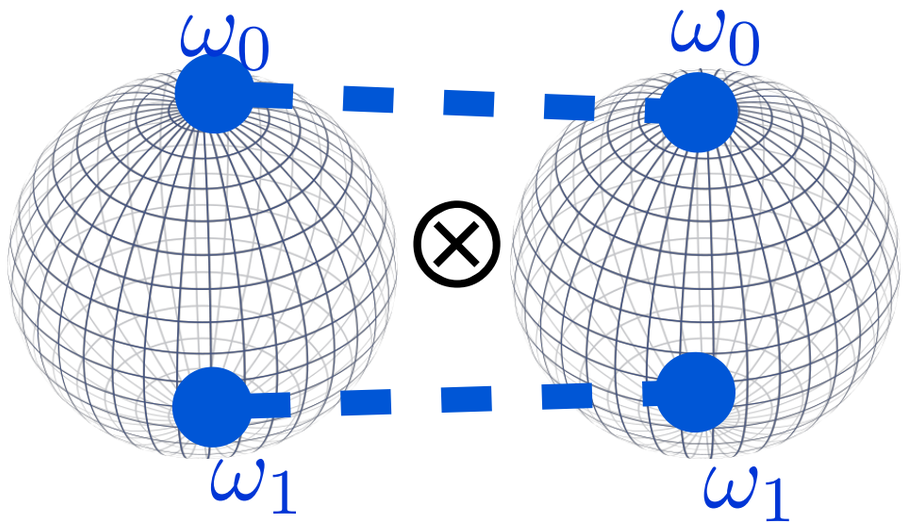
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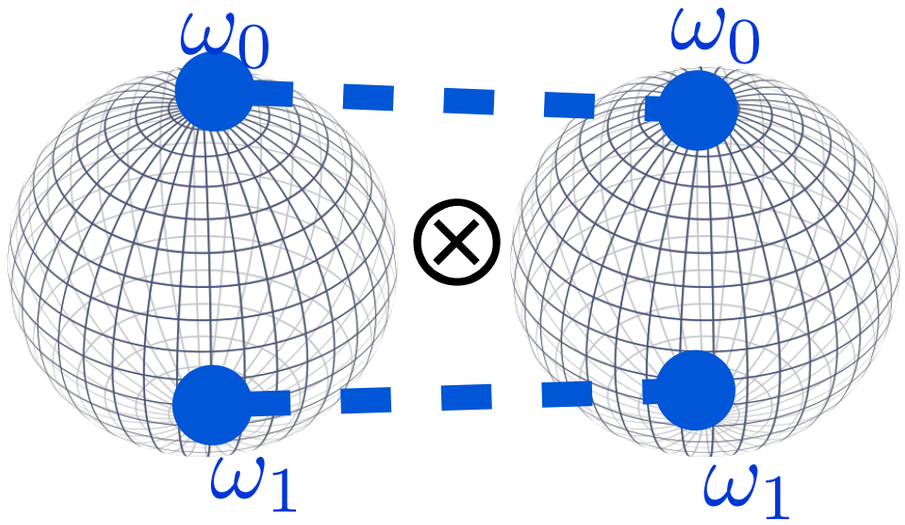


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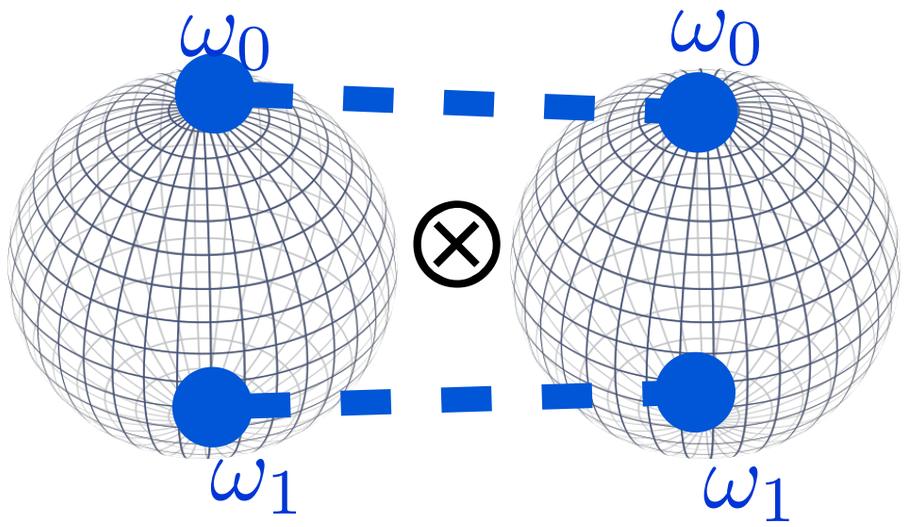
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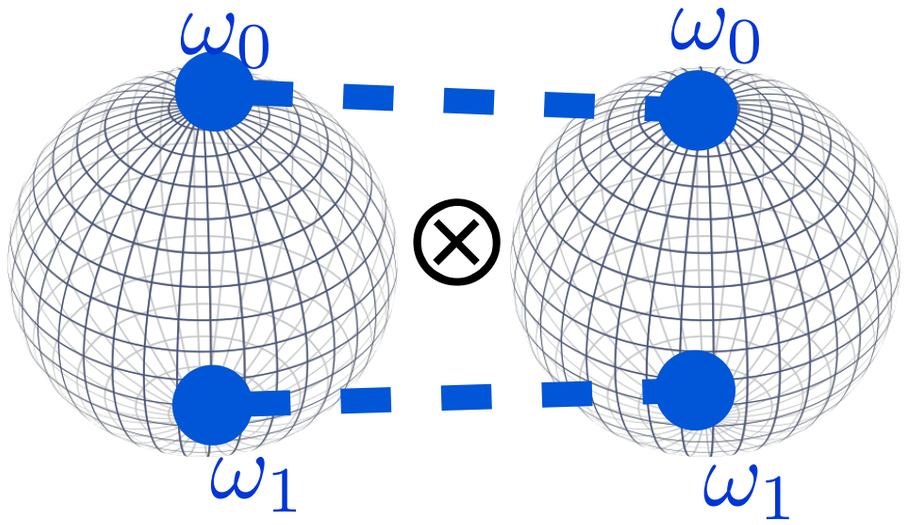
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Map 3-vectors to Hermitian matrices: $L(\omega) := \frac{1}{2}\left(\mathbf{1} + \sum_{i=1}^{3} \omega_i \sigma_i\right)$

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- Wigner's theorem
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Theorem: Every theory satisfying Axioms I, II, III (other than CPT) is equivalent to $(\Omega_N, \mathcal{G}_N)$, where

- Ω_N are the density matrices on $\mathbb{C}^N$,
- $\mathcal{G}_N$ is the group of unitaries, acting by conjugation,
- the measurements are exactly the POVMs.

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1. Contextuality: the Kochen-Specker theorem

2. Against the received view: QM as a theory of probability

3. Quantum theory from simple principles

4. Spekkens' contextuality in a nutshell (if time allows)

5. Conclusions

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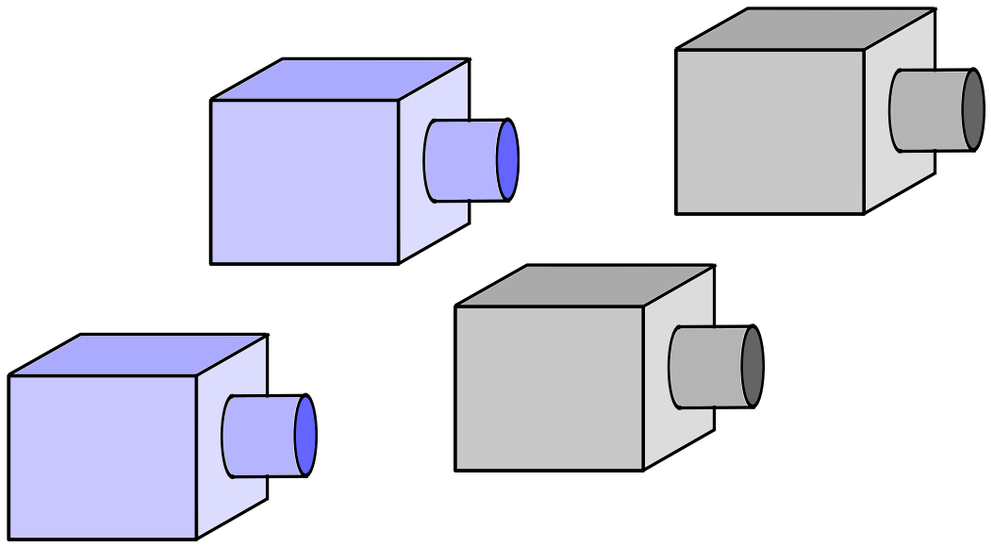
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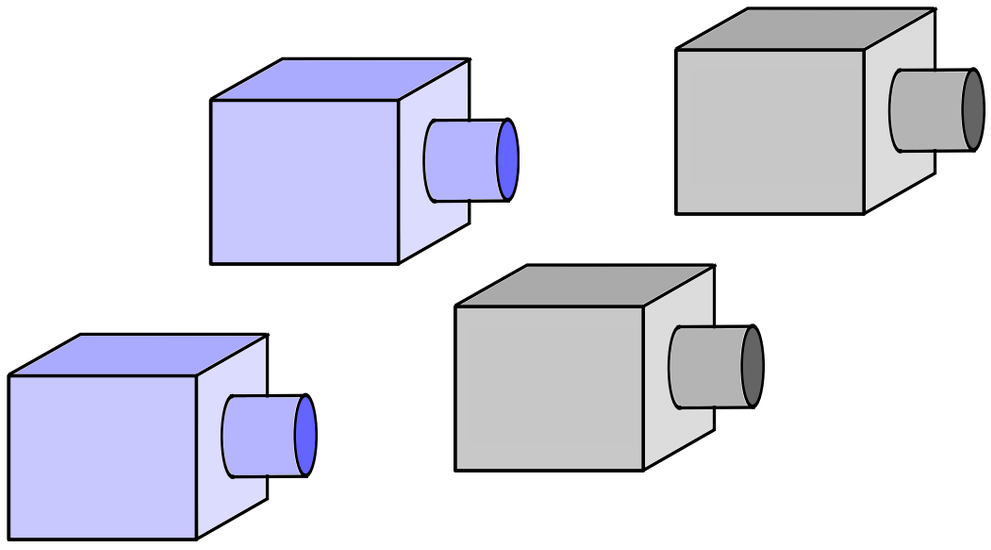
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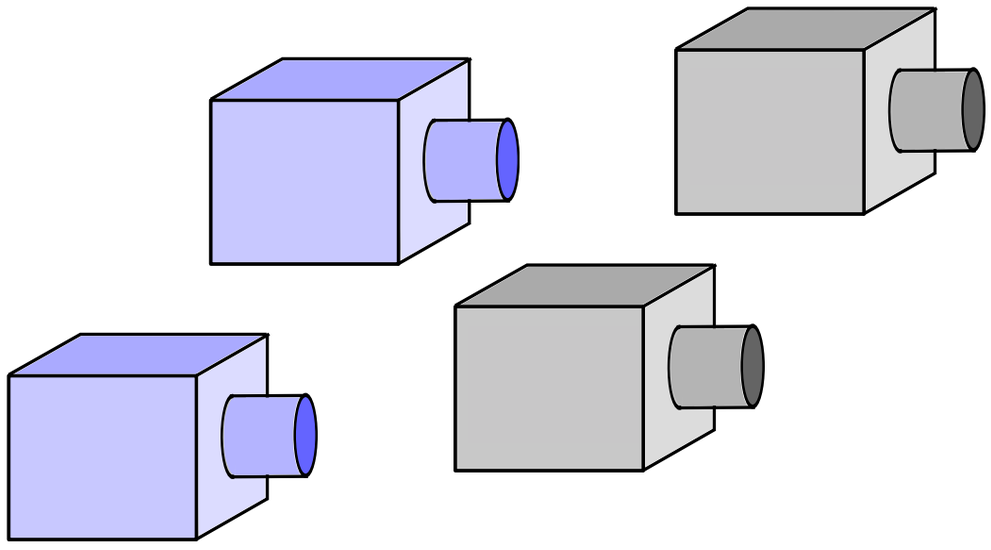
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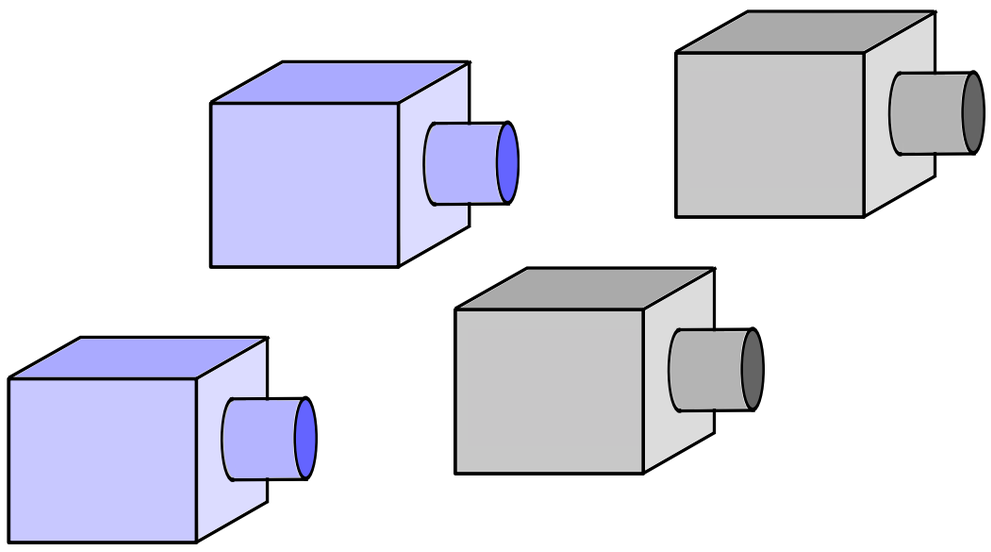


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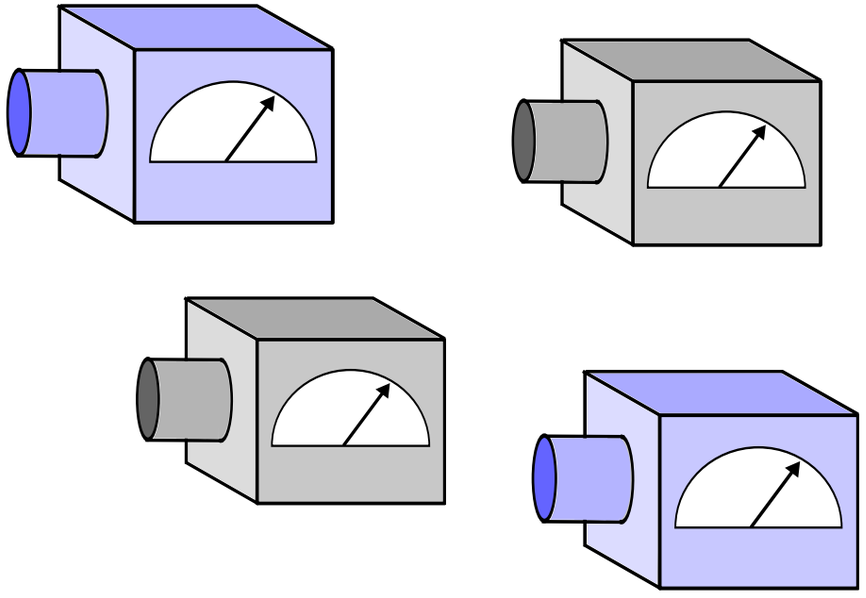
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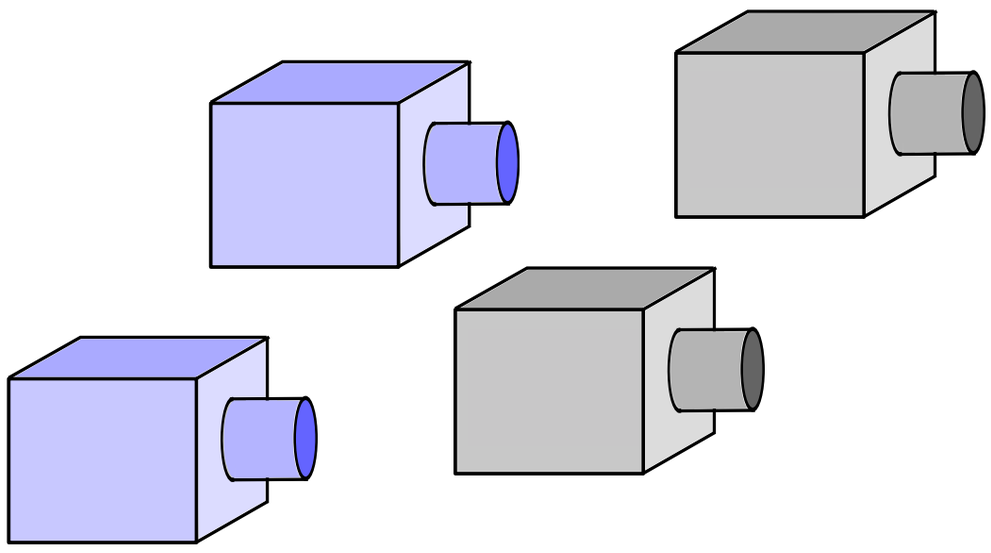


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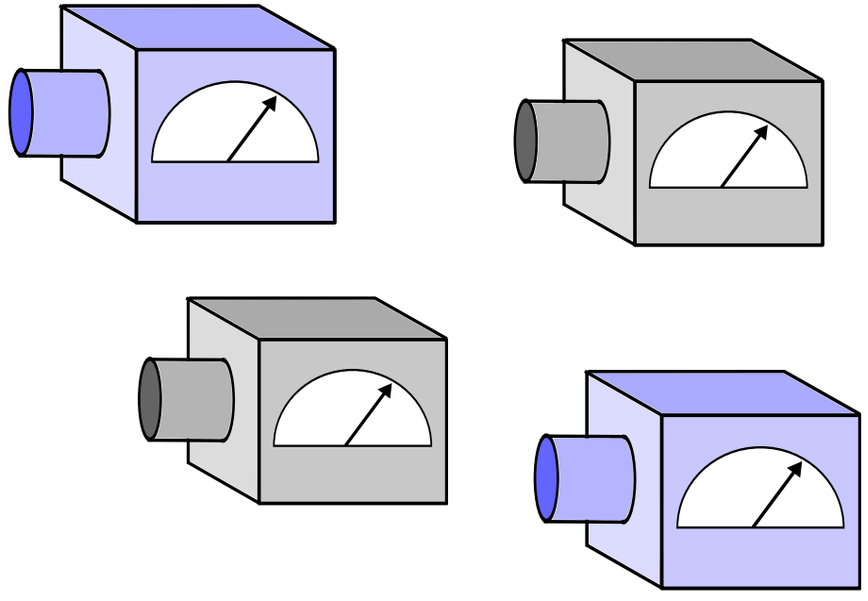
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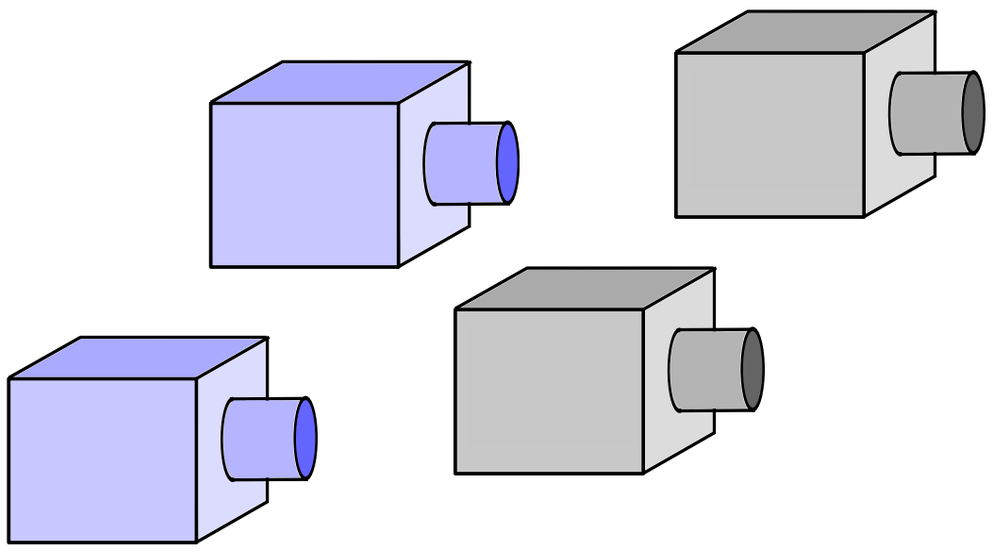
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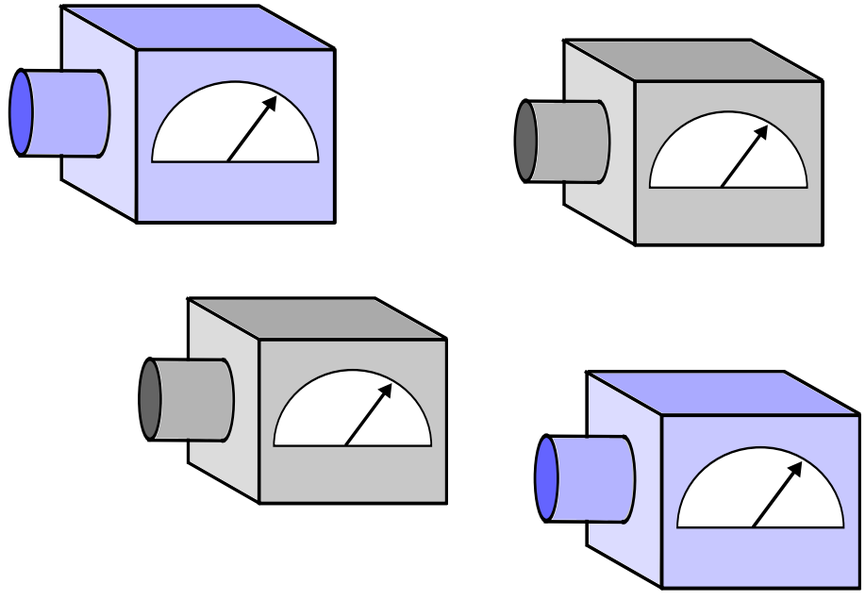
$(k_1, M_1) \sim (k_2, M_2)$ if $\mathrm{Prob}(k_1|M_1, P) = \mathrm{Prob}(k_2|M_2, P)$ for all accessible preparations P.

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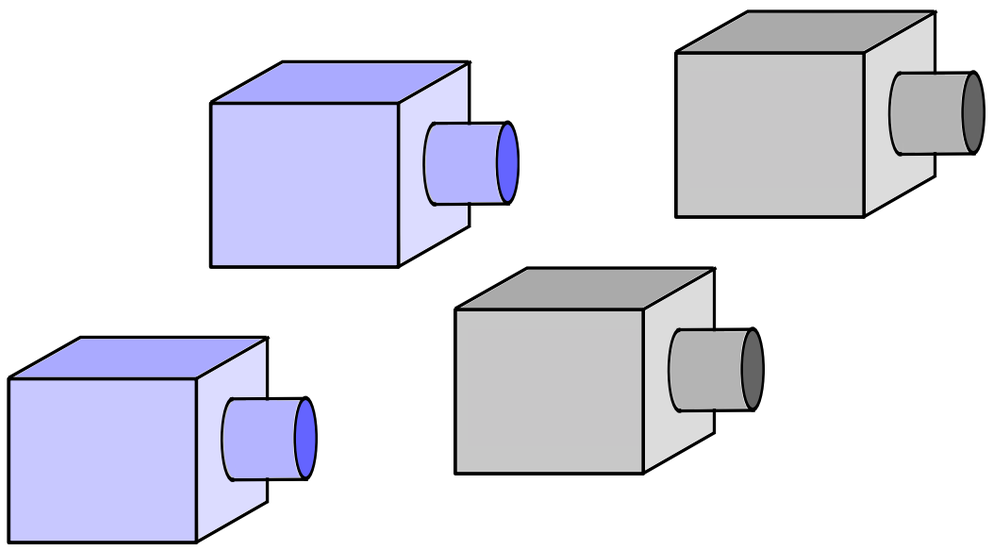
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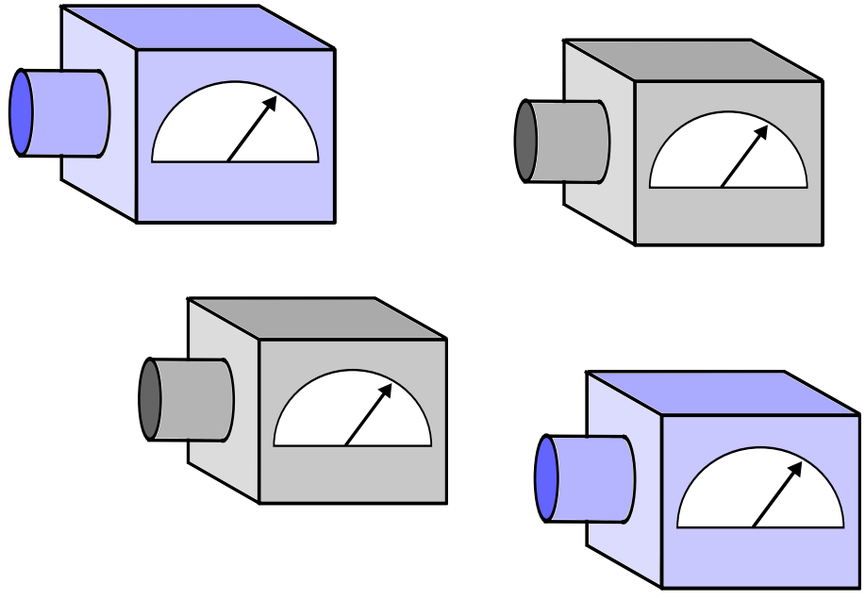
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Operational theories



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$$\mathrm{Prob}(k|P, M) = \langle \omega_P, e_{k,M} \rangle \qquad (e_{k,M} \in A, \omega_P \in A^*).$$

Spekkens' notion of noncontextuality

Contextuality for preparations, transformations and unsharp measurements

R. W. Spekkens*
Perimeter Institute for Theoretical Physics, 35 King St. North, Waterloo, Ontario N2J 2W9, Canada
(Dated: Feb. 25, 2005)



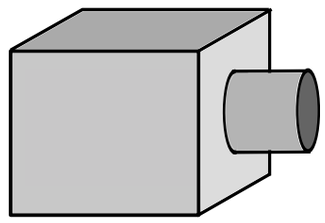
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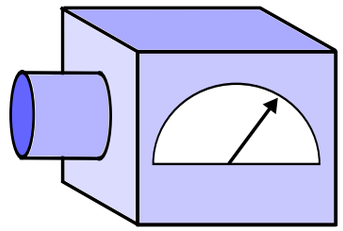
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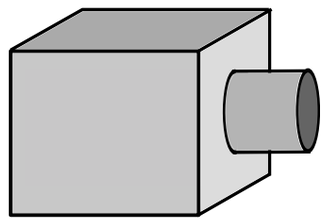
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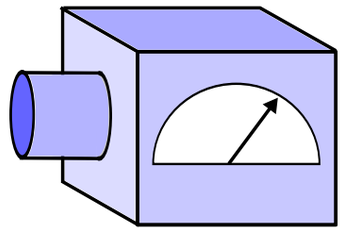
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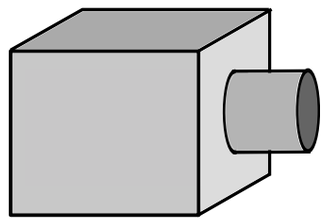
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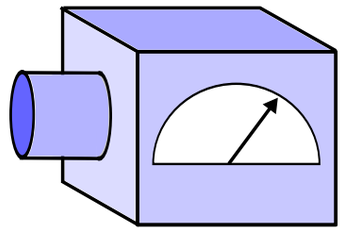
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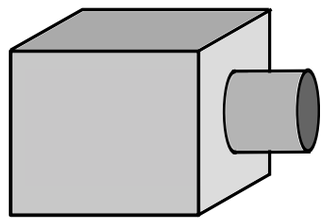
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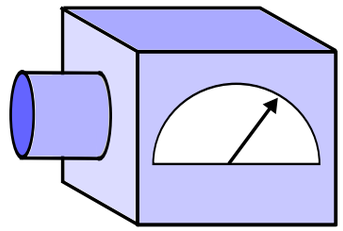
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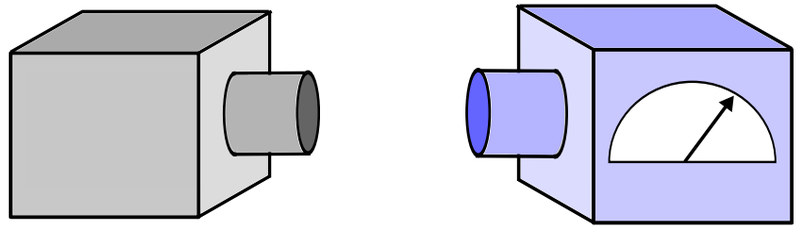
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such that $$\text{Prob}(k|P,M) = \int_\Lambda d\lambda \mu_P(\lambda)\chi_{M,k}(\lambda).$$

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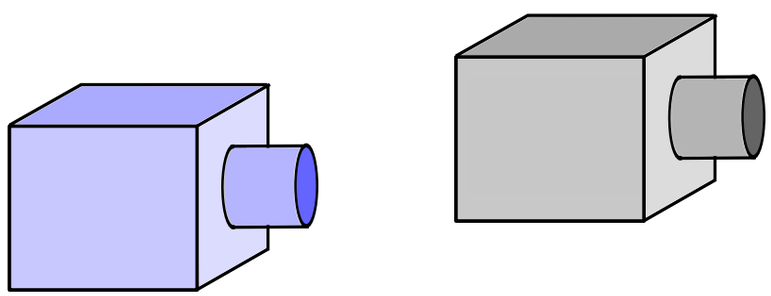
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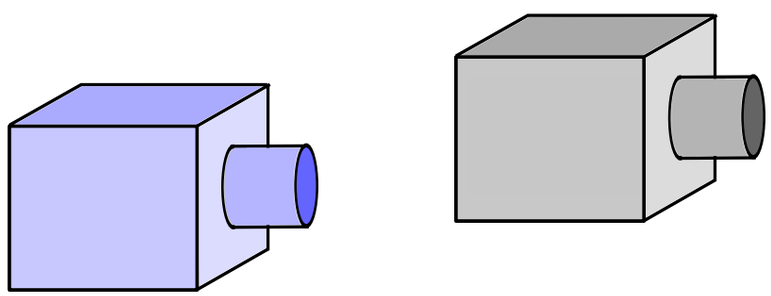


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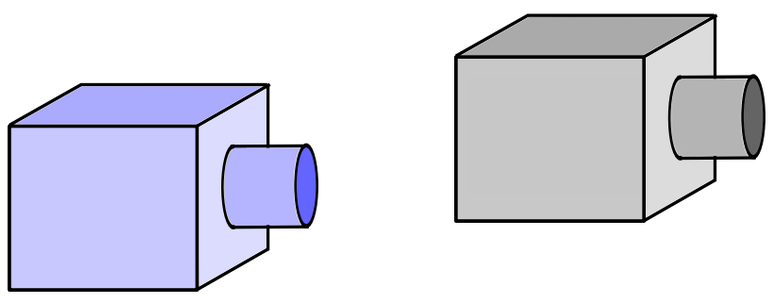
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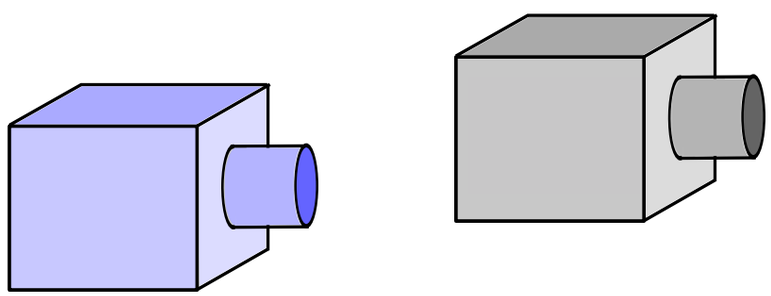
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Intuition: Contextual models are implausible because they are **fine-tuned**: operationally, $P \sim P'$, but ontologically, $\mu_P \neq \mu_{P'}$.

An instance of Leibniz' principle of the "identity of the indiscernibles".

Spekkens' notion of noncontextuality

Significant advantages over Kochen-Specker notion:
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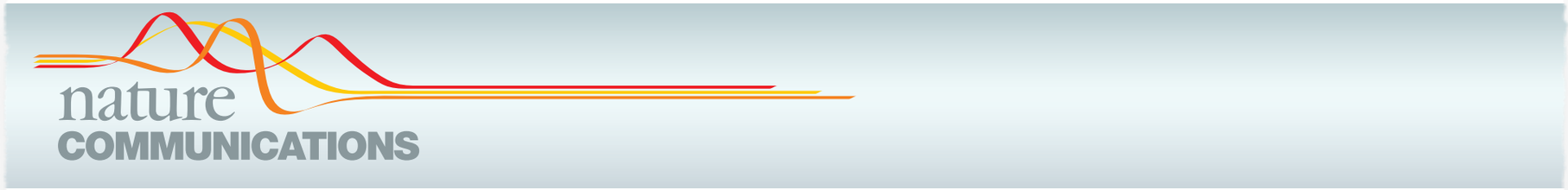
DOI: 10.1038/ncomms11780 OPEN

An experimental test of noncontextuality without unphysical idealizations

Michael D. Mazurek[1,2], Matthew F. Pusey[3], Ravi Kunjwal[4], Kevin J. Resch[1,2] & Robert W. Spekkens[3]

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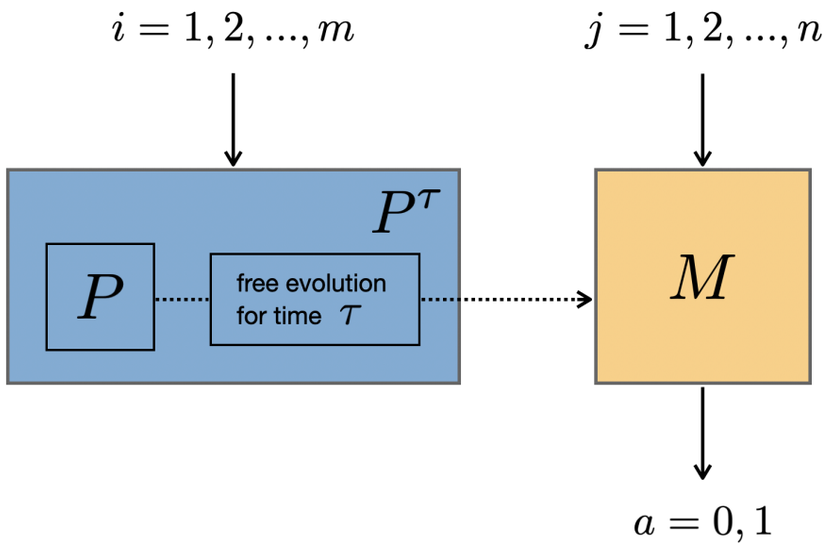
Notion of nonclassicality	Proves that HV model cannot preserve the…	Assumes QT?	Experimentally robust?
Kochen-Specker contextuality	logical structure / structure of propositions	Yes	No
Spekkens' contextuality	statistical mixing structure	No	Yes
Bell nonlocality	causal structure	No	Yes

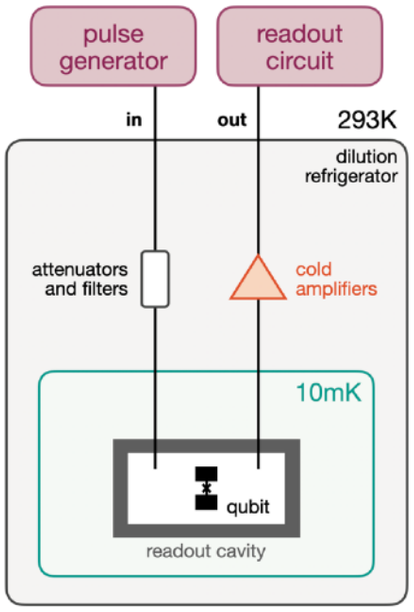
Theory-independent contextuality on a superconducting qubit



Matteo Fadel (ETH Zurich)

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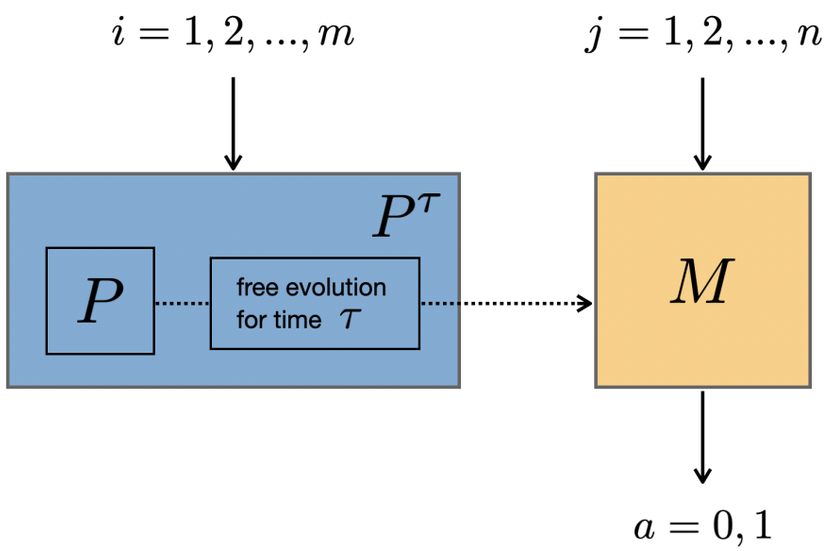


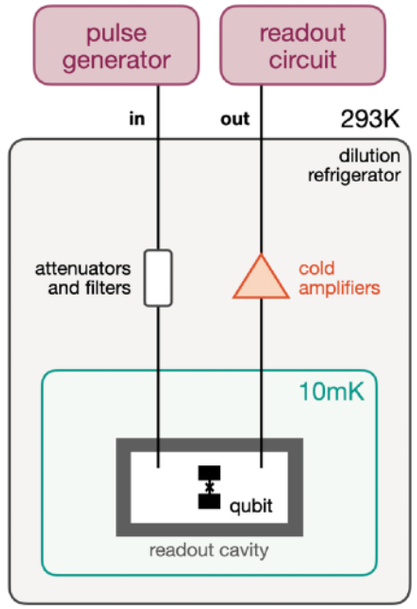




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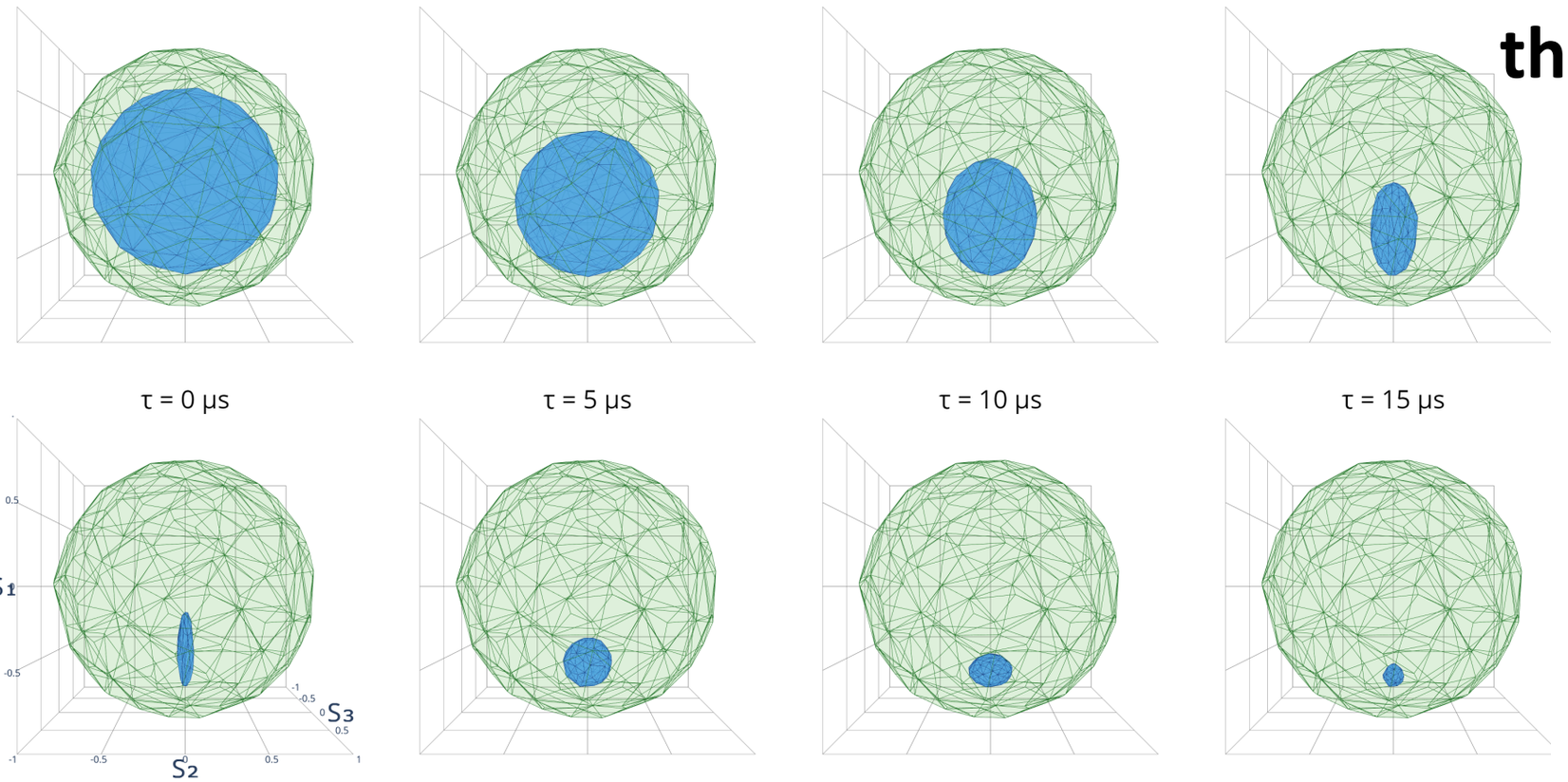




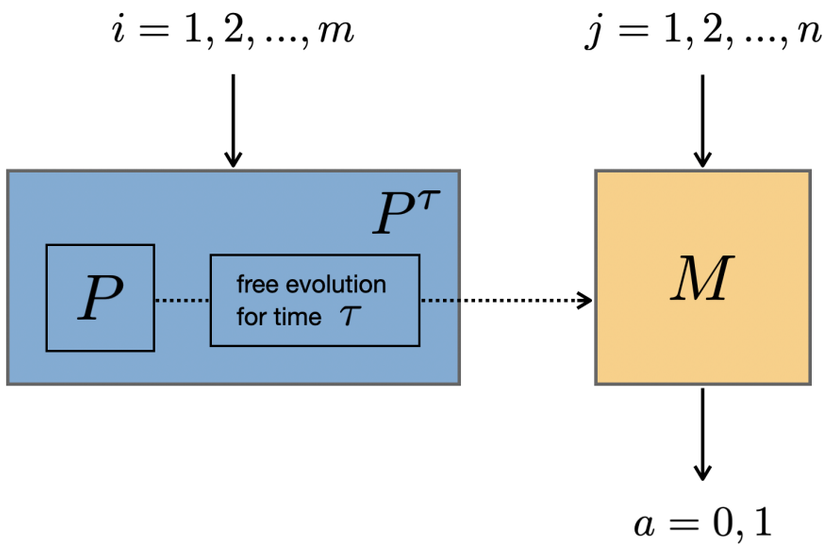
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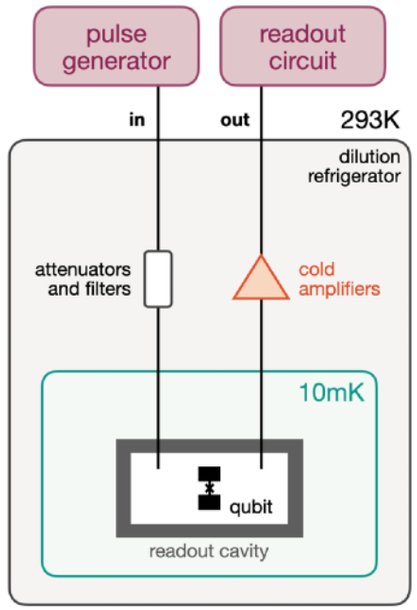
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theory-agnostic “movie”



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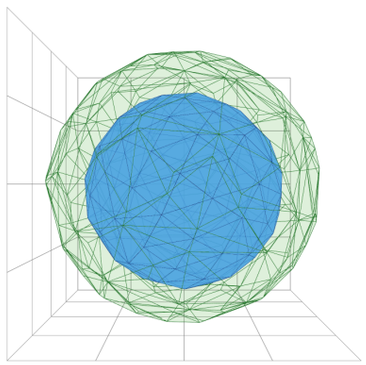




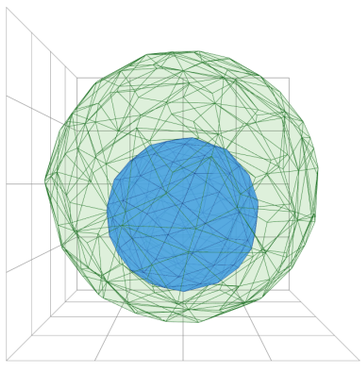


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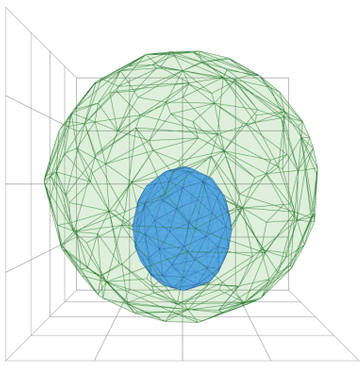
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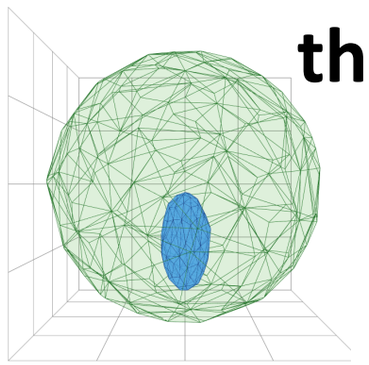
τ = 0 μs



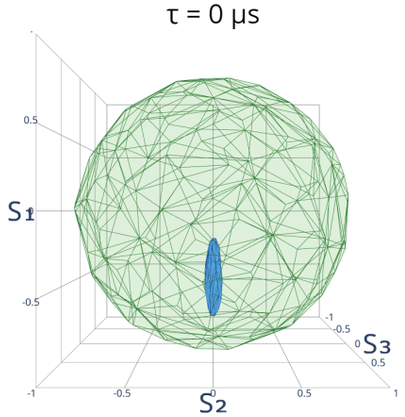
τ = 5 μs



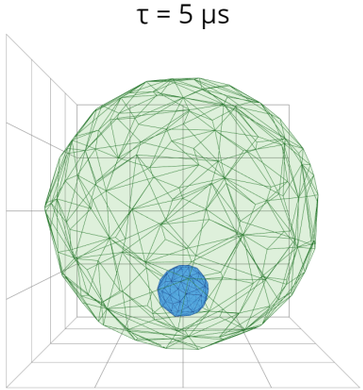
τ = 10 μs



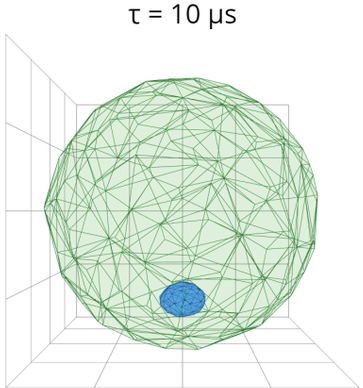
τ = 15 μs



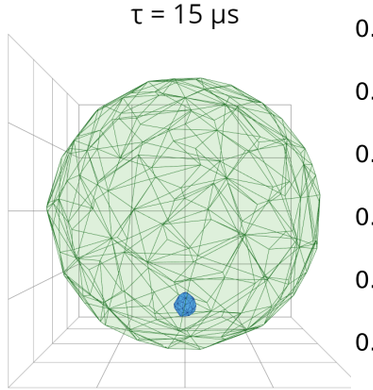
τ = 20 μs



τ = 30 μs



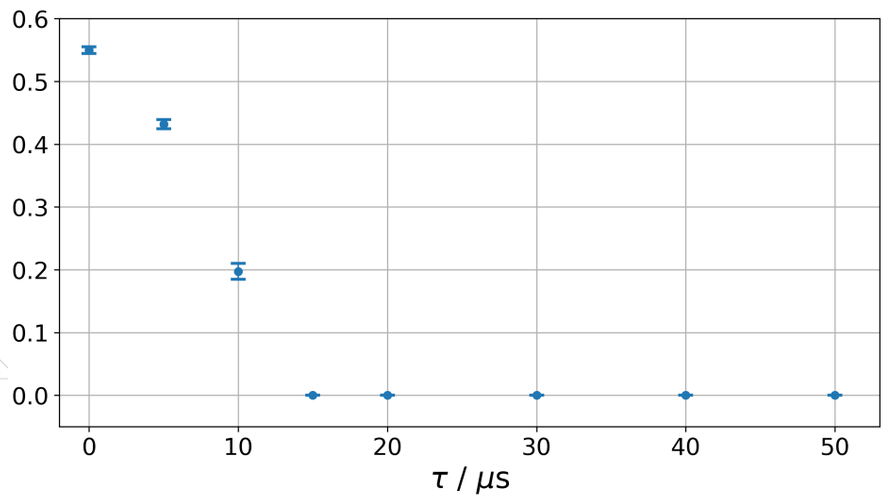
τ = 40 μs



τ = 50 μs

theory-agnostic “movie”

Contextual until 15μs:



Overview

1. Contextuality: the Kochen-Specker theorem

2. Against the received view: QM as a theory of probability

3. Quantum theory from simple principles

4. Spekkens' contextuality in a nutshell (if time allows)

5. Conclusions

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My own view: Quantum states are catalogues of **probabilities** of a “self’s” **future experiences**. (*Not themselves* “states of consciousness”, cf. Q-IIT.) QM fundamental, q. states all there is => the self and its probabilities are fundamental, the world is (statistically) emergent. (Algorithmic) **idealism**.

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  Talk and more papers on mpmueller.net/ai