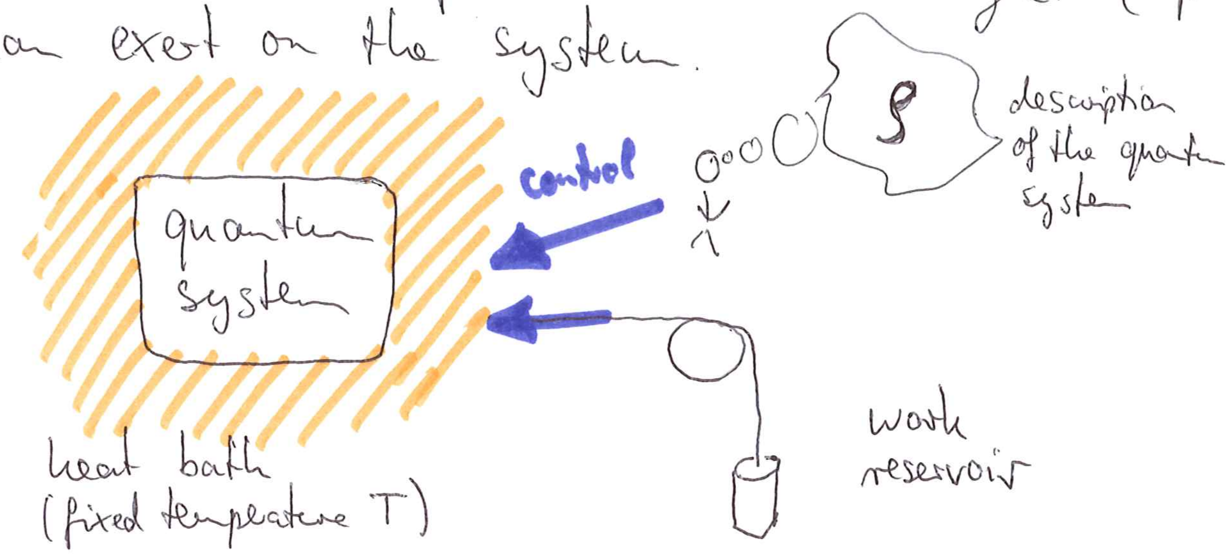
Single-shot thermo, lecture 1

Introduction to Maxwell's Demon, Landauer erasure and Szilard engines: see links online.

Single-shot thermodynamics: take an "agent-centric" point of view.
Center stage is not the time evolution of a physical system, but the amount of control that an agent (physicist) can exert on the system.



Some **simple rules** determine the amount of control:
- Energy-preserving reversible transformations can be implemented,
- subsystems can be discarded,
- arbitrary heat baths of temperature T can be used.

Some **major questions** that are attempted to be answered:
- How much work can the agent extract in a <u>single run</u> of the experiment with near certainty (not just on average)?

- What state transitions are possible / impossible?
- How is this affected by resources like quantum coherence or correlations?

The standard thermodynamic identities (free energy etc.) are recovered in the thermodynamic limit.

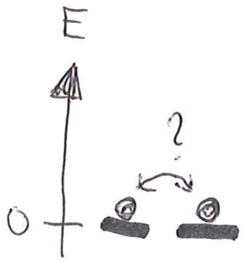
Many things we learn here have applications in other areas:

- information theory and data compression,
- "single-shot" entropies: quantum cryptography, black-hole physics, AdS/CFT-correspondence, decoupling in quantum information etc.
- large overlap with quantum information theory in general.
- Majorization / Lorenz curves: economics (cf. Gini index).

1. Landauer's Principle: rigorous derivations

1.1. Folklore derivation

Suppose we have two degenerate energy levels $|0\rangle$ and $|1\rangle$ (energy $E=0$ in both cases), and a particle described by $\rho_0 = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| = \frac{1}{2}\mathbb{1}$



System is embedded in a heat bath of temperature T.

Inverse temperature: $\beta = \frac{1}{k_B \cdot T}$.

At time $t=0$, Hamiltonian is $H_0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

At integer time steps $t = i \cdot \Delta t$ $(i \in \mathbb{N})$

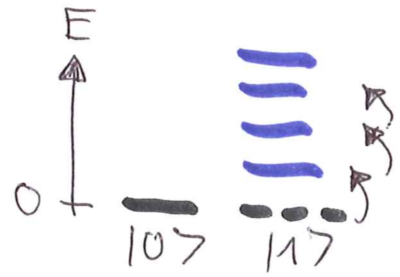
If Δt is large, state will "thermalize" and change to

$$\rho_i = \frac{e^{-\beta H_i}}{Z}, \text{ with } Z := \mathrm{tr}\left(e^{-\beta H_i}\right)$$

(Gibbs state / thermal state).

We can use this to "erase" = reset the bit:

- **Raise** the energy level $|1\rangle$ in ΔE-steps to infinity ($\longrightarrow$ thermalization will make the state "fall down" to $|0\rangle$), alternating with thermalization steps.



$H_0 \to H_1 \to H_2 \to \ldots$
$\rho_0 \to \rho_1 \to \rho_2 \to \ldots$
where $H_k = \begin{pmatrix} 0 & 0 \\ 0 & \mathbf{k \cdot \Delta E} \end{pmatrix}$

- Quickly lower level $|1\rangle$ back down to $E=0$. Before thermalizing again, this will restore H_0, and give $\rho = |0\rangle\langle 0|$.

Average work cost:

$$\langle W \rangle = \mathrm{tr}\left[\rho_0 (H_1 - H_0)\right] + \mathrm{tr}\left[\rho_1 (H_2 - H_1)\right] + \ldots$$
$$= p_0 \Delta E + p_1 \Delta E + p_2 \Delta E + \ldots$$

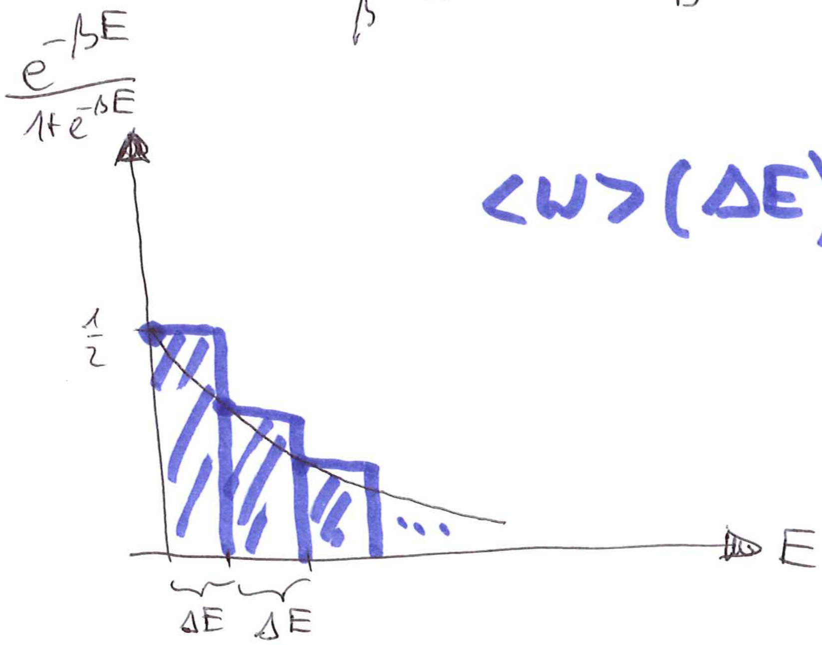
where $\rho_k = \begin{pmatrix} 1-p_k & 0 \\ 0 & p_k \end{pmatrix}$, $\quad p_k = \dfrac{e^{-\beta k \Delta E}}{e^{-\beta 0} + e^{-\beta k \Delta E}}$

Step k costs ΔE with probability p_k.

$$\Rightarrow \langle W \rangle = \Delta E \cdot \sum_{k=0}^{\infty} \frac{e^{-\beta k \Delta E}}{1 + e^{-\beta k \Delta E}}$$

$$\geq \lim_{\Delta E \to 0} \Delta E \sum_k \frac{e^{-\beta k \Delta E}}{1 + e^{-\beta k \Delta E}} = \int_0^\infty \frac{e^{-\beta E}}{1 + e^{-\beta E}} dE$$

$$= \frac{1}{\beta} \ln 2 = k_B T \ln 2$$



$\langle W \rangle (\Delta E) =$ blue area

$$\geq \int_0^\infty \rho(E) dE$$

Is this optimal?
Formulating thermodynamics as a resource theory (as is done in single-shot thermodynamics) will allow to "optimize over all possible strategies" and prove optimality.