Single-shot thermo, lecture 14

- Jakob gives back last corrected exercises.

Today (partly with projector): peripheral results / relation to majorization, single-shot entropies; current research.

4.2. Attempts of unification with fluctuation-dissipation theorems

Crooks' theorem:

- forward protocol: parameter $\lambda(t)$ determines time-dependent Hamiltonian $H(\lambda(t)) =: H_t$.
  At $t = -T$: start in state $\gamma_{-T} = e^{-\beta H_{-T}}/Z$.
  Interactions with heat bath. Not quasistatic $\Rightarrow$ system leaves equilibrium
  Stop at $t = T$ (possibly let thermalize to γ_T).

e.g. forward: compression; reverse: expansion.

- reverse protocol: run backwards.

Repeat many times $\Rightarrow$ prob. density $P_{fwd}(W)$ over required work W, $P_{rev}(W)$ over gained work W.

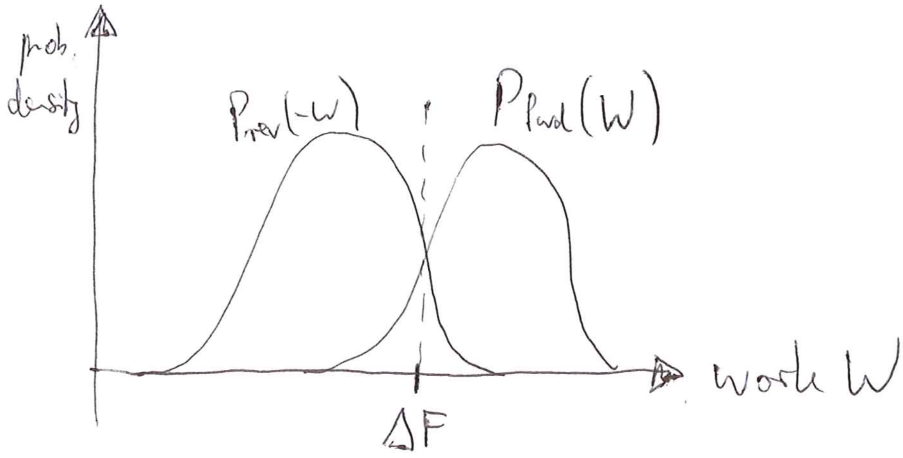
Crooks: $$\boxed{\frac{P_{fwd}(W)}{P_{rev}(-W)} = e^{\beta(W - \Delta F)}}$$

$\Delta F = F(\gamma_T) - F(\gamma_{-T})$ free energy difference. $-\frac{1}{\beta}\log Z_T$

$$\Rightarrow \int P_{fwd}(W)\, e^{-\beta W}\, dW = \int P_{rev}(-W)\, e^{-\beta \Delta F}\, dW$$

$$\boxed{\langle e^{-\beta W} \rangle_{fwd} = e^{-\beta \Delta F}}$$ Jarzynski's Equality

Can be used to determine equilibrium quantity ΔF from out-of-equilibrium measurements. $\to$ 1.pdf



prob. distribution over work (cf. Landauer erasure etc.)

Lemma: $\frac{1}{\beta} D\big(P_{rev}(-W) \,\|\, P_{fwd}(W) \big) = \Delta F - \langle W \rangle_{rev}$

where $\langle W \rangle_{rev}$ = average work gain from reverse process.

Note: $D(\cdot \| \cdot) \geq 0 \implies \langle W \rangle_{rev} \leq \Delta F$,

one obtains less work if $P_{rev}(-W)$ and $P_{fwd}(W)$ are more distinguishable (irreversibility).

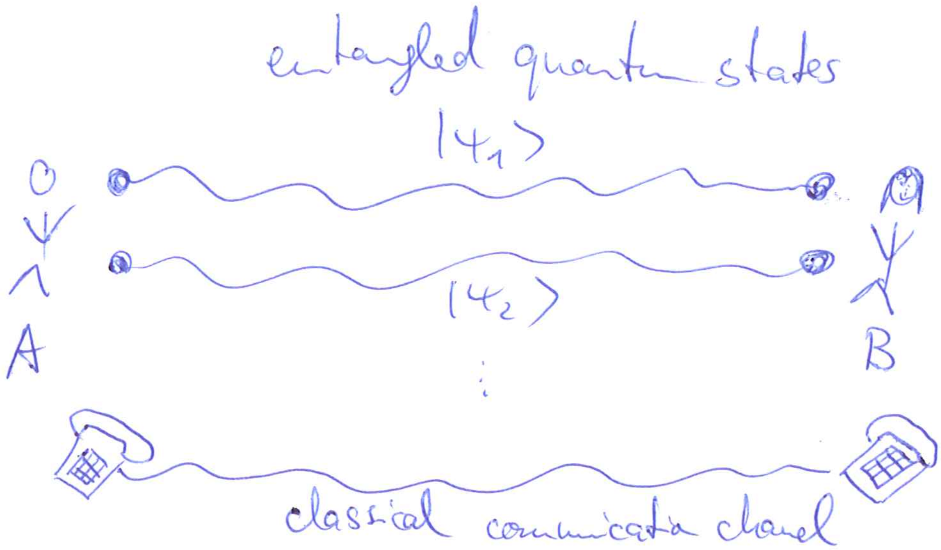
Lemma: W_{min} := the least work that any reverse trial can output. Then

$$\frac{1}{\beta} \underset{\underset{D_{max}}{\shortparallel}}{D_\infty} \big(P_{rev}(-W) \,\|\, P_{fwd}(W) \big) = \Delta F - W_{min}$$

Recall $D_{min} \leq D \leq D_{max}$

More details: arXiv: 1409.3878.

5. Majorization and entanglement: Nielsen's Theorem



LOCC: local operations and classical communication.

$\rightarrow$ cannot create entanglement.

But they can for example distinguish the Bell states

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right)$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B\right)$$

Local reduced states are $\rho_A = \rho_B = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$.

But: • Alice measures qubit ($\rightarrow$ 0 or 1), sends result to Bob, Bob measures $\rightarrow$ they know!

Question: Given $|\psi\rangle, |\varphi\rangle \in \mathbb{C}^d \otimes \mathbb{C}^d$, can A and B transform $|\psi\rangle$ into $|\varphi\rangle$ via LOCC?

(Intuition: only if $|\varphi\rangle$ "less entangled than" $|\psi\rangle$.)

→ Resource theory of entanglement!

<u>Free states</u>: product states $|\psi_A\rangle \otimes |\psi_B\rangle$

<u>Free transf.</u>: LOCC

<u>Resources</u>: Entangled states.

Every $|\Psi_{AB}\rangle$ can be written

$$|\Psi_{AB}\rangle = \sum_{i=1}^{d} \sqrt{\lambda_i}\, |i_A\rangle \otimes |i_B\rangle$$

"Schmidt coefficients"

local ONB

$\Rightarrow$ both ρ_A and ρ_B have eigenvalues $\lambda_1, \dots, \lambda_d$.

<u>Nielsen's Theorem</u>: $|\psi\rangle \to |\varphi\rangle$ by LOCC if and only if $\lambda \prec \lambda'$, where λ and λ' are the Schmidt coefficients of $|\psi\rangle$ resp. $|\varphi\rangle$.

<u>Example</u>: $|\psi_{AB}\rangle = |\psi_A\rangle \otimes |\psi_B\rangle \in \mathbb{C}^2 \otimes \mathbb{C}^2$

$\Rightarrow \lambda = (1, 0$[redacted]$)$ useless state

$$|\varphi_{AB}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$\Rightarrow \lambda' = \left(\frac{1}{2}, \frac{1}{2}\right)$ very valuable (Bell) state

$\Rightarrow |\varphi\rangle \to |\psi\rangle$ but not the other way round.

$\Rightarrow$ Resource th. of entanglement analogous, but "inverse" to res. th. of nonuniformity:

	res. th. of entanglement	res. th. of nonuniformity
useless state	$\|\psi\rangle = \|\psi_A\rangle \otimes \|\psi_B\rangle$, i.e. $\lambda = (1,0,\dots,0)$	$p = (\frac{1}{d}, \dots, \frac{1}{d})$
most valuable state in d dim.	max. entangled state with $\lambda = (\frac{1}{d}, \dots, \frac{1}{d})$	$q = (1,0,\dots,0)$ pure bit/trit/...
transf. possible iff...	$\|\psi\rangle \longrightarrow \|\varphi\rangle$ $\Leftrightarrow$ $\lambda_\psi \prec \lambda_\varphi$	$p \longrightarrow q$ $\Leftrightarrow$ $p \succ q$
transition rate	$\|\psi\rangle^{\otimes n} \longrightarrow \|\varphi\rangle^{\otimes m_n}$ $\frac{m_n}{n} \longrightarrow \frac{S(\text{[redacted]}\,\rho_A)}{S(\rho_A')}$ where $\rho_A = Tr_B \|\psi\rangle\langle\psi\|$, $\rho_A' = Tr_B \|\varphi\rangle\langle\varphi\|$ Proof: asymptotic equip. property.	$p^{\otimes n} \longrightarrow q^{\otimes m_n}$ ← as large as possible $\frac{m_n}{n} \longrightarrow \frac{\log d_p - H(p)}{\log d_q - H(q)}$

$\Rightarrow$ entanglement entropy $S(Tr_B |\psi\rangle\langle\psi|)$ gives asymptotically the number of "Bell pairs"

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

that can be distilled from $|\psi\rangle$ via LOCC.
(or are needed to form $|\psi\rangle$).