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Classical probability theory: (B, Ω_B, E_B), where

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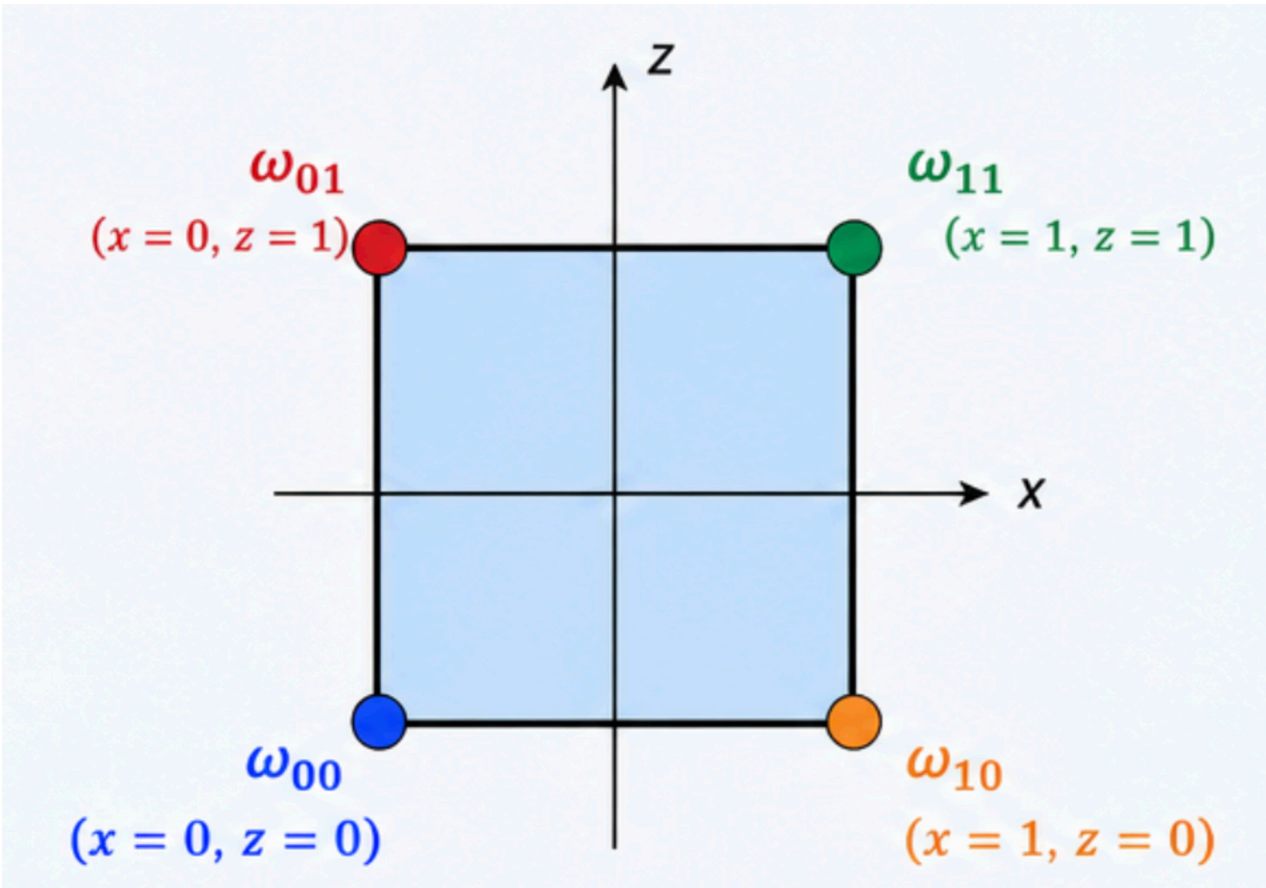
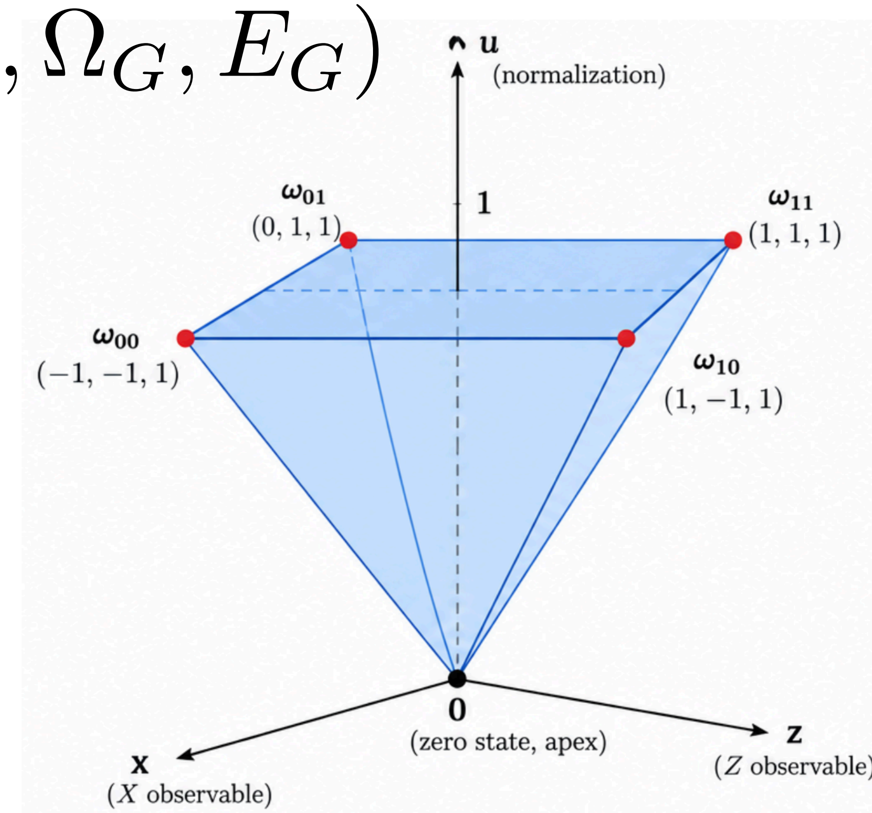


$$(G, \Omega_G, E_G)$$



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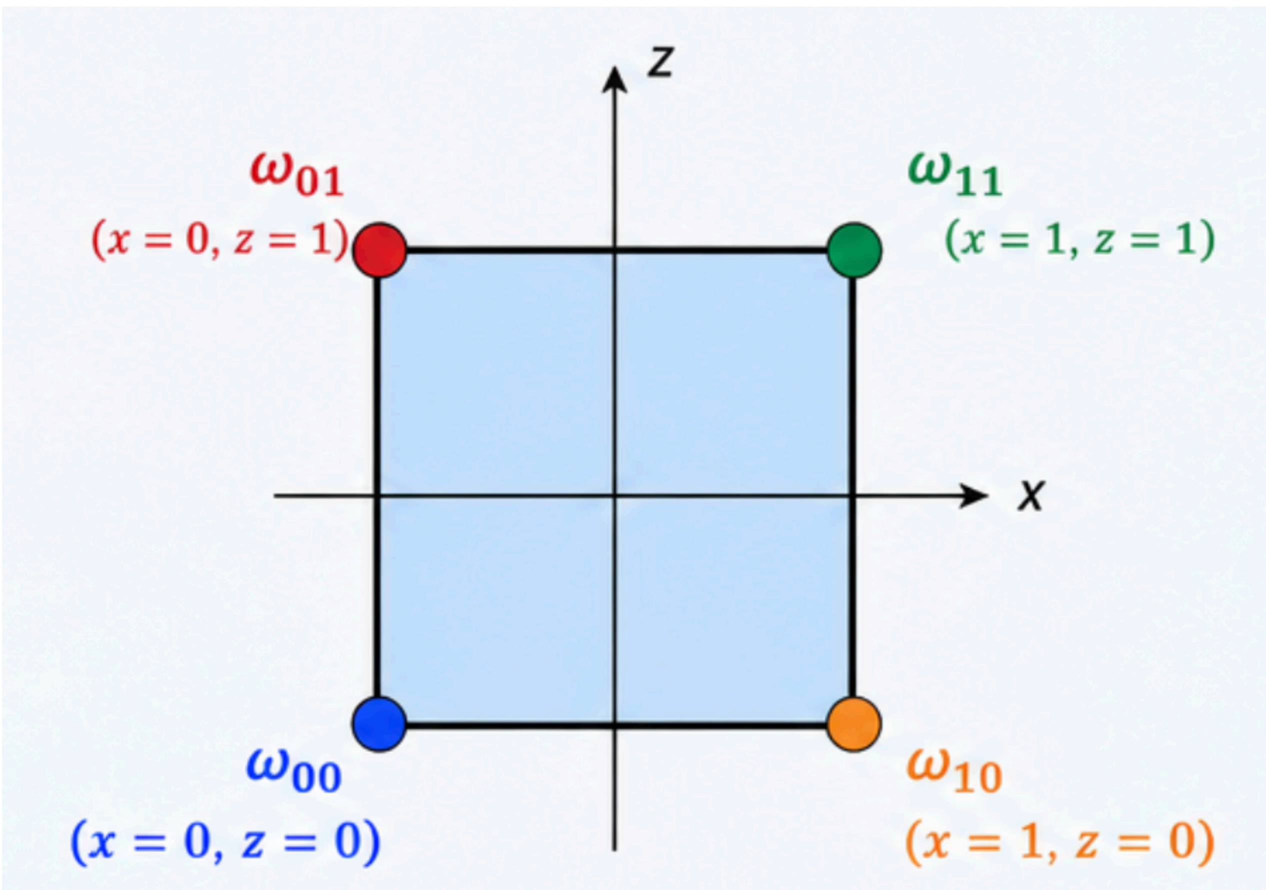

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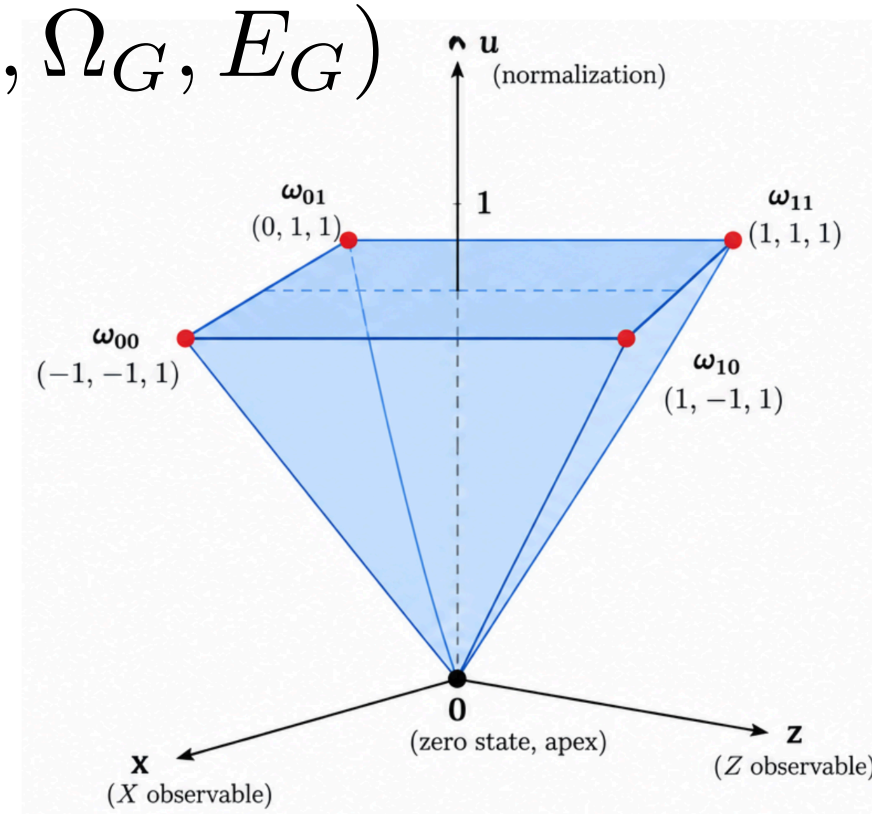

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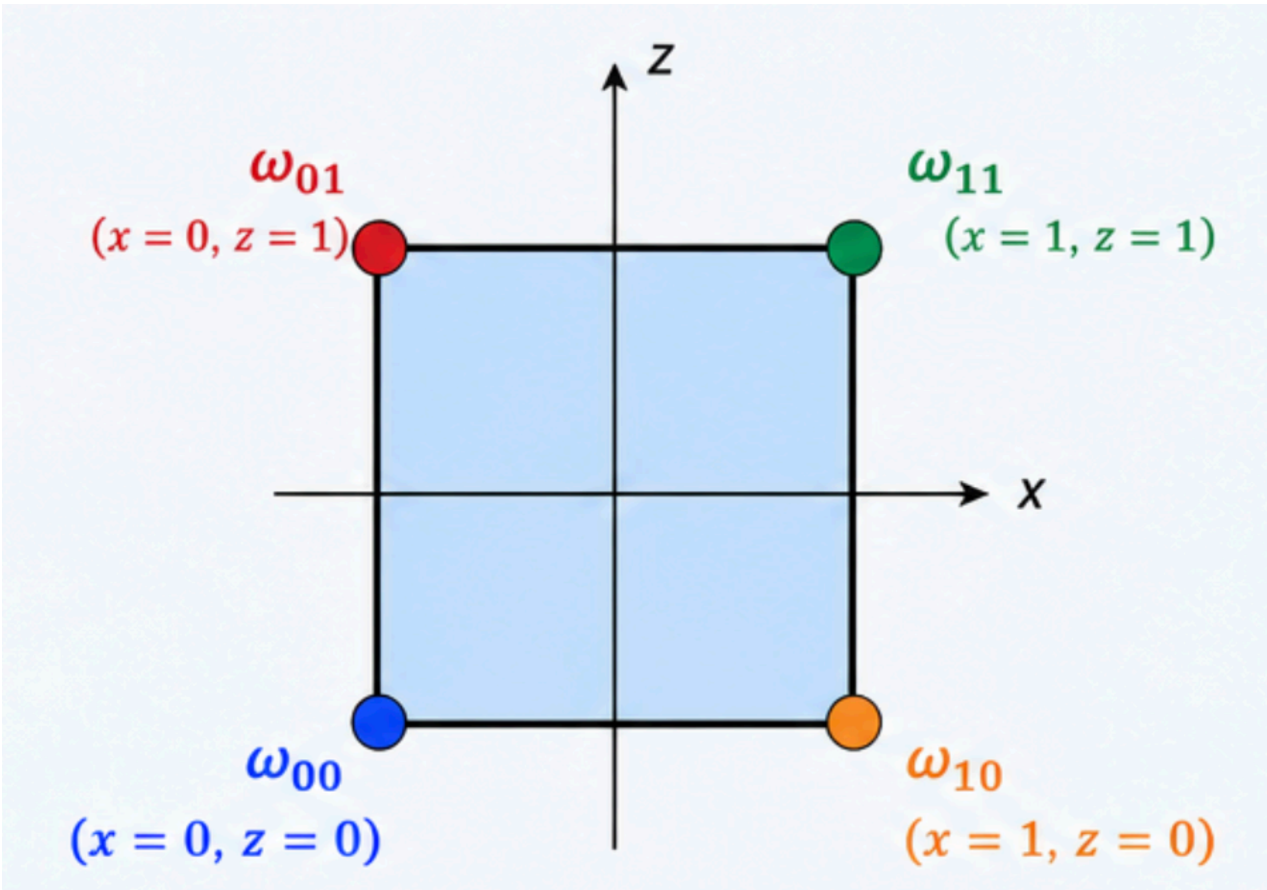

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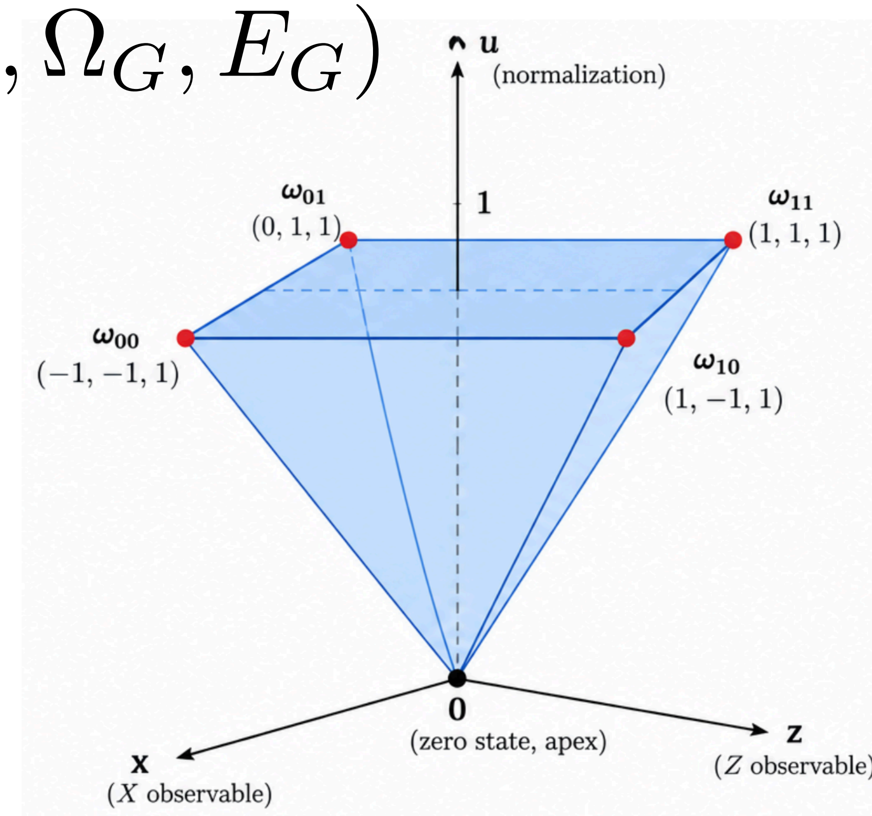

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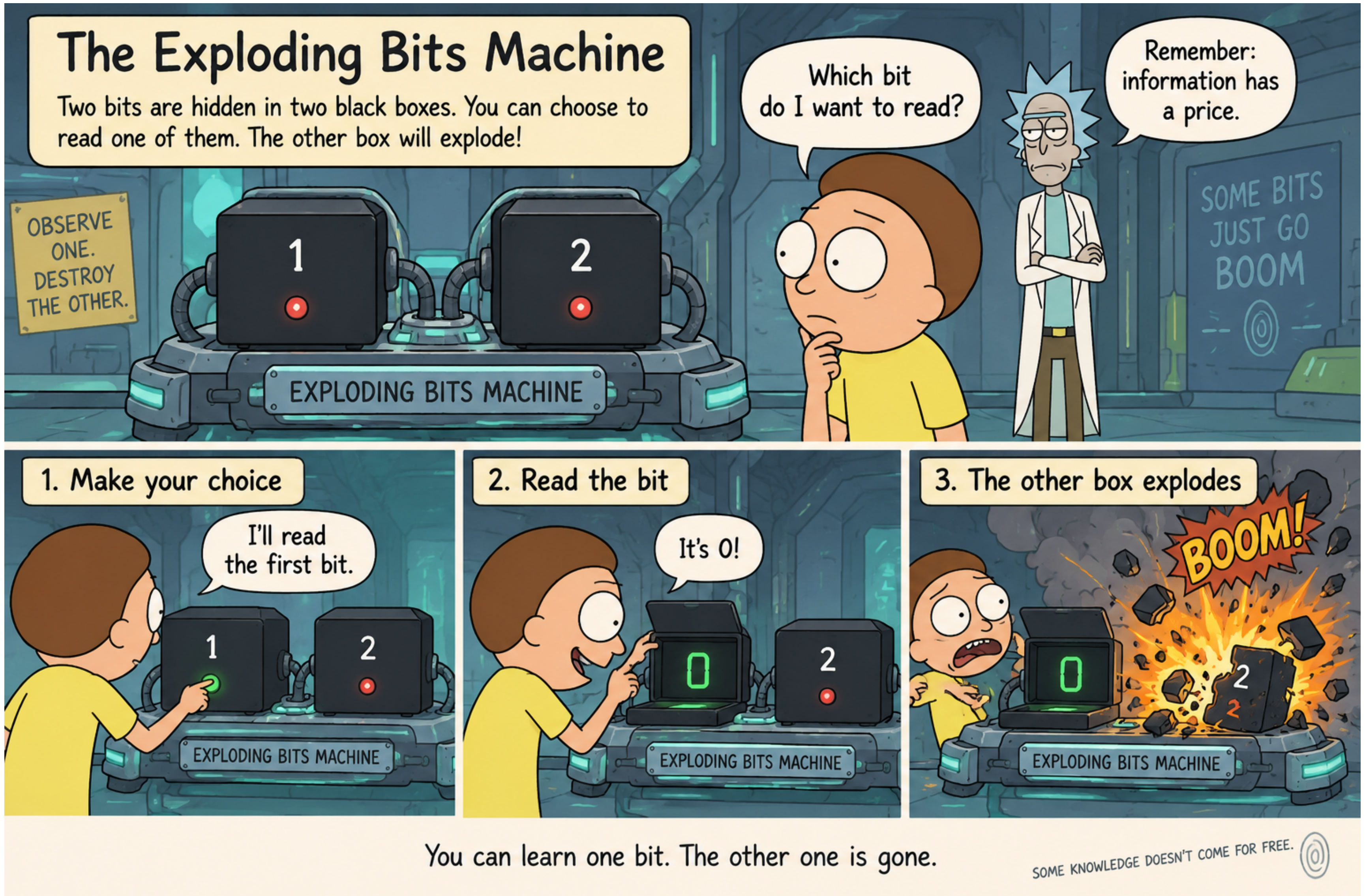
It turns out that E_G is the convex hull of 0, u_G, and

$$x = \frac{1}{2}(1, 0, 1), \;\; \bar{x} = \frac{1}{2}(-1, 0, 1), \;\; z = \frac{1}{2}(0, 1, 1), \;\; \bar{z} = \frac{1}{2}(0, -1, 1).$$

Classical simulation of the gbit (sort of...)



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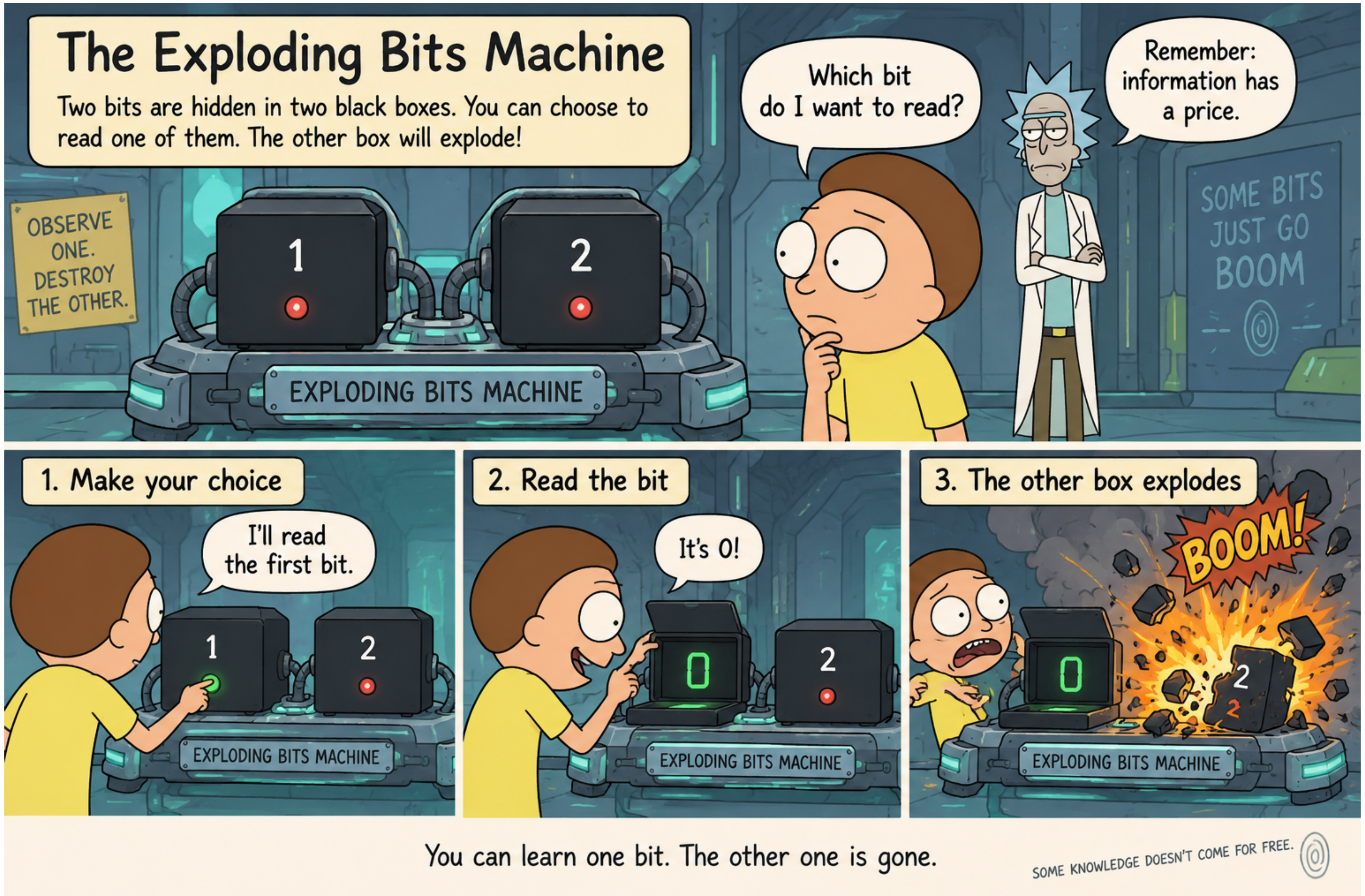

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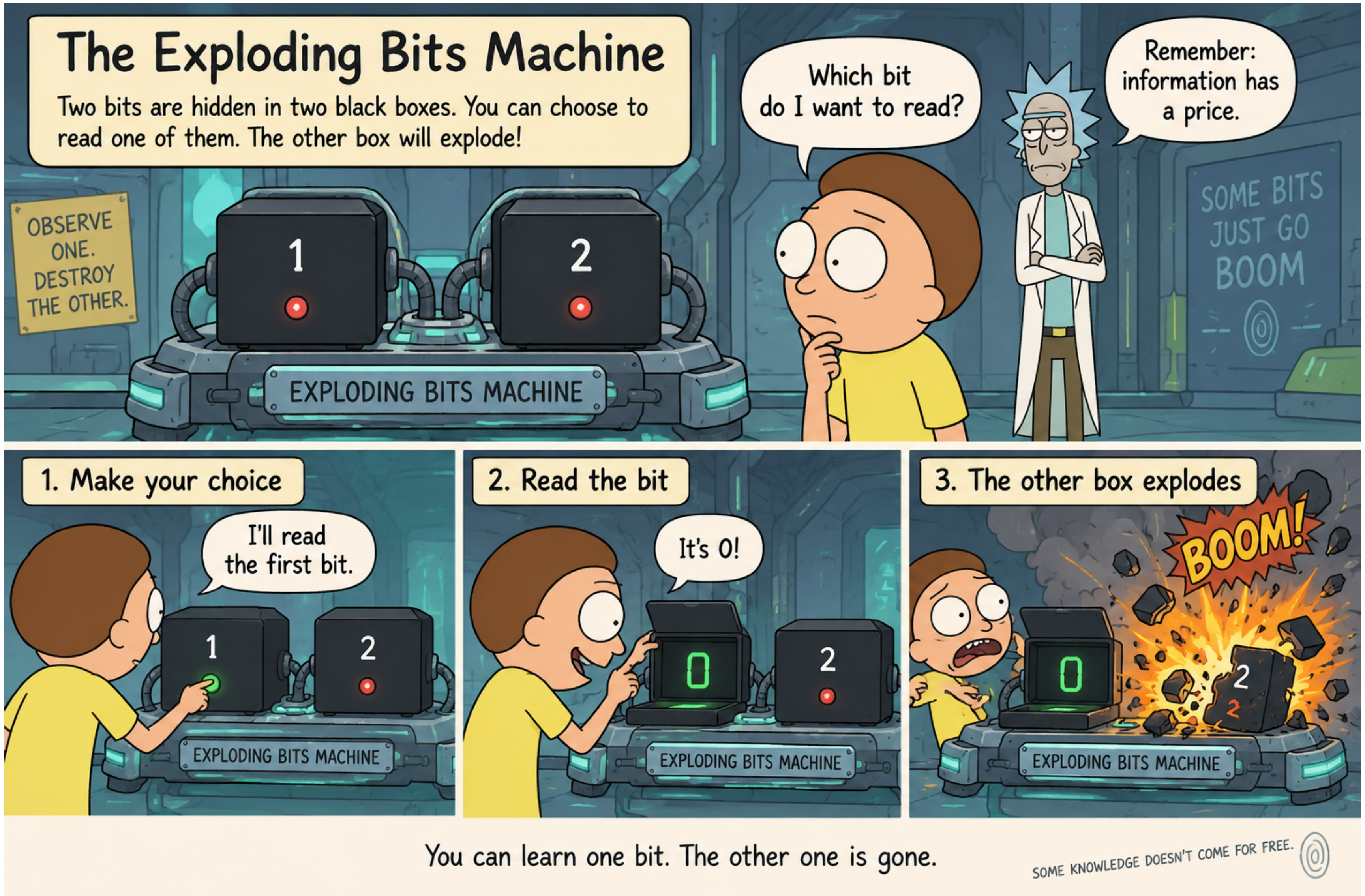

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Then $\frac{1}{2}P_{0,0} + \frac{1}{2}P_{1,1}$

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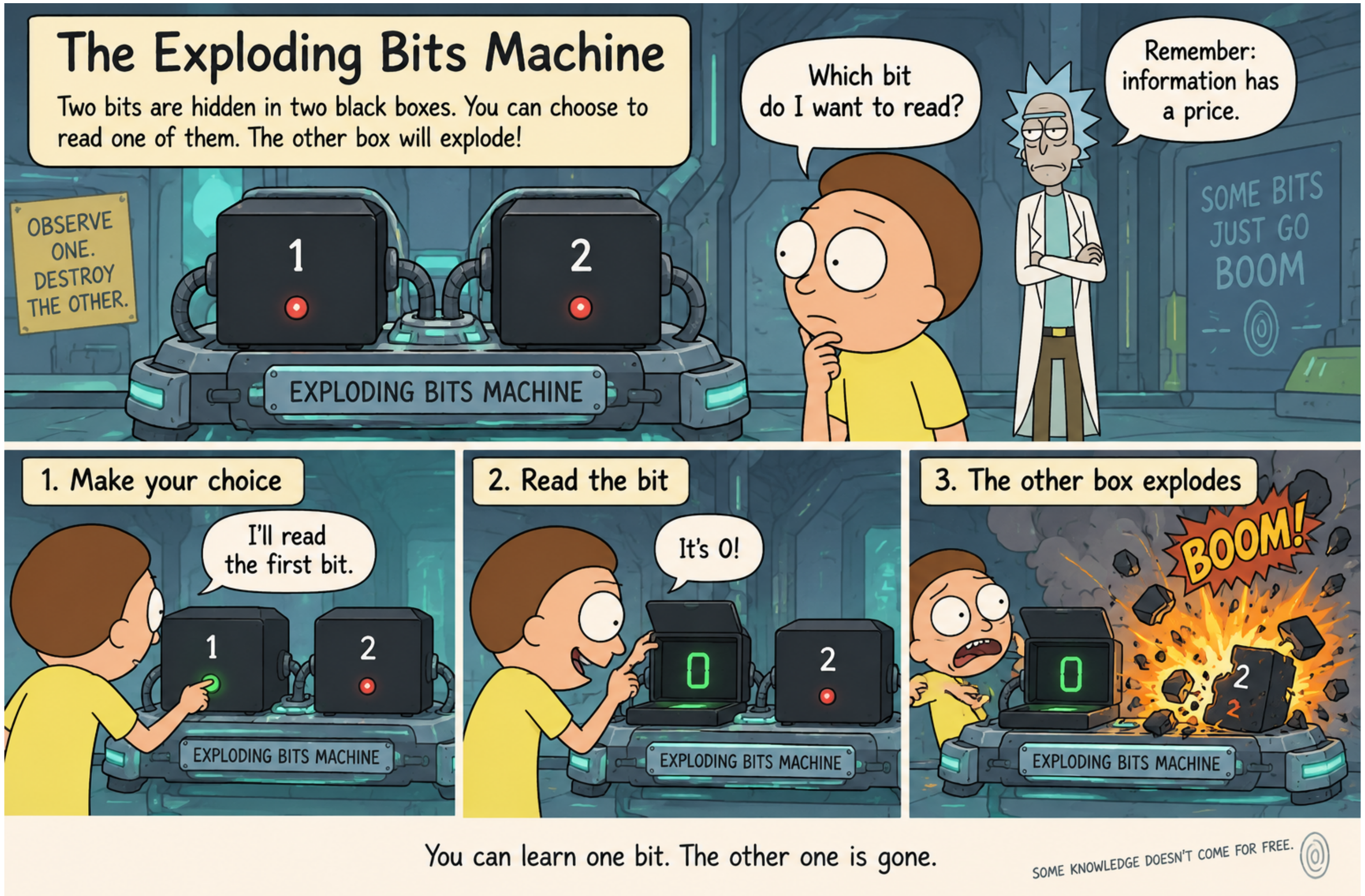

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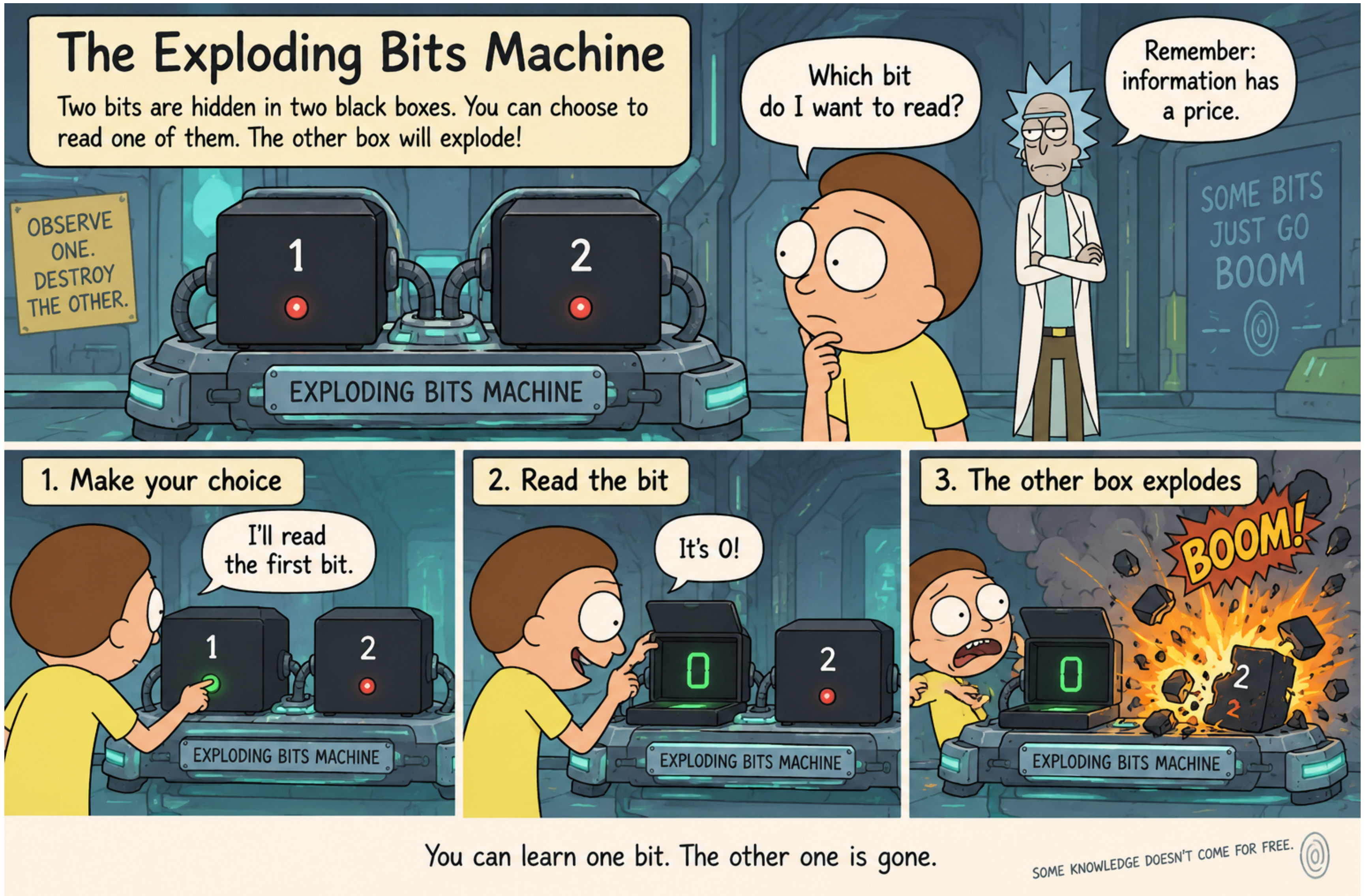

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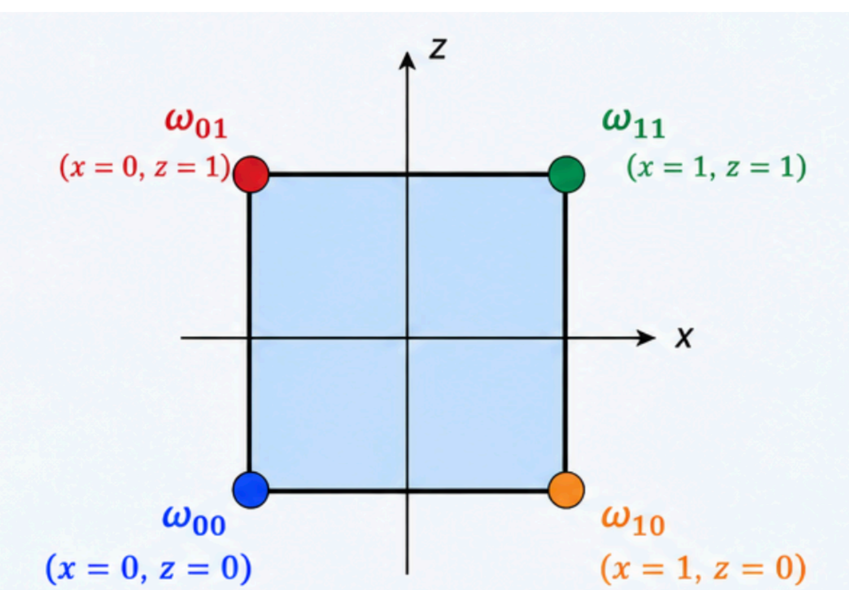

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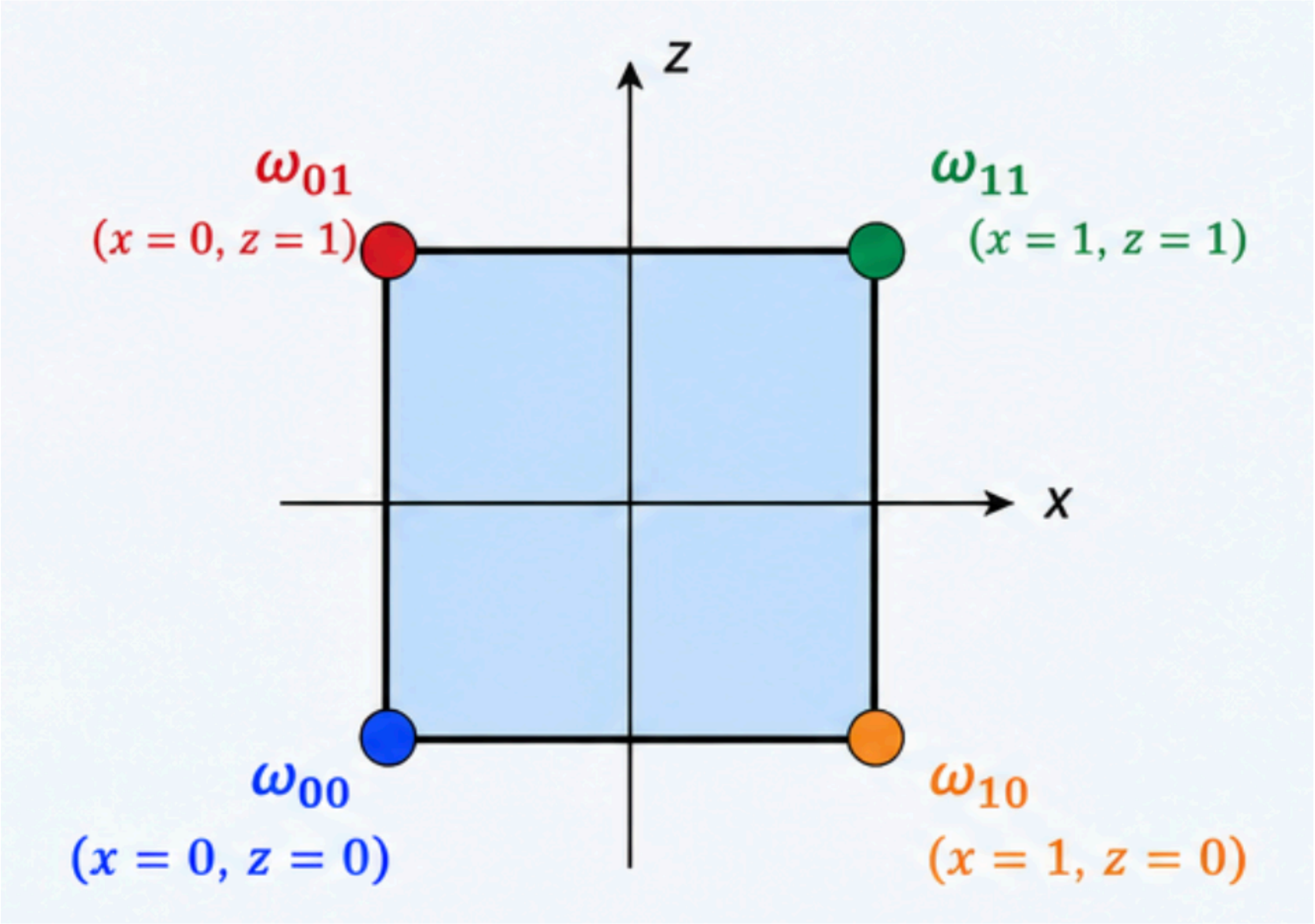

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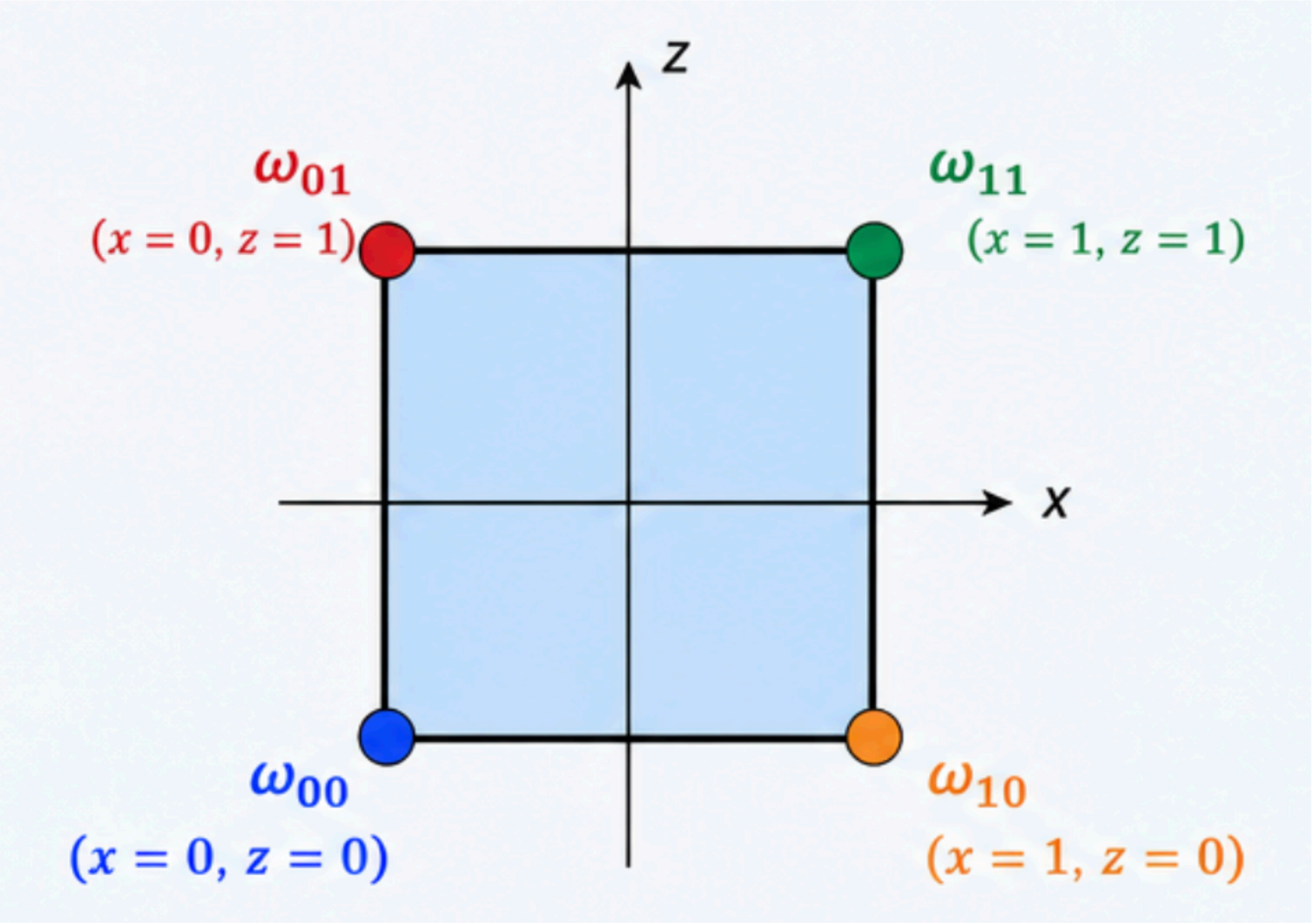

No time to talk about:

- The gbit has N=2 and not more states that can be perfectly distinguished in any single measurement, hence the name g**bit**.
- Composite systems of 2 gbits admit **"PR-box"- states** with superstrong nonlocality.

Contextuality

Warum-Up: Kochen-Specker Contextuality

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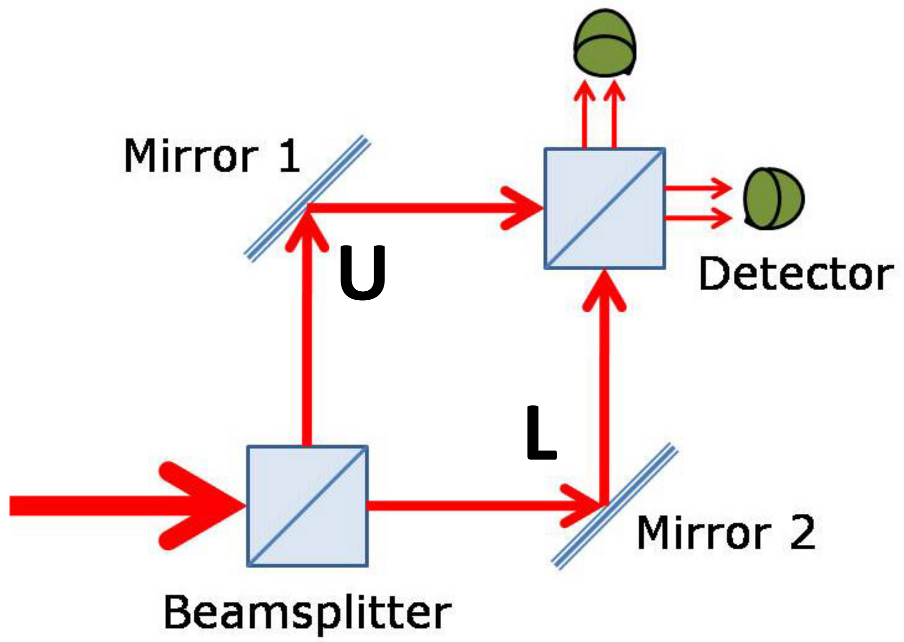

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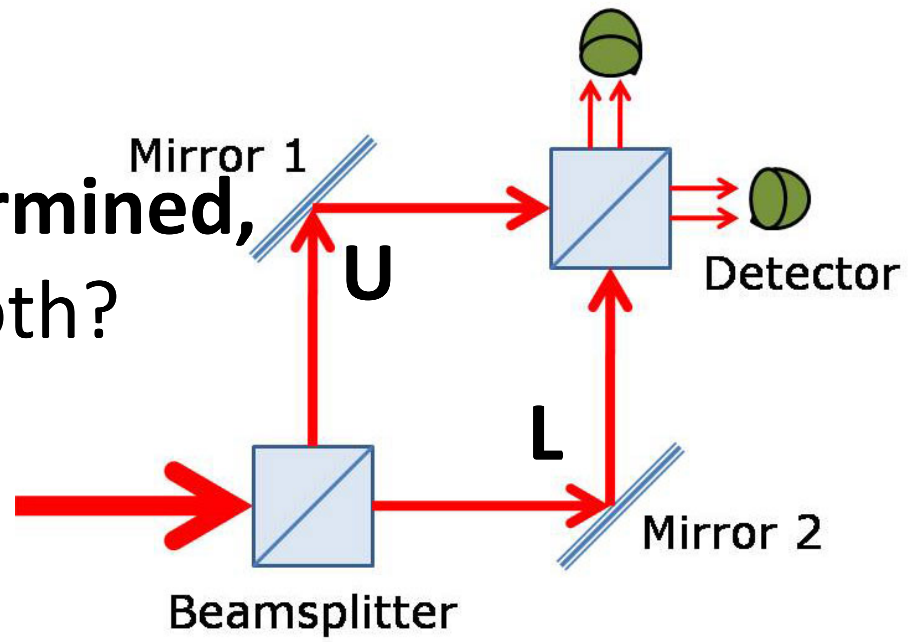
Could it be that the outcome (U or L) is **predetermined,** even though QM predicts 50% probability for both?



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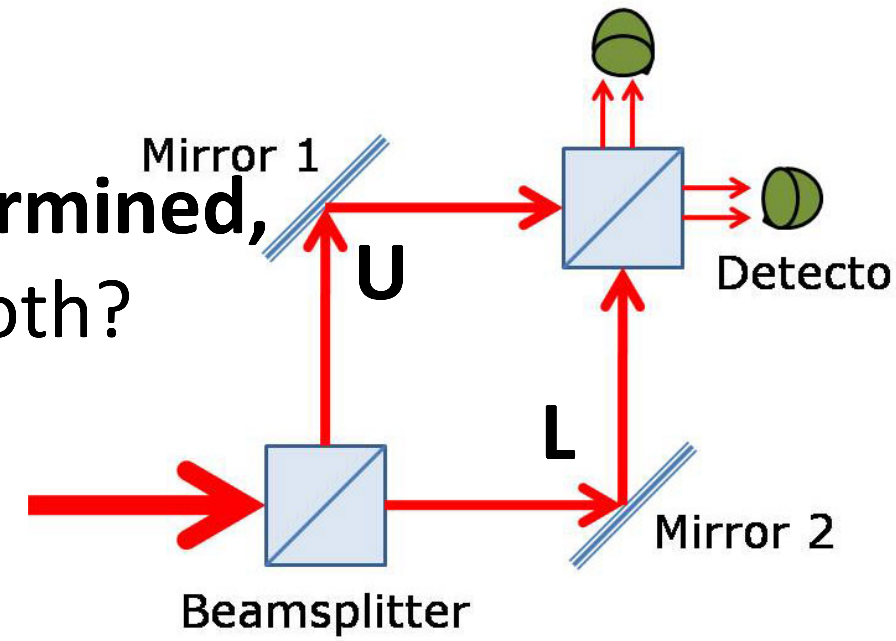

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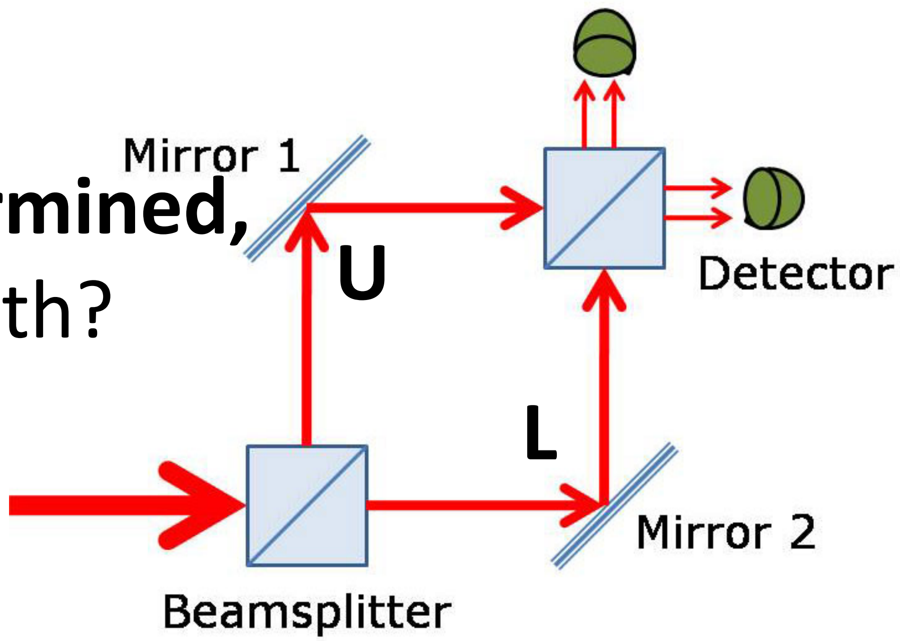

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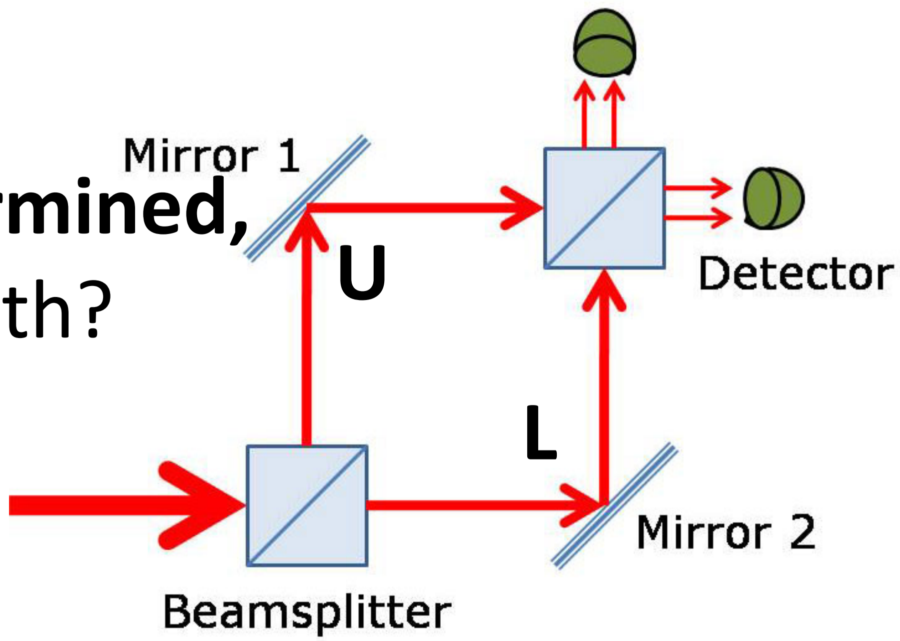

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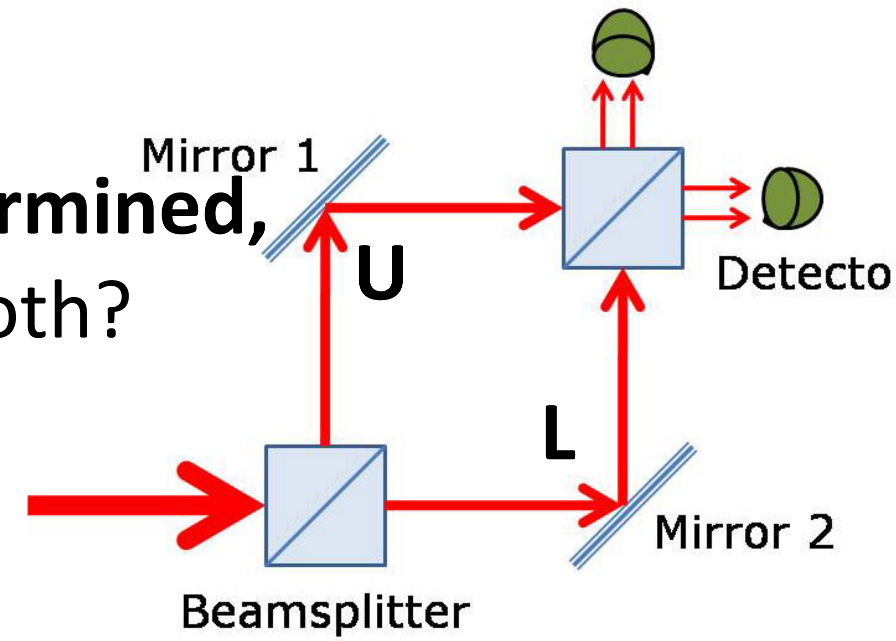

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But for three and more dimensions, it gets more interesting…

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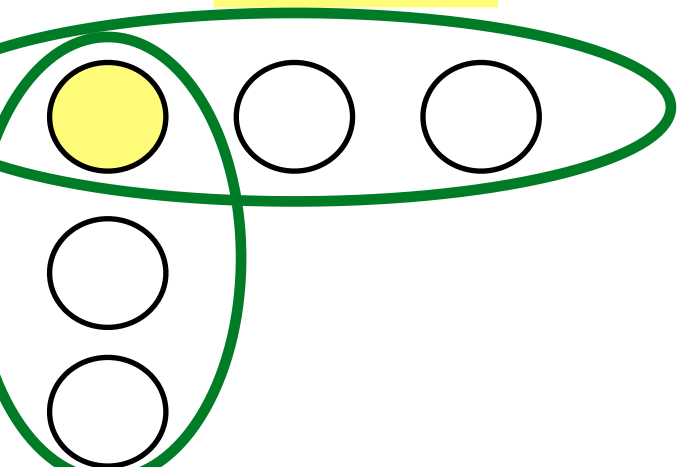

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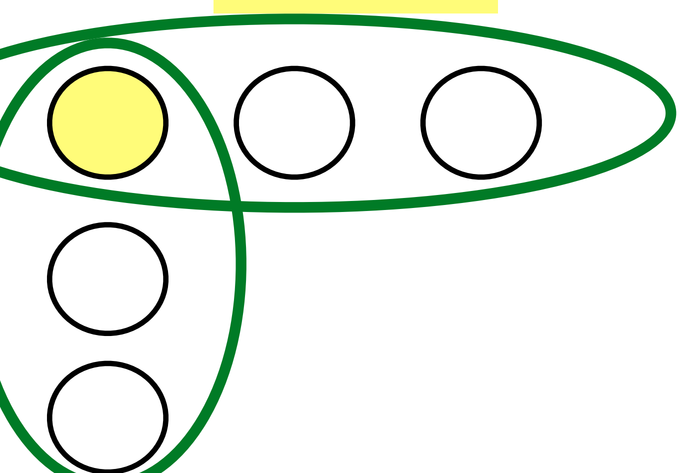

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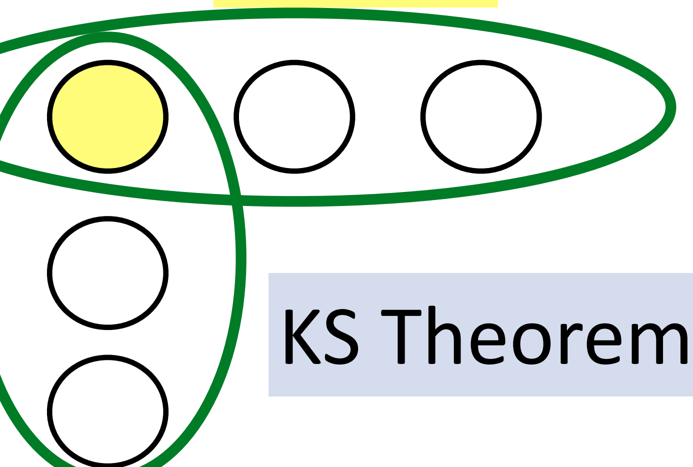

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KS Theorem rules out the existence of **noncontextual** hidden variables.

Simplified proof for d=4 due to Cabello

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(1, 1, 0, 0)	(1, 0, 1, 0)	(1, 1, 0, 0)	(1, 0, −1, 0)	(1, 0, 0, 1)	(1, 0, 0, −1)	(1, −1, 0, 0)	(1, 0, 1, 0)	(1, 0, 0, 1)
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Equal color = same vector = same truth value.

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(0, 0, 1, 0)	(0, 1, 0, 0)	(1, −1, −1, 1)	(1, 1, 1, 1)	(0, 1, 0, 0)	(1, 1, 1, 1)	(1, 1, 1, −1)	(−1, 1, 1, 1)	(−1, 1, 1, 1)
(1, 1, 0, 0)	(1, 0, 1, 0)	(1, 1, 0, 0)	(1, 0, −1, 0)	(1, 0, 0, 1)	(1, 0, 0, −1)	(1, −1, 0, 0)	(1, 0, 1, 0)	(1, 0, 0, 1)
(1, −1, 0, 0)	(1, 0, −1, 0)	(0, 0, 1, 1)	(0, 1, 0, −1)	(1, 0, 0, −1)	(0, 1, −1, 0)	(0, 0, 1, 1)	(0, 1, 0, −1)	(0, 1, −1, 0)

Equal color = same vector = same truth value.

Every column defines a measurement. Truth value 1 appears **9 times**.

Simplified proof for d=4 due to Cabello

Denote projections by the vectors on which they project (dropping normalization).

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(0, 0, 0, 1)	(0, 0, 0, 1)	(1, –1, 1, –1)	(1, –1, 1, –1)	(0, 0, 1, 0)	(1, –1, –1, 1)	(1, 1, –1, 1)	(1, 1, –1, 1)	(1, 1, 1, –1)
(0, 0, 1, 0)	(0, 1, 0, 0)	(1, –1, –1, 1)	(1, 1, 1, 1)	(0, 1, 0, 0)	(1, 1, 1, 1)	(1, 1, 1, –1)	(–1, 1, 1, 1)	(–1, 1, 1, 1)
(1, 1, 0, 0)	(1, 0, 1, 0)	(1, 1, 0, 0)	(1, 0, –1, 0)	(1, 0, 0, 1)	(1, 0, 0, –1)	(1, –1, 0, 0)	(1, 0, 1, 0)	(1, 0, 0, 1)
(1, –1, 0, 0)	(1, 0, –1, 0)	(0, 0, 1, 1)	(0, 1, 0, –1)	(1, 0, 0, –1)	(0, 1, –1, 0)	(0, 0, 1, 1)	(0, 1, 0, –1)	(0, 1, –1, 0)

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(0, 0, 1, 0)	(0, 1, 0, 0)	(1, –1, –1, 1)	(1, 1, 1, 1)	(0, 1, 0, 0)	(1, 1, 1, 1)	(1, 1, 1, –1)	(–1, 1, 1, 1)	(–1, 1, 1, 1)
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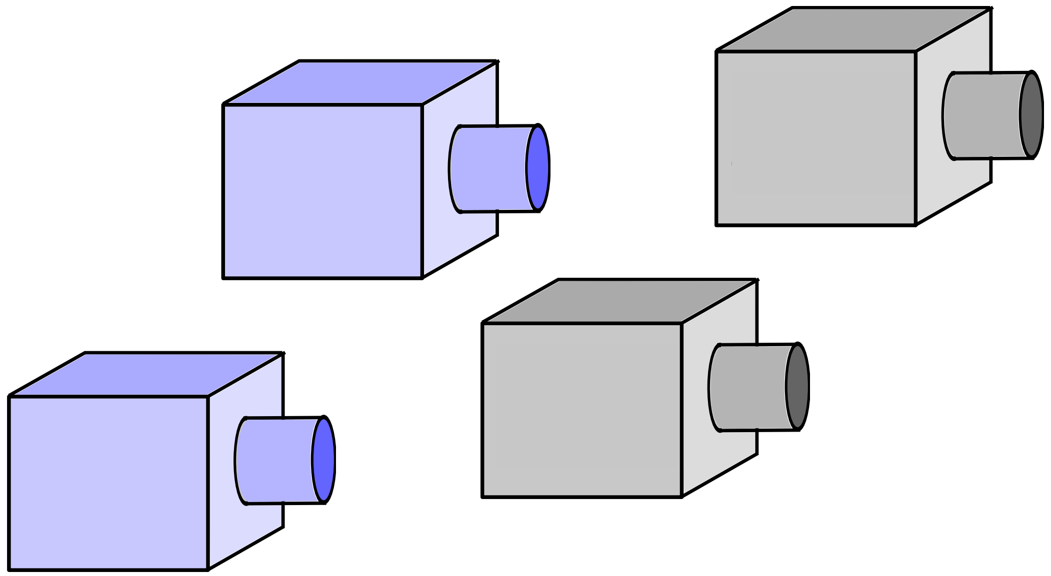
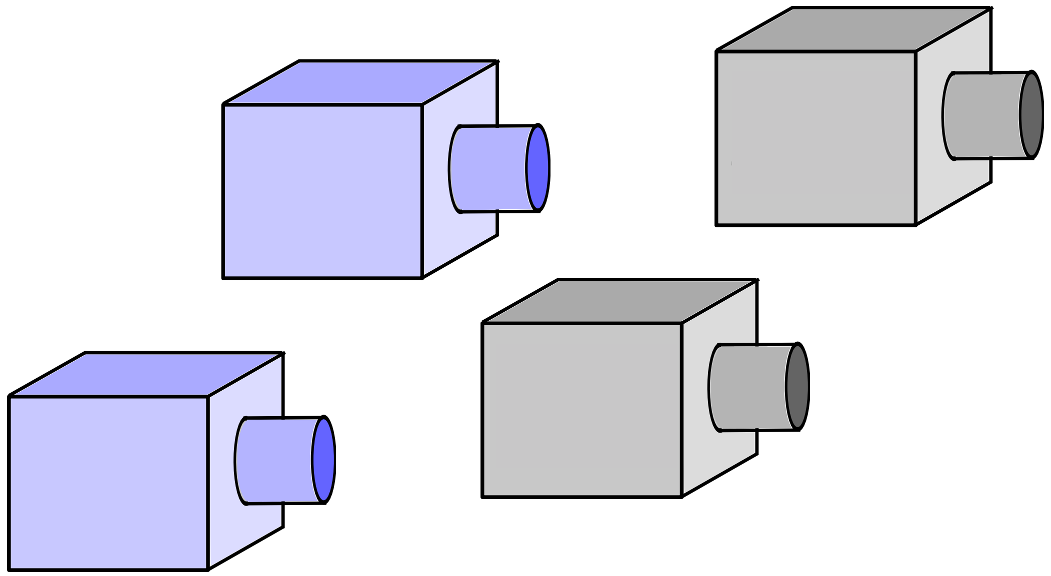
Contradiction. Hence, no such truth value assignment can exist.

Operational theories



(all accessible preparation procedures)

Operational theories



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$P_1 \sim P_2$ if they give identical probabilities for all outcomes of all accessible measurements.

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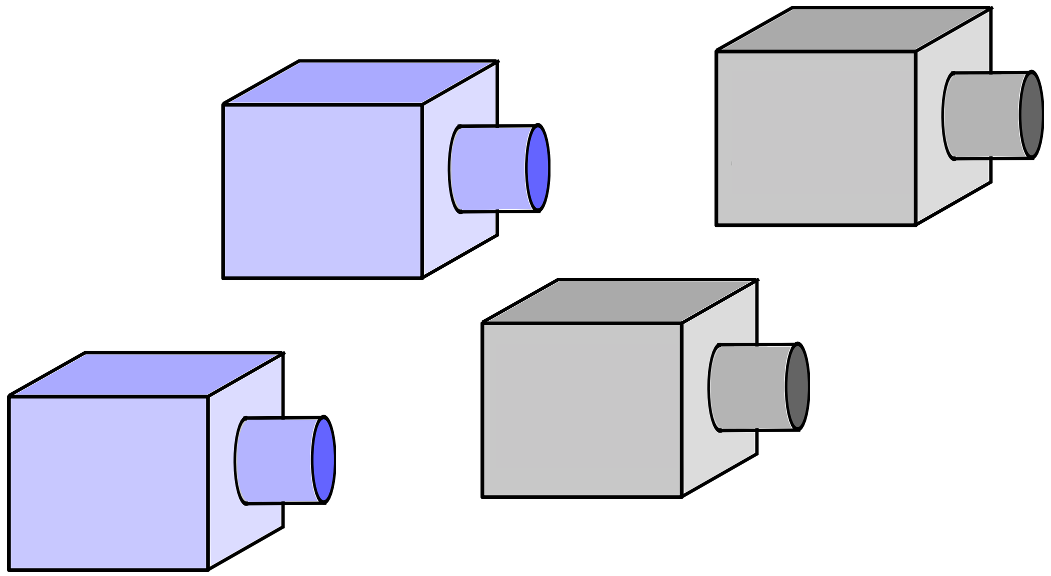

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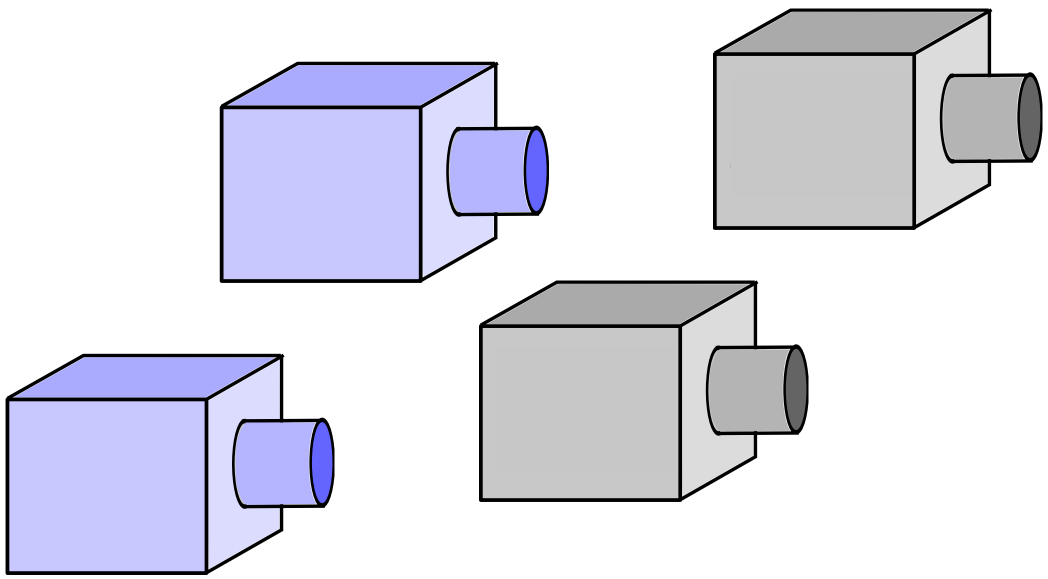

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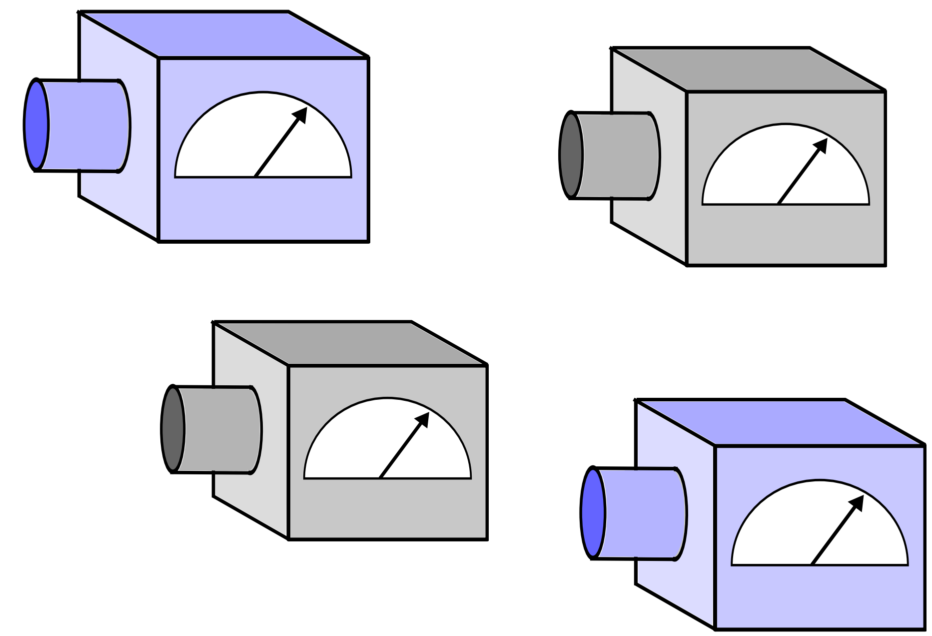

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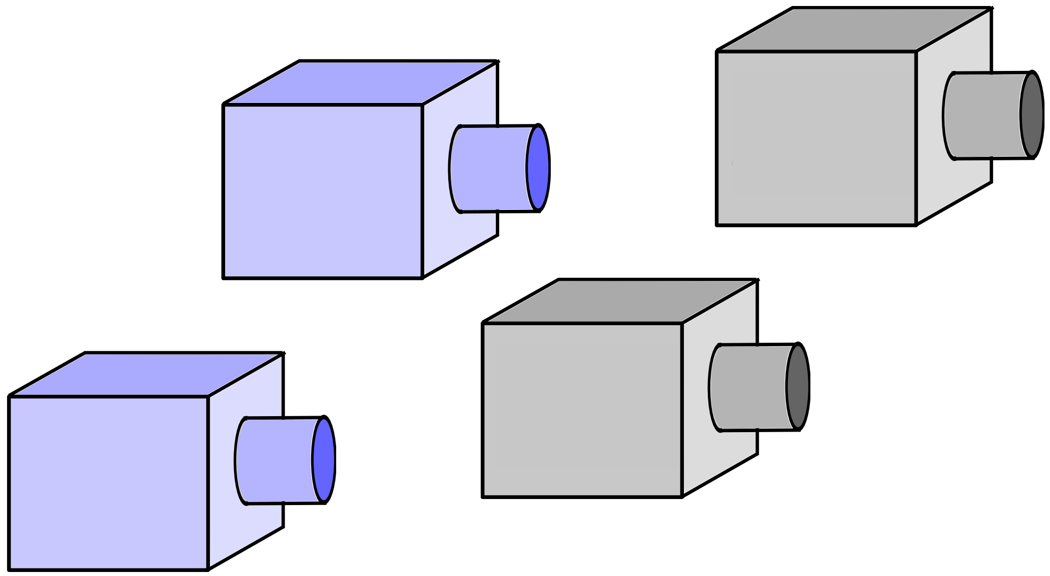

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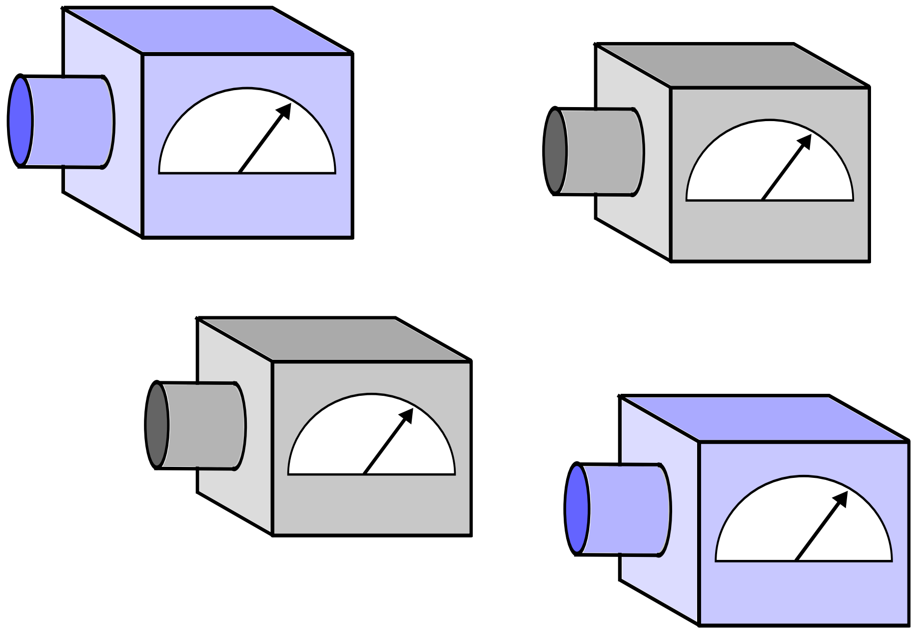

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$(k_1, M_1) \sim (k_2, M_2)$ if $\mathrm{Prob}(k_1|M_1, P) = \mathrm{Prob}(k_2|M_2, P)$ for all accessible preparations P.

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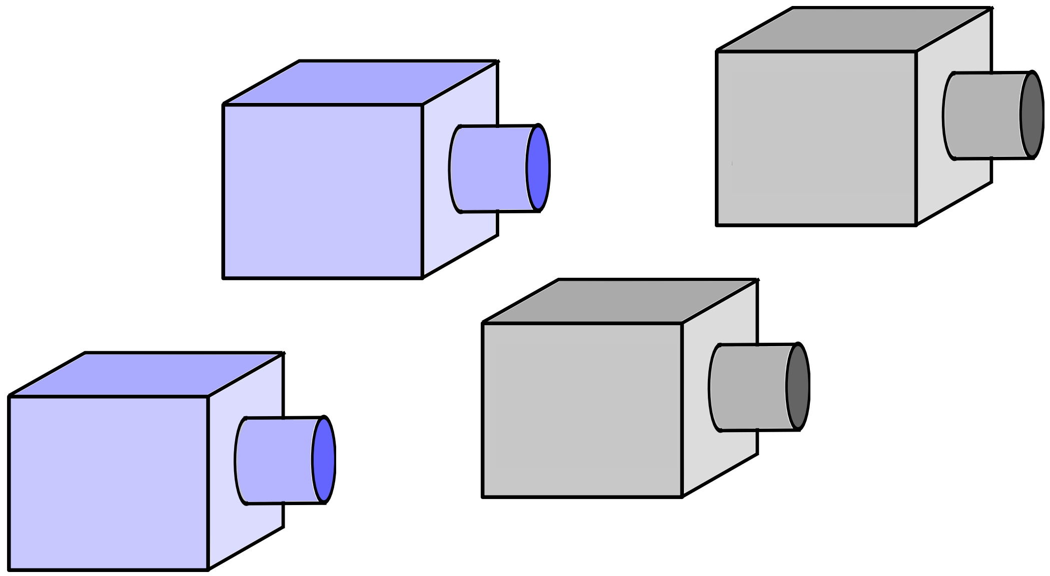

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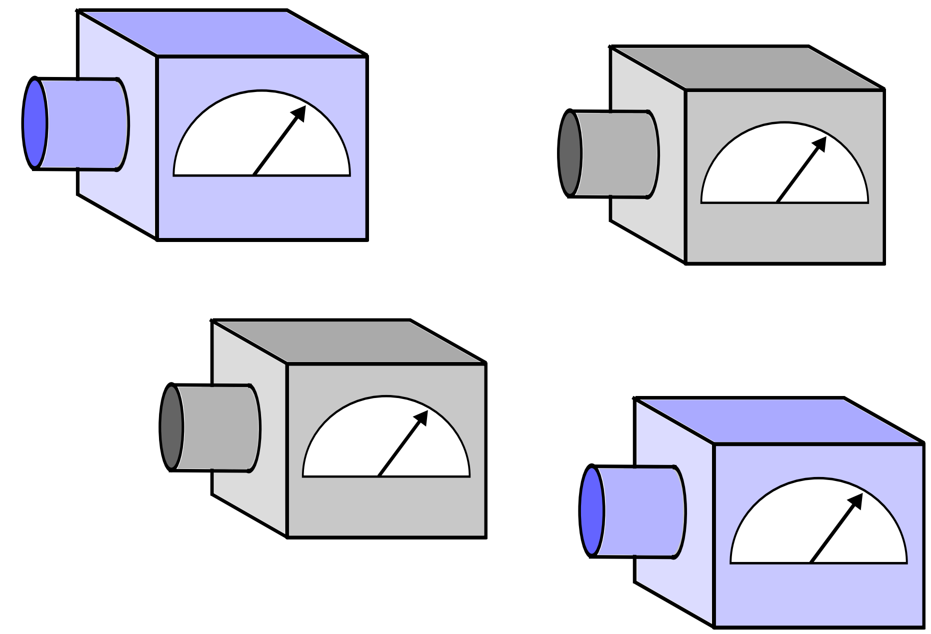

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Effect $e_{k,M}$ = equivalence class of outcome-measurement pairs

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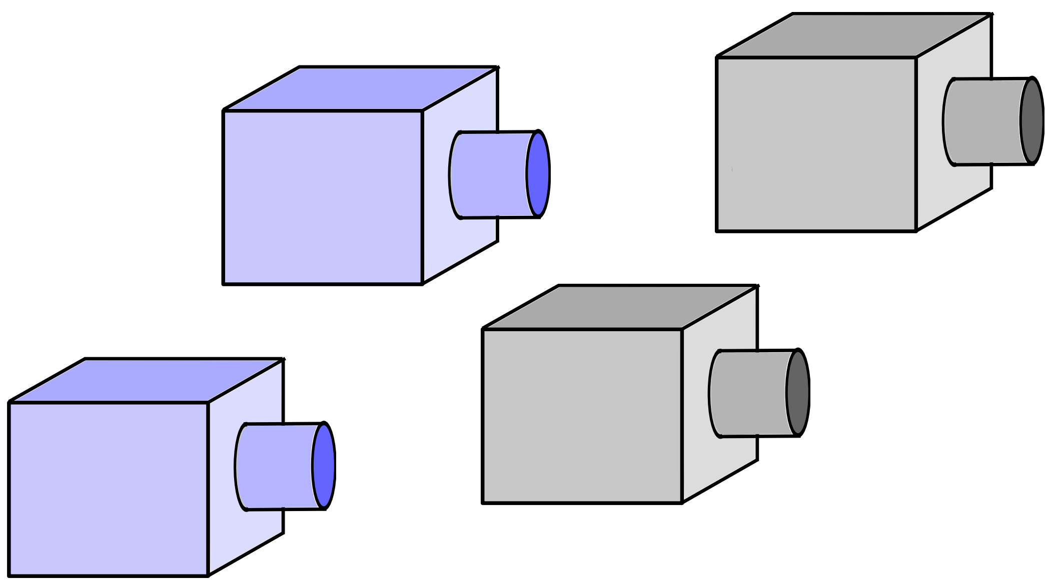

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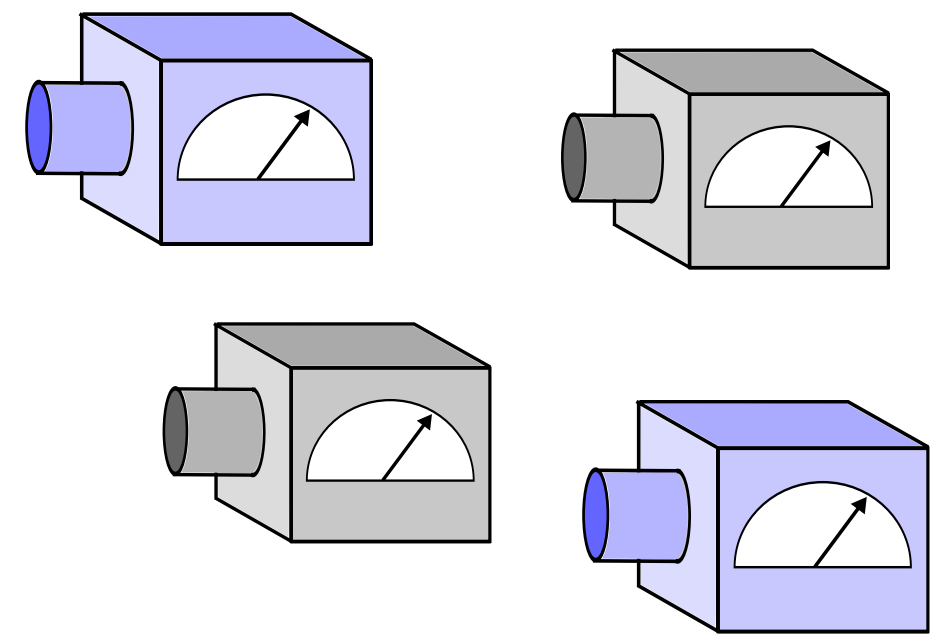

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$$\mathrm{Prob}(k|P, M) = \langle \omega_P, e_{k,M} \rangle \qquad (e_{k,M} \in A, \omega_P \in A^*).$$

Spekkens' notion of noncontextuality

Contextuality for preparations, transformations and unsharp measurements

R. W. Spekkens*

Perimeter Institute for Theoretical Physics, 35 King St. North, Waterloo, Ontario N2J 2W9, Canada

(Dated: Feb. 25, 2005)

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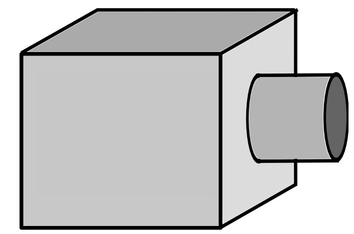

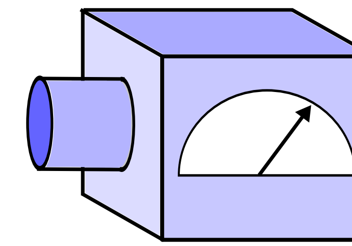

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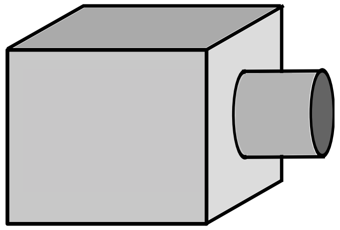

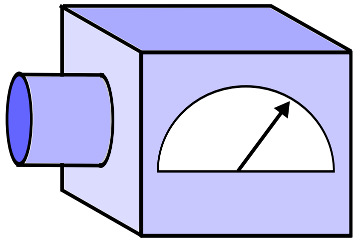

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A set of classical variables Λ.

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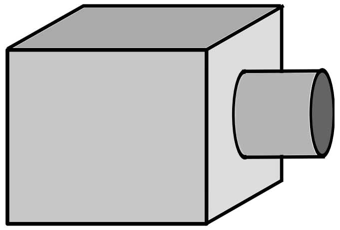

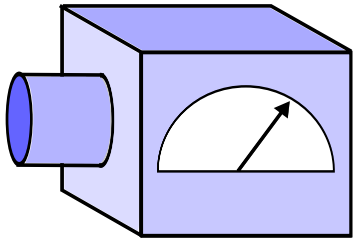

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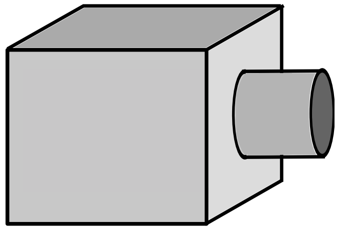

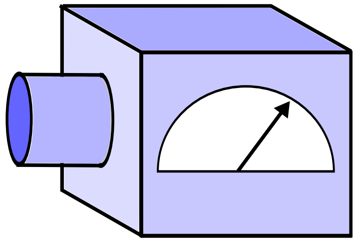

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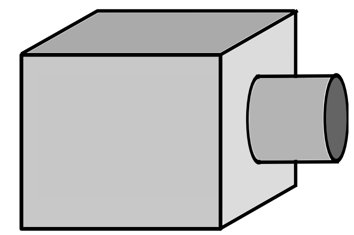

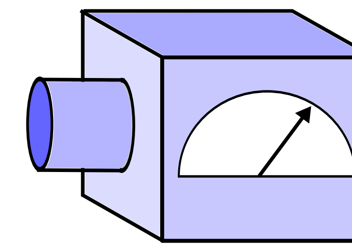

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such that

$$\mathrm{Prob}(k|P,M) = \int_\Lambda d\lambda \mu_P(\lambda)\chi_{M,k}(\lambda).$$

Spekkens' notion of noncontextuality

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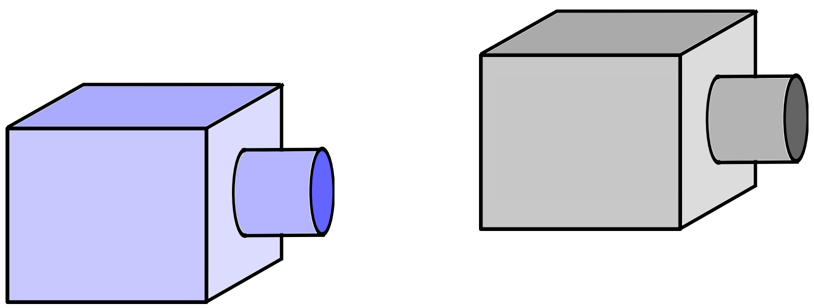

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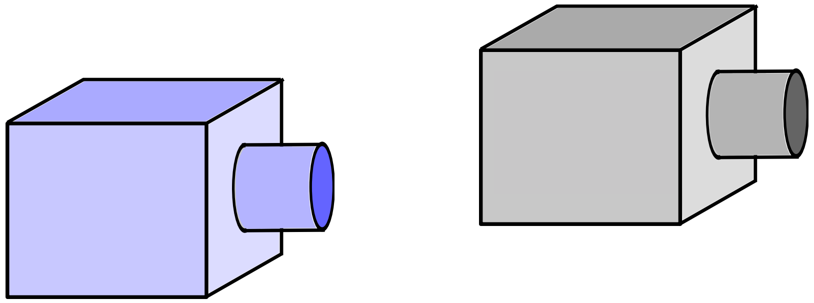

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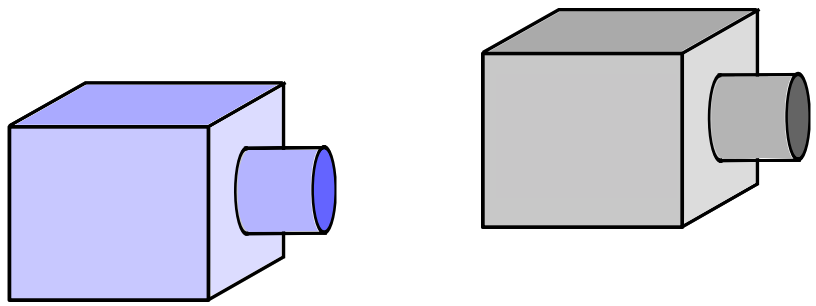

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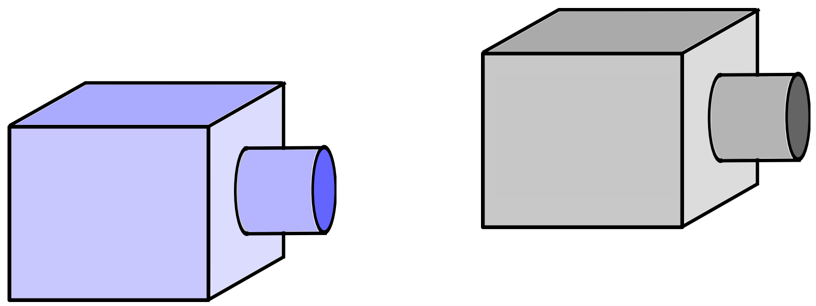

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<u>Intuition</u>: Contextual models are implausible because they are **fine-tuned**: operationally, $P \sim P'$, but ontologically, $\mu_P \neq \mu_{P'}$.

An instance of Leibniz' principle of the "identity of the indiscernibles".

Spekkens’ notion of noncontextuality

Significant advantages over Kochen-Specker notion: experimentally robust, does not assume validity of QM.

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nature COMMUNICATIONS

ARTICLE

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An experimental test of noncontextuality without unphysical idealizations

Michael D. Mazurek[1,2], Matthew F. Pusey[3], Ravi Kunjwal[4], Kevin J. Resch[1,2] & Robert W. Spekkens[3]

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Notion of nonclassicality	Proves that HV model cannot preserve the…	Assumes QT?	Experimentally robust?
Kochen-Specker contextuality	logical structure / structure of propositions	Yes	No
Spekkens' contextuality	statistical mixing structure	No	Yes
Bell nonlocality	causal structure	No	Yes

Generalized contextuality as a very clear notion of nonclassicality

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Why interference phenomena do not capture the essence of quantum theory

Lorenzo Catani[1], Matthew Leifer[2], David Schmid[3], and Robert W. Spekkens[4]

[1]Electrical Engineering and Computer Science Department, Technische Universität Berlin, 10587 Berlin, Germany
[2]Institute for Quantum Studies and Schmid College of Science and Technology, Chapman University, One University Drive, Orange, CA, 92866, USA
[3]International Centre for Theory of Quantum Technologies, University of Gdansk, 80-308 Gdansk, Poland
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No time for details, unfortunately...

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Editors' Suggestion

Aspects of the phenomenology of interference that are genuinely nonclassical

Lorenzo Catani,[1,*] Matthew Leifer,[2] Giovanni Scala,[3] David Schmid,[3] and Robert W. Spekkens[4]
[1]*Electrical Engineering and Computer Science Department, Technische Universität Berlin, 10587 Berlin, Germany*
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Interference phenomena are often claimed to resist classical explanation. However, such claims are undermined by the fact that the specific aspects of the phenomenology upon which they are based can in fact be reproduced in a noncontextual ontological model [Catani *et al.*, arXiv:2111.13727]. This raises the question of what *other* aspects of the phenomenology of interference *do* in fact resist classical explanation. We answer this question by demonstrating that the most basic quantum wave-particle duality relation, which expresses the precise tradeoff between path distinguishability and fringe visibility, cannot be reproduced in any noncontextual model. We do this by showing that it is a specific type of uncertainty relation and then leveraging a recent result establishing that noncontextuality restricts the functional form of this uncertainty relation [Catani *et al.*, Phys. Rev. Lett. **129**, 240401 (2022)]. Finally, we discuss what sorts of interferometric experiment can demonstrate contextuality via the wave-particle duality relation.

Simplex-embeddability of the stabilizer qubit

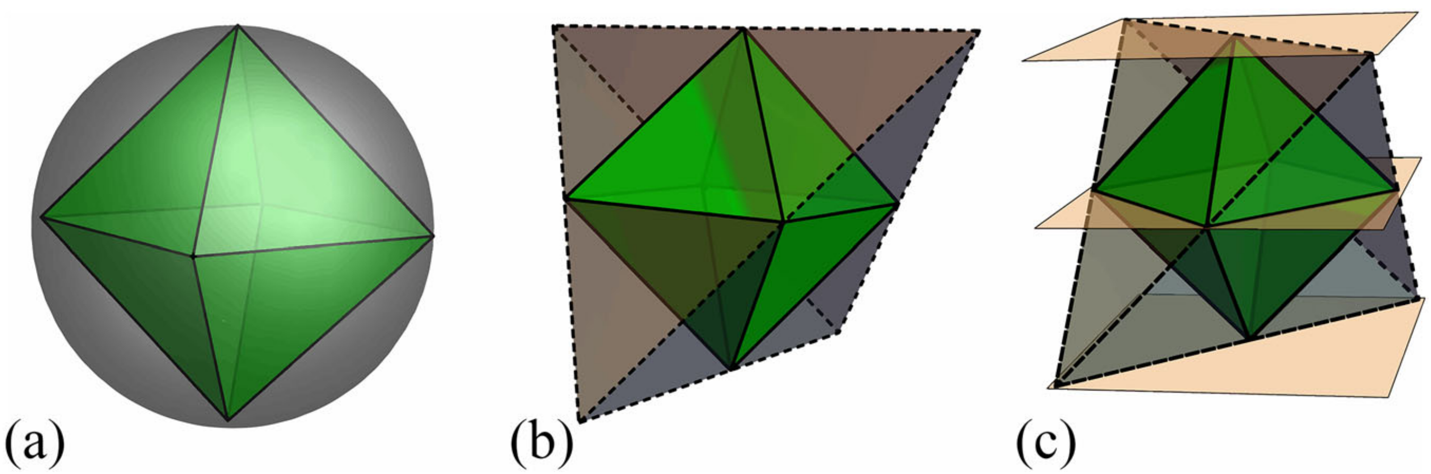

Fig. 3 | Relations between the state spaces of the qubit, stabilizer qubit[31] and classical four-level system. a The state space of the stabilizer qubit (in green) is embedded in the state space of the qubit, given by the gray ball; **b** the state space of the stabilizer qubit (in green) is embeddable within a tetrahedron (in gray), the state space of a classical four level system; **c** the embedding of the stabilizer qubit into the tetrahedron pictured here is such that any valid effect on the stabilizer qubit is also a valid effect on the tetrahedron. This requirement prevents, for example, the embeddability of the qubit into the tetrahedron (we know that the qubit is none-mbeddable because it is contextual). Here the orange planes represent the effect corresponding to the +1 outcome of the Pauli Z measurement: the upper plane intersects all states giving probability 1 for the outcome +1, the central plane those giving probability $\frac{1}{2}$ and the lower plane those giving probability 0.

A. Aloy, M. Fadel, T. D. Galley, C. L. Jones, and M. P. Müller, Nat. Commun. **17**, 2474 (2026).